STRETCH PROCESSING OF SIMULTANEOUS, SEGMENTED BANDWIDTH LINEAR FREQUENCY MODULATION IN COHERENT LADAR

Thesis
Submitted to
The School of Engineering of the UNIVERSITY OF DAYTON

In Partial Fulfillment of the Requirements for
The Degree
Master of Science in Electro-Optics

By
Robert L. Brown

UNIVERSITY OF DAYTON
Dayton, Ohio
May, 2011
STRETCH PROCESSING OF SIMULTANEOUS, SEGMENTED
BANDWIDTH LINEAR FREQUENCY MODULATION IN COHERENT
LADAR

Name: Brown, Robert Loren
APPROVED BY:

________________________
Peter E. Powers, Ph. D.
Advisory Committee
Professor, Physics
& Electro-Optics

________________________
Matthew P. Dierking, Ph. D.
Chairman Committee Member
Technical Advisor
AFRL/RYJM, WPAFB, OH

Joseph W. Haus, Ph. D.
Committee Member
Professor & Director,
Electro-Optics

________________________
John G. Weber, Ph.D.
Associate Dean
School of Engineering

________________________
Tony E. Saliba, Ph. D.
Dean
School of Engineering
ABSTRACT

STRETCH PROCESSING OF SIMULTANEOUS, SEGMENTED BANDWIDTH LINEAR FREQUENCY MODULATION IN COHERENT LADAR

Name: Brown, Robert L.
University of Dayton
Advisor: Dr. Joseph W. Haus

In stretch processing (SP) both the local oscillator (LO) and the transmitted signal are linearly frequency modulated (LFM). A heterodyne detection process is performed using the LO and the received echo signal, which create a detected signal at a single difference-frequency. The frequency is proportional to the distance the received echo signal travels relative to the LO signal, and the range resolution is inversely proportional to the bandwidth making large bandwidth LFM chirps favorable. However, it is difficult to maintain linearity over a larger bandwidth LFM chirp. On the other hand small bandwidth LFM chirps can be easily produced, so the idea of segmenting the transmitted pulse into multiple small non-overlapping frequency LFM chirps was conceived. The extended frequency bandwidth is recovered in post processing. This
technique is called multi-frequency stretch processing (MFSP). The procedure outlined is a practical method to achieve greater range resolution using less expensive technology. Another advantage of this technique is the similar modulation noise on each LFM chirp. The multiple signals are processed using an algorithm developed for extracting the additional bandwidth information. The range resolution is related to the time span and bandwidth of the LFM pulses. For π transmitted LFM chirped signals the range resolution is nearly π times longer. Moreover the required detection bandwidth of the echo signal is lower than for other LFM processing systems without a chirped LO signal.
ACKNOWLEDGMENTS

Without the guidance, help, support, pushing and prodding of others this thesis could not have been. The first person I would like to thank is Joseph Haus who extended me the offer to join LOCI. Dr. Haus took a lot of time out of his busy schedule to help me even if that meant weekends. I also would like to think Matthew Deirking of Air Force Research Labs for developing the concept behind my project and for always making sure my work was up to par. Peter Powers is another person whose guidance I could not have finished my project without. A special thanks to Nancy Wilson who always made sure I had the correct paper work in order, even if that meant someone calling me at home. I also would like to thank Paul McManamon, Bradley Duncan, Nicholas Miller and everyone else at LOCI for their support.

I would like to thank my mother Cathy Brown for her constant advice on subjects that she knew nothing about, but somehow was usually correct. I would like to thank my father Terry Brown for his lifelong support and understanding. Thanks to my four siblings who always made sure I was fed, had a place to stay, and believed in me. Without their love and encouragement I would not have had this opportunity to pursue my dreams. Finally I would like to thank my soon to be wife Meritt Rollins. She has been there for me throughout this entire experience, with her love she kept level headed and...
focused. She was also the secret to my thesis going from a jumbled up group of words
and graphs to a grammatically correct thesis in a week.

This effort was supported in part by the U.S. Air Force through contract number
FA8650-06-2-1081, and the University of Dayton Ladar and Optical Communications
Institute (LOCI). The views expressed in this article are those of the author and do not
reflect on the official policy of the Air Force, Department of Defense or the U.S.
Government.
# Table of Contents

ABSTRACT ........................................................................................................................................ iii

ACKNOWLEDGMENTS ..................................................................................................................... v

Table of Contents ............................................................................................................................ vii

Table of Figures ............................................................................................................................... ix

Chapter 1: Introduction ..................................................................................................................... 1

Chapter 2: Stretch And Multi-frequency Stretch Processing ................................................................. 7

  2.1 Stretch Processing ....................................................................................................................... 7

  2.1.1 Analytical Model .................................................................................................................... 7

  2.2 Multi-Frequency Stretch Processing .......................................................................................... 11

  2.2.1 Conceptual Theory ................................................................................................................. 11

  2.2.2 Analytical Model .................................................................................................................... 15

  2.2.3 Post Processing ....................................................................................................................... 19

Chapter 3: Experimental Demonstration ............................................................................................ 26

  3.1 Experimental Setup ..................................................................................................................... 26

  3.2 Post Processing .......................................................................................................................... 28

  3.2.1 Stretch Processing .................................................................................................................. 29

  3.2.2 Multi-frequency Stretch Processing $n = 2, 3, 4$ ................................................................ 30

Chapter 4: Results .............................................................................................................................. 34

  4.1 Increased Resolution ................................................................................................................... 34

  4.2 Experimental Artifacts ................................................................................................................. 36

    4.2.1 Polarization Maintaining Fiber ............................................................................................ 36

    4.2.2 AOM Operation ....................................................................................................................... 38

    4.2.3 Polarization Leakage And Attenuation .................................................................................... 42

Chapter 5: Conclusion ......................................................................................................................... 44
5.1 Future Work .................................................................................................................. 45
Works Cited...................................................................................................................... 46
Appendix A: Matlab code................................................................................................. 47
List of Figures

Figure 1.1: At optical frequency an LFM chirped LO represented by a solid line and LFM chirped echo signal represented by the dashed line interfere when mixed by detector creating a beat frequency represented by the gray line at $df$ in radio frequency. $T$ is the length of the signal, $B$ is the bandwidth of the chirp, $df$ is the difference frequency detected, and $t_1$ is equal to the time delay between to LO and echo signals. .................. 3

Figure 2.1: The ideal case is to use a single modulator to simultaneously LFM chirp both signals, used for MFSP, to produce identical modulation and noise. The boxes containing differently dashed lines attached to the corner of the LO identifier boxes and transmitting identifier boxes illustrate the different frequency laser lines before and after modulation. ................................................................. 12

Figure 2.2: In the optical frequency regime an LFM chirped LO represented by a solid line and a dual laser line LFM chirped echo signal represented by the two dashed lines interfere when mixed by a photodiode detector creating beat frequencies represented by the gray lines at $df_1$ and $df_2$ in radio frequency. $T$ is the length of the signals, $B$ is the bandwidth of the chirps, $df_1$ and $df_2$ are the difference frequencies detected, and $t_1$ is equal to the time delay between to LO and echo signals. .................................................. 15

Figure 2.3: Graphical simulation of an MFSP detected signal. Each peak is a sinc function created by the major frequency laser lines of the detected signal. ................................. 19
Figure 2.4: Entire signal Fourier transformed detected MFSP signal when the number of
difference frequency laser lines equals 2 ($n = 2$). .......................................................... 20

Figure 2.5: Entire signal Fourier transformed detected MFSP signal when separated by
major frequencies. The separation of the two is denoted by different colors in the graph.
.............................................................................................................................................. 21

Figure 2.6: The frequency response after shifting the signals by $DF_{1,n}$. ......................... 22

Figure 2.7: Time signal of frequencies aligned for optimal $\Delta R$ ....................................... 23

Figure 2.8: Spectra comparison of a SP and MFSP $n = 2$ after post processing. .............. 24

Figure 2.9: Pictorial representation of how the post processing procedure of MFSP
affects the detected signal and the optical situation that would produce the same
results. $T$ is the pre-processed length of the signal, $B$ is the post processed bandwidth,
$df_1$ is the difference frequency detected, and $t_1$ is equal to the time delay between to
LO and echo signals................................................................................................................ 25

Figure 3.1: Block diagram of experimental setup. Splitters and couplers are shown; PR is
a polarization rotator and AOM denotes an acousto-optic modulator that generates the
frequency chirp. ......................................................................................................................... 28

Figure 3.2: Block Diagram of SP system.................................................................................. 29

Figure 3.3: Spectra comparison of signal on the detector of an SP system and the
numerical simulation created in Matlab................................................................................... 30

Figure 3.4: Resulting peaks when time signal is $\pi$ out of phase of optimal $\Delta R$............. 32

Figure 4.1: Final Result of MFSP experiment when the echo signal contains two
frequencies. ................................................................................................................................. 35
Figure 4.2: Cross section of PANDA fiber [7] ................................................................. 37

Figure 4.3: Input signal from the function generator to drive the AOM [6]. .................. 39

Figure 4.4: Frequency output of the AOM when using a 4us saw tooth input. (Black line
added for clarity).............................................................................................................. 40

Figure 4.5: Portion of usable frequency difference between the LO and echo signal. Echo
signals which contains multiple LFM chirps is represented by a single line for simplicity.
.................................................................................................................................................. 41

Figure 4.6: Signal detected by the MFSP experiment before post processing.............. 43
Chapter 1: Introduction

Pulse compression was conceived during World War II and has been implemented at radio frequencies for over sixty years to enhance radar signals. The word RADAR was derived from the acronym: RAdio Detection And Ranging; radar detects and ranges objects using radio frequency waves. Continuing with the same theme LADAR is the acronym for LAser Detection And Ranging; LADAR detects and ranges objects using optical frequency waves. Many of the same principles apply at both radio and optical wavelengths but stretch processing (SP). The optical region has been plagued with difficulty generating high bandwidth linear frequency modulated (LFM) chirps which contain enough energy to be used on a target at range. This phenomenon is also called laser radar or lidar (LIght Detection And Ranging).

There are different forms of pulse compression techniques to improve range resolution over non-frequency-modulated signal radar and for better resolution long-pulse radar [1]. The first pulse compression method and likely the most abundant is LFM [2]. A signal whose frequency linearly rises or falls over a period of time is referred to as a chirped pulse. In SP a single laser frequency is linearly modulated for and then split to be used as both the local oscillator (LO) and transmitted signals. The transmitted signal is scattered off a target at range. The received scattered signal, known as an echo signal,
is combined with the LO before being mixed by the photodiode detector. The mixing results in a beat signal whose frequency can be related to the path length difference between the LO and the echo signal.

The SP heterodyne detection process at optical frequencies is illustrated in Figure 1.1. A LFM chirp pulse of time $T$ and bandwidth $B$ is used as the LO and echo signals. A photodiode detector acts as a mixer of the combined LO and echo signals upon detection. The result of the mixing is a RF beat frequency of the two signals. This process is a form of heterodyne detection and it requires a coherence time over the span of the pulse. The path length difference of the two signals is the distance to and from the target which time delays the echo by what will be referred to as $t_1$ and can be shown as

$$t_1 = \frac{2R}{c},$$

(1.1)

where $R$ is the range of the target and $c$ is the speed of light. The angular frequency slope of the chirp, $\beta$, is two pi multiplied by the bandwidth over the time period,

$$\beta = \frac{2\pi B}{T},$$

(1.2)

where $B$ is the bandwidth of the LFM chirp and $T$ is the time period of the signal. The beat frequency, $df$, is directly proportional to $t_1$. If $\beta$ is linear $df$ will be equal to

$$df = \frac{\beta t_1}{2\pi},$$

(1.3)
where \( t_1 \) is the time delay between the signals. \( df \) can also be related to the \( B \) and \( T \) by Eq. 1.2 and Eq. 1.3 as

\[
df = \frac{B}{T} t_1. \tag{1.4}
\]

If \( \beta \) is not linear \( df \) will vary, rendering the SP technique useless. It is much easier to maintain linearity over a short \( T \) LFM chirp, but this limits \( B \).

Figure 1.1: At optical frequency an LFM chirped LO represented by a solid line and LFM chirped echo signal represented by the dashed line interfere when mixed by detector creating a beat frequency represented by the gray line at \( df \) in radio frequency. \( T \) is the length of the signal, \( B \) is the bandwidth of the chirp, \( df \) is the difference frequency detected, and \( t_1 \) is equal to the time delay between LO and echo signals.

Equation 1.1 can be combined with Eq. 1.4 to find the relationship between \( R \) and \( df \),
\[ R = \frac{cTdf}{2B} \]  

(1.5)

where \( T \) is time period of the signal, \( c \) is the speed of light and \( B \) is the bandwidth of the signal. There is an uncertainty in \( df \) which limits how accurately \( R \) can be measured known as the range resolution, \( \Delta R \). Eq. 1.5 can be modified to include \( \Delta R \) resulting in

\[ R \pm \Delta R = \frac{cT(df \pm \Delta df)}{2B}, \]  

(1.6)

where \( \Delta df \) is the uncertainty of \( df \). By simplifying Eq. 1.6 it is shown that the \( \Delta R \) is inversely proportional to bandwidth of the signal as shown by,

\[ \Delta R = \frac{cT\Delta df}{2B}. \]  

(1.7)

As earlier discussed others have used different techniques to create large bandwidth chirps. In reference 3 Piracha et al. discusses temporally stretching a sub-picosecond pulse created by a mode-locked laser with wavelength 1553 nm using a chirped fiber Bragg grating. The technique creates 746 GHz of bandwidth which is orders of magnitudes greater than the \( \sim 70 \text{ MHz} \) produced by the experiment in this thesis [3]. Comparatively the \( \Delta R \) of the stretched pulse is over 10,000 times finer than what is achievable by our experiment. The difficulty with temporally stretched pulses is that the transmitted pulse would have to be amplified to high peak power to make free space applications practical.

Using continuous wave (CW) LFM signals could contain the power needed to be applied to a ranging system but creating linear high time bandwidth LFM chirps is
difficult [4]. Chimenti et al. discusses creating high bandwidth linear chirps by using
different frequency continuous wave (CW) laser lines to simultaneously generate
spectrally separate LFM chirps and combining the bandwidth. This process is known as
sparse frequency linear frequency modulation, SF-LFM [5]. Using a similar technique to
SF-LFM a transmitting signal containing multiple non-frequency-overlapping chirps can
be introduced into SP. The heterodyne detection is done with a single laser line LFM
chirped LO and a multiple laser line LFM chirped echo signal resulting in a detected
signal containing multiple frequencies. The multiple frequencies of the detected signal
can be processed to take advantage of the bandwidth created by the multiple laser lines
of the echo signal. This technique will be referred to as multi-frequency stretch
processing (MFSP). One advantage to this practice is that known ways to create LFM
chirps using CW lasers can be used along with MFSP to simulate larger bandwidth
pulses. These signals will contain enough signal energy to measure an echo signal at
range, making it suitable for SP at optical wavelengths.

There are other obstacles to overcome when considering SP or MFSP at optical
wavelengths. For instance, coherence has to be maintained over the duration of the
pulse to successfully extract the $R$ and $\Delta R$. Another consideration is nonlinearities and
variations in noise that arise when creating high time bandwidth optical LFM chirps.

Chapter two begins with a brief analytical description of SP which is followed by
a section conceptually describing MFSP detection. The next section is an analytical
derivation of the detection process of MSFP. Chapter two ends with an explanation of
the post processing procedure of MFSP which results in better $\Delta R$. Chapter three begins with the experimental setup and concludes with an outline of the algorithm used for MFSP post processing. The results of the experiment are presented in the first section of chapter four. The next and final section of chapter four characterizes the signal and identifies difficulties created by the equipment used in the experiment. The final chapter, chapter five, concludes the thesis and contains a section for future work that could be done.
Chapter 2: Stretch And Multi-frequency Stretch Processing

The following chapter discusses the analytical models for SP and MFSP after heterodyne detection. The models will explain the frequency components appearing in the spectra of SP and MFSP when mixed by the photodiode detector. The conceptual theory of MFSP is discussed at the beginning of the MFSP section presented in the same manner the conceptual theory of SP was in chapter one. The MFSP section ends with a discussion of how the detected signal can be post processed to improve $\Delta R$ compared to SP.

2.1 Stretch Processing

Stretch processing has been conceptually described in the introduction of the thesis and illustrated by Figure 1.1. The analytical model is the only remaining description of SP that is significant to this thesis.

2.1.1 Analytical Model

SP uses a single LFM chirped LO signal and a single LFM chirped transmitted signal. The separated LO and the echo signals are represented by

$$E(t)_{LO} = \bar{A}_1 \left[ e^{i(2\pi f_{LO}t + \frac{1}{2}\beta t^2 + \varphi_1)} + C.C. \right]$$  (2.1)
and

\[
E(t)_{\text{echo}} = \tilde{A}_2 \left[ e^{j(2\pi(f_1)(t-t_1) + \frac{1}{2}\beta(t-t_1)^2 + \phi_2)} + C.C. \right],
\]  

(2.2)

where the initial frequency of the laser line is \(f_1\) which equals the carrier frequency of the LO \(f_{LO}\), \(t\) is the time variable of the signal, \(t_1\) is the time delay accrued while the echo signal is propagating to and from the target, \(\tilde{A}_1\) and \(\tilde{A}_2\) are complex amplitudes of the signals, \(\beta\) is the angular frequency slope of the chirp, and \(C.C.\) is the complex conjugate [5,6]. The phase difference between the LO and echo signals are unknown and will be analytically simulated by a phase term \(\phi_n\). The frequency chirp is created by the quadratic phase function \(\beta t^2\) induced by the modulator.

The two signals are coherently combined before the detector with a resulting complex field of

\[
E(t) = \tilde{A}_1 \left[ e^{j(2\pi(f_1)t + \frac{1}{2}\beta t^2 + \phi_1)} + C.C. \right] + \tilde{A}_2 \left[ e^{j(2\pi(f_1)(t-t_1) + \frac{1}{2}\beta(t-t_1)^2 + \phi_2)} + C.C. \right].
\]

(2.3)

The photodiode detector acts as a mixer sensing the magnitude squared of the signal shown as

\[
u(t) = |E(t)|^2.
\]

(2.4)

The expanded form of \(u(t)\) is
\[ u(t) = \frac{1}{2} \left[ |\tilde{A}_1|^2 + |\tilde{A}_2|^2 + \tilde{A}_1^2 e^{j2(2\pi(f_1)t+\frac{1}{2}\beta t^2+\varphi_1)} + \tilde{A}_2^2 e^{j2(2\pi(f_1)(t-t_1)+\frac{1}{2}\beta(t-t_1)^2+\varphi_2)} \right. \\
+ 2\tilde{A}_1\tilde{A}_2 e^{j\left[2\pi(f_1)(2t-t_1)+\beta t^2-\beta tt_1+\frac{1}{2}\beta t_1^2+\varphi_1+\varphi_2\right]} \\\n+ 2\tilde{A}_1\tilde{A}_2 e^{j\left[-2\pi(f_1)t_1-\beta tt_1+\frac{1}{2}\beta t_1^2-\varphi_1+\varphi_2\right]} + C. C.] \]

(2.5)

The detector responds differently to terms of Eq. 2.5 according to frequency. The first two terms do not contain frequency components therefore are voltage offsets in the detected signal. The third, fourth and fifth terms of Eq. 2.5 are at a frequency of \( f_1 \) or higher which are filtered out by the photodiode detector. The frequency of the sixth term is \( \beta t_1 \) which is related to range, \( R \), by Eq. 1.3 and Eq. 1.5. The frequency of the sixth term is the only one considered in the remainder of the derivation because it is the only non-DC frequency the photodiode detector does not filter out. The detected signal is represented by

\[ u(t) = 2Re\{E(t)_{\text{echo}}E(t)_{\text{LO}}\}. \]

(2.6)

In a real world situation the amplitude of the LO will be much greater than the amplitude of the echo,

\[ |\tilde{A}_1| \gg |\tilde{A}_2|, \]

(2.7)

and for simplicity the amplitude of the LO will be considered as unity,

\[ |\tilde{A}_1| = 1. \]

(2.8)
Inserting Eq. 2.3 into Eq. 2.6 yields

\[ u(t)^\prime = e^{j(-2\pi(f_1)t_1 - \beta t t_1 + \frac{1}{2} \beta t_1^2 - \varphi_1 + \varphi_2)}. \]  

(2.9)

The constants can be collected into a single phase term,

\[ \Delta \Phi = -2\pi(f_1)t_1 - \beta t t_1 + \frac{1}{2} \beta t_1^2 - \varphi_1 + \varphi_2. \]  

(2.10)

By combining the constants the important frequency, $\beta t_1$ is easily extracted

\[ u(t)^\prime = e^{j(\beta t_1 t + \Delta \Phi)}, \]  

(2.11)

where $\beta$ is the angular frequency slope defined by Eq. 1.2, $t$ is time of propagation, and $\Delta \Phi$ is a constant phase term. Equation 2.11 is the time signal at the detector of a working SP system.

The processing takes place in the frequency domain therefore the Fourier Transform, $FT$, of $u^\prime(t)$ is taken,

\[ FT[u(t)^\prime] = U(\omega)|^T_{t_1}. \]  

(2.12)

The limits of the transform are from the moment the LO and the echo signal interfere, $t_1$, to the time the LO ends $T$ (refer to Fig. 1.1). $U(\omega)$ is the image point response, IPR, of the target in frequency space and is shown to be

\[ U(\omega)|^T_{t_1} = \int_{t_1}^{T} e^{j(\beta t_1 t + \Delta \Phi)} e^{-j2\pi(f) t} dt, \]  

(2.13)
where the limits, $T$ and $t_1$ are set by the overlapping sections of the LO and echo signals, $\beta$ is the angular frequency slope of the LFM chirps, $t_1$ is the time delay of the echo, $\Delta \Phi$ is the constant phase of the detected signal, and $e^{-j2\pi(f)t}$ is the FT kernel. The result of the integration, $U(\omega)$ or IPR, is a sinc function in frequency space centered at $\beta t_1$:

$$U(\omega) = 2\pi (T - t_1)e^{j\Delta \Phi} \text{Sinc} \left( \frac{(\omega - \beta t_1)(T-t_1)}{2} \right).$$

(2.14)

Knowing $\beta t_1$, the major frequency in the IPR, $R$ and $\Delta R$ can be found using Eq. 1.3 and Eq. 1.6.

### 2.2 Multi-Frequency Stretch Processing

We introduce a concept of MFSP to achieve improved $\Delta R$ by combining two or more, smaller bandwidth frequency chirped signals to emulate one larger bandwidth frequency chirped signal. Many of the same parameters such as $T$, $B$, $\beta$ and $t_1$ apply to both SP and MFSP.

#### 2.2.1 Conceptual Theory

Multi-frequency stretch processing begins with a non-modulated single frequency LO and non-modulated transmitting signal containing multiple laser lines at different frequencies. All laser lines possess the same temporal time span, $T$. The multiple laser lines of the transmitting signal are separated enough spectrally that when the signal is modulated they will not overlap. An LFM chirp is simultaneously imposed...
on the multiple-frequency transmitting signal and single frequency LO signal which results in the angular frequency slope, $\beta$, and bandwidth, $B$, of all the LFM chirps to be equal. The ideal case is to use a single modulator on both signals which would result in identical noise along the length of each LFM chirp. Figure 2.1 is a block diagram illustrating the ideal case described. The laser line(s) of the signals are represented by the differently textured lines contained in the boxes attached to the boxes identifying the transmitted and LO signals.

![Figure 2.1: The ideal case is to use a single modulator to simultaneously LFM chirp both signals, used for MFSP, to produce identical modulation and noise. The boxes containing differently dashed lines attached to the corner of the LO identifier boxes and transmitting identifier boxes illustrate the different frequency laser lines before and after modulation.](image)

The transmitting signal is scattered off a target and the echo signal is received. A heterodyne signal detection technique is used at the receiver to extract multiple RF beat frequencies.
Figure 2.2 pictorially demonstrates the received signal using a two laser line transmitting signal MFSP system. As in a SP system a MFSP system heterodyne detects the echo signal by using the photodiode detector as a mixer. Unlike SP the heterodyne detection of MFSP results in multiple different frequency laser lines from the mixing of the LFM chirped LO (solid black line Figure 2.2) and multiple frequency LFM chirped laser lines of the echo (two black dashed lines Figure 2.2). The spectral difference between the un-modulated LO and un-modulated laser line will be represented by $DF_{1,n}$, where $n$ is the $n^{th}$ number laser line in the transmitting signal when order lowest to highest by frequency farthest from the LO. $DF_{1,n}$ can be shown to be;

\begin{align}
DF_{1,1} &= f_1 - f_{LO} = 0, \\
DF_{1,2} &= f_2 - f_{LO}, \\
& \text{and} \\
DF_{1,n} &= f_n - f_{LO}
\end{align}

where $f_{LO}$ is the un-modulated frequency of the LO. $f_1$, $f_2$, $f_n$ are the un-modulated frequencies of the first, second, $n^{th}$ laser lines in the echo signal. Commonly the same laser line is used to create the LO and the first laser line of the echo signal therefore $f_{LO} = f_1$. The frequencies of the detected laser lines ($df_1$, $df_2$, $\ldots$, $df_n$) can be shown mathematically to be

\begin{align}
df_1 &= \frac{B}{2\pi} t_1,
\end{align}
\[ df_2 = DF_{1,2} + \frac{\beta}{2\pi} t_1, \]  

(2.19)

and

\[ df_n = DF_{1,n} + \frac{\beta}{2\pi} t_1, \]  

(2.20)

where \( \beta \) is the angular frequency of the LFM chirp, \( t_1 \) is the delay accumulated by the echo signal while travelling to and from the target at range.

The number of frequency laser lines present in the detected signal is equal to the number of laser lines contained in the echo signal. Figure 2.2 illustrates the detected signal for a two laser line echo signal MFSP system. The LO is represented by the solid line in Figure 2.2. The echo signal frequency laser lines are time delayed by \( t_1 \) and they are represented in Figure 2.2 by the black long dashed and the black short dashed line. The signal bandwidth, \( B \), is illustrated by a vertical line in the figure. The difference frequencies between the LO and the echo signal laser lines are \( df_1 \) and \( df_2 \) as illustrated by the vertical lines in Figure 2.2.
The analytical model of the MFSP detection process will be presented in this section. Multi-frequency stretch processing and SP use heterodyne detection making the derivations similar. Equation 2.1 below is the expression used for the LO in both the SP and MFSP scenarios. The echo signal of MFSP takes the same form of the echo signal SP but contains multiple laser lines and can be represented by Eq. 2.22

\[ E(t)_{LO} = \bar{A}_1 e^{i(2\pi f_{LO}t + \frac{1}{2}Bt^2 + \phi_1)} + C.C \]  

\[ (2.1) \]
\[ E(t)_{\text{echo}} = \tilde{A}_2 e^{i \left( 2\pi (f_1(t-t_1) + \frac{1}{2} \beta(t-t_1)^2 + \varphi_1) \right)} + \tilde{A}_3 e^{i \left( 2\pi (f_2(t-t_1) + \frac{1}{2} \beta(t-t_1)^2 + \varphi_3) \right)} \]
\[ + \tilde{A}_n e^{i \left( 2\pi (f_n(t-t_1) + \frac{1}{2} \beta(t-t_1)^2 + \varphi_n) \right)} + C.C. \quad (2.22) \]

where \( \tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \) and \( \tilde{A}_n \) are complex amplitude of the signals, the frequency of the pre-modulated laser lines are \( f_{LO}, f_1, f_2, \) and \( f_n \) as before \( f_{LO} = f_1, \) \( t \) is the time variable of the signal, \( t_1 \) is the time delay accrued while the echo signal is propagating to and from the target, \( \beta \) is the angular frequency slope of the chirp, \( \varphi_1, \varphi_2, \varphi_3, \) and \( \varphi_n \) are unknown phases of each laser line, and \( C.C. \) is the complex conjugate. As in SP the two signals combine together before detection,

\[ E(t) = \tilde{A}_1 e^{i \left( 2\pi (f_1 t + \frac{1}{2} \beta t^2 + \varphi_1) \right)} + \]
\[ \tilde{A}_2 e^{i \left( 2\pi (f_1(t-t_1) + \frac{1}{2} \beta(t-t_1)^2 + \varphi_2) \right)} + \]
\[ \tilde{A}_3 e^{i \left( 2\pi (f_2(t-t_1) + \frac{1}{2} \beta(t-t_1)^2 + \varphi_3) \right)} + \ldots \]
\[ \tilde{A}_n e^{i \left( 2\pi (f_n(t-t_1) + \frac{1}{2} \beta(t-t_1)^2 + \varphi_n) \right)} + C.C. \quad (2.23) \]

The detector mixes the two signals resulting in the magnitude squared of the signal shown earlier in Eq. 2.4, and rewritten here again for convenience

\[ u(t) = |E(t)|^2. \quad (2.4) \]

Again the LO’s amplitude will be much stronger than the amplitudes of the echo signals,
\[ |\tilde{A}_1| \gg |\tilde{A}_2|, |\tilde{A}_3|, \ldots, |\tilde{A}_n|. \] (2.24)

This means when taking the magnitude squared of \( u(t) \), the amplitude of the frequencies resulting in the multiplication of two laser lines from the echo signal will be insignificant. Just as in the SP case the detector will filter out higher frequencies, therefore the only remaining frequencies will be the echo signal mixed with the complex conjugate of the LO shown earlier in Eq. 2.6,

\[ u(t)' = 2Re\{E(t)_{\text{echo}}E(t)^*_{\text{LO}}\}. \] (2.6)

Without loss of generality the complex amplitude of the LO will be set to unity as in Eq. 2.8. Taking into account Eq. 2.24 and Eq. 2.8 the expansion of Eq. 2.6 in the MFSP case reveals

\[
u(t)' = e^{i\left(-2\pi(f_1)t_1 - \beta tt_1 + \frac{1}{2}\beta t_1^2 - \varphi_1 + \varphi_2\right)} + e^{i\left(2\pi(f_2-f_1)t - 2\pi f_2 t_1 - \beta tt_1 + \frac{1}{2}\beta t_1^2 - \varphi_1 + \varphi_3\right)} + e^{i\left(2\pi(f_n-f_1)t - 2\pi f_n t_1 - \beta tt_1 + \frac{1}{2}\beta t_1^2 - \varphi_1 + \varphi_n\right)}. \] (2.25)

Equation 2.25 can be simplified inserting equations 2.15, 2.16, 2.17 and combining the constant terms in the following ways,

\[
\Delta \Phi_1 = -2\pi(f_1)t_1 - \beta tt_1 + \frac{1}{2}\beta t_1^2 - \varphi_1 + \varphi_2, \] (2.26)

\[
\Delta \Phi_2 = -2\pi(f_2)t_1 - \beta tt_1 + \frac{1}{2}\beta t_1^2 - \varphi_1 + \varphi_3, \] (2.27)

and

\[
\Delta \Phi_n = -2\pi(f_n)t_1 - \beta tt_1 + \frac{1}{2}\beta t_1^2 - \varphi_1 + \varphi_n. \] (2.28)
The simplification of Eq. 2.25 reveals the frequency of each laser line created when the LO and the echo signal of MFSP are heterodyne detected,

\[ u(t) = e^{j(-\beta t_1 + \Delta \Phi_1)} + e^{j((2\pi DF_{1,2} - \beta t_1)t + \Delta \Phi_2)} + e^{j((2\pi DF_{1,n} - \beta t_1)t + \Delta \Phi_n)}. \]  

(2.29)

By inserting equations 2.18, 2.19 and 2.20, \( u'(t) \) can be further simplified as

\[ u(t) = e^{j(-2\pi df_1 t + \Delta \Phi_1)} + e^{j(2\pi df_2 t + \Delta \Phi_2)} + e^{j(-2\pi df_n t + \Delta \Phi_n)}. \]  

(2.30)

When the Fourier Transform, \( FT \), is taken of the MFSP detected signal, \( u'(t) \), the product is the IPR \( U(\omega) \). For MFSP \( U(\omega) \) is multiple sinc functions centered at the major frequencies, \( df_1, df_2, ..., df_n \),

\[ U(\omega) = 2\pi(T - t_1) \left( e^{j\Delta \Phi_1 \text{Sinc}} \left( \frac{(\omega - \beta t_1)(T-t_1)}{2} \right) + e^{j\Delta \Phi_2 \text{Sinc}} \left( \frac{(\omega-(2\pi DF_{1,2} - \beta t_1))(T-t_1)}{2} \right) + \ldots e^{j\Delta \Phi_n \text{Sinc}} \left( \frac{(\omega-(2\pi DF_{1,n} - \beta t_1))(T-t_1)}{2} \right) \right). \]  

(2.14)

The IPR of an MFSP detected signal is shown graphically by Figure 2.3.
Figure 2.3: Graphical simulation of an MFSP detected signal. Each peak is a sinc function created by the major frequency laser lines of the detected signal.

Individually the peaks do not contain better range resolution, $\Delta R$, than a Fourier Transformed SP detected signal.

### 2.2.3 Post Processing

The MFSP detected signal must be post processed to take advantage of the increased bandwidth to achieve the gain in $\Delta R$. This section will explain how the post processing is preformed on the MFSP detected signal. Post processing can be sufficiently described as if the echo signal contained two different frequency laser lines, $n = 2$. 
Post processing begins by Fourier Transforming the heterodyned signal. The complete Fourier Transform of the heterodyned signal is shown in Figure 2.4. The two peaks in the signal represent the detected laser lines at the frequencies $df_1$ and $df_2$ that is created during heterodyning.

Figure 2.4: Entire signal Fourier transformed detected MFSP signal when the number of difference frequency laser lines equals 2 ($n = 2$).

The signal is segmented into multiple frequency spans. Figure 2.5 demonstrates which portion of the data points are windowed to encompass each peak created by the multiple laser lines. The portion which is separated is chosen as the minimum between the two adjoining peaks. The more different frequency laser lines the detected signal
contains, the smaller the allotted window for each frequency peak can be. In the graph there is a 30 dB minimum between the two signal segments.

![Separated Signals](image)

**Figure 2.5**: Entire signal Fourier transformed detected MFSP signal when separated by major frequencies. The separation of the two is denoted by different colors in the graph.

The separate sections of the data are transformed back into time domain. A frequency offset of $DF_{1,n}$, which is found by Eq. 2.17, is multiplied with each corresponding time signal. This multiplication of $DF_{1,n}$ translates the center frequencies of all the peaks to overlap at the frequency $\beta t_1$ or $2\pi df_1$ (Eq. 1.3). The result of the translation in frequency domain is shown in Figure 2.6.
Once spectrally overlapped the data is temporally aligned. The time delay, $t_1$, acquired by each of the multiple frequency laser lines of the echo signal are taken into account for the processed signal to create an accurate representation of range to the target. Matching the phases of the signals consist of aligning the translated frequencies to include $t_1$ between each time signal. The correct phase between the wavelengths is numerically determined to ensure the best possible $\Delta R$. Figure 2.7 displays the time domain signal produced by processing. The dotted line is the phase that would have been acquired during $t_1$ if the signal was continuous.
Figure 2.7: Time signal of frequencies aligned for optimal $\Delta R$

The processed signal is then transformed back to the frequency domain. The MFSP signal contains $n$-times the bandwidth as the SP signal, so an improvement of $n$-times is expected in the $\Delta R$. Figure 2.8 compares a stretch processed signal to a MFSP signal when the echo signal contains two different frequency laser lines, $n = 2$. The widths of the peaks at -3dB reveal improved range resolution.
The radio frequency range of Figure 2.9 illustrates what post processing of MFSP does to the detected signal. In the optical frequency regime, Figure 2.9 demonstrates what type of pre-heterodyned optical signal would create the same detected affect. If Figure 2.2 is considered the signal before processing Figure 2.9 is the signal post processing. The detected time of MFSP after post processing is $n$-times as long as the detected time for SP with the same pulse length, $T$ (Figure 1.1 and Figure 2.9).
Figure 2.9: Pictorial representation of how the post processing procedure of MFSP affects the detected signal and the optical situation that would produce the same results. $T$ is the pre-processed length of the signal, $B$ is the post processed bandwidth, $d_{f_1}$ is the difference frequency detected, and $t_1$ is equal to the time delay between the LO and echo signals.
Chapter 3: Experimental Demonstration

3.1 Experimental Setup

The versatile experimental equipment implemented for this thesis has been used in the development of two preceding thesis’s. Chimenti et al. and Bailey et al. laid the groundwork for the development of MFSP through development of SF-LFM and sparse frequency Doppler processing respectively [5,6]. With the exception of polarization rotating equipment and a polarization beam splitter, Chimenti et al. and Bailey et al. used the same equipment [1, 8].

The experiment uses two laser sources with a frequency offset of $D F_{1,2}$ between them. A schematic of the system is illustrated in Figure 3.1. The figure shows a block diagram of the experimental setup, along with polarization orientation of the signals denoted by the circles containing arrows. The small boxes attached to the polarization circles denote frequency versus time of each laser line in the signal. The laser 1 signal passes through a splitter to create two laser lines. One of the lines of the split signals is designated as the LO. For the experiment to work as described, the LO must be a single laser line therefore it must be separated from the transmitting signal. In order to
separate it from the transmitted signal, its polarization is rotated by 90° (denoted as PR in Figure 3.1). It is important to note that by orthogonally polarizing the two signals they can propagate down a single fiber independently and simultaneously. The other branch of the split signal is mixed with the output of Laser 2 and together they become the transmitted signal. The two signals are combined with the LO, but do not interfere with it due to the orthogonal polarizations. The resulting signals pass through an acousto-optic modulator, AOM, where the frequencies of the signals are chirped, each receiving identical modulation and noise.

Once the combined signals are chirped they pass through a polarization beam splitter. One output from the polarizer contains the signal that will be transmitted to the target, while the other output will be the LO. The transmitted signal is scattered off the target and the echo from the target is received. The LO signal’s polarization is rotated to match the echo signal’s polarization. The LO and the echo are mixed together for heterodyne detection. The detector’s output is digitized and processed using a Fourier Transform to analyze the range and range resolution.
The polarization maintaining fiber is not designed to propagate two orthogonally polarized optical signals. When the orthogonally polarized LO and transmitted signals are propagating in the same fiber an unintended mixing occurs between the two signals. In the final experiment a parasitic peak appears at $2\pi D F_{1,2}$ as a result of this unintended mixing. As shown in the results the parasitic peak at $2\pi D F_{1,2}$ did not influence the proof that MFSP is a viable option for increasing $\Delta R$.

### 3.2 Post Processing

Just as Chapter 2 constructs the analytical model of MFSP upon the analytical model of SP the remainder of this chapter will explain the post processing beginning with SP and then transitioning into MFSP.
3.2.1 Stretch Processing

As discussed before when a SP signal is heterodyned the result is a beat frequency signal. This single frequency can be converted into a range and range resolution by Eq. 1.6. Any beat frequency created is already or can be made lower than the original frequency; this means less expensive technology can be used in detection. These factors make SP a very robust and straightforward method to finding the range of a target. Post processing for the experiment began by creating Matlab code to simulate SP. This program began with the LO and echo signals so it could later be modified to MFSP. A SP system was assembled to compare the simulated results with measured results.

The SP system is illustrated in Figure 3.2. A laser line was created by a single laser at a wavelength of 1550 nm. The laser line was chirped using an AOM. The chirped signal was then split into the LO and the transmitting signal. The transmitting signal is scattered off a target and the received echo signal is mixed with the LO creating an RF beat frequency on the detector. In Figure 3.2 is a box diagram of the SP system with the frequency forms shown in smaller boxes.

![Figure 3.2: Block Diagram of SP system.](image-url)
Being able to compare a measured output to a numerical simulation of an SP system was critical to our understanding of the actual experimental signal characteristics. Figure 3.3 compares the measured and calculated results of the SP system. The agreement between the spectra in most details confirms our SP range resolution by the width of the central peak.

![Spectra comparison of signal on the detector of an SP system and the numerical simulation created in Matlab.](image)

**Figure 3.3:** Spectra comparison of signal on the detector of an SP system and the numerical simulation created in Matlab.

### 3.2.2 Multi-frequency Stretch Processing $n = 2, 3, 4...$

Once the Matlab code was completed for SP the program it could be easily adapted for MFSP $n = 2$ (the number of frequencies the experimental echo contained) by adding a frequency to the echo signal. With the experimental setup the frequencies...
of the lasers could be changed therefore $DF_{1,2}$ would change. This was an advantage because one could align the laser line at $df_2$ to a desirable frequency. The variability of $df_2$ also caused a problem for the program to create an accurate simulation. This problem was fixed by initially calculating $DF_{1,2}$ using the measured signal and setting that as the offset for the numerical simulation. Knowing $DF_{1,2}$ was also essential to spectrally overlapping the frequencies.

The numerically simulated signal and measured signal at this point are ideally the same, so the remainder of the post processing applies to each. The upper and lower frequencies containing the major peaks at $2\pi df_1$ and $2\pi df_2$ were separated by windowing a set of data points around the $df$'s (Figure 2.6). The number of data segments is determined by the number of different frequency laser lines in the detected signal. Once the data segments are separated the inverse Fourier transform of the data points is taken. The upper frequency signal is frequency shifted by multiplying the complex amplitude of the signal by $e^{-j2\pi DF_{1,2}t}$. The signals are then aligned in time by taking the array containing the lower frequencies and extending it by the array containing the upper shifted frequencies. This time shift aligns the frequencies but the phase of the two signals must also be aligned. Through iteration the program finds the narrowest central peak to match the phase function as shown in Figure 2.7. This produces signal with the best estimation of $\Delta R$. The signal with the estimated $\Delta R$ is shown in Figure 2.8.
It is interesting to note that when the phase is changed by $\pi$ from optimal $\Delta R$, the spectrum at the frequency $df_1$ has a dip as shown in Figure 3.4. This is an interesting observation because the -3dB width of the valley is much narrower than the -3dB width of the peak at phase of best $\Delta R$. More about this will be discussed in the future experiment section contained in the conclusion of this thesis.

![Graph showing resulting peaks when time signal is $\pi$ out of phase of optimal $\Delta R$.](image)

**Figure 3.4:** Resulting peaks when time signal is $\pi$ out of phase of optimal $\Delta R$.

The program for MFSP when $n = 2$ was written with any number of frequencies in mind. The section of the program which calculated $DF_{1,2}$ was extended to calculate $DF_{1,2}, DF_{1,3}, \ldots DF_{1,n}$. The difference between each $DF$ was also calculated to determine how close each major frequency was to one another. The closer the major
peaks are together the fewer data points one can allot for each frequency, or the smaller the window around each frequency will be.

The data points containing the multiple major frequency peaks were separated and Fourier transformed into multiple time domain signals. Each time signal is then multiplied by a time signal containing the corresponding frequency $DF$; e.g.

$$e^{j2\pi (DF_{1,n} + \beta t_1)}e^{-j2\pi (DF_{1,n})t} = e^{j2\pi (\beta t_1)t}.$$  

The frequencies are the same for all signals but the phase of each have to be matched. The phase alignment processes are much like what is described for MFSP $n = 2$ only the iterations are increased by $n$ times. The alignment begins by finding the phase of optimal $\Delta R$ of two signals. Once the correct phase is found the two signals are considered one and the process begins again until all the signals are linked together and correctly phased. The linked time signal now contains $n$ times the temporal bandwidth of the initial stretch processed signal.
Chapter 4: Results

Now that the theory behind MFSP has been investigated the results of the experiment may be discussed. As with any new research venture, unexpected challenges were presented during the experiment. Fortunately each problem that presented itself had a solution which could be adapted to the experimental setup. This chapter will also report the challenges and their resolutions.

4.1 Increased Resolution

The purpose of this thesis was to develop an SP system that could take advantage of the increased bandwidth created by SF-LFM. Using multiple shorter LFM chirps to simulate a longer LFM chirp a gain in range resolution, $\Delta R$, is achieved. Doing this exploits the fact that it is easier to maintain linearity over shorter time bandwidth CW LFM chirps. Another advantage is noise created by the modulator being duplicated over the length of the multiple LFM chirps; making this noise periodic in the processed signal.

The experiment succeeded in proving that a gain in resolution can be achieved by MFSP over SP. As theorized the gain in resolution is proportional to the number of laser lines in the echo signal. The calculated gain in resolution can be observed in
Figure 4.1 by the -3dB width difference of the two dashed lines. The dashed lines are the numerical models of the frequency response of SP and MFSP. The calculated MSFP is when there are two different frequency laser lines in the echo signal as it is in the experiment. The black line in Figure 4.1 is the detected and processed results of MFSP with a dual laser line echo.

![Graph of Figure 4.1](image)

**Figure 4.1: Final Result of MFSP experiment when the echo signal contains two frequencies.**

It can be found, using Figure 4.1 and Eq. 1.7, that the $\Delta R$ of the numerical model of SP is 115.6 m. The numerical model of MFSP was found to have a $\Delta R$ of 56.9 m. The $\Delta R$ of the MFSP system was found to be 51 m. The higher than expected side lobes was due to polarization issues when propagating two orthogonal signals down a single
PM fiber. This forced the echo signal to be attenuated, making the heterodyned signal just above the noise. The low signal to noise ratio, SNR, broadened the side lobes which led to compression of the main lobe resulting in better than expected $\Delta R$.

The single fiber multiple signal approach to modulation was the most intriguing and suitable solution for this experiment. A better method of controlling and maintaining the polarization would also result in a better SNR. There are multiple ways to separate the LO and the transmitting signal that would not rely on polarization; one example is to use two AOMs.

### 4.2 Experimental Artifacts

#### 4.2.1 Polarization Maintaining Fiber

The first and most important task was to separate the single frequency LO and the multi-frequency transmitting signal. The experiment contained a single AOM that would be used to modulate both signals. Even if multiple AOMs were available, the difference in noise and modulation between the AOMs would create unwanted frequencies. The initial thought was to use an optical filter such as a Fabry-Perot to separate the two signals after modulation. This concept was ultimately unachievable because it required an optical filter with line-widths below 100 MHz. This would allow a single LFM chirped frequency to be separated and used as the LO.

Ultimately we adopted the idea of broadcasting two orthogonally polarized signals through a single polarization maintaining (PM) fiber and into the AOM to
identically modulate them. This idea contained many obstacles to overcome but compared to other solutions, the benefits outweighed the drawbacks.

PM fiber only maintains linear polarization along one axis but will partially maintain linear polarization along the orthogonal axis. Polarization-maintaining and absorption-reducing (PANDA) fiber was originally created for the telecommunications industry. Stress rods are fabricated to strain the fiber core making its material have birefringent properties. Figure 4.2 is a cross section of a PANDA fiber showing the stress rods and core. [7]

![Figure 4.2: Cross section of PANDA fiber [7]](image)

The birefringence creates a fast and slow axis. The light propagating along the slow axis maintains its polarization while the orthogonal fast axis partially maintains polarization. The polarization of light traveling along the fast axis will vary with changing temperature and stresses. The varying polarization will allow the light propagating along the fast axis to leak into the orthogonal axis, mixing the two signals.

The transmitting signal mixing with the LO (LO propagating along the slow axis) would be detrimental to the experiment because the process of separating the single frequency LO and the multiple frequency transmitting signal would have failed. The LO
mixing with the transmitting signal (transmitting signal along the slow axis) does not affect the outcome of the experiment. The single frequency contained in the LO already exists in the transmitting signal therefore it will not affect the transmitting signal. The idea of a fiber that would maintain both polarizations separately is an interesting one to discuss in the future work section of the conclusion.

4.2.2 AOM Operation

The AOM works by vibrating an optically clear medium using acoustic waves. The acoustic wave distorts the medium’s index of refraction in a wave pattern therefore creating a diffraction grating. [8] An AOM can only be as dynamic as the speed of which the substance can react to the vibrations. Undesirable variations in the linear frequency chirp begin to form as the bandwidth of the chirped signal grows; therefore the shortest chirp possible of 4μs was chosen. The experiment uses a 4μs and 0 to 5 volt sawtooth signal from a waveform generator illustrated in Figure 4.3.
The waveform changes the diffraction grating of the AOM creating a change in output frequency. However the AOM has a $2\mu s$ rise and fall time resulting in a triangular sweep of the frequency. The spectrogram of the signal upon leaving the AOM is shown in Figure 4.4, the black line was added for visualization.
The LO and the echo receive the same modulation but do not align completely due to the time delay, $t_1$, the echo experiences as it travels to and from the target. Figure 4.5 demonstrates the alignment. The experiment is only valid when the LO and the echo overlap with the same slope; this area is labeled “usable area” in Figure 4.5. The multiple frequencies of the echo signal will be represented as a single line in Figure 4.5.
Figure 4.5: Portion of usable frequency difference between the LO and echo signal. Echo signals which contains multiple LFM chirps is represented by a single line for simplicity.

Only 800 ns of the overlap can be used due to the shape of the modulation and the offset time of the echo, approximately 1 μs or 25% of the chirp length. The width of useable area is the length of time that can be used without introducing non-linearities in the chirped signal resulting in unwanted frequency broadening. The length of overlap time used directly corresponds to the ΔR achievable by the system as shown in Chapter 2. In radar different LO signal delay techniques are used to increase the overlap time. One simple technique is to introduce a time delay in the LO before heterodyning by using a fiber span. The time delay would increase the overlap time by decreasing the time difference between the LO and echo slopes. It has also been suggested that an
electronic-optic modulator (EOM) can generate higher bandwidth LFM CW chirps [9].

With increase in the LO bandwidth our experiments usable bandwidth increases by

\[ Usable \ Bandwidth = LO \ Bandwidth \times \left( n - n \frac{t_1}{T} \right), \]  \hspace{1cm} (2.1)

where \( n \) is the number of different frequency laser lines in the echo signal, \( t_1 \) is the time delay, \( T \) is the length of the signal and \( n \frac{t_1}{T} \) accounts for the delay acquired by each frequency.

### 4.2.3 Polarization Leakage And Attenuation

A major loss in LO power was anticipated due to the PM fiber. This loss was minimized by allowing the fiber to acclimate to the temperature and the fiber was not disturbed by moving it. The echo signal still needed to be attenuated. This insures the beat frequency created by its two echo laser line frequencies would not dominate the beat frequencies created by the LO and echo signal. Parasitic peaks still appear in the results from polarization leakage. The attenuation produces a signal power close to the noise floor resulting in a low signal to noise ratio (SNR). Figure 4.6 shows the unprocessed detected signal with arrows to emphasize which peaks contain information and the parasitic peak circled.
Figure 4.6: Signal detected by the MFSP experiment before post processing.
Chapter 5: Conclusion

The procedure outlined in this thesis lessens the complexity of generating long bandwidth optical LFM chirps. Linearity is easily maintained over shorter LFM pulses and the simultaneous modulation of multiple laser lines result in periodic noise after MFSP post processing. The use of SF-LFM along with the MFSP is a practical technique to utilize both of these advantages.

Multi-frequency stretch processing significantly improves $\Delta R$ when compared to SP systems with the same modulation abilities. The cost of this improvement is the extra processing done to increase usable bandwidth. During post processing of MFSP attention has to be paid to the phasing when extending the detected signal. Some suggested post processing could be simplified by performing all calculations in the frequency domain; this type of algorithm was not investigated due to time constraints.

Heterodyne detection works in part because a photodiode detector is too slow to detect the higher frequencies. That being said, there is a limit on the number of laser lines the echo signal contain. That limit is the modulation bandwidth multiplied by the highest detectable frequency of the diode.

The experiment performed verified that MFSP is a viable way to increase $\Delta R$. Overcoming the challenges presented in the experiment required some innovative solutions which could be further investigated on their own.
5.1 Future Work

The next obvious step for the immediate experiment is to create a transmitting signal in which the number of different frequency laser lines is increased which would result in more usable bandwidth. The Matlab program used in post processing was written with expansion in mind and can be mildly modified to except any number of $\pi$. One promising procedure to increase $\pi$ is to loop a single laser line through a fiber Bragg-Grating. Using higher diffraction orders as the upper frequencies in the transmitting signal while continuing to separate the LO using the orthogonal polarization technique.

The best thing about research sciences is the number of new experiments presented with each experiment performed. Many of the challenges presented in this thesis had solutions that could be expanded into individual experiments. The first proposal is about the vacancy of a peak when the translated detected frequencies are $\pi$ out of phase. Figure 3.4 and Figure 2.8 demonstrates the -3dB width of the non-peak being significantly narrower than -3dB of the optimal phase peak.

The final interesting concept is of propagating two separate non-interfering signals through a single fiber. Even though this solution was not ideal given the available equipment, the experiment still proved that it could be done. If a fiber could be created that would propagate two independent signals maintaining each signal’s polarization the potential applications would be numerous.
Works Cited


Appendix A: Matlab code.

%% Shows a movie of the range profile of a single chirp vs. an N chirp synthetic range profile.
close all;
clear all;
cclc;
warning off;

%Waveform Period, s
T = 4e-6;

%Sample rate, Samples/s
Sampling_rate=.5e9;

%Target delay, s
t1 = 1.5*200/3e8;

%Chirp bandwidth, Hz
B = 2*12.67e6;

%Max number of chirps
Num_chirps = 2;

%Difference frequency between lasers
DF = B;

%Center of overlap between LO and signal, s
t0 = (T-t1)/2+t1;
%Length of overlap between LO and signal, s
T1 = T-t1;

%Chirp coefficient, rad/s^2
beta = 2*pi*B/(T);

%Datapoints for original waveform
Datapoints=T*Sampling_rate;

%Datapoints non-overlap
DP=Sampling_rate*t1;
NT1=(T-t1)*Sampling_rate; % number of data points in delayed time

%Time step
deltaT=T/Datapoints;

time=0:deltaT:T;
tshift=(time-t1);
tau=tshift(DP+1:Datapoints);

%Setting theory signal array
zerol=zeros(1,DP+1);

target_signal=[zerol,exp(1j*(beta*(tau).^2)/2)+exp(-1j*2*pi*DF*(tau)).*exp(1j*(beta*(tau).^2)/2)];

% local oscillator no delay.
theory_LO = exp(1j*(beta*(time).^2)/2);

% The time average or signal at detector
signal=target_signal.*conj(theory_LO);

% zero array created for padding of signal
numzero=20;
zero=zeros(1,numzero*length(signal));

% padding signal for FFT
signal=[zero, signal, zero];

%taking the Fourier Transform of signal
fft_theory=fftshift(fft(signal));
S=fft_theory;
S=S/max(S);
S1=S;
DFmax = Sampling_rate / 2;

frequency = linspace(-DFmax, DFmax, length(S));

upper = frequency < -(DF - B / T * t1) & frequency > - max(frequency);
lower = frequency < max(frequency) & frequency > -(3 * B / T * t1);

upper = S1.*upper;
lower = S1.*lower;

frequency = linspace(-DFmax, DFmax, length(S1));
frequency1 = linspace(-DFmax, DFmax, length(lower));
frequency2 = linspace(-DFmax, DFmax, length(upper));

% Figure 2.3
figure
hold on;
plot(frequency, 10*log10(S1), 'k')
title('Entire Signal');
set(gca, 'XTick', -2*10^8:10^8:2*10^8)
set(gca, 'XTickLabel', {'-2', '-1', '0', '1', '2'})
xlabel('frequency (10^8)')
ylabel('dB')

% Figure 2.4
figure
hold on;
plot(frequency1, 10*log10(lower), 'c')
plot(frequency2, 10*log10(upper), 'k')
set(gca, 'XTick', -2*10^8:10^8:2*10^8)
set(gca, 'XTickLabel', {'-2', '-1', '0', '1', '2'})
title('Separated signals');
xlabel('frequency (10^8)')
ylabel('dB')

% Translating upper major frequency to overlap lower frequency

% Inverse fft both upper and lower frequencies
upper = ifft(upper);
lower = iffft(lower);

% Frequency shifting
new_time = linspace(0, (2*numzero+1)*T, length(upper));
shift = exp(1j*2*pi*DF*(new_time));
upper = upper.*shift;
% Insuring both signals are equal length
kill = [zero, ones(1, length(time)), zero];
lower = kill .* lower;
upper = kill .* upper;

% Normalizing signals
lower = lower / max(lower);
upper = upper / max(upper);

nonzerolower = find(lower);
nonzeroupper = find(upper);

lower = lower (nonzerolower);
upper = upper (nonzeroupper);

upper = [zero, upper, zero];
lower = [zero, lower, zero];

frequency1 = linspace(-DFmax, DFmax, length(lower));
frequency2 = linspace(-DFmax, DFmax, length(upper));

lower = abs(fft(lower) / max(fft(lower)));
upper = abs(fft(upper) / max(fft(upper)));

% Figure 2.5
figure
hold on;
plot(frequency1, 10 * log10(lower), 'c')
plot(frequency2, 10 * log10(upper), 'k')
set(gca, 'XTick', -2 * 10^8:10^8:2 * 10^8)
set(gca, 'XTickLabel', {'-2', '-1', '0', '1', '2'})
title('Peaks after translation')
legend('Lower Peak', 'Translated Peak')
xlabel('frequency (10^8)')
ylabel('dB')
% Shows a movie of the range profile of a single chirp vs. an N chirp synthetic range profile.
close all;
clear all;
clc;
warning off;

%Waveform Period, s
T = 4e-6;

%Sample rate, Samples/s
Sampling_rate=.5e9;

%Target delay, s
t1 = 1.5*200/3e8;

%Chirp bandwidth, Hz
B = 12.67e6/2;

%Max number of chirps
Num_chirps = 2;

%Difference frequency between lasers
DF = 70e6;

%Center of overlap between LO and signal, s
t0 = (T-t1)/2+t1;

%Length of overlap between LO and signal, s
T1 = T-t1;

%Chirp coefficient, rad/s^2
beta = 2*pi*B/(T);

% Datapoints for original waveform
Datapoints=T*Sampling_rate;

% Datapoints non-overlap
DP=Sampling_rate*t1;
NT1=(T-t1)*Sampling_rate; % number of data points in delayed time

% Time step
deltaT=T/Datapoints;

time=0:deltaT:T;
tshift=(time-t1);
tau=tshift(DP+1:Datapoints);

% Setting theory signal array
zerol=zeros(1,DP+1);

% Signal created by first chirped offset
signal1=[zerol, exp(1j*(beta*(tau).^2)/2)];

% Signal created by second chirped offset
signal2=[zerol, exp(-1j*beta*(T-t1)^2)*exp(-1j*2*pi*DF*(tau)).*exp(1j*(beta*(tau).^2)/2)];

% Signal created after shift has been made
theory_signal = [zerol, exp(1j*(beta*(tau).^2)/2)+exp(1j*2*pi*DF*(tau)).*exp(-1j*2*pi*DF*(tau)).*exp(1j*(beta*(tau).^2))];

% Local oscillator no delay.
theory_LO = exp(1j*(beta*(time).^2)/2);

signall_processed=signal1.*conj(theory_LO);

% Creation of double time array
double_time=0:deltaT:2*T;

double_tshift=(double_time-t1);

double_tau=double_tshift(DP+1:2*Datapoints);
%double length local oscillator
double_LO = exp(1j*(beta*(double_time).^2)/2);

phasemovie = avifile('phasemovie.avi');
phasemovie.fps = 2;

double_signal1 = [zero1, exp(1j*(beta*(double_tau).^2)/2)];
double_signal1 = double_signal1.*conj(double_LO);

%creating a matrix of ones and zeros to delete center part of signal
first_zero_in_signal = [zero1, ones(1, length(tau)), zero1, zeros(1, length(tau)-1)];
second_zero_in_signal = [zero1, zeros(1, length(tau)), zero1, ones(1, length(tau)-1)];
third_zero_in_signal = [zero1, zeros(1, length(tau)), ones(1, length(zero1)), zeros(1, length(tau)-1)];

%Deleting center part of signal
first_double_signal = first_zero_in_signal.*double_signal1;
second_double_signal = second_zero_in_signal.*double_signal1;
third_double_signal = third_zero_in_signal.*double_signal1;

black = zeros(1, length(double_signal1));

%figure 2.6
hold on;
plot(double_time, first_double_signal, 'K')
plot(double_time, second_double_signal, 'c')
plot(double_time, third_double_signal, ':K')
plot(double_time, black, 'k')
legend('Lower Frequency signal', 'Translated Signal', 'Ideal phase')
%% Shows a movie of the range profile of a single chirp vs. an N chirp synthetic range profile.
close all;
clear all;
clc;
warning off;

%Waveform Period, s
T = 4e-6;

%Sample rate, Samples/s
Sampling_rate=4e9;

%Target delay, s
t1 = 1.5*200/3e8;

%Chirp bandwidth, Hz
B = 2*12.67e6;

%Max number of chirps
Num_chirps = 2;

%Difference frequency between lasers
DF = 70e6;

%Center of overlap between LO and signal, s
t0 = (T-t1)/2+t1;

%Length of overlap between LO and signal, s
T1 = T-t1;
% Chirp coefficient, rad/s^2
beta = 2*pi*B/(T);

% Datapoints for original waveform
Datapoints = T*Sampling_rate;

% Datapoints non-overlap
DP = Sampling_rate*t1;
NT1 = (T-t1)*Sampling_rate; % number of data points in delayed time

% Time step
deltaT = T/Datapoints;

time = 0:deltaT:T;
tshift = (time-t1);

% Setting theory signal array
zerol = zeros(1,DP+1);

% Signal created by first chirped offset
signal1 = [zerol, exp(1j*((beta*(tau).^2)/2))];

% Signal created by second chirped offset
signal2 = [zerol, exp(-1j*beta*(T-t1)^2)*exp(-1j*2*pi*DF*(tau)).*exp(1j*((beta*(tau).^2)/2))];

% Signal created after shift has been made
theory_signal = [zerol, exp(1j*((beta*(tau).^2)/2))+exp(1j*2*pi*DF*(tau)).*exp(-1j*2*pi*DF*(tau)).*exp(1j*((beta*(tau).^2))];

% Local oscillator no delay.
theory_LO = exp(1j*(beta*(time).^2)/2);

signall_processed = signal1.*conj(theory_LO);

% Creation of double time array
double_time = 0:deltaT:2*T;

double_tshift = (double_time-t1);

% Double tau goes from t1 to 2*T
double_tau = double_tshift(DP+1:2*Datapoints);

% double length local oscillator
double_LO = exp(1j*(beta*(double_time).^2)/2);

phasemovie = avifile('phasemoviethrough.avi');
phasemovie.fps = 8;

double_signal1 = [zero1, exp(1j*((beta*(double_tau).^2)/2))];
double_signal1 = double_signal1.*conj(double_LO);

% creating a matrix of ones and zeros to delete center part of signal
zero_in_signal = [zero1, ones(1,length(tau)), zero1, ones(1,length(tau)-1)];

% Deleting center part of signal
double_signal1 = zero_in_signal.*double_signal1;

a = 0;
num = 1;
for n = 1:num

    phase_in_signal = [zero1, ones(1,length(tau)), zero1, ones(1,length(tau)-1)...
    *exp(1j*2*pi/n)];

    % correcting phase in signal
    double_signal1 = phase_in_signal.*double_signal1;

    zero = zeros(1,10*length(double_signal1));
    double_signal2 = [zero, double_signal1, zero];
    fft_doublesignal1 = fftshift(fft(double_signal2));

    DS1 = (abs(fft_doublesignal1));
    DS1 = DS1/max(DS1);
zero=zeros(1,10*length(signal1_processed));
signal2_processed=[zero, signal1_processed, zero];
fft_signal1=fftshift(fft(signal2_processed));
DFmax=Sampling_rate/2;
SP1=(abs(fft_signal1));
SP1=SP1/max(SP1);
central=peak(DS1,1,1,1);
for m = central:length(DS1)
    if DS1(m)<DS1(m+1)
        break
    end
end
LDS1=DS1(m:length(DS1));
side=peak(LDS1,.05,5,1);
frequency1=linspace(-DFmax,DFmax,length(signal2_processed));
frequency2=linspace(-DFmax,DFmax,length(double_signal2));
hold on;
plot(frequency1,10*log10(SP1),'k:');
plot(frequency2,10*log10(DS1),'k');
legend('Stretch Processing','Mutli-Stretch')
xlabel('Frequency')
ylabel('dB')
axis([-9e6 -3e6 -30 0])
X=getframe;
phasemovie=addframe(phasemovie,X);
end
% DoublechirpnoDF.m
% 3/27/10
% Robert Brown
% University of Dayton - Ladar and Optical Communications Institute
% This M-file contains the matlab code to create a simulation
% of a multiple chirp signal n=2 with a single chirped LO.
% This matlab creates figure 2.8 of "Linear frequency Modulation
% Signal and Local Oscillator Coherent Optical Detection"

close all;
clear all;
clc;
warning off;

%Waveform Period, s
T = 4e-6;

%Sample rate, Samples/s
Sampling_rate=4e9;

%Target delay, s
t1 = 1.5*200/3e8;

%Chirp bandwidth, Hz
B = 2*12.67e6;

%Max number of chirps
Num_chirps = 2;

%Difference frequency between lasers
DF = 30e6;

%Center of overlap between LO and signal, s
t0 = (T-t1)/2+t1;

%Length of overlap between LO and signal, s
T1 = T-t1;

%Chirp coefficient, rad/s^2
beta = 2*pi*B/(T);
% Datapoints for original waveform
Datapoints = T * Sampling_rate;

% Datapoints non-overlap
DP = Sampling_rate * t1;
NT1 = (T - t1) * Sampling_rate; % number of data points in delayed time

% Time step
deltaT = T / Datapoints;

% Time vector
time = 0:deltaT:T;

% Time vector shift due to delay
tshift = (time - t1);
tau = tshift(DP + 1:Datapoints);

% Setting theory signal array
zero1 = zeros(1, DP + 1);

% Signal created by first chirped offset
signal1 = [zero1, exp(1j * ((beta * (tau).^2) / 2))];

% Signal created by second chirped offset
signal2 = [zero1, exp(-1j * beta * (T - t1).^2) * exp(-1j * 2 * pi * DF * (tau)) .* exp(1j * ((beta * (tau).^2) / 2))];

% Signal created after shift has been made
theory_signal = (signal1 + signal2);

% Local oscillator no delay.
threey_LO = exp(1j * (beta * (time).^2) / 2);

% Signal modulated with LO
signal1_processed = (signal1 + signal2) .* conj(threey_LO);

% Creation of double time array
double_time = 0:deltaT:2 * T;

% Shifted double time array
double_tshift = (double_time - t1);

% Double tau goes from t1 to 2 * T
double_tau = double_tshift(DP + 1:2 * Datapoints);

% Double length local oscillator
double_LO = \exp(1j*(\beta*(\text{double\_time})^2)/2);

% creating a signal twice as long as the first
double_signal1 = [zero1, \exp(1j*((\beta*(\text{double\_tau})^2)/2))];

% signal modulated with LO
double_signal1 = double_signal1.*\text{conj}(double\_LO);

% creating a matrix of ones and zeros to delete center part of signal
zero_in_signal = [zero1, ones(1, length(tau)), zero1, ones(1, length(tau) - 1)];

% Deleting center part of signal
double_signal1 = zero_in_signal.*double_signal1;

% Creating zero array for padding
zero = zeros(1, 10*length(double_signal1));

% Padding signal
double_signal1 = [zero, double_signal1, zero];

% Fourier transforming the signal
fft_doublesignal1 = \text{fftshift}(\text{fft}(double_signal1));

% creating a zero array for padding
zero = zeros(1, 10*length(signal1_processed));

% Padding signal
signal1_processed = [zero, signal1_processed, zero];

% Fourier transforming single signal
fft_signal1 = \text{fftshift}(\text{fft}(signal1\_processed));
SP1 = (\text{abs}(fft\_signal1));
SP1 = SP1/\text{max}(SP1);

% Creating frequency space
DFmax = Sampling\_rate/2;
frequency1 = linspace(-DFmax, DFmax, length(signal1\_processed));

figure;
hold on;
plot(frequency1, 10*log10(SP1), 'k')
xlabel('frequency')
ylabel('dB')
title('Experimental Theory')
axis([-6e7 0 -30 0])
% chirp synthetic range profile.
close all;
clear all;
clc;
warning off;

%Waveform Period, s
T = 4e-6;

%Sample rate, Samples/s
Sampling_rate=4e9;

%Target delay, s
t1 = 1.5*200/3e8;

%Chirp bandwidth, Hz
B = 2*12.67e6;

%Max number of chirps
Num_chirps = 2;

%Difference frequency between lasers
DF = 30e6;

%Center of overlap between LO and signal, s
t0 = (T-t1)/2+t1;

%Length of overlap between LO and signal, s
T1 = T-t1;

%Chirp coefficient, rad/s^2
beta = 2*pi*B/(T);
% Datapoints for original waveform
Datapoints = T * Sampling_rate;

% Datapoints non-overlap
DP = Sampling_rate * t1;
NT1 = (T - t1) * Sampling_rate; % number of data points in delayed time

% Time step
deltaT = T / Datapoints;

% Time vector
time = 0:deltaT:T;

% Time vector shift due to delay
tshift = (time - t1);
tau = tshift(DP + 1:Datapoints);

% Setting theory signal array
zerol = zeros(1, DP + 1);

% Signal created by first chirped offset
signal1 = [zerol, exp(1j * (beta * (tau).^2) / 2)];

% Signal created by second chirped offset
signal2 = [zerol, exp(-1j * beta * (T - t1).^2) * exp(-1j * beta * (tau).^2) * exp(-1j * 4 * pi * DF * (tau)) * exp(1j * (beta * (tau).^2) / 2) + exp(-1j * beta * (T - t1).^2) * exp(-1j * 6 * pi * DF * (tau)) * exp(1j * (beta * (tau).^2) / 2)];

% Signal created after shift has been made
theory_signal = (signal1 + signal2);

% Local oscillator no delay.
theory_LO = exp(1j * (beta * (time).^2) / 2);

% Signal modulated with LO
signal1_processed = (signal1 + signal2) .* conj(theory_LO);

% Creation of double time array
double_time = 0:deltaT:2 * T;

% Shifted double time array
double_tshift = (double_time - t1);

% Double tau goes from t1 to 2 * T
double_t1 = double_tshift(DP:t+2*Datapoints);

%double length local oscillator
double_LO = exp(1j*(beta*(double_time).^2)/2);

%creating a signal twice as long as the first
double_signal1 = [zero1, exp(1j*(beta*(double_tau).^2)/2)];

% signal modulated with LO
double_signal1 = double_signal1.*conj(double_LO);

%creating a matrix of ones and zeros to delete center part of signal
zero_in_signal = [zero1, ones(1, length(tau)), zero1, ones(1, length(tau)-1)];

%Deleting center part of signal
double_signal1 = zero_in_signal.*double_signal1;

%Creating zero array for padding
zero = zeros(1, 10*length(double_signal1));

%Padding signal
double_signal1 = [zero, double_signal1, zero];

%fourier transforming the signal
fft_doublesignal1 = fftshift(fft(double_signal1));

%creating a zero array for padding
zero = zeros(1, 10*length(signal1_processed));

%padding signal
signal1_processed = [zero, signal1_processed, zero];

%fourier transforming single signal
fft_signal1 = fftshift(fft(signal1_processed));
SP1 = abs(fft_signal1);
SP1 = SP1 / max(SP1);

%creating frequency space
DFmax = Sampling_rate/2;
frequency1 = linspace(-DFmax, DFmax, length(signal1_processed));

% Figure 2.9
figure;
hold on;
plot(frequency1, 10*log10(SP1), 'k')
xlabel('frequency')
ylabel('dB')
title('Experimental Theory')
axis([ -9e7 0 -30 0])
close all;
clear all;
c1c;
warning off;
% Waveform Period, s
T = 4e-6;

% Sample rate, Samples/s
Sampling_rate=.5e9;

% Target delay, s
t1 = 1.5*200/3e8;

% Chirp bandwidth, Hz
B = 25.668e6;

% Max number of chirps
Num_chirps = 1;

% Center of overlap between LO and signal, s
t0 = (T-t1)/2+t1;

% Length of overlap between LO and signal, s
T1 = T-t1;

% Chirp coefficient, rad/s^2
beta = 2*pi*B/T;

% Datapoints for original waveform
Datapoints = T * Sampling_rate;

% Datapoints for overlap
DP = Sampling_rate * T1;

% Time step
deltaT = T1 / DP;

% Time vector, s
time = t1:deltaT:T;

% Setting theory signal array
theory_signal = zeros(1, DP + 1);

% Sumation of time shifts of each frequency
for i = 0:Num_chirps - 1;
    theory_signal = theory_signal +
        exp(1j * (beta * t1 + 2 * pi * i * B) * time));
end

% Creating a time array to compensate for delay
time1 = zeros(1, Datapoints - length(theory_signal) + 1);

% Compensating for delay
theory_signal = [time1, theory_signal];
zero = zeros(1, length(theory_signal));
theory_signal = [zero, theory_signal, zero];

DF = Sampling_rate / Datapoints;

DFmax = Sampling_rate / 2;

fft_theory = fftshift(fft(theory_signal));

frequency = -DFmax:DF:DFmax;

S = abs(fft_theory(Datapoints + 2:2 * Datapoints + 2));
S = S / max(S);

% Legend(['Sampling Rate = ', sprintf('%0.5g', (Sampling_rate))])

% Load Matched Filter
hinfo = hdf5info('difchirp4us.h5');
mf = hdf5read(hinfo.GroupHierarchy.Groups(1).Datasets(1));
mf=double(mf);
mf=mf-mean(mf);

% time truncation begins
b=1.2e-6;

% time truncation ends
e=2e-6;

timed=0:T/(length(mf)-1):T;
timeused=(timed>=b & timed<e);

mf=timeused.*mf;
mf=mf(find(mf));
timed=timed(find(timed));

length(mf)
zero=zeros(1,abs(length(mf)-length(frequency)));
mf=[zero,mf];

fftfm=fftshift(fft(fftshift(mf)));

% Figure 3.3
hold on;
plot(frequency/3,10*log10(S),'k:')
plot(frequency/3,10*log10(abs(fftfm)/abs(max(fftfm))),'k');

axis([0 20e6 -30 0]);
title('4us single chirp stretch processing')
xlabel('frequency')
ylabel('dB')
legend('Calculated','Measured')
%% Shows a movie of the range profile of a single chirp vs. an N chirp synthetic range profile.
close all;
clear all;
clc;
warning off;

%Waveform Period, s
T = 4e-6;

%Sample rate, Samples/s
Sampling_rate=4e9;

%Target delay, s
t1 = 1.5*200/3e8;

%Chirp bandwidth, Hz
B = 2*12.67e6;

%Max number of chirps
Num_chirps = 2;

%Difference frequency between lasers
DF = 70e6;

%Center of overlap between LO and signal, s
t0 = (T-t1)/2+t1;

%Length of overlap between LO and signal, s
T1 = T-t1;

%Chirp coefficient, rad/s^2
beta = 2*pi*B/(T);

%Datapoints for original waveform
Datapoints=T*Sampling_rate;

%Datapoints non-overlap
DP=Sampling_rate*t1;
NT1=(T-t1)*Sampling_rate;  % number of data points in delayed time

%Time step
deltaT=T/Datapoints;

time=0:deltaT:T;
tshift=(time-t1);
tau=tshift(DP+1:Datapoints);

%Setting theory signal array
zero1=zeros(1,DP+1);

%signal created by first chirped offset
signal1=[zero1, exp(1j*((beta*(tau).^2)/2))];

%signal created by second chirped offset
signal2=[zero1, exp(-1j*beta*(T-t1)^2)*exp(-1j*2*pi*DF*(tau)).*exp(1j*((beta*(tau).^2)/2))];

%signal created after shift has been made
theory_signal = [zero1, exp(1j*((beta*(tau).^2)/2))+exp(1j*2*pi*DF*(tau)).*exp(-1j*2*pi*DF*(tau)).*exp(1j*((beta*(tau).^2)))];

% local oscillator no delay.
theory_LO = exp(1j*(beta*(time).^2)/2);

signall_processed=signall.*conj(theory_LO);

% creation of double time array
double_time=0:deltaT:2*T;

%shifted double time array
double_tshift=(double_time-t1);
double tau goes from t1 to 2*T
double_tau=double_tshift(DP+1:2*Datapoints);

double length local oscillator
double_LO=exp(1j*(beta*(double_time).^2)/2);

phasemovie=avifile('phasemoviethrough.avi');
phasemovie.fps=8;
double_signal1 = [zero1,exp(1j*((beta*(double_tau).^2)/2))];
double_signal1 = double_signal1.*conj(double_LO);

%creating a matrix of ones and zeros to delete center part of signal
zero_in_signal=[zero1,ones(1,length(tau)),zero1,ones(1,length(tau)-1)];

%Deleting center part of signal
double_signal1=zero_in_signal.*double_signal1;

a=0;
num=1;

for n = 1:num

phase_in_signal=[zero1,ones(1,length(tau)),zero1,ones(1,length(tau)-1)...
*exp(1j*2*pi/2)];

%correcting phase in signal
double_signal1=phase_in_signal.*double_signal1;

zero=zeros(1,10*length(double_signal1));
double_signal2=[zero, double_signal1, zero];
fft_doublesignal1=fftshift(fft(double_signal2));
DS1=(abs(fft_doublesignal1));
DS1=DS1/max(DS1);

zero=zeros(1,10*length(signal1_processed));
signal2_processed=[zero, signal1_processed, zero];
fft_signal1=fftshift(fft(signal2_processed));

DFmax=Sampling_rate/2;
SP1=(abs(fft_signal1));
SP1=SP1/max(SP1);

central=peak(DS1,1,1,1);
for m = central:length(DS1)
    if DS1(m)<DS1(m+1)
        break
    end
end

LDS1=DS1(m:length(DS1));
side=peak(LDS1,.05,5,1);

frequency1=linspace(-DFmax,DFmax,length(signal2_processed));
frequency2=linspace(-DFmax,DFmax,length(double_signal2));

hold on;
plot(frequency1,10*log10(SP1),'k:');
plot(frequency2,10*log10(DS1),'k')
legend('Stretch Processing (SP)', 'Multi-SP \pi/2 out of phase')
xlabel('Frequency')
ylabel('dB')

axis([-9e6 -3e6 -30 0])
X=getframe;
phasemovie=addframe(phasemovie,X);
end
phasemovie=close(phasemovie);
close all;
clear all;
clc;
warning off;

%% 4us Chirp

%Import data
hinfo = hdf5info('chirp4us.h5');
mf = hdf5read(hinfo.GroupHierarchy.Groups(1).Datasets(1));

%Generate and display spectragram.
specgram(mf,[],4e9)
axis([0 4e-6 7.5e8 8.5e8])
colormap bone
title('AOM frequency output at 4 us')
close all;
clear all;
clcl;
warning off;

%Waveform Period, s
T = 4e-6;

%Sample rate, Samples/s
Sampling_rate=4e9;

%Target delay, s
t1 = 1.5*200/3e8;

%Chirp bandwidth, Hz
B = 23.75e6;

%Max number of chirps
Num_chirps = 2;

%Difference frequency between lasers

%Center of overlap between LO and signal, s
t0 = (T-t1)/2+t1;

%Length of overlap between LO and signal, s
T1 = T-t1;

%Chirp coefficient, rad/s^2
beta = 2*pi*B/(T);
% Datapoints for original waveform
Datapoints = T * Sampling_rate;

% Datapoints non-overlap
DP = Sampling_rate * t1;
NT1 = (T - t1) * Sampling_rate;  % number of data points in delayed time

% Time step
deltaT = T / Datapoints;

% time vector, s
time = 0:deltaT:T;
tshift = (time - t1);
tau = tshift(DP + 1:Datapoints);

% Setting theory signal array
zero1 = zeros(1, DP + 1);

%% Experimental signal chirp signal

% load Matched Filter
hinfo = hdf5info('polarized800ns.h5');
mf = hdf5read(hinfo.GroupHierarchy.Groups(1).Datasets(1));
mf = double(mf);
mf = mf - mean(mf);

% time truncation begins
b = 0e-6;

% time truncation ends
e = .8e-6;

timed = 0:T/(length(mf) - 1):T;
timeused = (timed >= b & timed < e);

mf = timeused .* mf;

timed = timed .* timeused;
mf = mf(find(mf));
timed = [timed(find(timed))];

length(mf)
% zero array created for padding of signal
numzero = 10;

% Padding signal
zero = zeros(1, numzero * length(mf));

zero2 = zeros(1, abs(length(mf) - length(time)));
mf = [zeros(1, length(zero2)/2), mf, zeros(1, length(zero2)/2 + 1)];

zero = zeros(1, 160010);
mf = [zero, mf, zero];

length(mf)

% Creating frequency space for transformed signal
DFmax = Sampling_rate/2;
frequency = linspace(-DFmax, DFmax, length(mf));
fftmf = fftshift(fft(mf))/max(fft(mf));

%% Cutting and translating signal
mfupper = frequency > 2.2e8 & frequency < 2.75e8;
mflower = frequency > 0.5e7 & frequency < 8e7;

mfupper = fftmf.*mfupper;
mflower = fftmf.*mflower;

number = 100;

z = linspace(-(number)/2, (number)/2, number + 1);
k = fftmf == max(mflower);
k = find(k);
\begin{verbatim}
\texttt{k=sum(frequency(z+k))/length(z);}
\texttt{l=fftmf==max(mfupper);} \texttt{l=find(l);} \texttt{l=sum(frequency(z+l))/length(z);} 
\texttt{beta=2*pi*k/1e-6;} \texttt{DF=l-k;}

\texttt{mfupper=ifft(mfupper);} \texttt{mflower=ifft(mflower);} 
\texttt{new\_time=linspace(0,(2*numzero+1)*T,length(mfupper));} 
\texttt{shift = exp(-1j*2*DF*pi*(new\_time));} \texttt{mfupper=shift.*mfupper;}

\texttt{kill = [zeros(1,length(zero)/2),ones(1,length(time)+length(zero)),zeros(1,length(zero)/2)];} \texttt{mflower=kill.*mflower;} \texttt{mfupper=kill.*mfupper;}

\texttt{nonzerolower=find(mflower);} \texttt{nonzeroupper=find(mfupper);} 
\texttt{mflower=mflower(nonzerolower);} \texttt{mfupper=mfupper(nonzeroupper);} 
\texttt{% Normalizing the the two peaks} 
\texttt{mflower=[zero,mflower,zero];} \texttt{mfupper=[zero,mfupper*exp(1j*pi*5/9),zero];} 
\texttt{mflower=mflower/max(mflower);} \texttt{mfupper=mfupper/max(mfupper);} 

\texttt{mflowernew = abs(mflower)>0.1;} \texttt{LK=mflowernew;} \texttt{mflower=mflower.*mflowernew;} 
\texttt{mfuppernew = abs(mfupper)>0.1;} \texttt{UK=mfuppernew;}
\end{verbatim}
mfupper = mfupper.*mfuppernew;

mfupper1 = mfupper(find(mfupper));

% combining the two signals to create better resolution
mfS = [mflower, mfupper];
mfSnew = abs(mfS) > .1;
mfS = mfS.*mfSnew;
mfS = mfS(find(mfS));

%% Creating theory signal and chirp

% signal created by modulation of combined signal and target echo
target_signal = [zero1, exp(1j*(beta*(tau).^2)/2) + exp(-1j*2*pi*DF*(tau)).*exp(1j*(beta*(tau).^2)/2)];

% local oscillator no delay.
theory_LO = exp(1j*(beta*(time).^2)/2) + exp(-1j*2*pi*DF*(time)).*exp(1j*(beta*(time).^2)/2);

% The time average or signal at detector
signal = conj(target_signal).*(theory_LO);

% padding signal for FFT
signal = [zero, signal, zero];

% taking the Fourier Transform of signal
fft_theory = fftshift(fft(signal));
S = fft_theory;
S = S / max(S);

frequency = linspace(-DFmax, DFmax, length(S));

% making a 1 or 0 matrix to split the upper and lower peak
upper = frequency > 10e7 & frequency < 30e7;
lower = frequency > 0 & frequency < 10e7;

% splitting the upper and lower peak
upper = S.*upper;
lower = S.*lower;

hold on;
plot(frequency,10*log10(abs(S)),'k')
plot(frequency,10*log10(abs(fftmf)),'k')

% returning to split portions to time domain
upper=(ifft(upper));
lower=(ifft(lower));

% creating a new time to shift the upper peak with the lower
new_time1=linspace(0,(2*numzero+1)*T,length(upper));

% creating a shift array with a random phase
shift1 = exp(-1j*2*pi*DF*(new_time1))*exp(1j*2*pi*rand);

% shifting the upper peak down
upper=upper.*shift1;

% Making sure the matrix are the same size
kill = [zero,ones(1,length(time)),zero];
lower=kill.*lower;
upper=kill.*upper;

nonzerolower=find(lower);
nonzeroupper=find(upper);

lower=lower(nonzerolower);
upper=upper(nonzeroupper);

% Normalizing the the two peaks
lower=lower/max(lower);
upper=upper/max(upper);

lowerdiff=abs(length(LK)-length(lower))/2;
lower=LK.*[zeros(1,lowerdiff),lower,zeros(1,lowerdiff)];
lower=lower(find(lower));

upperdiff=abs(length(UK)-length(upper))/2;
upper=UK.*[zeros(1,upperdiff),upper,zeros(1,upperdiff)];
upper=upper(find(upper));

%combining the two signals to create better resolution
double_signal1=[lower,upper];
num=60;
for n = 1:num

phase_in_signal=[ones(1,length(mflower1)),ones(1,length(mfupper1))]*exp(1j*2*pi*(n/(n+1)));
% correcting phase in signal
nmfS=phase_in_signal.*mfS;

mfS2=[zero, nmfS, zero];
fft_mfS=(fft(mfS2));

mfS1=(abs(fft_mfS));
mfS1=mfS1/max(mfS1);
DFmax=Sampling_rate/2;
central=peak(mfS1,1,1,1);
for m = central:length(mfS1)
    if mfS1(m)<mfS1(m+1)
        break
    end
end
LmfS1=mfS1(m:length(mfS1));
side=peak(LmfS1,.05,5,1);
if abs(LmfS1(side)-.48)<.03
    break
end
end

% Using iterations to correct the phase in the theory signal
num=60;
for n = 1:num

    phase_in_signal=[ones(1,length(lower)),ones(1,length(upper))\n*exp(1j*2*pi*(n/(n+1)))];

    % correcting phase in signal
    double_signall=phase_in_signal.*double_signall;

    zero=zeros(1,numzero*length(double_signall));
    double_signal2=[zero, double_signall, zero];
    fft_doublesignal1=(fft(double_signal2));
    DS1=(abs(fft_doublesignal1));
    DS1=DS1/max(DS1);
    DFmax=Sampling_rate/2;
    central=peak(DS1,1,1,1);
    for m = central:length(DS1)
        if DS1(m)<DS1(m+1)
            break
        end
    end
    LDS1=DS1(m:length(DS1));
side=peak(LDS1,.05,5,1);
if abs(LDS1(side)-.33)<.05
    break
end
end

%Plotting results
double_signal2=[zero, double_signal1, zero];
fft_double_signal1=(fft(double_signal2));

DS1=(abs(fft_double_signal1));
DS1=DS1/max(DS1);

upper=[zero, upper, zero];
lower=[zero, lower, zero];
mfupper=[zero, mfupper, zero];
mflower=[zero, mflower, zero];

frequency=linspace(-DFmax,DFmax,length(DS1));
frequency1=linspace(-DFmax,DFmax,length(lower));
frequency2=linspace(-DFmax,DFmax,length(upper));
mffrequency=linspace(-DFmax,DFmax,length(mfS1));
mffrequency1=linspace(-DFmax,DFmax,length(mflower));
mffrequency2=linspace(-DFmax,DFmax,length(mfupper));

lower=abs(fft(lower)/max(fft(lower)));
upper=abs(fft(upper)/max(fft(upper)));
mflower=abs(fft(mflower)/max(fft(mflower)));
mfupper=abs(fft(mfupper)/max(fft(mfupper)));

figure
hold on;
plot(frequency1,10*log10(lower),'--k')
plot(frequency,10*log10(DS1),'--c')
plot(frequency2,10*log10(upper),'r')
plot(mffrequency,10*log10(mfS1),'k')
plot(mffrequency1,10*log10(mflower),'m')
plot(mffrequency2,10*log10(mfupper),'y')
axis([.5e7 3.5e7 -16 0])
colormap gray
%title('Signal after post processing')
legend('Stretch Processing','Calc. Multi','Measured Multi');
xlabel('frequency')
ylabel('dB')