OPTICAL TWEEZERS USING CYLINDRICAL VECTOR BEAMS

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By
Chenchen Wan
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ABSTRACT

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Name: Wan, Chenchen
University of Dayton

Advisor: Dr. Qiwen Zhan

Optical trapping by a highly focused laser beam has been extensively used for the manipulation of submicron-size particles and biological structures. Usually the gradient force will support a stable trapping while the scattering force will push the particles away and destroy the stable trapping. Metallic particles are generally considered difficult to trap due to strong scattering and absorption forces. As one class of spatially variant polarized beams, cylindrical vector beam (CV beam) is proven to have advantage for metallic particle trapping because the axial scattering force is identical zero.

In this work, CV beam is generated and applied to trap metallic nanoparticles while the expected stable trapping is not observed. Several possible reasons are examined and the argument that a curl scattering force is responsible for the trapping difficulty is proposed. The curl scattering force is usually neglected in previous research for linearly polarized beam trapping. Numerical simulation shows the curl scattering force can be significant when vectorial beams such as CV beams are used for trapping. After realizing the significance of the curl force, innovative methods to engineer the forces are proposed and investigated to find the fields distribution which could support three dimensional stable trapping.
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CHAPTER 1
INTRODUCTION

Optical trapping by highly focused laser beams has been extensively studied and applied for the manipulation of micro particles and biological structures since Arthur Ashkin demonstrated the first practical laser trapping. In the early 1970s, he and his colleagues showed that micron-sized particles could be accelerated by the forces of radiation pressure from a single cw laser beam and be trapped in a stable optical potential well in a series of pioneering research papers [1]. He also developed a stable three dimensional trapping based on counter propagating laser beams [2]. Eventually Ashkin developed stable optical trapping based on single-beam gradient force, which is now commonly known as optical tweezers. Since then, optical tweezers have been successfully employed to a wide-range series of experiments from the cooling and trapping of neutral atoms [3] to manipulating live bacteria and viruses [4-6].

For metallic Rayleigh particles (particle size is much smaller than the trapping wavelength), stable trapping is generally considered difficult to achieve due to strong scattering force. Theoretical simulation shows that the cylindrical vector beam has advantage for stable three dimensional trapping of metallic particles through elimination of the scattering force. In this thesis, experiments are performed to demonstrate this advantage. In Chapter 2, the mechanics of optical trapping especially the optical forces of Rayleigh particles are derived. Then CV beams are introduced and their potential advantage for metallic nanoparticles trapping is presented. In Chapter 3, experimental methods to generate CV beams in free space and by few-mode optical fiber are introduced. Also the expression of the electric fields at the focal region of a high
numerical aperture (NA) objective lens are derived and used to numerically calculate the optical forces. Experimental results are discussed in Chapter 4. Radially polarized beam at 1064 nm is generated with a piece of few-mode optical fiber and used to trap golden nanoparticles. The trapping results show that stable trapping of polymer beads can be easily achieved while stable trapping for metallic particles remains difficult. One of the possible reasons of this difficulty in trapping metallic particles may arise from the existence of a newly discovered curl scattering force, which is often ignored in the previous research due to its usually small magnitude in the trapping by linearly polarized beam. In Chapter 5, detailed derivation and simulation of optical forces especially the curl scattering force is given. During the simulations it is found that the curl force could have negative value under certain condition. The negative scattering force may be engineered to some desired distributions that could actually support stable trapping of metallic particles or be used in some other applications such as the “tractor beams” that are usually seen in science fictions. The whole thesis is summarized and future possible researches are given in the last Chapter.
CHAPTER 2
PRINCIPLES OF OPTICAL TRAPPING AND CYLINDRICAL VECTOR BEAMS

2.1 Principles of Optical Trapping

Optical tweezers are formed by a laser beam tightly focused by a high NA objective lens so that a microscope can be used to build a trapping system. Due to the nature that photons have linear momentum, when a micron-sized particle located near the focal region interacts with photons, the exchange of linear momentum between particle and photons will create an optical force on the particle. If there is an equilibrium point of the force distribution, the particle can be trapped at the equilibrium point. Follow the tradition the optical force can be decomposed into two components: a scattering force acts “pushing” the particle along the light propagation direction and a gradient force that is along the direction of the spatial gradient of the optical intensity. Generally the gradient force supports the trapping while the scattering force destroys the trapping. The scattering force can be explained as the forward moving photons strike the particle and are scattered back or any other directions with a net forward linear momentum being transferred to the particle. The gradient force arises from the fact that a dipole in an inhomogeneous electric field experiences a force in the direction of the intensity gradient. In an optical trap, the focused laser field induces fluctuating dipoles in the dielectric particle. These dipoles interact with the electric field and produce the gradient force. The gradient force is proportional to both the polarizability of the dielectric particle and the optical intensity gradient at the particle location. To establish stable trapping in all three directions, the axial component of the gradient force pulling the particle towards the focal region must exceed the axial component of the scattering force pushing the particle away from that region. This condition requires a high
NA objective lens to provide a very steep gradient in the electric field within the focal volume. The balance between the forces forms an equilibrium position where the particle can be trapped. When the particle moves slight off the equilibrium position the gradient force can be approximately treated proportional to the displacement of the particle, which acts like a Hookean force that restores the particle to the trapping point.

Typically two extreme situations are considered in optical trapping. When the particle size is much larger than the trapping wavelength, ray optics can be used to describe the origins of optical forces. For the cases with particle size much smaller than the trapping wavelength, the forces can be calculated under dipole approximation.

2.2 Ray Optics Illustrations of Optical Forces

When the trapped particle size is much larger than the wavelength of the trapping laser, i.e., the radius $a \gg \lambda$, the conditions for Mie scattering are satisfied, and optical forces can be illustrated from simple ray optics [7].

Figure 1. Ray optics description of the optical force. (a) Light intensity profile is asymmetric, resulting in a net force towards downright. (b) Symmetric light intensity produces a net force that is downward and points to the focus.
In Figure 1(a), a spherical transparent particle is illuminated by a parallel beam of light with an intensity increasing from left to right. Two representative rays of light of different intensities (represented by black lines of different thickness) from the beam are shown. The momentum gained by the particle has the same amount but in the opposite direction as momentum lost by photons due to momentum conservation. The corresponding forces are presented as gray arrows in the figure. The net force on the bead points downright and its transverse component is in the direction of the intensity gradient. In Figure 1(b), the particle is illuminated by a focused beam of light with a radial intensity gradient. Two representative rays are again refracted by the particle. However the change in momentum leads to a net force towards the focus and decreases away from the focus. If the bead moves in the focal region the imbalance of optical forces will draw it back to the equilibrium position so the bead could be stable trapped. From Figure 1, one can see that the scattering force and the gradient force essentially both originate from the transfer of linear momentum from photons to particles.

2.3 Optical Force for Rayleigh Particles

Optical forces on small (Rayleigh) particles are one of the main topics of this thesis and will be derived in details under dipole approximation [8]. From the point of view of classical electrodynamics, the net force exerted on an arbitrary object is entirely determined by Maxwell’s stress tensor $T$ [9]. For simplicity, let’s consider an object in vacuum and in the presence of a harmonic electromagnetic field of frequency $\omega$. The time averaged force can be written as:

$$\langle F \rangle = \int d^3r \nabla \langle T(\vec{r}) \rangle = \int_A \langle T \rangle \cdot \vec{n} dA,$$

(2.1)

where $A$ is any arbitrary closed surface enclosing the object and:

$$\nabla T = \epsilon_0 \vec{E} \left( \nabla \cdot \vec{E} \right) + \epsilon_0 \left( \nabla \times \vec{E} \right) \times \vec{E} + \mu_0 \vec{H} \left( \nabla \cdot \vec{H} \right) + \mu_0 \left( \nabla \times \vec{H} \right) \times \vec{H},$$

(2.2)

with:
\[ T_{ij} = \epsilon_0 E_i E_j - \mu_0 H_i H_j - \delta_{ij} \frac{1}{2} \left( \epsilon_0 |\vec{E}|^2 + \mu_0 |\vec{H}|^2 \right), \]  

(2.3)

where the electric and magnetic field vectors, \( \vec{E} \) and \( \vec{H} \) correspond to the total electromagnetic fields including both the external and induced fields. For a small particle, the forces can also be expressed in terms of the external fields. The total fields in the vacuum outside the particle, \( \vec{E} \) can be written as the sum of external (polarizing or incoming) \( \vec{E} \) and induced polarization. For a spherical Rayleigh particle with radius \( a \) and relative permittivity \( \epsilon(\omega) \) and located at \( \vec{r} = \vec{r}_i \), the total electric field is

\[ \vec{E} = \vec{E}(\vec{r}) + \vec{G}(\vec{r} - \vec{r}_i) \cdot \vec{p} = \vec{E}(\vec{r}) + \alpha \vec{G}(\vec{r} - \vec{r}_i) \vec{E}(\vec{r}_i), \]  

(2.4)

where \( \vec{G} \) is the free space Green function,

\[ G_0(\vec{r}) = \left( k_0^2 \delta_{ij} + \partial_i \partial_j \right) g(r); \quad g(r) = \frac{e^{ik_0r}}{4\pi\epsilon_0 r}, \]  

(2.5)

\( k_0 = \omega/c \) is the wave number, \( \vec{p} = \alpha \vec{E}(\vec{r}) \) is the induced dipole, and the polarizability \( \alpha \) is given by

\[ \alpha = \frac{\alpha_0}{1 - i\alpha_0 k_0^3/(6\pi\epsilon_0)}, \quad \alpha_0 = 4\pi\epsilon_0 a^3 \frac{\epsilon - 1}{\epsilon + 2}. \]  

(2.6)

The time averaged force [Eq.(2.1)] can be rewritten in terms of the dipole moment and the external polarizing field as

\[ \langle \vec{F}(\vec{r}_i) \rangle = \frac{1}{2} \text{Re} \left\{ \sum_i p_i \nabla E_i^* (\vec{r}_i) \right\} = \frac{1}{2} \text{Re} \left\{ \sum_i \alpha E_i (\vec{r}_i) \nabla E_i^* (\vec{r}_i) \right\}. \]  

(2.7)

For harmonic field solutions of Maxwell’s equations, there is \( \nabla \times \vec{E} = i\omega \mu_0 \vec{H} \) and using the identity

\[ \sum_i E_i \nabla E_i^* = (\vec{E} \cdot \nabla) \vec{E}^* + \vec{E} \times (\nabla \times \vec{E}^*), \]  

(2.8)
the dipole force in Eq.(2.7) can be rewritten as the summation of three terms:

$$\langle F \rangle = \frac{1}{4} \text{Re}\{\alpha \} \nabla |E|^2 + \sigma \frac{1}{2} \text{Re}\left\{ \frac{1}{c} E \times H^* \right\} + \sigma \frac{1}{2} \text{Re}\left\{ \frac{\varepsilon_0}{k_0} (E \cdot \nabla) E^* \right\},$$  \hspace{1cm} (2.9)

where the total cross section of the particle is defined as $\sigma \equiv k_0 \text{Im}(\alpha)/\varepsilon_0$. The first term is proportional to the gradient of the field intensity so that it corresponds to the gradient force. The second term is proportional to the time averaged Poynting vector $\left( 2 \langle \hat{S} \rangle = \text{Re}[E \times H^*] \right)$ and is identified as the radiation pressure scattering force. The third term is usually neglected in the discussions of optical forces on small particles since it is zero when the field only has a single plane wave component. It has not been received special attention until recently.

Considering the following identity,

$$-2i \text{Im}\left[ (E^* \cdot \nabla) E \right] = (E \cdot \nabla) E^* - (E^* \cdot \nabla) E = \nabla \times (E \times E^*),$$  \hspace{1cm} (2.10)

which is valid for the external field with $\nabla \cdot E = 0$, one can then rewrite the third term in Eq. (2.9) as

$$\sigma \frac{1}{2} \text{Re}\left\{ \frac{\varepsilon_0}{k_0} (E \cdot \nabla) E^* \right\} = \sigma c \nabla \times \left\{ \frac{\varepsilon_0}{4\omega_0} (E \times E^*) \right\}.$$  \hspace{1cm} (2.11)

Defining the time averaged spin density of a transverse electromagnetic field [8]:

$$\langle \hat{L}_s \rangle \equiv \frac{\varepsilon_0}{4\omega_0} (\hat{E} \times \hat{E}^*),$$  \hspace{1cm} (2.12)

one can finally write the total force on a Rayleigh particle as

$$\langle \vec{F} \rangle = \text{Re}\{\alpha\} \left\{ -\nabla \frac{1}{4} |E|^2 \right\} + \sigma \left\{ \frac{1}{c} \langle \hat{S} \rangle \right\} + \sigma \left\{ c \nabla \times \langle \hat{L}_s \rangle \right\}.$$  \hspace{1cm} (2.13)

The combination of the second term and the third term is called scattering force. The two contributions constitute the traditional radiation pressure term that is proportional to the time averaged Poynting vector and a curl scattering force that is associated to the nonuniform distribution of the spin density of the light field. By definition, the latter term corresponds to a
non-conservative force. When the light has axial symmetric polarization state in both amplitude and phase, this curl scattering force becomes comparable or even larger than the radiation pressure. So the curl scattering force cannot be neglected and has to be included in optical force calculation for radially polarized beam trapping. In this thesis, curl force maybe one factor that leads to the difficulty in metallic nanoparticle trapping experiment.

2.4 Cylindrical Vector Beams

Polarization is one important property of light. Utilization of the vectorial nature of light has attracted great interests to build innovative optical devices and systems [10]. Recently there is an increasing interest in light beams with spatially variant state of polarization within the beam cross section. One particular example is laser beam with cylindrical symmetry in polarization, the so called CV beams. Since the polarization state is spatial dependent the full vector wave equation for the electric field must be considered:

$$\nabla \times \nabla \times \vec{E} - k^2 \vec{E} = 0, \quad (2.14)$$

with assumption of an $\exp(-i\omega t)$ time dependence of the electric field and $k=\omega/c$, an axially symmetric beamlike vector solution with the electric field aligned in the azimuthal direction should have the form of

$$\vec{E}(r,z) = U(r,z) \exp[i(kz - \omega t)] \hat{e}_\phi, \quad (2.15)$$

where $\hat{e}_\phi$ is the unit azimuthal unit vector. Under the paraxial and slow varying envelope approximation, $U(r,z)$ satisfies the following equation:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial U}{\partial r} \right) - \frac{U}{r^2} + 2ik \frac{\partial U}{\partial z} = 0. \quad (2.16)$$

The solution that obeys azimuthal polarization symmetry has the trial solution

$$U(r,z) = E_0 j_1 \left( \frac{\beta r}{1 + iz/z_0} \right) \exp \left[ -i\beta^2 z/(2k) \right] u(r,z), \quad (2.17)$$
where \( u(r, z) \) is the fundamental Gaussian solution given by

\[
u(r, z) = \frac{w_0}{w(z)} \exp\left[-i\varphi(z)\right]\exp\left[i \frac{k}{2q(z)} r^2 \right], \tag{2.18}\]

and \( J_1(x) \) is the first order Bessel function of the first kind. In Eq.(2.18), \( w(z) \) is the beam size, \( w_0 \) is the beam size at beam waist, \( q(z) = z - iz_0 \) is the complex beam parameter, \( z_0 = \frac{\pi w_0^2}{\lambda} \) is the Rayleigh range, \( \varphi(z) = \tan^{-1}\left(\frac{z}{z_0}\right) \) is the Gouy phase shift. Similarly, a transverse magnetic field solution \( H(r, z) \) in the azimuthal direction should exist. For this azimuthal magnetic field solution, the corresponding electric field in the transverse plane is aligned in the radial direction.

The mode described in Eq.(2.17) is also called Bessel-Gaussian beam. As comparison, the common solutions to scalar Helmholtz equation:

\[
\left(\nabla^2 + k^2\right)E = 0, \tag{2.19}
\]

are also given. In Cartesian coordinates, the solution is Hermite-Gauss (HG\(_{mn}\)) beam and has the form of

\[
u(x, y, z) = E_0 H_m \left(\sqrt{2} \frac{x}{w(z)}\right) H_n \left(\sqrt{2} \frac{y}{w(z)}\right) \exp\left[-i\varphi_{mn}(z)\right]\exp\left[i \frac{k}{2q(z)} r^2 \right], \tag{2.20}
\]

where \( H \) is the Hermite polynomials that satisfies the equation:

\[
\frac{d^2H_m}{dx^2} - 2x \frac{dH_m}{dx} + 2mH_m = 0, \tag{2.21}
\]

\( \varphi_{mn}(z) = (m + n + 1)\tan^{-1}\left(\frac{z}{z_0}\right) \) is the Gouy phase shift, \( q(z) \) is the complex beam parameter defined above, note when \( m = n = 0 \), HG\(_{00}\) mode has the same form of the fundamental Gaussian mode as in Eq.(2.18).

Another solution of Eq. (2.19) in the cylindrical coordinates follows the form:
\[ u(r, \phi, z) = E_0 \left( \sqrt{2\pi} \frac{r}{w(z)} \right)^l L_p^l \left( 2 \frac{r^2}{w^2(z)} \right) \frac{w_0}{w(z)} \exp \left[ \frac{-i\varphi_{pl}(z)}{2q(z)} \right] \exp (i\phi) \]  
\[ (2.22) \]

where \[ L_p^l(x) \] donates the Laguerre polynomial which satisfies the following equation:

\[ x \frac{d^2 L_p^l}{dx^2} - (l + 1 - x) \frac{dL_p^l}{dx} + pL_p^l = 0, \]  
\[ (2.23) \]

\[ \varphi_{pl}(z) = (2p + l + 1) \tan^{-1}(z/z_0) \] is the Gouy phase. Similarly, when \[ p=l=0, \] the solution reduces to the fundamental Gaussian form shown in Eq. (2.18). The spatial distributions and the polarization states of these modes are illustrated in Figure 2.

Figure 2. Spatial distributions of instantaneous electric vector field for several conventional modes and CV modes: (a) \( x \)-polarized fundamental Gaussian mode; (b) \( x \)-polarized HG\(_{10} \) mode; (c) \( x \)-polarized HG\(_{01} \) mode; (d) \( y \)-polarized HG\(_{01} \) mode; (e) \( y \)-polarized HG\(_{10} \) mode; (f) \( x \)-polarized LG\(_{01} \) mode; (g) radially polarized mode; (h) azimuthally polarized mode; (i) generalized CV beams as a linear superposition of (g) and (h).
In many applications, a simpler distribution of the field can be used instead of the vector Bessel-Gaussian solution described in Eq.(2.17). For very small $\beta$, the vector Bessel-Gauss beam at the beam waist can be approximated as

$$E(r,z) = Ar \exp\left(-\frac{r^2}{w^2}\right) \hat{e}_i, \quad i = r, \phi.$$ \hspace{1cm} (2.24)

The amplitude profile is exactly the same as LG$_{01}$ mode without the vortex phase term $\exp(i\phi)$. It is easy to show that CV beams can also be expressed as the superposition of orthogonally polarized Hermite-Gauss HG$_{01}$ and HG$_{10}$ modes:

$$\vec{E}_r = HG_{10}\vec{e}_x + HG_{01}\vec{e}_y,$$

$$\vec{E}_\phi = HG_{01}\vec{e}_x + HG_{10}\vec{e}_y.$$ \hspace{1cm} (2.25) (2.26)

The decomposition is illustrated in Figure 3.

Figure 3. CV beams as superposition of orthogonally polarized HG modes. It is obvious that a radially polarized beam could be decomposed to an $x$-polarized HG$_{01}$ mode plus a $y$-polarized HG$_{10}$ mode. Similarly, an azimuthally polarized beam is superposition of a $y$-polarized HG$_{01}$ mode and an $x$-polarized HG$_{10}$ mode.
CHAPTER 3

METHODS OF CYLINDRICAL VECTOR BEAMS GENERATION

3.1 Passive Generation of CV Beams in Free Space

Passive generation of CV beams in free space usually requires a passive device to convert the polarization state of a commonly polarized light such as linearly or circularly polarized beam to CV beams. For example, a segmented spatially variant half wave plate which has several sector half wave plates in different orientations so that it could convert linear polarization into CV polarization. Since the sector half wave plates are discrete, the output beam is roughly radially or azimuthally polarized and the quality of the generated CV beam is related to the number of the sectors. Another device used in this work has a localized polarization transmission axis aligned along either radial or azimuthal direction. This type of device is commonly known as radial analyzer or radial polarizer. The input beam for the radial polarizer is circular polarization. Through a coordinate transformation from Cartesian coordinates to polar coordinates, a circular polarization can be decomposed into a radial and an azimuthal polarization component with a $\pi/2$ phase shift between them as:

\[
E_{in} = \vec{e}_x + i\vec{e}_y = \left( \cos \phi \vec{e}_r - \sin \phi \vec{e}_\phi \right) + i \left( \sin \phi \vec{e}_r + \cos \phi \vec{e}_\phi \right) = e^{i\phi} \left( \vec{e}_r + i\vec{e}_\phi \right),
\]

(3.1)

Notice the decomposition results in an extra spiral phase term $e^{i\phi}$ depending on the azimuthal angle known as Berry’s phase, which needs an additional treatment to be canceled out in order to produce pure CV beams. The $\vec{e}_x, \vec{e}_y$ are the unit vectors in Cartesian coordinates and $\vec{e}_r, \vec{e}_\phi$ are the
unit vectors in the polar coordinates system. After the circular polarized beam passes through a radial analyzer, if only the radial component survives, then we have:

$$\mathbf{E}_{\text{out}} = e^{i \phi} \mathbf{e}_r.$$  \hspace{1cm} (3.2)

To get a pure radially polarized beam, a spiral phase element (SPE) with the opposite helicity is necessary to compensate for the Berry’s phase. Figure 4 shows the experiment layout of the generation of CV beams using radial analyzer and SPE. The input is a circular polarized beam and the two half wave plates can rotate the polarization pattern to any desired general CV beams.

![Figure 4. Generation CV beams using radial analyzer and SPE. The input is circular polarization, the SPE is used to opposite the helicity after the beam passing through the radial analyzer, the two cascaded half-wave plates rotate the polarization to any general CV beams.](image)

### 3.2 Passive CV Beams Generation Using Optical Fiber

It is known that a multimode step index optical fiber can support the TE$_{01}$ and TM$_{01}$ annular modes with TE$_{01}$ mode being azimuthally polarized and the TM$_{01}$ mode being radially polarized (Figure 5). To generate CV beams with an optical fiber, the TE$_{01}$ or TM$_{01}$ mode needs to be selected. From the mode theory for circular waveguides [11], the single mode fiber condition is that the normalized frequency $V$ satisfies:

$$V = \frac{2 \pi a}{\lambda} \sqrt{n_1^2 - n_2^2} \leq 2.405.$$  \hspace{1cm} (3.3)

The TE$_{01}$ and TM$_{01}$ mode are degenerated to LP$_{11}$ mode. In order to excite the LP$_{11}$ mode in the optical fiber efficiently, $V$ need to be greater than 2.405 and smaller than the cutoff condition of.
LP$_{21}$ mode, which is 3.83. This type of fiber is also known as few-mode fiber. In this experiment, to generate CV beams at 1064 nm, Thorlab 1310BHP select cut-off single mode fiber with the second mode cutoff is 1260 nm is employed. The normalized frequency at 1064 nm can be calculated:

$$V = \frac{2\pi a}{1260nm} \sqrt{n_1^2 - n_2^2} = 2.406 \Rightarrow V = \frac{2\pi a}{1064nm} \sqrt{n_1^2 - n_2^2} = 2.848.$$  (3.4)

Thus this fiber satisfies the few-mode condition. Through careful adjustment of the coupling angle and position of the fiber, the desired CV beams output from the fiber can be realized when the coupling efficiency of TE$_{01}$ or TM$_{01}$ mode is maximized.

3.3 Tightly Focusing of CV Beams

In the optical trapping with CV beams, high NA microscope objective lens is required to provide large intensity gradient so that the gradient force could overcome the destabilizing scattering forces. In order to estimate the optical force, it is necessary to develop methods to numerically calculate the electric field near the focal region. The geometry of the problem is shown in Figure 6 [12].
Figure 6. Geometry of focusing by a high NA lens. After the lens, plane wave front (plane 0) becomes spherical wave front (plane 1) with the state of polarization changing from \((g_0, k)\) to \((g_1, s_1)\), the maximum of \(\theta\) is determined by the NA of the lens.

The incident field is assumed to have a planar wave front at plane 0 which is the entrance pupil of the optical system; the incident field could have any prescribed spatial amplitude and polarization distribution. The focusing lens is assumed to be an aplanatic lens which produces a converging, spherical wave. \(g_0\) is the unit vector of the radial direction of the incident beam that is perpendicular to the optical axis and can be expressed in Cartesian coordinates as:

\[
g_0 = \cos \phi \hat{e}_x + \sin \phi \hat{e}_y,
\]

where \(\phi\) is the azimuthal angle in plane 0 with respect to the \(x\)-axis. The azimuthal unit vector could be denoted by \(\hat{g}_0 \times \hat{k}\), where \(\hat{k}\) represents the unit vector along optical axis. The electric field of the incident CV beams could be written as the combination of the radial and azimuthal components:

\[
\vec{E}_0 = l_0 \left[ e_r^{(0)} \hat{g}_0 + e_\phi^{(0)} (\hat{g}_0 \times \hat{k}) \right],
\]

where \(l_0\) is the amplitude of the incident beam which is assumed to be cylindrically symmetric about the optical axis. Following Richards and Wolf [13], the electric field near the focus can be expressed as integral over the spherical plane 1 with radius \(f\) as:
\[ \vec{E}_s = -\frac{ik}{2\pi} \int_{\text{plane}1} \vec{a}_i(\theta, \phi) e^{ik(s_i \cdot \vec{r})}, \quad (3.7) \]

where \( k \) is the wave number, \( s_i \) is the direction of the refracted ray, \( \vec{r} \) is from plane 1 to the observation point, \( \vec{a}_i \) is the vector field amplitude can be expressed as:

\[ \vec{a}_i = f \cos^{1/2}(\theta) l_0(\theta) \left[ e^{(0)}_r \vec{g}_i + e^{(0)}_\phi (\vec{g}_i \times \vec{s}_i) \right]. \quad (3.8) \]

From the above equation, one can show that after the objective lens the radial component of the incident beam becomes along \( \vec{g}_i \) direction and the azimuthal component is along \( \vec{g}_i \times \vec{s}_i \) direction. \( \vec{g}_i \) can be expressed in the Cartesian coordinates as:

\[ \vec{g}_i = \sin \theta \vec{e}_z + \cos \theta \vec{g}_0 = \sin \theta \vec{e}_z + \cos \theta \left( \cos \phi \vec{e}_x + \sin \phi \vec{e}_y \right). \quad (3.9) \]

Also, for \( \vec{s}_i \) we have:

\[ \vec{s}_i = \cos \theta \vec{e}_z - \sin \theta \vec{g}_0 = \cos \theta \vec{e}_z - \sin \theta \left( \cos \phi \vec{e}_x + \sin \phi \vec{e}_y \right). \quad (3.10) \]

If the observation point is \( \vec{r} = (\rho, \phi, z) = \rho \cos \phi \vec{e}_x + \rho \sin \phi \vec{e}_y + z \vec{e}_z \), then:

\[ \vec{s}_i \cdot \vec{r} = -\rho \sin \theta \left( \cos \phi \cos \phi + \sin \phi \sin \phi \right) + z \cos \theta \]
\[ = z \cos \theta - \rho \sin \theta \cos (\phi - \phi) \quad . \quad (3.11) \]

In the case of radially polarized beam illumination, we take \( e^{(0)}_\phi = 0 \) in Eq.(3.6), the electric field near the focal region expressed in Cartesian coordinates is:

\[ \vec{E}_s = \begin{bmatrix} e_s^{(x)} \\ e_s^{(y)} \\ e_s^{(z)} \end{bmatrix} = -\frac{ikf}{2\pi} \int_0^\alpha \int_0^{2\pi} \sin \theta \cos^{1/2}(\theta) l_0(\theta) e^{ik(\rho \cos \theta - \rho \sin \theta \cos (\phi - \phi))} \begin{bmatrix} \cos \theta \cos \phi \\ \cos \theta \sin \phi \\ \sin \theta \end{bmatrix} d\phi d\theta, \quad (3.12) \]

where \( \alpha \) is the limit of \( \theta \), \( \alpha \) is determined by the NA of the focusing lens.

By using the coordinate transformation:
\(e_{\phi}^{(s)} = e_y^{(s)} \cos \phi_x - e_x^{(s)} \sin \phi_x\)
\(e_{\rho}^{(s)} = e_x^{(s)} \cos \phi_x + e_y^{(s)} \sin \phi_x,\) (3.13)

\(\bar{E}_x\) could be rewritten in cylindrical coordinates as:

\[
\bar{E}_{\rho}^{(s)} = \frac{-ikf}{2\pi} \int_0^\alpha \int_0^{2\pi} \sin \theta \cos \theta \cos^{1/2} \theta \cos(\phi - \phi_x) \theta_0(\theta) e^{ik\left(z_\theta x \cos \theta - \rho \sin \theta \cos(\phi - \phi_x)\right)} d\phi d\theta
\]
\(\bar{E}_\phi^{(s)} = 0\) \hspace{1cm} (3.14)
\(\bar{E}_z^{(s)} = \frac{-ikf}{2\pi} \int_0^\alpha \int_0^{2\pi} \sin^2 \theta \cos^{1/2} \theta \theta_0(\theta) e^{ik\left(z_\theta x \cos \theta - \rho \sin \theta \cos(\phi - \phi_x)\right)} d\phi d\theta\)

The integral over \(\phi\) could be accomplished by the identity of Bessel function:

\[
\int_0^{2\pi} \cos(n\phi) e^{ic\cos \phi} d\phi = 2\pi i^n J_n(x).
\]

So Eq. (3.14) can be simplified as:

\[
\bar{E}_{\rho}^{(s)} = kf \int_0^\alpha \sin \theta \cos \theta \cos^{1/2} \theta \theta_0(\theta) e^{ik\left(z_\theta x \cos \theta - \rho \sin \theta \cos \phi_x\right)} J_1(k \rho \sin \theta) d\theta
\]
\(\bar{E}_z^{(s)} = -ikf \int_0^\alpha \sin^2 \theta \cos^{1/2} \theta \theta_0(\theta) e^{ik\left(z_\theta x \cos \theta - \rho \sin \theta \cos \phi_x\right)} J_0(k \rho \sin \theta) d\theta\) \hspace{1cm} (3.15)

An example of the numerical results of focused radially polarized beam is shown in Figure 7-Figure 9. In the simulation, an objective lens with NA=0.98 is used, and \(l_0(\theta)=1\) is assumed. The axis is in the unit of wavelength in vacuum.

Figure 7. Normalized intensity of transverse (radial) component at focus and through focus of a radial polarized beam focused by high NA (1.3) oil immersion objective lens.
Similarly, for azimuthally polarized illumination, taking $e_r^{(0)} = 0$ in Eq.(3.6), the field near the focal region is:

$$E_\phi^{(x)} = -ikf\int_0^\pi \sin^2 \vartheta \cos^{1/2} \vartheta \partial_\vartheta \left( \theta \right) e^{ik(\varphi_0 - 2\vartheta)} J_0 \left( k\rho \sin \theta \right) d\theta.$$  \hspace{1cm} (3.16)

An example of the numerical results is shown in Figure 10. The same parameters are used as the radial polarization above.
As conclusion, the focused azimuthally polarized beam produces only transverse electric field along azimuthal direction through the entire focal region with a donut shape. While for radially polarized beam, there are both longitudinal and radial transverse electric field components and the axial field is about four times in magnitude as the radial components.
CHAPTER 4
TRAPPING EXPERIMENTS WITH CYLINDRICAL VECTOR BEAMS

4.1 Generation of CV Beams at 1064 nm

When linear polarization is used for trapping, the axial scattering and absorption force will push the particle away from the focus. However, as shown in Figure 7, the transverse component of focusing CV beam at \( z \)-axis is zero resulting in a diminishing axial time-averaged Poynting vector along the \( z \)-axis. Consequently, the axial radiation pressure force that is proportional to the time averaged Poynting vector is also zero along \( z \)-axis. Moreover, the large axial gradient force due to the dominant non-propagating longitudinal electric field provides strong restoring force to support more stable 3D trapping. These properties make it very attractive to use radial polarization for the trapping of metallic particles. The CV beam used in the trapping experiments is generated by mode selection from a few-mode optical fiber at 1064 nm (CrystaLaser CL-2000). The experiment setup is shown in Figure 11.

Figure 11. Setup of CV beam generation using optical fiber. A 1064 nm cw laser is coupled into a few-mode fiber by a fiber coupler, the coupling efficiencies of each mode can be changed by adjusting the fiber angle and position.
The output of the laser is coupled in to the few-mode fiber (ThorLabs 1310 BHP select Cut-Off SM fiber) through the optical fiber coupler which is made of an objective lens to focusing the laser beam into the optical fiber. As shown in Figure 5, the CV beams (TM$_{01}$ or TE$_{01}$ modes) have zero intensity at the center and the intensity distribution is circular symmetric, which is also known as „donut“ shape. In the experiment, after getting the highest fundamental Gaussian beam coupling efficiency, one need to change the position of the fiber end and tilt the incident angle until the intensity profile is a donut shape, as shown in Figure 13. Please note the distortion of the donut is due to the lens I placed between the fiber output end and the Spiricon camera (shown in Figure 12), the beam itself is not distorted.

After a donut intensity profile is coupled out from, a linear polarizer is inserted after the fiber output and rotated to check the polarization state of the output. The beam after the polarizer
should be a two-lobe pattern. However, the HE$_{21}$ mode is also a donut shape and after passing through a linear polarizer, it also generates a two-lobe pattern. To ensure the output is pure CV beams, the polarizer is rotated and the rotation of the light pattern after the polarizer is observed. If the pattern after the polarizer rotates at opposite direction of the rotation of the polarizer, it indicates that the output contains other hybrid mode; otherwise, the output is CV beams. Figure 14 is the donut beam after the linear polarizer as polarizer rotates, confirming the generation of a radially polarized beam with the few-mode fiber.

Figure 14. The CV beam generated from the optical fiber after a rotating linear polarizer. The black arrows indicate the orientation of the transmission axis of the linear analyzer. In this case, a radially polarized beam is generated.

4.2 Experimental Setup of Optical Trapping and Sample Preparation Method

The optical trapping experiment setup is based on a Nikon eclipse TE2000-U microscope, which is illustrated in Figure 15.
The dichroic BS has the same reflectance for both TE and TM waves, so that the laser beam will remain its polarization state after reflection. The illumination could be either form top or bottom. The microscope objective lens is 100x oil immersion lens with variable NA from 0.5 to 1.3. A CCD camera is used for capturing trapping videos or images.

Initially, the samples were prepared with simply place a drop solution of particles on a cover glass. The problem of this method is that the solution drop gets dry very quickly. A better sample preparation method needs to be employed in order for longer trapping experiment time. The tools and materials used in the sample preparation include microscope glass slide, cover glass,
finger nail polish and tapes. The preparation process has mainly four steps. First, on the slide, tape two or three layers of tape on two different positions. The distance between them should be slightly smaller than the width of the cover glass, as shown in Figure 16.

Figure 16. Step 1 of the sample preparation process. Two or three layers of tape are attached on the slide at two locations.

Then the cover glass is placed across two tapes and the cover glass is sealed to the tape with finger nail polish carefully (Figure 17). Notice that the nail polish is transparent which is not very obvious in these pictures. After five to ten minutes until the nail polish dries, a pipette is used to take one drop of the solution and drip the drop into the space under the cover glass from one side gently until the liquid fill the space (Figure 18). Figure 19 shows the last step to seal all the sides of the cover glass with nail polish and waiting for the nail polish to dry completely.

Figure 17. Step 2 of the sample preparation process. Finger nail polish is pasted on both tapes.

Figure 18. Step 3 of the sample preparation process. A cover glass is put across the tapes and sealed with tapes by nail polish.
One drawback of this method is sometimes the tape maybe not very impermeable especially at the edger or between layers of tapes. One solution to solve this problem is to replace the tapes with nail polish as spacer between the cover glass and the glass slide. No tape is used in this method but one need to be careful to control the quality of finger nail polish between the slide and the cover glass in order to control the space gap. In the experiments, the samples prepared by this method are quiet durable that could be used for at least one or two days which is enough for a single tapping experiment.

4.3 Experimental Results of Trapping Polystyrene Spherical Beads

After the CV beam is generated, it is send into the TE-2000U Nikon inverted microscope. Fluorescence 50 nm diameter particles dispersed in solution are used in the trapping system and the fluorescence images are captured by a CCD camera to confirm the generation of CV beams. Figure 20 shows the fluorescence images of focused radially and azimuthally polarized beams which confirm well with the simulation results. The focused radially polarization beam produces a solid spot field (shown in Figure 20 (a)) while the focused azimuthally polarized beam creates a light ring (shown in Figure 20 (b)).
Polystyrene (PS) beads are easily trapped and manipulated in focused laser field because the scattering force is small. Figure 21 is two frames from a video of trapping of an 800 nm PS beads when moved transversely. The bead is relatively moved in $x$-$y$ plane by moving the sample solution.

Stable trapping in axial direction is also shown in Figure 22, the $x$-$y$ position is fixed, the objective lens is moved up and down which indicates the laser focus is moving up and down inside the sample. As shown in the figure, the PS bead stays in focus while the background being in and out of focus, this demonstrates that the trapping in $z$ direction is stable. From these observations, it can be concluded that the PS bead is stable trapped in all three dimensions.
Figure 22. Stable trapping of 800 nm PS bead in axial direction. The trapped bead stays in focus while the solution is moving up and down along axis direction.

4.4 Attempts to Trap Metallic Particles

Spherical golden particles with diameter about 50 nm are also inserted in the radially polarized beam trapping system but stable three dimensional trapping was not observed. Instead of stable trapping, some other phenomenon is observed. When particles are in high density in the solution, the particles are rapidly moving around like swarms of bees or insects due to the Brownian motion. The laser focus creates a forbidden area near the focal point where no particles will occupy in. When I tried to move some of the particles which stuck on the cover glass into the laser focus, I observed that a large bubble is created from the particle and increasing the size until it reaches a certain size. This burning of particles I observed indicates that a large amount of heat is produced during the trapping.

This confliction between theoretical prediction and experimental is observed in variety of trapping attempts including using 800 nm or 1064 nm lasers. A possible explanation of this confliction could be the heat created in trapping. A recent paper [14] about plasmonic coupling heating effect in optical trapping of metallic nanoparticle also confirms the burning phenomenon in our experiment. Another possible reason may be the large curl scattering force. In next chapter, I numerically calculate and study the property of the curl force under focused radially polarized circumstance and try to find if there is any way to eliminate the curl force or maybe make use of
it. Other possible reason could be the problem of the sample that was obtained from a medical company. Little knowledge of the sample particles is known so the sample itself could lead to unstable trapping.
CHAPTER 5

CURL SCATTERING FORCE AND ITS ENGINEERING

For Rayleigh particles, the optical forces can be calculated under dipole approximation, as shown in Eq. (2.9). Besides the gradient force and the traditional scattering which is proportional to the time averaged Poynting vector, another type of scattering force called curl scattering force that is proportional to the time averaged spin density as defined in Eq. (2.12). This curl force, which is often ignored in linear polarization case due to its small magnitude, can be on the same order of magnitude of the traditional scattering force for radially polarized beams.

5.1 Derivation of Curl Force in Focused Radially Polarized Beams

In order to derive the expression of the optical force in focal region of the radially polarized beam [15], the electric and magnetic fields around the objective focal region can be rewritten as [16]:

\[
\begin{align*}
E &= \frac{ikf}{2} e^{-ikf} E_0 \left[ i(I_1 - I_2) \cos \varphi, i(I_1 - I_2) \sin \varphi, 4I_0 \right] \\
H &= \frac{ikf}{2} e^{-ikf} E_0 \left[ -i(I_1 + 3I_2) \sin \varphi, i(I_1 + 3I_2) \cos \varphi, 0 \right],
\end{align*}
\]

(5.1)

where

\[
\begin{align*}
I_0(\rho, z) &= \int_0^\alpha \sqrt{\cos \theta} l_0(\theta) \sin^2 \theta J_0(k \rho \sin \theta) e^{ikz \cos \theta} d\theta \\
I_1(\rho, z) &= \int_0^\alpha \sqrt{\cos \theta} l_0(\theta) \sin \theta (1 + 3 \cos \theta) J_1(k \rho \sin \theta) e^{ikz \cos \theta} d\theta \\
I_2(\rho, z) &= \int_0^\alpha \sqrt{\cos \theta} l_0(\theta) \sin \theta (1 - \cos \theta) J_1(k \rho \sin \theta) e^{ikz \cos \theta} d\theta.
\end{align*}
\]

(5.2)

The axial component of radiation pressure \( F^r_p \) can be rewritten as:
Whereas the axial component of the curl scattering force is:

\[ F_{z}^{\text{sc}} = \sigma c \nabla \times \left\{ \frac{\varepsilon_{0}}{4 \omega i} \left( \mathbf{E} \times \mathbf{E}^\ast \right) \right\} = \sigma \nabla \times \{ \mathbf{E} \times \{ \mathbf{E} \} \}. \]  

(5.4)

In the Cartesian coordinate system, the time averaged spin density \( \langle L_z \rangle \) can be expressed as:

\[
\langle L_z \rangle = \frac{\varepsilon_0 k^2 f^2 E_0^2}{16 \omega i} \left\{ -4i \left( I_1 - I_2 \right) I_0^* \sin \varphi - 4i \left( I_1 - I_2 \right)^* I_0 \sin \varphi, \right\}
\]

\[
4i \left( I_1 - I_2 \right) I_0^* \cos \varphi + 4i \left( I_1 - I_2 \right)^* I_0 \cos \varphi, 0 \}
\].

(5.5)

After transformed into cylindrical coordinates, \( \langle L_z \rangle \) is

\[
\langle L_z \rangle_r = \langle L_z \rangle_r \cos \varphi + \langle L_z \rangle_y \sin \varphi = 0
\]

\[
\langle L_z \rangle_\varphi = -\langle L_z \rangle_x \sin \varphi + \langle L_z \rangle_y \cos \varphi = \frac{\varepsilon_0 k^2 f^2 E_0^2}{2 \omega} \operatorname{Re} \left[ I_0^* \left( I_1 - I_2 \right) \right]
\]

(5.6)

And Eq. (5.4) in cylindrical coordinates is

\[
F_{z}^{\text{sc}} = \sigma c \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \langle L_z \rangle_\varphi \right) - \frac{1}{r} \frac{\partial}{\partial \varphi} \langle L_z \rangle_r \right] = \sigma c \frac{1}{r} \left( \langle L_z \rangle_\varphi + \frac{\partial}{\partial r} \langle L_z \rangle_\varphi \right)
\]

\[
= \sigma \frac{\varepsilon_0 k^2 E_0^2}{2} \left[ \frac{1}{r} \operatorname{Re} \left( I_0^* \left( I_1 - I_2 \right) \right) + \frac{\partial}{\partial r} \left( \operatorname{Re} \left[ I_0^* \left( I_1 - I_2 \right) \right] \right) \right].
\]

(5.7)

5.2 Numerical Results of the Optical Forces

The optical forces discussed in the above section are numerically calculated here. The laser wavelength is 1064 nm, laser power is 100 mW and beam diameter is 12 mm, the particle is spherical gold particle with radius of 50 nm. To simplify the simulation, the electric field of the laser is assumed to uniform across the whole beam cross section. The high NA (NA=1.3) oil immersion objective lens is used to focus the laser beam and the immersion oil has refractive
index of 1.5. The calculated constituting optical forces are shown in Figure 23-Figure 25. The total axial force is plotted in Figure 26.

Figure 23. Axial ($z$-) radiation pressure force of focused radially polarized beam.

Figure 24. Axial ($z$-) curl scattering force of focused radially polarized beam.
From Figure 23-Figure 26, it can be seen that the axial gradient force produces a potential well that could support trapping. The axial radiation pressure has zeros value along z-axis while the axial curl scattering force is always positive and has larger magnitude than the gradient force. The total axial force along z-axis is always positive. As a result, the positive force will push the particle away from the focus thus destroy the stable trapping. This curl force might be part of the reasons that I failed in the metallic particle trapping experiments.
To construct stable three dimensional trapping, the properties of the curl scattering force need to be further studied. If there are some methods to eliminate the curl force or at least create equilibrium positions in the total force, stable trapping of metallic particles is still possible. Several parameters could be tuned to engineer the curl force which include the laser wavelength, integral range of $\theta$, applying one or two binary step function, or the ray projection function $P(\theta)$. The details are discussed in the following sections.

5.3 Innovative Uses of Curl Scattering Force

As discussed above, the curl force pushes the particles away and destroys the trapping. The initial thought is to eliminate the force along optical axis so that the remaining forces could still support a stable trapping. A large number of numerical simulations of the curl force under varieties of circumstances have been conducted, such as changing the integral range of $\theta$ either upper or lower limits, changing several different types of ray projection functions, applying one or two binary filters or changing multiple conditions together. All numerical results indicate that the curl force always exists and its magnitude remains dominant among all type of forces.

Thus it appears to be unlikely to eliminate the curl force in optical trapping. However, during the simulation, negative curl scattering force is observed and draw our attention. The negative force will pull the particles towards the laser source. This interesting phenomenon could be potentially used as “laser tractor” which usually appears in science fictions, Star Trek in particular. Recently, NASA becomes interested in this laser tractor beam because it provides a solution to remotely capture planetary or atmospheric particles and to deliver them to a robotic rover [17]. A pair of research groups have studied and discussed the theory behind two possible approaches, using multiple plane waves [18] and Bessel beam [19].

In order to make use this negative curl scattering force, the first parameter to optimize is the wavelength of laser. Based on Eq.(2.13), the curl scattering force is proportional to total cross
section of the particle $\sigma \equiv k_0 \text{Im}(\alpha)/\varepsilon_0$, to maximize the curl scattering force the optimized wavelength is found to be around 500 nm. In the following simulations, a laser wavelength of 532 nm will be used, which is different from the 1064 nm wavelength we used in the experiment. The gold sphere has a radius of 50 nm and its refractive index at 532 nm is $0.467 + 2.4083i$ [21]. The laser beam has a power of 100 mW with a beam diameter of 12 mm and uniform amplitude distribution.

5.4 Adding a Binary Filter

Starting with the same ray projection function, a binary filter, i.e., adding a sign function $\text{sign}(\theta - \theta_0)$ as illustrated in Figure 27, is applied to the integral Eq. (5.2), where

$$\text{sign}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases} \quad (5.8)$$

Figure 27. Illustration of $\text{sign}(\theta - \theta_0)$ and the pupil of objective lens for $\theta_0 = 55^\circ$

Figure 28 are the plots of axial optical forces at optical axis vs. the binary filter step position. Since the axial radiation pressure is null on the optical axis, only axis curl scattering force and axis gradient force are plotted in the following simulation. The effective NA is assumed to be close 1, $l(\theta)$ is 1 as well, the y-axis denote $\theta_0$ in $\text{sign}(\theta - \theta_0)$. 

34
The ray projection function is defined as the relation of the light distribution before and after the focusing lens [20] as shown in Figure 6, in which the light at radius \( r \) is \( P(r) \) and the corresponding distribution after the focusing lens is \( P(\theta) \). For a high NA lens, the difference between \( P(r) \) and \( P(\theta) \) cannot be negligible so that the ray projection function need to be considered.

The design condition used in Eq.(3.8) is sine condition:

\[
P(\theta) = P(r) \sqrt{\cos \theta}.
\]  

(5.9)

To find the position of the binary filter, a scanning of \( \theta_0 \) from 0 to 90 degree is calculated. The curl scattering force keeps positive around focus until when \( \theta_0 \) reaches nearly 1 rad. However, as shown in Figure 28, the negative force is only about one tenth in the magnitude as the positive force. An equilibrium point can be found but the trapping is not stable because the potential well is not deep enough.

![Figure 28](image.png)

Figure 28. Force distribution when adding a binary filter. The y-axis is \( \theta_0 \) is in the sign function \( \text{sign}(\theta-\theta_0) \). z is the optical axis. The left is the axial curl scattering force; the right is the net axial force.
In Figure 29, an equilibrium point can be found when a binary filter with step at \( \theta_0 = 0.97 \) rad. However, the trapping potential is not high because the positive force is about 5 times as the negative force. In order to pursue one or more equilibrium points, the magnitude of the negative curl force need to be increased which could be realized by adjusting the ray projection function.

For an objective lens that obeys the Helmholtz condition:

\[
g(\theta) = \tan \theta
\]

\[
P(\theta) = P(r) \left( \frac{1}{\cos \theta} \right)^3.
\]  

(5.10)

If Eq. (5.2) has the form as

\[
I_0(\rho, z) = \int_0^\alpha \left( \frac{1}{\cos \theta} \right)^3 I_0(\theta) \sin^2 \theta J_0(k \rho \sin \theta) e^{ikz \cos \theta} d\theta
\]

\[
I_1(\rho, z) = \int_0^\alpha \left( \frac{1}{\cos \theta} \right)^3 I_0(\theta) \sin \theta (1 + 3 \cos \theta) J_1(k \rho \sin \theta) e^{ikz \cos \theta} d\theta
\]

\[
I_2(\rho, z) = \int_0^\alpha \left( \frac{1}{\cos \theta} \right)^3 I_0(\theta) \sin \theta (1 - \cos \theta) J_1(k \rho \sin \theta) e^{ikz \cos \theta} d\theta.
\]

(5.11)

Note the \( \sqrt{\cos \theta} \) term is replaced by \( \left( \frac{1}{\sqrt{\cos \theta}} \right)^3 \) due to the change of the ray projection function.

Similar to Figure 28, the axial curl force and the total axial force along optical axis now are:
In Figure 30, the integral of $\theta$ is from 0 to 85 degree, which means the NA of the objective lens need to be very close to 1. In this case, if the binary step filter is placed near $\theta=1.3$ rad (74.5 degree), the corresponding forces are shown in Figure 31.

It can be seen that the positive force is about twice as the negative force, which means it could support a more stable trapping along $z$ direction than sine condition of ray projection function. Besides adding a binary filter, other techniques such as adding two binary filters or using other type of ray projection functions are also explored. So a lot of parameters and degrees of freedom can be controlled to engineer the curl force to the desired distribution for different applications.
In sum, the optical forces can be designed by changing the shape of the focal region. The negative curl force provides potential to form a stable trapping of metallic nanoparticles. Adding binary filter is illustrated as one method to manipulate the focal electric field. More complicated phase mask may be used to provide better results.
CHAPTER 6
CONCLUSIONS AND SUMMARY

In this thesis, CV beams were introduced from its mathematic origin, wave equation. Numerical method based on Richards-Wolf theory was also introduced to calculate the electric field of a laser beam focused by a high NA objective lens. The purpose of this project was to construct a stable trapping of Rayleigh metallic particles. The experiment started from generating the CV beams in free space and using few-mode optical fiber. The trapping system using CV beams was also built based on a Nikon microscope.

In the experiment, three dimensional trapping of polymer beads was established. However, attempts to trap 50 nm gold particles failed because the metallic particles were observed to always be pushed away from the focus. To find out the reason, the optical forces of Rayleigh particles at the electric fields of a focused CV beam were calculated. During the calculation, we found that the curl scattering force was large enough in CV beam trapping system to overcome the trapping force and destroy the trapping.

The initial idea of eliminate the curl force in the trapping system did not work after numbers of numerical simulations. However, it is found that the curl scattering force could be negative at certain condition and the negative force could be applied for trapping or other applications. Methods to engineer the curl force through managing the ray projection function of the lens and applying binary filter could provide potential possibilities of trapping metallic Rayleigh particles are found based on numerical calculations of the forces.

Although the stable trapping was not observed in this experiment, extensive researches of trapping of golden nanoparticles [22] and carbon nanotubes [23] have shown the advantage of
trapping using CV beams. These experiments demonstrate that even though the improvement of using CV beams for trapping is reduced due to the curl scattering force, the advantage still exists due to the minimization of radiation pressure force. Most importantly, CV beams for trapping allows us to manipulate the optical force by shaping the focal region so that the optical force for specific application can be designed.
REFERENCES


