WET ETCHING OPTICAL FIBERS TO SUB-MICRON DIAMETERS
FOR SENSING APPLICATION

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ABSTRACT

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In this thesis we explore a novel technique to fabricate sub-micron diameter tapered fibers for sensor applications. Physically the light propagating in a tapered fiber has an evanescent field that extends into the medium surrounding the fiber containing an analyte. A sub-micron diameter taper can expels most the electromagnetic energy into the medium thus increasing the sensitivity of the measurement. The tapering process we develop enables us to have precise control over the final diameter of the taped fiber’s waist. The tapered single mode fiber sensors (TSMFs) are fabricated using a two-step procedure. First, a single mode fiber is tapered to about 10 microns using Vytran Glass Processing System. Second, we etch the TSMFs with 6:1 buffered HF solution to a controlled sub-micron size. During the etching process we monitor the fiber’s progress by measuring the transmittance characteristics. The in situ measurements are made by
connecting a laser at one fiber end and using a photodetector to measure the transmittance at the other end. We find a temporal modulation of the transmittance during the etching process, which is due to the changes in the propagation constants of the fiber modes. The details of this device are described and its optical properties are examined in this thesis.

To better understand the transmission characteristics recorded in the experiment we develop a simulation of the optical power propagating through the tapered fiber to calculate the transmitted power. We apply a Beam Propagation Method (BPM) to simulate the light wave passing through the tapered fiber sensor. We numerically analyze the transmittance characteristics of the beam oscillating inside the TSMFs. Our simulations are applied to validate the experimental results.
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CHAPTER 1

INTRODUCTION

Background

Fiber optic sensors (FOS) are developed to measure chemical, biological or physical properties of the environment they are placed in. In the past 30 years, fiber optic sensors attracted increasing attention of researchers in many different fields and in many cases the research has advanced from the experimental stage to commercial applications [1]. Optical fibers sensors can be used to measure temperature, strain, pressure, humidity, refractive index (RI) and many other quantities [2-5]. Compared with other types of sensors, fiber optic sensors hold many attractive advantages, including small size, high sensitivity, multiplexing ability and remote sensing capability [6].

Generally, fiber optic sensors can be classified in two groups: extrinsic and intrinsic fiber optic sensors [7]. Extrinsic fiber optic sensors consist of optical fibers that lead up to and out of a ‘black box’ that modulates the light beam passing through it in response to an environmental effect; on the contrary, the intrinsic fiber optic sensors rely on the light beam propagating through the optical fiber being modulated by the environmental effect either directly or through the environmentally
induced optical path length changes in the fiber itself [8-9]. In the first group, one of the most popular measurement modalities is based on evanescent wave field (EWF) interaction [10].

In the development of optical sensors, an important parameter to evaluate their performance is sensitivity. In most evanescent wave based sensors, sensitivity is determined by the fraction of EWF that is immersed in the environment, which by definition is the optical power travelling outside cladding. The greater the EWF power outside the fiber, the higher its detection sensitivity. Unfortunately the standard optical fiber is insensitive to the surrounding medium since the EWF outside the cladding is negligible. Hence, researchers work on techniques to increase the fraction of external EWF intensity and this is a central issue. The natural outcome of this line of reasoning is the development of tapered optical fiber sensor (TOFs) precisely to force the field outside the fiber cladding. By tapering a portion of a fiber, the waist size can be small enough such that most of the optical power travel out of the cladding, thus the intensity of EWF is enhanced dramatically, therefore the sensitive to the refractive index of surrounding medium can be highly increased [11-13].

By now, plenty of methods for fabricating tapered optical fiber sensors (TOFs) have been developed and published. There is a vast series of papers on the subject. We can only devote our attention to a small subset of the research done on this optics and explore the next steps in the development. In 2004, M. Sumetsky et al. fabricated a TOFs
by drawing a standard fiber and heated by a CO$_2$ laser in a micro-furnace [14]. In 2006, Shi et al. reported a new fabrication method for submicron-diameter fibers, which has a total optical loss of about 0.1dB/cm. The fabrication process is fulfilled by heating a single-mode fiber (SMF) with an electrical strip heater that was designed specifically to stretch the fiber when it softened near the glass transition temperature [15]. After four years, a new paradigm was developed; a model that simulated the fabrication process of tapered fibers with complex shapes; more specifically Pricking et al. showed that tapered fibers with a modulated waist of the tapered fibers had a sinusoidal pattern [16]. In the same year, an etching method based on surface tension driven flows of hydrofluoric acid micro-droplets was published by Zhang et al. to fabricate a subwavelength-diameter TOFs which has a super low loss, less than 0.1dB/mm [17].

**Objective and Organization of Thesis**

The objective of this research is to fabricate and model a transmission-based single-mode tapered fiber sensor (SMTFs) with a taper waist diameter of around 1 µm. The fabrication process has two steps: first, we taper the fiber by using a Vytran glass processing system after which the waist size of fiber decreases to approximate 10 µm, the tapered profile will be discussed in Chapter 3. Second, we use a buffered HF solution with HF concentration of 6:1 to etch the tapered region of the fiber until the taper waist size is reduced to about 1µm. In this process, a tunable c.w. laser is used as input power, the relation between output intensity and etching time is recorded by using a computer.
program. Finally, we use the beam propagation method (BPM) to model the tapered fiber, simulate the etching process and make a comparison with the experimental result.

Chapter 1 provides a basic introduction to the fast-developing optical sensor. In Chapter 2, we first explain some basic concepts of circular optical fiber which will set a stage for the rest part of this thesis, and then introduce a numerical algorithm which is called beam propagation method. BPM is a numerical method to calculate the field launched in a waveguide with which the dynamic field distribution can be easily and accurately determined as the beam propagates through a waveguide with any shape.

The fabrication process of a SMTFs is discussed in Chapter 3. This process can be achieved by tapering a single mode fiber through heating and pulling as well as etching with HF acid solution. Some basic knowledge of how to taper a fiber is introduced at the beginning. Second, since the objective of taper waist is 1µm, which cannot be completely fulfilled by tapering, we need etching the tapered fiber with chemical solutions, HF solution is chosen. The etch rates of HF solution is measured and calculated by using scanning electron microscope (SEM). It can be seen that the etch rate of HF solution with fiber will not be altered by the shape of fibers; the initial waist region has a stable shape with time and the etch rate increases as concentration goes up.

In order to ensure that the etching experiment is controllable and accurate enough, the 6:1 buffered HF solution is used. In the etching process, the input power is stabilized at 1mW and worked at λ=1550nm; the tapered fiber is dipped into the buffered HF solution
for a period of time, the transmitted power is measured and recorded in computer by using a Labview program. An interesting phenomenon that the output power decreases with significant oscillation with time is found in this experiment. After etching, the tapered fiber is removed from HF solution and dipped into water to halt the etch process. Then we sweep the input wavelength from 1.48 $\mu m$ to 1.64$\mu m$ and record the output signal. We find that the transmitted power also oscillates with wavelength.

In Chapter 4, a theoretical explanation as well as mathematical derivation is discussed to clarify this phenomenon. Also, extensive numerical simulations by using BPM are performed in modeling and simulating the tapered fiber. The simulation results of output power versus etching time as well as wavelength show some similar trend as indicated in the etching experiment, which proved that the experimental results are reasonable. However, the accuracy of the program needs to be further improved.

In Chapter 5, the results of this thesis are summarized and conclusions are drawn. Some potential future solutions of increasing the simulation accuracy are also discussed.
A description of modes inside a circular fiber is discussed in this chapter. We will concentrate on the fundamental mode $HE_{11}$ which is supported by any step-index fiber. The Gaussian shape of $HE_{11}$ mode profile are also discussed and used in our beam propagation method (BPM) simulations. In addition, we describe and implement the numerical BPM technique, which is used in simulating the transmission characteristic of a tapered fiber in chapter 3.

2.1 Modes of step-index circular waveguides

We consider an electromagnetic wave propagating along the $z$-axis in a cylindrically symmetric fiber. The wave equation can be decomposed into angular frequency coordinates, $\omega$, and the azimuthal angle, $\phi$, is expanded in a Fourier series with index $v$. The solution of the vector wave equation in cylindrical coordinates $(r, \phi, z)$ in a step-index circular waveguide is expressed for the electric and magnetic field vector components by [18]:
\[ E_z(r, \phi, z) = A J_v(\kappa r) e^{j\nu \phi} e^{-j\beta z}, \]

\[ E_r(r, \phi, z) = -\frac{j\beta}{\kappa^2} \left[ A K'_v(\kappa r) + \frac{j\omega \nu}{\beta r} B J_v(\kappa r) \right] e^{j\nu \phi} e^{-j\beta z}. \]

\[ E_\phi(r, \phi, z) = -\frac{j\beta}{\kappa^2} \left[ \frac{ju}{r} A J_v(\kappa r) - \frac{\omega \mu}{\beta} B K'_v(\kappa r) \right] e^{j\nu \phi} e^{-j\beta z}. \]

\[ H_z(r, \phi, z) = B J_v(\kappa r) e^{j\nu \phi} e^{-j\beta z}, \]

\[ H_r(r, \phi, z) = -\frac{j\beta}{\kappa^2} \left[ B K'_v(\kappa r) - \frac{j\omega \nu}{\beta r} B J_v(\kappa r) \right] e^{j\nu \phi} e^{-j\beta z}. \]

\[ H_\phi(r, \phi, z) = -\frac{j\beta}{\kappa^2} \left[ \frac{ju}{r} B J_v(\kappa r) + \frac{\omega \mu}{\beta} B K'_v(\kappa r) \right] e^{j\nu \phi} e^{-j\beta z}. \]

(2-1)

In the core region \((r<a)\) and by

\[ E_z(r, \phi, z) = C K_v(\gamma r) e^{j\nu \phi} e^{-j\beta z}, \]

\[ E_r(r, \phi, z) = \frac{j\beta}{\gamma^2} \left[ C \gamma K'_v(\gamma r) + \frac{j\omega \nu}{\beta r} D K_v(\gamma r) \right] e^{j\nu \phi} e^{-j\beta z}. \]

\[ E_\phi(r, \phi, z) = \frac{j\beta}{\gamma^2} \left[ \frac{ju}{r} C K_v(\gamma r) - \frac{\omega \mu}{\beta} D \gamma K'_v(\gamma r) \right] e^{j\nu \phi} e^{-j\beta z}. \]

(2-2)

\[ H_z(r, \phi, z) = D K_v(\gamma r) e^{j\nu \phi} e^{-j\beta z}, \]

\[ H_r(r, \phi, z) = \frac{j\beta}{\gamma^2} \left[ D \gamma K'_v(\gamma r) - \frac{j\omega \nu}{\beta r} C K_v(\gamma r) \right] e^{j\nu \phi} e^{-j\beta z}. \]

\[ H_\phi(r, \phi, z) = \frac{j\beta}{\gamma^2} \left[ \frac{ju}{r} D K_v(\gamma r) + \frac{\omega \mu}{\beta} C \gamma K'_v(\gamma r) \right] e^{j\nu \phi} e^{-j\beta z}. \]

In the cladding region \((r>a)\), where \(\beta\) is the longitudinal wave vector, also
called the propagation constant, the variable $a$ represents the core radius. $\kappa$ is the transverse wave vector and $\gamma$ is related to the penetration depth of the evanescent wave in the cladding. $\mu$ is the magnetic permeability of the medium. $j_v(\gamma r)$ and $K_v(\gamma r)$ are Bessel functions of the first and second kind, respectively.

At $r=a$, the four tangential components: $E_z$, $E_\phi$, $H_z$ and $H_\phi$ are continuous at the core-cladding interface. Plugging the boundary condition in Eq. (2-1) and Eq. (2-2), we obtain [19]:

$$\begin{bmatrix}
    J_v(\kappa a) & 0 & -K_v(\gamma a) & 0 \\
    0 & J_v(\kappa a) & 0 & -K_v(\gamma a) \\
    J_v(\kappa a) & \frac{j \omega \mu}{\kappa} J'_v(\kappa a) & K_v(\gamma a) & \frac{j \omega \mu}{\gamma} K'_v(\gamma a) \\
    -\frac{j \omega \varepsilon_{\text{core}}}{\kappa} J'_v(\kappa a) & J_v(\kappa a) & -\frac{j \omega \varepsilon_{\text{clad}}}{\gamma} K'_v(\gamma a) & \frac{\beta \nu}{\Delta \gamma^2} K_v(\gamma a)
\end{bmatrix} \begin{bmatrix}
    A \\
    B \\
    C \\
    D
\end{bmatrix} = 0. \quad (2-3)$$

The four coefficients $A$, $B$, $C$ and $D$ can be determined by solving Eq. (2-4):

$$C = \frac{J_v(\kappa a)}{K_v(\gamma a)} A. \quad (2-4a)$$

$$D = \frac{J_v(\kappa a)}{K_v(\gamma a)} B. \quad (2-4b)$$

$$B = \frac{j \nu \beta}{\omega \mu a} \left[ \frac{J'_v(\kappa a)}{K'_v(\gamma a)} + \frac{K'_v(\gamma a)}{\gamma K_v(\gamma a)} \right]^{-1} A. \quad (2-4c)$$

$$B = \frac{j \omega a}{\beta \nu} \left[ \frac{n_{\text{core}}^2}{\kappa} J'_v(\kappa a) + \frac{n_{\text{clad}}^2}{\gamma} K'_v(\gamma a) \right] \left[ \frac{1}{\gamma^2} + \frac{1}{\kappa^2} \right]^{-1} A. \quad (2-4d)$$
Eq. (2-4c) and Eq. (2-4d) are derived by applying the continuity of $E_\phi$ and $H_\phi$, respectively.

The determinant of matrix should be equal to zero for non-trial solutions, which yields the following equation:

$$\frac{\beta^2 \nu^2}{a^2} \left[ \frac{1}{\nu^2} + \frac{1}{\kappa^2} \right]^2 = \left[ \frac{J_0'(\nu\alpha)}{\kappa J_0(\nu\alpha)} + \frac{\kappa_0'(\gamma\alpha)}{\gamma K_0'(\gamma\alpha)} \right] \left[ \frac{k_0^2 n_\text{core}^2 J_0'(\nu\alpha)}{\kappa J_0(\nu\alpha)} + \frac{k_0^2 n_\text{clad}^2 K_0'(\gamma\alpha)}{\gamma K_0'(\gamma\alpha)} \right]. \quad (2-5)$$

Eq. (2-5) is called the characteristic equation for step-index waveguide. The integer $\nu$ is named as angular mode number, which represents the number of nodes that exist in the field distribution.

Eq. (2-5) sets a stage of deciding the types of mode inside a circular waveguide. There are four possible types of mode in circular waveguides: transverse electric (TE), transverse magnetic (TM), EH and HE modes.

2.1.1 TE and TM modes

If $\nu = 0$, then Eq. (2-5) simplifies as:

$$\left[ \frac{J_0'(\nu\alpha)}{\kappa J_0(\nu\alpha)} + \frac{K_0'(\gamma\alpha)}{\gamma K_0'(\gamma\alpha)} \right] \left[ \frac{k_0^2 n_\text{core}^2 J_0'(\nu\alpha)}{\kappa J_0(\nu\alpha)} + \frac{k_0^2 n_\text{clad}^2 K_0'(\gamma\alpha)}{\gamma K_0'(\gamma\alpha)} \right] = 0. \quad (2-6)$$

If the first term in Eq. (2-6) is zero, then $A$ should also be zero in order to keep the magnitude $B$ in Eq. (2-4c) finite. Then the longitudinal component of electric field
will be zero, thus the solution will therefore be a TE mode. Similarly, if the second term in Eq. (2-6) is zero, then B should also be zero such that the magnitude A in Eq. (2-4d) is finite. Then the solution will be a TM mode.

The Bessel function relations are used in simplifying the characteristics equation, which are given as [18]:

\[
\frac{j_ν'}{\kappa j_ν} = \pm \frac{j_{ν+1}'}{\kappa j_ν} \mp \frac{v}{\kappa^2}. \tag{2-7a}
\]

\[
\frac{k_ν'}{\gamma K_ν} = \pm \frac{k_{ν+1}'}{\gamma K_ν} \mp \frac{v}{\gamma^2}. \tag{2-7b}
\]

Substituting Eq. (2-7) into Eq. (2-5) and setting the first term in Eq. (2-5) equal to zero, arrives at:

\[
\frac{j_1'(\kappa a)}{\kappa j_1(\kappa a)} + \frac{k_1(\gamma a)}{\gamma K_0(\gamma a)} = 0. \tag{2-8}
\]

This is the eigenvalue equation for TE modes. Similarly, the eigenvalue equation for TM modes can be derived by setting the second term of Eq. (2-5) to be zero:

\[
\frac{k_0^2 n^2_{core} j_1'(\kappa a)}{\kappa j_0(\kappa a)} + \frac{k_0^2 n^2_{clad} K_1'(\gamma a)}{\gamma K_0(\gamma a)} \bigg|_{\gamma=0} = 0. \tag{2-9}
\]

\[
\frac{n^2_{core} j_1(\kappa a)}{\kappa j_0(\kappa a)} + \frac{n^2_{clad} K_1(\gamma a)}{\gamma K_0(\gamma a)} = 0. \tag{2-10}
\]
The two eigenvalue equations above can be solved numerically, in this way all the possible values of propagation number $\beta$ can be determined. These modes are interesting because of their cylindrically symmetric polarization states, but will not be further studied here.

2.1.2 Hybrid modes

If $v \neq 0$, the longitudinal components of fields $E_z$ and $H_z$ in Eq. (2-1) and Eq. (2-2) will both be non-zero. Generally, these modes are called EH or HE modes depending on dominant amplitude between the $E_z$ and $H_z$ field components:

If $|A| > |B|$, then the mode is called an HE mode since $E_z$ dominant $H_z$;

If $|A| < |B|$, then the mode is called an EH mode since $H_z$ dominant $E_z$;

Both HE and EH modes are called hybrid modes since the longitudinal components of both electric field and magnetic field are non-zero.

2.1.3 Cut-off condition and normalized frequency

In order to know find out many modes that can be supported in a waveguide, we need solving the characteristic equations Eq. (2-5). Without proof, the cutoff conditions for each mode are given as [18]:

11
TE_{0m} Modes: \( \kappa_{\text{max}} a > m^{th} \) root of \( J_0(\kappa a) \)

HE_{1m} Modes: \( \kappa_{\text{max}} a > m^{th} \) root of \( J_1(\kappa a) \)

EH_{um} Modes: \( \kappa_{\text{max}} a > m^{th} \) root of \( J_v(\kappa a) \)

HE_{um} Modes: \( \left( \frac{\varepsilon_{\text{core}}}{\varepsilon_{\text{clad}}} + 1 \right) J_{v-1}(\kappa a) = \frac{\kappa a}{v-1} J_{v-1}(\kappa a) \)

Where \( \varepsilon_{\text{core}} = n_{\text{core}}^2 \) and \( \varepsilon_{\text{clad}} = n_{\text{clad}}^2 \) are the core and cladding dielectric permittivities. Note that \( \kappa_{\text{max}} a \) plays an important role in determine how many modes existed in a fiber, hence we can define a parameter named normalized frequency or V number to characterize a fiber, which is defined as:

\[
V_{\text{number}} = \kappa_{\text{max}} a = \frac{2\pi a}{\lambda_0} \sqrt{n_{\text{core}}^2 - n_{\text{clad}}^2}. \tag{2-11}
\]

Where \( a \) is the radius of the core and \( \lambda_0 \) is the wavelength in vacuum.

The normalized frequency provides an easy way to calculate the mode number in a fiber since the cut-off conditions of each mode are determined by \( V \).

Figure 2-1 illustrates the four lowest orders of Bessel function of first kind, the cut-off condition of some modes are also plotted.
It is clear from Figure 1 that the first root of $J_1(\kappa a)$ is $\kappa = 0$, which means the HE$_{11}$ mode has no cut-off condition, thus any step-index fiber can hold this mode. The first zero of $J_0(\kappa a)$ is locate at $V_c=2.405$, hence the cut-off condition for single-mode fibers is $V < 2.405$. If the normalized frequency increases, then some higher modes can be launched into the fiber.

Accordingly, the cut-off wavelength of a single-mode fiber is defined as:

$$
\lambda_c = \frac{2\pi a}{V_c} \sqrt{n_{core}^2 - n_{clad}^2}. 
$$

(2-12)

If $\lambda < \lambda_c$, the single-mode fiber will suddenly become multi-mode fiber.
2.1.4 Fundamental HE\textsubscript{11} mode

As discussed in Section 2.1.3, the HE\textsubscript{11} mode has no cut-off condition so that every step-index waveguide can support this mode. For a single-mode fiber, all higher-order modes are cut off at the operation wavelength, thus HE\textsubscript{11} mode is also called fundamental mode of fibers.

The field distribution of HE\textsubscript{11} mode can be obtained by using Eq. (2-1) and Eq. (2-2). If the fraction index change at core-cladding interface is small enough, the longitudinal components of field will be negligible. Therefore HE\textsubscript{11} mode can be regarded as linearly polarized for weekly guiding fiber (i.e. \( n_{\text{core}} \approx n_{\text{clad}} \)). Hence the transverse field can be treated as a superposition of two linearly polarized fields polarized in x-axis and y-axis, respectively. Taking \( E_x \) for example, the electric field is given as [20]:

\[
E_x = E_0 [J_0(\kappa r)/J_0(\kappa a)]; \quad r \leq a. \quad (2-13a)
\]

\[
E_x = E_0 [K_0(\kappa r)/K_0(\kappa a)]; \quad r > a. \quad (2-13b)
\]

However, since the field distribution in core has a Bessel function form which is cumbersome to manipulate, a Gaussian distribution is often used as an approximation, which is given by:
\[ E_x = E_0 e^{-\left(\frac{x^2 + y^2}{w_0^2}\right)} e^{-j\beta z}. \] (2-14)

Where \( w_0 \) is the waist radius of the Gaussian distribution which is often referred to as the spot size.

For a fiber with a core radius of \( a \), the spot size \( w \) can be determined by [21]:

\[ w \approx a \left( 0.65 + 1.619V^{-\frac{3}{2}} + 2.879V^{-6} \right). \] (2-15)

Note that the value \( 2w \) is defined as the mode field diameter (MFD) since the amplitudes of electric field and magnetic field reduced to 1/e of their maximum values.

When connecting two single mode fibers, it is critical to make sure that the MFDs of the two fibers are close to each other such that the loss is insignificant.

2.2 Beam Propagation Method

Beam propagation method (BPM) is a powerful numerical technique to investigate linear and nonlinear wave propagation phenomenon in axially varying waveguides such as S-shaped bent waveguides, curvilinear directional couplers, branching and combining waveguides as well as tapered waveguides [22]. We will discuss the implementation of our FFT-based BPM procedure in this chapter.
2.2.1 FFT-based BPM

The three-dimensional Helmholtz equation is expressed as [23]:

\[ \nabla^2 E(x, y, z) + k^2 n^2(x, y, z) E(x, y, z) = 0. \]  \hspace{1cm} (2-16)

Where \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \) is the Laplacian operator expressed in Cartesian coordinates, \( E(x, y, z) \) is the electric field, \( k \) is the wave number in vacuum, and \( n(x, y, z) \) is the refractive index, which is a function of position.

We assume that the electric field \( E(x, y, z) \) could be separated into two parts: the axially slowly varying envelope term \( \varphi(x, y, z) \) as well as the rapidly varying phase term \( \exp(-j k n_0 z) \). Here, we choose the \( n_0 = n_{\text{clad}} \) as the reference index of refraction. However, we can use either \( n_{\text{clad}} \) or \( n_{\text{core}} \) since the difference between them is very small. Then the wave equation can be expressed by:

\[ E(x, y, z) = \varphi(x, y, z) \exp(-j k n_0 z). \]  \hspace{1cm} (2-17)

The first derivative of the wave equation with respect to direction \( z \) is:

\[ \frac{\partial}{\partial z} E(x, y, z) = \frac{\partial \varphi}{\partial z} \exp(-j k n_0 z) j k n_0 \varphi(x, y, z) \exp(-j k n_0 z). \]  \hspace{1cm} (2-18)

The second derivative of the wave equation with respect to direction \( z \) is:

\[ \frac{\partial^2}{\partial z^2} E(x, y, z) = \left( \frac{\partial^2 \varphi}{\partial z^2} - 2 j k n \frac{\partial \varphi}{\partial z} - k^2 n^2 \varphi \right) \exp(-j k n_0 z). \]  \hspace{1cm} (2-19)
The second derivatives of wave equation with respect to $x$ and $y$ are:

\[
\frac{\partial^2}{\partial x^2} E(x, y, z) = \frac{\partial^2 \varphi}{\partial x^2} \exp(-j kn_0 z). \tag{2-20}
\]

\[
\frac{\partial^2}{\partial y^2} E(x, y, z) = \frac{\partial^2 \varphi}{\partial y^2} \exp(-j kn_0 z). \tag{2-21}
\]

Assuming that the envelope function of the wave in the fiber varies slowly, the second derivative of the envelope function is ignored in the propagation direction, which is expressed as:

\[
\frac{\partial^2 E}{\partial z^2} \cong 0. \tag{2-22}
\]

Substituting Eq. (2-18), Eq. (2-19), Eq. (2-20) and Eq. (2-21) in Eq. (2-16), we obtain:

\[
\nabla^2 \varphi - 2jn_0 \frac{\partial \varphi}{\partial z} + k^2 (n^2 - n_0^2) \varphi = 0. \tag{2-23}
\]

Where the transverse Laplacian operator is

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \tag{2-24}
\]

Eq. (2-23) is the main result used in our simulations. The first term on the right hand side of Eq. (2-23) represents light propagated in a medium with a refractive index of $n_0$ whilst the second term describes refractive effects, which contains the refractive index variations that confine the wave to the core region.
Eq. (2-23) can be rewritten as operator-form as:

\[
\frac{\partial \varphi}{\partial z} = (\hat{D} + \hat{S}) \varphi.
\]  

(2-25)

Where

\[
\hat{D} = -j \frac{1}{2k\eta_0} \nabla_\perp^2.
\]  

(2-26a)

\[
\hat{S} = -j \frac{k}{2\eta_0} (n^2 - n_0^2).
\]  

(2-26b)

Operator \( \hat{D} \) is the linear differential operator that accounts for diffraction effect, which also called the diffraction operator; whilst \( \hat{S} \) is the space-dependent inhomogeneous operator. Generally, the two operators act together on the electric field \( \varphi \). If we assume operators \( \hat{S} \) and \( \hat{D} \) are \( z \)-independent, then the operator-form solution of Eq. (2-25) is given by:

\[
\varphi(x, y, z + \Delta z) = \exp[(\hat{D} + \hat{S})\Delta z] \varphi(x, y, z).
\]  

(2-27)

In general, according to Baker-Campbell-Hausdorff theorem [24], for two non-commutative operators \( \hat{A} \) and \( \hat{B} \) we have the following expression:

\[
\exp(\hat{A})\exp(\hat{B}) = \exp \left[ \hat{A} + \hat{B} + \frac{1}{2} [\hat{A}, \hat{B}] + \frac{1}{12} [\hat{A} - \hat{B}, [\hat{A}, \hat{B}]] + \cdots \right].
\]  

(2-28)

Where \([\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}\) represents the commutation of operators \( \hat{A} \) and \( \hat{B} \).
If $\Delta z$ is small enough, we can neglect the higher order term than $O(h^2)$. As a result, we can approximate as:

$$
\exp[(\hat{D} + \hat{S})\Delta z] = \exp(\hat{D}\Delta z)\exp(\hat{S}\Delta z) + O(h^2)
$$

$$
\cong \exp(\hat{D}\Delta z)\exp(\hat{S}\Delta z).
$$

Eq. (2-29) indicates that the diffraction and the inhomogeneous operators can be regarded independently of each other. Substituting Eq. (2-29) in Eq. (2-27), the formal solution of Eq. (2-25) is expressed by:

$$
\varphi(x, y, z + \Delta z) = \exp(\hat{S}\Delta z)\exp(\hat{D}\Delta z)\varphi(x, y, z).
$$

It is clarified from Eq. (2-30) that the electric field $\varphi(x, y, z)$ is first propagated in free space (operator $\hat{D}$) over a distance of $\Delta z$, and then the phase retardation of this distance $\Delta z$ is taken into consideration at the end of space (operator $\hat{S}$) to get the final result $\varphi(x, y, z + \Delta z)$.

Besides, it is known that the accuracy of Eq. (2-30) can be further improved to $O(h^3)$ by rewritten as:

$$
\exp[(\hat{S} + \hat{D})\Delta z] = \exp\left(\frac{1}{2}\hat{D}\Delta z\right)\exp(\hat{S}\Delta z)\exp\left(\frac{1}{2}\hat{D}\Delta z\right) + O(h^3).
$$

Similarly, if we substitute Eq. (2-31) in Eq. (2-25), the new formal solution of Eq. (2-11) is simply given by:
\[ \varphi(x, y, z + \Delta z) = \exp\left(\frac{1}{2} \hat{D} \Delta z\right) \exp(S \Delta z) \exp\left(\frac{1}{2} \hat{D} \Delta z\right) \varphi(x, y, z). \] (2-32)

It should be noted that Eq. (2-32) can be realized from Eq. (2-30) by choosing a reasonable value of \( \Delta z \). In our simulations, Eq. (2-30) will be used and we use a small enough value of \( \Delta z \) to insure convergence. To speed up our programs Eq. (2-32) could be implemented.

Eq. (2-30) can be solved by taking the Fourier transform of both sides of it. The 2-D Fourier transform as well as inverse Fourier transform are defined by:

\[ F(k_x, k_y; z) = F\{f(x, y; z)\} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y; z) \exp(j k_x x + j k_y y) \, dx \, dy. \] (2-33)

\[ f(x, y; z) = F^{-1}\{F(k_x, k_y; z)\} = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(k_x, k_y; z) \exp(-j k_x x - j k_y y) \, dk_x \, dk_y. \] (2-34)

Substitute Eq. (2-33) and Eq. (2-34) in Eq. (2-30), we obtain:

\[ F\{\exp(\hat{D} \Delta z) \varphi\} = \exp \left[ \frac{j(k_x^2 + k_y^2)}{2k} \Delta z \right] F\{\varphi\}. \] (2-35)

Taking the inverse Fourier transform of Eq. (2.35), yield:

\[ \exp(\hat{D} \Delta z) \varphi = F^{-1}\left\{ \exp \left[ \frac{j(k_x^2 + k_y^2)}{2k} \Delta z \right] F\{\varphi\} \right\}. \] (2-36)
Finally the solution of Eq. (2-30) is expressed by plugging Eq. (2-36) into it:

$$\exp[(\mathcal{S} + \mathcal{D})\Delta z] = \exp\left(-j \frac{k}{2n_0} (n^2 - n_0^2) \Delta z\right) F^{-1}\left\{\exp\left[i \frac{(k_x^2 + k_y^2)}{2k} \Delta z\right] F\{\varphi\}\right\}. \tag{2-37}$$

Eq. 2-37 indicates that the field at position $z + \Delta z$ can be calculated if the field at position $z$ is given. If $\mathcal{S}$ and $\mathcal{D}$ are fixed at a given plane we are able to calculate the field intensity distribution at any plane step by step.

In summary, the procedures of FFT-BPM of order $O(h^2)$ can be demonstrated in the following steps:

1. Taking the Fourier transform of the field distribution on the input surface to get the wave function in frequency domain $\tilde{\varphi}(z)$:

$$\tilde{\varphi}(k_x, k_y, z) = F\{\varphi(x, y, z)\}. \tag{2-38}$$

2. Multiply $\tilde{\varphi}(k_x, k_y, z)$ by the phase factor $\exp\left[i \frac{(k_x^2 + k_y^2)}{2k} \Delta z\right]$, yields:

$$\exp\left[i \frac{(k_x^2 + k_y^2)}{2k} \Delta z\right] F\{\varphi(x, y, z)\}. \tag{2-39}$$
3. Taking the inverse Fourier transform of the factor obtained in step 2 and multiplying the index operator $\exp[-jk(n^2 - n_0^2)\Delta z]$, the field distribution at $z + \Delta z$ is then given by:

$$\varphi(x, y, z + \Delta z) = \exp[-jk(n^2 - n_0^2)\Delta z]F^{-1}\left\{\exp\left[i\left(k_x^2 + k_y^2\right)/2k\right]F\{\varphi}\right\} \quad (2-40)$$

For a given waveguide, the total length can be divided into a series of $\Delta z$, for each distance we can repeat steps 1-3 to propagate the beam through the waveguide, through this way we can find out the field distribution on any target plane. The numerical procedure is illustrated in the Figure 2-2.

![Figure 2-2 An illustration of the calculation procedure of BPM](image)
2.2.2 Test of FFT-BPM

In this section, Matlab programs of FFT-based beam propagation method are made and tested by simulating a Gaussian beam propagating in a fiber with several centimeters in length. The parameter of this fiber used in simulation is given in table 2-1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_{\text{core}}$</td>
<td>1.48</td>
</tr>
<tr>
<td>$n_{\text{clad}}$</td>
<td>1.4735</td>
</tr>
<tr>
<td>$n_{\text{water}}$</td>
<td>1.33</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.55 $\mu$m</td>
</tr>
<tr>
<td>$V$</td>
<td>2.4</td>
</tr>
<tr>
<td>$d_{\text{core}}$</td>
<td>9 $\mu$m</td>
</tr>
<tr>
<td>$d_{\text{clad}}$</td>
<td>125 $\mu$m</td>
</tr>
</tbody>
</table>

Here, we set the input wavelength $\lambda$ to be 1.55 $\mu$m, the normalized frequency $V$ can be calculated by Eq. (2-11), here we have $V$=2.4. In this program, we assume that the fiber is put straightly along z direction, the length of the fiber is 3 cm, which is the same
as the length of the tapered fiber. Since the length is short enough so that the dispersion and nonlinear effects of the fiber are negligible. As discussed in chapter 2-1, the fundamental HE_{11} mode can be approximately regarded as a Gaussian beam, should the input beam be perfectly coupled into the fiber, the mode field diameter (MFD) of the input beam must be determined by Eq. (2-14). When the Gaussian beam is launched in the fiber with the correct waist radius \( w_0 \), it will propagate essentially without changing its shape, however, if \( w_0 \) is wrong, then the beam will reshape itself and shed power into the cladding. Thus, in this way we can test the accuracy of this program by using the correct waist size and an incorrect waist size.

First, we use the correct waist diameter. The length of the transverse window in simulation is 0.128 mm, the sampling number in x and y direction is 512 whilst 20000 in z direction. The correct waist radius in this case is simply 5.15 \( \mu m \). The input beam profile as well as the output beam profile generated by this program is illustrated in figure 2-3.
Figure 2-3 Beam profile at input plane and output plane (with correct waist): (a) Input beam profile; (b) Output beam profile.
Figure 2-3 (a) shows the input beam profile whilst the output beam profile is shown in figure 2-3 (b). It is clearly that the output beam almost remains its initial profile after travelling through the fiber. However some noise in the cladding region can also be noticed which is possibly as a result of the inaccuracy of the program.

Next, if the input waist size is far bigger than the MFD, for instance 50 μm in this situation, then the output field will reshape itself and the optical power will flows from the core region into the cladding as well as outer medium. The beam profiles are illustrated in figure 2-4.
As shown in figure 2-4 (b), the field distribution on the output plane doesn’t have perfect Gaussian profile as on the input plane. Further increasing the waist size will lead to more clumsy of the output beam profile.

Additionally, we want to test the accuracy of the program. Since the profile of output beam should not change too much after travelling a distance through the fiber, hence, the transmittance in the fiber core should also be stabled. The transmittance in the core region is given by:

$$T(\%) = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{\int_0^R |\psi_{\text{out}}|^2 r dr}{\int_0^R |\psi_{\text{in}}|^2 r dr} \times 100\%. \quad (2.41)$$
Here, we will use the transmittance measuring the accuracy of the program. The accuracy of the program is influenced mainly by the following factors: transverse step size $\Delta x$, transverse windows size $L$, longitudinal step size $\Delta z$ and longitudinal windows size $Z$. These four parameters work together on the precision of the program. The longitudinal step length $\Delta z$ should be chosen wisely depending on the waveguide structure as well as the input wavelength. In order to keep the BPM validity, $\Delta z$ should satisfy the following equation [25]:

$$\Delta z \ll 6k_0(k_e + k_w)^{-2}.$$  \hspace{1cm} (2-42)

Where $k_e$ is the largest relevant nonzero transverse component of the electric field, whilst $k_w$ is the largest spatial frequency required to describe the index profile. In this case we estimate, $\Delta z$ should be of order 0.15 micrometer.

Here, we fix the longitudinal windows size $Z$ to be 3 cm, and the transverse windows size to be $L = 128 \, \mu m$, the transverse sampling number $N$ is set to be 512 and 1024, respectively. Hence, the corresponding transverse step sizes $\Delta x$ are simply $0.25 \, \mu m$ and $0.125 \, \mu m$, respectively. By adjusting the value of longitudinal sampling number $M$, the transmittance in power will also change. The transmittance and corresponding running time are given in table 2-2 and illustrated in figure 2-4.
Table 2-2 Simulation results of Gaussian beam travelling in single mode fibers using BPM

<table>
<thead>
<tr>
<th>Grid size L in transverse windows (μm)</th>
<th>Transverse sampling number N</th>
<th>Longitudinal sampling number M</th>
<th>Longitudinal step size Δz (μm)</th>
<th>Transmittance (%)</th>
<th>Approximate running time (s)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>54</td>
<td></td>
</tr>
<tr>
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<tr>
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<td>57.115905</td>
<td>348</td>
<td></td>
</tr>
<tr>
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<td>15000</td>
<td>2</td>
<td>83.763389</td>
<td>496</td>
<td></td>
</tr>
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<td>99.594450</td>
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<td>M</td>
<td>Transmittance</td>
<td>Error</td>
<td></td>
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</table>

It is clear from figure 2-2 that if \( N = 512 \), the transmittance will reach nearly 100% after \( M = 20000 \) with some fluctuation of error, while if \( N = 1024 \) however, the
transmittance fluctuated from 60% to 100% after when $M = 15000$. Meanwhile, it is noted that the running time increases linearly with the sampling number $M$ and approximately proportion to $N^2$.

Figure 2-5 Test of accuracy and efficiency of the program: (a) Transmittance dependence on longitudinal sampling number $M$; (b) Running time vs. $M$
In order to make a compromise between the accuracy and running efficiency (time consumption), we will choose $L = 128 \, \mu m$ and $N = 512$. For the untapered fiber, it is accurate enough to choose a value of 30000 for the sampling number $M$. However, for tapered fiber, $M$ should be chosen as large as possible. This part will be discussed in Chapter 4.
CHAPTER 3

EXPERIMENTAL RESULTS

In this chapter a tapered single mode fiber sensor is fabricated by Vytran Glass Processing System. A pigtail SP12FCSM with cladding/core diameters of 125/9\(\mu\)m is used in the fabrication procedure. The tapered fiber is then etched with buffered HF solution. Meanwhile, the transmission characteristic of the tapered fiber is also measured and discussed in this chapter.

3.1 Tapered fiber fabrication

The fabrication procedure is done by taper a single mode fiber by using the Vytran Glass Processing System (VGPS), which can be divided into three steps: cleaving, splicing and tapering.
To begin with, both ends of the pigtail (See Figure 3-1) used in this experiment are cleaved so that they can be spliced together. The fiber cleaver is shown in figure 3-2. In order to decrease the slicing loss, the cleave angle is set to be 0° so that the two end surfaces of the fibers should perfectly perpendicular to each other.

![Figure 3-2 Fiber cleaver](image)

After cleaving, the two fibers are connected by splicing the two end surfaces together. This procedure is done by using the Vytran glass processing system shown in figure 3-3.

![Figure 3-3 Glass Processing System GPX-3000](image)
A typical splicing process is shown in Figure 3-4. The fibers are aligned in both transverse and longitudinal directions, and then heated and fused to be spliced together. The splicing loss can also be calculated by the VGPS. In this experiment, the averaged splicing loss is around 0.1dB to 0.2 dB, so that the transmittance is close to 100%.

Figure 3-4 Splicing process: (a) Before splicing; (b) After splicing; (c) Loss estimation
In the tapering step, the protective jacket is removed for a length of about 10cm. The tapering procedure is normally fulfilled by pulling and heating the optical fiber. Argon gas is used in heating to provide a local inert atmosphere. Upon reaching around 1800 Celsius degree, both materiel of core and cladding will become liquid states, as a result, the fiber can be pulsed resulting in a change in its length and as a result it becomes much thinner at its waist. By controlling the argon flow rate and transition speed, the fiber can be tapered as thin as ten micrometer in diameter. A schematic model of the tapered fiber profile used in this experiment is shown in figure 3-5.

![Figure 3-5 A sketch figure of tapered fiber profile with uniform waist](image)

After the taper process the fiber consists of three contiguous segments: one taper waist segment with uniform diameter is located in the middle region, which is surrounded by two transition regions, which we call the up-taper and down-taper regions.
whose diameters are gradually changed. The ends of the conical transition segments are connected to the fiber without taper. In this experiment, the transition length is set to be 5 mm while the length of waist segment is 2 cm. A typical sample of tapered fiber is illustrated in figure 3-6.

Figure 3-6 illustrates the profile of a tapered single mode fiber. It is clarified that since the core and cladding are mixed together after heating and pulling so that they cannot be clearly classified. What’s more, since the VGPS is not able to consistently reproduce taper profiles, the tapered fiber in reality is not perfectly axial symmetric around the middle of the waist, i.e., the transition segments from two sides are slightly different in length. Besides, the waist in reality is not exactly 10 μm which fluctuated in about ±1 μm in diameter.

Figure 3-6 Parts of the tapered region observed by optical microscope (both tapered and untapered parts of the fiber)
3.2 Etching experiment

3.2.1 Etch rate

Since sensitivity plays a significant role in determining the performance of optical sensors, which to some extent limits the detection accuracy of an optical sensor, thus, high sensitivity is always desired. In order to further enhance the sensitivity of the tapered single mode fiber sensor, the waist size should be as small as possible. Our target is 1μm in diameter. This target is achieved by etching the tapered fiber with acid solution. To etch silica a Hydrofluoric (HF) acid solution is required.

In order to make sure that the etching experiment will be controllable and stable, etch rate of HF solution should be measured and the concentration of HF solution should be chosen wisely. The etch rate that measures the removing speed of material in an etching process is defined as the ratio of difference in diameter and etch time. The etch rate can be influenced by many factors such as flowing gas, pressure change, electrical field as well as temperature.

First, concentrated HF (49%) solution is tested. Four untapered fibers are held on Teflon and dipped into the HF solution for 5 min, 10 min, 15 min and 20 min, respectively, and then are removed from HF solution and dipped into water to remove any remaining HF solution from the fiber surface. Second, we remove the fibers from water and measure the diameter of the fibers after etching. For accurate measurement, the etching procedure is done at room temperature (around 25 degree Celsius) and the
solution is kept still. The diameter of etched fibers as well as unetched fibers is measured by using Scanning Electron Microscope (SEM). The results are shown in figure 3-7.
Figure 3-7 Fiber diameters after etching, measured by using SEM: (a) Etching 5 min; (b) Etching 10 min; (c) Etching 15 min; (d) Etching 20 min; (e) Unetched fiber

The diameter of fibers after etching measured in this experiment as well as a fitting curve is plotted in figure 3-8.

As indicated in figure 3-8, the etch rate of HF (49%) solution with fiber is nearly a constant since the etched length is nearly the same for the same etching duration. The averaged etch rate of HF (49%) solution calculated in figure 4-8 is about $2.7716 \pm 0.0417$ μm/min, which means a tapered fiber with 10 μm in diameter will possibly
gone by etching 3-4 min, which is too rapid to accurately control the fiber size. In order to slow down the etch speed, a buffered HF (BHF) solution, also known as buffered oxide etch (BOE) solution would be more suitable as a wet etchant with fiber. [26]

Buffered oxide etch solution is a mixture of ammonium fluoride (NH₄F) and hydrofluoric acid (HF). The 6:1 BOE solution is commonly used in etching silicon oxide which comprises a 6:1 volume ratio of NH₄F (40%) in water to HF (49%) in water. The etch rate of 6:1 BOE solution is referred in ref. [27] It is claimed that the 6:1 BOE solution has an etch rate of approximately 0.12 $\mu$m per minute at 25 degree Celsius when etching oxides, which means, etching a tapered fiber with 10 $\mu$m of waist diameter will last about 30-40 min. Additionally, the etch rate will not vary significantly as used in experiment since the solution is buffered, the concentration will not change significantly, which give us a good choice for controllable etching procedure of tapered fibers. [26] Hence, we choose the 6:1 BOE solution as the etching solution.
3.2.2 Experimental result

The experiment can be divided into two parts: measuring the transmittance dependence upon wavelength as well as on etching time. In this experiment, a tunable laser source is used as the input power; one end of the tapered fiber is connected to the laser source while the other end is connected to the Light wave Measurement System (LMS) “Agilent 8164A”. The LMS is connected to a computer so that we can control the experiment by computer. The BOE solution is stored in a plastic container; the tapered part of the fiber is bended and held on a box which is fixed on a translation axis. Thus the tapered fiber can be dipped into the BOE solution with controllable depth by moving the translation axis. A bottle of water is also used in this experiment for clearance. The experimental setup is illustrated in figure 3-9.

Figure 3-9 Experimental setups
(a) Transmittance dependence upon wavelength

Before etching, the fiber was dipped into water. The input laser power is fixed at 1 mW. We observed that the output power is stabled at approximately 0.45 mW. The wavelength of input beam is sweep from 1.475 μm to 1.565 μm by using a Labview program in the computer. The output power is measured and displayed in figure 3-10.

![Figure 3-10 Output power vs. wavelength (Before etching)](image)

It is clear that the output power shows a sinusoidal behavior with wavelength, which is possibly due to the interference of each mode since the output field is a superposition of several modes.

Next, we remove the fiber out of water and dip it into BOE solution, thus, the etching process begins. The values of output power and corresponding etching time are stored in the computer.
After etching the fiber about 15 min, we remove it from the BOE solution and re-dip it into the water. And then sweep the wavelength and measure the output power again.

We repeat the two steps above several times and measure the wavelength dependence of output power until the output power reduces to about 1 nW, nearly $1/450000$ the value of initial, which in our opinion indicates that the fiber is broken with a high possibility. The dependence of output power on wavelength at different etching time is illustrated in figure 3-11.
Figure 3-11 Output power vs. wavelength at different etching time: (a) Etching 15 min; (b) Etching 21 min; (c) Etching 26 min; (d) Etching 29 min; (e) Etching 35 min
As shown in figure 3-11, as the etching procedure continues, the output power decreases significantly from 0.24 mW in figure 3-11 (a) to 1.2 μW in figure 3-11 (e) (measured at \( \lambda = 1.55 \mu m \)). Besides, the curve of output power becomes smooth (the sinusoidal variations disappear). In addition, the fluctuation in output power also becomes less significant as the output power reduces. There are a couple of physical processes at play in the gradual disappearance of the transmission oscillations. One reason is that the higher modes are reaching their cutoff condition and have greater loss per unit length as the fiber section is etched. Since the total output power decreases, the relative power transmitted through the waist segment by each mode also decreases. Hence, the power transferred from each mode became insignificant compared with the total power.

(b) Transmittance dependence upon etching time

The output power is also measured and recorded in the computer; the corresponding values of time are also stored. The relationship between output power and etching time is shown in figure 3-12.
Figure 3-12 Output power dependence on entire etching procedure

Figure 3-11 shows the development of output power on the entire etching duration. Compared with figure 3-10, we clearly observed that the output power decreases dramatically with etching time. Meanwhile, a big fluctuation in power is also illustrated. Besides, several discontinuous points in power can also be seen in this figure at time points where the fiber is removed from the solution; this is as a result of the change in refractive index of surrounding medium (from BOE to air). The change in refractive index will lead to the change in $V$ number, which has an influence on the mode profiles, their cutoff conditions and their affect by surface scattering losses, which we observe in a change in the output power.

In order to analyze the dependence of output power on etching time, a new experiment is performed with higher accuracy. The dependence of output power on
entire etching time is illustrated in figure 3-13; more specific details are plotted in figure 3-14.
Figure 3-14 A more specific plot of output power vs. time. (a) T: 140s-240s; (b) T: 300s-400s; (c) T: 520s-620s; (d) T: 730s-830s; (e) T: 820s-920s; (f) T: 1060s-1160s; (g) T: 1280s-1380s; (h) T: 1480s-1580s; (i) T: 1680s-1780s; (j) T: 1780s-1880s; (k) T: 2170s-2270s

Compare each plot in figure 3-14, we can make several assumptions:

(1) The output electric field can possibly be written as a superposition of a series of sinusoidal function can be defined as:

\[ E = \sum_{n=1}^{N} E_n \sin(\omega t - \phi_n) e^{(-a_n h t)}. \]  

(3-1)

Where \( E_n \) is the amplitude of the nth mode out of N total modes; \( a \) is an attenuation constant for mode \( n \); \( h \) is the etch rate of BOE solution and \( \phi_n \) is the phase term of each mode accumulated through the tapered region, which is defined as:

\[ \phi_n = \int_0^L \beta_n dz. \]  

(3-2)
Where $L$ is the taper length; $\beta_n$ is the effective wave number of each mode;

(2) The period of fluctuation first decreases from at $t=140s$ to $t=910s$ and then grows sharply from at $t=910s$ to $t=1060s$, and then reduces again from then on;

(3) The waist diameter decreases with time, the number of N also decreases as time passes, since the V number who determines the numbers of modes supported by a fiber is proportional to waist size;

(4) After $t=2100s$, the output curve becomes a smooth curve, which means that only the fundamental mode HE$_{11}$ mode still exists inside the fiber, other hybrid modes are gone.

A pure sinusoidal variation of the output power indicates that primarily two modes are participating in the observed interference. The frequencies of the interference pattern are connected to the difference of propagation constants between the dominant HE$_{11}$ mode and the excited cladding modes. Micron or nanometer scale fiber sensors will be designed based on the combination of fiber tapering and acid etching using the results studied here.
CHAPTER 4

SIMULATION RESULTS

In this chapter the transmittance characteristics of the tapered single mode fiber is simulated by using FFT-based Beam Propagation Method. The basic equations and conditions on convergence were discussed in Chapter 2. The model of the SMTFs is established here and results are presented. The simulations show some basic features found in the experimental results, but it is not yet at a level where quantitative comparisons can be made. In the future the accuracy of the simulation should be further improved to become a more quantitative indicator of the tapered fiber’s optical characteristics.

4.1 Model of the SMTFs

The parameters of the fiber are the same as we discussed in Chapter 2. The model of the tapered single mode fiber is illustrated in figure 4-1. It is shown that the tapered fiber consists of three contiguous segments: one taper waist segment with uniform diameter, two transition regions whose diameters are gradually changed and two untapered segments. Here, we set the transition or taper length to be 5 mm and a 2 cm waist length. The waist diameter is set to be 10μm. In this model, we assume that the
fiber core is also tapered during the taper process, which means the profile of core is pretty similar comparing with that of the cladding. Hence, we assume that the radius of the core stands at 0.7μm in the tapered region and increases gradually along the transition segment from 0.7 μm at one end to 4.5μm at the other end. The parameters of the fiber are the same as we discussed in Chapter 2. The model of the tapered single mode fiber is illustrated in figure 4-1. The untapered fiber has an outer radius of 62.5μm.

![3-D Model of the initial TSMFs](image)

**Figure 4-1 3-D Model of the initial TSMFs**

**4.2 Simulation result**

In the simulation, we assume that the tapered fiber is put straightly along z axis and is etched by buffered HF solution. We assume that the etching experiment takes place only in the taper region. The etch rate is set to be 0.12 μm/min which is constant
during the entire etch time and etching occurs uniformly in the tapered region. The transverse window size $L$ in simulation is $128 \mu m$, whilst the transverse sampling number $N$ is 512.

As described in Chapter 2, the step size $\Delta z$ should satisfy Eq. (2-41):

$$\Delta z \ll 6k_0(k_e + k_w)^{-2}.$$  \hspace{1cm} (2-41)

Here, $k_e$ and $k_w$ can be approximated as:

$$k_e = \frac{\pi}{d x} = \frac{N \pi}{L} = 12.6 \mu m^{-1}.$$ \hspace{1cm} (4-1a)

$$k_w = m \frac{\pi}{r_{core}} = 0.349 \mu m^{-1}.$$ \hspace{1cm} (4-1b)

Where $m$ is a positive integer that defines how many Fourier components should be used in the simulation.

Plugging Eq. (4-1a) and Eq. (4-1b) into Eq. 2-42, and set $m = 1$, the step size should satisfy: $dz < 0.146 \mu m$, which means that for a taper length of 3 cm, the longitudinal sampling number $M$ should be as large as 205500. This number is too big for us such that getting one group of result will consume at least 3 hours. Instead, we will compromise the accuracy for speeding up the turn-around times of the simulations and choose a smaller number of sample points in the simulation, $M = 60000$. The
simulations still require a computational times that are several hours and the etch times are spaced by 1 second.

(a) Transmittance dependence upon etch time

Here, we will simulate the etching experiment. Since the entire etching procedure is too long so that too many data need to be recorded. In order to reduce the memory occupying, we simulate several segments of etch time. The transmission function is not continuous over the entire etch time; instead we skip from one time span to another by changing the fiber profile to correspond to the desired etch time. During the etching process the radius of the fiber is given by:

\[ r' = r - hT. \]  \hspace{1cm} (4-2)

Where \( h \) is the etch rate, \( T \) is the etching time and \( r \) is the initial fiber radius for the segment; the fiber has already been etched for a period leading to different initial values for \( r \). The sampling rate in this simulation is one point per second. We assume that the etching process is first took place at the cladding, and after several minutes the cladding were removed entirely so that from then on the core was etched. In every “time period”, the field distribution on the input plane as well as on the output plane are recorded, the dependence of transmittance on etching time during ten different “time segments” are illustrated in figure 4-2(a)-(j). The last illustration in figure 4-2(k) is the synthesis of all seven segments together over the total time span. In these simulations the variations
between points are disjoint, rather than forming a smooth curve. Nevertheless, the trend in the transmission versus etch time is qualitatively similar to the experiments reported in chapter 3.
Figure 4-2 Transmittance dependence on etching time at different time duration. (a) T: 0-80s; (b) T: 310-380s; (c) T: 600-660s; (d) T: 800-880s; (e) T: 1000-1070s; (f) T: 1300-1360s; (g) T: 1400-1460s; (h) T: 1620-1680s; (i) T: 1700-1760s; (j) T: 1800-1860s; (k) T: 0-1900s
Generally, it can be shown from figure 4-2 that the transmittance patterns share some basic features of those shown in the experimental result.

- The transmitted power decreases with etching time;
- The output power fluctuates as the fiber is being etched;
- The magnitude of the power fluctuations decreases for longer etch time;

However, some features in the simulation patterns are not closely meeting the experimental results:

- The fluctuation in power does not show good sinusoidal form that we observe in the experiments;
- The magnitude of fluctuation are larger than found in the experiment results;

(b) Transmittance dependence upon wavelength

The output power is also recorded by sweeping the input wavelength at an arrested etch time. The wavelength ranges from 1.525 $\mu m$ to 1.565 $\mu m$. Similarly, the initial fiber profile is also adjusted by the changing the etching time $T$, the relationship between transmittance and wavelength at different location of etching time are plotted in figure 4-3.
Similarly, the transmittance dependence on wavelength shows some similar trends as shown in Figure 4-2:

- The transmittance fluctuated on wavelength;
- When the fiber is etched longer, the transmittance shows a decreasing trend;
- The magnitude of fluctuation in power also decreases when the etching procedure continues.

Although the transmittance patterns do show some similarity with the experimental results, the accuracy of the program should, however, be further improved.

Hence, we take two steps to fulfill this goal. First, we reduce the sampling time from one point per second to three point per second. Second, the longitudinal sampling number $M$ is increased to 180000. With these two changes, new simulations are performed. The transmittance dependence on etching time is illustrated in figure 4-4.
Figure 4-4 Transmittance dependence on etching time at different time duration. (a) T: 100-160s; (b) T: 500-560s

As shown in figure 4-4, the new transmittance curve of etching time becomes much smoother, yet we found no oscillation characteristics of the output power in our simulations. The results indicate that reducing the longitudinal step size alone will not increase the accuracy of the program significantly;
The field distribution on the output surface is plotted in figure 4-5. It is shown that a great deal of power contains within the core and cladding region, which is reasonable as we expected, in addition, the optical field does extends into the outer medium with a reducing magnitude when away from the core. Generally, the results indicate that our simulations are at least working in the right direction.

![Propagated Gaussian beam](image)

**Figure 4-5 Field distribution on the output plane**

The unsuccessful simulation results indicate that reducing the longitudinal step size will not increase the accuracy of the program to a large extent; there must be other factors that limit the accuracy. The transverse step size was subsequently analyzed in chapter 5.
CHAPTER 5

SUMMARY

Optical fiber sensors can be used to measure temperature, strain, pressure, humidity, refractive index (RI) and many other quantities. Compared with other types of sensors, fiber optic sensors hold many attractive advantages such as small size, cheap price, high sensitivity and remote sensing capability. Tapered optical fiber sensor (TOFs) is one kind of fiber sensors with pretty high sensitivity, which are based on evanescent wave field (EWF) interaction. TOFs force the optical field outside the fiber core. Sensitivity of tapered optical fiber sensor is the change in output signal detected by the tapered region. If most of the power contained outside the core, then even a small change of the refractive index in the outer medium may be recorded as a change in output power. Hence, the sensor design is a compromise between a small the fiber radius and the requirement for interference between several modes in the waist region to achieve the highest sensitivity of the device. By tapering a portion of a fiber, the waist size can be small enough so that most of optical power of at least one mode is displaced into the cladding. In this way the intensity of EWF is enhanced dramatically, and therefore the sensitive to the refractive index of surrounding medium can be highly increased. Our
target in this thesis is to fabricate a tapered single mode fiber sensor with sub-micron waist diameter and study its transmittance characteristics. We achieve the ultimate nanoscale fiber diameter by using a wet etching technique.

In this thesis, a tapered single mode fiber with sub-micron of waist diameter is fabricated by tapering and wet etching a single-mode fiber with core/cladding diameter of $9/125\mu m$. The fiber is initially tapered by locally heating and stretching it using the Vytran Glass Processing System. The tapered fiber has three components: a waist with thickness of $10\mu m$ and length of $2cm$, two transition segments with gradually varied diameter from $10\mu m$ to $125\mu m$ and two untapered components with radius of $62.5\mu m$. The tapered fiber is then be etched by HF solution with the purpose of reducing the waist size to nanometer sizes and thus increasing the sensitivity. We use a 6:1 BOE solution which is chosen as the wet etchant that gives a controllable etch rate.

The transmittance characteristics are measured while the wet etching experiment is in progress. A tunable laser is used for a signal source and the output power is measured synchronized with the etch time and recorded in computer. As a result, we can either acquire the transmittance dependence on etching time or the transmittance can be measured as a function of wavelength. In the first instance, we set the input wavelength to $1.55\mu m$ and measure the output power while the tapered fiber is dip into BOE solution. The transmittance spectrum is measured after we remove the fiber from acid and dip it into water to stop the etch process. Then a tunable laser is used to sweep the input wavelength from $1.48 \mu m$ to $1.56\mu m$. For each case, the input power is stable at
1mW so that the transmittance can be easily calculated by recording the output power.

During the etching experiment, we found that the transmittance fluctuated with wavelength. In addition, the magnitude of fluctuation reduces when the fiber is further etched. This phenomenon has been explained by several researchers. [28-32]

We note that when the initial fiber is tapered down to a diameter of around 10um using the Vytran system, the radius of core becomes too thin so that the core is negligible and the cladding modes are excited in the waist region. Hence, the tapered fiber essentially becomes a uniform material with outer medium acting as its new cladding and the cladding acting as its new core. Since the refractive index of outer medium is far smaller than those of cladding and core, the mode cut-off frequency for the cladding modes of the tapered fiber will grow sharply. As a result, the tapered single-mode fiber can support multiple propagation modes. [33] As discussed in Chapter 3, the optical field on the output plane can be written as a superposition of fundamental HE\textsubscript{11} mode as well as several hybrid modes, the number of modes is determined by the V number:

\[ E = \sum_{n=1}^{N} E_n \sin(\omega t - \phi_n). \] (5-1)

Where \( \omega \) is the frequency of the input frequency, \( \phi_n \) is the optical phase of each modes accumulated when travelling through the tapered region. Since different modes will accumulate different optical phases as they travel along the tapered fiber, as they come back to the end of the tapered region, they interfere with each other. Hence, when we fix the time and sweep the input wavelength (frequency), the output power
which proportional to the square of optical field will becomes an oscillation function of wavelength. Higher the $V$ number is, more complicated the oscillation pattern becomes.

The intensity on the output plane for two-mode case is written as:

$$ I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \Delta \phi. \quad (5-2) $$

Where $\Delta \phi$ is the phase different between the two modes which take the following form:

$$ \Delta \phi = \phi_1 - \phi_2 = \int_0^L \Delta \beta \, dz. \quad (5-3) $$

Where $\Delta \beta$ is the difference between the propagation constants of the two modes.

The transmittance dependence on etching time shows similar trend as it shows on wavelength. A typical result of output power versus etch time is illustrated in figure 5-1.

![Figure 5-1 Output dependence on etching time](image-url)
Some general characteristics of transmitted power can be listed:

- The transmitted power shows a decreasing trend with etching time;
- The output power fluctuates as the fiber is being etched;
- The magnitude of the power fluctuations decreases for longer etch time;

Similarly, the oscillation characteristic of transmittance with etching time can also be explained by the interference of each mode. Assume that the fiber is being etched with a constant etch rate, the radius of tapered fiber reduces linearly with time. Before etching, the optical phase differences between each mode are constant so that the output power is stabled. When the fiber is being etched, the optical phases of each mode will change periodically which results in oscillations of the output power. When the fiber is further etched, the power carried by each mode will decrease, so the amplitude of oscillation will also decrease. In addition, since the radius of fiber is reduced, the number of modes also reduces as the V number decreases. As a result, the transmittance pattern becomes regular and even sinusoidal when the number of modes is two. When the fiber etching continues long enough only a single cladding mode will be supported by the fiber and the transmission oscillations will cease.

In order to verify the experimental results, we used FFT-based Beam Propagation Method (BPM) to simulate the transmittance dependence on wavelength and etch time. The BPM is easy to implement for a scalar system, but for our nanoscale fiber taper system is does present a great challenge for getting useful results.

To begin with, the transverse and longitudinal sampling number are set to be
N=512 and M=60000, respectively. The transverse windows size is L=0.128mm. The simulation results show that the transmittance dependence on wavelength is reasonable by comparing them with the experimental results. This result indicates that our simulation method is at least in the right direction. For the etching experiment, however, the simulation results are not faithful enough to capture fine scale features of the oscillations; although they do show some general characteristics found in the experiment. The disagreement between the simulation results and those of experiment can be traced to the discretization of the transverse space. The program still needs to be modified to improve the results, which is left as a future project. To capture the essential features we estimate that both transverse sampling number as well as longitudinal sampling number should be increased.

The longitudinal sampling number was increased to 180000 while other parameters were kept the same. This time the transmittance curve of etching time becomes much smoother, yet we found no oscillation characteristics of the output power in our simulations. The results indicate that reducing the longitudinal step size alone will not increase the accuracy of the program significantly; the transverse step size was subsequently analyzed.

In the experiment the output power sample rate is one point per second, the etch rate is 0.12\(\mu\)m/min, then the change in fiber radius is given by

\[
\Delta r = \frac{0.12}{60} = 0.002\mu m. \tag{5-4}
\]
The step size in transverse plane is defined as the ratio of transverse windows size and sampling number

\[ \Delta x = \frac{L}{N} = \frac{128}{512} = 0.25 \mu m. \]  \hspace{1cm} (5-5)

Note that the transverse step size is far bigger than the change in fiber radius. The change in integration area when calculating the output power is much smaller than the grid size. To be sensitive to fiber changes on the scale of 1 second etch times we must make sure that the step size \( \Delta x \) no larger than the radius change \( \Delta r \).

Since the sample rate should not less than one point per second, and the transverse window should not less than 0.128mm, then the transverse sampling number should satisfies

\[ N \geq \frac{L}{\Delta r} = \frac{128}{0.002} = 64000. \]  \hspace{1cm} (5-6)

From this result we concluded that we would require a finer transverse discretization by two orders of magnitude. Adding this many points to our simulation is impossible due to the memory and run time limitations.

Some potential future solutions of solving this problem are listed below:

- A simulation using a one-dimensional Hankel transform could be introduced. This approach saves memory by using the radial symmetry of the modes. The fast Hankel transform programs are not as numerically efficient as the FFT programs, but the program would reduce memory
requirements and capture the fine changes in the refractive index of the fiber. [34-35]

- Other software, such as COMSOL, which uses Finite Element Method (FEM), could be used to implement appropriate boundary conditions, such as perfectly matched layers, to refine the transverse scale to meet the demands of our fiber etching on the nanoscale. [36]

- Other numerical methods are also available, such as, Coupled Mode Theory (CMT), [37-39] and Finite Difference Time Domain Method (FDTD). [40] The CMT has been applied by Ighor Idehenre for describing tapered fibers similar to those in this thesis. [41] However, CMT has not been applied to time-dependent experiments that we studied here. FDTD is a numerically intensive numerical method that also can be adapted to follow nanoscale changes in the fibers.
BIBLIOGRAPHY


