PREDICTION OF PRESSURE DROP IN VERTICAL AIR/WATER FLOW IN
THE PRESENCE/ABSENCE OF SODIUM DODECYL SULFATE AS A
SURFACTANT

Thesis
Submitted to

The School of Engineering of the
UNIVERSITY OF DAYTON

In Partial Fulfillment of the Requirements for
The Degree
Master of Science in Chemical Engineering

By
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Dayton, Ohio
August, 2013
PREDICTION OF PRESSURE DROP IN VERTICAL AIR/WATER FLOW IN
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SURFACTANT

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ABSTRACT

PREDICTION OF PRESSURE DROP IN VERTICAL AIR/WATER FLOW IN THE PRESENCE/ABSENCE OF SODIUM DODECYL SULFATE AS A SURFACTANT

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The aim of this thesis is to develop a better approach for predicting pressure gradient in vertical multiphase flow with and without use of Sodium dodecyl sulfate (SDS) as a surfactant and to develop a program for the prediction of pressure drop by using Microsoft Visual Basic in Excel. Data was collected from four fixed liquid superficial velocity at different ranges of gas superficial velocity in a 0.052m i.d. and 10m long, clear PVC pipe. Results indicate that the addition of SDS resulted in reducing surface tension between phases from 72 to 64 mN/m, decreasing pressure drop by approximately 26% and also Hasan and Kabir model for Air/DI water and Hagedorn and Brown model in the presence of SDS in the mixture is the best model and leads to a reasonably accurate pressure gradient according to measured pressure drop.
ACKNOWLEDGMENTS

I would like to thank my thesis advisor, Dr. Robert Wilkens who was helpful and supportive and shared with me a lot of his time and experience during this work.

I would like to thank Miss. Jing Zhou and Mr. Innocent Akor with whom I worked in the lab during this work. The assistance of Mr. Michael Green in constructing the test equipment is appreciated.

I would like to express my gratitude to my parents for their endless support, and dedicate this work to them.
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1. INTRODUCTION

Accurately predicting pressure drop in wells is a broadly recognized problem in petroleum industries. This is related to the fact that large number of variables involved for predicting pressure even for limited conditions. Under some conditions, velocity of liquid along the pipe can change in a short distance and cause a different friction loss. Under other conditions, the liquid influence on the friction is negligible when it is entrained thoroughly in the gas. Also under some other conditions, liquid moves at a lower velocity than the gas. Therefore, density that would be obtained from the produced liquid-gas ratio is less than the in-situ density of liquid-gas mixture. Other problems related to flow patterns with different geometry, instabilities of the fluid in interfaces. The difference in geometry and velocity of the two phases strongly affect pressure drop. Pressure drop prediction is necessary because it effects on production rates, corrosion rates, operation costs and shear stresses between piping network.

Predicting pressure drop in multiphase flow is complex due to a secondary pressure drop which is results from the slippage. In two phases, differences between velocities cause slippage. Unlike single-phase flow, in multiphase flow pressure drop
does not always increase with an addition in rate of production or a reduction in the tubing size.

Many studies have been done to correlate friction loss and slippage drop in terms of a single energy loss factor similar to the factor used for single-phase flow, although many important variables such as surface tension, liquid velocity, and gas-liquid ratio are not completely accounted for.

Other investigations have been done to correct the static gradient. The remaining drops were related to friction factor. After applying correlations to data taken in a long pipe, the observed pressure drops are much less than calculated pressure drops. In many cases pressure drops obtained from corrected static gradients surpass the observed total pressure drops. This shows fractional volume for the long pipe which is occupied by liquid is smaller than the short distance pipe.

Drag reduction as a phenomenon happens when adding certain amount of drag reducing agent, such as polymers, surfactants, or fibers. Polymers increase the shear viscosity of a fluid. Surfactants can decrease the surface tension of a liquid. Fibers move in the main direction of the flow to decrease drag.

Drag reducers have two main effects on vertical flow. They decrease the frictional component of the pressure gradient but can also alter flow pattern and liquid holdup.
Since the flow is vertical, the increase in the liquid holdup increases the hydrostatic component of the pressure gradient.

Developing a better approach for predicted pressure gradient in vertical multiphase flow with and without use of drag reducing agent is the primary objective of this study. Also, using water and SDS surfactant solutions at concentration 96ppm and how it may lead to reduced pressure drop due to change in flow patterns and/or hold up is discussed and collected laboratory data is used with different models to develop an Excel VBA program that predicts pressure drop in vertical pipes with different flow patterns.

The approach taken to fulfill this objective is as follows:

1. To experimentally study the effects of surfactant on pressure drop in vertical pipes with different flow patterns.
2. To identify available models for pipeline calculations and to determine the best models for wellbore model design.
3. To develop an Excel VBA program to compute the model predictions of part 2.
4. To compare the program output with laboratory data both with and without surfactant (SDS).
2. LITERATURE REVIEW

2.1 Single-phase flow

In single-phase pipe flow, interactions between pipe wall and fluid can cause pressure drop. Depletion of pressure through pipeline may produce multiphase flow, even if just one phase is generated from reservoir.

Pressure-gradient equation

A pressure-gradient equation for steady state condition is obtained by adding conservation of momentum equation:

$$\frac{\partial}{\partial t} (\rho v) + \frac{\partial}{\partial L} (\rho v^2) = -\frac{\partial p}{\partial L} - \tau \frac{\pi d}{A} - \rho g \sin \theta \quad (2-1)$$

and conservation of mass equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial L} = 0 \quad (2-2)$$
Mass accumulation cannot occur in steady state condition. So

$$\frac{\partial (\rho v)}{\partial L} = 0 \quad (2 - 3)$$

By solving Eq.2-1 and 2-2 the pressure gradient in steady state situation obtains

$$\frac{dp}{dL} = -\tau \frac{\pi d}{A} - \rho g \sin \theta - \rho v \frac{dv}{dL} \quad (2 - 4)$$

Where $p$ is pressure, $L$ length, $\tau$ shear stress, $d$ diameter, $A$ flow area, $\rho$ density, $g$ gravity, $\theta$ inclination angle from vertical, and $v$ velocity.

Eq. 2-4 is also called the mechanical energy balance equation. From this equation the total pressure gradient in a steady state condition is the of sum of three components,

$$\left( \frac{dp}{dL} \right)_t = \left( \frac{dp}{dL} \right)_f + \left( \frac{dp}{dL} \right)_h + \left( \frac{dp}{dL} \right)_a \quad (2 - 5)$$

Where:

$$\left( \frac{dp}{dL} \right)_f = \frac{f \rho v^2}{2d} \quad \text{(Frictional pressure gradient)} \quad (2 - 6)$$

$$\left( \frac{dp}{dL} \right)_h = \rho g \sin \theta \quad \text{(Hydrostatic pressure gradient)} \quad (2 - 7)$$

$$\left( \frac{dp}{dL} \right)_a = -\rho v \frac{dv}{dL} \quad \text{(Acceleration pressure gradient)} \quad (2 - 8)$$
2.2 Multiphase flow

Behavior when there is two or more phases flow in the pipe is much more complex than single phase flow. Because of immiscibility and density differences, the phases tend to be separated. Shear stress for each phase in pipe flow is different because of fluid velocities and properties. The volumetric gas flow rate will be increased by decreasing pressure along the pipe and increasing the volume occupied by the compressible gas phase. Therefore, liquid and gas phases do not move at the same velocity. In a vertical pipe, liquid tends to flow at a lower velocity than the less dense, less viscous and more compressible gas. The gas often flows slower than the liquid in downward flow due to buoyancy.

Understanding of the basic flow pattern definitions, as well as parameters including fluid properties, velocities, and holdup is important to handle with the complex nature of multiphase flow.

2.2.1 Fluid properties

The two phases typically are considered in multiphase flow correlations are gas and liquid. There are many equations to explain the physical properties of liquid and gas mixtures.
Slip mixture density can be defined by

\[ \rho_s = \rho_L H_L + \rho_G (1 - H_L) \]  

(2 - 9)

And no-slip mixture density is defined

\[ \rho_n = \rho_L\lambda_L + \rho_G (1 - \lambda_L) \]  

(2 - 10)

Because of dynamic processes such as flow patterns, bubble size etc., there are different equations for mixture viscosity.

\[ \mu_s = \mu_L H_L + \mu_G (1 - H_L) \]  

(2 - 11)

\[ \mu_s = \mu_L^{H_L}\mu_G^{(1-H_L)} \]  

(2 - 12)

\[ \mu_n = \mu_L\lambda_L + \mu_G (1 - \lambda_L) \]  

(2 - 13)

2.2.2 Velocity

Velocities for gas or liquid phase are completely different. In the case of a homogeneous mixture such as high velocity annular flow regime and high turbulent dispersed bubble flow, the phase velocities are equal. For all other cases, slippage can happen between liquid and gas phases. Superficial velocity is expressed as the velocity if the fluid were to occupy the entire pipe area. Superficial gas velocity is given by:

\[ v_{SG} = \frac{q_G}{A} \]  

(2 - 14)
And for liquid

\[ v_{SL} = \frac{q_L}{A} \]  \hspace{1cm} (2 - 15)

The mixture or total velocity can be defined as the sum of liquid and gas superficial velocities.

\[ v_m = \frac{q_L + q_G}{A} = v_{SL} + v_{SG} \]  \hspace{1cm} (2 - 16)

Liquid and gas can flow at the mixture velocity when no slip occurs between phases. If there is slip between liquid and gas phases, the gas flows at a higher velocity and liquid at a lower velocity than the mixture velocity. When liquid holdup is known, time and space-averaged velocities or actual phase velocities are given by:

\[ v_L = \frac{q_L}{A_L} \]  \hspace{1cm} (2 - 17)

\[ v_G = \frac{q_G}{A_G} \]  \hspace{1cm} (2 - 18)

The slip velocity can be expressed as a difference between the actual liquid and gas velocities.

\[ v_s = v_G - v_L \]  \hspace{1cm} (2 - 19)
2.2.3 Holdup

Liquid holdup \((H_L)\) is defined as the proportion volume of the pipe that is occupied by the liquid phase. It is obtained experimentally by averaging the volume of gas or liquid in pipe versus volume of pipe. Determining liquid holdup is important for calculating liquid and gas viscosities, mixture density and pressure gradient. If volume fraction is occupied by gas phase, the holdup is expressed in terms of void fraction or gas holdup.

\[
H_L = 1 - H_G
\]  

(2 – 20)

Liquid holdup is a function of different variables such as pipe diameter, liquid and gas properties and flow patterns and should be obtained from empirical correlations.

In the concurrent flowing gas-liquid pipe flow, the gas normally moves faster than the liquid and it causes a slippage between the liquid and gas phase. The in-situ liquid volume fraction cannot be determined from input conditions because of this slippage. Determining hydrostatic head losses is important to predict liquid holdup.

Holdup generally is recognized between volume average,

\[
H_a = \frac{V_a}{V}
\]  

(2 – 21)
Line average

\[ H_a = \frac{L_a}{L} \quad (2 - 22) \]

And area average

\[ H_a = \frac{A_a}{A} \quad (2 - 23) \]

These three methods can give the same fluid fraction for homogeneous mixture.

In some places such as subsea pipelines, calculating fluid fractions and estimation of the liquid holdup are difficult. Therefore, no-slip liquid and gas fractions can be calculated with known volumetric flow rates.

\[ \lambda_L = \frac{q_L}{q_L + q_G} \quad (2 - 24) \]
\[ \lambda_G = \frac{q_G}{q_L + q_G} \quad (2 - 25) \]

Slip exists when there are different phase velocities. The slip ratio is given by the ratio between phase velocities.

\[ S = \frac{v_G}{v_L} \quad (2 - 26) \]
When slip exists the fluid fractions cannot be determined from equations (2-24) through (2-26) so they must be calculated by using:

\[
H_L = \frac{A_L}{A} = \frac{A_L}{A_L + A_G} = \frac{q_L}{q_L + \frac{1}{S}q_G} \quad (2 - 27)
\]
\[
H_G = \frac{A_G}{A} = \frac{A_G}{A_L + A_G} = \frac{q_G}{q_G + \frac{1}{S}q_L} \quad (2 - 28)
\]

Where \(H_L = \lambda_L, H_G = \lambda_G,\) and \(S=1.\)

### 2.2.4 Flow patterns

In general there are two flow regimes in single phase flow, turbulent and laminar flow. Multiphase flow can adopt various geometric formations, which are called flow patterns. In a vertical pipe with multiphase flow of liquid and gas, four flow patterns are commonly observed: bubble flow, slug flow, churn flow, and mist flow as shown in Figure 1.

#### 2.2.4.1 Bubble flow

Bubble flow occurs when liquid flows at high velocity and gas flows at low to high velocity. The liquid is almost totally occupies the pipe and moves up with an equal velocity and different density. The gas bubbles have approximately the same size
throughout but different velocities and are randomly distributed in the pipe cross-section. This flow pattern has a little effect on the pressure gradient.

2.2.4.2 Slug flow

Slug flow occurs when liquid flows at low to mediocre velocity and gas flows at mediocre velocity. This flow pattern includes large bullet shaped gas-bubbles and small ones flowing in between. The gas-bubbles unify and create bubbles with the same shape and size; however the liquid phase is still the continuous phase. The velocity of the bubble is greater than velocity of the liquid. At the beginning of flow, slugs have irregular length at indefinite intervals. By increasing gas flow rate, slugs flow at regular intervals and identical length.

2.2.4.3 Churn flow

Froth or churn flow happens when liquid flows at low to mediocre velocity and gas flows at high velocity. It is a very unstable and chaotic flow in which the gas bubbles and the liquid slugs appear to be more distorted. This flow could be recognized from slug flow on the basis of fast moving, distorted, and short length slugs. The liquid also churns as it surges upwards and back down.
2.2.4.4 Annular/mist flow

Annular or mist flow happens when gas flows at very high velocity. It consists of an annular film of liquid on the wall and gas with small liquid droplets in the center of the pipe. In this flow, when liquid drops are moved from the liquid film through the core of the flow, a phenomenon which is called entrainment can occur. When the drops return to the well it is called deposition.

*Figure 1: Two-phase vertical flow patterns [24]*
Accurately predicting flow pattern is necessary for calculating holdup and predictions of pressure drop. Flow pattern characterizations are obtained from a flow pattern map which relates flow regimes to fluid properties, geometry and flow rates.

2.2.5 Pressure-gradient correlations

Two phase pressure drop in a pipe is generally a sum of three $\Delta P$ terms including a friction, gravity and acceleration pressure drop. The acceleration pressure drop is normally negligible and only is considered when there are high flow velocities.

$$\Delta P = \Delta P_f + \Delta P_g$$  \hspace{1cm} (2-29)

The friction term is based on changes in viscosity and inter-phase friction between gas-liquid phases. The gravity or elevation term depends on the two phase mixture density. Most of the pressure drops are encompassed by this term in vertical flow except for high velocity conditions.

There are many different pressure drop correlations. In this project, six multiphase flow models or correlations (Hagedorn and Brown, Duns and Ros, Orkiszewski, Aziz, Mukherjee and Brill and Hasan and Kabir) were used for predicting the pressure drop in vertical multiphase flow.
2.2.5.1 Hagedorn and Brown Method

This method is developed from data calculated in a 1500 ft two-phase vertical well created to study pressure gradients. Experimental analysis was done along the gas phase (air), with four different liquid phases: oil with viscosities between 10 and 110sp in 80° F and water. Tubing diameters were between 1 and 2in. The resulting method is independent of flow patterns. In this method, correlations were developed from the test results to accurately predict pressure gradients for a wide range of flow rates, tubing sizes and liquid properties. These correlations apply to flow in all pipe orientations by calculating the hydrostatic pressure difference using only the vertical elevation of the pipe segment and the friction pressure drop based on the total pipeline length. However, they did not measure liquid holdup. Rather, Hagedorn and Brown developed an equation that guessed a friction factor correlation, allowed calculating of pseudo liquid holdup for each measurement in order to match measured pressure gradient.

The following results were obtained in this method:

- By defining a Reynolds number from a friction factor diagram, a two-phase flow friction factor can be determined in the same way as single-phase friction factor.
- The generalized correlation was developed without separating two-phase flow into the different flow regimes.
- The pressure drop due to a change in kinetic energy can account for a percentage of the total pressure drops, especially near top of the well.
The pressure drops are accurately predicted for different tubing sizes ranging from 1-1.5in. Pressure drop is over predicted by further increasing in tube size more than 1.5 in.

The pressure drops are accurately predicted for a wide range of water-cuts.

Some modifications have been done to improve this method over the years.

- Using Griffith and Wallis correlations for accurately calculating bubble flow pressure gradients.
- Using Duns and Ros acceleration pressure gradient instead of Hagedorn and Brown one.
- Replacing liquid holdup values with no-slip holdup when bubble flow is predicted to occur and revised hold up for slug flow pattern.

2.2.5.2 Duns and Ros Method

The Duns and Ros method is developed for vertical liquid-gas flow in wells. It is a result of 4000 two-phase flow tests in a 185 ft. vertical pipe with diameters ranging from 1.26 to 5.60in. in which pressure drop and liquid holdup were measured. Most of the experiments were done at atmospheric conditions with water as the liquid and air as gas phase. There are three flow patterns in this method and for each of them friction factor and liquid holdup correlations were developed. This method is valid for varying ranges of gas-oil mixture and flow patterns. It is applicable for both dry and wet gas/oil mixtures.
The following results were obtained in this method:

- This method is not suitable for three phase of gas-oil -water. It works well in bubble, slug, and churn flow patterns for water-cut less than 10%.
- For a wide range of gas-liquid ratio, the pressure drop is over predicted and errors are more than 20 percent for gas-liquid ratio more than 5000.
- The pressure drop is over predicted for a range of pipe diameters from 1 to 3in.

There are two main modifications to this method. The Ros field method includes modifications which were obtained from seventeen gas-liquid ratio vertical oil wells and the Moreland-Mobil-Shell method in which holdup correlations obtained for bubble and slug flow are simpler than those obtained in Duns and Ros method.

2.2.5.3 Orkiszewski Method

The Orkiszewski method is proper only for vertical two-phase flow and valid for different flow patterns. The bubble flow correlation was improved with the Griffith and Wallis method, the slug flow with Hagedorn and Brown method and the mist flow with the Duns and Ros method. The main differences of the Orkiszewski method over most others are that pressure drop is related to flow pattern changes and geometrical distribution of gas and liquid phases, the holdup is obtained from observed physical phenomena, and it describes a good comparison of what occurs in the pipe.
The following results were obtained in this method:

- The pressure drop is over predicted for pipe sizes more than 2 in. For a range of pipe sizes from 1 to 2 in, this method performs well.
- The accuracy of this method is very good for gas-liquid ratio up to 5000 and errors are more than 20 percent for gas-liquid ratio more than 5000.

Orkiszewski calculated the liquid distribution coefficient, \( \Gamma \), from Hagedorn and Brown data. There are two modifications for this method.

The first modification is to replace the equation which is defined for calculating the liquid distribution coefficient, \( \Gamma \), with following equation.

\[
\Gamma = \frac{0.013 \log \mu_L}{d^{1.38}} - 0.287 - 0.162 \log v_m - 0.428 \log d
\]  

(2 – 30)

This modification can affect the accuracy of result.

This coefficient \( \Gamma \), sometimes becomes a large negative number for high flow rates according to large values of mixture velocity. In this case, slip density should be replaced with no-slip density and friction pressure gradient should be calculated from following equations instead of the equation which is defined in appendix (Orkiszewski correlations).

\[
\left( \frac{dP}{dZ} \right)_f = \frac{f \rho_L v_m^2}{2d} \left[ \frac{(v_m + v_b)}{v_m + v_b} + \Gamma \right]
\]  

(2 – 31)
2.2.5.4 Aziz Method

“They proposed a multiphase flow method that depended on the flow pattern for predicting pressure drop in wells producing oil and gas at gas-oil ratios between 150 and 1600 scf/bbl. They compared actual field data on 48 wells with different empirical prediction methods including Hagedorn and Brown, Duns and Ros and Orkiszewski and a proposed method based on the mechanism of flow, the proposed method gave results with accuracy equal to the best of the empirical methods. Most of the test data showed that single-phase (liquid), bubble and slug flow were encountered respectively in the lower, middle and upper reaches of the pipe. Four flow patterns are considered in this method including: bubble, slug, transition, and annular-mist. They presented original correlations of predicting pressure drop for the bubble and slug flow patterns and used the method of Duns and Ros for the transition and annular-mist flow patterns.” [4]

Aziz et al. multiphase method can be developed by replacing its flow-regime map with Duns and Ros flow-pattern map. “A significant improvement has been observed giving an overall absolute average percent deviation of 2.16% compared with 5.33% for the original correlation. Also, with this combination the relative performance factor has been reduced to 1.33 from 2.90 for the original Aziz et al. correlation. The efficiency of the Aziz et al. and Duns and Ros combinations at high oil production rates, when compared with the original Aziz et al. correlation, was further examined by noticing the
improvement gained in statistical measures at higher levels of superficial liquid velocity.”[2]

2.2.5.5 Mukherjee and Brill Method

In the Mukherjee and Brill method pressure drop behavior in two phase inclined flow was studied and modifications used to improve the Beggs and Brill method. “Their test facility consisted of an inverted U-shaped 3.8-cm id nominal steel pipe. The closed end of the U-shape test sections could be raised or lowered to any angle from 0 to ±91 degree from horizontal. Each leg of the U was 17 m long with 6.7 m entrance lengths followed by 9.8 m long test sections on both uphill and downhill sides. Pressure taps 9.3 m apart were located in each test section to permit measuring absolute and differential pressure. Approximately 1,000 pressure drop measurements were obtained for a broad range of gas and liquid flow rates.”[15]

They reached to following conclusions:

- This method is valid for both downward and upward two phase flow.
- Their method depended on flow pattern for each angle of inclination.
2.2.5.6 Hasan and Kabir Method

Hasan and Kabir developed correlations which are dependent on flow patterns for predicting two-phase oil/gas pressure drop in vertical wells. In this method, flow patterns such as bubble, dispersed bubble, slug, churn and mist are considered while developing appropriate correlations for predicting pressure drop and void fraction in each flow pattern. 115 field experiments were performed in this method. They compared their correlations with Aziz et al, Duns and Ros, Hagedorn and Brown and Orkiszewski methods.

The following conclusions were obtained in Hasan and Kabir method:

- When vertical multiphase flow is influenced by any single phase including bubble and slug flow patterns, this method is consistent with Aziz et al and Orkiszewski methods.
- This method has better performance in churn and mist flow patterns.
- It was suggested that the correlations used for slug flow also be used for churn flow.

All of these correlations are used in chapter 4 to predict pressure drop in vertical two phase flow and also develop an Excel VBA program to predict pressure drop in vertical pipes with different flow patterns and by different methods, which is mentioned in Appendix B.
2.2.6 Drag reducing agents

Many wells are placed in remote sites such as subsea. Furthermore transportation of multiphase mixtures, which often includes water-gas-oil in large-diameter pipelines, are more expensive than smaller-diameter ones in which pressure drop can be significant. The benefits of drag reducing agent are reduced operation costs such as power required for pumps, reduced frictional pressure drop, increased production without change in mechanical operations, and improved refinery. Both pipeline diameter reduction and lower pumping costs are the benefits of adding drag reducing agent in design of new systems.

Drag reducing agent is usually polymers such as polyethylene oxide (PEO), polymethylmethacrylate (PMMA) and polymethylacrylate (PMA).

There are very few studies on the effect of DRAs in vertical multiphase flow. For example,

“Kang and Jepson considered the effect of polymeric drag reducing agents on pressure gradient in multiphase vertical flows for superficial gas/liquid velocities of 0.4 to 2 m/s and 0.5 to 10 m/s with light condensate oil in a 10 cm pipe diameter. The average pressure drop increased with increasing superficial liquid velocity for the same superficial gas velocity. This is due to the increase in the liquid volume in pipe as the superficial liquid velocity increases. The average pressure drop decreased with increasing
superficial gas velocities for all cases. The increased void fraction in the flow reduced the density of the flowing mixture and hence the pressure drops due to the hydrostatic head. The addition of DRA had little or no effect on the average pressure drop. DRA did help "smooth" the flow by reducing the levels of pressure fluctuations. The DRA is more effective in churn and slug flow where a DRA concentration of 50 ppm produces a much larger decrease in the pressure fluctuations. However, some benefit was seen at 10 ppm of DRA. There are small reductions in bubble flows. The DRA does not seem effective at the high superficial liquid velocity where the hydrostatic head is very large. The DRA also was not effective at high superficial gas velocities when annular flow was present.\textsuperscript{[12]}

The effectiveness of a drag reduction agent can be defined:

\[
\%DR = \frac{\Delta P_{\text{without RA}} - \Delta P_{\text{with DRA}}}{\Delta P_{\text{without DRA}}} \times 100\% \tag{2 - 32}
\]
3. EQUIPMENT, METHODOLOGY

Pressure drop data for vertical air/water flow rates was obtained in a 0.052 m i.d. and 30 m long, clear PVC pipe. Flow rates studied ranged from 0.12 to 0.72 m/s liquid superficial velocity and from 0.33 to 13.25 m/s gas superficial velocity.

3.1 Equipment

The equipment used in this study was shown in Figure 2. It is capable of measuring air superficial velocities in the range of 0.35 to 15 m/s, and liquid superficial velocities in the range of 0.03 to 1.25 m/s. It includes a 200 gallon tank, which holds the test liquid and acts as a final separator at the end of primary separator drain. Air is drawn from an air compressor which operates at 100 psi and passes through a metering section on its way to the pipe inlet. Pressure and air flow rate are controlled by a pressure regulator and a rotameter valve.

The liquid is pumped by a 3hp centrifugal pump through a globe valve that regulated flow rate. The pipe is approximately 10 m long. The liquid flow empties into the separation tank which is open to atmosphere.
Figure 2: Schematic diagram of the multiphase flow loop
Three differential pressure transducers which are shown in Figures 3 and 4 were used to measure the pressure drop of the flow across a 10 m long PVC pipe. They are one Keller Valueline pressure transmitter with 5±0.0025 psi pressure range, 4-20 mA output and two GE Druck UNIK5000 pressure transmitter PTX5012 series with 5±0.002 psi pressure range, 4-20mA output and ¼” MNPT process connection.

*Figure 3: Keller Valueline Pressure Transmitter [25]*
The bottom pressure transducer is about 1 m above the air/liquid mixing nozzle. The distance between two other pressure transducers is 1.55 m.

An Omega FTB106 liquid turbine flowmeter and an omega FL904P liquid Rotameter were used to measure liquid flow rate. The turbine range measurement is between 4 to 60 gpm with an accuracy of ±0.5% and the range of rotameter is between 0 to 34.5 gpm with an accuracy of ±2.0%.
Three gas rotameters are used in parallel in the system. An Omega FL7722A gas rotameter has the highest range up to 250 scfm, with an accuracy of ±2.0% which is calibrated at 70°F and 100 psig.

The electrical capacitance sensor which, is shown in Figure 5, measures the resistance of an electrical system and includes parallel wires with thin diameter and two electrodes. Calibration of this sensor was mentioned in Appendix C.
3.2 Methodology

For the first part of the experiment considering of air and water, deionized water was added to the separation tank and in different ranges of liquid and gas flow rates, pressure drops were measured.

For the second part of experiment, for water, air and SDS with concentration 96 ppm, the deionized water with appropriate surfactant (in this study sodium dodecyl sulfate or SDS as a surfactant) added to tank. The initial surfactant amount was estimated from the amount of water in the separation tank. Solutions were mixed for five minutes. After that, by getting sample from tank, surface tension, viscosity and surfactant concentration according to Appendix C are measured. Concentration was adjusted by adding surfactant or water as required. Liquid and gas flow rate were adjusted to desired value and by recording holdup, pressure, gas and liquid flow rate from PC, pressure drop was predicted by using different correlation which are mentioned in the Appendix A.
4. RESULTS AND DISCUSSION

The pressure drop results from the two pressure transducers $P_{s1}$ and $P_{s2}$ was measured at fixed superficial liquid velocities (vsl) of 0.12, 0.24, 0.48 and 0.72 m/s with gas superficial velocity (vsg) in the range of 0.3 to 9.0 m/s. The gas used was air and the liquid was DI water (both with and without SDS). Measured pressure drop was compared to six prediction pressure drops of different models including Hagedorn and Brown (H&B), Duns and Ros (D&R), Orkiszewski, Aziz, Mukherjee and Brill (M&B), and Hasan and Kabir (H&B). The overall model performances were compared by calculating relative performance factor. Flow pattern and pressure drop of different models and measured pressure drop at different fixed liquid superficial velocity and different ranges of gas superficial velocity are shown in Tables 3 through 6 in Appendix E.
The pressure drop measured at a fixed superficial liquid velocity of 0.12 m/s at gas superficial velocities of 0.50 to 8.27 m/s is shown in Figure 6. Total pressure drop decreased with increasing superficial gas velocity due to reduced hydrostatic head which is the most dominant part of the total pressure drop (whereas frictional pressure drop is a small part of that in many cases). For example, measured pressure drop at a vsg of 0.50 m/s was 5299 Pa/m while only 2041 Pa/m at 8.27 m/s superficial gas velocity. For superficial gas velocity less than 3.66 m/s, Hasan and Kabir, Aziz, Mukherjee and Brill and Orkiszewski predictive models had the same trend and they were consistent with measured pressure drop. From 3.66 to 8.27 m/s, Hasan and Kabir was the only model consistent with measured pressure drop, however, Mukherjee and Brill, Aziz and Orkiszewski varied due to the fact that Mukherjee & Brill predicted bubble/slug, Orkiszewski slug/churn and Aziz slug/churn flow pattern as compared to Hasan and Kabir that predicted all slug. The Hagedorn and Brown prediction was very low at this fixed superficial liquid velocity as compared to the rest models. This could be attributed to the fact that this model did not predict the pressure drop from the flow patterns but from correlations on which case the no-slip holdup was used rather than the slip hold up. The Duns and Ros model gave the lowest pressure drop from gas superficial velocity of 1.66 to 8.27 m/s due to change in flow pattern from slug to churn. Duns and Ros and Hagedorn and Brown were poor and under-predicted models at fixed liquid superficial velocity of 0.12 m/s air/DI water flow. Hasan and Kabir led to a reasonably accurate pressure gradient according to measured pressure drop for fixed 0.12 m/s vsl. The Aziz model could be improved if it gets modified by using predicted pressure drop of slug.
flow instead of transition region from 3.66 to 8.27 m/s vsg.

Plot of measured and predicted pressure drop at fixed 0.12 m/s vsl for water with SDS is shown in Figure 7. The sodium dodecyl sulfate (SDS) added to deionized water at 96 ppm. The protocol for calculating the SDS concentration is given in the Appendix D. The pressure drop was measured at a fixed liquid superficial velocity flow rate of 0.12 m/s at different gas flow rates with gas superficial velocities of between 0.50 to 8.28 m/s. Pressure drop decreased by adding SDS, which reduces the interfacial molecular force (surface tension) between phases, thereby decreasing the holdup and reducing pressure loss. The surface tension decreased from 72 N/m to 64 mN/m after adding SDS. For example, measured pressure drop at a vsg of 0.50 m/s was 3411 Pa/m while only 1290 Pa/m at 8.28 m/s superficial gas velocity. This is a 35% decrease relative to just Air/DI water. The Duns and Ros model was best up to 1.68 m/s which predicted slug flow, but from 1.66 to 8.28 m/s there was a reduction with this model due to the changing flow patterns from slug to churn and it was under-predicted by the model. The Hagedorn and Brown model had the best pressure drop prediction from 1.66 to 8.28 m/s of superficial gas velocity. Other models were poor and inconsistent with measured pressure drop. They had the same trend up to 3.66 m/s vsg and from 3.66 to 8.28 m/s different trends because of changing in flow patterns. Orkiszewski and Hasan & Kabir were over-predicted pressure drop at fixed 0.12 m/s vsl SDS. Duns and Ros was the best model up to 1.68 m/s, and then H&B was the best model at fixed 0.12 m/s vsl SDS.
Figure 6: Plot of measured and predicted pressure drop at fixed 0.12 m/s vsl air/DI water flow.
Figure 7: Plot of measured and predicted pressure drop at fixed 0.12 m/s vs. SDS
Plot of measured and predicted pressure drop at fixed 0.24m/s vsl air/DI water flow is shown in Figure 8. The liquid superficial velocity was fixed at 0.24m/s while the gas superficial velocity varied from 0.33 to 7.62m/s. Total pressure drop decreased with increasing superficial gas velocity due to reduced hydrostatic head which is the most dominant part of the total pressure drop. Measured pressure drop at 0.33m/s was 6412 Pa/m while only 2469 Pa/m at 7.62m/s superficial gas velocity. It is observed that as the liquid flow rate increased, there was a slight increase in the pressure drop as compared to 0.12m/s superficial liquid velocity. The predictive models, Hasan and Kabir, Aziz, Orkiszewski and Mukherjee and Brill were accurate models up to 3.66m/s vsg and more than 3.66m/s only Hasan and Kabir and Mukherjee and Brill were accurate models due to the fact that Aziz predicted slug/transition region and Orkiszewski predicted slug/churn flow pattern from 3.66 to 7.62 m/s as compared to Hasan and Kabir and Mukherjee and Brill that predicted all slug. The Hagedorn and Brown model was very low at this fixed superficial liquid velocity as compared to the rest of the models. This could be attributed to the fact that the Hagedorn and Brown model did not predict the pressure drop from the flow patterns but from correlations on which case the no-slip holdup was used rather than the slip holdup. The Duns and Ros model gave the lowest pressure drop from superficial gas velocity of 0.99 to 7.62 m/s due to change in flow pattern from slug to churn. Duns and Ros and Hagedorn and Brown were poor and under-predicted models at fixed liquid superficial velocity of 0.24m/s air/DI water flow. The Hasan and Kabir and Mukherjee and Brill lead to a reasonably accurate pressure gradient according to measured pressure drop for fixed 0.24m/s. vsl. The Aziz model could be improved if it gets modified by
using predicted pressure drop of slug flow instead of transition region from 3.66 to 7.62 m/s vsg.

The pressure drop measured at a fixed liquid superficial velocity flow rate of 0.24 m/s for water with SDS at different gas flow rates with gas superficial velocities of between 0.33 to 7.62 m/s as shown in Figure 9. The pressure drop decreased by adding SDS which was confirmed from the reduced calculated measured pressure drop in Figure 9. For example, measured pressure drop at a vsg of 0.50 m/s was 5785 Pa/m while only 1696 Pa/m at 7.62 m/s superficial gas velocity. This is a 33% decrease relative to just Air/DI water. The Duns and Ros model was best up to 1.00 m/s which predicted slug flow, but from 1.00 to 7.62 m/s there was a reduction with this model due to the changing flow pattern from slug to churn and it was under-predicted model at this velocity. The Hagedorn and Brown had the best prediction of pressure drop from 1.00 to 7.62 m/s of superficial gas velocity. Other models were poor and inconsistent with measured pressure drop. They had the same trend up to 3.65 m/s vsg and from 3.65 to 8.28 m/s, they gave a different trend because of changing in flow pattern. Orkiszewski, Mukherjee and Brill and Hasan and Kabir were over-predicted models at fixed 0.24 m/s vsl SDS. The Duns and Ros method was the best model up to 1.00 m/s, and then Hagedorn and Brown was the best model at fixed 0.24 m/s vsl SDS.
Figure 8: Plot of measured and predicted pressure drop at fixed 0.24 m/s vsl air/DI water flow
Figure 9: Plot of measured and predicted pressure drop at fixed 0.24 m/s vs. SDS
The plot of pressure drop vs. superficial gas velocity is shown in Figure 10. From this plot, total pressure drop decreased with increasing superficial gas velocity due to reduced hydrostatic head which is the most dominant part of the total pressure drop (whereas frictional pressure drop is a small part of that in many cases). The liquid superficial velocity was fixed at 0.48m/s while the gas superficial velocity varied from 0.33 to 7.64m/s. Measured pressure drop at a vsg of 0.33m/s was 7183 Pa/m while only 2974 Pa/m at 7.64m/s superficial gas velocity. It was observed that as the liquid flow rate increased, there was a slight increase in the pressure drop as compared to 0.24m/s superficial liquid velocity. The predictive models, Hasan and Kabir, Aziz, Orkiszewski and Mukherjee and Brill were accurate models up to 2.33m/s vsg and at this gas superficial velocity Orkiszewski varied due to change in flow pattern from slug to churn. From 2.33 to 3.66m/s, Hasan and Kabir, Aziz and Mukherjee and Brill had the same trend with measured pressure drop and were good models, however, Mukherjee & Brill varied due to change in flow pattern from slug to churn. From 3.66 to 4.98m/s, Hasan and Kabir and Aziz models predicted same trend with measured pressure drop and were good models, however, Aziz varied due to change in flow pattern from slug to churn as compared to Hasan and Kabir that predicted all slug. The Hagedorn and Brown prediction was very low at this fixed superficial liquid velocity as compared to the rest of the models. This could be attributed to the fact that this model did not predict the pressure drop from the flow patterns but from correlations in where the no-slip holdup was used rather than the slip holdup. The Duns and Ros model was consistent with measured pressure drop at low gas superficial velocity up to 0.66m/s and varied at 0.66m/s due to
the change flow pattern from bubble to slug. It gave the lowest pressure drop from gas superficial velocity of 1.66 to 7.64m/s due to change in flow pattern from slug to churn. Duns and Ros and Hagedorn and Brown were poor and under-predicted models at fixed liquid superficial velocity of 0.48m/s air/DI water flow. The Hasan and Kabir model led to a reasonably accurate pressure gradient according to measured pressure drop for fixed 0.48m/s. vsl. Aziz model could be improved if it gets modified by using predicted pressure drop of slug flow instead of transition region from 4.97 to 7.64m/s vsg.

Plot of measured and predicted pressure drop at fixed 0.48m/s vsl SDS vs. the superficial gas velocity which was ranged between 0.33-7.63m/s is shown in Figure 11. The liquid superficial velocity was fixed at 0.48m/s in the same manner as the air/water. It was observed that pressure drop decreased by adding SDS which reduces the interfacial molecular force (surface tension) between phases. For example, measured pressure drop at a vsg of 0.33m/s was 7172 Pa/m while only 2227 Pa/m at 7.63m/s superficial gas velocity. This is an 18% decrease relative to just Air/DI water. At a low gas superficial velocity of 0.33 to 0.66m/s, Aziz, Mukherjee and Brill and Hasan and Kabir models were consistent with measured pressure drop. But from 0.66 to 7.63m/s, these models became inconsistent with measured pressure drop due to changes in flow pattern. Orkiszewski had the same trend with measure pressure drop from 0.33 to 0.66m/s which predicted bubble flow. It varied at 0.66m/s due to changes in flow pattern from bubble to slug. It became consistent again with measured pressure drop from 3.65 to 7.63m/s. It varied in flow pattern from slug to churn at 3.65m/s. Duns and Ros was good model up to 1.67m/s
which predicted slug flow, but from 1.67 to 7.63 m/s there was a reduction with this model due to changes in flow pattern from slug to churn and under-predicted model at this velocity. Hagedorn and Brown was the best model and leaded to a reasonably accurate pressure gradient according to measured pressure drop for fixed 0.48 m/s. vsl SDS.
Figure 10: Plot of measured and predicted pressure drop at fixed 0.48m/s air/DI water flow
Figure 11: Plot of measured and predicted pressure drop at fixed 0.48 m/s vsl SDS
The plot of pressure drop vs. superficial gas velocity at fixed 0.72 m/s vsl air/DI water flow is shown in Figure 12. From this plot, total pressure drop decreased with increasing superficial gas velocity because of reduced hydrostatic head which is the most dominant part of the total pressure drop. The liquid superficial velocity was fixed at 0.72 m/s while the gas superficial velocity varied from 0.33 to 6.29 m/s. Measured pressure drop at a vsg of 0.33 m/s was 7828 Pa/m and 3444 Pa/m at 6.29 m/s superficial gas velocity. It was observed that as the liquid superficial velocity was being increased, there was a slight increase in the pressure drop as compared to liquid superficial velocity of 0.48 m/s. From the predictive models, Hasan and Kabir, Aziz, Orkiszewski and Mukherjee and Brill were good models. However, Aziz varied from 5.64 to 6.29 m/s, Mukherjee & Brill from 4.32 to 6.29 m/s and Orkiszewski from 2.99 to 6.29 m/s. This could be due to changes in flow pattern from slug to churn for these three as compared to Hasan and Kabir that predicted all slug. Duns and Ros and Hagedorn and Brown models were poor at this fixed liquid superficial velocity of 0.72 m/s air/DI water flow. The Hasan and Kabir model led to a reasonably accurate pressure gradient according to measured pressure drop for fixed 0.72 m/s vsl. Aziz model could be improved if it gets modified by using predicted pressure drop of slug flow instead of transition region from 5.64 to 6.29 m/s vsg.
Plot of measured and predicted pressure drop at fixed 0.72m/s vsl SDS is shown in Figure 13. The liquid superficial velocity was fixed at 0.72m/s in the same manner as the air/water while the superficial gas velocity was between 0.33-6.30m/s. It was observed that pressure drop decreased by adding SDS which reduces the interfacial molecular force (surface tension) between phases. This is confirmed from the Figure 13 with the reduced calculated measured pressure drop. For example, measured pressure drop at a vsg of 0.33m/s was 7688 Pa/m while only 2801 Pa/m at 6.30m/s superficial gas velocity. This is a 19% decrease relative to just Air/DI water. The Duns and Ros model was accurate model up to 1.67m/s which predicted slug flow, but from 1.67 to 6.30m/s there was a reduction with this model due to changes in flow pattern from slug to churn. The Duns and Ros model was an under-predicted model at this velocity. The Hagedorn and Brown model was the best model and led to a reasonably accurate pressure gradient according to measured pressure drop for fixed 0.72m/s vsl SDS. Other models were poor, except Orkiszewski, which was consistent with measured pressure drop from 2.99m/s in which changing flow pattern from slug to churn happened.
Figure 12: Plot of measured and predicted pressure drop at fixed 0.72 m/s vsl air/DI water flow
Figure 13: Plot of measured and predicted pressure drop at fixed 0.72 m/s vs. SDS
Hagedorn and Brown, Duns and Ros, Orkiszewski, Aziz, Mukherjee and Brill and Hasan and Kabir methods are used for the comparison which, is done by defining a relative performance:

$$R_{rp} = \frac{|E_1| - |E_{1min}|}{|E_{1max} - |E_{1min}|} + \frac{E_2 - E_{2min}}{E_{2max} - E_{2min}} + \frac{E_3 - E_{3min}}{E_{3max} - E_{3min}} + \frac{|E_4| - |E_{4min}|}{|E_{4max} - |E_{4min}|} + \frac{E_5 - E_{5min}}{E_{5max} - E_{5min}} + \frac{E_6 - E_{6min}}{E_{6max} - E_{6min}}$$

(4–1)

In which $E_1$ is average percent error, $E_2$ absolute average percent error, $E_3$ percent standard deviation, $E_4$ average error, $E_5$ absolute average error, $E_6$ standard deviation. The maximum and minimum possible values for relative performance factor are 6 and 0, respectively.

The relative performance factors of six models in different fixed liquid superficial velocity for DI Water and SDS is shown in Table 1. According to this table, Hasan and Kabir method has the best performance for DI water for different fixed liquid superficial velocity and performance of Hagedorn and Brown method is the best for SDS. For slug flow, the performance of Hasan and Kabir method is the best for DI water. For slug flow in presence of SDS, for 0.12 and 0.48 m/s vsl Mukherjee and Brill method, for 0.24 m/s vsl Duns and Ros method and for 0.72 m/s vsl Orkiszewski method is superior.
Table 1: Relative performance factors

<table>
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<th>Model</th>
<th>DI Water EDB</th>
<th>SNH</th>
<th>SDS EDB</th>
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EDB = entire data, SNH = all cases with 100% slug flow pattern without Hagedorn & Brown model.
5. CONCLUSIONS

Total pressure drop decreased with increasing superficial gas velocity due to hydrostatic head which is the most dominant part of the total pressure drop whereas frictional pressure drop is a small part of the pressure drop in many cases.

Sodium dodecyl sulfate (SDS) as a surfactant which reduces the interfacial molecular force (surface tension) between phases from 72 N/m to 64 mN/m and changing the flow pattern, decreased pressure drop by approximately 26%.

The influence of surfactant on pressure gradients is greater at lower liquid superficial velocities than at higher ones.

The pressure gradients of surfactant solutions are lower than pressure gradients of DI water especially in slug and slug/churn flow patterns.

The Hasan and Kabir model is the best model and leads to a reasonably accurate pressure gradient according to measured pressure drop for these four fixed liquid superficial velocity air/DI water flows at different ranges of gas superficial velocity.

The Hagedorn and Brown model is the best model and leads to a reasonably accurate pressure gradient according to measured pressure drop for these four fixed liquid
superficial velocities at different superficial gas velocity ranges after the surfactant is added to the liquid phase.

The Duns and Ros method is accurate method at low gas superficial velocities from 0.33 to 1.00 m/s for these four fixed liquid superficial velocities in the presence of SDS in mixture.

Models are missing something which is related to the liquid holdup, not a lot to the reduction of surface tension
6. RECOMMENDATIONS

The Aziz model could lead to a reasonably accurate pressure gradient if it is modified by using predicted pressure drop of slug flow instead of the transition region.

The Duns and Ros model can be improved by considering following modifications:

- Using friction factor obtained from a moody diagram as a function of a Reynolds number instead of the friction equation mentioned in this method.
7. REFERENCES


27. http://www.fishersci.com/ecom.YELLOW/servlet/fsproductdetail_10652_1660329-1_0
APPENDIX A

A-1 Hegedorn and Brown correlations

(1) Calculate Liquid velocity number:

\[ N_{LV} = \sqrt[4]{\frac{\rho_L}{\sigma_L g} v_{SL}} \]  \hfill (A-1)

(2) Calculate Gas velocity number

\[ N_{gV} = v_{sg} \]  \hfill (A-2)

(3) Calculate Pipe diameter number:

\[ N_d = d \sqrt[4]{\frac{\rho_L g}{\sigma_L}} \]  \hfill (A-3)

(4) Calculate Liquid viscosity number:

\[ N_L = \mu_L \sqrt[4]{\frac{g}{\rho_L \sigma_L^3}} \]  \hfill (A-4)
By determining a value of $N_{LC}$, which is correlated with $N_L$ from Figure 14 and a secondary correction factor $\psi$, which is correlated with $\frac{(N_{Lv}N_L^{0.380})}{N_{D14}^2}$ from Figure 15. In most cases $\psi$ will be equal to 1.0. Liquid holdup has been determined from Figure 16. The factor $\psi$ was included to fit some of the data where it was postulated that a transition would occur before mist flow begins.

Figure 14: Hagedorn and Brown correlation for $N_{LC}$
Figure 15: Hagedorn and Brown correlation for $\psi$

Figure 16: Hagedorn and Brown correlation for $H_L/\psi$

Correlation based on tubing sizes of 1 to 2 in. viscosities of 0.86 to 110 cp
(5) Calculate Reynolds number:

\[ N_{Re} = \frac{\rho_n v_m d}{u_s} \]  \hspace{1cm} (A-5)

Where:

\[ \rho_n = \rho_L \lambda_L + \rho_g (1 - \lambda_L) \]  \hspace{1cm} (A-6)

\[ \mu_s = \mu_L^H \lambda_L \mu_g^{(1-H_L)} \]  \hspace{1cm} (A-7)

\[ v_m = v_{SL} + v_{sg} \]  \hspace{1cm} (A-8)

\[ v_{SL} = \frac{q_L}{A} \]  \hspace{1cm} (A-9)

\[ v_{sg} = \frac{q_g}{A} \]  \hspace{1cm} (A-10)

(5) Calculate friction factor:

\[ f = \frac{16}{Re} \text{ Laminar} \]  \hspace{1cm} (A-11)

\[ f = 0.046Re^{-0.2} \text{ Turbulent Smooth (Re < 100,000)} \]  \hspace{1cm} (A-12)

\[ f = 0.0014 + \frac{0.125}{Re^{0.32}} \text{ Turbulent/Smooth (Re < 3,000,000)} \]  \hspace{1cm} (A-13)

(6) Calculate total pressure gradient:

\[ \left( \frac{dP}{dZ} \right)_t = \left( \frac{dP}{dZ} \right)_f + \left( \frac{dP}{dZ} \right)_{el} \]  \hspace{1cm} (A-14)
Where:

\[
\left( \frac{dP}{dz} \right)_f = \frac{f v^2}{2 \rho d} \quad (A-15)
\]

\[
\left( \frac{dP}{dz} \right)_{el} = \rho g \sin \theta \quad (A-16)
\]

Where:

\[
\theta = 90^\circ
\]

\[
\rho_s = \rho_L H_L + \rho_g (1 - H_L) \quad (A-17)
\]

A-2 Duns and Ros correlations

Duns and Ros identified twelve variables that were potentially important in the prediction of pressure gradient. They developed the flow pattern map which is shown in Figure 17. Regions I through III are bubble, slug and mist flow.
For prediction pressure drop, it is necessary to determine flow patterns.

Region I. bubble flow, plug flow and part of froth flow regime exist.

Region II. The region thus covers slug flow and the remainder of the froth flow regime.

Region III. The mist flow pattern exists.

(1) Calculate flow-pattern boundaries:

Bubble/slug boundary

\[ N_{g v_{B/S}} = L_1 + L_2 N_{L v} \quad (A-18) \]
Slug/transition boundary

\[ N_{gv_{S/Tr}} = 50 + 36 N_{Lv} \]  \hspace{1cm} (A-19)

Transition/mist boundary

\[ N_{gv_{Tr/M}} = 75 + 84 N_{Lv}^{0.75} \]  \hspace{1cm} (A-20)

\( N_{Lv} \) is calculated from equation (A-1). \( L_1 \) and \( L_2 \) are obtained from Figure 18 as function of \( N_d \) which is calculated from equation (A-3).

![Figure 18: Duns and Ros bubble/slug transition parameters](image)

For calculating liquid-holdup, a dimensionless slip-velocity number, \( S \), should be calculated.

(2) Calculate slip-velocity number

For bubble flow \( (N_{gv} < N_{gv_{B/S}}) \):

\[ S = F_1 + F_2 N_{Lv} + F_3 \left( \frac{N_{gv}}{1 + N_{Lv}} \right)^2 \]  \hspace{1cm} (A-21)
Where $F_1$, $F_2$, $F_3$, and $F_4$ are obtained from Figure 19.

\[ F'_3 = F_3 - \frac{F_4}{N_d} \quad (A-22) \]

Figure 19: Duns & Ros bubble flow, slip velocity parameters

For Slug Flow ($N_{ gvB/S } < N_{ gv } < N_{ gvS/TR }$):

\[ S = (1 + F_5) \frac{(N_{ gv })^{0.982} + F'_6}{(1 + F_7 N_{ lv })} \quad (A-23) \]

Where $F_5$, $F_6$, and $F_7$ are obtained from Figure 20.

\[ F'_6 = 0.029 N_d + F_6 \quad (A-24) \]

$N_d$ is obtained from equation (A-3).

For Mist flow ($N_{ gv } > N_{ gvTR/M }$):

\[ S = 0, \ v_s = 0, \text{ and } H_L = \lambda_L \]
(3) Calculate slip velocity:

\[ v_S = \frac{S}{4\sqrt{\frac{\rho_L}{g\sigma_L}}} \]  \hspace{1cm} (A-25)

(4) Calculate liquid-holdup:

\[ H_L = \frac{v_S - v_m + \sqrt{(v_m - v_S)^2 + 4v_S v_{SL}}}{2v_S} \]  \hspace{1cm} (A-26)

(5) Calculate friction factor for bubble and slug flows:

Friction factor is a function of Reynolds number which is obtained from equation (A-27).
\[ N_{Re} = \frac{\rho_L v_{sl} d}{\mu_L} \quad (A-27) \]

Where \( v_{sl} \) is obtained from equation (2-9).

\[ f = f_1 f_2 f_3 \quad (A-28) \]

Where \( f_1 \) is obtained from equations A-11 through A-13, \( f_2 \) is obtained from Figure 21.

\[ f_3 = 1 + \frac{f_1}{4} \sqrt{\frac{v_{sg}}{50 v_{SL}}} \quad (A-29) \]

\[ \text{Figure 21: Duns & Ros bubble and slug flow friction factor parameter} \]

The calculation of friction factor for slug flow is the same as bubble flow.

(6) Calculate friction factor for Mist flow:

\[ N_{Reg} = \frac{\rho_g v_{sg} d}{\mu_g} \quad (A-30) \]
Where $v_{sg}$ is obtained from equation (A-10).

\[ N_{we} = \frac{\rho_g v_{sg}^2 \varepsilon}{\sigma_L} \quad (A-31) \]
\[ N_\mu = \frac{\mu_L^2}{\rho_L \sigma_L \varepsilon} \quad (A-32) \]

If $N_{we}N_\mu \leq 0.005$; \( \frac{\varepsilon}{d} = \frac{0.0749 \sigma_L}{\rho_g v_{sg}^2 d} \) \quad (A-33)

If $N_{we}N_\mu > 0.005$; \( \frac{\varepsilon}{d} = \frac{0.3713 \sigma_L}{\rho_g v_{sg}^2 d} \left( N_{we} N_\mu \right)^{0.302} \) \quad (A-34)

\[ f = 4 \left\{ \frac{1}{\left[ 4 \log_{10} \left( \frac{0.27 \varepsilon}{d} \right) \right]^2} + 0.067 \left( \frac{\varepsilon}{d} \right)^{1.73} \right\} \quad (A-35) \]

(7) Calculate total pressure gradient:

For bubble and slug flows:

\[ \left( \frac{dP}{dZ} \right)_t = \left( \frac{dP}{dZ} \right)_f + \left( \frac{dP}{dZ} \right)_{el} \quad (A-36) \]

For Mist flow:

\[ \left( \frac{dP}{dZ} \right)_t = \frac{\left( \frac{dP}{dZ} \right)_{el} + \left( \frac{dP}{dZ} \right)_f}{1 - E_k} \quad (A-37) \]

Where:

\[ \left( \frac{dP}{dZ} \right)_f = \frac{f \rho_L v_{SL} v_m}{2d} \quad (A-38) \]
\begin{align}
    \left( \frac{dP}{dz} \right)_{el} &= \rho_s g \sin \theta \quad (A-39) \\
    E_k &= \frac{v_m v_S g \rho_n}{p} \quad (A-40)
\end{align}

For Transition region \((N_{gvS/Tr} < N_{gv} < N_{gvTr/M})\):

\begin{align}
    \left( \frac{dP}{dz} \right)_t &= A \left( \frac{dP}{dz} \right)_{stu} + (1 - A) \left( \frac{dP}{dz} \right)_{mist} \quad (A-41)
\end{align}

Where:

\begin{align}
    A &= \frac{N_{gvTr/M} - N_{gv}}{N_{gvTr/M} - N_{gvS/Tr}} \quad (A-42)
\end{align}

**A-3 Orkiszewski correlations**

(1) Calculate friction pressure gradient for bubble flow:

Bubble flow exists if \(\lambda_g = 1 - \lambda_L \leq \lambda_{gB/S}\)

Where:

\begin{align}
    \lambda_L &= \frac{q_L}{q_L + q_g} \quad (A-43)
\end{align}

\begin{align}
    \lambda_{gB/S} = L_B = 1.071 - 0.2218 \frac{v_m^2}{d} \quad (A-44)
\end{align}

Liquid holdup is determined from below equation (A-45).
\[ H_L = 1 - \frac{1}{2} \left[ 1 + \frac{v_m}{v_s} - \sqrt{\left(1 + \frac{v_m}{v_s}\right)^2 - 4 \frac{v_{sg}}{v_s}} \right] \] (A-45)

\[
\frac{dP}{dZ} = f \frac{\rho_L \left(\frac{v_{SL}}{H_L}\right)^2}{2d} \] (A-46)

Where:
\[ f \] is obtained from equations (A-11) through (A-13), \( v_{SL} \) from equation (A-9) and \( H_L \) from equation (A-45).

Reynolds number is calculated from equation (A-47).

\[
N_{Re} = \frac{\rho_L \left(\frac{v_{SL}}{H_L}\right)d}{\mu_L} \] (A-47)

(2) Calculate friction pressure gradient for slug flow:

Slug flow exists if \( \lambda_g > \lambda_{gb/s} \) and \( N_{gv} < N_{gvst/tr} \).

Slip density is calculated from below equation.

\[
\rho_s = \frac{\rho_L (v_{SL}+v_b)+\rho_g v_{sg}}{v_m+v_b} + \rho_L \Gamma \] (A-48)

Where:
\( v_{SL}, v_{sg}, v_m \) are obtained from equations (A-9), (A-10), and (A-8).

The procedure for calculating \( v_b \) follows.

1. Estimate a value of \( v_b \) by guessing.

\[
v_b = 0.5 \sqrt{gd} \] (A-49)
2. Calculate $N_{Re_b}$

$$N_{Re_b} = \frac{\rho_L v_b d}{\mu_L}$$  \hspace{1cm} (A-50)

3. Calculate $v_b$ from one of the below equations.

If $N_{Re_b} \leq 3,000$, $v_b = (0.35 + 8.74 \times 10^{-6} N_{Re_L})\sqrt{gd}$ \hspace{1cm} (A-51)

If $N_{Re_b} \leq 8,000$, $v_b = (0.35 + 8.74 \times 10^{-6} N_{Re_L})\sqrt{gd}$ \hspace{1cm} (A-52)

If $3,000 < N_{Re_b} < 8,000$, $v_b = \frac{1}{2} \left( v_{bs} + \sqrt{v_{bs}^2 + \frac{13.59\mu_L}{\rho_L\sqrt{d}}} \right)$ \hspace{1cm} (A-53)

Where:

$$N_{Re_L} = \frac{d v_b \rho_L}{\mu_L}$$  \hspace{1cm} (A-54)

$v_{bs} = (0.251 + 8.74 \times 10^{-6} N_{Re_L})\sqrt{gd}$ \hspace{1cm} (A-55)

4. Compare the values of $v_b$ in steps 1 and 3. If two values are close use the value of $v_b$ in step 3 otherwise continue these three steps to obtain the close values.

$\Gamma$ is obtained from below equation for water.

$$\Gamma = \frac{0.013 \log \mu_L}{d^{1.38}} - 0.287 - 0.162 \log v_m - 0.428 \log d$$  \hspace{1cm} (A-56)

$$\left( \frac{dP}{dz} \right)_f = \frac{f \rho_L v_m^2}{2d} \left[ \left( \frac{v_{SL}+v_b}{v_m+v_b} \right) + \Gamma \right]$$  \hspace{1cm} (A-57)
(3) Calculate total pressure gradient:

\[
\left( \frac{dp}{dz} \right)_t = \left( \frac{dp}{dz} \right)_f + \left( \frac{dp}{dz} \right)_{el} \tag{A-58}
\]

Where:

\[
\left( \frac{dp}{dz} \right)_f \text{ for bubble flow is obtained from equation (A-46) and for slug flow from equation (A-57).}
\]

\[
\left( \frac{dp}{dz} \right)_{el} = \rho_s g \sin \theta \tag{A-59}
\]

The total pressure gradient for mist flow or the transition region between slug and mist flow is predicted from Duns & Ros method.
Flow pattern map which is used by Aziz is shown in Figure 22.

\[ N_x = \nu_{sg} \left( \frac{\rho_g}{0.0764} \right)^{1/3} \left[ \left( \frac{g}{\sigma_L} \right) \left( \frac{\rho_L}{62.4} \right) \right]^{1/4} \]  
(A-60)

\[ N_y = \nu_{SL} \left[ \left( \frac{g}{\sigma_L} \right) \left( \frac{\rho_L}{62.4} \right) \right]^{1/4} \]  
(A-61)

\[ N_1 = 0.51 \left( 100N_y \right)^{0.172} \]  
(A-62)
\[ N_2 = 8.6 + 3.8N_y \]  \hspace{1cm} (A-63)

\[ N_3 = 70(100N_y)^{-0.152} \]  \hspace{1cm} (A-64)

(1) Calculate holdup and friction pressure gradient for different flow patterns:

Bubble flow exists when \( N_x < N_1 \)

1- Calculate bubble in static liquid velocity:

\[ v_{bs} = 1.41 \left[ \frac{\sigma_L g (\rho_L - \rho_g)}{\rho_L^2} \right]^{1/4} \]  \hspace{1cm} (A-65)

2- Calculate bubble in flowing liquid velocity:

\[ v_{bf} = 1.2v_m + v_{bs} \]  \hspace{1cm} (A-66)

3- Calculate holdup:

\[ H_L = 1 - \frac{v_{sg}}{v_{bf}} \]  \hspace{1cm} (A-67)

4- Calculate Reynolds number:

\[ N_{Re} = \frac{\rho_L v_m d}{\mu_L} \]  \hspace{1cm} (A-68)

5- Calculate friction pressure gradient:

\[ \left( \frac{dp}{dz} \right)_f = \frac{f \rho_s v_m^2}{2d} \]  \hspace{1cm} (A-69)

Slug flow exists when \( N_1 < N_x < N_2 \) for \( N_y < 4 \) or \( N_1 < N_x < 26.5 \) for \( N_y \gg 4 \)
1- Calculate bubble in static liquid velocity:

\[ v_{bs} = C \sqrt{\frac{gd(\rho_L - \rho_g)}{\rho_L}} ] \quad \text{(A-70)}

Where:

\[ C = 0.345 \left[ 1 - e^{(-0.029N_v)} \right] \left[ 1 - e^{\left(\frac{3.37 - N_E}{m}\right)} \right] ] \quad \text{(A-71)}

\[ N_E = \frac{gd^2(\rho_L - \rho_g)}{\sigma_L} ] \quad \text{(A-72)}

\[ N_v = \frac{\sqrt{g\rho_L(\rho_L - \rho_g)d^3}}{\mu_L} ] \quad \text{(A-73)}

m is determined from

\[
\begin{align*}
N_v & \quad \text{m} \\
\geq 250 & \quad 10 \\
250 > N_v > 18 & \quad 69N_v^{-0.35} \\
\leq 250 & \quad 25
\end{align*}
\]

2- Calculate bubble in flowing liquid velocity:

\[ v_{bf} = 1.2v_m + v_{bs} ] \quad \text{(A-74)}

3- Calculate holdup:

\[ H_L = 1 - \frac{v_{sg}}{v_{bf}} ] \quad \text{(A-75)}

4-Calculate Reynolds number:

\[ N_{Re} = \frac{\rho L v_m d}{\mu_L} \]  

(A-76)

5-Calculate friction pressure gradient:

\[ \left( \frac{dp}{dZ} \right)_f = \frac{f \rho_L L v_m^2}{2d} \]  

(A-77)

Mist flow exists when \( N_x > N_3 \) for \( N_y < 4 \) or \( N_x > 26.5 \) for \( N_y > 4 \)

Pressure gradient for mist flow is calculated by using Duns & Ros mist flow method.

Transient Region exists when \( N_2 < N_x < N_3 \) for \( N_y < 4 \)

\[ \left( \frac{dp}{dZ} \right)_t = A \left( \frac{dp}{dZ} \right)_{slug} + (1 - A) \left( \frac{dp}{dZ} \right)_{mist} \]  

(A-78)

Where:

\[ A = \frac{N_3 - N_x}{N_3 - N_2} \]  

(A-79)

(2) Calculate total pressure gradient:

\[ \left( \frac{dp}{dZ} \right)_t = \left( \frac{dp}{dZ} \right)_f + \left( \frac{dp}{dZ} \right)_{el} \]  

(A-80)
(1) Determine transition regions:

There are two transitions including bubble/slug transition and slug/mist transition in vertical uphill flow. The bubble/slug transition was found to be linear and at 45 degree with the axis.

Bubble/slug transition:

\[ N_{Lv B/s} = 10^x \]  \hspace{1cm} (A-81)

Where:

\[ x = log N_{gv} + 0.940 + 0.074 \sin \theta - 0.855 \sin^2 \theta + 3.695 N_L \]  \hspace{1cm} (A-82)

Slug/mist transition:

\[ N_{gv B/s} = 10^{(1.401 - 2.694 N_L + 0.521 N_{Lv}^{0.329})} \]  \hspace{1cm} (A-83)

Where \( N_L \) and \( N_{Lv} \) are obtained from equations (A-4) and (A-1).

(2) Calculate liquid holdup:

Three liquid holdup correlations were developed using regression analysis. One of these was for uphill flow which is shown in equation (A-84).

\[ H_L = \exp \left[ (C_1 + C_2 \sin \theta + C_4 N_L^2) \left( \frac{N_{gv}^{C_s}}{N_{Lv}^{C_s}} \right) \right] \]  \hspace{1cm} (A-84)
Where regression coefficients are obtained from Table 2.

**Table 2: Mukherjee & Brill empirical coefficients for uphill flow**

| \( C_1 \) | -0.380113 |
| \( C_2 \) | 0.129875 |
| \( C_3 \) | -0.119788 |
| \( C_4 \) | 2.343227 |
| \( C_5 \) | 0.475686 |
| \( C_6 \) | 0.288657 |

(3) Calculate total pressure gradient:

**Bubble and slug flow:**

\[
\left( \frac{dp}{dL} \right)_t = \frac{f \rho_s \frac{v_m^2}{2d} + \rho_s g \sin \theta}{1 - E_k} \quad (A-85)
\]

Where:

\[
E_k = \frac{\rho_s v_m v_{sg}}{p} \quad (A-86)
\]

**Mist Flow:**

\[
\left( \frac{dp}{dL} \right)_t = \frac{f \rho_s \frac{v_m^2}{2d} + \rho_s g \sin \theta}{1 - E_k} \quad (A-87)
\]
Where friction factor is obtained from equations (A-11) through (A-13) and Reynolds number from below equation.

\[ N_{Re} = \frac{\rho_n \nu_m d}{\mu_n} \]  

(A-88)

A-6 Hasan and Kabir correlations

(1) Calculate superficial gas velocity for different flow transition:

Bubbly/slug flow transition: \( v_{SGB/S} > v_{sg} \) (Equation: A-10)

This transition occurs when the gas void fraction reaches about 25 percent.

\[ v_{SGB/S} = \frac{\sin \theta}{4-c_0} \left( c_0 v_{SL} + v_s \right) \]  

(A-89)

Where:

\[ c_0 = \begin{cases} 
1.2 \text{ if } d < 0.12 \text{ m or if } v_{SL} > 0.02 \frac{m}{s} \\
2.0 \text{ if } d > 0.12 \text{ m and if } v_{SL} < 0.02 \frac{m}{s} 
\end{cases} \]  

(A-90)

Dispersed bubbly flow transition: \( v_{m_{DB}} > v_m \) (Equation: A-8)

If large bubbles at high liquid rates scattered into small bubbles, transition to slug flow is inhibited even though the gas void fraction exceeds 25 percent. When the gas void fraction exceeds 52 percent, bubble coalescence cannot be prevented and transition to slug, churn or annular flow must occur.
Annular flow transition:

In annular flow, the high velocities keep the liquid droplets in suspension. This transition is modeled by balancing the drag forces on the liquid droplets and the gravitational forces acting on them.

\[ v_m^{1.12} = 4.68d^{0.48} \left[ \frac{g(\rho_L - \rho_g)}{\sigma_L} \right]^{0.5} \left( \frac{\sigma_L}{\rho_L} \right)^{0.6} \left( \frac{\rho_L}{\mu_L} \right)^{0.08} \]  

(A-91)

\[ v_{sg} = 3.1 \left[ \sigma_L g (\rho_L - \rho_g) / \rho_g^2 \right]^{0.25} \]  

(A-92)

Figure 23 shows flow pattern map.

Figure 23: Hasan & Kabir flow pattern map
(2) Calculate liquid holdup:

\[ H_L = 1 - \frac{v_{sg}}{c_0 v_m + v_s} \quad (A-93) \]

Where:

For Bubble and dispersed bubble flow:

\[ v_s = 1.53 \left[ \frac{g \sigma_L (\rho_L - \rho_g)}{\rho_L^2} \right]^{1/4} \quad \text{and} \quad C_0 \ \text{from equation} \ (A-90). \]

For slug and churn flow:

\[ v_s = 0.35 \left( gd \frac{\rho_L - \rho_g}{\rho_L} \right)^{0.5} \sqrt{\sin \theta} \ (1 + \cos \theta)^{1.2} \quad \text{and} \quad C_0 = 1.2 \ \text{for slug flow} \]

and 1.15 for churn flow.

For annular flow:

\[ H_L = (1 + X^{0.8})^{-0.378} \quad (A-94) \]

Where:

\[ X = \left( \frac{1 - x_g}{x_g} \right)^{0.9} \sqrt{\frac{\rho_g}{\rho_L}} \left( \frac{\mu_L}{\mu_g} \right)^{0.1} \quad (A-95) \]

(3) Calculate total pressure gradient:

For bubble, slug and churn flow:

\[ \left( \frac{dp}{dz} \right)_t = \left( \frac{dp}{dz} \right)_f + \left( \frac{dp}{dz} \right)_{el} \quad (A-96) \]
Where:

\[
\left( \frac{dp}{dz} \right)_f = \frac{f \rho_s v_m^2}{2d} \quad \text{(A-97)} \quad \text{and} \quad \left( \frac{dp}{dz} \right)_{el} = \rho_s g \sin \theta \quad \text{(\(\rho_s\) is obtained from equation (A-17) and \(\theta = 90^\circ\)).}
\]

For annular flow:

\[
\left( \frac{dp}{dz} \right)_f = \frac{f_c \rho_c}{2d} \left( \frac{v_{SG}}{1-H_L} \right)^2 \quad \text{(A-98)}
\]

Where:

\[
f_c = 0.046 \left( \frac{\rho_g v_{SG} d}{\mu_g} \right)^{-0.2} (1 + 75H_L) \quad \text{(A-99)}
\]

\[
\rho_c = \frac{v_{SG} \rho_g + v_{SL} \rho_L F_E}{v_{SG} + v_{SL} F_E} \quad \text{(A-100)}
\]

\[
F_E = \begin{cases} 
0.0055 v_{crit}^{2.86} & \text{if } v_{crit} < 4 \\
0.857 \log_{10} v_{crit} - 0.20 & \text{if } v_{crit} > 4 
\end{cases} \quad \text{(A-101)}
\]

\[
v_{crit} = 10000 \frac{\mu_g v_{SG}}{\sigma_L} \left( \frac{\rho_g}{\rho_L} \right)^{1/2} \quad \text{(A-102)}
\]
APPENDIX B

B-1  Modeling flow pattern with VBA Excel

Option Explicit

Dim d As Double, L As Double, A As Double, vsl As Double, vsg As Double
Dim sigma As Double, rhol As Double, rhog As Double, mul As Double, mug As Double
Dim g As Double, vm As Double, Ngv As Double, Nlv As Double, NgvTrM As Double
Dim Nd As Double, Nl As Double, NgvBS As Double, NgVsTr As Double, Nx As Double
Dim Ny As Double, N1 As Double, N2 As Double, N3 As Double, landal As Double
Dim landag As Double, Lb As Double, Vs As Double, C0 As Double, vsgb As Double
Dim vmdb As Double, x As Double, NLvBS As Double, NgvSM As Double
Dim y As Double, NgBS As Double, theta As Double, NLvST As Double, z As Double
Dim vsgan As Double

'Nomenclature for global variables

' A  m²    Pipe area
' C0  -    Flow coefficient
'd  m  Pipe inner diameter
'g  m/s²  Gravity
'L  m  Pipe Length
'landal  L³/L³  No-slip liquid volume fraction
'landag  L³/L³  No-slip gas volume fraction
'LB  -  Parameter in Eq.4.59
'mug  m/s  Viscosity of gas phase
'muL  m/s  Viscosity of liquid phase
'Nd  -  Pipe diameter number
'Ngv  -  Gas velocity number
'NgBS  -  Slug/mist transition gas velocity number
'NgvSM  -  Bubble/slug transition gas velocity number
'NgvBS  -  Bubble/slug boundary
'NgVsTr  -  Slug/transition boundary
'NgvTrM  -  Transition/mist boundary
'N1  -  Flow-pattern transition in Eq.4.87
'N2  -  Flow-pattern transition in Eq.4.88
'N3  -  Flow-pattern transition in Eq.4.89
'Nl  -  Liquid viscosity number
'Nlv  -  Liquid velocity number
'NlvBS  -  Bubble/slug transition Liquid velocity number
'NlvST  -  Slug transition Liquid velocity number
'Nx             Flow-pattern coordinate in Eq.4.85
'Ny             Flow pattern coordinate in Eq.4.86
'rhog           kg/m$^3$ Density of the gas phase
'rhoL           kg/m$^3$ Density of the liquid phase
'sigma         N/m Liquid surface tension
'theta         degree Inclination angle from Vertical
'vsL           m/s Superficial liquid velocity
'vsg           m/s Superficial Gas velocity
'vsgb          m/s Superficial Gas velocity in Eq.4.240
'Vm            m/s Mixture velocity
'vmdb          m/s Dispersed-bubble flow velocity
'Vs            m/s Slug velocity
'x              Parameter in Eq.4.129
'y              Parameter in Eq.4.132
'z              Parameter in Eq.4.134

Sub Button5_Click()

Call getinput

Call initialcalc

Call HegedornBrown

Call DunsRos

Call Orkiszewski
Call aziz
Call MukherjeeBrill
Call hasankabir
End Sub

Private Sub getinput()
  vsl = Range("D3").Value
  vsg = Range("D4").Value
  d = Range("D5").Value
  theta = Range("D6").Value
  rhol = Range("D7").Value
  rhog = Range("D8").Value
  mul = Range("D9").Value
  mug = Range("D10").Value
  sigma = Range("D12").Value
  L = Range("D15").Value
  g = Range("D11").Value
End Sub

Private Sub initialcalcs()
  A = 3.14 * (d / 2) ^ 2
  landal = vsl / (vsl + vsg)
landag = 1 - landal
vm = vsg + vsl
Lb = 1.071 - 0.7277 * ((vm) ^ 2) / d
Ngv = vsg * (rhol / g / sigma) ^ 0.25
Nlv = vsl * (rhol / g / sigma) ^ 0.25
Nd = d * (rhol * g / sigma) ^ 0.5
Nil = mul * (g / rhol / (sigma ^ 3)) ^ 0.25
NgvBS = 2 + 0.4 * Nlv
NgVsTr = 50 + 36 * Nlv
NgvTrM = 75 + 84 * (Nlv ^ 0.75)
Nx = vsg / 0.3048 * (rhog / 1.2) ^ (1 / 3) * (72 / 72 * rhol / 1000) ^ 0.25
Ny = vsl / 0.3048 * (72 / 72 * rhol / 1000) ^ 0.25
N1 = 0.51 * (100 * Ny) ^ 0.172
N2 = 8.6 + 3.8 * Ny
N3 = 70 * (100 * Ny) ^ (-0.152)

x = Log(Ngv) * 0.4343 + 0.94 + 0.074 * Sin((theta / 180) * 3.14) - 0.855 * Sin((theta / 180) * 3.14)) ^ (2) + 3.695 * Nil
NLvBS = 10 ^ x
NgvSM = 10 ^ (1.401 - 2.694 * Nil + 0.521 * Nlv ^ (0.329))
y = 0.431 - 3.003 * Nil - 1.138 * Log(Nlv) * 0.4343 * Sin((theta / 180) * 3.14) - 0.429 * (Log(Nlv) * 0.4343) ^ (2) * Sin((theta / 180) * 3.14) + 1.132 * Sin((theta / 180) * 3.14)
NgBS = 10^y

z = 0.321 - 0.017 * Ngv - 4.267 * Sin((theta / 180) * 3.14) - 2.972 * Nl - 0.033 * (Log(Ngv) * 0.4343)^(2) - 3.925 * Sin((theta / 180) * 3.14)^(2)

NLvST = 10^z

Vs = (1.53 * (g * sigma * (rhol - rhog) / rhol^2)^(0.25))

If d >= 0.12 And vsl <= 0.02 Then
  C0 = 2
ElseIf d < 0.12 And vsl > 0.02 Then
  C0 = 1.2
End If

vsgb = Sin((theta / 180) * 3.14) * (C0 * vsl + Vs) / (4 - C0)

vmdb = (0.468 * d^0.48 * (g * (rhol - rhog) / sigma)^(0.5) * (sigma / rhol)^(0.6) * (rhol / mul)^(0.08))^(1 / 1.12)

vsgan = 3.1 * (g * sigma * (rhol - rhog) / rhog^2)^(0.25)

End Sub

Private Sub HegedornBrown()

Range("H14") = "NA"

End Sub

Private Sub DunsRos()

If Ngv < NgvBS Then

End If
Range ("H15") = "BUBBLE"

ElseIf NgvBS < Ngv < NgVsTr Then
Range ("H15") = "SLUG"

ElseIf Ngv > NgvTrM Then
Range ("H15") = "MIST"

ElseIf NgVsTr < Ngv < NgvTrM Then
Range ("H15") = "TRANS"

End If

End Sub

Private Sub Orkiszewski()

If landag <= Lb Then
Range ("H16") = " BUBBLE"

ElseIf Ngv < NgVsTr And landag > Lb Then
Range ("H16") = "SLUG"

ElseIf Ngv > NgvTrM Then
Range ("H16") = "MIST"

ElseIf NgVsTr < Ngv < NgvTrM Then
Range ("H16") = "Trans"

End If

End Sub
Private Sub aziz()

If Nx < N1 Then

Range ("H17") = "BUBBLE"

ElseIf N1 < Nx < N2 And Ny < 4 Then

Range ("H17") = "SLUG"

ElseIf N1 < Nx < 26.5 And Ny >= 4 Then

Range ("H17") = "SLUG"

ElseIf Nx > N3 And Ny < 4 Then

Range ("H17") = "MIST"

ElseIf Nx > 26.5 And Ny > 4 Then

Range ("H17") = "MIST"

ElseIf N2 < Nx < N3 And Ny < 4 Then

Range ("H17") = "TRANS"

End If

End Sub

Private Sub MukherjeeBrill()

If Ngv < NgBS Then

Range ("H18") = "BUBBLE"

ElseIf Ngv > NgBS Then

Range ("H18") = "SLUG"

ElseIf Ngv >= NgvSM Then
Range ("H18") = "MIST"
End If
End Sub

Private Sub hasankabir()
If vsg < vsgb Then
Range ("H19") = "BUBBLE"
ElseIf vmdb < vm Then
Range ("H19") = "SLUG"
ElseIf vsg > vsgan Then
Range ("H19") = "Annular"
End If
End Sub

**B-2  Prediction pressure drop with VBA Excel**

Option Explicit

Dim vsl As Double, vsg As Double, d As Double, theta As Double, rhol As Double
Dim rhog As Double, mul As Double, mug As Double, sigma As Double, L As Double
Dim g As Double, Nlv As Double, Ngv As Double, Nd As Double, Nl As Double
Dim E1 As Double, E2 As Double, E3 As Double, E4 As Double, E5 As Double, d1 As Double
Dim S As Double, Vs As Double, vm As Double, hl As Double, Re As Double, f1 As Double
Dim f2 As Double, f3 As Double, f As Double, parameter1 As Double
Dim rhos As Double, dpdzel As Double, dpdzf As Double, dpdzt As Double, Vbs As Double
Dim vbf As Double, landal As Double, mun As Double, C0 As Double
Dim rhon As Double, F5 As Double, F6 As Double, F7 As Double, F8 As Double
Dim Vb As Double, Reb As Double, Vbsi As Double, T As Double, Ne As Double
Dim Nv As Double, C As Double, m As Double, vbsa As Double, vbfa As Double
Dim A As Double, B As Double, Ek As Double, dpdzts As Double, dpdztm As Double
Dim Res As Double, fs As Double, hls As Double, rhoss As Double, dpdzfs As Double
Dim dpdzels As Double, dpdzelts As Double, AT As Double, mus As Double
Dim dpdzmeas As Double, fm As Double, hlm As Double, rhosm As Double
Dim dpdzfm As Double, dpdzelm As Double, dpdzelm As Double, NgvTrM As Double
Dim NgVsTr As Double, NLC As Double, HL_om As Double, omega As Double
Dim Nx As Double, Ny As Double, N1 As Double, N2 As Double, N3 As Double
Dim P1 As Double, P2 As Double, pressure As Double, parameter3 As Double
Dim parameter4 As Double, Resl As Double, fsl As Double, Vcrit As Double
Dim FE As Double, Landalc As Double, fc As Double, rhoc As Double, vsgb As Double
Dim vmdb As Double, B1 As String, B2 As String, B3 As String, B4 As String, B5 As String
Dim S1 As String, S2 As String, S3 As String, S4 As String, S5 As String, M1 As String
Dim M2 As String, M3 As String, M4 As String, M5 As String, T1 As String, T2 As String
Dim T3 As String, NA As String, fp As String

'Statement for global variables

'A - Obtained parameter of multiplication of Eq.4.47 and 4.48
'AT - Parameter in Eq.4.56
'B - Parameter in Eq. 4.49 and 4.50
'B1 - Duns & ros bubble flow
'B2 - Orkiszewski bubble flow
'B3 - Aziz bubble bubble flow
'B4 - Mukherjee & brill bubble flow
'B5 - Hasan & kabir bubble flow
'C - Correction factor
'C0 - Flow coefficient
'd m - Pipe inner diameter
'd1 m - distance between two sensors
'dpdz Pa/m - Total pressure gradient
'dpdztm Pa/m - Mist total pressure gradient
'dpdzts Pa/m - Slug total pressure gradient
'dpdzel Pa/m - Elevation pressure gradient
<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p_{d}p_{dzelm})</td>
<td>(\text{Pa/m})</td>
<td>Mist elevation pressure gradient</td>
</tr>
<tr>
<td>(p_{d}p_{dzels})</td>
<td>(\text{Pa/m})</td>
<td>Slug elevation pressure gradient</td>
</tr>
<tr>
<td>(p_{d}p_{dzf})</td>
<td>(\text{Pa/m})</td>
<td>Friction pressure gradient</td>
</tr>
<tr>
<td>(p_{d}p_{dzfm})</td>
<td>(\text{Pa/m})</td>
<td>Mist friction pressure gradient</td>
</tr>
<tr>
<td>(p_{d}p_{dzfs})</td>
<td>(\text{Pa/m})</td>
<td>Slug friction pressure gradient</td>
</tr>
<tr>
<td>(E_{1})</td>
<td>-</td>
<td>Functions of the liquid velocity number</td>
</tr>
<tr>
<td>(E_{2})</td>
<td>-</td>
<td>Functions of the liquid velocity number</td>
</tr>
<tr>
<td>(E_{3})</td>
<td>-</td>
<td>Functions of the liquid velocity number</td>
</tr>
<tr>
<td>(E_{4})</td>
<td>-</td>
<td>Functions of the liquid velocity number</td>
</tr>
<tr>
<td>(E_{5})</td>
<td>-</td>
<td>Functions of the liquid velocity number</td>
</tr>
<tr>
<td>(E_{k})</td>
<td>-</td>
<td>Dimensionless kinetic-energy pressure gradient</td>
</tr>
<tr>
<td>(f)</td>
<td>-</td>
<td>Friction</td>
</tr>
<tr>
<td>(f_{p})</td>
<td>-</td>
<td>Flow pattern</td>
</tr>
<tr>
<td>(f_{s})</td>
<td>-</td>
<td>Slug friction factor</td>
</tr>
<tr>
<td>(f_{m})</td>
<td>-</td>
<td>Mist friction factor</td>
</tr>
<tr>
<td>(f_{1})</td>
<td>-</td>
<td>Friction factor</td>
</tr>
<tr>
<td>(f_{2})</td>
<td>-</td>
<td>Correction for the in-situ gas/liquid ratio</td>
</tr>
<tr>
<td>(f_{3})</td>
<td>-</td>
<td>Second-order correction factor for liquid viscosity and in-situ gas/liquid ratio</td>
</tr>
<tr>
<td>(F_{5})</td>
<td>-</td>
<td>Function of the liquid viscosity number</td>
</tr>
<tr>
<td>(F_{6})</td>
<td>-</td>
<td>Function of the liquid viscosity number</td>
</tr>
<tr>
<td>(F_{7})</td>
<td>-</td>
<td>Function of the liquid viscosity number</td>
</tr>
</tbody>
</table>
'F8       -    Function of the liquid viscosity number
'g        m/s^2  Gravity
'h_l      -    Liquid holdup
'h_lm     -    Mist liquid holdup
'HL_om    -    Parameter in Fig.4.2
'h_ls     -    Slug liquid holdup
'L        m     Pipe Length
'landal   L^3/L^3 No-slip liquid volume fraction
'm        -    Parameter in Eq.4.96
'M1       -    Duns & Ros mist flow
'M2       -    Orkiszewski mist flow
'M3       -    Aziz bubble mist flow
'M4       -    Mukherjee & brill mist flow
'M5       -    Hasan & Kabir mist flow
'mug      m/s   Viscosity of gas phase
'muL      m/s   Viscosity of liquid phase
'mun      m/s   No-slip viscosity
'mus      m/s   Slip viscosity
'Nd       -    Pipe diameter number
'Ne       -    Dimensionless number in Eq. 4.97
'Ngv      -    Gas velocity number
'NgVsTr   -    Slug/transition boundary
'NgvTrM    -    Transition/mist boundary

'N1          -    Flow-pattern transition in Eq.4.87

'N2          -    Flow-pattern transition in Eq.4.88

'N3          -    Flow-pattern transition in Eq.4.89

'Nl           -    Liquid viscosity number

'NLC          -    Corrected liquid-viscosity number

'Nlv            -    Liquid velocity number

'Nv            -    Dimensionless velocity number

'Nx            -    Flow-pattern coordinate in Eq.4.85

'Ny            -    Flow pattern coordinate in Eq.4.86

'omega        -    Parameter in fig. 4.4

'P1              psig    Pressure in test section 1

'P2              psig    Pressure in test section 1

'paremeter1 - f1 * vsg * Nd ^ (2 / 3)) / vsl

'paremeter3 - (Nlv * NLC) / ((Ngv ^ (0.575)) * Nd)) * ((pressure + 14.696) / 14.696) ^ 0.1

'paremeter4 - (Ngv * (NLC ^ (0.38)) / (Nd ^ (2.14))

'pressure        psig    Average pressure

'Re           -    Reynolds number

'Reb          -    Bubble Reynolds number

'Res            -    Slug Reynolds number

'rhog            kg/m^3    Density of the gas phase
'\(\rho_L\) kg/m\(^3\) Density of the liquid phase

'\(\rho_n\) kg/m\(^3\) No-slip mixture density

'\(\rho_s\) kg/m\(^3\) Slip density

'\(\rho_{sm}\) kg/m\(^3\) Mist slip density

'\(\rho_{ss}\) kg/m\(^3\) Slug slip density

'S - Slip-velocity number

'S1 - Duns & Ros slug flow

'S2 - Orkiszewski slug flow

'S3 - Aziz bubble slug flow

'S4 - Mukherjee & brill slug flow

'S5 - Hasan & kabir slug flow

'\(\sigma\) N/m Liquid surface tension

'T - Liquid distribution coefficient

'T1 - Duns & Ros transition region

'T2 - Orkiszewski transition region

'T3 - Aziz bubble transition region

'\(\theta\) degree Inclination angle from Vertical

'\(v_{sL}\) m/s Superficial liquid velocity

'\(v_{sg}\) m/s Superficial Gas Velocity

'\(v_b\) m/s Bubble-rise velocity

'\(v_{bs}\) m/s Bubble in static liquid velocity

'\(v_{bsa}\) m/s Bubble in static liquid velocity (Aziz Method)
' vbsi  m/s  Bubble in static liquid velocity (Orkiszewski Method)
' vbf  m/s  Bubble in flowing liquid velocity
' vbfa m/s  Bubble in flowing liquid velocity (Aziz Method)
' Vm  m/s  Mixture velocity
' Vs  m/s  Slug velocity

Private Sub Button2_Click()
    Call getinput
    Call initialcalcs
    If fp = "B1" Then
        Call calcdunsrospbubmflow
    ElseIf fp = "B2" Then
        Call calcOrkiszewskibubbleflow
    ElseIf fp = "B3" Then
        Call calcAzizbubbleflow
    ElseIf fp = "B4" Then
        Call calcMukherjeeBrillbubbleflow
    ElseIf fp = "B5" Then
        Call calchasanKabirbubbleflow
    ElseIf fp = "S1" Then
        Call calcdunsrosslugflow
    ElseIf fp = "S2" Then
Call calcOrkiszewskislugflow
ElseIf fp = "S3" Then
    Call calcAzizslugflow
ElseIf fp = "S4" Then
    Call calcMukherjeeBrillslugflow
ElseIf fp = "S5" Then
    Call calcHasanKabirslugflow
ElseIf fp = "M1" Then
    Call calcDunsrosmistflow
ElseIf fp = "M2" Then
    Call calcOrkiszewskimistflow
ElseIf fp = "M3" Then
    Call calcAzizmistflow
ElseIf fp = "M4" Then
    Call calcMukherjeeBrillmistflow
ElseIf fp = "M5" Then
    Call calcHasanKabirmistflow
ElseIf fp = "T1" Then
    Call calcDunsrostransitionregion
ElseIf fp = "T2" Then
    Call calcOrkiszewskitransitionregion
ElseIf fp = "T3" Then
Call calcAziztransitionregion

ElseIf fp = "NA" Then

    Call calcHagedornBrown

End If

Call getoutput

End Sub

Private Sub getinput()

    vsl = Range("D3").Value
    vsg = Range("D4").Value
    d = Range("D5").Value
    theta = Range("D6").Value
    rhol = Range("D7").Value
    rhog = Range("D8").Value
    mul = Range("D9").Value
    mug = Range("D10").Value
    g = Range("D11").Value
    sigma = Range("D12").Value
    P1 = Range("D13").Value
    P2 = Range("D14").Value
    L = Range("D15").Value
Private Sub initialcalcs()
Nlv = vsl * (rhol / g / sigma) ^ 0.25
Ngv = vsg * (rhol / (g * sigma)) ^ 0.25
Nd = d * ((rhol * g) / sigma) ^ 0.5
Nl = mul * (g / (rhol * sigma ^ (3))) ^ 0.25
If fp = "NA" Then
landal = vsl / (vsl + vsg)
NLC = 0.001649017 * (Log(Nl) * 0.4343) ^ 6 + 0.01595006 * (Log(Nl) * 0.4343) ^ 5 +
0.06106662 * (Log(Nl) * 0.4343) ^ 4 + 0.118372 * (Log(Nl) * 0.4343) ^ 3 + 0.1254921 *
(Log(Nl) * 0.4343) ^ 2 + 0.07677237 * (Log(Nl) * 0.4343) + 0.02875255
Pressure = (P1 + P2) / 2
parameter3 = ((Nlv * NLC) / ((Ngv ^ (0.575)) * Nd)) * ((pressure + 14.696) / 14.696) ^
0.1
If parameter3 < 0.000001 Then
\( HL_{om} = 0.05 \)

Else

\[
HL_{om} = 0.00009817827 \times (\log(\text{parameter3}))^6 + 0.005236729 \times (\log(\text{parameter3}))^5 + 0.1123527 \times (\log(\text{parameter3}))^4 + 1.23187 \times (\log(\text{parameter3}))^3 + 7.219152 \times (\log(\text{parameter3}))^2 + 21.38365 \times (\log(\text{parameter3})) + 25.99029
\]

End If

\[
\text{parameter4} = (Ngv \times (Nl^{0.38})) / (Nd^{2.14})
\]

If \( \text{parameter4} \leq 0.01 \) Then

\( \omega = 1 \)

ElseIf \( 0.01 < \text{parameter4} < 0.09 \) Then

\[
\omega = 9.509747 \times \log(\text{parameter4})^5 + 73.26576 \times \log(\text{parameter4})^4 + 220.4116 \times \log(\text{parameter4})^3 + 323.1868 \times \log(\text{parameter4})^2 + 231.7226 \times \log(\text{parameter4}) + 67.05032
\]

End If

\( \rho_h = \rho_{hl} \times \lambda_d + \rho_{hg} \times (1 - \lambda_d) \)

ElseIf \( \text{fp} = \text{"B1"} \) Then

\[
E1 = -0.321 \times (\log(Nl) \times 0.4343)^4 - 2.3 \times (\log(Nl) \times 0.4343)^3 - 5.61 \times (\log(Nl) \times 0.4343)^2 - 4.99 \times (\log(Nl) \times 0.4343) + 0.647
\]

\[
E2 = -0.0554 \times (\log(Nl) \times 0.4343)^6 - 0.2939 \times (\log(Nl) \times 0.4343)^5 - 0.3065 \times (\log(Nl) \times 0.4343)^4 + 0.3671 \times (\log(Nl) \times 0.4343)^3 + 0.1684 \times (\log(Nl) \times 0.4343)^2 - 0.3808 \times (\log(Nl) \times 0.4343) + 0.7938
\]
E3 = -0.7232 * (Log(Nl) * 0.4343) ^ 3 - 2.8112 * (Log(Nl) * 0.4343) ^ 2 - 0.9616 * (Log(Nl) * 0.4343) + 4.5696
E4 = -0.2696 * (Log(Nl) * 0.4343) ^ 3 - 1.3422 * (Log(Nl) * 0.4343) ^ 2 - 1.2475 * (Log(Nl) * 0.4343) + 1.4511
E5 = E3 - E4 / Nd
S = E1 + E2 * Nlv + E5 * (Ngv / (1 + Nlv)) ^ 2
Vs = S / (rhol / (g * sigma)) ^ 0.25
Re = (rhol * vsl * d) / mul
If Re < 100000 Then
f1 = (0.046 * Re ^ (-0.2)) * 4
ElseIf Re > 100000 Then
f1 = (0.0014 + 0.125 / Re ^ (0.32)) * 4
End If
parameter1 = (f1 * vsg * Nd ^ (2 / 3)) / vsl
If parameter1 < 0.5 Then
f2 = 1
ElseIf 0.5 <= parameter1 < 100 Then
f2 = 0.005713103 * (Log(parameter1) * 0.4343) ^ 5 - 4.234367E-28 * (Log(parameter1) * 0.4343) ^ 4 + 0.0384202 * (Log(parameter1) * 0.4343) ^ 3 + 0.2362309 * (Log(parameter1) * 0.4343) ^ 2 - 0.679861 * (Log(parameter1) * 0.4343) + 0.7784401
End If
f3 = 1 + (f1 / 4) * (vsg / (50 * vsl)) ^ 0.5

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ElseIf fp = "B2" Then

E1 = -0.321 * (Log(Nl) * 0.4343) ^ 4 - 2.3 * (Log(Nl) * 0.4343) ^ 3 - 5.61 * (Log(Nl) * 0.4343) ^ 2 - 4.99 * (Log(Nl) * 0.4343) + 0.647
E2 = -0.0554 * (Log(Nl) * 0.4343) ^ 6 - 0.2939 * (Log(Nl) * 0.4343) ^ 5 - 0.3065 * (Log(Nl) * 0.4343) ^ 4 + 0.3671 * (Log(Nl) * 0.4343) ^ 3 + 0.1684 * (Log(Nl) * 0.4343) ^ 2 - 0.3808 * (Log(Nl) * 0.4343) + 0.7938
E3 = -0.7232 * (Log(Nl) * 0.4343) ^ 3 - 2.8112 * (Log(Nl) * 0.4343) ^ 2 - 0.9616 * (Log(Nl) * 0.4343) + 4.5696
E4 = -0.2696 * (Log(Nl) * 0.4343) ^ 3 - 1.3422 * (Log(Nl) * 0.4343) ^ 2 - 1.2475 * (Log(Nl) * 0.4343) + 1.4511
E5 = E3 - E4 / Nd
S = E1 + E2 * Nlv + E5 * (Ngv / (1 + Nlv)) ^ 2
Vs = S / (rhol / (g * sigma)) ^ 0.25

ElseIf fp = "B3" Then

vm = vsl + vsg
Vbs = (1.41 * (sigma * g * (rhol - rhog) / 0.0624 / (rhol / 0.0624) ^ 2) ^ 0.25)
vbf = 1.2 * vm + Vbs

ElseIf fp = "B4" Then

vm = vsg + vsl
landal = vsl / (vsl + vsg)
mun = mul * landal + mug * (1 - landal)

rhon = rhol * landal + rhog * (1 - landal)

ElseIf fp = "B5" Then

Vs = 1.53 * ((g * sigma * (rhol - rhog)) / (rhol ^ 2)) ^ 0.25

If d >= 0.12 And vsl <= 0.02 Then

C0 = 2

ElseIf d < 0.12 And vsl > 0.02 Then

C0 = 1.2

End If

ElseIf fp = "S1" Then

F5 = -0.2063075 * (Log(Nl) * 0.4343) ^ 6 - 1.77845 * (Log(Nl) * 0.4343) ^ 5 - 5.473126 *

* (Log(Nl) * 0.4343) ^ 4 - 6.547501 * (Log(Nl) * 0.4343) ^ 3 - 1.005135 * (Log(Nl) *

0.4343) ^ 2 + 1.911352 * (Log(Nl) * 0.4343) + 1.099639

If Nl <= 0.02 Then

F6 = 1.4269 * (Log(Nl) * 0.4343) ^ (2) + 5.3753 * (Log(Nl) * 0.4343) + 4.9138

ElseIf 0.02 < Nl <= 0.1 Then

F6 = 1.4632 * (Log(Nl) * 0.4343) ^ (2) + 7.154 * (Log(Nl) * 0.4343) + 7.8307

ElseIf Nl > 0.1 Then

F6 = 0.9622 * (Log(Nl) * 0.4343) ^ 4 - 1.0541 * (Log(Nl) * 0.4343) ^ 3 + 0.473 *

(Log(Nl) * 0.4343) ^ 2 + 0.1453 * (Log(Nl) * 0.4343) + 1.7205

End If
\[ F7 = 0.02006777 \times (\log(Nl) \times 0.4343)^6 + 0.1759328 \times (\log(Nl) \times 0.4343)^5 + \\
0.5398718 \times (\log(Nl) \times 0.4343)^4 + 0.6932904 \times (\log(Nl) \times 0.4343)^3 + 0.3869378 \times \\
(\log(Nl) \times 0.4343)^2 - 0.4274677 \times (\log(Nl) \times 0.4343) - 0.1498071 \]

\[ F8 = 0.029 \times Nd + F6 \]

\[ S = ((1 + F5) \times ((Ngv^{0.982} + F8) / (1 + F7 \times Nlv)^2)) \]

\[ Vs = S / (\rho_l / (g \times \sigma))^{0.25} \]

\[ Re = (\rho_l \times v_{sl} \times d) / (\mu_l) \]

If \( Re < 100000 \) Then

\[ f1 = (0.046 \times Re^{-0.2}) \times 4 \]

ElseIf \( Re > 100000 \) Then

\[ f1 = (0.0014 + 0.125 / Re^{0.32}) \times 4 \]

End If

\[ \text{parameter1} = (f1 \times v_{sg} \times Nd^{2/3}) / v_{sl} \]

If \( \text{parameter1} < 0.5 \) Then

\[ f2 = 1 \]

ElseIf \( 0.5 \leq \text{parameter1} < 100 \) Then

\[ f2 = 0.005713103 \times (\log(\text{parameter1}) \times 0.4343)^5 - 4.234367E-28 \times (\log(\text{parameter1}) \times 0.4343)^4 + 0.0384202 \times (\log(\text{parameter1}) \times 0.4343)^3 + 0.2362309 \times \\
(\log(\text{parameter1}) \times 0.4343)^2 - 0.679861 \times (\log(\text{parameter1}) \times 0.4343) + 0.7784401 \]

End If

\[ f3 = 1 + (f1 / 4) \times (v_{sg} / (50 \times v_{sl}))^{0.5} \]

ElseIf \( fp = "S2" \) Then
vm = vsg + vsl

Re = (rhol * vm * d) / mul

Vbsi = (0.251 + 0.00000874 * Re) * (g * d) ^ 0.5

Vb = 0.5 * (g * d) ^ 0.5

Reb = (rhol * Vb * d) / mul

If Reb <= 3000 Then

Vb = (0.546 + 0.00000874 * Re) * ((g * d) ^ 0.5)

ElseIf Reb >= 8000 Then

Vb = (0.35 + 0.00000874 * Re) * ((g * d) ^ 0.5)

ElseIf 3000 < Reb < 8000 Then

Vb = 0.5 * (Vbs + (Vbsi ^ (2) + ((13.59 * mul) / (rhol * d ^ 0.5))))

End If

Reb = (rhol * Vb * d) / mul

T = 0.013 * Log(mul * 1000) * 0.4343 / (d / 0.3048) ^ (1.38) - 0.287 - 0.162 * Log(vm / 0.3048) * 0.4343 - 0.428 * Log(d / 0.3048) * 0.4343

ElseIf fp = "S3" Then

vm = vsl + vsg

Ne = g * (d) ^ 2 * (rhol - rhog) / (sigma)

Nv = (d ^ (3) * g * rhol * (rhol - rhog)) ^ 0.5 / mul

If Nv <= 18 Then

m = 25

ElseIf Nv >= 250 Then
m = 10

Elseif 18 < Nv < 250 Then

m = 69 * Nv ^ (-0.35)

End If

C = 0.345 * (1 - Exp(-0.029 * Nv)) * (1 - Exp((3.37 - Ne) / m))

vbsa = C * ((g * d * (rhol - rhog)) / rhol) ^ (0.5)

vbfa = 1.2 * vm + vbsa

ElseIf fp = "S4" Then

vm = vsg + vsl

landal = vsl / (vsl + vsg)

mun = mul * landal + mug * (1 - landal)

ElseIf fp = "S5" Then

Vs = 0.35 * (g * d * (rhol - rhog) / rhol) ^ (0.5) * (Sin(theta / 180) * 3.14) ^ (0.5) * (1 + Cos(theta / 180 * 3.14)) ^ (1.2)

C0 = 1.2

ElseIf fp = "M1" Then

A = (rhog * vsg ^ (2) * mul ^ (2)) / (rhol * sigma ^ (2))

If A <= 0.005 Then

B = (0.033984 * sigma) / (rhog * vsg ^ (2) * d)
ElseIf A > 0.005 Then

B = (0.1684 * sigma * A ^ (0.302)) / (rhog * vsg ^ (2) * d)

End If

vm = vsg + vsl
landal = vsl / (vsl + vsg)
rhon = rhol * landal + rhog * (1 - landal)
Ek = (vm * vsg * rhon) / 101325

ElseIf fp = "M2" Then

A = (rhog * vsg ^ (2) * mul ^ (2)) / (rhol * sigma ^ (2))

If A <= 0.005 Then

B = (0.033984 * sigma) / (rhog * vsg ^ (2) * d)

ElseIf A > 0.005 Then

B = (0.1684 * sigma * A ^ (0.302)) / (rhog * vsg ^ (2) * d)

End If

vm = vsg + vsl
landal = vsl / (vsl + vsg)
rhon = rhol * landal + rhog * (1 - landal)
Ek = (vm * vsg * rhon) / 101325

ElseIf fp = "M3" Then

A = (rhog * vsg ^ (2) * mul ^ (2)) / (rhol * sigma ^ (2))
If A <= 0.005 Then

B = (0.033984 * sigma) / (rhog * vsg ^ (2) * d)

ElseIf A > 0.005 Then

B = (0.1684 * sigma * A ^ (0.302)) / (rhog * vsg ^ (2) * d)

End If

vm = vsg + vsl

landal = vsl / (vsl + vsg)

rhon = rhol * landal + rhog * (1 - landal)

Ek = (vm * vsg * rhon) / 101325

ElseIf fp = "M4" Then

vm = vsg + vsl

landal = vsl / (vsl + vsg)

mun = mul * landal + mug * (1 - landal)

ElseIf fp = "M5" Then

vm = vsg + vsl

Resl = (rhol * vm * d) / mul

If Resl < 100000 Then

fsl = (0.046 * Resl ^ (-0.2))

ElseIf Resl > 100000 Then

fsl = (0.0014 + 0.125 / Resl ^ (0.32))

Else

vm = vsg + vsl

landal = vsl / (vsl + vsg)

mun = mul * landal + mug * (1 - landal)

Resl = (rhol * vm * d) / mul

If Resl < 100000 Then

fsl = (0.046 * Resl ^ (-0.2))

ElseIf Resl > 100000 Then

fsl = (0.0014 + 0.125 / Resl ^ (0.32))
ElseIf Resl <= 2000 Then

fsl = 16 / Resl

End If

Vcrit = 10000 * (vsg * mug / sigma) * (rhog / rhol) ^ (0.5)

If Vcrit < 4 Then

FE = 0.0055 * Vcrit ^ (2.86)

ElseIf Vcrit > 4 Then

FE = 0.857 * (Log(Vcrit) * 0.4343) - 0.2

End If

Landalc = (FE * vsl) / (FE * vsl + vsg)

fc = 0.046 * (rhog * vsg * d / mug) ^ (-0.2) * (1 + 75 * Landalc)

rhoc = (vsg * rhog + vsl * rhol * FE) / (vsg + vsl * FE)

Vs = 1.53 * ((g * sigma * (rhol - rhog)) / (rhol ^ (2))) ^ 0.25

If d >= 0.12 And vsl <= 0.02 Then

C0 = 2

ElseIf d < 0.12 And vsl > 0.02 Then

C0 = 1.2

End If

vsgb = Sin((theta / 180) * 3.14) * (C0 * vsl + Vs) / (4 - C0)

vmdb = (0.468 * d ^ (0.48) * (g * (rhog - rhog) / sigma) ^ (0.5) * (sigma / rhol) ^ (0.6) *

(rhol / mul) ^ (0.08)) ^ (1 / 1.12)

If vmdb < vm Then
Vs = 0.35 * (g * d * (rhol - rhog) / rhol) ^ (0.5) * (Sin(theta / 180) * 3.14) ^ (0.5) * (1 + 
Cos(theta / 180 * 3.14)) ^ (1.2)
C0 = 1.2
Elseif vsg < vsgb Then
Vs = 1.53 * ((g * sigma * (rhol - rhog)) / (rhol ^ (2))) ^ 0.25
If d >= 0.12 And vsl <= 0.02 Then
C0 = 2
ElseIf d < 0.12 And vsl > 0.02 Then
C0 = 1.2
End If
End If
ElseIf fp = "T1" Then
F5 = -0.2063075 * (Log(Nl) * 0.4343) ^ 6 - 1.77845 * (Log(Nl) * 0.4343) ^ 5 - 5.473126
* (Log(Nl) * 0.4343) ^ 4 - 6.547501 * (Log(Nl) * 0.4343) ^ 3 - 1.005135 * (Log(Nl) *
0.4343) ^ 2 + 1.911352 * (Log(Nl) * 0.4343) + 1.099639
If Nl <= 0.02 Then
F6 = 1.4269 * (Log(Nl) * 0.4343) ^ (2) + 5.3753 * (Log(Nl) * 0.4343) + 4.9138
ElseIf 0.02 < Nl <= 0.1 Then
F6 = 1.4632 * (Log(Nl) * 0.4343) ^ (2) + 7.154 * (Log(Nl) * 0.4343) + 7.8307
ElseIf Nl > 0.1 Then

\[
F_6 = 0.9622 \times (\log(N_l) \times 0.4343)^4 - 1.0541 \times (\log(N_l) \times 0.4343)^3 + 0.473 \times \\
(\log(N_l) \times 0.4343)^2 + 0.1453 \times (\log(N_l) \times 0.4343) + 1.7205
\]

End If

\[
F_7 = 0.02006777 \times (\log(N_l) \times 0.4343)^6 + 0.1759328 \times (\log(N_l) \times 0.4343)^5 + \\
0.5398718 \times (\log(N_l) \times 0.4343)^4 + 0.6932904 \times (\log(N_l) \times 0.4343)^3 + 0.3869378 \times \\
(\log(N_l) \times 0.4343)^2 - 0.4274677 \times (\log(N_l) \times 0.4343) - 0.1498071
\]

\[
F_8 = 0.029 \times N_d + F_6
\]

\[
S = ((1 + F_5) \times ((N_{gv} \times (0.982) + F_8) / (1 + F_7 \times N_{lv})^2))
\]

\[
V_s = S / (\rho_l / (g \times \sigma))^0.25
\]

\[
Res = (\rho_l \times v_s \times d) / (\text{mul})
\]

If Res < 100000 Then

\[
f_1 = (0.046 \times \text{Res}^{-0.2}) \times 4
\]

ElseIf Res > 100000 Then

\[
f_1 = (0.0014 + 0.125 / \text{Res}^{0.32}) \times 4
\]

End If

\[
\text{parameter1} = (f_1 \times v_s \times N_d^2 / 3) / v_s
\]

If parameter1 < 0.5 Then

\[
f_2 = 1
\]

ElseIf 0.5 <= parameter1 < 100 Then

\[
f_2 = 0.005713103 \times (\log(\text{parameter1}) \times 0.4343)^5 - 4.234367E-28 \times (\log(\text{parameter1}) \times \\
0.4343)^4 + 0.0384202 \times (\log(\text{parameter1}) \times 0.4343)^3 + 0.2362309 \times \\
(\log(\text{parameter1}) \times 0.4343)^2 - 0.679861 \times (\log(\text{parameter1}) \times 0.4343) + 0.7784401
\]

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End If

\[ f_3 = 1 + \left( \frac{f_1}{4} \right) \times \left( \frac{v_{sg}}{50 \times v_{sl}} \right)^{0.5} \]

\[ f_s = f_1 \times f_2 \div f_3 \]

\[ v_m = v_{sg} + v_{sl} \]

\[ h_{ls} = \frac{(V_s - v_m + ((v_m - V_s)^2 + 4 \times V_s \times v_{sl})^{0.5})}{2 \times V_s} \]

\[ \rho_{oss} = \rho_{ol} \times h_{ls} + \rho_{og} \times (1 - h_{ls}) \]

\[ \frac{d}{p_d}z_els = \rho_{oss} \times g \times \sin(\theta / 180 \times 3.14) \]

\[ \frac{d}{p_d}z_fs = f_s \times \rho_{ol} \times v_{sl} \times v_m \div (2 \times d) \]

\[ \frac{d}{p_d}zts = \frac{d}{p_d}z_els + \frac{d}{p_d}z_fs \]

\[ A = \frac{\rho_{og} \times v_{sg}^2 \times \mu \times ^2}{\rho_{ol} \times \sigma^2} \]

If \( A \leq 0.005 \) Then

\[ B = \frac{0.033984 \times \sigma}{\rho_{og} \times v_{sg}^2 \times d} \]

ElseIf \( A > 0.005 \) Then

\[ B = \frac{0.1684 \times \sigma \times A^{0.302}}{\rho_{og} \times v_{sg}^2 \times d} \]

End If

\[ v_m = v_{sg} + v_{sl} \]

\[ \lambda_{nal} = v_{sl} \div (v_{sl} + v_{sg}) \]

\[ \rho_{on} = \rho_{ol} \times \lambda_{nal} + \rho_{og} \times (1 - \lambda_{nal}) \]

\[ E_k = \frac{(v_m \times v_{sg} \times \rho_{on})}{101325} \]

\[ R_e = \frac{(\rho_{og} \times v_{sg} \times d)}{\mu g} \]

If \( B > 0.05 \) Then

\[ f_m = (0.067 \times B^{1.73} + (1 / (4 \times \log(B \times 0.27) \times 0.4343)^2)) \]
ElseIf B < 0.05 And Re < 100000 Then
  fm = (0.046 * Re ^ (-0.2))
ElseIf Re > 100000 And B < 0.05 Then
  fm = (0.0014 + 0.125 / Re ^ (0.32))
End If

hlm = landal
rhosm = rhol * hlm + rhog * (1 - hlm)

dpdzfm = fm * rhog * vsg ^ (2) / 2 / d

dpdzelm = rhon * g * Sin(\theta / 180 * 3.14)

dpdztm = (dpdzelm + dpdzfm) / (1 - Ek)

NgvTrM = 75 + 84 * Nlv ^ (0.75)
NgVsTr = 50 + 36 * Nlv

AT = (NgvTrM - Ngv) / (NgvTrM - NgVsTr)

ElseIf fp = "T2" Then
  vm = vsg + vsl
  Re = (rhol * vm * d) / mul
  Vbsi = (0.251 + 0.00000874 * Re) * (g * d) ^ 0.5
  Vb = 0.5 * (g * d) ^ 0.5
  Reb = (rhol * Vb * d) / mul
  If Reb <= 3000 Then
    Vb = (0.546 + 0.00000874 * Re) * ((g * d) ^ 0.5)
ElseIf Reb >= 8000 Then

\[ V_b = (0.35 + 0.00000874 \times R_e) \times ((g \times d)^{0.5}) \]

ElseIf 3000 < Reb < 8000 Then

\[ V_b = 0.5 \times (V_b + (V_b \times (2) + (13.59 \times \mu) / (r_h \times d^{0.5}))) \]

End If

\[ Reb = (r_h \times V_b \times d) / \mu \]

\[ T = 0.013 \times \log(\mu \times 1000) - 0.4343 / (d / 0.3048)^{(1.38)} - 0.287 - 0.162 \times \log(v_m / 0.3048) - 0.428 \times \log(d / 0.3048) \times 0.4343 \]

If Re < 100000 Then

\[ f = (0.046 \times R_e^{(0.2)}) \times 4 \]

ElseIf Re > 100000 Then

\[ f = (0.0014 + 0.125 / R_e^{(0.32)}) \times 4 \]

End If

\[ r_h = (r_h \times (v_s + V_b) + r_h \times v_s) / (v_m + V_b) + (r_h \times T) \]

\[ \frac{d_p d_z F}{\mu} = ((f \times r_h \times v_m^{(2)}) / (2 \times d)) \times (((v_s + V_b) / (v_m + V_b)) + T) \]

\[ d_p d_z e l = r_h \times g \times \sin(\theta / 180 \times 3.14) \]

\[ d_p d_z t s = d_p d_z e l + d_p d_z f \]

\[ d_p d_z m e a s = (P_1 - P_2) / 14.696 \times 101325 / d_1 \]

\[ A = (r_h \times v_s^{(2)} \times \mu^{(2)}) / (r_h \times \sigma^{(2)}) \]

If A <= 0.005 Then

\[ B = (0.033984 \times \sigma) / (r_h \times v_s^{(2)} \times d) \]

ElseIf A > 0.005 Then
B = (0.1684 * sigma * A ^ (0.302)) / (rhog * vsg ^ (2) * d)

End If

vm = vsg + vsl

landal = vsl / (vsl + vsg)

rhon = rhol * landal + rhog * (1 - landal)

Ek = (vm * vsg * rhon) / 101325

Re = (rhog * vsg * d) / mug

If B > 0.05 Then

fm = (0.067 * B ^ (1.73) + (1 / (4 * Log(B * 0.27) * 0.4343) ^ 2))

ElseIf B < 0.05 And Re < 100000 Then

fm = (0.046 * Re ^ (-0.2))

ElseIf Re > 100000 And B < 0.05 Then

fm = (0.0014 + 0.125 / Re ^ (0.32))

End If

hlm = landal

rhosm = rhol * hlm + rhog * (1 - hlm)

dpdzfm = fm * rhog * vsg ^ (2) / 2 / d

dpdzelm = rhon * g * Sin(theta / 180 * 3.14)

dpdztm = (dpdzelm + pdzfm) / (1 - Ek)

NgvTrM = 75 + 84 * Nlv ^ (0.75)

NgVsTr = 50 + 36 * Nlv

AT = (NgvTrM - Ngv) / (NgvTrM - NgVsTr)
ElseIf fp = "T3" Then

A = (rhog * vsg ^ (2) * mul ^ (2)) / (rhol * sigma ^ (2))

If A <= 0.005 Then

B = (0.033984 * sigma) / (rhog * vsg ^ (2) * d)

ElseIf A > 0.005 Then

B = (0.1684 * sigma * A ^ (0.302)) / (rhog * vsg ^ (2) * d)

End If

vm = vsg + vsl

landal = vsl / (vsl + vsg)

rhon = rhol * landal + rhog * (1 - landal)

Ek = (vm * vsg * rhon) / 101325

Re = (rhog * vsg * d) / mug

If B > 0.05 Then

fm = (0.067 * B ^ (1.73) + (1 / (4 * Log(B * 0.27) * 0.4343) ^ 2))

ElseIf B < 0.05 And Re < 100000 Then

fm = (0.046 * Re ^ (-0.2))

ElseIf Re > 100000 And B < 0.05 Then

fm = (0.0014 + 0.125 / Re ^ (0.32))

End If

hlm = landal

rhosm = rhol * hlm + rhog * (1 - hlm)

dpdzfm = fm * rhog * vsg ^ (2) / 2 / d
dpdzelm = rhon * g * Sin(theta / 180 * 3.14)

dpdztm = (dpdzelm + dpdzfm) / (1 - Ek)

vm = vsl + vsg

Ne = g * (d) ^ 2 * (rhol - rhog) / (sigma)

Nv = (d ^ (3) * g * rhol * (rhol - rhog)) ^ 0.5 / mul

If Nv <= 18 Then
  m = 25
ElseIf Nv >= 250 Then
  m = 10
ElseIf 18 < Nv < 250 Then
  m = 69 * Nv ^ (-0.35)
End If

C = 0.345 * (1 - Exp(-0.029 * Nv)) * (1 - Exp((3.37 - Ne) / m))

vbsa = C * ((g * d * (rhol - rhog)) / rhol) ^ (0.5)

vbfa = 1.2 * vm + vbsa

vm = vsg + vsl

hls = 1 - vsg / vbfa

Res = rhol * vm * d / mul

If Res < 100000 Then
  fs = 0.046 * Res ^ (-0.2)
ElseIf Re > 100000 Then
  fs = 0.0014 + 0.125 / Res ^ (0.32)

End If

rhoss = rhol * hls + rhog * (1 - hls)

dpdzfs = fs * rhol * (vm ^ (2)) * hls / 2 / d

dpdzels = rhoss * g * Sin(theta / 180 * 3.14)

dpdzts = dpdzels + dpdzfs

Nx = vsg / 0.3048 * (rhog / 1.2) ^ (1 / 3) * (72 / 72 * rhol / 1000) ^ (1 / 4)

Ny = vsl / 0.3048 * (72 / 72 * rhol / 1000) ^ (1 / 4)

N1 = 0.51 * (100 * Ny) ^ 0.172

N2 = 8.6 + 3.8 * Ny

N3 = 70 * (100 * Ny) ^ -0.152

A = (N3 - Nx) / (N3 - N2)

End If

End Sub

Private Sub calcHagedornBrown()

vm = vsl + vsg

hl = HL_om * omega

If hl >= landal Then

hl = HL_om * omega

ElseIf hl < landal Then

hl = landal

End If

mus = mul * hl + mug * (1 - hl)
rhos = rhol * hl + rhog * (1 - hl)

Re = (vm * rhon * d) / mus

If Re < 100000 Then

f = 0.046 * Re ^ (-0.2)

ElseIf Re > 100000 Then

f = 0.0014 + 0.125 / Re ^ (0.32)

End If

dpdzf = 4 * f * (rhon ^ (2)) * (vm ^ (2)) / 2 / rhos / d

dpdzel = rhos * g * Sin(theta / 180 * 3.14)

dpdzt = dpdzf + dpdzel

dpdzmeas = (P1 - P2) / 14.696 * 101325 / d1

End Sub

Private Sub calcdunsrosbubbleflow()

vm = vsg + vsl

hl = (Vs - vm + ((vm - Vs) ^ 2 + 4 * Vs * vsl) ^ 0.5) / (2 * Vs)

Re = (rhol * vsl * d) / mul

f = f1 * f2 / f3

rhos = rhol * hl + rhog * (1 - hl)

dpdzel = rhos * g * Sin(theta / 180 * 3.14)

dpdzf = f * rhol * vsl * vm / (2 * d)

dpdzt = dpdzel + dpdzf

dpdzmeas = (P1 - P2) / 14.696 * 101325 / d1
End Sub

Private Sub calcOrkiszewskibubbleflow()

vm = vsg + vsl

hl = 1 - 0.5 * (1 + vm / Vs - ((1 + vm / Vs) ^ 2 - (4 * vsg / Vs)) ^ 0.5)

rhos = rhol * hl + rhog * (1 - hl)

Re = rhol * vsl * d / (hl * mul)

If Re < 100000 Then
    f = (0.046 * Re ^ (-0.2)) * 4
ElseIf Re > 100000 Then
    f = (0.0014 + 0.125 / Re ^ (0.32)) * 4
End If

dpdzf = f * rhol * (vsl / hl) ^ 2 / (2 * d)

dpdzel = rhos * g * Sin(theta / 180 * 3.14)

dpdzt = pdpdzel + pddzf

dpdzmeas = (P1 - P2) / 14.696 * 101325 / d1

End Sub

Private Sub calcAzizbubbleflow()

vm = vsg + vsl

hl = 1 - vsg / vbf

Re = rhol * vm * d / mul

If Re < 100000 Then
    f = 0.046 * Re ^ (-0.2)
End If
ElseIf Re > 100000 Then

\[
f = 0.0014 + 0.125 / \text{Re}^{0.32}
\]

End If

rhos = \text{rhol} \times hl + \text{rhog} \times (1 - hl)

dpdzf = f \times rhos \times (\text{vm}^2) / 2 / d

dpdzel = \text{rhos} \times g \times \text{Sin(\theta / 180 \times 3.14)}

dpdzt = pdzel + pdzf

dpdzmeas = (\text{P1} - \text{P2}) / 14.696 \times 101325 / \text{d1}

End Sub

Private Sub calcMukherjeeBrillbubbleflow()

\[
\text{vm} = \text{vsg} + \text{vsl}
\]

\[
\text{hl} = \text{Exp}((-0.380113 + 0.1298758 \times \text{Sin(90 / 180 \times 3.14)}) - 0.119788 \times (\text{Sin(90 / 180 \times 3.14)})^2 + 2.343227 \times Nl^2) \times (\text{Ngv} \times (0.475686) / \text{Nlv} \times (0.288657)))
\]

\[
\text{Re} = \text{rhon} \times \text{vm} \times \text{d} / \text{mun}
\]

If Re < 100000 Then

\[
f = 0.046 \times \text{Re}^{-0.2} \times 4
\]

ElseIf Re > 100000 Then

\[
f = 0.0014 + 0.125 / \text{Re}^{0.32} \times 4
\]

End If

rhos = \text{rhol} \times hl + \text{rhog} \times (1 - hl)

dpdzf = f \times rhos \times \text{vm}^2 / (2 \times d)

dpdzel = \text{rhos} \times g \times \text{Sin(\theta / 180 \times 3.14)}
dpdzt = dpdzel + dpdzf

dpdzmeas = (P1 - P2) / 14.696 * 101325 / d1

End Sub

Private Sub calchasanKabirbubbleflow()

vm = vsg + vsl

Vs = (1.53 * (g * sigma * (rhol - rhog) / rhol ^ (2)) ^ 0.25)

If d >= 0.12 And vsl <= 0.02 Then

C0 = 2

ElseIf d < 0.12 And vsl > 0.02 Then

C0 = 1.2

End If

hl = 1 - (vsg / (C0 * vm + Vs))

Re = (rhol * vm * d) / mul

If Re < 100000 Then

f = 0.046 * Re ^ (-0.2)

ElseIf Re > 100000 Then

f = 0.0014 + 0.125 / Re ^ (0.32)

End If

rhos = rhol * hl + rhog * (1 - hl)

dpdzf = f * rhos * vm ^ (2) / (2 * d)

dpdzel = rhos * g * Sin(theta / 180 * 3.14)

dpdzt = dpdzel + dpdzf'
dpdzmeas = (P1 - P2) / 14.696 * 101325 / d1

End Sub

Private Sub calcDunsrosslugflow()

vm = vsg + vsl

hl = (Vs - vm + ((vm - Vs) ^ 2 + 4 * Vs * vsl) ^ 0.5) / (2 * Vs)

Re = (rhol * vsl * d) / mul

f = f1 * f2 / f3

rhos = rhol * hl + rhog * (1 - hl)

dpdzel = rhos * g * Sin(theta / 180 * 3.14)

dpdzf = f * rhol * vsl * vm / (2 * d)

dpdzt = dpdzel + dpdzf

dpdzmeas = (P1 - P2) / 14.696 * 101325 / d1

End Sub

Private Sub calcOrkiszewskislugflow()

vm = vsg + vsl

hl = 0

Re = (rhol * vm * d) / mul

If Re < 100000 Then

f = (0.046 * Re ^ (-0.2)) * 4

ElseIf Re > 100000 Then

f = (0.0014 + 0.125 / Re ^ (0.32)) * 4

End If
rhos = ((rhol * (vsl + Vb) + rhog * vsg) / (vm + Vb)) + (rhol * T)
dpdzf = ((f * rhol * vm ^ (2)) / (2 * d)) * (((vsl + Vb) / (vm + Vb)) + T)
dpdzel = rhos * g * Sin(theta / 180 * 3.14)
dpdzt = dpdzel + dpdzf
dpdzmmeas = (P1 - P2) / 14.696 * 101325 / d1

End Sub

Private Sub calcAzizslugflow()
vm = vsg + vsl
hl = 1 - vsg / vbfa
Re = rhol * vm * d / mul
If Re < 100000 Then
f = 0.046 * Re ^ (-0.2)
ElseIf Re > 100000 Then
f = 0.0014 + 0.125 / Re ^ (0.32)
End If
rhos = rhol * hl + rhog * (1 - hl)
dpdzf = f * rhol * (vm ^ (2)) * hl / 2 / d
dpdzel = rhos * g * Sin(theta / 180 * 3.14)
dpdzt = dpdzel + dpdzf
dpdzmmeas = (P1 - P2) / 14.696 * 101325 / d1
End Sub
Private Sub calcMukherjeeBrillslugflow()

    hl = Exp((-0.380113 + 0.1298758 * Sin(90 / 180 * 3.14) - 0.119788 * (Sin(90 / 180 * 3.14))^2 + 2.343227 * Nl^2) * (Ngv^(0.475686) / Nlv^(0.288657)))

    landal = vsl / (vsl + vsg)

    mun = mul * landal + mug * (1 - landal)

    rhon = rhol * landal + rhog * (1 - landal)

    Re = rhon * vm * d / mun

    If Re < 100000 Then
        f = 0.046 * Re^(-0.2) * 4
    ElseIf Re > 100000 Then
        f = 0.0014 + 0.125 / Re^0.32 * 4
    End If

    rhos = rhol * hl + rhog * (1 - hl)

    dpdzf = f * rhos * vm^2 / (2 * d)

    dpdzel = rhos * g * Sin(theta / 180 * 3.14)

    dpdzt = dpdzel + dpdzf

    dpdzmeas = (P1 - P2) / 14.696 * 101325 / d1

End Sub

Private Sub calchasanKabirslugflow()

    vm = vsg + vsl

    hl = 1 - (vsg / (C0 * vm + Vs))

    Re = (rhol * vm * d) / mul
If $Re < 100000$ Then

\[ f = (0.046 \times Re^{-0.2}) \]

ElseIf $Re > 100000$ Then

\[ f = (0.0014 + 0.125 / Re^{0.32}) \]

End If

\[ \rho_s = \rho_l \times h_l + \rho_g \times (1 - h_l) \]

\[ \frac{dP}{dz} = f \times \rho_s \times \frac{v_m^2}{2 \times d} \]

\[ \frac{dP}{z \cos \theta} = \rho_s \times g \times \sin \left( \frac{\theta}{180 \times 3.14} \right) \]

\[ \frac{dP}{z} = \frac{dP}{z \cos \theta} + \frac{dP}{dz} \]

\[ \frac{dP}{z} = \frac{(P_1 - P_2)}{14.696 \times 101325 / d_1} \]

End Sub

Private Sub calcdunsrosmistflow()

\[ v_m = v_{sg} + v_{sl} \]

\[ h_l = landal \]

\[ Re = (\rho_g \times v_{sg} \times d) / \mu_g \]

If $B > 0.05$ Then

\[ f = (0.067 \times B^{1.73}) + \frac{1}{(4 \times \log(B \times 0.27) \times 0.4343)^2} \]

ElseIf $B < 0.05$ And $Re < 100000$ Then

\[ f = (0.046 \times Re^{-0.2}) \]

ElseIf $Re > 100000$ And $B < 0.05$ Then

\[ f = (0.0014 + 0.125 / Re^{0.32}) \]

End If
rhos = rhol * hl + rhog * (1 - hl)

dpdzf = f * rhog * vsg ^ (2) / 2 / d

dpdzel = rhon * g * Sin(theta / 180 * 3.14)

dpdzt = (dpdzel + dpdzf) / (1 - Ek)

dpdzmeas = (P1 - P2) / 14.696 * 101325 / d1

End Sub

Private Sub calcOrkiszewskimistflow()

vm = vsg + vsl

hl = landal

Re = (rhog * vsg * d) / mug

If B > 0.05 Then
f = (0.067 * B ^ (1.73) + (1 / (4 * Log(B * 0.27) * 0.4343) ^ 2))
ElseIf B < 0.05 And Re < 100000 Then
f = (0.046 * Re ^ (-0.2))
ElseIf Re > 100000 And B < 0.05 Then
f = (0.0014 + 0.125 / Re ^ (0.32))
End If

rhos = rhol * hl + rhog * (1 - hl)

dpdzf = f * rhog * vsg ^ (2) / 2 / d

dpdzel = rhon * g * Sin(theta / 180 * 3.14)

dpdzt = (dpdzel + dpdzf) / (1 - Ek)

End Sub
dpdzmeas = (P1 - P2) / 14.696 * 101325 / d1

End Sub

Private Sub calcAzizmistflow()

vm = vsg + vsl

hl = landal

Re = (rhog * vsg * d) / mug

If B > 0.05 Then
f = 4 * (0.067 * B ^ (1.73) + (1 / (4 * Log(B * 0.27) * 0.4343) ^ 2))
ElseIf B < 0.05 And Re < 100000 Then
f = (0.046 * Re ^ (-0.2)) * 4
ElseIf Re > 100000 And B < 0.05 Then
f = (0.0014 + 0.125 / Re ^ (0.32)) * 4
End If

rhos = rhol * hl + rhog * (1 - hl)

dpdzf = f * rhog * vsg ^ (2) / 2 / d

dpdzel = rhon * g * Sin(theta / 180 * 3.14)

dpdzt = (dpdzel + pdzf) / (1 - Ek)

dpdzmeas = (P1 - P2) / 14.696 * 101325 / d1

End Sub

Private Sub calchasankabirmistflow()

vm = vsg + vsl

hl = 1 - (vsg / (C0 * vm + Vs))
Re = (rhol * vm * d) / mul

f = 0.046 * (rhog * vsg * d / mug) ^ (-0.2) * (1 + 75 * Landalc)

rhos = rhol * hl + rhog * (1 - hl)

dpdzf = (f * rhoc / 2 / d) * (vsg / (1 - hl)) ^ (2)

dpdzel = rhos * g * Sin(theta / 180 * 3.14)

dpdzt = pdzel + pdzf

dpdzmeas = (P1 - P2) / 14.696 * 101325 / d1

End Sub

Private Sub calcMukherjeeBrillmistflow()

hl = Exp((-0.380113 + 0.1298758 * Sin(90 / 180 * 3.14) - 0.119788 * (Sin(90 / 180 * 3.14)) ^ 2 + 2.343227 * Nl ^ (2)) * (Ngv ^ (0.475686) / Nlv ^ (0.288657)))

landal = vsl / (vsl + vsg)

mun = mul * landal + mug * (1 - landal)

rhon = rhol * landal + rhog * (1 - landal)

Re = rhon * vm * d / mun

If Re < 100000 Then

f = 0.046 * Re ^ (-0.2)

ElseIf Re > 100000 Then

f = 0.0014 + 0.125 / Re ^ (0.32)

End If

rhos = rhol * hl + rhog * (1 - hl)

dpdzf = 4 * f * rhon * vm ^ (2) / (2 * d)
dpdzel = rhos * g * Sin(theta / 180 * 3.14)
dpdzt = dpdzel + dpdzf
dpdzmeas = (P1 - P2) / 14.696 * 101325 / d1

End Sub

Private Sub calcdunsrotransitionregion()

hl = 0
Re = 0
f = 0
rhos = 0
dpdzel = 0
dpdzf = 0
dpdzt = AT * dpdzts + (1 - AT) * dpdztm

dpdzmeas = (P1 - P2) / 14.696 * 101325 / d1

End Sub

Private Sub calcOrkiszewskitransitionregion()

hl = 0
Re = 0
f = 0
rhos = 0
dpdzel = 0
dpdzf = 0
dpdz = AT * dpdzts + (1 - AT) * dpdztm

dpdzmeas = (P1 - P2) / 14.696 * 101325 / d1

End Sub

Private Sub calcAziztransitionregion()

hl = 0

Re = 0

f = 0

rhos = 0

dpdzel = 0

dpdzf = 0

dpdzt = A * dpdzts + (1 - A) * dpdztm

dpdzmeas = (P1 - P2) / 14.696 * 101325 / d1

End Sub

Private Sub getoutput()

If fp = "B1" Then

    Range("j2") = "Duns & Ros_Bubble Flow"

ElseIf fp = "B2" Then

    Range("j2") = "Orkiszewski_Bubble Flow"

ElseIf fp = "B3" Then

    Range("j2") = "Aziz_Bubble Flow"

ElseIf fp = "B4" Then

    Range("j2") = "Mukherjee & Brill_Bubble Flow"

ElseIf fp = "B5" Then

    Range("j2") = "Other_Bubble Flow"

End Sub
Range("j2") = "Hassan & Kabir_Bubble Flow"

ElseIf fp = "S1" Then
    Range("j2") = "Duns & Ros_Slug Flow"

ElseIf fp = "S2" Then
    Range("j2") = "Orkiszewski_Slug Flow"

ElseIf fp = "S3" Then
    Range("j2") = "Aziz_Slug Flow"

ElseIf fp = "S4" Then
    Range("j2") = "Mukherjee & Brill_Slug Flow"

ElseIf fp = "S5" Then
    Range("j2") = "Hassan & Kabir_Slug Flow"

ElseIf fp = "M1" Then
    Range("j2") = "Duns & Ros_Mist Flow"

ElseIf fp = "M2" Then
    Range("j2") = "Orkiszewski_Mist Flow"

ElseIf fp = "M3" Then
    Range("j2") = "Aziz_Mist Flow"

ElseIf fp = "M4" Then
    Range("j2") = "Mukherjee & Brill_Mist Flow"

ElseIf fp = "M5" Then
    Range("j2") = "Hassan & Kabir_Mist Flow"

ElseIf fp = "T1" Then
    Range("j2") = "Duns & Ros_Transition Region"
    ElseIf fp = "T2" Then
        Range("j2") = "Orkiszewski_Transition Region"
    ElseIf fp = "T3" Then
        Range("j2") = "Aziz_Transition Region"
    End If

    Range("J3") = vm
    Range("J4") = hl
    Range("J5") = Re
    Range("J6") = f
    Range("J7") = rhos
    Range("J8") = dpdz
    Range("J9") = dpdz
    Range("J10") = dpdz
    Range("J11") = dpdz
    End Sub
APPENDIX C

C-1 Calibration of sensors

For the calibration of sensors as shown in Figure 24, a wooden stand and two rubber stoppers which were located at the end of sensor were used. One of the stoppers was located at one end of it in order to add water. After that, excess water was removed by placing another stopper. Then the water in sensor was poured in graduated cylinder and the water volume is measured to obtain the maximum liquid volume that sensor can keep. In different angles (0°, 90°, 180°, and 270°) which are marked on each sensor and vertical position, the maximum volume is determined. The maximum liquid volume was 537mL. After that sensor which was still connected to capacitance meter, is filled in 5 percent of maximum volume in order to obtain capacitance in vertical position. This method is done repeatability at 5 percent until 100 percent for both sensors. This process also is repeated for SDS and water to get capacitance.

Figure 24: Wooden stand and rubber stoppers used in calibration of sensors
Figures 25 and 26 show calibration of each electrical capacitance sensor versus percent volume for different configurations (0° to 270° and vertical position).

*Figure 25: Horizontal capacitance for DI water and SDS in different positions*
As shown in Figures 25 and 26, there is about 10% error for each sensor in DI water and SDS.
D-1 Determination of SDS concentration by titration method

Titration method is used to determine the concentration of SDS in the tank. In this method, $\text{C}_{20}\text{H}_{18}\text{BrN}_3$ (Dimidium Bromide), $\text{C}_{27}\text{H}_{31}\text{N}_2\text{NaO}_6\text{S}_2$ (Diulfine Blue VN 150), a cationic and anionic indicator and $\text{CH}_2\text{CL}_2$ (Methylene Chloride) are used. These are added to sample and a pink color is appeared in the mixture because of dimidium in organic phase. In order to titrate solution $\text{C}_{27}\text{H}_{42}\text{ClNO}_2$ (Hyamine 1622) which is a cationic surfactant is added to the sample until the color of sample change to blue and organic layer is transparent. After that the amount of consumed Hyamine and the grams of SDS in the sample are determined to calculate the concentration of SDS.

6 g of Sodium dodecyl sulfate was titrated by using 5ml of Hyamine 0.0004N which is equal to 0.0004mol/L. The molecular weight of Sodium dodecyl sulfate is 288.4g/mol. The density of water was used in calculation.

\[ 5(\text{ml}) \times 0.0004 \left( \frac{\text{mol}}{\text{L}} \right) \times 0.001 \left( \frac{\text{L}}{\text{ml}} \right) = 0.000008 \text{ moles of Hyamine used} \]

\[ 0.000008(\text{mol}) \times 288.4 \left( \frac{\text{g}}{\text{mol}} \right) = 0.000576767 \text{ g of SDS in sample} \]

\[ \frac{0.000576767(\text{g SDS})}{6(\text{g})} \times 1000 \left( \frac{\text{g}}{\text{L}} \right) \times 1000 \left( \frac{\text{mg}}{\text{g}} \right) = 96 \left( \frac{\text{mg}}{\text{L}} \right) = 96 \text{ppm} \]
D-2 Measurement of SDS surface tension

Fisher surface Tensiometer, shown in Figure 27, was used to determine surface tension of SDS in the sample. “Sample was placed in the glass vessel with a diameter of at least 45mm. After that, vessel was placed on stand and ring was released by unclipping the lever arm. The sample table was raised until ring was immersed in the sample at least an 1/8in. scale was adjusted on zero and knob on the right of the case was adjusted until the ring broke the surface of the sample”.[27] The surface tension of SDS was recorded 64 mN/m.

Figure 27: Fisher Surface Tensiometer
D-3  Measurement of viscosity

The Brookfield programmable DV-II+ viscometer, shown in Figure 28, was used to measure the viscosity and temperature of SDS in sample. The measurement accuracy is ±11%. The viscosity of sample was 0.94cP and temperature 20.2°C.

Figure 28: The Brookfield programmable DV-II+ viscometer
Table 3: Flow pattern, pressure drop of different models and measured pressure drop at fixed vs_l = 0.12 m/s

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<tr>
<th>Air/Dis/Water</th>
<th>D&amp;R</th>
<th>Orkiszewski</th>
<th>Aziz</th>
<th>M&amp;B</th>
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