DAMPING PARAMETER STUDY OF A PERFORATED PLATE WITH BIAS FLOW

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DAMPING PARAMETER STUDY OF A PERFORATED PLATE WITH BIAS FLOW

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ABSTRACT

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One of the main impediments to successful operation of combustion systems in industrial and aerospace applications including gas turbines, ramjets, rocket motors, afterburners (augmenters) and even large heaters/boilers is the dynamic instability also known as thermo-acoustic instability. Concerns with this ongoing problem have grown with the introduction of Lean Premixed Combustion (LPC) systems developed to address the environmental concerns associated with the conventional combustion systems.

The most common way to mitigate thermo-acoustic instability is adding acoustic damping to the combustor using acoustic liners. Recently damping properties of bias flow initially introduced to liners only for cooling purposes have been recognized and proven to be an asset in enhancing the damping effectiveness of liners. Acoustic liners are currently being designed using empirical design rules followed by build-test-improve steps; basically by trial and error. There is growing concerns on the lack of reliability associated with the experimental evaluation of the acoustic liners with small
size apertures. The development of physics-based tools in assisting the design of such liners has become of great interest to practitioners recently. This dissertation focuses primarily on how Large-Eddy Simulations (LES) or similar techniques such as Scaled Adaptive Simulation (SAS) can be used to characterize damping properties of bias flow. The dissertation also reviews assumptions made in the existing analytical, semi-empirical, and numerical models, provides a criteria to rank order the existing models, and identifies the best existing theoretical model.

Flow field calculations by LES provide good insight into the mechanisms that led to acoustic damping. Comparison of simulation results with empirical and analytical studies shows that LES simulation is a viable alternative to the empirical and analytical methods and can accurately predict the damping behavior of liners. Currently the role of LES for research studies concerned with damping properties of liners is limited to validation of other empirical or theoretical approaches. This research has shown that LES can go beyond that and can be used for performing parametric studies to characterize the sensitivity of acoustic properties of multi–perforated liners to the changes in the geometry and flow conditions and be used as a tool to design acoustic liners. The conducted research provides an insightful understanding about the contribution of different flow and geometry parameters such as perforated plate thickness, aperture radius, porosity factors and bias flow velocity. While the study agrees with previous observations obtained by analytical or experimental methods, it also quantifies the impact from these parameters on the acoustic impedance of perforated plate, a key parameter to determine the acoustic performance of any system.

The conducted study has also explored the limitations and capabilities of commercial tool when are applied for performing simulation studies on damping properties of liners. The overall agreement between LES results and previous studies
proves that commercial tools can be effectively used for these applications under certain conditions.
For my family, my dear wife Faezeh and my beautiful daughter Melika, my dear parents who taught me the importance of life-long learning
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LIST OF ABBREVIATIONS AND NOTATIONS

\( Q \) = Volume flow

\( P \) = Pressure

\( \mu \) = Viscosity

\( \nu \) = Dynamic viscosity

\( Sr \) = Strouhal number

\( Re \) = Reynolds Number

\( I_1, K_1 \) = The modified Bessel functions

\( f \) = Excitation frequency

\( \rho \) = Density

\( a \) = Radius of aperture

\( T/R \) = Thickness Radius ratio

\( T \) = Thickness of aperture

\( K \) = Acoustic conductivity

\( \gamma \) = Real part of acoustic conductivity

\( \delta \) = Imaginary part of acoustic conductivity

\( w \) = Shed vorticity

\( \omega \) = Angular frequency

\( \omega_n \) = natural Angular frequency

\( u \) = Velocity

\( v \) = Acoustic particle velocity

\( A \) = Cross section area of aperture
\( d \) = Hole spacing distance
\( M \) = Mach number
\( C \) = Speed of sound
\( \sigma \) = Porosity
\( Z \) = Impedance
\( R \) = Real part of impedance
\( X \) = Imaginary part of impedance
\( t \) = Time
\( \Delta t \) = Time step size
\( \Delta y \) = Grid size
CFL = Courant numbers (Courant-Friedrichs-Levi)
\( Q \) = Volume flow
\( l \) = Length
\( m \) = Mass
\( \phi \) = Phase
\( A \) = Amplitude
\( \lambda \) = Wave length
\( x \) = Axial distance
\( r \) = Radial distance
\( k \) = Wave number
\( Hn \) = Helmholtz number
\( R\omega \) = Acoustic Mach number
\( B \) = Bernoulli enthalpy
\( HM \) = Howe Model
\( MHM \) = Modified Howe Model
\( JSM \) = Jing Sun Model
LPC = Lean Premixed Combustion

LHS = Left Hand side

RHS = Right Hand side

j = Complex Number

**SUBSCRIPTS**

R  Raleigh

in  Inlet

M  Modified

0  Initial

f  Amplitude

th  Thickness

bf  Bias flow

t  Total

e  effective

h  hole

p  plate

**Superscripts**

-  Mean component of a variable

^  Fluctuated component of a variable

+  Upstream

-  Downstream

°  Rate
CHAPTER 1
INTRODUCTION

Demands for more environmentally friendly gas turbine combustion systems have increased in the recent past years. Environmental agencies throughout the world have mandated a reduction in NO\textsubscript{x} emissions. This has been a great motive for development of Dry Low NOx (DLN) or Dry Low Emissions (DLE) combustors to meet these strict emission requirements. Low emissions and higher efficiency associated with these combustors have made them increasingly popular in energy conversion applications. LPC (Lean Premixed Combustion) which operates at temperatures lower than conventional gas turbine combustor is currently the main technology that has demonstrated its ability to reduce emissions while maintaining high efficiency. Combustion process at low temperatures involves mixing the fuel and excessive amount of air beforehand and then delivering a lean, unburned fuel-air mixture with equivalence ratio close to the blow-out limit, to the combustion chamber [1]. Operation at low equivalence ratio\textsuperscript{1}, makes the flame sensitive to fluctuations in F/A ratio which in turn can lead to fluctuation in energy release, excessive noise and flame instability [2,3, and 4]. These fluctuations propagate away from the flame in the form of acoustic

\textsuperscript{1} lower Fuel/Air (F/A) than Stoichiometric
waves (sound) and if the combustion process occurs in a free field, the generated sound simply radiates away. However, when sound is generated in a confined region (such as a combustor) the acoustic waves reflect from the boundaries and can interfere with the combustion process at certain frequencies [5] (see Figure 1.1).

![Figure 0.1 Interacting acoustic waves and unsteady heat release](image)

As shown in Figure 1.2 these flow field variations in feedback manner coupling the heat release dynamics with acoustics. As in most feedback systems, under certain circumstances could become unstable resulting in thermo-acoustic instability of the combustion chamber.

![Figure 0.2 Flame instability, noise and fluctuation in energy release](image)
1.1 Methods of Instability Treatments

Combustion instability, if not treated can become a serious issue and may result in adverse effects on system performance, degrade operating life causing structural damages. This can offset the inherent benefits of using LPC technology for emission reduction. Methods for eliminating combustion instabilities generally fall into the two main categories of passive and active control. Active control methods have not been favorable to the practitioner mainly due to their complexity and potential instability issues [6]. The alternative passive methods are widely used for combustors despite the fact that they also have their own deficiencies. There are two ways for implementing passive methods: one by reducing or eliminating the source of variation by modifying the introduction of heat; and the other one is by removing the acoustic energy. The simplest strategy using the first approach is to shift natural frequencies of the system by changing the overall dimensions of the combustor. Other strategies involve modification of the combustor hardware such as fuel injection or flame holder geometry. The second approach is manly based on acoustic damping. It involves removing energy from the acoustic field. Adding resonators, baffles or liners are some of the practical techniques that have shown success in adding acoustic damping to the system. Among all of the passive devices, acoustic liners have been most effective for over three decades in turbo-machinery applications.

1.2 Acoustic Liner

An acoustic liner consists of a perforated screen lining the combustor ducts and a back imperforated screen. The cavity between the two screens is either divided into individual cells by a honeycomb core or is a common cavity for all holes (see Figure
1.3.a). An arrangement of an acoustic liner in a typical application is shown in Figure 1.3.b [7].

![Figure 0.3 Acoustic liner [7] a) A schematic of acoustic liner b) Liner placement in an aircraft engine](image)

An acoustic liner can also be viewed as an array of Helmholtz resonators [7]. A typical Helmholtz resonator simply made of a cavity and a neck is shown in Figure 1.3. A Helmholtz resonator absorbs sound at frequencies near its resonance by experiencing large amplitudes of acoustic oscillation in its neck and if connected to an acoustic system it can absorb the oscillation of the system [8].

![Figure 0.4 A Helmholtz resonator](image)

Damping of Helmholtz resonators are mostly created by the fluid-wall interaction due to the viscous effects or by the means of nonlinear convection of vorticity. The latter takes place in the form of interaction of vortex shedding at the neck-duct interface with acoustic waves [13, 18]. The damping behavior of a liner is mainly expressed in
terms of its acoustic impedance of Equation (1.1) which is a frequency-dependent complex quantity [11, 12 and 13]:

\[ Z = R + jX \]  

(1.1)

the resistance, \( R \), is related to the dissipative processes (visco-thermal acoustic damping or acoustic-vortex shedding interaction) and \( X \), the reactance, represents the inertia of the fluid fluctuating in and out of the orifices thru the plate thickness. The reactance affects mainly the resonant frequency; the thinner the plate the higher the resonant frequency. The goal in designing an effective acoustic liner damping device is to enhance the absorption of acoustic energy through increasing \( R \) and tuning the liner to the frequency of interest (through adjusting \( X \)). Mathematical analysis of Helmholtz resonator dynamics shows that maximum acoustic power absorption is achieved at \( X = 0 \) satisfied at the resonator resonance frequency [14]; the same is true for acoustic liners. The problem with damping devices designed based on the Helmholtz resonators is that they provide damping only within a small bandwidth.

1.3 Liners with Bias Flow

Air flow required for cooling the walls of the combustor is called bias flow (Figure 1.6 [11]). In addition to cooling the liner, this air flow blown through the perforation has interesting damping properties. For instance, increasing bias mean flow velocity increases the resistance and decreases the reactance of a perforated plate. It also extends the absorption coefficient and widens its effective frequency range by increasing the damping bandwidth of the liner [16, 17, and 52]. These unique properties are very attractive in enhancing passive damping attributes of the liner and when applied properly can become an effective damping mechanism for mitigation of thermo-acoustic instabilities.
1.4 Damping Mechanism of Liners with Bias Flow

Generally, vorticity is produced due to the viscous effects at the boundaries in the shear layers of various types. The rate of production is greatest in regions where the velocity in the primary flow changes rapidly, such as the sharp edges of an aperture. When an incoming sound wave meets vortices generated at the rims of a liner perforation, it diffuses vortices and as result of this interaction, sound is dissipated into heat. The basic underlying physical phenomenon for dissipation is the interaction between sound waves and vorticity [14, 15, 16 and 52].

In the absence of bias flow, acoustic energy dissipation by vortex-shedding is not significant and actually, the strength of the vortex sheet depends on the strength of the bias flow.
Dissipation by vortex-shedding is a nonlinear process and it depends on the convection and break up of vorticity by means of acoustic field itself. Considering that the relatively small aperture size, limits the produced vorticity to the regions near the edges of the apertures so as the interaction between vorticity and sound waves, the subsequent result is weak damping. However when a bias flow enters the orifices, a linear contribution from convection of vorticity is added to the resistance $R$ described in Equation (1.2). This linear contribution tends to have a dominant effect over the nonlinear effect when the bias flow velocity is sufficiently larger than the acoustic velocity. Bias flow sweeps away the shed vortices at the boundary edges (Figure 1.7) so that more interaction between shed vorticity and acoustic wave occurs and thus more kinetic energy is permanently lost to the acoustic wave. The linear and nonlinear damping behaviors of vortex shedding are both summarized by Equation (1.2) [17].

\[ \Pi \approx \rho \int w \times u \cdot v dV \] \hspace{1cm} (1.2)

In the absence of mean flow, a sound wave still produces and sheds vortices “$w$” but mean velocity “$u$” has the same order of magnitude as the local velocity fluctuations “$v$”; as such the damping is relatively weak. However, when there exists an independent and substantial mean flow both $u$ and $w$ can have large mean components independent of the incident sound hence a substantial linear damping is added to the system.

1.5 The Early Work

The acoustic properties of perforated plates with bias flow have been studied intensely in the past 10-30 years in order to comprehend the acoustic-vortex interaction mechanism that occurs in the lined acoustic medium causing sound absorption. In the
early work by Rayleigh [18] the acoustic conductivity of a single aperture is defined by
the ratio of mass flow rate to pressure drop across the hole. Ingard [19] found a linear
relation between pressure and velocity amplitudes for oscillatory flow with sufficiently
low amplitudes and a square-law relation for large amplitudes. Howe in 1979 [16]
derived an analytical expression for Rayleigh conductivity of an aperture in a thin and
infinite wall for a high-Reynolds low-Mach-number bias flow, as a function of
Strouhal number. Work of Howe and later Bechert [14] established the basis for
understanding the physical process of sound attenuation of an aperture with mean flow.
Hughes & Dowling [20] verified Howe’s model by a series of experiments and
expanded his model to a very thin perforated screen backed by a rigid wall and showed
rigid backing wall increases the interaction between the sound and screen which leads
to more absorption of sound. Jing and Sun’s [11] experimental work revealed that the
thickness of the screen impacts the acoustic properties of a perforated liner with bias
flow. They modified Howe’s model with an additional term to account for plate
thickness. Later in (2000) they derived the governing equation for a model with finite
thickness and solved the governing equation by numerical boundary-element method.
Bellucci et al. (2004) presented a semi-empirical model to predict the acoustic
impedance of a perforated screen with considerable aperture interactions [12, 13]. He
started from the axi-symmetric incompressible Navier-Stokes equations to find the
pressure loss and then added a term to his formulation to better fit his experimental
data. He calibrated this term by conducting a series of experiments covering a range of
plate thicknesses, aperture diameters, pressure amplitudes and bias flows.

Most works available on damping effect of liners are conducted using analytical,
empirical or semi-empirical approaches. While analytical approaches suffer from
simplifying assumption which makes them inadequate in addressing the more realistic conditions encountered in industrial applications, the empirical approaches suffer from accuracy and face many challenges and limitations in a real physical lab environment. A more sophisticated methodology that can permit parametric study on acoustic liners is clearly needed and in light of this shortcoming, numerical simulations have become very attractive. Such simulations readily allow for parametric study of flow and geometry as well, and will eventually lead to the development of tool for designing acoustic liners.

1.6 Recent Efforts

The study of combustion instabilities has entered a new phase with recent advancements in the computer hardware and software technologies which allowed the use of fine grid sizes, fine time step sizes, and large number of time steps when numerically solving Navier Stokes equations. However, Direct Numerical Simulation (DNS) for capturing all scales of the turbulent fluctuation including small and large is still out of reach for the foreseeable future [21].

The next approximation level known as Large Eddy Simulation (LES) is more promising for industrial applications. LES unlike DNS only calculates the turbulent scales larger than the grid size and filters the smaller scales. LES uses mathematical models for calculation of smaller scales instead of direct discretization. Because of the fine grid size and time step requirements, as well as the large number of time steps required to generate statistically meaningful correlations for the fluctuating components LES approach is also impractical for many engineering applications.

The method with the highest level of approximation for time dependent flows is the Unsteady Reynolds-averaged Navier-Stokes (URANS). This method is applied for
unsteady flows when the turbulent level is not very large. Even for small turbulent level, LES clearly can show more small-scale unsteady vortical structures than the URANS model. This is because the URANS only represent the ensemble averaged flow field and hence can only exhibit periodic large-scale unsteadiness while the LES represents an instantaneous snapshot of the flow field.

New unsteady approaches for performing accurate CFD have been developed by researchers with the intention to preserve the accuracy while avoiding putting extreme demands on computer resources. These intermediate models are called Hybrid and their approximation ranks between LES and URANS. Scaled Adaptive Simulation (SAS) and Detached Eddy Simulation (DES) are two of such methods. Hybrid model solves the URANS equations in the attached boundary region and switches to a LES model for detached flow regions [22]. DES is the first industrial model of high Re flows with LES content, it is an explicit mix of URANS and LES, it performs the switch from the URANS to LES by a comparison of the turbulent length-scale calculated by URANS; this produces grid dependency in the model. SAS was developed to overcome this concern by allowing the model to automatically adapt to the length scale present in the flow [23]. This is substantially different from a DES simulation as it does not explicitly splits up the flow in RANS and LES region.

Despite the development of high power computers and application of CFD methods on many industrial flows, these methods are still considered a new avenue for studying damping properties of multi-perforated liners. One study in this area that has used LES for confirming damping properties of bias flow liners was performed by Mendez et al [24, 25]. Their results showed good agreement with the numerical model of Jing & Sun [28], which accounts for the finite thickness and Belluci’s experimental
model [12, 13]. Another significant work in this area is done by Eldrige et al [26] who used LES for calculating Rayleigh conductivity of the aperture with bias flow. Their model compared well with the modified model of Howe [11] at lower frequencies and suggested more in-depth analysis to further reveal the acoustic properties of aperture. In addition to the two numerical works mentioned above there are a few other examples which have used LES for studying damping properties of liners but their contribution is not very significant.

1.7 Tools, Methods and Techniques

Most previous LES works in this area were developed by in-house codes. In this effort a commercial package, ANSYS CFX, is selected as the analysis tool. Both CFD methods of SAS and LES are applied for performing simulation studies on perforated plates. Note that in-house codes offer higher order differencing schemes, both in space and time and the most that commercial packages offer are second order differencing schemes. Despite this limitation, commercial CFD tools offer robustness which may encourage the use of LES by the practitioners who prefer robustness over high order differencing capabilities.

1.8 Thesis Objective

LES has mostly been used for verification of experimental and theoretical studies. One of the objectives of this study is to expand the use of LES beyond its current role and introduce it as a tool applicable for conducting characterization studies on perforated plates. Due to the limitation of experimental and theoretical methods, only a limited work is currently available on characterizing the acoustic impedance of the perforated plates and there is certainly a need for thorough investigation of the
impacts of geometric and flow parameters on such characterization. Utilizing LES as a tool for expressing the damping properties of perforated plate in terms of flow and geometry variables is one of the main goals of this research.

1.9 Thesis Outline

Chapter 2 reviews the existing theoretical models that quantify acoustic Raleigh conductivity, some of the previous LES work on perforated plates and some of the most recent parametric studies conducted by experimental methods.

Chapter 3 discusses a preliminary study conducted on a widely known configuration in the literature namely as Belluci configuration\textsuperscript{2}. Results of the simulation from the preliminary study are compared with the theoretical data. Preliminary study is performed to develop methods and procedures required to administer and/or automate different stages of the work including model development, processing, as well as acquisition and evaluation of the results.

Chapter 4 discusses an asymptotic study conducted on a 1-D convective wave equation to show that at low Mach numbers the compressible flow exhibits incompressible behavior. This chapter also discusses different cases that can emerge from the general case.

Chapter 5 assesses and evaluates capabilities and limitations of CFX solver for different flow conditions. Results of a case study are presented and compared with outcomes of the theoretical study performed in chapter 4.

Chapters 6, discusses the grid convergence study performed for a single perforation described in chapter 3. Several cases are subjected to a comparative study to investigate the sensitivity of the solution to the grid size. After establishing the

\textsuperscript{2} This configuration has also been used by Mendez and several other researchers
adequacy of the computational grid, the research continues on chapter 7 by studying the effects of design parameters on the acoustic attributes of a perforation. This parametric study is performed varying geometry and flow parameters in a non-dimensional form including thickness/radius ratio, Strouhal number and porosity.

Finally, Chapter 8 summarizes the results and proposes some future works.
CHAPTER 2
REVIEW OF PREVIOUS WORK

The ratio of rate of change of mass flow rate \( m^{\infty} \) to pressure drop \( \Delta P \) across a single aperture is defined as acoustic conductivity \( K_R \) [8] and it is usually expressed in frequency domain; see Equation (2.1). Note that \( \rho \) is density and \( \omega \) is frequency in the equation.

\[
K_R = \frac{m^{\infty}}{\Delta P} = j \frac{\rho \omega \dot{Q}}{\Delta P}
\]  

(2.1)

\( K_R \) is inversely related to acoustic impedance \( Z \), defined by Equation (2.2).

\[
Z = \frac{\Delta \dot{P}}{\dot{Q}} = j \frac{\rho \omega}{K_R}
\]  

(2.2)

Inviscid and incompressible flows are the two core assumptions made when developing an analytical formulation of \( K_R \). These assumptions are justified\(^3\) due to the fact that the flow thru the aperture is presumably high Reynolds, low Mach number. Navier-Stoke equation for inviscid and incompressible flow, known as Euler equation is shown by Equation (2.3).

\[
\frac{\partial u}{\partial t} + (u, \nabla)u + \frac{\nabla p}{\rho} = 0
\]  

(2.3)

\(^3\) The validity of the incompressible assumption is presented in more detail in Chapter 4.
In the absence of bias flow, contribution from the nonlinear term is negligible and Euler Equation simplifies to

$$\frac{\partial u}{\partial t} + \nabla p + \frac{\rho}{\rho} = 0$$

(2.4)

By some manipulation, the relationship between pressure and mass flow rate, i.e., Raleigh conductivity $K_R$ can be obtained from equation (2.4) and is found to be the ratio of area to the length of the aperture:

$$K_R = \frac{A}{l_e}$$

(2.5)

$K_R$ for a circular hole of zero thickness is equal to the aperture diameter (Appendix A).

### 2.1 The Howe Model (HM)

Thru the use of vector properties Howe [17] converted Euler equation into a form shown by Equation (2.6) that contains vorticity term

$$\frac{\partial u}{\partial t} + \nabla \left( \frac{1}{2} u^2 + \frac{p}{\rho} \right) = u \times \omega$$

(2.6)

By defining Bernoulli enthalpy as $B = u^2/2 + p/\rho$, applying the divergence on Equation (2.6) and using the interchangeability property between partial and divergence, Equation (2.6) converts to

$$\frac{\partial}{\partial t} (\nabla u) + \nabla^2 B = \nabla(u \times \omega)$$

(2.7)

Taking into account the incompressibility of the fluid, one arrives at

$$\nabla^2 B = \nabla(u \times \omega)$$

(2.8)

Howe linearized Equation (2.8) using the perturbation theory (by viewing the variables as the sum of their fluctuating and mean values, i.e. $\omega = \hat{\omega} + \bar{\omega}$ and $u = \hat{u} + \bar{u}$).

Equation (2.9) shows the linearized formulation.
\[ \nabla^2 B = \nabla(\vec{u} \times \hat{\omega}) \]  

(2.9)

Approximating the shed vorticity is harmonic, i.e. 
\[ \hat{\omega} = \sigma k \delta (r - \alpha) \exp(-j \omega t - \frac{x}{u}) \]

and placing it into Equation (2.9) Howe finally came up with an inhomogeneous, axi-symmetric equation for Bernoulli enthalpy. By solving the equation he derived a relationship between mass flow rate and pressure in the presence of bias flow which enabled him to find the analytical expression for Raleigh conductivity by

\[ K_X = \gamma - i \delta = 1 + \frac{\pi}{2} I_1(S_r) e^{-S_r} - j K_i(S_r) \sinh(S_r) \]  

(2.10)

\[ S_r = \frac{2 \pi f a}{U} \]  

(2.11)

Howe’s model was developed based on series of assumptions, including linear acoustic sound absorption theory, zero perforation thickness and large aperture spacing/radius (no interaction between apertures). Howe’s model has been extensively studied by many researchers working in the field of acoustic damping and put to test by a series of experiments.

2.2 Modified Howe Model (MHM)

The impact of thickness on the acoustic properties of perforation has been realized first in the modified model of Howe’s introduced by Jing and Sun [11]. They added impedance due to the thickness (Z_{th}) to the impedance due to the bias flow (Z_{bf}) and defined the total impedance Z_t as

\[ Z_t = Z_{th} + Z_{bf} \]  

(2.12)
Through the relationship between the impedance and conductivity, the mathematical expression for modified Howe conductivity ($K_{R_{M}}$) results as

\[
\frac{1}{K_{R_{M}}} = \frac{1}{K_{R_{H}}} + \frac{1}{K_{R_{B}}} \tag{2.13}
\]

$K_{R_{H}}$ and $K_{R_{B}}$ are the corresponding acoustic conductivities due to the thickness and bias flow, respectively. Substituting $K_{R_{H}}$ from Equation (2.10) and $K_{R_{B}}$ from Equation (2.5) in Equation (2.13) results in modified Raleigh conductivity of Equation (2.14)

\[
K_{R_{M}} = 2a\left(\frac{1}{\gamma - j\delta}\right) + \frac{2 l}{\pi a} \tag{2.14}
\]

MHM is widely used due to its simplicity despite the fact that it is not built based on an insightful theoretical justification. The real and imaginary parts of the Raleigh conductivity predicted by HM and MHM are depicted in Figure 2.1 for a perforation with thickness radius ratio of T/R=0.5.

![Figure 0.1 K_R of a single aperture by HM and MHM](image)

In comparison, the two models of HM and MHM have similar behavior except for the fact that MHM underpredicts the real term of acoustic conductivity for Sr>1 and also has shifted the peak in imaginary part more to the right. These changes are due to the inertia effect of thickness accounted for in the MHM.
2.3 Jing Sun Numerical Model (JSM)

Subsequent to modifying Howe’s model to account for the perforation thickness, Jing and Sun [28] took a more in-depth approach to formulating $K_R$. Their model has two fundamental differences with the original work of Howe. First, they applied the governing equation to a model with finite thickness and secondly they assumed a realistic form for the jet profile and not a simple cylindrical vortex sheet assumed in HM. They used the geometry of the free streamline obtained from the experimental data of Rouse and Abol [29] (see Figure 2.2). All the other assumptions are similar to those of Howe’s. Due to the complex jet profile assumption, Jing and Sun were not able to solve the governing equation analytically. They used boundary value method and numerically solved the equation for $T/R= 0, 0.5, 1.0, 1.5$ and 2. Figure 2.3 depicts the graphical representation of these solutions. Their results reduce to those of Howe’s in the limit of zero thickness.

Figure 0.2 Bias flow configuration and the jet profile

![Bias flow configuration and the jet profile](image)

Figure 0.3 Real and imaginary parts of $K_R$ for different thickness/radius ratios [28]

![Real and imaginary parts of $K_R$](image)
Jing and Sun compared their results with the one predicted by the MHM as well as experimental data. The experimental data agreed better with their numerical data than with the MHM. A noticeable, sharp peak in $K_R$ predicted by JSM (see Figure 2.3) is absent in HM and MHM which is a drastic behavioral change and distinguishes JSM from previous models. The frequency and sharpness of this peak seem to be geometry dependent and a function of T/R ratio. They provided no explanation for the frequency shift and variation of sharpness of this peak.

2.4 Overview of Previous LES Works

LES has been used for modeling many complex flows such as the ones associated with combustion instabilities. However, building complicated models of industrial sized liners that includes all the complexities associated with the geometry and flow of multiple interacting holes with jet flow is still too demanding and requires excessive CPU time and computer memory. As such, in most previous LES work associated with studying the acoustic damping of liners, it has been assumed that the perforations are spaced out far enough from each other so that their interactions can be ignored. With this assumption, only one perforation is normally modeled and via the use of periodic boundary condition, the outcome of the study is extended to a liner with multiple holes. Two LES works mentioned earlier in chapter 1 are discussed and reviewed in this section. The first LES work is the study conducted by Mendez et al. [24, 25] for confirming the damping properties of bias flow. The configuration used in Mendez et al. study is shown in Figure 2.4. Among HM, MHM, JSM and also Belluci experimental data [12, 13], Mendez et Al. LES data show the most agreement with numerical model developed by JSM.
Mendez linked the good agreement between JSM data and LES model to the good correlation they observed between the jet profile obtained by LES and the jet profile used in JSM. While their work clearly shows how numerical simulation can effectively be used to examine the validity of assumptions, it also underlines the importance of the accurate jet profile which was ignored in HM.

In the above and few other available LES studies performed on the perforated plates with bias flow, similar to the experimental works, reflection and absorption coefficient were used in evaluating the damping performance of the perforation\(^4\). Note that reflection and absorption coefficient can easily be traced back to Rayleigh conductivity. One unique work that extracts Rayleigh conductivity directly from the LES model is the study done by Eldrige et al.[26]. The geometry used in their analysis is substantially different from those used in previous analytical and experimental studies and is more in line with practical film cooling liners (see Figure 2.5). They used a tilted aperture with high aspect ratio (T/R=8). Their model includes grazing turbulent

\(^4\text{This is due to the difficulties of measuring acoustic conductivity in laboratory which requires measuring flow rate and pressure drop at the aperture}\)
flows in the regions above and below the aperture. They only compared their data with MHM as no JSM data is available in the literature for T/R>2.

Figure 0.5 Tilted cylindrical aperture used by Eldrige et. al [26]

Although Eldrige et. al. [26] showed good agreement at lower frequencies with MHM, but their work cannot provide a solid conclusion since the geometry and flow used in their model is different from the ones used in other studies. The good agreement with MHM can be due to the high aspect ratio used in the model. This implies that thickness may play a more dominant role for perforations with higher aspect ratios and the notion of adding $Z_{dh}$ and $Z_{bf}$ used in MHM might be justified for models with high aspect ratios.

2.5 Recent Parametric Studies on Perforated Plate

On most of the previous experimental, numerical, or theoretical works, the focus was more on the understanding of the damping mechanism associated with bias flow and less emphasis was placed on capturing the impact of different parameters that contribute to the damping mechanism. These parameters may include number, size, shape and distribution of the orifices, wall thickness, hole inclination angle, cavity volume, bias and grazing flow velocities and signal frequency.
More recently researchers have started conducting parametric studies amongst which the work by Heuwinkel et al. [54] is notable; it provides a comprehensive experimental data. They investigated the influence of several geometric, acoustic and fluid dynamic parameters on damping performance of multi-perforated liners. They studied eleven liner configurations at seven bias flow conditions and three grazing flow conditions to establish an extensive database. Table 2.1 shows the overview of configurations used in their study.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Reference 1</th>
<th>Reference 2</th>
<th>Orifice Diameter $D_k$ mm</th>
<th>Number of Orifices $n$</th>
<th>Porosity $\phi$ %</th>
<th>Wall Thickness $t$ mm</th>
<th>Cavity Volume $V$ cm$^3$</th>
<th>Single Skin</th>
<th>Double Skin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping Liner</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
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The configurations used in Heuwinkel et al. [54] study were based on a realistic liners used in engines. Moreover, similar to real engine conditions they considered the contribution of bias flow by varying the pressure ratio across the liner. They also compared the content of their experimental database with a one-dimensional low-order thermo-acoustic model proposed by Stow and Dowling [60], and observed a good agreement for a wide range of geometries, Strouhal and bias flow Mach numbers.
Another great recent work in this area is done by Andreini, et al. [55]. Although most of this work discusses the implementation and assessment of a set of available numerical tools used for analyzing perforated plates, but they also provided a good extensive parametric study on a number of variables. For their assessment study, they used three levels of complexity: a 1-D network model based on Howe’s Rayleigh conductivity, FEM to solve 3D acoustic fields in the frequency domain with a superimposed mean flow field, and finally LES. Their study revealed the possible convenience of using all the three selected methodologies at different phases of the design flow.

Most noteworthy findings of these two studies which are directly related to our parametric study are reviewed and discussed in this section. Heuwinkel et al. [54] work revealed that variation of the orifice diameter and the number of orifices is insignificant. As far as the shape of the orifice concerns, they showed while the design of the orifice edge (Figure 2.6) affects the dissipation, the orifice shape has only a minor impact. The work of Andreini et al. [55] revealed that the effect of hole inclination is important and increase in hole inclination can contribute to more damping effect however this compromises the cooling performance of the liner which can be better achieved with smaller hole inclinations. Both works showed that thickness has mainly negative effects on the damping and reduces the ability of broadband absorption of bias flow liner. Their study on the porosity distribution (hole pattern as shown in Figure 2.7), showed similar results for non-uniform and uniform cases.
Based on the extensive study on several flow and geometry parameters, both works concluded that only porosity and the pressure ratio (or bias flow velocity) are the dominant parameters. Their experimental works shows higher porosity and higher bias flow improves damping performance (Figure 2.8).
From the design standpoint the combination of these two is important, as bias flow velocity varies inversely with the porosity for the same mass flow rate. So in order to improve the damping performance, by increasing porosity, the mass flow rate should be increased to maintain the same pressure ratio/bias flow.
CHAPTER 3
PRELIMINARY MODEL

A preliminary model based on the configuration used by Mendez (see Figures 2.4 and 3.1) is built and studied. In the preliminary study Raleigh conductivity across a single aperture is extracted by calculating spatial average pressure and velocity over the aperture (similar to the work performed by Eldridge [26]; see section 2.4). Data are compared with the theoretical models discussed in Chapter 2. Consistent with theoretical models, flow is simulated as incompressible.

3.1 Model Set-up

Boundary condition assumptions for simulation are the same as those used in theoretical models. Periodicity boundary conditions are applied in both tangential directions. By definition periodicity implies that the velocity components repeat themselves in space as shown by equations (3.1) [49].

Periodicity assumption reduces the computational domain to only one perforation and also ignores aperture interactions and impact from the surrounding walls so and also ignores aperture interactions and impact from the surrounding walls so

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5 Disclaimer: This chapter is presented only to discuss assumptions and outline procedures which will be used later during the parametric study. Results presented in this chapter can be used only qualitatively and not quantitatively since the data are not subjected to the grid refinement and temporal study with a constant time step. Data in this chapter were produced based on local time stepping (constant CFL number) which is not recommended for unsteady problems and will lead to higher temporal errors.
the model can simulate infinite configuration. The same assumption was used in the

\[
\begin{align*}
    u(\vec{r}) &= u(\vec{r} + l) = u(\vec{r} + 2l) = \\
    v(\vec{r}) &= v(\vec{r} + l) = v(\vec{r} + 2l) = \\
    w(\vec{r}) &= w(\vec{r} + l) = w(\vec{r} + 2l) = 
\end{align*}
\]

development of the theoretical models of Howe, modified Howe and Jing-Sun. Model
can be excited in two different ways [51]:

1. Modulating the pressure difference across the plate and computing the resulting
   oscillations in volume flow rate.
2. Modulating the imposed bias flow and computing the resulting pressure
   oscillations.

*Figure 0.1 Model configuration, plate dimension and flow conditions*

\[ T = 1.5 \text{ mm}; \ R = 3 \text{ mm}; \ d = 35 \text{ mm}; \ \sigma = \frac{\pi d^2}{d^2} = 0.0231; \ u_{in} = 0.115 \text{ m/s}; \ u_h = u_{in}/\sigma = 5 \text{ m/s}; \text{ Flow} \]

conditions: \( M = 0.015 \) and \( Re = 2055 \) (Note: Periodicity applied on all 4 side walls)

Excitation of the model by mass flow instead of pressure provides leverage when
assessing model quality by means of mass conservation theory. Thus mass flow
fluctuation of 2\% is added to the mean flow at the inlet in the form of equation (3.2)
\[ Q = \bar{Q} + \hat{Q}\cos(\omega t) \]  

\text{(3-2)}

Pressure differential across the plate follow the same harmonic variation, i.e.

\[ P^+ - P^- = \Delta \bar{P} + \Delta \hat{P}\cos(\omega t + \phi) \]  

\text{(3-3)}

where \( P^\) is the spatial average of pressure at entrance of the hole, \( P^+ \) is the spatial average of pressure at exit of the hole and \( \nu \) is the phase difference of pressure drop signal with respect to the mass flow input. The average value is removed when calculating the Rayleigh conductivity thus the dynamic pressure drop across the hole and volume flow are simply defined by Equations (3.4) and (3.5).

\[ P^+ - P^- = \Delta \hat{P}\Re\{e^{i(\omega t + \phi)}\} \]  

\text{(3-4)}

\[ Q = \hat{Q}\Re\{e^{i\omega t}\} \]  

\text{(3-5)}

\[
\begin{array}{c}
\text{Flow} \\
\bar{P}^- \\
\bar{P}^+ \\
\end{array}
\]

Figure 0.2 Pressure drop definition

\( K_R \) is calculated by substituting for mass flow rate and pressure drop in Equation (2.3) and cancelling \( e^{i\omega t} \), see Equation (3.6)

\[ K_R = \frac{j\omega \rho \hat{Q}}{P^+ - P^-} = j \frac{\omega \rho \hat{Q}}{\Delta \bar{P}e^{i\phi}} = \frac{\omega \rho \hat{Q}}{\Delta \bar{P}} (j \cos \phi + \sin \phi) \]  

\text{(3-6)}

A procedure for calculating the phase and amplitude from CFD data is developed and provided in the APPENDIX B. Due to the frequency dependency of \( K_R \), the frequency of harmonic perturbation added to the inlet needs to vary. Neither CFX nor any other CFD tool can provide the computation in the frequency domain, so for each frequency point a simulation in time domain is needed to collect \( \hat{Q} \), \( \Delta \bar{P} \) and \( \theta \). A
sinusoidal input signal with the frequencies varying from 50-850 Hz, in steps of 100 Hz, is applied at the inlet so that a total of nine transient simulations are performed. Simulation time is chosen based on physical time of 15 periods for each run. Statistical convergence in the data is achieved after 10 periods. During data extraction the transitory part is ignored and the data from the last 5 cycles is used. Time step is another important simulation parameter which needs to be specified very cautiously. CFX as an implicit code can converge on CFL\(^6\) greater than one (large time step sizes). However, for accurate transient calculations and not to overly damp the turbulence a small CFL is advised but it should not be unnecessarily too small to require more CPU time. Extensive computational settings re-evaluations indicate that a CFL of 0.2-0.25 would accomplish acceptable accuracy in an affordable CPU time.

As to the element type, from available mesh types in CFX (full hexahedral, full tetrahedral and tetrahedral/prism\(^7\)), only full hex mesh seems to produce faster and better results with fewer cells\(^8\). Typical mesh plots used in the simulation are displayed in Figure 3.3.

![Mesh plots of the computational domain, different cross section views](image)

Figure 0.3  Mesh plots of the computational domain, different cross section views

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\(^6\)Courant–Friedrichs–Lewy condition is defined on \(CFL = (C\Delta t)/\Delta y\)

\(^7\)For a hybrid mesh prism is used for the boundaries

\(^8\)Increased number of edges in the tetrahedral does not improve the accuracy, but does lengthen solution time [49, 50]. In acoustics waves do not typically propagate as well on tetrahedral as on hex [49]. For turbulence, very few LES simulations have been published on tetrahedral [50].
CFX provides second order central differencing scheme in space and 2\textsuperscript{nd} order backward scheme in time. The lower order scheme offered by CFX is compensated for by adding more grid points to get good results [26, 50].

3.2 Results and Discussion on the Preliminary Model

Methods for evaluation of model quality are briefly discussed in this section before reviewing the results. Evaluation of the CFD data is based on two principles:

1. Continuity and conservation of mass
2. Incompressibility assumption used in the simulation

Meeting the continuity condition is measured thru introducing a parameter $Err_1$ which is defined as the difference between the „volume flow fluctuation” at the inlet and at the aperture (hole) and formulated by Equation (3.7)

$$Err_1 = 1 - \frac{\dot{Q}_h}{\dot{Q}_m}$$

(3-7)

Ideally for incompressible flow, fluctuation level at the inlet and aperture should be equal [53] and therefore $Err_1$ should be zero. Moreover, for the incompressible flow there should be no phase difference for volume flow when it travels from inlet to aperture since the forces are transferred instantaneously [48]. The incompressibility assumption is measured by $Err_2$ and is defined as phase deviation of volume flow at the aperture from that at the inlet; see equation (3.8). Ideally $Err_2$ should equal zero also.

$$Err_2 = \theta_{\dot{Q}_n} - \theta_{\dot{Q}_h}$$

(3-8)

Fluid quantities directly measured from the CFD results are listed in Table 3.1. The two checks mentioned earlier provide useful means for assessing model quality. Nonzero values of $Err_1$ indicate the introduction of flow at the side wall boundaries. This is due to the discretization error. For very weak forcing, the error in global
conservation of mass may become comparable to the forcing amplitude, but if physics is correctly implemented, this error should be reduced by using a finer grid. The 5-6% offset measured for $Err_1$ seems to be consistent for different pulsation frequencies which reinforce the idea of $Err_1$ being initiated due to the discretization error. Nonzero values of $Err_2$ indicate deviation from incompressibility; it is worth noting that although for all frequencies $Err_2$ is less than 1 degree but as the frequency increases this deviation increases.

<table>
<thead>
<tr>
<th>$f$ (Hz)</th>
<th>$\Delta P$ (Pa)</th>
<th>$\theta_{\Delta P}$ (°)</th>
<th>$Err_1$ %</th>
<th>$Err_2$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.18</td>
<td>-180.3</td>
<td>6</td>
<td>-0.2</td>
</tr>
<tr>
<td>150</td>
<td>1.13</td>
<td>-172.1</td>
<td>6</td>
<td>-0.1</td>
</tr>
<tr>
<td>250</td>
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<td>6</td>
<td>-0.2</td>
</tr>
<tr>
<td>350</td>
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</tr>
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<td>550</td>
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</tr>
<tr>
<td>650</td>
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<td>5</td>
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<tr>
<td>850</td>
<td>2.98</td>
<td>-103.5</td>
<td>5</td>
<td>-0.6</td>
</tr>
</tbody>
</table>

Table 3.1 provides the data required by equation (3.5) for calculating the real and the imaginary components of Rayleigh conductivity. Also clear from the data in Table 3.1, pressure drop increases and phase difference between flow and pressure signal decreases with increase in the forcing frequency. Deriving mathematical equations for phase and amplitude in case of a simple geometry with uniform section has shown that this behavior is expected; as will be shown in Chapter 4 the equations show the pressure amplitude is proportional to the frequency and the phase is negatively impacted by the frequency (See chapter 4).
Data shown in Table 3.1 are obtained from typical graphs of Figure 3.4 that depict flow rate in the aperture and the pressure drop fluctuations across the aperture at the sample pulsation of 250 Hz. The two traces depict the CFD outputs (blue solid line) and its curve fits (dashed line) for the sample excitation frequency of 250 Hz.

![Figure 0.4](image1)

**Figure 0.4** Typical flow rate and pressure drop at a sample pulsation of 250 Hz

Data depicted in Figure 3.5 are normalized to the Rayleigh conductivity of no bias flow. Figure 3.6 depicts normalized specific impedance of a single aperture for different Strouhal numbers. Clear from the graphs of Figure 3.5 and Figure 3.6 CFD data show the best agreement with JSM.

![Figure 0.5](image2)

**Figure 0.5** Normalized real and imaginary term of acoustic conductivity and specific impedance of a single aperture

a) acoustic conductivity vs. $Sr$  

b) specific impedance vs. $Sr$
3.3 Physical Insight from CFD Simulation

The basic components of the classic HM are an inertia term (due to the presence of mass in the vicinity of the orifice) and a dissipation term that has no frequency dependence. Using electro-acoustic analogy for the inertia and a locally-linearized flow characteristics one can model the impedance of an orifice as that of shown by Equation (3.9)

\[
Z = \frac{P}{Q} = j\omega \frac{\rho L_{\text{eff}}}{A} + \frac{1}{2C_d} \frac{U}{A}
\]

(3-9)

Where \( L_{\text{eff}} \) is the effective length of the orifice accounting for flanged-end corrections \((2 \times 0.79a, \text{where } a \text{ is the orifice radius})\), and \( C_d \) is the discharge coefficient \(-0.62\). A simple MATLAB script yields the Rayleigh conductivity plot shown in Figure 3.7 which is nearly identical to the Howe model results.

![Figure 0.6 Normalized acoustic conductivity of a single aperture versus Sr](image-url)
It is fair to say that the conversion of acoustic energy into shedding vorticity (the premise of HM for the additional dissipation due to flow) is phenomenologically modeled by the second term of the equation and, furthermore, it has no frequency dependence. The presumption is that CFD or JSM variations from this distribution are due to jet dynamics of the bias flow interacting with the acoustic forcing.

The frequency shift and sharper peaks of the LES and JSM can be explained by the better alignment of natural jet fluid dynamic modes with acoustic forcing. In order to investigate the frequency dependency of the jet dynamics, the same preliminary model was run for a wider range of frequencies (70 to 1090 Hz) and the velocity field was plotted at different frequencies (Figure 3.8).

![Figure 0.7 Velocity field, jet shape at different pulses](image)

Figure 0.7 Velocity field, jet shape at different pulses
As the plots of Figure 3.8 show jet shape is different at each frequency and as $S_r$ gets closer to unity the tail of the jet extends and as it passes the unity it starts shrinking. Plots of Figure 3.8 also show the most peak value of acoustic conductivity to be around $S_r = 1$. Hence, the role of $S_r$ is somewhat similar to the non-dimensional frequency $\omega/\omega_n$, suggesting that $U/r$ can be viewed as vortex shedding frequency$^9$ of the jet exiting the aperture. Strouhal number in this sense represents the ratio of two mechanisms that work against each other in sound attenuation (a produced vorticity by sound and the jet flow). When jet flow is dominant ($r\omega < U$) or ($S_r < 1$) the transmitted sound is reduced or more damping properties are expected by the aperture and when produced vorticity by sound is dominant ($r\omega > U$) or ($S_r > 1$), the jet flow has negligible influence on the sound.

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$^9$ This is different than the natural frequency
CHAPTER 4
LOW MACH NUMBER ACOUSTIC FLOW

Theoretically, acoustics is considered as a particular form of fluid dynamics. In general numerical treatments of acoustic flows are a significant challenge for any CFD code and in particular when the flow speed is an order of magnitude smaller than the sound speed\(^{10}\). In these situations it is not an easy task to capture the sound propagation and fluid movement both at the same time since each phenomenon has different physical behaviors and occur in different time scales and with different magnitudes. Due to these physical characteristics, numerical treatment for low Mach number oscillating flows is difficult. In this chapter some of the concerns associated with these kinds of flow are discussed and a method widely used in aero-acoustic community for low speed flow applications is introduced and conditions that make this method applicable are discussed.

4.1 Problem of Different Time Scales and Different Magnitudes

Sound propagates much faster than the moving fluid. This results in acoustic pressure reaching equilibrium quickly everywhere. For a typical flow of \(M=0.01\), if a time step is solely chosen based on the wave propagation, for a CFL condition of 0.5 and a grid size of 100 points, 20000 time steps would be required to move the fluid

\(^{10}\) In fluid dynamic this is often referred as incompressible flow
along the domain. On the other hand, if the time step is chosen based on the advection, it would be 100 times larger than the time step required for wave propagation (acoustics). Even if not face with numerical stability problems, the use of such large time scale will result in numerical accuracy issues. Hence, it is clear that it is not feasible to have time steps that obey the CFL condition for resolving the fluid motion and at the same time capture the sound propagation efficiently and accurately. The illustrated scaling problem is a multi-time scale (stiffness) problem known as one of the main difficulties in computational field of aero-acoustics. Problems of this kind, where advection is occurring at much smaller rate than the propagation are referred as stiff problems [30, 31].

Another difficulty in resolving low speed acoustic flows is that acoustic pressure fluctuations are much smaller than those of the hydrodynamic pressure; there is often no chance to capture the aero-acoustics noise within a flow calculation in an appropriate way. In these cases due to the propagation to long distances and inherent dissipation and dispersion associated with all CFD solvers, numerical errors usually produce more noise than physics does. Most robust CFD solvers are incapable of simulating acoustic flows and even advanced schemes still cannot overcome the limits of current computational capabilities.

### 4.2 Common Approach for Low Mach Acoustic Flows

Despite all the above mentioned difficulties, however, there are situations in which we are interested in flows that are near zero Mach number. A vortex dominated flow that exits from an aperture in a perforated plate is a clear example of such situation where the flow is largely incompressible and unsteady. In general, when flow is nearly incompressible and not influenced by sound waves, acoustic and flow are solved
In these situations rapid modes of the problem quickly relax to a secular equilibrium (i.e. quasi-equilibrium) and the interest is on the much longer time scale evolution of the system. Hence CPU resources are not wasted on the short time scale modes that are related to the quasi equilibrium. The common approach in the aero-acoustic field for low Mach number applications is to combine incompressible flow computations and vortex sound theory [33, 34 and 35]. This is done by first decoupling the flow from the acoustic problem and then using an acoustic code to study the propagation. The incompressible solver basically calculates unsteady velocity and pressure fields which are used as a source for acoustic propagation. Acoustic solver then computes noise levels some distance from the source.

4.3 Perforated Plate Bias Flow Problem

Sound or noise is understood as the pressure fluctuation in a medium. Acoustics as a science is only concerned about the propagation of sound in a medium and not on the creation of it. The great importance in regard to perforated plate with bias flow problem is not the propagation of sound waves but a formation of the jet at the aperture. In the underlying problem, the region of interest (vicinity of the orifice) is much smaller than the acoustic wavelength and the acoustic fluctuations created by incident sound are almost spatially uniform. This ensures incompressible treatment of this region when subjected to oscillatory pressure. All the theoretical models were established based on incompressible assumption and several other lead researchers working on this problem\textsuperscript{11} have also made the same assumption. Despite all of the above, acoustic problems are oscillatory compressible flow in nature and treating them as incompressible flow needs further justification. Furthermore assumptions allowing for

\textsuperscript{11}This assumption has also been used by Howe [16], Eldrige [24], Lee [57] and several others when establishing a theoretical model for flow thru perforated plates.
this justification need to be well understood. To reveal conditions required for the flow and geometry so the problem can be treated as an incompressible flow, an unsteady 1-D flow problem with a simple geometry is modeled and solved for different cases of non-convective compressible, convective compressible and incompressible. The imposed boundary conditions for 1-D problem are shown on Figure 4.1. The model is excited at one end and is connected to open environment at the other end (x=L).

\[ x = 0 \]
\[ u(0,t) = u_0 + u_p \sin(\omega t) \]
\[ x = L \]
\[ P(x,0) = P_0 f(x) \]
\[ P(L,t) = 0 \]
\[ u(x,0) = u_0 \]

Figure 0.1 1-D problem

4.4 Incompressible Solution

Equations (4-1) and (4-2) show the governing fluid flow equations for inviscid-incompressible 1D flow.

\[ \frac{\partial u}{\partial x} = 0 \]  
\[ \rho \frac{\partial u}{\partial t} + \frac{\partial P}{\partial x} = 0 \]  

Note that the convective term has been eliminated from Equation (4.2) because of the continuity. Therefore for the specific case of 1-D incompressible, inclusion of mean flow will not make a difference on the governing equations. Taking the divergence of Equation (4.2) in conjunction with equation (4.1) results in Equation (4-3):

\[ \frac{\partial^2 P}{\partial x^2} = 0 \]
The general solution for pressure field is linear in space as shown by Equations (4.4).

\[ P(x,t) = x\phi(t) + \psi(t) \]  (4-4)

Substituting for \( P(x,t) \) from Equation (4.4) into Equation (4.2), velocity field is found as

\[ u(x,t) = \frac{-\int \phi(t)dt}{\rho} + h(x) \]  (4-5)

Applying initial conditions \( P(x,0) = P_0 f(x) \) and \( u(x,0) = u_0 \)

\[ P_0 f(x) = x\phi(0) + \psi(0) \]  (4-6)

\[ h(x) = \frac{\int \phi(t)dt}{\rho} \bigg|_{t=0}^{t=0} + u_0 \]  (4-7)

Applying boundary condition at \( P(L,t)=0 \)

\[ L\phi(t) + \psi(t) = 0 \]  (4-8)

Hence Equation (4.6) becomes

\[ P_0 f(x) = x\phi(0) - L\phi(0) \]  (4-9)

Or \( s(0) = P_0 \) and \( f(x) = x - L \)

Applying boundary condition at \( (x=0) \) on Equation (4.5)

\[ u_0 + u_f \sin(\alpha x) = \frac{-\int \phi(t)dt}{\rho} + \frac{\int \phi(t)dt}{\rho} \bigg|_{t=0}^{t=0} + u_0 \]  (4-10)

\( \Phi(t) \) is found by (4.11) and \( P_0 = u_f \rho \omega \)

\[ \phi(t) = -u_f \rho \omega \cos(\alpha x) \]  (4-11)

Finally the unique solution for pressure field becomes

\[ P(x,t) = (L-x)u_f \rho \omega \cos \alpha x \]  (4-12)

**4.5 Compressible Solution**

The governing fluid flow equations in the case of inviscid-compressible for 1-D flow are written by Equations (4-13) and (4-14).
\[ \frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x} = 0 \]  
(4-13)

\[ \rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial P}{\partial x} = 0 \]  
(4-14)

Since acoustics processes are nearly isentropic, relationship between density and pressure becomes \((\partial P = C^2 \partial \rho)\) so that density in (4.13) is replaced by pressure

\[ \frac{\partial P}{\partial t} + \rho C^2 \frac{\partial u}{\partial x} + u \frac{\partial P}{\partial x} = 0 \]  
(4-15)

By splitting the mean and fluctuating parts of the variables

\[ u = \bar{u} + \tilde{u} \]
\[ P = \bar{P} + \tilde{P} \]
\[ \rho = \bar{\rho} + \tilde{\rho} \]  
(4-16)

and assuming the mean pressure and velocity to be constant and multiplication of the two fluctuated component to be zero, a new system of equations are obtained

\[ \frac{\partial \tilde{P}}{\partial t} + \bar{\rho} C^2 \frac{\partial \tilde{u}}{\partial x} + \bar{u} \frac{\partial \tilde{P}}{\partial x} = 0 \]  
(4-17)

\[ \bar{\rho} \frac{\partial \tilde{u}}{\partial t} + \bar{\rho} \frac{\partial \tilde{u}}{\partial x} + \tilde{P} \frac{\partial \tilde{P}}{\partial x} = 0 \]  
(4-18)

A wave equation for a moving media\(^{12}\) is then derived [8] by using the definition of Mach number \((M = \bar{u}/C)\).

\[ \frac{\partial^2 P}{\partial t^2} + 2MC \frac{\partial^2 P}{\partial t \partial x} - C^2 (1 - M^2) \frac{\partial^2 P}{\partial x^2} = 0 \]  
(4-19)

For simplicity the fluctuated symbol is dropped for the rest of this section and it is assumed that variation occurs from the initial value therefore the initial value is the

\(^{12}\) Note that Equation (4.19) converts into incompressible equation (4.3) when \((C \rightarrow \infty)\) and \(M\) becomes zero by definition, emphasizing the fact that the mean flow has no impact on the 1-D incompressible problem as noted previously in section 4.4.
same as the mean value. Equation (4.19) is solved by the Method of Characteristics for two cases of \( (M=0) \) and \( (M\neq 0) \).

**Case 1 when \( (M=0) \)**

That is when no mean flow is introduced \( (u_0=0) \) and the governing equation becomes

\[
\frac{\partial P}{\partial t} + \rho_0 C^2 \frac{\partial u}{\partial x} = 0 \\
\rho_0 \frac{\partial u}{\partial t} + \frac{\partial P}{\partial x} = 0
\]

(4-20)

(4-21)

Equation (4.19) converts into the well known classical wave equation.

\[
\frac{\partial^2 P}{\partial t^2} - C^2 \frac{\partial^2 P}{\partial x^2} = 0
\]

(4-22)

By using new coordinates along the Characteristic curves of \( \xi=x+Ct \) and \( \eta=x-Ct \) the canonical form of Equation (4.22) is obtained

\[
\frac{\partial^2 P}{\partial \eta \partial \xi} = 0
\]

(4-23)

The general solution of (4.23) is \( P(x,t)=\Phi(\eta)+\Psi(\xi) \), in which the solution in terms of the primitive variable \( x \) and \( t \) becomes \( P(x,t)=\Phi(x-Ct)+\psi(x+Ct) \). For a wave traveling to right or left with the same speed of \( C \), \( \Psi \) and \( \Phi \) are assumed to be equal, so that

\[
\psi(x) = \phi(x) = \frac{P_0}{2} \sin \left( \frac{\omega}{C} (x-L) \right) = \frac{P_0}{2} f(x)
\]

(4-24)

By replacing \( \Psi \) and \( \Phi \) into (4.26) pressure fields become

\[
P(x,t) = \frac{P_0}{2} [\sin(\omega \tau + \frac{\omega}{C} (x-L)) + \sin(\omega \tau + \frac{\omega}{C} (x-L))] \]

(4-25)

Solving for velocity field and applying \( u(0,t)=0 \), the unique solution is obtained as

\[
P(x,t) = u_0 \rho_0 C \frac{\sin \left( \frac{\omega}{C} (L-x) \right)}{\cos \left( \frac{\omega}{C} L \right)} \cos \omega \tau
\]

(4-26)
Case: 2 when \((M \neq 0)\)

When convective term in the governing equation (4.19) is not ignored, one wave travels by \(-u_0\) and the other one travels by \(+u_0\) in opposite directions. The characteristic curves of \(\xi\) and \(\eta\) for this case become \(\xi = x + C(1-M)t\) and \(\eta = x - C(1+M)t\). The canonical form of the Equation (4.19) is similar to the no mean flow case and written as

\[
\frac{\partial^2 P}{\partial \eta \partial \xi} = 0
\]  

(4-27)

The general solution in terms of \(x\) and \(t\) becomes \(P(x,t) = \Phi(x-C(1+M)t) + \psi(x+C(1-M)t)\).

The Following are assumed for general functions \(\Phi(x)\) and \(\psi(x)\):

\[
\phi(x) = \frac{P_0}{2} \sin \left( \frac{\omega}{C(1+M)} x - \frac{\omega}{C(1+M)} L \right)
\]

\[
\psi(x) = \frac{P_0}{2} \sin \left( \frac{\omega}{C(1-M)} x - \frac{\omega}{C(1-M)} L \right)
\]

(4-28)

Note that \(\Phi(x)\) and \(\psi(x)\) are constituents of \(f(x)\) and are related to waves moving in opposite directions with one traveling by speed of \(+u_0\) and the other one travelling with speed of \(-u_0\). Therefore in the case of mean flow, \(\Phi\) and \(\psi\) become different functions as opposed to the no mean flow, where they were identical. Hence, pressure fields become

\[
P(x,t) = \frac{P_0}{2} \left[ \sin \left( -\alpha x + \frac{\omega}{C(1+M)} x - \frac{\omega}{C(1+M)} L \right) + \sin \left( \alpha x + \frac{\omega}{C(1-M)} x - \frac{\omega}{C(1-M)} L \right) \right]
\]

(4-29)

Finally similarly to the no mean flow case by applying the boundary conditions\(^{13}\), the particular solution is obtained as

\(^{13}\) In this case, one has to drive an equation of velocity similar to the pressure equation (4.19) and solve the velocity equation by the same methods of characteristics equation and then can apply the boundary condition at \(x=0\) which then will obtain \(P_0\).
One implication from introducing mean flow into the classic wave equation is the creation of phase in pressure which is a function of location, frequency and mean flow velocity. The second implication is that the amplitude of oscillation has also become a function of Mach number which in the no mean flow case it is only a function of location, frequency. This drastic behavioral change is the result of including the convective term in the governing flow equations.

4.6 Asymptotic Study of 1D Wave Problem

The low Mach number limit of compressible flow should converge to the incompressible solution. The notion of recovering the incompressible solution from the compressible solution by applying principals of asymptotic study will justify the treatment of acoustic flow at low Mach number as a simple oscillatory problem. Taking the limit as \( M \to 0 \) on (4.30) the compressible solution conveniently converges to no mean flow compressible case.

\[
P(x,t) = u_j \rho C \left[ \frac{\sin \left( \frac{\omega}{C} (L - x) \right)}{1 - M^2} \right] \cos(\omega t - \frac{\omega}{C} (L - x) \frac{M}{1 - M^2})
\] (4-30)

Taylor expansion at \( x=L \) is applied on (4.31).

\[
P(x,t) = u_j \rho C \left[ \frac{\omega L}{C} \left( 1 - \frac{x}{L} \right) \frac{\omega t}{C} \left( 1 - \frac{x}{L} \right)^{1/2} \right] \cos(\omega t)
\] (4-32)
For $\omega L/C <<1$, all the higher order terms in (4.32) will be ignored and $\cos\left(\frac{\omega}{C}L\right) \approx 1$

$$P(x,t) \approx u_j \rho_0 \alpha(L - x) \cos(\alpha t) \quad (4-33)$$

As shown the compressible solution converges to the incompressible solution when both $(M= (u_0/C) \to 0)$ and $(\omega L/C <<1)$ are met. Note that for incompressible flow, these conditions are satisfied automatically. In incompressible flow case, density doesn’t change in the flow, and speed of sound becomes infinite therefore both $(M= (u_0/C) \to 0)$ and $(\omega L/C <<1)$.

At this point it is more convenient to introduce another parameter called “acoustic Mach number”\(^{14}\) defined as:

$$M_\omega = \frac{\omega L}{C} = \frac{2\pi L}{C} = \frac{2\pi L}{\lambda} \quad (4-34)$$

The convective Mach number ($M_v$) is defined by using the mean flow velocity as the characteristic velocity.

$$M_v = \frac{u_0}{C} \quad (4-35)$$

This view of $C$ going to infinity only works for the fluids in which their density cannot change inherently. However for compressible fluid, density can also remain constant if flow moves at low Mach numbers. In this case $M_v$ and $M_\omega$ become zero because of small numerator, i.e. $u \to 0$ or $\omega L <<1$ instead of large denominator i.e. $C \to \infty$. As shown, to meet the incompressibility conditions both $M_v$ and $M_\omega$ have to be sufficiently low. Using $M_\omega <<1$ along with Equation (4.34) determine for which frequencies and distances downstream $(L-x)$, the incompressibility assumption are valid. It also states that if length is sufficiently small compared to a wave length

\(^{14}\)This is equivalent to the Helmholtz number or wave number times length however for clarity a use of the term acoustic Mach number is more related to this discussion and is preferred. This term is also used in classical fluid dynamic text by Panton [48].
$L/\lambda<<1/2\pi$ then incompressible assumption becomes a valid physical assumption. This conclusion directly relates to the wave propagation thru holes in a perforated plate in which the flow inside the hole is assumed to be incompressible.

For an isentropic flow which is valid for acoustic process, density changes can be expressed as a function of Mach number [63].

$$\frac{\rho}{\rho_0} = (1 + \frac{\gamma - 1}{2} M_v^2)^{\frac{1}{\gamma - 1}}$$  \hspace{1cm} (4-36)

Note that $\gamma$ is ratio of specific heat equal to 1.4 for air. Density changes ($\Delta \rho/\rho$) less than 5% is widely acceptable for flow to be considered incompressible [48]. Solving Equation (4.36) gives $M_v < 0.3$. Subsequently, Taylor’s expansion can be used to compute pressure changes ($\Delta P/P_0$) in terms of ($\Delta \rho/\rho$) [62].

$$\frac{\Delta P}{P_0} = \gamma \frac{\Delta \rho}{\rho_0} + \frac{1}{2} \gamma (\gamma - 1) \left(\frac{\Delta \rho}{\rho_0}\right)^2 + ...$$  \hspace{1cm} (4-37)

By using only the first term of Taylor series in Equation (4.50) a pressure change of approximately 7% is obtained. Therefore, so long as the density and pressure changes in flow normalized by their stagnation values are less than 5% and 7%, respectively, it will be considered that the incompressible equations can be used to model the flow.

### 4.7 Illustrative Numerical Study Using MATLAB

The asymptotic study performed in previous section is simulated in MATLAB [47]. Solution of compressible and incompressible cases at different frequencies of (150, 250, 350, 450 Hz) are plotted for different values of ($L$=1, 0.1, 0.01) and different values of ($M_v$ =0.01 to 0.2). The amplitude of pressure for each case is depicted in Figure 4.2. On each plot the solid symbols represent the compressible solutions and the open symbols represent the incompressible solutions. Each plot has a label for $L$, $M_v$ and $M_\omega$. Plots on the LHS of the figure correspond to $M_v$=0.01 and plots on the RHS of
the figure correspond to $M_v=0.2$. Since, $M_{\omega}$, is a function of length and frequency so for each “L” several $M_{\omega}$ is calculated. Examining these plots show that increasing $M_v$ increases the deviations of the two solutions but doesn’t cause drastic changes, but $M_{\omega}$ contribution is more of a behavioral impact and increasing $M_{\omega}$ may change the form of a graph from linear into a sinusoidal. Consistent with the asymptotic study, the best agreement between compressible and incompressible solution happens when both $M_{\omega}$ and $M_v$ are very low (See Figure 4.2.e).

Figure 0.2 Compressible and incompressible solution for different Length domain and different frequency
Compressible solution (solid symbol) and incompressible solution (open symbol)
4.8 Criteria for Incompressibility

To implement studies in previous sections, three different cases are considered to explore flow and geometry conditions required for assuming incompressibility. The parameters of study are length, frequency and Mach number. Each time only one variable is changed and the other two are maintained constant. For a perforated plate problem domain length is the same as plate thickness. In plots displayed in Figure 4.3-5, pressure amplitudes are presented for both compressible and incompressible solution. Error is measured by $\Delta P/P_0$.

**Case 1: $M_\nu$ and $L$ are constant and frequency is varying**

In the plots displayed in Figure 4.3, $M_\nu$ is assigned a low value of 0.01 so that only the change in $M_\omega$ can be investigated. $M_\omega$ is varied thru the change in frequency. These plots clearly show that the two solutions of compressible and incompressible flow have more agreement for smaller $M_\omega$ or $L/\lambda$. Plots also show that the incompressible solution starts breaking away from the compressible solution as $M_\omega$ increases, however it preserves its linear behavior till $M_\omega=1$. From that critical point the
drastic behavioral changes emerge and sinusoidal behavior begins to replace the linear behavior.

As shown during Taylor expansion studies (Equation 4.32), error is a decreasing function of \((L-x)\omega/C\) indicating the locations closer to open end exhibit more incompressible behavior as all plots in Figure 4.3 follows that trend. The error plots show that when the two solutions have the most agreement the error is below 2% throughout the entire domain (Figure 4.3.a). Error plot of Figure 4.3.b shows that for Mach number of 0.3 the error is 7% as the analytical derivation in section 4.6 showed as a criterion to assume incompressibility. Plots also show as \(L/\lambda\) or \(M\omega\) increase, the error increases (Figure 4.3.c and d).

Figure 4.3 Compressible solution (solid symbol) and incompressible solution (open symbol) (constant \(L\), \(M_c\) and various \(M_\omega\))
Case 2: Frequency and $L$ are constant and $M_v$ is varying

In plots displayed in Figure 4.4, $M_\omega$ and $L$ are assigned constant value and pressure amplitude is plotted for different values of $M_v$ (0.01, 0.3, 0.5, and 0.9). Length and frequency are chosen for small $M_\omega$ so that only the impact of $M_v$ is investigated. The most agreement is shown for $M_v=0.01$ (Figure 4.4.a). The error for this case remains below 3% throughout the entire domain. For $M_v=0.3$ (Figure 4.4.b), error reaches 8.5% although Taylor analysis in section 4.6 has predicted 7%. This can be due to the fact that other parameters may also contribute to errors. Errors start growing with the increase in $M_v$ and similar to the previous case, error decreases for locations closer to the open end. Figure 4.3.c and d show that the deviation of the two solutions increases with the increase in $M_v$ until it reaches the critical value of unity. From that point drastic behavioral appears in the compressible solution and errors become significantly high. This case study demonstrates that for $M_v<0.3$, incompressible solution can sufficiently provide accurate results if $M_\omega$ and $L/\lambda$ are kept small.

![Figure 4.4 Compressible solution (solid symbol) and incompressible solution (open symbol) (constant $L$, $M_\omega$, and various $M_v$)](image)

50
Case 3: $M_\nu$ and frequency are constant and $L$ is varying

Finally, the impact of geometry on the incompressible assumptions is investigated by varying the length of domain and maintaining frequency and speed of flow constant (Figure 4.5). In this case, $M_\omega$ changes due to the variation of $L$. Data are reported for length domain of (0.03, 0.06, 0.12 and 0.15). Best agreement is shown for smaller domain length ($L/\lambda=0.032$) which is equivalent to lower $M_\omega$ (Figure 4.5.a). Plots of case 1 and 3 are very similar indicating that low values for $M_\omega$ is equivalent to having low ratio for $L/\lambda$ or small length domain.
The study of the three cases presented, as more of a graphical representation of mathematical derivation performed in 4.5. They depicted the solution of the governing equations for a simple problem and showed that to justify the incompressible assumption; certain flow and geometry conditions should be met. Proper choice of frequency, mean flow velocity and dimensions should allow the use of incompressible equations to represent compressible flows.
CHAPTER 5
VERIFICATION STUDY

Numerical characteristics of governing equations for compressible flow are fundamentally different from the ones for incompressible flow [36]. Compressible flow partial differential equations (PDE) are classified as parabolic-hyperbolic system and incompressible flow PDEs are classified as parabolic-elliptic system. CFD developers for the last 50 years have dedicated their efforts to implement methods that can cover a wide range of speed flows so it can be applied to both compressible and incompressible flows. Methods suitable for the compressible equations cannot be applied to incompressible equations without adaption. If a compressible flow solver is used at the incompressible limit without adaptation, computational efficiency and accuracy will become a significant issue [37, 38].

5.1 Singular Limit of Compressible Solver near Zero Mach Number

Mathematically speaking a singular limit occurs when the type of the equations change and number of variables is reduced. This definition fits well when Mach number goes to zero in which the governing equations convert from hyperbolic into elliptic and the number of variables drops from four to three. The zero Mach number singular limits pose a significant challenge in numerical solutions. At the limit of zero Mach number,
density is constant and independent of pressure so the time derivative vanishes from the continuity equation. Since pressure is not related to density anymore, there will be no means for updating pressure and thus the time-marching schemes become singular. Considering that the three momentum equations are the logical choice for velocity equations, one would look at the pressure-independent continuity equation as the equation for pressure. This phenomenon is commonly referred to in the literature as weak coupling of velocity and pressure fields. The singularity in the incompressible Navier-Stokes Equations leads to an ill-conditioned system of continuity Equation (5.1) and momentum Equation (5.2) [39, 40 and 41].

\[
\nabla u = 0 \quad \quad (5-1)
\]
\[
\rho \frac{\partial u}{\partial t} + \rho u \nabla u = -\nabla P + \mu \nabla^2 u \quad \quad (5-2)
\]

Or in a matrix form:

\[
\begin{bmatrix}
A_u & \nabla(.) \\
\nabla(.) & 0
\end{bmatrix}
\begin{bmatrix}
u \\
p
\end{bmatrix}
= \begin{bmatrix} 0 \\
0
\end{bmatrix} \quad \quad (5-3)
\]

In this chapter, a summary of methods developed to remove the singularity problem from zero Mach number flows are briefly discussed. Results of a case study performed using CFX are presented and compared with the theory to examine the effectiveness of CFX in dealing with such applications.

### 5.2 Different Flow Solvers

There are two major methodologies currently found in the literature for solving flow problems, primitive variable approach \((u, v, w, p \text{ or } \rho)\) and non-primitive variable approach where velocities are replaced by vorticity and the stream function. CFX works with primitive variables. Based on the choice for primitive variables, this method is
divided into pressure-based and density-based methods. CFX is a pressure-based code. Historically speaking, the pressure based method was developed for low speed incompressible flows and density based method was mainly developed for high-speed compressible flows. However, recently both methods have been extended and reformulated to solve a wide range of flow conditions [42].

**Density based solvers**

The underlying idea behind density-based methods is to remove the numerical stiffness by means of artificial compressibility such that standard numerical techniques applicable to high-speed flows also work for low Mach number flows [43].

The most well known method for density based approach is pseudo compressibility. In this method, an artificial compressibility term is added to the continuity equation and the unsteady terms are returned in the momentum equations. Hence, the system of equations becomes hyperbolic shown by Equation (5.4) and traditional techniques developed for solving subsonic compressible flows can be applied.

\[
\frac{1}{\beta} \frac{\partial P}{\partial t} + \frac{\partial u_i}{\partial x_i} = 0
\]

\[
\frac{\partial u}{\partial t} = \frac{\partial P}{\partial x_i} + H_i
\]

(5-1)

The artificial compressibility parameter (\(\beta\)) can be selected to maintain the pseudo-acoustic time scale at the same order as that of the convective time scale, reducing stiffness and optimizing convergence. Note that \(H_i\) in Equation (5.4) represents the convection and viscous terms. At steady state the pressure term in first equation drops out and thus incompressibility is recovered. For time accurate, the \(\partial t\) in the pressure equation is replaced with \(\partial \tau\), and artificial time step which is iterated on to steady state in \(\tau\). [A \((\partial u / \partial \tau)\) term is added to momentum as well].
Pressure based solvers

Incompressible flow codes have traditionally used the Poisson Equation (5.5) for pressure. This elliptic equation is derived from continuity in order to satisfy the divergence-free constraint for the velocity field. Taking the divergence of the momentum equation and applying continuity lead to the Poisson equation [44, 45 and 46].

\[ \nabla^2 P = -\rho \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \]  

(5-2)

The role of pressure in pressure-based method is to enforce continuity and for this class of equations is more of a mathematical variable than a physical one. The governing equations for pressure based methods are shown by Equation (5.6). Note that \( H_i \) represents the convection and viscous terms.

\[
\rho \frac{\partial u_i}{\partial t} = -\rho \frac{\partial u_i u_j}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_i} - \frac{\partial P}{\partial x_i} = H_i - \frac{\partial P}{\partial x_i}
\]

(5-3)

Pressure and velocity are now coupled by the system of equations (5.6). From this point pressure-based methods take two different paths. One group of methods including pressure correction and projection methods or fractional step, account for the coupling between pressure and velocity by a set of segregated equations. The other approach is based on directly solving coupled equations. This has recently gained interest due to the increase in computers memory. CFX only uses coupled approach. The pressure-based approaches are discussed in more detail in APPENDIX E.
5.3 Case Study

A test case is developed based on the 1-D unsteady problem discussed in chapter 4 and is used to confirm the accuracy of CFX pressure-based method for low Mach applications. The problem domain and imposed boundary conditions are shown on Figure 5.1. The model is excited at different frequencies of 150, 250, 350, 450 Hz and is solved for 10 cycles at each frequency. A sufficiently low flow fluctuation level i.e. 2% is chosen to reduce the nonlinearity effects in the model.

Density 1.185 kg/m³ and $C = 343$ m/sec are used for the compressible case. Periodic boundary conditions are prescribed at the side walls to eliminate any errors from boundary closure schemes that might otherwise contaminate the solution and hinder an accuracy analysis of the methods. The following two flow conditions are simulated:

1. Low Mach: Mean flow of $u_0=5$ m/sec ($M=0.015$); $u_f=0.1$ m/sec
2. Moderate Mach: Mean flow of $u_0=100$ m/sec ($M=0.3$); $u_f=2$ m/sec

Each flow case is simulated as incompressible and compressible resulting in the following four simulation cases:

1. Mean flow with low Mach number simulated as incompressible
2. Mean flow with moderate Mach number simulated as compressible
3. Mean flow with low Mach number simulated as incompressible
4. Mean flow with moderate Mach number simulated as compressible

At the end, comparing the simulation results with the corresponding analytical solution provides insight on the proper way of simulating flow for each of the flow conditions. For data collection, one data point is chosen in the incompressible region and one data point is intentionally chosen in the compressible region. This way the response of a model when a violation in the incompressibility condition occurs can be captured and compared with the data that is in the incompressible region. These data points are labeled as point 1 \( (x_1=0.045 \, \text{m}) \) and 2 \( (x_2=0.03 \, \text{m}) \) in Figures 5.2-3. During the post processing, area averaged values of pressure and velocity are collected at these points and are compared to their corresponding theoretical values.

5.4 CFD Results at \( M=0.015 \)

Pressure amplitudes calculated by simulation at points 1 \( (x_1=0.045 \, \text{m}) \) and 2 \( (x_2=0.03 \, \text{m}) \) for low Mach flows are depicted in Figures 5.2. Both theory and simulation (compressible/incompressible) depict the pressure amplitude as an increasing function of frequency.

![Figure 0.2 Pressure amplitude for points of 1 and 2 by theory and CFD (M=0.015)](image-url)
Error plots (Figure 5.3) show as the frequency increases the error of incompressible simulations grows while the error on the compressible flow remains almost constant. However, errors remain below 7% for both points \((x_1=0.045; x_2=0.03)\) at all the frequencies used in the simulations and for either case of compressible and incompressible simulations. This indicates that the incompressible simulation can sufficiently capture the behavior of low Mach number flows as long as the conditions for incompressibility are not violated.

![Error plots](image)

Figure 0.3 Error plots for points of 1 and 2 (M=0.015)

Plots of Figure 5.4 display the phase difference between points 1 and 2. Analytical solution of chapter 4 revealed that pressure phase should remain constant throughout the domain and should also be independent of the frequency. Incompressible simulation calculates a lower phase drop of about \((0.02^\circ-0.04^\circ)\) compared to the compressible simulation of \((0.04^\circ-0.1^\circ)\).
5.5 CFD Results at M=0.3

Pressure amplitudes calculated by simulation for points \(x_1=0.045; x_2=0.03\) m in the case of a moderate Mach number flow are depicted in Figure 5.5. The two simulations show similar trend as they show increase in pressure amplitude with the increase in frequency as the theory expects. However examining the error plots show that the compressible simulation outperforms the incompressible simulation in this regard.
Error plots show the compressible simulation has errors in range of (2-4 %), while the incompressible simulation has significantly higher errors in range of (6-16 %). Similar to the low Mach simulation results, for the incompressible case, errors grow rapidly with the increase in frequency increases, while the error on the compressible case remains almost constant. Also note that the error for compressible case for M=0.3 is almost half of the error for compressible case when the M=0.01 (See Figure 5.3). This indicates significant improvement when compressible simulations are applied for higher Mach number flows.

The phase difference between points 1 and 2 is displayed on Figure 5.7. A false trend is predicted by incompressible simulation showing a constant phase while theory expects a frequency dependent phase when there is a substantial mean flow. As the plots show the compressible simulation also shows a very good correlation with theory.
5.6 Conclusion on Test Case Study

The test case study shows that the overall performance of CFX is satisfactory and a good agreement with the theory is to be expected if proper simulation type is chosen based on the flow conditions. As the results of the test study show, the low Mach applications flows can be simulated as incompressible and compressible simulation only adds complexity and CPU time to the simulation and doesn’t lead to any better results. On the other hand moderate Mach flows are better represented by compressible simulation. Other factors that might have caused the difference between theory and CFD solutions for the test problem can be attributed to the issues such as not accounting for viscosity in the theoretical solution, discretization error associated with CFD in general and also a notion that case study was not a true 1-D model and a 3D computational domain was used for representing 1-D behavior.

The shortcoming of incompressible simulation at higher frequencies is due to the fact that $M_o \omega$ increases with increase in frequency and, as it was learned in Chapter 4, it is a limiting factor for incompressibility consideration. On the other hand the
compressible simulation suffers from the fact that it is not suited for computation at incompressible limit ($M$, being too low).
CHAPTER 6
GRID DEPENDENCY STUDY

Development of a proper computational grid that maintains a good balance between efficiency and accuracy is a challenging task especially when dealing with LES of a perforated plate. Turbulence produced by the bias flow in and around the aperture, where the jet is established, is the mechanism known to introduce damping to the system. The computational grid in this region (i.e., in and around the aperture) should have enough resolution so it can accurately predict the flow. This chapter discusses mesh refinement strategies and mesh sensitivity analysis for perforated plates.

6.1 Mesh Refinement Studies

Two mesh refinement approaches of a) uniform grid sizing which assigns refinement globally and b) non-uniform grid sizing which applies refinement only locally at the region of interest are commonly used. The concern with applying the uniform mesh refinement strategy to perforations is that when adding more division points in and around the aperture, the model size increases very rapidly leading to disk space and runtime memory issues. On the other hand non-uniform meshing is more susceptible to accuracy issues. The perception is that having a coarser mesh in one area of the model may impact the overall accuracy of the model since errors can simply propagate from one region to another.
To evaluate the performance of the two meshing approaches, six different meshes are developed for the model used during the preliminary study. Table 6.1 provides details of the six mesh configurations. The two parameters “div” and “TR” listed in the table are the number of division points thru the hole and the ratio of global/local\(^\text{15}\) grid sizes, respectively. TR controls the global refinement and is also a measure of mesh non-uniformity in the model while div controls the local refinement at the hole.

Models labeled as Sys 1-3 are built by uniform meshing and models labeled as Sys 4-6 are built by non-uniform meshing. Sys4 and Sys6 have similar local settings (div=4) but Sys6 has a finer mesh in the regions outside the aperture (TR=1.66). Sys5 is set to have the finest mesh in the aperture region and coarsest mesh everywhere else.

A criterion for the evaluation of mesh quality is the conformity with the theoretical models (Howe, modified Howe and Jing-Sun). For comparison purposes the impedance of a single perforation for the SAS and LES solution is displayed along with the impedance predicted by HM, MHM, and JSM. Solution for each case is provided for different levels of mesh refinements. In case of uniform mesh the solutions are labeled as 2 divisions, 3 divisions and 4 divisions and in case of non-uniform mesh the solutions are labeled as Sys4, Sys5, and Sys6.

<table>
<thead>
<tr>
<th>Grid type</th>
<th>Configuration</th>
<th>Number of division point “div”</th>
<th>Global/Local Size ratio “TR”</th>
<th>Number of grid points in the model</th>
<th>Size on the disk (MB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>Sys1</td>
<td>2</td>
<td>1</td>
<td>71,365</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>Sys2</td>
<td>3</td>
<td>1</td>
<td>528,482</td>
<td>16.0</td>
</tr>
<tr>
<td></td>
<td>Sys3</td>
<td>4</td>
<td>1</td>
<td>2,369,717</td>
<td>75.5</td>
</tr>
<tr>
<td>Non-uniform</td>
<td>Sys4</td>
<td>4</td>
<td>3.33</td>
<td>146,507</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>Sys5</td>
<td>7</td>
<td>4</td>
<td>482428</td>
<td>14.2</td>
</tr>
<tr>
<td></td>
<td>Sys6</td>
<td>4</td>
<td>1.66</td>
<td>683,789</td>
<td>22.4</td>
</tr>
</tbody>
</table>

\(^{15}\) Grid size assigned to the regions inside and outside of the hole

65
6.2 SAS Results

Figures 6.1 and 6.2 present the SAS solutions of the uniform and non-uniform mesh, respectively. Real part of the uniform mesh solution indicates considerable improvements toward JSM when one division point is added while the non-uniformed mesh solution shows less mesh dependency. Clear from the plots the imaginary part is not very sensitive to mesh for both cases of uniform and non-uniform mesh.

Figure 0.1 Impedance calculated by SAS with different uniform-grid sizing

Figure 0.2 Impedance calculated by SAS with different non-uniform-grid sizing
6.3 LES Results

Figures 6.3 and 6.4 represent the real and imaginary parts of acoustic impedance of the perforation extracted from the LES solutions of uniform and non-uniform mesh, respectively. Many similarities between SAS and LES solutions can be observed. For instance the imaginary part of acoustic impedance is not sensitive to mesh and only the real part of impedance shows mesh dependency behavior. Also similar to the SAS solution of uniform mesh, adding one division point across the hole results in significant improvement of data, judged by comparison of the results with the theoretical solution of JSM. However there are slight differences between the two. In case of uniform mesh, the mesh converged solution is achieved over $Sr<1.5$ and $Sr>2.5$ while for the SAS solution mesh converged solution was achieved only for $Sr<1.5$. For non-uniform mesh, LES has less mesh dependency across the whole range of Sr number; this is not the case for SAS.

![Normalized Specific Impedance](image)

Figure 0.3 Impedance calculated by LES with a different uniform-grid sizing
6.4 Conclusions on Mesh Refinements

Results of both SAS and LES simulations presented in Figures 6.1 thru 6.4 show better agreement with the theoretical model of Jing-Sun [28], for all different mesh configurations, than with Howe [17] and modified Howe [11]. Also numerical solutions with different mesh configurations show that the real part of impedance is more mesh dependent than the imaginary part is. The uniform mesh is more sensitive to grid size than the non uniform mesh is. Overall LES non-uniform solution shows a better mesh converged solution compared to the LES uniform, SAS non-uniform and SAS uniform.

All different mesh models show that when the refinement is continued, mesh converged solution cannot be maintained and the solution starts breaking up when Sr number passes 1.5. However the deteriorated solution tends to preserve its pattern and still agrees better with JSM.
Grid dependency study indicates that at higher Sr numbers the numerical calculation is more sensitive to grid size and the best results in terms of less mesh dependency can be expected at $Sr < 1.5$. Also looking at the data, the number of divisions in the aperture should be 4 or more, as for $Sr <$ about 2, to get fairly good agreement in terms of grid refinement.

6.5 Time Resolution Studies

A discrepant behavior is observed for all cases of uniform, non-uniform, LES and SAS at higher values of Sr number as pointed out in previous section i.e., the mesh refinement does not essentially result in the improvement of the data. This may be explained thru the role of time marching. When numerically solving hyperbolic part of differential equations of wave propagation problems, the CFL number $(C\Delta t/\Delta y)$ which ties the mesh size to the time step size becomes a very critical parameter. The basic idea behind the concept of CFL condition is that the distance a wavelike disturbance travels in a time step has to be smaller than the grid size $(C\Delta t/\Delta y < 1)$. Thus when the grid point separation is reduced, the upper limit for the time step also has to decrease.

For implicit codes like CFX, meeting CFL condition is not a requirement since stability is not a concern, however the time step should be chosen wisely for accuracy; note that reducing the time step increases the resources required for a simulation. The proper compromise choice of a time step is not easy to obtain. The time step size needs to be chosen in order to correctly describe the important physical phenomena which are being simulated. As spatial resolution is increased, by either a finer grid or higher fidelity spatial scheme, finer-scale turbulent structures are captured. For consistency, the time step must be adjusted in order to accommodate evolution and convection of these finer structures. Therefore, successful outcome of a LES or SAS simulation
solution relies on both the grid size and the accuracy of time-marching. If time marching is not done accurately, an error initiated from time-stepping scheme may remove any potential benefit from using a finer grid size. This is the area that leaves room for numerical experimentation.

Temporal studies are not commonly performed for LES due to the excessive computational cost and resources it demands. As such in a perforated plate problem, it is carried out on one of the coarser meshes described in Table 6.1 (uniform mesh with 2 division points across the hole). The results of this study are reflected in Figure 6.5. Although the model under temporal study is not highly accurate, since it is performed on a coarse mesh, however it is adequate for explaining the trend observed during the mesh refinement study and it offers guidance on how to treat the model at higher Sr numbers. Results show that computation must be adjusted according to the wave length and as more time points are added, higher frequencies tend to become more in compliance with the theoretical models. At higher frequencies, sound waves have shorter wave lengths, therefore the same mesh model show more discrepant behavior at higher Strouhal numbers.

Figure 0.5 Temporal study on impedance carried out on Sys1 model using LES
Acoustic attributes of perforated plates depend on a number of variables. While the damping concept has been proven, however it is still not possible to accurately quantify the damping performance for a given configuration. The lack of knowledge on the impact of different variables on damping effectiveness has led to lengthy trial and error based methods for designing liners. These variables are often classified as “flow variables” including the extent and Mach number of grazing and bias flow and “geometry variables” including plate thickness, aperture diameter, apertures spacing distance, apertures inclination, apertures distribution pattern, apertures shape, and apertures orientation. From previous experimental and numerical studies which was reviewed in section 2.5, it was learned that from many of these “geometric variables” some are insignificant; some may harm the cooling properties of liners\textsuperscript{16} and some be irrelevant due to their practicality issues despite their damping benefits. So for the most part, variables such as orifice inclination angle, orifice shape (which can be rectangular, cylindrical, and conical with diverging or converging nozzle), the orifice edge shape (chamfered, crossed), orifice distribution and number of orifices fit the above criteria are ignored and not included in the parametric study.

\textsuperscript{16} The primary reason for introducing bias flow is the cooling benefits and its damping property are the secondary object
This chapter aims to address only the practical factors which are known to have the most impact on the performance of dampers. In the case of bias flow the orifice mean flow velocity, thickness radius ratio and radius to orifice spacing ratio (square root of porosity) [54, 57 and 57] are the most important flow and geometry variables. Results of multiple simulations studies performed on various configurations are presented and used to discuss the effect of these major parameters.

### 7.1 Configurations

A parametric study is performed by calculating acoustic impedance of a perforation with circular orifice for thickness/radius ratios varying from 0.5 to 2 and porosity factors varying from .01 to .025, over a Strouhal number range of 0.4 to 1. LES simulation is limited to the Strouhal number range smaller than unity since as described in Chapter 6, such simulation provides less reliable data for Sr>1 unless extra computer resources are spent. Thickness/radius ratio and porosity are investigated for various perforation radii. Details of the configurations used in parametric study for thickness/radius ratio and for porosity are listed in Table 7-1 and 7-2, respectively.

<table>
<thead>
<tr>
<th>Configuration A Aperture radius (mm)</th>
<th>Porosity (σ)</th>
<th>Mach number in the aperture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Configuration B Hole spacing distance (mm)</td>
<td>3</td>
<td>.023</td>
</tr>
<tr>
<td>Configuration C Frequency (Hz)</td>
<td>3.6</td>
<td>.015</td>
</tr>
<tr>
<td></td>
<td>4.2</td>
<td>Mach number in the aperture</td>
</tr>
<tr>
<td></td>
<td>1.8, 3.6, 5.4, 7.2</td>
<td>Configuration #</td>
</tr>
<tr>
<td></td>
<td>2.1, 4.2, 6.3, 8.4</td>
<td>Aperture radius (mm)</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>Thickness (mm)</td>
</tr>
<tr>
<td></td>
<td>42</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>49</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>Sr</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>85</td>
<td>128.0</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>171.0</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.90</td>
</tr>
</tbody>
</table>
Table 0.2 Parameters used for porosity study

<table>
<thead>
<tr>
<th></th>
<th>Configuration D</th>
<th>Configuration E</th>
<th>Configuration F</th>
<th>Configuration #</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aperture radius (mm)</td>
<td>3</td>
<td>3.6</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>Input velocity (m/sec)</td>
<td>.115</td>
<td>.08</td>
<td>.06</td>
<td></td>
</tr>
<tr>
<td>porosity σ=.023</td>
<td>35</td>
<td>42</td>
<td>49</td>
<td>d = spacing distance (mm)</td>
</tr>
<tr>
<td>porosity σ=.016</td>
<td>42</td>
<td>50.4</td>
<td>58.8</td>
<td></td>
</tr>
<tr>
<td>porosity σ=.012</td>
<td>49</td>
<td>58.8</td>
<td>68.6</td>
<td></td>
</tr>
<tr>
<td>aperture Mach number:</td>
<td>.015</td>
<td>Frequency (Hz)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>100</td>
<td>85</td>
<td>Strouhal Number Sr</td>
</tr>
<tr>
<td>Mass flow rate:</td>
<td>180</td>
<td>150</td>
<td>128.0</td>
<td>0.68</td>
</tr>
<tr>
<td>.000166 kg/s</td>
<td>240</td>
<td>200</td>
<td>171.0</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Note that when studying different geometry configurations (different radii) comparison should be made based on similar flow conditions (Strouhal number and aperture Mach number).

Strouhal number is controlled by adjusting the frequency. During both thickness and porosity parametric studies, when a larger radius is used the excitation frequency is reduced to keep the Strouhal number constant and vise versa.

For Mach number, different adjustment is required. That is, during the thickness/radius ratio study when larger radius is used, the spacing distance between the holes is increased to keep the plate porosity and the resulting aperture Mach number constant and during the porosity study, the input velocity at the inlet is to increase to keep the mass flow rate in the aperture and the resulting aperture Mach number constant.

The results of the parametric study are presented by impedance plots, on subsequent pages of this chapter. All the impedance data are normalized by $\rho CM$ (where $\rho$ is the density, $M$ is the aperture Mach number and $C$ is the speed of sound).
7.2 Perforation Thickness Variations

The simplest analytical model that accounts for thickness effect, when it cannot be ignored, is the “modified Howe model”, MHM [11]. MHM only captures the inertia effects of thickness by adding a frequency dependent term to the imaginary part of the impedance predicted by the Howe model [16]. This view of thickness simply ignores its acoustic resistance properties. Figure 7.1 depicts the real and imaginary parts of acoustic impedance, as a function of $T/R$, calculated by MHM for a perforation. The impedance values calculated at each excitation frequency are labeled in terms of Helmholtz number\(^{17}\) in these plots. As various plots in Figure 7.1 show, the real component of impedance doesn’t change when thickness varies. This indicates the inadequacy of MHM in predicting resistive properties of thickness. However, as expected the reactance (imaginary part of the impedance) of the perforation increases with the increase in thickness; note that the reactance is proportional to the mass of fluid inside the perforation.

![Figure 7.1 Impedance of a perforation vs. $T/R$ predicted by MHM](image)

\(^{17}\) Helmholtz number is defined as $H_n = ka$ where $a$ is the radius of the perforation (the characteristic length) and $k$ is the wave number.
Another model that accounts for the thickness but unlike MHM captures its impact on both real (resistance) and imaginary (reactance) parts of the acoustic impedance are the JSM [28]. Figure 7.2 depicts the impedance of a perforation with bias flow for different thickness/radius ratios based on the JSM. While the imaginary part follows the same trend as the one predicted by the MHM, the real term shows different damping properties for different thickness models. Clear from these plots resistance decreases as the plate thickness increases.

The negative impact of thickness on resistance properties of a perforated plate with bias flow is also demonstrated by CFD simulation. Figures 7.3, 7.4 and 7.5 depict real and imaginary parts of impedance obtained by CFD simulation for different radii of 3, 3.6 and 4.2 mm. Resistance plots show behavior similar to the one predicted by JSM; the plots show for all different radii, resistance decrease with the increase in thickness. Reduction in resistance term of impedance results in less acoustic energy dissipation. This trend has also been observed experimentally [56], comparing the acoustic dissipation of a 3 mm thick perforation with that of 1 mm thick perforation both backed by the same size cavity and exposed to the same flow conditions.

Figure 0.2 Impedance of a perforation vs. $T/R$ predicted by JSM
Figure 0.3 Impedance of the 3.0 mm perforation vs. T/R at various Helmholtz numbers (JS: solid symbols; CFD: open symbols)

Figure 0.4 Impedance of the 3.6 mm perforation vs. T/R at various Helmholtz numbers (JS: solid symbols; CFD: open symbols)
The higher resistance of the perforation with smaller $T/R$ ratios can be attributed to the decrease in viscous effect\textsuperscript{18} of the flow for the smaller perforation. In order to illustrate the viscosity effects shear strain rate is plotted for $T/R$ of 0.5 and 2 with the same aperture radius of 3 mm (Figures 7.6 and 7.7). Flow field calculations for shear stain show that the $T/R$ of 2 has relatively higher shear strain rate compared to $T/R$ of 0.5 under similar conditions suggesting more viscous effects for $T/R$ of 2. Comparing plots of Figures 7.6 and 7.7 also show that flow is more reattached to the wall for $T/R$ of 2.

From physical standpoint when the perforation thickness increases, the exposure of the flow inside the perforation to the viscosity effects of the walls increases. This in turn makes the flow less turbulent resulting in less severe vortex shedding around the edges of the perforation leading to less energy dissipation. Therefore benefit that may

\textsuperscript{18} Viscosity of the fluid is defined as the ratio of the shear stress to the strain rate.
be achieved by viscous damping is negligible compared to the adverse impacts that it has on the damping produced by vorticity effect.

Figure 0.6 Shear strain rate field ($T/R$ of 0.5 and 2) at frequency of 120 Hz

Figure 0.7 Shear strain rate field ($T/R$ = 0.5 and 2) at frequency of 240 Hz
7.3 Perforation Radius Variations

In this section, the liner impedance of different perforation radii but the same thickness-radius ratios are compared. Figure 7.8 compares the real part of the acoustic impedance, obtained by CFD for various aperture radii at the same Strouhal number, and thicknesses to radius ratios. Bias Flow conditions are maintained constant for all different configurations. The impedance data predicted by JSM are also added to the plots for comparison purposes. CFD results compare well with the analytical data especially at smaller \( T/R \) shown by Figure 7.8. Resistance (the terms representing the damping) seems to be fairly independent of radius and a function of thickness/radius ratio only. These plots clearly show that the impact of the perforation size (radius) on the resistive component of the impedance is not significant. Experimental works by Heuwinkel et al. [54] also reported that the variation of the orifice diameter had little impact on acoustic performance.

Figure 0.8 Real component of the specific acoustic impedance of perforations with various radii and \( T/R \)
Similar comparisons are made on the imaginary part of the impedance. The imaginary part of the specific acoustic impedance (the reactance) is mainly the inertia term of the mass within and at the vicinity of the perforation. Considering that the inertia term is proportional to the "effective length" of the flow divided by the cross sectional area of the flow, the reactance data for different perforation radii are normalized by $T/R^2$ to remove the dependence of reactance on the perforation size. Figure 7.9 depicts the imaginary parts of impedance for various aperture sizes normalized by $T/R^2$. As expected different aperture sizes show very similar normalized acoustic reactance; the slight variation is mainly due to the use of thickness of the perforation instead of the "effective length" of the flow, as the normalization factor. Note that the similar normalized reactance indicates that the reactance of the larger perforation is smaller than that of the smaller perforation.

Figure 0.9  Scaled imaginary component of the specific acoustic impedance of perforations with various radii and $T/R$

---

19 Comprised of the perforation thickness and the jet length
20 Note that JS data cannot be produced for the scaled imaginary plots of Figure 7.9 since the JS data are produced only based on the $T/R$ ratio and no actual data for radius and thickness size is available to produce the $T/R^2$
7.4 Strouhal Number Variations

Bias flow velocity, or in its dimensionless from, aperture Mach number has been mentioned earlier as one of the major parameters impacting the damping performance of liners; however for parametric study, it is replaced by Strouhal number as the flow parameter of interest. The fact that Strouhal number, as shown in equation (7.1) encompasses Mach number makes the use of Mach number as a separate parameter in this study, unnecessary.

\[
S_r = \frac{\omega r}{u_b} = \frac{\omega r}{u_b} \frac{1}{C} = \frac{\omega r}{M_b}
\]

(7.1)

This is also consistent with the other studies which have viewed aperture Mach number only in the context of Strouhal number [54 and 57]. Many other non-dimensional numbers such as, convective Reynolds number \((Re)\), acoustic Reynolds number \((R_w)\), and Helmholtz number \((H_n)\) defined in section 7.2 can also be viewed as the constituents of the Strouhal number as shown in equation (7.2) and (7.3), making \(S_r\) number the all encompassing parameter for this study.

\[
S_r = \frac{H_n}{M_b}
\]

(7.2)

\[
S_r = \frac{\omega r^2 \sqrt{c}}{Vr} = \frac{R_w}{Re}
\]

(7.3)

Plots of Figures 7.10-12 indicate that for all different thickness sizes and different aperture radii more resistive properties are to be expected at lower Strouhal numbers or lower frequencies. Figure 7.10 depicts the real and imaginary parts of impedance for the perforation radius size of 3 mm, Figure 7.11 depicts the real and imaginary parts of impedance for the perforation radius size of 3.6 mm and Figure 7.12 depicts the real
and imaginary parts of impedance for the perforation radius size of 4.2 mm. These plots all show very similar trends.

Figure 0.10  Impedance of the 3.0 mm perforation vs. \( S_r \) for various \( T/R \)

Figure 0-11 Impedance of the 3.6 mm perforation vs. \( S_r \) for various \( T/R \)

Figure 0-12 Impedance of the 4.2 mm perforation vs. \( S_r \) for various \( T/R \)
MHM (Figure 7.13) and JSM (Figure 7.14) also predict trends similar to the computation simulation, and show more damping effectiveness at lower Strouhal numbers. This attribute of perforations with bias flow is very helpful for practical aspects. Considering that at low frequencies waves have large wavelengths in which require acoustic damping with high level of effectiveness.

Howe contributed the increase in damping performance of perorated plates at lower Strouhal numbers to the fact that at higher frequencies the length scales of the unsteady vorticity is small. Therefore vorticity production by the sound should have negligible influence on the flow [17].

Figure 0.13 Impedance of a perforation vs. $S_r$ for various $T/R$ predicted by MHM

Figure 0.14 Impedance of a perforation vs. $S_r$ for various $T/R$ predicted by JSM
7.5 Porosity Variation

In all of the previous configurations porosity was not varied when different aperture radii were used. That is managed by changing the aperture spacing distance according to the radius of the aperture. One of the observations from those simulations is the fact that normalized impedance doesn’t change for configurations of different radii and aperture spacing distances as long as similar porosity factors and thickness/radius ratios are used. While the importance of the porosity on the damping performance of liners is already known, this observation is also a clear indication that porosity is another dimensionless parameter which needs its own independent study.

It is already a known fact that the impedance of the perforated plate \( Z_p \) is related to porosity \( \sigma \) by equation (7.4); see APPENDIX D.

\[
Z_p = \frac{Z_h}{\sigma} \tag{7.4}
\]

The goal of this study is to show the dependency of a hole impedance \( Z_h \) on the porosity, itself. By that the the impedance of the perforated plate is better represented by Equation (7.5).

\[
Z_p = \frac{Z_h(\sigma)}{\sigma} \tag{7.5}
\]

Studying porosity in this section is done only on a single hole configuration and the periodicity assumption in space is used to expand the results of this study to the multi-hole configurations. Although studying porosity in this fashion, is very simplistic as it assumes the holes are all in phase and there is a perfect symmetry however it can be viewed as the first step toward building a multi-hole configurations.

A key parameter for studying porosity is the spacing (distance between apertures). As in the thickness study, three different aperture radii of 3, 3.6 and 4.2 mm are used
and for each radius 3 different spacing distances between apertures are used. The spacing distances for each aperture radius are selected in a way that similar porosities are archived for each radius size (See Table 7.2). This provides a convenient way for comparing the perforation impedance data from configurations with different radii but similar porosity and different porosity data and similar radii. For producing data of similar porosity and different radii, spacing distance is increased to maintain the porosity constant. Data are plotted as a function of $S_r$ number for both cases.

Other geometric parameters notably, thickness/radius ratio do not change throughout porosity study; only $T/R=1$ is used. Flow parameter (aperture Mach number and Strouhal number) are maintained constant so that the comparison is made under similar conditions.

**Constant aperture radii and different porosity**

The first group of plots presented in Figures 7.15-17, displays the impedance data corresponding to configurations with same aperture radii but different porosity. Figure 7.15 depicts the impedance for aperture radii of 3 mm, Figure 7.16 depicts the impedance for aperture radii of 3.6 mm and Figure 7.17 depicts the impedance for aperture radii of 4.2 mm.

Figure 0.15  Impedance of the 3.0 mm perforation for various porosities
Figures 7.15-17 all show very similar trends for different aperture radii. They all show that increase in porosity, results in increase in both inertia effects and resistive properties. The increase in inertia effects is related to the fact that porosity increase means decrease in the spacing distance between apertures which in turn results in the increase of the mass within and attached to the apertures. The increase in resistance needs further investigation although it agrees with previous experimental observations [54].

Flow field calculations depicted on Figures 7.18-20 are employed again to explain this observation the same way it is used to explain the impact of thickness on the resistance component of the perforation impedance. Shear strain rates for
configurations with aperture radii of 3 mm, is displayed on Figures 7.18, shear strain rates for configurations with aperture radii of 3.6 mm, is displayed in Figures 7.19 and shear strain rates for configurations with aperture radii of 4.2 mm is displayed in Figures 7.20. Comparing flow field calculation for various porosities with same radius size and same $T/R$ ratio show that when porosity increases, shear strain rate decreases or in other word the increase in porosity decreases the viscous effects. This observation can be explained by the fact that when holes are spaced out closer, the surface area of the plate which contributes to viscous forces is decreased and as mentioned during the study of thickness on section 7.2, viscous effects adversely impacts the jet created by the aperture. Thus when porosity is increased, or the spacing distance is reduced, in fact the viscous forces are reduced so the net effect is observed as an increase in the resistance properties of the hole.

Figure 0.18 Shear strain rate field ($T/R=1$ and $\sigma=0.012$) at frequency of 120 Hz
Figure 0.19 Shear strain rate field \((T/R=1 \text{ and } \sigma=0.016)\) at frequency of 120 Hz

Figure 0.20 Shear strain rate field \((T/R=1 \text{ and } \sigma=0.023)\) at frequency of 120 Hz

**Constant porosity and different radii**

The same data provided for plots on Figures 7.15-17 are used to depict the impedance data corresponding to the configurations with same porosity but different radii. Figures 7.21 depicts the calculated impedance for \(\sigma=0.012\) and various radii, Figures 7.22 depicts the calculated impedance for \(\sigma=0.016\) and various radii and Figure 7.22 depicts the calculated impedance for \(\sigma=0.024\) and various radii.
Comparing the resistance terms of various configurations with similar porosity factors but different radii shows that the real part of perforation impedance is less size dependent at lower $S_r$ numbers than it is at higher $S_r$ numbers. It should be noted that
this dependency at higher Strouhal number is very consistent and all different porosities show very similar trends.

The reactance data, i.e. the imaginary term of impedance data, are normalized by $T/R^2$ to remove the size dependency of inertia effects; see section 7.3 for justification of using $T/R^2$ as the normalizing parameter. The normalized reactance plots of Figures 7.21-23 show very similar trends for all different porosities, i.e. different aperture sizes exhibit very similar normalized acoustic reactance.
A novel parametric study was presented for exploring acoustic damping properties of perforated plates using CFD techniques. Study takes advantages of the simplicity of a configuration with a single perforation while its results can be extended to the multi perforated plates.

In Chapter 2, the theoretical models of MH, MHM and JSM developed by their corresponding authors to quantify the damping properties of perforated plates were reviewed. This review was used later for building our numerical model. Also, previous parametric studies were reviewed and learned that from so many variables that may be considered for parametric study, only a few have major contributions to the damping properties.

In Chapter 3, numerical results were presented along with the results obtained from the theoretical models which showed that the best theoretical model that predicts the damping properties of a given configuration is JSM. It showed that it is important to take into account the jet dynamics of the bias flow in the calculation which necessitates the use of CFD-based methods.

In Chapter 4, mathematical difficulties that pose significant challenges to model low speed flow applications were discussed. A mathematical study revealed conditions
required for the flow and geometry so the problem can be treated as an incompressible flow. In Chapter 5, a numerical tool (ANSYS CFX) used throughout the course of this dissertation was evaluated. Theoretical study from Chapter 4 was used for this assessment and showed good agreement between theory and simulation at low Mach numbers when flow is simulated as incompressible. It also revealed that modeling unsteady flows at low Mach numbers as compressible may not produce any improvement in results.

In Chapter 6, results of mesh refinement studies were presented and temporal studies were briefly discussed. This study showed that there is not significant mesh sensitivity at lower Strouhal numbers and the mesh sensitivity grows when $S_r > 1.5$. The best numerical results may be expected when $S_r < 1$. Divisions in the aperture appear to be important and should be looked at in more detail in future. Temporal study has revealed the importance of time step size which should be chosen in accordance with mesh grid size to increase the quality of the model at higher $S_r$ numbers. However lack of results or the potential impact on the validity at higher $S_r$ is not an issue. First in most applications the $S_r$ is relatively low and the most damping benefits from bias flow is achieved at lower $S_r$.

In Chapter 7, parametric study explored the relevant aspects of thickness, aperture radius, Strouhal number and porosity separately on the performance of acoustic dampers. Results of the parametric study demonstrated that the use of suitable non-dimensional quantities would result in similar damping properties for perforations. The similar performance observed from perforated plates with similar porosity, thickness/radius ratio and Strouhal number indicates that these non-dimensional numbers in fact are proper normalized parameters in this parametric study.
Results of parametric study show agreement with previous researches conducted in this area, and it underlines the negative impact of thickness and signifies the positive contribution from bias flow velocity and porosity on the damping properties of perforations.

One of the lessons learned during the course of this study is that the computational cost in terms of memory and time is still a very challenging issue with all the advancements in the computer technology even for conducting CFD of a single jet. This can easily become an overwhelming issue when more than one aperture or interaction effects are considered. However it is recommended for the next phase of the study, another hole be added to the current configuration which has only a single hole. By putting two holes in the model, the spacing distance between the two holes can be defined as a new parameter and by varying this distance; the porosity impact can be measured and compared with the results of a single perforation model. This will provide the critical spacing distance that allows for assumption of no interaction effects. This also introduces another variable for defining the damping characteristics of perforated plate which will be the hole interaction effects. Also by making measurement of acoustic impedance of perforated plate and acoustic impedance of a hole, the relationship between the perforated plate and hole acoustic impedance will come under further investigation. Another benefit of having two holes in the model would be removing the enforced symmetry to the breakdown of the jet, which putting two holes in would allow one to check.

Finally, it is recommended that the grazing flow be studied and included in the bias flow study. Also, it is encouraged to conduct further investigation by using the results of the presented parametric study to develop co-relations that relate the acoustic impedance of a single perforation to the non-dimensional parameters such as
thickness/radius ratio, Strouhal number, and porosity factor. A high performance parallel computing system is encouraged to provide data more efficiently so the database provided by our parametric study can be extended to a wider range of flows and geometries so a wider database can be used when developing a mathematical function for impedance of perforations.

The numerical results presented here are promising and we hope that the work which we have presented here be continued and replace the current trial and error based methods used for design of bias flow liners and find a place in designing acoustic liners.
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APPENDIX A
RAYLEIGH CONDUCTIVITY WITH NO MEAN FLOW

The Rayleigh acoustic conductivity is calculated directly from the governing equation. In the no mean flow condition, Rayleigh acoustic conductivity reaches its maximum value, knowing this value is useful to set a basis for Rayleigh acoustic conductivity.

\[
\rho \frac{\partial u}{\partial t} + \nabla P = 0 \quad (A-1)
\]

Volume integration of equation (A-1) and applying the Gauss theorem:

\[
\rho A \frac{\partial u}{\partial t} = (P - P_0)A \quad (A-2)
\]

Considering mass flow rate of \( m^e = \rho Q \) and volume flow rate of \( Q^e = A \frac{\partial u}{\partial t} \), mass flow rate becomes:

\[
m^e = \rho A \frac{\partial u}{\partial t} \quad (A-3)
\]

Combing equations A-2 and A-3 results in:

\[
l_e m^e = (P - P_0)A \quad (A-4)
\]

By definition, Rayleigh acoustic conductivity is a ratio of pressure drop to mass flow rate. For a circular hole with \( A = \pi a^2 \) and effective length defined as \( l_e = \frac{\pi a}{2} \), Rayleigh acoustic conductivity is then found to be equal to the diameter of the hole.
APPENDIX B

PHASE AND AMPLITUDE CALCULATION

A method used for extracting phase and amplitude is discussed. It was assumed that the flow is perturbed with an input signal of the form:

\[ Q = \hat{Q}\cos(2\pi ft) \]  

(B-1)

By adopting linear theory, it is assumed that pressure drop (or output signal) also follows the same form of input signal with some phase delay of \( \phi \):

\[ \Delta P = A\cos(2\pi ft + \phi) \]  

(B-2)

The goal is to find amplitude and phase of the output signal with respect to the input signal. In our approximate representation of the output signal, it is assumed that after some transitory behavior the output signal stabilizes and the amplitude and phase remain constant through the time. Let’s say \( \Delta P_1, \Delta P_2, ..., \Delta P_N \) are pressure drop across the aperture measured at discrete time points: \( t_1, t_2, ..., t_N \). Hence at each time point the pressure drop can be written as: \( \Delta P_i = A\cos(2\pi ft_i + \phi) \)

\[ \Delta P_1 = A\cos(2\pi ft_1 + \phi) \]

\[ \Delta P_2 = A\cos(2\pi ft_2 + \phi) \]

\[ \vdots \]  

(B-3)

\[ \Delta P_N = A\cos(2\pi ft_N + \phi) \]

In expanded from:
\[ \Delta P_1 = A[\cos(2\pi f_1) \cos \phi - \sin(2\pi f_1) \sin \phi] \]
\[ \Delta P_2 = A[\cos(2\pi f_2) \cos \phi - \sin(2\pi f_2) \sin \phi] \]
\[ \vdots \]
\[ \Delta P_N = A[\cos(2\pi f_N) \cos \phi - \sin(2\pi f_N) \sin \phi] \]

By the use of matrix algebra and assuming:

\[ Y_{N \times N} = \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_N \end{bmatrix}, \quad M_{N \times 2} = \begin{bmatrix} \cos(2\pi f_1) & -\sin(2\pi f_1) \\ \cos(2\pi f_2) & -\sin(2\pi f_2) \\ \vdots & \vdots \\ \cos(2\pi f_N) & -\sin(2\pi f_N) \end{bmatrix} \quad \text{and} \quad \phi_{2 \times 1} = \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix}. \]

The relation between phase and amplitude measured at different discrete time points is expressed as:

\[ \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_N \end{bmatrix} = A \begin{bmatrix} \cos(2\pi f_1) & -\sin(2\pi f_1) \\ \cos(2\pi f_2) & -\sin(2\pi f_2) \\ \vdots & \vdots \\ \cos(2\pi f_N) & -\sin(2\pi f_N) \end{bmatrix} \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix} \]

In Matrix notation, it is simply written as:

\[ Y_{N \times 1} = AM_{N \times 2}X_{2 \times 1} \] (B-6)

Both sides are multiplied by the transpose of M:

\[ M^T Y = AM^T M \phi \] (B-7)

Amplitude is found as the norm of \((M^T M)^{-1} M^T Y\):

\[ A = \| (M^T M)^{-1} M^T Y \| \] (B-8)

Phase is found by solving the equation above for \(\phi\):

\[ \phi = (AM^T M)^{-1} M^T Y \] (B-9)
APPENDIX C
RALEIGH CONDUCTIVITY AND IMPEDANCE

Raleigh conductivity, the ratio fluctuating flow by the velocity potential, is formulated by Howe for a perforation with zero thickness as:

\[ K_R = 2a(\gamma - j\delta) \]  \hspace{1cm} (C-1)

Acoustic impedance is defined as the ratio of fluctuating pressure difference across a perforation by the volume flux (volume velocity) thru the perforation. Equation C-2 shows the impedance of a perforation and its relationship with Raleigh conductivity.

Substituting for the Raleigh conductivity in the impedance equation (C-2) establishes the relationship between different terms of the impedance and different terms of the Raleigh conductivity as presented in Equations C-3 thru C-5.

\[ Z = \frac{\Delta \tilde{p}}{\tilde{Q}} = j\frac{\rho A \omega}{K_R} \]  \hspace{1cm} (C-2)

\[ Z = R + jX = \frac{\rho A \omega (-\delta + j\gamma)}{2a(\delta^2 + \gamma^2)} \]  \hspace{1cm} (C-3)

\[ R = -\frac{\rho A \omega \delta}{2a} \frac{\delta}{(\gamma^2 + \delta^2)} \]  \hspace{1cm} (C-4)

\[ X = \frac{\rho A \omega \gamma}{2a} \frac{\gamma}{(\gamma^2 + \delta^2)} \]  \hspace{1cm} (C-5)
APPENDIX D
PERFORATED PLATE IMPEDANCE

Impedance of a perforated plate can be calculated from the impedance of a hole thru the use of Raleigh conductivity. Impedance of perorated plate with aperture spacing distance of “d” is written in form of equation (D-1)

\[ Z_p = j\rho\omega \frac{d^2}{K_R} \]  \hspace{1cm} (D-1)

Spacing distance is related to the porosity and aperture area by:

\[ d^2 = A_h / \sigma \]  \hspace{1cm} (D-2)

Replacing spacing distance in equation (D-1) results in equation (D-3)

\[ Z_p = j\rho\omega \frac{A_h}{K_R\sigma} \]  \hspace{1cm} (D-3)

By definition \( j\rho\omega \frac{A_h}{K_R} \) is \( Z_h \), therefore plate impedance and aperture impedance are related thru the porosity by equation (D-4)

\[ Z_p = \frac{Z_h}{\sigma} \]  \hspace{1cm} (D-4)
APPENDIX E
PRESSURE BASED SOLVERS AND CFX

Different way of integration of governing partial differential equation (5.6) for pressure based methods may lead to different solution strategy. These may be classified as explicit, semi-implicit and fully implicit method.

Explicit methods

A forward–difference formula with space derivatives evaluated at the known time level \( n \) is known as explicit method:

\[
(\rho u_i)^{n+1} - (\rho u_i)^n = \Delta t (H_i^n - P_i^n) \tag{E-1}
\]

\[
 \partial_j (\partial p_j)^n = \partial_j H_i^n \tag{E-2}
\]

Explicit methods might be the natural choice but they face many numerical difficulties because of restrictive conditions for the step size.

Semi-implicit methods

Most commercial solvers overcome the restrictive CFL conditions by the use of semi-implicit discretization method. In semi-implicit methods, discretized equations are solved by decoupling the pressure and velocity field, thru an intermediate velocity field obtained from the momentum equations and initial guess for pressure field. This velocity field cannot satisfy the continuity equation so it has to be corrected [43]. A velocity correction is calculated based on a new pressure field, obtained from a Poisson
equation whose source terms involve the intermediate velocity. Pressure and velocity field then are updated until a divergence-free velocity field is reached [43]. Once this procedure is over, the correct pressure-velocity coupling is recovered and solution is advanced in time. This class of solvers are referred as segregated solvers since the 3 momentum equations (i.e. velocity) and the updated velocity field that are used to calculate the pressure correction equation for continuity are solved sequentially. One of the most notorious algorithms in this group is called SIMPLE (Semi-Implicit Method for Pressure-Linked Equations) is shown in below. Other variants of this approach are SIMPLEC, PISO, SIMPLER.

![Flow chart of a segregation algorithm](image)

**Fully implicit methods**

Fully implicit method evaluates all the space derivatives at the unknown time level $n+1$:

$$
(\rho u_i)^{n+1} - (\rho u_i)^n = \Delta t(H_i^{n+1} - P_j^{n+1})
$$

(E-3)
In a fully implicit code the 3 momentum equations and the pressure equation are solved simultaneously and pressure and velocity are treated as a single vectorial unknown. This approach requires more time per iteration and more memory as the matrix is bigger but it converges much faster compared to segregated approaches. This method is known as coupled solver. CFX solver uses a fully implicit coupled solver.

\begin{equation}
\partial_j \left( \partial_p \right)_i^{n+1} = \partial_j H_i^{n+1}
\end{equation}

\[ (E-4) \]

Figure E.2: Flow chart of a coupled algorithm

**CFX Solver**

For unsteady, incompressible, Newtonian fluid, the governing equations are given by:

\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} \left( \rho u_j \right) = 0 \]  
\[ (E-5) \]

\[ \frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_j} (\rho u_i u_j) = \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] - \frac{\partial P}{\partial x_i} \]  
\[ (E-6) \]
These equations in the general conservative form are known as general scalar transport equations and can be expressed by conservation laws:

\[
\frac{\partial}{\partial t} (\rho \phi) + \nabla (\rho \vec{V} \phi) - \nabla \cdot (\Gamma \nabla \phi) = Q
\]

(E-7)

For the continuity equation, \( \phi \) and \( \Gamma \) stands for 1 and for momentum equation, \( \phi \) stands for \( u_i \), and \( \Gamma \) stands for \( \mu \) and \( Q \) is the source term and is zero for the continuity equation. Integrating Equation (E.7) over the control volume \( \Delta V \) and using divergence theorem to convert volume integral into surface integral:

\[
\int_{\partial V} \frac{\partial}{\partial t} (\rho \phi) d\Gamma + \int_{\partial V} \rho \vec{V} \phi d\vec{A} - \int_{\partial V} \Gamma \nabla \phi d\vec{A} = \int_{\partial V} Q dV
\]

(E-8)

The discretized form of the governing equations using finite volume method becomes:

\[
\frac{(\rho \phi)^{new} - (\rho \phi)^{old}}{\Delta t} \Delta V + \sum_{faces} (\rho V_f \phi_f - \Gamma_f \nabla \phi_f ) A_f = Q \Delta V
\]

(E-9)

The obtained discretized equation has a mixed representation of the variables such that some are expressed at the cell centers and some at the cell faces. Some codes express scalar quantities (pressure) at the cell centers and the vector quantities (velocities) at the cell faces; this is known as staggered grid approach. However, CFX uses a co-located grid. In which the values of all the variables are calculated at the cell centers. Thus all the variables at the control volume faces should be expressed in terms of nodal values by interpolation. Despite so many advantages the co-located approach provides, however this method cause a decoupling between pressure and velocity field giving a chequerboard effect. In staggered grid, the location of each of the three momentum control volumes are shifted relative to one another and to the continuity, the grid staggering overcomes the decoupling of pressure that can otherwise arise. This is overcome by applying an alternative discretization for the mass flows proposed by Rhie Chow. The one-dimensional representation of mass conservation can be written as:
Comparing to the differential form of continuity, it is evident that the resulting numerical representation of continuity is a second order central difference approximation to the first derivative in velocity modified by a fourth derivative in pressure which acts to redistribute the influence of the pressure. Whereas the second order first derivative numerical representation of continuity, equation (E-10), admits a decoupled pressure field, the addition of the fourth derivative pressure term to the first derivative provides a well behaved numerical continuity equation that does not admit pressure field decoupling. As the grid is refined, the magnitude of the second term in equation (E-10) goes to zero at the rate $\Delta x^3$ relative to the velocity derivative, so that the desired differential form of continuity is quickly recovered. Applying interpolation schemes to interpolate the pressure values at the faces in terms of the nodal values and substituting corresponding values of $\Theta$, $\Gamma$ and $Q$ for momentum and obtaining the pressure equation from the continuity equation combined by Rhie-Chow interpolation complete the set of equation. Old values of $\Theta$ are used as a source term and all the convection and diffusion and pressure gradient terms in the momentum are treated implicitly so a linear set of equations that arise by applying the Finite Volume Method to all elements is written in the form:

$$\sum_{nb} a_{i}^{nb} \phi_{i}^{nb} = b_{i}$$

(E-11)

Where $\phi$ is the solution, $b$ the right hand side, the coefficients of the equation, $i$ is the identifying number of the finite volume or node in question, and $nb$ means “neighbor”.
The set of these, for all finite volumes constitutes the whole linear equation system. For the coupled, 3D mass-momentum equation set, they are a \((4 \times 4)\) matrix or a \((4 \times 1)\) vector, which can be expressed as:

\[
a_{i}^{nb} = \begin{bmatrix}
    a_{uu} & a_{uv} & a_{uw} & a_{up} \\
    a_{vu} & a_{vv} & a_{vw} & a_{vp} \\
    a_{wu} & a_{wv} & a_{ww} & a_{wp} \\
    a_{pu} & a_{pv} & a_{pw} & a_{pp}
\end{bmatrix}^{nb} \\
\phi_{i}^{nb} = \begin{bmatrix}
    u^{nb} \\
    v^{nb} \\
    w^{nb} \\
    p^{nb}
\end{bmatrix}, \quad b_{i} = \begin{bmatrix}
    b_{u} \\
    b_{v} \\
    b_{w} \\
    b_{p}
\end{bmatrix}
\]