FLEXURAL ANALYSIS AND COMPOSITE BEHAVIOR OF PRECAST CONCRETE SANDWICH PANEL

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ABSTRACT

FLEXURAL ANALYSIS AND COMPOSITE BEHAVIOR OF PRECAST CONCRETE SANDWICH PANEL

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Several experimental studies have shown the effect of core shear contribution in precast concrete sandwich wall panels. Due to the complex nature of such construction, quantifying the contribution of the core on the behavior of the precast concrete sandwich wall panel subjected to lateral load and in-plane loads is still a challenge. Based on engineering judgment and experience, current design practices assume a certain percentage in composite action between the faces (wythes) of the sandwich panel. In this study, a general equation for the deflection of a simply supported sandwich panel under in-plane and lateral loads was developed. The formulated equation includes all mechanical properties of the core and the thick similar faces. Methods for calculating bending moments and stresses to design the precast concrete sandwich wall panel were developed and validated. The proposed equations allow for parametric studies without limitations regarding reinforcements, core shear mechanical properties, and geometrical dimensions.
Dedicated to my parents
ACKNOWLEDGEMENTS

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<td>PCSP</td>
<td>Precast Concrete Sandwich Panel</td>
</tr>
<tr>
<td>I</td>
<td>Moment of Inertia</td>
</tr>
<tr>
<td>E</td>
<td>Modulus of Elasticity</td>
</tr>
<tr>
<td>$E_f$</td>
<td>Modulus of Elasticity of the Faces</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of Elasticity of the Core</td>
</tr>
<tr>
<td>t</td>
<td>Thickness of the Faces</td>
</tr>
<tr>
<td>c</td>
<td>Thickness of the Core</td>
</tr>
<tr>
<td>Q</td>
<td>Shear force</td>
</tr>
<tr>
<td>s</td>
<td>First Moment of Area</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Shear Stress</td>
</tr>
<tr>
<td>M</td>
<td>Applied Moment</td>
</tr>
<tr>
<td>q</td>
<td>Uniformly Distributed Transverse Load per Unit Area</td>
</tr>
<tr>
<td>$G_{yz}$</td>
<td>Shear Modulus of the Core in $yz$ Plane</td>
</tr>
<tr>
<td>$G_{zx}$</td>
<td>Shear Modulus of the Core in $zx$ Plane</td>
</tr>
<tr>
<td>$G_{xy}$</td>
<td>Shear Modulus of the Faces in $xy$ Plane</td>
</tr>
<tr>
<td>$E_x$</td>
<td>Modulus of Elasticity of the Faces in $x$ Direction</td>
</tr>
<tr>
<td>$E_y$</td>
<td>Modulus of Elasticity of the Faces in $y$ Direction</td>
</tr>
<tr>
<td>$\nu_y$</td>
<td>Poisson’s Ratio of the Faces in $y$ Direction</td>
</tr>
<tr>
<td>$\nu_x$</td>
<td>Poisson’s Ratio of the Faces in $x$ Direction</td>
</tr>
</tbody>
</table>
$W$    Deflection of the PCSP
$a$    Length of the PCSP
$b$    Width of the PCSP
$n$    Mode Number
$m$    Mode Number
$a_{mn}$    Amplitude
$U_c$    Shear Strain Energy in the Core
$\mu$    Parameter to Account for Shear Flexibility of the Core
$\lambda$    Parameter to Account for Shear Flexibility of the Core
$U_{fm1}$    Membrane Strain Energy in the Upper Face
$U_{fm2}$    Membrane Strain Energy in the Bottom Face
$U_{fm}$    Total Membrane Strain Energy
$E_{x1}$    Modulus of Elasticity of the Upper Face in $x$ Direction
$t_1$    Thickness of the Upper Face
$E_{y1}$    Modulus of Elasticity of the Upper Face in $y$ Direction
$v_{y1}$    Poisson’s Ratio of the Upper Face in $y$ Direction
$G_{xy1}$    Shear Modulus of the Upper Face in $xy$ Plane
$E_{x2}$    Modulus of Elasticity of the Bottom Face in $x$ Direction
$t_2$    Thickness of the Bottom Face
$E_{y2}$    Modulus of Elasticity of the Bottom Face in $y$ Direction
$v_{y2}$    Poisson’s Ratio of the Bottom Face in $y$ Direction
$G_{xy2}$    Shear Modulus of the Bottom Face in $xy$ Plane
$U_{fb1}$    Bending Strain Energy in the Upper Face
$U_{fb2}$    Bending Strain Energy in the Bottom Face
\( U_{fb} \)  Total Bending Strain Energy

\( N_x \)  Tensile/Compressive Edge load in \( x \) Direction

\( N_y \)  Tensile/Compressive Edge load in \( y \) Direction

\( \nu_1 \)  Potential Energy Due to the Uniformly Distributed Transverse Load

\( \nu_2 \)  Potential Energy of the Applied Edge Loads

\( E_s \)  Modulus of Elasticity of the Steel

\( E_c \)  Modulus of Elasticity of the Concrete

\( \nu_s \)  Poisson’s Ratio of the Steel

\( \nu_c \)  Poisson’s Ratio of the Concrete

\( G_s \)  Shear Modulus of the Steel

\( G_c \)  Shear Modulus of the Concrete

\( Q_{ij} \)  Stiffness Transformation Matrix

\( A_{ij} \)  Extensional Stiffness

\( B_{ij} \)  Coupling Stiffness

\( I_{cx} \)  Moment of Inertia of the Concrete Material Where \( x \) Is Constant

\( I_{sx} \)  Moment of Inertia of the Steel Reinforcement Where \( x \) Is Constant

\( I_{cy} \)  Moment of Inertia of the Concrete Material Where \( y \) Is Constant

\( I_{sy} \)  Moment of Inertia of the Steel Reinforcement Where \( y \) Is Constant
CHAPTER I

INTRODUCTION

Precast Concrete Sandwich Panel (PCSP) systems are composed of two concrete wythes separated by a single or multiple layers of insulation. The reason the sandwich panel is being used is that the flexural strength and flexural rigidity can be improved in comparison with a homogeneous plate of material without increasing weight. Consider a single skin structure, as shown in Fig. 1.1, one can apply bending to this beam and calculate weight, bending stiffness, and bending strength and set them into unity. Now one can cut the beam from the middle and separate parts with insulation (core), and one more time calculate weight, bending stiffness, and bending strength. The more the distance between the two parts is increased, the bigger value for flexural strength and flexural rigidity will be obtained [1]. The relative properties of each beam are provided in table 1.1.

![Fig. 1.1 Comparison between homogeneous and sandwich cross section beam (taken from [1])](image)
Table 1.1 Properties of homogeneous and sandwich beam

<table>
<thead>
<tr>
<th>Thickness of core</th>
<th>Weight</th>
<th>Flexural rigidity</th>
<th>Bending strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2t</td>
<td>1</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>4t</td>
<td>1</td>
<td>48</td>
<td>12</td>
</tr>
</tbody>
</table>

The concrete wythes are connected through the insulation by shear connectors, such as fiber composite pin connectors, transverse welded wire ladder connectors, or welded wire trusses and M-ties [2]. Shear connectors and non-shear connectors (or plain insulation foam) are used to construct the sandwich panel. Non-shear connectors, as shown in Fig. 1.2, transfer only normal tension between the two wythes, while shear connectors, which may resist shear in one or two perpendicular directions, transfer normal tension forces and horizontal shear forces between the wythes [3]. One-way shear connectors and two-ways shear connectors are shown in Fig. 1.3 and Fig. 1.4, respectively.

Fig. 1.2 Non-shear connector (taken from [4])
Fig. 1.3 Continuous one-way shear connector (taken from [4])

Fig. 1.4 Two-way shear connector (taken from [4])
Depending on the degree of composite action achieved, PCSP are divided into three categories [2, 5]:

**Fully composite panel:**

A panel is considered fully composite when the two concrete wythes act like a single unit, and this is achieved by providing enough shear connectors between the two wythes. In this case, connectors are transferring required longitudinal shear so that the strain and bending stress distribution remains linear across the panel thickness as shown in Fig. 1.5(a).

**Semi-composite panel:**

A panel is considered semi-composite when shear connectors transfer only a fraction of the longitudinal shear as required for a fully composite panel. This fraction could range from zero to one hundred percent of the total longitudinal shear required for fully composite action. Note that quantifying the composite action in PCSP is based on experience and engineering judgment. The strain and bending stress distribution for a partially composite panel is shown in Fig. 1.5(b).

**Non-composite panel:**

In this special case, shear connectors do not have enough capacity to transfer longitudinal shear, and as a result, the two wythes act independently. In some cases, both concrete wythes have same stiffness and reinforcement; therefore, each wythe resists half of the applied load. However, in most cases, only one wythe called the structural wythe will resist the total load. The stress and strain distributions for the former and latter case are shown in Fig. 1.5(c) and Fig. 1.5(d), respectively.
(a) Fully composite  (b) partially-composite  (c) Non-composite,  (d) Non-composite,

Two structural wythe  one structural wythe

Fig. 1.5 Stress and strain distribution in PCSP under flexure

Moment of inertia, flexural rigidity, and shear stress for sandwich beam:

Fig. 1.6 shows a cross section of a sandwich beam with two faces, each with the thickness of $t$, separated by a core of low density material and thickness $c$. For a simple beam, the flexural rigidity is defined as the product of moment of inertia and modulus of elasticity ($EI$) while in a sandwich beam, the flexural rigidity is the sum of flexural rigidities of faces and core about neutral axes.

Fig. 1.6 Cross section of sandwich beam
Where \( E_f \) and \( E_c \) are the moduli of elasticity of the faces and the core, respectively.

Based on Allen’s sandwich beam theory, the first term in the flexural rigidity \( (D) \) is less than 1% of the second term when [6]:

\[
E_f \frac{b t^3}{6} < \frac{1}{100} E_f \frac{b t d^2}{2}
\]

\[
3 \left( \frac{d}{t} \right)^2 > 100
\]

\[
\frac{d}{t} > 5.77
\]

Sandwich beams with thin faces usually satisfy the above assumption.

Also, the third term in the equation (1.1) is less than 1% of the second term when [6]:

\[
E_c \frac{b c^3}{12} < \frac{1}{100} E_f \frac{b t d^2}{2}
\]

\[
\frac{E_f t}{E_c} \left( \frac{d}{c} \right)^2 > 100
\]

Shear stress for a sandwich beam:

\[
\tau = \frac{Q}{D b} \sum (SE)
\]
Where $Q$ is the shear force, $D$ is the flexural rigidity of the entire section, $b$ is sandwich width and $S$ is equal to the first moment of area. Shear stress at level $z$ in the core of the sandwich beam shown in Fig. 1.6 is equal to the following:

$$\sum (SE) = E_f \frac{bd}{2} + E_c b \left( \frac{c}{2} - z \right) \left( z + \frac{c - z}{2} \right)$$

$$\sum (SE) = E_f \frac{bd}{2} + E_c b \left( \frac{c}{2} - z \right) \left( \frac{z}{2} + \frac{c}{4} \right)$$

$$\sum (SE) = E_f \frac{bd}{2} + E_c \frac{b}{2} \left( \frac{c}{2} - z \right) \left( \frac{c}{2} + z \right)$$

And

$$\tau = \frac{Q}{D} \left\{ E_f \frac{td}{2} + E_c \frac{c^2}{4} \left( \frac{c^2}{4} - z^2 \right) \right\}$$

(1.5)

The maximum core shear stress at $z = 0$:

$$\tau_{max} = \frac{Q}{D} \left\{ E_f \frac{td}{2} + E_c \frac{c^2}{4} \right\}$$

The minimum core shear stress at $z = \pm c/2$:

$$\tau_{min} = \frac{Q}{D} \left\{ E_f \frac{td}{2} \right\}$$

The ratio of the maximum core shear stress to minimum core shear stress is [6]:

$$\frac{\tau_{max}}{\tau_{min}} = 1 + \frac{E_c}{E_f} \frac{c^2}{4td}$$

This equation is within 1% of unity provided that
If equation (1.6) is satisfied, the shear stress is constant over the thickness of the core. Equation (1.3) is similar to equation (1.6); thus,

“When a core is too weak to provide significant contribution to the flexural rigidity of the sandwich, the shear stress may be assumed constant over the depth of the core” [6].

Therefore, for a weak core

\[ E_c = 0 \]

Not only is this true for the sandwich beam but one can also extend the above assumption to a sandwich panel.

In this study, PCSP consists of one layer of insulation (could be reinforced/unreinforced) sandwiched between two layers of reinforced concrete slabs (inner and outer wythes). Current design practice consists of calculating the applied moment for a PCSP under a uniform load \( q \), and a vertical span \( l \), using \( M = ql^2/8 \), which is the maximum moment for a homogeneous simple beam (or strip in a slab). This approach is conservative and assumes one wythe resisting the lateral load. Such is the case in a non-composite panel. In addition, common design softwares (such as LecWall, Losch Software Ltd) calculate the capacity at 0% composite and 100% composite, then finding a ratio based on the percent of composite specified to plot the stresses in the inner and outer wythe. As an example, based on LecWall design software, the moment of the inertia for 70% composite action is the following:

\[ MI (0\%) + (MI (100\%) - MI (0\%)) \times 0.70 \]  

\[ (1.7) \]
Where, MI (0%) and MI (100%) are moment of inertia at 0% composite and 100% composite, respectively.

The basic relations for moments in an orthotropic plate are given by [7]:

\[ M_x = - \left( D_x \frac{\partial^2 w}{\partial x^2} + D_{xy} \frac{\partial^2 w}{\partial y^2} \right) \]  \[ (1.8) \]

\[ M_y = - \left( D_y \frac{\partial^2 w}{\partial y^2} + D_{xy} \frac{\partial^2 w}{\partial x^2} \right) \]  \[ (1.9) \]

As shown in equations (1.8) and (1.9) the moments are directly related to the curvature and deflection of the panel. Hence, an accurate prediction of the deflections in the outer and inner wythes will better estimate the internal resisting moment, and therefore, a more realistic design of the panel.

According to the sandwich plate theory, the transverse shear deformation significantly affects the performance of the panel. Therefore, considering the shear modulus of the core \( G_{yz} \) and \( G_{zx} \) to predict the deflections, and hence to predict the internal moments, will lead to a more accurate and economical design.

The main thrust behind this research is to develop an analytical design method to accurately predict the resisting internal moments and stresses in a PCSP under lateral or flexure load. The mechanical properties of top/bottom plates and shear modulus of the core and all related parameters such as \( G_{yz}, G_{zx}, G_{xy}, E_x, E_y, \nu_x \) and \( \nu_y \) will be considered.

The reinforced concrete panels (top/bottom facings) will be designed for the resisting moments calculated due to the uniform applied load, satisfying the below equation:

\[ M_{\text{applied}} < M_{\text{resisting}} \]
Literature Survey:

Thomas D. Bush and Zhiqi Wu (1998) worked on the flexural analysis of a precast concrete sandwich panel with truss connectors. They introduced a closed form theory which is based on Allen’s beam equation to estimate the service load deflections and bending stresses of non-loadbearing, semi-composite sandwich panels. The reason behind using beam theory was that typical sandwich panels often have an aspect ratio of two or greater, so one-way bending behavior was assumed for this type of PCSP [8].

Bush’s method is restricted to a sandwich panel with the aspect ratio of two and greater. In addition, it is only applicable to out-of-plane loading and non-loadbearing sandwich wall panels. In this study, a more general closed form theory based on Allen’s thick skins sandwich panel were developed and presented. Analytical design equations were formulated and can accurately predict the behavior of PCSP. Two main advantages of employing Allen’s sandwich panel theory rather than beam theory exist. First, there is no restriction regarding the aspect ratio of the panel, and secondly, the method is applicable for both a load bearing and non-load bearing composite sandwich panel.

The complex behavior of PCSP due to its material non-linearity, the uncertain rule of the shear connectors, and the interaction between various components has led researchers to rely on experimental investigations backed by simple analytical studies [5]. However, in this study, analytical design methods were developed to accurately predict the resisting internal moment and stresses in a PCSP under lateral or flexural load, considering all the mechanical properties of core and faces. This accurate analytical design method will lead to a better prediction of the resisting internal moment and stresses which makes predicting the behavior of PCSP based on degree of composite action pointless.
The approach is valid for a concrete sandwich panel with thick faces (concrete wythes) and a weak core (insulation layer). In the past, the thick skin did not capture too much attention in composite researches due to the lack of analytical analysis and consequently, relying on experimental data; however, in this analytical thesis, because of using a thick reinforced concrete slab as an upper and bottom wythe, thick skin sandwich panel theory is applied in the whole research.

The method is readily programmable in a spreadsheet but cumbersome for hand calculation. However, it is practicable to perform calculations by hand if it is desired to determine either amplitude of the first mode \((m = n = 1)\) or internal moment corresponding to the dominant mode. If, on the other hand, obtaining accurate results for deflection and stresses by taking the sum of the deflection and stresses in different modes is desired, relying on computer programs such as MATLAB is crucial [6].
CHAPTER II

PROBLEM FORMULATION

In this chapter a general equation for the deflection of a simply supported orthotropic sandwich panel with thick similar faces (wythes) is described. The solution formulation will be based on the energy method. This section covers the sandwich panel theory discussed by Allen [6] and extrapolated to cover the PCSP for thick sandwich facings. The sandwich theory with thick facings (wythe) is used to account for local bending of the top/bottom wythe with respect to their own axis. The neutral axis is located in the middle of the cross section of the PCSP. Outer and inner wythes are made of the same materials and have the same thickness. Typical concrete sandwich panel with length "a" and width "b" is shown in Fig. 2.1.

Fig. 2.1 Concrete sandwich panel
It is assumed that deflection of the sandwich panel simply supported on all four edges may be expressed in the form of [6]:

\[ w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right) \]  

(2.1)

Where \( w \) is deflection, \( a_{mn} \) is an undetermined factor, "\( a \)" and "\( b \)" are dimensions of the sandwich panel, and \( n \) and \( m \) are number modes considered (example: \( n = m = 1 \) is mode number one). Obviously from the above equation all parameters are known except \( a_{mn} \), so the following steps describe the method to find the appropriate equation for \( a_{mn} \), and therefore, the deflection of the orthotropic sandwich panel.

The shear strain energy in the core of the composite panel is in the form of [6]:

\[ U_c = \left( \frac{c}{2} \right) \int_{0}^{a} \int_{0}^{b} \left[ G_{zx}(1 - \lambda)^2 \left( \frac{\partial w}{\partial x} \right)^2 + G_{yz}(1 - \mu)^2 \left( \frac{\partial w}{\partial y} \right)^2 \right] dy dx \]  

(2.2)

Where \( c \) is the core thickness, \( \lambda \) and \( \mu \) are parameters to account for the shear flexibility of the core (insulated layer), and \( G_{zx} \) and \( G_{yz} \) are shear moduli of the core.

The membrane strain energy in the upper face is the following [6]:

\[ U_{fm1} = \frac{t_1}{2g_1} \int_{0}^{a} \int_{0}^{b} \left[ E_{x1} \left( \frac{c}{2} + \frac{t_1}{2} \right)^2 \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + E_{y1} \left( \frac{c}{2} + \frac{t_1}{2} \right)^2 \left( \frac{\partial^2 w}{\partial y^2} \right)^2 \right. \\
+ 2E_{x1}v_{y1} \left( \frac{c}{2} + \frac{t_1}{2} \right) \left( \frac{c}{2} + \frac{t_1}{2} \right) \left( \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right) \right] dy dx \]  

(2.3)

\[ + \frac{t_1}{2} \int_{0}^{a} \int_{0}^{b} G_{xy1} \left( \frac{c}{2} + \frac{t_1}{2} \right)^2 \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \] dy dx

The membrane strain energy in the bottom face is the following [6]:
Where and are elastic moduli of the faces, is shear moduli of the faces, and is the core thickness.

Note that , which is the thickness for the outer wythe, is equal to , which is the thickness of the inner wythe equal to . Also to account for the longitudinal and transverse reinforcement in each wythe, both wythes are considered orthotropic with similar and .

Therefore: and is equal to the following:

\[
U_{f_{m2}} = \frac{t_2}{2g_2} \int_{0}^{a} \int_{0}^{b} \left[ E_{x2} \left( \frac{c}{2} + \frac{t_2}{2} \right)^2 \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + E_{y2} \left( \frac{c}{2} + \frac{t_2}{2} \right)^2 \left( \frac{\partial^2 w}{\partial y^2} \right)^2 \right. \\
+ 2E_{x2}v_{y2} \left( \frac{c}{2} + \frac{t_2}{2} \right) \left( \frac{c}{2} + \frac{t_2}{2} \right) \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] dydx \\
+ \frac{t_2}{2} \int_{0}^{a} \int_{0}^{b} G_{xy2} \left( \frac{c}{2} + \frac{t_2}{2} \right)^2 \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 dydx \\
\tag{2.4}
\]

The bending strain energy in the outer wythe is the following [6]:

\[
U_{f_{m}} = U_{f_{m1}} + U_{f_{m2}} \\
= \frac{t}{g} \int_{0}^{a} \int_{0}^{b} \left[ E_{x} \left( \frac{c}{2} + \frac{t}{2} \right)^2 \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + E_{y} \left( \frac{c}{2} + \frac{t}{2} \right)^2 \left( \frac{\partial^2 w}{\partial y^2} \right)^2 \right. \\
+ 2E_{x}v_{y} \left( \frac{c}{2} + \frac{t}{2} \right) \left( \frac{c}{2} + \frac{t}{2} \right) \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] dydx \\
+ t \int_{0}^{a} \int_{0}^{b} G_{xy} \left( \frac{c}{2} + \mu \frac{c}{2} + \frac{t}{2} \right)^2 \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 dydx \\
\tag{2.5}
\]

The bending strain energy in the outer wythe is the following [6]:

\[
U_{fb1} = \frac{t_{1}^{3}}{24g_{1}} \int_{0}^{a} \int_{0}^{b} \left[ E_{x1} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + E_{y1} \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + 2E_{x1}v_{y1} \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] dydx \\
+ \frac{t_{1}^{3}}{6} \int_{0}^{a} \int_{0}^{b} G_{xy1} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 dydx \\
\tag{2.6}
\]
The bending strain energy in the inner wythe is the following [6]:

\[
U_{fb2} = \frac{t_2^3}{24g_2} \int_0^a \int_0^b \left[ E_{x2} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + E_{y2} \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + 2E_{x2}\nu_{y2} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] dy dx
\]

\[
+ \frac{t_2^3}{6} \int_0^a \int_0^b G_{xy2} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 dy dx
\]

Note that \( t_1 = t_2 = t \). Also the inner and outer faces are orthotropic with similar \( E_x, E_y, \nu_y, \nu_x \), and \( G_{xy} \). Therefore \( U_{fb} = U_{fb1} + U_{fb2} \) and is equal to the following:

\[
U_{fb} = \frac{t^3}{12g} \int_0^a \int_0^b \left[ E_x \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + E_y \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + 2E_x\nu_y \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] dy dx
\]

\[
+ \frac{t^3}{3} \int_0^a \int_0^b G_{xy} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 dy dx
\]

The potential energy of the applied forces is in the form of the following [6]:

\[
v_1 = -q \int_0^a \int_0^b w dy dx
\]

\( q \) is the uniformly distributed transverse load per unit area, and \( w \) is the deflection (equation 2.1). Assume also that the panel is loadbearing, and the PCSP is subjected to a tensile/compressive edge load \( N_x \) and \( N_y \) per unit length in the \( x \) and \( y \) direction, respectively (See Figure 2.1). The potential energy of the applied edge loads can be written in the form of the following [6]:

\[
v_2 = \frac{N_x}{2} \int_0^a \int_0^b \left( \frac{\partial w}{\partial x} \right)^2 dy dx + \frac{N_y}{2} \int_0^a \int_0^b \left( \frac{\partial w}{\partial y} \right)^2 dy dx
\]

The total energy in the system is equal to the following:
\[ U + V = U_c + U_{fm} + U_{fb} + v_1 + v_2 \]  

(2.11)

Below are the simplifications leading to the equation of total potential energy [6]:

\[
w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \left( \frac{m \pi x}{a} \right) \sin \left( \frac{n \pi y}{b} \right)\]

\[
\int_{0}^{a} \int_{0}^{b} (\frac{\partial w}{\partial x})^2 dy dx = a_{mn} \frac{\pi^2 m^2}{a^2} \cdot \frac{ab}{4} = a_{mn} \frac{b_5}{4}
\]

\[
b_5 = G_{xx} b_5
\]

Similarly

\[
\int_{0}^{a} \int_{0}^{b} (\frac{\partial w}{\partial y})^2 dy dx = a_{mn} \frac{\pi^2 n^2}{b^2} \cdot \frac{ab}{4} = a_{mn} \frac{b_6}{4}
\]

\[
b_6 = G_{yz} b_6
\]

So the shear strain energy in the core could be written in the form of

\[
U_c = \left( \frac{C}{2} \right) \int_{0}^{a} \int_{0}^{b} \left[ G_{xx} (1 - \lambda)^2 \left( \frac{\partial w}{\partial x} \right)^2 + G_{yz} (1 - \mu)^2 \left( \frac{\partial w}{\partial y} \right)^2 \right] dy dx
\]

\[
= \left( \frac{C}{2} \right) a_{mn} \frac{2}{(1 - \lambda)^2 b_5 + (1 - \mu)^2 b_6}
\]

(2.12)

In a same way

\[
\int_{0}^{a} \int_{0}^{b} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 dy dx = a_{mn} \frac{\pi^4 m^4}{a^4} \cdot \frac{ab}{4} = a_{mn} \frac{b_1}{4}
\]

\[
b_1 = \frac{E_x}{g} i_1
\]
So the total membrane strain energy in the wythes (faces) could be written in the form of the following:

\[
U_{fm} = \frac{t}{g} \int_0^a \int_0^b \left[ E_x \left( \lambda \frac{c}{2} + \frac{t}{2} \right)^2 \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + E_y \left( \mu \frac{c}{2} + \frac{t}{2} \right)^2 \left( \frac{\partial^2 w}{\partial y^2} \right)^2 \right. \\
+ 2E_x v_y \left( \lambda \frac{c}{2} + \frac{t}{2} \right) \left( \mu \frac{c}{2} + \frac{t}{2} \right) \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \left. \right] \, dydx \\
+ t \int_0^a \int_0^b G_{xy} \left( \lambda \frac{c}{2} + \mu \frac{c}{2} + t \right)^2 \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \, dydx = \\
ta_{mn}^2 \left\{ \left( \lambda \frac{c}{2} + \frac{t}{2} \right)^2 b_1 + \left( \mu \frac{c}{2} + \frac{t}{2} \right)^2 b_2 + 2 \left( \lambda \frac{c}{2} + \frac{t}{2} \right) \left( \mu \frac{c}{2} + \frac{t}{2} \right) b_3 \\
+ \left( \lambda \frac{c}{2} + \mu \frac{c}{2} + t \right)^2 b_4 \right\} \quad (2.13)
\]
Similarly, the total bending strain energy in the wythes (faces) could be written in the form of the following:

\[
U_{fb} = \frac{t^3}{12g} \int_a^b \int_0^a \left[ E_x \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + E_y \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + 2E_xv_y \left( \frac{\partial w}{\partial x} \right) \left( \frac{\partial w}{\partial y} \right) \right] dy dx
\]

\[
+ \frac{t^3}{3} \int_0^a \int_0^b \left[ G_{xy} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] dy dx
\]

\[
= \frac{t^3}{12} a_{mn}^2 \{ b_1 + b_2 + 2b_3 + 4b_4 \} \quad (2.14)
\]

Also, the potential energy of a uniformly distributed transverse load \( q \) could be written in the form of the following [6]:

\[
v_1 = -q \int_0^a \int_0^b w \ dy \ dx
\]

It is assumed that

\[
\int_0^a \int_0^b \sin \left( \frac{m \pi x}{a} \right) \sin \left( \frac{n \pi y}{b} \right) \ dy \ dx = i_7 = \frac{4ab}{mn \pi^2} (m, n \ both \ odd, \ otherwise \ zero)
\]

Therefore

\[
v_1 = -a_{mn} i_7 = -a_{mn} \frac{4ab}{mn \pi^2} (m, n \ both \ odd, \ otherwise \ zero) \quad (2.15)
\]

In a same way, the potential energy of the edge loads \( (N_x, N_y) \) could be written in the form of the following:

\[
v_2 = \frac{N_x}{2} \int_0^a \int_0^b \left( \frac{\partial w}{\partial x} \right)^2 \ dy \ dx + \frac{N_y}{2} \int_0^a \int_0^b \left( \frac{\partial w}{\partial y} \right)^2 \ dy \ dx = \frac{N_x}{2} a_{mn}^2 i_5 + \frac{N_y}{2} a_{mn}^2 i_6 \quad (2.16)
\]

So the total energy of the system is equal to the following:
\[
\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{ a_{mn}^2 f(\lambda, \mu) + \frac{N_x}{2} a_{mn}^2 i_5 + \frac{N_y}{2} a_{mn}^2 i_6 - qa_{mn} \frac{4ab}{mnp^2} \right\}
\] (2.17)

Where \( f(\lambda, \mu) \) is

\[
f(\lambda, \mu) = \left( \frac{c}{2} \right) \left( (1 - \lambda)^2 b_5 + (1 - \mu)^2 b_6 \right) + t \left\{ \left( \frac{\lambda c}{2} + \frac{t}{2} \right)^2 b_1 + \left( \frac{\mu c}{2} + \frac{t}{2} \right)^2 b_2 + 2 \left( \frac{\lambda c}{2} + \frac{t}{2} \right) \left( \frac{\mu c}{2} + \frac{t}{2} \right) b_3 \right. \]

\[
+ \left. \left( \frac{\lambda c}{2} + \mu c + t \right)^2 b_4 \right\} + \frac{t^3}{12} (b_1 + b_2 + 2b_3 + 4b_4) \] (2.18)

For equilibrium, it is crucial that \( U + V \) should be stationary with respect to any unknown variables such as \( \lambda \) and \( \mu \). Thus, there are two equations to be satisfied:

\[
\begin{align*}
\frac{\partial (U + V)}{\partial \lambda} &= 0 \\
\frac{\partial (U + V)}{\partial \mu} &= 0
\end{align*}
\] (2.19)

It is useful to write \( f(\lambda, \mu) \) in the form of the following:

\[
f(\lambda, \mu) = B_{xx} \lambda^2 + B_{yy} \mu^2 + B_{xy} \lambda \mu + B_{x} \lambda + B_{y} \mu + B_0
\] (2.20)

Where

\[
B_{xx} = \frac{c}{2} b_5 + \frac{tc^2}{4} (b_1 + b_4)
\]

\[
B_{yy} = \frac{c}{2} b_6 + \frac{tc^2}{4} (b_2 + b_3)
\]

\[
B_{xy} = 2 \frac{tc^2}{4} (b_3 + b_4)
\]
\[ B_x = 2 \left[ -\frac{c}{2} b_5 + \frac{ct^2}{4} (b_1 + b_3 + 2b_4) \right] \]

\[ B_y = 2 \left[ -\frac{c}{2} b_6 + \frac{ct^2}{4} (b_2 + b_3 + 2b_4) \right] \]

\[ B_0 = \frac{c}{2} (b_5 + b_6) + \frac{t^3}{3} \{ b_1 + b_2 + 2b_3 + 4b_4 \} \]

Then, by equation (2.19)

\[
\begin{align*}
\frac{\partial f}{\partial \lambda} &= 2B_{xx}\lambda + B_{xy}\mu + B_x = 0 \\
\frac{\partial f}{\partial \mu} &= 2B_{yy}\mu + B_{xy}\lambda + B_y = 0
\end{align*}
\]

And then

\[
\times \lambda \left[ \frac{\partial f}{\partial \lambda} = 2B_{xx}\lambda + B_{xy}\mu + B_x = 0 \right] = 2B_{xx}\lambda^2 + B_{xy}\lambda\mu + B_x\lambda = 0
\]

\[
\times \mu \left[ \frac{\partial f}{\partial \mu} = 2B_{yy}\mu + B_{xy}\lambda + B_y = 0 \right] = 2B_{yy}\mu^2 + B_{xy}\mu\lambda + B_y\mu = 0
\]

By adding these two equations and dividing by two one gets the following:

\[ B_{xx}\lambda^2 + B_{yy}\mu^2 + B_{xy}\mu\lambda + \frac{B_x}{2}\lambda + \frac{B_y}{2}\mu = 0 \quad (2.21) \]

Finally, by plugging this equation into equation (2.20) one obtains the following:

\[ f(\lambda, \mu) = \frac{B_x}{2}\lambda + \frac{B_y}{2}\mu + B_0 \quad (2.22) \]

Solving the two equations with two unknowns \((\lambda, \mu)\) of the equation (2.19) and plugging the result into the equation (2.22) leads to an equation for \(f(\lambda, \mu)\) without \(\lambda\) and \(\mu\) parameters. The MATLAB program was used to compute and solve equation (2.19), and after simplification
\[
\mu = - \left( \frac{-A1 - A7 + A9 - A10 + A12 + \frac{t}{c} (A2 + A3 + A4 + A5 + A6 + A8 + A12 + 2A9)}{A1 + A2 + A3 + A4 + A5 + A6 + A7 + A8 + A9 + A10} \right)
\]

\[
\lambda = - \left( \frac{-A1 - A8 - A9 + A10 + A11 + \frac{t}{c} (A2 + A3 + A4 + A5 + A6 + A7 + A11 + 2A10)}{A1 + A2 + A3 + A4 + A5 + A6 + A7 + A8 + A9 + A10} \right)
\]

Where

\[A1 = 4b_5b_6\quad A7 = 2b_1b_6ct\]

\[A2 = -b_3^2c^2t^2\quad A8 = 2b_2b_5ct\]

\[A3 = b_1b_2c^2t^2\quad A9 = 2b_4b_5ct\]

\[A4 = b_1b_4c^2t^2\quad A10 = 2b_4b_6ct\]

\[A5 = b_2b_4c^2t^2\quad A11 = 2b_3b_6ct\]

\[A6 = -2b_3b_4c^2t^2\quad A12 = 2b_3b_5ct\]

By plugging the values of \(\lambda\) and \(\mu\) into the equation (2.22), the resulting equation for \(f(\lambda, \mu)\) without \(\lambda\) and \(\mu\) parameters is called \(f_{\text{min}}\). Therefore:

\[
f_{\text{min}} = \frac{B_x}{2} \lambda + \frac{B_y}{2} \mu + B_0
\]

\[
f_{\text{min}} = f_{\text{min}}(t, c, v_y, v_x, G_{xy}, G_{yz}, G_{xz}, E_x, E_y, a_{mn})
\]

The resulting equation for the total energy may now be written in the form of the following:

\[
U + V = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{ a_{mn}^2 f_{\text{min}} + \frac{N_x}{2} a_{mn}^2 i_5 + \frac{N_y}{2} a_{mn}^2 i_6 - qa_{mn} \frac{4ab}{mn\pi^2} \right\}
\]

(2.23)
Additionally, the total energy of the system should be stationary with respect to \(a_{mn}\). Therefore

\[
\frac{\partial (U + V)}{\partial a_{mn}} = 0
\]

\[
\frac{\partial (U + V)}{\partial a_{mn}} = 2a_{mn}f_{\text{min}} + N_xa_{mn}i_5 + N_ya_{mn}i_6 - q \frac{4ab}{mnpz} = 0
\]  

\[ (m, n \text{ both odd, otherwise zero}) \]

Finally

\[
a_{mn} = \frac{q \frac{4ab}{mnpz}}{2f_{\text{min}} + N_xi_5 + N_yi_6}
\]

\[ (2.25) \]

Note that if one considers \(N_x = N_y = 0\), the final answer for the deflection amplitude is

\[
a_{mn} = \frac{q \frac{4ab}{mnpz}}{2f_{\text{min}}}
\]

\[ (2.26) \]

\[
a_{mn} = a_{mn}(t, c, \nu_y, \nu_x, G_{xy}, G_{yz}, G_{zx}, E_x, E_y)
\]

\[ (m, n \text{ both odd, otherwise zero}) \]

By plugging equation (2.26) into equation (2.1), the general form of deflection of PCSP will be obtained:

\[
w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{q \frac{4ab}{mnpz}}{2f_{\text{min}}} \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right)
\]

\[ (2.27) \]

We see that deflection of PCSP is dependent on all parameters related to the core and faces such as \(t, c, \nu_y, \nu_x, G_{xy}, G_{yz}, G_{zx}, E_x, E_y\).
Here is the MATLAB program for calculating an equation for the deflection of PCSP:

%calculating amn

syms landa eta c t Ex Ey g b a pi Vy Gxy Gzx Gyz q

b1=(Ex/g)*((pi^4*n^4)/a^4)*(a*b/4);
b2=(Ey/g)*((pi^4*n^4)/b^4)*(a*b/4);
b3=(Ex*Vy/g)*(((pi^4*n^2*m^2)/(a^2*b^2)))*(a*b/4);
b4=(Gxy)*(((pi^4*n^2*m^2)/(a^2*b^2)))*(a*b/4);
b5=(Gzx)*(((pi^4*m^2)/(a^2*b^2)))*(a*b/4);
b6=(Gyz)*(((pi^4*m^2)/(b^2)))*(a*b/4);

Bxx=(c/2)*b5+t*c^2/4*(b1+b4);
Byy=(c/2)*b6+t*c^2/4*(b2+b4);
Bxy=(2*t*c^2/4)*(b3+b4);
Bx=2*((-c/2)*b5+c*t^2/4*(b1+b3+2*b4));
By=2*((-c/2)*b6+c*t^2/4*(b2+b3+2*b4));
B0=(c/2)*(b5+b6)+(t^3/3)*(b1+b2+2*b3+4*b4);

f=Bxx*landa^2+Byy*eta^2+Bxy*landa*eta+Bx*landa+By*eta+B0;
[eta,landa]=solve(diff(f,landa),diff(f,eta),landa,eta);
f=(Bx/2)*landa+(By/2)*eta+B0
amn=(q*(4*a*b)/(m*n*pi^2))/(2*f)
CHAPTER III

MODEL VALIDATION

In the first part of this chapter, the equation (2.27), which accounts for an orthotropic core \((G_{zx} \neq G_{yz})\) with orthotropic wythes (faces) \((E_x \neq E_y, \nu_x \neq \nu_y)\) is compared to the deflection equation of the special case of a simply supported sandwich panel with identical isotropic thick wythes (faces) \((E_x = E_y = E, \nu_x = \nu_y = \nu, G = E/2(1 + \nu) and g = 1 - \nu^2)\) and an isotropic core \((G_{zx} = G_{yz} = G)\).

This approach is based on applying the following assumptions for an orthotropic case and then comparing the result with the isotropic case to validate the results discussed in Chapter II.

\[
E_x = E_y = E
\]

\[
\nu_x = \nu_y = \nu
\]

\[
G_{xy} = \frac{E}{2(1 + \nu)}
\]

\[
G_{zx} = G_{yz} = G
\]

and \(g = 1 - \nu^2\)

The amplitude of the \((m, n)\)th mode due to a uniform transverse pressure \(q\) of a simply supported sandwich panel with an identical isotropic thick wythes (faces) and isotropic core is in the form of [6]:

24
\[ a_{mn} = \frac{16q b^4}{\pi^4 mnD_2} \frac{\rho}{\omega^2} \]

Where

\[ D_2 = \frac{E t d^2}{2g} \]

\[ \frac{1}{\rho} = \left[ 1 + \frac{t^2}{1 + \varphi \omega} + \frac{t^2}{3d^2} \right] \]

\[ \omega = m^2 \frac{b^2}{a^2} + n^2 \]

\[ \varphi = \frac{\pi^2}{b^2} \frac{E ct}{G 2g} \]

and consequently:

\[ w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{16q b^4}{\pi^4 mnD_2} \frac{\rho}{\omega^2} \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right) \]

(3.1)

Both cases are solved using MATLAB and the following numerical example showed excellent agreement in the deflection equation for both cases.

\[ E_x = E_y = E = 10000 \quad \text{Mpa} \]

\[ G_{zx} = G_{yz} = G = 330 \quad \text{Mpa} \]

\[ \nu_x = \nu_y = \nu = 0.3 \]

\[ G_{xy} = \frac{E}{2(1 + \nu)} = \frac{10000}{2(1 + 0.3)} = 3846.2 \quad \text{Mpa} \]

\[ g = 1 - \nu^2 = 1 - 0.3^2 = 0.91 \]
\[ n = m = 1 \quad \text{Mode one} \]

\[ q = 100 \, \text{MPa} \quad \text{Uniform transverse pressure} \]

\[ b = 1 \, \text{m} \quad \text{Panel dimension} \]

\[ a = 2 \, \text{m} \quad \text{Panel dimension} \]

\[ c = 0.03 \, \text{m}, \, t = 0.02 \, \text{m} \quad \text{Core and face thickness} \]

The MATLAB program for the computing amplitude for a simply supported sandwich panel with identical isotropic thick wythes (faces) and isotropic core is as follows:

```matlab
n=1; m=1; q=100; b=1; a=2; E=10000; c=0.03; t=0.02; d=c+t; V=0.3; g=1-V^2; G=330; fe=(pi^2*E*c*t)/(b^2*G*2*g); omega=((m^2*b^2)/a^2)+n^2; Ro=1/((1/(1+fe*omega))+(t^2/(3*d^2))); D=(E*t*d^2)/(2*g); amn=(16*q*b^4*Ro)/(pi^6*m*n*D*omega^2)
```

By running the above program, the calculated value for \( a_{mn} \) is 4.1088, and the equation for deflection yields

\[ w = 4.1088 \sin \left( \frac{\pi x}{2} \right) \sin \left( \frac{\pi y}{1} \right) \]

Note that, the above calculations were performed for the first mode \( m = n = 1 \). The MATLAB program for computing the amplitude for the special case of a simply supported sandwich panel with orthotropic thick wythes (faces) and orthotropic core is as follows:
By running the above program, the deflection equation has the form of

\[ w = 4.1088 \sin \left( \frac{\pi x}{2} \right) \sin \left( \frac{\pi y}{1} \right) \]

As shown above, the method derived in chapter II yields the same value as the isotropic case.
In the second part of this chapter, the deflection of a simply supported PCSP using one mode \((m = n = 1)\) is compared to the deflection using six modes \((m = 1, 3, n = 1, 3, 5)\). The following is a numerical study used in both cases and Fig. 3.1 shows the maximum deflection for the first and second case along the x axis where \(y = b/2\).

\[
E_x = 21400 \; \text{Mpa} \quad E_y = 20000 \; \text{Mpa}
\]

\[
G_{xy} = 6000 \; \text{Mpa} \quad g = 1 - \nu_x \nu_y
\]

\[
\nu_y = 0.28 \quad \nu_x = 0.3
\]

\[
c = 0.08 \; \text{m} \quad t = 0.05 \; \text{m}
\]

\[
G_{yz} = 50 \; \text{Mpa} \quad G_{zx} = 50 \; \text{Mpa}
\]

\[
b = 4 \; \text{m} \quad a = 6 \; \text{m}
\]

\[
q = 0.002 \; \text{Mpa}
\]
As shown in Fig. 3.1, the maximum deflection for the first mode is $7.5 \times 10^{-4}$ m, and the maximum deflection using the six modes is $7 \times 10^{-4}$ m. By subtracting these two numbers, one gets a difference of $0.5 \times 10^{-4}$ m, which is small enough to conclude that the first mode (one term of series) gives a reasonable deflection prediction of the PCSP. Because of this small amount of changes between one and several modes, we intend to use the first mode and avoid complexity in all equations.

In the last part of this chapter, a parametric study is conducted by varying only the $G_{yz}$ while other parameters are fixed. At the end, influence of varying this parameter on the deflection of PCSP will be investigated.

The following numerical example is used in this analysis:

\[
E_x = 21400 \text{ Mpa} \quad E_y = 20000 \text{ Mpa}
\]

\[
G_{xy} = 6000 \text{ Mpa} \quad g = 1 - \nu_x\nu_y
\]

\[
\nu_y = 0.28 \quad \nu_x = 0.3
\]

\[
c = 0.08 \text{ m} \quad t = 0.04 \text{ m}
\]

\[
G_{yz} = [15, 30, 60, 90, 120] \text{ Mpa} \quad G_{zx} = 20 \text{ Mpa}
\]

\[
b = 4 \text{ m} \quad a = 6 \text{ m}
\]

\[
q = 0.002 \text{ Mpa}
\]

Fig. 3.2 shows deflection variations with respect to $G_{yz}$ along the $x$ axis where $y = b/2$. Also, values of $G_{yz}$ accompanied with their related maximum deflections are listed in Table 3.1.
Fig. 3.2 Significance of a partially composite panel varying $G_{yz}$

<table>
<thead>
<tr>
<th>$G_{yz}$ (Mpa)</th>
<th>15</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deflection (mm)</td>
<td>1.7476</td>
<td>1.349</td>
<td>1.1178</td>
<td>1.0352</td>
<td>0.9928</td>
</tr>
</tbody>
</table>

Table 3.1 Variation of deflection based on changing $G_{yz}$

These results show that by increasing $G_{yz}$, hence by adding reinforcement in $yz$ plane in the core, or having a high density foam insulation layer, the smaller value for deflection of
PCSP will be obtained and vice versa. One can conclude that the deflection of PCSP is really dependent on parameters such as $G_{yz}$, $G_{zx}$ and $G_{xy}$. 
CHAPTER IV

DESIGN APPROACH

The method of analysis and finding an equation for the deflection of an orthotropic sandwich panel with thick similar faces discussed in this study is related to the composite panel, in which one layer of orthotropic insulation is sandwiched between two layers of orthotropic thick faces/wythes. Since the orthotropic faces considered in this study is a reinforced concrete slab with $x$ and $y$ directed reinforcement steel bars, additional analysis and consideration is required for calculating $E_x, E_y, G_{xy}, v_x$ and $v_y$.

Considering just the upper slab, that the concrete slab is equally divided into two sections is assumed, where the bottom section contains reinforcement in the $x$ direction and the upper section contains reinforcement in the $y$ direction as shown in Fig. 4.1.

![Fig. 4.1 Concrete slab with $x$ and $y$ steel reinforcement layers](image-url)
The bottom and upper sections are individually shown in Fig. 4.2 and Fig. 4.3, respectively.
By assuming this model, to figure out the unknown parameters in each section of the slab using a simple “rule-of-mixture” relationship is possible as follows [9]:

\[ E_1 = E_s V_s + E_c (1 - V_s) \]  \hspace{1cm} (4.1)

Where

\[ V_s = \frac{A_s}{A}, \quad V_c = \frac{A_c}{A} \quad \text{and} \quad V_s + V_c = 1 \]

Note that, subscripts \( s \) and \( c \) denote reinforcement steel bar and concrete, respectively.

Similarly [9]

\[ \nu_{12} = \nu_s V_s + \nu_c (1 - V_s) \]  \hspace{1cm} (4.2)

\[ E_2 = \frac{E_s E_c}{[E_c V_s + E_s (1 - V_s)]} \]  \hspace{1cm} (4.3)

\[ G_{12} = \frac{G_s G_c}{[G_c V_s + G_s (1 - V_s)]} \]  \hspace{1cm} (4.4)

Where

\[ G_s = \frac{E_s}{2(1 + \nu_s)} \]

\[ G_c = \frac{E_c}{2(1 + \nu_c)} \]

Since the area of the reinforcement steel bar in the bottom section is different than the upper section, the volume fractions of the steel and concrete, denoted by \( V_s \) and \( V_c \), in the bottom ply is different than the volume fractions in the upper ply. As a conclusion, we have two sets of equations (4.1-4.4), one for each ply, so \( E_1, E_2, \nu_{12} \) and \( G_{12} \) are related to the bottom ply and \( E_1', E_2', \nu_{12}' \) and \( G_{12}' \) are related to the upper ply.
Under a state of plane stress, the constitutive relations for an orthotropic material take the form of the following [9]:

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_6
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\
-\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\
0 & 0 & \frac{1}{G_{12}}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_6
\end{bmatrix}
\]  
(4.5)

By inverting this relationship, we get

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_6
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_6
\end{bmatrix}
\]  
(4.6)

Where

\[
Q_{11} = \frac{E_1}{(1 - \nu_{12}^2 \frac{E_2}{E_1})} \quad Q_{12} = \nu_{12} Q_{22}
\]

\[
Q_{22} = \frac{E_2}{(1 - \nu_{12}^2 \frac{E_2}{E_1})} \quad Q_{66} = G_{12}
\]

For the bottom concrete section (0° ply), \( E_1 \gg E_2 \), then

\[
Q_{11} = \frac{E_1}{(1 - \nu_{12}^2 \frac{E_2}{E_1})} = E_1 
\]  
(4.7.a)

\[
Q_{22} = \frac{E_2}{(1 - \nu_{12}^2 \frac{E_2}{E_1})} = E_2 
\]  
(4.7.b)

\[
Q_{12} = \nu_{12} Q_{22} = \nu_{12} E_2 
\]  
(4.7.c)

and
For the upper concrete section (90° ply), $E_1' \gg E_2'$ (it is always assumed that fiber direction is parallel to 1 direction), then the following is true:

\[
Q_{11}' = \frac{E_1'}{1 - \nu_{12}' \frac{E_2'}{E_1'}} = E_1' \tag{4.8.a}
\]

\[
Q_{22}' = \frac{E_2'}{1 - \nu_{12}' \frac{E_2'}{E_1'}} = E_2' \tag{4.8.b}
\]

\[
Q_{12}' = \nu_{12}'Q_{22}' = \nu_{12}'E_2' \tag{4.8.c}
\]

\[
Q_{66}' = G_{12}' \tag{4.8.d}
\]

Stiffness transformation relations in the x-y coordinate system are as follows [9]:

\[
Q_{xx} = Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4
\]

\[
Q_{xy} = (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4)
\]

\[
Q_{xx} = mn[(Q_{11} - Q_{12} - 2Q_{66})m^2 - (Q_{22} - Q_{12} - 2Q_{66})n^2] \tag{4.9}
\]

\[
Q_{yy} = Q_{11}n^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}m^4
\]

\[
Q_{ys} = mn[(Q_{11} - Q_{12} - 2Q_{66})n^2 - (Q_{22} - Q_{12} - 2Q_{66})m^2]
\]

\[
Q_{ss} = (Q_{11} + Q_{22} - 2Q_{12})m^2n^2 + Q_{66}(m^2 - n^2)^2
\]

Where

\[
m = \cos(\theta) \quad n = \sin(\theta)
\]

So for the 0° ply
And the transformation relations are the following:

\[ Q_{xx}^{(0)} = Q_{11} m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4 = Q_{11} \]

\[ Q_{xy}^{(0)} = (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4) = Q_{12} \]

\[ Q_{xs}^{(0)} = mn[(Q_{11} - Q_{12} - 2Q_{66})m^2 - (Q_{22} - Q_{12} - 2Q_{66})n^2] = 0 \]  \hspace{1cm} (4.10)

\[ Q_{yy}^{(0)} = Q_{11}n^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}m^4 = Q_{22} \]

\[ Q_{ys}^{(0)} = mn[(Q_{11} - Q_{12} - 2Q_{66})n^2 - (Q_{22} - Q_{12} - 2Q_{66})m^2] = 0 \]

\[ Q_{ss}^{(0)} = (Q_{11} + Q_{22} - 2Q_{12})m^2n^2 + Q_{66}(m^2 - n^2)^2 = Q_{66}m^4 = Q_{66} \]

And for the \(90^0\) ply:

\[ m = \cos(90) = 0 \hspace{1cm} n = \sin(90) = 1 \]

Transformation relations are these:

\[ Q_{xx}^{(90)} = Q_{11}' m^4 + 2(Q_{12}' + 2Q_{66}')m^2n^2 + Q_{22}'n^4 = Q_{22}' \]

\[ Q_{xy}^{(90)} = (Q_{11}' + Q_{22}' - 4Q_{66}')m^2n^2 + Q_{12}'(m^4 + n^4) = Q_{12}' \]

\[ Q_{xs}^{(90)} = mn[(Q_{11}' - Q_{12}' - 2Q_{66}')m^2 - (Q_{22}' - Q_{12}' - 2Q_{66}')n^2] = 0 \]  \hspace{1cm} (4.11)

\[ Q_{yy}^{(90)} = Q_{11}'n^4 + 2(Q_{12}' + 2Q_{66}')m^2n^2 + Q_{22}'m^4 = Q_{11}' \]

\[ Q_{ys}^{(90)} = mn[(Q_{11}' - Q_{12}' - 2Q_{66}')n^2 - (Q_{22}' - Q_{12}' - 2Q_{66}')m^2] = 0 \]

\[ Q_{ss}^{(90)} = (Q_{11}' + Q_{22}' - 2Q_{12}')m^2n^2 + Q_{66}'(m^2 - n^2)^2 = Q_{66}'n^4 = Q_{66}' \]
That the top concrete slab is a [90/0] laminate (crossply laminate) is assumed. Therefore, based on lamination theory, extensional stiffnesses, relating in-plane loads to in-plane strains, are in the form of [9]:

\[ A_{ij} = \sum_{k=1}^{n} Q_{ij}^k (h_k - h_{k-1}) \]  

(4.12)

Therefore,

\[ A_{xx} = \sum_{k=1}^{2} Q_{xx}^k (h_k - h_{k-1}) = Q_{xx}^{(1)}(h_1 - h_0) + Q_{xx}^{(2)}(h_2 - h_1) = \frac{t}{2} (Q_{xx}^{(0)} + Q_{xx}^{(90)}) \]

Based on the equations (4.10) and (4.11)

\[ Q_{xx}^{(0)} = Q_{11} \text{ and } Q_{xx}^{(90)} = Q_{22}' \]

So:

\[ A_{xx} = \frac{t}{2} (Q_{11} + Q_{22}') \]

Also, based on equation (4.7.a) and (4.8.b)

\[ Q_{11} = E_1 \text{ and } Q_{22}' = E_2' \]

So

\[ A_{xx} = \frac{t}{2} (E_1 + E_2') \]  

(4.13)

Similarly,

\[ A_{xy} = \sum_{k=1}^{2} Q_{xy}^k (h_k - h_{k-1}) = Q_{xy}^{(1)}(h_1 - h_0) + Q_{xy}^{(2)}(h_2 - h_1) = \frac{t}{2} (Q_{xy}^{(0)} + Q_{xy}^{(90)}) \]
Based on equations (4.10) and (4.11)

\[ Q_{xy}^{(0)} = Q_{12} \text{ and } Q_{xy}^{(90)} = Q_{12}' \]

So

\[ A_{xy} = \frac{t}{2} (Q_{12} + Q_{12}') \]

Also, based on equation (4.7.c) and (4.8.c)

\[ Q_{12} = \nu_{12}E_2 \text{ and } Q_{12}' = \nu_{12}'E_2' \]

So

\[ A_{xy} = \frac{t}{2} (\nu_{12}E_2 + \nu_{12}'E_2') \quad (4.14) \]

In a same way

\[ A_{yy} = \sum_{k=1}^{2} Q_{yy}^k (h_k - h_{k-1}) = Q_{yy}^{(1)}(h_1 - h_0) + Q_{yy}^{(2)}(h_2 - h_1) = \frac{t}{2} (Q_{yy}^{(0)} + Q_{yy}^{(90)}) \]

Based on equations (4.10) and (4.11)

\[ Q_{yy}^{(0)} = Q_{22} \text{ and } Q_{yy}^{(90)} = Q_{11}' \]

So

\[ A_{yy} = \frac{t}{2} (Q_{22} + Q_{11}') \]

Also, based on equation (4.7.b) and (4.8.a)

\[ Q_{22} = E_2 \text{ and } Q_{11}' = E_1' \]
So

\[ A_{yy} = \frac{t}{2}(E_2 + E_1') \]  

(4.15)

\[ A_{xs} = \sum_{k=1}^{2} Q_{xs}^k (h_{k} - h_{k-1}) = Q_{xs}^{(1)}(h_1 - h_0) + Q_{xs}^{(2)}(h_2 - h_1) = \frac{t}{2}(Q_{xs}^{(0)} + Q_{xs}^{(90)}) \]

Based on equations (4.10) and (4.11)

\[ Q_{xs}^{(0)} = 0 \text{ and } Q_{xs}^{(90)} = 0 \]

So

\[ A_{xs} = 0 \]  

(4.16)

\[ A_{ys} = \sum_{k=1}^{2} Q_{ys}^k (h_{k} - h_{k-1}) = Q_{ys}^{(1)}(h_1 - h_0) + Q_{ys}^{(2)}(h_2 - h_1) = \frac{t}{2}(Q_{ys}^{(0)} + Q_{ys}^{(90)}) \]

Based on equations (4.10) and (4.11)

\[ Q_{ys}^{(0)} = 0 \text{ and } Q_{ys}^{(90)} = 0 \]

So

\[ A_{ys} = 0 \]  

(4.17)

And finally,

\[ A_{ss} = \sum_{k=1}^{2} Q_{ss}^k (h_{k} - h_{k-1}) = Q_{ss}^{(1)}(h_1 - h_0) + Q_{ss}^{(2)}(h_2 - h_1) = \frac{t}{2}(Q_{ss}^{(0)} + Q_{ss}^{(90)}) \]

Based on equations (4.10) and (4.11)
\[ Q_{ss}^{(0)} = Q_{66} \text{ and } Q_{ss}^{(90)} = Q_{66}' \]

So

\[ A_{ss} = \frac{t}{2} (Q_{66} + Q_{66}') \]

Also, based on equation (4.7.d) and (4.8.d)

\[ Q_{66} = G_{12} \text{ and } Q_{66}' = G_{12}' \]

So

\[ A_{ss} = \frac{t}{2} (G_{12} + G_{12}') \quad (4.18) \]

Note that, in all above equations (4.13-4.18), \( t/2 \) is the thickness of one lamina and is half of the total thickness of the upper concrete slab, \( t \).

Similarly,

\[ B_{ij} = \frac{1}{2} \sum_{k=1}^{n} Q_{ij}^k \left( h_k^2 - h_{k-1}^2 \right) \quad (4.19) \]

Where \( B_{ij} \) are coupling stiffnesses, relating in-plane loads to curvatures. Therefore, the overall load-deformation relations are in the form of the following:

\[
\begin{bmatrix}
  N_x \\
  N_y \\
  N_z
\end{bmatrix} =
\begin{bmatrix}
  A_{xx} & A_{xy} & 0 & B_{xx} & B_{xy} & 0 \\
  A_{xy} & A_{yy} & 0 & B_{xy} & B_{yy} & 0 \\
  0 & 0 & A_{ss} & 0 & 0 & B_{ss}
\end{bmatrix}
\begin{bmatrix}
  \varepsilon_x \\
  \varepsilon_y \\
  \gamma_{xy} \\
  k_x \\
  k_y \\
  k_z
\end{bmatrix} \quad (4.20)
\]

Because the in-plane/flexure coupling laminate moduli, the relation between the in-plane loads to curvatures, have already been assumed in the Ritz method (see chapter II): we
can neglect the coupling stiffness and write the load-deformation relations in the following form:

\[
\begin{bmatrix}
N_x \\
N_y \\
N_s
\end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\
A_{xy} & A_{yy} & 0 \\
0 & 0 & A_{ss}
\end{bmatrix} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

(4.21)

The above equation for load-deformation stiffness is similar to the case of symmetric balanced laminate, so simple relations can be obtained for engineering properties as a function of laminate stiffnesses for the special case of symmetric balanced laminate [9]:

\[
E_x = \frac{1}{t} \left[ A_{xx} - \frac{A_{xy}^2}{A_{yy}} \right]
\]

(4.22)

\[
E_y = \frac{1}{t} \left[ A_{yy} - \frac{A_{xy}^2}{A_{xx}} \right]
\]

(4.23)

\[
\nu_{xy} = \frac{A_{xy}}{A_{yy}}
\]

(4.24)

\[
\nu_{yx} = \frac{A_{xy}}{A_{xx}}
\]

(4.25)

\[
G_{xy} = \frac{A_{ss}}{t}
\]

(4.26)

Once the deflection equation for the orthotropic PCSP is derived, one can calculate the moment in the top or bottom concrete slab using the theory of plates. The main idea is that the upper/bottom concrete slabs and the insulation layer act together as a single unit (fully composite action); hence, the deflection equation is valid for each of these three parts separately. Therefore, if one considers the upper (or bottom) concrete slab, and assuming that the deflection equation for this part is the same as the deflection equation for the whole PCSP,
one can get the applied moment on the reinforced concrete using any of these two following methods based on the theory of plate for orthotropic materials.

**Basic relations for orthotropic plate [7]:**

\[
M_x = - \left( D_x \frac{\partial^2 w}{\partial x^2} + D_{xy} \frac{\partial^2 w}{\partial y \partial x} \right) \quad (4.27)
\]

\[
M_y = - \left( D_y \frac{\partial^2 w}{\partial y^2} + D_{xy} \frac{\partial^2 w}{\partial x \partial y} \right) \quad (4.28)
\]

Where

\[
D_x = \frac{E_x}{1 - \nu_x \nu_y} l_x
\]

\[
D_y = \frac{E_y}{1 - \nu_x \nu_y} l_y
\]

\[
D_{xy} = \frac{E_y \nu_x}{1 - \nu_x \nu_y} l_x
\]

\[
D_{xy} = \frac{E_y \nu_x}{1 - \nu_x \nu_y} l_y
\]

**Timoshenko equation for specific case of reinforced concrete slab [10]:**

\[
M_x = - \left( D_x \frac{\partial^2 w}{\partial x^2} + D_{xy} \frac{\partial^2 w}{\partial y \partial x} \right) \quad (4.29)
\]

\[
M_y = - \left( D_y \frac{\partial^2 w}{\partial y^2} + D_{xy} \frac{\partial^2 w}{\partial x \partial y} \right) \quad (4.30)
\]

Where
\[ \begin{align*}
D_x &= \frac{E_c}{1 - \nu_c^2} \left[ I_{cx} + (n - 1)I_{sx} \right] \\
D_y &= \frac{E_c}{1 - \nu_c^2} \left[ I_{cy} + (n - 1)I_{sy} \right] \\
D_{xy} &= \nu_c \left( D_x D_y \right)^{\frac{1}{2}}
\end{align*} \]

Let \( E_s \) be the Young’s modulus of steel, \( E_c \) the Young’s modulus of concrete, \( \nu_c \) the Poisson’s ratio for the concrete, and \( n \) equal to \( E_s / E_c \). Note that, \( I_{cx} \) is the moment of inertia of the slab material, and \( I_{sx} \) is the moment of inertia of steel reinforcement, both taken with respect to the neutral axis in the section where \( x \) is constant and \( I_{cy} \) and \( I_{sy} \) are the respective values for the section where \( y \) is constant.

The stress equations in the panel are in the form of [7]:

\[ \begin{align*}
\sigma_x &= -z \left( \frac{E_x}{1 - \nu_x \nu_y} \frac{\partial^2 w}{\partial x^2} + \frac{E_x \nu_y}{1 - \nu_x \nu_y} \frac{\partial^2 w}{\partial y^2} \right) \\
\sigma_y &= -z \left( \frac{E_y}{1 - \nu_x \nu_y} \frac{\partial^2 w}{\partial y^2} + \frac{E_x \nu_y}{1 - \nu_x \nu_y} \frac{\partial^2 w}{\partial x^2} \right)
\end{align*} \]  

(4.31) 

(4.32)

Where \( z \) is the distance from natural axis.
CHAPTER V

CONCLUSIONS

The moments and stresses in a precast concrete sandwich panel (PCSP) are directly related to the curvature and consequently, to the deflection of the panel. Accurate prediction of the deflections in the outer and inner wythes will better estimate the internal resisting moment and stresses.

In this study, a general equation for the deflection of a simply supported sandwich panel under in-plane and lateral loads was developed. The formulated equation includes all mechanical properties of the core and the thick similar faces. Methods for calculating bending moments and stresses to design the precast concrete sandwich wall panel were developed and validated. The proposed equations allow for parametric studies without limitations, regarding reinforcements, core shear mechanical properties, and geometrical dimensions. The following conclusion can be drawn from this research work:

1. Design equations were developed to capture the composite action in the PCSP panel. A design program/algorith was developed to aid in designing load-bearing/non load-bearing sandwich panels. The proposed equations allow for parametric studies without limitation, regarding wythe reinforcements, core shear mechanical properties, and geometric dimensions.
2. Failure to account for the reinforced core contribution in a concrete sandwich panel can force the thinner wythe to deflect as much as the load-bearing wythe when subjected to thermal bowing or lateral load. Consequently this will lead to cracking of the thinner wythe, affecting serviceability of the aesthetic look of the panel.

3. Deflection of the PCSP is dependent on all core parameters and skin parameters such as \( t \), \( c \), \( v_y \), \( v_x \), \( G_{xy} \), \( G_{yz} \), \( G_{xz} \), \( E_x \) and \( E_y \).

4. It was found that the first mode (one term in the series) provides an accurate deflection prediction of the PCSP; hence, for simplicity, the first mode was assumed in all equations provided in this study.

5. The deflection of PCSP is strongly dependent on core parameters such as \( G_{yz} \) and \( G_{zz} \). By increasing the shear modulus of the core (\( G_{yz} \) or \( G_{zz} \)), hence by adding longitudinal and transverse reinforcement in the \( yz \) or \( zz \) plane in the core, or having a high density foam insulation layer, a smaller value for the deflection of PCSP will be obtained and vice versa.

6. Methods such as lamination theory and “rule-of-mixture” were implemented in the design program for calculating mechanical properties (\( E_x \), \( E_y \), \( G_{xy} \), \( v_x \) and \( v_y \)) in outer or inner concrete wythes.

7. The current design practice for calculating applied moment on the PCSP is highly conservative in comparison with the presented methods in this study.
REFERENCES:


APPENDIX

NUMERICAL AND DESIGN CASE STUDY

Design Example:

Determine the maximum applied moment and stresses on an upper concrete slab of a simply supported PCSP shown in Fig. A.1.

![Diagram of precast concrete sandwich panel](image)

Fig. A.1 Precast concrete sandwich panel reinforced both in x and y direction

Given:

\[ E_s = 29 \times 10^6 \text{ psi} \]

\[ f'_{c} = 4000 \text{ psi} \]

\[ G_{yz} = G_{zx} = 560 \text{ psi} \]

\[ v_s = 0.3 \]
\[ \nu_c = 0.17 \]

\[ m = n = 1 \quad \text{Assuming just first mode} \]

\[ a = 7 \text{ ft} = 7 \times 12 = 84 \text{ in} \]

\[ b = 5 \text{ ft} = 5 \times 12 = 60 \text{ in} \]

\[ t = 3 \text{ in} \quad \text{Thickness of the upper and bottom concrete slab} \]

\[ c = 2 \text{ in} \quad \text{Thickness of the insulation core} \]

\[ q = 30 \text{ psf} = 0.208 \text{ psi} \quad \text{Transverse pressure (due to wind or inertial forces)} \]

Considering just the upper concrete slab and by assuming that upper concrete slab consists of [90/0] laminate, one can get properties of each ply using simple Rule-of-Mixture relations:

For the 0° ply as shown in Fig. A.2:

\[ V_s = \frac{A_s}{A} = \frac{0.2}{12 \times 1.5} = 0.011 \]

\[ G_s = \frac{E_s}{2(1 + \nu_s)} = 1.1154 \times 10^7 \text{ psi} \]
For the ply as shown in Fig. A.3:

\[ G_c = \frac{E_c}{2(1 + \nu_c)} = 1.5406 \times 10^6 \text{ psi} \]

\[ E_1 = E_s V_s + E_c (1 - V_s) = 3.8872 \times 10^6 \text{ psi} \]

\[ v_{12} = v_s V_s + v_c (1 - V_s) = 0.1714 \]

\[ E_2 = \frac{E_s E_c}{[E_c V_s + E_s (1 - V_s)]} = 3.6404 \times 10^6 \text{ psi} \]

\[ G_{12} = \frac{G_s G_c}{[G_c V_s + G_s (1 - V_s)]} = 1.5555 \times 10^6 \text{ psi} \]

For the 90° ply as shown in Fig. A.3:

\[ V_s = \frac{A_s}{A} = \frac{0.3}{12 \times 1.5} = 0.0167 \]

\[ G_s = \frac{E_s}{2(1 + \nu_s)} = 1.1154 \times 10^7 \text{ psi} \]

\[ G_c = \frac{E_c}{2(1 + \nu_c)} = 1.5406 \times 10^6 \text{ psi} \]

\[ E'_1 = E_s V_s + E_c (1 - V_s) = 4.0282 \times 10^6 \text{ psi} \]
\[ v_{12}' = v_s V_s + v_c (1 - V_s) = 0.1722 \]

\[ E_2' = \frac{E_s E_c}{[E_c V_s + E_s (1 - V_s)]} = 3.6584 \times 10^6 \text{ psi} \]

\[ G_{12}' = \frac{G_s G_c}{[G_c V_s + G_s (1 - V_s)]} = 1.5630 \times 10^6 \text{ psi} \]

The extensional stiffnesses are in the form of the following:

\[ A_{xx} = \frac{t}{2} (E_1 + E_2') = 1.1318 \times 10^7 \]

\[ A_{xy} = \frac{t}{2} (v_{12} E_2 + v_{12}' E_2') = 1.8810 \times 10^6 \]

\[ A_{yy} = \frac{t}{2} (E_2 + E_1') = 1.1503 \times 10^7 \]

\[ A_{ss} = \frac{t}{2} (G_{12} + G_{12}') = 4.6778 \times 10^6 \]

\[ A_{xs} = 0 \]

\[ A_{ys} = 0 \]

The engineering properties for a concrete slab as a function of laminate stiffnesses are in the form of the following:

\[ E_x = \frac{1}{t} \left[ A_{xx} - \frac{A_{xy}^2}{A_{yy}} \right] = 3.6703 \times 10^6 \]

\[ E_y = \frac{1}{t} \left[ A_{yy} - \frac{A_{xy}^2}{A_{xx}} \right] = 3.7301 \times 10^6 \]

\[ v_{xy} = \frac{A_{xy}}{A_{yy}} = 0.1635 \]

\[ v_{yx} = \frac{A_{xy}}{A_{xx}} = 0.1662 \]
Below is the MATLAB program for all above calculations:

% Lamination Theory

% dimension

\[ t = 3; \] % inch (total height)
\[ b = 12; \] % inch
\[ As1 = 0.2; \] % inch^2/foot (area in 0 direction)
\[ As2 = 0.3; \] % inch^2/foot (area in 90 direction)
\[ At = b \times (t/2); \] % inch^2/foot

% concrete and steel properties

\[ Es = 29 \times 10^6; \] % psi
\[ fc = 4000; \] % psi
\[ Ec = 57000 \times \sqrt{fc}; \] % psi
\[ Vs = 0.3; \] % (Poisson’s ratio for steel)
\[ Vc = 0.17; \] % (Poisson’s ratio for concrete)
\[ Gs = Es/(2*(1+Vs)); \] % psi
\[ Gc = Ec/(2*(1+Vc)); \] % psi

% unidirectional composite for 0 lamina

for Vf = As1/At
    \[ E1 = Es \times Vf + Ec \times (1-Vf); \] % E1' (E1 prime)
    \[ E2 = Es \times Ec / (Ec \times Vf + Es \times (1-Vf)); \] % E2' (E2 prime)
    \[ V12 = Vs \times Vf + Vc \times (1-Vf); \] % V12' (V12 prime)
    \[ G12 = Gs \times Gc / (Gc \times Vf + Gs \times (1-Vf)); \] % G12' (G12 prime)
end

% unidirectional composite for 90 lamina

for Vf = As2/At
    \[ EE1 = Es \times Vf + Ec \times (1-Vf); \] % E1' (E1 prime)
    \[ EE2 = Es \times Ec / (Ec \times Vf + Es \times (1-Vf)); \] % E2' (E2 prime)
    \[ VV12 = Vs \times Vf + Vc \times (1-Vf); \] % V12' (V12 prime)
    \[ GG12 = Gs \times Gc / (Gc \times Vf + Gs \times (1-Vf)); \] % G12' (G12 prime)
end

\[ Axx = (t/2) \times (E1 + EE2); \]
\[ Axy = (t/2) \times (V12 \times E2 + VV12 \times EE2); \]
\[ Ayy = (t/2) \times (E2 + EE1); \]
\[ Ass = (t/2) \times (G12 + GG12); \]

\[ Ex = (Axx - Axy^2/Ayy) / t; \]
\[ Ey = (Ayy - Axy^2/Axx) / t; \]
\[ Gxy = Ass / t; \]
\[ Vxy = Axy / Ayy; \]
\[ Vy = Axy / Axx; \]
Once the engineering properties of the reinforced concrete are calculated, one can use the following MATLAB program to find the deflection equation of the PCSP:

```matlab
% calculating amn
syms landa eta
m=1;
n=1;
Ex =3.6703*10^6;  \% psi
Ey =3.7301*10^6;  \% psi
Gxy=1.5593*10^6;  \% psi
Vx =0.1635;
Vy =0.1662;
g=1-Vx*Vy;
a=7*12;  \% in
b=5*12;  \% in
Gzx=560;  \% psi
Gyz=560;  \% psi
t=3;  \% in
c=2;  \% in
q=0.208;  \% psi

b1=(Ex/g)*((pi^4*m^4)/a^4)*(a*b/4);
b2=(Ey/g)*((pi^4*n^4)/b^4)*(a*b/4);
b3=(Ex*Vy/g)*((pi^4*n^2*m^2)/(a^2*b^2))*(a*b/4);
b4=(Gxy)*((pi^4*n^2*m^2)/(a^2*b^2))*(a*b/4);
b5=(Gzx)*((pi^2*m^2)/a^2)*(a*b/4);
b6=(Gyz)*((pi^2*n^2)/b^2)*(a*b/4);

Bxx=(c/2)*b5+t*c^2/4*(b1+b4);
Byy=(c/2)*b6+t*c^2/4*(b2+b4);
Bxy=(2*t*c^2/4)*(b3+b4);
Bx=2*(-c/2)*b5+c*t^2/4*(b1+b3+2*b4));
By=2*(-c/2)*b6+c*t^2/4*(b2+b3+2*b4));
B0=(c/2)*(b5+b6)+(t^3/3)*(b1+b2+2*b3+4*b4);

f=Bxx*landa^2+Byy*eta^2+Bxy*landa*eta+Bx*landa+By*eta+B0;
[eta, landa]=solve(diff(f, landa), diff(f, eta), landa, eta);
f=(Bx/2)*landa+(By/2)*eta+B0;
amn=(q*(4*a*b)/(m*n*pi^2))/(2*f);
amn=vpa(amn,3)

This MATLAB program resulted in a value of 0.0011 for the amplitude \(a_{mn} = 0.00105\). Thus, the deflection equation will be (see Fig. A.4) the following:

\[
w = 0.0011 \sin \left( \frac{\pi x}{84} \right) \sin \left( \frac{\pi y}{60} \right)
\]
The following is a finite element approximation performed using ABAQUS, predicting the deflection in the middle of the concrete slab equal to 0.0012.
Basic relations for orthotropic plate

Assuming just the upper concrete slab, one has the following:

\[ w_{xx} = 0.0011 \left( \frac{\pi}{84} \cos \left( \frac{\pi}{84} \right) \sin \left( \frac{\pi}{60} \right) \right) \]

\[ w_{xx} = -0.0011 \left( \frac{\pi}{84} \right) \left( \frac{\pi}{84} \sin \left( \frac{\pi}{84} \right) \sin \left( \frac{\pi}{60} \right) \right) \]

\[ w_{yy} = 0.0011 \left( \frac{\pi}{60} \sin \left( \frac{\pi}{84} \right) \cos \left( \frac{\pi}{60} \right) \right) \]

\[ w_{yy} = -0.0011 \left( \frac{\pi}{60} \right) \left( \frac{\pi}{84} \sin \left( \frac{\pi}{84} \right) \sin \left( \frac{\pi}{60} \right) \right) \]

\[ M_x = - \left( D_x \frac{\partial^2 w}{\partial x^2} + D_{xy1} \frac{\partial^2 w}{\partial y^2} \right) \]

\[ M_y = - \left( D_y \frac{\partial^2 w}{\partial y^2} + D_{xy2} \frac{\partial^2 w}{\partial x^2} \right) \]

Where

\[ D_x = \frac{E_x}{1 - \nu_x \nu_y} l_x \]

\[ D_y = \frac{E_y}{1 - \nu_x \nu_y} l_y \]

\[ D_{xy1} = \frac{E_x \nu_y}{1 - \nu_x \nu_y} l_x \]

\[ D_{xy2} = \frac{E_y \nu_x}{1 - \nu_x \nu_y} l_y \]

For calculating \( M_x \)

\[ l_x = \frac{bh^3}{12} = \frac{12 \times 5 \times 3^3}{12} = 135 \text{ in}^4 \]
\[ D_x = \frac{E_x}{1 - v_x v_y} \, l_x = 5.0933 \times 10^8 \text{ lb} - \text{in}^2 \]

\[ D_{xy1} = \frac{E_x v_y}{1 - v_x v_y} \, l_x = 8.4651 \times 10^7 \text{ lb} - \text{in}^2 \]

So the \( M_x \) is going to be the following:

\[
M_x = - \left( 5.0933 \times 10^8 \times \left( -0.0011 \left( \frac{\pi}{84} \right) \left( \frac{\pi}{84} \right) \sin \left( \frac{\pi x}{84} \right) \sin \left( \frac{\pi y}{60} \right) \right) \right) + 8.4651 \times 10^7
\times \left( -0.0011 \left( \frac{\pi}{60} \right) \left( \frac{\pi}{60} \right) \sin \left( \frac{\pi x}{84} \right) \sin \left( \frac{\pi y}{60} \right) \right)
\]

The maximum moment occurs at \( x = \frac{a}{2} = 42 \) and \( y = \frac{b}{2} = 30 \); thus \( M_{x,\text{max}} \) will have the form of the following:

\[
M_{x,\text{max}} = - \left( 5.0933 \times 10^8 \times \left( -0.0011 \left( \frac{\pi}{84} \right) \left( \frac{\pi}{84} \right) \right) \right) + 8.4651 \times 10^7
\times \left( -0.0011 \left( \frac{\pi}{60} \right) \left( \frac{\pi}{60} \right) \right) = 1038.95 \text{ lb} - \text{in}
\]

Fig. A.6 shows the distribution of \( M_x \) in upper concrete slab:
Similarly for $M_y$

$$I_y = \frac{ah^3}{12} = \frac{12 \times 7 \times 3^3}{12} = 189 \text{ in}^4$$

$$D_y = \frac{E_y}{1 - v_x v_y} I_y = 7.2468 \times 10^8 \text{ lb - in}^2$$

$$D_{xy} = \frac{E_y v_x}{1 - v_x v_y} I_y = 1.1849 \times 10^8 \text{ lb - in}^2$$

So the $M_y$ will take the form as follows:

$$M_y = -\left(7.2468 \times 10^8 \times \left(-0.0011 \left(\frac{\pi}{60}\right) \left(\frac{\pi}{60}\right) \sin\left(\frac{\pi x}{84}\right) \sin\left(\frac{\pi y}{60}\right)\right) + 1.1849 \times 10^8 \right)$$

$$\times \left(-0.0011 \left(\frac{\pi}{84}\right) \left(\frac{\pi}{84}\right) \sin\left(\frac{\pi x}{84}\right) \sin\left(\frac{\pi y}{60}\right)\right)$$

The maximum moment in the $y$ direction will occur at $x = \frac{a}{2} = 42$ and $y = \frac{b}{2} = 30$, so one has the following:

$$M_{y_{\text{max}}} = -\left(7.2468 \times 10^8 \times \left(-0.0011 \left(\frac{\pi}{60}\right) \left(\frac{\pi}{60}\right)\right) + 1.1849 \times 10^8 \right)$$

$$\times \left(-0.0011 \left(\frac{\pi}{84}\right) \left(\frac{\pi}{84}\right)\right) = 2367.74 \text{ lb - in}$$

Fig. A.7 shows distribution of $M_y$ on the upper concrete slab:
The following is the MATLAB program for calculating the applied moment based on the basic relations for orthotropic plate:

```
syms x y

%given

%Amn=0.0011;               %in
a=7*12;                   %in
b=5*12;                   %in
Ex =3.6703*10^6;          %psi
Ey =3.7301*10^6;          %psi
Gxy=1.5593*10^6;          %psi
Vx =0.1635;
Vy =0.1662;
h=3;                      %in

%Equation for Deflection

w=Amn*sin(pi*x/a)*sin(pi*y/b);
e1=diff(w,x,2)
e2=diff(w,y,2)
```

Fig. A.7 Distribution of $M_y$
The Timoshenko equation for the specific case of reinforced concrete slab is as follows:

\[ M_x = - \left( D_x \frac{\partial^2 w}{\partial x^2} + D_{xy} \frac{\partial^2 w}{\partial y^2} \right) \]

\[ M_y = - \left( D_y \frac{\partial^2 w}{\partial y^2} + D_{xy} \frac{\partial^2 w}{\partial x^2} \right) \]

Where

\[ D_x = \frac{E_c}{1 - \nu_x^2} \left[ I_{cx} + (n - 1)l_{sx} \right] \]

\[ D_y = \frac{E_c}{1 - \nu_y^2} \left[ I_{cy} + (n - 1)l_{sy} \right] \]

\[ D_{xy} = \nu_c (D_x D_y)^{1/2} \]

For calculating \( M_x \)

\[ A_s = 0.2 \frac{in^2}{ft} \]

Calculating the neutral axis, which is shown in the cross section in Fig. A.8
Then

Assuming that the concrete below the neutral axis is cracked and does contribute to the stiffness, one can write as follows:

\[ d = 3 - \text{cover} - 0.5(\text{bar diameter}) \]

\[ d = 3 - 0.75 - 0.5(0.5) = 2 \text{ in} \]

\[ n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000 \times \sqrt{4000}} = 8 \]

\[ 12x\left(\frac{x}{2}\right) = n \times 0.2(2 - x) \]

\[ 6x^2 + 1.6x - 3.2 = 0 \]

Then

\[ x = 0.61 \]

Assuming that the concrete below the neutral axis is cracked and does contribute to the stiffness, one can write as follows:

\[ I_{cx} = \frac{bh^3}{12} + Ad^2 = \frac{12 \times 0.61^3}{12} + (12 \times 0.61)\left(\frac{0.61}{2}\right)^2 = 0.91 \text{ in}^4/ft \]

\[ I_{cx} = 0.91 \times 5 = 4.54 \text{ in}^4 \]
\[ I_{sx} = n \times A d^2 = 8 \times 0.2(2 - 0.61)^2 = 3.1 \text{ in}^4 \]

\[ I_{sx} = 3.1 \times 5 = 15.46 \text{ in}^4 \]

\[ D_x = \frac{E_c}{1 - v_c^2} \left[ I_{cx} + (n - 1)I_{sx} \right] = \frac{57000 \times \sqrt{4000}}{1 - 0.17^2} \left( 4.54 + 7 \times 15.46 \right) = 4.19 \times 10^8 \text{ lb - in}^2 \]

For calculating \( M_y \)

\[ A_s = 0.3 \text{ in}^2/ft \]

Calculating the neutral axis, which is shown in the cross section in Fig. A.9

![Neutral Axis Diagram](neutral_axis_diagram.png)

Fig. A.9 Cross section of PCSP

\[ d = 3 - \text{cover} - 0.5(\text{bar diameter}) \]

\[ d = 3 - 0.75 - 0.5(0.5) = 2 \text{ in} \]

\[ n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000 \times \sqrt{4000}} = 8 \]
12x\left(\frac{x}{2}\right) = n \times 0.3(2 - x)

6x^2 + 2.4x - 4.8 = 0

So:

x = 0.72

Assuming that the concrete below the neutral axis is cracked and does contribute to the stiffness, one can write the following:

\[
I_{cy} = \frac{bh^3}{12} + Ad^2 = \frac{12 \times 0.72^3}{12} + (12 \times 0.72)\left(\frac{0.72}{2}\right)^2 = 1.493 \frac{in^4}{ft}
\]

\[
I_{cy} = 1.49 \times 7 = 10.45 \text{ in}^4
\]

\[
I_{sy} = n \times Ad^2 = 8 \times 0.3(2 - 0.72)^2 = 3.93 \frac{in^4}{ft}
\]

\[
I_{sy} = 3.93 \times 7 = 27.53 \text{ in}^4
\]

\[
D_y = \frac{E_c}{1 - \nu_c^2}\left[I_{cy} + (n - 1)I_{sy}\right] = \frac{57000 \times \sqrt{4000}}{1 - 0.17^2}(10.45 + 7 \times 27.53)
\]

\[
= 7.54 \times 10^8 \text{ lb} - \text{in}^2
\]

\[
D_{xy} = \nu_c(D_x + D_y)^\frac{1}{2} = 0.17 \sqrt{((4.19 \times 10^8) \times (7.54 \times 10^8))} = 9.55 \times 10^7 \text{ lb} - \text{in}^2
\]

When the values of \(D_x, D_y\) and \(D_{xy}\) are calculated, one can easily calculate the moment in the top concrete slab (outer wythe) based on the basic relationship for the orthotropic plate:

\[
M_x = -\left(D_x \frac{\partial^2 w}{\partial x^2} + D_{xy} \frac{\partial^2 w}{\partial y^2}\right)
\]
Thus the $M_x$ takes the form as follows:

$$M_x = -\left(4.19 \times 10^8 \times \left(-0.0011 \left(\frac{\pi}{84}\right)^2 \left(\frac{\pi}{84}\right) \sin \left(\frac{\pi x}{84}\right) \sin \left(\frac{\pi y}{60}\right)\right) + 9.55 \times 10^7 \right)$$

$$\times \left(-0.0011 \left(\frac{\pi}{60}\right)^2 \left(\frac{\pi}{84}\right) \sin \left(\frac{\pi x}{84}\right) \sin \left(\frac{\pi y}{60}\right)\right)$$

The maximum moment will occur at $x = \frac{a}{2} = 42$ and $y = \frac{b}{2} = 30$, so we have the following:

$$M_{x_{max}} = -\left(4.19 \times 10^8 \times \left(-0.0011 \left(\frac{\pi}{84}\right) \left(\frac{\pi}{84}\right)\right) + 9.55 \times 10^7 \times \left(-0.0011 \left(\frac{\pi}{60}\right) \left(\frac{\pi}{60}\right)\right)\right)$$

$$= 938 \text{ lb} - \text{in}$$

And $M_y$ is going to be as follows:

$$M_y = -\left(7.54 \times 10^8 \times \left(-0.0011 \left(\frac{\pi}{60}\right)^2 \left(\frac{\pi}{84}\right) \sin \left(\frac{\pi x}{84}\right) \sin \left(\frac{\pi y}{60}\right)\right) + 9.55 \times 10^7 \right)$$

$$\times \left(-0.0011 \left(\frac{\pi}{84}\right)^2 \left(\frac{\pi}{84}\right) \sin \left(\frac{\pi x}{84}\right) \sin \left(\frac{\pi y}{60}\right)\right)$$

The maximum moment will occur at $x = \frac{a}{2} = 42$ and $y = \frac{b}{2} = 30$, so we have the following:

$$M_{y_{max}} = -\left(7.54 \times 10^8 \times \left(-0.0011 \left(\frac{\pi}{60}\right) \left(\frac{\pi}{60}\right)\right) + 9.55 \times 10^7 \times \left(-0.0011 \left(\frac{\pi}{84}\right) \left(\frac{\pi}{84}\right)\right)\right)$$

$$= 2436 \text{ lb} - \text{in}$$

Below is the MATLAB program for calculating the applied moment based on Timoshenko’s method:

$$M_y = -\left(D_y \frac{\partial^2 w}{\partial y^2} + D_{xy} \frac{\partial^2 w}{\partial x \partial y}\right)$$
syms x y

given

Amn=0.0011; % in
a=7*12; % in
b=5*12; % in
Es=29*10^6; % psi
fc=4000; % psi
Ec=57000*sqrt(fc); % psi
n=Es/Ec;
Vc=0.17; % (poisson’s ratio for concrete)

calculating:

Icx=4.54; % in^4
Isx=15.46; % in^4
Icy=10.45; % in^4
Isy=27.53; % in^4

Equation for Deflection

w=Amn*sin(pi*x/a)*sin(pi*y/b);
e1=diff(w,x,2);
e2=diff(w,y,2);

Dx=Ec*(Icx+(n-1)*Isx)/(1-Vc^2);
Dy=Ec*(Icy+(n-1)*Isy)/(1-Vc^2);
Dxy=Vc*sqrt(Dx*Dy);

Mx=-(Dx*e1+Dxy*e2);
My=-(Dy*e2+Dxy*e1);
subs(subs(Mx,x,42),y,30)
subs(subs(My,x,42),y,30)

The current method for calculating applied moment [11]:

\[ M_y = \frac{(ql_2)l_1^2}{8} \]

Where

\[ q = 0.208 \text{ psi} \]
Thus, the applied moment is as follows:

\[ M_y = \frac{(ql_2)l_1^2}{8} = \frac{(0.208 \times 60)(84)^2}{8} = 11008 \text{ lb - in} \]

Note that each concrete slab resists half of the above value for moment which is approximately 5500 \text{ lb - in}.

The stress equations can be written as the following:

\[
\sigma_x = -z \left( \frac{E_x}{1 - \nu_x \nu_y} \frac{\partial^2 w}{\partial x^2} + \frac{E_x \nu_y}{1 - \nu_x \nu_y} \frac{\partial^2 w}{\partial y^2} \right)
\]

\[
\sigma_y = -z \left( \frac{E_y}{1 - \nu_x \nu_y} \frac{\partial^2 w}{\partial y^2} + \frac{E_x \nu_y}{1 - \nu_x \nu_y} \frac{\partial^2 w}{\partial x^2} \right)
\]

Where

\[ E_x = 3.6703 \times 10^6 \]
\[ E_y = 3.7301 \times 10^6 \]
\[ \nu_x = 0.1635 \]
\[ \nu_y = 0.1662 \]
\[ z = 3 + 1 = 4 \]

By plugging the above values in the stress equations and setting \( x = \frac{a}{2} = 42 \) and \( y = \frac{b}{2} = 30 \), one has the following:

\[
\sigma_{x_{\text{max}}} = -z \left( \frac{E_x}{1 - \nu_x \nu_y} \frac{\partial^2 w}{\partial x^2} + \frac{E_x \nu_y}{1 - \nu_x \nu_y} \frac{\partial^2 w}{\partial y^2} \right) = 30.7838 \text{ psi}
\]
Below is the MATLAB program for calculating stress:

```matlab
syms x y 
z=4; %in
Ex =3.6703*10^6; %psi
Ey =3.7301*10^6; %psi
Vx =0.1635;
Vy =0.1662;
Amn=0.0011;
a=7*12; %in
b=5*12; %in
w=Amn*sin(pi*x/a)*sin(pi*y/b);
e1=diff(w,x,2);
e2=diff(w,y,2);
Stressx=-z*((Ex/(1-Vx*Vy))*e1+(Ex*Vy/(1-Vx*Vy))*e2);
subs(subs(Stressx,x,42),y,30)
Stressy=-z*((Ey/(1-Vx*Vy))*e2+(Ex*Vy/(1-Vx*Vy))*e1);
subs(subs(Stressy,x,42),y,30)
```

Fig. A10 shows the magnitude of stress ($\sigma_y$) versus the depth of the panel in the middle of the PCSP where $x = \frac{a}{2} = 42 \text{ in}$ and $y = \frac{b}{2} = 30 \text{ in}$.
Fig. A.11 shows stress at $z$ equal to 4 ($\sigma_y @ z = 4$) in the middle of the panel versus different multiples of shear moduli of the core. Note that the original shear modulus is $G_{yz} = G_{zx} = G = 560 \text{ Pst}.$

One can see that by increasing the shear moduli of the core, hence by adding reinforcement in $yz$ and $zx$ plane in the core, or having a high density foam insulation layer, the smaller value for the bending stress of PCSP will be obtained and vice versa.
Fig. A.12 shows the maximum stresses in the middle of the panel where $z$ is equal to 4, versus a different $t/c$ ratio for two different values of shear modulus of the core. Note that the total depth of the PCSP remains constant during the analysis.

It can be inferred from the figure that, if one desires to decrease the stress in the panel by increasing the $t/c$ ratio, it is pointless to exceed the ratio of $t/c = 2$ which leads to an almost constant stress line.