ANOMALY DETECTION AND MICROSTRUCTURE CHARACTERIZATION IN FIBER REINFORCED CERAMIC MATRIX COMPOSITES

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ABSTRACT

ANOMALY DETECTION AND MICROSTRUCTURE CHARACTERIZATION IN FIBER REINFORCED CERAMIC MATRIX COMPOSITES

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Ceramic matrix composites (CMCs) have the potential to replace current superalloys being used in hot components of jet engines. CMCs with continuous fiber reinforcement exhibit significant strength retention beyond temperatures at which Nickel based superalloys approach their melting temperature (900° C). While ceramics typically exhibit brittle failure modes making them unsuitable for use in dynamic systems, fiber reinforcement increases fracture toughness, crack growth resistance and strength. Differences in weave type, processing technique, and chemical makeup, however, result in a broad range of material microstructures each with a high degree of variability. Little is known about how the variation and imperfections within the microstructure affect the material properties. It is theorized that stress concentrations exist at certain abnormal microstructural configurations, resulting in either crack nucleation or propagation. Due to the amount of data available and the amount of variation in the microstructure, it is impractical to hope to discover the relationship between microstructural organization and cracking simply by observation. Instead, it is thought that the areas of greatest importance are those that do not adhere to the typical behavior
of the material. These areas can be highlighted for analysis via anomaly detection methods for any measurable feature.

In this thesis, two features are developed to describe the microstructure: fiber orientation and orientation gradient. Because fiber reinforcements are the primary method for strength enhancement, the features defined in this work both describe fibers, though the anomaly detection algorithm can be applied to other material constituents. Various image pre-processing techniques are implemented to prepare the feature field for anomaly detection. Novel techniques for segmentation of individual material phases are described. An ellipse detection algorithm for identification of fibers is described, as well as a subsequent fiber tracking algorithm.

The orientation and orientation gradient fields are described in detail. Fiber orientation refers to the geometric interpretation of individual fibers embedded in ceramic matrix. The orientation gradient of fibers describes the relative changes in orientation in a neighborhood of fibers. Eigen-analysis of the orientation gradient reveals the geometric distortion of fiber orientations with position. This affect is similar to an affine transformation with shear and scaling. It is shown that by modeling the normal behavior of a microstructure, anomalies can be identified and described. Here, it is shown that anomalies of the orientation gradient can be identified and are commonly linked to expansion/contraction at fiber tow edges. This is a large step in correlating microstructure organization with damage, and ultimately optimizing material design.
I would like to express my gratitude to Dr. Craig Przybyla and Dr. Jeff Simmons for their guidance and continued support over the course of this project. Dr. Simmons provided support and encouragement on development of the mathematics behind orientation and orientation gradient. I am especially grateful to Dr. Przybyla and the Air Force Research Lab for providing funding for the project and my degree. I would like thank Dr. Russell Hardie for his guidance during the writing process, and for supporting the development of anomaly detection techniques. I would also like to thank Dr. Partha Banerjee, for agreeing to serve on the committee and his input on the thesis, Nicholas Engel, a Wright State student with whom I worked on development of MATLAB image processing algorithms, and my family for their love and support. Finally, I would like to thank my wife Sarah for her continued support and encouragement over the course of my degree.
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CHAPTER I

INTRODUCTION - A BACKGROUND OF CERAMIC MATRIX COMPOSITES

1.1 Ceramic Matrix Composites

Traditional operating temperatures for hot jet engine components have been limited by Ni based superalloys to approximately 900 °C. Higher performance necessitates higher operating temperatures and has led to the need for materials that retain strength at higher temperatures. High temperature ceramics have been used in aerospace applications beginning in the 1960s for their high temperature capabilities. Ceramics retain strength up to temperatures exceeding 2000 °C making them attractive for leading edges in hypersonic vehicles and as next generation jet engine components. Ceramics are also significantly less dense than superalloys currently used in hot components, making them an even more compelling option for their use in aviation. While ceramics have lower densities and favorable temperature limitations compared to metals, they have a greatly reduced fracture toughness and require some type of added reinforcement to mitigate damage. Material reinforcement significantly increases toughness of ceramics, but complex microstructures with many features that may contribute to material properties. It is theorized by materials scientists that it is the anomalies of these features that hold the greatest impact on material strength.
1.2 Reinforcement Mechanisms

Homogeneous ceramics exhibit catastrophic failure due to brittle fracture modes. When induced stress is too high, a single crack propagates entirely through the material due to the stress concentration at the crack tip. The purpose of matrix reinforcement is to reduce the stress concentration at the crack tip to limit crack growth and prevent sudden material failure. The matrix reinforcement can be composed of particles, whiskers, platelets, or fibers.

In ceramic matrix composites, defects are introduced through processing, producing stress concentrations which nucleate cracks. Matrix reinforcements are designed to limit the propagation of these cracks through several mechanisms: transformation toughening, microcracking, crack deflection, crack bridging, and fiber reinforcement [1]. Transformation toughening utilizes a stress induced phase transformation in ceramics. When a crack tip causes high stress concentrations, the crystallographic phase changes causing a slight volume increase and transfer of stress from the crack tip to surrounding material, increasing the crack propagation resistance of the ceramic [1].

Microcracks are small separate cracks that open in a region near the crack tip [1]. The onset of a microcrack relieves elastic energy, de-stressing the crack tip and increasing crack propagation resistance. Crack deflection refers to direction deviation caused by stress fields around embedded particles. This results in increased fracture toughness, because fracture toughness is greater in microstructures featuring non-planar cracks rather than simple planar cracks. Crack bridging mechanisms utilize long particles that remain embedded in the matrix on both sides of a crack and continue bearing stress after the crack tip passes. Fiber reinforcement has had the most success in increasing fracture toughness because it incorporates microcracking, crack deflection, and crack bridging mechanisms.

The interface boundary between fibers and the matrix is also a key element in increasing toughness [1]. Instilling a weak boundary allows microcracks to form in the fiber coating before the crack
tip reaches a fiber and prevents the crack from propagating directly through the fiber. As a crack widens, the fiber continues to bear load until it breaks. Often though, the fiber does not break at the location of the crack, but somewhere still embedded in the matrix. Energy is then further diffused as the fiber is pulled through the matrix away from the fiber break. Because of the multiple toughening mechanisms present with fiber reinforcement, they are the leading design for next generation materials. Many variations of fiber reinforced composites exist with varying processing techniques that will be briefly described in Section 1.3.

1.3 CMC Processing Techniques

Fiber reinforced composites have microstructure organization at several length scales. Individual fibers are grouped into tows or unidirectional groups which can be laid up in unidirectional laminates or woven into fabrics. Many weave structures can be employed for specific attributes. Weaves that are confined to a single plain hold the greatest in-plain tensile properties [2]. Three dimensional weaves offer better strength in the out-of-plain direction, but a percentage of the fibers are forced to high bending at certain points in the weave and do not exhibit the same in-plain strength as 2D weaves. The satin weave is commonly used for its high tensile properties, good fiber wet out (resin filling), and easy conformation to complex shapes. Non-symmetric weaves such as the satin weave, however, have differing primary strength directions along opposite surfaces.

Every weave produces a different microstructure, ultimately responsible for different material properties. Furthermore, defects are introduced during material processing making the microstructure variability even higher. Common processing techniques include slurry infiltration, polymer impregnation and pyrolysis (PIP), melt infiltration (MI), and chemical vapor infiltration/deposition (CVI/CVD) [1]. Slurry infiltration impregnates a fiber preform (woven textile) by passing it through a liquid containing a carrier liquid, matrix powder, and organic binder. The slurry can be driven
through the textile by a pressure gradient to increase infiltration. Polymer impregnation and pyrolysis first impregnates the fiber preform with an organo-metallic polymer. The pyrolysis step then uses thermal treatment and hot pressing to decompose the organic components of the polymer resulting in a ceramic matrix. This technique can result in significant shrinkage and cracking if the appropriate precursor polymer and pyrolysis conditions are not met. PIP does however, require lower processing temperature than other techniques, and offers easier infiltration and shape forming. Melt infiltration relies on chemical reactions at high temperatures while one chemical component of the matrix, most commonly Si, is melted and infiltrates the preform by capillary action. Typically, the preform is a porous ceramic media that will react with the liquid infiltration to form the ceramic matrix. In the case of SiC matrix, the preform is pre-coated with carbon by CVI. Melt infiltration often has the lowest residual porosity. In CVI/CVD, a vapor is infiltrated into the fiber preform and either deposited as a solid or reacted with the surface to form a solid. The process can be allowed to proceed by diffusion, or driven by thermal or pressure gradients. CVI can be applied to complex shapes and does not require high temperatures that can lead to fiber degradation, but processing can take weeks and final porosity is quite high.

1.4 Microstructure Variability

Due to the huge amount of variability within ceramic composites, it is difficult to identify how local microstructures contribute to material properties. Also, the volume of data to be analyzed is continually growing, especially with the advancement of automated data collection. While the huge amount of data available affords important advantages, it often creates bottlenecks in processing and analysis. Often, measurements such as volume fraction or grain size must be made manually, making it impossible to manually obtain these measurements for an entire data-set. These measurements also fail to capture local variations because they are computed as an average. Understanding more complex qualities of the microstructure becomes a daunting task when faced with so much
data. In a uniformly heterogeneous material, damage initiation and propagation has no preferential
pathway. Real composite microstructures, however, are not uniformly heterogeneous, and defects
are introduced due to fiber weave structure and processing techniques.

This thesis looks at several methods for automated extraction of features, as well as methods for
narrowing the microstructure search to events that potentially have the greatest detrimental affect
on material properties. To that end, any set of features such as fiber orientation, size, coating thick-
ness, etc., can be defined and modeled with an estimated probability density function. Estimating
the probability density enables the classification of normal behavior for that particular set of fea-
tures. Modeling the normal behavior allows us to identify specific feature elements that are rare or
anomalous and thus should have the most significant impact on overall material properties.

1.4.1 Extreme Value Distributions

Previous work on the variability within composite microstructures focuses on probabilities as-
associated with extreme values [3]. Extreme value statistics refers to probability problems that deal
with the largest and smallest values of sets of random variables. Evaluation of component safety and
material strength often focuses on extreme values because of their role in limiting ultimate failure
strength. For a set of observations, a probability density function governs the likelihood of occur-
rence for all values. A single set of observations exhibits a specific minimum and maximum, but
with repeated sets, the observed minima and maxima will differ. With many repeated observed sets,
the minima and maxima are observed to be random variables with distributions associated with that
of the original variable and dependent on the number of observations. Traditional techniques for
determining the high cycle fatigue life of superalloys involved extensive experimentation to obtain
statistically significant samples which was then used to predict component life with a given level of
risk [4]. Such extensive testing is time and resource intensive, and often the predictions vary with
changing sample sizes. Utilizing knowledge of how the extreme values within a microstructure
affect the fatigue strength allows development of probabilistic failure models that do not require
destructive testing of large numbers of samples.

1.4.2 Anomaly Testing

Identifying the areas of behavior that likely conspire to form weak points in the material can
be accomplished through pattern recognition and data classification techniques. The data can be
thought of as either belonging to the normal behavior of the material or not. Generally, two ap-
proaches can be used in data classification: supervised and unsupervised learning. Supervised ap-
proaches require the presence of training data to establish the parameters of the classifier. Training
data refers to a sample where the individual classes are already known and the data has been labeled
according to its class. Testing data may either refer to data to be classified after training the classi-
fier, or a separate set of labeled data used to test the accuracy of the trained classifier. In anomaly
detection applications, often a pure data-set, or sample containing only normal data, is used for the
training data. For anomalies within CMC microstructure, anomalous structures are unknown and
may not even be intuitively visible, making the use of training data infeasible. A set of data cannot
be pre-divided into normal and anomalous classes, and it is impossible to know whether a sub-
sample is absent of anomalies. Unsupervised classification approaches are used in instances where
the initial classification of the training data is either too laborious or too subjective. Established
techniques in anomaly detection will be discussed within this section, with special attention given
to unsupervised approaches.

Much previous work in anomaly detection pertains to the areas on computer network defense
and hyperspectral imaging [5, 6, 7, 8]. In network defense, two main approaches exist: signature
based intrusion detection, and anomaly based intrusion detection [8]. Signature based detection
utilizes a bank of known signatures for various intrusion types. While very effective at detecting
intrusions with known signatures, frequent updates are needed to stay effective against new attacks.
The alternative, anomaly based intrusion detection, trains the classifier on the system during normal operation. Once the normal behavior of the system is learned, subsequent anomalous behavior is flagged as an intrusion. Anomaly based intrusion detection allows detection of new attacks because the detector makes no distinction between new or old attacks. Several techniques used for anomaly detection in network defense are statistical anomaly detection, machine learning approaches, and data mining approaches [8].

Statistical anomaly detection methods utilize a number of collected measures or dimensions of the training data and calculate statistics for each measure independently. Each new measurement is classified as normal or anomalous by a rule based method and assigned an anomaly score. Testing points with an anomaly score above a threshold are labeled anomalous and reported.

Machine learning approaches utilize training data to form a model which 'learns' optimal parameters for correct classification from the labeled training data. While machine learning refers to a large number of techniques and models, a prevalent method is artificial neural networks [5]. A neural network uses a series of network of interconnected nodes to simulate the decision making abilities of the human brain. During operation, inputs to a node are summed, and a weighted output is sent to other nodes. The result is a fast network of multiplications and summations that produce outputs used to classify the data. Because the network is just a series of nodes, however, the neural network functions as a black box, and it is impossible to know exactly how it is functioning. Additionally, neural networks cannot be implemented in an unsupervised scenario because the labeled data is necessary in training the network. Initially, arbitrary weights are assigned to the nodes and the training data classified by the network. The result of the classification is compared to the true labels, and the weights are adjusted iteratively to minimize the error. While the classification process is fast, training the neural network can be very slow especially for more complex neural networks.
Data mining refers to the process of identifying patterns within large datasets, and is often employed as a pre-processing step before other anomaly detection methods. In the case of intrusion detection, certain system calls may be associated others, and certain orders may be more common than others. The preprocessing steps described in Chapter II can be viewed as manual data mining procedures.

Data classification within image processing has many applications including surveillance, target detection in military applications, mass detection in medical images, and spectroscopy. Hyperspectral imaging refers to the collection of images at a series of wavelengths beyond the spectrum of visible light. Differences in absorption and reflectance of specific wavelengths of light for different materials creates different spectra that can be used to classify them. This is particularly useful in distinguishing man-made targets from a natural background [6] and any other abnormal spectral behavior. Each pixel contains an intensity value for each collected wavelength and is represented by a point in multidimensional space. This multidimensional 'cloud' can then be fit using a number of models. The most common technique for modeling the data is via a multivariate Gaussian. Gaussian fitting becomes hard to visualize beyond two dimensions; though, the multivariate Gaussian equation does not change with higher dimensionality. The covariance is a square matrix of size $K \times K$, where $K$ is the number of measurements or dimensions recorded for each point. For example, hyperspectral data collected by AVIRIS, an airborne optical sensor, features 224 contiguous spectral channels with wavelengths from 400 to 2500 nanometers. Estimation of models with such high dimensions is computationally expensive and requires very high numbers or data-points to be accurate. To simplify modeling, principle component analysis is implemented to transform the data and utilize only the components with high variance. By eliminating components with low variance, most of the collected information is retained, and the number of dimensions is significantly reduced.
Implementation of a classification algorithm requires a feature or set of features to evaluate. Previous microstructure characterization has focused either on the fiber scale or the fiber tow scale. Fiber scale features include fiber size, orientation, and fiber coatings. Tow features include tow cross section and affects of weave constraints on the shape of tow cross section. To the best of the authors knowledge, anomaly detection of ceramic composite microstructure has never been accomplished, nor has a feature space been developed that lies between the fiber and fiber tow scales.
CHAPTER II

PRE-PROCESSING STEPS

The goal of this work is to develop tools for the characterization of normal microstructural behavior, to pinpoint areas that may be of interest because of their non-conformity, and to intuitively show why specific areas are flagged as anomalous. Before the normal behavior of the CMC can be modeled, various pre-processing steps are employed to obtain a continuous 3D representation of the material. The following sections detail the steps of how the images are captured, translated into useful data, and modeled. Pre-processing techniques are presented in the following order: image acquisition and registration, segmentation, feature extraction, and feature tracking.

2.1 Image Acquisition and Registration

The first step in producing a full 3D representation of the sample is image stitching and registration. Samples are prepared via serial sectioning with the RoboMet.3D autonomous serial sectioning system shown in Figure 2.1a, manufactured by UES Inc. 4401 Dayton-Xenia Rd. Dayton, OH 45432. For ceramic matrix composites, scanning electron microscopy and computed tomography images often lack significant contrast between material phases. For this reason, images were captured with a Zeiss Observer.Z1M optical microscope. The RoboMet.3D robot polishes a sample to remove a thin layer, accurate to 0.25 microns, before moving the sample to the microscope for automatic imaging. To achieve the resolution necessary for feature identification and quantification,
a mosaic of high magnification images was collected with a 10\% overlap on all neighbors. The microscope stage automatically collects images for a specific mosaic pattern and records the stage positions for each image.

![Figure 2.1: a) Robomet.3D serial sectioning robot. b) Mosaic pattern used to collect images with 10\% overlap](image)

Aligning the images can be approached in one of may well established ways, though only two techniques are presented here; block matching and normalized cross-correlation. Block matching is a very simple approach to alignment that assumes image shift is purely translational. The stage shifts recorded by the microscope are utilized, along with image size, to define a window of probable overlap between neighboring images as seen in Figure 2.1b. The window of the reference image is set to the full size of the overlap area, while the window in the comparison image is reduced in size to ensure that it indeed lies completely within the reference image. If the windows are small enough or sufficiently close in size, an exhaustive search can be made between pixels in the reference and comparison windows for all possible shifts of the comparison image. For each potential shift, the mean squared error
\begin{align*}
MSE &= \frac{1}{NM} \sum_{i=1}^{N} \sum_{j=1}^{M} (C_{ij} - R_{ij})^2 
\end{align*}

is computed for the window, where \( N \) is the window width, \( M \) is the window height, and \( C_{ij} \) and \( R_{ij} \) are the comparison and reference pixels respectively. To reduce computation time with large windows, the windows can be coarsely aligned via block matching on a down-sampled image and the resulting vector shift implemented as a start point for progressively higher resolution images [9]. Use of an approximate starting point allows the block matching algorithm to search a smaller window but still converge to a global MSE minimum.

For normalized cross-correlation, both windows are first normalized by subtracting the local mean from every pixel and dividing by the standard of deviation, ensuring that both windows have a mean of zero and the same dynamic range [10]. This eliminates errors in matching due to intensity differences over large areas caused by illumination variability. The normalized comparison window is then shifted across the normalized reference window, and the sum of the pixel by pixel product is recorded. Shifts that have a high cross-correlation value correspond to a good match.

The resulting vector shifts from either the block matching algorithm or normalized cross correlation are utilized to stitch the images into a single, large mosaic. After images from each serial section are stitched, the resultant images are also registered. The assumption inherent to both of these techniques when applied to serially sectioned samples is that the underlying structure does not change significantly between images. While stitching images within a single slice, this assumption holds true. In the case of fiber reinforced composites, particularly multi-directional laminates, the sectioning orientation determines the difference in position between slices for fiber cross sections. In the sample presented here, multiple fiber tows are oriented 0° and 90° alternating such that between slices, the fibers of each orientation are displaced in opposite directions, according to Figure 2.2. It is important that fibers from both tows are present in approximately equal numbers within the
registration window so that the recorded registration shifts do not adhere to the actual shift present in a single tow. For the data presented here, a reference window of 900 x 900 pixels was taken from the target image and registered with a 1,000 x 1,000 pixel square from the reference image.

Figure 2.2: Orientation of the imaging plane with respect to fiber weave directions.

2.2 Image Segmentation

The next step in pre-processing is segmenting the images into the physical phases present in the material. Often this includes one or two matrix phases, fibers, and a fiber coating. While it is quite a simple task for a human to differentiate between the phases, it is a substantial problem in computer vision. The simplest method for dividing the constituent phases is to pick threshold values based on the image histogram such as seen in Figure 2.3b. Each of the phases present in the sample shown in Figure 2.3a corresponds to an intensity peak in the histogram. By choosing thresholds between peaks, such as the vertical lines in Figure 2.3b, and classifying pixels by which which partition they fall within, the majority of pixels are correctly separated into the appropriate phases. Regions where peaks overlap, such as the fiber coating phase and matrix phase in Figure 2.3b, have pixels belonging to both phases, and any hard decision boundary will result in considerable misclassification. The resulting image in Figure 2.3 exhibits rough edges and speckling due to
the nature of hard thresholding. Applying morphological operations or filters such as opening and median filtering can reduce the presence of speckling, but they do not guarantee the preservation of edges.

Figure 2.3: a) Gray-scale image of the S200 SiC/SiC CMC b) Histogram and Peaks fit using EM/MPM

2.2.1 EM/MPM Segmentation

The expectation maximization/maximization of posterior marginals algorithm for segmentation proposed by Comer et al. applies a Bayesian forward model to minimize the expected number of misclassifications[11, 12]. Each peak in the histogram corresponds to a material phase in the sample and is modeled with a single Gaussian, characterized by mean and variance. The expectation maximization (EM) algorithm is employed for estimation of Gaussian model parameters, mean and variance, to produce the model peaks in Figure 2.3b. Between iterations of the EM algorithm, the maximization of posterior marginals (MPM) developed by Marquin [13] is used for segmentation. The MPM algorithm is a Bayesian technique based on the Ising ferromagnet model that computes
classification probabilities for each pixel. The boundary between any two phases is assigned an energy proportional to the number of 'bonds' broken along a boundary divided by the length of the boundary. The 'bonds' are characterized by a strength parameter $\beta$ that favors straight edge boundaries at high values and has zero effect at $\beta = 0$. The value for $\beta$ must be adjusted with care taken that only segmentation noise is smoothed and not real features of the microstructure. Ideally, long boundaries are smoothed and preserved, while speckling is completely removed. The MPM algorithm uses a Monte Carlo algorithm to iteratively classify pixels according to the best fit of the interface smoothness of the segmentation and the gray-scale intensity of the original image.

### 2.2.2 Entropy Segmentation

Often, imaging techniques fail to produce images with significant contrast between material phases. In this case, a technique useful for segmentation of textured images utilizes an entropy filter and morphological processing to distinguish a specific feature from the background. Because fibers do not react when the matrix is infiltrated, fibers have a uniform chemical composition and the cross section of a fiber has a very uniform value. Chemical reactions during processing give the matrix a variable chemical makeup, resulting in high frequency gray-scale intensity variation when imaged. Employing an entropy filter [14] yields high entropy along boundaries, and within the matrix due to the speckled composition. The resulting entropy image exhibits 'holes' where objects of uniform value exist, and can be thresholded to remove high entropy pixels. The original image is then segmented by thresholding and compared via a logical AND with the entropy holes to eliminate pores. Prior knowledge of fiber size is used to exclude very large objects which may correlate to secondary matrix phases, or areas where the matrix features a uniform intensity. The watershed algorithm is then applied to remaining objects that are still deemed too large to be single fibers and thus may be several fibers that are connected as a result of segmentation. At this point, an entropy segmentation mask for all the fibers exists, but the fiber edges are incorrect. To minimize the use of
the watershed algorithm the entropy threshold is chosen very low to produce fewer interconnected objects. Entropy rises approaching edges, which, when combined with low thresholding, results in objects that correspond with the center of a uniform object and tend to be smaller than their original size. To refine the edge boundaries, each object is considered individually. Using the center of each object as a starting point, a series of rays are cast outward uniformly all the way around the object. As the ray moves outward, gray-level intensities are recorded via interpolation from the original image. The maximum gradient is used to define a boundary at the edge of a fiber. This boundary is used to define the true edge of a fiber. In the case of coated fibers, the ray encounters two boundaries: fiber to coating, and coating to matrix. The dual boundary constraint in this case helps to get an exact measure of the fiber boundary. This method has been found to robustly find accurate fiber segmentations in images with poor contrast between material phases.

2.3 Ellipse Extraction

Once the physical material structure is separated into its constituent phases, any number of features can be extracted. In the case of fiber reinforced composites, the fibers are usually the focus of feature extraction because of their affect on material properties. Often of interest are volume fraction, fiber size, fiber coating thickness and connectivity, and matrix secondary phases. For this work, fiber orientation is extracted and used for microstructure characterization. Fiber orientation of serially sectioned samples presumes knowledge of fiber positions within each layer. This can be achieved by means of geometric shape matching techniques. In the case that fibers are oriented 0° and 90°, such as with the data presented here, the sectioning plane can be oriented perpendicular to plane of the fiber weave and at an angle of 45° to both fiber orientations as previously seen in Figure 2.2. The sectioning results in 45° angle cross-sections of cylindrical fibers which can be modeled as ellipses.
2.3.1 Modified Hough Transform

The Hough transform (HT) is a well known technique for detection of lines and parametrically described shapes [16]. The Hough transform was originally introduced by Hough for detection of lines and has since been generalized to a variety of shapes [17, 18, 19, 20], particularly circles and ellipses. The HT defines a mapping between the image space, and a parameter space with number of dimensions equal to the number of shape parameters. The HT then considers all possible ranges of the parameters and accumulates votes for the best match in the parameter space. Increasing the number of shape parameters increases computation time exponentially, and many techniques have been explored for decreasing the number of parameters needed to define ellipses [18]. Here, every separate object found during segmentation is considered sequentially to reduce accumulator space size. The x and y centers \((x_0, y_0)\) along with major axis \(a\), minor axis \(b\), and rotation from x-axis \(\theta\) , are are used to uniquely define an ellipse. First, an edge detection algorithm such as the Sobel method [21] is applied to a binary mask of each object to eliminate interior pixels. Two algorithms are then applied in succession to determine the five ellipse parameters. The first, introduced by Niezgoda and Kalidindi [22], utilizes a complex filter for size invariant circle detection. The algorithm was later adapted by Przybyla for size invariant ellipse detection [23]. The complex phase coded annulus filter,

\[
\begin{align*}
\nu_{mn} &= \begin{cases} 
\frac{1}{2\sqrt{m^2+n^2}}e^{2\pi i \phi_{mn}} & \text{while } (r_{\text{min}})^2 \leq m^2 + n^2 \leq (r_{\text{max}})^2 \\
0 & \text{otherwise}
\end{cases} \\
\phi_{mn} &= \frac{\sqrt{m^2 + n^2 - r_{\text{min}}}}{r_{\text{max}} - r_{\text{min}}},
\end{align*}
\tag{2.2}
\]

is used to identify the major axis for the ellipse. The parameters \(m\) and \(n\) are the spatial coordinates of the filter, where the filter is a square of size \(2r_{\text{max}}\). By limiting the radius of the filter with predetermined constants \(r_{\text{min}}\) and \(r_{\text{max}}\), only circles with a radius between \(r_{\text{min}}\) and \(r_{\text{max}}\) are
considered by the accumulator. The magnitude, $\sqrt{m^2 + n^2 - r_{min}^2}$, of $\nu_{mn}$ ensures that circles of all radii within $r_{min}$ and $r_{max}$ are weighted equally during convolution.

The annulus specified in Equation 2.2, shown in Figure 2.4, has the appearance of a doughnut of intensity varying with radius between the minimum and maximum radii. Convolution of the fiber edge image with the filter results in constructive interference while the filter is within an ellipse of appropriate size. Because the filter is circular while the fibers are elliptical, the convolution produces a high intensity line along the major axis of the ellipse, as seen in Figure 2.5. A series of lines intersecting the intensity peaks of the convolution are considered as candidates for the major axis of the ellipse. The lines connecting the intensity peaks are extended until they reach edge pixels produced by the Sobel filter. Both edge intersections are assumed to be the endpoints of the major axis, with the midpoint being the center of the candidate ellipse.

The second algorithm uses the candidate ellipse centers in a modified ellipse HT proposed by Xie and Ji [24]. The modified HT takes all sets of edge points, $(x_1, y_1), (x_2, y_2)$, that are co-linear with the candidate ellipse centers and computes the size of the major axis, $a = 1/2\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, as well as the angle of rotation, $\theta = tan^{-1}[(y_2 - y_1)/(x_2 - x_1)]$. A third, non-collinear point, $(x, y)$, is chosen from the edge of the ellipse, and used to calculate a potential minor axis $b = \sqrt(a^2d^2\sin^2\tau)/(a^2 - d^2\cos^2\tau)$. The parameter $d$ is the distance from
the ellipse center, \((x_0, y_0)\), to one of the major axis endpoints, \((x_1, y_1)\). Variable \(f\) is the distance between the major axis endpoint and the non-co-linear point, \((x, y)\). The parameter \(\tau\) is the angle from the major axis to the non-collision point \((x, y)\). The minor axis sizes for all co-linear sets are accumulated, and the minimum is said to be the true minor axis. Calculating ellipse parameters for candidate ellipses rather than including them in the accumulator drastically limits the search space because the accumulator only holds one parameter. Readers interested in the Hough transform are directed to the references [16, 17, 18].

### 2.4 Feature Tracking

After ellipse extraction, the fiber information exists as a collection of 2 dimensional points. Utilizing the vector shifts from the registration process and the depth information from the microscope,
the data can be arranged as a 3D collection of points, but each fiber point is not associated with the corresponding fiber centers of subsequent layers. To perform analysis of 3D fiber statistics, the fibers must be individually constructed from the 2-dimensional data. This is approached as an object tracking problem where object identification has already been completed separately by the HT.

For this example, a simple rule-based object tracker is implemented. Starting at the first serial section, $z = 0$, each fiber, $i$, is compared to fibers in the next slice, $z+1$, within a predefined window of width comparable to average fiber diameter. If only a single fiber lies within the window, it is concatenated to fiber $i$, removed from further consideration, and the search is incremented to fiber $i+1$ within slice $z$. In the case of multiple fibers found within the window, Euclidean distance from $i$ to each of the candidates is calculated and the fiber with the minimum distance is considered a match.

Figure 2.6: Ellipses overlayed on the segmented image. Note missed detections and false detections, due to segmentation error.
A degree of complexity is added if one considers the possibility of failed fiber location within some slices, such as seen in Figure 2.6, due to poor segmentation and ellipse extraction. If an empty fiber track is encountered, that is, existing fiber $i$ has no match in the current slice, a predicted fiber position is used to define the search window. Predicted position is calculated as a linear projection of the fiber from previous known positions, though if gap size grows too large the fiber is considered lost and further matches are not sought. If a match is found for a projected fiber, the missing segment of fiber is filled by linear interpolation. To incorporate fibers that may be detected after the first slice, at the end of each layer, positions that have not been matched are added as new fiber tracks.

After looping all layers, many spurious fiber tracks may exist due to false ellipse detections, evidenced in Figure 2.6. False detections are typically associated with the secondary matrix phase having a gray-scale intensity close to that of the fibers, but are inconsistent from slice to slice and generally short lived. To remove these spurious fiber tracks, a maximum interpolation percentage criteria is implemented to remove fibers that are based heavily on interpolation. A minimum length
criteria eliminates short fiber tracks found in secondary phases as well as ellipses that appear in only one layer. To increase accuracy, the above algorithm is implemented twice. In the first fiber track, only fibers that are present in the first layer are tracked to prevent false ellipse detections from being matched to a real fiber. The second fiber track allows addition of new fibers, but greatly restricts the search window, further penalizing nonlinear behavior associated with spurious tracks. The resulting fiber tracks can be seen in Figure 2.7 where color is indicative of direction.
CHAPTER III

METHODS

After the images have been segmented, fiber identified, and fiber paths mapped in 3D, the behavior of fiber orientation and fiber neighborhood interaction can be examined. Fiber neighborhoods are of interest because they characterize the microstructure on a scale larger than the individual fibers but smaller than that of conventional statistical techniques. These meso-scale properties are believed to hold a large impact on the formation and propagation of damage in fibrous composites.

3.1 Orientation

The orientation field of fibers in the composite can be likened to a velocity field in fluid dynamics where fibers are represented by fluid streamers. Each fluid streamer is analogous to a fiber embedded in the material, where \( \mathbf{r} = (x, y, z) \) is the position vector of the streamer, \( \frac{\partial}{\partial \tau} \mathbf{r} \) describes the orientation or instantaneous ‘velocity’ of the streamer, and \( \tau \) refers to a length traveled along the streamer. In this analogy, the fiber centers function as discrete samples of the continuous orientation vector field. Any point in the field can be modeled as a fluid streamer who’s position varies smoothly along its length. A subtle point to be distinguished here is that while the serial section data is in an orthogonal coordinate system that is readily converted to \( (x, y, z) \) coordinates, the fiber ‘velocity’ is described by the distance traveled along each fiber. In other words, the sample representation is in the "Eulerian space," whereas the fibers are in the "Lagrangian space." This section describes
the transformations between these two coordinate systems and will be used as a foundation for the computation of orientation gradients, which will be used for anomaly detection.

Lagrangian coordinates are often used to describe fluid flows and conservation laws as they apply to a specific particle in a fluid [25]. Instead of a stationary coordinate system where the field is observed from a fixed point, Lagrangian coordinates move with a particle in a fluid field as it moves in space and time, or in the case of an embedded fiber, along the length of the fiber. The fiber centers, then, give discrete samples of the continuous orientation vector field. Quantitatively, a specific point on a streamer can be denoted by \( \mathbf{\rho} = (\xi, \eta, \tau) \) in a Lagrangian coordinate system, where \( \xi \) and \( \eta \) pinpoint a particular streamer, and \( \tau \) indexes length traveled along the path of flow.

Finding the orientation, \( \frac{d}{d\tau} \mathbf{r} \), is not trivial because the serial section data is collected in an orthogonal "laboratory reference frame," which is converted to Cartesian coordinates, \((x, y, z)\), which are the basis of the Eulerian coordinate system. Because the system is to be analyzed in Lagrangian coordinates and the data is in Eulerian coordinates, it is necessary to develop the transformation between the two systems. To accomplish this, following standard methods of multivariate calculus [26], we define the mapping between \((\xi, \eta, \tau)\) to \((x, y, z)\) in terms of implicitly defined functions as follows:

\[
\begin{align*}
x &= \Psi(\xi, \eta, \tau) \\
y &= \Phi(\xi, \eta, \tau) \\
z &= Z(\xi, \eta, \tau),
\end{align*}
\]  

(3.1)

and
\[ \xi = F(x, y, z) \]
\[ \eta = G(x, y, z) \]
\[ \tau = H(x, y, z). \] (3.2)

The transformation to Lagrangian space via the functions \( F, G, \) and \( H \) can be expanded into a Taylor series about some reference point, \((\xi_i, \eta_i, \tau_i)\),

\[ \xi = \xi_i + \frac{\partial F}{\partial x} \Delta x + \frac{\partial F}{\partial y} \Delta y + \frac{\partial F}{\partial z} \Delta z + O(|\Delta r|^2) \]
\[ \eta = \eta_i + \frac{\partial G}{\partial x} \Delta x + \frac{\partial G}{\partial y} \Delta y + \frac{\partial G}{\partial z} \Delta z + O(|\Delta r|^2) \]
\[ \tau = \tau_i + \frac{\partial H}{\partial x} \Delta x + \frac{\partial H}{\partial y} \Delta y + \frac{\partial H}{\partial z} \Delta z + O(|\Delta r|^2), \] (3.3)

where \( O(|\Delta r|^2) \) represents the higher powers of the Taylor’s series, and \( i \) indexes some point in space.

The parameters \( \Delta x, \Delta y, \) and \( \Delta z \) represent changes in position within Euler space from the expansion point \( i: \Delta x = x - x_i, \Delta y = y - y_i \) and \( \Delta z = z - z_i. \) The three equations can be combined in matrix form

\[
\begin{bmatrix}
\Delta \xi \\
\Delta \eta \\
\Delta \tau
\end{bmatrix} = \begin{bmatrix}
\frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \\
\frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} & \frac{\partial G}{\partial z} \\
\frac{\partial H}{\partial x} & \frac{\partial H}{\partial y} & \frac{\partial H}{\partial z}
\end{bmatrix} \begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix} + O(|\Delta r|^2) \] (3.4)

where \( \Delta \xi = \xi - \xi_i, \Delta \eta = \eta - \eta_i, \) and \( \Delta \tau = \tau - \tau_i. \) This can be simplified by using the symbols, \( \Delta \rho = [\Delta \xi, \Delta \eta, \Delta \tau]^T \) and \( \Delta r = [\Delta x, \Delta y, \Delta z]^T \) to describe the corresponding change in position for Lagrangian and Eulerian coordinates respectively. By letting the Jacobian transformation matrix,
\[
J_{F,G,H} = \begin{bmatrix}
\frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \\
\frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} & \frac{\partial G}{\partial z} \\
\frac{\partial H}{\partial x} & \frac{\partial H}{\partial y} & \frac{\partial H}{\partial z}
\end{bmatrix},
\]  \tag{3.5}

Equation 3.4 can be represented

\[
\Delta \rho = J_{F,G,H} \Delta \mathbf{r} + O(\| \Delta \mathbf{r} \|^2). \tag{3.6}
\]

In this work, the fibers do not have significant curvature, and it can be assumed that between sections the residual term, \(O(\| \Delta \mathbf{r} \|^2)\), may be neglected, yielding the final coordinate transformation equation

\[
\Delta \rho = J_{F,G,H} \Delta \mathbf{r}. \tag{3.7}
\]

The coordinate transformation defined by Equation 3.7 holds true as long as the determinant of the Jacobian is non-zero, and the fibers are continuous. This is justified by assuming that (1) fibers are independent and unique, so that each fiber \(i\) has its own pair of coordinates, \((\xi_i, \eta_i)\) for a given \(\tau_i\), and (2) there are no breaks in the fibers. The first is virtually guaranteed by the process, which does not allow fibers to join, and the second is typically correct. A break in a fiber would result in an anomaly condition.

Since traveling along a fiber is represented by changing \(\tau\), each fiber can be uniquely identified by its \((\xi, \eta)\) coordinates. The track of a fiber can be determined by indexing \(\tau\) down its length and holding both \(\xi\) and \(\eta\) constant. To convert this to measurable coordinates, the Lagrangian coordinates of the fiber must be converted to the Eulerian coordinates of the laboratory. To accomplish this, Equation 3.7 is rearranged to solve for \(\Delta \mathbf{r}\).

\[
J_{F,G,H}^{-1} \Delta \rho = \Delta \mathbf{r}, \tag{3.8}
\]
where $J^{-1}_{F,G,H}$ is the inverse of the Jacobian transformation matrix. Mathematically, $J^{-1}_{F,G,H} = J^T_{F,G,H}$ [26], but in practice it is determined from the data and Equations 3.2 are never evaluated explicitly.

The orientation, $\frac{d}{d\tau} r$, can then be found by taking the limit of $\Delta r/\Delta \tau$ as $\Delta \tau$ goes to zero. By replacing $\Delta r$ with its Lagrangian counterpart from Equation 3.8, the limit becomes

$$\lim_{\Delta \tau \to 0} \frac{\Delta r}{\Delta \tau} = \lim_{\Delta \tau \to 0} \frac{J^{-1}_{F,G,H} \Delta \rho}{\Delta \tau}.$$  

(3.9)

Because the Jacobian does not depend on $\Delta \tau$, it can be moved outside the limit while $\Delta \tau$ is distributed to the components of $\Delta \rho$, yielding the general equation for orientation,

$$\frac{\partial r}{\partial \tau} = J^{-1}_{F,G,H} \lim_{\Delta \tau \to 0} \left[ \begin{array}{c} \Delta \xi \\ \Delta \eta \\ 1 \end{array} \right] = J^{-1}_{F,G,H} \left[ \begin{array}{c} \frac{\partial \xi}{\partial \tau} \\ \frac{\partial \eta}{\partial \tau} \\ 1 \end{array} \right],$$

(3.10)

for any arbitrary change in $\xi$ and $\eta$. For a particular fiber, $i$, $\Delta \xi_i$ and $\Delta \eta_i$ are zero. The orientation of a fiber can then be described as, $\frac{\partial r_i}{\partial \tau} = J^{-1}_{F,G,H} [0, 0, 1]^T$, or the last column of the inverse Jacobian. Because the Jacobian is a transformation from Eulerian to Lagrangian coordinates, the inverse Jacobian is the transformation from Lagrangian to Eulerian. The Taylor series expansion for Equation 3.1 can be written in the same way as Equation 3.3 resulting in the inverse Jacobian,

$$J^{-1}_{F,G,H} = \frac{\partial (\Psi, \Phi, Z)}{\partial (\xi, \eta, t)} = \left[ \begin{array}{ccc} \frac{\partial \psi}{\partial \xi} & \frac{\partial \psi}{\partial \eta} & \frac{\partial \psi}{\partial \tau} \\ \frac{\partial \phi}{\partial \xi} & \frac{\partial \phi}{\partial \eta} & \frac{\partial \phi}{\partial \tau} \\ \frac{\partial z}{\partial \xi} & \frac{\partial z}{\partial \eta} & \frac{\partial z}{\partial \tau} \end{array} \right].$$

(3.11)

For a specific fiber, the orientation becomes the last column of $J^{-1}_{F,G,H}$, according to Equation 3.10.

### 3.1.1 Geometric Interpretation of the Lagrangian Space

Due to serial sectioning and ellipse detection, the orientation field is only sampled at fiber locations for discrete increments in $z$. To effectively reduce expressions to two dimensions, fibers
are assumed to belong to two primary orientations of equal and opposite angles 45° with respect to vertical. Under this constraint, all fibers advance by the same length, \( \tau \), between serial sections. Figure 3.1 details how the individual \([x, y, z]\) components of \( \mathbf{r} \) can be defined in terms of \( \tau \). The vector,

\[
\mathbf{r} = \begin{bmatrix}
\tau \sin \theta \cos \phi \\
\tau \sin \theta \sin \phi \\
\tau \cos \theta
\end{bmatrix} = \begin{bmatrix}
x \\
y \\
z
\end{bmatrix},
\]

(3.12)
can then be used to define the orientation of the fiber,

\[
\frac{\partial \mathbf{r}}{\partial \tau} = \begin{bmatrix}
\sin \theta \cos \phi \\
\sin \theta \sin \phi \\
\cos \theta
\end{bmatrix},
\]

(3.13)

where \( \cos \theta \) is a constant for all fibers. In practice, the \( x \) and \( y \) components of orientation are calculated by assuming that a fiber remains linear over a stretch of several serial sections. Zenith angle, \( \theta \), remains constant for all fibers, resulting in \( \sin \theta = \cos \theta = \sqrt{2}/2 \). The remaining components \( \cos \phi \) and \( \sin \phi \) can be approximated for each fiber by

\[
\cos \phi \approx \frac{x}{P}, \quad \sin \phi \approx \frac{y}{P},
\]

(3.14)

where \( P \) is the projection of \( \mathbf{r} \) into the \((x, y)\)-plane. For \( \theta = 45^\circ \), the magnitude of vector \( P \) is the same as \( z \). This allows equation Equation 3.13 to be written

\[
\frac{\partial \mathbf{r}}{\partial \tau} \approx \begin{bmatrix}
\sqrt{2} \frac{x}{z} \\
\sqrt{2} \frac{y}{z} \\
\sqrt{2} \frac{z}{z}
\end{bmatrix},
\]

(3.15)

which can be easily evaluated for a section of fiber. To reduce noise in the computation of orientation, \( x/z \) and \( y/z \) are computed from several slices about the current slice instead of between single
slices. While the measured values theoretically must be multiplied by \( \sin \theta \), it remains constant for all fibers and simply has a scaling affect on any subsequent analysis, and thus can be neglected.

3.2 Orientation Gradient

The features of the orientation field that describe neighborhood behavior collectively, such as eddies, boundaries, or other non-uniformity in the flow are characterized by the gradient of the orientation field. Assuming a smoothly varying orientation field, the gradient of the orientation can be used to describe how the orientation is changing with respect to each of the Eulerian coordinates, \((x, y, z)\).

In practice, the orientation gradient can not be directly calculated. Instead, a Taylors series expansion of orientation,
\[ \Delta \frac{\partial \mathbf{r}}{\partial \tau} = \nabla \frac{\partial \mathbf{r}}{\partial \tau} \Delta \mathbf{r} + O(|\Delta \mathbf{r}|^2), \tag{3.16} \]

is used to express the change in orientation via the orientation gradient and a change in position, \( \Delta \mathbf{r} \). The difference in orientation between two points, \( \Delta \frac{\partial \mathbf{r}}{\partial \tau} \), corresponds to the same three-dimensional difference in position as \( \Delta \mathbf{r} \), related through the orientation gradient. Again, the residual term, \( O(|\Delta \mathbf{r}|^2) \), can be neglected due to the assumption of linearity. Equation 3.16 can be represented between two arbitrary points, \( \mathbf{r}_0 \), a reference point, and \( \mathbf{r}_1 \). By replacing \( \Delta \frac{\partial \mathbf{r}}{\partial \tau} \) and \( \Delta \mathbf{r} \) with the resulting components, the general equation for the Taylor series becomes

\[ \frac{\partial \mathbf{r}_1}{\partial \tau} - \frac{\partial \mathbf{r}_0}{\partial \tau} = \nabla \frac{\partial \mathbf{r}_0}{\partial \tau} \begin{bmatrix} x_1 - x_0 \\ y_1 - y_0 \\ z_1 - z_0 \end{bmatrix}, \tag{3.17} \]

which can be utilized to solve for the gradient using measured values of \( \Delta \mathbf{r} \) and \( \Delta \frac{\partial \mathbf{r}}{\partial \tau} \). The orientation gradient at \( \mathbf{r}_0 \) can be expressed fully as

\[ \nabla \frac{\partial \mathbf{r}}{\partial \tau} = \begin{bmatrix} \frac{\partial}{\partial x} \left( \frac{\partial \Psi}{\partial \tau} \right) & \frac{\partial}{\partial y} \left( \frac{\partial \Psi}{\partial \tau} \right) & \frac{\partial}{\partial z} \left( \frac{\partial \Psi}{\partial \tau} \right) \\ \frac{\partial}{\partial x} \left( \frac{\partial \Phi}{\partial \tau} \right) & \frac{\partial}{\partial y} \left( \frac{\partial \Phi}{\partial \tau} \right) & \frac{\partial}{\partial z} \left( \frac{\partial \Phi}{\partial \tau} \right) \\ \frac{\partial}{\partial x} \left( \frac{\partial Z}{\partial \tau} \right) & \frac{\partial}{\partial y} \left( \frac{\partial Z}{\partial \tau} \right) & \frac{\partial}{\partial z} \left( \frac{\partial Z}{\partial \tau} \right) \end{bmatrix}, \tag{3.18} \]

a \([3 \times 3]\) matrix defining how the Lagrangian components of orientation change with respect to changes in the Eulerian coordinates. The gradient can then be broken into symmetric and anti-symmetric components according to the equation

\[ A = \frac{1}{2} (A + A^T) + \frac{1}{2} (A - A^T) = A_{\text{sym}} + A_{\text{anti}}, \tag{3.19} \]

where the symmetric part of the matrix, \( A_{\text{sym}} \), contains the linear deformation components of the matrix, and the anti-symmetric part, \( A_{\text{anti}} \), contains the rotational components. Splitting the matrix
into symmetric and anti-symmetric parts allows it to intuitively represent the physical behavior of the transformation. The symmetric component of the orientation gradient, then, describes how the fibers are collectively moving towards or away from one another, as well as shearing past each other.

### 3.2.1 Measurement of the Orientation Gradient

The S200 sample features measurements of the continuous orientation field only at the fiber positions for increments in the $z$-plane. The Taylor series of orientation can then be evaluated for any pair of fiber centers in the sample. Because the Taylor series is a linear approximation, only fiber centers that are near one another are used in calculations. For computational ease, differences in position are always evaluated for fiber pairs in the same plane.

In practice, measurement of the gradient is made in two steps. First, the linear approximation of fiber orientation is calculated, as in Section 3.1.1. The only change between fibers is $\cos \phi$ and $\sin \phi$, which can be approximated using Equation 3.15. Second, the linear approximation of the gradient is computed using measured values of position and the calculated values of linear orientation. Here, the vector for orientation is represented,

$$\frac{\partial \mathbf{r}}{\partial \tau} = \begin{bmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{bmatrix} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix},$$

(3.20)

to simplify the expression of following equations. Utilizing the substitution from Equation 3.20, the left hand side of Equation 3.17 can be replaced, yielding

$$\begin{bmatrix} V_{x_i} - V_{x_j} \\ V_{y_i} - V_{y_j} \\ V_{z_i} - V_{z_j} \end{bmatrix} = \nabla \frac{\partial \mathbf{r}}{\partial \tau} \begin{bmatrix} x_i - x_j \\ y_i - y_j \\ z_i - z_j \end{bmatrix},$$

(3.21)

where index, $i$ and $j$, denote evaluation at a reference point and some nearby point, respectively.

As previously stated, for the sample presented here, the zenith angle $\theta$, is assumed to be the same for all fibers. This results in $V_{z_i} = V_{z_j}$, causing the third row of $\Delta \frac{\partial \mathbf{r}}{\partial \tau}$ to be zero. Serial
sectioning results in all measurements in a section having the same $z$, resulting in the third row of $\Delta \mathbf{r}$ also being zero. Because the $z$ component, $V_z$, of orientation is constant, the derivative is zero, making the third row of the gradient matrix zero. This leads to the reduced expression,

$$
\begin{bmatrix}
V_{x_i} - V_{x_j} \\
V_{y_i} - V_{y_j} \\
0
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\partial}{\partial x_i} (V_{x_i}) & \frac{\partial}{\partial y_i} (V_{x_i}) & \frac{\partial}{\partial z_i} (V_{x_i}) \\
\frac{\partial}{\partial x_i} (V_{y_i}) & \frac{\partial}{\partial y_i} (V_{y_i}) & \frac{\partial}{\partial z_i} (V_{y_i}) \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_i - x_j \\
y_i - y_j \\
0
\end{bmatrix},
$$

(3.22)

where, upon multiplication, the third column of the gradient disappears due to the zero in $\Delta \mathbf{r}$. The result is in an effective 2D orientation gradient Taylor series,

$$
\begin{bmatrix}
V_{x_i} - V_{x_j} \\
V_{y_i} - V_{y_j} \\
0
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\partial}{\partial x_i} (V_{x_i}) & \frac{\partial}{\partial y_i} (V_{x_i}) \\
\frac{\partial}{\partial x_i} (V_{y_i}) & \frac{\partial}{\partial y_i} (V_{y_i}) \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
x_i - x_j \\
y_i - y_j \\
0
\end{bmatrix},
$$

(3.23)

that is used to calculate the $(x, y)$-plane orientation gradient for fibers.

The gradient can be calculated by replacing the values of $\Delta \mathbf{r}$ and $\Delta \partial \mathbf{r}/\partial \tau$ with known data associated with a fiber and its neighbor. Calculation of the gradient based on a single neighbor, however, is very noisy because fibers provide sparse sampling of the orientation field, and the Taylor series assumes small changes in location.

### 3.3 Noise Reduction

Noise reduction can be approached through the addition of neighboring points to make an over-determined set of equations. The 2D Taylor series from Equation 3.23 is computed for every fiber, $i = 1, 2, 3...I$. A neighborhood is defined around each fiber, $i$, to include every fiber within a given radius. Each fiber has a different different number of neighbors, indexed by $j = 1, 2, 3...J_i$, where $J_i$ is the number of neighbors of $i$. Equation 3.23 can be written as a transpose and expanded to include all neighboring fibers,
\[
\begin{bmatrix}
\Delta V_x(i, 1) & \Delta V_y(i, 1) \\
\Delta V_x(i, 2) & \Delta V_y(i, 2) \\
\vdots & \vdots \\
\Delta V_x(i, J_i) & \Delta V_y(i, J_i)
\end{bmatrix}
= 
\begin{bmatrix}
\Delta x(i, 1) & \Delta y(i, 1) \\
\Delta x(i, 2) & \Delta y(i, 2) \\
\vdots & \vdots \\
\Delta x(i, J_i) & \Delta y(i, J_i)
\end{bmatrix}
\begin{bmatrix}
\frac{\partial V_{xi}}{\partial x} & \frac{\partial V_{yi}}{\partial x} \\
\frac{\partial V_{xi}}{\partial y} & \frac{\partial V_{yi}}{\partial y}
\end{bmatrix}
\]

where \( \Delta V_x(i, j) = V_{xi} - V_{xj} \) and \( \Delta V_y(i, j) = V_{yi} - V_{yj} \). Parameter \( B_i \) is the matrix of in-plane orientation change between all \( j = 1, 2, 3...J_i \) neighbors of \( i \) and fiber \( i \). The parameter \( A_i \) is the change in position matrix between fiber \( i \) and its \( J_i \) neighbors, and \( G_i \) is the orientation gradient at fiber \( i \). Both \( \hat{A}_i \) and \( \hat{B}_i \) are known and can be solved for \( \hat{G}_i \) as an overdetermined set of linear equations via least squares, \( \hat{G}_i = (\hat{A}_i^T \hat{A}_i)^{-1} \hat{A}_i^T \hat{B}_i \). The gradient then, contains information of how the orientation is changing with position in a single \((x, y)\)-plane. The gradient, \( \hat{G}_i \), can then be split into its rotational and linear components according to Equation 3.19. In planar laminates, the rotational component of the gradient is assumed to be minimal, thus the symmetric gradient, \( \hat{G}_{sym} \), is the focus of further analysis. More complex layup geometries may require the analysis of anti-symmetric components of the gradient alongside the symmetric gradient. The symmetric gradient itself represents a combination of possible scaling or shear in the \( x \) and \( y \) directions. In the case of pure shear, the transformation matrix in

\[
\begin{bmatrix}
\hat{x} \\
\hat{y}
\end{bmatrix} = \begin{bmatrix}
a & s_1 \\
s_2 & b
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix},
\]

exhibits a diagonal equal to one and \( s_1 \) and \( s_2 \) are equal and non-zero, where the axis of shear is always an eigenvector of the matrix. For bidirectional shear, the primary shear direction corresponds to the eigenvector associated with the larger eigenvalue, and the other eigenvector corresponds to the secondary shear direction. While shear lies on the off-diagonal of the transformation matrix and directly correlates to eigenvectors, expansion and contraction are represented by the magnitudes \( a \)
and $b$ in the diagonal. In the case of expansion or contraction in a given direction, the corresponding coefficient is greater than or less than one respectively, and the eigenvectors lie along the axes of original coordinate system. In the event of multiple factors, which is often the case in real samples, the calculated eigenvectors are a combination of bidirectional shear and scaling.

### 3.4 Gaussian Mixture Modeling

For the purpose of modeling, the data is represented by a feature vector

\[
S_i = [S_{k1}, S_{k2}, ..., S_K]^T, \tag{3.26}
\]

where $S_{k1}, S_{k2}, ..., S_K$ can be any number of measurable features. A model for each of the classes is formed such that they produce the minimum number of expected classification errors. The model generated for the training data is then applied to remaining unknown data to make classifications. In the case of anomaly detection, particularly in materials science, the classes are not known and cannot be labeled prior to modeling, thus unsupervised approaches must be considered. Unsupervised machine learning fits the data as a whole and applies the same model to find anomalous behavior.

An assumption of this method is that the anomalies represent a negligible portion of the data, and that the model is a fit of the normal behavior. If the anomaly contamination is too high when creating the model, any abnormal behavior will be included in the normal model and considered normal.

In this work, the data was assumed to be Gaussian, though due to the presence of multiple fiber tows in the sample resulting in a multi-modal set, a mixture of Gaussians was necessary to model data accurately. In the case of all fiber tows being perfectly aligned in the weave plane, two Gaussians would be sufficient for modeling. While this condition is almost met, there is enough out of plane miss-alignment between tows to merit additional Gaussian components. In practice, one
Gaussian per tow is sufficient for accurate models. The Gaussian mixture model (GMM) is defined as

\[ p(v) = \sum_{m=1}^{M} w_m f_m(v | \mu_m, \Sigma_m), \]  

(3.27)

where \( p(V) \) is the probability density (pdf) function of the modeled feature vector. Estimating the total number of components, \( M \) can be computationally expensive and, as previously stated, each tow can be modeled with a single Gaussian. The parameter \( w_m \) is the weight associated with each Gaussian, and

\[ f_m(v | \mu_m, \Sigma_m) = \left(\frac{2^{-k/2} |\Sigma_m|^{-1/2}}{2\pi^{K/2}}\right) \times \exp \left[ -\frac{1}{2} (v - \mu_m)^T \Sigma_m^{-1} (v - \mu_m) \right] \]  

(3.28)

refers to each Gaussian component. The parameters \( \mu_m \) and \( \Sigma_m \) are the mean and covariance of the \( m^{th} \) component, respectively. The mean is a vector of size \( 1 \times K \) and covariance is of size \( K \times K \) where \( K \) is the length of the feature vector. Estimation of the model is accomplished via the expectation maximization (EM) algorithm [27]. Anomalies are then classified based on the probability density calculated by the EM algorithm. Figure 3.2 shows an example of a Gaussian mixture model fit to a feature vector of \( x \) and \( y \) fiber orientation.

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Figure 3.2: Gaussian mixture model fit to orientation

The height at a given point specifies its likelihood of occurrence. An arbitrary threshold can be chosen, and adjusted to yield the desired number of anomalies. A given likelihood threshold, however, lacks physical meaning. Instead a probability bound is chosen to capture a certain percentage (98%) of the normal data. A fine grid is created and the likelihood values are calculated at all points. Iterative integration is then applied to find the likelihood threshold that captures the appropriate percentage of the pdf. Each sample in the real feature vector is then evaluated for its likelihood and compared to the threshold. A sample is declared anomalous if its likelihood is less than the threshold. A more intuitive representation is obtained by taking the $-\log$ of Equation 3.28,

$$-ln[f_m(\mathbf{v}|\mu_m, \Sigma_m)] = -ln(2\pi^{-k/2}|\Sigma_m|^{-1/2}) \times \left[\frac{1}{2}(\mathbf{v} - \mu_m)^T \Sigma_m^{-1}(\mathbf{v} - \mu_m)\right], \tag{3.29}$$
so that the exponential drops out, and the remaining terms are a constant multiplied with a Mahalanobis distance (M-dist). Mahalanobis distance is the same as Euclidean distance when the covariance matrix is the identity, or the distribution is circular. In the case of non-zero covariance between components, the M-dist measures the distance away from the center of the distribution corrected by the covariance matrix so that points not along the principle orientation have a larger M-dist. Anomalies then, are points that exceed a certain sum of distances from the center of any Gaussian.
CHAPTER IV

RESULTS

4.1 Visualization of Fields

Visualization of orientation field and orientation gradient is attained via the HSV color-model. Hue (H) in the HSV color-model changes cyclically with angle as shown in Figure 4.1. Saturation (S) refers to the intensity of the color and value (V) refers to the darkness. When the HSV color-model is applied to visualization, Figure 4.1 b) accurately reflects the correlation between angle and hue as it is implemented.

Figure 4.1: Color visualization achieved via the HSV model

(a) 3D representation of the HSV color-model.
(b) Implementation of the HSV color-model.
4.1.1 Orientation Visualization

To visualize orientation, the \((x,y)\) vector of fiber position change between layers is transformed into polar coordinates, \((r,\phi)\), where the direction of fiber shift, \(\phi\), is represented as hue. Saturation is set to the normalized magnitude of shift, \(r\). Value remains constant at 0 so that orientation visualization features no variability in color darkness.

Visualization and anomaly testing are initially performed on the synthetic data (phantom) seen in Figure 4.2. Each fiber, modeled as an ellipse, is pasted to a hexagonally packed grid to form a tow. To avoid perfect rows, a small amount of white noise is added to the hexagonal grid positions. Several groups of fibers (tows) are placed one on top of another, and a uniform horizontal shift is then assigned to all fibers in a tow, with neighboring tows receiving equal and opposite orientations. Horizontal arrows are placed in the center of each tow denoting the direction of fiber displacement between slices.

The same synthetic image is shown in Figure 4.3 with each fiber being colored according to its difference in position between slices. Three anomalies were introduced to the phantom; a sink, hill, and a fiber oriented such that it moves through two tows (trans-tow fiber). The sink, seen in the left-center of the phantom, is a radius in which fibers are pulled uniformly toward the center. The hill, evidenced in the right-center of the image, behaves as a radius of uniform fiber repulsion from the center. The trans-tow fiber can be seen in the upper-left portion of the image. In the original gray-scale image of the phantom, the inserted anomalies are difficult to spot, particularly the trans-tow fiber. The colored figure immediately highlights fibers associated with the anomalies and gives insight within the 2D image as to their 3D orientation.
Applying the described methods for visualization and anomaly detection of real CMCs affords similar results. Figure 4.4 shows the results of coloring individual fibers by orientation. Fibers that fail to be detected with the Hough transformation algorithm do not receive a color, highlighting regions of poor segmentation and feature extraction. For the region of interest displayed here, most
fibers have been accurately captured by the ellipse extraction methods. Based on the observed fiber colors, candidates for anomalous fiber orientation emerge.

Figure 4.4: S200 sample colored by fiber orientation

4.1.2 Orientation Gradient Visualization

Orientation gradient is also visualized according to the HSV color-model in Figure 4.1 b), though only angles from 0 to 180° are used. Eigenvectors that have a negative angle are rotated 180° so that they fall within the specified range. Because the orientation gradient is computed locally for each fiber, the most intuitive approach for visualizing the orientation gradient field is with a Voronoi tessellation about the fiber centers. Individual Voronoi cells are constructed such that every point within the cell is closer to the fiber it is centered on than any of its neighbors [27]. Alternatively, the lines constructing a cell all bisect the distance between neighboring fibers, and are perpendicular to the line connecting fiber centers. Each Voronoi cell is then colored with the HSV model according to the eigenvalues and eigenvectors of the symmetric orientation gradient (Equation 3.19). Hue is assigned by the direction of the eigenvector associated with the larger eigenvalue. Saturation denotes the normalized difference in eigenvalues, while the normalized sum of eigenvalues is represented by value.
The phantom is again employed to validate the visualization techniques and Gaussian mixture model anomaly testing. The colored Voronoi tessellation is displayed atop the original grayscale image, so that the underlying fibers/tows can be seen. The most noticeable aspect of the orientation gradient Voronoi image is the colored bands along tow boundaries. The width of the color bands corresponds to the neighborhood radius used in computing the gradient. Along the tow boundaries, where the neighborhood radius includes fibers of another tow, the eigenvalues of the symmetric orientation gradient are of opposite sign. Near the center of a fiber tow, the difference in eigenvalues is very small due to relatively uniform fiber movement. Saturation, then, is low in the middle of a tow and high along tow edges creating the color banding due to shear deformation seen in Figure 4.5. The color of the shear band is determined by the angle of shear. For the phantom, tow boundary shearing is in the ±45° directions depending on the ordering of the tows (which tow is on top). The value of a cell is large when both eigenvalues are positive with large magnitude and small when both eigenvalues are negative with large magnitude. As with saturation, fibers of uniform neighborhood orientation have zero orientation gradient, and the sum of eigenvalues is zero. Areas in the gradient image where value deviates from zero represent contraction or expansion. The gradient image of the known anomalies within the phantom appropriately reflects the behavior of the fibers. The sink exhibits a darker color indicative of fibers converging, while the hill has a whiter color indicating divergence. The stray fiber features a region of divergence preceding the fiber as the tow opens to accommodate it, and a region of convergence behind the fiber as the neighboring fibers settle back into position.

Implementation of the Voronoi tessellation on S200 SiC/SiC CMC again produces results similar to the phantom, though significantly more noise is present. Figure 4.6 shows a cropped region of the SiC/SiC sample after orientation gradient calculation. Due to the discrete sampling of the orientation field, increasing the number of points used in calculating the gradient requires expanding the
fiber inclusion radius during gradient computation. The result of a larger radius can be seen in Figure 4.7. While the noise of the orientation gradient is smoothed producing a Voronoi diagram with less fluctuation between cells, too large of a neighborhood size risks smoothing out real features of the orientation gradient.

Figure 4.5: Orientation gradient representation of the phantom data.

Figure 4.6: Orientation gradient representation of the S200 sample.
4.2 Anomaly Detection of the Fields

Anomaly detection is implemented via Gaussian mixture modeling, as it is described in Section 3.4. Results of the GMM anomaly detection are displayed by numbering points atop the field visualization that have been labeled anomalous.

4.2.1 Orientation Anomalies

Anomaly detection applied to the phantom offers proof of concept; the methods presented are indeed capable of identifying known anomalies within the orientation field. Modeling the distribution of fiber orientations by means of a Gaussian mixture model with one Gaussian per fiber tow yields the model pdf seen in Figure 4.8. The bimodal distribution present in the model is a result of assigning opposite shifts to the tows. The construction of the phantom with a limited number of possible shifts between layers results in a very discretized plot of the orientations. While probability density is not observed because the points lay atop each other, the Gaussian model in 4.8 shows the relative density of orientations. A decision threshold is applied to the pdf such that it accounts for 98% of the data, and the points lying outside the boundary are marked as anomalies. Figure
4.9 shows the fibers that have been classified as anomalous by orientation. Fibers that have been identified as anomalies closely correlate to the anomalies manually inserted into the phantom.

![Estimated PDF](image)

Figure 4.8: Model pdf of the orientation field for the phantom.

![Anomalies](image)

Figure 4.9: Anomalies of the orientation field for the phantom.

When applied to the S200 SiC/SiC CMC, a limited number of orientation anomalies are identified. As with the phantom, two primary orientations are evident in the estimated PDF shown in
Figure 4.10, due to the orthogonal weave structure. Each tow however, results in a slightly different cluster within the two primary groups, and leads to modeling the PDF with one Gaussian per tow. Most anomalous orientation fibers are classified anomalies due to high angle of orientation, meaning that they shift position significantly between slices. These fibers typically reside on the edge of a tow, as seen in Figure 4.11, and are less constrained than a fiber in the center of a tow. Several fibers are classified anomalous due to a lack of shift between slices. These fibers are generally on the interior of a tow. Anomalies of the orientation field do not tend to form close groups, but rather are spread sparsely throughout the sample. This implies that the effects of anomalies in the orientation field are likely negligible.

Figure 4.10: Model pdf of the orientation field for the S200 SiC/SiC.
4.2.2 Orientation Gradient Anomalies

Anomaly detection on the orientation gradient of the phantom, seen in Figure 4.12, exhibits several issues with the orientation gradient. First, edge effects caused by neighborhood computation where the neighborhood is single sided result in large areas of anomaly classification where no intentional anomaly is present. The behavior of the synthetic microstructure along the tow edges is anomalous and should be classified as an anomaly, though it is not of interest and is usually cropped out. In practice, large areas must be utilized to minimize the influence of edge effects. Secondly, noise in the fiber positions results in a substantial number of anomalies that have no neighboring anomalies and only exist within a single slice. Because the fibers are rigid, a true anomaly is likely to be classified through a number of layers. Additionally, computation of anomalies for a finite neighborhood size casts doubt on anomalies with no neighbors. Despite the problems with the orientation gradient, the anomalies within the phantom are correctly classified as anomalous. Techniques for improving the accuracy of the anomaly detection will be discussed.

Figure 4.12 shows anomalies based on the feature vector \( \mathbf{S} = [\lambda_1 - \lambda_2, \lambda_1 + \lambda_2] \) where \( \lambda_1 \) and \( \lambda_2 \) are the eigenvalues of the symmetric gradient. Anomalies inserted into the phantom are correctly classified by the Gaussian model. Additionally, anomalies corresponding to neighborhood
Figure 4.12: Anomalies of the orientation gradient field applied to the phantom data. Notice anomalies along edges.

behavior, such as expansion and contraction, are identified more accurately by means of the orientation gradient than the orientation field. Noise is more prevalent in the gradient field due to the neighborhood computation based on a relatively small number of samples. The noisy gradient field results in classifications of point anomalies where no anomaly exists in the phantom. The number of point anomalies can be directly reduced by increasing the number of neighborhood fibers used in computation as discussed in Section 4.1.2, or by introducing a more restrictive threshold to the anomaly detector. Increasing the neighborhood size, however, has a significant blurring effect on the orientation gradient, and can smooth away real features. Reducing the threshold for the anomaly detector also risks eliminating genuine anomalies.

True anomalies, however, are present in multiple layers due to the rigidity of the fibers. To emphasize the consistency of anomalies, anomalies are clustered based on average neighborhood distance, and colored according to the number of layers in which they are present. Blue signifies the presence of an anomaly while red indicates high consistency through multiple layers. Figure 4.13 shows the grouped anomalies for the gradient field. The grouped anomaly figure exhibits three main anomaly groupings. Anomalies within the real microstructure center on regions where neighboring tows separate due to the weave. Often the separation of tows as a result of weave architecture results
in matrix dense regions which open into a pore. Figure 4.13 features an anomaly on the right side where the end of a tow has created space for a pore. The anomaly visualization indicates high shear between the tows as well as expansion as the edge of the tow recedes, opening a gap between fiber tows occupied by a pore. In the case of the anomaly grouping on the left-center of the image, a matrix dense region is seen preceding the presence of a pore.

Figure 4.13: Anomaly groupings based on neighborhood and depth consistency. Color between tow boundaries indicates high shear in the gradient, while light or dark regions indicates expansion and contraction respectively.

Before grouped anomalies can be confidently called real microstructural anomalies, the fiber extraction and tracking must be examined for all layers contributing to the anomaly classification. Fiber location errors, such as fiber labels swapping during tracking, result in abnormal fiber tracks that the detector classifies as anomalies. While these false fiber tracks are indeed anomalies, they do not correspond to anything real in the microstructure and should be eliminated. To verify the accuracy of the fiber track, a sequence of images with fiber tracks uniquely numbered is examined to be sure each track follows a single fiber. Figure 4.14 shows the rightmost example of one of the anomaly regions from Figure 4.13 with numbered fiber tracks. Color is assigned to the numbers to aid in identifying fiber swapping. The region around each anomaly group was verified by inspection to have consistent fiber tracks with no significant fiber position jumping.
After verifying the presence of a true anomaly, it is helpful in understanding the microstructure to know why a region was classified as an anomaly. This is accomplished by numbering anomaly points on both the orientation gradient image as well as the plot of the feature vector $\mathbf{S}$ such as with Figures 4.15 and 4.16. Each anomaly within Figure 4.15 corresponds to a point in Figure 4.16. It can be seen that the anomalies lying below zero on the vertical axis correspond to contraction anomalies, and those above zero are expansion anomalies. Anomalies from the cross-tow fiber exhibit expansion in front of the fiber and contraction behind; though, some of the numbered anomalies (7, 11, 12, 16) can also be attributed to high magnitude of shear. Pairing the anomaly plot with the visualization of the orientation gradient field enables clear determination of why points are anomalous.
Figure 4.15: Numbered anomalies displayed on the orientation gradient of the phantom.

Figure 4.16: Numbered anomalies corresponding to the numbered anomalies of the orientation gradient.
It can be seen in Figures 4.15 and 4.16 that the anomalies can be split into three general regions denoted by the dashed lines. Anomalies that lie above the bulk of the data (horizontal line) can be attributed to expansion. This is distinctly shown with anomalies 21, 24-28. Though the points are not particularly close to one another with respect to shear, they all exhibit similar expansion. The points corresponding to the compression anomaly can be seen below the horizontal axis of the plot, as expected for compression. The stray fiber features expansion anomalies as neighboring fibers open to accommodate the fiber and compression anomalies as they close in behind the fiber. Interestingly, the stray fiber also results in a number of fibers with high shear that lie in the rightmost region in the anomaly plot.

Similar regions can be defined for the S200 sample to aid in understanding the behavior of the local microstructure. Figures 4.17 and 4.18 show the numbered anomalies of the orientation gradient on the gradient image and feature vector plot. The circled region in the orientation gradient image corresponds to the tightly clustered points circled in the anomaly plot. This region of contraction corresponds to the edge of a receding tow, leaving a pore between neighboring tows.

Figure 4.17: Numbered anomalies displayed on the orientation gradient of the S200 sample.
Figure 4.18: Numbered anomalies corresponding to the anomalies of the orientation gradient for the S200 sample.
CHAPTER V

CONCLUSIONS

Anomaly detection via Gaussian mixture modeling performed on material microstructures pinpoints areas believed to be of interest in determining overall material strength. Because the feature vector used as the basis of anomaly detection can be any combination of measurable features, anomaly detection can be applied to a variety of materials. Often, extensive pre-processing such as described in this chapter may be necessary to extract the desired features.

For the features presented here, it is theorized that the anomalies established by the detector affect the growth of cracks within the CMC. Areas where the microstructure features anomalous contraction are of interest because it is thought that high interconnections of fiber coatings, associated with overly dense fiber packing, correspond to easy crack growth pathways.

Anomalies based on the velocity gradient are also valuable for their potential in predicting the emergence of certain microstructural features. Due to the stiffness (local linearity) of the fibers, anomalies of the velocity gradient highlight emergence of either dense or sparse microstructures before they are within the field of view. Because destructive techniques are used during imaging, it is important for experimentation purposes to know a defect is present before it is destroyed. Upon detection of the suspected defect, destructive imaging can be halted for experimentation to see if damage corresponds to anomalies of the velocity gradient.
The ultimate goal of anomaly detection within CMC’s is to correlate crack initiation and propagation with specific attributes of the microstructure which should then be adjusted or eliminated during processing. Cracks close upon unloading and cannot be imaged. This makes evaluation of crack pathways through sample depth difficult because it requires the subsequent loading, imaging, unloading, and sectioning of the sample without introducing further damage. Development of techniques for imaging 3D volumes in such a way that cracks are visible through the thickness would allow analysis of anomaly and crack correlation. Currently, anomalies are most often seen near tow edges where pores are created, suggesting processing changes to tow weave or infiltration techniques that effect porosity may have a large impact on overall strength. It is also the aim of future research to apply these anomaly detection methods to fiber coating connectivity and orientation to test the correlation between connected coatings and velocity gradient anomalies.
REFERENCES


