ANALYSIS OF JOINT EFFECTS OF REFRACTION AND TURBULENCE ON
LASER BEAM PROPAGATION IN THE ATMOSPHERE

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ANALYSIS OF JOINT EFFECTS OF REFRACTION AND TURBULENCE ON LASER
BEAM PROPAGATION IN THE ATMOSPHERE

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ABSTRACT

ANALYSIS OF JOINT EFFECTS OF REFRACTION AND TURBULENCE ON LASER BEAM PROPAGATION IN THE ATMOSPHERE

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Experimental data obtained from recently conducted long-range laser beam propagation experiments has revealed inconsistencies with analytic and numeric simulations results based on classical Kolmogorov turbulence theory. This inconsistency may be related with not accounting for refraction effects caused by refractive index variation with elevation and presence of large-scale atmospheric structures which introduce refractive index gradients and can alter the trajectory of optical wave energy flux. In this thesis, atmospheric refraction effects are studied using a ray tracing technique. Due to refraction a ray propagating in the atmosphere doesn’t follow a straight line and may not arrive to a desired location. In this thesis the ray tracing technique was applied for analysis of optical propagation over a 150 km propagation path. It was shown that due to refraction the ray trajectory may deviate from the geometric straight line by 60m in the middle of the path. We also considered the impact of refraction on atmospheric propagation of laser beams with different wavelengths (\(\lambda=0.532\mu m\), \(\lambda=1.064\mu m\), and \(\lambda=1.550\mu m\)) which were launched at the same angle. Due to the difference in refractive index of
air for different wavelengths, the ray’s paths follow different trajectories. It was shown that at the end of the propagation path, the distance between ray trajectories can be as long as ~4.1m for the 0.532µm and the 1.064µm rays, and ~4.3m for 0.532µm and 1.550µm rays. Besides traditional ray tracing technique we also introduced a new computational method that allows analysis of combined refraction and turbulence effects on laser beam propagation. In this method, traditional beam propagation using the well-known split step operator method is combined with ray tracing. In this technique the atmospheric volume is represented as a set of thin phase screens that obey Kolmogorov turbulence statistics. The ray tracing technique is applied to describe optical wave propagation between phase screens. At each screen, the turbulence-induced random tip and tilt wave-front phase component is added to the ray angle. In this way, the ray trajectory is no longer deterministic, but it has a turbulence induced uncertainty. It was shown that at the end of a 150km propagation path, the turbulence induced deviation on ray trajectory can be on the order of 5m. These results show that for correct analysis of laser beam propagation over long distances in the atmosphere, refraction and turbulence effects should be considered jointly. The proposed numerical simulation technique allows this joint analysis.
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CHAPTER 1

INTRODUCTION

Atmospheric turbulence causes index of refraction inhomogeneities such as different size eddies which affect optical wave propagation through the atmosphere. These refractive index inhomogeneities cause image jitter and blurring in imaging systems, and intensity scintillations, and focal spot wander and widening in laser systems [1]. The analysis of atmospheric turbulence has led to a statistical model of the refractive index fluctuations [2].

Recent data from long range propagation experiments has suggests that numerical simulations incorporating statistical models such as Kolmogorov theory is not sufficient for long (from tens to hundreds km) propagation distances. Over long propagation distances it is necessary to also take into account refraction from large scale structures in the atmosphere. Large scale coherent structures can create strong refractive index gradients which can greatly alter the trajectory of optical wave energy flux. New analytical and numerical techniques need to be developed which take these phenomena into account.

Analysis that accounts for both small scale turbulence eddies and large scale refractive structures is challenging because of the large range of scales involved. The conventional numerical techniques that are commonly applied to analysis of laser propagation in turbulence is based on separate treatment of refractive (ray tracing) and turbulence (wave-optics) effects. The turbulence effects are accounted for by subdividing the volume of atmosphere into several thin layers that
impact only optical wave phase. In this so-called split-operator approach [3], the diffraction effects are accounted by integrating the corresponding free-space propagation equation (parabolic equation) between the phase distorting layers. The split-operator technique is not sufficient to simulate optical wave propagation along extended range paths that include large scale refractive structures that cause combined effect of optical axis bending due to refraction and turbulence-induced phase aberrations. Since the number of pixels used in numerical simulations to sample optical field is limited, there is a tradeoff between grid size and pixel size. If the grid size is large to capture large scale refractive index inhomogeneities, then the resolution is not enough to sample small turbulent eddies. To solve this problem, we suggest a hybrid approach in which large scale refraction, and small scale turbulent eddies are accounted by combining ray tracing and wave-optics techniques. In the presented theses a new numerical technique which simultaneously uses both ray tracing and wave optics to simulate optical wave propagation through the atmosphere is described. The developed numerical simulation approach is used to create new software that is used to study laser beacon propagation over long range propagation distances including a 150 km path between two Hawaiian Islands.
CHAPTER 2

ATMOSPHERIC OPTICAL REFRACTION EFFECTS AND RAY PROPAGATION

In order to simulate ray propagation through the atmosphere, an atmospheric index of refraction profile is needed. Section 2.1 describes the US 1976 standard atmospheric profile which is used for simulations. Section 2.2 discusses ray tracing through large scale refractive index variations.

2.1 Atmospheric Refractive Index Profile

2.1.1 Index of Refraction: Basic Equations

The index of refraction is directly proportional to the density of the air at that particular point in space. The refractivity of the air is given by [4]

\[ n - 1 = A_D(\lambda) \frac{P(h)}{T(h)}, \]  

(2.1)

where \( P \) is the air pressure in hPa and \( T \) is the temperature in Kelvin and \( A_D \) is the so-called reduced refractivity coefficient for air given in hPa\(^{-1}\)K. Several different models for \( A_D \) have been developed which agree at the level of 1 part in 10\(^4\)[4]. In this study we use, Edlin’s Sellmeier [5] form equation:

\[ A_D(\lambda) = 10^{-10} \left[ \frac{8342.13}{130 - \frac{\lambda_0^2}{\lambda^2}} + \frac{2406030}{38.9 - \frac{\lambda_0^2}{\lambda^2}} + \frac{15997}{1013.25} \right] \]  

(2.2)
where $\lambda_0$ is 1.0µm. The refractivity of air decreases for longer optical wavelengths under normal dispersion conditions, i.e. over a wavelength range where there are no major absorption bands. Figure 2.1 shows the reduced refractivity $A_D$ as a function of wavelength.

![Figure 2.1 Reduced refractivity $A_D$ vs. wavelength for dry air](image)

If air is considered to be an ideal gas, then the temperature and pressure in equation (2.1) are related to each other by the differential equation [4]

$$
\frac{dP}{dh} = -\frac{mN_A g}{R} \frac{P(h)}{T(h)},
$$

(2.3)

where $m$ is the molecular mass of dry air in grams $N_A$ is Avogadro’s number, $R$ is the universal gas constant and $g$ is the gravitational acceleration. Once the temperature profile is known, the pressure profile can be found using equation (2.3), and the refractive index profile can then be calculated. Atmospheric profiles can be based on deterministic models, measured data, or even meteorological prediction models. For most examples presented here, the US 1976 standard atmospheric profile is used.
2.1.2 The US 1976 Standard Atmosphere Temperature Profile

The US 1976 standard atmosphere vertical temperature profile assumes constant temperature gradients for each of a set of atmospheric layers. Figure 2.2.a shows a plot of the US 1976 standard atmospheric temperature altitude profile.

Each circled point in this figure represents a boundary in the layered model. This profile is strictly a function of altitude, so the temperature and refractive index gradient only vary in the vertical direction. For the lowermost layer, the troposphere, the gradient is -6.5 K/km. The temperature at sea level is an adjustable parameter. The temperature profile for the troposphere in the 1976 US standard atmosphere is given by

\[ T = -\alpha h + T_0, \]

where \( \alpha \) is 6.5K per kilometer, \( h \) is altitude in km, and \( T_0 \) is the average sea level temperature in degrees Kelvin. The global average temperature at sea level is 288.15 K. Substituting into
equation (2.3) and solving for the air pressure, one can obtain the pressure profile as a function of altitude:

\[ P(h) = \frac{P_0}{T_0} \left( \frac{T_0}{T_0 + \alpha h} \right)^{\frac{Mg}{\alpha R}} \]

(2.5)

where \( R = 8.3144 \text{ J mol}^{-1} \text{K}^{-1} \) is the universal gas constant, \( g = 9.8 \text{ m/s}^2 \), and \( k = 1.38065 \text{ m}^2 \text{kgs}^{-2} \text{K}^{-1} \) is Boltzmann’s constant. Knowing both the pressure and temperature profile, the refractive index altitude profile can be found using equation (2.1):

\[ n(h) = 1 + A_D(\lambda) \frac{P_0}{T_0} \left( \frac{T_0}{T_0 + \alpha h} \right)^{\frac{Mg}{\alpha R + 1}}. \]

(2.6)

Figure 2.3 shows refractivity vs. altitude for the troposphere in the 1976 US standard atmosphere. As the density increases near the earth’s surface, so does the refractive index. The refractive index gradient, which is important for ray tracing calculations, is given by

\[ \frac{dn}{dh} = -A_D(\lambda) P_0 T_0 \frac{Mg}{R} \left( \frac{Mg}{R} + \alpha \right) \left( T_0 + \alpha h \right)^{\frac{Mg-2}{\alpha R}}. \]

(2.7)
The 1976 US standard atmosphere is one of the simplest atmospheric models as its only variable parameter is sea level air temperature. It represents a worldwide average and does not take into account any local effects. Other meteorological models, or even measured data can be utilized to create a more site-specific refractive index profile. Although the refractive index gradient for the 1976 standard atmosphere has only an altitude dependence, in general, for 3D meteorological models, the gradient can be varied in any arbitrary direction.

2.2 Ray Tracing Technique

Ray tracing [6] is based on the geometric optics approximation to optical wave propagation equation. In this approximation, diffraction effects are neglected which results in a representation of optical wave propagation in the form of ray trajectories. Ray optics can be understood as the limit of wave optics as the wavelength of light approaches zero.

In wave optics, the Helmholtz equation [1] describes evolution of the optical field complex amplitude in space and is given by
\[ \nabla^2 U + k^2 n^2(r) U = 0, \]  
\hspace{1cm} (2.8)

Where \( k \) is the wavenumber and \( n \) is the index of refraction. The complex amplitude, \( U \), can be written in the form

\[ U(r) = a(r) e^{i\phi(r)}. \]  
\hspace{1cm} (2.9)

where \( a(r) \) is the amplitude and \( \phi(r) \) is the phase. Now, suppose there is a monochromatic wave in a medium whose refractive index \( n(r) \) varies slowly enough in space to be considered locally homogeneous. The complex amplitude can be written as

\[ U(r) = a(r) e^{-jk_0 S(r)}. \]  
\hspace{1cm} (2.10)

Here, the wave fronts are specified by surfaces \( S(r) = \text{constant} \). This surface of constant phase is called the eikonal surface. Substituting equation (2.10) into the Helmholtz equation yields [6]

\[ k_0^2 \left[ n^2 - |\nabla S|^2 \right] a + \nabla^2 a - jk_0 \left[ 2\nabla S \cdot \nabla a + a\nabla^2 S \right] = 0. \]  
\hspace{1cm} (2.11)

Setting the real part of equation (2.11) to zero and substituting \( 2\pi/\lambda \) for \( k_0 \) gives

\[ |\nabla S|^2 = n^2(r) + \left(\frac{\lambda_0}{2\pi}\right)^2 \frac{\nabla^2 a(r)}{a(r)}. \]  
\hspace{1cm} (2.12)

Since \( a(r) \) varies slowly over the distance of a wavelength, the derivative term on the right hand side is small. If the limit of that term is taken as \( \lambda_0 \) goes to zero, the entire term vanishes, and what’s left is the well-known eikonal equation [5]:

\[ |\nabla S|^2 = n^2(r), \]  
\hspace{1cm} (2.13)

where \( n \) is the index of refraction spatial distribution. The scalar function \( S(r) \) represents a surface of constant phase of an optical wave propagating through a volume with the refractive index field \( n(r) \). A ray propagates in the direction normal to \( S \) which is given by the gradient of
the eikonal. In other words, the ray propagation angle is pointed in the direction in which the rate of change of the phase front is greatest.

2.2.1 Ray Tracing Equation

Starting with the eikonal equation, which is the foundation of geometric optics, the ray tracing equation can be developed [6], [7]. This equation can be solved to find the trajectory of a ray through a volume of propagation medium that is characterized by space varying index of refraction.

First consider a ray that is characterized by a position vector \( r \) and denote \( s \) as a scalar distance along the ray trajectory. The unit vector in the direction of the ray trajectory at a point \( r \) is then given by

\[
\hat{s} = \frac{dr}{ds}.
\]  

(2.14)

Now, the magnitude of the gradient of the eikonal equation is given by

\[
\nabla S = n\hat{s},
\]  

(2.15)

where \( n \) is the spatial distribution of the index of refraction. Taking the derivative of both sides of this equation yields

\[
\frac{d}{ds}(n\hat{s}) = \frac{d}{ds}(\nabla S).
\]  

(2.16)

Plugging equation (2.14) into the left side of this equation gives

\[
\frac{d}{ds}\left(n\frac{dr}{ds}\right) = \frac{d}{ds}(\nabla S).
\]  

(2.17)

Now we can use the chain rule,
\[ \frac{d}{ds} = \frac{\mathbf{dr}}{ds} \cdot \nabla, \]  
(2.18)

to expand the right hand side of equation (2.17), which gives

\[ \frac{d}{ds} \left( n \frac{dr}{ds} \right) = \frac{dr}{ds} \cdot \nabla (\nabla S) \]  
(2.19)

Substituting for \( \frac{dr}{ds} \) from equation (2.14);

\[ \frac{d}{ds} \left( n \frac{dr}{ds} \right) = \frac{1}{n} \nabla S \cdot \nabla (\nabla S), \]  
(2.20)

which can be written as:

\[ \frac{d}{ds} \left( n \frac{dr}{ds} \right) = \frac{1}{2n} \nabla (\nabla S)^2. \]  
(2.21)

Now, using the eikonal equation (2.21) becomes

\[ \frac{d}{ds} \left( n \frac{dr}{ds} \right) = \frac{1}{2n} \nabla n^2, \]  
(2.22)

which is simplified to

\[ \frac{d}{ds} \left( n \frac{dr}{ds} \right) = \nabla n. \]  
(2.23)

Equation (2.23) is known as the ray tracing equation. It relates the trajectory of the ray to the scalar distance along the ray and the spatial distribution of the refractive index. This equation is the basis for calculation of ray trajectories in propagation medium with space varying refractive index.
2.2.2 Numerical Integration of the Ray Tracing Equations

In most cases the ray tracing equation cannot be solved analytically, but rather requires numerical integration. This numerical integration is commonly achieved by transitioning into a pair of first order differential equations

\[
\frac{dr}{ds} \quad \frac{d}{ds} n = \frac{T}{n}, \quad \text{and} \quad (2.24)
\]

\[
\frac{dT}{ds} = \nabla n. \quad (2.25)
\]

These equations can be solved using standard numerical techniques [8]. In the Euler’s first order approximation method the derivative of the functions are approximated by

\[
\frac{dT}{ds} \approx \frac{T(s + h) - T(s)}{h}. \quad (2.26)
\]

Equations (2.24), and (2.25) can be represented using Euler’s method by

\[
r = r_0 + h T / n, \quad \text{and} \quad (2.27)
\]

\[
T = T_0 + h \nabla n. \quad (2.28)
\]

Where \( h \) is the step size, \( r \) is the position vector and \( T \) is initially given by the direction cosines for the ray direction i.e. \( T = (\cos(\theta_i) \hat{x}, \sin(\theta_i) \hat{y}) \), where \( \theta_i \) is the initial angle. So, given an initial position, and direction, the ray trajectory can be calculated. The ray is built up step by step where each new section of the ray is a vector with magnitude \( h \). The new ray position is a function of both the refractive index, and its gradient.
Figure 2.4 Vector diagram illustrating numeric integration of the ray tracing equation

Since this is a first order method, the local error is on the order of $h^2$, and a sufficiently small step size must be used to achieve a high level of accuracy.[8] For the sake of simplicity, this method was implemented with a small step size. To speed up the simulation, more complex numerical techniques such as the ‘Adams-Bashforth’ method can be used to achieve accurate results with much less computational expense[8].

2.2.3 Implementation in C++

This section provides a brief overview of the implementation of ray tracing that was developed.

Since C++ is an object oriented programming language, the process of ray tracing is implemented by identifying classes and specifying relations between them. A class is simply a ‘blueprint’ for an object, and it contains both data, and functions which relate to that object. In the case of ray tracing, there are two basic classes which were identified and created; ray and index profile. The class ray stores the physical parameters of the ray as well as the iterative algorithm used for ray tracing which is given by

$$r_{i+1} = r_i + hT_i / n$$

(2.29)
\[ T_{i+1} = T_i + h \nabla n. \]  

(2.30)

where \( i \) is an integer representing the step number.

The class \textit{index profile} contains the atmospheric refractive index model from which the index of refraction and its gradient are computed. The most basic and essential function in \textit{ray} is the \textquote{next step} function which propagates the ray by one step using the algorithm derived from the ray tracing equation. Figure 2.5 depicts the basic architecture used for ray tracing. The function \textit{next step} uses an atmospheric model from the class \textit{index profile} to compute the index of refraction and its gradient, which are then used to propagate the ray from its current position, to the next position.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{ray_tracing_diagram.png}
\caption{Diagram illustrating implementation of ray tracing equations numerical integration}
\end{figure}

This functionality can be placed in a \textit{for} loop to iteratively propagate over any path length which is an integer product of the step size. After each new step, the current position can be stored and saved to a file \textit{data.txt} as shown in the following code snippet.
for(int i=0; i<num_steps; i++) {
    // loop over the path length
    ray.next_step(step_size, index_profile)
    fprintf('data.txt', ray.r) // store results in file
}

The trajectory can then be plotted by accessing the file data.txt.

A benefit of dividing the program up in this way is that any number of models can be used to create the index of refraction profile without changing the rest of the code. This allows flexibility for easily adapting the simulation for more complex models than the US 1976 profile.

2.3 Ray Tracing Atmospheric Optics: Examples

In this section, two examples of ray tracing through different atmospheric refractive profiles are presented. The first example is ray tracing through a simple layered atmosphere with a temperature profile that exhibits wave guiding of rays. The second example is a simulation supporting the COMBAT experiment performed between two Hawaiian Islands. Before the examples are presented, the process of plotting the ray trajectories is explained in detail.

Impact of Earth’s curvature

For long horizontal propagation paths, the curvature of the earth cannot be neglected when computing the trajectory of the ray, since the refractive index profile is primarily dependent on the ray’s altitude above the earth’s surface. Figure 2.6 shows an example of how the altitude of a ray changes due to the curvature of the earth.
In the following examples the refractive index profile is strictly a function of altitude, so the gradient is always in the direction perpendicular to the earth’s surface. This means that the ray will not deviate in a side to side motion, but only up and down in a single cross sectional plane of the Earth. For this reason, a two dimensional coordinate system can be used where the surface of the earth is modeled as a circle with a radius of 6,371 km. Since the ray tracing algorithm was developed in Cartesian coordinates, the ray trajectory positions are stored in Cartesian coordinates and the height is calculated at each step in order to find the index of refraction at that point. So, at each point, the height above the earth’s surface is calculated by

\[ h = \sqrt{x^2 + y^2} - r_{\text{earth}}, \]

where \( x \), and \( y \) are the position coordinates in Cartesian coordinate system which is centered at the center of the earth, and \( r_{\text{earth}} \) is 6,371 km.
2.3.1 Optical Ray Propagation through Stratified Layers

In this section, we consider the effects of a vertically stratified temperature profile. Temperature layers create a refractive boundary which alters the trajectory of a ray passing through the layers. A cool layer of air trapped between two warm layers may act as a waveguide. Similar to a graded index fiber, the cool air has a higher index of refraction acting as the ‘core’ while the outer warm layers have a lower index. If the index gradient between the layers is strong enough, a ray can become trapped inside the layer.

To investigate this effect, a simple artificial temperature profile was used to create an atmospheric refractive index profile. The US 1976 standard temperature profile was modified by simply adding a sinusoidal term. This temperature profile is given by

\[ T = -\alpha h + T_0 + t_s \sin(h/h_p) \]  

(2.32)

where \( t_s \) is the amplitude of the temperature variation inside the layer, and \( h_p \) is the period in meters. This temperature profile represents a simplistic model, but it gives insight into how a vertically layered atmosphere may effect ray propagation. With \( t_s \) set to 0.5K and \( h_p \) set to 30 meters, the ray trajectories are shown in Figure 2.7

![Figure 2.7 Ray Trajectories through stratified layers. (a) Rays launched inside wave-guide structure (b) Rays launched beneath wave-guide layer](image)
In figure 2.7(a), ten rays were launched at different angles. The grayscale image behind the rays represents the density profile which is proportional to the index of refraction profile for the modified atmosphere. Darker areas are more dense, so the index of refraction is relatively higher in those regions. Due to the refractive index gradient, the rays begin to bend. Since they bend towards the higher refractive index in the layered atmosphere, they are guided along a constant altitude. At certain points along the path, the rays are focused to a small region where the ray density is high while at other points along the path the ray density is low. The intensity of the light measured by a receiver would greatly vary depending on the location of the receiver along the optical axis. In figure 2.7(b), the rays are launched below the waveguide structure. In this scenario, some of the rays are guided by the higher density layer, while the lower rays are not trapped. These rays refract below the layer and get separated from the wave-guided rays.

Another effect of the higher density layer is that aiming rays to a target on the other side of the dense air pocket becomes difficult. With a normal atmosphere, all rays propagate in predictable straight lines with only slight curvature due to the index of refraction gradient of the standard atmosphere. When a denser air layer is added, rays become trapped in the higher index of refraction layer and do not follow their expected trajectories. Figure 2.8 illustrates this scenario.
In figure 2.8(b), it can be seen that all the rays miss below the target point. The highest, middle and lowest rays are shown in color to more clearly track how the ray bundle is mixed up. It might make sense to then aim the bundle of rays to a higher angle to hit the target, but upon closer inspection of the figure it can be seen that the rays which were initially aimed higher actually missed below the target by the greatest amount. These rays were trapped by the upper boundary of the waveguide but escaped out of the lower boundary. This simple example illustrates how refractive layers can greatly alter the trajectory of rays and cause confusion as to aiming and alignment of sources and detectors.

2.3.2 Mirage Formation

Another example of atmospheric refraction is the formation of mirages. A mirage is a false image created by bended rays due to atmospheric refraction. The most commonly seen mirages are called inferior mirages because the mirage image is located below the actual object. These mirages are typically formed near the Earth’s surface due to a localized layer hot air at ground boundary layer. Direct sunlight heats the ground much faster than the air surrounding it. The air directly in contact with the ground is much hotter than the air a few feet above the ground and because of this, the index of refraction is lower closest to the ground. This causes rays at
nearly grazing angles to bend back upward. These rays then appear to be coming from below the horizon and may look like a lake or stream. Figure 2.9(a) shows an image of a mirage and 2.9(b) gives a ray diagram explaining this phenomenon.

![Image of inferior mirage over desert (a) and ray illustration of mirage formation (b)](image)

*Figure 2.9 Image of inferior mirage over desert (a) and ray illustration of mirage formation (b)*

In order to simulate this effect, the US 1976 atmosphere was considered in the ray tracing calculations, but a 5 degree spike in temperature was added at a layer near to the Earth’s surface. A group of rays was then launched from a point with some of the rays grazing the surface. Figure 2.10 shows the results of this simulation as well as the index of refraction profile used.
The rays which are incident at small angles with respect to the earth’s surface are refracted upwards due to the strong reverse index of refraction gradient. Since the index of refraction anomaly is only located near the surface of the earth, other rays above this point are not affected by the change and they propagate as would normally be expected. Within the region where the two groups or rays intersect, a mirage of the point would be seen by the viewer. The actual image would appear to come directly from the point where the rays originate, and the mirage image would appear to come from a point below the earth’s surface where the refracted rays appear to come from.

### 2.3.3 Analysis of the Coherent Optical Multi-Beam Atmospheric Transmission (COMBAT) Experiment Using Ray Tracing Technique

Another situation in which refraction of optical waves plays an important role is propagation of laser beams over very long paths (>100km). For these paths, refraction with the standard

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*Figure 2. 10 Simulation of inferior mirage using ray tracing (left) and index of refraction altitude profile used in simulations (right)*
atmosphere is enough to cause a significant deviation of the beam from the geometric path. Experimental data for such a long path was recorded in the recently done COMBAT test.

**Description of COMBAT Experiment**

In 2010, a long range laser beam propagation experiment was performed over a 149.2 km path between two Hawaiian Islands; Mauna Loa and Haleakala [9]. A laser beacon was aligned to illuminate a 3.6 meter telescope which was used as a receiver. The beacon was located at an altitude of 3397m on Haleakala, and the receiver telescope was located at an altitude of 3058m on Mauna Loa. Figure 2.11 shows the layout of the experiment with the curvature of the Earth included. [9]

![Figure 2.11 Elevation profile along the propagation path from the Mauna Loa NOAA observatory to the AEOS telescope on Haleakala](image)

The lowest point on the geometric projection between the transmitter and receiver is located towards the middle of the path due to the curvature of the earth.
The laser beacon was comprised of three single mode fiber collimators with wavelengths of 1.55\(\mu\)m, 1.064 \(\mu\)m, and 0.532 \(\mu\)m. The three fiber collimators were attached to a gimbal and aimed at the receiver.

At the receiver end, the three beams are collected by a 3.6 meter telescope simultaneously and directed to a beam divider. From there, the three individual beams are filtered using narrow pass filters, and illuminate CCD sensor. Figure 2.12 shows a schematic of the transmitter-receiver set up. [9]

![Figure 2.12 COMBAT Experimental Set Up](image)

**Shifted coordinate system for viewing ray tracing plots**

Before presenting the ray tracing trajectories for the COMBAT simulation, a brief description of the coordinate axis is addressed.
The Earth centered global coordinate system is not always the best system for plotting the trajectories of rays because large difference in scales which must be viewed. In a Cartesian system, the coordinate axes are on the scale of km due to the length of the propagation path, and the change in altitude from transmitter to receiver. In this reference frame it is impossible to see the changes in trajectory due to refraction which are on the order of 10’s of meters. It is more understandable to plot the deviation of the ray relative to some reference such as the geometric path between two points. For the 150 km path between two Hawaiian Islands, the deviation of the ray trajectory caused by atmospheric refraction is most easily visualized in reference to the deviation from the geometric path. In order to do this, the horizontal coordinate axis was oriented along the geometric path. This corresponds to a coordinate transformation where the origin of the new coordinates is the ray launch point, and the axes are rotated so that the new $x$ axis is along the line connecting the start and end point as shown in Figure 2.13.

![Figure 2.13 Illustration of the coordinate transformation used for ray tracing calculations.](image-url)
The new coordinate system $x'$ and $y'$ is defined by the following equations

\[ x' = x \cos(\varphi) + (y - r_{\text{earth}}) \sin(\varphi) \]  \hspace{1cm} (2.33)  

\[ y' = -x \sin(\varphi) + (y - r_{\text{earth}}) \cos(\varphi) \]  \hspace{1cm} (2.34)

This coordinate system allows better visualization of the atmospheric refraction effects.

**Ray Trajectories for Optical waves with Different Wavelengths**

The ray tracing method was used to find the ray trajectory for the long range propagation path between two Hawaiian Islands. The ray starts at Mauna Loa with an altitude of 3397 m and ends at Haleakala at an altitude of 3058 m. As in the COMBAT experiment, three beams with wavelengths of 0.532 μm, 1.064 μm, and 1.55 μm were considered for ray tracing. Figure 2.14 shows the simulation geometry before using the shifted coordinate system equation (2.33), (2.34) which has its origin at sea level on the transmitter side. The two islands are simply represented as two triangles. The curvature of the earth causes the ray to pass through its lowest altitude in the middle of the propagation path.
When a ray is propagated through the atmosphere with a given refractive index profile, the gradient of the refractive index results in the beam to bend towards the denser air which has a higher index of refraction. The curvature causes the ray to deviate from the geometric path between the two points, and miss its target. In order for the ray to reach its predefined end point, the laser beam must be aimed slightly above the target to correct for atmospheric refraction. The ray deviation from the geometric path is shown in Figure 2.15. The coordinate system in this plot is the same as illustrated in Figure 2.13.
In figure 2.15, four rays with different wavelengths were launched from the transmitter at the same angle such that the ray corresponding to $\lambda=0.532\mu m$ is properly aimed on target whereas the other rays miss the target due to refractive effects. All rays deviate from the geometric path by $\sim 60$ m in the middle of the propagation path. This refraction effect is not accounted for in traditional wave optics modeling approaches. In the standard wave optics simulations, the optical axis is taken to be along the geometric path, and the wavelength difference induced separation of optical beam trajectories due to refraction is not considered.

Another important feature exhibited by this simulation is the separation of ray trajectories for optical waves with different wavelengths – the effect that can be associated with atmospheric dispersion. The best way to visualize the difference in trajectories for optical waves with different wavelengths is to plot the ray separation along the propagation path with respect to a reference ray. Any of the rays can be used as a reference. Figure 2.16 shows the ray deviations from the reference ray corresponding optical wave with $\lambda=0.532 \ \mu m$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2_15}
\caption{Ray height above geometric path between two islands for three wavelengths}
\end{figure}
Figure 2. 16 Ray deviations from the reference ray corresponding to 532nm wavelength

In this figure it is seen that the separation between the rays increases as the propagation path increases. For the 150 km path simulated, the rays are misaligned by about 4 meters with respect to the $\lambda=0.532 \, \mu\text{m}$ ray. This beam separation distance is significant because it is on the order of the diffraction-limited beam footprint. If the beams do not overlap at the receiver, then their characteristics cannot be simultaneously measured by the same telescope.

2.4 Limitations of Ray Tracing Approach

Since the ray tracing method represents an approximation of the Helmholtz equation without accounting for diffraction effects, ray tracing does not explicitly yield information about such important characteristics of optical waves as phase and intensity distributions. Also, in ray
tracing, the energy in a ray does not dissipate. For these reasons, ray tracing is not adequate for analysis of laser beam propagation. A thorough analysis of beam propagation must include impact of turbulence and diffraction effects.
3.1 Atmospheric Turbulence

Atmospheric turbulence causes random index of refraction fluctuations because of variations of temperature and pressure which arise from turbulent air flow. Turbulent air flow is due to differential heating by sunlight on the earth’s surface. Thermal convection results in large scale temperature fluctuations as the hotter less dense air rises and cooler air takes its place. Another cause of turbulent mixing of air is wind shear due to drag on air moving close to the earth’s surface. The different wind speeds and directions cause mixing and turbulent flow.

There is much interest in understanding and modeling atmospheric turbulence because of its effect on optical wave propagation. Astronomers and engineers want to understand how an optical system will perform in atmospheric conditions in order to optimize their designs. The foundation of the modern statistical representation of the atmospheric index of refraction came from Kolmogorov’s study of turbulent flow. The theory has proved to agree with experimental data under proper conditions. [9]

The refractive index of air in the earth’s atmosphere is a non-uniform function which can be given in the form

\[ n(r, t) = n_0 + n_t(r, t), \]  

(3.1)
where \( n_0 \) is the average index of refraction, and \( n_f \) is a small deviation from the average caused by turbulence. Kolmogorov used the structure function, \( D_n(r_1, r_2) \), to describe the expected variance in the refractive index between two points given by [2]

\[
D_n(r_1, r_2) = \langle [n(r_1) - n(r_2)]^2 \rangle,
\]

where \( r \) is the spatial coordinate. Over some inertial range, this function is assumed to be isotropic and is given by

\[
D_n(r) = C_n r^{2/3},
\]

where \( r \) is the scalar distance between two points, and \( C_n^2 \) is the refractive index structure parameter. [10] The inertial range where Equation (3.3) is valid is bounded by the inner scale \( l_0 \), and the outer scale \( L_0 \), which are physically the average size of the largest eddies and smallest eddies in the turbulent flow. For scales larger than \( L_0 \), the flow is anisotropic and the structure function does not obey the \( 2/3 \) power proportionality. The spectral representation of the structure function equation is given by

\[
\Phi^K_n(\kappa) = 0.033C_n^2\kappa^{-11/3},
\]

where \( \kappa \) is the spatial frequency in radians per meter.

**Optical Parameters of the Atmosphere**

Since \( C_n^2 \) characterizes only local turbulence, it is not always the best descriptor of turbulence strength for a particular path. \( C_n^2 \) may change over a long path length, and also has an altitude dependence. For this reason, other parameters are more conventional for describing turbulence strength over an optical path. One such parameter is called the Fried parameter or atmospheric coherence length. This parameter is found using the integration of \( C_n^2 \) along the path, and is
therefore a better indicator of turbulence strength for a particular path. The Fried parameter is given by

\[ r_0 = \left( \frac{0.423 k_0}{L} \int_0^L C_n^2(h) \, dz \right)^{-3/5}, \quad (3.5) \]

where \( k_0 \) is the optical wavenumber, \( z \) is the length along the path, and \( h \) is the altitude [11]. The Fried parameter is really describing the aperture diameter for which the wave-front distortions are uniform tilts. An aperture which is larger than this will not provide any more resolution because the atmospheric turbulence effect is limiting the resolution[1]. Therefore, a small \( r_0 \) value indicates strong turbulence and poor resolution for imaging telescopes, and a large \( r_0 \) value means weaker turbulence.

Many different atmospheric turbulence profiles have been developed for simulation of vertical and slant propagation paths. These models represent average turbulent conditions and do not have much flexibility for including local conditions. One of the most popular is the Hufnagel Valley model which is given by [1]

\[ C_n^2(h) = 5.94 \times 10^{-23} h^{16} e^{-h} \left( \frac{W}{27} \right)^2 + 2.7 \times 10^{-16} e^{-2h/3} + Ae^{-10h} \quad (3.6) \]

where \( h \) is the altitude, and \( W \) and \( A \) are adjustable parameters. Standard values of these parameters are \( A = 1.7 \times 10^{-14} \) and \( W = 21 \), which lead to the HV5/7-model turbulence profile. Figure 3.1 shows a plot of the HV5/7 model \( C_n^2 \) altitude profile.
3.2 Optical Wave Propagation.

3.2.1 Helmholtz Equation

Optical wave propagation can be described by the wave equation given by,

\[ \nabla^2 u = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} \]  

where \( u \) is the wavefunction which is a function of both time and space and \( c \) is the speed of light in a vacuum [1]. For a monochromatic wave, the solution can be represented in the form

\[ u(r, t) = U_0(r)e^{-j\omega t} \]  

where \( \omega \) is the angular frequency and \( U_0 \) is the complex amplitude of the wave. By separating the temporal and spatial components, equation (3.7) can be simplified by substituting equation (3.8) for \( u(r, t) \). The result of this substitution is the time-independent wave equation called the Helmholtz equation given by
where $k$ is the wavenumber [1].

### 3.2.2 Parabolic Equation

Equation 3.9 can be further simplified by making some approximations based on typical beam propagation geometry. Since the longitudinal propagation distance is much longer than the transverse spreading of the beam, paraxial approximation is valid.

Writing $n(r)=n_0+n_1(r)$, where $n_1$ is the medium refractive index and $n_0=1$, the parabolic equation is given by

$$2ik \frac{\partial U(r,z)}{\partial z} + \nabla_T^2 U(r,z) + 2k^2 n_1(r,z) U(r,z) = 0$$

where $U$ is the complex amplitude of the optical wave, $\nabla_T^2$ is the Laplacian operator in the transverse direction, and $k$ is the wavenumber. This equation describes the complex amplitude in a medium with refractive index fluctuations $n_1(r,z)$.

A common way to solve this equation numerically is to use the split-operator (split-step) technique[3]. This method separates free space diffraction of the optical wave and the turbulence impact. In atmospheric simulations this means alternating between free-space propagation and propagation through a thin phase screen that represents impact of refractive index fluctuations inside a relatively short length volume layer (slab). Figure 3.2 shows the thin phase screen representation of the atmospheric turbulence volume.
3.2.3 Split Step Method: Free-Space Propagation

Mathematically, free space propagation between thin phase screens in the split-operator method is described by the parabolic equation (Equation 3.10) with \( n_1 = 0 \):

\[
2ik \frac{\partial U(r,z)}{\partial z} + \nabla_r^2 U(r,z) = 0
\]

(3.11)

The solution to this equation can be expressed through the Fresnel integral [12]:

\[
U(x_2, y_2, z) = \frac{e^{jkz}}{j\lambda z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(x_1, y_1) e^{j\frac{k}{2z}[(x_2-x_1)^2+(y_2-y_1)^2]} \, dx_1 \, dy_1
\]

(3.12)

where \( U(x_1, y_1) \) is the complex amplitude in the source plane \((x_1, y_1)\). If the quadratic exponential term is expanded, we get another form of the Fresnel integral:

\[
U(x_2, y_2) = \frac{e^{jkz}}{j\lambda z} e^{jkz} \left( e^{j\frac{k}{2z}(x_2^2+y_2^2)} \right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(x_1, y_1) e^{j\frac{k}{2z}(x_1^2+y_1^2)} e^{-j\frac{2\pi}{\lambda z}(x_1x_2+y_1y_2)} \, dx_1 \, dy_1
\]

(3.13)

This integral can easily be evaluated numerically using fast Fourier transform.

3.2.4 Split Step Method: Refraction

As stated earlier, the parabolic equation can be solved using the split step method where free space diffraction is performed using the Fresnel integral and impact of refraction effects is
described by an additional phase term added to the wave-front phase. Propagation through sequential slabs of volume atmosphere is computed in the split-operator method consecutively by the following equation in operator notation

\[
U(\Delta z, r) = \hat{D}(\Delta z / 2) \hat{R}(\Delta z) \hat{D}(\Delta z / 2) U_0(r),
\]

(3.14)

where \( D \) represents an operator given by equation (3.16), and operator \( R \) represents impact to the field complex amplitude by thin phase screens, and \( \Delta z \) is the length of the subsection (slab) of the path. This procedure is repeated for the entire propagation length. At each step, \( R \) represents a change in the field \( U_0 \) given by

\[
U(z + \Delta z, r) = U_0(z, r) \exp \left( -jk \int_{z}^{z+\Delta z} n(z) dz \right),
\]

(3.15)

where the phase term represents turbulence effects. To actually create phase screens, the spectral approach is used in conjunction with Kolmogorov turbulence theory. The correlation function for the phase in the transverse plane over a distance \( \Delta z \) is given by

\[
B_S(\rho, \Delta z) = \int_{-\infty}^{\infty} \int \Phi_s(\kappa_\perp, \Delta z) e^{i\kappa_\parallel \rho} d\kappa_\perp = 2\pi k^2 \Delta z \int_{-\infty}^{\infty} \Phi_n(\kappa_\perp, 0) e^{i\kappa_\parallel \rho} d\kappa_\perp
\]

(3.16)

where \( \Phi_s \) is the phase spectrum, and \( \Phi_n \) is the refractive index fluctuation spectrum. Now, pseudo-random phase screens must be created which satisfy equation (3.19).
CHAPTER 4

MERGING RAY TRACING AND WAVE OPTICS APPROACHES

4.1 Basic Ideas of Merging Ray Tracing and Wave Optics Approaches

Ray tracing simulations have shown that for long propagation paths and for atmospheric conditions exhibiting strong refractive index gradients, the trajectory of an optical beam can significantly deviate from the geometric straight line path between the transmitter and receiver. Ray tracing allows us to find an average beam trajectory that can be associated with coordinates of beam centroid for propagation in a medium with smooth refractive index variations which scale significantly exceeds laser beam localization area (laser beam footprint). The ray tracing approach cannot be directly applied to propagation in turbulence. Contrarily, the described wave optics technique based on split-operator method describes impact of small scale refractive index inhomogeneities on optical wave propagation, but does not take into account large-scale refractive structures which affect the trajectory of the beam. Both approaches yield important information about the atmospheric effects on optical wave propagation, so they should be considered simultaneously.

Refraction on large-scale refractive index inhomogenities and turbulence-induced effects are not independent of one another and should be taken into account jointly. The turbulence
strength depends on the altitude, and hence on the beam centroid coordinates, which are calculated using ray tracing. Thus, ray tracing calculations are important for correct accounting for turbulence effects. On the other hand, turbulent layers introduce random wave-front tip and tilts which affect the beam centroid trajectory and should be accounted in ray tracing simulations.

Here we introduce an approach that allows joint consideration of optical wave refraction and turbulence-induced effects. This approach is illustrated in Figure 4.1 as a diagram of sequential iterative process of accounting for either atmospheric turbulence or refraction effects. In the following section, this approach is described in more details.

![Conceptual diagram describing fusion of ray tracing and wave optics](image)

**Figure 4.1 Conceptual diagram describing fusion of ray tracing and wave optics**

The method is based on sequential application of split-operator and ray tracing techniques. The propagation medium is subdivided into a number of slabs and turbulence effects are accounted for only at slabs boundaries using thin phase screens. Propagation between phase screens is described in the framework of ray tracing that accounts for the impact of large-scale refractive index variations but not for turbulence. At the same time, the impact of turbulence effects on ray tracing...
is accounted by computing the turbulence-induced wave-front tip and tilts and shift of beam centroid coordinates at the exit plane of each slab. As a result the ray tracing inside the next slab starts with the “corrected” by turbulence beam centroid coordinates and ray angle as shown in Figure 4.2.

![Figure 4.2 Illustration of ray tracing correction at each phase screen](image)

Propagation of ray inside atmospheric slab results in change of beam centroid elevation that in its turn impacts the turbulence strength. Thus the new phase screen generation depends on results of ray tracing. The described technique of sequential accounting for turbulence and refractive effects can be referred to as wave-optics-ray-tracing (WORT) iterative algorithm

**Wave-Optics Ray-Tracing (WORT) Iterative Technique**

In WORT the simulation begins with the choice of a beam intensity profile, altitude and propagation direction. First, a ray is started from the initial location and propagates through the first atmospheric slab until it reaches the location of the first phase screen. The turbulence strength is calculated using the end coordinates of the ray trajectory, and a phase screen is generated accordingly. After passing the screen, the wave-front tilt and centroid shift is calculated and used to update the ray position and angle. The updated ray then propagates through the next atmospheric slab to the next screen, and the entire process is repeated until the beam reaches the receiver plane.
The fusion of the two simulation approaches in WORT comes from the creation of the phase screens that are dependent on turbulence impact, and the updating of the ray position and angle dependent on impact of turbulent phase screens at the atmospheric slab boundaries. The following paragraphs will cover the new functionality in detail starting with the process of the phase screen generation and updating the ray parameters.

**Phase screen generation in WORT**

At each step of the WORT split step propagation method, a single thin phase screen is generated to represent the volume turbulence for a particular air volume along the propagation path. The strength of turbulence depends on the trajectory of the beam centroid. Since there is an altitude dependence on the $C_n^2(h)$, the trajectory of the beam affects the strength of the turbulence. Ray tracing can be used to estimate $C_n^2(h)$ at the end of each atmospheric slab and generate the corresponding turbulent phase screen.

**Ray Tracing in WORT**

Correction in ray launching angle at the atmospheric slab boundary in WORT is based on computation of laser beam wave-front tip and tilt right after phase screen. Since wave-front phase can contain $2\pi$-phase cuts and branch points, calculation of wave-front tip and tilt directly from phase function (as phase function first moments) can lead to significant errors. In WORT the tip and tilt estimation is performed using an auxiliary (virtual) lens with infinite aperture and focal distance $F$, which is placed behind phase screen. The presence of wave-front tip and tilt phase aberrations results in shift of focal spot centroid. With knowledge of the lens focal distance $F$ the tip and tilt can be calculated by computing the centroid shift distance in the focal plane of a lens:
\[ x^F_c = F \tan(\theta_x) \quad \text{and} \quad y^F_c = F \tan(\theta_y), \] where \( x^F_c, y^F_c \) are the centroid coordinates, \( F \) is the focal distance of the lens, and \( \theta_x, \theta_y \) are the tip and tilt angles. These parameters are shown in figure 4.3.

\[ r_n = r_{n-1} + c \] (4.2)

\[ \theta^x_n = \theta^x_{n-1} + \theta_x, \quad \text{and} \quad \theta^y_n = \theta^y_{n-1} + \theta_y \] (4.3)

where \( c \) is the turbulence-induced centroid shift and \( \theta_x, \theta_y \) are the tip and tilt induced deviations in ray launching angle.
The described WORT technique is illustrated in Figure 4.4.

\[ r_n = r_{n-1} + c \]

\[ \theta_n^x = \theta_{n-1}^x + \theta_x \]

**Figure 4. 4 WORT Technique**
CHAPTER 5

LASER BEACON ANALYSIS USING WORT TECHNIQUE

This chapter presents some examples of laser beam propagation analysis using WORT simulation technique.

5.1 Simulation Parameters

As in already discussed COMBAT wave propagation geometry, a 150 km path between two Hawaiian Islands is investigated. At the transmitter, the beam is given as a collimated Gaussian beam with radius 10 cm and a wavelength of 1.064µm. The numerical grid is 2048 x 2048 pixels with an entire grid size of 5.0 m. The propagation path is divided into 80 slabs with one phase screen for each section. The phase screens are Kolmogorov spectrum generated using the HV57 Cn2 profile. The index of refraction profile used is the US 1976 standard atmosphere with sea level temperature set to the global average of 288.15 K.

5.2 WORT Simulation Results

First the results from the ray tracing output are presented. Since there is a random component of the tilt angle at each phase screen, the ray trajectory through the US 1976 atmosphere is no longer a constant-length path. The ray trajectory which is computed using the WORT technique deviates from the trajectory computed with conventional ray tracing method. Figure 5.1 shows an exaggerated picture of the turbulence influenced ray deviating from the reference ray. The
The reference ray trajectory is simply computed using standard ray tracing with no turbulence induced tilt correction.

The deviation from the reference ray can be plotted along the propagation path $z$. Figure 5.2 shows this deviation for several realizations of ray tracing with turbulence induced tilts for a selected sets of turbulent screens.

*Figure 5.1 Reference ray and deviated ray*
Figure 5.2 Ten realizations of ray deviations from reference ray

The rays all start at the exact same point, but as they pass through the turbulence phase screens, they begin to randomly deviate from the reference ray because of the turbulence-induced tilt phase components impact ray trajectory. Over the propagation distance, the deviation tends to accumulate so that at the end of the propagation path, there is the greatest uncertainty at the final ray destination point. Figure 5.3 shows the average and standard deviation of the ray deviation distance.
On average, the rays end up at the same target point as the reference ray, but the uncertainty increasingly grows along the propagation path. At the end of the 150 km path, the standard deviation is +/- about 2 meters. The small tilt additions to the ray end up being significant after propagation over a long distance. Turbulence effects are not insignificant when using ray tracing for simulation of refraction.

Conclusions

Both atmospheric refraction and turbulence effects are important for simulations of optical wave propagation especially over long paths. Traditional methods based on Kolmogorov theory do not accurately predict the observed experimental results for such long paths. Ray tracing simulations presented have shown that atmospheric refraction has a great impact on the trajectory of the beam.
It has also been shown that ray tracing should not be considered independent of atmospheric turbulence properties. Wave-front tilt caused by turbulence has a significant impact on the trajectory of rays. Small tilt angles propagated over a long distance can change the end aim point by several meters. Turbulence causes an uncertainty in the trajectory computed with ray tracing alone. The results presented show that both atmospheric refraction and turbulence effects need to be accounted for in the simulation of long range laser beam propagation. The WORT technique allows for joint analysis of both effects.
REFERENCES


