IDEAL PROCESS DESIGN APPROACH FOR HOT METAL WORKING

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IDEAL PROCESS DESIGN APPROACH FOR HOT METAL WORKING

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ABSTRACT

IDEAL PROCESS DESIGN APPROACH FOR HOT METAL WORKING

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The overall objective for this research is to develop an ideal process design approach for designing robust hot metal working process. This ideal process design approach is based on literature because of the limited capability on experimental conditions. In this research, a hot metal process consists of material system, process deformation system, and equipment system. Material system consists of workpiece material and its behavior under processing conditions. Deformation process system consists of tooling, preform geometry, and lubrication. Equipment system consists of metalworking machine and furnace including controls for temperature, ram speed, and applied force. Several exiting analyses methods are used to determine ideal process conditions for each system.

The Dynamic material model approach is used to determine a window of process conditions for ideal material system behavior and ideal forming theory with methods of analyses are used to calculate the ideal uniform deformation conditions for deformation process system. Moreover, a robust open-loop control system for metalworking machine
is designed by effective model-based approach. The relationships between each system are represented by some valid empirical constitutive equations. In this research, an axiomatic design methodology is also used to analysis the efficiency of the overall ideal process design approach. This ideal process design approach is illustrated by a steel hot rolling process case study.
ACKNOWLEDGEMENTS

I would like to first thank God who has provided me with the necessary tools and circumstances I needed to succeed.

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NOMENCLATURES

FRs............................................................................................ Functional requirements

DPs.................................................................................................... Design parameters

PVs.................................................................................................... Process variables

EPs ................................................................................................. Equipment parameters

[A]........................................................................................................ Design matrix

[B]........................................................................................................ Design matrix

[C]........................................................................................................ Design matrix

\(m\)................................................................................................. Strain rate sensitivity parameter

\(s\) .................................................................................................... Temperature sensitivity parameter

\(\sigma\) .................................................................................................. Flow Stress (MPa)

\(\sigma_s\) ............................................................................................ Steady flow Stress (MPa)

\(\varepsilon\) ................................................................................................ Plastic strain

\(\dot{\varepsilon}\) .......................................................................................... Strain rate (s\(^{-1}\))

\(T\) ................................................................................................. Temperature (K)
$Q$ ................................................................. Apparent activation energy (KJ/mol)

$Q_s$ ............................................. Activation energy for static recrystallization (KJ/mol)

$Q_{gg}$ ............................................. Activation energy for grain growth (KJ/mol)

$Q$ ................................................................. Activation energy (KJ/mol)

$Q^*$ ................................................................. Desired activation energy (KJ/mol)

$R$ ................................................................. Universal gas constant, 8.314 J/(g·mol·K)

$t$ ................................................................. Operation time (s)

$t_{0.5}$ ............................................. Required time for 50 volume percent of the static recrystallization (s)

$t_s$ ................................................................. Dwell time (s)

$\eta$ ............................................. Efficiency of power dissipation respect to metallurgical processes (%)

$d(j)$ ................................................................. Final grain size ($\mu$m)

$d(j-1)$ ................................................................. Initial grain size ($\mu$m)

$ds$ ................................................................. Sub grain size ($\mu$m)

$d_{dyn}$ ................................................................. Dynamic recrystallized grain size ($\mu$m)

$d_{stat}$ ................................................................. Static recrystallized grain size ($\mu$m)

$X_{rxn}$ ................................................................. Volume fraction of dynamic recrystallized grain (%)

$X_{stat}$ ................................................................. Volume fraction of static recrystallized grain (%)

$\varepsilon_c$ ................................................................. Critical strain

$\varepsilon_{0.5}$ ................................................................. Plastic strain for 50 volume percent recrystallization
$T_W$ .............................................................. Working temperature (K)

$T_B$ .............................................................. Billet temperature (K)

$H_0$ .............................................................. Initial sheet height (m)

$r$ .............................................................. Percent reduction in height

$V$ .............................................................. Roll velocity (m/s)

$R$ .............................................................. Roll radius (m)

$Z$ .............................................................. Zener Holloman parameter (s$^{-1}$)
CHAPTER I

INTRODUCTION

Manufacturing processes such as rolling, forging and heat treatment have been developed for a long time to manufacture metallic products. These processes transform workpiece material into a desired geometry shape. Meanwhile, properties of products depend upon the entire manufacturing procedure. Hence, the knowledge of material behavior under processing conditions is significant for designing a robust process with a high production rate and optimized properties. In addition to pressures from the marketplace, the complexity of process design relationships among material behavior, process mechanics, and dynamic equipment characteristics create major challenges for process designers.

1.1 Challenges of Process Design

Today, process designers face major challenges from their customers, such as faster delivery, stringent cost requirements and complex geometrical shapes. Some specific problems that metalworking companies experienced such as:

1. Eliminating internal defects in products which are caused by poor material behavior and non-uniform deformation.
2. Eliminating surface defects of products caused by poor lubrication, die design and temperature control.

3. Utilizing full capabilities of metalworking equipment.

4. Eliminating the costly trial-and-error approach to die design.

Existing process design methods can be improved to more explicitly consider material behavior and optimize the effects of primary process parameters such as workpiece temperature and deformation rates on the final product properties. For example, computer-aid process design has been established for decades. This approach has been extensively used for evaluating process design variables by using computer simulation software before setting up a real process. This approach could significantly reduce the cost of process design. However, this approach still relies on the process design database which built on prior experience. Mechanical properties of product depend upon microstructure, defects, and finished part shape which are achieved by manufacturing process. Thus, material properties can be controlled by the process parameters. However, computer based process design approach lack specific relationships between process parameters and material characteristics. This situation hinders enhancement of product quality and design of new product. For instance, process engineers can not apply existing computer process design database for effectively designing products with new geometrical shape and precisely controlled microstructure.
1.2 Design Process Relationships

Basically, the purpose of design is to determine set of specified inputs that produce the desired outputs. According to Suh,⁹,¹⁰ these inputs and outputs can be defined in terms of functional requirements (FRs), design parameters (DPs) and constraints. Suh’s design world consists of consumer domain, functional domain, physical domain, and process domain. The mapping between the consumer domain and functional domain is considered as product design. Similarly, the mapping between the physical domain and process domain is considered as process design. Figure.1 shows the framework of Suh’s design world. The typical design process starts with customer needs for a product. Designer needs to consider these desired features and define them in terms of specific requirements in the functional domain.

![Figure.1 Suh’s design world](image)

Then, a set of DPs must be created to satisfy the FRs in the physical domain. Furthermore, to satisfy the DPs, a manufacturing process can be established in process domain. In a typical material process design, mechanical properties, microstructures and deformation
mechanics need be considered. According to Suh’s method, mechanical properties are considered to be a set of FRs in the functional domain. For example, these mechanical properties can be strength, modulus and hardness. From material science, the mechanical properties are based upon the material microstructure and defects. Thus, microstructures can be assumed to be a set of DPs in the physical domain. To achieve the required microstructures, we need to adjust the process variables (PVs) such as strain, strain rate, temperature and dwell time in the processing domain. Figure.2 shows the design process for a deformation process.

Figure.2 Design process for a hot deformation process

Indeed, the relationships among material behavior, process mechanics and dynamic equipment characteristics are complex. Material behavior under hot working conditions is generally nonlinear, stochastic, and dynamic with respect to flow stress, workability, and microstructure development. Process mechanics for forging, rolling and extrusion involves nonlinear changes in workpiece geometry and complex stress states. Metalworking equipment possesses specific dynamic characteristics and limitations.
Clear understanding these relationships between these domains is significant to design a reliable metallurgical process.

1.3 Objective

In order to design a robust manufacturing process with a product rate and desired quality, the relative process parameters must be studied. Since computer has been invented, several computer-aid process design methodologies have been established in past decades. In the meantime, more empirical data has become available, a large databases linking material properties and process parameters have been accumulated to expand selection of process to obtain desired product. However, most existing process design models are based upon empirical data and evaluated by computer. Thus, those models only can be employed for a specific process with a certain material. Therefore, it is necessary to develop a new process design methodology based on material scientific principles which can clearly evaluate material behavior under the manufacturing process. Such a method would give highly accurate microstructure control and eliminate the iteration work for the process design.

The specific objectives of the research described herein are as follows:

1. Explore an ideal design approach that more explicitly considers the relationships among process parameters, material microstructure and material properties.
2. Design a robust steel hot rolling process through this overall ideal process design approach.
CHAPTER II

APPROACH

2.1 Overall Systems Approach for Ideal Process Design

A typical manufacturing process involves a series of thermomechanical operations. For a certain thermomechanical operation, it consists of several aspects, including equipment, material, and deformation systems. With existing complex process design relationships, a new systems approach is developed in this work to explore an ideal

Figure 3 Schematic representation of “Ideal” process design as system consisted of material, deformation and equipment
process design approach that aimed at reducing complexity and producing desired microstructures on a repeatable basis. The representation of the new systems approach for the “ideal” process design is shown in Figure.3. Material system consists of workpiece material and its behaviour under processing conditions. Deformation process system consists of tooling, preform geometry, and lubrication. Equipment system consists of metalworking machine and furnace including controls for temperature, ram speed, and applied force.

2.2 Ideal Material System Behavior

For ideal process design, the material system behavior would be predictable and controllable. Dynamic material model approach\(^\text{12}\) is used to determine a window of process conditions for controlling microstructure and avoiding manufacturing defects.

Gegel\(^\text{12,13}\) and Prasad\(^\text{12}\) developed a methodology called “dynamic material modeling” (DMM). This method plays a key role in unifying the relationship among constitutive equations, hot workability and microstructure development. DMM was developed in response to the need for a macroscopic description of flow, fracture and workability of complex engineering materials under hot deformation conditions.\(^\text{13}\) Base on the DMM map, Malas\(^\text{14,15}\) introduced stability theory into the dynamic material modeling to give a stable process range. According to the stability criteria, if the working condition can satisfy the stable condition, the material system can reach a steady state which consume lower energy and yield higher efficiency without fracture. Through this
DMM approach, a robust process design window for an ideal range of strain rates and temperatures can be determined: 14,15

- Stability Analyses for determining safe processing condition
- Activation energy analyses for determining where deformation mechanisms are operative and dominant
- Sensitivity analyses for determining desired processing range for deterministic and robust microstructure control

Figure 4 Schematic diagram of hot working process map. Desired activation energy is identified as Q*. Shaded region satisfies all stability criteria (From ref.15)

As an example shown schematically in Figure 4, the processing window of a given metallic system is defined as the domain of processing conditions under which material behavior is stable and a desirable and relatively constant value of apparent activation energy exists.
2.3 Ideal Deformation Process System Behavior

For ideal process design, the deformation flow behavior would be more uniform, and predictable. The simplifying assumptions given in analytical process models are used to determine the geometry of workpiece, tooling and their boundary conditions for more fundamental process design solutions that idealize the deformation mechanics.

Richmond\textsuperscript{17,18} et al. developed “Ideal Forming” methodology. According to this work, the minimum work path is considered to be an ideal forming path if the final product can be achieved by homogeneous deformation. To simplify the ideal process design, some simplifying assumptions for deformation process have been used. Some of these assumptions listed as below:\textsuperscript{24}

- Lower degrees of freedom for material flow are more predictable and controllable.
- Strain or stress state is well defined (e.g. planer, axisymmetric).
- The working piece material is a homogeneous, isotropic continuum.
- The onset of yielding in tension and compression is identical.
- The material volume remains constant for plastic deformation
Figure 5 (a) Simple shear deformation. (b) Comparison of effective strains between the minimum work path and simple shear. (From ref. 19)

Figure 5 shows a simple shear deformation for isotropic material. The material points move in a straight line from initial position to final position. Because of a continuous principal material lines change during simple shear deformation, straight movements of material points to their end positions do not generally represent a minimum work path. The minimum work path for homogenous deformation is achieved when the principal material lines are kept fixed during deformation and the ratio of principal true strain rates is constant.

2.4 Ideal Equipment System Behavior

For ideal process design, the control system of the metal forming equipment would provide the environmental and dynamic conditions for reliably producing high quality products with controlled microstructures and properties. An effective model-
based approach for designing an open loop control system would avoid robustness issues of system with minimal output measurement capabilities.

In this research, the hot working system was controlled by an open loop control system. This open loop system uses thermodynamic kinetic models of metallurgical mechanisms to determine the optimum deformation path for achieving the desired microstructure and a process model to determine the necessary process variables for achieving near ideal conditions throughout the workpiece. The required process parameters are set up to regulate the metal working equipment to yield the final product. Feedback improvements between the metalworking equipment and the process model are not feasible for control system because the microstructure and internal defects cannot be measured in real time. If the process is set up a safe DMM region where the process is robust naturally, the microstructure is not sensitive to the process variations.\textsuperscript{20,21} Hence, the process parameters do not need to be controlled with a high accuracy degree. In this open loop control system, a simulation feedback from process model can calculate the ideal inputs.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{open_loop_control_system}
\caption{Schematic diagram of open-loop control system for ideal process design}
\end{figure}
Figure 6 represents the whole open loop system for this ideal process design. The material model is constrained by material workability and metallurgical. The process model is constrained by workpiece geometry, tooling geometry and boundary conditions. Through the two models, the equipment parameters to achieve the desired microstructure can be determined which also constrained by equipment capacities (e.g. applied force, ram velocity). If inputs follow the safe DMM region that could avoid defects and achieve desired microstructure, the process would yield high quality product with specified properties on repeatable basis.

2.5 Illustration of Ideal Process Design Approach Using Hot Metal Rolling Process

In order to illustrate the overall ideal design approach, it is important to use this approach to solve a given problem. In this research, a hot metal rolling process has been chosen as a case study. In case study, a hot rolling process designed by using the overall ideal design approach. In this case, plain C-Mn steel has been chosen as the work piece material. The rolling process relationships and microstructure evolution relationships were adopted from published literature.

In this research, Suh’s axiomatic design approach\(^\text{10}\) which based upon design principles has been employed to help verify this ideal process design approach. According to Suh’s Independence Axiom\(^\text{10}\), a good design should be functional
Figure 7 Schematic diagram of a plasticating extruder (From ref. 10)

independent with the minimum information. As an example shown in Figure 7, the screw speed affects both the temperature and the pumping rate of the extrudate. Moreover, the barrel temperature also affects both the temperature and the pumping rate. As a result, this design is coupled which could reduce the product quality and increase the waste. The solution to this problem is to separate these two functions by adding a gear pump. As shown in Figure 8, the gear pump is set up to control the pumping rate, and screw speed is adjusted to control the extrudate temperature, respectively. The solution decoupled the function of flow rate and temperature which would give more precisely control to the process.

Figure 8 Schematic diagram of a pump-assisted plasticating extruder (From ref. 10)
This axiomatic design method not only provides a solution to reduce the complexity of an existing design but also could be used to analyses the efficiency of a process design. Hence, this method was used to improve the control system of the ideal process design.
CHAPTER III

DETERMINISTIC MATERIAL BEHAVIOR FOR IDEAL PROCESS DESIGN

Under ideal material behavior conditions, the workpiece material would have good workability and controllable microstructure; the material behavior would be steady and deterministic. However, under non-ideal material behavior conditions, defects such as shear bands, voids and fractures would be produced and can compromise product quality; material behavior would be non-steady and stochastic. As briefly discussed in Chapter II, Dynamic Material Model (DMM) approach\textsuperscript{12,13} is used to determine the process window for ideal process design. By using the DMM approach, processing conditions for stable and controllable material processing behavior can be found. According to the stability criteria, a “safe” (i.e. zero material defects) process range can be determined from the DMM. Analyzing the value of the apparent activation energy over range of temperature and strain rate is used for identifying and targeting desirable mechanisms which give controlled microstructure. If the workpiece material is processed within the stable process window which is a region of desired and nearly constant activation energy, microstructural and mechanical properties will have low sensitivity to small variations in processing conditions. In addition, the microstructure evolution models are more accurate under this condition. Finally, a robust process can be achieved by the above approach.
3.1 Dynamic Material Model Stability Criteria

Malas\textsuperscript{14,15} introduced stability theory into the dynamic material modeling to establish a stable process range. Stability is an important characteristic of the transient behavior of a material system. The DMM stability criteria are:

\begin{align*}
0 < m & \leq 1 \\
\frac{\partial m}{\partial (\log \dot{\varepsilon})} & < 0 \\
s & \geq 1 \\
\frac{\partial s}{\partial (\log \dot{\varepsilon})} & < 0
\end{align*}

Where $m$ is the strain rate sensitivity parameter and $s$ is the temperature sensitivity parameter. $m$ and $s$ are defined as:

\begin{equation}
\begin{align*}
m & = \left[ \frac{\partial (\ln \sigma)}{\partial (\ln \dot{\varepsilon})} \right] \varepsilon, T \\
s & = \frac{1}{T} \left[ \frac{\partial (\ln \sigma)}{\partial \left( \frac{1}{T} \right) } \right] \varepsilon, \dot{\varepsilon}
\end{align*}
\end{equation}

Table.1 summarized unstable phenomena which violate the stability criteria and Table.2 list some ideal phenomena which obey the stability criteria.

The range of strain rate sensitivity value $m$ for stable material flow (equation 1) is derived from theoretical considerations of the maximum rate of power dissipation by material system and experimental observations.\textsuperscript{15} Metals and alloys satisfy this criterion under hot working conditions. Negative values of the strain rate sensitivity parameter are obtained only under conditions promoting dynamic strain aging. Another mechanism that
can lead to negative $m$ value is the dynamic propagation of pre-existing or newly formed microcracks in the workpiece which lead to fracture of the workpiece.\textsuperscript{14} As the value of $m$ increase, the tendency for localized deformation decreases, enabling extensive elongation of specimens under tensile loading without necking. Similarly, the occurrence of shear band formation can be prevented very effectively at high $m$ value. When $m = 1$, it present ideal superplastic behavior resulting from the Newtonian flow typical of a glass material.

Table. 1 Unstable Phenomena

<table>
<thead>
<tr>
<th>Stability Criteria</th>
<th>Possible Metallurgical Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m &lt; 0$</td>
<td>Dynamic strain aging, Shear bands</td>
</tr>
<tr>
<td>$\frac{\partial m}{\partial (\log \dot{\varepsilon})} &gt; 0$</td>
<td>Fracture</td>
</tr>
<tr>
<td>$0 &lt; s &lt; 1$</td>
<td>Stochastic dynamic recovery</td>
</tr>
<tr>
<td>$\frac{\partial s}{\partial (\log \dot{\varepsilon})} &gt; 0$</td>
<td>Significant thermal softening</td>
</tr>
</tbody>
</table>

The stability criterion relating to the variation of $m$ with $\log \dot{\varepsilon}$ (equation 2) stems from the theoretical requirement for the material system to continuously lower its total energy. If fracture stress is assumed to be independent to strain rate, then increasing $m (\dot{\varepsilon})$

Table. 2 Ideal Phenomena

<table>
<thead>
<tr>
<th>Stability Criteria</th>
<th>Possible Metallurgical Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m = 1$</td>
<td>Ideal superplastic behavior</td>
</tr>
<tr>
<td>$\frac{\partial m}{\partial (\log \dot{\varepsilon})} &lt; 0$</td>
<td>Uniform stress distribution</td>
</tr>
<tr>
<td>$s \geq 1$</td>
<td>Flow stress is low sensitive to temperature</td>
</tr>
<tr>
<td>$\frac{\partial s}{\partial (\log \dot{\varepsilon})} &lt; 0$</td>
<td>Controllable material flow</td>
</tr>
</tbody>
</table>

will probably cause catastrophic failure at high strain rate. In contrast, a decreasing $m (\dot{\varepsilon})$ has a lower probability of inducing fracture in the workpiece. Moreover, this stability
criterion leads to more uniform stress fields across the workpiece and decreases tendency for strain localization.

The lower limit of $s$ value (equation 3) is derived from the premise that the net entropy production rate associated with irreversible processes must be positive for stable conditions. It is well established that when dynamic recovery alone operates as a softening mechanism, the temperature dependence of flow stress is relatively weak. In contrast, when both dynamic recovery and dynamic recrystallization are in operation, the flow stress varies markedly with temperature. Thus low values of $s$ indicated the dynamic recovery processes, while high values of $s$ are usually associated with dynamic recrystallization processes.

The stability criterion relating to the variation of $s$ with $\log \dot{\varepsilon}$ (equation 4) arises from the necessity for the material system to continuously lower its total energy. As the strain rate increases the extent of adiabatic heating in local regions increases significantly. Hence, in any particular region of the workpiece, if the local strain rate increases above the nominal value, then it is accompanied by a reduction in the local flow stress. However, if $s(\dot{\varepsilon})$ increases with $\dot{\varepsilon}$, then a very significant thermal softening will be encountered in the regime of high strain rates, which will produce severe strain localization and adiabatic shear bands. This process is likely to lead to severe cracking of the workpiece, if the material has poor resistance to nucleation and growth of cracks. In contrast, decreasing $s(\dot{\varepsilon})$ has a mitigating influence upon tendency for flow localization.
The four stability criteria of the DMM approach provide the necessary conditions for avoiding instabilities such as shear band and fracture. It can be regarded as a probabilistic indictor of the behavioral trends exhibited during hot working. Violations of the DMM stability criteria could affect the predictability of the microstructure development, which can yield non-uniform microstructure and deformation. Figure 9 shows an example of DMM stability map with contours of apparent activation energy. Through this process map, process designer can easily determine a stable process range with a desired metallurgical mechanism for controlling microstructure and properties.

Figure 9 DMM stability map for stainless steel 316L. The values of activation energy are represented as contours in kJ/mol (From ref.21)
3.2 Activation Energy Analysis for Identifying Desirable Mechanism

Deformation mechanisms can be identified by the amount of potential free energy required for their activation. Microstructural transformations require atomic mobility; and for activation of this mobility, an increase in energy must be provided to transport the atoms from one site to another. The movement of the atom from the initial state to the final state is opposed by an energy barrier; therefore, until the atom can temporarily acquire the extra energy necessary to carry it over the barrier, it must remain in the initial state. The minimum increment of energy which will allow the atom to go over the barrier is the activation free energy of the metallurgical reaction.

The apparent activation energy, $Q$, can be determined from computation of $m$ and $s$.

$$Q = \frac{sRT}{m} \quad (6)$$

Where $R$ is gas constant and $T$ is the working temperature. Different deformation mechanism required different activation energy. Table.3 shows metallurgical mechanisms of some metals with the associate activation energy under hot working condition.
Where the desirable deformation mechanisms are operative and dominant from DMM map is determined through the activation energy analysis.

In some cases, instead of apparent activation energy, $Q$, the efficiency of power dissipation parameter, $\eta$, is a more indirect, but useful parameter for identifying deformation mechanisms. It is fundamentally related to strain-rate sensitivity and is useful to identify only phenomena affected by strain-sensitivity changes over ranges of strain rate and temperature. $\eta$ is defined as below:

$$\eta = \frac{2m}{m + 1}$$  \hspace{1cm} (7)
where \( m \) is the strain rate sensitivity parameter. Activation energy analysis is preferred but processing maps with contours of \( m \) are prevalent in the published literature including a compendium of processing maps\(^{29}\) with over 200 metal alloy systems. Therefore, this research includes the use of efficiency contours as a valid alternative approach to identifying desirable mechanisms and for evaluating material behavior sensitivity to internal and external perturbations in the process.

### 3.3 Sensitivity Analysis for Robust Process Design

In order to design a robust process, it is important to find out an insensitive processing range to the internal and external variations such as strain rate, and temperature. From the DMM stability criteria which was presented in section 3.1 provide the sensitivity analysis. In the low sensitivity region (i.e. a region of low and nearly constant activation energy), material deformation behaviors are not sensitive to perturbations in the ideal thermomechanical path. In the unstable regions, the material flow paths are generally sensitive to small variations in process variations such as temperature and strain rate.\(^{21}\) The process naturally robust in the stable region. A perturbation in the temperature and strain rate during processing will not cause significant changes in the microstructure and properties.

In the stable process region, material microstructure evolution models would be more accurate and valid. Here Figure.10 is a simple plot of measured grain size of stainless steel 316L in a wide process range. The grain size of stainless steel 316L within the design window is nearly constant that is non-sensitive to process variations. In contrast, grain size varies at other regions.
3.4 Summary

In order to achieve ideal material behavior conditions, DMM approach has been used to determine a stable process region. Then desirable mechanisms can be determined to control microstructure through the apparent activation energy analysis and the efficiency of power dissipation analysis. Finally, according to sensitivity analysis, a non-sensitivity region with essentially constant apparent activation energy values or desired efficiency of power dissipation can be found. Material system is the core of the overall ideal process design approach. It is the priority of deformation process system and equipment control system because the ability to achieve desired mechanical properties
and geometry shapes which depend greatly on the material selection and its pedigree.

Figure.11 shows the basic input/output and concentric relationships of the three systems.

Figure.11 Schematic diagram of three systems in ideal process design
CHAPTER IV

IDEAL FORMING THEORY AND SIMPLIFYING ASSUMPTIONS FOR IDEAL PROCESS DESIGN

Under ideal forming conditions, the deformation of material workpiece would be uniform; the material flow would be steady and predictable. Non-ideal deformation conditions such as significant flow localization (shear bands), and rigid body rotations (turbulence) would produce strain, strain rate, and temperature gradients with heterogeneous microstructures and residual stresses; the material flow would be non-steady and stochastic. In order to design an ideal deformation process, knowledge of material flow under deformation conditions is required. “Ideal forming theory” provides such a useful starting point for deformation process design. In this theory, materials elements are prescribed to deform along the minimum plastic work path, assuming that the material have optimum workability in this path. Then the ideal forming processes are obtained with the most uniform strain distributions. However, even using the ideal forming theory, there still have some uncertainties such as workpiece dimensions changes in the processing and the initial internal defects in billets that would increase the complexity of process design. Therefore, some methods of analysis have been used to idealizing the process design solution in this research.
4.1 Ideal Forming Principles

Richmond\textsuperscript{17,18,19} et al. developed Ideal Forming methodology. In this theory, ideal homogeneous deformation is defined as the path which produces a desired homogeneous deformation without redundant plastic work (i.e. minimum plastic work).\textsuperscript{19} The Figure.12 shows the comparison between an ideal homogeneous deformation and redundant deformation. In ideal homogeneous deformation, the strain distribution would be uniform. If the process applied redundant plastic deformation, the strain would be non-uniform, as showed in Figure.12 (bottom), the surface layers are sheared relative to the center.

Figure.12 Comparison of ideal homogeneous deformation (top) and redundant plastic deformation (bottom) (From ref.33)

The ideal forming theory provides the basic principles to designing an ideal deformation process. Workpiece material follow ideal deformation process can achieve the final shape with a uniform strain distribution. Figure.13 shows two extrusion dies. As an example of non-ideal process, in shear die, the material flow is non-steady during the process and produces steep gradients of strain and strain rate throughout deformation.
zone. In contrast, as an example of near ideal deformation process, the material flow is steady and uniform in stream lined die.

![Figure 13 Schematic representation of (a) shear die and (b) stream lined die for extrusion](image)

(From ref.24)

**4.2 Simplifying Assumptions for Ideal Process Design**

Generally, simpler design solutions are better than more complex design solutions. In process designs, the complexity of the solution increases with greater degrees of nonlinear or probabilistic interactions among inputs and outputs. Simpler process design solutions would tend to produce more uniform velocity fields in the deformation zone and essentially linear response to changes in ram speed.

A variety of analysis methods for deformation processes can be used to model input/output interactions and provide fundamental insights into ideal process design solutions. These analysis methods differ based on varying degrees of simplifying assumptions such as those listed as below:24
- Lower degrees of freedom for material flow are more predictable and controllable.
- Strain or stress state is well defined (e.g. planer, axisymmetric).
- The workpiece material is a homogeneous, isotropic continuum.
- The onset of yielding in tension and compression is identical.
- The material volume remains constant for plastic deformation

In this research, ideal process design solutions are derived from simplified models of the deformation process by imposing constraints that limit or minimize material flow interactions. Process design solutions with simplifying assumptions tend to be ones with the most uniform strain, strain rate, and temperature distributions. These ideal cases can be modeled and controlled accurately. Therefore, the field quantities, initial conditions for designing ideal deformation process can be guided by existing methods of analysis.

The methods of analysis, in increasing order of complexity and ability to predict fine detail, are:\textsuperscript{28}

1. \textit{The slab method}- assumes homogeneous deformation.
2. \textit{Uniform deformation energy method}- calculated average forming stress from the work of plastic deformation.
3. \textit{Slip line field theory}- permits point by point calculation of stress for plane strain conditions only.
4. \textit{Upper and lower bound solutions}- based on the theory of limit analysis, uses reasonable stress and velocity fields to calculate the bounds within which the actual forming load must lie.
4.3 Summary

Ideal forming theory is based upon the concepts that ideal deformation was defined as the path which produces a desired homogeneous deformation with minimum plastic work. The boundary conditions associated with simplified process analysis methods are often consistent with the assumptions used in ideal forming concepts. Therefore, the simplifying assumptions built into the methods of analysis actually provide good design criteria for ideal process design. Since the simpler design solution is better than complex ones, it is desirable to find design solutions for which a simplified analysis is valid. For example, it is generally desirable to design a 3D metal deformation process so that it is predominantly a 2.5D forming process.\textsuperscript{16}
CHAPTER V

OPEN LOOP CONTROL SYSTEM FOR IDEAL PROCESS DESIGN

The control system for hot working process has been developed for decades. Now, the control system can cover the whole manufacturing system. However, the development of advanced control system is still active because the market demand to improve quality has become increasingly severe. In metal hot working process, the product mechanical properties highly depend upon the internal structure features such as defects and grain size, but the instant measurement of internal defects and grain size is still a major challenge to existing detection technology. Moreover, if the process is carried out within the design window (Chapter III), and the microstructure development models are accurate in the design window, then the hot working process would be robust. As a result, It is not necessary to have a feedback between working machine and process control model in this case. Hence, an effective open loop control system is essential to the ideal design approach for hot metal working process.

5.1 Open Loop vs. Close Loop

In an open loop control system, there is no feedback from the output of the system to improve the input. Consequently, open loop control system is useful for well-defined systems where the relationship between the input and output can be modeled by a high
accurate mathematical formula or it is difficult to obtain an instant feedback from output (e.g. hot metal working process). As a result, the open loop control system is simple, economical and stable. Figure.14 shows a simple open loop control system.

![Simple Open Loop Control System Diagram](image)

**Figure.14** Schematic diagram of simple open-loop control system

In a close loop control system, there is a feedback from output which is used to calibrate the input to improve the future output. Figure.15 shows a simple close loop control system. Obviously, the feedback could reduce the errors and adjust the input to give better output. In hot metal working, the main challenge for designing a close loop control system is the lack of effective sensors for real-time measurement of internal microstructural characteristics (e.g. grain size, sub grain size, and dislocation). Indeed, there are many detection methods have been applied for sensors such as X-Ray diffraction, electron diffraction, and ultrasonic wave, but none of existing detection

![Simple Close Loop Control System Diagram](image)

**Figure.15** Schematic diagram of a simple close-loop control system
technology can penetrate more than 1mm from the surface.\textsuperscript{31} Therefore, existing sensors cannot provide internal measurement of workpiece in most hot metal working processes.\textsuperscript{40} Hence, close loop control system is preferred in most cases but it is not suitable in this case because the potential for useful real-time feedback is low at this point in time.

5.2 Summary

As discussed earlier in this chapter, open loop control system could be successfully used in hot metal working process for producing quality products. This control system required high accurate microstructure development models and the deformation process should be designed from robust region (i.e. process design window). The microstructure characteristics of workpiece can be determined indirectly from process parameters (i.e. strain, strain rate, and temperature) by using the microstructure evolution models. Moreover, these process parameters are determined by equipment parameters such as ram speed, furnace temperature, and load force. Hence, dynamic models of equipment control system are useful for determining the desired adjustable parameters settings for coincident tracking of the equipment response with optimized inputs.\textsuperscript{32} As an example, Figure.16 shows a block diagram constructed using software (Simulink) to model the 1000 ton forge press including the accumulator, pump, servo-manifold, fluid dynamic effects, and the ram. Using this model for dynamic system analysis could significant improve the precision of ram speed control and the smoothness
Figure 16 Simulink block diagram model of 1000-ton forging press (From ref. 32)

Figure 17 A comparison of state-of-the-art forge press control capability and improved dynamic system analysis approach (From ref. 32)

of servo-valve response, as shown in Figure 17. Indeed, dynamic modeling of equipment system is an important part of open loop control system since it is central to realizing optimized, time-varying material processing conditions. 32
CHAPTER VI

APPROACH ILLUSTRATION THROUGH CASE STUDY

In this chapter, hot steel rolling process has been chosen to illustrate this overall ideal process design approach. Hot steel rolling process was selected because C-Mn steel and hot rolling process have been studied extensively in published literature. The hot rolling process has been subject to analysis for a long time because it is the major process to produce sheet and plate. Even the hot rolling process is matured and relatively simple, but yielding a product with desired microstructure, non-defects, and reproducibility is

Figure.18 Schematic diagram of microstructure development under hot rolling process

(From ref.39)
still a challenge since there are metallurgical phenomena such as dynamic recovery, dynamic recrystallization, and grain growth that occur during the process. Complex interactions among metallurgical mechanisms increase the complexity of process design. Figure.18 shows a single hot rolling process with possible interactions among metallurgical phenomena. In this chapter, a steel hot rolling process has been analyzed to illustrate the ideal process design approach.

6.1 Case Study: Steel Hot Rolling Process Designed by Ideal Process Design Approach

In this case, a steel hot rolling process is designed upon the ideal process approach. In the overall ideal process approach, the first step is using the DMM approach to determine a process design window where the material behavior would be stable, controllable, and robust. The process map for plain C-Mn steel is shown in Figure.19, the shaded region violate the stability criteria. Also shown on the map is the contours of efficiency of power dissipation, $\eta$, which describes the amount of total power applied to the material system which is partitioned to metallurgical processes.\textsuperscript{14}To determine the process design window the apparent activation energy, $Q$ contours are preferred, but $\eta$ contours are more prevalent in the published literature and can be useful too. Table.4 summarized the reported\textsuperscript{29} metallurgical interpretation in the process map range.
Figure. 19 Process design map of plain C-Mn steel. Contour numbers represent efficiency of energy dissipation. Shaded region correspond to flow instability. (From ref. 29)

Table. 4 Metallurgical Phenomena in Process Map Range (From ref. 29)

<table>
<thead>
<tr>
<th>Phenomena</th>
<th>Temperature, °C(°F)</th>
<th>Strain rate, s⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic recrystallization</td>
<td>1000-1200 (1832-1292)</td>
<td>0.04-1</td>
</tr>
<tr>
<td>Wedge cracking</td>
<td>900-1000 (1652-1832)</td>
<td>0.01-0.1</td>
</tr>
<tr>
<td>Flow instability</td>
<td>900(1652); 1000-1200(1832-2192)</td>
<td>&gt; 0.3</td>
</tr>
</tbody>
</table>

From the analysis of η contours, a robust process design window can be determined in the process map. By identifying stable region with desired η values the process design window is shown in Figure. 20. The temperature range for the design window is form 1080°C to 1180°C and the strain rate range is from 0.1 s⁻¹ to 0.3 s⁻¹.
In the design window, dynamic recrystallization is dominating. The microstructure development in the design range is represented and predicted by some microstructure evolution models. The kinetics of recrystallization in plain C-Mn steel has been studied by several researchers,\cite{1,32,39} and key concepts are described in Appendix A. In this research, Yada’s equations\cite{1} for dynamic recrystallization (DRX) of plain C-Mn steel were used extensively. Yada’s equations are highly reviewed by other researchers,\cite{1} these equations provide a highly accurate prediction of microstructure development in the process design window range. According to Yada’s equations, microstructure characteristics such as grain size \([d(j)]\) and volume fraction of recrystallization \([X_{r,xn}]\) can be predicted as a function of temperature \([T_W]\), plastic strain \(\varepsilon\), strain rate \(\dot{\varepsilon}\), critical strain \(\varepsilon_c\) and initial grain size\([d(j-1)]\). Yada’s equation listed as follow:
\[
\varepsilon_c = 4.76 \times 10^{-4} \exp\left(\frac{8000}{T_W}\right) \tag{10}
\]
\[
d(j) = 22,600[\dot{\varepsilon}\exp\left(\frac{Q}{RT_W}\right)]^{-0.27} \tag{11}
\]
\[
X_{rxn} = 1 - \exp[-0.693\left(\frac{\varepsilon - \varepsilon_c}{\varepsilon_{0.5}}\right)^2] \tag{12}
\]
\[
\varepsilon_{0.5} = 1.144 \times 10^{-5} d_{(j-1)}^{0.28} \dot{\varepsilon}^{0.05} \exp\left(\frac{6420}{T_W}\right) \tag{13}
\]

Where \( Q = 267.1 \text{ KJ/mol}, R \) is the gas constant which is 8.314 J/(g·mol·K) and \( \varepsilon_{0.5} \) is the plastic strain for 50 volume percent recrystallization. The units of \( T_W, \dot{\varepsilon} \) and \( d \) are kelvin, per second, and micrometers, respectively.

Since the robust process window has been determined and the microstructure development can be predicted with the corresponding process parameters, the second step is to design a rolling process by using the ideal forming concepts and simplifying assumptions. For the rolling process, roll radius and roll gap play a key role to produce uniform deformation. If the sheet or slab is rolling with small rolls or at low reduction per pass, it could produce residual stresses on the surface. Similarly, if the sheet or slab is rolling with large rolls or at high reduction per pass, it could produce residual stresses in the center. Those residual stresses could lead to defects (e.g. edge crack, wavy edges).

In an ideal deformation process, the material flow would be steady and deterministic. For rolling process, the usual assumption of ideal homogeneous deformation is that planes remaining planes during a rolling pass. Therefore, the roll
radius and roll gap must be studied for ideal deformation process. An important parameter for designing the desirable roll radius and roll gap is called the shape factor\textsuperscript{33}, which defined as the ratio of the average thickness of the rolled metal to the projected contact length, $h/L$. As shows the Figure.21 (a), at a high value of $h/L \gg 1$, deformation is concentrated in zones that adjacent to the rolls and dose not penetrate the full thickness of the workpiece.\textsuperscript{33} At intermediate values of $h/L \gg 1$, the deformation zone could be complex, dead metal zone extend partway into the body of the workpiece from the sticking zone on the contact surface. Intense deformation occurs between these two sticking zones (one on each side). Minor plastic deformation takes place outside the entry and exit of the roll pass. All these deformation zones shows in the Figure.21 (b), corresponded to intermediate $h/L$ values.

![Figure.21](image.png)

Figure.21 (a) Highly inhomogeneous plane-strain deformation at large $h/L$ ratios. (b) Deformation zones at intermediate $h/L$ ratios (I) Dead metal zone; (II) Intense deformation zone; (III and IV) Indirect deformation zones. (From Ref.33)
The combined effects of inhomogeneous deformation and friction on roll forces are calculated and shows in Figure.22. In Figure.22, Y-axis is a factor which is defined by the ratio of the average interface pressure, $p$, and the plane-strain flow stress of the workpiece material, $2k$. The study of $h/L$ ratio provides a useful guidance to designing the roll radius and roll gap for an ideal rolling process.

![Figure.22 Pressure multiplying factor as a function of h/L ratio (From ref.33)](image)

Some other simplifying assumptions and method of analysis have been used to help determining ideal rolling process. The simple plane strain analysis method\(^3^\) assumes that the strain distribution is uniform and the width of rolling sheets is constant ($\Delta$width = 0) during processing. Other assumptions such as the product of initial height and initial velocity equal to the product of final height and final velocity ($h_0 \times v_0 = h_f \times v_f$); the heat distribution in the billet assumed to be uniform.

The process parameters are controlled by the rolling equipment and temperature is controlled by furnace in this case. The plastic strain ($\varepsilon$) is defined as:

$$\varepsilon = \ln(1 - r)$$

(14)
Where $r$ is the height reduction for the workpiece, it can be controlled by roll gap. Strain rate for rolling process is determined by roll velocity ($V$), roll radius ($R$), and percent reduction in height ($r$). The relationship is represented as:

$$\dot{\varepsilon} = V \sqrt{\frac{r}{RH_0}}$$  \hspace{1cm} (15)

Where $H_0$ is the initial height of workpiece. The real time heat generation in the hot rolling process is complicated; the empirical relation for working temperature is known to depend up on roll velocity ($V$), roll radius ($R$), and initial work piece temperature ($T_B$).

$$T_W = kT_B^{a}V^{b}$$  \hspace{1cm} (16)

Where $k$, $a$, and $b$ are coefficients depend upon material and process.

**6.2 Example: Design of Hot Rolling Process for Producing Steel Sheets with Desired Microstructure**

The purpose of this section is to demonstrate how the ideal process design approach solves a given problem. In this example the initial conditions for workpiece are:

- Initial grain size, $d(0) = 100\mu m$
- Initial sheet thickness, $H_0 = 10mm$

In the example, the required design objectives are:

- Average final grain size, $d(1) = 50\mu m$
- Dynamic recrystallization, $X_{\text{rxn}} \geq 70\%$. 

41
According to the DMM, the temperature range for the design window is from 1080°C to 1180°C and the stain rate range is from 0.1s\(^{-1}\) to 0.3s\(^{-1}\). The isothermal process is assumed \(T_w = T_B = 1080°C\). By using the equation (11), the \(\dot{\epsilon}\) correspond to required grain size can be determined, which is 0.33 s\(^{-1}\). The DRX is determined from \(T_w, \varepsilon_c, \varepsilon_{0.5},\) and \(\varepsilon\). By using equation (10), (12), and (13), the \(\varepsilon\) for 70\%\(X_{\text{rxn}}\) is 0.182. Thus, the required process variables are:

- Working temperature, \(T_w = 1080°C\)
- Average strain rate, \(\dot{\varepsilon} = 0.33\) s\(^{-1}\)
- Minimum strain, \(\varepsilon = 0.182\)

To determine the ideal roll radius for this hot rolling process, the ideal ratio for \(h/L\) would be \(\leq 1\). Since the minimum required \(\varepsilon\) is 0.182, that the height reduction \(r\) can be determined from equation (14). The temperature raise \(\Delta T\) due to deformation heat is only 11 K which would not affect the result. The height reduction \(r\) is found to be 0.167, \(H_f = 8.33 mm\) Here, \(h= \frac{(H_0+H_f)}{2} = 9.165 mm\) and assume \(h/L = 0.4\) that \(L=22.9125 mm\). The roll radius can determined from the relationship between \(R, L,\) and \(\Delta H\): \(R^2=L^2 + (R - \frac{\Delta H}{2})^2\), where \(\Delta H = (H_f - H_0)\). The ideal \(R= 315 mm.\) and Roll gap \(G=L= 22.9125 mm\). From equation (15), the roll velocity \(V\) is determined to be 45 mm/s. The desired process variables are:

- Roll radius, \(R=315 mm\)
- Roll gap, \(G=22.9125 mm\)
- Roll velocity, \(V=45 mm/s\)
In summary, a single hot rolling process was designed for achieving the required microstructural parameters by using the ideal process design approach. In the example, DMM approach was used to determine a stable process region and Yada’s equations were used to determine the process parameters corresponded to the desired microstructure. The related rolling equipment variables are determined by rolling process equations. In Appendix B, plain C-Mn steel microstructure development equations were built into decoupled design matrix through Suh’s axiomatic design approach. Through these design matrixes, the process design could be more effective.
CHAPTER VII

CONCLUSION

An ideal process design approach for hot metal working was developed in this research. Ideal process design approach provides solution for desired microstructure and geometry shape with uniform deformation and robust process range. The hot steel rolling process case study shows the overall ideal process design approach has the capability to yield an ideal solution for a given problem. This approach uses some ideal assumptions and mathematical models to calculate the solutions for ideal process conditions. Hence, the accuracy of this approach highly depends upon the empirical constitutive models for each system. However, the models never exactly describe the metallurgical phenomena; there are always assumptions which the analysts impose to make the task tractable. For example, the initial internal defects might be ignored to make the microstructure development model more tractable. The set of assumptions form the idealization of the system. Even though these assumptions could lead to differences between predicted result and the real result, this ideal process design still provide an effective solution to design a robust metal working process.

Future work will involve statistical analyses and experimental validation of ideal steel hot rolling process. Also, the potential application of ideal design approach to other material process could be investigated.
REFERENCES


34. Sellars, C. M.; Whiteman, J.A. *Metal Science* **1979** 5.187-194


APPENDIX A

MICROSTRUCTURE EVOLUTION MODELS FOR HOT WORKING

Metallurgical phenomena such as Dynamic recovery and dynamic recrystallization occur during hot deformation operations. The microstructures obtained from these dynamic metallurgical phenomena are unstable. After hot deformation operations, further microstructure development is achieved by static recovery, static recrystallization and grain growth. The mechanisms and kinetics of these processes as well as the association of changes in size, morphology, volume fraction and composition of phases are determined by the macroscopic heat flow and material flow processes. Moreover, material flow and thermal history also affected the distribution of strain, strain rate and effective stress. Hence, the microstructure development during hot working is dependent upon both heat flow and material flow.

The microstructure development is the core of designing thermomechanical operations. The microstructure of workpiece material significantly influences the mechanical properties in the operation and final product. Thus, it is necessary to build a reliable microstructure evolution model under hot working condition, especially for multi-stage thermomechanical processes. Figure.23 shows the microstructure evolution during hot working.
The dynamic recovery and recrystallization during hot working have been studied extensively and reviewed comprehensively by Jonas\textsuperscript{36}, Sellars\textsuperscript{34}, and Hodgson\textsuperscript{37}. These two phenomena have significant influence on the microstructure development during hot working conditions. If dynamic recovery occurs during hot working, the flow stress initially rises with strain up to a steady state value, \(\sigma_s\), which is determined by a balance between the accumulation and elimination of dislocations by climb or cross-slip.\textsuperscript{34} For metals and alloys which have a face-centered-cubic (FCC) crystalline structure, dynamic recovery is relatively slow, the sub-grain boundaries are formed abnormally, and sufficient strain energy is stored until a critical strain, \(\varepsilon_c\), is reached where dynamic recrystallization occurs.

Figure 23 Schematic of microstructure development during hot working (From Ref.35)
Dynamic Recovery Relationship in Hot Working

In dynamic recovery, a steady sub-grain structure tends to develop at deformation over the critical plastic strain, \( \varepsilon_c \), which increase with an increase in imposed \( \dot{\varepsilon} \) or a decrease in temperature \( T \). The combined influence of \( \dot{\varepsilon} \) and \( T \) on \( \varepsilon_m \) can be represented as:

\[
\varepsilon_c = \rho Z^{0.2}
\]  

(17)

Where \( \rho \) is a constant \( \approx 2 \times 10^{-3} \) and \( Z = \dot{\varepsilon} e^{\frac{Q}{R T}} \), \( Q \) is activation energy for dynamic recovery and \( R \) is the universal gas constant. The sub-grain size, \( d_s \), is determined by:

\[
d_s^{-1} = \frac{\sigma_s - \sigma_0}{K}
\]  

(18)

Where \( \sigma_0 \) and \( K \) are constants. The steady state flow stress can be expressed in terms of the function temperature compensated strain.

Dynamic Recrystallization Relationship in Hot Working

Under hot working condition, when the plastic strain exceeds a critical strain value, \( \varepsilon_c \), dynamically recrystallized grains nucleate through a mechanism involving strain induced grain boundary migration. With the continuation of deformation, additional nuclei form until all grain boundary sites are exhausted. A standard Avrami equation for the fraction of recrystallization, \( X_{rxn} \), can be shown as:

\[
X_{rxn} = 1 - e^{[\xi (\varepsilon - \varepsilon_c)^
u]}
\]  

(19)

51
Where $\nu$ and $\xi$ are constants, $\nu$ is considered to be independent of $\dot{\varepsilon}$, $T$, and $d_0$; $\xi$ is considered as a kinetic dependence parameter. $\xi$ increase with a decrease in $d_0$ or in $Z$. The critical strain, $\varepsilon_c$, decrease steadily with $Z$, reaching a minimum value at very low $Z$. The grain growth for new recrystallized grains is very limited. The recrystallized grain, $d_{dyn}$, increase very slightly as $X_{rxn}$ increase and is independent of $d_0$.

**Static Recrystallization Relationship in Hot Working**

During static recrystallization, the volume fraction of recrystallization grains behavior as a function of time after deformation:

$$X_{stat} = 1 - \exp(-k_s t^{n_s})$$  \hspace{2cm} (20)

Where $k_s$ and $n_s$ are constants. $k_s$ contains the dependence upon prior strain, temperature and grain size. The reference time, $t_{0.5}$, the required time for 50 volume percent of the static recrystallization has been shown an empirical dependence:

$$t_{0.5} = A e^{-a d_0^b Z^{-c}} \exp \left( \frac{Q}{RT} \right)$$  \hspace{2cm} (21)

Where $a$, $b$ and $c$ are constants. The volume fraction of static recrystallization can be expressed in terms of $t_{0.5}$:

$$X_{stat} = 1 - \exp \left[ -0.693 \left( \frac{t}{t_{0.5}} \right)^{n_s} \right]$$  \hspace{2cm} (22)

The grain size obtained at the completion of static recrystallization (SRX) is highly dependent upon the prior strain, $\varepsilon$, and the prior grain size, $d_0$: 

52
\[ d_{stat} = A + B e^{-a d_0^b Z^{-c}} \] (23)

Where the exponents a and b are in range 0.5-1.0 and c < 0.1. A and B are constants \( Z \) is Zener-Holloman parameter.

**Microstructure Evolution Model of Plain C-Mn Steel**

The microstructure evolution of plain C-Mn steel under hot working condition has been developed for decades. Devadas\(^1\) reviewed some microstructure evolution model from different researchers in Table.5. This table presented the microstructure evolution models developed by Sellars, Yada, Saito, and Perdrix are shown for dynamic recrystallization (DRX), static recrystallization (SRX) and grain growth.

In this research, The Yada’s equations for DRX of plain C-Mn steel were used extensively. This DRX model provides a comprehensive description of the relationship between microstructure and process variables.
Table 5 Microstructure Evolution Equations for Plain C-Mn Steel (From ref. 1)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic Recrystallization</td>
<td></td>
<td></td>
<td>not incorporated</td>
</tr>
<tr>
<td>$\varepsilon = 4.9 \times 10^{-4} \exp \left( \frac{8000}{T} \right)$</td>
<td>$\varepsilon = 4.76 \times 10^{-2} \exp \left( \frac{8000}{T} \right)$</td>
<td>$\varepsilon = 5.58 \times 10^{-2} \exp \left( \frac{8000}{T} \right)$</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_c = 0.8\varepsilon$</td>
<td>$\varepsilon_c = 0.8\varepsilon$</td>
<td>$\varepsilon_c = 0.8\varepsilon$</td>
<td></td>
</tr>
<tr>
<td>$Z = \varepsilon \exp \left( \frac{Q}{RT} \right)$</td>
<td>$Z = \varepsilon \exp \left( \frac{Q}{RT} \right)$</td>
<td>$Z = \varepsilon \exp \left( \frac{Q}{RT} \right)$</td>
<td></td>
</tr>
<tr>
<td>$Q = 312 \text{ kJ/mol}$</td>
<td>$Q = 267.1 \text{ kJ/mol}$</td>
<td>$Q = 312 \text{ kJ/mol}$</td>
<td></td>
</tr>
<tr>
<td>(metadynamic)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X = 1 - \exp \left( -0.693 \left( \frac{T}{T_m} \right) \right)$</td>
<td>$X_m = 1 - \exp \left( -0.693 \left( \frac{T}{T_m} \right) \right)$</td>
<td>$X = 1 - \exp \left( -0.693 \left( \frac{T}{T_m} \right) \right)$</td>
<td></td>
</tr>
<tr>
<td>$e_m = 1.44 \times 10^{-2} \exp \left( \frac{6420}{T} \right)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Static Recrystallization</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X = 1 - \exp \left( -0.693 \left( \frac{T}{T_m} \right) \right)$</td>
<td>$X = 1 - \exp \left( -0.693 \left( \frac{T}{T_m} \right) \right)$</td>
<td>$X = 1 - \exp \left( -0.693 \left( \frac{T}{T_m} \right) \right)$</td>
<td>$X = 1 - \exp \left( -0.693 \left( \frac{T}{T_m} \right) \right)$</td>
</tr>
<tr>
<td>$\varepsilon &lt; 0.8\varepsilon_c$</td>
<td>$\varepsilon &lt; 0.8\varepsilon_c$</td>
<td>$\varepsilon &lt; 0.8\varepsilon_c$</td>
<td>$\varepsilon &lt; 0.8\varepsilon_c$</td>
</tr>
<tr>
<td>$t_{xs} = 2.2 \times 10^{-13} \exp \left( \frac{Q_{xs}}{RT} \right)$</td>
<td>$t_{xs} = 2.2 \times 10^{-13} \exp \left( \frac{Q_{xs}}{RT} \right)$</td>
<td>$t_{xs} = 2.2 \times 10^{-13} \exp \left( \frac{Q_{xs}}{RT} \right)$</td>
<td>$t_{xs} = 3.67 \times 10^{-13} \exp \left( \frac{Q_{xs}}{RT} \right)$</td>
</tr>
<tr>
<td>$t_{xs} = 2.5 \times 10^{-9} \exp \left( \frac{30000}{T} \right)$</td>
<td>$t_{xs} = 2.5 \times 10^{-9} \exp \left( \frac{30000}{T} \right)$</td>
<td>$t_{xs} = 2.5 \times 10^{-9} \exp \left( \frac{30000}{T} \right)$</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon = 0.8\varepsilon_c$</td>
<td>$\varepsilon = 0.8\varepsilon_c$</td>
<td>$\varepsilon = 0.8\varepsilon_c$</td>
<td></td>
</tr>
<tr>
<td>$t_{xs} = 1.06 \times 10^{-5} \exp \left( \frac{Q_{xs}}{RT} \right)$</td>
<td>$t_{xs} = 1.06 \times 10^{-5} \exp \left( \frac{Q_{xs}}{RT} \right)$</td>
<td>$t_{xs} = 1.06 \times 10^{-5} \exp \left( \frac{Q_{xs}}{RT} \right)$</td>
<td>$t_{xs} = 272 \exp \left( \frac{3000}{T} \right)$</td>
</tr>
<tr>
<td>$Q_{xs} = 300 \text{ kJ/mol}$</td>
<td>$Q_{xs} = 300 \text{ kJ/mol}$</td>
<td>$Q_{xs} = 300 \text{ kJ/mol}$</td>
<td>$Q_{xs} = 300 \text{ kJ/mol}$</td>
</tr>
<tr>
<td>$d_m = 0.5d_{m0} \exp \left( \frac{5482}{T} \right)$</td>
<td>$d_m = 0.5d_{m0} \exp \left( \frac{5482}{T} \right)$</td>
<td>$d_m = 0.5d_{m0} \exp \left( \frac{5482}{T} \right)$</td>
<td>$d_m = 18.51 \ln \left( \frac{T}{T_m} \right)$</td>
</tr>
<tr>
<td>$d_m = 0.5d_{m0} \exp \left( \frac{5482}{T} \right)$</td>
<td>$d_m = 0.5d_{m0} \exp \left( \frac{5482}{T} \right)$</td>
<td>$d_m = 0.5d_{m0} \exp \left( \frac{5482}{T} \right)$</td>
<td>$d_m = 303 \text{ kJ/mol}$</td>
</tr>
<tr>
<td>$\varepsilon^* = 0.07 d_{m0} \varepsilon$</td>
<td>$\varepsilon^* = 0.07 d_{m0} \varepsilon$</td>
<td>$\varepsilon^* = 0.07 d_{m0} \varepsilon$</td>
<td></td>
</tr>
<tr>
<td>Grain Growth</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d^{10} = d_0^{10} + At \exp \left( -\frac{Q_{xs}}{RT} \right)$</td>
<td>$d^{10} = d_0^{10} + At \exp \left( -\frac{Q_{xs}}{RT} \right)$</td>
<td>$d^{10} = d_0^{10} + At \exp \left( -\frac{Q_{xs}}{RT} \right)$</td>
<td>$d = d_m \left( 1 + a \ln \frac{T}{T_m} \right)$</td>
</tr>
<tr>
<td>$T &gt; 1100 \text{ °C}$</td>
<td>$T &gt; 1100 \text{ °C}$</td>
<td>$T &gt; 1100 \text{ °C}$</td>
<td></td>
</tr>
<tr>
<td>$A = 3.87 \times 10^{10}$</td>
<td>$A = 3.87 \times 10^{10}$</td>
<td>$A = 3.87 \times 10^{10}$</td>
<td></td>
</tr>
<tr>
<td>$Q_{xs} = 400 \text{ kJ/mol}$</td>
<td>$Q_{xs} = 400 \text{ kJ/mol}$</td>
<td>$Q_{xs} = 400 \text{ kJ/mol}$</td>
<td></td>
</tr>
<tr>
<td>$T &lt; 1100 \text{ °C}$</td>
<td>$T &lt; 1100 \text{ °C}$</td>
<td>$T &lt; 1100 \text{ °C}$</td>
<td></td>
</tr>
<tr>
<td>$A = 1.31 \times 10^{11}$</td>
<td>$A = 1.31 \times 10^{11}$</td>
<td>$A = 1.31 \times 10^{11}$</td>
<td></td>
</tr>
<tr>
<td>$Q_{xs} = 914 \text{ kJ/mol}$</td>
<td>$Q_{xs} = 914 \text{ kJ/mol}$</td>
<td>$Q_{xs} = 914 \text{ kJ/mol}$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 24 shows the Yada’s DXR model under a temperature range from 1073K to 1473K with a corresponding critical strain range form 0.8-0.1.
Figure 24 Critical strain values for Yada’s DRX equations from a temperature range from 1073K to 1473K.

According to Yada’s DXR equation of recrystallized grain size, the grain size is only determined by temperature and strain rate. Figure 25 shows the effect of different strain rate on the grain size under same temperature.
Figure 25 Dynamic recrystallized grain size values obtained by different strain rate at a temperature range from 1073K to 1473K, predicted by Yada’s equation
APPENDIX B

AXIOMATIC APPROACH FOR PROCESS DESIGN

Designing a robust thermomechanical process is a challenge. It requires evaluation of a series of alternative operations and processing variables to get the most effective solution. Concurrently, the processes also subjected to several constraints such as cost, production rate, material workability. Therefore, the process design activity is difficult because of the diversification of knowledge involved. Furthermore, an understanding of the material behavior under processing conditions is required to design a robust process.

As briefly mentioned in Introduction, according to Suh, a design can be divided into four domains: Desire Attributes, Functional Requirements (FRs), Design Parameters (DPs) and Process Variables (PVs). The relationships between these four domains already have been discussed in Introduction. There are two fundamental design axioms established by Suh for systematic design.

1) Axiom1: The independence axiom.
2) Axiom2: The information axiom.

The Axiom1 states that an optimized design always maintains the independence of FRs, while Axiom2 states that the best design is a functionally independent design (satisfy Axiom1) with the minimum information content.
Mathematical Presentation of Design Relationships

Since the relationships between the four domains of design process have been defined, these relationships can be characterized mathematically. First, a set of independent function requirements [FRs] have been decided by designer. These FRs can be treated as a vector FR with M components. To achieve FRs, a right set of design parameters [DPs] need be established and can be treated as a vector DP with N components. Here, the relationships between FRs and DPs can be expressed as:

\[
\{\text{FR}\} = [A] \{\text{DP}\}
\]

Where \([A]\) is design matrix and \(\{\text{FR}\}\) is functional requirement vector, \(\{\text{DP}\}\) is design parameter vector, respectively. Similarly, the relationships between four domains can be expressed as:

\[
\{\text{CF}\} \xrightarrow{[A]} \{\text{FR}\} \xrightarrow{[B]} \{\text{DP}\} \xrightarrow{[C]} \{\text{PV}\}
\]

Where \([A]\), \([B]\) and \([C]\) are design matrix and \(\{\text{CF}\}\), \(\{\text{FR}\}\), \(\{\text{DP}\}\) and \(\{\text{PV}\}\) are design domain vector.

For a hot working process, the whole process system can be divided into four domains. The required material properties such as yield strength, ultimate tensile strength, fracture toughness considered as FRs (functional requirements). These FRs was determined by microstructure states. In this dissertation, the relationship between mechanical properties and microstructure features is beyond the scope and is not attempted. Table.6 shows some design vectors and associated symbols for process design.
Table 6 Design Vectors for Process Design

<table>
<thead>
<tr>
<th>Design vector</th>
<th>Function</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRs</td>
<td>Mechanical properties</td>
<td>$\sigma, E, K$</td>
</tr>
<tr>
<td>DPs</td>
<td>Microstructure</td>
<td>$d, X, A$</td>
</tr>
<tr>
<td>PVs</td>
<td>Process Variables</td>
<td>$T_W, \varepsilon, \dot{\varepsilon}$</td>
</tr>
<tr>
<td>EPs</td>
<td>Equipment Parameters</td>
<td>$R, V, T_B$</td>
</tr>
</tbody>
</table>

Design Matrix for Hot Working

In general, material microstructure states can be characterized by grain size ($d$), volume fraction of recrystallization ($X_{rxn}$), volume fraction of second phase ($X_p$), grain size of second phase ($d_p$), the ratio between matrix and second phase ($A$). Those microstructure features were achieved by thermomechanical processes and the initial previous states. Relevant thermomechanical process variables are workpiece temperature ($T_W$), dwell time ($t_s$), strain ($\varepsilon$) and strain rate ($\dot{\varepsilon}$). The microstructure features are considered as DPs and the associated process variables are considered as PVs.

The design matrix which based upon the constitutive equations can be developed for relating DPs to PVs; for hot deformation process, the design matrix is expressed as below:

$$
\begin{pmatrix}
    d(j) \\
    X_{rxn}(j) \\
    X_p(j) \\
    d_p(j) \\
    A(j)
\end{pmatrix} =
\begin{bmatrix}
    0 & \times & \times & \times & 0 & 0 & 0  \\
    0 & \times & \times & \times & \times & \times & 0  \\
    \times & \times \times 0 & 0 & \times & 0  \\
    0 & \times & \times & 0 & 0 & \times & 0  \\
    0 & \times & \times & 0 & 0 & 0 & \times
\end{bmatrix}
\begin{pmatrix}
    \varepsilon \\
    \dot{\varepsilon} \\
    T_W \\
    d(j_{-1}) \\
    X_{rxn}(j_{-1}) \\
    X_p(j_{-1}) \\
    d_p(j_{-1}) \\
    A(j_{-1})
\end{pmatrix}
$$

(24)
Where \( j \) and \( j_{-1} \) represent the \( j^{\text{th}} \) state and the previous state, respectively. In the matrix, \( \times \) represents a nonzero element and \( 0 \) represents a zero element.

**Dynamic Recrystallization of Plain C-Mn Steel**

The kinetics of recrystallization in plain C-Mn steel has been studied by several researchers, which already presented in Appendix A. According to Table.9, the activation energy for dynamic recrystallization has been reported to a range from 267 to 312 KJ/mol. In this research, Yada’s equations\(^1\) were adopted to build a design matrix because the completeness and generality in describing the dynamic recrystallization process. By using Yada’s microstructure evolution model, microstructure characteristics such as grain size \([d(j)]\) and volume fraction of recrystallization \([X_{\text{rxn}}]\) can be predicted as a function of temperature \([T_W]\), plastic strain \((\varepsilon)\), strain rate \((\dot{\varepsilon})\), critical strain \((\varepsilon_c)\) and initial grain size\([d(j_{-1})]\). Yada’s equation listed as follow:

\[
\varepsilon_c = 4.76 \times 10^{-4} \exp \left( \frac{8000}{T_W} \right) \quad (10)
\]

\[
d(j) = 22,600 \left[ \dot{\varepsilon} \exp \left( \frac{Q}{RT_W} \right) \right]^{-0.27} \quad (11)
\]

\[
X_{\text{rxn}} = 1 - \exp \left[ -0.693 \left( \frac{\varepsilon - \varepsilon_c}{\varepsilon_{0.5}} \right)^2 \right] \quad (12)
\]

\[
\varepsilon_{0.5} = 1.144 \times 10^{-5} d(j_{-1})^{0.28} \dot{\varepsilon}^{0.05} \exp \left( \frac{6420}{T_W} \right) \quad (13)
\]
Where $Q = 267.1$ KJ/mol, $R$ is the gas constant which is 8.314 J/(g·mol·K) and $\varepsilon_{0.5}$ is the plastic strain for 50 volume percent recrystallization. The units of $T_w$, $\dot{\varepsilon}$ and $d$ are kelvin, per second, and micrometers, respectively.

In order to put those nonlinear equations into a design matrix, it is necessary to linearize terms by using a logarithmic operation. The equation (11) can be rewritten as:

$$\ln[d(j)] = \ln(22,600) - 0.27 \ln \dot{\varepsilon} - 0.27 \left(\frac{Q}{RT_w}\right)$$  \hspace{1cm} (25)

For evaluation of changes in grain size with strain rate at a fixed temperature, equation (25) is differentiated as follows:

$$\frac{\partial \ln[d(j)]}{\partial \ln \dot{\varepsilon}} = -0.27 \hspace{1cm} (26)$$

For evaluation of changes in grain size with temperature at a fixed strain rate, equation (25) is differentiated as follows:

$$\frac{\partial \ln[d(j)]}{\partial \frac{1}{T_w}} = -0.27 \left(\frac{Q}{R}\right) = 8674.16 \hspace{1cm} (27)$$

For the evaluation of volume fraction of recrystallization($X_{rxn}$), it is convenient to consider the volume fraction of \textit{not} dynamic recrystallized($X_{not}$), where

$$X_{not} = 1 - X_{rxn} = \exp[-0.693\left(\frac{\varepsilon - \varepsilon_c}{\varepsilon_{0.5}}\right)^2]$$ \hspace{1cm} (28)

Taking the logarithm of equation (28) gives
\[ \ln(X_{not}) = -0.693 \left( \frac{\varepsilon - \varepsilon_c}{\varepsilon_{0.5}} \right)^2 \]  

(29)

Taking the logarithm of equation (29) gives

\[ \ln(-\ln(X_{not})) = \ln(0.693 + 2\ln(\varepsilon - \varepsilon_c) - 2\ln\varepsilon_{0.5}) \]  

(30)

Where \(\ln\varepsilon_{0.5}\) can be expressed by logarithm of equation (13) which is

\[ \ln(\varepsilon_{0.5}) = \ln(1.144 \times 10^{-5}) + 0.28 \ln d_{(j-1)} + 0.05 \ln \dot{\varepsilon} + \frac{6420}{T_W} \]  

(31)

To evaluate the sensitivity of \(X_{not}\) with \(\dot{\varepsilon}, T_W, d_{(j-1)},\) and \(\varepsilon - \varepsilon_c\), the equation (30) combined with equation (31) is differentiated as follows:

\[ \frac{1}{\ln(-\ln(X_{not}))} \cdot \frac{\partial}{\partial \ln \dot{\varepsilon}} \ln(-\ln(X_{not})) = -2(0.05) = -0.1 \]  

(32)

\[ \frac{1}{\ln(-\ln(X_{not}))} \cdot \frac{\partial}{\partial \frac{1}{T_W}} \ln(-\ln(X_{not})) = -2(6420) = -12840 \]  

(33)

\[ \frac{1}{\ln(-\ln(X_{not}))} \cdot \frac{\partial}{\partial \ln d_{(j-1)}} \ln(-\ln(X_{not})) = -2(0.28) = -0.56 \]  

(34)

\[ \frac{1}{\ln(-\ln(X_{not}))} \cdot \frac{\partial}{\partial \ln(\varepsilon - \varepsilon_c)} \ln(-\ln(X_{not})) = 2 \]  

(35)

After the equations have been linearized, a design matrix can be built upon the equations (25)-(35). In this design matrix, the grain size and volume fraction of not dynamically recrystallized grain are defined as DPs. The corresponding PVs are defined as temperature, strain, strain rate, and the initial grain size. The matrix can be written as:
\[
\begin{align*}
\begin{bmatrix}
\ln[d(j)] \\
\ln[-\ln(X_{not})]
\end{bmatrix} &=
\begin{bmatrix}
-0.27 & -8.674 & 0 & 0 \\
-0.10 & 12.840 & 2.0 & -0.56
\end{bmatrix}
\begin{bmatrix}
\ln\dot{\varepsilon} \\
\frac{1}{T_W} \\
\ln(\varepsilon - \varepsilon_c) \\
\ln d_{(j-1)}
\end{bmatrix}
\end{align*}
\] (36)

The design matrix can be decoupled by fixing the temperature and initial grain size. Then the matrix can be reduced to:

\[
\begin{align*}
\begin{bmatrix}
\ln[d(j)] \\
\ln[-\ln(X_{not})]
\end{bmatrix} &=
\begin{bmatrix}
-0.27 & 0 \\
-0.10 & 2.0
\end{bmatrix}
\begin{bmatrix}
\ln\dot{\varepsilon} \\
\ln(\varepsilon - \varepsilon_c)
\end{bmatrix}
+\begin{bmatrix}
A \\
B
\end{bmatrix}
\end{align*}
\] (37)

Where \( A = \ln(22,600) - 0.27 \frac{R}{T_W} \) and \( B = \ln(0.693) - 2 \ln(1.144 \times 10^{-5}) - \frac{12,840}{T_W} - 0.56 \ln d_{(j-1)} \)

**Static Recrystallization of Plain C-Mn Steel**

Similarly, a design matrix for static recrystallization of plain C-Mn steel is built based on Saito’s equations:

\[
d(j) = 0.5d_{(j-1)}^{0.67}\varepsilon^{-1}
\] (38)

\[
X_{rxn} = 1 - \exp[-0.693(\frac{t}{t_{0.5}})^2]
\] (39)

\[
t_{0.5} = 2.5 \times 10^{-19}d_{(j-1)}^2\varepsilon^{-4}\exp\left(\frac{Q_s}{RT}\right)
\] (40)
Where $Q_s = 300\text{KJ/mol}$, $R=8.314 \text{J/(g\cdot mol\cdot K)}$, and $t_{0.5}$ is the time interval for 50 volume percent recrystallization. The units of $T$, $t$ and $d$ are Kelvin, seconds and micrometers, respectively.

Those Saito’s equations can be put in the design matrix by using the same mathematical approach which employed for dynamic recrystallization. In this design matrix, the grain size and volume fraction of not recrystallized grain are considered as DPs. The corresponding PVs are defined as initial grain size, plastic strain, and temperature and dwell time.

Taking the logarithm of equation (38) gives:

$$\ln d(j) = \ln 0.5 + 0.67\ln d(j_{-1}) - \ln \varepsilon$$  \hspace{1cm} (41)

For evaluation of changes in grain size with plastic strain and initial grain size equation (41) is differentiated as follows:

$$\frac{\partial \ln[d(j)]}{\partial \ln \varepsilon} = -1.0$$  \hspace{1cm} (42)

And

$$\frac{\partial \ln[d(j)]}{\partial \ln[d(j_{-1})]} = 0.67$$  \hspace{1cm} (43)

For the evaluation of volume fraction of not dynamic recrystallized, $X_{not}$, equation (39) can be converted as:
Taking the logarithm of equation (44) gives

\[ \ln(X_{not}) = -0.693\left(\frac{t}{t_{0.5}}\right)^2 \]  

Then taking the logarithm of equation (45) gives

\[ \ln[-\ln(X_{not})] = \ln 0.693 + 2 \ln(t) - 2 \ln t_{0.5} \]  

Where \( \ln t_{0.5} \) can be express by logarithm of equation (40) which is

\[ \ln(t_{0.5}) = \ln(2.5 \times 10^{-19}) + 2 \ln d_{(j-1)} - 4 \ln \varepsilon - \frac{Q_s}{RT} \]  

Where \( Q_s = 300 \text{KJ/mol}, R = 8.314 \text{ J/(g·mol·K)}. \) To evaluate the sensitivity of \( X_{not} \) with \( \varepsilon, T, d_{(j-1)}, \) and \( t, \) the equation(46) combined with equation(47) is differentiated as follow:

\[ \frac{1}{\ln[-\ln(X_{not})]} \cdot \frac{\partial \ln[-\ln(X_{not})]}{\partial \ln \varepsilon} = -2(-4) = 8 \]  

\[ \frac{1}{\ln[-\ln(X_{not})]} \cdot \frac{\partial \ln[-\ln(X_{not})]}{\partial \frac{1}{T}} = -2(36,083) = -72,166 \]  

\[ \frac{1}{\ln[-\ln(X_{not})]} \cdot \frac{\partial \ln[-\ln(X_{not})]}{\partial \ln d_{(j-1)}} = -2(2) = -4 \]  

\[ \frac{1}{\ln[-\ln(X_{not})]} \cdot \frac{\partial \ln[-\ln(X_{not})]}{\partial \ln t} = 2 \]  

According to equations (41) to (51), the matrix can be written as:
The design matrix can be decoupled by fixing the temperature and initial grain size. Then the matrix can be decoupled to:

\[
\begin{pmatrix}
\ln[d(j)] \\
\ln[-\ln(X_{not})]
\end{pmatrix} =
\begin{bmatrix}
0.67 & -1.0 & 0 & 0 \\
-4.0 & 8.0 & -72,133 & 2.0
\end{bmatrix}
\begin{pmatrix}
\ln d_{(j-1)} \\
\ln \varepsilon \\
\frac{1}{T} \\
\ln t
\end{pmatrix}
\]  

(52)

The design matrix can be decoupled by fixing the temperature and initial grain size.

Then the matrix can be decoupled to:

\[
\begin{pmatrix}
\ln[d(j)] \\
\ln[-\ln(X_{not})]
\end{pmatrix} =
\begin{bmatrix}
-1.0 & 0 \\
8.0 & 2.0
\end{bmatrix}
\begin{pmatrix}
\ln \varepsilon \\
\ln t
\end{pmatrix}
+ \begin{pmatrix}
C \\
D
\end{pmatrix}
\]  

(53)

Where \( C = \ln 0.5 + 0.67 \ln d_{(j-1)} \) and \( D = \ln(0.693) - 2 \ln(2.5 \times 10^{-19}) - 2\left(\frac{36.083}{T}\right) \).

**Grain Growth of Plain C-Mn Steel**

When the recrystallization events completed, the workpiece material is still under hot working temperature, recrystallized grains will start growth. A design matrix for the grain growth behavior is derived from Yada’s grain growth equations:

\[
d^2 = d_0^2 + 1.44 \times 10^{12} t \times \exp\left(-\frac{Q_{gg}}{RT}\right)
\]  

(54)

Where \( \frac{Q_{gg}}{R} = 32,100 \).

From equation (54), grain growth behavior depends upon the initial grain size, grain growth time and temperature. Taking the logarithm of equation (54) gives:

\[
\ln(d^2 - d_0^2) = \ln 1.44 \times 10^{12} + \ln t - \frac{32,100}{T}
\]  

(55)
Here the initial grain size $d_0$ is assumed to be known. For evaluation of changes in grain size with grain growth time and temperature equation (55) is differentiated as follows:

$$\frac{\partial \ln(d^2 - d_0^2)}{\partial \ln t} = 1.0$$

(56)

And

$$\frac{\partial \ln(d^2 - d_0^2)}{\partial \ln \frac{1}{T}} = -32,100$$

(57)

According to equations (55) to (57), the matrix can be written as:

$$\{ \ln(d^2 - d_0^2) \} = [-32,100 \quad 1.0] \begin{bmatrix} \frac{1}{T} \\ \ln t \end{bmatrix} + \{ \ln 1.44 \times 10^{12} \}$$

(58)

**Design Matrix for Hot Rolling Process**

In order to precisely control the process parameters, a design matrix is built for connecting the \{FQs\} associated with hot deformation processes to the \{PVs\} associated with hot rolling process. In a typical hot rolling process, the \{PVs\} are determined by the equipment parameters \{EPs\} which including velocity of the rolls ($V$), roll radius ($R$), percent reduction in height ($r$), and initial work piece temperature ($T_B$). Here, \{PVs\} are plastic strain ($\varepsilon$), strain rate ($\dot{\varepsilon}$) and working temperature ($T_W$). The plastic strain ($\varepsilon$) is defined as:

$$\varepsilon = \ln(1 - r)$$

(14)
Strain rate for rolling process is determined by roll velocity, roll radius and height reduction, the relationship is represented as:

\[ \dot{\varepsilon} = V \sqrt{\frac{r}{R H_0}} \]  

(15)

The empirical relation for working temperature is known to depend upon roll velocity \( V \), roll radius \( R \), and initial work piece temperature \( T_B \). Where \( k, a, b \) are coefficients depend upon material and process. Natural logarithm relationships tend to be convenience to decoupling. However, this is not always the case, such as equation (14). In some condition, the relationship can be reasonable simplified into linear relationship. Here, the relationship between \( \varepsilon \) and \( r \) if \( \varepsilon \leq 0.2 \), then:

\[ \varepsilon \approx r \]  

(59)

In rolling process, the strain is relatively low; this simplified assumption is considered to be valid. Taking the logarithm of equation (59) gives:

\[ \ln \varepsilon = \ln r \]  

(60)

For analysis of changes in strain with \( r \) equation (60) is differentiated as follows:

\[ \frac{\partial \ln \varepsilon}{\partial \ln r} = 1.0 \]  

(61)

Taking the logarithm of equation (15) gives:

\[ \ln \dot{\varepsilon} = \ln V + 0.5 \ln r - 0.5 \ln (R h_0) \]  

(62)
For analysis of changes in strain rate with \( V \), and \( r \) equation (62) is differentiated as follows:

\[
\frac{\partial \ln \dot{\varepsilon}}{\partial \ln r} = 0.5
\]  

(63)

And

\[
\frac{\partial \ln \dot{\varepsilon}}{\partial \ln V} = 1.0
\]  

(64)

Similarly, taking the logarithm of equation (16) gives:

\[
\ln T_W = \ln k + \ln T_B + a \ln r + b \ln V
\]  

(65)

Equation (65) is differentiated to evaluate the sensitivity of \( T_W \) with \( T_B \), \( r \), and \( V \)

\[
\frac{\partial \ln T_W}{\partial \ln T_B} = 1.0
\]  

(66)

\[
\frac{\partial \ln T_W}{\partial \ln r} = a
\]  

(67)

and

\[
\frac{\partial \ln T_W}{\partial \ln V} = b
\]  

(68)

According to equations (60) to (68), the design matrix for rolling can be written as:

\[
\begin{bmatrix}
\ln \varepsilon \\
\ln \dot{\varepsilon} \\
\ln T_W
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
0.5 & 1 & 0 \\
a & b & 1
\end{bmatrix} \begin{bmatrix}
\ln r \\
\ln V \\
\ln T_B
\end{bmatrix} + \begin{bmatrix}
0 \\
-0.5 \ln (R\rho_0) \\
\ln k
\end{bmatrix}
\]  

(69)