A PARAMETRIC STUDY FOR PANEL BUCKLING SENSITIVITY
OF COMPOSITE SANDWICH WIND TURBINE BLADES

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A PARAMETRIC STUDY FOR PANEL BUCKLING SENSITIVITY OF COMPOSITE SANDWICH WIND TURBINE BLADES

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ABSTRACT

A PARAMETRIC STUDY FOR PANEL BUCKLING SENSITIVITY OF COMPOSITE SANDWICH WIND TURBINE BLADES

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A parametric study for generating buckling results and performance for composite wind turbine blade preliminary design with thin symmetric laminated sandwich rectangular panels subjected to uniform axial shell edge compression loads and with four edges elastically restrained as simple supported is presented. This research is mainly focused on buckling behavior at critical buckling strain level and with nondimensional core material parameters (transverse core shear modulus and core thickness) for rectangular sandwich strips of sufficiently long aspect ratio. In order to represent different regions of the structures, the cross sections of the sandwich panels are both flat and curved model. Extensive buckling-design trends are plotted based on a fixed set of laminate designs and critical buckling loads, in order to provide insight on optimal core solutions. Several typical buckling response sweeps presented and discussed in this paper to provide useful and readable design data. The application of these results in practical design optimization would involve assessing the cost and weight of various core products at the indicated optimal thickness values, then comparing the cost and weight of the various solutions toward the design
objectives. The analytical solutions are verified by closed-form numerical calculations and the trends generated are based on results using the ABAQUS 6.10.
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LIST OF SYMBOLS

\(a\) = length of the model, m.
\(b\) = width of the mode, m.
\(c\) = core (foam) thickness, m.
\(d\) = distance between centre-lines of opposite faces =\(c + t\), m.
\(k\) = panel curvature ratio, \(\%\) (arch height over panel width)
\(l\) = curve length, m.
\(r\) = radius, m
\(t\) = facing thickness, m
\(h\) = overall thickness of sandwich.
\(m\) = 1, 2, 3, ......
\(n\) = 1, 2, 3, ......
\(w_2\) = deformation, m.
\(x, y, z\) = rectangular coordinates.
\(C_0\) = normalized core thickness (core thickness over total facing sheet thickness)
\(R, Z, T\) = cylindrical coordinates.
\(Z\): direction along the length
\(T\): radial direction
\(R\): curve angle direction
\(U_1, U_2, U_3\) = displacement in \(x, y, z\) direction
\(U_r, U_z, U_t\) = displacement in \(R, Z, T\) direction
I: second moment of area, especially total value of faces about centroid of sandwich plate.

\( P_{cr} \) = critical buckling load (=eigenvalue), N/m

\( P_x \) = critical buckling value (=eigenvalue), N/m

\( \sigma_{cr} \) = critical buckling stress, N/m²

\( \varepsilon_{cr} \) = critical buckling strain, %

\( \lambda \) = critical buckling load (eigenvalue), N/m

\( E_x, E_y, E_z (E_1, E_2, E_3) \) = Moduli of elasticity

\( D_{xy}, D_x, D_y \) = flexural rigidity.

\( D_{Qx}, D_{Qy} \) = core shear stiffnesses, per unit width

\( G_{xz} (G_{13}) \) = Transverse shear modulus, Pa

\( G_{yz} (G_{23}) \) = Transverse shear modulus, Pa

\( \nu_1, \nu_2, \nu_3 \) = Poisson's ratio

\( Q_x, Q_y \) = shear forces,

\( N_x \) = Uniform compressive edge load, N/n
CHAPTER I
INTRODUCTION

Over the past several years renewable energy sources have been responsible for a higher share of the global energy production. This trend is mostly covered by wind energy and is not only the result of more turbines being installed but also of increasing the power output by increasing the blade length. With the blade length increasing, composite materials are used in the blade design. Since the composite wind turbine blade is becoming thin-walled to save the weight, the buckling analysis has to be addressed [1]. There is a large potential for reducing the amount of material in the blade, and thus its cost and weight.

The modern wind energy industry was established in the late 1970s, since then the wind industry has been growing steadily [2]. Driven by the requirement of a higher energy output per turbine unit, the dimension of large wind turbine blades under development is also increasing. It is because the wind turbine power output is dependent on the blade length, wind speed, and mechanical efficiency, the size of the wind turbines has increased rapidly to match the higher power demand [3]. It was announced in March, 2011 that a 7 MW wind turbine with blade tip radius of 82 m would be constructed for field analyze [4]. With the increasing size of the wind turbine blade, the structure requires small weight, large stiffness and simplicity of assembling. As such, the traditional materials and blade structures are no longer reasonable to use. It has been recognized that sandwich structures are used increasingly in applications requiring high bending stiffness and strength combined
with low weight and high fatigue life compared to wood and metals [5][6]. Sandwich plates, usually consisting of two stiff, strong face sheets and a light weight core can be designed to possess a high bending stiffness and strength at a low weight, which exactly meet the structural requirements of wind turbine blade.

The design of composite wind turbine blades is a challenging work due to the need for pushing the material utilization to the limit in order to obtain light and cost effective structures [7]. As a result of the minimum material design strategy, the composite wind turbine blade structures are becoming thin-walled, such that buckling problems must be addressed.

With the development of finite element analysis (FEA) field, new analysis methods and software are found and launched. The result will be an upper limit for the real strength value and ensures the structure is safe to use. This parametric study herein is based on a FEA tool ABAQUS, code 6.10 to find the panel buckling sensitivity of composite sandwich wind turbine blades, with multiple core materials and different number of facing layers.

Wind turbine blades are thin-wall structures and are sensitive to geometric design parameters. The structural performance of the blade structures mainly depends on the properties of the skin and core materials. Also, the strength of the sandwich structure is influenced by the bonding characteristics of the skin and core and the geometrical dimensions of the components. The largest single role of the sandwich core is to assure adequate stability of the large panel regions against buckling. Toward that goal, the most significant attributes of the core material are the transverse shear modulus and the core thickness [6]. The performance of a parametric study at a basic level can be conducted using simple uniform-width panel geometries to represent the various sandwich regions of a blade. The analyzed models consist of
four typical core materials within a given design domain. Considering the blade design model, the analyzed thickness of the core is ranging from 20mm to 45mm with minimum one facing layer up to maximum five facing layers. The analyzed plate model is a high aspect ratio model which provides buckling loads and wavelengths characteristic of very long compressively loaded plate elements. The width of the plate is ranging from 1m up to 5m to represent different characteristic regions of sandwich laminates in blades. Another analyzed model is a symmetrically laminated infinitely long plate with different initial curvatures from 5% up to 25% with uniformly compressed axial shell edge load. In order to compare the result in the same scale, the two analyzed models are using the same boundary conditions i.e. two long sides simple-supported and two short sides simple-supported. All the FEA models and the analyze results are validated by the closed-form solution from the studies of Allen [8] and Gambhir [9].

Many non-dimensional parameters are introduced in order to find generalized design curves. The critical buckling strain is an ideal non-dimensional design parameter. It is often the practice in structural design to characterize material loading and performance in terms of strain levels rather than stress levels, because strain compatibility must exist across the interface between spar caps, shells, and shear webs, in the blade span direction, whereas stress levels will be discontinuous due to the variety of composite constructions used in the different composite regions.

The analysis process can be divided into three steps: (1) set up the model and use ABAQUS to find the critical buckling load (buckling eigenvalue); (2) rerun the same model in a static analysis using the critical buckling load to find the material strain; (3) repeat the model with different core materials and the number of facing layers and generate the plots and trends.
The results shown in design plots can clearly provide a certain core material and corresponding thickness to meet material to meet the structural design requirement or compare the material weight/cost. Also, the result shows how the thickness of composite core and core shear modulus effects the panel buckling. For low-stiffness core materials, or discretely reinforced core products, local buckling of facings is a potential failure mode. In local areas of some blade designs, there are also significant transverse shear and/or through-thickness core stresses that must be considered.

Setting aside these less prevalent structure design considerations, there is merit in performing this parametric study of the buckling performance of core materials on the basis of number of facing layers and core thickness, which can provide insight on optimal core solutions. It is because prevalent core materials and products are generally available in a range of weights, with corresponding ranges of mechanical properties and cost.

The results from this thesis are practical in design which cannot only provide a better pre-design plan but would also involve assessing the cost and weight of various core products at the indicated optimal thickness values, then comparing the cost and weight of the various solutions toward the design objectives.
CHAPTER II
REVIEW OF RELATED RESEARCH AND LITERATURE

References [4, 10-17, 20] present general concepts of the composite structures and introduce wind turbine structures. The design for thin-walled structures and the methods of buckling analysis are also referred. They also summarize the information about the present and future of the composite wind industry. Reference [19] provided a detailed cost study for large wind turbine blades, including material cost, labor cost, indirect manufacturing cost, overhead cost, development cost, and facilities cost. It shows the cost study for 30m, 50m, and 70m large wind turbine blade. The concept of composite material structures cost also referred in reference [20].

The composite materials for wind turbine blades have been discussed in reference [21] and the new blade optimize methods are also introduced including: Discrete Material Optimization (DMO) approach proposed by Stegmann and Lund (2005), and Nondestructive Inspection (DNI) methods. A survey of optimal design of laminated plates and shells can be found in, e.g., Abrate [22]. The exhaustive steps of Discrete Material Optimization (DMO) approach is presented in reference [7]. The DMO approach can convert a discrete design optimization problem to a continuous problem (introducing continuous weight functions to convert discrete problem to continuous model) to obtain light and cost effective thin-walled shell structures. It is because in composite structures materials ply angles and layer thicknesses are often employed as continuous design variables. However, in practice the layer thickness for each material is usually fixed and fiber orientation angles of the fiber-reinforced
materials are often limited to a discrete set. This DMO approach is based on ideas from multi-phase topology optimization where the material stiffness is computed as a weighted sum of candidate materials, thus making it possible to solve discrete optimization problems using gradient based techniques and mathematical programming.

The DMO method is applied for buckling design of laminated multi-material shell structures, and in the current work the approach is investigated for buckling design of a wind turbine blade analyzes section. The later work related to this method is presented in reference [23] to find more improved models for both buckling and cracking analysis along interfaces in material joints.

In reference [2], a parametric finite element model (created by MSC/Patran) is used to analyze two basic designs with single skin and sandwich flanges in large wind turbine blades (110m- 125m blade), respectively. It compares traditional single-skin blade design with the one composite sandwich panels are used as load carrying elements, a significant weight reduction (43%) and increased buckling capacity is obtained. The related later work was presented in Reference [24], in which the same process was done for the 180m wind turbine blade to analysis the single skin and sandwich flanges respectively.

Reference [6.25] present a cost-effective study for wind turbine using preliminary design calculations for a 3.0 MW rotor blade, quantify the potential benefits in stiffness and decreased gravity loading by replacement of a baseline fiberglass spar with carbon-fiberglass hybrid material, because the wind turbine application for composite materials is very cost sensitive. Also in reference [42] a sandwich core trade-off study is generated for both blade skin and shear web sub-structures. However, early in this effort anomalies were encountered in the modeling
results, so the verification calculations were only performed for buckling of simple flat plates.

Weaver and Nemeth [26] analyzed for buckling resistance of long rectangular orthotropic plates with simply supported or clamped edges and subjected to uniform axial compression, uniform shear, or pure in-plane bending loads. Nondimensional parameters and equations governing the buckling behavior of rectangular symmetrically laminated plates are presented that can be used to represent the buckling in a general manner amenable to the development of concise design data. The study use classical composite construction equations and focus on how to generate the practical upper and lower bounds of basically the nondimensional parameters in order to gain insight into bounds on the buckling resistance of rectangular plates made of existing materials. A similar process was presented in reference [27] but for an infinitely long symmetric plate model focusing on the effects of flexural orthotropy and flexural anisotropy on plates subjected to various combined loading conditions. Those processes were not based on finite element code but for a purpose parametric analysis. It is due to the cost and effort usually involved in generating a large number of FE results with a general purpose code. In reference [28] Nemeth also presented parametric study for infinite long, flexural anisotropic plates, with the same boundary conditions as presented above.

In reference [29,30], extensive and accurate numerical results are presented for the critical buckling loads of all edges simply supported, rectangular, laminated composite plates subjected to five types of loading conditions: (1) uniaxial, (2) hydrostatic biaxial, (3) compression-tension biaxial, (4) positive shear and (5) negative shear. The rectangular plate has an aspect ratio (a/b) of 2 with three selected core materials and a fixed facing material. The study method is the Ritz method with a
set of displacement equations and the displacement equations assumed is in the form of a double sine series. Experimental analysis and flammability analyzes carried out on sandwich structures in reference [31] with four kinds of rigid composite core and six different skin materials have been reported. The result shows how the core-facing material properties would enhance the flexural property and the failure mechanisms.

Morovvati [32] presented his study in the analyze 2011 American Society of Composites (ASC) 26th Annual Technical Conference Reference. The study deals with the theoretical prediction of buckling loads for cyclic sandwich shells under axial compression with laminated facings and foam core. Various materials are employed to provide comparative data that can be used in design. Parameter studied in the stability analysis includes the radius to thickness ratio. The analytical solution is verified by finite element (FE) analysis and results are generated by the computer code ABAQUS, code 6.10. Reference [33], also explores on the theoretical prediction of buckling loads for cyclic sandwich shells under axial compression but with only isotopic material shell and closed-from solution numerical study. Reference [34] presented for simulating simply supported boundary conditions in axial cylinder buckling experiments but the model does not induce the rotation.

Reference [35] presents rectangular plates with symmetric layups using the Ritz Method to specify the coordinate functions in the general form irrespective of particular boundary conditions. The critical buckling stress is then found for general case and then is specified for typical combinations of three basic types of the boundary conditions, i.e., for free edges, simply supported edges and clamped edges. The rectangular plate studied is subjected to bi-axial in-plane tension or compression, in-plane shear, in-plane bending, and combined loading. The study presented is the
unification and supplementation of existing analytical approaches for composite plate buckling analysis. The analysis procedure is programmed into an analysis routine that is part of Spirit AeroSystems (SAS).

Reference [36] is based on the discrete-structural theory (similar as DMO approach) of thin plates and shells and variant of the equations of buckling stability, interfacial contact study is presented for a buckling stability of multilayer plates and shells of thin-walled structures, while reference [37] shows a linear buckling problem of orthotropic inhomogeneous rectangular plates under in-plane compression. The concept of this study is introducing the newly defined position of the reference plane to construct nonlinear buckling equations from linearized von Karman plate model.

The other lamination parametric researches are presented by Fukunaga [38], who studied buckling optimization of orthotropic laminated cylindrical shells under combined loadings using lamination parameters and a mathematical programming method. More work is done in reference [39-41], showing the approach for the lamination parameter method. Introduction of lamination parameters is efficient for stiffness optimization because the stiffness components of laminated composites are expressed as linear functions with respect to lamination parameters. However, the lamination parameter method requires closed-form analytical formulation of appropriate lamination parameters which has so far only been achieved for relatively simple geometries, i.e., not in case of general shell structures.

Webcore [42] and GEC did research on the buckling analysis for the entire blade for 25% and 50% span station by built FE model with bricks model. The load end boundary condition for the research is pinned and clamped. The study ended up with a local buckling study with a rectangular plate model and the research goal was only partly completed due to the complications in validation of the modeling approach,
particularly the selection of the finite element type for modeling the effects of bucking and shear deformation.

**SUMMARY OF THE LITERATURE REVIEW**

Review of literature reveals that there is a limitation for the buckling analysis for the thin walled sandwich structures. Most of the early buckling analysis works were based on anisotropic plate not sandwich structure. Also, these models are all plate models, which are not representative of caved panel in structures. At last, many research are based on numerical closed form research not computer program data. It is because the effort and time to generate the large data base.

With the blade length increasing, more and more researchers focus on the skin and core cost-effective study, which is one of the most important issues in blade design. Also, many works have been done with the buckling of thin walled cylindrical structures study. However, the result is still not practical in a real blade design or is not easy for engineer to read. The buckling analysis models built were limited to rectangular plate or infinitely long plate with no initial curvature.

Hence, an attempt has been made here to study the infinitely long models with curvatures and the goal is to find trends directly related to the core material thickness in shell and the design requirement. Those parameters are nondimensional. As we know material thickness can be easily turned into the production cost. This study here provides a practical and useful buckling analysis method in composite wind turbine blade design.
Also, since the lacking closed-form solution for shear buckling in sandwich composite panel, the models presented in this thesis are only for the compression loads.
CHAPTER III

FINITE ELEMENT ANALYSIS

In setting up the model, we consider the panel in the blade skin is found to be virtually a flat plate. We assumed that all layers of the panel are perfectly bonded together and thus the displacements and strains are continuous throughout the thickness. The strain results were chosen as basis for force distribution due to its linear variation throughout the thickness. Interlaminar effects were not taken into account.

Fig.1 General boundary conditions of the infinitely long strip of the shell [1]
Part one: Infinitely long symmetrically laminated flat plate FEA

I built the ABAQUS model with elements of S4R type. S4R means: A 4-node doubly curved thin or thick shell, reduced integration, hourglass control, finite membrane strains [46]. There are total 1111 number of nodes and total 1000 number of elements.

Fig 2. Flat sandwich panel model layout
The boundary conditions for flat panel are: (1) at the load edge, we set $U_2 = U_2 = 0$; (2) at the panel end, we set $U_1 = U_2 = U_3 = 0$; (3) at the both long side we set $U_2 = U_2 = 0$.

- $U_1$: along x direction
- $U_2$: along y direction
- $U_3$: along z direction
Fig. 4 Boundary conditions for flat panel

Fig. 5 Mesh for flat panel
The buckling wave and results are:

Fig. 6 ABAQUS buckling wave result for flat panel
Part two: Infinitely long symmetrically laminated curved plate validation

To avoid local buckling at the load edge, we set two rigid steel edges at the ends of the model. There are total number of 1313 nodes and 1200 elements in the model.

Fig.7 Curved sandwich panel model layout
Fig. 8 Curved plate model with rigid steel edges
Fig. 9 ABAQUS model layout-2
**Boundary Conditions for curved model**

Uz: direction along the length

Ut: radial direction

Ur: angle change direction

Fig. 10 Boundary conditions for curved panel
Fig. 11 Mesh for curved model
Fig. 12 ABAQUS result for finitely long curved sandwich panel model with no rigid ends
Fig. 13 ABAQUS result for finitely long curved sandwich panel model with rigid ends

Table 1: Candidate material properties:

<table>
<thead>
<tr>
<th></th>
<th>(E_1)</th>
<th>(E_2)</th>
<th>(E_3)</th>
<th>(\nu_{12})</th>
<th>(\nu_{13})</th>
<th>(\nu_{23})</th>
<th>(G_{12})</th>
<th>(G_{13}(G_1))</th>
<th>(G_{23}(G_2))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
</tr>
<tr>
<td>E_TLX 5500 (face sheet)</td>
<td>2140</td>
<td>1000</td>
<td>0</td>
<td>0.4</td>
<td></td>
<td></td>
<td></td>
<td>6000</td>
<td>3740</td>
</tr>
<tr>
<td>M1</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>0.33</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>M2</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>M3</td>
<td>284</td>
<td>250</td>
<td>210</td>
<td>0.39</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>146</td>
<td>108</td>
</tr>
<tr>
<td>M4</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>250</td>
<td>250</td>
</tr>
</tbody>
</table>
Description of the candidate material and analysis parameters

**E_TLX 5500:** The face sheet E_TLX 5500 [48] is E-glass material commonly used as the composite reinforcement wind turbine blade skin, which is certificated by Det Norske Veritas (DNV) and Germanischer Lloyd (GL).

![E-TLX 5500](image)

Fig.14 E-TLX 5500 facing sheet of wind turbine blade (courtesy of Vectorply.com)

Since in thin-walled structures buckling analysis, the core transverse shear modulus (G) will be the dominant parameter and the moduli of elasticity (E) is not significant to affect the results. We set the range of the transverse shear modulus from 20 Mpa to 250 Mpa.

**M1:** The material M1 is typically representative of the low modulus material such as low density poly vinyl chloride (PVC) materials.

**M2:** The material M2 is an engineering core with a higher modulus than M1.

**M3:** The material M3 is also an engineering core. It is representative of the most commonly used materials in the wind industry.
M4 [49]: The material M4 is in the range of high modulus core materials, such as high density balsa. Balsa is also a common core material in wind turbine blade industry but the core shear modulus ranges from mid to high.

**Load:** Uniformly distributed edge compression load; \( N_x = 1 \) (Newton);

**Face sheet thickness:** \( t = 0.0015 \) m; five layers symmetric

**Normalized core thickness:** \( C_0 = c/t \);

**Core thickness range:** 20mm-45mm

**Eigenvalue:** In buckling analysis we solve for the eigenvalues [50] that are scale factors that multiply the applied load in order to produce the critical buckling load. In general, only the lowest buckling load is of interest, since the structure will fail before reaching any of the higher-order buckling loads [45]. Therefore, only the lowest eigenvalues are computed from ABAQUS [46] results here.
CHAPTER IV

CLOSED-FORM SOLUTION EQUATIONS

Thin-walled structures made of laminated composite materials are being increasingly used in aerospace, automotive, marine and other technical applications. This is primarily because of the large values of specific strength and stiffness which may be obtained in comparison with conventional materials, such as metals. Laminated composite plates are often fabricated in cross-ply or alternating angle-ply sequences. The stacking of plies is usually done symmetrically with respect to the mid-plane of the plate to avoid bending-stretching coupling effects which may seriously diminish the stiffness [43]. When such plates are subjected to in-plane loading conditions, which cause internal stresses which are compressive, buckling may occur.

Many classical closed form solutions have been done by Whitney [44] for the laminated rectangular and cylindrical plate under uniaxial compression or pure shear with simple-supported or clamped boundary condition. Kollár [47] also presented closed form solution on buckling for composite structures including more boundary conditions and long plate study. Yet, not many closed form solutions exist for infinitely long plate with initial curvatures.

The presented work here is based on the ABAQUS analysis result data from two symmetrically laminated models: one is infinitely long flat plate; another is infinitely long curved plate. For the first model is validated by the closed form solutions from Allen [8] and only one closed form solutions for infinitely long curved
plate with isotropic material is found in Gambhir [9]. Symmetrically laminated here refers to plates in which every lamina above the plate midplane has a corresponding lamina located the same distance below the plate midplane, with the same thickness, material properties, and fiber orientation.

**Part one: Infinitely long symmetrically laminated flat plate validation**

In Kollár [22], as long as the length of a rectangular plate $a$ is more than five time of its width $b$ ($a/b > 5$), it is a long plate. Since we can not model an infinitely long plate, the analyzed model for closed form solution here is have a $a/b$ ratio of 5.

The closed form solution for infinitely long symmetrically laminated flat plate is obtained from Allen [8] for simply-supported sandwich orthotropic panel with thin faces and thick core. The critical buckling load is given by the following equations:

The transverse deformation $w_2$ is associated with shear strain in the core and therefore with the shear forces $Q_x$ and $Q_y$. The relationship between $w_2$ and the shear forces may be defined in terms of core shear stiffnesses, $D_{Qx}$ and $D_{Qy}$, as follows:

$$Q_x = D_{Qx} \frac{\partial w_2}{\partial x}; \quad Q_y = D_{Qy} \frac{\partial w_2}{\partial x};$$

(1)

From equation (1) may be derived the curvatures and rate of twist due to the deformation $w_2$:

$$\frac{\partial^2 w_2}{\partial x^2} = \frac{1}{D_{Qx}} \frac{\partial Q_x}{\partial x}; \quad \frac{\partial^2 w_2}{\partial y^2} = \frac{1}{D_{Qy}} \frac{\partial Q_y}{\partial y};$$

(2)

$$\frac{\partial^2 w_2}{\partial x \partial y} = \frac{1}{2D_{Qx}} \frac{\partial Q_x}{\partial y} + \frac{1}{2D_{Qy}} \frac{\partial Q_y}{\partial x};$$

(3)

Give definition to $g$ as:
Expressions for the various bending, twisting and shearing stiffnesses are given below without proof. The simplest cases are listed first.

The following is for orthotropic faces and core; faces are equal thickness and similar material:

\[
D_x = \frac{E_x t d^2}{2}, \quad D_y = \frac{E_y t d^2}{2}, \quad D_{xy} = \frac{G_{xy} t d^2}{2}, \quad D_{Qx} = D_{Qy} = \frac{G_x d^2}{c},
\]

\[
\nu_x E_x = \nu_y E_y.
\]

\[
D_{Qx} = \frac{G_{zx} d^2}{c}, \quad D_{Qy} = \frac{G_{zy} d^2}{c},
\]

Where \( G_{xy} \) refers to the faces; \( G_{zx} \) and \( G_{zy} \) to the core.

Precise expressions for the stiffnesses of a corrugated-core sandwich are given by Libove and Hubka, the result which follow are approximations:

\[
D_x = \frac{E_j t d^2}{2} + E_c I_c,
\]

\[
D_y = \frac{E_j t d^2 / 2}{1 - \frac{E_j t d^2}{2E_c I_c}} - \frac{\nu_j^2}{1 + \frac{E_j t d^2}{2E_c I_c}},
\]

\[
D_{xy} = \frac{E_j t d^2}{2(1 + \nu_j)}, \quad D_{Qx} = \infty;
\]

\[
D_{Qy} = \frac{S d E_c}{1 - \nu_x^2} \left( \frac{t_x}{d_c} \right)^3;
\]

\[
\nu_x = \nu_y, \quad \nu_y = \nu_j D_y / D_x;
\]
Consider a simply-supported orthotropic sandwich panel with edge load $N_x, N_y,$ $N_{xy}$, and a sinusoidally distributed transverse load $q(x,y)$ where:

$$q(x,y) = q_{mn} \sin \alpha x \sin \beta y;$$  \hspace{1cm} (13)

Where,

$$\alpha = \frac{m\pi}{a}; \quad \beta = \frac{n\pi}{b}; \quad m = 1, 2, 3\ldots; \quad n = 1, 2, 3\ldots,$$

It will be assumed that a solution for the transverse displacements can be found in the form:

$$w = a_{mn} \sin \alpha x \sin \beta y;$$  \hspace{1cm} (14)

It will also be assumed that $\lambda'$ and $\mu'$ are constants, independent of $x$ and $y$, so that can give the result:

$$\lambda'(\frac{D_y}{g} \alpha^2 + \frac{D_{xy}}{2} \beta^2 + D_{Q_x}) + \left(\frac{D_{xy}}{g} + \frac{D_y}{2}\right) \beta^2 - D_{Q_y} = 0;$$  \hspace{1cm} (15)

$$\lambda'(\frac{D_y}{g} \alpha^2 + \frac{D_{xy}}{2}) \alpha^2 + \mu'(\frac{D_y}{g} \beta^2 + \frac{D_{xy}}{2} \alpha^2 + D_{Q_y}) - D_{Q_x} = 0;$$  \hspace{1cm} (16)

The solution of equation (15) and (16) may be written in the following from:

$$\lambda' = \frac{\delta_1}{\Psi}; \quad \mu' = \frac{\delta_2}{\Psi};$$  \hspace{1cm} (17)

$$B_{Q_x} = b^2 D_{Q_x}, \quad B_{Q_y} = b^2 D_{Q_y};$$  \hspace{1cm} (18)

Where $\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \Psi$ can be defined as fellow:

$$\delta_1 = \frac{D_{xy}}{2B_{Q_y}} \frac{m^2b^2}{a^2} - \left(\frac{D_{xy}}{B_{Q_x}} - \frac{D_{xy}}{B_{Q_y}} + \frac{g}{2} \frac{D_{xy}}{B_{Q_x}} \frac{n^2\pi^2}{g} + 1\right);$$
\[
\delta_3 = -\left( \frac{q_{yy} P_x}{2B_{Q_y}} - \frac{q_{mm} D_y}{B_{Q_y}} + \frac{g D_{xy} m^2 \pi^2 b^2}{2 B_{Q_y} g a^2} + \frac{D_{xy} n^2 \pi^2}{2B_{Q_y}} \right) + 1;
\]

\[
\delta_5 = \frac{\pi^4}{2gB_{Q_y}} \left\{ D_{xy} n^2 \left[ D_y (D_y - D_{xy} D_{xy}) \right] + \frac{m^2 b^2 \pi^2}{a^2} \left( D_y + gD_{xy} \right) \right\} + \frac{D_{xy} n^2 \pi^2}{g} \left( D_y + gD_{xy} \right); 
\]

\[
\delta_4 = \frac{\pi^4}{2gB_{Q_y}} \left\{ D_{xy} n^2 \left[ D_y (D_y - D_{xy} D_{xy}) \right] + \frac{m^2 b^2 \pi^2}{a^2} \left( D_y + gD_{xy} \right) \right\} + \frac{D_{xy} n^2 \pi^2}{g} \left( D_y + gD_{xy} \right); 
\]

\[
\psi = \frac{\pi^4}{2gB_{Q_y}} \left\{ D_{xy} n^2 \left[ D_y (D_y - D_{xy} D_{xy}) \right] + \frac{m^2 b^2 \pi^2}{a^2} \left( D_y + gD_{xy} \right) \right\} + \frac{D_{xy} n^2 \pi^2}{g} \left( D_y + gD_{xy} \right) + 1;
\]

The result is:

\[
D_y \Phi_0 = \frac{\pi^4}{b^4} \frac{q_{mm} P_x}{a^2} - \frac{m^2 \pi^2}{a^2} - \frac{P_y n^2 \pi^2}{b^2} = 0 
\]

(19)

Where,

\[
\Phi_0 = \frac{\delta_3}{\psi}; 
\]

(20)

A final re-arrangement of the equation (19) provides the desired expression for the amplitude of the transverse displacements:

\[
a_{mm} = \frac{q_{mm} b^4}{\pi^2 D_y \Theta}; 
\]

(21)

Where,

\[
\Theta = \Phi_0 \pi^2 \left( \frac{m^2 b^2}{D_y a^2} + \frac{P_y n^2}{P_x} \right); 
\]

(22)
\[ P_x = P_{x_{cr}} = \frac{\Phi_0 \pi^2 D_x}{b^2 \left\{ m^2 b^2 / a^2 + (P_y / P_x) n^2 \right\}}; \]

The critical value \( P_x \) is:

\[ P_{x_{cr}} = \frac{\pi^2 D_x}{b^2} \frac{K_3}{g}; \quad (23) \]

Where,

\[ K_3 = \frac{g \Phi_0}{(m^2 b^2 / a^2) + n^2 (P_y / P_x)}; \quad (24) \]

The analyzed result (see table 2) and equations are computed in Excel and compare the calculation with ABAQUS FEA results (S4R element), which is quite match, so the FEA model is validated.
The FEA data in table 2 is generated in a model with aspect ratio (panel length over width) of 5 and the same boundary condition as described above. The data of the closed-Form Solutions are generated from the equations described above and selected the lowest results (eigenvalue).

Table 2: FEA result compare with closed-form solutions, flat model (critical buckling load)

<table>
<thead>
<tr>
<th>multiplier</th>
<th>GPa</th>
<th>FEA Pcr (10⁹ N/m)</th>
<th>Closed-Form Solutions Pcr (10⁹ N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>Gxz*0.01</td>
<td>Low shear modulus core, local buckling</td>
<td>69</td>
</tr>
<tr>
<td>0.1</td>
<td>Gxz*0.1</td>
<td>Low shear modulus core, local buckling</td>
<td>155</td>
</tr>
<tr>
<td>1</td>
<td>Baseline</td>
<td>578</td>
<td>762</td>
</tr>
<tr>
<td>10</td>
<td>Gxz*10</td>
<td>1000</td>
<td>1069</td>
</tr>
<tr>
<td>100</td>
<td>Gxz*100</td>
<td>1095</td>
<td>1110</td>
</tr>
</tbody>
</table>
Fig. 15 Flat model FEA result compare with the closed form solution

**Part two: Infinitely long symmetrically laminated curved plate validation**

The definition of the ratio of curvature $k\%$ is:

$$k\% = \frac{H}{b}$$

where:

- $H$: height of the arc
- $b$: width of the panel
For the infinitely long curved plate with all four edges simple supported and uniform axial shell edge load, the figure is:

Fig. 17 The infinitely long curved plate model
The equation for the critical buckling load for infinitely long isotropic material curved panel is given by Gambhir [9].

\[ P_{cr} = \frac{E}{6(1-v^2)} \left\{ \left[ 12(1-v^2) \left( \frac{t}{r} \right)^2 + \left( \frac{\pi t}{b} \right)^2 \right]^{1/2} + \left( \frac{\pi t}{b} \right)^2 \right\}; \tag{25} \]

Since this is only for isotropic material, we use steel and high shear modulus rigid ends.

In the infinitely long symmetrically laminated curved plate model, we add a rigid edge to the two short ends to prevent initial local buckling. The following plots show the comparison for (1) closed form solutions; (2) infinitely long symmetrically laminated curved plate model with no rigid edge at short ends; (3) infinitely long symmetrically laminated curved plate model with rigid edge at short ends. The results show that the FEA critical buckling load matches well the closed form solutions regardless if there is rigid end. However, the uniform distributed buckling wave appears in the rigid end model and local buckling occurred at the load edge appear in the model with no rigid ends.

The results show below:

Table 3: FEA result compare with closed-form solutions, curved model

<table>
<thead>
<tr>
<th>Critical Load</th>
<th>Core Thickness</th>
<th>closed-form</th>
<th>FEA-1 No rigid ends</th>
<th>FEA-2 with rigid ends</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01m</td>
<td>1.3793</td>
<td>1.0070</td>
<td>1.3710</td>
<td></td>
</tr>
<tr>
<td>0.02m</td>
<td>2.8605</td>
<td>2.5859</td>
<td>2.7740</td>
<td></td>
</tr>
<tr>
<td>0.03m</td>
<td>4.4488</td>
<td>4.2090</td>
<td>4.2377</td>
<td></td>
</tr>
<tr>
<td>0.04m</td>
<td>6.1493</td>
<td>5.7360</td>
<td>5.7758</td>
<td></td>
</tr>
<tr>
<td>0.05m</td>
<td>7.9669</td>
<td>7.3346</td>
<td>7.3894</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 18 Curved model FEA result compare with the closed form solution

Fig. 19 Curved sandwich model validation result FEA-1
Fig. 20 Curved sandwich model validation result FEA-2
BUCKLING SENSITIVITY TRENDS AND PLOTS

Local Buckling

1) The core thickness and shear modulus must be adequate to prevent the panel from buckling under end compression loads

![Local Buckling Image](image1.png)

Fig. 21 Global panel buckling

2) The core thickness and shear modulus must be adequate to prevent the core from prematurely failing in shear under end compression loads (low shear modulus core)

![Local Buckling Image](image2.png)

Fig. 22 Core instability (Shear crimping)
3) The compressive modulus of the facing skin and the core compression strength must both be high enough to prevent a skin wrinkling failure.

$$\sigma_{cr} = \frac{G_{cc} h_t}{2 t_f}$$

Fig. 23 Skin wrinkling

$$\sigma_{cr} = \frac{2 t_f}{3 h_c (1-\nu^2)} E_c E_f$$
With a higher transverse shear modulus; the critical buckling load of the core reaches a higher level and with the thickness of the core increasing, the critical values increasing faster in the higher transverse shear modulus core. The trends are shown in the following figures.

The definition of the normalized core thickness $C_0$ is the core thickness ‘c’ over the total thickness of the facing layers ‘t’, i.e., $C_0 = c/t$. 
Fig. 24 Critical buckling load versus core thickness for core material M1. Flat panel.

1m width
Fig. 25 Critical buckling load versus core thickness for core material M2. Flat panel.

1m width
Fig. 26 Critical buckling load versus core thickness for core material M3. Flat panel.

1m width
Fig. 27 Critical buckling load versus core thickness for core material M4. Flat panel.
1m width
Fig. 28 Critical buckling load versus core thickness for 1-layer face sheet. Flat panel.

1m width
Fig. 29 Critical buckling load versus core thickness for 2-layer face sheet. Flat panel.

1m width
Fig. 30 Critical buckling load versus core thickness for 3-layer face sheet. Flat panel.

1m width
Fig. 3.1 Critical buckling load versus core thickness for 4-layer face sheet. Flat panel.

1m width
Fig. 32 Critical buckling load versus core thickness for 5-layer face sheet. Flat panel.

1m width
Fig. 33 Critical buckling load versus core thickness for all five facing layers and all four core materials. Flat panel. 1m width.
Fig. 34 Core thickness versus critical buckling load for all five facing layers and all four core materials. Flat panel. 1m width.
With a higher transverse shear modulus; the critical buckling load of the core reaches a higher level and with normalized core thickness increasing, the critical values increasing faster in the higher transverse shear modulus core. The trends are shown in the following figures.

Fig. 35 Critical buckling load versus normalized core thickness $C_0$ for 1-layer and all four core materials. Flat panel. 1m width
Fig. 36 Critical buckling load versus normalized core thickness $C_0$ for 2-layer and all four core materials. Flat panel. 1m width.
Fig. 37 Critical buckling load versus normalized core thickness $C_0$ for 3-layer and all four core materials. Flat panel. 1m width.
Fig. 38 Critical buckling load versus normalized core thickness $C_0$ for 4-layer and all four core materials. Flat panel. 1m width.
Fig. 39 Critical buckling load versus normalized core thickness $C_0$ for 5-layer and all four core materials. Flat panel. 1m width
Fig. 40 Critical buckling load versus normalized core thickness $C_0$ for all five facing layers and all four core materials. Flat panel. 1m width.
With a higher transverse shear modulus; the critical buckling stress of the core reaches a higher level and with the thickness of the core increasing, the critical values increasing faster in the higher transverse shear modulus core. The trends are shown in the following figures.

The critical buckling stresses are calculated by the critical bulking loads over the total sandwich facing layers’ thicknesses. It is because the modulus of the facings is much higher than the cores and stresses are mostly taken by the facing materials.
Fig. 41 Critical buckling stress versus core thickness for core material M1. Flat panel.

1 m width
Fig. 42 Critical buckling stress versus core thickness for core material M2. Flat panel.

1m width
Fig. 43 Critical buckling stress versus core thickness for core material M3. Flat panel.

1m width
Fig. 44 Critical buckling stress versus core thickness for core material M4. Flat panel.

1m width
With a higher transverse shear modulus, the critical buckling stress of the core reaches a higher level and with the core thickness and the number of face sheet increasing, the critical values increasing faster in the higher transverse shear modulus core. The trends are shown in the following figures.

Fig. 45 Critical buckling stress versus core thickness for 1-layer face sheet. Flat panel.

1m width
Fig. 46 Critical buckling stress versus core thickness for 2-layer face sheet. Flat panel.

1m width
Fig. 47 Critical buckling stress versus core thickness for 3-layer face sheet. Flat panel.

1m width
Fig. 48 Critical buckling stress versus core thickness for 4-layer face sheet. Flat panel.  

1m width
Fig. 49 Critical buckling stress versus core thickness for 5-layer face sheet. Flat panel.

1m width
Fig. 50 Critical buckling stress versus core thickness for all five facing layers and all four core materials. Flat panel. 1m width
Fig. 51 Core thickness versus critical buckling stress for all five facing layers and all four core materials. Flat panel. 1m width
With a higher transverse shear modulus; the critical buckling stress of the core reaches a higher level and with the normalized core thickness and the number of face sheet increasing, the critical values increasing faster in the higher transverse shear modulus core. The trends are shown in the following figures.

![Graph showing critical buckling stress versus normalized core thickness for 1-layer and all four core materials.

Fig.52 Critical buckling stress versus normalized core thickness $C_0$ for 1-layer and all four core materials. Flat panel. 1m width]
Fig. 53 Critical buckling stress versus normalized core thickness $C_0$ for 2-layer and all four core materials. Flat panel. 1m width.
Fig. 54 Critical buckling stress versus normalized core thickness $C_0$ for 3-layer and all four core materials. Flat panel. 1m width.
Fig. 55 Critical buckling stress versus normalized core thickness $C_0$ for 4-layer and all four core materials. Flat panel. 1m width.
Fig. 56 Critical buckling stress versus normalized core thickness $C_0$ for 5-layer and all four core materials. Flat panel. 1m width.
Fig. 57 Critical buckling stress versus normalized core thickness $C_0$ for all five facing layers and all four core materials. Flat panel. 1m width.
With a higher transverse shear modulus; the critical buckling strain of the core reaches a higher level and with the core thickness and the number of face sheet increasing, the critical values increasing faster in the higher transverse shear modulus core. The trends are shown in the following figures.

![Graph](image)

Fig. 58 Critical buckling strain versus core thickness for core material M1. Flat panel. 1m width
Fig. 59 Critical buckling strain versus core thickness for core material M2. Flat panel.

1 m width
Fig. 60 Critical buckling strain versus core thickness for core material M3. Flat panel.

1m width
Fig. 61 Critical buckling strain versus core thickness for core material M4. Flat panel.

1 m width
Fig. 62 Critical buckling strain versus core thickness for 1-layer face sheet. Flat panel.

1m width
Fig. 63 Critical buckling strain versus core thickness for 2-layer face sheet. Flat panel.

1m width
Fig. 64 Critical buckling strain versus core thickness for 3-layer face sheet. Flat panel.

1m width
Fig. 65 Critical buckling strain versus core thickness for 4-layer face sheet. Flat panel.

1m width
Fig. 66 Critical buckling strain versus core thickness for 5-layer face sheet. Flat panel.

1m width
Fig. 67 Critical buckling strain versus core thickness for all five facing layers and all four core materials. Flat panel. 1m width
Fig. 68 Core thickness versus critical buckling strain for all five facing layers and all four core materials. Flat panel. 1m width
In 3m panel width model, with a higher transverse shear modulus; the critical buckling load of the core reaches a higher level and with the core thickness and the number of face sheet increasing, the critical values increasing faster in the higher transverse shear modulus core. The trends are shown in the following figures.

Fig. 69 Critical buckling load versus core thickness for core material M1. Flat panel. 

3m width.
Fig. 70 Critical buckling load versus core thickness for core material M2. Flat panel.

3m width.
Fig. 71 Critical buckling load versus core thickness for core material M3. Flat panel.

3m width.
Fig. 72 Critical buckling load versus core thickness for core material M4. Flat panel.

3m width.
Fig. 73 Critical buckling load versus core thickness for 1-layer face sheet. Flat panel.

3m width.
Fig. 74 Critical buckling load versus core thickness for 2-layer face sheet. Flat panel.

3m width.
Fig. 75 Critical buckling load versus core thickness for 3-layer face sheet. Flat panel. 3m width.
Fig. 76 Critical buckling load versus core thickness for 4-layer face sheet. Flat panel.

3m width.
Fig. 77 Critical buckling load versus core thickness for 5-layer face sheet. Flat panel.

3m width.
Fig. 78 Critical buckling load versus core thickness for all five facing layers and all four core materials. Flat panel. 3m width.
Fig. 79 Core thickness versus critical buckling load for all five facing layers and all four core materials. Flat panel. 3m width.
In 3m panel width model, with a higher transverse shear modulus; the critical buckling load of the core reaches a higher level and with the core thickness and the number of face sheet increasing, the normalized critical values increasing faster in the higher transverse shear modulus core. The trends are shown in the following figures.

Fig. 80 Critical buckling load versus normalized core thickness $C_0$ for 1-layer and all four core materials. Flat panel. 3m width.
Fig. 81 Critical buckling load versus normalized core thickness $C_0$ for 2-layer and all four core materials. Flat panel. 3m width.
Fig. 82 Critical buckling load versus normalized core thickness $C_0$ for 3-layer and all four core materials. Flat panel. 3m width.
Fig. 83 Critical buckling load versus normalized core thickness $C_0$ for 4-layer and all four core materials. Flat panel. 3m width.
Fig. 84 Critical buckling load versus normalized core thickness $C_0$ for 5-layer and all four core materials. Flat panel. 3m width.
Fig. 85 Critical buckling load versus normalized core thickness $C_0$ for all five facing layers and all four core materials. Flat panel. 3m width.
In 5m panel width model, with a higher transverse shear modulus; the critical buckling load of the core reaches a higher level and with the core thickness and the number of face sheet increasing, the critical values increasing faster in the higher transverse shear modulus core. The trends are shown in the following figures.

Fig. 86 Critical buckling load versus core thickness for core material M1. Flat panel.

5m width.
Fig. 87 Critical buckling load versus core thickness for core material M2. Flat panel.

5m width.
Fig. 88 Critical buckling load versus core thickness for core material M3. Flat panel.

5m width.
Fig. 89 Critical buckling load versus core thickness for core material M4. Flat panel.

5m width.
Fig. 90 Critical buckling load versus core thickness for 1-layer face sheet. Flat panel. 5m width.
Fig. 91 Critical buckling load versus core thickness for 2-layer face sheet. Flat panel.

5m width.
Fig. 92 Critical buckling load versus core thickness for 3-layer face sheet. Flat panel.

5m width.
Critical buckling load versus core thickness for 4-layer face sheet. Flat panel.

5m width.
Fig. 94 Critical buckling load versus core thickness for 5-layer face sheet. Flat panel.

5m width.
Fig. 95 Critical buckling load versus core thickness for all five facing layers and all four core materials. Flat panel. 5m width.
Fig. 96 Core thickness versus critical buckling load for all five facing layers and all four core materials. Flat panel. 5m width.
Adding the number of facing layers would benefit more in higher transverse shear modulus core. In lower transverse shear modulus core, the critical buckling load/stress increase slower when the number of facing layers are added. It indicates that: for low shear modulus core it is worthless to improve its buckling resistance by wasting facing material. It is because when buckling occurs, the low shear modulus core cannot work with the strong facing. The low shear modulus core actually shears or slips inside the blade skin. For high shear modulus core, the core and facing can work together better.

Fig.97 Critical buckling load versus normalized core thickness $C_0$ for 1-layer and all four core materials. Flat panel. 5m width.
Fig. 98 Critical buckling load versus normalized core thickness $C_0$ for 2-layer and all four core materials. Flat panel. 5m width.
Fig. 99 Critical buckling load versus normalized core thickness $C_0$ for 3-layer and all four core materials. Flat panel. 5m width.
Fig. 100 Critical buckling load versus normalized core thickness $C_0$ for 4-layer and all four core materials. Flat panel. 5m width.
Fig. 101 Critical buckling load versus normalized core thickness $C_0$ for 5-layer and all four core materials. Flat panel, 5m width.
Fig. 102 Critical buckling load versus normalized core thickness $C_0$ for all five facing layers and all four core materials. Flat panel. 5m width.
Fig. 103 Critical buckling load versus core thickness for core material M1. Flat panel.

1m, 3m, 5m width.
Fig. 104 Critical buckling load versus core thickness for core material M2. Flat panel.

1m, 3m, 5m width.
Fig. 105 Critical buckling load versus core thickness for core material M3. Flat panel.

1m, 3m, 5m width.
Fig. 106 Critical buckling load versus core thickness for core material M4. Flat panel. 1m, 3m, 5m width.
The result here also shows that with the analyze model geometry expand for the width form 1m to 5 m, the critical buckling load/stress decrease very quickly. It is reasonable that larger thinner panel is easier to

Fig. 107 Critical buckling load versus core thickness for 1-layer face sheet. Flat panel. 1m, 3m, 5m width.
Fig. 108 Critical buckling load versus core thickness for 2-layer face sheet. Flat panel.

1m, 3m, 5m width.
Fig. 109 Critical buckling load versus core thickness for 3-layer face sheet. Flat panel.

1m, 3m, 5m width.
Fig. 110 Critical buckling load versus core thickness for 4-layer face sheet. Flat panel.

1m, 3m, 5m width.
Fig. 111 Critical buckling load versus core thickness for 5-layer face sheet. Flat panel.

1m, 3m, 5m width.
With the width of the panel increase from 1m to 5m, the critical buckling value drops faster with the core shear modulus increase. The trends go to flat when the width increasing to 3m. The suggestion could be made that it is not suitable to build a thin-walled panel with the width larger than 3m. In that case, whatever the core shear modulus is or adding the number of face sheet, the critical buckling value will not increase.

Fig.112 Critical buckling load versus panel width for material M1 in 20mm core thickness. Flat panel.
Fig. 113 Critical buckling load versus panel width for material M1 in 30mm core thickness. Flat panel.
Fig. 114 Critical buckling load versus panel width for material M1 in 40mm core thickness. Flat panel.
Fig. 115 Critical buckling load versus panel width for material M1 in 20, 30, 40mm core thickness. Flat panel.
Fig. 116 Critical buckling load versus panel width for material M2 in 20mm core thickness. Flat panel.
Fig.117 Critical buckling load versus panel width for material M2 in 30mm core thickness. Flat panel.
Fig. 118 Critical buckling load versus panel width for material M2 in 40mm core thickness. Flat panel.
Fig. 119 Critical buckling load versus panel width for material M2 in 20, 30, 40mm core thickness. Flat panel.
Fig. 120 Critical buckling load versus panel width for material M3 in 20mm core thickness. Flat panel.
Fig. 121 Critical buckling load versus panel width for material M3 in 30mm core thickness. Flat panel.
Fig. 122 Critical buckling load versus panel width for material M3 in 40mm core thickness. Flat panel.
Fig. 123 Critical buckling load versus panel width for material M3 in 20, 30, 40mm core thickness. Flat panel.
Fig. 124 Critical buckling load versus panel width for material M4 in 20mm core thickness. Flat panel.
Fig. 125 Critical buckling load versus panel width for material M4 in 30mm core thickness. Flat panel.
Fig. 126 Critical buckling load versus panel width for material M4 in 40mm core thickness. Flat panel.
Fig. 127 Critical buckling load versus panel width for material M4 in 20, 30, 40mm core thickness. Flat panel.
Fig. 128 Critical buckling load versus panel width for all four materials in 20mm core thickness. Flat panel.
Fig. 129 Critical buckling load versus panel width for all four materials in 30mm core thickness. Flat panel.
Fig. 130 Critical buckling load versus panel width for all four materials in 40mm core thickness. Flat panel.
With the curvature ratio of the panel increase, the critical strain also increases. When the curvature ratio increase to a certain point, add curvature ratio will not increase panel buckling resistance. The ‘critical curvature ratio’ for the low shear modulus core is low and for the high shear modulus core is high.

For a fixed core material, there is a certain core thickness, in which that the critical buckling strain will be the same regardless of the number of facing layers. It is very valuable point because it the ‘optimal value thickness’ for a certain core material. In that ‘optimal value’ point, we can minimize the facing material, i.e. only 1 layer.

Finding the ‘optimal value thickness’ thickness of core material is worth the effort to do more data research. The result here shows that the lower transverse shear modulus a core material has, the ‘optimal value thickness’ is lower. As long as the material design strain meet the structure requirement, we suggest use the lower ‘optimal value thickness’ core material to minimize both core thickness and the number of facing layers. As is shown below:
Fig 131. Critical buckling load versus core thickness for core material M1. 5%, 10%, 25% Curved panel.
Fig. 132 Critical buckling load versus core thickness for core material M2. 5%, 10%, 25% Curved panel.
Fig. 133 Critical buckling load versus core thickness for core material M3. 5%, 10%, 25% Curved panel.
Fig. 134 Critical buckling load versus core thickness for core material M4. 5%, 10%, 25% Curved panel.
Fig. 135 Critical buckling load versus core thickness for all four core materials. 5%.

Curved panel.
Fig. 136 Critical buckling load versus core thickness for all four core materials. 10%.
Curved panel.
Fig. 137 Critical buckling load versus core thickness for all four core materials. 25%.

Curved panel.
Fig. 138 Critical buckling strain versus core thickness for all four core materials. 5%.

Curved panel.
Fig. 139 Critical buckling strain versus core thickness for all four core materials. 10%.

Curved panel.
Fig. 140 Critical buckling strain versus core thickness for all four core materials. 25%.

Curved panel.
Fig. 141 Critical buckling load versus panel curvature ratio for core material M1 in 20mm core thickness and all five facing layers. Curved panel.
Fig. 142 Critical buckling load versus panel curvature ratio for core material M1 in 30mm core thickness and all five facing layers. Curved panel.
Fig. 143 Critical buckling load versus panel curvature ratio for core material M1 in 40mm core thickness and all five facing layers. Curved panel.
Fig. 144 Critical buckling load versus panel curvature ratio for core material M1 in 20, 30, 40mm core thickness and all five facing layers. Curved panel.
Fig. 145 Critical buckling load versus panel curvature ratio for core material M2 in 20mm core thickness and all five facing layers. Curved panel.
Fig. 146 Critical buckling load versus panel curvature ratio for core material M2 in 30mm core thickness and all five facing layers. Curved panel.
Fig. 147 Critical buckling load versus panel curvature ratio for core material M2 in 40mm core thickness and all five facing layers. Curved panel.
Fig. 148 Critical buckling load versus panel curvature ratio for core material M2 in 20, 30, 40mm core thickness and all five facing layers. Curved panel.
Fig. 149 Critical buckling load versus panel curvature ratio for core material M3 in 20mm core thickness and all five facing layers. Curved panel.
Fig. 150 Critical buckling load versus panel curvature ratio for core material M3 in 30mm core thickness and all five facing layers. Curved panel.
Fig. 151 Critical buckling load versus panel curvature ratio for core material M3 in 40mm core thickness and all five facing layers. Curved panel.
Fig. 152 Critical buckling load versus panel curvature ratio for core material M3 in 20, 30, 40mm core thickness and all five facing layers. Curved panel.
Fig. 153 Critical buckling load versus panel curvature ratio for core material M4 in 20mm core thickness and all five facing layers. Curved panel.
Fig. 154 Critical buckling load versus panel curvature ratio for core material M4 in 30mm core thickness and all five facing layers. Curved panel.
Fig. 155 Critical buckling load versus panel curvature ratio for core material M4 in 40mm core thickness and all five facing layers. Curved panel.
Fig. 156 Critical buckling load versus panel curvature ratio for core material M4 in 20, 30, 40mm core thickness and all five facing layers. Curved panel.
Fig. 157 Critical buckling load versus panel curvature ratio for all four core materials in 20mm core thickness and all five facing layers. Curved panel.
Fig. 158 Critical buckling load versus panel curvature ratio for all four core materials in 30mm core thickness and all five facing layers. Curved panel.
Fig 159. Critical buckling load versus panel curvature ratio for all four core materials in 40mm core thickness and all five facing layers. Curved panel.
It is also shown that the ‘optimal value curvature’ exists for a fix core material then its core thickness is known. For the low shear modulus core, the ‘optimal value curvature’ occurs early than then high shear modulus core. In the practical design, it reveals that it would be no use to increase the buckling resistance for the plate by adding its curvature after a certain ‘optimal value curvature’.

For the critical buckling strain in the curved model, the result shows that the strain goes down when the curvature of the plate increase. For the certain core material with fix core thickness, when its plate curvature reaches a point named the ‘curvature strain reverse point’, the strain actually bump up very quickly. The less number of layers the plate have, the critical strain grows faster after the ‘curvature strain reverse point’. The weaker transverse shear modulus the core material has, the earlier the ‘curvature strain reverse point’ occurs with the increasing of the plate curvature. In practical design of the skin, it means that in order to meet the strain design of the structure requirement, a certain number of layers of facing are needed after reaches the core material’s ‘curvature strain reverse point’.
Fig. 160 Critical buckling strain versus panel curvature ratio for core material M1 in 20mm core thickness and all five facing layers. Curved panel.
Fig. 161 Critical buckling strain versus panel curvature ratio for core material M1 in 30mm core thickness and all five facing layers. Curved panel.
Fig. 162 Critical buckling strain versus panel curvature ratio for core material M1 in 40mm core thickness and all five facing layers. Curved panel.
Fig. 163 Critical buckling strain versus panel curvature ratio for core material M1 in 20, 30, 40mm core thickness and all five facing layers. Curved panel.
Fig. 164 Critical buckling strain versus panel curvature ratio for core material M2 in 20mm core thickness and all five facing layers. Curved panel.
Fig. 165 Critical buckling strain versus panel curvature ratio for core material M2 in 30mm core thickness and all five facing layers. Curved panel.
Fig. 166 Critical buckling strain versus panel curvature ratio for core material M2 in 40mm core thickness and all five facing layers. Curved panel.
Fig. 167 Critical buckling strain versus panel curvature ratio for core material M2 in 20, 30, 40mm core thickness and all five facing layers. Curved panel.
Fig. 168 Critical buckling strain versus panel curvature ratio for core material M3 in 20mm core thickness and all five facing layers. Curved panel.
Fig. 169 Critical buckling strain versus panel curvature ratio for core material M3 in 30mm core thickness and all five facing layers. Curved panel.
Fig. 170 Critical buckling strain versus panel curvature ratio for core material M3 in 40mm core thickness and all five facing layers. Curved panel.
Fig. 171 Critical buckling strain versus panel curvature ratio for core material M3 in 20, 30, 40mm core thickness and all five facing layers. Curved panel.
Fig. 172 Critical buckling strain versus panel curvature ratio for core material M4 in 20mm core thickness and all five facing layers. Curved panel.
Fig. 173 Critical buckling strain versus panel curvature ratio for core material M4 in 30mm core thickness and all five facing layers. Curved panel.
Fig. 174 Critical buckling strain versus panel curvature ratio for core material M4 in 40mm core thickness and all five facing layers. Curved panel.
Fig. 175 Critical buckling strain versus panel curvature ratio for core material M4 in 20, 30, 40mm core thickness and all five facing layers. Curved panel.
Fig. 176 Critical buckling strain versus panel curvature ratio for all four core materials in 20mm core thickness and all five facing layers. Curved panel.
Fig. 177 Critical buckling strain versus panel curvature ratio for all four core materials in 30mm core thickness and all five facing layers. Curved panel.
Fig. 178 Critical buckling strain versus panel curvature ratio for all four core materials in 40mm core thickness and all five facing layers. Curved panel.
Fig. 179 Critical buckling strain versus core transverse shear modulus in 20, 30, 40mm core. 1 layer. Flat panel.
Fig. 180 Critical buckling strain versus core transverse shear modulus in 20, 30, 40mm core. 2 layers. Flat panel.
Fig. 181 Critical buckling strain versus core transverse shear modulus in 20, 30, 40mm core. 3 layers. Flat panel.
Fig. 182 Critical buckling strain versus core transverse shear modulus in 20, 30, 40mm core. 4 layers. Flat panel.
Fig.183 Critical buckling strain versus core transverse shear modulus in 20, 30, 40mm core. 5 layers. Flat panel.
Fig. 184 Critical buckling strain versus core transverse shear modulus in 20, 30, 40mm core. All 5 layers. Flat panel. 1m width
Fig. 185 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 20mm core, Curved panel, 1 layer
Fig. 186 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 30mm core, Curved panel, 1 layer
Fig. 187 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 40mm core, Curved panel, 1 layer
Fig. 188 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 20mm core, Curved panel, 2 layers
Fig. 189 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 30mm core, Curved panel, 2 layers
Fig. 190 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 40mm core, Curved panel, 2 layers
Fig. 191 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 20mm core, Curved panel, 3 layers
Fig. 192 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 30mm core, Curved panel, 3 layers
Fig. 193 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 40mm core, Curved panel, 3 layers
Fig. 194 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 20mm core, Curved panel, 4 layers
Fig. 195 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 30mm core, Curved panel, 4 layers
Fig. 196 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 40mm core, Curved panel, 4 layers
Fig. 197 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 20mm core, Curved panel, 5 layers
Fig. 198 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 30mm core, Curved panel, 5 layers
Fig. 199 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 40mm core, Curved panel, 5 layers
Fig. 200 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 20mm core, Curved panel, all 5 layers
Fig. 201 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 30mm core, Curved panel, all 5 layers
Fig. 202 Critical buckling strain versus transverse shear modulus in 5%, 10%, 25% curvature, 40mm core, Curved panel, all 5 layers
CHAPTER VI

DESIGN STUDY

This chapter presented here shows the approach for the usage of the thesis’ result curves to assess the cost and weight of various core products.

The cost of wind industry production include: (1) Material Cost; (2) Labor Cost; (3) Overhead Cost; (4) Development Cost; and (5) Facilities Cost; (6) Facilities Cost [19]. With the blade length increasing, the material cost weigh more and more and has large space to optimize.

Fig. 203 30m blade overall cost [19]
Fig. 204 50m blade overall cost [19]

Fig. 205 70m blade overall cost [19]
The study from reference [19] also shows that with the blade length increasing, the weight of fiberglass and core grows. The fiberglass and core grows are the most expensive part of blade and the price of which can be transfer into thickness.

The material cost includes: (1) Gelcoat; (2) Continuous Strand Mat; (3) Double-Bias E-Glass Fabric; (4) Unidirectional E-Glass Fabric; (5) Core;  (6) Resin; (7) Promotor; (8) Catalyst

![Fig.206 30m blade bill of materials cost breakdown [19]](image-url)
Fig. 207 50m blade bill of materials cost breakdown [19]

Fig. 208 70m blade bill of materials cost breakdown [19]
Examples-1:

During former effort we have found the critical buckling load/strain with composite core thickness curves.

Fig.209 Example-1. Core thickness versus critical buckling strain for all five facing layers and all four core materials. Flat panel.

The example here shows that if we have a design critical buckling strain of 0.5%, we can choose: (1) M4 for 32mm core and 1 layer of facing; (2) M3 for 35mm core and 1 layer of facing (3) M2 for 38mm core and 2 layers of facing; or (4) M1 for 40mm core and 4 layers of facing.

When that composite blade skin geometry design transferred into weight, and optimized price is obtained.

Examples-2:
We also have found the relationship of critical buckling load and the panel width. Suppose we have a design load of 1.5 MN/m for a fixed core material M3, the best design plane could be (1) 40mm core with 3 layers of facing; or (2) 30mm core with 5 layers of facing. Considering the cost of face sheet, the design option could be made.

The same approach can be used in the design options for the core thickness, core material choice, panel width, panel curvature ratio, number of face sheets needed et al.

![Critical buckling load versus panel width for material M3 in 20, 30, 40mm core thickness. Flat panel.](image)

**Fig.210 Example-2.** Critical buckling load versus panel width for material M3 in 20, 30, 40mm core thickness. Flat panel.
Conclusion:

The effort presents a parametric study on the buckling sensitivity of infinitely long symmetric laminated composite plates under static uniform axial compression conditions of loading. In this study we have both infinitely long flat plate and curved plate model. The goal of this study is to find the buckling performance based on the number of facing layers, core thickness and material strain, which can provide insight on optimal core material choice and blade skin design, which can save the structure weight and lower the production cost.

The work presented here is only focus on the optimal buckling sensitivity study for composite blade skin. The reasons why not also launch a study for the buckling sensitivity on blade web or spar is: (1) the area of the blade shells (skins) is at least twice as much as the area of the shear webs, and so it has the biggest benefit for optimization; (2) The spar caps are solid laminate, and so they are not subject to optimization of core properties.

A finite element analysis (buckling analysis for Eigenvalue and linear static analysis) was carried out to model the problem using FE method. The analytical results are obtained using a program developed in ABAQUS, code 6.10 and the results are validated by closed form solutions. Results are produced for critical buckling load/strain with respect to core thickness and number of facing layers.
Based on the results of the nondimensional analysis carried out on the core thickness, layer number of the skin, and plate shape, the following conclusions were derived.

(1) With a higher transverse shear modulus; the critical buckling load/stress of the core reaches a higher level and with the thickness of the core increasing, the critical values increasing faster in the higher transverse shear modulus core. In return, the material strain grows in the same trend.

(2) Adding the number of facing layers would benefit more in higher transverse shear modulus core. In lower transverse shear modulus core, the critical buckling load/stress increase slower when the number of facing layers are added. It indicates that: for low shear modulus core it is worthless to improve its buckling resistance by wasting facing material. It is because when buckling occurs, the low shear modulus core cannot work with the strong facing. The low shear modulus core actually shears or slips inside the blade skin. For high shear modulus core, the core and facing can work together better.

(3) With the width of the panel increase from 1m to 5m, the critical buckling value drops faster with the core shear modulus increase. The trends go to flat when the width increasing to 3m. The suggestion could be made that it is not suitable to build a thin-walled panel with the width larger than 3m. In that case, whatever the core shear modulus is or adding the number of face sheet, the critical buckling value will not increase.

(4) With the curvature ratio of the panel increase, the critical strain also increases. When the curvature ratio increase to a certain point, add curvature ratio will not increase panel buckling resistance. The ‘critical curvature ratio’ for the low shear modulus core is low and for the high shear modulus core is high.
(5) For a fixed core material, there is a certain core thickness, in which that the critical buckling strain will be the same regardless of the number of facing layers. It is very valuable point because it the ‘optimal value thickness’ for a certain core material. In that ‘optimal value’ point, we can minimize the facing material, i.e. only 1 layer.

Finding the ‘optimal value thickness’ thickness of core material is worth the effort to do more data research. The result here shows that the lower transverse shear modulus a core material has, the ‘optimal value thickness’ is lower. As long as the material design strain meet the structure requirement, we suggest use the lower ‘optimal value thickness’ core material to minimize both core thickness and the number of facing layers.

(6) The result here also shows that with the analyze model geometry expand for the width from 1m to 5 m, the critical buckling load/stress decrease very quickly. It is reasonable that a wider, thinner panel is easier to buckle.

(7) For the curvature plate model, the result shows clearly that the buckling resistance increase quickly with curvature of the plate increase. It is a pre-buckle phenomenon that the plate is stiffer when the curvature exists.

(8) It is also shown that the ‘optimal value curvature’ exists for a fix core material then its core thickness is known. For the low shear modulus core, the ‘optimal value curvature’ occurs early than then high shear modulus core. In the practical design, it reveals that it would be no use to increase the buckling resistance for the plate by adding its curvature after a certain ‘optimal value curvature’.

(9) For the critical buckling strain in the curved model, the result shows that the strain goes down when the curvature of the plate increase. For the certain core material with fix core thickness, when its plate curvature reaches a point named the ‘curvature strain reverse point’, the strain actually bump up very quickly. The less
number of layers the plate have, the critical strain grows faster after the ‘curvature strain reverse point’. The weaker transverse shear modulus the core material has, the earlier the ‘curvature strain reverse point’ occurs with the increasing of the plate curvature. In practical design of the skin, it means that in order to meet the strain design of the structure requirement, a certain number of layers of facing is needed after reaches the core material’s ‘curvature strain reverse point’.

(10) In the plots 160-163, when critical buckling stress trend goes flat because the face wrinkling occur (reference [45] page.325.) In that case, the core and the face sheets no longer work together as a panel. The core will fail with shear and local buckling will occur in the face sheets. It would of no use to increase the number of facing sheet to increase panel buckling resistance.

According to the results, the analytical method presented here can be a practical solution and optimal tool for future large composite wind turbine blades design.

This project will help build more exact models to analysis, and the data from the research will help the future of the work.
Future

The study brought promising findings and thus it would be beneficial to continue the work in this area.

Since adding more kinds of core materials is not a beneficial solution for improving performance of this study, the efforts for improvement shall continued with introducing different material of the facing of the skin that will also show the buckling sensitivity.

Though save the weight for the blade skin has the biggest benefit for optimization, it is also worth the effort to study the shear web buckling sensitivity the similar approach as presented here.

More finite element models shall be found for more complicated structures.
REFERENCE/ SELECTED BIBLIOGRAPHY


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## APPENDIX

### ABAQUS DATA

**TABLE 4: ABAQUS data**

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