SUSPENSION OF SOLID MIXTURES
BY MECHANICAL AGITATION

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SUSPENSION OF SOLID MIXTURES
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ABSTRACT

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Agitation is a critical aspect of many processes, such as food production, mineral processing, and water treatment, with liquid-solid agitators representing a significant portion of all agitation installations. Improper mixing operation in liquid-solid agitators can result in negative financial and environmental issues. Over-mixing may damage solid particles and erode impeller blades as well as waste energy. On the other hand, under-mixing may allow solids to settle down on the base of the tank, which may cause solids to adhere to one another and bring about difficulties of solids removal from the vessel. The purpose of this work is to develop a design guideline that can be used to successfully predict the agitation speed required to suspend a solid mixture in just-suspended condition, in which no solid remains on the base of tank for longer than 1-2 seconds, based on a knowledge of agitation speeds required to suspend the individual components in the solids mixture. The primary design guideline investigated is summing the powers required to suspend the individual solids alone to predict the power required to suspend the solids mixture. The secondary design guideline that is investigated is that the speed required to suspend the solids mixture is equal to the speed required to suspend the more difficult to suspend solid alone.
All binary solids mixtures can be categorized into three different groups in this work based on the magnitude of specific gravity of the solids in each system. It is found that speed predicted based on the sum of powers required to suspend the individual solids is normally higher than the actual speed at which a solids mixture is at the just-suspended condition in the case of low-density systems where the specific gravities of both solids are below 1.5 grams per cubic centimeter. In other cases, including mixed-density system, which is a solid with low density (below 1.5 grams per cubic centimeter) plus a solid with high density (above 2.4 grams per cubic centimeter), and high-density system in which both solids have densities above 2.4 grams per cubic centimeter, the prediction speed found by summing the powers required for suspension of each individual component in a solid mixture is approximately equal to that necessary to suspend solids mixture. However, a few systems diverge from these typical behaviors, possibly due to the unusual characteristics of one solid – olivine sand. Results from those solids mixtures involving olivine sand are not consistent with typical conclusion obtained from the sum of powers hypothesis. Adding olivine sand always reduces mixture suspension speed from the speed predicted by summing of powers estimation. A reasonable explanation of these atypical phenomena should be investigated in future studies. In addition, two systems that consisted of three different solids were tested and it was found that the speed based on summing the powers required for suspension of individual components provided a reasonable prediction of the suspension speed of those ternary systems.
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>C</td>
<td>Impeller off-bottom clearance, m</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Dimensionless drag coefficient</td>
</tr>
<tr>
<td>$d_p$</td>
<td>Particle size, m</td>
</tr>
<tr>
<td>D</td>
<td>Impeller diameter, m</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational acceleration, m s$^{-2}$</td>
</tr>
<tr>
<td>k</td>
<td>Constant of Equation 2</td>
</tr>
<tr>
<td>n</td>
<td>Scale-up exponent</td>
</tr>
<tr>
<td>N</td>
<td>Impeller rotational speed, s$^{-1}$</td>
</tr>
<tr>
<td>$N_{js}$</td>
<td>Just-suspended speed, subscript “m” denotes solids mixture, subscript “1” and “2” denotes individual solids, s$^{-1}$</td>
</tr>
<tr>
<td>$N_P$</td>
<td>Dimensionless impeller power number, subscript “m” denotes solids mixture, subscript “1” and “2” denotes individual solids</td>
</tr>
<tr>
<td>P</td>
<td>Impeller power draw, W</td>
</tr>
<tr>
<td>S</td>
<td>Parameter that is a function of impeller type and system geometry</td>
</tr>
<tr>
<td>T</td>
<td>Tank diameter, m</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Tank diameter in a reference scale, m</td>
</tr>
<tr>
<td>$u_t$</td>
<td>Terminal settling velocity for a particle, m sec$^{-1}$</td>
</tr>
<tr>
<td>X</td>
<td>Solids loading (i.e. weight of solid per weight of liquid times 100), per cent</td>
</tr>
<tr>
<td>Z</td>
<td>Liquid level, m</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Density of the solid, kg m$^{-3}$</td>
</tr>
<tr>
<td>$\rho_l$</td>
<td>Density of the liquid, kg m$^{-3}$</td>
</tr>
<tr>
<td>v</td>
<td>Liquid kinematic viscosity, m$^2$ sec$^{-1}$</td>
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CHAPTER 1

INTRODUCTION

1.1 Background and Literature

Nienow (1992) noted that agitation of liquid-solid stirred tanks is involved in many aspects of industrial processes, such as solids dissolution, catalytic chemical reaction, and crystallization. Due to the difference in densities between solids and liquid in mixing operation, there are two main classes in liquid-solids agitation – suspension of settling solids (solids are more dense than liquid) and dispersion of floating solids (solids are less dense than the liquid or solids float before being wetted by the liquid but settle after being wetted). Further, there are three primary levels of agitation for settling solids suspension – bottom or corner fillets, just complete suspension, and homogeneous suspension – in order of increasing agitation intensity. In case of bottom or corner fillets, a small portion of solid particles is permitted to settle in relatively stagnant region on the tank bottom or in corners. This level of agitation could possibly lead to many problems, such as undesired reaction byproducts and non-representative slurry withdrawal from a tank. However, this agitation level has the advantage of significant reduction in power consumption if complete solids suspension is not necessary for adequate agitation performance. Just complete suspension occurs when no solid particles rest on the tank
base for longer than one to two seconds such that all solids are suspended in liquid phase and the maximum surface area is accessible for desired process requirements, such as mass or heat transport and chemical reaction. This condition minimizes power consumption required to suspend all solids but the solids distribution through the vessel may not be uniform since this level of agitation is concerned only with solids on tank base instead of throughout the vessel. For homogeneous suspension, the distribution of solids is uniform throughout the vessel at the expense of a considerably higher speed and power demanded in comparison with just complete suspension. However, this level of agitation is desirable when a continuous solids flow from the system is required.

Zwietering (1958) carried out the pioneering investigation of solids suspension in liquid-solid stirred tank and defined minimum stirrer speed for complete suspension (i.e. just-suspended speed) at which no solid particles rest on the bottom of vessel for longer than one to two seconds. In his study, the parameters that influence the just-suspended speed, such as tank diameter \( T \) and impeller diameter \( D \), were experimentally tested to develop the empirical correlation in which the just-suspended speed was related to each independently varied factor.

\[
N_{js} = \frac{S^0.1}{d_p^0.2 \left( \frac{\rho_s - \rho_l}{\rho_l} \right)^{0.45}} X^{0.13}
\]

(1)

Zwietering’s correlation has been extensively applied to predict just-suspended speed of uniform solid in many cases, and it has also effectively helped engineers to understand the influence of the primary factors on the performance of agitators in solids suspension applications. However, the emphasis of this work was on investigations of solids
suspension with uniform solid (solid particles with same shape, size \(d_p\), and density \(\rho_s\)) rather than solids mixtures. In the case of uniform solid particles, the just-suspended speed was expressed as a function of the particle diameter, \(d_p\), and density, \(\rho_s\), that are easy to determine, whereas it is quite difficult to determine the solids mixture characteristics that affect the mixture just-suspended speed. Therefore, Zwietering’s correlation can’t be directly used to successfully predict the just-suspended speed of solid mixtures with different physical characteristics. However, Zwietering’s work led to a reliable criterion of just-suspended condition – no solids particles rest on the vessel base for longer than one to two seconds, and just-suspended condition has become the most commonly encountered level of agitation in research of solids suspension in liquid-solid stirred tank.

Corpstein et al. (1994) studied the primary parameters exerting significant influence on the minimum solids-suspension agitator speed required to achieve the just-suspended condition in liquid-solid stirred vessels (i.e. the just-suspended speed, \(N_{js}\)). The variables, such as liquid and solid physical properties, solids loading and geometry (e.g. \(D/T\)), were separately examined in connection with the just-suspended speed, and then the correlation of the just-suspended speed was developed by means of combining these independently varied parameters. The influence of physical properties (liquid and solid density, solids size, etc.) on the just-suspended speed was described in terms of the adjusted settling velocity that is the product of the terminal particle settling velocity, \(u_t\), and the normalized density difference between solid and liquid, \(\frac{\rho_s - \rho_l}{\rho_l}\).
As was the case with Zwietering’s correlation, the above relationship can only be applied to estimate the just-suspended speed of uniform solid (solid particles with same shape, size, and density) rather than solids mixtures since it is not known how to define the adjusted settling velocity of a solid mixture.

The previous work on solid suspension in liquid-solid stirred tank was based on investigations with an uniform solid. The correlation for the uniform solids just-suspended speed ($N_{js}$) was experimentally developed and also used as a general design procedure to estimate the agitator performance. Recently, two studies have begun the investigation of the just-suspended speeds of solid mixtures with different physical characteristics.

Etchells et al. (2010) investigated whether the just-suspended speed of a binary solids mixture could be reasonably predicted as the sum of powers required to suspend the individual solids alone. Only a single solids mixture, composed of sand with density of 2.656 grams per cubic centimeter and 425 μm mass-averaged particle size plus urea formaldehyde resin with density of 1.323 grams per cubic centimeter and 200 μm mass-averaged particle size, was tested in this work. All experiments were performed in a cylindrical flat-bottomed tank of 11.5 inches diameter and 17.5 inches height. A Lightnin A310 impeller of 3.54 inches diameter (D/T=0.31) was studied at an off-bottom clearance of 0.310 of the tank diameter (C/T=0.31). The just-suspended speeds of solid mixtures were determined by visual observation throughout the experiments. It was found that the approach that estimated the mixture just-suspended speed by summing the powers

\[
N_{js} = k \left[ \left( \frac{\rho_s - \rho_l}{\rho_l} \right) u_t \right]^{0.28} f(X) f \left( \frac{D}{T} \left( \frac{T}{T_0} \right)^{-n} \right)
\]
required to suspend the individual solids generally under predicted the mixture suspension speed. However, this prediction method worked reasonably well, having an average error of seven percent for the twenty-two data points at various ratios of resin mass to sand mass. The novelty of this work is the transition in emphasis from the just-suspended speed of uniform solids to the mixture just-suspended speed of solids with different physical characteristics. However, the one solids mixture studied in this work is not sufficient for understanding the behavior of all solid mixtures that will be encountered industrially in liquid-solid mixing operations.

Ayranci and coworkers (2010, 2011a, 2011b) proposed three design guidelines for estimating the just-suspended agitation speed of five solids mixtures studied over a range of impeller off-bottom clearance (C/T=0.15, 0.25, 0.33) with two impellers (pitched-blade turbine and Lightnin A310). Four studied solids mixtures were composed of two high-density solid materials (both solids with density above 2.4 grams per cubic centimeter) while only one was a mixed-density mixture, with one solid of density below 1.5 grams per cubic centimeter plus another solid of density above 2.4 grams per cubic centimeter. No low-density solids mixtures, where the densities of both solids are below 1.5 grams per cubic centimeter, were studied in this work. For their initial work (2010), they proposed that the just-suspended speed of solids mixture is simply equal to that of the more difficult to suspend solid alone (i.e. addition of less difficult to suspend solid does not affect the just-suspended speed). It was found that this design guideline does not accurately predict the solid mixtures just-suspended speed except for only one solids mixture. In general, the just-suspended speed of studied solids mixtures exceeded the more difficult to suspend solid just-suspended speed alone for both tested impellers at all
three clearances. Thus it was found that the effect of the easier to suspend solid on the solids mixture just-suspended speed must be taken into consideration. In their more recent work (2011a, 2011b), they proposed two other design guidelines for estimating the just-suspended speed of solids mixture. One is the momentum model in which the just-suspended momentum of solids mixture is equal to the sum of the momentums required to suspend the individual solids, and the other is the power model in which the just-suspended power of solids mixture is equal to the sum of the powers required to suspend the individual solids. Comparison of the momentum and power models indicated that the speeds predicted by the sum of momentum approach averaged ten percent higher than those predicted by the sum of powers approach. Generally, the just-suspended speed of solids mixture was over predicted by the momentum model. In comparison with the momentum model, the solids mixture just-suspended speed was reasonably predicted by the power model. There was only one solids mixture, small glass-nickel, where the sum of powers approach substantially over predicted the mixture just-suspended speed by more than twenty percent. Both small glass and nickel were quite small (no more than 100 μm) and highly dense (both densities above 2.4 grams per cubic centimeter) and their particular physical characteristics may have lead to atypical behavior. In conclusion, Ayranci and coworkers found that the predictive model of summing powers to suspend the individual solids alone was generally reliable for predicting the mixture just-suspended speeds of both high-density and mixed-density solid systems; however, they did not study mixtures of low-density solids that may be encountered in industrial practice.
Scully and Frawley (2011) simulated solids suspension in liquid-solid stirred tank with only one solid mixture – granular $\alpha$ crystals and needle-like $\beta$ crystals of L-glutamic acid – via computational fluid dynamics (CFD) software. It was proposed that the crucial factor in liquid-solid flow within a stirred tank is drag force that the fluid exerts on the solid and that is characterized using the drag coefficient, $C_D$. The drag coefficient is dependent on liquid and solid physical properties (density, shape, and kinematic viscosity) and can be used to model suspension of the particular solids in CFD simulation. For this work, the experimental results of solids suspension were successfully matched by the proposed simulation method in many aspects, such as cloud height (the maximum height of solids suspension relative to the overall tank height) and solids volume fraction.

Though this study didn’t focus on the agitator speed required for suspension of a solids mixture at just-suspended condition, its significance was to begin the investigation of the potential of CFD simulation to model the suspension of not only uniform solids but also solids mixtures based on solids characteristics and reduce the dependence on extensive experimentation to develop design procedures.

In conclusion, the correlation of just-suspended speed of uniform solids (solids particles with same size, shape, and density) has been well developed in terms of many primary parameters (e.g. liquid and solid physical properties and agitator geometry) exerting influence on the performance of solids suspension in liquid-solid stirred tank. However, in these correlations, the just-suspended speed is directly dependent on the physical characteristics of a given solid (e.g. density, shape, particle-settling velocity). For a solids mixture, it is rather difficult to describe the combined physical properties of a solids mixture. Consequently, the correlations for estimating the just-suspended speed of
uniform solids may not extend to the case of solids mixture, but establish reliable
knowledge of the agitation levels required to suspend uniform solids in just-suspended
condition. Recently, a few studies have emphasized predicting the just-suspended speed
of a binary mixture from the just-suspended speeds on the individual components.
Though these studies investigated a limited number of solid mixtures or incomplete range
of solids properties, it was found that the method of summing the powers required to
suspend the individual solids may reasonably predict the just-suspended speed of a solid
mixture.

1.2 Objective

This research is concerned with solids suspension in a liquid-solid stirred tank at
one particular agitation level – just-suspended condition in which no solids rest on the
tank base for longer than one to two seconds. The novelty of this work is that though
there have been many studies on the just-suspended speed of uniform solid (solid
particles with same shape, size, and density), there has been very little work in the
industrially important area of mixtures of solids with different physical properties. For
this work, the sum of powers approach of estimating the just-suspended speed of solids
mixture will be specifically investigated with a broader range of solids physical
properties than previous studies in the area of solids mixture suspension in liquid-solid
stirred tank. The ultimate goal of this work is to develop a design guideline that can be
used to reasonably predict the agitation speed required to suspend a solid mixture at just-
suspended condition based on knowledge of just-suspended speed of the individual
components.
1.3 Sum of Powers Just-Suspended Speed Prediction Method

The design hypothesis tested in this study is whether the mixture suspension power is equal to the sum of the powers required to suspend the individual solids in a binary system. Equation 3 is the expression of this design hypothesis in which the mixture suspension power, \( P_m \), is set equal to the sum of powers to individually suspend solid 1, \( P_1 \) and solid 2, \( P_2 \).

\[
P_m = P_1 + P_2 \quad (3)
\]

The connection of just-suspended speed, \( N_{js} \), with power, \( P \), comes from a dimensionless parameter, the power number, \( N_p \), that is used to characterize impeller power draw. Although there are other parameters like impeller diameter to tank diameter ratio, \( D/T \), that can affect power number, it is primarily a function of the impeller type and Reynolds number, \( N_{Re} \). Equation 4 is the power number definition, and it indicates that power draw is proportional to speed cubed.

\[
N_p = \frac{P}{\rho N^3 D^5} \quad (4)
\]

Equation 5 results when the power number is used to express the just-suspended powers of the proposed design criterion of Equation 3.

\[
N_{pm} \rho_m N_{js,m}^3 D^5 = N_{p1} \rho_1 N_{js,1}^3 D^5 + N_{p2} \rho_2 N_{js,2}^3 D^5 \quad (5)
\]

In this work, the impeller always operated in the turbulent regime in which the power number is essentially constant, so the power numbers in a solid mixture and two individual solids were equal to each other. In addition, the slurry density was assumed to be constant no matter how much solid mass was added in an agitated vessel. Also, the
impeller was fixed throughout the experiments meaning that impeller diameter, $D$, can be canceled out of Equation 5. Then, Equation 5 can be simplified to Equation 6 that predicts the mixture just-suspended speed from the just-suspended speeds of the individual solids in a binary solids mixture.

$$N_{js,m}^3 = N_{js,1}^3 + N_{js,2}^3$$  \hspace{1cm} (6)

Equation 7 is the expression of the mixture just-suspended speed prediction obtained by taking the cube root of Equation 6.

$$N_{js,m} = \left( N_{js,1}^3 + N_{js,2}^3 \right)^{\frac{1}{3}}$$  \hspace{1cm} (7)
CHAPTER 2
EXPERIMENTAL SETUP AND PROCEDURE

2.1 Experimental Apparatus

All experiments were performed in a clear acrylic cylindrical flat-bottomed tank of 11.5 inches diameter and 15 inches height. Four straight baffles of 1 inch width and 13 inches length that attached to the wall of tank were employed to eliminate swirling. The impeller, which was a pitched-blade turbine with four blades inclined at a 45 degree angle to the horizontal, was 4-inch diameter and placed on a straight shaft. The impeller was set in the center of tank with an off-bottom clearance of one-fourth of the tank diameter (C/T=0.25). Tap water was always used as liquid phase and the liquid level to tank diameter ratio was equal to unity (Z/T=1). An electric motor was used as a power source to provide shaft and impeller rotation by means of a gear head that connected to the agitator drive. The variable speed gear box allowed the rotational speed to be changed until solids slurry achieved just-suspended conditions in stirred vessel then the speed was measured by a direct contact tachometer with a digital display. In addition, an inclined square mirror was placed beneath the tank bottom to allow observers to take note of solids movement on the base of stirred vessel. The following figures illustrate the pitched-blade turbine and one tested solid mixture – green plastic and olivine sand.
Due to the subjective nature of just-suspended speed measurement, it was possible to deviate from the true mixture just-suspended speed even though it was determined by agreement of two observers throughout the experiments. To maintain the highest possible degree of consistency, it was essential to set up a tank as a reference next to the test tank. Further, the agitator geometry (D/T, C/T etc.) and impeller type of the reference tank was set equal to the test tank. During a set of experiments, a fixed mass of one solid was added to the reference tank and kept in just-suspended condition while measuring the mixture suspension speed in test tank. The observers were able to obtain high quality measurements of the mixture just-suspended speed by adjusting the speed in
the test tank until the test tank solids behavior matched the just-suspended speed behavior of the solids in the reference tank.

2.2 Solid Materials

A variety of solid materials were studied throughout this work and they had different properties (shape, particle size, and density). In this research, all solids were classified into two groups based on specific gravities. Low-density solid was considered as solids with density below 1.5 grams per cubic centimeter while high-density solid was regarded as solids with density above 2.4 grams per cubic centimeter. Table 1 provides information about physical properties of all low-density solids, and Table 2 provides characteristics of all high-density solids.

Table 1: Low-density solid characteristics

<table>
<thead>
<tr>
<th>Material</th>
<th>Size (μm)</th>
<th>Density (g/cm³)</th>
<th>Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>White Resin</td>
<td>780</td>
<td>1.053</td>
<td>Spheres</td>
</tr>
<tr>
<td>Brown Resin</td>
<td>620</td>
<td>1.23</td>
<td>Spheres</td>
</tr>
<tr>
<td>Red Acrylic</td>
<td>2950</td>
<td>1.18</td>
<td>Rectangular Cylinders</td>
</tr>
<tr>
<td>Green Acrylic</td>
<td>3000</td>
<td>1.028</td>
<td>Rectangular Cylinders</td>
</tr>
<tr>
<td>Green Plastic</td>
<td>2900</td>
<td>1.32</td>
<td>Ellipsoid Cylinders</td>
</tr>
<tr>
<td>Melamine</td>
<td>200</td>
<td>1.45</td>
<td>Thin Flakes</td>
</tr>
</tbody>
</table>

Table 2: High-density solid characteristics

<table>
<thead>
<tr>
<th>Material</th>
<th>Size (μm)</th>
<th>Density (g/cm³)</th>
<th>Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>Olivine Sand</td>
<td>85</td>
<td>3.50</td>
<td>Spheres</td>
</tr>
<tr>
<td>Fine Sand</td>
<td>200</td>
<td>2.78</td>
<td>Granules</td>
</tr>
<tr>
<td>Glass</td>
<td>600</td>
<td>2.44</td>
<td>Spheres</td>
</tr>
<tr>
<td>Aluminum Oxide</td>
<td>900</td>
<td>3.89</td>
<td>Spheroids</td>
</tr>
<tr>
<td>Small Silicon Carbide</td>
<td>400</td>
<td>3.13</td>
<td>Granules</td>
</tr>
<tr>
<td>Large Silicon Carbide</td>
<td>1000</td>
<td>3.13</td>
<td>Granules</td>
</tr>
</tbody>
</table>
The following figures show the appearance of solids studied in this work. Figure 3 shows three individual low-density solids of large size.

**Figure 3**: Green acrylic (left), red acrylic (middle), and green plastic (right)

Figure 4 illustrates three smaller solids in the low-density group. The first two solids, white resin and brown resin, are porous and their reported densities are when the pores are water filled.

**Figure 4**: White resin (left), brown resin (middle), and melamine (right)

Figure 5 shows high-density solids of small particle size.

**Figure 5**: Olivine sand (left), fine sand (middle), and small silicon carbide (right)
Figure 6 is the larger high-density solids.

**Figure 6**: Glass (left), aluminum oxide (middle), and large silicon carbide (right)

### 2.3 Experimental Procedure

Just-suspended condition during solids suspension in stirred vessel is where none of the solids particles remain stationary on the base of vessel for longer than one to two seconds. Just-suspended condition requires the lowest power consumption to expose all of solid surface to the liquid and is thus often the desired operating condition for solids suspension. In this work, measurement of speed at which particular solids particles get to just-suspended condition is the primary experimental task.

Based on the developed design guideline of Equation 7, the speed based on the sum of powers for the individual solids suspension alone can be used to predict the solids mixture just-suspended speed. Thus, the just-suspended speeds required to suspend individual components in a binary system as well as the mixture just-suspended speeds need to be determined in each experiment.

As a rule, within one set of experiments, a fixed mass of solid 1 was first added in test tank and the speed required to suspend fixed solid 1 mass was determined by agreement of two observers with no more than five percent variation. Then, the varied amounts of solid 2 masses were progressively poured into the test tank, and a series of
speeds required for suspension of specific masses of solids 2 alone were measured. After measuring just-suspended speeds for the individual solids in a solids mixture, it was possible to estimate the solids mixture just-suspended speed by means of the design guideline of Equation 7. To investigate the validity of design guideline, the mixture just-suspended speeds were measured with the fixed mass of solid 1 and the various masses of solid 2 that were previously studied individually.

A reference tank was operated at the same time when measuring just-suspended speed of a solid mixture. The fixed amount of solid 1 was added to the reference tank, and speed required for suspension of solid 1 alone was set to that determined in the previous individual solid test.

In test tank, with the fixed mass of solid 1, a series of speeds required to suspend mixtures of solid 1 and solid 2 were measured with progressively increasing mass of solid 2. Once just-suspended speeds of the solid mixture had been determined, the prediction suspension speeds by the sum of powers required for suspension of the individual solids alone were compared to the measured mixture just-suspended speeds.

After running the experiment, solids were taken out of both test and reference tanks, and separated by sieve due to the difference in size of solid particles. Solids could then be used for further research when they were completely dried.
CHAPTER 3

RESULTS AND DISCUSSION

All the raw experimental data in this study is tabulated in the appendices. The tested mixtures of solids with different properties (particle size, shape, and specific gravity) are categorized into three different groups for analysis based on the specific gravities of individual solids in each system: systems where the specific gravities of both solids are below 1.5 grams per cubic centimeter (i.e. low-density system), both solid densities are above 2.4 grams per cubic centimeter (i.e. high-density system), and solids of mixed densities - that is, a solid with low density (below 1.5 grams per cubic centimeter) plus a solid with high density (above 2.4 grams per cubic centimeter). Two design guidelines are used to analyze the results obtained from each solid mixture: 1) just-suspended speed of a solids mixture is equal to that required to suspend the more difficult to suspend solid alone; 2) just-suspended speed of a solid mixture is equal to that based on summing the powers required for suspension of the individual solids alone. In this study, the first design guideline is not in accord with results from most experiments. Thus, the main focus of the following discussion is on the second guideline that estimates the mixture just-suspended speed by summing powers required to suspend the individual solids alone.
3.1 High-Density Systems

Seven solids mixtures were tested with the combination of two solids of density above 2.4 grams per cubic centimeter. Table 3 presents the details of these high-density mixtures.

Table 3: High-density solid mixture characteristics

<table>
<thead>
<tr>
<th>Solid 1</th>
<th>Solid 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine Sand, 1000 grams</td>
<td>Glass, 50-4000 grams</td>
</tr>
<tr>
<td>SG=2.78 g/cm³, Size=200 μm</td>
<td>SG=2.44 g/cm³, Size=600 μm</td>
</tr>
<tr>
<td>$N_{js}=363$ rpm</td>
<td>$N_{js}=282-606$ rpm</td>
</tr>
<tr>
<td>Aluminum Oxide, 200 grams</td>
<td>Fine Sand, 50-2500 grams</td>
</tr>
<tr>
<td>SG=3.89 g/cm³, Size=900 μm</td>
<td>SG=2.78 g/cm³, Size=200 μm</td>
</tr>
<tr>
<td>$N_{js}=465$ rpm</td>
<td>$N_{js}=260-422$ rpm</td>
</tr>
<tr>
<td>Small Silicon Carbide, 500 grams</td>
<td>Glass, 50-4000 grams</td>
</tr>
<tr>
<td>SG=3.13 g/cm³, Size=400 μm</td>
<td>SG=2.44 g/cm³, Size=600 μm</td>
</tr>
<tr>
<td>$N_{js}=431$ rpm</td>
<td>$N_{js}=282-606$ rpm</td>
</tr>
<tr>
<td>Large Silicon Carbide, 300 grams</td>
<td>Small Silicon Carbide, 50-4000 grams</td>
</tr>
<tr>
<td>SG=3.13 g/cm³, Size=1000 μm</td>
<td>SG=3.13 g/cm³, Size=400 μm</td>
</tr>
<tr>
<td>$N_{js}=460$ rpm</td>
<td>$N_{js}=315-688$ rpm</td>
</tr>
<tr>
<td>Glass, 200 grams</td>
<td>Olivine Sand, 50-5000 grams</td>
</tr>
<tr>
<td>SG=2.44 g/cm³, Size=600 μm</td>
<td>SG=3.50 g/cm³, Size=85 μm</td>
</tr>
<tr>
<td>$N_{js}=319$ rpm</td>
<td>$N_{js}=244-388$ rpm</td>
</tr>
<tr>
<td>Aluminum Oxide, 200 grams</td>
<td>Olivine Sand, 100-10,000 grams</td>
</tr>
<tr>
<td>SG=3.89 g/cm³, Size=900 μm</td>
<td>SG=3.50 g/cm³, Size=85 μm</td>
</tr>
<tr>
<td>$N_{js}=465$ rpm</td>
<td>$N_{js}=254-432$ rpm</td>
</tr>
<tr>
<td>Large Silicon Carbide, 300 grams</td>
<td>Olivine Sand, 100-10,000 grams</td>
</tr>
<tr>
<td>SG=3.13 g/cm³, Size=1000 μm</td>
<td>SG=3.50 g/cm³, Size=85 μm</td>
</tr>
<tr>
<td>$N_{js}=460$ rpm</td>
<td>$N_{js}=254-432$ rpm</td>
</tr>
</tbody>
</table>

3.1.1 Fine Sand – Glass System

For this experiment, sand was set to a fixed mass of 1000 grams, and just-suspended speed for this amount of sand was measured at 363 rpm. Then, glass with various amount of mass in a range from 50 to 4000 grams was progressively poured into agitated vessel, and a series of speeds for suspension of glass alone were determined to be
282 to 606 rpm. After that, with 1000 grams fine sand, the mixture just-suspended speeds were measured with increasing mass of glass. Figure 7 is the experimental data set of fine sand – glass system. It is shown that speed required for suspension of various masses of glass alone as red square, speed necessary to suspend 1000 grams of fine sand alone as brown line, and speed required to suspend the solid mixture as blue diamond. The green curve is the prediction suspension speed of the solid mixture based on the sum of suspension powers of glass and fine sand alone.

Figure 7: Experimental data for fine sand – glass system

As shown in Figure 7, in the case of fine sand – glass system, the prediction speed based on the sum of powers for suspension of the individual solids alone is approximately equal to the mixture just-suspended speed in a range of glass mass from 400 to 3000 grams with only one percent average variation, although the mixture just-suspended speed for the last data point with 4000 grams of glass is four percent higher than the prediction speed. On the other hand, at low glass mass between 50 and 400 grams, the
sum of powers approach always over predicts the just-suspended speed of fine sand – glass mixture with an average of six percent difference. In addition, it is seen that, at small amount of glass mass between 50 and 400 grams, just-suspended speed of fine sand alone is higher than that of glass alone; consequently, fine sand becomes the more difficult to suspend solid when glass mass is below 400 grams. At higher amounts of glass mass, the more difficult to suspend solid turns out to be glass since just-suspended speed of glass alone exceeds that of fine sand alone. In general, the prediction speed from the more difficult to suspend solid alone (either fine sand at small glass mass or glass at its higher mass) always under predicts the mixture just-suspended speed for all experimental data. However, as more glass mass is added, just-suspended speed of glass alone progressively approaches the mixture just-suspended speeds. For instance, for glass mass with 3000 grams, the deviation of the glass alone prediction speed from the measured mixture suspension speed is only six percent that is much lower than that of eleven percent for 500 grams of glass mass. For fine sand – glass system, comparison of two prediction approaches indicates that the sum of powers method is the better approach to estimate the just-suspended speed of binary mixture.

3.1.2 Aluminum Oxide – Fine Sand System

In the system of aluminum oxide and fine sand, aluminum oxide was fixed at 200 grams while fine sand varied with mass in the range from 50 to 2500 grams. Speed required to suspend 200 grams of aluminum oxide was measured at 465 rpm, higher than that for suspension of 2500 grams of fine sand that was 422 rpm, so all speeds required to suspend various fine sand masses were lower than that required to suspend 200 grams
aluminum oxide. This means that aluminum oxide is much more difficult to suspend even at small amount of mass, than fine sand. Figure 8 is a data representation for aluminum oxide and fine sand system. This plot includes the speeds required for suspension of fine sand with various masses alone (red square), the speed necessary to suspend 200 grams aluminum oxide alone (brown line), and the speeds required to suspend solids mixture (blue diamond). The green curve is the prediction speed based on the sum of suspension powers of individual solids in binary system.

![Figure 8: Experimental data for aluminum oxide – fine sand system](image)

In the case of aluminum oxide – fine sand system, the prediction guideline based on the sum of powers for suspension of the individual solids alone is essentially equal to the measured just-suspended speed of the solids mixture for fine sand mass between 500 and 2500 grams. However, for the first seven low fine sand masses, it is noted that the measured mixture just-suspended speed averages approximately two percent higher than the prediction speed from summing the individual solids suspension powers. In
comparison with the sum of powers approach, although aluminum oxide is much more
difficult to suspend than fine sand, the just-suspended speed of aluminum oxide alone is
always lower than the measured mixture just-suspended speed. It is found that the just-
suspended speed of aluminum oxide alone is seven percent lower than the measured
mixture just-suspended speed at the first fine sand mass of only 50 grams. For aluminum
oxide – fine sand system, compared with the more difficult to suspend solids approach,
the sum of powers approach provides more reasonable estimate of the mixture just-
suspended speed.

3.1.3 Small Silicon Carbide – Glass System

In this binary system, fixed mass of small silicon carbide was 500 grams and glass
mass progressively increased from 50 to 4000 grams. Just-suspended speed of 500 grams
of small silicon carbide was determined at 431 rpm while the range of just-suspended
speed for various glass masses alone was from 282 to 606 rpm. Figure 9 is an
experimental data plot for small silicon carbide – glass system. The just-suspended
speeds of glass alone are highlighted as red square, the speed necessary to suspend small
silicon carbide alone is shown in brown line, and the speeds required to suspend the
solids mixture are expressed as blue diamond. The green curve is the prediction speed of
solids mixture based on the sum of suspension powers of the glass and silicon carbide
alone.
**Figure 9:** Experimental data for small silicon carbide – glass system

As shown in Figure 9, in this binary system, the prediction method based on the sum of powers required to suspend the individual solids alone is valid for estimating the mixture just-suspended speed throughout this experiment. It is determined that the difference between the measured mixture just-suspended speed and the prediction speed from summing the individual solids suspension powers averages about one percent for all tested experimental data. In addition, for glass masses less than 1000 grams, it is observed that the small silicon carbide is the harder to suspend solid since its just-suspended speed is higher than that of glass alone. However, the prediction speed from the more difficult to suspend solid alone approach is almost twelve percent average lower than the measured mixture just-suspended speed in a range of glass mass from 50 to 1000 grams, indicating the effect of the easier to suspend glass on the mixture just-suspended speed should take into consideration. At higher amount of glass mass, the glass becomes
the more difficult to suspend solid and the harder to suspend material prediction approach still under predicts the mixture just-suspended speed with thirteen percent average variation. For small silicon carbide – glass system, again, the sum of powers approach is the better method to estimate the mixture just-suspended speed in comparison with the more difficult to suspend solid approach.

3.1.4 Large Silicon Carbide – Small Silicon Carbide System

In this mixture system, the density of the individual solids is the same, and the particle size of those solids is different – large silicon carbide of 1000 μm and small silicon carbide at 400 μm. With 300 grams of large silicon carbide alone, its just-suspended speed was measured at 460 rpm. In comparison, just-suspended speed of 300 grams small silicon carbide was 412 rpm that was lower than speed of larger one by more than ten percent. Figure 10 is an experimental data set for this binary system. The plot includes the speed required for suspension of small silicon carbide alone (red square), the speed necessary to suspend large silicon carbide alone (brown line), and the speeds required to suspend the solids mixture (blue diamond). The green curve is the prediction suspension speed of solids mixture based on the sum of powers required for suspension of large silicon carbide and small silicon carbide alone.
Figure 10: Experimental data for large silicon carbide – small silicon carbide system

As shown in Figure 10, the prediction approach based on summing the suspension powers of the individual solids in binary system over estimates the measured mixture just-suspended speed by no more than five percent average difference for all tested experimental data. It is also determined that the difference between the mixture just-suspended speed and the prediction speed from the sum of powers approach averages over six percent in a range of small silicon carbide mass from 50 to 300 grams. For small silicon carbide mass over 300 grams, the variation between the predicted and measured speeds averages less than four percent. Thus, once small silicon carbide mass is above 300 grams, the sum of powers approach reasonably estimates the just-suspended speed of the solids mixture. On the other hand, for small silicon carbide masses less than 750 grams, it is seen that just-suspended speed of large silicon carbide alone is higher than that of small silicon carbide, indicating the large silicon carbide is the more difficult to
suspend solid. The prediction speed from the more difficult to suspend solid alone is always lower than the measured mixture just-suspended speed with an average of ten percent difference for small silicon carbide mass from 50 to 750 grams. At higher amount of small silicon carbide mass, the more difficult to suspend solid turns out to be small silicon carbide and its just-suspended speed alone is always less than the measured mixture just-suspended speed with eleven percent average variation. Also, it is shown in the plot that the just-suspended speed of small silicon carbide alone gradually approaches the mixture just-suspended speed as its mass fraction goes up. For large silicon carbide – small silicon carbide system, the mixture just-suspended speed could be reasonably predicted by the sum of powers approach, although this estimate may be conservative in most instances. The more difficult to suspend solids approach significantly under predicts the mixture just-suspended speed as it has with other high-density solid mixtures.

3.1.5 Glass – Olivine Sand System

In this binary system, olivine sand of 85 μm, which is the smallest tested solid, is studied with glass. The mass of glass was set to 200 grams while the mass of olivine sand was varied from 50 to 5000 grams. Speed for suspension of 200 grams glass was measured at 319 rpm; in comparison, speed required for suspension of 200 grams of olivine sand was 268 rpm that was lower than the suspension speed of glass by more than fifteen percent. Figure 11 is an experimental data set for mixture system of glass and olivine sand. This plot consists of the speed required for suspension of olivine sand with various masses alone (red square), and the speed necessary to suspend 200 grams glass alone (brown line), and the speed required for suspension of solids mixture (blue
diamond). The green curve is the prediction speed of solids mixture based on the sum of powers required to suspend the olivine sand and glass alone.

![Graph showing just-suspended speed vs. olivine sand mass](image)

**Figure 11:** Experimental data for glass – olivine sand system

As shown in Figure 11, the prediction method based on the sum of powers required to suspend the individual solids in a binary system always substantially over predicts the measured mixture just-suspended speeds. It is determined that the difference between the measured mixture just-suspended speed and the sum of powers prediction speed averages thirteen percent for the tested data points. Glass – olivine sand system becomes the first atypical high-density solids mixture in which the sum of powers approach is not valid for estimating the mixture just-suspended speed. In addition, at low olivine sand masses (the first four data points), the just-suspended speed of glass is essentially equal to the mixture just-suspended speed. As the olivine sand mass is increased above that of the first four data points, the just-suspended speed of the solids mixture turns out to be higher than that of either solid alone, but lower than the sum of
powers estimate. Further, as olivine sand mass increases, the just-suspended speed of olivine sand alone progressively approaches the mixture just-suspended speed; particularly for the last tested olivine sand mass. For glass – olivine sand system, the sum of powers approach does not predict the mixture just-suspended speed, perhaps due to the atypical physical characteristic of olivine sand (smallest size). Also, the more difficult to suspend solids approach does not hold true except at the lower and highest olivine sands mass.

3.1.6 Aluminum Oxide – Olivine Sand System

In the test of aluminum oxide and olivine sand system, aluminum oxide was fixed at 200 grams while olivine sand mass varied from 100 to 10,000 grams. Speed required for suspension of 200 grams aluminum oxide was 465 rpm and was significantly higher than that required for suspension of 200 grams olivine sand that was measured at 268 rpm. Further, it was found that even 10,000 grams of olivine sand is easier to suspend than 200 grams of aluminum oxide. Figure 12 is an experimental data set for the solids mixture of aluminum oxide and olivine sand. The plot consists of the speed required for suspension of olivine sand alone (red square), the speed necessary to suspend 200 grams aluminum oxide alone (brown line), and the speed required to suspend solids mixture (blue diamond). The green curve is the prediction speed based on the sum of powers required to suspend the individual solids alone in the binary system.
**Figure 12:** Experimental data for aluminum oxide – olivine sand system

As shown in Figure 12, the measured just-suspended speed of the solids mixture is not only much lower than the speed predicted from summing the individual solids suspension powers, but also lower than that required for suspension of aluminum oxide (the more difficult to suspend solid) alone except for the last olivine sand mass. Neither prediction method is valid in this binary system though the just-suspended speed of aluminum oxide is approximately equal to the mixture just-suspended speed as olivine sand mass alone exceeds 3000 grams. It is shown that the addition of small olivine sand mass makes the aluminum oxide become easier to suspend in the stirred tank, and the reductions in the mixture just-suspended speed in comparison with the just-suspended speed of the aluminum oxide alone average nearly ten percent for the first six olivine sand masses. This is a significant reduction and is not observed with any other tested systems. For aluminum oxide – olivine sand system, the atypical physical properties of the olivine sand (most likely its small size and/or spherical shape) significantly affects the
suspension behavior of solids mixture studied, so that the two design guidelines are not reasonably valid for predicting the mixture just-suspended speed.

3.1.7 Large Silicon Carbide – Olivine Sand System

In this binary system, the mass of silicon carbide was fixed at 300 grams while olivine sand mass varied from 100 to 10,000 grams. Speed required to suspend 300 grams large silicon carbide was found to be 460 rpm, significantly higher than just-suspended speed of the same amount of olivine sand that was measured at 278 rpm. Also, even 10,000 grams of olivine sand is easier to suspend than 300 grams of large silicon carbide similar to the preceding aluminum oxide – olivine sand system. Further, silicon carbide of 1000 μm is substantially larger than olivine sand of 85 μm. Figure 13 is an experimental data set for a mixture of silicon carbide and olivine sand. This plot includes the speed required for suspension of olivine sand alone (red square), the speed necessary to suspend large silicon carbide alone with a fixed mass of 300 grams (brown line), and the speed required for suspension of solids mixture (blue diamond). The green curve is the prediction speed of solids mixture based on summing the individual suspension powers alone.
Figure 13: Experimental data for large silicon carbide – olivine sand system

As shown in Figure 13, the measured just-suspended speed of large silicon carbide – olivine sand system is much lower than the prediction method of summing the individual solids suspension powers, averaging over eleven percent for all experimental data. On the other hand, the prediction method based on the more difficult to suspend solid (large silicon carbide here) alone is approximately valid in this case. A small reduction in the mixture just-suspended speed occurred when adding small masses of olivine sand, averaging three percent for the first five olivine sand masses in comparison with just-suspended speed of large silicon carbide alone. This reduction is much lower than that observed with aluminum oxide – olivine sand system in which the decrease averages nearly ten percent for the first six olivine sand masses. For large silicon carbide – olivine sand system, the sum of powers approach is not valid for estimating the mixture just-suspended speed while the more difficult to suspend solid approach may be preferable for predicting mixture just-suspended speed with olivine sand.
3.2 Low-Density Systems

Eight solids mixtures were tested with the combination of two solids of density below 1.5 grams per cubic centimeter. Table 4 presents a list of information about each tested solid in the binary systems.

Table 4: Low-density solid mixture characteristics

<table>
<thead>
<tr>
<th>Solid 1</th>
<th>Solid 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green Plastic, 200 grams</td>
<td>Red Acrylic, 50-3000 grams</td>
</tr>
<tr>
<td>SG=1.32 g/cm³, Size=2900 μm</td>
<td>SG=1.18 g/cm³, Size=3000 μm</td>
</tr>
<tr>
<td>N_J=202 rpm</td>
<td>N_J=146-286 rpm</td>
</tr>
<tr>
<td>Red Acrylic, 1000 grams</td>
<td>Brown Resin, 250-6000 grams</td>
</tr>
<tr>
<td>SG=1.18 g/cm³, Size=3000 μm</td>
<td>SG=1.23 g/cm³, Size=620 μm</td>
</tr>
<tr>
<td>N_J=218 rpm</td>
<td>N_J=155-295 rpm</td>
</tr>
<tr>
<td>Red Acrylic, 500 grams</td>
<td>Brown Resin, 125-5000 grams</td>
</tr>
<tr>
<td>SG=1.18 g/cm³, Size=3000 μm</td>
<td>SG=1.23 g/cm³, Size=620 μm</td>
</tr>
<tr>
<td>N_J=185 rpm</td>
<td>N_J=155-265 rpm</td>
</tr>
<tr>
<td>Brown Resin, 500 grams</td>
<td>Red Acrylic, 50-3000 grams</td>
</tr>
<tr>
<td>SG=1.23 g/cm³, Size=620 μm</td>
<td>SG=1.18 g/cm³, Size=3000 μm</td>
</tr>
<tr>
<td>N_J=176 rpm</td>
<td>N_J=147-286 rpm</td>
</tr>
<tr>
<td>Green Acrylic, 500 grams</td>
<td>White Resin, 100-2000 grams</td>
</tr>
<tr>
<td>SG=1.028 g/cm³, Size=3000 μm</td>
<td>SG=1.053 g/cm³, Size=780 μm</td>
</tr>
<tr>
<td>N_J=106 rpm</td>
<td>N_J=95-135 rpm</td>
</tr>
<tr>
<td>Green Acrylic, 1000 grams</td>
<td>Brown Resin, 50-2000 grams</td>
</tr>
<tr>
<td>SG=1.028 g/cm³, Size=3000 μm</td>
<td>SG=1.23 g/cm³, Size=620 μm</td>
</tr>
<tr>
<td>N_J=125 rpm</td>
<td>N_J=142-218 rpm</td>
</tr>
<tr>
<td>Red Acrylic, 500 grams</td>
<td>White Resin, 500-5000 grams</td>
</tr>
<tr>
<td>SG=1.18 g/cm³, Size=3000 μm</td>
<td>SG=1.053 g/cm³, Size=780 μm</td>
</tr>
<tr>
<td>N_J=185 rpm</td>
<td>N_J=116-161 rpm</td>
</tr>
<tr>
<td>Melamine, 500 grams</td>
<td>Brown Resin, 50-2000 grams</td>
</tr>
<tr>
<td>SG=1.45 g/cm³, Size=200 μm</td>
<td>SG=1.23 g/cm³, Size=620 μm</td>
</tr>
<tr>
<td>N_J=178 rpm</td>
<td>N_J=142-218 rpm</td>
</tr>
</tbody>
</table>

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3.2.1 Green Plastic – Red Acrylic System

In this binary system, the particle sizes of both solid materials are essentially the same but the density of green plastic is higher than that of red acrylic. The just-suspended speed of 200 grams green plastic is 202 rpm, higher than the 163 rpm just-suspended speed of the same amount of red acrylic. Figure 14 is an experimental data set for green plastic and red acrylic system. This plot contains the speed required to suspend the various red acrylic masses alone (red square), the speed required to suspend 200 grams green plastic alone (brown line), and the speed required to suspend the solids mixture at various masses of red acrylic (blue diamond). The green curve is the prediction speed of solids suspension based on the sum of powers required to suspend the individual solids alone.

![Graph showing experimental data for green plastic - red acrylic system](image)

**Figure 14:** Experimental data for green plastic – red acrylic system
As shown in Figure 14, the prediction method of summing individual solids suspension powers exceeds the measured mixture just-suspended speed for all experimental data. It is determined that the deviation of prediction speeds from the measured mixture just-suspended speeds averages almost ten percent. However, as red acrylic mass increases, the mixture just-suspended speeds approach the prediction speeds from summing the individual solids suspension powers. Particularly for the last red acrylic mass of 3000 grams, the difference between the measured and predicted mixture suspension speed is only three percent. On the other hand, the prediction method based on the more difficult to suspend solid alone determining mixture just-suspended speed is only valid at low red acrylic mass. It is shown that the measured mixture just-suspended speeds are essentially equal to the just-suspended speed of 200 grams green plastic alone for the four lowest red acrylic masses, averaging no more than two percent variation. As red acrylic mass is added over 200 grams, this prediction method no longer holds true. For green plastic – red acrylic system, the sum of powers approach does not reasonably predict the just-suspended speed of solid mixture though it approximates the prediction speed at high red acrylic mass (e.g. 3000 grams). In comparison with the sum of powers approach, the more difficult to suspend solid approach does successfully predict the mixture just-suspended speed at low red acrylic mass but fails once more red acrylic mass is added.

3.2.2 Red Acrylic – Brown Resin System (1)

In the system of red acrylic and brown resin, the specific gravities of both solid materials are similar to each other, but the particle size of red acrylic is substantially
larger than that of brown resin. In this set of experiments, red acrylic mass was fixed at 1000 grams while brown resin mass varied in the range of 250 to 6000 grams. The speed required to suspend 1000 grams red acrylic was measure at 218 rpm and it was higher than the 194 rpm required for suspension of brown resin at the same mass. Figure 15 is an experimental data set for this binary system. The plot includes the speed required to suspend brown resin alone (red square), the speed required to suspend 1000 grams red acrylic alone (brown line), and the speed required to suspend solids mixture (blue diamond). The green curve is the prediction speed of solids mixture based on the sum of powers required to suspend the individual solids alone.

![Figure 15: Experimental data for red acrylic – brown resin system (1)](image)

As shown in Figure 15, the prediction speed based on summing the suspension powers of individual solids always over predicts the measured mixture suspension speed, with the error of prediction speeds to measured mixture suspension speeds averaging
approximately eleven percent for brown resin masses from 250 to 4000 grams. However, for the three highest brown resin masses, the measured mixture just-suspended speeds approach the prediction speeds with only five percent average difference. On the other hand, for brown resin mass less than 2000 grams, red acrylic is the more difficult to suspend solid in that just-suspended speed of 1000 grams red acrylic is higher than that of brown resin alone. It is found that the prediction method based on the more difficult to suspend solid alone determining the mixture just-suspended speed is only valid for the three lowest brown resin masses. For brown resin masses between 1000 and 2000 grams, just-suspended speed of red acrylic alone is seven percent less than the measured just-suspended speed. Besides, for brown resin mass higher than 2000 grams, red acrylic becomes the more easier to suspend solid due to lower just-suspended speed in comparison with brown resin. It is determined that the more difficult to suspend solid alone approach is also less than the measured mixture just-suspended speed, averaging ten percent variation for brown resin mass from 2000 to 6000 grams. For red acrylic – brown resin system, the sum of powers approach always over predicts the mixture just-suspended speeds though the mixture just-suspended speeds approximate the prediction speeds for brown resin mass over 4000 grams. The more difficult to suspend solid alone approach is only valid for brown resin masses less than 1000 grams, after that, the prediction speed always under predicts the mixture just-suspended speeds.

3.2.3 Red Acrylic – Brown Resin System (2)

In this binary system, the mass of red acrylic was set to 500 grams while the various masses of brown resin are in the range of 125 through 5000 grams. Just-
suspended speed of 500 grams red acrylic was determined as 185 rpm, and just-suspended speed was 174 rpm for suspension of the same amount of brown resin mass. Figure 16 is an experimental data set for red acrylic and brown resin system. It contains the speed required to suspend brown resin with various masses alone (red square), the speed necessary to suspend 500 grams red acrylic alone (brown line), and the speed required to suspend the solids mixture (blue diamond). The green curve is the prediction speed based on summing the individual solids suspension powers.

![Figure 16](image)

**Figure 16:** Experimental data for red acrylic – brown resin system (2)

In this system of red acrylic and brown resin, the prediction method of summing the individual solids suspension powers is not valid throughout the experiments with the difference of prediction speed from the measured mixture suspension speed averaging about fifteen percent. For the three highest brown resin masses, the variations between the predicted and measured speed average nearly twelve percent; consequently, the
mixture just-suspended speed does not approach the prediction speed even at highest brown resin masses. On the other hand, for brown resin mass less than 750 grams, red acrylic is the harder to suspend solid as just-suspended speed of 500 grams for red acrylic is greater than that of brown resin. It is observed that, for the first three brown resin masses, the more difficult to suspend solid alone approach reasonably predicts the mixture just-suspended speed with only about three percent average difference. Also, for brown resin mass greater than 750 grams, it turns out that brown resin is more difficult to suspend solid as a result of higher solids suspension speed relative to red acrylic. It is found that for this situation the more difficult to suspend solid alone approach also reasonably estimates the mixture just-suspended speed with about three percent average variation. For red acrylic – brown resin system (2), the sum of powers approach does not provide a reasonable estimate of just-suspended speed of the solids mixture while the more difficult to suspend solid alone approach reasonably predicts the mixture just-suspended speed.

3.2.4 Brown Resin – Red Acrylic System (3)

In this binary system, the brown resin mass was fixed to 500 grams while the red acrylic masses were varied from 50 to 3000 grams. Brown resin has smaller size and higher specific gravity relative to red acrylic. Speed for suspension of 500 grams brown resin was 176 rpm, and speed for suspension of the same amount of red acrylic mass was 193 rpm. Figure 17 is an experimental data set for this binary system. The plot contains the speed required to suspend red acrylic (red square), the speed required to suspend 500 grams brown resin alone (brown line), and the speed required to suspend the mixture of
solids at various proportions of two solids (blue diamond). The green curve is the prediction speed of the solids mixture based on the sum of powers required to suspend the individual solids alone.

![Graph showing the relationship between red acrylic mass and just-suspended speed with various data points and curves representing different conditions.](image)

**Figure 17:** Experimental data for brown resin – red acrylic system (3)

As shown in Figure 17, the prediction method of the sum of powers required to suspend the individual solids alone always over predicts the mixture just-suspended speed, averaging nearly nine percent higher. However, it is seen that for red acrylic mass greater than 1500 grams, the prediction speeds from summing the individual solids suspension powers approach the mixture just-suspended speed with only about five percent average difference. On the other hand, for red acrylic masses less than 400 grams, just-suspended speed of brown resin alone is greater than that of red acrylic, indicating that brown resin is more difficult to suspend solid at low red acrylic mass. In the range of red acrylic mass from 50 to 300 grams, just-suspended speed of 500 grams brown resin
under predicts the measured mixture just-suspended speed except for the first red acrylic mass, averaging seven percent lower. For red acrylic masses over 400 grams, red acrylic becomes the more difficult to suspend solid. It is found that just-suspended speeds of red acrylic alone averages nearly five percent lower than the mixture just-suspended speeds for red acrylic masses from 1000 to 3000 grams. For brown resin – red acrylic system (3), neither the sum of powers approach or the more difficult to suspend solid alone approach is valid for estimating the mixture just-suspended speed for all experimental data, but both prediction approaches could reasonably predict the mixture just-suspended speed for brown resin mass greater than 1500 grams.

3.2.5 Green Acrylic – White Resin System

In the test of this binary system, the two solid materials with the lowest densities are studied. The mass of green acrylic was fixed at 500 grams while white resin masses varied from 50 to 2000 grams. Speed required for suspension of 500 grams green acrylic was only 106 rpm. Figure 18 is an experimental data set for green acrylic and white resin system. The plot contains the speed required to suspend white resin with various masses alone (red square), the speed required to suspend green acrylic alone (brown line), and the speed required to suspend the solids mixture (blue diamond) at various proportions of two solids. The green curve is the prediction speed of solids mixture based on the sum of powers required to suspend the individual solids alone.
As shown in Figure 18, the prediction speeds of summing the individual solids suspension powers are always substantially higher than the measured mixture just-suspended speeds with an average of eighteen percent error. In addition, for white resin masses less than 250 grams, it is shown that just-suspended speed of 500 grams green acrylic is greater than that of white resin alone, indicating that green acrylic is harder to suspend solid at low white resin masses. The more difficult to suspend solid alone approach is essentially valid for estimating the mixture just-suspended speed with only two percent average variation for white resin masses from 50 to 250 grams. For white resin masses over 300 grams, white resin turns out to be the more difficult to suspend solid, and it is found that just-suspended speed of white resin alone averages about three percent lower than the mixture just-suspended speed in a range of white resin mass from 300 to 2000 grams. For green acrylic – white resin system, the sum of powers approach
does not provide a reasonable estimate of the mixture just-suspended speed throughout the experiment. In comparison with the sum of powers approach, the more difficult to suspend solid alone approach more accurately predicts the mixture just-suspended speed.

3.2.6 Green Acrylic – Brown Resin System

In the system of green acrylic and brown resin, the mass of green acrylic was set to 1000 grams while the masses of brown resin were studied in the range of 50 to 2000 grams. The speed required to suspend 1000 grams green acrylic was only 125 rpm, much lower than the 194 rpm required to suspend the same brown resin mass. Further, it is observed that just-suspended speed of 1000 grams green acrylic is even lower than that of brown resin with the smallest mass (50 grams), indicating brown resin is the more difficult to suspend solid throughout this test. Figure 19 is an experimental data set for green acrylic and brown resin system. It includes the speed required to suspend various masses of brown resin alone (red square), the speed required to suspend 1000 grams green acrylic alone (brown line), and the speed required for suspension of solids mixture (blue diamond). The green curve is the prediction speed of summing the individual solids suspension powers.
**Figure 19:** Experimental data for green acrylic – brown resin system

In the case of this low-density system, as shown in Figure 19, the prediction speed based on the sum of powers to suspend the individual solids alone is approximately equal to the mixture suspension speed for brown resin masses from 400 to 2000 grams, averaging only one percent variation. For brown resin masses less than 400 grams, the prediction speed always over predicts the measured mixture just-suspended speed by an average of about nine percent. On the other hand, the more difficult to suspend solid (brown resin) alone approach always under predicts the measured mixture just-suspended speed with about seven percent average difference, with the difference being greater at higher brown resin masses. For green acrylic – brown resin system, the sum of powers approach can reasonably predict the mixture just-suspended speed for brown resin masses greater than 400 grams while the more difficult to suspend solid alone approach can
provide an acceptable estimate of the mixture just-suspended speed, particularly for brown resin masses from 50 to 200 grams.

### 3.2.7 Red Acrylic – White Resin System

In the system of red acrylic and white resin, the red acrylic mass was fixed at 500 grams and it required 185 rpm to achieve the just-suspended condition. The white resin mass varied from 100 to 5000 grams, and the speed required to suspend 500 grams white resin was 116 rpm, indicating red acrylic is always the more difficult to suspend solids in this low-density system. Figure 20 is an experimental data set for this binary system. The plot contains the speed required to suspend various masses of white resin alone (red square), the speed necessary to suspend red acrylic alone (brown line), and the speed required to suspend the mixture of solids at various proportions of the two solids (blue diamond). The green curve is the prediction speed of solids mixture based on the sum of powers required to suspend the individual solids alone.
Figure 20: Experimental data for red acrylic – white resin system

As shown in Figure 20, the prediction method of summing the individual solids suspension powers always over predicts the measured mixture just-suspended speed with over eleven percent average error. On the other hand, the prediction speed based on the more difficult to suspend solid (red acrylic) alone determining the mixture just-suspended speed is valid for estimating the mixture just-suspended speed, averaging about two percent variation for all experimental data. This indicates that the effect of addition of white resin on the just-suspended speed can be neglected, even for the highest white resin masses of 5000 grams. For red acrylic – white resin system, the sum of powers approach does not provide a reasonable estimate of the mixture just-suspended speed while the more difficult to suspend solid alone approach is more acceptable method than the sum of powers approach.
3.2.8 Melamine – Brown Resin System

In the system of melamine and brown resin, the particle size of melamine is smaller than that of brown resin but melamine has higher density than brown resin. It is found that the 176 rpm required for melamine suspension is essentially equal to the 178 rpm required for brown resin at 500 grams for each solid. Figure 21 is an experimental data set for melamine and brown resin system. It contains the speed required to suspend various masses of brown resin alone (red square), the speed required to suspend 500 grams melamine alone (brown line), and the speed required to suspend the mixture of solids at various solids proportions (blue diamond). The green curve is the prediction speed of solids mixture based on the sum of powers required to suspend the individual solids alone.

![Graph showing experimental data for melamine – brown resin system](image)

**Figure 21:** Experimental data for melamine – brown resin system

In the case of this low-density system, as shown in Figure 21, the prediction speeds based on the sum of powers for the individual solids suspension alone are always
higher than the measured mixture just-suspended speeds with nearly ten percent average variation throughout the experiment. As brown resin mass increases, the prediction speeds approach the measured mixture just-suspended speeds, being only four percent higher at the highest brown resin masses. On the other hand, for brown resin masses less than 750 grams, it is seen that just-suspended speed of melamine alone is greater than that of brown resin alone, meaning melamine is more difficult to suspend solid at low brown resin masses. Also, for the first four brown resin masses, speed of the more difficult to suspend melamine alone is essentially equal to the measured mixture just-suspended speed with only three percent average difference. For brown resin masses higher than 750 grams, brown resin becomes the more difficult to suspend solid. It is found that, for the last four brown resin masses, speed of the more difficult to suspend brown resin alone averages about twelve percent lower than the mixture just-suspended speed. For melamine – brown resin system, the sum of powers approach always over predicts the mixture just-suspended speed though the prediction speed approaches the mixture suspension speed at the two highest brown resin masses. In comparison with the sum of powers approach, the more difficult to suspend solid alone is only valid for estimating the mixture just-suspended speed for small brown resin masses.

3.3 Mixed-Density Systems

Four solids mixtures were tested with the combination of one solid with density below 1.5 grams per cubic centimeter plus the other with density above 2.4 grams per cubic centimeter. Table 5 presents information about each tested solid in these binary systems.
### Table 5: Mixed-density solid mixture characteristics

<table>
<thead>
<tr>
<th>Solid 1</th>
<th>Solid 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aluminum Oxide, 200 grams</strong></td>
<td><strong>Green Plastic, 200-2500 grams</strong></td>
</tr>
<tr>
<td>SG=3.89 g/cm³, Size=900 μm</td>
<td>SG=1.32 g/cm³, Size=2900 μm</td>
</tr>
<tr>
<td><em>N</em>=465 rpm</td>
<td><em>N</em>=202-342 rpm</td>
</tr>
<tr>
<td><strong>Glass, 200 grams</strong></td>
<td><strong>Melamine, 50-7000 grams</strong></td>
</tr>
<tr>
<td>SG=2.44 g/cm³, Size=600 μm</td>
<td>SG=1.45 g/cm³, Size=200 μm</td>
</tr>
<tr>
<td><em>N</em>=319 rpm</td>
<td><em>N</em>=150-450 rpm</td>
</tr>
<tr>
<td><strong>Green Plastic, 1000 grams</strong></td>
<td><strong>Fine Sand, 50-2500 grams</strong></td>
</tr>
<tr>
<td>SG=1.32 g/cm³, Size=2900 μm</td>
<td>SG=2.78 g/cm³, Size=200 μm</td>
</tr>
<tr>
<td><em>N</em>=269 rpm</td>
<td><em>N</em>=260-422 rpm</td>
</tr>
<tr>
<td><strong>Green Plastic, 1000 grams</strong></td>
<td><strong>Olivine Sand, 50-5000 grams</strong></td>
</tr>
<tr>
<td>SG=1.32 g/cm³, Size=2900 μm</td>
<td>SG=3.50 g/cm³, Size=85 μm</td>
</tr>
<tr>
<td><em>N</em>=269 rpm</td>
<td><em>N</em>=244-388 rpm</td>
</tr>
</tbody>
</table>

#### 3.3.1 Aluminum Oxide – Green Plastic System

In the system of aluminum oxide and green plastic, the mass of aluminum oxide was fixed at 200 grams while green plastic masses varied from 400 to 2500 grams. It is observed that just-suspended speed was measured at 465 rpm for suspension of 200 grams aluminum oxide while just-suspended speed of the highest green plastic mass of 2500 grams is about twenty six percent less than that of 200 grams aluminum oxide, indicating the aluminum oxide is the much more difficult to suspend component in this binary system. Figure 22 is an experimental data set for aluminum oxide and green plastic system. The plot contains the speed required for suspension of various green plastic masses (red square), the speed necessary to suspend 200 grams aluminum oxide alone (brown line), and the speed required to suspend solids mixture (blue diamond). The green curve is the prediction of solid mixture suspension speed based on the sum of suspension powers of the individual solids alone.
Figure 22: Experimental data for aluminum oxide – green plastic system

As shown in Figure 22, the prediction method of summing the individual solids suspension powers alone is valid in this case except for the last data point with 2500 grams of green plastic, with the variations between the predicted and measured speeds averaging only about two percent for all experimental data. On the other hand, although green plastic is much easier to suspend than aluminum oxide in this solids mixture, as seen in Figure 20, just-suspended speed of 200 grams aluminum oxide is always lower than the measured mixture just-suspended speed with nearly seven percent average difference. However, the more difficult to suspend solid alone approach is valid for estimating the mixture just-suspended speed for low green plastic masses, averaging only three percent variation for the first three green plastic masses. At green plastic mass higher than 1000 grams, the mixture just-suspended speed increases above the aluminum oxide suspension speed alone by an the average of about eight percent. For aluminum oxide – green plastic system, the sum of powers approach provides a reasonable estimate
of the mixture just-suspended speed while the more difficult to suspend speed approach is only valid for green plastic masses less than 1000 grams. Comparison of the sum of powers approach and the more difficult to suspend solid alone approach indicates that the sum of powers approach is preferable to predict just-suspended speed of the solids mixture.

3.3.2 Glass – Melamine System

In the set of experiments for glass and melamine system, the glass mass was set to 200 grams while the melamine mass varied from 50 to 7000 grams. In this case of mixed-density systems, in comparison to melamine, glass has not only bigger particle size but also higher density. For example, just-suspended speed of 200 grams glass was 319 rpm whereas it was only 159 rpm for suspension of the same amount of melamine mass. Figure 23 is an experimental data set for glass and melamine system. The following plot includes the speed required for suspension of the various melamine masses alone (red square), the speed necessary to suspend the fixed glass mass alone (brown line), and the speed required to suspend the solids mixture (blue diamond). The green curve is the prediction of solid mixture suspension speed based on the sum of suspension powers of the melamine and glass alone.
**Figure 23:** Experimental data for glass – melamine system

As shown in Figure 23, in glass – melamine system, the prediction method of the sum of powers for the individual solids suspension alone is approximately equal to the measured mixture just-suspended speed, averaging only two percent difference. However, it is seen that most mixture just-suspended speeds are slightly higher than the prediction speeds for melamine masses from 200 to 4500 grams. On the other hand, for melamine masses less than 5000 grams, it is seen that just-suspended speed of 200 grams glass alone is always higher than that of various melamine masses, indicating glass is the more difficult to suspend solid. The more difficult to suspend solid alone approach is only valid for predicting the mixture just-suspended speed for the four lowest melamine masses, and not for melamine masses greater than 200 grams. For melamine masses above 4500 grams, melamine becomes the more difficult to suspend solid as the just-suspended speed of melamine alone is higher than glass. The more difficult to suspend
solid alone approach under predicts the measured mixture just-suspended speed although the prediction speed is close to the measured speed for the highest melamine mass, being only five percent lower. For glass – melamine system, the sum of powers approach is reasonably valid for predicting the mixture just-suspended speed throughout the experiment while the more difficult to suspend solid alone approach generally under predicts the mixture just-suspended speed although it provides a reasonable estimate of the mixture just-suspended speed at lower melamine masses.

3.3.3 Green Plastic – Fine Sand System

In the system of green plastic and fine sand, the green plastic mass was set to 1000 grams while the fine sand mass measured from 50 to 2500 grams. The speed to suspend 1000 grams of green plastic alone was 269 rpm, the same as for suspension of only 100 grams fine sand due to the higher density of fine sand compared to green plastic. Figure 24 is an experimental data set for the mixed-density system of green plastic and fine sand. The following plot includes the speed required for suspension of the various fine sand masses alone (red square), the speed necessary to suspend the fixed green plastic mass alone (brown line), and the speed required to suspend the solids mixture (blue diamond). The green curve is the prediction of solids mixture suspension speed based on the sum of powers to suspend the individual solids.
Figure 24: Experimental data for green plastic – fine sand system

In the case of this mixed-density system, the prediction speed based on the sum of powers for the individual solids suspension alone is valid for the mixture just-suspension speed estimation, with the average error being only one percent. In addition, as shown in Figure 24, for fine sand mass greater than 100 grams, just-suspended speed of various fine sand alone is higher than that of green plastic alone, indicating fine sand is the more difficult to suspend solid. It is found that prediction speeds from the more difficult to suspend solid alone approach are always below the solids mixture just-suspended speeds, but are gradually approaching the measured mixture speed as fine sand mass increases. For the highest fine sand mass of 2500 grams, the difference between the prediction speed of fine sand alone and measured mixture speed is eight percent. For green plastic – fine sand system, comparison of the sum of powers approach and the more difficult to
suspend solid alone approach indicates that the sum of powers approach is preferred to estimate the just-suspended speed of the binary mixture.

3.3.4 Green Plastic – Olivine Sand System

In the system of green plastic and olivine sand, the green plastic mass was set to 1000 grams total while the olivine sand varied from 50 through 5000 grams. The speed required to suspend 1000 grams of olivine sand was 342 rpm which was substantially higher than the 269 rpm at which 1000 grams of green plastic was in just-suspended condition. Figure 25 is an experimental data set for green plastic and olivine sand system. The following plot includes the speed required for suspension of the various olivine sand masses alone (red square), the speed necessary to suspend the fixed green plastic mass alone (brown line), and the speed required to suspend the solids mixture (blue diamond). The green curve is the prediction of solids mixture suspension speed based on the sum of powers of the green plastic and olivine sand alone.
Figure 25: Experimental data for green plastic – olivine sand system

In this case of this mixed-density system, the method of summing the individual solids suspension powers always over predicts the solids mixture just-suspended speeds for all experimental data, with the difference between the predicted and measured speeds averaging eight percent that is significant in comparison with the results from the previous work in mixed-density systems. On the other hand, for olivine sand masses more than 200 grams, just-suspended speed of olivine sand alone is greater than that of 1000 grams green plastic, indicating that olivine sand is the more difficult to suspend solid. Just-suspended speeds of olivine sand alone are always lower than the measured mixture just-suspended speeds for olivine sand mass from 200 to 5000 grams with eight percent average difference. However, it is also observed that, for the two highest olivine sand masses, the more difficult to suspend solid alone approach is approximately valid for mixture just-suspended speed estimation with only four percent average error. For green plastic – olivine sand system, neither the sum of powers approach nor the more
difficult to suspend solid alone approach provides a good estimate of the mixture just-
suspended speed, though the more difficult to suspend solid alone approach is
approximately valid at the higher olivine sand masses.

3.4 Ternary Systems

Most experiments in this work are associated with systems consisting of two
solids with individual physical properties; however, limited study of two ternary systems
is carried out to investigate whether the prediction method of summing the individual
solids suspension powers can provide a reasonable estimate of just-suspended speed of
these more complex solids mixtures. One ternary system consisted of green plastic, fine
sand, and aluminum oxide while the other was large silicon carbide, fine sand, and glass.
For the first solids system, each of these solids has been tested as a two solid pair, and it
was found that the sum of powers approach successfully fits the measured mixture just-
suspended speed for any combination of two solids. For the second solids system in
which densities of all solids are higher than 2.4 grams per cubic centimeter, it is
reasonable to expect that the sum of powers approach can reasonably predict the mixture
just-suspended speed based on the previous work on high-density systems, though the
individual solids were not studied in all possible binary combinations. Table 6 presents
information about each tested solid in the ternary systems.
Table 6: Ternary solid mixture characteristics

<table>
<thead>
<tr>
<th>Solid 1</th>
<th>Solid 2</th>
<th>Solid 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Green Plastic</strong></td>
<td><strong>Aluminum Oxide</strong></td>
<td><strong>Fine Sand</strong></td>
</tr>
<tr>
<td>1000 grams</td>
<td>200 grams</td>
<td>200-2500 grams</td>
</tr>
<tr>
<td>SG=1.32 g/cm³, Size=2900 μm</td>
<td>SG=3.89 g/cm³, Size=900 μm</td>
<td>SG=2.78 g/cm³, Size=200 μm</td>
</tr>
<tr>
<td>N&lt;sub&gt;j&lt;/sub&gt;=269 rpm</td>
<td>N&lt;sub&gt;j&lt;/sub&gt;=465 rpm</td>
<td>N&lt;sub&gt;j&lt;/sub&gt;=291-422 rpm</td>
</tr>
<tr>
<td><strong>Large Silicon Carbide</strong></td>
<td><strong>Fine Sand</strong></td>
<td><strong>Glass</strong></td>
</tr>
<tr>
<td>300 grams</td>
<td>1000 grams</td>
<td>300-4000 grams</td>
</tr>
<tr>
<td>SG=3.13 g/cm³, Size=1000 μm</td>
<td>SG=2.78 g/cm³, Size=200 μm</td>
<td>SG=2.44 g/cm³, Size=600 μm</td>
</tr>
<tr>
<td>N&lt;sub&gt;j&lt;/sub&gt;=460 rpm</td>
<td>N&lt;sub&gt;j&lt;/sub&gt;=364 rpm</td>
<td>N&lt;sub&gt;j&lt;/sub&gt;=341-606 rpm</td>
</tr>
</tbody>
</table>

3.4.1 Green Plastic – Aluminum Oxide – Fine Sand System

In the system of green plastic, aluminum oxide, and fine sand, the masses of green plastic and aluminum oxide were fixed at 1000 and 200 grams, respectively, while the fine sand mass varied from 200 to 2500 grams. Just-suspended speed of green plastic and aluminum oxide system was 493 rpm. It is observed that, for the highest fine sand masses, just-suspended speed of fine sand alone is still less than that of green plastic and aluminum oxide system. Figure 26 is an experimental data set for this ternary system.

The plot contains the speed required to suspend the various fine sand masses alone (red square), the speed required to suspend the solids slurry of green plastic and aluminum oxide alone (brown line), and the speed required for suspension of ternary system (blue diamond). The green curve is the prediction of solid mixture suspension speed based on the sum of powers required to suspend the individual solids alone.
Figure 26: Experimental data for green plastic – aluminum oxide – fine sand system

In this ternary system, the prediction speeds based on the sum of powers for the individual solids suspension alone is approximately equal to the speeds required to suspend the three-component mixture for all experimental data though the sum of powers approach always slightly over predicts the measured just-suspended speeds. The difference between the predicted and measured speeds averages less than two percent throughout the experiment.

3.4.2 Large Silicon Carbide – Fine Sand – Glass System

In this ternary system, the masses of large silicon carbide and fine sand were set to 300 and 1000 grams, respectively, while the glass mass varied from 300 to 4000 grams. Speed for suspension of large silicon carbide and fine sand system was measured at 513 rpm. Figure 27 is an experimental data set for this ternary system. The plot contains the
speed required to suspend the various glass masses alone (red square), the speed required to suspend the solids slurry of silicon carbide and fine sand alone (brown line), and the speed required for suspension of ternary system (blue diamond). The green curve is the prediction of solid mixture suspension speed based on the sum of powers required to suspend the individual solids alone.

**Figure 27:** Experimental data for large silicon carbide – fine sand – glass system

As shown in Figure 27, the prediction method of summing the individual solids suspension powers is valid for estimating just-suspended speed of this ternary solids mixture throughout the experiment, with the average error between the predicted and measured speeds being only one percent. However, for the highest glass mass of 4000 grams, the measured mixture just-suspended speed increases somewhat abruptly and exceeds the prediction speed by three percent.
CHAPTER 4
CONCLUSION

The purpose of this work was the study of suspension of solids mixtures with a broad range of physical characteristics in a liquid-solid stirred tank at one specific agitation level – the just-suspended condition. The just-suspended condition is the primary agitation level for settling solids suspension, when no solid particles rest on the bottom of tank for longer than one to two seconds. It allows all solids to be suspended in liquid phase and provides the maximum surface area for desired process requirements, such as mass or heat transport and chemical reaction, with the minimum power consumption. In this work, all tested binary combinations of solids with different physical characteristics are classified into three groups based on the specific gravities of solids in each system, and limited study of two ternary systems is also carried out. Two proposed design guidelines, the sum of powers approach and the more difficult to suspend solid approach, have been simultaneously studied to predict the agitation speed required to suspend a solid mixture at just-suspended condition. The following paragraphs are a brief summary of results for mixture just-suspended speed estimation based on these two studied design guidelines.

Seven high-density solids systems in which the densities of the individual solids are higher than 2.4 grams per cubic centimeter were studied in this work. Figure 28
presents a parity plot that compares the measured just-suspended speeds of high-density systems to predicted speeds by summing the individual solids suspension powers.

**Figure 28**: Parity plot of measured and predicted just-suspended speeds of seven high-density solids systems

In general, as seen in Figure 28, for first four high-density systems, those that do not contain olivine sand, the data are generally distributed on the diagonal line of the predicted – measured just-suspended speed graph, indicating prediction speeds from the sum of powers approach are approximately equal to measured just-suspended speeds for all experimental data. On the other hand, for last three high-density systems involving olivine sand, the sum of powers approach always substantially over predicts the measured
mixture just-suspended speeds, with an average deviation of predicted speeds from measured speeds being over ten percent. Compared to the sum of powers approach, the more difficult to suspend solid alone approach always significantly under predicts just-suspended speeds of high-density systems that do not contain olivine sand while, for olivine sand systems, this approach is valid for estimating just-suspended speeds of both glass – olivine sand and aluminum oxide – olivine sand systems for the lowest and highest olivine sand masses, and is approximately valid for the large silicon carbide – olivine sand system. Further, for aluminum oxide – olivine sand system as well as large silicon carbide – olivine sand system, it is observed that the addition of small olivine sand masses makes the mixture just-suspended speed less than just-suspended speed of the more difficult to suspend solid alone (either aluminum oxide or large silicon carbide). This reduction is not observed with any other tested high-density systems in which the mixture just-suspended speed increases with the addition of the second solid. The atypical suspension behavior of olivine sand systems is perhaps due to the physical properties of olivine sand (small size and high density) as well as its low Archimedes number \( \text{Ar} = \frac{d^3 g (\rho_s - \rho_i)}{v_s^2 \rho_i} \). Table 7 presents Archimedes number of the individual solids studied in this work.
Table 7: Archimedes number of tested solids

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<th>Material</th>
<th>Size (µm)</th>
<th>Density (g/cm³)</th>
<th>Archimedes Number</th>
</tr>
</thead>
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</tr>
<tr>
<td>Melamine</td>
<td>200</td>
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</tr>
<tr>
<td>Fine Sand</td>
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<td>Brown Resin</td>
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<td>530</td>
</tr>
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<td>3006</td>
</tr>
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<td>Green Acrylic</td>
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<td>1.028</td>
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</tr>
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<td>Aluminum Oxide</td>
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<td>Large Silicon Carbide</td>
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<td>3.13</td>
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</tr>
<tr>
<td>Red Acrylic</td>
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<td>1.18</td>
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</tr>
<tr>
<td>Green Plastic</td>
<td>2900</td>
<td>1.32</td>
<td>75000</td>
</tr>
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</table>

Molerus and Latzel (1987) postulated that solids with Archimedes number less than forty are completely immersed in the viscous sub-layer near the vessel base and wall and boundary layer effects are essential for prediction of complete suspension speed of these solids. Since Archimedes number of olivine sand is only fifteen, this significant characteristic may be responsible for the atypical behavior of olivine sand systems. Possible support for this view is that Ayranci and coworkers (2011b) observed that the sum of powers approach significantly over estimated just-suspended speeds of one solids mixture in which the Archimedes numbers of the individual solids were fifteen and forty-one.

Eight low-density solids mixtures in which the densities of the individual solids are less than 1.5 grams per cubic centimeter were examined in this study. The novelty of this work is no other researchers have investigated mixtures of low-density solids, only high-density mixtures and a few mixed-density mixtures. Figure 29 presents a parity plot comparing the measured just-suspended speeds of low-density systems to predicted speeds by summing the individual solids suspension powers.
Figure 29: Parity plot of measured and predicted just-suspended speeds of eight low-density solids systems

As shown in the graph, it is found that the prediction speeds from summing the individual solids suspension powers are almost always greater than the measured speeds, with the average difference between the predicted and measured speeds typically being ten percent or more. For only one exceptional case, brown resin – green acrylic system, the sum of powers approach is valid for estimating the mixture just-suspended speed for brown resin masses from 400 and 2000 grams, averaging only one percent variation. For the other seven solids mixtures, the sum of powers approach is a conservative estimate of
the mixture just-suspended speeds. On the other hand, compared to the sum of powers approach, the more difficult to suspend solid alone approach is sometimes valid for estimating the just-suspended speed of low-density solids mixtures, particularly for the red acrylic – white resin system. In red acrylic – white resin system, the mixture just-suspended speeds closely agree with the more difficult to suspend red acrylic suspension speed alone, averaging about two percent variation for all experimental data. For the other seven solids mixtures, in general, the more difficult to suspend solid alone approach provides a reasonable estimate of the mixture just-suspended speed at the lowest added masses of the second solid. Also, for some solids systems (i.e. red acrylic – brown resin (2)), at the highest added solid masses, the prediction speeds from the more difficult to suspend solid alone approach are approximately equal to the measured mixture just-suspended speeds.

Four mixed-density solids systems, consisting of one solid with density below 1.5 grams per cubic centimeter plus another solid with density above 2.4 grams per cubic centimeter were tested in this work. Figure 30 presents a parity plot comparing the measured just-suspended speeds of mixed-density systems to predicted speeds by summing the individual solids suspension powers.
Figure 30: Parity plot of measured and predicted just-suspended speeds of four mixed-density solids systems

For three solid mixtures without olivine sand, the data basically lie on the diagonal line in Figure 30, implying the sum of powers approach provides a reasonable estimate of the mixture just-suspended speed for all experimental data. However, it is also found that some of the measured mixture just-suspended speeds are slightly higher than the prediction speeds from summing the individual solids suspension powers, indicating the sum of powers approach is not always a conservative estimate as was generally observed for high-density and low-density solids systems. This was in agreement with observation from Etchells et al.'s (2010) work. They found the sum of powers approach under predicts the measured just-suspended speed of one mixed-density system, with the
predicted speeds averaging seven percent lower than the measured speeds. Although the Archimedes number of melamine is less than forty, it does not behave like olivine sand that makes the sum of powers approach significantly over predict the mixture just-suspended speed. It was found that the sum of powers approach for the glass – melamine system still reasonably estimates the mixture just-suspended speed throughout the experiment. For only one mixed-density system, that contains olivine sand (green plastic – olivine sand system) and shown as red squares in the graph, the sum of powers approach significantly over predicts the mixture just-suspended speed throughout the experiment in disagreement with the other mixed-density systems. As with the high-density systems containing olivine sand, this smallest solid with the lowest Archimedes number of all tested solid materials may be responsible for the atypical solids suspension behavior of green plastic – olivine sand system. The more difficult to suspend solid alone approach is not acceptable to estimate the mixture just-suspended speed of mixed-density systems since the prediction speeds of the more difficult to suspend solid alone are generally significantly less than the measured mixture just-suspended speeds, although this prediction approach is valid for estimating the mixture just-suspended speed at the highest olivine sand masses for the green plastic – olivine sand system.

Only two ternary systems were studied in this work, and it was found that the sum of powers approach is reasonably valid for estimating the mixture just-suspended speed for this limited experimental data.

In summary, the sum of powers approach can reasonably predict the just-suspended speed of both high-density and mixed-density solids systems, except those with olivine sand. The more difficult to suspend solid alone approach is valid for
estimating the just-suspended speed of low-density solids systems in limited instances, particularly for the lowest and highest added masses of the second solid while the sum of powers approach is almost always a conservative estimate of low-density systems just-suspended speeds. Future research in this area needs to carefully investigate the effect of atypical solids characteristics (i.e. small solid particles with high density) on solids mixture suspension as well as suspension behaviors of low-density solids systems to develop a complete design guideline for successfully estimating the agitation speed required to suspend a solid mixture at just-suspended condition.


APPENDIX A

JUST-SUSPENDED SPEEDS OF SOLIDS ALONE

Table A-1: Just-suspended speed of low-density solids as a function of mass

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<th>Mass (gram)</th>
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<th>Red Acrylic $N_{js}$ (rpm)</th>
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Table A- 3: Just-suspended speed of high-density solids as a function of mass

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<th>Olivine Sand $N_{js}$ (rpm)</th>
<th>Glass $N_{js}$ (rpm)</th>
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APPENDIX B

JUST-SUSPENDED SPEEDS OF SOLIDS MIXTURES

Table B-1: Just-suspended speed of high-density solid mixtures

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<th>Solid 2 Mass (gram)</th>
<th>Fine Sand (1) – Glass (2) System N_{js} (rpm)</th>
<th>Aluminum Oxide (1) – Fine Sand (2) System N_{js} (rpm)</th>
<th>Small Silicon Carbide (1) – Glass (2) System N_{js} (rpm)</th>
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Table B-3: Just-suspended speed of low-density solid mixtures

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