SEMI-ACTIVE CONTROL OF AIR-SUSPENDED
TUNED MASS DAMPERS

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SEMI-ACTIVE CONTROL OF AIR-SUSPENDED TUNED MASS DAMPERS

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ABSTRACT

SEMI-ACTIVE CONTROL OF AIR-SUSPENDED TUNED MASS DAMPERS

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Extensive studies have been carried out, in recent years, to find methods to mitigate the unwanted structure vibration caused by human excitation, machinery, and winds. Modern structures such as floors and bridges using high strength materials, and extending across long spans are very flexible with negligible damping. Vibration control devices and strategies are constantly being developed to eliminate/dissipate the unwanted vibration and to increase the serviceability level of such structures. One such method for abating the vibration is tuned mass damping.

In this dissertation, semi-active control of air-suspended tuned mass dampers with pendulum configuration was explored. A novel semi-active Air Sprung PTMD was designed, built, and evaluated, analytically and experimentally. The dynamics and control of such PTMD were evaluated, and its effectiveness was compared with that of the conventional passive PTMD.
The main reason for introducing semi-active control to a TMD is to enable the TMD to adapt itself, robustly, to the primary structure’s parameters (mainly mass and stiffness) changes and maintain its tuning.

Following extensive analytical work, simulation, and experimentation it was found that the velocity feedback can modify the stiffness of the semi-active air-suspended tuned mass damper. Positive velocity feedback increases the stiffness while negative velocity feedback decreases it. Moreover, pressure and the acceleration feedback adjust the damping of the semi-active TMD.

The air spring used in this work is of convoluted type. This type of air springs, because of their particular geometry, experiences a rather severe change in their cross-sectional area, as they contract and expand. It was found that to properly account for the impact of this important parameter on the inner-working of the air spring, one needs to consider two areas for the air spring, namely, the effective area and the geometric area. The effective area of the air spring is the area used to calculate the exerted force by the air spring while the geometric area is the cross-sectional area used to calculate the rate of change of the volume of the air enclosed in the air spring. The use of these two areas resulted in an accurate model of the air spring.

The model of the semi-active PTMD was verified using the experimental setup. Both the simulation and experimental demonstrate the effectiveness of the semi-actively controlled air-suspended PTMD in adjusting its tuning frequency as well as damping.
I dedicate this thesis to my parents who I owe everything to them, especially my father who died in January 2011. May Allah have mercy upon his soul and may he grant him the highest level in Paradise.

إلي من أحمل إسمك بكل فخر .. إلي من يرتعش قلبي لذكرك ...
التي من كان ملاذي وملجئ في صعوبات الدراسة و الغربه إلي من علمني أن العلم أجمل شي في الحياة ... إلي من فقدته وأنا بعيد عنه أعمل على هذا البحث ...
إلي من أودعتني الله ... أهديك هذا البحث أبي.
إلي ينبع الصبر و التفاؤل و الأمل إلي كل من في الوجود بعد الله ورسوله إلي أمي الغالي.
إلي سندي و قوتي إلي من أثارتي على نفسها إلي من تدفعني إلي الإمام دوما إلي زوجتي الغالية أماني.
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CHAPTER I

INTRODUCTION

This chapter begins with the motivation for the work that is presented in this dissertation. After stating the objectives and approach to this research, it covers the contributions of this study. The chapter ends with an outline of this dissertation.

1.1 Problem Statement

Extensive studies in recent years have been carried out to find methods to mitigate the unwanted vibration in a structure caused by human activities or machinery operation. The ways that architects and engineers design modern structures such as buildings, towers, and bridges using new lightweight materials in large open spaces create very flexible and underdamped structures. Human and environmental excitations can cause these types of structures to vibrate. These vibrations certainly cause occupants discomfort and might also have an influence on the reliability of the structure which may cause structural failure. Incorporating devices into such structures that can absorb the dynamic load and thus reduce the unwanted vibration is of great interest to engineers.

The transmission of steady-state vibrations from machinery to the support structure could be reduced using isolation techniques. However, in some cases, some
machinery could not be isolated. Furthermore, humans whose activities induce vibrations to a structure cannot be isolated from the structure. Considering that human induced excitation and machinery are the main sources of structural vibrations, remedial measured for reducing unwanted vibrations of such structures are essential. When the structure could not be modified either by increasing or decreasing mass properties, stiffness properties, damping properties, and the number of degrees of freedom, passive, active, semi-active and hybrid control systems are used to abate the unwanted vibration in a structure.

Tuned mass dampers rank high amongst the structural control devices. However, passive tune mass dampers (TMDs) are effective only in a narrow frequency range, and tend to be detuned as the dynamic characteristics of structures vary with time. Active tuned mass dampers have been considered as an effective alternative to mitigate structural vibrations, but disadvantages such as cost and the need for frequent maintenance limit their use.

The main goal of this research is to investigate the performance of a novel semi-active tuned mass damper, as an alternative to active TMDs, for reducing unwanted vibrations of a structure. The proposed semi-active TMD has a pendulum configuration, which includes an inertia and an air spring. The stiffness and damping of the air spring are adjusted by a servovalve under the control of a supervisory controller. Semi-active systems are excellent solutions in many engineering problems because they possess the simplicity of passive systems and the flexibility of active systems.
1.2 Research Scope and Objectives

This research has developed a new semi-active TMD capable of in-situ adjustment of its stiffness and damping, using an air spring, which surpasses the performance of a passive TMD. It has also assessed the comprehensive dynamics of the new tuned damping system. The main objectives of this study are summarized as follows:

1. Model the dynamics of the novel semi-active air spring TMD, followed by design and build of the device to verify the model.
2. Evaluate, analytically and experimentally, the dynamics of semi-active air spring TMDs and compare it with the performance of the conventional passive TMDs.
3. Investigate the ability of semi-active air spring TMDs to robustly adapt to the primary structures’ parameter changes, which is critical in determining the effectiveness of semi-active TMDs to structural vibration control.
4. Suggest effective guidelines for the use of air spring semi-active TMDs in structural vibration applications.

1.3 Approach

There are two distinct phases in this investigation. The first phase is numerical modeling of the semi-active single-degree-of-freedom (SDOF) TMD. The numerical investigation emphasizes the realization of semi-active control methods to regulate the damping and stiffness of the air spring suspended TMD. The numerical model serves as the basis for numerical parametric studies of such TMDs for further evaluation of their
dynamic performance. The parametric study covers the effects of damping and stiffness, as well as system parameter variations (detuning).

The second phase of this study is an experimental investigation, involving the use of a laboratory set-up to present the semi-active TMD. Using this test setup, a series of tests were conducted to a) evaluate the effectiveness of the semi-active TMD, and b) verify the model developed in phase 1. Moreover, the experiments conducted served as a “proof-of-concept” of the effectiveness of the air spring suspended TMD for structural vibration abatements.

1.4 Contributions

The findings of this research will contribute to the development of a new class of TMDs, namely semi-active air suspended, for reducing vibrations in structures and mechanical systems. These semi-active air spring TMDs exhibit superior performance in reducing the primary structure’s vibration levels as well as robustness to system parameter changes than their equivalent passive TMDs. Moreover, the semi-active air spring TMD is more cost-effective and maintenance-free than a comparable active TMD.

More specific contributions of this research include:

- Establishing an experimentally verified numerical model for the semi-active TMD.
- Finding the most suitable control algorithm (in terms of both performance and robustness) and establishing a procedure for the best control parameter.

The above-listed contributions were experimentally demonstrated.
1.5 Outline of the Dissertation

Chapter 2 presents the background and literature review. It covers some of past studies on structural vibrations and tuned mass dampers. It then discusses the necessary background information on air spring, semi-active control, and servovalve. Chapter 3 presents the dynamic analysis of the proposed pendulum tuned mass damper (PTMD). This chapter derives the mathematical models of

- a pendulum tuned mass damper using Lagrange equation,
- an air spring by using energy conservation laws, and
- a servovalve using the International Society of Automation (ISA) model.

Chapter 4 discusses the physical and numerical properties of the major components of the semi-active PTMD system such as air spring and the servovalve used in this study. Knowledge of these properties is essential in creating and developing an accurate simulation model in chapter 5. The parametric studies of the mathematical model of the dynamics of semi-active TMD are presented also in chapter 5. The parametric studies include the effects of change in damping, as well as system natural frequency. The experimental components of this research are discussed in Chapter 6. The last section of chapter 6 discusses the experimental test results. Finally, chapter 7 provides a summary of the work. Significant results are highlighted, and recommendations for future research are also discussed.
CHAPTER II

LITERATURE REVIEW AND BACKGROUND

This chapter reviews the earlier studies of structural vibrations and tuned mass damper. In addition to providing a literature review, it also provides the needed background information on air spring. The chapter concludes with an overview of semi-active control systems.

2.1 Structural Vibrations

Modern structures are made of advanced, lightweight materials and as such they are flexible and more oscillatory. Structures have multiple degrees of freedom with an infinite number of modes of vibration. For each mode of vibration, there is a corresponding natural frequency, mode shape, and damping ratio. However, like most of the real physical systems, only a limited number of modes of vibration within a specific frequency range are of great concern. The first few modes, which are normally in the frequency range of less than 10 Hertz (Hz) in civil engineering structures, are dominant and contain most of the vibration energy of the structure [1]. The first, and the most important, mode called the fundamental mode has the lowest natural frequency and vibrates with the largest amplitude.
2.1 Remedial Methods

Structural engineers have many codes and criteria for estimating vibration extent and lowering it to below the acceptable level before designing and constructing a structure. However, there are still many vibrations with excessive levels which manifest themselves after the completion of construction. Vibration control techniques used to reduce such annoying vibrations could be grouped into the three categories of 1) reduction of the vibration at the source, 2) isolation of the source, and 3) reduction of vibration at the vibration receiver [2].

2.1.1 Reduction at the Vibration Source

Reducing the vibration source is done when the source can be modified and made better. For example, when vibration is caused by excessive eccentricities in rotating shafts in a machine, balancing those shafts would lower the vibration. Also, avoiding excessive clearances in bearings and paying attention to other details during manufacturing of an equipment could help reduce the vibration of that equipment [2].

2.1.2 Isolation

Vibration isolation can be done by placing properly selected resilient devices between the source of the vibration and the structure supporting it, to reduce the transmission of the vibration from the source to the structure or vice versa [2].
2.1.1.3 Reduction at the Vibration Receiver

The easiest and the lowest cost corrective measure to reduce the vibration response at the receiver is relocation of the vibration sources. Locating a gym on the ground level instead of the middle stories, or installing an equipment close to the columns and walls as opposed to the middle of a floor are examples of such corrective measure.

Increasing the natural frequency of a structure by modifying the structural stiffness or mass can reduce vibration. This can be done by changing the layout of the structure either by adding new column supports, cover plates, or rods to the supporting joists and girders.

Increasing the damping of the structure is another method used to reduce the vibration of a structure. For example, the presence of non-structural elements such as furniture, computers and partition as well as the occupants on the floor increases the damping of the floor. When increases in damping using these measures are not adequate tuned mass dampers (TMDs) can be used to minimize the vibration of the structure. A TMD is usually made up of a mass, a restoring mechanism and dissipative element which act as narrow-band damper lowering vibration at its tuned frequency.

2.1.2 Vibration Control Strategies

Irrespective of the remedial methods considered, the strategies for lowering the unwanted vibration could be classified into three categories of passive, active, and semi-active.
2.1.2.1 Passive Systems

A passive system is a system which does not require an external power source to function [3]. Passive systems reduce forces that are resulted from the excitation of the structure at the location of their installation. The simplicity of passive devices is one of their advantages. Moreover, the energy in the structure cannot be increased by passive systems resulting in guaranteed stability and higher reliability. Passive damping strategies include vibration absorbers, base isolation systems, viscoelastic dampers, and tuned mass dampers.

The effectiveness of passive tuned devices depends strongly on how accurately they are tuned. This makes them unable to adapt to changes in structural parameters.

2.1.2.2 Active Systems

An active vibration control system requires an external power source to operate. The electrohydraulic or electromechanical actuators which are frequently used in active systems require power to control the dynamic forces applied to the structure. The control forces produced by active system devices are based on the readout of the sensors measuring the response of a structure. These measurements allow the active systems to evaluate and produce the control forces. These forces can be used to dissipate the energy in the structure to achieve an acceptable response. The disadvantages of the active system are relatively high cost, relatively high energy consumption, periodic maintenance, complex control laws and components, and the potential for instability [3].
2.1.2.3 Semi-Active Systems

Semi-active systems are passive systems with adjustable parameters such as stiffness, damping, or both. Such adjustability enables semi-active systems to enhance the effectiveness of passive system. Semi-active systems have been received favorably by designers and researcher of structural vibration control schemes. The use of energy in semi-active devices is low. When the adjustable component is properly adjusted, semi-active systems are highly effective [4] [5]. The presence of passive component in semi-active systems would guarantee higher reliability in case if the active part failed. The cost of a semi-active vibration control systems is less than the equivalent active systems.

In summary, several vibration control techniques and strategies are available but each one has its own advantage and disadvantage. Cost, efficiency, reliability and the types of vibration are significant factors on making decision as to what is the most suitable method to abate vibration. Semi-active strategies have demonstrated high performance controlling vibration. They combine the simplicity and stability of passive systems and the effectiveness of active systems without needing excessive energy and incurring excessive cost.

2.2 Tune Mass Damper

This section provides a discussion about Tuned Mass Damper (TMD), including its concept and development. It also provides a short literature review on its applications to structures such as floor system and high-rise buildings.
2.2.1 Passive Tuned Mass Damper

Vibration control devices and strategies are constantly being developed to eliminate the unwanted vibration issues and to increase the serviceability of structural systems. Chief among such devices is tuned mass damper which is a single degree of freedom subsystem coupled with main structure to dampen the unwanted vibration at a certain frequency.

The dynamic absorption proposed by Frahm in 1909 is the principle behind the inner working of tuned mass dampers. It consists of a mass, a damper, and a spring and is tuned to the natural frequency of a single mode of the structure targeted for damping. Usually, a TMD is installed at the location of the highest amplitude of vibration. The TMD dissipate the vibrational energy of the structure corresponding to a certain mode by transferring that energy to the damping element of the TMD.

Den Hartog provided a simple formula for obtaining the parameters of TMDs, i.e. damping ratio, stiffness, and tuning frequency to optimally control the vibration of undamped systems under a harmonic excitation [6]. Since then, tuned mass damping has become one of the most investigated fields in controlling structural vibration.

2.2.2 Active Tuned Mass Damper

Passive TMDs have been used successfully and widely to mitigate excessive vibrations. However, TMD in certain applications could be detuned; note that a passive TMD cannot adapt to the changes in the structure’s parameters which could occur over time.
In an effort to increase the effectiveness of tuned mass damper Chang and Soong [7] suggested the use of active control strategies where an active force acts between the TMD and the structure. The result they got from the study showed that the active tuned mass damper is highly effective in reducing vibration. Chang Yang [8] proposed a closed-loop, full feedback control algorithm for an active tune mass damper to control a building modeled as a single degree of freedom system. The stiffness and the damping ratio of the TMD were kept constant. The control gains were calculated optimally, minimizing the vibration displacement of the structure.

2.2.3 Semi-Active Tuned Mass Damper

The high cost and lack of stability robustness are the main disadvantages of the active TMDs. Moreover, the needs of an external power to operate the active devices are also viewed as a major problem in case of seismic events resulting in power outage. To solve these problems, many attempted to develop new designs by combining passive and active TMDs into semi-active, and hybrid TMDs. Housner et al. [9], Symans and Constantinou [3], Soong and Spencer [10], Spencer & Nagarajaiah [11], Kurata & Kobori [12], and Fisco & Adeli [13] detailed a full review on active, and semi-active TMDs.

Semi-active vibration control is realized mainly by adjusting the damping via the use of variable-orifice dampers, variable-friction dampers, and controllable-fluid (magnetorheological) dampers, and/or modifying the stiffness of the TMD.

Hrovat, et al., [14] proposed a semi-active tuned mass damper (TMD) to control vibration of tall building due to wind. He integrated the concept of semi-active control in vehicle suspension system into semi-active TMD (SATMD). He introduced an adjustable
damper between the building and the TMD which consists of hydraulic cylinder supplemented by a control valve. The performance of the semi-active TMD has been compared with those of passive and active TMD systems where the results indicated that significant improvements could be achieved by using SATMD. Since then, there have been a large number of studies regarding the use of semi-active TMDs; see for examples Wang and Kim [15].

Masato Abe [16] discussed a semi-active strategy to utilize the initial TMD displacement and the variable damping to find the best control law. He found the impulse response of the structure is the best control algorithms for SATMD which utilizes the initial TMD displacement and the variable damping. His simulation results showed that SATMD is more efficient than passive TMD.

Djajakesukma, et al., [17] evaluated the effectiveness of various control laws for the semi-active stiffness damper (SASD) including resetting control, switching control, linear quadratic regulator (LQR), and modified LQR. A semi-active stiffness damper (SASD) consists of a hydraulic damper connected to a bracing frame. They found that all control laws demonstrated effectiveness, to various degrees in abating vibration. They concluded that the modified LQR control was capable of optimizing its performance against variation in structure and the perturbation type.

Setareh [18] discussed ground-hook’ tuned mass dampers (GHTMD)\(^1\) which is a TMD with the ground-hook semi-active damper as its damping element. He used GHTMD to reduce the floor vibrations due to human movements as recommended by Murray et al.[1]The TMD Setareh introduced used a continuously variable, adjustable

\(^1\) where the damper is coupled with the ground
\(^2\) The auxiliary chamber is used to either lower the stiffness or add damping or both
damper (ground-hook damper) containing a magnetically responsive fluid in which micron-sized, magnetizable particles are suspended. This fluid can be used in valve mode, with the fluid flowing through an orifice, or in shear mode, with the fluid between two surfaces moving in relation to each other. In the absence of a magnetic field applied across the gap the fluid occupies, the fluid flows freely. The GHTMD is applied to a single degree of freedom structure representing of floor system. The optimum values of control parameters were found based on the minimization of the acceleration response of the floor for different GHTMD mass ratios and floor damping ratios. He claims that GHTMDs can perform up to 14\% better than their equivalent TMDs.

Analytical and experimental studies of a pendulum tuned mass damper (PTMD) to control vibrations were presented by Setareh [19, 20]. The PTMD used in his study acts as a passive tuned mass damper where the damping element installed under the vibratory mass. He also investigated the effects of off-tuning of the PTMD due to the variations in the structure’s mass. His study results indicated that a properly tuned PTMD can significantly reduce excessive vibrations.

Nagarajaiah, et al., [21] proposed variable stiffness tuned mass dampers to reduce the response of the main structure under several types of excitations. They used semi-active variable stiffness devices (SAIVS) which consist of four spring elements arranged in a plane rhombus configuration with pivot joints at the vertices. A linear electromechanical actuator configures the aspect ratio of the rhombus configuration of SAIVS device. The aspect ratio changes between the joints leads to adjustment in stiffness. They developed a control algorithm based on real-time frequency tracking of excitation signal by short-time Fourier transform analysis to actuate the linear
electromechanical actuator such that if the fundamental frequency changes, due to damage or wear of the main structure, the TMD will not be de-tuned.

2.3 Air Spring

This section provides an overview of air spring used as the adjustable element of the semi-active TMD in this research. It begins with brief history of air spring and explains how an air spring behaves. This section also introduces different types of air springs.

In the late 1930’s air springs were developed for use in vehicle suspensions. An air spring is a reinforced rubber fabric bellow which contains a column of compressed air and sealed by two bead plates on the bottom and top. The bead plates allow for rigid attachment of the air spring to the mounting surfaces of a structure. The column of air inside the air spring, not the rubber bellow, plays the major role in providing force and supporting the load [22].

Air spring has many advantages over conventional mechanical coil spring, including low weight and low cost. Air spring provides low friction in all directions. The ability to maintain the height around the design point by adjusting the air pressure as load is changed without changing the air spring’s features is another advantage that air spring has over conventional mechanical coil spring.

Equation (2.1) shows the relationship between the load applied on the air spring and the pressure inside the air spring. Variation in applied load could be made up for by modifying the air spring pressure or its diameter or a combination of both. Note that, changing the two parameters of effective area and pressure is easier than changing the air
spring itself, if the load changes. This feature is a major advantage that air springs have over conventional coil springs.

\[ \text{force} = \text{pressure} \times \text{effective area} \]  
\[ \text{effective area} = \pi \times \frac{\text{effective diameter}^2}{4} \]  

Equation (2.1)

Effective area used in Equation (2.1) is an important parameter of an air spring and will be defined in chapter 3.

### 2.3.1 Air Spring Types

Air spring exists in two common styles of reversible sleeve with almost cylinder shape which has fixed area and adjustable height and a convoluted style with donut shape which has adjustable height and area; see Figure 2.1. Both air spring types have been used in vibration isolation. The basic principle behind the air spring is the same for both types of air springs. The gas is compressed inside the air spring which produces a reaction force to sustain the load carried by it. As stated earlier in addition to pressure, the load carried by the air spring depends on the effective area of the air spring. Therefore, the diameter of the air spring plays a major role in determining air pressure needed for supporting that load.
2.3.2 Literature Review on Air Spring

Air spring has been used widely in vibration control for isolation purpose. The literature review shows many methods have been developed to model air springs. Most of the available models of the air spring are developed either by using the thermodynamics, heat transfer, or fluid mechanic principles. Many of these models are customized for air springs used in vehicle suspensions or for isolation purposes.

Grayson Dixon [24] presented the modeling and experimental work for an automatic closed-loop controlled air spring used to maintain its mean height when a change occurs in the vehicle weight.

Toyofuku, et al., [25] studied the relation between vibration frequency and pressure in auxiliary chambers\(^2\) connected to an air spring. They divided their system into air spring, auxiliary chamber and connecting pipes, and then used the equations of continuity, motion, state of gas, and energy to model their system. In their study, they

\(^2\) The auxiliary chamber is used to either lower the stiffness or add damping or both
found that the length and the diameter of the pipes mainly affect the air suspension system, while the auxiliary chamber has negligible effects, at high frequencies, and on the dynamic spring constant of the total suspension system.

Sorli and Quaglia [26] studied the effect of the air spring parameters on a pneumatic suspension with an auxiliary reservoir shown in Figure 2.2 They derived the stiffness of the air spring by studying the relation between the force and the displacement. They studied the force resulting from the air spring pressure and the effective area by building a mathematical model which presents the change of the air spring pressure with the effective area. Also, they considered the flow restriction between the auxiliary volume and the air spring by including the flow rate within the pipe. They were able to achieve various resonant frequencies and retune the air spring by changing the diameter of the tube connecting the air spring to the auxiliary chamber which provides a resistance to the flow of the air between the two volumes.

Shimozawa, Tohtake [27] studied an air spring system for railway vehicles consisting of a diaphragm, an orifice, an additional spring and an auxiliary air reservoir. The diaphragm and the auxiliary air reservoir were connected by the orifice. The study examined a non-linear model for the vertical motion focusing on two basic characteristics of the air spring, i.e., the compressibility of the air and the damping characteristic related to the pressure loss in the air passing thru the orifice. The study found that at large displacements, it is necessary to consider the non-linearity of the pressure-receiving area because there is a difference between the measurement and prediction based on linear approximation. Moreover, the volume rate change does not depend on the air spring initial pressure and the displacement and it is nearly equal to the effective area change.
On the other hand at low frequencies, the dynamic stiffness is nearly equal to the stiffness without the orifice but is larger than the static stiffness. It was found that the damping characteristic of the air spring was neither viscous (linear) nor quadratic but proportional to velocity.

S. J. Lee [28] developed a general analytic model of an air spring connected to a model of pneumatic systems designed to control air spring height. The model represented the main characteristics of the air spring’s stiffness. The mathematical model was established on the basis of thermodynamics with the assumptions that the thermodynamic parameters do not vary with the height of the air spring. The model showed that the stiffness is affected by the volume variation, the heat transfer, the variation of the air mass, and the effective area. In particular, it was revealed that the increase of the volume due to the variation in cross-sectional area increases the stiffness, while the increase of the volume due to the other reason decreases it.
Lee, et al., [29] studied the deformation of diaphragm-type air springs which consisted of nylon fiber-reinforced rubber, composite rubber linings and steel wire bead parts. The analysis was carried out with a finite element code developed to consider the orthotropic properties of fiber-reinforced rubber lamina, geometric nonlinearity by large deformation, and contact between an air bag and a bead ring. It was noted that an air bag had different modes of deformation depending on the cord angle and could have different modes of change in the outer diameter and the fold height with deformation.

Fengxiang, et al., [30] confirmed the feasibility FEA simulation of air springs. The Simulation of static test of their model displayed that initial pressure, cord angle, and auxiliary chamber volume play an important role in air spring’s static vertical mechanical performance. However, only initial pressure and cord angle revealed visible effect on its static transverse mechanical performance. Dynamic simulation claimed that dynamic stiffness of air springs was quite sensitive to the load frequency.

2.4 Servovalve

Servovalve is a control directional valve directing the flow of a fluid to different parts of a fluidic actuator (e.g. a cylinder). Assumption of ‘isothermal’ or ‘adiabatic’ for the analysis of air in closed pneumatic systems is an issue in pneumatic processes modeling [31]. The fundamental equations of both models are similar; however, the adiabatic process has a process-dependent constant of 1.4 while in the isothermal process the constant is 1. Some researchers recommended using the adiabatic process in the analyses after they become dis-satisfied using the isothermal assumption first in their model [31]. Kawakami, et al. [31], investigated the differences between isothermal and
adiabatic processes in a closed pneumatic system. Their research concluded that there are no significant differences between the two assumptions. They also, found out that linear models are poor predictors of the actual performance of a pneumatic system. A detailed nonlinear model of the dynamics of a proportional valve in was presented in Vaughan and Gamble [32], were the open-loop behavior of the valve was predicted accurately.

Three flow models are in the literature characterizing the behavior of a servovalve: the National Fluid Power Association (NFPA) model which approximates flow thru an orifice [33], a similar approximation presented by Esposito [34], and a model proposed by the International Society of Automation (ISA) [35]. The models are almost similar to each other in a number of ways. In each, flow is proportional to the supply pressure and a flow coefficient ($C_V$). Each model also contains two regions for valve flow. In the subsonic flow region, the flow rate increases as the ratio of downstream pressure to upstream pressure decreases. In the choked flow region, the flow through the valve is sonic and does not increase as the downstream pressure drops. In the ISA model, this critical pressure ratio is determined by a valve design parameter $X_T$. In the other models, the critical pressure ratio is calculated from the ratio of specific heats, and is found to be 0.528 for air. Experimental data suggest that the assumption of constant flow in the choked flow region is not valid.

2.5 Summary of Literature Review

Passive, active, and semi-active vibration control technique have been briefly reviewed in the earlier sections. Significant research in the area of air springs, as well as passive and semi-active TMD systems has been carried out either numerically or
experimentally by a number of researchers but only in rare instances have the researchers investigated the use of air springs in a TMD. The majority of air spring related previous research is concentrated on the use of air springs in isolation or vehicle suspension applications. Plenty of research have been conducted on the application of semi-active control strategies to vibration dampening applications, but relatively little research has focused on semi-active control of vibration via change in stiffness. The study presented in this work explores the possibility, suitability and effectiveness of using a novel semi-active control strategy for tuned damping of structural vibration using a tuned mass damper with air spring as its restoring element.
CHAPTER III

MATHEMATICAL MODEL

The mathematical model of passive TMD is described in Section 1 of this chapter. Then, the mathematical model of the pendulum tune mass damper (PTMD) is derived using Lagrange equations. The air spring dynamic model is described and derived using the ideal gas law in Section 3. The air spring is the restoring element in the proposed TMD. Section 4 examines the proportional valve using the three mathematical models, discussed in Chapter 2 to find the most suitable one for modeling the flow thru the proportional valve used in this research.

3.1 Tune Mass Damper

The spring-mass-damper system shown in Figure 3.1 is frequently used in the literature as representation of a tuned mass damper system appended to a one degree of freedom structure. The goal of the TMD, as described earlier, is to dissipate the vibrational energy of the structure. In order to achieve that goal the TMD needs to be tuned to the resonant frequency of the vibrating structure. The system mathematical model are derived using the equations of motion of the system and presented in Equation (3.1) and (3.2)
Figure 3.1: The Schematic of TMD Applied to a Structure

\[ m_1 \ddot{x}_1 + (c_1 + c_2) \dot{x}_1 + (k_1 + k_2)x_1 = \dot{x}_2 c_2 + x_2 k_2 + f(t) \]  \hspace{1cm} (3.1)

\[ m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 = \dot{x}_1 c_1 + x_1 k_1 \]  \hspace{1cm} (3.2)

where

\( x_1, \dot{x}_1, \text{ and } \ddot{x}_1 \) are the vibrating structure’s displacement, velocity, and acceleration,

\( x_2, \dot{x}_2, \text{ and } \ddots \) are the TMD’s displacement, velocity, and acceleration,

\( m_1, k_1, \text{ and } c_1 \) are the mass, stiffness, and damping coefficient of the structure,

and

\( m_2, k_2, \text{ and } c_2 \) are the mass, stiffness, and damping coefficient of the TMD.
When designing a TMD four variables need to be determined; the natural frequency of the vibrating structure $\omega_{ns}$, the ratio of TMD natural frequency to structure natural frequency known as the frequency ratio $f_{on}$, the ratio of TMD mass over structure mass known as mass ratio $\mu$, and the damping ratio of the TMD $\zeta$. The mass ratio is preferred to be within the range of 1/50 to 1/10 of the structure mass; the larges the mass ratio the more effective the TMD.

The natural frequency of the structure is related directly to its stiffness and mass

$$\omega_{ns} = \sqrt{\frac{k_s}{m_s}}$$  \hspace{1cm} (3.3)

The natural frequency of the TMD is

$$\omega_{nt} = \sqrt{\frac{k_t}{m_t}}$$  \hspace{1cm} (3.4)

The mass of the TMD as a percentage of the structure mass, i.e., mass ratio is

$$\mu = \frac{m_t}{m_s}$$  \hspace{1cm} (3.5)

The damping ratio of the structure which describes the damping coefficient scaled by the critical damping coefficient is

$$\zeta_s = \frac{c_s}{2\omega_{ns}m_s}$$  \hspace{1cm} (3.6)

The damping ratio of the TMD is

$$\zeta_t = \frac{c_t}{2\omega_{nt}m_t}$$  \hspace{1cm} (3.7)

The frequency ratio is
\[ f_{on} = \frac{\omega_t}{\omega_{ns}} \] (3.8)

In the special case of, both the TMD and the structure having no damping (\(\zeta=0\)). The TMD is called “tuned vibration absorber” and is traditionally used for abating the forced vibration of the structure at a non-varying forcing frequency.

Den Hartog [6] studied the case where the TMD is used to add damping to an undamped structure and proposed the optimal tuning frequency ratio and optimal damping ratio \(f_{opti}\) and \(\zeta_{opti}\), presented by Equations (3.9) and (3.10)

\[ f_{opti} = \frac{\omega_t}{\omega_{ns}} = \frac{1}{1 + \mu} \] (3.9)

\[ \zeta_{opti} = \frac{c_t}{2\omega_t m_t} = \frac{3\mu}{2\sqrt{8(1 + \mu)^3}} \] (3.10)

3.2 Pendulum Tune Mass Damper

This section presents the system of equations for pendulum tuned mass damper (PTMD) shown in Figure 3.2. Following the formulation of the equations of motion of a PTMD equipped with the more traditional spring and damper system, the modeling effort will be extended to an air spring suspended semi-active PTMD with adjustable stiffness and damping.

A two-degrees-of-freedom (2DOF) model is used to present the dynamic behavior of the structure coupled with a PTMD as shown in Figure 3.2. The hinge of the beam is the origin of the coordinate system. All the parameters in the equation of motion could vary with time with the exception of the beam length and the spring location.
Figure 3.2: PTMD Configuration

Figure 3.3: A Structure Coupled with a Pendulum Tune Mass Damper (PTMD)
The parameters of the 2 DOF system shown in Figure 3.3 are listed below:

\[ \theta, \dot{\theta}, \text{ and } \ddot{\theta} \text{ are the PTMD angular displacement, angular velocity, and angular acceleration.} \]

\[ L \text{ is the length of the PTMD beam from the hinge to the center of the mass.} \]

\[ L_2 \text{ is the distance from the air spring to the PTMD hinge.} \]

The Lagrange formulation of Equation (3.11) is used to find the equations of motion for this coupled structure and PTMD

\[
\frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = f \tag{3.11}
\]

where \( T \) and \( U \) are the kinetic and potential energy of the system and \( q_i \) are the general coordinates of \( x_1 \) and \( \theta \).

Equation (3.12) presents the kinetic energy (KE) of the structure coupled with the PTMD

\[
T = KE = \frac{1}{2} m_1 \dot{x}_1 + \frac{1}{2} I \left( \frac{\dot{x}_1 + L \dot{\theta}}{L} \right)^2 \tag{3.12}
\]

where \( I \) is the mass moment of inertia of the TMD presented by Equation (3.13)

\[
I = (m_{tmd} + \frac{1}{3} m_{beam}) L^2 = m_2 L^2 \tag{3.13}
\]

The potential energy (PE) of the structure coupled with PTMD is described by Equation (3.14)

\[
U = PE = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 L^2 \theta^2 \tag{3.14}
\]
Substituting the KE and PE of Equation (3.12) and (3.14) into the Lagrange formulation of Equation (3.11) results in Equations (3.15) and (3.16) describing the motion of the system.

\[(m_1 + m_2) \ddot{x}_1 + m_2 L \ddot{\theta} + k_1 x_1 = f - c_1 \dot{x}_1 \quad (3.15)\]

\[m_2 L^2 \dddot{\theta} + m_2 L \dddot{x}_1 + k_2 L^2 \theta = -c_2 L \dddot{\theta} \quad (3.16)\]

Evident from Equation (3.16) rearranged in terms of \(\theta\), i.e.

\[\dddot{\theta} + \frac{c_2 L^2}{m_2 L^2} \dddot{\theta} + \frac{k_2 L^2}{m_2 L^2} \theta = 0 \quad (3.17)\]

the natural frequency of the PTMD is

\[\omega_{nPTMD} = \frac{L_2}{L} \sqrt{\frac{k_{PTMD}}{m_{PTMD}}} \quad (3.18)\]

Moreover, the damping ratio of the PTMD is

\[\zeta_{PTMD} = \left(\frac{L_2}{L}\right)^2 \frac{c_{PTMD}}{2 \omega_{nPTMD} m_{PTMD}} \quad (3.19)\]

A close inspection of the natural frequency of both TMD and PTMD shows that, the natural frequency of the PTMD is proportional to the length ratio of the arm. In other words, to make the PTMD to have a similar frequency as the TMD when the same spring is used, the PTMD’s mass should be changed by a factor of \(\left(\frac{L_2}{L}\right)^2\) of the TMD’s mass.

When the same spring and mass is used in both PTMD and TMD, the natural frequency of the PTMD will decrease by \(\left(\frac{L_2}{L}\right)\) when \(L_2\) is less than \(L\), while the PTMD’s natural frequency is going to increase by \(\left(\frac{L_2}{L}\right)\) when \(L_2\) is greater than \(L\).
3.3 Air Spring

As mentioned earlier, air spring is used to provide stiffness and damping to the proposed PTMD. Air spring is modeled using conservation laws. A single convoluted Firestone air spring shown in Figure 3.4 was used in this study. An opening placed at the center of the lower bead plate allows the air to flow in and out of the air spring. The flow of air in and out of the air spring is controlled by a servovalve.

![Figure 3.4: Firestone Style 115 Air Spring [23]](image)

In this analysis, air is assumed to behave as an ideal gas undergoing an adiabatic process. Derivation of the equation governing the time rate of change of the pressure inside the air spring requires a brief review of thermodynamics including ideal gas law. The ideal gas law is commonly expressed as

\[ PV = mRT \]  \hspace{1cm} (3.20)

Alternately, the ideal gas law may be expressed in terms of density at the standard condition.

\[ P = \rho RT \quad \text{or} \quad \frac{P}{R} = \frac{m}{v} T = \rho T \]  \hspace{1cm} (3.21)

The universal gas constant \( R \) is related to the heat capacity of the gas at constant volume \( c_v \) and constant pressure \( c_p \); see Equation (3.22). The heat capacity ratio \( k \), which equals 1.4 for air, is shown in Equation (3.23).
\[ R = c_p - c_v \]  \hspace{1cm} (3.22)

\[ k = \frac{c_p}{c_v} \]  \hspace{1cm} (3.23)

The space inside the air spring is treated as a control volume \( V \). The air in the spring has an internal energy that is a function of the temperature, \( T \).

\[ E = c_v \rho VT \]  \hspace{1cm} (3.24)

In an adiabatic process, the time rate of change in the internal energy is equal to the rate of energy added to the control volume by the incoming air flow, minus the rate of work of the control volume performs on the air spring; see Equation (3.25)

\[ \frac{d}{dt} (c_v \rho VT) = c_p \dot{m} T - P \dot{V} \]  \hspace{1cm} (3.25)

Applying Equation (3.21) to (3.25) yields,

\[ \frac{d}{dt} \left( \frac{c_v}{R} PV \right) = c_p \dot{m} T - P \dot{V} \]  \hspace{1cm} (3.26)

Rearranging Equation (3.26) results in Equation (3.27)

\[ \frac{c_v}{c_p RT} \frac{d}{dt} (PV) + \frac{P \dot{V}}{c_p T} = \dot{m} \]  \hspace{1cm} (3.27)

Expanding the derivative of \( (P \dot{V}) \), with Equation (3.23) being applied, leads to Equation (3.28)

\[ \frac{1}{kRT} (\dot{P} V) + \left( \frac{1}{kRT} + \frac{1}{c_p T} \right) (P \dot{V}) = \dot{m} \]  \hspace{1cm} (3.28)

Rearranging Equation (3.28) in term of \( k, R, \) and \( T \) as shown in Equations (3.22) and (3.23) results in

\[ \frac{1}{kRT} (\dot{P} V) + \frac{1}{RT} (P \dot{V}) = \dot{m} \]  \hspace{1cm} (3.29)
Expressing Equation (3.29) in terms of $\dot{p}$

$$\dot{p} = \left(\frac{kRT}{V}\right)m - \left(\frac{kP}{V}\right)\dot{V}$$

(3.30)

Equation (3.30) is the general form for the time rate of change of pressure in the air spring. The mass flow rate $m$ is a nonlinear function of the supply and air spring pressures.

Replacing $m$ (with $\rho q$ where $q$ is the volumetric flow of the air thru the servovalve in and out the air spring) and pressure $P$ from Equation (3.21) in Equation (3.30) results in Equation (3.31)

$$\dot{p} = \left(\frac{kRT\rho}{V}\right)q - \left(\frac{kRT\rho}{V}\right)\dot{V}$$

(3.31)

Equation (3.31) shows the rate of change in dynamic pressure of the air inside the air spring as a function of air flow rate and the rate of change of the volume of air enclosed in the air spring. The sign of the flow is positive when the air flows into the air spring and negative when air leaves the air spring.

Using the definition of speed of sound $c = \sqrt{kRT}$, allows for the rearrangement of Equation (3.31) as

$$\dot{p} = \frac{\rho c^2}{V} (q - \dot{V})$$

(3.32)

Note that contrary to the air in a pneumatic cylinder experiencing large changes in volume, the change in the volume of the air inside the air spring is relatively small; note that air springs operate at a nearly constant static pressure with a relatively small oscillation in their pressure. Although the variation in air volume inside the air spring
might be small, its derivative might not be. Thus considering the air volume as a constant, Equation (3.32) can be rearranged in integral form as

$$P = \frac{c^2}{V} \int \rho q \, dt - \frac{\bar{\rho} c^2}{V} \int \dot{V} \, dt = \frac{c^2}{V} \int \dot{m} \, dt - \frac{1}{\bar{c}} \int \dot{V} \, dt$$  \hspace{1cm} (3.33)

where $\bar{c} = \frac{V}{\bar{\rho} c^2}$ is the average compliance and $\bar{\rho}$ is the average density of the air enclosed in the air spring; $\frac{1}{\bar{c}}$ can be viewed as the volumetric stiffness of the air spring relating the change in volume of enclosed air to change in its pressure. The pressure can be varied by either adjusting the compliance of the air spring or manipulating the flow of air in and out of the air spring. These mechanisms are the basis for semi-active control of the air-suspended PTMD.

As stated earlier, the inverse of compliance is viewed as the fluidic stiffness of the enclosed air presented by Equation (3.34)

$$k_2 = \frac{1}{\bar{c}} = \frac{\rho c^2}{V} ;$$  \hspace{1cm} (3.34)

The volume of the convoluted air spring used in this model does not have a linear relation with neither the area nor the height of the air spring. Thus the manufacturer’s data of the air spring are used to relate the volume of the air spring to its height.

### 3.3.1 Geometric and Effective Areas

There are two areas in the model of convoluted air spring: one is referred to by the geometric area and the other by the effective area. The geometric area is the actual cross sectional area of the air spring governed by the diameter of the air spring bellow. The effective area, on the other hand, is an imaginary area relating the air spring pressure to
the load (force) it supports. The geometric area of the bellow varies since it depends on the volume and their height of the air spring. The effective area has been assumed constant by some researchers to make the model more straight forward; see [26]. However, the modeling effort is better served not making such assumption and calculating the instantaneous effective area using Equation (2.1) by measuring the instantaneous load and the corresponding pressure dividing them by each other.

3.4 Servovalve

In the semi-active PTMD a servovalve is used to vary the amount of air flowing in and out of the air spring. The mathematical model for this servovalve is needed as a component of the simulation model of the PTMD system.

In addition to the flow coefficient, the flow rate thru a servovalve depends on a number of variables; including supply pressure, air spring pressure as well as the temperature of the air. Three existing, commonly used, mathematical models characterizing the flow coefficient of a valve are examined to find the most suitable one for our application. These models are Esposito [34], National Fluid Power Association (NFPA) [33], and International Society of Automation (ISA) [35]. These models shown in Equations (3.35), (3.36), and (3.38) calculate the volumetric flow rate in standard cubic feet per minute.

The Esposito model:
The NFBA model:

\[
Q_{SCFM} = \begin{cases} 
22.67 \times C_v \sqrt{\frac{(P_{UP} - P_{DN})P_{DN}}{T}} ; & \left(\frac{P_{DN}}{P_{UP}} \right) > \left(\frac{P_{DN}}{P_{UP}}\right)_{crit} \\
11.32 \times C_v \frac{P_{UP}}{\sqrt{T}} ; & \left(\frac{P_{DN}}{P_{UP}}\right) \leq \left(\frac{P_{DN}}{P_{UP}}\right)_{crit}
\end{cases}
\]  

(3.35)

where the critical pressure drop ratio for Esposito and NFPA models is

\[
\left(\frac{P_{DN}}{P_{UP}}\right)_{crit} = \left(\frac{2}{k + 1}\right)^{k/k-1}
\]  

(3.37)

The ISA model:

\[
Q_{SCFM} = \begin{cases} 
22.48 \times C_v \sqrt{\frac{(P_{UP} - P_{DN})P_{DN}}{T}} ; & \left(\frac{P_{DN}}{P_{UP}}\right) > \left(\frac{P_{DN}}{P_{UP}}\right)_{crit} \\
11.22 \times C_v \frac{P_{UP}}{\sqrt{T}} ; & \left(\frac{P_{DN}}{P_{UP}}\right) \leq \left(\frac{P_{DN}}{P_{UP}}\right)_{crit}
\end{cases}
\]  

(3.36)

where

\[
X = \frac{P_{UP} - P_{DN}}{P_{UP}} = 1 - \frac{P_{DN}}{P_{UP}}
\]  

(3.39)

\(P_{UP}\) is upstream pressure, \(P_{DN}\) is the downstream pressure, \(X\) is the pressure ratio, \(X_T\) is the critical pressure drop ratio for ISA model, and \(Q_{SCFM}\) is the volumetric flow rate.

All these models determine the flow rate as a function of critical pressure ratio \(\left(\frac{P_{DN}}{P_{UP}}\right)_{crit}\), defined as the ratio of the downstream pressure to upstream pressure.
when the flow thru the valve is choked. However, the method of calculating the critical pressure ratio in Esposito and NFPA models are different than that of ISA. The critical pressure ratio in Esposito and NFPA model is calculated based on Equation (3.37). In ISA model critical pressure ratio is calculated based on Equation (3.39). Note that, the flow of the air through the orifice is unchoked or subsonic when the flow corresponds to a pressure ratio smaller than the critical pressure ratio. The flow through the orifice is sonic or choked, i.e. decreasing the downstream pressure or increasing the upstream pressure does not affect the flow rate, when it corresponds to a pressure ratio equal or more than the critical pressure ratio.

With the specific heat ratio $k=1.4$, the critical pressure ratio for air is 0.528 for Esposito and NFPA models indicating that the air flow rate will not change when the ratio of the downstream pressure to upstream pressure is less than 0.528. This ratio can be used as a guide as to which part of the equation in either Esposito or NFPA equations should be used.

The ISA model shown in Equation (3.38) is based on the ASTM standard reference number S75.01. It differs from the previous two models since its critical pressure ratio is based on the geometry of the valve itself. Moreover, the ISA model shows that two valves with identical flow coefficients can exhibit different flow rates under identical pressure conditions. In this model the critical pressure ratio is not only depends on the ratio of upstream pressure to downstream pressure but also on the critical pressure drop ratio $X_T$. To find the critical pressure drop ratio $X_T$ a series of experimental tests are requires [35].
In the Esposito and NFPA equations, choked flow occurs when the ratio of downstream to upstream pressures drops below the critical ratio predicted by Equation (3.37). The critical ratio for ISA equation, on the other hand, varies according to the valve design. This kind of performance may be tribute to the flow thru a series of orifices in line with each other. In the case of steady flow conditions, it is obvious from the principle of the conservation of mass that the flow rate thru each orifice is identical [37]. For example, the pressure drop across each orifice is not fundamentally equal. It is possible for flow thru one orifice to be choked, while instantaneously unchoked in another. In other words, if a valve is considered not as a single orifice, but as a set of orifice-like obstructions to flow, then it is logical to argue that the choked-flow pressure ratio of the valve, as a whole, is not necessarily 0.528 [37].

The definition of the maximum flow rate of a valve by ISA is “the flow rate at which, for a given upstream pressure, a decrease in downstream pressure will not produce an increase in flow rate” [35]. The maximum flow rate of the valve needs to be measured in order to find the value of $X_T$. The conduct of such test for a valve with a large flow capacity requires a most amount of compressed air.

As will be discussed in Chapter 4, the ISA model turned out to be the most suitable for predicting the flow attributes of the servovalve used in this work.
CHAPTER IV

EXPERIMENTAL SET UP

This chapter discusses the characteristics of the major components, mainly the air spring and the servo valve, used in the making of the semi-active PTMD discussed in this dissertation. Understanding their behaviors and dynamics at the component level is necessary in building an accurate system model for the semi-active tuned mass damper.

4.1 Air Spring

A Firestone style 115 convoluted air spring shown in Figure 4.1 was used in this study. Convoluted air springs are fabric-reinforced rubber bellows sealed by two bead plates on the bottom and top part of their surrounded rubber. The diameter of the lower and upper bead plate is 6.3 inches while the diameter of the air spring bellow is 10.1 inches when pressurized at 100 psig.
One of the most important attributes of a convoluted air spring is its effective area which is the area that when multiplied by the pressure results in the air spring load (force).

At a constant load the pressure and the effective area are proportional, Equation (2.1) describe the relation between the pressure and the effective area. Table 4.1 and Table 4.2 show the force and pressure at different air spring’s height. This Firestone air information is used to calculate the air spring effective area. The effective area for different corresponding heights and pressures for the Firestone air spring used in this research are presented in Table 4.3.

Table 4.1: Applied Forces and Pressure under Different Firestone Style 115 Air Spring Height [23]
Table 4.2: Dynamic Characteristics of Firestone Style 115 Air Spring at 4.5 in. Design Height [23]

<table>
<thead>
<tr>
<th>Gage Pressure (PSIG)</th>
<th>Load (lbs.)</th>
<th>Spring Rate (lbs./in.)</th>
<th>Natural Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>1.210</td>
<td>900</td>
<td>162</td>
</tr>
<tr>
<td>60</td>
<td>1.890</td>
<td>1.264</td>
<td>154</td>
</tr>
<tr>
<td>80</td>
<td>2.540</td>
<td>1.638</td>
<td>151</td>
</tr>
<tr>
<td>100</td>
<td>3.270</td>
<td>2.027</td>
<td>148</td>
</tr>
</tbody>
</table>

Table 4.3: Firestone Style 115 Air Spring’s Effective Area (in²)

<table>
<thead>
<tr>
<th>Pressure (psi)</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (in)</td>
<td>3</td>
<td>4</td>
<td>4.5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>41</td>
<td>43</td>
<td>43.83</td>
<td>43.88</td>
<td>44.4</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>35.75</td>
<td>36.33</td>
<td>36.75</td>
<td>37.5</td>
</tr>
<tr>
<td>4.5</td>
<td>29.5</td>
<td>30.25</td>
<td>31.33</td>
<td>31.75</td>
<td>32.7</td>
</tr>
<tr>
<td>5</td>
<td>23.5</td>
<td>24</td>
<td>25.33</td>
<td>26</td>
<td>26.9</td>
</tr>
</tbody>
</table>

Figure 4.2 presents five effective areas as a function of pressure and air spring height. Figure 4.2 shows that the effective area increases when the air spring pressure increases at a fixed height. Moreover, when the pressure is fixed the effective area increases when the air spring height decreases. The volume of the air spring is 192 in³ at the recommended height of 4.5 inches; see Table 4.2. Table 4.1 shows the static volume of the air spring at 3, 4, and 5 inches. The volume of the air spring at other heights can be obtained by interpolating the information in Table 4.1. Using the information presented in Figure 4.2 the instantaneous effective area of the air spring can be obtained using 2D linear interpolation of the air spring pressure and height.
4.2 Proportional Servovalve

A Norgren VP60 proportional valve controls the flow of air in and out of the air spring used in this study. The valve comes with internal electronics that provides closed-loop control of the spool position. The valve accepts 0-10 volt control signal. Table 4.4 lists the significant characteristics of the valve as published by the manufacturer. Figure 4.3 is the pneumatic symbol of the Norgren VP60 valve, showing the ports and their connections. Figure 4.4 shows the published flow rate, in normalized liters per minutes “NL/min”, through the valve as a function of upstream pressure (P1) and setpoint value. The setpoint of the valve is adjusted by setting the upper and lower level of command voltage signal; in this research the setpoint is set at 0 volts. Thus, at 100% of the setpoint the command signal will be ±5 volts depending on the desired flow direction and at 0%
the signal will be 0 volt. Note that Figure 4.4 is separated into two regions: one for flow from port 1 to port 2; and the other for flow from port 1 to port 4.

Figure 4.3: Norgren VP60 Proportional Servovalve Pneumatic Symbol[36]

Table 4.4: Norgren VP60 Proportional Servovalve Properties

<table>
<thead>
<tr>
<th>Model Number</th>
<th>VP6010L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>5/3 proportional directional control valve</td>
</tr>
<tr>
<td>Input Voltage</td>
<td>±5.0..10V</td>
</tr>
<tr>
<td>Max flow for</td>
<td>1200 Nl/min</td>
</tr>
</tbody>
</table>

P1=6bar, P2, P4=5bar

Figure 4.4 Norgren VP60 Proportional Servovalve Flow Rate as a Function of Setpoint Value and P1 [36]
As indicated in Chapter 3 to build an accurate mathematical model for semi-active PTMD it is essential to have an accurate model for the flow through the Norgren VP60 proportional valve. There are many variables such as upstream pressure, downstream pressure, and temperature affects the flow rate.

Figure 4.5 compares the three flow models Esposito Equation (3.35), NFPA Equation (3.36), and ISA Equation (3.38), discussed in Chapter 3 using five different values for \(X_T\) at the supply pressure of 50 psi and room temperature of 20°C (528°F). The value of 1.4 is used for the flow coefficient \(C_V\) for all this three flow models. Figure 4.5 shows that the prediction of flow rate by Esposito model is close to that of the NFPA model, which was expected considering the only difference between the two is less than 1% variation in the constant coefficient of their equation.

The measurements of the flow rate through the Norgren VP60 valve demonstrate the Esposito flow and NFPA equations are not accurate for this valve. The use of NFPA model or Esposito model would not help to find an accurate model for the valve since the valve does not choke around the critical pressure. The experimental data in Figures 4.8 does not show the choked-flow region predicted by either of these models. A visual comparison of Figure 4.8 with Figure 4.5 supports the conclusion that neither NFPA model nor Esposito model cannot accurately predict flow thru the Norgren VP60 proportional valve. Furthermore, Figure 4.5 also demonstrates how the value of \(X_T\) affects the flow rate using ISA model.
Figure 4.6 shows the test setup for measuring the flow coefficient of the Norgren VP60 valve [37]. Two tests were conducted with two different tube sizes. In the first test, tubes with 8 mm outside diameter (1/4” tubing) were used to do the plumbing while in the second test tube with 10 mm outside diameter (5/16” tubing) were used. Two pressure sensors were installed about ten inches away from the valve to measure the pressure drop across the valve. The flow rate of the air spring thru the valve was measured by a rotameter downstream of the valve exhausting the air. A fixed signal was sent to the valve to adjust the spool position. The rotameter was kept fully open to read the maximum flow rate establishing the critical pressure ratio corresponding to that signal.
To measure the pressure drop across the valve, the air flow needs to be modified by adjusting the opening of the rotameter. Depending on the extent of flow, two rotameters with different capacities were used: one with a capacity of 200 to 1800 SCFH, and the other with a capacity of 50 to 400 SCFH.

The flow rate data collected from the first test through the Norgren VP60 valve using 1/4” tubing is presented in Figure 4.7. This data shows that the flow does not reach its maximum value published by the valve manufacturer. However, the data shows that the critical pressure ratio is around 0.5. Multiple curves on Figures 4.7 show the flow rate thru the servovalve for various pressure drops, as function of voltage. This family of plots shows that the air flow does not change when the signal to the valve is less than 3.75 volts corresponding to 25% valve opening\(^3\). This means that the air flow gets choked at that signal level and any further increase of the valve opening by decreasing the voltage level does not increase the flow rate. The choking of flow at lower values of opening pointed to the tubing being too small.

\(^3\)“At the command signal of 5V, the valve is completely closed, while 0V the valve is fully opened”.
Figure 4.7: Norgren VP60 Proportional Servovalve Flow Rate as a Function of Voltage for Various Pressure Drop, using 1/4” Tubing

The tubing was changed to 5/16” and the flow measured thru the servovalve repeated. The volumetric flow rate using a supply pressure of 50 psig for the flow from port 1 to port 4, as a function of the command voltage to the Norgren VP60 proportional valve is shown in Figure 4.8. A drop of the pressure at the regulator was noticed as the flow rate increased. The reason for this drop was the head loss in the length of tubing connecting the regulator to the supply air. A correction has been made for the pressure to correct the flow rate in Figure 4.8. The correction is made by multiplication of the ratio of the pressure at no flow to the measured supply pressure under flow; shown by Equation (4.1).

\[ Q_{corr} = \left( \frac{P_{noFlow}}{P_{Meas}} \right) Q_{Meas} \]  

(4.1)
The uncorrected ratio of flow rate to supply pressure as a function of the pressure ratio is shown in Figure 4.9.

Fitting the ISA model to the measured flow data results in the values of the flow coefficient $C_V$ and the critical pressure drop ratio $X_T$, 0.24 and 0.9, respectively. Figure 4.10 compares the ISA model predicted as well as the measured flow, using a supply pressure of 50 psig and a temperature of 528°R for the corrected data set corresponding to 4 V command signal. The predicted and measured data, shown in Figure 4.10 comparison, favorably indicating the suitability of ISA equation in modeling the flow attributes of Norgren VP60 Proportional Servovalve. These parameters ($C_V$ and $X_T$) are evaluated by minimizing the root mean square error “RMS” between the measured data and the model predicted data.

Figure 4.8: Norgren VP60 Proportional Servovalve Volumetric Flow Rate for Port 1 to 4 as a Function of Pressure Ratio
The values of the flow coefficient $C_V$ and the critical ratio $X_T$ are evaluated for the other command voltages by using the same method of minimizing the root mean
square error “RMS”. At each command voltage tested, the best-fit value for the critical ratio $X_T$ was found to be 0.9. The flow coefficient as a function of the command voltage using a critical ratio $X_T$ of 0.9 is presented in Table 4.5 and Figure 4.11.

The ISA equation may be used to calculate the maximum theoretical flow rate through the valve. The limits of the valve’s capacity are 16 bars to vacuum in pressure, and the flow rate of 4900 NL/min at full scale voltage command. Since the maximum supply pressure available in the lab was 6 bars, the experiments could not be conducted at the rated flow. In addition to the supply pressure, the supply flow was also limited. In other words, the free flow thru the valve could not be maintained at any pressure difference higher than 50psi (3.3 bars).

<table>
<thead>
<tr>
<th>Volt</th>
<th>$C_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.41</td>
</tr>
<tr>
<td>4</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>0.38</td>
</tr>
<tr>
<td>2</td>
<td>0.34</td>
</tr>
<tr>
<td>1</td>
<td>0.24</td>
</tr>
<tr>
<td>0.75</td>
<td>0.17</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0997</td>
</tr>
<tr>
<td>0.3</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Figure 4.11: Norgren VP60 Proportional Servovalve Flow Coefficient as a Function of Command Voltage (1 → 4) (a) and Coefficient as a Function of Command Voltage (b)
Figure 4.12 depicts the flow of the valve as function of the setpoint and pressure ratio. The graph shows the percentage of the flow rate for a given signal level for a certain pressure ratio. For example for the valve spool signal of 3.0 volts and the pressure ratio is 0.5, the flow rate would be 50% of the maximum flow rate (corresponding to minimum pressure ratio and maximum spool voltage).

Figure 4.12: Norgren VP60 Proportional Servovalve Flow Rate Ratio as Function of Setpoint and Pressure Ratio [36]
CHAPTER V

SIMULATION STUDIES

The component models discussed in chapter 4 are used in the makeup of the system model discussed in chapter 3. This chapter discusses the development of system model of the semi-active PTMD.

5.1 PTMD Model

The schematic of semi-active air-suspended PTMD is shown in Figure 5.1. Air is flown in and out of the air spring via a pneumatic servo valve. The semi-active control scheme developed in this work changes the parameters, including the stiffness and damping, of the PTMD via velocity feedback and acceleration feedback respectively. In addition, feeding back the air pressure adjusts the damping, albeit to a lesser degree than the acceleration feedback does.

Depending on the feedback configuration and control algorithm, both stiffness and damping can be adjusted. The simulation model of the semi-active PTMD is done in Matlab Simulink environment.
The semi-active PTMD system with horizontal pendulum configuration, shown in Figure 5.1, is made up of an inertia element, one air spring, an I-beam, a servovalve, compressed air supply, and a controller. The I-beam is used to build up the arm of the pendulum which is hinged at one end to the PTMD frame using two ball bearings. This arrangement allows for only rotational motion and prevents the PTMD from having lateral motion. The mass which makes up the inertia element is installed at the free end of the I-beam. The air spring is positioned between the mass and the supporting hinge of the I-beam.
The controller adjusts the servovalve such that a) the static height of the air spring is maintained, and b) the stiffness and damping of the air spring is dynamically controlled. The sensory information to the feedback controller are provided by acceleration, displacement, and pressure sensors.

The nonlinear differential equations describing the dynamics of the PTMD coupled with a structure was derived in chapter 3. Figure 5.2 (a) shows the block diagram model of the PTMD coupled with a structure. The Simulink model of this system is shown in Figure 5.2(b). The components of this block diagram are detailed in the following sections.
Figure 5.2: The PTMD/Structure Model (a) and the PTMD Structure Simulink Model (b)
5.1.1 The Vibrating Structure Model

A 2nd order system presenting a single mode of the vibrating structure is shown in Figures 5.3. A perturbing (excitation) force and the PTMD rotational acceleration are the inputs to this model which is based on Equation (3.15) which presents the equation of motion of the structure. The excitation/perturbation force which could be caused by human activities, machine operation, wind, etc. is the exogenous input to and acceleration is the output of the structure model.

Figure 5.3: The Compact (a) and Expanded (b) Block Diagram Model of the Vibrating Structure
5.1.2 Pendulum Tune Mass Damper Model

The block diagram of the PTMD is shown in Figure 5.4. The inputs to this block are instantaneous dynamic pressure of the air spring, the torque input from the vibrating structure, and the instantaneous effective area of the air spring. The outputs of this block are the angular displacement, velocity, and acceleration of the pendulum.

![Diagram of PTMD](image)

(a)

![Expanded Block Diagram Model](image)

(b)

Figure 5.4: The Compact (a) and Expanded (b) Block Diagram Model of the PTMD
In Equation (3.16), the mechanical system of the PTMD is described using Lagrange equation; however, the equation did not include the moment created due the gravitational force exerted on the vibratory mass nor the mechanical effect of the air spring. Including the static force of the air spring with the effect of the inertia exerted by the weight into equation (3.16) results in

\[ m_2 L^2 \ddot{\theta} + c_2 L_2^2 \dot{\theta} + k_2 L_2^2 \theta = m_2 g L - P_s A_{\text{eff}} L_2 \]  

(5.1)

where \( g \) is the acceleration of gravity, \( P_s \) is the static pressure of the air spring, and \( A_{\text{eff}} \) is the effective area of the air spring.

The static pressure is selected to support the inertia of the I-beam with the TMD mass. Note that, statically the moment caused by the weight (\( m_2 g L \)) cancels the moment due the force resulting from the static pressure (\( P_s A_{\text{eff}} L_2 \)). The reason that the inertia and the torque of the air spring do not cancel each other (add up to zero) is that the effective area in the air springs, especially the ones with convoluted shape, is not constant and varies with the air spring’s height which varies with the motion of the mass.

To build the block diagram model of the PTMD block model shown in Figure 5.4, Equation (5.1) rearranged in terms of the PTMD angular acceleration, as shown in Equation (5.2),

\[ \ddot{\theta} = - \frac{c_2 L_2^2 \dot{\theta}}{m_2 L^2} - \frac{k_2 L_2^2 \theta}{m_2 L^2} + \frac{m_2 g L}{m_2 L^2} - \frac{P_s A_{\text{eff}} L_2}{m_2 L^2} \]  

(5.2)

where the viscous damping coefficient (\( c_2 \)) is assumed to represent the unknown inherent damping of the PTMD. Replacing the stiffness of the system (\( k_2 \)) with the air spring
dynamic pressure which provides the PTMD with the stiffness and the extra damping through the work performed on the PTMD, turns Equation (5.2) to Equation (5.3).

\[
\ddot{\theta} = -\frac{c_2 L_2^2 \dot{\theta}}{m_2 L^2} - \frac{P_d A_{eff} L_2}{m_2 L^2} + \frac{m_2 g}{m_2 L} - \frac{P_s A_{eff} L_2}{m_2 L^2}
\]  

Equation (5.3) indicates that the dynamic pressure with effective area is the variables of the air spring which provides the PTMD with dynamic attributes of stiffness and the damping. Note that the linearized stiffness of the air spring is the derivative of force relationship with respect to the displacement as shown in Equation (5.4).

\[
K_2 = \frac{dF}{dx}
\]  

By replacing the force of the air spring in Equation (5.4) with Equation (2.1) and taking the derivative, the stiffness is formulated as in Equation (5.5)

\[
K_2 = \frac{A_{eff} dP_d}{L_2 d\theta} + \frac{P_d dA_{eff}}{L_2 d\theta}
\]

showing the impact of dynamic pressure and effective area on the stiffness.

5.1.3 Air Spring Model

The air spring block diagram model is shown in Figure 5.5. The inputs to this block are instantaneous air spring height and the controlled air flow (q) and the outputs of this block are the instantaneous effective area, dynamic pressure, and air spring pressure.
Figure 5.5: The Compact (a) and Expanded (b) Block Diagram Model of the Firestone Air Spring

The total pressure of the air spring is the dynamic pressure added to the static pressure as shown in Equation (5.6).

\[ P = P_s + P_d \]  \hspace{1cm} (5.6)

60
The static pressure is the recommended pressure of the air spring for the specified load and height which can be found using the technical information of the air spring (see chapter 4). The dynamic pressure of the air spring can be calculated using Equation (3.32) which represents the time rate of change of dynamic pressure that consists of two major variables, one which represent the volumetric flow rate \( q \) adjusted by the servovalve and the other is the work performed resulting in the rate of change of volume \( \dot{V} \). The density in Equation (3.32) depends on the pressure of the air spring, as shown in Equation (5.7)

\[
\rho = \left(\frac{P_{ATM} + P}{P_{ATM}}\right) \rho_{ATM}
\]  

(5.7)

Considering the isothermal nature of the process, the speed of sound \( c \) in Equation (3.32) is considered constant.

Static volume of the air spring \( V_0 \) as the function of height is provided by the manufacturer. The dynamic volume for a small displacement variation can be found by multiplying the displacement variation by the geometric area. The total volume of the air spring \( V \) is the sum of the static volume \( V_0 \) and the dynamic volume \( XA_{geo} \) as shown in Equation (5.8)

\[
V = V_0 + XA_{geo}
\]  

(5.8)

where \( X \) is the net relative displacement of the air spring measured from the static height, and \( A_{geo} \) is the geometric area of the air spring at 100 psi. The rate of change of volume of the air spring is found by taking the derivative of Equation (5.8) with respect to time.

\[
V = XA_{geo} = (X_t - X_0)A_{geo}
\]  

(5.9)
5.1.4 Controller

The Controller of Figure 5.2(b) is shown in detailed in Figure 5.6. The inputs to this controller are the dynamic pressure, the angular displacement, velocity, and acceleration which can be presented by the conventional state variables $P_d, \theta, \dot{\theta},$ and $\ddot{\theta}$, respectively. The output of the controller is the command signal driving the servovalve.

![Controller Diagram]

Figure 5.6: The Compact (a) and Expanded (b) Block Diagram Model of the Controller
5.1.5 Servovalve Model

The servovalve of Figure 5.2(b) is shown in more detail in Figure 5.11. The inputs to the servovalve are instantaneous command signal, instantaneous air spring pressure, and the supply pressure. The output of servovalve is the controlled air flow (q) to the air spring.

![Diagram of servovalve inputs and outputs]

Figure 5.7: The Compact (a) and Expanded (b) Block Diagram Model of Norgren VP60 Proportional Servovalve
The flow rate of air into air spring \((q)\) is a nonlinear function of the supply pressure, air spring pressure, and the valve spool position. The flow rate of air out of the air spring is estimated using one of the models presented in chapter 3; see Equations (3.36-3.38). The critical pressure ratio and the flow coefficient of the servovalve are evaluated, experimentally. Moreover, the values of \(P_{UP}\) and \(P_{DN}\) in Equations (3.47-3.51) are necessary in constructing the flow model. Air spring pressure \((P)\) could be either upstream or downstream pressure depending on the direction of servovalve while supply pressure \(P_{supply}\) can only be upstream pressure (when air spring pressure \(P\) is the downstream pressure). On the other hand, the atmospheric pressure \(P_{ATM}\) is downstream pressure when the air spring pressure \((P)\) is the upstream pressure. Table 5.1 shows the switching logic used to determine \(P_{UP}\) and \(P_{DN}\) for the air spring.

### Table 5.1: Switching Logic for Norgren VP60 Proportional Servovalve to Determine \(P_{UP}\) and \(P_{DN}\) to be Used in the ISA Flow Model

<table>
<thead>
<tr>
<th>(X_{SPOOL})</th>
<th>Pressure relationship</th>
<th>(P_{UP})</th>
<th>(P_{DN})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>(P &lt; P_{supply})</td>
<td>(P_{supply})</td>
<td>(P)</td>
</tr>
<tr>
<td>Negative</td>
<td>(P &gt; P_{ATM})</td>
<td>(P)</td>
<td>(P_{ATM})</td>
</tr>
</tbody>
</table>
The flow coefficient $C_v$ of the valve (see section 3.4) is a function of the spool position $X_{\text{SPOOL}}$. Considering that the servovalve has its own internal feedback control with the voltage command sent to the valve as the reference input, the spool position is almost identical to the voltage command ($V_{\text{COM}}$)

$$x_{\text{spool}} = V_{\text{com}} \quad (5.10)$$

The value of the command voltage which is the only adjustable modifying the dynamics of the system is determined by summing all feedback loop signals which are proportional to relative angular displacement, velocity, acceleration, and air spring’s pressure ($\theta, \dot{\theta}, \ddot{\theta}$, and $P$); see Equation (5.11). The actual value of $V_{\text{COM}}$ which controls the valve is the summation of the feedback loop

$$V_{\text{com}} = k_a\ddot{\theta} + k_v\dot{\theta} + k_x\theta + k_p P_d \quad (5.11)$$

where $k_a$, $k_v$, $k_x$ and, $k_p$ are the proportional gains of the acceleration, velocity, displacement, and pressure.

The block diagram model of the servovalve is presented in Figure 5.12. The valve output is affected by the command voltage ($V_{\text{COM}}$), the supply pressure, and the air spring pressure. Note that the objective of semi-active control is varying $P_d$, by adjusting the flow of air in and out of the air spring, with the goal of modifying its stiffness and damping attributes.

The model of the PTMD is evaluated using numerical simulation, the results of which are experimentally verified. In section 5.2, the simulation results are presented
5.2 Simulation Results

One of the important reasons for performing simulation tests in this study is to evaluate the stiffness and the damping present in the test rig with and without semi-active control. The free response of the TMD and PTMD models are used to determine their corresponding natural frequencies at certain operating conditions. The results of these simulations are compared to a) the manufacturer provided information and b) the experimental data.

The simulations are performed using Simulink/Matlab computational platform as explained in section 5.1. The parameters of the simulation model are the mass, air spring height, and the locations of the air spring as well as the mass of a percentage of the length of the pendulum. In this section, three examples are presented to show how the servovalve can control the air spring to modify the natural frequency and the damping ratio of the TMD. The first example compares a passive traditional TMD and a passive pendulum TMD (with no servovalve in the pneumatic circuit) and shows that a TMD with the pendulum configuration (PTMD) has a lower natural frequency than a tuned mass damper (TMD) with the same mass and stiffness. The second example discusses the damping adjustment of the PTMD when the acceleration or the pressure feedback controls is introduced. Finally, the third example introduces the velocity feedback control to adjust the stiffness of air spring and thus the natural frequency of the PTMD. Table 5.2 shows the system parameters used in there three examples. Note that the TMD is only present in the first example.
Table 5.2: Parameters of the TMD and the PTMD

<table>
<thead>
<tr>
<th></th>
<th>PTMD</th>
<th>TMD</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass (m₂)</td>
<td>290 (kg)</td>
<td>615 (kg)</td>
</tr>
<tr>
<td>mass location (L)</td>
<td>120.6 (cm)</td>
<td>N/A</td>
</tr>
<tr>
<td>Air spring location (L₂)</td>
<td>56.5 (cm)</td>
<td>N/A</td>
</tr>
<tr>
<td>Air spring height (h₀)</td>
<td>11.63 (cm)</td>
<td>11.63 (cm)</td>
</tr>
<tr>
<td>Air spring static pressure (P₀)</td>
<td>318.5 (kPa) {46.2 (psi)}</td>
<td>318.5 (kPa) {46.2 (psi)}</td>
</tr>
</tbody>
</table>

5.2.1 TMD vs. PTMD

The first example highlights the differences between passive (uncontrolled) TMD and passive PTMD. The goal of this example is to show how the pendulum configuration of the tuned mass damper (PTMD) has a lower natural frequency compared to the tuned mass damper (TMD). Figure 5.8 compares the displacement power spectra of a one degree of freedom (1-DOF) TMD and a 1-DOF pendulum TMD with similar mass and stiffness, to a common perturbation. From Figure 5.8, the natural frequency of the PTMD is at 1.9 Hz while for the TMD is at 2.75 Hz. It can be noted that the reduction in the natural frequency of the PTMD compared to TMD is around 31%.
The static pressure of the air spring is selected to support the weight of the vibratory mass in the TMD and PTMD. Equation (5.12) shows the relationship between the TMD load and the pressure inside the air spring. Equation (5.13) shows the same relationship for the PTMD.

\[ PA = m_t g \]  \hspace{1cm} (5.12)

\[ PAL_2 = m_{ptmd} g L \]  \hspace{1cm} (5.13)

Dividing Equation (5.12) by Equation (5.13) gives the relationship between the mass of PTMD and TMD as shown in Equation (5.14).

\[ m_{ptmd} = m_t \frac{L_2}{L} \]  \hspace{1cm} (5.14)

Clear from Equation (5.14), to make the air spring pressure in both cases equal, the vibratory mass of the PTMD should be modified, compared to the TMD mass, by a factor of the arm length of the air spring divided by the arm length of the mass location.
The relationship between the natural frequencies of the PTMD and TMD is shown in Equation (5.15).

\[ \omega_{n_{PTMD}} = \omega_{nt} \sqrt{\frac{L_2}{L}} \]  

That is, the natural frequency of the pendulum configuration is reduced by a factor of \((L_2/L)^{1/2}\), compared to that of a similar TMD.

The free response of air spring pressure of the TMD and PTMD configuration are shown in Figure 5.9. The air spring pressure fluctuates around the static pressure of 318.5 kPa. The reason that air spring pressure does not fluctuate around the static pressure is due to the fact that the moment caused by the weight \((m_2gL)\) does not cancel the moment resulted from the static pressure \((P_sA_{eff}L_2)\) in the air spring. As stated earlier the reason that these two moments do not cancel each other (add up to zero) is that the effective area \(A_{eff}\) in the air spring, especially the ones with convoluted shapes, is not constant and varies with the air spring height as the system vibrates. The fluctuation of the air spring pressure, the dynamic pressure, is due to the change of the volume of the air enclosed in the air spring as represented in Equation (3.32). This equation presents the dynamic pressure of the air inside the air spring as a function of air flow rate from the valve and the rate of change of volume of the air enclosed in the air spring.

Figure 5.10 shows the dynamic pressure fluctuation versus time. The dynamic pressure of the TMD is higher compared to that of the PTMD by 6.8%. Air spring’s net displacements of the TMD and PTMD configuration are shown in Figure 5.11. Clear from Figure 5.11 the deflections of the air spring in both cases are similar and oscillate with the amplitude of 0.08 mm around the static height of the air spring.
Figure 5.9: Air Spring Pressure of the TMD and PTMD

Figure 5.10: Air Spring Dynamic Pressure of the TMD and PTMD
Figure 5.11: Air Spring Net Displacement of the TMD and PTMD

Figure 5.12 shows the relationship between the air spring dynamic pressure versus its effective area. The relationship is linear with positive slope indicating that when the air spring is contracting, the pressure inside the air spring increases while the expansion of the air spring causes the pressure to decrease. Figure 5.13 depicts the air spring net displacement versus its effective area which is also linear but with a negative slope. According to Figure 5.13 the height of the air spring decreases with increase in the effective area and vice versa.
Figure 5.12: Air Spring Pressure of the TMD and PTMD vs. the Effective Area

Figure 5.13: Air Spring Net Displacement of the TMD and PTMD vs. the Effective Area
The force excreted by the air spring in both TMD and PTMD, evaluated by multiplying the air spring’s pressure with the effective area, is shown in Figure 5.14. A close inspection of this figure shows that the air springs in both configurations generate the same force of 6247 N.

![Graph showing force excreted by the air spring in TMD and PTMD over different durations.](image)

Figure 5.14: Air Spring Force in TMD and PTMD over Long (a) and Short (b) Durations
Figure 5.15 shows the relationship between the air spring height and its force. In this figure, the force of the air spring decreases while the net displacement increases and vice versa. Note that the slope of the line is negative. The relationship between the air spring height and its force is non-linear over an extended range of height. However, as shown in Figure 5.15, the relationship is linear over a small range of displacement, e.g. -1 mm to +1 mm.

![Figure 5.15: Air Spring Exerted Force of the TMD and PTMD vs. the Air Spring Displacement](image)

**5.2.2 Damping Control**

In the previous section, a passive TMD and PTMD using an air spring as the restoring element were presented. In this section, feedback control is introduced to adjust the pressure of the air spring with the goal of controlling its damping and stiffness. When the TMD is excited, the pressure in the air spring oscillates due to the vibration of the
TMD. Although not substantial, the pressure feedback control of the air spring introduces a modest amount of active damping to the system. As mentioned earlier and discussed in more detailed, below, larger amount of damping can be realized by feeding back the acceleration of the air spring.

Power spectra of displacement shown in Figure 5.16 highlight the effect of the pressure feedback and the acceleration feedback on the natural frequency and the damping of the system. The system is perturbed by sending a pulse signal to the servovalve to disturb the pressure inside the air spring.

Evident from the height of the peaks in Figure 5.16, the uncontrolled TMD has lower damping compared to the controlled TMD using pressure and acceleration feedback. Also, the additional damping lowers the frequency of the peaks slightly.

Figure 5.16: Air Spring Displacement Power Spectra for Uncontrolled and Damping Controlled PTMD
Figure 5.17 shows the time traces of air spring pressure of the uncontrolled PTMD and controlled PTMD using pressure feedback and combination of pressure and acceleration feedback. From Figure 5.17, it can be noted that pressure feedback lowers the amplitude of pressure oscillation of the air spring while maintaining the same frequency as that of the uncontrolled system. Despite the damping effect of pressure feedback, the settling time of the system with control is not drastically lower than that of the uncontrolled system. However, cascading acceleration feedback with pressure feedback increases the damping and lower the settling time, compared to the controlled PTMD using pressure feedback only.

The dynamic pressure of the air spring for uncontrolled PTMD and controlled PTMD using pressure feedback and combination of pressure and acceleration feedback are shown in Figure 5.18. Note that the dynamic pressure of the controlled PTMD is resulted from the combination of a) the air flow from the servovalve and b) the air spring volume change (due to the change in height of air spring). Figure 5.19 shows the dynamic pressure caused by the air flow thru the servovalve only, for the controlled PTMD. Although, the difference between the flow rates of the servovalve using pressure feedback and using the combination of the pressure and acceleration feedback is not large but it still has a noticeable effect on the damping of the system.

As mentioned earlier, the main goal of the pressure feedback control is to maintain the static height of the air spring; the damping attribute of pressure feedback is an added benefit.
Figure 5.17: Air Spring Pressure of Uncontrolled and Damping Controlled PTMD

Figure 5.18: Air Spring Dynamic Pressure of Uncontrolled and Damping Controlled PTMD
Figure 5.19: Air Spring Dynamic Pressure Due the Air Flow through the Servo valve of Controlled PTMD

Figure 5.20 shows the displacement of the uncontrolled PTMD, the controlled PTMD using the pressure feedback, and the controlled PTMD using the combination of the pressure and acceleration feedback. The displacement of both, the controlled PTMD using the pressure feedback and the combination of the pressure and acceleration feedback, start at the static height. This is because the excitation method used to perturb the system is a short pulsation of the static pressure of the air spring. While the uncontrolled PTMD starts at 1 mm because the excitation method is the 1 mm initial condition on the displacement.
From Figure 5.20, similar to dynamic pressure variation shown in Figure 5.18-12, the displacement settling time is shorter in controlled system compared to that of uncontrolled system.

![Air Spring Displacement of Uncontrolled and Damping Controlled PTMD](image)

Figure 5.20: Air Spring Displacement of Uncontrolled and Damping Controlled PTMD

Figure 5.21 shows the relationship between the air spring pressure versus the effective area of the air spring for the uncontrolled and controlled PTMD. The pressure changes linearly with the effective area in the uncontrolled PTMD. However, in the controlled PTMD, either using pressure feedback or combination of pressure and acceleration feedback, the pressure/effective area relationship is nonlinear. The higher damping effectiveness of combined pressure and acceleration feedback compared to that of pressure feedback only, is also evident from pressure/effective area variation shown in Figure 5.21.

Figure 5.22 shows the air spring deflection (around the static height) versus the effective area of the air spring in the uncontrolled and controlled PTMD. Comparison of
Figures 5.21 and 5.22 points to similar behavior of the air spring pressure and deflection versus the effective area. However in Figure 5.22, the deflection of the controlled PTMD with the combination feedback of the pressure and acceleration decreases until it settled around the static height and, the controlled PTMD with the pressure feedback keeps oscillating around the static height. Note that the deflection in the uncontrolled PTMD varies linearly with the effective area for a small range of displacement even though it is nonlinear on wider.

Figure 5.21: Air Spring Pressure of Uncontrolled and Damping Controlled PTMD vs. the Effective Area.

The force exerted by the air spring of the uncontrolled PTMD and the controlled PTMD is shown in Figure 5.23. A close inspection of this Figure shows that the controlled air spring exhibits less force compared to the uncontrolled one. This is expected since the pressure of the controlled PTMD is lower. However, the force oscillation of the air spring with combined feedback control fades faster since it has a higher damping, as mentioned earlier.
Figure 5.22: 115 Air Spring Displacement of Uncontrolled and Damping Controlled PTMD vs. the Effective Area.

Figure 5.23: Air Spring Exerted Force of Uncontrolled and Damping Controlled PTMD
5.2.3 Stiffness Control

In the previous section, the controlled PTMD using pressure feedback and combination of pressure and acceleration feedback were presented to show their effects on damping and to a lesser extent on natural frequency of the system. The results show that the pressure feedback has moderate and acceleration feedback has significant impact on damping while their effect on the natural frequency of the system is negligible.

In this section, the feedback of the velocity of the air spring is discussed as a mechanism to modify the stiffness of the air spring. The air spring velocity is feedback through either a positive or negative gain resulting in increase or decrease in stiffness of the air spring, respectively.

Figure 5.24 shows the power spectra of displacement of the uncontrolled and controlled (using positive and negative velocity gains) PTMD, perturbed by a short-duration pressure pulsation in air spring. Comparison of the three traces in this figure shows that positive gain of the velocity feedback increase the natural frequency from that of the passive PTMD while negative gain of the velocity feedback decreases it. This indicates that velocity feedback with positive gains increases the air spring’s stiffness and with negative gains decreases the stiffness in the controlled air spring from that of passive air spring with similar static pressure.

To enhance the stability robustness of the PTMD under velocity feedback control, the acceleration feedback loop (providing damping) is closed, as well.

Figures 5.25 and 5.26 show the pressure and dynamic pressure of the uncontrolled and velocity feedback controlled air spring, respectively. Close inspection of Figure 5.26 shows that pressure oscillation of controlled air spring with negative velocity gain (with
lower stiffness) starts below the static pressure. Moreover, it does not approach the static pressure, as rapidly as the pressure in the uncontrolled and controlled air spring using positive velocity gain.

Figure 5.27 shows the air spring deflection for the controlled PTMD using velocity feedback as well as the uncontrolled PTMD. From this figure it can be noted that the controlled PTMD with positive velocity gain increases the height of the air spring compared to the uncontrolled PTMD and controlled PTMD with negative velocity gain. Consider that positive gains on the velocity feedback make the air spring stiffer, explains the smaller amplitude of oscillation. On the other hand, using the negative gain on the velocity feedback makes the air spring softer which in turn makes the amplitude of oscillation larger compared to that of the positive gain.

Figure 5.24: Air Spring Displacement Power Spectra for Uncontrolled and Stiffness Controlled PTMD
Figure 5.25: Air Spring Pressure of Uncontrolled and Stiffness Controlled PTMD

Figure 5.26: Air Spring Dynamic Pressure of Uncontrolled and Stiffness Controlled PTMD
Figure 5.27: Air Spring Displacement of Uncontrolled and Stiffness Controlled PTMD

Figure 5.28 presents the acceleration of the air-suspended PTMD mass with and without velocity feedback. As shown in Figure 5.28 the acceleration of the controlled PTMD using velocity feedback is lower than the uncontrolled PTMD. However, the acceleration of the controlled PTMD with negative velocity feedback gain is relatively closer to the controlled PTMD with positive velocity feedback gain. That means the system is experiencing more damping when a negative velocity feedback gain is used. The same behavior can also be seen in Figure 5.29 which presents the air spring force exerted on the system.

Figure 5.30 shows the relationship between the pressure and the effective area of the air spring of controlled PTMD using velocity feedback. Clear from Figure 5.30, the
variation of the effective area of the negative velocity feedback gain is higher compared to that of the positive velocity feedback gain. This means a small change in the air spring pressure in the controlled PTMD using negative velocity feedback gain leads to a rapid change in the effective area. Also, a large change in the air spring pressure in the controlled PTMD using a positive velocity feedback gain leads to a smaller change in the effective area.

The air spring’s net displacement versus the effective area is shown in Figure 5.31. For the uncontrolled PTMD and controlled PTMD using the velocity feedback, the effective area increases when the net displacement of the air spring decreases.

![Graph showing the acceleration of uncontrolled and stiffness controlled PTMD](image.png)

Figure 5.28: Acceleration of Uncontrolled and Stiffness Controlled PTMD
Figure 5.29: Air Spring Force of Uncontrolled and Stiffness Controlled PTMD

Figure 5.30: Air Spring Pressure of Uncontrolled and Stiffness Controlled PTMD vs. the Effective Area
Figure 5.31: Air Spring Displacement of Uncontrolled and Stiffness Controlled PTMD vs. the Effective Area
CHAPTER VI
EXPERIMENTAL SETUP AND PROCEDURES

This chapter presents a detailed description of the test setups for all experiments conducted in this dissertation. The chapter begins with a description of the primary and secondary components of the semi-active PTMD setup. Followed by the software used to perform the test. Then, a description of the excitation procedure to perturb the system is furnished. Experimental results are presented and compared with the simulation results.

6.1 Components of the Semi-Active PTMD

This section presents the primary and the secondary components of the test device of the semi-active PTMD assembled in the research laboratory.

6.1.1 The Primary Components of the Semi-Active PTMD

A 2-DOF test device was built to experimentally evaluate the effectiveness of the semi-active PTMD using an air spring as the combined restoring and damping elements. The physical representation of the experimental semi-active PTMD is shown in Figure 6.1(a). The schematic of a 2-DOF structure coupled with a PTMD is shown in Figure 6.1(b). The semi-active PTMD consists of the following primary components:
Figure 6.1: The Schematic of a Structure Coupled with a PTMD (a) and the Semi-Active PTMD Test Device (b)

- The Semi-active PTMD mass shown in Figure 6.2
- The Air-spring that represents the Semi-active PTMD’s stiffness and damping elements shown in Figure 6.3
- The servovalve that represent the Semi-active PTMD’s actuator shown in Figure 6.4
Figure 6.2: Semi-Active PTMD Test Device Mass

Figure 6.3: Semi-Active PTMD Test Device Air Spring

Figure 6.4: Semi-Active PTMD Test Device Servovalve
The semi-active PTMD’s mass consisting of a concentrated mass at the end of the I-beam making up the pendulum arm and a portion of the I-beam mass. The concentrated mass shown in Figure (6.2) is made of 21 steel plates weighing the total with the I-beam of 700 lbs. (318 Kg) are installed at the free end of the I-beam. The other end of the I-beam is hinged on the semi-active PTMD frame by two ball bearings with a cross bar. This is to secure the I-beam with the semi-active PTMD frame and to allow only rotational movement and prevent the semi-active PTMD from having lateral motion. The horizontal pendulum is coupled with the air spring at the middle of the I-beam where the air spring is mounted on the PTMD frame. The maximum weight that the semi-active PTMD can support depends on the size of the air spring used in the model; the air spring in this dissertation can support load up to 3270 lbs., at a static height of 4.5 inches and pressure of 100 psi.

The air spring is style 115 manufacture by Firestone. The dimensions of the air spring are shown in Figure (4.1). The valve used in this study is a Norgren VP60 1/4” high speed servovalve. The servovalve and the air springs were discussed earlier in chapter 4.

6.1.2 Hardware Secondary Components of the Semi-Active PTMD

The hardware secondary components of the Semi-active PTMD system include:

- One displacement sensor LVDT,
- an accelerometer,
- a pressure sensor, and
- a dSPACE control board, residing inside a computer.
The linear variable differential transformer, (LVDT model VL7A AC-AC made by Honeywell) measures the air spring height. The LVDT requires a power supply for position measurements of ±5V. It has the sensitivity of 197 mV/in. A PCB model U352L65 ICP accelerometer (sensitivity of 97.9 mV/g) measures the acceleration of the semi-active PTMD. The pressure sensor (ISE40 made by SMC) is used to measure the air spring pressure. Due to the space limitation the sensor is located within 25 cm from the air spring.

The controller board used was a dSPACE DS1102 Digital Signal Processor (DSP) with four input channels and four output channels. The controller board receives all the measurements signals of the PTMD variables and sends the output signal to the servovalve. The servovalve internal controller activates the servovalve’s spool. PTMD’s control signal made up of individual or combination of pressure, velocity, and acceleration feedback is the reference input to the servovalve’s internal controller.

6.2. Software Components

The controller board can be interfaced with Simulink, the simulation package that runs under Matlab. The controller block diagrams shown in Figure (6.2) is constructed inside Simulink and then downloaded, using real-time workshop toolbox of Matlab, into the DSP board. The DSP board and its user interface software allows for a real-time processing, real-time viewing of the selected signals, and real-time parameter changes (gain changes). The dSPACE ControlDesk software is the user interface for this experiment. Within the dSPACE ControlDesk, the data could be viewed and controller
parameters modified, in real-time. A snapshot of the dSPACE ControlDesk Software User Interface is shown in Figure 6.5.

![Snapshot of the dSPACE ControlDesk Software User Interface](image)

Figure 6.5: Snapshot of the dSPACE ControlDesk Software User Interface

Block diagram of Figure 6.6 shows the semi-active PTMD control algorithm. The four inputs to the controller residing in the DSP Board are coming from the air spring’s pressure, the acceleration, and the displacement of the PTMD with respect to the structure. To remove the DC offset the accelerometer signal is passed through an AC coupler with a pole at $s= -\pi/5$. Then the signal is multiplied by a calibration factor. The signal is used to adjust the damping of the PTMD by multiplying it by the corresponding gain. Also, the accelerometer signal is fed into the Kalman estimator. Displacement signal is also fed to the DSP board, where after being processed by its calibration factor,
is fed to the Kalman estimator. The Kalman estimator uses the acceleration and displacement as the input and estimates the velocity. The third signal from the air spring is pressure which after passing through a low pass filter and its calibration factor, is used as the feedback signal in the pressure control loop.

Prior to sending the control signal out of the DSP board (to the servovalve), a short duration pulse signal is added to it. This signal is used to pulsate the air spring pressure and consequently perturb the PTMD; see Figure 6.5. This perturbation makes the semi-active PTMD to vibrate at its natural frequency. In addition to providing small amount of damping, the pressure feedback maintains the static pressure.

In various experiment runs, the sensory data of interest were collected and used for presentation purpose.

Figure 6.6: Block Diagram of the Semi-Active PTMD Controller
6.3 Kalman Estimator

It is well understood that in a 2\textsuperscript{nd} order system the feedback force proportional to velocity and displacement results in stiffness and damping control, respectively. Due to the integration dynamics of the air spring, discussed in the section 5.1, stiffness control can be realized by feeding back the velocity (the integral of which becomes displacement) and damping control can be realized by feeding back the acceleration (the integral of which becomes velocity) of the mounted mass. In other words, making the flow rate proportional to velocity and acceleration of the mounted mass results in controlling the stiffness and damping; respectively.

Feedback control action proportional to the relative velocity of the PTMD with respect to the base will make the air spring to exhibit the behavior of a softer or stiffer spring than the uncontrolled air spring, depending on the sign of the feedback control scheme. The proper velocity measurement is required to achieve the desired stiffness value. The velocity sensors which are either magnetic or inductive based are not practical in measuring the velocity of the PTMD scheme with a low natural frequency. Neither the integration of the acceleration nor differentiations of displacement signals are appropriate for providing an acceptable velocity signal.

In this dissertation, the velocity is robustly and simply estimated using a kinematic Kalman estimator. Contrary to the more commonly known dynamic Kalman estimator which in addition to the input and outputs measurements, requires the dynamics of the system to be able to achieve a good estimate of the states. Kinematic Kalman estimator only requires the input and the output measurements. The mechanics of Kalman estimator is based on integrating the acceleration in two stages which results in velocity
and displacement. Note that a double integrator can be viewed as a second order system with the two states of displacement and velocity; the input is the acceleration and the measured output is the displacement. The kinematic Kalman estimator estimates the velocity state of this double integrator system using the measured acceleration as the input and the measured displacement as the output. The general state space equation of the kinematic Kalman estimator

\[
\dot{\theta} = A\theta + Bu \\
y = C\theta + Du \\
\hat{\theta} = A\hat{\theta} + Bu + L(y - C\hat{\theta} - Du)
\]  

(6.1)

Where

\(\theta\) is the state variable matrix of the model  
\(u\) is the known input of the model which is the acceleration \(\ddot{\theta}\)  
\(\hat{\theta}\) is the state estimate matrix

Equation 6.2 present the state space model of the kinematic Kalman estimator equation.

\[
A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \end{bmatrix} \\
C = \begin{bmatrix} 1 & 0 \end{bmatrix}, D = [0]
\]  

(6.2)

The Kalman estimator matrix gain \((L)\) can be found by using optimal estimation theory.

6.4 Experimental Results

In the previous chapter, the simulation results of the controlled PTMD using pressure, acceleration, and velocity feedback were presented. In this section the experimental evaluation of the passive (uncontrolled) and semi-active (controlled) PTMD are presented. Refer to Figure 6.1 for the schematic of the test set up used in the experimental work. The primary system parameters of the PTMD are presented in Table 6.1.
Table 6.1: Experimental PTMD’s Primary Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>The vibratory mass (m&lt;sub&gt;2&lt;/sub&gt;)</td>
<td>290 (kg)</td>
</tr>
<tr>
<td>The vibratory mass location (L)</td>
<td>120.6 (cm)</td>
</tr>
<tr>
<td>Air spring location (L&lt;sub&gt;2&lt;/sub&gt;)</td>
<td>56.5 (cm)</td>
</tr>
<tr>
<td>Sensor location (L&lt;sub&gt;s&lt;/sub&gt;)</td>
<td>84.5 (cm)</td>
</tr>
<tr>
<td>Air spring height (h&lt;sub&gt;0&lt;/sub&gt;)</td>
<td>11.63 (cm)</td>
</tr>
<tr>
<td>Air spring static pressure (P&lt;sub&gt;0&lt;/sub&gt;)</td>
<td>317.1 (kpa) {46 (psi)}</td>
</tr>
<tr>
<td>Supply pressure (P&lt;sub&gt;1&lt;/sub&gt;)</td>
<td>561.9 (kpa) {81.5 (psi)}</td>
</tr>
<tr>
<td>Air spring’s natural frequency</td>
<td>1.87 Hz</td>
</tr>
</tbody>
</table>

6.4.1 Pressure Control

The aim of this experiment is to find the lowest gain that can be used in the PTMD pressure feedback to maintain the static pressure and height of the air spring without changing the natural frequency of the system. Figure 6.7(a) shows the power spectra of displacement (free response to a pressure pulsation perturbation on the air spring) of the uncontrolled and controlled air spring using pressure feedback. Figure 6.7(b) which shows a zoomed shot of the Figure 6.7(a), show that a) the uncontrolled PTMD natural frequency is 1.87 Hz and b) increasing the pressure feedback gain of the controlled PTMD does not change, the natural frequency of the system significantly. However, the increases in gain add damping to the system.

Figure 6.8 shows the air spring pressure of the controlled PTMD using different pressure feedback gains. Close inspection of Figure 6.8 shows that when the pressure feedback gain is increased, the air spring pressure is maintained closer to the reference
(static) pressure. Figure 6.9 shows the dynamic pressure of the air spring. From this figure, it can be noted that as the pressure feedback gain increases the air spring pressure is controlled more tightly and held closer to the reference pressure.

The air spring displacement of the controlled PTMD using different pressure feedback gains is shown in Figure 6.10. Again, increase in pressure feedback gain shows that when the pressure feedback gain is increased, the air spring displacement is maintained closer to the reference (static) displacement.

![Figure 6.9](image)

**Figure 6.7:** Air Spring Displacement Power Spectra for Uncontrolled and Pressure-Controlled PTMD (a) and Zoomed Shot of the Power Spectra (b)
Figure 6.8: Air Spring Pressure of Uncontrolled and Pressure-Controlled PTMD

Figure 6.9: Air Spring Dynamic Pressure of Uncontrolled and Pressure-Controlled PTMD
6.4.2 Stiffness Control

The simulation results presented in section 5.2.3, show that natural frequency could be modified, via increase in stiffness of the air spring, by adding velocity feedback to the system. The experimental results for the positive and negative velocity feedback gains, in response to a pressure pulsation perturbation on the air spring, are presented in this section.

6.4.2.1 Positive Velocity Feedback

Figure 6.11 presents the power spectra of the displacement of PTMD under positive velocity feedback control using the gains of 0.02, 0.1, and 0.2. It can be noted
that when the velocity gain is increased the natural frequency is also increased. That is positive velocity feedback gain changes the stiffness of the air spring. However, modifying the system natural frequency lowers the stability margins of the system necessitating the need for added damping. Acceleration feedback is cascaded to the velocity feedback to introduce additional damping to the system.

![Graph showing the effect of different velocity feedback gains on the air spring displacement power spectra.](image)

**Figure 6.11: Air Spring Displacement Power Spectra for Positive Velocity Controlled PTMD**

The air spring pressure and dynamic pressure of the controlled PTMD using different positive velocity feedback gains are shown in Figure 6.12 and 6.13. From Figure 6.12, by increasing the velocity feedback gain, the air spring pressure is still behaving similar to that of pressure controlled as discussed earlier in section 5.2.1. In other words,
with the pressure feedback loop closed, the velocity feedback gain does not affect the air spring’s pressure.

Figure 6.12: Air Spring Pressure of Positive Velocity Feedback Controlled PTMD

Figure 6.13: Air Spring Dynamic Pressure of Positive Velocity Feedback Controlled PTMD
Figure 6.14 shows the displacement of the air spring with velocity feedback control. From this figure, the displacement decreases when the velocity feedback gain is increased and it oscillates more closely to the static height.

![Figure 6.14: Air Spring Displacement of Positive Velocity Feedback Controlled PTMD](image)

**6.4.2.2 Negative Velocity Feedback**

Figure 6.15 shows of power spectra of the displacement of the PTMD with velocity feedback for multiple negative gains, in response to a control pressure pulsation on the air spring. It can be noted that when the velocity gain is decreased, the natural frequency is decreased, i.e. the negative velocity feedback lowers the stiffness. Furthermore, the negative velocity feedback also increases the damping of the system.
This is expected since the damping is inversely proportional to the square root of the stiffness as shown in Equation (6.3)

\[ \zeta_t = \frac{c}{2\sqrt{km}} \]  

(6.3)

Figures 6.16 and 6.17 show the air spring pressure and the dynamic pressure of the negative velocity feedback controlled PTMD. From Figure 6.16, it can be noted that the velocity negative feedbacks gains reduce the static pressure compared to the TMD. However, the velocity negative feedback gain causes minimal change in the air spring pressure.

The air spring height using the negative velocity feedback is presented in Figure 6.18. Various traces in this figure show that the negative velocity feedback gain reduces the static height of the air spring initially, but by making the system operate for a longer time the static height return to its steady state value.

![Figure 6.15: Air Spring Displacement Power Spectra for Negative Velocity Feedback Controlled PTMD](image-url)
Figure 6.16: Air Spring Pressure of Negative Velocity Feedback Controlled PTMD

Figure 6.17: Air Spring Dynamic Pressure of Negative Velocity Feedback Controlled PTMD
6.5 Comparison of Simulation and Experimental Results

The aim of this section is to provide a comparison between the numerical simulation and the experimental results for the semi-active PTMD. The comparison are limited to the safely achieved (with adequate robustness against instability) maximum and minimum values of natural frequencies and the corresponding parameters in both simulation and experimental results for the semi-active PTMD. Table 6.2 shows the gains of the pressure, positive velocity, and negative velocity feedback loops used in the experimental and simulation tests.

Throughout the comparisons, presented in Figures 6.19 thru 6.23, the solid lines present the experimental results and the dotted lines present the simulation results.
Table 6.2: Experimental and Simulation PTMD Feedback Gain Values

<table>
<thead>
<tr>
<th>Feedback Technique</th>
<th>Experimental</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G_p$</td>
<td>$G_v$</td>
</tr>
<tr>
<td>Pressure Gain</td>
<td>-0.05</td>
<td>0</td>
</tr>
<tr>
<td>Positive Velocity Gain</td>
<td>-0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>Negative Velocity Gain</td>
<td>-0.05</td>
<td>-0.045</td>
</tr>
</tbody>
</table>

In Figure 6.19 the power spectra of the displacement of the simulated and the experimental PTMD are compared. It can be noted that the natural frequency of the PTMD using only pressure feedback is 1.8 Hz for both the experimental and the simulation results. Also, it can be noted that the velocity positive feedback gain increases the natural frequency and the velocity negative feedback gain decreases the natural frequency from the natural frequency of passive PTMD in both simulation and experimental results. However, the simulation produces slightly lower values compared to the experimental results with the maximum difference is around 0.05 Hz.

In Figure 6.20 the air spring pressure of the simulated and the experimental PTMD are compared. The experimentally measured pressure of the semi-active PTMD using positive and negative velocity feedback gains oscillate above the static pressure of 317 kPa. However, the simulation pressure results of the semi-active PTMD oscillate around the static pressure. Moreover, by close inspection of Figure 6.21, which presents the air spring dynamic pressure of the simulated and the experimental semi-active PTMD, the dynamic pressure of the experimental and the simulation model resemble each other very closely.
Figure 6.19: Air Spring Displacement Power Spectra of Simulated and Experimental Controlled PTMD
Figure 6.20: Air Spring Pressure of Simulated and Experimental Controlled PTMD
Figure 6.21: Air Spring Dynamic Pressure of Simulated and Experimental Controlled PTMD
Figure 6.22 highlights the air spring displacement of the simulated and the experimental PTMD with pressure, as well as positive and negative velocity feedback gains. From this figure, the displacement result of the simulated and the experimental tests for the positive gain of velocity feedback are in phase and have a difference of 0.75 mm. This result resembles that of the air spring pressure discussed in Figure 6.20. However, the negative gain of the velocity feedback for the simulated and the experimental results have a small difference. Furthermore, the simulated and the experimental results for the displacement of the pressure feedback test for the PTMD in Figure 6.22 are almost identical.

The average height oscillation of the simulated and the experimental results for the semi-active PTMD are shown in Figure 6.23. It can be noted that the positive gain of velocity for both the simulated and the experimental results are in phase after certain amount time but have different amplitudes. Both experimental and simulated results show that damping has been added to the system however the simulated one shows more damping than the experimental one. The main reason causing the discrepancies between the simulation and experimental results is the uncertainties in the servovalve model in the small input signal amplitudes ($\pm 100$ millivolts).
Figure 6.22: Air Spring Displacement of Simulated and Experimental Controlled PTMD
Figure 6.23: Air Spring Net Displacement of Simulated and Experimental Controlled PTMD
CHAPTER VII
SUMMARY AND CONCLUSIONS

7.1 Conclusions

Extensive studies have been carried out, in recent years, to find methods to mitigate the unwanted structure vibration caused by human excitation, machinery, and winds. Modern structures such as floors and bridges using high strength materials, and extending across long spans are very flexible with negligible damping. Vibration control devices and strategies are constantly being developed to eliminate/dissipate the unwanted vibration and to increase the serviceability level of such structures. One such method for abating the vibration is tuned mass damping.

In this dissertation, semi-active control of air-suspended tuned mass dampers with pendulum configuration was explored. A novel semi-active Air Sprung PTMD was designed, built, and evaluated, analytically and experimentally. The dynamics and control of such PTMD were evaluated, and its effectiveness was compared with that of the conventional passive PTMD.

The main reason for introducing semi-active control to a TMD is to enable the TMD to adapt itself, robustly, to the primary structure’s parameters (mainly mass and stiffness) changes and maintain its tuning.
Following extensive analytical work, simulation, and experimentation it was found that the velocity feedback can modify the stiffness of the semi-active air-suspended tuned mass damper. Positive velocity feedback increases the stiffness while negative velocity feedback decreases it. Moreover, pressure and the acceleration feedback adjust the damping of the semi-active TMD.

The air spring used in this work is of convoluted type. This type of air springs, because of their particular geometry, experiences a rather severe change in their cross-sectional area, as they contract and expand. It was found that to properly account for the impact of this important parameter on the inner-working of the air spring, one needs to consider two areas for the air spring, namely, the effective area and the geometric area. The effective area of the air spring is the area used to calculate the exerted force by the air spring while the geometric area is the cross-sectional area used to calculate the rate of change of the volume of the air enclosed in the air spring. The use of these two areas resulted in an accurate model of the air spring.

The model of the semi-active PTMD was verified using the experimental setup discussed in Chapter 6. Both the simulation and experimental demonstrate the effectiveness of the semi-actively controlled air-suspended PTMD in adjusting its tuning frequency as well as damping.

7.2 Future Work and Recommendations

The following is recommended for future work:

- The use of air springs with less variation in their area, e.g. rolling sleeve air springs as the adjustable element of PTMD.
- Extension of the control algorithm beyond proportional control.
- Extending the semi-active strategy to multi degree of freedom TMDs.
- Exploring other application such as automotive air suspension.
REFERENCES


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