FEASIBILITY STUDY FOR TESTING THE DYNAMIC STABILITY OF BLUNT BODIES WITH A MAGNETIC SUSPENSION SYSTEM IN A SUPersonic WIND TUNNEL

by

ABIGAIL E. SEVIER

Submitted in partial fulfillment of the requirements for the Degree of Masters of Science

Department of Mechanical and Aerospace Engineering

CASE WESTERN RESERVE UNIVERSITY

May 2017
CASE WESTERN RESERVE UNIVERSITY

SCHOOL OF GRADUATE STUDIES

We hereby approve the thesis of:

Abigail E. Sevier

Candidate for the degree of Master of Science*:

Committee Chair

Paul Barnhart, Ph.D.

Committee Member

Joseph Prahl, Ph.D.

Committee Member

James T’ien, Ph.D.

Date of Defense

March 31, 2017

*We also certify that written approval has been obtained for any proprietary material contained therein.
TABLE OF CONTENTS

LIST OF TABLES ............................................................................................................. viii
LIST OF FIGURES: ............................................................................................................. x
ACKNOWLEDGEMENTS ................................................................................................. xx
LIST OF SYMBOLS: ......................................................................................................... xxi
ABSTRACT ..................................................................................................................... xxvii
CHAPTER I INTRODUCTION ...................................................................................... 1
CHAPTER II BLOCKAGE TESTING ............................................................................. 5
  2.1 INTRODUCTION ..................................................................................................... 5
  2.2 METHODOLOGY ................................................................................................... 6
    2.2.1 Facility ............................................................................................................ 6
    2.2.2 Instrumentation ............................................................................................. 7
    2.2.3 Models .......................................................................................................... 8
    2.2.4 Test Procedure: ........................................................................................... 10
  2.3 RESULTS ............................................................................................................... 11
    2.3.1 Mach 2.5 Nozzle Block with Axisymmetric Test Section ....................... 12
    2.3.1.1 Blockage Data Variation with Boundary Layer in Axisymmetric Test Section ........................................................................................................................................... 13
    2.3.2 Mach 2.5 Nozzle Block with Square Test Section at 18.7 cm ............. 16
    2.3.2.1 Blockage Data Variation from Mach 2.5 Axisymmetric to Square Configuration ........................................................................................................................................... 17
    2.3.3 Mach 2 Nozzle Block with Square Test Section at 18.7 cm ............... 19
    2.3.4 Mach 3 Nozzle Block with Square Test Section at 18.7 cm ............... 19
2.3.5 Mach Number versus Blockage Relation ........................................20
2.3.6 Lowest Reynolds Number for All Configurations ......................21
2.3.7 Starting Loads Analysis ...............................................................22

2.4 SUMMARY .....................................................................................23

CHAPTER III WALL PROXIMITY TESTING .............................................. 26

3.1 INTRODUCTION .............................................................................26

3.2 TEST METHODOLOGY ....................................................................26

3.3 RESULTS: .......................................................................................28

3.3.1 Mach 2.5 Axisymmetric Configuration at 50.8 cm from the Nozzle ....................................................28
3.3.2 Mach 2.5 Axisymmetric Configuration at 10.2 cm from the Nozzle .......................................................30
3.3.3 Mach 2 Square Configuration at 18.7 cm from the Nozzle ............33
3.3.4 Mach 2.5 Square Configuration at 18.7 cm from the Nozzle ........35
3.3.5 Mach 3 Square Configuration at 18.7 cm from the Nozzle ............36

3.4 SUMMARY .......................................................................................37

CHAPTER IV VIDEO ANALYSIS .............................................................. 39

4.1 INTRODUCTION .............................................................................39

4.2 INSTRUMENTATION .......................................................................39

4.3 VALIDATION EXPERIMENT METHODOLOGY ......................... 40

4.3.1 Camera Mount ............................................................................40
4.3.2 Cam Design and Construction ...................................................42
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3.3 Tracker Program</td>
<td>43</td>
</tr>
<tr>
<td>4.4 VIDEO ANALYSIS PROGRAM DEVELOPMENT</td>
<td>45</td>
</tr>
<tr>
<td>4.4.1 Tracking Program</td>
<td>45</td>
</tr>
<tr>
<td>4.4.2 Stereo Camera Calibration</td>
<td>48</td>
</tr>
<tr>
<td>4.4.3 Position and Orientation Analysis</td>
<td>49</td>
</tr>
<tr>
<td>4.5 STEREO CAMERA RESULTS</td>
<td>51</td>
</tr>
<tr>
<td>4.5.1 Pitch Case Example</td>
<td>51</td>
</tr>
<tr>
<td>4.5.2 Pitch and Yaw Case</td>
<td>52</td>
</tr>
<tr>
<td>4.6 SINGLE Camera VIDEO ANALYSIS METHOD</td>
<td>54</td>
</tr>
<tr>
<td>4.6.1 Pitch Test With Single Camera</td>
<td>54</td>
</tr>
<tr>
<td>4.6.2 Single Camera Calibration</td>
<td>54</td>
</tr>
<tr>
<td>4.6.3 Angle of Attack Extraction</td>
<td>55</td>
</tr>
<tr>
<td>4.6.4 Results of Pitch Only Test</td>
<td>55</td>
</tr>
<tr>
<td>4.7 SUMMARY</td>
<td>56</td>
</tr>
<tr>
<td>CHAPTER V ONE DEGREE OF FREEDOM TEST</td>
<td>58</td>
</tr>
<tr>
<td>5.1 INTRODUCTION:</td>
<td>58</td>
</tr>
<tr>
<td>5.2 1DOF TEST APPARATUS DESIGN</td>
<td>59</td>
</tr>
<tr>
<td>5.2.1. Design Requirements</td>
<td>59</td>
</tr>
<tr>
<td>5.2.2 Model Selection</td>
<td>60</td>
</tr>
<tr>
<td>5.2.3 Stress Analysis</td>
<td>62</td>
</tr>
<tr>
<td>5.2.4 Test Models</td>
<td>70</td>
</tr>
<tr>
<td>5.2.5 Retraction Mechanism</td>
<td>72</td>
</tr>
</tbody>
</table>
5.3 VIDEO ANALYSIS METHOD

5.3.1 Camera Installation

5.3.2 Camera Calibration

5.3.3 Video Analysis Program

5.4 DESIGN MODIFICATIONS

5.4.1 Original Test-Matrix

5.4.2 1DOF Design Modification

5.4.3 1DOF Modified Test Matrix

5.5 BEARING FRICTION ANALYSIS

5.5.1 Derivation Including Bearing Damping

5.5.2 Damping Results

5.6 1DOF TEST RESULTS

5.6.1 Expected Frequency of Oscillation

5.6.2 Test Results

5.7 ERROR ANALYSIS

5.8 SUMMARY

CHAPTER VI CONCLUSIONS

FIGURES

APPENDIX A METHOD OF SOLUTION

APPENDIX A.1 MARKER TRACKING PROGRAM

APPENDIX A.2 VERIFICATION OF TRACKING PROGRAM PIXEL SELECTION
APPENDIX A.3 ANGLE EXTRACTION PROGRAM FOR TWO CAMERA TEST

APPENDIX A.4 ANGLE EXTRACTION PROGRAM FOR ONE CAMERA TEST

APPENDIX A.5 TRACKING PROGRAM FOR 1DOF TESTING

APPENDIX A.6 ANGLE EXTRACTION CODE FOR 1DOF TEST

APPENDIX A.7 SHO FIT PROGRAM FOR 1DOF TEST

APPENDIX B SOLIDWORKS DRAWINGS

BIBLIOGRAPHY:
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 2.1: Boundary-Layer and Displacement Thickness at Different Axial Locations</td>
<td>14</td>
</tr>
<tr>
<td>Table 2.2: Boundary-Layer Blockage for Axisymmetric Configuration</td>
<td>15</td>
</tr>
<tr>
<td>Table 2.3: Comparing Total Blockage for the Mach 2.5 Axisymmetric Configuration</td>
<td>16</td>
</tr>
<tr>
<td>Table 2.4: Blockage Comparison between Square and Axisymmetric</td>
<td>18</td>
</tr>
<tr>
<td>Table 2.5: Reynolds Numbers for Tunnel Unstart</td>
<td>21</td>
</tr>
<tr>
<td>Table 2.6: Drag on Model During Tunnel Start</td>
<td>22</td>
</tr>
<tr>
<td>Table 2.7: Drag on Model During &quot;Steady State&quot; Tunnel Operation</td>
<td>23</td>
</tr>
<tr>
<td>Table 3.1: Steady-State Total Pressure During Wall Proximity Test</td>
<td>27</td>
</tr>
<tr>
<td>Table 3.2: Wall Proximity Test Results</td>
<td>38</td>
</tr>
<tr>
<td>Table 4.1: Sony Cybershot Frame Rate Specifications</td>
<td>40</td>
</tr>
<tr>
<td>Table 4.2: Combined Results of Video Analysis Feasibility Tests</td>
<td>57</td>
</tr>
<tr>
<td>Table 5.1: Largest Models That Start In Square Test Section</td>
<td>61</td>
</tr>
<tr>
<td>Table 5.2: 63</td>
<td>64</td>
</tr>
<tr>
<td>Table 5.3: Loading on Sting</td>
<td>65</td>
</tr>
<tr>
<td>Table 5.4: Lift Force During Steady State Operation</td>
<td>65</td>
</tr>
<tr>
<td>Table 5.5: Loading Analysis for Pin</td>
<td>67</td>
</tr>
<tr>
<td>Table 5.6: Loading Analysis for Diamond Strut</td>
<td>68</td>
</tr>
<tr>
<td>Table 5.7: Loading Calculation for Bolts in Assembly</td>
<td>69</td>
</tr>
<tr>
<td>Table 5.8: List of Center of Gravity Locations for Flight Vehicles</td>
<td>70</td>
</tr>
<tr>
<td>Table 5.9: Moment of Inertia for Test Models</td>
<td>71</td>
</tr>
<tr>
<td>Table 5.10: Support Removal Times</td>
<td>74</td>
</tr>
</tbody>
</table>
Table 5.11: Mach 2 1DOF Test Matrix
Table 5.12: Mach 2.5 1DOF Test Matrix
Table 5.13: Mach 3 1DOF Test Matrix
Table 5.14: Maximum Deflection Compared for New and Original Sting
Table 5.15: Modified 1DOF Test Matrix
Table 5.16: Variables from SHO Fit for Video Data
Table 5.17: Change in Bearing Coefficient Due to Pitch Damping Coefficient
Table 5.18: Expected Frequencies of Oscillation for Test Models
Table 5.19: Model Properties Used For Calculation
Table 5.20: Mach 2 1DOF Individual Test Results
Table 5.21: Mach 2.5 1DOF Individual Test Results
Table 5.22: Mach 3 1DOF Individual Test Results
Table 5.23: Mach 2.5 1DOF Reverse Direction
Table 5.24: Averaged Frequency Results (Hz) Compared against Original SHO Results
Table 5.25: Moment Slope Coefficient Results Compared with Empirical Data
Table 5.26: Frequency (Hz) Compared with SHO Results, Outliers Removed
Table 5.27: Moment Slope Coeff. Compared with Empirical Data, Outliers Removed
Table 5.28: One Pixel Error in Degrees for All Tests
LIST OF FIGURES:

Figure 1.1: Ballistic Range at Eglin Air Force Base................................. 105
Figure 1.2: Magnetic Suspension and Balance System developed by MIT.......... 105
Figure 2.1: CAD representation of model with iron core ............................ 106
Figure 2.2: Test section configuration options ........................................... 107
Figure 2.3: GRC 225 cm² axisymmetric testing set-up................................. 108
Figure 2.4: 3D-Printed models with semi-vertex angles of 45, 60, and 70-degrees..... 109
Figure 2.5: Literature review results ......................................................... 109
Figure 2.6: Backshell of model with epoxied threaded rod ......................... 110
Figure 2.7: Pressure tap readings during test for model 7006 ......................... 110
Figure 2.8: Schlieren Stills for 6007.5 prior to SWT start a) and after start b) .... 111
Figure 2.9: Mach 2.5 Axisymmetric Test Section a) 50.8 cm b) 10.2 cm ............ 112
Figure 2.10: Displacement thickness as a function of axial location................... 113
Figure 2.11: Displacement thickness as a function of Reynolds number ............ 114
Figure 2.12: Blockage Plot for Mach 2.5 Square Test Section ......................... 115
Figure 2.13: Blockage Plot for Mach 2 Square Test Section ......................... 115
Figure 2.14: Blockage Plot for Mach 3 Square Test Section ......................... 116
Figure 2.15: Literature Review with Blockage Test Data Added ..................... 116
Figure 2.16: Blockage Study Results with High and Low Bound for Each Geometry . 117
Figure 3.1: Downstream View of Pressure Tap Configuration in Axisymmetric .... 118
Figure 3.2: Downstream View of Pressure Tap Configuration in Square ............ 118
Figure 3.3: Mach 2.5 Axi. Configuration (50.8 cm): 0.00 cm from Centerline .......... 118
Figure 3.4: Mach 2.5 Axi. Configuration (50.8 cm): 0.25 cm from Centerline .......... 119
Figure 3.5: Mach 2.5 Axi. Configuration (50.8 cm): 0.50 cm from Centerline............ 119
Figure 3.6: Mach 2.5 Axi. Configuration (50.8 cm): 0.75 cm from Centerline............ 120
Figure 3.7: Mach 2.5 Axi. Configuration (50.8 cm): 1.00 cm from Centerline............ 120
Figure 3.8: Mach 2.5 Axi. Configuration (50.8 cm): 1.25 cm from Centerline............ 121
Figure 3.9: Mach 2.5 Axi. Configuration (50.8 cm): 1.50 cm from Centerline............ 121
Figure 3.10: Mach 2.5 Axi. Configuration (50.8 cm): 1.75 cm from Centerline............ 122
Figure 3.11: Mach 2.5 Axi. Configuration (50.8 cm): 2.00 cm from Centerline............ 122
Figure 3.12: Mach 2.5 Axi. Configuration (50.8 cm): 2.25 cm from Centerline............ 123
Figure 3.13: Mach 2.5 Axi. Configuration (50.8 cm): 2.50 cm from Centerline............ 123
Figure 3.14: Mach 2.5 Axi. Configuration (50.8 cm): 2.75 cm from Centerline............ 124
Figure 3.15: Mach 2.5 Axi. Configuration (50.8 cm): 3.00 cm from Centerline............ 124
Figure 3.16: Mach 2.5 Axi. Configuration (50.8 cm): 3.25 cm from Centerline............ 125
Figure 3.17: Mach 2.5 Axi. Configuration (50.8 cm): 3.50 cm from Centerline............ 125
Figure 3.18: Mach 2.5 Axi. Configuration (50.8 cm): 3.75 cm from Centerline............ 126
Figure 3.19: Mach 2.5 Axi. Configuration (50.8 cm): 4.00 cm from Centerline............ 126
Figure 3.20: Mach 2.5 Axi. Configuration (50.8 cm): 4.25 cm from Centerline............ 127
Figure 3.21: Mach 2.5 Axi. Configuration (50.8 cm): 4.50 cm from Centerline............ 127
Figure 3.22: Mach 2.5 Axi. Configuration (50.8 cm): 4.75 cm from Centerline............ 128
Figure 3.23: Mach 2.5 Axi. Configuration (50.8 cm): 5.00 cm from Centerline............ 128
Figure 3.24: Mach 2.5 Axi. Configuration (50.8 cm): 5.25 cm from Centerline............ 129
Figure 3.25: Mach 2.5 Axi. Configuration (50.8 cm): 5.50 cm from Centerline............ 129
Figure 3.26: Mach 2.5 Axi. Configuration (50.8 cm): 5.75 cm from Centerline............ 130
Figure 3.27: Mach 2.5 Axi. Configuration (50.8 cm): 6.00 cm from Centerline............ 130
Figure 3.28: Mach 2.5 Axi. Configuration (10.2 cm): 0.00 cm from Centerline .......... 131
Figure 3.29: Mach 2.5 Axi. Configuration (10.2 cm): 0.25 cm from Centerline .......... 131
Figure 3.30: Mach 2.5 Axi. Configuration (10.2 cm): 0.50 cm from Centerline .......... 132
Figure 3.31: Mach 2.5 Axi. Configuration (10.2 cm): 0.75 cm from Centerline .......... 132
Figure 3.32: Mach 2.5 Axi. Configuration (10.2 cm): 1.00 cm from Centerline .......... 133
Figure 3.33: Mach 2.5 Axi. Configuration (10.2 cm): 1.25 cm from Centerline .......... 133
Figure 3.34: Mach 2.5 Axi. Configuration (10.2 cm): 1.50 cm from Centerline .......... 134
Figure 3.35: Mach 2.5 Axi. Configuration (10.2 cm): 1.75 cm from Centerline .......... 134
Figure 3.36: Mach 2.5 Axi. Configuration (10.2 cm): 2.00 cm from Centerline .......... 135
Figure 3.37: Mach 2.5 Axi. Configuration (10.2 cm): 2.25 cm from Centerline .......... 135
Figure 3.38: Mach 2.5 Axi. Configuration (10.2 cm): 2.50 cm from Centerline .......... 136
Figure 3.39: Mach 2.5 Axi. Configuration (10.2 cm): 2.75 cm from Centerline .......... 136
Figure 3.40: Mach 2.5 Axi. Configuration (10.2 cm): 3.00 cm from Centerline .......... 137
Figure 3.41: Mach 2 Square Configuration (18.7 cm): 0.00 cm from Centerline ....... 137
Figure 3.42: Mach 2 Square Configuration (18.7 cm): 0.25 cm from Centerline ....... 137
Figure 3.43: Mach 2 Square Configuration (18.7 cm): 0.50 cm from Centerline ....... 138
Figure 3.44: Mach 2 Square Configuration (18.7 cm): 0.75 cm from Centerline ....... 138
Figure 3.45: Mach 2 Square Configuration (18.7 cm): 1.00 cm from Centerline ....... 138
Figure 3.46: Mach 2 Square Configuration (18.7 cm): 1.25 cm from Centerline ....... 139
Figure 3.47: Mach 2 Square Configuration (18.7 cm): 1.50 cm from Centerline ....... 139
Figure 3.48: Mach 2 Square Configuration (18.7 cm): 1.75 cm from Centerline ....... 139
Figure 3.49: Mach 2 Square Configuration (18.7 cm): 2.00 cm from Centerline ....... 140
Figure 3.50: Mach 2 Square Configuration (18.7 cm): 2.25 cm from Centerline ....... 140
Figure 3.51: Mach 2 Square Configuration (18.7 cm): 2.50 cm from Centerline .......... 140
Figure 3.52: Mach 2 Square Configuration (18.7 cm): 2.75 cm from Centerline .......... 141
Figure 3.53: Mach 2 Square Configuration (18.7 cm): 3.00 cm from Centerline .......... 141
Figure 3.54: Mach 2 Square Configuration (18.7 cm): 3.25 cm from Centerline .......... 141
Figure 3.55: Mach 2.5 Square Configuration (18.7 cm): 0.00 cm from Centerline ...... 142
Figure 3.56: Mach 2.5 Square Configuration (18.7 cm): 0.25 cm from Centerline ...... 142
Figure 3.57: Mach 2.5 Square Configuration (18.7 cm): 0.50 cm from Centerline ...... 142
Figure 3.58: Mach 2.5 Square Configuration (18.7 cm): 0.75 cm from Centerline ...... 143
Figure 3.59: Mach 2.5 Square Configuration (18.7 cm): 1.00 cm from Centerline ...... 143
Figure 3.60: Mach 2.5 Square Configuration (18.7 cm): 1.25 cm from Centerline ...... 143
Figure 3.61: Mach 2.5 Square Configuration (18.7 cm): 1.50 cm from Centerline ...... 144
Figure 3.62: Mach 2.5 Square Configuration (18.7 cm): 1.75 cm from Centerline ...... 144
Figure 3.63: Mach 2.5 Square Configuration (18.7 cm): 2.00 cm from Centerline ...... 144
Figure 3.64: Mach 2.5 Square Configuration (18.7 cm): 2.25 cm from Centerline ...... 145
Figure 3.65: Mach 2.5 Square Configuration (18.7 cm): 2.50 cm from Centerline ...... 145
Figure 3.66: Mach 2.5 Square Configuration (18.7 cm): 2.75 cm from Centerline ...... 145
Figure 3.67: Mach 2.5 Square Configuration (18.7 cm): 3.00 cm from Centerline ...... 146
Figure 3.68: Mach 2.5 Square Configuration (18.7 cm): 3.25 cm from Centerline ...... 146
Figure 3.69: Mach 2.5 Square Configuration (18.7 cm): 3.50 cm from Centerline ...... 146
Figure 3.70: Mach 2.5 Square Configuration (18.7 cm): 3.75 cm from Centerline ...... 147
Figure 3.71: Mach 3 Square Configuration (18.7 cm): 0.00 cm from Centerline ...... 147
Figure 3.72: Mach 3 Square Configuration (18.7 cm): 0.25 cm from Centerline ...... 147
Figure 3.73: Mach 3 Square Configuration (18.7 cm): 0.50 cm from Centerline ...... 148
Figure 3.74: Mach 3 Square Configuration (18.7 cm): 0.75 cm from Centerline ........ 148
Figure 3.75: Mach 3 Square Configuration (18.7 cm): 1.00 cm from Centerline ........ 148
Figure 3.76: Mach 3 Square Configuration (18.7 cm): 1.25 cm from Centerline ........ 149
Figure 3.77: Mach 3 Square Configuration (18.7 cm): 1.50 cm from Centerline ........ 149
Figure 3.78: Mach 3 Square Configuration (18.7 cm): 1.75 cm from Centerline ........ 149
Figure 3.79: Mach 3 Square Configuration (18.7 cm): 2.00 cm from Centerline ........ 150
Figure 3.80: Mach 3 Square Configuration (18.7 cm): 2.25 cm from Centerline ........ 150
Figure 3.81: Mach 3 Square Configuration (18.7 cm): 2.50 cm from Centerline ........ 150
Figure 3.82: Mach 3 Square Configuration (18.7 cm): 2.75 cm from Centerline ........ 151
Figure 3.83: Mach 3 Square Configuration (18.7 cm): 3.00 cm from Centerline ........ 151
Figure 4.1: Diagram Illustrating Relationship Between Focal Length and Distance .... 152
Figure 4.2: CAD Drawing of Video Mount Box Structure................................. 152
Figure 4.3: Fabrication of Video Mount using T-SLOT Aluminum Extrusions .......... 153
Figure 4.4: Constant Velocity (Triangle Wave) Cam Driven Oscillation Profile ....... 153
Figure 4.5: Cam, Follower, and Stepper Motor Mounted to Flotek 360 .................... 154
Figure 4.6: Top View of Cam Subassembly .................................................. 154
Figure 4.7: Different Marker Trials and Tracker Results .................................... 155
Figure 4.8: Two Camera Views in Validation Experiment ................................. 156
Figure 4.9: Diagram Illustrating Pitch Angle Calculation ................................... 156
Figure 4.10: Overlay of Cam Profile and Video Analysis Results From Tracker ....... 157
Figure 4.11: Target Image Example in MATLAB Tracking Function ......................... 157
Figure 4.12: Verification of Tracking Marker Center Location ............................ 157
Figure 4.13: Verification of Tracking Marker Center Location (Magnified) ............... 158
Figure 4.14: Stereo Camera Calibration App Corner Detection Results .................... 158
Figure 4.15: Stereo Camera Calibration Results for Test-Set Up........................... 159
Figure 4.16: Calculation of Pitch Angle using Coordinates from Front Camera .... 159
Figure 4.17: Calculation of Pitch Angle using Coordinates from Left Camera ....... 160
Figure 4.18: Sinusoidal Cam Driven Oscillations with 7.5 degree Amplitude......... 160
Figure 4.19: Distance from Rotation Point to Marker Location (Front, Pitch Model).. 161
Figure 4.20: Distance from Rotation Point to Marker Location (Left, Pitch Model).... 161
Figure 4.21: X Pixel Results from Front Camera for Pitch Only Test.................... 162
Figure 4.22: Y Pixel Results from Front Camera for Pitch Only Test.................... 162
Figure 4.23: X Pixel Results from Left Camera for Pitch Only Test..................... 163
Figure 4.24: Y Pixel Results From Left Camera for Pitch Only Test..................... 163
Figure 4.25: Pitch and Yaw Results for Pitch Only Test.................................... 164
Figure 4.26: Pitch and Yaw Test Set-up ............................................................. 164
Figure 4.27: Distance from Rotation Point to Marker (Front, Pitch and Yaw) .... 165
Figure 4.28: Distance from Rotation Point to Marker (Left, Pitch and Yaw) ....... 165
Figure 4.29: X Pixel Results for Pitch and Yaw Test (Front Camera) .................... 166
Figure 4.30: Y Pixel Results for Pitch and Yaw Test (Front Camera) .................... 166
Figure 4.31: X Pixel Results for Pitch and Yaw Test (Left Camera) ..................... 167
Figure 4.32: Y Pixel Results for Pitch and Yaw Test (Left Camera) ..................... 167
Figure 4.33: Pitch and Yaw Results for Pitch and Yaw Test............................... 168
Figure 4.34: Single Camera Calibration Corner Detection Results ....................... 168
Figure 4.35: Single Camera Calibration Results.................................................. 169
Figure 4.36: X Pixel Results for Single Camera Test.......................................... 169
Figure 4.37: Y Pixel Results for Single Camera Test.................................................. 170

Figure 4.38: Pitch Results for Single Camera Pitch Only Test................................. 170

Figure 5.1: SolidWorks Drawing of Initial 1DOF Sting concept.................................. 171

Figure 5.2: Force Diagram During Testing (Top View).............................................. 172

Figure 5.3: Image of Ring Pivoted at 30 Degrees...................................................... 172

Figure 5.4: Representative 70-degree MSBS Model.................................................. 173

Figure 5.5: Representative 45-degree MSBS Model.................................................. 173

Figure 5.6: Fabricated and Painted Models ............................................................... 174

Figure 5.7: SolidWorks Assembly of 1DOF Sting....................................................... 174

Figure 5.8: Fabricated 1DOF Support installed in SWT ............................................. 175

Figure 5.9: Motor that Pulls Pin.................................................................................. 175

Figure 5.10: Support Position During Tunnel Start-Up.............................................. 176

Figure 5.11: Retracted Support Position..................................................................... 176

Figure 5.12: CAD Model of Camera Box Support for SWT ....................................... 177

Figure 5.13: Installed Camera Box Mount for 1DOF Testing...................................... 177

Figure 5.14: Results of Camera Calibration for 1DOF Test........................................ 178

Figure 5.15: Extrinsic Calibration Image for 1DOF Test........................................... 178

Figure 5.16: Image of Deflected (Left) and Centered Sting (Right)............................. 178

Figure 5.17: Sting Deflection Force Diagram............................................................ 179

Figure 5.18: 7005 Model with Modified Center of Gravity ..................................... 179

Figure 5.19: Coordinate System of Blunt Body Equations of Motion¹.......................... 179

Figure 5.20: SHO Fit and Video Data for 1st Test of 7005 Model ............................... 180

Figure 5.21: SHO Fit and Video Data for 2nd Test of 7005 Model.............................. 180
Figure 5.22: SHO Fit and Video Data for 3\textsuperscript{rd} Test of 7005 Model............................................. 181

Figure 5.23: Results from Run 1 of 4505.5 Model at Mach 2, Dataset 1 ...................... 181

Figure 5.24: Results from Run 1 of 4505.5 Model at Mach 2, Dataset 2 ..................... 182

Figure 5.25: Results from Run 2 of 4505.5 Model at Mach 2, Dataset 1 ..................... 182

Figure 5.26: Results from Run 2 of 4505.5 Model at Mach 2, Dataset 2 ..................... 183

Figure 5.27: Results from Run 3 of 4505.5 Model at Mach 2, Dataset 1 ..................... 183

Figure 5.28: Results from Run 3 of 4505.5 Model at Mach 2, Dataset 2 ..................... 184

Figure 5.29: Results from Run 4 of 4505.5 Model at Mach 2, Dataset 1 ..................... 184

Figure 5.30: Results from Run 4 of 4505.5 Model at Mach 2, Dataset 2 ..................... 185

Figure 5.31: Results from Run 1 of 7005 Model at Mach 2, Dataset 1 ...................... 185

Figure 5.32: Results from Run 1 of 7005 Model at Mach 2, Dataset 2 ...................... 186

Figure 5.33: Results from Run 2 of 7005 Model at Mach 2, Dataset 1 ...................... 186

Figure 5.34: Results from Run 2 of 7005 Model at Mach 2, Dataset 2 ...................... 187

Figure 5.35: Results from Run 3 of 7005 Model at Mach 2, Dataset 1 ...................... 187

Figure 5.36: Results from Run 3 of 7005 Model at Mach 2, Dataset 2 ...................... 188

Figure 5.37: Results from Run 4 of 7005 Model at Mach 2, Dataset 1 ...................... 188

Figure 5.38: Results from Run 4 of 7005 Model at Mach 2, Dataset 2 ...................... 189

Figure 5.39: Results from Run 1 of 4505.5 Model at Mach 2.5, Dataset 1 ................. 189

Figure 5.40: Results from Run 1 of 4505.5 Model at Mach 2.5, Dataset 2 ............... 190

Figure 5.41: Results from Run 2 of 4505.5 Model at Mach 2.5, Dataset 1 ............... 190

Figure 5.42: Results from Run 2 of 4505.5 Model at Mach 2.5, Dataset 2 ............... 191

Figure 5.43: Results from Run 3 of 4505.5 Model at Mach 2.5, Dataset 1 ............... 191

Figure 5.44: Results from Run 3 of 4505.5 Model at Mach 2.5, Dataset 2 ............... 192
Figure 5.45: Results from Run 1 of 7005 Model at Mach 2.5, Dataset 1 ...................... 192
Figure 5.46: Results from Run 1 of 7005 Model at Mach 2.5, Dataset 2 ...................... 193
Figure 5.47: Results from Run 2 of 7005 Model at Mach 2.5, Dataset 1 ...................... 193
Figure 5.48: Results from Run 2 of 7005 Model at Mach 2.5, Dataset 2 ...................... 194
Figure 5.49: Results from Run 3 of 7005 Model at Mach 2.5, Dataset 1 ...................... 194
Figure 5.50: Results from Run 3 of 7005 Model at Mach 2.5, Dataset 2 ...................... 195
Figure 5.51: Results from Run 1 of 4505.5 Model at Mach 3, Dataset 1 ...................... 195
Figure 5.52: Results from Run 1 of 4505.5 Model at Mach 3, Dataset 2 ...................... 196
Figure 5.53: Results from Run 2 of 4505.5 Model at Mach 3, Dataset 1 ...................... 196
Figure 5.54: Results from Run 2 of 4505.5 Model at Mach 3, Dataset 2 ...................... 197
Figure 5.55: Results from Run 3 of 4505.5 Model at Mach 3, Dataset 1 ...................... 197
Figure 5.56: Results from Run 3 of 4505.5 Model at Mach 3, Dataset 2 ...................... 198
Figure 5.57: Results from Run 1 of 7005 Model at Mach 3, Dataset 1 ...................... 198
Figure 5.58: Results from Run 1 of 7005 Model at Mach 3, Dataset 2 ...................... 199
Figure 5.59: Results from Run 2 of 7005 Model at Mach 3, Dataset 1 ...................... 199
Figure 5.60: Results from Run 2 of 7005 Model at Mach 3, Dataset 2 ...................... 200
Figure 5.61: Results from Run 3 of 7005 Model at Mach 3, Dataset 1 ...................... 200
Figure 5.62: Results from Run 3 of 7005 Model at Mach 3, Dataset 2 ...................... 201
Figure 5.63: Results from Run 1 of 4505.5 Model at Mach 2.5, Dataset 1, Reverse ... 201
Figure 5.64: Results from Run 1 of 4505.5 Model at Mach 2.5, Dataset 2, Reverse ... 202
Figure 5.65: Results from Run 2 of 4505.5 Model at Mach 2.5, Dataset 1, Reverse ... 202
Figure 5.66: Results from Run 2 of 4505.5 Model at Mach 2.5, Dataset 2, Reverse .... 203
Figure 5.67: Results from Run 3 of 4505.5 Model at Mach 2.5, Dataset 1, Reverse .... 203
Figure 5.68: Results from Run 3 of 4505.5 Model at Mach 2.5, Dataset 2, Reverse .... 204

Figure 5.69: Frequency for 1DOF Test and SHO Simulation for 7005......................... 204

Figure 5.70: Frequency for 1DOF Test and SHO Simulation for 4505.5...................... 205

Figure 5.71: Moment Slope Coefficient: 1DOF Test and Empirical Data for 7005..... 205

Figure 5.72: Moment Slope Coefficient: 1DOF Test and Empirical Data for 4505.5... 206

Figure 5.73: Frequency for 7005 with Outliers Removed ...................................... 206

Figure 5.74: Frequency for 4505.5 with Outliers Removed ...................................... 207

Figure 5.75: Moment Slope Coefficient: 7005 with Outliers Removed...................... 207

Figure 5.76: Moment Slope Coefficient: 4505.5 with Outliers Removed.................... 208
ACKNOWLEDGEMENTS

I would like to thank my advisor Dr. Paul Barnhart, who initially arranged this project and provided invaluable insight on many aspects of this program. Dr. Barnhart’s role as an advisor, professor and mentor has made me a better student and engineer. I’m endlessly grateful for the time and devotion he gives to his students. I would like to thank Dr. David O. Davis for his advisement and support during the course of the design process and wind tunnel testing. The patience and expertise he brought to this research was unending and I’m so thankful for the opportunity I had to work with him. I would like to thank Mark Schoenenberger for advising the direction of this study and providing expertise particularly related to blunt body aerodynamics. He provided me with the challenging and fascinating project that I’ve been working on for nearly three years and has taught me so much in that time, about not only the subject area, but how to be a better researcher. I am thankful to have had these three as mentors and learn from them for as long as I did. I would like to thank Brent Seifert, Katelyn McCormick and Joe Puskas for their help in facilitating wind tunnel runs. I would like to thank my committee members Dr. Joseph Prahl and Dr. James T’ien. Their technical insight as professors and members of my committee has taught me so much in my time at Case. I would like to thank the Engineering Services Fabrication Center for their design insight into the final section of this project. Finally, I would like to thank both the Department of Mechanical and Aerospace Engineering at Case Western Reserve University and the NASA Science and Technology Research Fellowship for supporting this work. Without this generosity, much of the work would not have been possible.
**LIST OF SYMBOLS:**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Acceleration</td>
</tr>
<tr>
<td>$A$</td>
<td>Amplitude</td>
</tr>
<tr>
<td>$A_{BL}$</td>
<td>Blockage area due to presence of boundary layer</td>
</tr>
<tr>
<td>$A_{model}$</td>
<td>Maximum model cross-sectional area</td>
</tr>
<tr>
<td>$A_{pin}$</td>
<td>Cross-sectional area of pin</td>
</tr>
<tr>
<td>$A_{sting}$</td>
<td>Cross-sectional area of sting</td>
</tr>
<tr>
<td>$A_{test}$</td>
<td>Test section cross-sectional area</td>
</tr>
<tr>
<td>$c$</td>
<td>Distance to neutral axis in bending</td>
</tr>
<tr>
<td>$C$</td>
<td>Constant in least square curves fitting technique</td>
</tr>
<tr>
<td>$C_{BL}$</td>
<td>Corner boundary-layer growth correction constant</td>
</tr>
<tr>
<td>$C_A$</td>
<td>Axial force coefficient</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Drag coefficient</td>
</tr>
<tr>
<td>$C_L$</td>
<td>Lift coefficient</td>
</tr>
<tr>
<td>$C_{L\alpha}$</td>
<td>Lift force slope coefficient</td>
</tr>
<tr>
<td>$C_{m\alpha}$</td>
<td>Pitching moment slope coefficient</td>
</tr>
<tr>
<td>$C_{m\alpha}$</td>
<td>Pitching moment per pitch rate coefficient</td>
</tr>
<tr>
<td>$C_{mq}$</td>
<td>Pitch moment coefficient</td>
</tr>
<tr>
<td>$(C_{mq} + C_{m\alpha})$</td>
<td>Pitch damping coefficient</td>
</tr>
<tr>
<td>$C_N$</td>
<td>Normal force coefficient</td>
</tr>
<tr>
<td>$C_{N\alpha}$</td>
<td>Normal force slope coefficient</td>
</tr>
<tr>
<td>$d$</td>
<td>Diameter of test model</td>
</tr>
</tbody>
</table>
\( d_m \) Distance between marker and point of rotation
\( d_x \) Distance between in x-coordinates
\( d_y \) Distance between in y-coordinates
\( d_z \) Distance between in z-coordinates
\( D_{o, \text{mm}} \) Distance required between camera and object
\( E \) Modulus of Elasticity
\( f \) Frequency of oscillation
\( F \) Force
\( F(a) \) Fourier Transform of matrix “\( a \)”
\( F^{-1}(a) \) Inverse Fourier Transform of matrix “\( a \)”
\( f_b \) Bearing friction coefficient
\( F_{\text{drag}} \) Drag force
\( F_L \) Lift force
\( F_{\text{magnetic}} \) Magnetic force due to magnetic suspension and balance system
\( F_N \) Normal force
\( f_o \) Focal length of camera
\( g \) Gravitational acceleration constant
\( H \) Magnetic field flux density
\( H_{o, \text{mm}} \) Physical height of the object captured by camera
\( H_{o, \text{pix}} \) Height of the object in pixels captured by camera
\( h_{\text{side}} \) Height of rectangular prism cross-section
\( H_{s, \text{mm}} \) Camera sensor physical height
\( H_{s, \text{pix}} \) Camera sensor height in pixels
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>Moment of Inertia of Model</td>
</tr>
<tr>
<td>$i_1$</td>
<td>First point of intersection</td>
</tr>
<tr>
<td>$i_2$</td>
<td>Second point of intersection</td>
</tr>
<tr>
<td>$k$</td>
<td>Stress factor due to aspect ratio of beam</td>
</tr>
<tr>
<td>$k_{spring}$</td>
<td>Spring force constant</td>
</tr>
<tr>
<td>$L_{test}$</td>
<td>Length of side of square test section</td>
</tr>
<tr>
<td>$L_{sting}$</td>
<td>Length of sting</td>
</tr>
<tr>
<td>$\bar{m}$</td>
<td>Average magnetization of model</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach number</td>
</tr>
<tr>
<td>$M_b$</td>
<td>Bending moment</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure or Force</td>
</tr>
<tr>
<td>$q$</td>
<td>Dynamic pressure</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius of axisymmetric test section</td>
</tr>
<tr>
<td>$R_{Earth}$</td>
<td>Radius of Earth surface</td>
</tr>
<tr>
<td>$r_{pin}$</td>
<td>Radius of pin in bearing</td>
</tr>
<tr>
<td>$Re_D$</td>
<td>Reynolds number based on hydraulic diameter of test section</td>
</tr>
<tr>
<td>$r_\alpha$</td>
<td>Distance between marker and point of rotation in front camera plane</td>
</tr>
<tr>
<td>$r_\beta$</td>
<td>Distance between marker and point of rotation in left camera plane</td>
</tr>
<tr>
<td>$S$</td>
<td>Maximum cross-sectional area of model</td>
</tr>
<tr>
<td>$S_{sy}$</td>
<td>Yield shear strength of material</td>
</tr>
<tr>
<td>$S_u$</td>
<td>Tensile strength of material</td>
</tr>
<tr>
<td>$S_y$</td>
<td>Yield strength of a material</td>
</tr>
</tbody>
</table>
$t$  
Time

$T$  
Torsion

$T_f$  
Frictional torque

$V_{\text{core}}$  
Iron core volume

$V$  
Velocity

$\dot{V}$  
Change in velocity

$W$  
Weight

$w_{\text{side}}$  
Width of rectangular prism cross-section

$x$  
Horizontal coordinate

$X_{\text{cg}}$  
Non-dimensional distance from model nose to center of gravity location

$y$  
Vertical coordinate

$z$  
Span coordinate

**Greek Symbols**

$\alpha$  
Pitch angle

$\dot{\alpha}$  
Pitch angle rate

$\ddot{\alpha}$  
Pitch angle acceleration

$\Delta\alpha$  
Change in pitch angle

$\beta$  
Sideslip angle

$\gamma$  
Flight path angle

$\dot{\gamma}$  
Change in flight path angle

$\delta$  
Boundary-layer thickness

$\delta^*$  
Boundary-layer displacement thickness

$\delta_p$  
Phase shift constant
\( \delta_{\text{sting}} \) Sting deflection

\( \varepsilon \) Damping coefficient

\( \theta \) Blunt body angle from local horizontal

\( \dot{\theta} \) Angular rate of blunt body angle

\( \ddot{\theta} \) Angular acceleration of blunt body angle

\( \mu \) Friction coefficient

\( \rho \) Freestream density in test section

\( \sigma_{\text{axial}} \) Axial stress

\( \sigma_{\text{bending}} \) Bending stress

\( \tau \) Shear stress

\( \omega \) Angular frequency

**Subscripts**

BL Boundary-layer

cam Cam produced oscillations

d Diameter

f Friction

i Initial

i,marker Initial marker position

micro Microprobe Entry Vehicle

o Original

pin Pin

sting Sting

test Test Section
<table>
<thead>
<tr>
<th>vid</th>
<th>Video analysis method results</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>Plenum or Total conditions</td>
</tr>
<tr>
<td>1DOF</td>
<td>1 Degree-of-Freedom</td>
</tr>
</tbody>
</table>
Feasibility Study for Testing the Dynamic Stability of Blunt Bodies with a Magnetic Suspension System in a Supersonic Wind Tunnel

ABSTRACT

By

ABIGAIL E. SEVIER

The feasibility of a magnetic suspension and balance system (MSBS) for testing dynamic stability of atmospheric entry capsules in the NASA GRC 225 cm² Supersonic Wind Tunnel is investigated. The following are examined in the study: largest model size for testing in the MSBS, minimum proximity between wall and model, and analysis techniques using high-speed video images of model movement. Results indicate larger models can be tested in an axisymmetric test section and at locations closer to the nozzle exit resulting from lower boundary-layer blockage. Additionally, models contact the reflected shock from the boundary-layer at a 2.5 cm distance from centerline. Video analysis methods establish a measurement error of 0.6 degrees in pitch or yaw angle. Using these methods, a proof of concept demonstration for a one degree-of-freedom test in pitch simulating blunt body dynamic behavior is compared to ballistic range data for atmospheric entry vehicles.
CHAPTER I

INTRODUCTION

The dynamic stability of capsules during planetary entry is difficult to quantify as computational methods have yet to demonstrate accurate predictive capabilities and experimental methods cannot explicitly measure damping derivatives. Ballistic range testing has been used in the past to determine dynamic behavior of blunt body entry vehicles by firing test models from a gun down an instrumented range at low supersonic Mach numbers. Orthogonal shadowgraphs of the test model are taken during flight to measure the capsule's position and orientation. A 6 degree-of-freedom (6DOF) simulation is then fit using a maximum likelihood method to the data points captured by the shadowgraphs and the aerodynamic coefficients are identified using a parameter identification process. Difficulties in achieving repeatable initial conditions as well as rapid changes in freestream conditions over the span of each shot presents data reduction challenges for the ballistic range test method. This is made even more difficult by the fact that no instrumentation that measures the dynamic moments of the capsule can be used during ballistic range testing which makes it impossible to measure or determine damping coefficients explicitly. In addition, the primary test range used to capture data for the Mars Science Laboratory (MSL) entry vehicle, the Aerodynamic Test and Evaluation Facility (ATEF) at the Eglin Air Force Base which is shown in Figure 1.1, was recently decommissioned. Although other ranges such as the ballistic range at Aberdeen in Maryland still exist, the inherently expensive nature of ballistic range tests and the data reduction challenges motivate investigations into creating alternative test facilities to determine capsule dynamic behavior.
One proposed alternative method to determine dynamic stability of blunt body entry vehicles is the implementation of a magnetic suspension and balance system (MSBS) in a Supersonic Wind Tunnel (SWT.) The NASA Glenn Research Center (GRC) 225 cm² SWT is the facility of choice. The tunnel has both a square test section (with sides of 15.0 cm) and an axisymmetric test section (with a diameter of 17.0 cm) which is why the tunnel is named by the test section cross-sectional area. In the proposed experimental set-up, a magnetic suspension system will react against aerodynamic and gravitational forces to hold the model in a translationally fixed position in the test section, but allow the model to rotate about its center-of-mass freely so that the dynamic behavior can be observed and recorded. High-speed cameras will capture the model’s position and attitude over time and a trajectory will be fit using a maximum likelihood method to these data points in a similar data reduction method to that used for ballistic range testing. The nearly constant freestream conditions and the elimination of significant lateral translation will improve the fidelity to which dynamic stability derivatives can be determined. This method has the potential to measure damping coefficients more accurately than both traditional ballistic range testing and wind tunnel testing using stings to support a model.

This test concept is being pursued in a parallel effort at the NASA Langley Research Center (LaRC) to update an existing MSBS originally developed by the Massachusetts Institute of Technology (MIT) which is shown in Figure 1.2. The MIT MSBS was intended for use in a 6-inch low-speed wind tunnel with an octagonal cross-section. The original MSBS was designed in the late 1960s and used 6 DOF magnetic control to suspend slender cone test models.² These models were tracked using an Electronic Position Sensor (EPS) system for position feedback which also recorded the
flight dynamics of the model. The MIT MSBS is being updated for this project so that it can suspend blunt body models which are a much more difficult to track optically due to their low length to diameter ratio. The concurrent project at LaRC is being approached as an initial test that can provide beneficial design and implementation guidance to the development of the scaled MSBS for the GRC 225 cm² SWT. The MSBS for the supersonic tunnel will be designed at LaRC while the test section and wind tunnel components will be designed at GRC.

This research is a feasibility study of work conducted at NASA GRC to assess the initial challenges facing the implementation of a MSBS. The first area that is investigated is the optimum model size for testing using a MSBS in a SWT. A series of blockage tests are conducted over a test matrix of different test sections and model geometries at speeds spanning Mach 2 to Mach 3. The largest model that starts, maintains supersonic flow, in blockage tests is then used for a series of wall proximity tests in the SWT. This test characterizes the change in wind tunnel conditions and shock interactions with the walls as the model moves off centerline approaching the walls.

As envisioned, the model movement will be captured by high speed cameras in the SWT in the future MSBS system. The data reduction of video from the MSBS test will be similar to that done in ballistic range testing. A video analysis method, similar to that planned for use in the operational test set-up with the MSBS, is developed as part of this research. The video analysis programs developed include a tracking code, a method of camera calibration, and extraction of model position and attitude. These methods of video analysis are used in the final 1DOF testing in the SWT. Chapter V provides results
of these tests and documents the design process used for the wind tunnel sting, permitting
the model to pitch about a single axis.

This preliminary investigation identifies potential problems that may be
encountered in the design and operation of the MSBS. The video analysis methods
developed enable a more informed design thereby leading to a successful concept
implementation. Conclusions drawn from this research are representative of the various
design and analysis concerns that were encountered. The tests performed in this research
will be beneficial in subsequent efforts as the MSBS system and associated test facility
are developed in the future.
CHAPTER II

BLOCKAGE TESTING

2.1 INTRODUCTION

The blunt body test models tested in the MSBS will be comprised of a non-magnetic material surrounding a spherical iron core. A sample 70-degree test model is shown in Figure 2.1. In the figure, 70-degrees refers to the semi-vertex forebody angle of the test capsule. The nose radius of the model is a fraction of the model maximum diameter. For models tested, the nose radius fraction is 0.25. The backshell angle is a constant 43.4 degrees for all models tested. The geometries of the test models are chosen to be similar to the Mars Exploration Rover (MER) capsule. The semi-vertex angle and maximum model diameter are used to distinguish various design configurations. The symmetry of the spherical iron core minimizes magnetic moments on the model while suspended in the test section by the MSBS. This configuration limits the amount of iron material that can fit within the model. The magnetic suspension system must be powerful enough to maintain the position of smaller soft iron cores. There is a tradeoff between core size and magnetic field strength required. The maximum size of the model and the corresponding iron core determines field strength requirements necessary. Therefore, it is critical to determine the maximum model sizes and associated wind tunnel conditions over a range of Mach numbers so that the MSBS field strength is minimized. Wind tunnel blockage effects limit model size as a result of starting dynamics.

The design of the MSBS must operate at desired freestream dynamic pressures while still maintaining control of the model translational position, while allowing rotation in three directions. In order to determine the size of the design space for a given model
geometry, the forces on the model due to drag and the magnetic suspension are considered. Equation 2.1 shows the aerodynamic drag force on the model which scales with the reference area of the model, typically the maximum projected area. Equation 2.2 gives the magnetic force in the upstream axial direction which scales with respect to the magnetic core volume or the radius cubed. A smaller model is preferable for wind tunnel testing because blockage is minimized, but a larger model is desirable for the MSBS because it reduces the required magnetic field. Since the magnetic force has a greater reliance on model size, it is desirable to test the largest model possible. However, the model size must maintain stable flow conditions in the wind tunnel and allow for wind tunnel start.

\[ F_{\text{drag}} = qA_{\text{model}}C_D \]  
\[ F_{\text{magnetic}} = V_{\text{core}}(\vec{m} \times \nabla)H \]

Additional objectives investigated during the course of testing include: determining the effect of the boundary-layer and the blockage it induces; the difference in model blockage at different test section locations; and the difference in model blockage between square and axisymmetric test sections. The comparison between square and axisymmetric test sections can only be done at Mach 2.5.

2.2 METHODOLOGY

2.2.1 Facility

Models of various geometries and sizes are tested in the GRC 225 cm$^2$ SWT to determine the maximum size for a given model geometry and Mach number. Test conditions of interest for this study are Mach 2, 2.5, and 3 because dynamic instability is
typically observed at low supersonic speeds for these types of blunt bodies. The 225 cm$^2$ SWT is a continuous flow facility with Mach number set by interchangeable fixed-geometry convergent-divergent (C-D) nozzle blocks. The facility can be configured with either a 15.0 by 15.0 cm square test section or an axisymmetric test section with a 17.0 cm diameter. The resulting cross-sectional areas are 225 cm$^2$ and 227 cm$^2$ for the square and axisymmetric test section, respectively. The axisymmetric test-section is slightly larger in size, chosen for ease of fabrication that results in a cross-sectional area very nearly 225 cm$^2$. The square test section has windows for schlieren flow visualization capability while the axisymmetric test section does not. Refer to chapter 6.10 of Liepmann and Roshko for a description of a schlieren flow visualization system.$^3$ The tunnel supersonic Mach number capability is 1.4, 1.7, 2.0, 2.5, and 3.0 for the square test section, and 2.5 for the axisymmetric test section. The total temperature is nominally ambient temperature and not controlled. The wind tunnel is supplied with lab-wide 377.1 kPa combustion air upstream and downstream to lab-wide altitude exhaust maintained below 13.8 kPa. All pressures specified are absolute pressures. A drawing of the 225 cm$^2$ SWT configurations is shown in Figure 2.2. Comparisons are drawn in particular between the blockage results between the axisymmetric and square test section configurations at Mach 2.5 to see if the difference in test section shape creates a tangible disparity between the two sets of data. A cross-sectional view of a model installed in the test section is shown in Figure 2.3.

2.2.2 Instrumentation

Tunnel conditions are the main source of data collected during this test series. The tunnel plenum pressure, stagnation conditions, is measured with a quartz pressure
transducer which needs infrequent calibrations due to its extreme precision. The total temperature in the plenum is calculated from the average of four type-E thermocouples. The wall static pressure in the test section is measured by a series of Electronically Scanned Pressure (ESP) modules that record pressures in the test section. The ESP system is calibrated every hour using a quartz downhole pressure transducer to ensure accuracy. Wall pressure taps are located at half-inch increments along the round test section starting from the nozzle exit, at the entrance of the test section. Eighth-inch increments along the square test section window, beginning at 11.1 cm from the nozzle exit. The test section wall pressure distributions are used to indicate whether the test section with the model installed has started. A started condition is indicated if the wind tunnel maintains supersonic flow throughout the test section.

2.2.3 Models

A matrix of test models was three-dimensionally (3D) printed with varying cross sectional areas and geometries. Three different forebody geometries were chosen due to their resemblance to actual geometries flown on space missions. The chosen entry vehicles have nose cone semi-vertex angles of 45, 60, and 70-degrees which correspond to the angles often used for Venus, Earth, and Mars entries. The dimensions of the capsule are very similar to the MER capsule with the only difference being the change in semi-vertex angles. All model lengths scale with the model diameter. The different angles 45, 60, and 70-degrees can be seen in Figure 2.4.

When referring to each individual model a numbering system is used. The first two digits correspond to the forebody semi-vertex angle while the last two digits express the model maximum area as a percent of the test section cross-sectional area. For
example, the 6007.5 model is 7.5 percent of the tunnel test section cross-sectional area with a 60-degree forebody semi-vertex angle. This notation is used throughout as a shorthand way to describe the test models. It is anticipated that blunter models will have greater difficulty starting, so 70-degree models of a given area are tested first, followed by 60 and then 45-degree models when possible.

A literature study is conducted to identify data or theory to predict the maximum cross-sectional area. Figure 2.5 shows the results of this study and plots the correlation between the maximum blockage and Mach number from both wind tunnel data and empirical sources for the range of Mach 2 to 4. The solid lines with square and circular symbols represent data taken from trend lines in the NASA GRC 1 by 1 Foot SWT user manual for both a 35 and 60-degree forebody (page 42). The dotted line with triangle data points plots data from the Aeronautical Systems Division (ASD) High Temperature Hypersonic Gasdynamics Facility. This set of data shows several blockage curves depending on the models drag coefficient over a large Mach number range (page 12). The drag coefficient of 1.6 was chosen because it most closely represents the drag of a 70-degree model experienced at low supersonic speeds which has a value of 1.58. The line with dashes and dots denotes a theoretical blockage prediction from the High-Speed Wind Tunnel Testing Handbook based on Mach number which is shown in Equation 2.3.

$$\frac{A_{\text{model}}}{A_{\text{test}}} = 1 - \sqrt{5 + M^2 \left(\frac{7M^2 - 1}{216M^6}\right)^{2.5}}$$

The dashed lines with square and circle data points show data from the LaRC 4-foot Unitary Plan SWT. This experimental data was taken from a plot that showed a considerable range for model blockage so the lower and upper bound of this data was
plotted in Figure 2.5. The singular green diamond data point presents an extremely accurate blockage value for an isentropic compression cone at Mach 2 in the GRC 225 cm$^2$ SWT in the square test section. Since these data were taken in the same tunnel, it is considered the most realistic value upon which to base sizing estimations.

The literature study is analyzed with the understanding that data are taken from wind tunnels with different total pressures and flowrate capabilities as well as varying boundary-layer characteristics. Since the singular data point from the GRC 225 cm$^2$ SWT appeared to most closely align with the lower half of the data shown in Figure 2.5, the selection of testing areas for this study is chosen to vary from 4 to 14 percent by 0.5 percentage increments.

2.2.4 Test Procedure:

Each model is installed in the center of the test section for the test runs and is supported in the test section by a half-inch diameter steel rod. Each model has a short length of quarter-inch threaded rod epoxied into a hole in the back shell of the model, as shown in Figure 2.6, which screws into the support rod.

For each model tested, the wind tunnel total pressure is increased gradually until the test section normal shock wave passes the model or until the maximum mass-flow limit or total pressure limit is reached. Once a started condition occurs, the tunnel total pressure is then slowly decreased to assess if the tunnel can remain started at a lower tunnel total pressure. The tunnel total pressure at which the shock wave passes back upstream in front of the model which then causes tunnel unstart is also recorded.

Axial wall pressure distributions are examined for both the axisymmetric and square test section to determine if the model allows for a started wind tunnel. Figure 2.7
shows an example of pressure distributions under unstarted and started conditions for model 6007.5 at Mach 3.0 in the square test section. Figure 2.8 shows the corresponding schlieren images for the same capsule model shown in Figure 2.7. In Figure 2.8a, the shock is seen very clearly since the flow is still attached to the models surface. In addition, the boundary layer on the tunnel walls is very enlarged. In Figure 2.8b, the tunnel has started and the bow shock sits ahead of the model. The tunnel total pressure, displayed as “PT0” (in psia) is shown in the screen legend. Note that for the unstarted case, Figure 2.8a, the normal shock influences the C-D nozzle exit static pressure taps leading to an indicated Mach number, shown as “MACH”, less than the design Mach number.

2.3 RESULTS

Figures 2.9, 2.12, 2.13, and 2.14 show the results of blockage tests conducted during the study. Data points are shown as the Reynolds number based on test section hydraulic diameter ($Re_D$) when the model has started. Reynolds number is chosen to characterize the starting condition rather than total pressure because the ambient temperature of the SWT varies over the test data, impacting the boundary-layer growth, and as a result the model blockage. The total temperature is observed to vary 10 K. This in turn affects the kinematic viscosity and Reynolds number of the airflow. For each plot, a dashed horizontal line is drawn near the top indicating the $Re_D$ corresponding to maximum allowable flow rate or tunnel total pressure. Three sets of data on each plot, the square, circle, and triangle represent the 45, 60, and 70-degree models respectively. As expected, models with larger semi-vertex forebody angles prove more difficult to start. This is demonstrated by the fact that the 70-degree models typically have a smaller
allowable blockage or require a higher total pressure for tunnel start than the 60-degree and 45-degree models. This trend also proves true when comparing the 60-degree models to the 45-degree models.

It should be noted that the start condition is not a precise point. This is due in part to the very unsteady nature of the normal-shock/boundary-layer interaction. On occasion while sitting at an unstarted condition for some time, the shock passes without an increase in pressure. A perturbation in the supply air or altitude exhaust due to other facilities coming online or going offline could also affect the tunnel start/unstart point.

2.3.1 Mach 2.5 Nozzle Block with Axisymmetric Test Section

Figure 2.9 shows the blockage and associated $Re_D$ for tunnel start for several models in the Mach 2.5 axisymmetric configuration. Models are tested at two axial locations in this configuration to help determine the significance of boundary-layer blockage variance within the length of the test section.

Figure 2.9a shows models that are tested at 50.8 cm downstream from the nozzle exit which is near the back of the 58 cm long test section. The largest 70-degree model is approximately 6 percent blockage while the largest model size for the 45-degree and 60-degree models are approximately 6.5 percent blockage. Although the largest model size is the same for both the 45-degree and 60-degree model, the 45-degree model starts at a lower $Re_D$, approximately $0.75 \times 10^6$ lower. The blockage values shown on the horizontal axis are slightly less than their square defined value because the axisymmetric test section has a larger cross sectional area versus that of the square test section (227 versus 225 cm$^2$).
Figure 2.9b shows data when the model axial testing location is moved closer to the nozzle. Models tested at 10.2 cm from the nozzle exit either start right away at the lowest possible $Re_D$ or cannot start even at the high limits of tunnel total pressure (the open symbol indicates an unstarted condition at the tunnel flow limit). The largest models that can start are approximately 9 percent for both a 70- and 60-degree model and approximately 10.5 percent for a 45-degree model. The maximum model sizes seen at 10.2 centimeters are considerably larger than those observed in the blockage results taken at 50.8 cm from the nozzle exit.

2.3.1.1 Blockage Data Variation with Boundary Layer in Axisymmetric Test Section

Figure 2.9 illustrates that the allowable model blockage differs considerably between 10.2 and 50.8 centimeters from the nozzle exit. One hypothesis to explain this phenomenon is that there is a substantial difference in boundary-layer blockage which increases downstream from the nozzle. This theory is investigated to determine if the boundary-layer blockage is significant and if so, how much it differs depending on axial location.

In a previous study in the GRC 225 cm$^2$ SWT, the boundary layer and displacement thickness at three axial locations in the test section is measured at a $Re_D$ of 4.0 x $10^6$. The measurements are shown below in Table 2.1. These data are used to characterize an approximate profile for the boundary-layer growth experienced in the tunnel during blockage testing. This profile is determined by applying a quadratic curve through three data points. This allowed the displacement thickness to be estimated at any axial location along the tunnel. The quadratic curve equation is shown in Figure 2.10.
Table 2.1: Boundary-Layer and Displacement Thickness at Different Axial Locations

<table>
<thead>
<tr>
<th>$x$ (cm)</th>
<th>$\delta$ (cm)</th>
<th>$\delta^*$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.81</td>
<td>0.608</td>
<td>0.161</td>
</tr>
<tr>
<td>43.2</td>
<td>1.312</td>
<td>-0.334</td>
</tr>
<tr>
<td>66.0</td>
<td>1.465</td>
<td>0.389</td>
</tr>
</tbody>
</table>

Data used in a study conducted by Davis are taken in the Mach 2.5 axisymmetric configuration at a $Re_D$ of $4 \times 10^6$, which is a different $Re_D$ than used during blockage testing. Unfortunately, no information is available about the change in displacement thickness due to Reynolds number for the GRC 225 cm$^2$ SWT axisymmetric configuration. However, another paper by Davis supplies the displacement thickness at different $Re_D$ for the same SWT, but with the square test section installed. In Davis’ paper, the displacement thickness at an axial location of 36.56 cm is measured for 5 different Mach numbers at the 3 different $Re_D$ of 1.48, 2.84 and $3.69 \times 10^6$. The displacement thicknesses at Mach 2.5 as well as a quadratic curve placed through data points is shown in Figure 2.11a. In Figure 2.11a, the open circular symbol represents the estimated displacement thickness for the axisymmetric test section at a $Re_D$ of $4 \times 10^6$ at 36.56 cm from the nozzle exit. This estimated displacement thickness is calculated using the quadratic curve shown in Figure 2.10. By comparing the displacement thickness trend line of the square test section in Figure 2.11a with the extrapolated data from the axisymmetric test section, it can be seen that the displacement thickness is less for the axisymmetric test section at a $Re_D$ of $4 \times 10^6$.

In order to determine the relationship between $Re_D$ and displacement thickness at 36.56 cm for the axisymmetric test section, the quadratic curve for the square test section is adjusted to pass through the axisymmetric estimation. The result of this adjusted quadratic curve is shown as the dotted line in Figure 2.11b. This quadratic curve for the
axisymmetric configuration is used to predict a displacement thickness depending on the 
$Re_D$ at which each model is tested. The adjusted quadratic curve estimates the 
displacement thickness at an axial location of 36.56 cm rather than the actual testing 
locations of 10.2 cm and 50.8 cm from the nozzle exit. The displacement thickness can be 
approximated at these other testing locations by approximating the same axial boundary-
layer growth as shown in Figure 2.10 and multiplying by a scaling factor appropriate to 
either 10.2 or 50.8 cm from the nozzle exit.

Using this approximation of the displacement thickness, the blockage due to the 
presence of the boundary layer is calculated using Equation 2.4 for the axisymmetric test 
section at both testing locations at 10.2 and 50.8 cm from the nozzle exit. The results of 
this approximation are shown in Table 2.2 for the largest blockage model that is able to 
start for each geometry. The approximations used to determine the values in Table 2.2 are 
made with assumptions based on limited data in the GRC 225 cm$^2$ SWT. These 
assumptions enable conclusions that help inform the feasibility of the MSBS.

$$\frac{A_{BL}}{A_{test}} = \frac{2R\delta^* - \delta^{*2}}{R^2}$$  \hspace{1cm} (2.4)

<table>
<thead>
<tr>
<th>Back</th>
<th>Model</th>
<th>$Re_D$</th>
<th>$\delta^*$ at 36.56 cm</th>
<th>$\delta^*$ at 50.8 cm</th>
<th>Blockage</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>6.0%</td>
<td>2.82</td>
<td>0.32</td>
<td>0.36</td>
<td>8.37%</td>
</tr>
<tr>
<td>60</td>
<td>6.5%</td>
<td>4.57</td>
<td>0.30</td>
<td>0.33</td>
<td>7.72%</td>
</tr>
<tr>
<td>45</td>
<td>6.5%</td>
<td>3.71</td>
<td>0.31</td>
<td>0.34</td>
<td>7.91%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Front</th>
<th>Model</th>
<th>$Re_D$</th>
<th>$\delta^*$ at 36.56 cm</th>
<th>$\delta^*$ at 10.2 cm</th>
<th>Blockage</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>9.0%</td>
<td>0.68</td>
<td>0.38</td>
<td>0.22</td>
<td>5.09%</td>
</tr>
<tr>
<td>60</td>
<td>9.0%</td>
<td>0.68</td>
<td>0.38</td>
<td>0.22</td>
<td>5.09%</td>
</tr>
<tr>
<td>45</td>
<td>10.5%</td>
<td>0.68</td>
<td>0.38</td>
<td>0.22</td>
<td>5.09%</td>
</tr>
</tbody>
</table>

**Table 2.2**: Boundary-Layer Blockage for Axisymmetric Configuration
Now that the corresponding boundary-layer blockage is estimated for each model, the total blockage due to both the boundary layer and the model can be analyzed. Table 2.3 shows the total blockage for the largest model of each geometry tested at 10.2 cm and 50.8 cm.

Looking at the results shown in Table 2.3, the agreement between the total blockage at the front and back of the test section seems is with 0.2 percent for the 60- and 70-degree models. This approximation strongly suggests that the presence of the boundary layer is non-negligible. Since larger models are desired for testing with the MSBS, the models should likely be tested as close to the nozzle as possible to minimize blockage due to the boundary layer. The test section should be short in length, since the model should be tested close to the nozzle exit, to minimize the effect of boundary-layer thickness growth.

<table>
<thead>
<tr>
<th>Location</th>
<th>Cone</th>
<th>Model</th>
<th>BL Blockage</th>
<th>Total Blockage</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.2 cm</td>
<td>70</td>
<td>9.0%</td>
<td>5.09%</td>
<td>14.01%</td>
</tr>
<tr>
<td>50.8 cm</td>
<td>70</td>
<td>6.0%</td>
<td>8.27%</td>
<td>14.22%</td>
</tr>
<tr>
<td>10.2 cm</td>
<td>60</td>
<td>9.0%</td>
<td>5.09%</td>
<td>14.01%</td>
</tr>
<tr>
<td>50.8 cm</td>
<td>60</td>
<td>6.5%</td>
<td>7.72%</td>
<td>14.16%</td>
</tr>
<tr>
<td>10.2 cm</td>
<td>45</td>
<td>10.5%</td>
<td>5.09%</td>
<td>15.50%</td>
</tr>
<tr>
<td>50.8 cm</td>
<td>45</td>
<td>6.5%</td>
<td>7.91%</td>
<td>14.36%</td>
</tr>
</tbody>
</table>

**Table 2.3:** Comparing Total Blockage for the Mach 2.5 Axisymmetric Configuration

2.3.2 Mach 2.5 Nozzle Block with Square Test Section at 18.7 cm

The next set of blockage data is a significant configuration switch from the axisymmetric test section to the square test section. The axisymmetric test section has pressure taps spanning the entire test section, but the square test section only has pressure taps along the window of the test section. Because of this, the most forward location the
model could be tested and still be at the axial location of a pressure tap is 18.7 cm from the nozzle.

The blockage testing results for the Mach 2.5 square test section are very similar to that of the Mach 2.5 axisymmetric configuration for the 60- and 70-degree models. Figure 2.12 shows the largest model size for both square and axisymmetric is 6 percent. The 70-degree model requires a higher Reynolds number than the 60-degree model to start. A larger 45-degree model is able start in the square test section. For models larger than 6 percent, the $Re_D$ between each subsequent model size is relatively close (within 2.5 and $4 \times 10^6$) resulting in the gradual increase of the square symbols as seen in Figure 2.12.

2.3.2.1 Blockage Data Variation from Mach 2.5 Axisymmetric to Square Configuration

The benefits of having an axisymmetric versus square test section can be evaluated by comparing blockage data from the square and axisymmetric test section at Mach 2.5 to help determine the best test section design for the MSBS.

In order to evaluate the total blockage for the Mach 2.5 Square configuration, the boundary-layer blockage is approximated. The displacement thickness and $Re_D$ correlation shown in Figure 2.11 provides the displacement thickness at 36.56 cm based on the $Re_D$ at model start. However, models are tested in the square test section at 14.7 cm. By making a similar assumption as with the axisymmetric testing locations, the displacement thicknesses are scaled by assuming the same axial growth profile shown in Figure 2.10. The boundary-layer blockage is calculated using the following formula shown in Equation 2.5.
\[ \frac{A_{\text{BL}}}{A_{\text{test}}} = C_{\text{BL}} \frac{L_{\text{test}}^2}{L_{\text{test}}^2 - (L_{\text{test}} - 2 \delta^*)^2} \]  

\( (2.5) \)

<table>
<thead>
<tr>
<th>Location</th>
<th>Cone</th>
<th>Model</th>
<th>BL Blockage</th>
<th>Total Blockage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axi-10.2 cm</td>
<td>70</td>
<td>9.0%</td>
<td>5.09%</td>
<td>14.01%</td>
</tr>
<tr>
<td>Axi-50.8 cm</td>
<td>70</td>
<td>6.0%</td>
<td>8.27%</td>
<td>14.22%</td>
</tr>
<tr>
<td>Square-18.7 cm</td>
<td>70</td>
<td>6.0%</td>
<td>7.87%</td>
<td>13.87%</td>
</tr>
<tr>
<td>Axi-10.2 cm</td>
<td>60</td>
<td>9.0%</td>
<td>5.09%</td>
<td>14.01%</td>
</tr>
<tr>
<td>Axi-50.8 cm</td>
<td>60</td>
<td>6.5%</td>
<td>7.72%</td>
<td>14.16%</td>
</tr>
<tr>
<td>Square-18.7 cm</td>
<td>60</td>
<td>6.0%</td>
<td>8.09%</td>
<td>14.09%</td>
</tr>
<tr>
<td>Axi-10.2 cm</td>
<td>45</td>
<td>10.5%</td>
<td>5.09%</td>
<td>15.50%</td>
</tr>
<tr>
<td>Axi-50.8 cm</td>
<td>45</td>
<td>6.5%</td>
<td>7.91%</td>
<td>14.36%</td>
</tr>
<tr>
<td>Square</td>
<td>45</td>
<td>10.0%</td>
<td>7.88%</td>
<td>17.88%</td>
</tr>
</tbody>
</table>

**Table 2.4:** Blockage Comparison between Square and Axisymmetric

The growth of the boundary-layer is not known at the corners of the square test section, so a corner growth correction constant is included in Equation 2.5. This parameter is adjusted until the total blockage matches that of the 60- and 70-degree models in the axisymmetric test section. This correction \( C_{\text{BL}} \) was found to be 1.087. This adjustment agrees with the assumption that the total blockage for both test sections should be similar, thereby matching at Mach 2.5.

Table 2.4 compares the total blockage for the largest models of each geometry tested in the square and axisymmetric test section. The first three rows of Table 2.4 show that the total blockage is within 0.35 percent for all configurations for the 70-degree model. The second three rows of Table 2.4 show the total blockage estimated for the 60-degree models is within 0.15 percent. The total blockage for the 45-degree model is shown in the last three rows of Table 2.4 and has a variation of 3.5 percent.
Table 2.4 shows that the boundary-layer blockage estimate is within 0.4 percent for the 70-degree model when comparing the axisymmetric 50.8 cm location to the square 18.7 cm location. The same phenomenon is observed for the 60 and 45-degree models where the differences are 0.37 and 0.03 percent respectively. This indicates a larger boundary-layer blockage growth in the square test section relative to the axisymmetric, since the blockages are comparable at 32.1 cm difference in location. It is likely that the boundary-layer growth in corner regions of the square test section is the major contributing cause. Also contributing is the presence of contra-rotating vortex pairs along the center of the non-contoured nozzle walls. These approximations suggest that an axisymmetric test section may be advantageous since it will allow testing of larger models as a result of comparably thinner boundary-layer growth.

2.3.3 Mach 2 Nozzle Block with Square Test Section at 18.7 cm

Figure 2.13 shows a different \( Re_D \) limit than previously seen in Figure 2.9 because the tunnel mass flow limit is reached at lower total pressure. The 70- and 60-degree models have a blockage of 5.5 percent for the largest models. The 45-degree model has a blockage of 6 percent for the largest model.

2.3.4 Mach 3 Nozzle Block with Square Test Section at 18.7 cm

For the Mach 3 nozzle, the upper wind tunnel limit is not dictated by mass-flow but rather plenum total pressure which is limited to 310.3 kPa for safety considerations. Prior to Mach 3 tests, the maximum blockage for the 60- and 70-degree models is consistent within 0.5 percent, as seen in Figures 2.12 and 2.13, with the 45-degree model behaving differently showing larger maximum blockage. Figure 2.14 shows different
characteristics between the all three model semi-vertex angles. The largest possible model size for the 70-degree model is 6 percent, the same value as seen at Mach 2.5. The 60-degree model shows larger blockage up to 8 percent from the 6 percent previously seen at Mach 2.5. Finally, the 45-degree model reached a blockage of 11 percent, comparable to the 10 percent blockage at Mach 2.5. An 11.5 percent, 45-degree model was tested, but excessive vibration observed during testing resulted in a rapid shutdown.

2.3.5 Mach Number versus Blockage Relation

Figure 2.16 shows the largest models of each geometry tested in the square test section plotted along with the original literature study. The data falls between the low and high bound for the Unitary Plan Tunnel and, as expected, 70-degree models have the smallest possible blockage and 45-degree models have largest possible blockage. The 60- and 70-degree models have very similar blockage for Mach 2 and Mach 2.5, but diverge at Mach 3.

The blockage data is then re-plotted again in Figure 2.16. The blockage values at which the wind tunnel starts normally without the need for additional total pressure increases are recorded as the lower bound for each geometry. The upper bound shows the largest blockage models for each geometry that are able to start at each Mach number by raising the plenum stagnation pressure up to limits of safe operation.

The 45-degree models have the largest range of allowable model sizes, whereas the 70- and 60-degree models have a narrower range. As stated earlier, starting any SWT is not a precise practice and this figure should be used for approximate sizing only. This study provides a bound on the size range for models of three different forebody semi-vertex angles.
2.3.6 Lowest Reynolds Number for All Configurations

During the course of testing it is found that once tunnel start is achieved with a model, the wind tunnel \( Re_D \) can be decreased significantly while still maintaining supersonic flow in front of the model. This hysteresis is advantageous because a lower \( Re_D \) results in less drag force or axial force on the model which reduces magnetic field strength requirements for the MSBS.

If the wind tunnel starts, the plenum total pressure is decreased incrementally to see if supersonic flow is maintained at a lower operating \( Re_D \). The tunnel \( Re_D \) is decreased to the point where the tunnel unstarts. The unstart conditions are shown in Table 2.5. It is seen that the \( Re_D \) at which the tunnel unstarts for a given configuration is similar regardless of model size and geometry. One exception is in the Mach 2.5 Square configuration where a larger range of \( Re_D \) is observed over two days of testing several weeks apart. This may show variability of results depending on ambient temperature conditions. Data taken at 10.2 cm from the nozzle in the Mach 2.5 axisymmetric configuration is not included in Table 2.5 because in this configuration it is impossible to lower the total pressure since all models start at the minimum total pressure.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( Re_D \times 10^6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mach 2 Square at 18.7 cm</td>
<td>0.56 to 0.66</td>
</tr>
<tr>
<td>Mach 2.5 Axisymmetric at 50.8 cm</td>
<td>0.68 to 0.75</td>
</tr>
<tr>
<td>Mach 2.5 Square at 18.7 cm</td>
<td>0.60 to 1.14</td>
</tr>
<tr>
<td>Mach 3 Square at 18.7 cm</td>
<td>0.68 to 0.81</td>
</tr>
</tbody>
</table>

Table 2.5: Reynolds Numbers for Tunnel Unstart
2.3.7 Starting Loads Analysis

The dynamic pressure and resultant drag force at the starting $Re_D$ is shown in Table 2.6. Only the drag on the largest model for a given geometry in each configuration is shown, representing the greatest magnitude.

<table>
<thead>
<tr>
<th>Test Section</th>
<th>Mach</th>
<th>Angle</th>
<th>Size</th>
<th>$q$</th>
<th>$C_D$</th>
<th>$F_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square-18.7cm</td>
<td>2</td>
<td>70</td>
<td>5.5%</td>
<td>27.34</td>
<td>1.56</td>
<td>52.78</td>
</tr>
<tr>
<td>Square-18.7cm</td>
<td>2</td>
<td>60</td>
<td>5.5%</td>
<td>22.08</td>
<td>1.43</td>
<td>39.07</td>
</tr>
<tr>
<td>Square-18.7cm</td>
<td>2</td>
<td>45</td>
<td>6.0%</td>
<td>27.34</td>
<td>1.24</td>
<td>45.76</td>
</tr>
<tr>
<td>Square-18.7cm</td>
<td>2.5</td>
<td>70</td>
<td>6.0%</td>
<td>40.45</td>
<td>1.53</td>
<td>83.55</td>
</tr>
<tr>
<td>Square-18.7cm</td>
<td>2.5</td>
<td>60</td>
<td>6.0%</td>
<td>32.90</td>
<td>1.40</td>
<td>62.19</td>
</tr>
<tr>
<td>Square-18.7cm</td>
<td>2.5</td>
<td>45</td>
<td>10.0%</td>
<td>41.14</td>
<td>1.24</td>
<td>114.78</td>
</tr>
<tr>
<td>Axi-50.8cm</td>
<td>2.5</td>
<td>70</td>
<td>6.0%</td>
<td>39.27</td>
<td>1.53</td>
<td>81.11</td>
</tr>
<tr>
<td>Axi-50.8cm</td>
<td>2.5</td>
<td>60</td>
<td>6.5%</td>
<td>43.52</td>
<td>1.40</td>
<td>89.10</td>
</tr>
<tr>
<td>Axi-50.8cm</td>
<td>2.5</td>
<td>45</td>
<td>6.5%</td>
<td>35.34</td>
<td>1.24</td>
<td>64.10</td>
</tr>
<tr>
<td>Axi-10.2cm</td>
<td>2.5</td>
<td>70</td>
<td>9.0%</td>
<td>6.47</td>
<td>1.53</td>
<td>20.06</td>
</tr>
<tr>
<td>Axi-10.2cm</td>
<td>2.5</td>
<td>60</td>
<td>9.0%</td>
<td>6.47</td>
<td>1.40</td>
<td>18.35</td>
</tr>
<tr>
<td>Axi-10.2cm</td>
<td>2.5</td>
<td>45</td>
<td>10.5%</td>
<td>6.47</td>
<td>1.24</td>
<td>18.97</td>
</tr>
<tr>
<td>Square-18.7cm</td>
<td>3</td>
<td>70</td>
<td>6.0%</td>
<td>30.89</td>
<td>1.52</td>
<td>63.38</td>
</tr>
<tr>
<td>Square-18.7cm</td>
<td>3</td>
<td>60</td>
<td>8.0%</td>
<td>38.14</td>
<td>1.35</td>
<td>92.69</td>
</tr>
<tr>
<td>Square-18.7cm</td>
<td>3</td>
<td>45</td>
<td>11.0%</td>
<td>33.36</td>
<td>1.24</td>
<td>102.39</td>
</tr>
</tbody>
</table>

Table 2.6: Drag on Model During Tunnel Start

The drag experienced during tunnel start is large and will likely be out of the performance capability of the MSBS which may be limited to 40 N.\(^{12}\) As discussed earlier in this chapter the $Re_D$ can be reduced after tunnel start to values shown in Table 2.5. The drag on the models is recalculated for the resulting lower total pressures thereby reducing the dynamic pressure. The estimated total pressure corresponding to the reduced dynamic pressure is determined to be 48.2 kPa for the Mach 2 square configuration, 62.05 kPa for both the Mach 2.5 square and axisymmetric configuration, and 82.74 kPa for the Mach 3 configuration. The new drag for steady state tunnel operation is shown in Table 2.7.
Table 2.7: Drag on Model During "Steady State" Tunnel Operation

By comparing Table 2.6 and Table 2.7 it is seen that drag is reduced in each case other than models tested at 10.2 cm from the nozzle exit in the Mach 2.5 axisymmetric configuration. The dynamic pressure for the Mach 2.5 axisymmetric configuration at 10.2 cm from the nozzle exit is increased to 12.74 kPa to provide an adequate margin avoiding supply air perturbations and resulting unstart. The drag force values in Table 2.7 are now within the anticipated performance of the MSBS. A solution to mitigate the high drag experienced during tunnel start is use of a support sting that can be removed once the dynamic pressure is reduced to steady-state operating conditions.

2.4 SUMMARY

The first step to determine the feasibility of the magnetic suspension system in the GRC 225cm² SWT is defining the allowable size of the model. The model size should be as large as possible so that the magnetic field and amperage required by the MSBS can be
minimized. However, the model also must be small enough to allow for tunnel starting. This allowable blockage is affected by the model geometry, axial testing location, and $Re_D$ for a given Mach number. A test matrix of models is tested in a Mach 2, 2.5, and 3 with a square test section and at Mach 2.5 with an axisymmetric test section.

Several important conclusions result from this testing. First, the model should be tested as close as possible to the nozzle exit to minimize blockage effects due to boundary-layer growth. By comparing blockage data at 10.2 and 50.8 cm in the Mach 2.5 Axisymmetric configuration, it is determined that the boundary-layer growth downstream is significant and adversely affects the allowable model blockage. Secondly, the blockage results from the axisymmetric and square test section at Mach 2.5 are compared. By using data from two different former studies in the GRC 225 cm$^2$ SWT, the boundary-layer blockages in the square test section and axisymmetric test section are approximated. The approximated boundary-layer blockage in the square test section is determined to be similar to the value estimated at the back of the axisymmetric test section even though it is located considerably closer to the nozzle. This shows that the square test section has more significant boundary-layer blockage then the axisymmetric test section due to the boundary-layer corner effects. These two results confirm that a short in length, axisymmetric test section allows the largest models to be tested. Finally, the blockage results are compiled in Figure 2.16 comparing allowable blockage over a range of Mach numbers. Although it is known that the plenum total pressure at which a model starts can vary up to 6.9 kPa from run to run, this blockage study provides operational bounds for a refined estimation needed when sizing models and the MSBS.
The total pressure may be decreased after tunnel start occurs with the model installed. This allows the tunnel to operate at lower $Re_D$ than necessary for model starting. The drag force is calculated both at tunnel start and a lower operational steady-state $Re_D$. The drag at tunnel start is greater than steady-state by at least a factor of 2 and may be greater than MSBS capabilities. It may be advantageous to size the MSBS magnetic field performance to match drag at steady-state conditions and use a removable model support during tunnel start.
CHAPTER III

WALL PROXIMITY TESTING

3.1 INTRODUCTION

In an idealized MSBS set-up, the EPS system maintains the model position permanently at the centerline while allowing it to rotate in three axes. The rotations of the model in pitch and yaw, however, produce a heaving motion as the pitch and sideslip angle changes. This motion puts the model off the centerline temporarily, but the EPS system will be designed so that the magnetic fields are adjusted in order to return the model position back to the centerline position. Despite responsive feedback control, the model movement off the centerline can impact the test by inducing wall interference effects in turn further influencing the model behavior. The extent of the wall influence as a function of its proximity must be assessed since it may be impossible to prevent all movement off centerline completely in a dynamic test. The results of this portion of the study aim to determine a distance the model can move without risking wall effects that can influence the model motion, including unstarting the wind tunnel.

3.2 TEST METHODOLOGY

The allowable wall proximity to the model is tested in the SWT at Mach 2, 2.5 and 3 as a coupled experiment with the blockage tests discussed in the previous chapter. The same models are tested in the axisymmetric test section at 10.2 and 50.8 cm from the nozzle exit and in the square test section at 18.7 cm from the nozzle exit. During each test, the model is placed at centerline and the plenum total pressure is increased until the wind tunnel starts. The total pressure is then decreased to lower operational conditions
shown in Table 3.1. Once the steady-state condition is reached, a pressure reading is taken at the centerline and then again each time the model moves a 0.25 cm increment toward the tunnel wall. The wall proximity tests are performed at a total pressure of 62.05 kPa for Mach 2 and Mach 2.5 in the square test section at 18.7 cm from the nozzle exit as well as Mach 2.5 tests at 50.8 cm from the nozzle exit in the axisymmetric test section. The wall proximity tests at the front of the axisymmetric test section at 10.2 cm from the nozzle exit, however, have a total pressure of 41.71 kPa.

<table>
<thead>
<tr>
<th>Mach Number</th>
<th>Total Pressure (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>62.05</td>
</tr>
<tr>
<td>2.5</td>
<td>62.05/51.71</td>
</tr>
<tr>
<td>3</td>
<td>81.35</td>
</tr>
</tbody>
</table>

Table 3.1: Steady-State Total Pressure During Wall Proximity Test

Pressure taps are axially distributed along the length of each test section, but the layout differs between the square and axisymmetric test section. The axisymmetric test section has a series of pressure taps at 90 degree increments around the perimeter for the first half of the test section (until 33 cm from the nozzle exit) which is shown in the left image in Figure 3.1. The second half of the test section has pressure taps along the top and bottom which is shown in the right-hand image in Figure 3.1. Figure 3.2 shows that the square test section only has pressure taps along the bottom of the test section. The square test section has one pressure tap in at the nozzle exit, but the pressure taps in the test section begin at the axial location of the test section windows at 11.1 cm from the nozzle exit. Schlieren still images are taken at every pressure reading to accompany the data taken in the square test section and provide a visual diagnostic.

Only one model is tested per Mach number and test section configuration in the wall proximity tests due to the relatively long duration of each position sweep. The
largest 60-degree model that started during blockage testing for a particular configuration is chosen for wall testing. The largest 60-degree models for the axisymmetric test section are 6 and 9 percent for 50.8 and 10.2 cm from the nozzle exit. The largest 60-degree models for the square test section are 5.5, 5.5 and 8 percent for Mach 2, 2.5 and 3 respectively.

3.3 RESULTS:

3.3.1 Mach 2.5 Axisymmetric Configuration at 50.8 cm from the Nozzle

The first wall test is conducted at 50.8 cm from the nozzle exit in the axisymmetric test section with a 60-degree, 6 percent model. Figure 3.3 shows the initial wall pressure taps readings when the model is at centerline. The horizontal axis shows the axial location of the pressure taps along the test section, relative to the nozzle exit. The vertical axis shows the wall pressure at that axial location normalized by the plenum total pressure. The vertical black lines represent the leading and trailing edges (nose and backshell) of the test model. The legend labels each individual set of pressure taps. PPBBF and PPAAF represent the pressure taps along the bottom and top of the test section while PPCSF and PPCPF dictates the right and left set of pressure taps if looking downstream at the test section.

Since the model is on centerline in Figure 3.3, all the individual sets of wall pressure taps read very similar values due to their symmetrical spacing along the wall. Figure 3.3 shows the wall pressure until the model leading edge is a flat horizontal line which indicates supersonic flow front of the model. The pressure taps reach a maximum value of 0.15 after the trailing edge of the model. This increase in pressure is due to a bow shock that stands off the front surface of the blunt body model. Figure 3.4 shows that
as the model moves towards the bottom of the test section the flow is no longer symmetric and as a result PPAAF and PPBBF start to diverge in value. Figures 3.5 to 3.11 display the bow shock boundary-layer interaction moving forward towards the nozzle exit as the model is positioned closer to the bottom wall. The bow shock boundary-layer interaction seen by the set of PPAAF pressure taps in these same figures, however, shows the shock moving downstream in the test section.

Since there is no schlieren capability in the axisymmetric test section, the flow is characterized from the pressure tap data alone. It becomes apparent in Figure 3.12 at 2.25 cm from the centerline that the point at the trailing edge of the model for the PPBBF pressure taps is comparatively much larger in magnitude than the other wall pressure readings. This point represents the peak wall pressure and is the most significant departure from the desired test conditions because it likely corresponds to the strongest shock in the test section.

Figures 3.13 to 3.15 show the bow shock no longer moving forward for the PPBBF pressure taps, but instead shows the original point of peak wall pressure seen in Figure 3.12 increasing in magnitude. Prior to Figure 3.16, the peak wall pressure was a distinct point as the other wall pressures were much smaller in magnitude. Figure 3.16 shows that at 3.25 cm from the centerline, the bow shock and wall interaction has shifted forward so that the wall pressure at two pressure taps near the model are nearly equivalent. Figure 2.17 shows one 0.25 cm increment further with the new peak wall pressure location now inside the model range. This location could that the shock interaction is becoming centered in the model zone. The peak wall pressure value with the model at 3.5 cm from centerline is now 20 percent greater than the peak wall pressure
of the test point when the model is at the centerline in Figure 3.3. From this point forward in the chapter, 20 percent peak pressure rise is used as an indicator to consistently evaluate the shock strength for other wall proximity tests. This indicator establishes a hypothetical demarcation between normal wall interactions seen in testing and when wall interactions can potentially influence undisturbed model behavior.

As the model continues to move beyond 3.5 cm from centerline, the peak wall pressure increases in magnitude but remains in the same location until Figure 3.21 when the model is 4.5 cm from centerline. This figure shows a point near the model leading edge become nearly equivalent in value to the current peak wall pressure. This point is the final location of the peak wall pressure. Subsequent tests presented in Figure 3.22 to 3.26 show the wall pressure at this point continuing to increase in magnitude. Figure 3.27 shows the final pressure tap reading for the test with the model at 6 cm from the centerline, nearly touching the wall. The model peak wall pressure at this location is twice what is measured when the model is at centerline.

3.3.2 Mach 2.5 Axisymmetric Configuration at 10.2 cm from the Nozzle

The next wall proximity test is performed again in the axisymmetric test section, but this time the model is tested at 10.2 cm from the nozzle exit. Blockage testing determined the largest 60-degree model that starts at 10.2 cm from the nozzle exit is 9 percent, significantly larger than the maximum blockage, 6 percent model tested at 50.8 cm from the nozzle exit. The larger model size limits the distance the model can move toward the tunnel bottom wall so fewer tests points are taken. Wall pressure changes in each 0.25 cm increment are more obvious in this test because the upstream location of the model causes the bow shock is captured by all four sets of pressure taps.
Figure 3.28 presents the first test point taken and when the model is at centerline. Only two pressure taps are before the leading edge of the model because the model is positioned close to the nozzle exit. The first two pressure taps measured a wall pressure of 0.06 which is the freestream pressure that is measured in the wall proximity test at 50.8 cm from the nozzle exit. Figure 3.28 shows the bow shock is downstream from the model trailing edge with a peak wall pressure of 0.16.

Although the furthest downstream data point is the actual peak wall pressure for this test point, it is not being considered. This point indicates that the shock train seen during tunnel start has not completely left the test section. The shock train is far enough downstream that it does not impact the model so the peak wall pressure is evaluated only until 45 cm from the nozzle exit.

Figure 3.29 and 3.30 show the wall pressures when the model is 0.25 and 0.5 cm from centerline. These figures show the bow shock boundary-layer interaction measured by the bottom set of pressure taps shifts upstream while the interaction measured by the top pressure taps shifts downstream. These two figures show the wall pressures measured by the left set of pressure taps (PPCPF) overlays the wall pressure measured by the bottom set of pressure taps (PPBBF.) The figures also show the wall pressure measured by the right set of pressure taps (PPCSF) overlays the wall pressure measured by the top set of pressure taps (PPAAF.) The overlay is no longer shown in Figure 3.31 when the model is 0.75 cm from centerline. The wall pressures measured by the four sets of pressure taps begin to separate and the bow shock and boundary-layer interaction appears axially staggered in the figure. The interaction is located furthest upstream in the wall pressures measured by the bottom set of pressure taps and then further downstream by the
left, right and top set of pressure taps, in order. Figures 3.32 through 3.36 show the model at 1.00 to 2.00 cm from centerline where the peak wall pressure is located at the trailing edge of the model. The figures also show the bow shock wall interaction measured from the left set of pressure tap moves downstream while the interaction measured by the right set of pressure taps moves upstream. The wall pressures measured by these two sets of pressure taps becomes close in value and show the interaction at a similar location.

Figure 3.36 shows that when the model is at 2 cm from centerline, a point upstream of the current peak wall pressure increases in value to a similar magnitude. This peak wall pressure progression between test points is similar to that seen between Figure 3.15 and 3.16 for the wall proximity test at 50.8 centimeters from the nozzle exit.

The upstream point in the model range is the new location of peak wall pressure for the subsequent tests beginning in Figure 3.37 when the model is at 2.25 cm from the centerline. The peak wall pressure when the model is 2.25 cm from the centerline is also 20 percent greater than the peak wall pressure measured when the model is at centerline. This approximate demarcation was determined in the previous wall proximity test and indicates that the strength of the bow shock wall interaction is too high and that wall effects at this test point are longer insignificant.

The final tests presented in Figures 3.38 to 3.40 show the peak wall pressure continuing to increase in magnitude at the same location. The final wall pressure tap measurements are shown in Figure 3.40 when the model is 3 cm from centerline. The peak wall pressure at this test point is 40 percent greater than the peak wall pressure measured when the model is at centerline.
3.3.3 Mach 2 Square Configuration at 18.7 cm from the Nozzle

Remaining wall proximity tests are performed in the square test section. The square test section has windows so the schlieren system is used. For the remaining wall proximity tests, both pressure tap measurements and schlieren still images are captured at each test point.

The first wall proximity test in the square test section is performed at Mach 2 with a 5.5 percent maximum blockage, 60-degree model. Figure 3.41 shows a schlieren still on the left and the corresponding wall pressures plotted in the right figure for when the model is at centerline. Only one set of pressure taps are in square test section so only one line of wall pressure data is plotted in the right hand figure.

The schlieren still in Figure 3.41 shows the bow shock stands off the model nose. The area where the bow shock contacts and reflects off the boundary-layer is shown distinctly. The wall pressure measurements plotted on the right show higher wall pressures in front of the model. The schlieren image still verifies that higher wall pressures indicate the location of the bow shock and wall interaction in the test section.

Wall pressures for a test point are captured by one line in the square test section. This allows data from one test point in the square test section to be easily compared with previous test points. Wall pressures for a particular test point are plotted against the wall pressure measurement when the model is at centerline as well as the previous test point starting when the model is 0.75 cm from centerline. This format is used for the remaining wall proximity figures and clearly shows how much the wall pressure changes between test points as well as compared to when the model is at centerline.
Figures 3.42 through 3.50 show the bow shock wall interaction moving upstream as the model is moved incrementally towards the bottom wall. This is the same behavior that is seen in the wall proximity tests in the axisymmetric test section. The accompanying schlieren image stills in these figures confirm that the wall pressure movement upstream is due to the bow shock hitting the wall of the test section at a further upstream location. The curvature of the bow shock causes the shock to impact the wall behind the model. When the model moves towards the bottom wall the location where the bow shock interacts with the boundary-layer moves upstream.

Figures 3.42 to 3.51 show minor increases in the peak wall pressure magnitude accompanied by minor shifts upstream between test points. Figure 3.52, however, shows a more significant increase in peak wall pressure between the points when the model is 2.50 cm and 2.75 cm from centerline. More significantly, the schlieren image still in Figure 3.52 shows that when the model is 2.75 cm from centerline the reflected bow shock off the boundary-layer contacts the model surface. It is also at this point that the peak wall pressure has a magnitude 20 percent greater than the peak wall pressure measured when the model is at centerline. This confirms that an off centerline model movement of 2.75 cm causes wall interactions that will interfere with model oscillations in MSBS testing.

Two more test points are taken in the wall proximity test series at Mach 2. Figures 3.53 and 3.54 show the peak wall pressure continuing to grow significantly in magnitude as more of the reflected shock contacts the model surface. The schlieren image still in Figure 3.54 shows that the bow shock is no longer visible which indicates the wind tunnel has unstarted. The final wall pressure recording taken in Figure 3.54 when the model is at
3.25 cm from centerline shows the peak wall pressure at this test point is 40 percent greater than the peak wall pressure when the model is at centerline.

### 3.3.4 Mach 2.5 Square Configuration at 18.7 cm from the Nozzle

The next wall proximity test is performed at Mach 2.5 using a 5.5 percent, 60-degree model. Figure 3.55 shows the model at centerline and the bow shock upstream from the model leading edge. Figures 3.56 through 3.60 show that as the model moves downward, the location where the bow shock intersects the boundary-layer moves upstream. This behavior is consistent with the previously performed wall proximity tests. In this test, the change in location and magnitude of the peak wall pressure differs very little between the test points. It is thereby difficult to determine a decisive increase in severity of the shock interactions with the wall so subtler changes are examined.

Figure 3.61 shows that when the model is 1.50 cm from the centerline the peak wall pressure at the trailing edge is considerably larger than the other wall pressures. The peak wall pressure at this point continues to increase at the same axial location in Figures 3.62 to 3.63. Figure 3.64, however, shows that when the model is 2.25 cm from centerline that the peak wall pressure location moves upstream. The peak wall pressure at this test point is greater than 20 percent of the peak wall pressure measured when the model is at centerline. The test point does not, however, correspond to when the reflected shock contacts the model surface or a significant increase in magnitude between test points as previously seen at Mach 2.

The schlieren image stills appear out of focus for the remainder of the wall proximity test so the boundary-layer and reflected shock cannot be clearly in Figures 3.65 to 3.70. If the location of the bow shock and boundary-layer interaction remained in the
same position shown in Figure 3.64, the reflected shock contacts the model surface when
the model is 2.75 cm from centerline. This model distance from centerline is the same
test point when the reflected shock contacted the model surface in the wall proximity test
at Mach 2. It can also be noted that downstream wall pressures from the model are lower
in magnitude than the freestream wall pressures. This is likely due to an expansion shock
after the model which causes the flow to accelerate.

Figure 3.68 presents the model at 3.25 cm from centerline and shows that the
difference in the magnitude of the peak wall pressure between test points is larger. This
substantial increase in peak wall pressure magnitude between tests points continues until
the final test when the model is 3.75 cm from centerline. This behavior shows a strong
contrast to the minor increases seen earlier in the test when the model is closer to
centerline. Figure 3.70 shows the model at the final test point when the model is 3.75 cm
from centerline. This test point has a peak wall pressure over 80 percent greater than
measured when the model is at centerline.

3.3.5 Mach 3 Square Configuration at 18.7 cm from the Nozzle

For the final test configuration an 8 percent maximum blockage, 60-degree model
is tested at Mach 3 in the square test section. This is a much larger model compared to 5.5
percent model that is tested at Mach 2 and 2.5 in the square test section. Figure 3.71
shows the schlieren image still and wall pressure distribution when the model is at
centerline.

Figure 3.73 shows that off center model movement of 0.50 cm from centerline
causes the location of the bow shock and wall interaction move more upstream compared
to previous wall proximity tests. The magnitude of the peak wall pressure holds relatively
constant in Figure 3.73 to 3.75. It is not until the model is 1.25 cm from the centerline that a large increase in peak wall pressure magnitude between tests points is seen. Figure 3.76 shows the test point that has a peak wall pressure over 20 percent greater than peak wall pressure measured at centerline. By comparing the previous wall proximity tests, this is the shortest model distance from centerline that a peak wall pressure 20 percent greater than measured at centerline is seen.

Figure 3.78 shows the magnitude of the peak wall pressure for the test at 1.75 cm is larger than the other wall pressures. This gives the wall pressure distribution a pointed appearance instead of the rounded profile seen in previous test points. This point continues to increase in magnitude in Figures 3.79 and 3.80. When the model is 2.50 cm from the centerline, the peak wall pressure location moves forward to the next upstream pressure tap. The schlieren still in Figure 3.81 also shows the reflected shock contact the model surface at this test point. The peak wall pressure is at this upstream location for the remaining test points and continues increasing as shown in Figures 3.82 and 3.83. The final model test point is at 3 cm from centerline and has a peak wall pressure that is 70 percent greater than the peak wall pressure recorded when the model is at centerline.

3.4 SUMMARY

A wall proximity test is conducted in the NASA Glenn 225 cm² SWT to determine the wall effects caused by off-center model movement in different test sections and at different Mach numbers. In the test, the model is moved at 0.25 cm increments towards the bottom of the test section and wall pressure readings are recorded. Schlieren image stills are also taken at each test point for tests performed in the square test section. The objective of the test is to determine how far off centerline a model may move before
the wall interactions influence test data. The results of the wall proximity tests show that this limit is difficult to quantify as the same model movement off centerline results in different wind tunnel conditions in different test configurations.

Table 3.2 shows the model distance off centerline for when a test point has a peak wall pressure greater than 20 percent of the peak wall pressure measured when the model is at centerline for that test. The table also shows the test point when the model surface contacts the reflected shock for tests in the square test section.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>20% Peak Wall Pressure (cm from centerline)</th>
<th>Contacts Reflected Shock (cm from centerline)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mach 2.5 Axisymmetric-50.8 cm</td>
<td>3.50</td>
<td>n/a</td>
</tr>
<tr>
<td>Mach 2.5 Axisymmetric-10.2 cm</td>
<td>2.25</td>
<td>n/a</td>
</tr>
<tr>
<td>Mach 2 Square-18.7 cm</td>
<td>2.75</td>
<td>2.75</td>
</tr>
<tr>
<td>Mach 2.5 Square-18.7 cm</td>
<td>2.25</td>
<td>2.75</td>
</tr>
<tr>
<td>Mach 3 Square-18.7 cm</td>
<td>1.50</td>
<td>2.50</td>
</tr>
</tbody>
</table>

**Table 3.2: Wall Proximity Test Results**

The current predictions for the electronic position system allows an off-center model movement of under 2.5 cm for the MSBS. If the 20 percent peak wall pressure indicator is used, Table 3.2 shows that this movement may cause wall effects to influence the model oscillations in future MSBS testing for half of the configurations, especially Mach 3. If the limit is determined to be when the model contacts the reflected shock, then wall effects will likely not influence model movement in the planned MSBS test facility.

It is difficult to predict the sensitivity of the model oscillations to wall effects without performing a dynamic test. Although no off center movement limit is established, the results of the wall proximity testing indicate that model movement off centerline is a concern for the MSBS test facility and must be further investigated.
CHAPTER IV

VIDEO ANALYSIS

4.1 INTRODUCTION

This chapter details the initial development of a video analysis method for use in the MSBS. The method of data analysis in ballistic range testing shares many similarities to the proposed data reduction method for the MSBS. Like ballistic range testing, images taken by high-speed cameras capture the model position and orientation throughout a test period. The location and attitude extracted from these images will comprise a trajectory that a 6DOF simulation is fit to. In the MSBS test facility, two orthogonal high-speed cameras will capture model motion in all three orientations. The high-speed video will then be reduced using a video analysis program detecting a marker on the model surface thereby determining the corresponding attitude of the capsule.

Before implementation of the video analysis system, a validation experiment is conducted. This experiment determines the accuracy of the video analysis method which aids in the development of the video analysis program. GRC has a low-speed Flotek 360 wind tunnel with similar test section dimensions comparable to the SWT. The Flotek 360 is well-suited for the validation experiment since modifications are inexpensive to perform and do not interfere with research occurring in the SWT.

4.2 INSTRUMENTATION

Two Sony Cybershot RX100MIV cameras are used to capture high-resolution high-speed video. These cameras have video frame rates of 240, 480 and 960 frames per second and capture video for 2 or 4 seconds depending on whether pixel resolution or a
longer video capture time is preferred. Pixel resolution is the priority since marker resolution determines the model position and angle. The pixel resolution for each frame rate is shown in Table 4.1. It can be seen that increasing frame rate causes a decrease in image resolution.

<table>
<thead>
<tr>
<th>Frame Rate (fps)</th>
<th>Pixels</th>
</tr>
</thead>
<tbody>
<tr>
<td>240</td>
<td>1,824 x 1,026</td>
</tr>
<tr>
<td>480</td>
<td>1,676 x 566</td>
</tr>
<tr>
<td>960</td>
<td>1,136 x 384</td>
</tr>
</tbody>
</table>

Table 4.1: Sony Cybershot Frame Rate Specifications

The cameras have a focal length range of 24 to 70 mm. In the test set-up, two separate infrared receivers for each camera detect a signal from a remote to begin video. This feature allows the cameras to be triggered almost simultaneously, within 83 milliseconds, which must be adjusted for in later analysis.

4.3 VALIDATION EXPERIMENT METHODOLOGY

The Flotek 360 wind tunnel is chosen for its similar test section size to the GRC 225 cm² SWT. The test section of the Flotek 360 is 15.2 by 15.2 cm while the length is 45.7 cm. The dimensions of the cross section are comparable to the square test section dimensions for the SWT, but it is shorter in length compared to the SWT. The Flotek 360 is vertically mounted from a past experiment and is located next to the SWT.

4.3.1 Camera Mount

The first task in developing the validation experiment is the design of a dual-camera mount. The cameras must be orthogonally positioned and at equal distances to the centerline of the wind tunnel test section. A design requirement for the camera mount is it
allowed for horizontal and vertical adjustment of the camera relative to the test section. This provides more versatility to capture different model test locations.

The necessary horizontal distance between the camera and test section is calculated using Equation 4.1 below.\textsuperscript{15} Figure 4.1 shows a diagram that details the basis of this calculation. The desired image frame width ($H_{o,\text{mm}}$) is 15.2 cm, the side of the test section. The focal length of the camera is 28 mm and the calculation assumes a frame rate setting of 240 fps.

$$D_{o,\text{mm}} = \frac{H_{o,\text{mm}} H_{s,\text{pix}}}{H_{s,\text{mm}} H_{o,\text{pix}}} f_o$$

$$D_{o,\text{mm}} = \frac{152 \times 1824}{13.2 \times 1824 \times \frac{28}{28}} = 322 \text{ mm}$$

This result provides the initial horizontal location of the camera relative to the test section which is used to begin designing the camera mount. This distance requires the camera to be supported off the table that holds the Flotek 360.

The camera mount is built from T-slot aluminum extrusions which are easily assembled. The camera mount is designed as a box structure which provides a sturdy base to support the cameras. The box structure is constructed with 2.54 by 2.54 cm cross section aluminum extrusions which are then bolted down to the table. To allow for the horizontal translation of the camera, 5.08 by 2.54 cm extrusions are clamped down to the box structure on the front and left sides of the camera mount. Tube holders compatible with T-Slot extrusions slide in the grooves of the 5.08 by 2.54 cm extrusions. These support two 1-inch diameter steel rods that are 45.72 cm in length. Two camera mounts, where were originally intended for bicycle handlebars, are clamped to the rod. The location of the mount can be adjusted to allow for vertical flexibility in the camera.
location. The SolidWorks™ design is shown in Figure 4.2 and the actual fabricated design is shown in Figure 4.3.

### 4.3.2 Cam Design and Construction

The next step is developing a mechanism to produce model pitch oscillations with known frequency and amplitude. The known oscillations produced are compared against results from the video analysis program to determine the attitude error. The mechanism must produce consistent oscillations as it is the control input to the experiment.

The mechanism design is comprised of a cam rotated by a stepper motor which in turn produces desired model oscillations about an axis. A test model is affixed to one end of a steel rod while a follower is affixed to the other. The follower moves along the cam profile as it rotates and the radial offset of the cam causes the model to oscillate in pitch. The cam is mounted to the backside of the test section wall through which a hole is drilled for the rod to penetrate. The frequency of oscillation is determined by the stepper motor rotation rate which is set to a constant 2.00 Hz. The amplitude is determined by the total radial offset of the cam along with the length of the follower from the shaft center.

Figure 4.4 shows the desired pitch oscillation produced by the first 3D-printed cam. The cam is designed using a constant angular velocity, or triangle, profile from the Cam Design Manual. The base circle of the cam is 1.27 cm and the follower is 4.57 cm, producing an oscillation amplitude of 5.5 degrees.

Figure 4.5 shows the assembly of the cam, follower and stepper motor. Both the cam and follower are 3D-printed which allowed both parts to be quickly and inexpensively fabricated. A spring is used to hold the follower against the cam, maintaining contact with the profile. Figure 4.6 shows the spring which is secured to a
square tube that is attached to the top of the follower as shown in Figure 4.6. The validation experiments in this chapter are performed with a 6 percent, 70-degree model that is also 3D-printed.

4.3.3 Tracker Program

A search for open source programs performing motion tracking is undertaken as a starting point for use in the video analysis study. This experiment is a proof of concept and it is desirable to find user-friendly software with satisfactory accuracy. The open source software, Tracker, a program developed for physics students as a modeling tool is selected. The first step in importing video into Tracker is selection of the x and y axes for the camera view. Secondly, a “calibration tape” must be defined to convert the pixel coordinates to physical coordinates \( x, y \). This is done by selecting two pixel locations in Tracker and defining the known linear distance in physical coordinates. Tracker can then scale the remaining portion of the image to the physical coordinates. The user then defines a “point mass” which is tracked automatically or manually. For automatic tracking, the program requires the user to provide a template of the object or portion of the image to be tracked. The template serves as a comparative target for the program to locate in the sequential bitmap frames.

The templates chosen for Tracker are two adhesive markers placed on the front and left of the model backshell. One marker is in line with the front facing camera while the other is in line with the left facing camera. Initial video analysis attempts determine which marker styles work better than others for Tracker to follow. The software is sensitive to changes in light in the video and large templates often cause the software to mistakenly mark the center of the target due to glare, shadows and reflection in the
image. Figure 4.7 shows three different types of markers tested and the corresponding tracking results. The upper image in Figure 4.7 shows the type of adhesive marker, where it is placed on the model, and the lower plot shows the tracking results produced by the software. It should be noted that the $x$ and $y$ coordinates are interchanged for the first plot. The leftmost model shows that Tracker misidentifies the marker center on the last oscillation. The tracking results for the other two models show the triangle wave distinctly, but the rightmost marker style produces the best tracking results. The lower axis range is small in scale which causes the tracking results to appear comparatively worse. The white dot is therefore used for the remaining tests.

Figure 4.8 shows the two orthogonal camera views for the validation experiment. The test captures video at a rate of 240 frames per second (fps) which is more than sufficient to resolve the model motion at 2.00 Hz. The data reduction is performed with the output data from Tracker for the left camera only. Because model motion occurs in one plane only, one camera is needed to define single-axis motion. Additionally, Tracker cannot analyze or combine two camera perspectives at once. The trigonometric relationship in Equation 4.2 is used to calculate the pitch angle of the model.

$$\Delta \alpha = \sin^{-1} \left( \frac{x - x_i}{d_m} \right)$$

(4.2)

$\Delta \alpha$ represents the change in pitch angle from the model initial attitude. Figure 4.9 shows the model view from the perspective of the front camera. A coordinate transform is applied to the video analysis results from the left camera since the $y$ coordinate in the image corresponds to the true $x$ coordinate of the wind tunnel. $x_i$ represents the initial or minimum value tracked in the test which corresponds to the minimum pitch angle in the tests. Pitch angles are calculated in reference to $x_i$ which is represented by “$dx$” in Figure
4.9 and in the numerator of Equation 4.2. The denominator $d_m$ in Equation 4.2 is the distance from the marker to the point of rotation from the perspective of the front camera shown in Figure 4.9. This distance is a constant and represents the hypotenuse of the right triangle calculation. This value is measured on the 3D-printed model using a steel ruler.

The results of the video tracking program are shown in Figure 4.10 where the known pitch angle produced by the cam and the results from Tracker are plotted together. The figure shows the results from Tracker are close in value to the actual movement driven by the cam. The amplitude of the cam is 5.5 degrees whereas the amplitude recorded by Tracker is 5.65 degrees. This degree of accuracy, 0.15 degrees, represents a promising first result produced by the video analysis method. The accuracy is limited by the precision of the calibration tape and pixel resolution of the camera.

A problem with Tracker is that the calibration tape does not accurately capture change in depth. This, along with difficulties using the autotracker feature, reveals the need to develop a more accurate tracking code and method of transforming individual pixels to corresponding physical coordinates.

4.4 VIDEO ANALYSIS PROGRAM DEVELOPMENT

4.4.1 Tracking Program

Alternate commercial software and more accurate open source programs are researched to find a replacement for Tracker. The program selected is a video tracking code created by Clara O’ Farrell at Jet Propulsion Laboratory. This code is originally developed for tracking the Mars Science Laboratory parachute in a wind tunnel test. The tracking code is created in MATLAB and uses many features provided by the Image
Processing Toolbox. The analysis procedure begins by separating high-speed video footage captured from a test into individual frames. The program reads in each frame and tracks the change in pixel location of the marker. The last step uses properties determined by a camera calibration to convert pixel to physical coordinates. The coordinates represent center of the marker during testing and determine the corresponding orientation of the parachute.

The JPL tracking code is used as a foundation to develop a video analysis tracking code for the Flotek 360 validation experiment. Appendix A.1 includes the tracking code that is modified from the program originally created by O’Farrell. Minimal adjustments are necessary with these circular markers in the validation experiment.

A key difference in the video set-up for the validation experiment is that two cameras are used to determine the position of the marker in three dimensions while only one camera is used in the TDT test. Therefore, video from the left and front camera must be analyzed by the tracking program to determine the model attitude in three orientations. Appendix A.1 shows the tracking code for the front camera. The program is for the left camera is nearly identical, but the output is a different variable name. This difference is necessary since both tracking program outputs are used later in the extraction step.

The tracking program in Appendix A.1 begins by synchronizing the images between the left and front camera. This difference in triggering times discussed earlier is resolved by having a blinking LED in the view of both cameras. The difference in frames between cameras where the LED turns on reveals the difference in triggering times. The variable “pic” shifts the initial camera input frame to synchronize the starting time.
The initial location of the center of the marker is identified first with the
“imshow” function and using the curser to identify the pixel location of the center. The
same window identifies the dimensions for the template image which are entered in the
variable “roi.” The “strel” function increases the contrast of the initial image and it is
cropped with the user-defined dimensions specified by “roi.” Smaller target images return
better tracking results so the image is cropped closely to the marker. An example of the
cropped target image is shown in Figure 4.11.

After the template is defined, remaining images are read in to the tracking
program through a loop. The program determines the new marker location by using the
correlation theorem, similar to the convolution theorem shown in Equation 4.3. The
equation shows the convolution of two matrices, represented as \( a \) and \( b \), in the spatial
domain is equivalent to the inverse of the product of the two Fourier transforms of the
matrices.\(^{18}\) This operation is an efficient way of computing the convolution for large
matrices.

\[
\begin{align*}
a \ast b &= F^{-1}(F(a) \ast F(b)) \\
\end{align*}
\]  

(4.3)

MATLAB interprets images as matrices so the Fourier Transform of the image is
easily performed by using the “fft” or Fast Fourier Transform Function. In order to locate
the new location of the marker, a correlation is performed between the initial image
template and the new image. An image correlation is the same as an image convolution
but the target image is rotated by 90 degrees.\(^{19}\) The results of the image correlation are
represented by the variable “corr_Output_f” in the program. This variable is normalized
to allow the tracking program to easily find the maximum correlation using the system
object “hFindMax.” The maximum value found in the correlation is the pixel location in
the image where the template matches. This determines the new location of the marker center.

The program uses the new location of the marker center to crop the next image in sequence around this point, using the variable “boxSize.” This reduces computation time and risk of misidentifying the target by performing the correlation for a smaller image. The location of the center of the marker for each image frame is saved as the variable “frontpix”, or in the case of the left camera, “leftpix.”

Appendix A.2 includes the pixel check program that is used to validate the program correctly identified the marker center. This pixel location of the marker center is plotted on the corresponding image. An example of the program output is shown in Figures 4.12 and 4.13. The pixel location of the marker center is shown in both figures.

4.4.2 Stereo Camera Calibration

The pixel location of the marker is converted into physical coordinates using the MATLAB Stereo Calibration Application. This application computes the intrinsic and extrinsic parameters of both cameras relative to each other. The intrinsic parameters such as the focal length and center of projection convert the pixel coordinates to normalized coordinates while the extrinsic parameters convert the normalized coordinates to physical coordinates. The extrinsic parameters also define the translation vector and rotational matrices of the coordinate system origin.

The MATLAB Stereo Calibration calculates the intrinsic and extrinsic parameters by detecting corners of a checkerboard pattern. The user inputs the physical width of each checkerboard square in millimeters and the program uses this define the associated image in physical coordinates. An example set of images used for calibration is shown in Figure
4.14. In order to calibrate two cameras simultaneously, the program requires images of the checkerboard from the left and front camera.

The default of the stereo camera calibration application places the origin of the coordinate system at the first camera, in this case the front camera. The final calibration result for the two cameras is shown in Figure 4.15. The mean pixel error determined from the calibration is 1.12 pixels.

### 4.4.3 Position and Orientation Analysis

The final step of the video analysis methodology is determining the pitch and sideslip attitude of the model during testing. Appendix A.3 contains the angle extraction code for this calculation. Pitch and sideslip angles are calculated by determining the marker location relative to the center of rotation. The point of rotation in the validation experiment is the 0.64 cm rod axis that connects the follower and model. The pixel location of this point is determined from images of the rod from both cameras and defined as “stingf” and “stingl” in the code. These two variables represent the three dimensional pixel location of the rotation axis at the model center which is converted to physical coordinates using the “triangulate” function. The “triangulate” function reads in the rod pixel coordinates and camera properties and returns the physical coordinates of the rotation axis at the model center. This same function also used converts the tracked locations of the marker center to physical coordinates.

Physical coordinates are then separated with respect to $x$, $y$ and $z$ coordinates. The change in pitch and yaw is calculated using right angle trigonometric relations. Figures 4.16 and 4.17 show diagrams of how the pitch and yaw angles are calculated from the perspective of the front and left camera.
Equations 4.4, 4.5 and 4.6 calculate the initial distance between the marker and rotation axis at the model center or sting, previously referred to as the rod. Equations 4.7 and 4.8 use this distance to calculate the hypotenus, \( r_\alpha \) and \( r_\beta \). The variables \( \alpha_0 \) and \( \beta_0 \) are the original pitch and yaw angle. These variables capture the inherent pitch and yaw angles due to the separation of the marker and center of rotation. The original pitch and sideslip angle (\( \alpha_0 \) and \( \beta_0 \)) are determined using the dimensions obtained from the CAD model in Equations 4.9 and 4.10. The actual pitch and sideslip angles of the model are then calculated from Equations 4.11 and 4.12.

\[
dx_i = x_{sting} - x_{i,marker} \quad (4.4)
\]
\[
dy_i = y_{sting} - y_{i,marker} \quad (4.5)
\]
\[
dz_i = y_{sting} - y_{i,marker} \quad (4.6)
\]
\[
r_\alpha = \sqrt{dx_i^2 + dy_i^2} \quad (4.7)
\]
\[
r_\beta = \sqrt{dx_i^2 + dz_i^2} \quad (4.8)
\]
\[
\alpha_0 = \tan^{-1} \frac{dy_i}{dx_i} \quad (4.9)
\]
\[
\beta_0 = \tan^{-1} \frac{dz_i}{dx_i} \quad (4.10)
\]
\[
\alpha = \sin^{-1} \frac{dy}{r_\alpha} - \alpha_0 \quad (4.11)
\]
\[
\beta = \sin^{-1} \frac{dz}{r_\beta} - \beta_0 \quad (4.12)
\]
4.5 STEREO CAMERA RESULTS

4.5.1 Pitch Case Example

The combination of the tracking and angle extraction, including calibration, codes (see Appendix A.1 and A.3) comprise a complete video analysis methodology to analyze the remaining tests in the validation study. Three cases are analyzed to determine the accuracy. The first test is a pitch-only case similar to the previously described testing using Tracker, but a new cam design and model are 3D-printed. The improved cam has a sinusoidal profile since it more closely represents oscillations experienced by entry capsules in flight.

The follower dimensions are unchanged, but since a higher oscillation amplitude is desired the radial offset of the cam is increased to 0.60 cm resulting in an oscillation amplitude of 7.5 degrees. The new oscillation profile produced by the cam is shown in Figure 4.18.

A new model is also 3D-printed for this test. A source of error in the previous tests results from the uncertainty in the marker location affixed to the model. The known physical location of the marker is important to calculate $\alpha_0$ and $\beta_0$. A new model is 3D-printed with indentations on the backshell surface to provide more precise marker locations. Figures 4.19 and 4.20 show dimensions as designed in the CAD model. The orientations from the shaft axis to the offset marker indentation center are at original pitch and sideslip angles of 55.27 and 55.23 degrees, $\alpha_0$ and $\beta_0$, respectively.

In order to determine the initial pitch angle, a digital level is placed on the backshell of the model. The cam is placed at the point of highest displacement (maximum pitch angle) and the initial pitch angle is measured to be approximately 11.5 degrees.
Figures 4.21 through 4.24 show the pixel tracking results for both the $x$ and $y$ dimensions of the front and left camera. Figures 4.21 through 4.23 show a sinusoidal movement while Figure 4.24 shows that the $y$-coordinate does not change for the left camera. Figure 4.25 plot the video analysis results on top of the known cam movement. The model only oscillates in pitch so it is expected that the video analysis software displays no sideslip motion. However, Figure 4.25 shows a small oscillatory yaw motion with an amplitude of 0.3 degrees. This is a result of the camera calibration because Figure 4.24 shows there is no pixel movement in the $y$-coordinate of the left camera.

The frequency in pitch oscillation determined by the video analysis system is accurate and matches the input frequency of 2.00 Hz, however, it has a slightly larger oscillation amplitude. The amplitude determined by the video analysis is 8.14 degrees while the cam driven amplitude is only 7.5 degrees, resulting in an error of approximately 0.6 degrees. This error results from pixel resolution limitations in addition to the method of camera calibration.

4.5.2 Pitch and Yaw Case

The next test analyzes model movement in pitch and yaw. As described previously, the rod that the model and follower are affixed to is perpendicular to the back panel of the test section. The only possible motion in this current construction is pitching motion. In order to create pitch and yaw, the back panel of the wind tunnel is rotated so it is no longer parallel to the $x$ and $y$ plane. Figure 4.26 shows the new position of the back panel as it is rotated back out of the wind tunnel frame. When viewing the panel from the front, the left side extends 7.62 cm in the $z$-direction. This provides an angle of rotation of 41 degrees to the original panel position.
The same model and cam are used for this test although the marker location is at front of the model to be in view of both cameras. This change corresponds to a new values for $\alpha_0$ and $\beta_0$ which are 0.0 and 27.46 degrees respectively and are calculated with the dimensions shown in Figure 4.27 and 4.28. The axes designated by SolidWorks do not correspond with those resulting from stereo camera calibration.

The cam driven amplitude of 7.5 degrees must be separated into components observed by the left and front camera which is calculated using trigonometric relationships due to the 41 degree change in panel orientation. The left camera observes an oscillation amplitude of 4.92 degrees while the front camera sees 5.66 degrees. The initial model orientation measured by the level is also separated into components. The initial orientation of the model at 8.3 degrees which results in an initial pitch angle of 6.26 degrees and an initial sideslip angle of 5.44 degrees.

Figures 4.29 through 4.32 show the tracked pixel locations of the marker for both $x$ and $y$ pixel coordinates of the front and left cameras. Figure 4.33 plots the pitch and yaw output from the video analysis program and the oscillations driven by the cam motion. The cam drive oscillations and video data do not overlay each other as they have different offsets from the $y$ axis. This may be a result of the initial, digital level measurement. The frequency of oscillation results from the video analysis program is 2.00 Hz for both pitch and yaw. The pitch oscillation amplitude determined from the video analysis program is 5.93 degrees which is 0.3 degrees from the cam driven oscillation amplitude of 5.66 degrees. The cam driven yaw oscillation amplitude is 4.92 degrees, but the video analysis program determines a slightly smaller amplitude of 4.43 degrees. This results in an error in yaw oscillation amplitude of 0.5 degrees.
The accuracy is within the same magnitude as the pitch-only case. It is likely that some error results from the measurement of the wind tunnel panel orientation, but the large sources of error continue to result from the method of camera calibration and limitations in pixel resolution.

4.6 SINGLE CAMERA VIDEO ANALYSIS METHOD

4.6.1 Pitch Test With Single Camera

The initially planned analysis method for the 1 Degree-of-Freedom (1DOF) test, described in the next chapter, used two cameras to capture model motion. It was later decided that only one camera is needed to capture the planar motion of the model. In order to ensure an accurate data collection method for the 1DOF test, a final validation study using the Flotek 360 and a single camera is undertaken. The method of data reduction is similar to the analysis for two cameras and uses a completely identical tracking program. The front camera is chosen to capture the model oscillations in the validation test because of its similar position to camera position in the SWT.

4.6.2 Single Camera Calibration

A single camera calibration application in MATLAB is used to determine the intrinsic and extrinsic parameters of the camera. The procedure is similar to the Stereo Camera Calibrator Application except a single set of images are required instead of two. The application determines camera properties by detecting the corners of the same checkerboard pattern used in stereo calibration. An example of a calibration image is shown in Figure 4.34. The results of the final calibration are shown in Figure 4.35 and the
accuracy of this calibration is 0.48 pixels. This is a pixel more accurate than stereo camera calibration result.

An origin must be determined by the user instead of the application automatically placing the origin at the first camera. The user determines the origin by placing the checkerboard corner at the desired location. Figure 4.34 shows the location of the chosen origin for the pitch only, single camera validation test. The rotation matrix and translation vector corresponding to this image are used for the conversion from pixel to physical coordinates.

4.6.3 Angle of Attack Extraction

Appendix A.4 includes the program that calculates the pitch angle of the model during testing. The differences between this program and the one used with two cameras are minor except that the function used to convert from pixel to physical coordinates is “pointsToWorld” instead of “triangulate.” This function inputs the pixel coordinates, rotation matrix, and camera parameters and returns the physical coordinates of the marker center. The angle of attack is computed using the same trigonometric relation shown in Figure 4.16 using Equation 4.11.

4.6.4 Results of Pitch Only Test

The same cam and follower mechanism is used for the validation test and the back panel of the wind tunnel is returned to the original position. The front position of the marker on the model is in line with the rotation axis at the model center so the value of \( \alpha_0 \) is zero. The initial pitch angle measured by the digital level is 6.7 degrees. Figure 4.36 and 4.37 show the tracked pixel results from video analysis program. Figure 4.36 shows
no movement in the $x$ coordinate of the marker while Figure 4.37 shows a choppy representation of sinusoidal movement in the $y$ coordinate. Figure 4.38 plots the pitch history determined by the video analysis method and the pitch oscillations driven by the cam mechanism. The results are the most accurate in this chapter with a measured pitch amplitude of 7.45 degrees, resulting in an error of 0.05 degrees. The frequency of oscillation determined by the video analysis method also matches the stepper motor frequency of 2.00 Hz. This reduced pitch amplitude error is due to the improvement in the calibration method. The accuracy of the method is still limited by the pixel resolution of the camera.

4.7 SUMMARY

This chapter details the development of a video analysis validation experiment, the creation of an oscillation mechanism, and the formation of a video analysis program. This chapter provides encouraging results that justify the video analysis method as a robust technique to track the blunt body movement in the planned MSBS system. Table 4.2 shows the compiled results of the amplitude and frequency obtained by the video analysis programs. These are shown next to the actual amplitude and frequency of the cam driven oscillations. Table 4.2 shows the frequency of oscillation determined from the video analysis method is accurate and measures 2.00 Hz for all test cases.

The most exact pitch amplitude determined from the video analysis method is in the single camera test while the least exact is the pitch-only, two camera test. This may be a result of the camera calibration because the stereo camera calibration has a mean pixel error of over 1 pixel where the single camera calibration has a pixel error of only 0.48.
The stereo camera calibration can be improved by capturing additional video frames. There are inherently more error calibrating two cameras simultaneously rather than one.

The accuracy of the video analysis system in all validation tests is limited due to pixel resolution. Pixel results that are plotted Figure 4.37, for example, exhibit a choppy characteristic and show large step sizes between points that occur because of the pixel resolution. This can be reduced by filming closer to the model or using a camera with a higher pixel resolution.

<table>
<thead>
<tr>
<th>Test Case</th>
<th>$\alpha_{\text{cam}}$</th>
<th>$\alpha_{\text{vid}}$</th>
<th>$\beta_{\text{cam}}$</th>
<th>$\beta_{\text{vid}}$</th>
<th>$f_{\text{cam}}$</th>
<th>$f_{\text{vid}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracker-Pitch Only</td>
<td>5.50</td>
<td>5.65</td>
<td>n/a</td>
<td>n/a</td>
<td>2.00 Hz</td>
<td>2.00 Hz</td>
</tr>
<tr>
<td>Stereo-Pitch Only</td>
<td>7.50</td>
<td>8.14</td>
<td>0</td>
<td>0.30</td>
<td>2.00 Hz</td>
<td>2.00 Hz</td>
</tr>
<tr>
<td>Stereo-Pitch and Yaw</td>
<td>5.66</td>
<td>5.93</td>
<td>4.92</td>
<td>4.43</td>
<td>2.00 Hz</td>
<td>2.00 Hz</td>
</tr>
<tr>
<td>Single-Pitch Only</td>
<td>7.50</td>
<td>7.45</td>
<td>n/a</td>
<td>n/a</td>
<td>2.00 Hz</td>
<td>2.00 Hz</td>
</tr>
</tbody>
</table>

**Table 4.2: Combined Results of Video Analysis Feasibility Tests**

The methods shown in this chapter will serve as a helpful foundation from which future video analysis methods can make improvements. The video analysis system provides a useful analysis methodology that may be built upon for the analysis method used in the MSBS. These results provide a demonstration with reasonable accuracy that give confidence as a proof of concept for the method intended for use in the MSBS test facility.
CHAPTER V

ONE DEGREE OF FREEDOM TEST

5.1 INTRODUCTION:

The results of the blockage and wall proximity tests determine the starting characteristics, wind tunnel conditions, and boundary-layer shock interaction behaviors for stationary models. A dynamic test is developed in this chapter to provide an initial examination of model movement in the MSBS. The pitch oscillation amplitude growth or decay is the primary behavior that will be assessed in the MSBS as it shows the model dynamic stability characteristics. Pitch amplitude oscillations can be analyzed to reduce the pitch damping coefficient which is observed to be nonlinear over Mach Number and angle of attack.

The data reduction of three-dimensional model rotation is a complex task. Before this is performed in the MSBS, an initial dynamic test with a simpler method of data reduction is planned. Schoenenberger shows that a one-dimensional rotation, free-to-oscillate test can be analyzed to determine the pitch damping coefficient. His analytical solution to the constant coefficient derivation of the blunt body planar equations of motion has the general form of a simple harmonic oscillator (SHO). The general SHO solution is shown in Equation 5.1 which calculates the model pitch angle as a function of time.

The analysis in a pitch-only test applies the fitting, described below, of a SHO solution to the video analysis data. The coefficients in the SHO solution contain the pitch damping coefficient in the exponential term, shown in Equation 5.2, and the moment slope coefficient in the angular velocity, shown in Equation 5.3. A one degree-of-
freedom (1DOF) test is thereby developed because of the comparative simplicity in analysis and design. This chapter details the results of the effort.

\[ \alpha = Ae^{-et} \cos(\omega t + \delta_p) \]  \hspace{1cm} (5.1)

\[ \varepsilon = \left( \frac{\rho VS}{4m} \right) \left[ \frac{d^2 m}{2I} \left( C_{m_\theta} + C_{m_\alpha} \right) + C_A \right] \]  \hspace{1cm} (5.2)

\[ \omega = \sqrt{-\frac{\rho V^2 SD}{2I} C_{m_\alpha}} \]  \hspace{1cm} (5.3)

5.2 1DOF TEST APPARATUS DESIGN

5.2.1. Design Requirements

Schoenenberger states a complication to mechanical, free-to-oscillate testing is that it is often difficult to identify authentic aerodynamic motion as a result of the sting behind the model supporting it in the test section.\(^5\) It is understood that no mechanically-supported dynamic test is truly free of sting interference, but it is the principal goal during design of this experiment to minimize this interference on model dynamic motion. The design objective is to construct a test apparatus enabling flight-like model movement. This determines the mechanical design consist of a fixed support in the test section that enables single axis model rotation. The support sting is designed to be as small as feasible to minimize interference with the model wake. This fixed support must also contain a bearing to reduce friction of the pitching motion.

A secondary design requirement is that multiple models may be tested using the same sting design. Blockage testing results show sizeable differences in model starting characteristics for different sizes and geometries at different Mach numbers. The sting design must also allow model range of motion. Plus or minus 30 degrees provides
sufficient range of motion for cases with both increasing and decreasing pitch amplitude oscillations.

Figure 5.1 shows an initial sketch for the 1DOF apparatus design. A simple journal bearing is used to enable pitching motion. The journal bearing construction involves a pin, that is press fit through a hole in the tip of the sting, and a bearing bronze ring is slip fit to the pin. Fine threads are cut into the outside cylindrical surface of the ring so that a model with internal threads can attach to the sting. This allows a model to be threaded onto the ring which then pivot together about the pin. The same test model geometries from blockage testing are used for 1DOF testing, but the internal threads in the model remove a portion of the backshell. The method of attachment between the threaded ring and model allows for the easy interchange of models in addition to allowing flexibility in setting the model pivot point. The pivot point relative to the model center of gravity can be adjusted by threading the model onto the ring to different depths. This last feature can be used to explore the change in dynamic stability due to the change in static margin by testing at multiple pivot points within the same model.

5.2.2 Model Selection

In order to determine the necessary strength of the 1DOF assembly, model geometry and sizes are selected. Blockage testing results are used to select model sizes for a particular geometry in the square test section at Mach 2, 2.5 and 3. The square test section is used rather than the axisymmetric because optical access to the model is required.

The test models for this test are chosen to represent the spectrum of potential model geometries that may be tested in the planned MSBS. The same geometries as
tested in blockage and wall proximity testing are chosen with forebody semi-vertex angles of 45, 60 and 70-degrees. Test model selections for the planned MSBS will likely be as large in size as blockage restrictions allow since a larger sized model reduces the magnetic field required from the MSBS. The test model size selections for the 1DOF test thereby approach size limitations set during blockage tests. Table 5.1 shows the largest models that are able to start in the square test section during blockage testing. The percentage listed is the maximum cross-sectional area of the model over the test section area for the SWT.

<table>
<thead>
<tr>
<th></th>
<th>Mach 2</th>
<th>Mach 2.5</th>
<th>Mach 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>70-degree</td>
<td>5.5%</td>
<td>6%</td>
<td>6%</td>
</tr>
<tr>
<td>60-degree</td>
<td>5.5%</td>
<td>6%</td>
<td>8%</td>
</tr>
<tr>
<td>45-degree</td>
<td>6%</td>
<td>~10%</td>
<td>~11%</td>
</tr>
</tbody>
</table>

**Table 5.1:** Largest Models That Start In Square Test Section

The blockage testing results in Table 5.1 are used to select model size for the 1DOF test. There will be more blockage in the test section in 1DOF testing due to the diamond strut so models that are a half percentage smaller than the largest possible size are chosen.

Table 5.1 shows larger models are able to start at higher Mach numbers. In order to test models near blockage limitations for a certain Mach number, two models are chosen for each geometry. For the 70-degree model, a 5.5 and 5 percent model are designed. The 5.5 percent is tested at Mach 2.5 and 3 while the 5 percent is tested at Mach 2. The 60-degree model shows more size variation so a 5 percent is designed to be tested at Mach 2 and 2.5 is designed and a 7.5 percent model is designed for Mach 3. An 8.5 percent, 45-degree model is designed for testing at Mach 2.5 and 3 while a 5.5 percent model is designed for Mach 2.
After the model design choices are made, the expected loading on the assembly is calculated. The loading used to design the 1DOF apparatus is the lift and drag force on the model at the plenum total pressure of 310.3 kPa. This loading represents anticipated aerodynamic forces and is used to ensure the apparatus supports all wind tunnel operating conditions when started.

The drag and lift coefficients shown in Table 5.2 are determined from test results for blunt body entry vehicles with the same forebody shape. The Mars Exploration Rover\textsuperscript{20} is consulted for the 70-degree models, the Stardust Capsule\textsuperscript{21} is consulted for the 60-degree models and the Mars Microprobe\textsuperscript{22} is used for the 45-degree model. The test results provide the aerodynamic properties in the form of the axial and normal force coefficients ($C_A$ and $C_N$) which are related to the corresponding lift and drag coefficients using Equations 5.4 and 5.5.\textsuperscript{23} A force diagram of the model in the SWT is shown in Figure 5.2.

\begin{align}
C_L &= C_N \cos \alpha - C_A \sin \alpha \\
C_D &= C_N \sin \alpha + C_A \cos \alpha
\end{align} (5.4, 5.5)

### 5.2.3 Stress Analysis

This stress analysis details the calculations used to justify the design, material selection, and corresponding dimensioning of the 1DOF testing apparatus to sustain anticipated loading on the models. Table 5.2 shows the anticipated axial loading on the models varies from 100 to 200 N. The 1DOF apparatus is designed to support 200 N of drag force on the model because it is the most extreme condition designed to. There is also a lift component on the support because the axial vector of the model changes with pitch angle. The maximum lift force occurs at 30 degrees for all models and has a value
of 65 N for the 70-degree model at Mach 2. Although the worst case lift force is 72.4 N, the model is ultimately not tested.

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
\text{Mach Number} & \text{Geometry} & \text{Size} & P_{01} & q & C_D & F_{\text{drag}} & C_L & F_L \\
\hline
2 & 70 & 5.0\% & 310.264 & 111.0 & 1.6 & 197.3 & 0.5 & 64.9 \\
2 & 60 & 5.0\% & 310.264 & 111.0 & 1.4 & 182.4 & 0.6 & 72.4 \\
2 & 45 & 5.5\% & 310.264 & 111.0 & 1.2 & 178.6 & 0.4 & 53.6 \\
2.5 & 70 & 5.5\% & 310.264 & 79.4 & 1.5 & 155.3 & 0.5 & 50.1 \\
2.5 & 60 & 5.0\% & 310.264 & 79.4 & 1.4 & 130.5 & 0.6 & 50.9 \\
2.5 & 45 & 8.5\% & 310.264 & 79.4 & 1.2 & 197.5 & 0.3 & 47.1 \\
3 & 70 & 5.5\% & 310.264 & 53.2 & 1.5 & 104.0 & 0.5 & 33.6 \\
3 & 60 & 7.5\% & 310.264 & 53.2 & 1.4 & 131.1 & 0.5 & 48.5 \\
3 & 45 & 8.5\% & 310.264 & 53.2 & 1.2 & 132.3 & 0.2 & 24.4 \\
\hline
\end{array}
\]

**Table 5.2:** Loading Calculation for 1DOF Test Models

5.2.3.1 Sting

The smallest test model and the model with the highest loading determines the sting design and material. The smallest model determines the sting diameter because it must be smaller than model diameter to minimize model wake interference. Previous free-to-oscillate testing uses the design criteria that the sting diameter must be less than 25 percent of the model diameter. The smallest models chosen for the 1DOF test are the 60 and 70-degree, 5 percent models. These models have a 3.78 cm diameter which determines the sting has a diameter of less than 0.95 cm. The final sting diameter chosen is 0.64 cm.

The other sting design requirement is that the model should be located as far upstream from the diamond strut as possible to minimize interference with the model behavior. A drawing of the wind tunnel panel in Appendix B.1 shows the diamond strut can be mounted at a maximum distance from the nozzle exit of 25.4 cm. This diamond
strut location results in a sting length of 20.32 cm. This leaves 5.08 cm of room upstream from the tip of the sting to ensure the model nose is within the test section.

Equation 5.6 shows the calculation that determines the axial stress on the sting. \( P \) is the maximum drag of 200 N while \( A_{\text{sting}} \) is the cross sectional area of the 0.64 cm sting. The axial stress is 6.32 MPa and is shown in Table 5.3 below.

\[
\sigma_{\text{axial}} = \frac{P}{A_{\text{sting}}}
\]  
Equation 5.6

Equation 5.7 represents the bending stress which occurs due to the lift force on the model. This bending stress is greatest at the largest moment arm which is the length of the sting, 20.32 cm. This results in a bending stress at the base of the sting of 525.43 MPa.

\[
\sigma_{\text{bending}} = \frac{M_{\text{bend}}}{I}
\]  
Equation 5.7

Table 5.3 lists the factors of safety for the axial and bending stress on the sting. O1 tool steel is chosen because of its high strength properties which is critical to withhold the large bending stress. The sting has a factor of safety of 2.8 in bending and over 200 in compression. The SolidWorks drawing of this component is located in Appendix B.2.

<table>
<thead>
<tr>
<th>Stress</th>
<th>Axial Stress</th>
<th>Bending Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6.32 MPa</td>
<td>525.43 MPa</td>
</tr>
<tr>
<td><strong>Material Strength</strong></td>
<td>1350 MPa\textsuperscript{24}</td>
<td>1500 MPa\textsuperscript{25}</td>
</tr>
<tr>
<td><strong>Factor of Safety</strong></td>
<td>213.77</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Table 5.3: Loading on Sting

5.2.3.2 Sting Deflection

The deflection of the sting due to model lift is calculated after the final sting geometry is determined. The deflection is calculated for steady state wind tunnel
conditions rather than at tunnel start. The chosen operating plenum total pressures for 1DOF testing are 62.05, 82.74, and 103.42 kPa for Mach 2, 2.5, and 3, respectively.

Table 5.4 shows the expected lift on the model assuming these wind tunnel conditions for models at the limit of pitch angle, 30 degrees.

<table>
<thead>
<tr>
<th>Mach</th>
<th>Geometry</th>
<th>Size</th>
<th>$P_o1$</th>
<th>$q$</th>
<th>$C_L$</th>
<th>$F_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>70</td>
<td>5.0%</td>
<td>62.1</td>
<td>22.2</td>
<td>0.5</td>
<td>13.0</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>5.0%</td>
<td>62.1</td>
<td>22.2</td>
<td>0.6</td>
<td>14.5</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
<td>5.5%</td>
<td>62.1</td>
<td>22.2</td>
<td>0.4</td>
<td>10.7</td>
</tr>
<tr>
<td>2.5</td>
<td>70</td>
<td>5.5%</td>
<td>82.7</td>
<td>21.2</td>
<td>0.5</td>
<td>13.4</td>
</tr>
<tr>
<td>2.5</td>
<td>60</td>
<td>5.0%</td>
<td>82.7</td>
<td>21.2</td>
<td>0.6</td>
<td>13.6</td>
</tr>
<tr>
<td>2.5</td>
<td>45</td>
<td>8.5%</td>
<td>82.7</td>
<td>21.2</td>
<td>0.3</td>
<td>12.6</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
<td>5.5%</td>
<td>103.4</td>
<td>17.7</td>
<td>0.5</td>
<td>11.2</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>7.5%</td>
<td>103.4</td>
<td>17.7</td>
<td>0.5</td>
<td>16.2</td>
</tr>
<tr>
<td>3</td>
<td>45</td>
<td>8.5%</td>
<td>103.4</td>
<td>17.7</td>
<td>0.2</td>
<td>8.1</td>
</tr>
</tbody>
</table>

Table 5.4: Lift Force During Steady State Operation

The maximum lift occurs for the 60-degree model at Mach 3 with a lift force of 16.2 N. Equation 5.8 calculates the maximum deflection which is at the tip of the 20.32 cm sting.\(^{26}\)

\[
\delta_{\text{sting}} = \frac{PL^3_{\text{sting}}}{3EI}
\]  

(5.8)

The modulus of elasticity given by the material data sheet is 190 GPa which results in a deflection of 0.24 cm.\(^{24}\) This is not determined to be significant during the initial design so not modifications to the sting are made.

5.2.3.3 Ring

The ring functions as part of the bearing as well as the threaded connector which fixes the model to the sting. The outside diameter of the ring is determined by the
maximum thread diameter that can fit in the test models. The inside diameter of the ring is determined by the clearance necessary for the model to rotate without hitting the outside surface of the sting. This distance depends on the height of the ring since a larger height causes the ring to impact the sting earlier. The height of the ring is determined to be 0.79 cm to ensure adequate thread engagement between the model and ring.

Figure 5.3 shows a diagram of the ring pivoted to an angle of 30 degrees. This figure shows the close proximity of the ring to the outside of the sting. The ring inner diameter in this figure is 1.11 cm which represents the minimum inner diameter that allows the ring to pivot 30 degrees fully. The inner diameter of the thread results in a ring outer diameter of 1.75 cm. This dimension is chosen to ensure the ring will be thick enough after the external threads are cut into the outside surface. This results in the outer thread size of 11/16-32 (inch) which is unconventional, but still a fine thread that enables more engagement between the ring and model and more precise adjustment in model pivot locations. Appendix B.3 shows the engineering drawing of the ring. Bearing bronze is chosen for the ring material because of the low friction coefficient with a steel pin.

5.2.3.4 Pin

The pin in the 1DOF assembly is designed to be as small as possible to minimize the contact surface between the ring and pin. It must be strong enough, however, to handle the loading on the model. The pin diameter that satisfies both these design requirements is 0.25 cm.

The pin experiences stress from shear loading and bending, where loading occurs at both ends of the pin where it supports the ring. The bending moment is calculated with the same formula as shown as in Equation 5.7, but the force causing the bending moment
is the drag force on the model. The bending stress is considered on one end of the pin. The bending moment is calculated from half the drag force (100 N.) The moment arm is 0.25 cm which is the distance from the ring inner diameter to the sting outside diameter. This results in a bending moment of 0.24 N-m and a corresponding bending stress of 179.63 MPa.

The shear stress, \( \tau \), experienced by the pin is calculated by Equation 5.9 which assumes a circular cross section.\(^{26}\) The shear force, \( V \), is equivalent to the drag force and \( A_{\text{pin}} \) is the circular cross-sectional area of the pin. The shear stress on the face of the pin has a magnitude of 59.88 MPa.

\[
\tau = \frac{4V}{3A_{\text{pin}}}
\]  
(5.9)

A dowel pin is chosen as the material because of the polished finish, small size options, and high strength properties. Table 5.5 shows the safety factors for both shear and bending stress on the pin and Appendix B.4 includes a SolidWorks drawing of the part. No information for the shear strength is available in the material datasheet for the dowel pin so that property is calculated using Equation 5.10.\(^{26}\)

\[
S_{sy} = \frac{\sqrt{3}}{3}S_y
\]  
(5.10)

<table>
<thead>
<tr>
<th>Stress</th>
<th>Bending Stress</th>
<th>Shear Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material Strength</td>
<td>862 MPa(^{27})</td>
<td>498 MPa</td>
</tr>
<tr>
<td>Factor of Safety</td>
<td>4.79</td>
<td>8.31</td>
</tr>
</tbody>
</table>

Table 5.5: Loading Analysis for Pin
5.2.3.5 Diamond Strut

The diamond strut connects the sting to the bottom panel of the test section. The diamond strut is secured to both the sting and wind tunnel panel using hardware. The hardware supports the shear and axial stress from the drag force, but the diamond strut supports the torsion due to the model lift. Equation 5.11 calculates the shear stress for torsion applied to a prismatic beam, a sufficient approximation for the geometry of the diamond strut.\(^{28}\) The torsion, \(T\), is calculated by multiplying the maximum model lift, 65 N, by the sting length of 20.32 cm. The stress factor, \(k\), is determined from the aspect ratio of the diamond strut.

\[
\tau = \frac{T}{kw_{\text{side}}h_{\text{side}}^2} \quad (5.11)
\]

Equation 5.11 calculates a shear stress of 71.46 MPa due to the torsion of 13.21 Nm. AISI 1020 Cold Rolled Steel is chosen for the diamond strut and Table 5.6 shows the factor of safety for the material. The yield strength of 350 MPa\(^{29}\) is the only material property available so the allowable shear yield strength is calculated with Equation 5.10.

<table>
<thead>
<tr>
<th>Shear Stress</th>
<th>71.46 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material Strength</td>
<td>202.07 MPa</td>
</tr>
<tr>
<td>Factor of Safety</td>
<td>2.83</td>
</tr>
</tbody>
</table>

**Table 5.6**: Loading Analysis for Diamond Strut

The design of the diamond strut is chosen to minimize blockage in the tunnel as possible. In addition, the diamond strut cannot cause shock waves that may interact with the model so the nose of the diamond strut is designed with a slender 10-degree angle. This results in a weak attached shock wave to the nose of the diamond strut. The dimensions of the diamond strut are shown in Appendix B.5 in the SolidWorks drawing.
5.2.3.6 Hardware and Other Components

In order to attach the diamond strut to the sting and wind tunnel panel, two sets of bolts are needed. The bolts are chosen to fit the size constraints due to the diameter of the sting and the width of the diamond strut. Each set of two bolts is designed to withstand shear loading due to the drag force on the model. Table 5.7 shows the loading analysis and resultant factor of safety for the bolt selections. Two 5-40 bolts are used to attach the sting to the diamond strut and two 10-24 bolts are used to attach the diamond strut to the wind tunnel panel. The loading analysis for both sets of bolts indicates only one bolt is necessary, but two bolts are used to help align the sting and diamond strut in the test section.

Only the tensile strength of 1172.11 MPa\textsuperscript{30} is shown in the steel bolt material properties. In order to get the shear strength of the hardware, Equations 5.12 and 5.10 are used to relate the tensile strength to the yield strength and then the shear strength as a function of the yield strength.

\[
S_y \approx 0.62S_u
\]  

<table>
<thead>
<tr>
<th>Bolt Type</th>
<th>5-40</th>
<th>10-24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Bolts</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Shear Stress</td>
<td>12.63</td>
<td>5.47</td>
</tr>
<tr>
<td>Material Strength</td>
<td>419.56 MPa</td>
<td>419.56 MPa</td>
</tr>
<tr>
<td>Factor of Safety</td>
<td>33.22</td>
<td>76.75</td>
</tr>
</tbody>
</table>

Table 5.7: Loading Calculation for Bolts in Assembly

The final component of the 1DOF support are the spacers that are located between the ring and sting. The distance between the ring inner diameter to the outer sting diameter is not fixed in the original sting design. This is problematic because it may
allow the model to move off centerline during testing. To prevent this, nylon spacers are placed around the pin and inside the inner diameter of the ring. This ensure that the ring is centered and that no translational movement of the pin and ring is allowed. The spacers are custom-made for the apparatus since the sizing tolerances are very high. The spacers must keep the ring centered, but not contribute as source of friction as the ring moves around the pin.

### 5.2.4 Test Models

The design of the 1DOF test models allows the model to pivot about a location that represents the center of gravity of past flight vehicles with similar geometries. The aerodynamics of past flight vehicles are well documented which is helpful to compare to results of the 1DOF test. The 70, 60 and 45-degree model will be compared with test data for the Mars Exploration Rover Entry\(^{20}\), Stardust Capsule\(^{21}\), and Mars Microprobe Entry vehicle.\(^{22}\) The semi-vertex forebody angle is the same for the 1DOF model geometries and the compared flight vehicle. Table 5.8 lists the non-dimensional center of gravity for the chosen flight vehicles which is used in the model design. It should be noted that the final test models do not actually share the same center of gravity as these vehicles, but can pivot about the center of gravity location of these flight vehicles.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Center of Gravity ((X_{cg}/D))</th>
</tr>
</thead>
<tbody>
<tr>
<td>70-degree, Mars Exploration Rover Capsule</td>
<td>0.27, 0.30 and 0.33</td>
</tr>
<tr>
<td>60-degree, Stardust Entry Capsule</td>
<td>0.36</td>
</tr>
<tr>
<td>45-degree, Mars Microprobe Entry Capsule</td>
<td>0.2477</td>
</tr>
</tbody>
</table>

**Table 5.8:** List of Center of Gravity Locations for Flight Vehicles

The test model size for the 1DOF test is intended to be representative of future MSBS test models. It is also desired to match the mass properties of models potential
models used in the MSBS. This is important because the mass properties of a test model affect the pitch oscillation behavior.

It is desired to match the moment of inertia for planned MSBS models since this parameter determines model oscillation behavior. Figure 5.4 and 5.5 show two potential MSBS model geometries. Figure 5.4 represents the 5 percent, 70-degree model and Figure 5.5 represents the 45-degree, 5.5 percent model. The core of MSBS models is iron, but the final outside material has not been selected. Two potential materials under consideration are nylon and aluminum so MSBS assemblies are created with both outside materials in SolidWorks. 1DOF test model design aims to have moment of inertia in the range of that expected for a nylon and aluminum MSBS model. The moment of inertia for the aluminum and nylon outside materials is shown in the second and third column from the right in Table 5.9. The rightmost column shows the moment of inertia for the 1DOF test models.

<table>
<thead>
<tr>
<th></th>
<th>MSBS-Aluminum (g*cm³)</th>
<th>MSBS-Nylon (g*cm³)</th>
<th>1DOF-with Brass Ring (g*cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7005</td>
<td>40.02</td>
<td>21.14</td>
<td>35.09</td>
</tr>
<tr>
<td>7005.5</td>
<td>50.78</td>
<td>26.81</td>
<td>44.39</td>
</tr>
<tr>
<td>6005</td>
<td>52.65</td>
<td>34.63</td>
<td>38.98</td>
</tr>
<tr>
<td>6007.5</td>
<td>145.25</td>
<td>95.66</td>
<td>106.72</td>
</tr>
<tr>
<td>4505.5</td>
<td>102.17</td>
<td>82.29</td>
<td>90.78</td>
</tr>
<tr>
<td>4508.5</td>
<td>305.55</td>
<td>242.17</td>
<td>285</td>
</tr>
</tbody>
</table>

**Table 5.9: Moment of Inertia for Test Models**

Table 5.9 demonstrates that the moment of inertia of all the 1DOF models falls within the range of the MSBS aluminum and nylon models. The 60 and 70-degree designs are fabricated with aluminum, but the 45-degree model is comprised of both steel and aluminum. 45-degree models have too low of a moment of inertia with only aluminum so they are fabricated in two separate pieces with the backshell out of steel and
the forebody with aluminum. Two materials are necessary because the 45-degree MSBS models fit the largest iron core as shown by Figure 5.5. The SolidWorks drawings are included in Appendix B.6 and B.7 for the 70-degree models and Appendix B.8 and B.9 for the 60-degree models. The 45-degree models have three drawings for each model. The first two drawings show the upper and lower model sections and the third drawing shows the assembly. The 5.5 percent model drawing is included in Appendix B.10, B.11 and B.12 and the 8.5 percent model is in Appendix B.13, B.14 and B.15.

The fabricated models are shown in Figure 5.6 after they are spray painted to eliminate reflectiveness to reduce issues seen in the tracking program. A CAD assembly of the 1DOF apparatus is shown in Figure 5.7 and the actual installed fabricated assembly is shown in Figure 5.8.

5.2.5 Retraction Mechanism

The pitch damping coefficient varies depending on the initial pitch angle of the model.1 In order to test this variability, three model supports are created to position the model at a different initial pitch angle. These supports also prevent model movement during tunnel start-up. The initial angles of 5, 15, and 25 degrees are chosen and a support for each angle is 3D-printed. The 3D-printed supports contact the ring than the model so each support works for every model configuration. The drawings for the 5, 15, and 25-degree sleeve are shown in Appendix B.17, B.18, and B.19.

A complication to the testing procedure is that the support will have to be removed quickly without interfering with the oscillations once the test began. The support cannot be removed manually since the test cell must be free of personnel during tunnel operation, so a mechanism to remove the support remotely is designed.
The mechanism that is designed uses a stepper motor to pull a pin. This allows the spring attached to the support to quickly retract it. The brass sleeve shown in Appendix B.16 is placed around the sting and two springs are attached to the sleeve as well as the diamond strut. The pin goes through a hole in the sleeve which initially holds it in place against the springs. The 3D-printed support pieces are clamped around the sleeve with six screws and nuts at the upstream end. To retract the support, the pin is pulled by the remotely triggered stepper motor, as shown in Figure 5.9, and the springs quickly pull the sleeve in the downstream direction away from the model. The initial and final positions of the sleeve and support are shown in Figure 5.10 and 5.11.

In order to select a spring that moves the sleeve quickly enough, the necessary acceleration and spring constant of the spring are calculated using Equations 5.13 and 14. The minimum time for complete removal is defined as the necessary time for the fastest pitching model (25 Hz) to complete a half an oscillation.

\[
F = k_{\text{spring}} x - \mu W = ma \tag{5.13}
\]

\[
a = \frac{k_{\text{spring}} x}{m} - \mu g \tag{5.14}
\]

\[
t = \sqrt{\frac{2d}{a}} < \frac{.5}{25 \text{ Hz}} = 0.02 \text{ sec} \tag{5.15}
\]

Table 5.10 shows springs that are considered for the apparatus. Two springs are used for the 1DOF apparatus. The spring needs to stretch a minimum of 2.54 cm. This is the maximum distance the support needs to move out of the way to not obstruct the model. The corresponding retraction times are calculated using Equation 5.15 with the acceleration from Equation 5.14. Based on the removal time requirement, the first spring is chosen for the design.
<table>
<thead>
<tr>
<th>Spring Selection</th>
<th>cm Stretch</th>
<th>N/m $k_{spring}$</th>
<th>sec</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.81</td>
<td>0.11</td>
<td>0.019</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3.81</td>
<td>0.09</td>
<td>0.021</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.54</td>
<td>0.12</td>
<td>0.023</td>
<td></td>
</tr>
</tbody>
</table>

**Table 5.10**: Support Removal Times for Spring Type

5.3 VIDEO ANALYSIS METHOD

5.3.1 Camera Installation

A camera mount is built out of T-Slot aluminum extrusions that positions the camera to record mode movement through the top window in the test section. The camera mount is designed with a box structure as shown in Figure 5.12 by repurposing components from the original Flotek 360 camera mount. The handlebar camera mount is connected to a section of the 2.54 cm steel rod which runs parallel to the test section. The box structure itself is built out of 45.72 cm, 5.08 by 2.54 cm extrusions and 39.37 cm, 2.54 by 2.54 cm extrusions that ran parallel to the test section. Four 60.96 cm, 2.54 by 2.54 cm extrusions provide the height above the test section and has room for adjustment so the optimal distance between the top window and camera is found. After the camera is positioned in a suitable location, the mount is clamped securely to the table so the camera does not move during testing. This is critical to ensure consistent analysis results.

The camera is re-installed daily at the beginning of test which causes some variability in camera field of view pointing, but a level is used to ensure the camera is parallel to the top window of the SWT. Figure 5.13 shows the assembled camera mount.
The camera is triggered synchronously with the mechanism that releases the retraction device. This ensures the camera captures the entire testing period after the model release.

5.3.2 Camera Calibration

The single camera calibration application from MATLAB is used for the 1DOF test in the SWT. The camera calibration is performed on the first day of testing and is used for the remaining days of wind tunnel testing. Only one calibration is necessary because the camera mount position is fixed and only a small change in camera view results from daily installation on the handlebar mount.

The top window in the SWT is 6.35 cm wide which prevents the original checkerboard used for calibration in the Flotek 360 from being viewed entirely. A new checkerboard pattern is created with 5.7 mm squares which is much smaller compared to the 14 mm squares in the original checkerboard pattern. The result of the camera calibration is shown in Figure 5.14. More calibration images of the checkerboard are taken so ensure the highest possible accuracy is attained. The location of the origin is indicated in Figure 5.15. The calibration results in a mean error of 0.29 pixels which is the smallest calibration error compared with those in the video analysis method development.

5.3.3 Video Analysis Program

The frames of data from the high-speed camera are reduced using the video analysis method developed in Chapter IV. Each image frame from testing is processed with the tracking program and is then converted into physical coordinates using the
camera calibration detailed in the earlier section. The marker physical coordinates are then analyzed to determine the resultant pitch angle.

An example of the tracking program analyzing the 1DOF video footage is shown in Appendix A.5. The tracking program is very similar to the code shown in Appendix A.1, but the program does not analyze every frame from the high-speed video. This is because the test period is not represented by the start and end of the camera footage, but rather by the beginning frame when the support moves and the final frame when the model has stilled. The starting point is determining the image where the retraction support first moves. This frame is represented by the variable “f1.” The end point, “f2”, is when the model has stopped oscillating.

Another change in the tracking program is that the best tracking results are achieved with a lower intensity for the “strel” function. This function applies a filter to the image to enhance the marker outline. This is likely due to the dimmer lighting conditions seen during the test.

The angle extraction portion of the code shown in Appendix A.6 is very similar except the trigonometric relationship that is used to find the pitch angle is tangent instead of sine. In addition, all oscillations observed during testing damped down so that the model eventually returns to a pitch angle of zero. This behavior determines the “a_corr” variable because it is known that the final model pitch angle is zero. This analysis resolves any discrepancies in initial marker location.

An element of error that is introduced in this method of analysis is the uncertainty in selecting the center of the pin in pixel coordinates. The camera is close enough to the pin about which the model rotates that the center is difficult to determine because of the
pixelated pin outlined. In addition, during the installation and removal of the camera, the rotation axis center location changes so the coordinates of the pin must be analyzed frequently.

The final data analysis step fits a Simple Harmonic Oscillator (SHO) function to the test data. The SHO fit determines the aerodynamic coefficients for a particular test using Equation 5.2 and 5.3. Appendix A.7 shows the code used to fit SHO function to the test data. The code reads in the variable “a” which corresponds to the reduced pitch history for a particular test. The frequency of the test is determined by where the pitch angle crosses the $x$ axis for a particular test. This is calculated by determining the frame before and after it crosses the $x$ axis and interpolating between the two frames to approximate the actual point of intersection. This first point of intersection ($i_1$) represents a quarter of an oscillation and the corresponding frequency can be calculated using Equation 5.16. The second point of intersection ($i_2$) is when the pitch angle crosses the $x$ axis again and goes from positive to negative. This point represents 0.75 of a full oscillation and a second frequency is calculated in Equation 5.17.

$$f_1 \left( \frac{\text{cycle}}{\text{sec}} \right) = \frac{.25 \text{ cycle} \cdot 480 \frac{\text{frames}}{\text{sec}}}{i_1 \text{ frames}} \tag{5.16}$$

$$f_2 \left( \frac{\text{cycle}}{\text{sec}} \right) = \frac{.75 \text{ cycle} \cdot 480 \frac{\text{frames}}{\text{sec}}}{i_2 \text{ frames}} \tag{5.17}$$

$$\omega = 2\pi f = 2\pi \frac{f_1 + f_2}{2} \tag{5.18}$$

These two frequencies of oscillation are then averaged to provide the best fit, final frequency as shown in Equation 5.18. The data is then broken up into half cycles. This
allows for the maximum amplitude to be found for each half cycle. The absolute
maximums of are used to determine the exponential term in the SHO equation.

In order to determine the exponential term for the SHO fit, the least squares fitting


technique is performed for an exponential curve, terms $A$ and $\varepsilon$ in Equation 5.19. The

functional form is shown in Equation 5.19 and the variables are calculated by using

summations shown in Equation 5.21 and 22.

\[
a = Ae^{\varepsilon t} \tag{5.19}
\]

\[
A = e^C \tag{5.20}
\]

\[
C = \frac{\sum_{i=1}^{n} (t_i^2 a_i) \sum_{i=1}^{n} (a_i \ln a_i) - \sum_{i=1}^{n} (t_i a_i) \sum_{i=1}^{n} (t_i a_i \ln a_i)}{\sum_{i=1}^{n} (a_i) \sum_{i=1}^{n} (t_i^2 a_i) - (\sum_{i=1}^{n} t_i a_i)^2} \tag{5.21}
\]

\[
\varepsilon = \frac{\sum_{i=1}^{n} (a_i) \sum_{i=1}^{n} (t_i a_i \ln a_i) - \sum_{i=1}^{n} (t_i a_i) \sum_{i=1}^{n} (a_i \ln a_i)}{\sum_{i=1}^{n} (a_i) \sum_{i=1}^{n} (t_i^2 a_i) - (\sum_{i=1}^{n} t_i a_i)^2} \tag{5.22}
\]

This is performed numerically in MATLAB and the best exponential fit is
determined. The phase of the Equation 5.23 is determined to be $\pi$ because SHO fit must
be shifted half a cycle. This is because the calculated amplitude is a positive value while
initial pitch angle is negative. The code returns all of the necessary variables to determine
a SHO fit having the functional form shown in Equation 5.23.

\[
\alpha = Ae^{-\varepsilon t} \cos(\omega t + \pi) \tag{5.23}
\]

5.4 DESIGN MODIFICATIONS

5.4.1 Original Test-Matrix

The original test matrix for the 1DOF test at Mach 2, 2.5 and 3 is shown in Tables
5.11, 5.12, and 5.13. Six models in total are fabricated so that one model of each
game is tested per Mach Number. Two different centers of gravity are chosen at
which the models are tested. The first center of gravity is chosen to match flight vehicle mass properties and the second center of gravity is the furthest back the ring can be positioned with maximum thread engagement. This position results in a much smaller static margin and the difference in the oscillations produced will be compared with the flight vehicle mass property location. The other adjustable configuration is the initial pitch angle of the model at either 5, 15 or 25 degrees angle of attack (AoA) which is achieved by changing out the 3D-printed support. These possible alterations provide 18 individual configurations per Mach Number. The testing process requires three tests for a particular configuration to ensure the repeatability of model oscillations. Each table shows the model of each geometry in italics as well as the two center of gravity points in the second and third columns.

<table>
<thead>
<tr>
<th>$X_{cg}/D$</th>
<th>0.27</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>AoA</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>7005</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>$X_{cg}/D$</td>
<td>0.36</td>
<td>0.4</td>
</tr>
<tr>
<td>AoA</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>6005</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>$X_{cg}/D$</td>
<td>0.25</td>
<td>0.31</td>
</tr>
<tr>
<td>AoA</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>4505.5</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

**Table 5.11:** Mach 2 1DOF Test Matrix
Table 5.12: Mach 2.5 1DOF Test Matrix

| $X_{cg}/D$ | 0.27 | 0.3 |
| AoA        | 5 15 25 | 5 15 25 |
| 7005.5     | X X X | X X X |

| $X_{cg}/D$ | 0.36 | 0.4 |
| AoA        | 5 15 25 | 5 15 25 |
| 6005       | X X X | X X X |

| $X_{cg}/D$ | 0.25 | 0.28 |
| AoA        | 5 15 25 | 5 15 25 |
| 4508.5     | X X X | X X X |

Table 5.13: Mach 3 1DOF Test Matrix

| $X_{cg}/D$ | 0.27 | 0.3 |
| AoA        | 5 15 25 | 5 15 25 |
| 7005.5     | X X X | X X X |

| $X_{cg}/D$ | 0.36 | 0.4 |
| AoA        | 5 15 25 | 5 15 25 |
| 6007.5     | X X X | X X X |

| $X_{cg}/D$ | 0.25 | 0.28 |
| AoA        | 5 15 25 | 5 15 25 |
| X X X | X X X | X X X |

5.4.2 1DOF Design Modification

The first day of testing revealed problems inherent in the design of the original sting. The first tests occur at Mach 2 with the 7005 model. The high speed video capture of test shows the model completes a quarter of an oscillation as it immediately returns to centerline once released. The sting is also deflecting a large portion due to the lift forces at the tip. Figure 5.16 shows two images from the 7005 model test at Mach 2 with the left image showing the model after release and the right hand image showing the model after it returned to centerline. The two images show the sting deflects considerably due to the model lift and as such, the sting cannot be correctly described as rigid.
Figure 5.17 shows the force diagram of the sting deflections and the effect on the model oscillations. The sting is deflecting at the pin location due to the lift on the model. This introduces a moment because the center of gravity of the model is forward of the pin location. It can be observed that as the model pitches up, a damping moment in the opposite direction is introduced as the sting deflects downwards. It is known that damping due to friction exists, but the damping due to sting deflection is unexpected and undesirable. The additional damping moment introduces another factor that obscures the aerodynamic data further so the sting and models are redesigned to reduce this effect.

The moment is eliminated by removing the moment arm between the model center of gravity and the pivot point. This is achieved by testing at the true center of gravity for the test models. This is only possible for the 45-degree models because the steel backshell results in a more aft center of gravity compared to the 60 and 70-degree models. The center of gravity is found in the models by taking the 1DOF sting out of the test section, rotating it 90 degrees and finding the balance point using gravity. Unfortunately, the 8.5 percent, 45-degree model balance point is only with 1.5 threads engaged so it is not able to be tested. The smaller 45-degree model balances with sufficient 4.5 threads engaged which results in a non-dimensional center of gravity of 0.415.

The 70-degree model center of gravity is forward of the possible pivot points so the model is modified. Eight holes are drilled into the back shell and 0.48 cm brass slugs that are 0.635 cm in length are affixed in the holes. This moves the model non-dimensional center of gravity to 0.275 from the model nose. A CAD image of the new 5
percent, 70-degree model with the brass inserts is shown in Figure 5.18. It is desired to adjust the center of gravity of more models, but no more funds are available to do so.

In addition to redesigning the models, the sting is shortened to reduce deflection due to model lift. Table 5.14 shows the deflection due to the 70-degree model at Mach 2 with the original sting and the modified sting that is shortened by 5.08 cm. The highest loading the sting will experience due to lift during steady state operation is 13 N and is with the 70-degree, 5 percent model at 25 degrees. The values in Table 5.14 are calculated using Equation 5.8.

<table>
<thead>
<tr>
<th>Sting Length</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.32 cm</td>
<td>0.192 cm</td>
</tr>
<tr>
<td>15.24 cm</td>
<td>0.075 cm</td>
</tr>
</tbody>
</table>

Table 5.14: Maximum Deflection Compared for New and Original Sting

The above table shows the sting deflection is reduced considerably from taking 5.08 cm off the length of the sting. The 1DOF sting assembly is returned to the Case Fabrication Center and both the sting and brass retraction sleeve are modified. The drawings for these modifications are included in Appendix B.20 and B.21.

5.4.3 1DOF Modified Test Matrix

Table 5.15 shows the modified test matrix after the model and sting modifications are made. Only two models, the 7005 and 4505.5, are able to be tested at the actual center of gravity which greatly reduces the size of the test matrix. The models are also only tested at an initial angle of attack of 15 and 25 degrees. This is because the bearing friction is significant enough that models tested at 5 degrees do not complete more than 0.25 of an oscillation. The 15 and 25 degree supports saw at least 0.75 of an oscillation which justified testing.
Table 5.15: Modified 1DOF Test Matrix

<table>
<thead>
<tr>
<th>Mach</th>
<th>AoA</th>
<th>AoA</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>15</td>
<td>7005</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>X</td>
</tr>
<tr>
<td>2.5</td>
<td>15</td>
<td>X</td>
</tr>
<tr>
<td>2.5</td>
<td>25</td>
<td>X</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>X</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>X</td>
</tr>
</tbody>
</table>

5.5 BEARING FRICTION ANALYSIS

5.5.1 Derivation Including Bearing Damping

It is known during the initial design phase that the bearing imparts some damping force that resists the model pitching motion. To characterize the severity of this damping, the bearing friction coefficient is determined by solving for the value for a test case with variable pitch damping coefficients. This determines the magnitude of bearing term.

The equation of motions from Schoenenberger “Limit Cycle Analysis” paper are re-derived with bearing friction included. In this paper, Schoenenberger begins with four equations of motion for the blunt body shown in Equation 5.24 through 5.27.

\[
\dot{V} = -\frac{\rho V^2 S C_D}{2m} - g \sin \gamma_0 \tag{5.24}
\]

\[
\dot{\gamma} = \frac{\rho V S C_L}{2m} - \left(\frac{g}{V} - \frac{V}{R_{\text{Earth}}}\right) \cos \gamma_0 \tag{5.25}
\]

\[
\dot{\theta} = \frac{\rho V^2 S d}{2l} \left( C_{m_q} \frac{\dot{\theta} d}{2V} + C_{m_\alpha} \frac{\dot{\alpha} d}{2V} + C_{m_\alpha} \alpha \right) \tag{5.26}
\]

\[
\theta = \alpha + \gamma \tag{5.27}
\]

Equation 5.24 is the sum of drag and gravitational forces on a blunt body in flight. Equation 5.25 represents the conservation of momentum equations due to the lift, gravity.
and centripetal forces. Equation 5.26 is the sum of static and dynamic moments on a blunt body that influence the rotation acceleration. Finally, Equation 5.27 represents the coordinate system shown in Figure 5.19. Another term is added to Equation 5.26 to account for bearing friction in the 1DOF test so accurate analysis of model movement can be performed.

Mungan represents frictional torque shown in the following form of Equation 5.28.\(^{32}\)

\[
T_f = f_b \dot{\theta}
\]  \hspace{1cm} (5.28)

This term is inserted into Equation 5.26 and the same derivation procedure by Schoenenberger is followed with this new term. Equation 5.29 shows the new equation for the rotational acceleration on the test model.

\[
\ddot{\theta} = \frac{\rho V^2 S d}{2l} (C_{m_d} \frac{\dot{\theta} d}{2V} + C_{m_a} \frac{\dot{\alpha} d}{2V} + C_{m_a} \alpha) - \frac{f_b \dot{\theta}}{I}
\]  \hspace{1cm} (5.29)

In the original derivation, Schoenenberger neglects the gravitational and centripetal terms in Equation 5.24 and 25 so the new sum of forces is shown in Equation 5.30 and the new derivative of the flight path angle is shown in Equation 5.31.

\[
\dot{\gamma} = -\frac{\rho V^2 S C_D}{2m}
\]  \hspace{1cm} (5.30)

\[
\dot{\gamma} = \frac{\rho V S C_L}{2m}
\]  \hspace{1cm} (5.31)

The left hand side (LHS) of Equation 5.29 is replaced with the second derivative of Equation 5.27. The derivative of Equation 5.25 replaces the resultant second derivative of the flight path angle in Equation 5.27.

\[
\ddot{\theta} = \ddot{\alpha} + \dot{\gamma} = \ddot{\alpha} + \frac{d}{d\alpha} \left( \frac{\rho V S C_L}{2m} \right) = \ddot{\alpha} + \frac{\rho V S C_{L,\alpha}}{2m} \dot{\alpha} + \frac{\rho V S C_L}{2m}
\]  \hspace{1cm} (5.32)
Equation 5.30 then replaces the velocity derivative shown in the Equation 5.32. Equation 5.33 shows the final second derivative of theta calculated from 5.27 which is then substituted back into the Equation 5.29. The new form of Equation 5.29 shown in Equation 5.34.

\[ \ddot{\theta} = \ddot{\alpha} + \frac{\rho V S C_{L\alpha} \dot{\alpha}}{2m} + \left( \frac{\rho V S}{2m} \right)^2 C_D C_{L\alpha} \alpha \]  

(5.33)

\[ \ddot{\alpha} + \frac{\rho V S C_{L\alpha} \dot{\alpha}}{2m} + \left( \frac{\rho V S}{2m} \right)^2 C_D C_{L\alpha} \alpha \]  

(5.34)

\[ = \frac{\rho V^2 S d}{2l} \left( C_{m_q} \frac{\dot{\theta}}{2V} + C_{m_a} \frac{\dot{\alpha} d}{2V} + C_{m_a} \alpha \right) - \frac{f_b \dot{\theta}}{l} \]

\[ \dot{\theta} = \dot{\gamma} + \dot{\alpha} \approx \dot{\alpha} \]  

(5.35)

The third term on the LHS of Equation 5.34 is neglected by Schoenenberger due to its small contribution to the angular acceleration of the model. The first derivative of theta in 5.34 is replaced by the first derivative of Equation 5.27. Equation 5.35 shows the change in flight path angle is not be significant so the first derivative of theta is approximated as the change in pitch angle.

After these simplifications, all terms are brought to the right hand side (LHS) of Equation 5.34. Its new form is shown below in Equation 5.36.

\[ \ddot{\alpha} - \frac{(\rho V S)^2}{2m} \left[ \frac{d^2 m}{2l} \left( C_{m_q} + C_{m_a} \right) - C_{L\alpha} - \frac{2f_b m}{1\rho V S} \right] \dot{\alpha} \]  

(5.36)

\[ - \frac{\rho V^2 S d C_{m_a}}{2l} \alpha = 0 \]

Schoenenberger then substitutes the lift force slope coefficient for the axial coefficient because these are nearly the same for low lift to drag vehicles at small angles of attack.
\[
\ddot{\alpha} - \left( \frac{\rho V S}{2m} \right) \left[ \frac{d^2 m}{2I} \left( C_{m_q} + C_{m_\alpha} \right) + C_A - \frac{2f_b m}{I \rho V S} \right] \dot{\alpha} - \frac{\rho V^2 S d C_{m_\alpha}}{2I} \alpha = 0
\] 

(5.37)

The form of this equation is the same as a simple harmonic oscillator (SHO) where the general form is shown in Equation 5.38 and the corresponding solution to this equation is shown in 5.39. The value of the coefficients for the solution to Equation 5.37 are shown in Equation 5.40 and 5.41.

\[
\ddot{\alpha} + 2\varepsilon \dot{\alpha} + \omega^2 \alpha = 0
\] 

(5.38)

\[
\alpha = A e^{-\varepsilon t} \cos(\omega t + \delta_p)
\] 

(5.39)

\[
\varepsilon = \left( \frac{\rho V S}{4m} \right) \left[ \frac{d^2 m}{2I} \left( C_{m_q} + C_{m_\alpha} \right) + C_A - \frac{2f_b m}{I \rho V S} \right]
\] 

(5.40)

\[
\omega = \sqrt{ - \frac{\rho V^2 S d C_{m_\alpha}}{2I}}
\] 

(5.41)

The new constant coefficients in the solution of Equation 5.37 are exactly the same as the original derivation the bearing friction is contained in the damping term in Equation 5.40. This is consistent with its physical effect imparting a damping moment. During video analysis, the solution shown in Equation 5.39 is fit to the reduced model pitch oscillations using the method described previously. The exponential term in the SHO best fit solution can be used to determine the bearing friction coefficient. This value can be solved for independently in model configurations with known pitch damping coefficients. Once the bearing term ‘\(f_b\)’ is defined, this term can be used to approximate the pitch damping coefficients for the remaining model configurations.
5.5.2 Damping Results

The damping results for the 70-degree, 5 percent model are assessed from three tests at Mach 2.5 with an initial pitch angle of 25 degrees. The data from the video and the resultant SHO fit for each test is plotted for each of the three tests in Figures 5.20, 5.21 and 5.22. The corresponding amplitude, frequency and damping that is returned from the SHO fit program is shown in Table 5.16.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Frequency</th>
<th>Damping</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.08</td>
<td>-34.69</td>
<td>29.06</td>
</tr>
<tr>
<td>2</td>
<td>24.51</td>
<td>-37.81</td>
<td>32.84</td>
</tr>
<tr>
<td>3</td>
<td>23.44</td>
<td>-34.72</td>
<td>32.77</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>24.01</td>
<td>-35.74</td>
<td>31.56</td>
</tr>
</tbody>
</table>

Table 5.16: Variables from SHO Fit for Video Data

These figures show that the damping portion of SHO fit does not match the test data precisely. It results in an initially larger amplitude than the first test data point and a shallower value for the smaller amplitude completing the first oscillation. In addition, the SHO fit has questionable accuracy for the remainder of test data. These results indicate that the damping coefficient is may not be a constant value and may need to be modeled as changing over time. The constant damping model is a suitable approximation to characterize the influence of the bearing friction.

The original strategy to determine the bearing friction coefficient is to solve for it using known pitch damping data from other model tests. Before this strategy is attempted, the influence on the damping term due to both the moments from the bearing friction and pitch damping coefficient is determined. This is performed by calculating the bearing friction coefficient for three different pitch damping coefficients, assuming a constant damping value. The damping value is chosen to be -35.74 which is the average.
of the SHO fit results in Table 5.16. The bearing term is solved for by re-arranging
Equation 5.40. The axial coefficient is set to zero since it is desired to only compared the
bearing friction and pitch damping moments. The re-arranged form is shown in Equation
5.42.

\[ f_b = \frac{\rho V S d^2}{4} \left( c_{m_q} + c_{m_a} \right) - 2 \epsilon l \] (5.42)

This bearing coefficient is solved for three separate times using different pitch
damping values of -5. 0 and 0.5. The magnitude of change in the bearing coefficient is
shown in Table 5.17.

<table>
<thead>
<tr>
<th>Pitch Damping</th>
<th>( f_b )</th>
<th>Delta</th>
<th>Friction Torque</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>2.42E-4</td>
<td>-3.4%</td>
<td>0.37</td>
<td>0.11</td>
</tr>
<tr>
<td>0</td>
<td>2.51E-4</td>
<td>0.0%</td>
<td>0.39</td>
<td>0.11</td>
</tr>
<tr>
<td>0.5</td>
<td>2.66E-4</td>
<td>5.9%</td>
<td>0.41</td>
<td>0.12</td>
</tr>
</tbody>
</table>

**Table 5.17**: Change in Bearing Coefficient Due to Pitch Damping Coefficient

The dimensional damping friction torque, or moment, is then calculated using
Equation 5.28 and the bearing friction coefficient in Table 5.17. The change in blunt
body angle is approximated as the change in pitch angle, following the simplification
detailed in Equation 5.35. The change in pitch angle is calculated by taking the derivative
with respect to time of Equation 5.39. The result is shown in Equation 5.43.

\[ \dot{\alpha} = (-\epsilon)Ae^{-\epsilon t} \cos(\omega t + \delta_p) + (-\omega)Ae^{-\epsilon t} \sin(\omega t + \delta_p) \] (5.43)

The average pitch rate is calculated from the 70-degree mass properties and wind
tunnel conditions at Mach 2.5 using Equation 5.43. This results in an average pitch rate of
19.04 rad/s. This result is used to calculate the friction moment for the different bearing
friction coefficients shown in Table 5.17.
The Coulomb damping friction coefficient, $\mu$, is calculated in the rightmost column in Table 5.17. This value is found by converting the bearing friction moment to a friction force and dividing by the normal force, or drag force, as shown in Equation 5.44.

The drag force for the model is determined to be 36.47 N assuming a plenum total pressure of 82.74 kPa. The calculated Coulomb friction coefficient is approximately 0.11. This value is consistent with the boundary lubrication friction coefficient range of 0.05 to 0.20 described in *Fundamentals of Machine Component Design*.\(^{26}\)

$$\mu = \frac{F_f}{F_N} = \frac{r_{pin}}{F_{drag}}$$  \hspace{1cm} (5.44)

Although the value of the bearing friction coefficient, $f_b$ is small, Table 5.17 shows that it changes very little due to different pitch damping values. The middle column calculates the difference in value of the bearing friction coefficient as a percent change relative to the value if there is no influence from the pitch damping term at all. A pitch damping coefficient of -0.5 changes the bearing term by a little over three percent while a pitch damping coefficient of 0.5 alters the result by roughly six percent. This indicates that the damping due to the bearing friction is accountable for roughly 90 percent of the damping and completely obscures the aerodynamic effects.

This result indicates that it is difficult to determine model aerodynamic due to the dominance from the bearing friction. For the remainder of testing, only the frequency of oscillation is evaluated since the pitch damping of the model is indecipherable. This means that only a cosine curve is plotted against the test data using an approximate manual matching of the frequency rather than by the previous SHO solution.
5.6 1DOF TEST RESULTS

5.6.1 Expected Frequency of Oscillation

Equation 5.41 calculates the angular frequency from the aerodynamic coefficients, mass properties and wind tunnel conditions. The expected angular velocities at Mach 2, 2.5, and 3 are calculated using Equation 5.41 and the resultant frequencies are shown in Table 5.18. The mass properties of the models used to calculate these values are shown in Table 5.19.

<table>
<thead>
<tr>
<th>Model</th>
<th>Mach 2</th>
<th>Mach 2.5</th>
<th>Mach 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>7005</td>
<td>21.93 Hz</td>
<td>21.42 Hz</td>
<td>19.60 Hz</td>
</tr>
<tr>
<td>4505.5</td>
<td>18.64 Hz</td>
<td>18.21 Hz</td>
<td>19.95 Hz</td>
</tr>
</tbody>
</table>

**Table 5.18:** Expected Frequencies of Oscillation for Test Models

<table>
<thead>
<tr>
<th>Model</th>
<th>d (m)</th>
<th>S (m²)</th>
<th>I (kg m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7005</td>
<td>0.0378</td>
<td>0.0011</td>
<td>4.88E-6</td>
</tr>
<tr>
<td>4505.5</td>
<td>0.0397</td>
<td>0.0012</td>
<td>9.08E-6</td>
</tr>
</tbody>
</table>

**Table 5.19:** Model Properties Used For Calculation

5.6.2 Test Results:

Tables 5.20, 5.21, and 5.22 show the frequency for each test at Mach 2, 2.5, and 3 and the corresponding figure number, respectively. Each figure for a certain test plots the video data against the resultant cosine approximation. Each test point is repeated four times at Mach 2 and three times at Mach 2.5 and 3. Only data using the initial 25-degree support is analyzed because every test case completed at least one oscillation. The 15-degree initial support only completes 0.75 of an oscillation at Mach 2 for both the 70 and 45-degree model which is not enough for a suitable cosine approximation.
The tables below indicate that two datasets for each configuration are analyzed to determine the frequency of oscillation. No test at any Mach number completes even two oscillations so it is difficult to determine a consistent starting point to represent as time zero. This value is important because it affects the time value of the point of intersection shown in Equations 5.16, 5.17, and 5.18. The first dataset starting point is defined when the pitch angle first begins to move after the support release. The results and associated set of figures for this dataset are shown in the 2nd and 3rd column from the left. The starting time in the second dataset is determined by analyzing the frames of video and determining when the support device begins to retract. The results of this data set are shown in the 1st and 2nd column from the right. It is observed that this data set generally has lower frequencies of oscillation compared to the first.

<table>
<thead>
<tr>
<th>Test</th>
<th>Frequency of Oscillation (1)</th>
<th>Figure Number (1)</th>
<th>Frequency of Oscillation (2)</th>
<th>Figure Number (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4505.5, Run 1</td>
<td>18.18</td>
<td>5.23</td>
<td>18.18</td>
<td>5.24</td>
</tr>
<tr>
<td>4505.5, Run 2</td>
<td>19.60</td>
<td>5.25</td>
<td>17.40</td>
<td>5.26</td>
</tr>
<tr>
<td>4505.5, Run 3</td>
<td>17.83</td>
<td>5.27</td>
<td>16.07</td>
<td>5.28</td>
</tr>
<tr>
<td>4505.5, Run 4</td>
<td>18.43</td>
<td>5.29</td>
<td>16.51</td>
<td>5.30</td>
</tr>
<tr>
<td>7005, Run 1</td>
<td>24.00</td>
<td>5.31</td>
<td>21.25</td>
<td>5.32</td>
</tr>
<tr>
<td>7005, Run 2</td>
<td>25.26</td>
<td>5.33</td>
<td>22.20</td>
<td>5.34</td>
</tr>
<tr>
<td>7005, Run 3</td>
<td>28.50</td>
<td>5.35</td>
<td>24.41</td>
<td>5.36</td>
</tr>
<tr>
<td>7005, Run 4</td>
<td>27.41</td>
<td>5.37</td>
<td>23.61</td>
<td>5.38</td>
</tr>
</tbody>
</table>

**Table 5.20**: Mach 2 1DOF Individual Test Results

<table>
<thead>
<tr>
<th>Test</th>
<th>Frequency of Oscillation (1)</th>
<th>Figure Number (1)</th>
<th>Frequency of Oscillation (2)</th>
<th>Figure Number (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4505.5, Run 1</td>
<td>23.18</td>
<td>5.39</td>
<td>20.67</td>
<td>5.40</td>
</tr>
<tr>
<td>4505.5, Run 2</td>
<td>23.17</td>
<td>5.41</td>
<td>20.60</td>
<td>5.42</td>
</tr>
<tr>
<td>4505.5, Run 3</td>
<td>23.55</td>
<td>5.43</td>
<td>20.94</td>
<td>5.44</td>
</tr>
<tr>
<td>7005, Run 1</td>
<td>27.54</td>
<td>5.45</td>
<td>24.08</td>
<td>5.46</td>
</tr>
<tr>
<td>7005, Run 2</td>
<td>28.31</td>
<td>5.47</td>
<td>24.51</td>
<td>5.48</td>
</tr>
<tr>
<td>7005, Run 3</td>
<td>26.69</td>
<td>5.49</td>
<td>23.44</td>
<td>5.50</td>
</tr>
</tbody>
</table>

**Table 5.21**: Mach 2.5 1DOF Individual Test Results
<table>
<thead>
<tr>
<th>Test</th>
<th>Frequency of Oscillation (1)</th>
<th>Figure Number (1)</th>
<th>Frequency of Oscillation (2)</th>
<th>Figure Number (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4505.5, Run 1</td>
<td>19.17</td>
<td>5.51</td>
<td>17.46</td>
<td>5.52</td>
</tr>
<tr>
<td>4505.5, Run 2</td>
<td>20.94</td>
<td>5.53</td>
<td>18.80</td>
<td>5.54</td>
</tr>
<tr>
<td>4505.5, Run 3</td>
<td>23.72</td>
<td>5.55</td>
<td>21.17</td>
<td>5.56</td>
</tr>
<tr>
<td>7005, Run 1</td>
<td>19.25</td>
<td>5.57</td>
<td>17.51</td>
<td>5.58</td>
</tr>
<tr>
<td>7005, Run 2</td>
<td>23.83</td>
<td>5.59</td>
<td>21.25</td>
<td>5.60</td>
</tr>
<tr>
<td>7005, Run 3</td>
<td>23.71</td>
<td>5.61</td>
<td>21.16</td>
<td>5.62</td>
</tr>
</tbody>
</table>

**Table 5.22: Mach 3 1DOF Individual Test Results**

All of the tests documented in Table 5.20, 5.21, and 5.22 are conducted with a negative initial pitch angle due to the left pointing orientation of the model nose. The final 1DOF test that is conducted reverse the support so the model has a positive initial pitch angle. This test occurs at Mach 2.5 with the 4505.5 model at a 25-degree initial angle of attack. The resultant frequencies of oscillation are shown in Table 5.23. These results show a lower frequency of oscillation than for the same model test with the negative initial pitch angle. This may be a resolute of an inconsistency in friction depending on the direction of angular movement or not enough test cases.

<table>
<thead>
<tr>
<th>Test</th>
<th>Frequency of Oscillation (1)</th>
<th>Figure Number (1)</th>
<th>Frequency of Oscillation (2)</th>
<th>Figure Number (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4505.5, Run 1</td>
<td>21.11</td>
<td>5.63</td>
<td>19.08</td>
<td>5.64</td>
</tr>
<tr>
<td>4505.5, Run 2</td>
<td>21.75</td>
<td>5.65</td>
<td>19.55</td>
<td>5.66</td>
</tr>
<tr>
<td>4505.5, Run 3</td>
<td>22.54</td>
<td>5.67</td>
<td>20.12</td>
<td>5.68</td>
</tr>
</tbody>
</table>

**Table 5.23: Mach 2.5 1DOF Reverse Direction**

The final frequency of oscillation is determined by averaging the frequency results from the two datasets analyzed for each test. This provides an approximation that relies both on quantitative and qualitative reasoning to define the test starting point. Table 5.24 shows the frequency results from the test data in comparison with the expected frequencies of oscillations shown in Table 5.18. This table reveals that the frequencies of oscillation extracted from the test data are within the same magnitude, but are higher than
the results predicted by Table 5.18. The frequencies from both the test results and the SHO model are plotted in Figure 5.69 for the 70-degree model and Figure 5.70 for the 45-degree model.

Figure 5.69 shows that the data generally trends in the direction predicted by the SHO model with the frequency decreasing as the Mach number increases. The 1DOF Mach 2.5 data point has a higher frequency of oscillation than at Mach 3 which is inconsistent with the SHO results. The 1DOF test results for the 45-degree model differ more from the SHO model. The SHO model indicates that frequency is expected to increase with the Mach number, but the 1DOF test data peaks at Mach 2.5. The triangular data point in Figure 5.70 shows the comparatively lower frequency at Mach 2.5 for the positive initial orientation test. This values of the two frequencies are shown in Table 5.24 in the same cell for Mach 2.5

<table>
<thead>
<tr>
<th>Model</th>
<th>Mach 2-SHO</th>
<th>Mach 2-1DOF</th>
<th>Mach 2.5-SHO</th>
<th>Mach 2.5-1DOF</th>
<th>Mach 3-SHO</th>
<th>Mach 3-1DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>4505.5</td>
<td>15.03</td>
<td>17.77</td>
<td>18.21</td>
<td>22.02/20.69</td>
<td>19.95</td>
<td>20.21</td>
</tr>
</tbody>
</table>

Table 5.24: Averaged Frequency Results (Hz) Compared against Original SHO Results

The extracted moment slope coefficient is then calculated from the frequencies in Table 5.24 using Equation 5.41. The results of this calculation are given below in Table 5.25. The expected moment slope coefficient is found from the MER ballistic range results where the moment slope coefficient of test models with three different center of gravities (0.27, 0.3 and 0.33) are included. The actual center of gravity of the 1DOF test model is 0.275 so the moment slope coefficient at this location is interpolated using the MER results. The ballistic range test data shows that the moment slope coefficient is constant over Mach number. The moment slope coefficient from the empirical data as
well as the test data for the 70-degree model is shown in Figure 5.71. This Figure displays that the test data has a similar moment slope coefficient for the Mach 2 and 3 test, the moment slope coefficient at Mach 2.5 is larger in magnitude.

The empirical moment slope coefficient for the 45-degree model is from the Mars Microprobe test data from past CFD, wind tunnel, and ballistic range results.\textsuperscript{21} The moment slope coefficient from this paper is given for a test model with a center of gravity at 0.248. This differs considerably from the 1DOF test model center of gravity at 0.415.

Equation 5.45 shows the moment slope coefficient at the 1DOF model center of gravity is found by adding the distance between the Microprobe and 1DOF center of gravity and multiplying by the normal force slope coefficient.

\[
(C_{m\alpha})_{1DOF} = (C_{m\alpha})_{micro} + x_{cg}C_{N\alpha}
\]  

(5.45)

Figure 5.72 plots the empirical moment slope coefficient from the Microprobe capsule against the moment slope coefficient calculated by the 1DOF test data. This figure shows the 1DOF test data trends correctly for Mach 2 and 2.5, but is deviates at Mach 3. This low value is a result of the low frequency of oscillation seen for the Mach 3 test.

<table>
<thead>
<tr>
<th>Model</th>
<th>Mach 2-Empirical</th>
<th>Mach 2-1DOF</th>
<th>Mach 2.5-Empirical</th>
<th>Mach 2.5-1DOF</th>
<th>Mach 3-Empirical</th>
<th>Mach 3-1DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>7005</td>
<td>-0.098</td>
<td>-0.124</td>
<td>-0.098</td>
<td>-0.1623</td>
<td>-0.098</td>
<td>-0.127</td>
</tr>
<tr>
<td>4505.5</td>
<td>-0.074</td>
<td>-0.104</td>
<td>-0.114</td>
<td>-0.167/0.147</td>
<td>-0.164</td>
<td>-0.062</td>
</tr>
</tbody>
</table>

Table 5.25: Moment Slope Coefficient Results Compared with Empirical Data

Table 5.26 shows the frequency of oscillation for the test models after removing the data outliers from the 1DOF test. The re-plotted frequency results are shown Figure 5.73 for the 70-degree model and Figure 5.74 for the 45-degree model. The 70-degree results are improved as the frequency of oscillation at Mach 2.5 is now is lower than at
Mach 2. The frequency of the 45-degree model at Mach 2.5 and Mach 2 is improved, but the Mach 3 frequency remains unchanged. The frequency results from this test are inconclusive because all three test runs had differing results so no outlier can be determined. A re-test of this configuration could clarify and correct this interpretation.

<table>
<thead>
<tr>
<th>Model</th>
<th>Mach 2-SHO</th>
<th>Mach 2-1DOF</th>
<th>Mach 2.5-SHO</th>
<th>Mach 2.5-1DOF</th>
<th>Mach 3-SHO</th>
<th>Mach 3-1DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>7005</td>
<td>21.93</td>
<td>25.98</td>
<td>21.42</td>
<td>25.76</td>
<td>19.60</td>
<td>23.77</td>
</tr>
<tr>
<td>4505.5</td>
<td>15.025</td>
<td>17.21</td>
<td>18.21</td>
<td>21.91/20.37</td>
<td>19.95</td>
<td>20.21</td>
</tr>
</tbody>
</table>

**Table 5.26**: Frequency (Hz) Compared with SHO Results, Outliers Removed

The moment slope is recalculated using the frequency results displayed in Table 5.26 and the results of the new calculation are shown in Table 5.27. Figure 5.75 shows the recalculated moment slope coefficient at Mach 2.5 is now closer to the values at Mach 2 and 3. This improvement results in the moment slope coefficient to appear more consistent over the Mach number range, similar to the empirical data from the MER ballistic range test. The 45-degree model shown in Figure 5.76 continues to trend consistently with the empirical data at Mach 2 and 2.5, but the Mach 3 data point remains inconclusive.

<table>
<thead>
<tr>
<th>Model</th>
<th>Mach 2-Empirical</th>
<th>Mach 2-1DOF</th>
<th>Mach 2.5-Empirical</th>
<th>Mach 2.5-1DOF</th>
<th>Mach 3-Empirical</th>
<th>Mach 3-1DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>7005</td>
<td>-0.124</td>
<td>-0.138</td>
<td>-0.162</td>
<td>-0.162</td>
<td>-0.127</td>
<td>-0.145</td>
</tr>
<tr>
<td>4505.5</td>
<td>-0.074</td>
<td>-0.0975</td>
<td>-0.114</td>
<td>-0.165/0.143</td>
<td>-0.164</td>
<td>-0.062</td>
</tr>
</tbody>
</table>

**Table 5.27**: Moment Slope Coeff. Compared with Empirical Data, Outliers Removed

5.7 ERROR ANALYSIS

The largest source of error of the video analysis procedure occurs within the tracking program. The tracking program can misidentify the center of the marker by a pixel depending on the contrast filter applied to the frames of data. This pixel results in a
different physical coordinate being identified as the marker center and a different
corresponding angle of attack. The difference due to an error of one pixel in the \( x \) and \( y \)
coordinate from the tracking program is evaluated for all 1DOF video analysis results.
The error is expressed as the difference between the correct pitch angle and the pitch
angle caused by the one-pixel error. The pitch angle error due to a one-pixel error in the \( x \)
and \( y \) direction is expressed separately in Table 5.28 where both the maximum and mean
error in \( x \) and \( y \) is shown.

<table>
<thead>
<tr>
<th>Test Configuration</th>
<th>Test No</th>
<th>Max Error (( x ))</th>
<th>Max Error (( y ))</th>
<th>Mean Error (( x ))</th>
<th>Mean Error (( y ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>M2, 4505.5</td>
<td>1</td>
<td>0.40</td>
<td>1.41</td>
<td>0.20</td>
<td>1.04</td>
</tr>
<tr>
<td>M2, 4505.5</td>
<td>2</td>
<td>0.43</td>
<td>1.38</td>
<td>0.16</td>
<td>1.21</td>
</tr>
<tr>
<td>M2, 4505.5</td>
<td>3</td>
<td>0.45</td>
<td>1.42</td>
<td>0.23</td>
<td>1.08</td>
</tr>
<tr>
<td>M2, 4505.5</td>
<td>4</td>
<td>0.64</td>
<td>1.65</td>
<td>0.31</td>
<td>1.34</td>
</tr>
<tr>
<td>M2, 7005</td>
<td>1</td>
<td>0.60</td>
<td>1.54</td>
<td>0.21</td>
<td>1.04</td>
</tr>
<tr>
<td>M2, 7005</td>
<td>2</td>
<td>0.58</td>
<td>1.50</td>
<td>0.22</td>
<td>1.27</td>
</tr>
<tr>
<td>M2, 7005</td>
<td>3</td>
<td>0.60</td>
<td>1.50</td>
<td>0.23</td>
<td>1.14</td>
</tr>
<tr>
<td>M2, 7005</td>
<td>4</td>
<td>0.60</td>
<td>1.50</td>
<td>0.20</td>
<td>1.15</td>
</tr>
<tr>
<td>M3, 4505.5</td>
<td>1</td>
<td>0.90</td>
<td>1.82</td>
<td>0.29</td>
<td>1.52</td>
</tr>
<tr>
<td>M3, 4505.5</td>
<td>2</td>
<td>1.08</td>
<td>1.82</td>
<td>0.41</td>
<td>1.54</td>
</tr>
<tr>
<td>M3, 4505.5</td>
<td>3</td>
<td>1.01</td>
<td>1.46</td>
<td>0.53</td>
<td>0.96</td>
</tr>
<tr>
<td>M3, 7005</td>
<td>1</td>
<td>0.91</td>
<td>2.10</td>
<td>0.53</td>
<td>1.44</td>
</tr>
<tr>
<td>M3, 7005</td>
<td>2</td>
<td>0.94</td>
<td>1.24</td>
<td>0.75</td>
<td>0.76</td>
</tr>
<tr>
<td>M3, 7005</td>
<td>3</td>
<td>0.94</td>
<td>1.24</td>
<td>0.65</td>
<td>0.66</td>
</tr>
<tr>
<td>M2.5, 7005</td>
<td>1</td>
<td>0.43</td>
<td>1.26</td>
<td>0.09</td>
<td>0.65</td>
</tr>
<tr>
<td>M2.5, 7005</td>
<td>2</td>
<td>0.57</td>
<td>1.46</td>
<td>0.14</td>
<td>0.79</td>
</tr>
<tr>
<td>M2.5, 7005</td>
<td>3</td>
<td>0.59</td>
<td>1.42</td>
<td>0.16</td>
<td>0.91</td>
</tr>
<tr>
<td>M2.5, 4505.5</td>
<td>1</td>
<td>0.50</td>
<td>1.25</td>
<td>0.18</td>
<td>0.97</td>
</tr>
<tr>
<td>M2.5, 4505.5</td>
<td>2</td>
<td>0.57</td>
<td>1.26</td>
<td>0.24</td>
<td>1.12</td>
</tr>
<tr>
<td>M2.5, 4505.5</td>
<td>3</td>
<td>0.54</td>
<td>1.26</td>
<td>0.21</td>
<td>1.00</td>
</tr>
<tr>
<td>M2.5, 4505.5 (R)</td>
<td>1</td>
<td>0.73</td>
<td>1.18</td>
<td>0.36</td>
<td>0.92</td>
</tr>
<tr>
<td>M2.5, 4505.5 (R)</td>
<td>2</td>
<td>0.72</td>
<td>1.20</td>
<td>0.37</td>
<td>0.99</td>
</tr>
<tr>
<td>M2.5, 4505.5 (R)</td>
<td>3</td>
<td>0.70</td>
<td>1.20</td>
<td>0.38</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Average:</strong></td>
<td></td>
<td><strong>0.57</strong></td>
<td><strong>1.41</strong></td>
<td><strong>0.24</strong></td>
<td><strong>1.03</strong></td>
</tr>
</tbody>
</table>

Table 5.28: One Pixel Error in Degrees for All Tests
Table 5.28 shows larger errors in pitch angle result from a one pixel error in the y-coordinate compared with the x-coordinate due to how the pitch angle is calculated. The average of the mean pitch angle error due to a one-pixel error in the y-coordinate is 1.03 degrees while the average due to a one-pixel error in the x-coordinates is 0.24 degrees.

5.8 SUMMARY

This chapter details the design, fabrication and test of a wind tunnel support that allows the model to oscillate in only pitch. This experiment provides many valuable insights that can be applied to future testing. One such insight is that the high damping results shown by the test data indicates the friction in the bearing design is too high to accurately determine model dynamic damping coefficient. The data demonstrates that a redesign of the bearing mechanism to reduce friction is necessary in order to provide useful pitch damping coefficients.

The video analysis results illustrate the difficulty in tracking and extracting model motion from the optical tracking technique. Although the data does result in the same order of magnitude as anticipated results, the error in the collection and analysis method can be reduced by improving the video analysis procedure. A higher frame rate reduces the uncertainty in the frequency calculation and a more precise method to define the center of rotation will improve the fidelity of the instantaneous pitch angle calculation. In addition, the camera set-up can be redesigned to allow for a more precise and reproducible installation since discrepancies resulting from camera repositioning during installation and removal.

These tests demonstrate a similar analysis procedure that will be used to analyze similar model motion in the planned MSBS test facility. The steps performed in this
chapter will provide a useful starting point that illuminates the difficulty in obtaining high-quality aerodynamic data. This chapter also shows aspects of the experiment that can be improved in future tests.
CHAPTER VI

CONCLUSIONS

This research investigates the feasibility of implementing a magnetic suspension and balance system in a supersonic wind tunnel to simulate the dynamic movement of blunt body entry vehicles. Several initial design issues are investigated to aid in the future design of the MSBS. The model size is a critical dimension because it determines the performance and power requirements for the MSBS. It is determined that a larger model is preferable since it requires a smaller magnetic field. Blockage tests are performed at Mach 2, 2.5, and 3 to identify the limitation in mode size for a certain geometry. The blockage test matrix is built from 3D-printed models at 0.5 percent of test section cross-sectional area size increments for three different forebody semi-vertex angles. The largest models that permit tunnel start for each geometry at a specific Mach number and test section axial location are determined. The largest 45, 60 and 70-degree models in the Mach 2.5 axisymmetric test section at 50.8 cm from the nozzle exit are 6.5, 6.5 and 6 percent of the test section cross-sectional area, respectively. The largest 45, 60 and 70-degree models that start at 10.2 cm from the nozzle exit are 10.5, 9 and 9 percent, respectively. This size difference is due to the increase in boundary-layer blockage with increasing distance downstream from the nozzle exit. The boundary-layer blockage at 50.8 cm is 8 percent while the boundary-layer blockage at 10.2 cm is 5 percent. This comparison shows that testing closer to the nozzle exit enables larger diameter models to start.

The largest models that started at Mach 2.5 in the square test section are 6 percent for the 60 and 70-degree models and 10 percent for the 45-degree model. The boundary-
layer blockage at this test axial location is 7.9 percent. This boundary-layer blockage is in
closer agreement with blockage calculated for the axisymmetric test section at 50.8 cm in
spite of testing at an axial location 30 cm closer to the nozzle. This shows boundary-layer
blockage is inherently higher in the square test section due to boundary-layer growth at
the corners and that a larger model will be able to start in an axisymmetric test section.

The result of blockage tests at Mach 2 determine the largest 45, 60, and 70-degree
model that enable wind tunnel start are 6, 5.5 and 5.5 percent, respectively. The largest
models that start at Mach 3 are 6 percent for the 70-degree model, 8 percent for the 60-
degree model and 11 percent for the 45-degree model. This demonstrates that larger
models are able to start at higher Mach numbers.

The Reynolds number based upon the hydraulic diameter of the test section at
which a model unstarts is also determined during blockage testing. It is observed that
models unstart at similar Reynolds numbers regardless of the size or geometry of a
specific configuration. The unstart Reynolds number is much lower than what is required
for a model start with a value of 0.56 to 1.11 x 10^6. Testing can therefore occur in the
MSBS at a lower steady state total pressure than that required for tunnel start. Tunnel
plenum total pressures of 62.05, 82.74, and 103.4 kPa are chosen for Mach 2, 2.5, and 3
as tunnel operational points that reduce model loading up to 60 percent compared to
loading at tunnel start.

The largest 60-degree models that start for each configuration in blockage testing
are evaluated in a wall proximity test. Model sizes of 6 and 9 percent are tested in the
Mach 2.5 axisymmetric test section at 50.8 cm and 10.2 cm from the nozzle exit. For the
square test section, the largest 60-degree models tested are 5.5, 5.5 and 8 percent for
Mach 2, 2.5, and 3, respectively. This wall proximity test records wall static pressures and schlieren image stills at each 0.25 cm increment as the model is positioned from centerline toward the bottom wall. The change in location and magnitude of the peak wall pressure between the test points, off centerline locations, are analyzed. The initial objective is to define an allowable off center movement for the model before wall proximity may affect its movement. Two different limit criterion are proposed. The first limit measures the difference in the peak wall pressure that is measured at each test increment compared to the original peak wall pressure measured at centerline. When the peak wall pressure at a test point exceeds 20 percent of the peak wall pressure measured at centerline, then the model is deemed too close to the wall, affecting model dynamic rotation free of wall interference. The other limit establishes that the model is too close when it comes into visual contact with the reflected bow shock from the boundary-layer shock interaction at the wall. The current conceived design for the electronic position sensor threshold is 2.50 cm of model lateral deviation off centerline. This movement threshold is too large for half the configurations if the first criterion is used. The movement is too great for the 8 percent model at Mach 3 which reaches the limitation at 1.5 cm from the centerline. The second limitation criterion shows the EPS movement threshold of 2.50 cm is satisfactory because the model contacts the reflected shock at 2.75, 2.75, and 2.50 cm at Mach 2, 2.5, and 3 in the square test section, respectively.

These limitations are established as guidelines since not all configurations produced conclusive results when a limit is reached with a large increase in peak wall pressure between test points. Further testing is recommended to observe and quantify discrepancies in model behavior using a dynamic, 1 degree-of-freedom, test at different
distances from centerline. Performing these additional tests will then determine the actual model sensitivity to off center movement.

An initial video analysis method is developed to perform data reduction for eventual use in the MSBS. A validation experiment determines the accuracy of the analysis method. The validation experiment uses a Flotek 360, subsonic wind tunnel with similar dimensions to the GRC 225 cm² SWT. A camera mount built to support two high speed cameras capture test model movement in three orientations, $x$, $y$, and $z$. A cam and follower mechanism oscillate the model at a known amplitude and frequency. This movement is compared to video analysis results quantifying errors in measured pitch and yaw angles.

Three validation tests using video analysis methods are performed. The first test has two cameras capturing model motion in pitch. The video analysis method determines a pitch angle amplitude within 0.6-degree accuracy and a yaw angle amplitude within 0.3-degree accuracy compared to the designed cam driven model movement. The second test analyzes model movement in both pitch and yaw with two cameras, yielding an oscillation amplitude within 0.3-degree in pitch and 0.5-degree in yaw. The sources of error in tests are due to camera calibration and limitations resulting from pixel resolution.

The final video analysis validation experiment analyzes model oscillations about a single axis using a single camera. The video analysis method results are within 0.05-degree accuracy compared to the designed cam driven pitch oscillation amplitude. The analysis method is still limited by the pixel resolution which results in an error in pitch amplitude. All three validation tests determine the model frequency to be 2.00 Hz which is the actual frequency of the stepper motor.
This series of programs is then used in the analysis of the 1DOF rotation tests in the SWT. A sting design allowing pitching motion of plus or minus 30 degrees is used to simulate 1 degree-of-freedom angular dynamic motion.

1DOF test data originally intended to characterize the pitch damping of the models, however, it is found that bearing friction dominates this behavior. This prevents the pitch damping coefficient from being measured directly or accurately so only the moment slope coefficient, $C_{m\alpha}$, is measured.

The frequency of oscillation for a 70-degree model is 4 Hz higher than predicted from the Simple Harmonic Oscillator (SHO) solution using MER Ballistic Range data. The overall trend of the 1DOF data of the 70-degree model is consistent with the MER Ballistic range data. The frequency of the 45-degree model is higher in magnitude by 2 and 4 Hz for Mach 2 and Mach 2.5, respectively, when compared to the SHO solution using the Mars Microprobe capsule data. The trend in the 45-degree model results is consistent at Mach 2 and 2.5 with empirical data, but Mach 3 tests are inconclusive.

The 1DOF test uses the video analysis method, planned for MSBS use, and validates the ability to capture model dynamic behavior with demonstrated accuracy. The test also represents the first use of supersonic wind tunnel test starting procedure that will be used. The tunnel is first started at higher total pressure and then brought down to a lower operating total pressure, at which point the video analysis is then synchronized with model release.

The results demonstrating feasibility tests provide information that will be used to design and size the MSBS capabilities. Tests provide an understanding of complexity required to capture aerodynamic behavior of blunt body models in free flight and lessons
learned as a result of preliminary testing enable successful future testing. Although the MSBS has not completed design nor yet ready for implementation, this research provides initial wind tunnel data demonstrating the feasibility of the system. Once designed and fabricated, the MSBS in the SWT will serve as a very useful research tool in the design and analysis of next generation atmospheric entry space vehicles.
**FIGURES**

**Figure 1.1:** Ballistic Range at Eglin Air Force Base

**Figure 1.2:** Magnetic Suspension and Balance System developed by MIT
Figure 2.1: CAD representation of model with iron core
Figure 2.2: Test section configuration options
Figure 2.3: GRC 225 cm² axisymmetric testing set-up
Figure 2.4: 3D-Printed models with semi-vertex angles of 45, 60, and 70-degrees

![3D-Printed models with semi-vertex angles of 45, 60, and 70-degrees](image)

Figure 2.5: Literature review results

![Literature review results](image)
Figure 2.6: Backshell of model with epoxied threaded rod

Figure 2.7: Pressure tap readings during test for model 7006
Figure 2.8: Schlieren Stills for 6007.5 prior to SWT start a) and after start b)
Figure 2.9: Mach 2.5 Axisymmetric Test Section a) 50.8 cm b) 10.2 cm
Figure 2.10: Displacement thickness as a function of axial location
Figure 2.11: Displacement thickness as a function of Reynolds number.
Figure 2.12: Blockage Plot for Mach 2.5 Square Test Section

Figure 2.13: Blockage Plot for Mach 2 Square Test Section
Figure 2.14: Blockage Plot for Mach 3 Square Test Section

Figure 2.15: Literature Review with Blockage Test Data Added
Figure 2.16: Blockage Study Results with High and Low Bound for Each Geometry
Figure 3.1: Downstream View of Pressure Tap Configuration in Axisymmetric Mach 2.5 Axi. Configuration (50.8 cm): 0.00 cm from Centerline

Figure 3.2: Downstream View of Pressure Tap Configuration in Square

Figure 3.3: Mach 2.5 Axi. Configuration (50.8 cm): 0.00 cm from Centerline
Figure 3.4: Mach 2.5 Axi. Configuration (50.8 cm): 0.25 cm from Centerline

Figure 3.5: Mach 2.5 Axi. Configuration (50.8 cm): 0.50 cm from Centerline
Figure 3.6: Mach 2.5 Axi. Configuration (50.8 cm): 0.75 cm from Centerline

Figure 3.7: Mach 2.5 Axi. Configuration (50.8 cm): 1.00 cm from Centerline
**Figure 3.8:** Mach 2.5 Axi. Configuration (50.8 cm): 1.25 cm from Centerline

**Figure 3.9:** Mach 2.5 Axi. Configuration (50.8 cm): 1.50 cm from Centerline
**Figure 3.10:** Mach 2.5 Axi. Configuration (50.8 cm): 1.75 cm from Centerline

**Figure 3.11:** Mach 2.5 Axi. Configuration (50.8 cm): 2.00 cm from Centerline
Figure 3.12: Mach 2.5 Axi. Configuration (50.8 cm): 2.25 cm from Centerline

Figure 3.13: Mach 2.5 Axi. Configuration (50.8 cm): 2.50 cm from Centerline
Figure 3.14: Mach 2.5 Axi. Configuration (50.8 cm): 2.75 cm from Centerline

Figure 3.15: Mach 2.5 Axi. Configuration (50.8 cm): 3.00 cm from Centerline
Figure 3.16: Mach 2.5 Axi. Configuration (50.8 cm): 3.25 cm from Centerline

Figure 3.17: Mach 2.5 Axi. Configuration (50.8 cm): 3.50 cm from Centerline
**Figure 3.18**: Mach 2.5 Axi. Configuration (50.8 cm): 3.75 cm from Centerline

**Figure 3.19**: Mach 2.5 Axi. Configuration (50.8 cm): 4.00 cm from Centerline
**Figure 3.20**: Mach 2.5 Axi. Configuration (50.8 cm): 4.25 cm from Centerline

**Figure 3.21**: Mach 2.5 Axi. Configuration (50.8 cm): 4.50 cm from Centerline
Figure 3.22: Mach 2.5 Axi. Configuration (50.8 cm): 4.75 cm from Centerline

Figure 3.23: Mach 2.5 Axi. Configuration (50.8 cm): 5.00 cm from Centerline
Figure 3.24: Mach 2.5 Axi. Configuration (50.8 cm): 5.25 cm from Centerline

Figure 3.25: Mach 2.5 Axi. Configuration (50.8 cm): 5.50 cm from Centerline
Figure 3.26: Mach 2.5 Axi. Configuration (50.8 cm): 5.75 cm from Centerline

Figure 3.27: Mach 2.5 Axi. Configuration (50.8 cm): 6.00 cm from Centerline
Figure 3.28: Mach 2.5 Axi. Configuration (10.2 cm): 0.00 cm from Centerline

Figure 3.29: Mach 2.5 Axi. Configuration (10.2 cm): 0.25 cm from Centerline
Figure 3.30: Mach 2.5 Axi. Configuration (10.2 cm): 0.50 cm from Centerline

Figure 3.31: Mach 2.5 Axi. Configuration (10.2 cm): 0.75 cm from Centerline
Figure 3.32: Mach 2.5 Axi. Configuration (10.2 cm): 1.00 cm from Centerline

Figure 3.33: Mach 2.5 Axi. Configuration (10.2 cm): 1.25 cm from Centerline
Figure 3.34: Mach 2.5 Axi. Configuration (10.2 cm): 1.50 cm from Centerline

Figure 3.35: Mach 2.5 Axi. Configuration (10.2 cm): 1.75 cm from Centerline
Figure 3.36: Mach 2.5 Axi. Configuration (10.2 cm): 2.00 cm from Centerline

Figure 3.37: Mach 2.5 Axi. Configuration (10.2 cm): 2.25 cm from Centerline
Figure 3.38: Mach 2.5 Axi. Configuration (10.2 cm): 2.50 cm from Centerline

Figure 3.39: Mach 2.5 Axi. Configuration (10.2 cm): 2.75 cm from Centerline
Figure 3.40: Mach 2.5 Axi. Configuration (10.2 cm): 3.00 cm from Centerline

Figure 3.41: Mach 2 Square Configuration (18.7 cm): 0.00 cm from Centerline

Figure 3.42: Mach 2 Square Configuration (18.7 cm): 0.25 cm from Centerline
Figure 3.43: Mach 2 Square Configuration (18.7 cm): 0.50 cm from Centerline

Figure 3.44: Mach 2 Square Configuration (18.7 cm): 0.75 cm from Centerline

Figure 3.45: Mach 2 Square Configuration (18.7 cm): 1.00 cm from Centerline
Figure 3.46: Mach 2 Square Configuration (18.7 cm): 1.25 cm from Centerline

Figure 3.47: Mach 2 Square Configuration (18.7 cm): 1.50 cm from Centerline

Figure 3.48: Mach 2 Square Configuration (18.7 cm): 1.75 cm from Centerline
Figure 3.49: Mach 2 Square Configuration (18.7 cm): 2.00 cm from Centerline

Figure 3.50: Mach 2 Square Configuration (18.7 cm): 2.25 cm from Centerline

Figure 3.51: Mach 2 Square Configuration (18.7 cm): 2.50 cm from Centerline
Figure 3.52: Mach 2 Square Configuration (18.7 cm): 2.75 cm from Centerline

Figure 3.53: Mach 2 Square Configuration (18.7 cm): 3.00 cm from Centerline

Figure 3.54: Mach 2 Square Configuration (18.7 cm): 3.25 cm from Centerline
Figure 3.55: Mach 2.5 Square Configuration (18.7 cm): 0.00 cm from Centerline

Figure 3.56: Mach 2.5 Square Configuration (18.7 cm): 0.25 cm from Centerline

Figure 3.57: Mach 2.5 Square Configuration (18.7 cm): 0.50 cm from Centerline
Figure 3.58: Mach 2.5 Square Configuration (18.7 cm): 0.75 cm from Centerline

Figure 3.59: Mach 2.5 Square Configuration (18.7 cm): 1.00 cm from Centerline

Figure 3.60: Mach 2.5 Square Configuration (18.7 cm): 1.25 cm from Centerline
Figure 3.61: Mach 2.5 Square Configuration (18.7 cm): 1.50 cm from Centerline

Figure 3.62: Mach 2.5 Square Configuration (18.7 cm): 1.75 cm from Centerline

Figure 3.63: Mach 2.5 Square Configuration (18.7 cm): 2.00 cm from Centerline
**Figure 3.64:** Mach 2.5 Square Configuration (18.7 cm): 2.25 cm from Centerline

**Figure 3.65:** Mach 2.5 Square Configuration (18.7 cm): 2.50 cm from Centerline

**Figure 3.66:** Mach 2.5 Square Configuration (18.7 cm): 2.75 cm from Centerline
Figure 3.67: Mach 2.5 Square Configuration (18.7 cm): 3.00 cm from Centerline

Figure 3.68: Mach 2.5 Square Configuration (18.7 cm): 3.25 cm from Centerline

Figure 3.69: Mach 2.5 Square Configuration (18.7 cm): 3.50 cm from Centerline
Figure 3.70: Mach 2.5 Square Configuration (18.7 cm): 3.75 cm from Centerline

Figure 3.71: Mach 3 Square Configuration (18.7 cm): 0.00 cm from Centerline

Figure 3.72: Mach 3 Square Configuration (18.7 cm): 0.25 cm from Centerline
Figure 3.73: Mach 3 Square Configuration (18.7 cm): 0.50 cm from Centerline

Figure 3.74: Mach 3 Square Configuration (18.7 cm): 0.75 cm from Centerline

Figure 3.75: Mach 3 Square Configuration (18.7 cm): 1.00 cm from Centerline
Figure 3.76: Mach 3 Square Configuration (18.7 cm): 1.25 cm from Centerline

Figure 3.77: Mach 3 Square Configuration (18.7 cm): 1.50 cm from Centerline

Figure 3.78: Mach 3 Square Configuration (18.7 cm): 1.75 cm from Centerline
Figure 3.79: Mach 3 Square Configuration (18.7 cm): 2.00 cm from Centerline

Figure 3.80: Mach 3 Square Configuration (18.7 cm): 2.25 cm from Centerline

Figure 3.81: Mach 3 Square Configuration (18.7 cm): 2.50 cm from Centerline
**Figure 3.82:** Mach 3 Square Configuration (18.7 cm): 2.75 cm from Centerline

**Figure 3.83:** Mach 3 Square Configuration (18.7 cm): 3.00 cm from Centerline
Figure 4.1: Diagram Illustrating Relationship Between Focal Length and Distance

Figure 4.2: CAD Drawing of Video Mount Box Structure
**Figure 4.3:** Fabrication of Video Mount using T-SLOT Aluminum Extrusions

**Figure 4.4:** Constant Velocity (Triangle Wave) Cam Driven Oscillation Profile
Figure 4.5: Cam, Follower, and Stepper Motor Mounted to Flotek 360

Figure 4.6: Top View of Cam Subassembly
Figure 4.7: Different Marker Trials and Tracker Results
Figure 4.8: Two Camera Views in Validation Experiment

Figure 4.9: Diagram Illustrating Pitch Angle Calculation
Figure 4.10: Overlay of Cam Profile and Video Analysis Results From Tracker

Figure 4.11: Target Image Example in MATLAB Tracking Function

Figure 4.12: Verification of Tracking Marker Center Location
Figure 4.13: Verification of Tracking Marker Center Location (Magnified)

Figure 4.14: Stereo Camera Calibration App Corner Detection Results
Figure 4.15: Stereo Camera Calibration Results for Test-Set Up

Figure 4.16: Calculation of Pitch Angle using Coordinates from Front Camera
**Figure 4.17:** Calculation of Pitch Angle using Coordinates from Left Camera

**Figure 4.18:** Sinusoidal Cam Driven Oscillations with 7.5 degree Amplitude
Figure 4.19: Distance from Rotation Point to Marker Location (Front, Pitch Model)

Figure 4.20: Distance from Rotation Point to Marker Location (Left, Pitch Model)
Figure 4.21: X Pixel Results from Front Camera for Pitch Only Test

Figure 4.22: Y Pixel Results from Front Camera for Pitch Only Test
Figure 4.23: X Pixel Results from Left Camera for Pitch Only Test

Figure 4.24: Y Pixel Results From Left Camera for Pitch Only Test
Figure 4.25: Pitch and Yaw Results for Pitch Only Test

Figure 4.26: Pitch and Yaw Test Set-up
Figure 4.27: Distance from Rotation Point to Marker (Front, Pitch and Yaw)

Figure 4.28: Distance from Rotation Point to Marker (Left, Pitch and Yaw)
Figure 4.29: X Pixel Results for Pitch and Yaw Test (Front Camera)

Figure 4.30: Y Pixel Results for Pitch and Yaw Test (Front Camera)
**Figure 4.31:** X Pixel Results for Pitch and Yaw Test (Left Camera)

**Figure 4.32:** Y Pixel Results for Pitch and Yaw Test (Left Camera)
Figure 4.33: Pitch and Yaw Results for Pitch and Yaw Test

Figure 4.34: Single Camera Calibration Corner Detection Results
Figure 4.35: Single Camera Calibration Results

Figure 4.36: X Pixel Results for Single Camera Test
Figure 4.37: Y Pixel Results for Single Camera Test

Figure 4.38: Pitch Results for Single Camera Pitch Only Test
Figure 5.1: SolidWorks Drawing of Initial 1DOF Sting concept
**Figure 5.2:** Force Diagram During Testing (Top View)

**Figure 5.3:** Image of Ring Pivoted at 30 Degrees
Figure 5.4: Representative 70-degree MSBS Model

Figure 5.5: Representative 45-degree MSBS Model
Figure 5.6: Fabricated and Painted Models

Figure 5.7: SolidWorks Assembly of 1DOF Sting
Figure 5.8: Fabricated 1DOF Support installed in SWT

Figure 5.9: Motor that Pulls Pin
Figure 5.10: Support Position During Tunnel Start-Up

Figure 5.11: Retracted Support Position
Figure 5.12: CAD Model of Camera Box Support for SWT

Figure 5.13: Installed Camera Box Mount for 1DOF Testing
Figure 5.14: Results of Camera Calibration for 1DOF Test

Figure 5.15: Extrinsic Calibration Image for 1DOF Test

Figure 5.16: Image of Deflected (Left) and Centered Sting (Right)
Figure 5.17: Sting Deflection Force Diagram

Figure 5.18: 7005 Model with Modified Center of Gravity

Figure 5.19: Coordinate System of Blunt Body Equations of Motion
Figure 5.20: SHO Fit and Video Data for 1st Test of 7005 Model

Figure 5.21: SHO Fit and Video Data for 2nd Test of 7005 Model
Figure 5.22: SHO Fit and Video Data for 3rd Test of 7005 Model

Figure 5.23: Results from Run 1 of 4505.5 Model at Mach 2, Dataset 1
Figure 5.24: Results from Run 1 of 4505.5 Model at Mach 2, Dataset 2

Figure 5.25: Results from Run 2 of 4505.5 Model at Mach 2, Dataset 1
**Figure 5.26**: Results from Run 2 of 4505.5 Model at Mach 2, Dataset 2

**Figure 5.27**: Results from Run 3 of 4505.5 Model at Mach 2, Dataset 1
Figure 5.28: Results from Run 3 of 4505.5 Model at Mach 2, Dataset 2

Figure 5.29: Results from Run 4 of 4505.5 Model at Mach 2, Dataset 1
**Figure 5.30**: Results from Run 4 of 4505.5 Model at Mach 2, Dataset 2

**Figure 5.31**: Results from Run 1 of 7005 Model at Mach 2, Dataset 1
Figure 5.32: Results from Run 1 of 7005 Model at Mach 2, Dataset 2

Figure 5.33: Results from Run 2 of 7005 Model at Mach 2, Dataset 1
Figure 5.34: Results from Run 2 of 7005 Model at Mach 2, Dataset 2

Figure 5.35: Results from Run 3 of 7005 Model at Mach 2, Dataset 1
Figure 5.36: Results from Run 3 of 7005 Model at Mach 2, Dataset 2

Figure 5.37: Results from Run 4 of 7005 Model at Mach 2, Dataset 1
Figure 5.38: Results from Run 4 of 7005 Model at Mach 2, Dataset 2

Figure 5.39: Results from Run 1 of 4505.5 Model at Mach 2.5, Dataset 1
Figure 5.40: Results from Run 1 of 4505.5 Model at Mach 2.5, Dataset 2

Figure 5.41: Results from Run 2 of 4505.5 Model at Mach 2.5, Dataset 1
Figure 5.42: Results from Run 2 of 4505.5 Model at Mach 2.5, Dataset 2

Figure 5.43: Results from Run 3 of 4505.5 Model at Mach 2.5, Dataset 1
Figure 5.44: Results from Run 3 of 4505.5 Model at Mach 2.5, Dataset 2

Figure 5.45: Results from Run 1 of 7005 Model at Mach 2.5, Dataset 1
Figure 5.46: Results from Run 1 of 7005 Model at Mach 2.5, Dataset 2

Figure 5.47: Results from Run 2 of 7005 Model at Mach 2.5, Dataset 1
Figure 5.48: Results from Run 2 of 7005 Model at Mach 2.5, Dataset 2

Figure 5.49: Results from Run 3 of 7005 Model at Mach 2.5, Dataset 1
Figure 5.50: Results from Run 3 of 7005 Model at Mach 2.5, Dataset 2

Figure 5.51: Results from Run 1 of 4505.5 Model at Mach 3, Dataset 1
Figure 5.52: Results from Run 1 of 4505.5 Model at Mach 3, Dataset 2

Figure 5.53: Results from Run 2 of 4505.5 Model at Mach 3, Dataset 1
**Figure 5.54:** Results from Run 2 of 4505.5 Model at Mach 3, Dataset 2

**Figure 5.55:** Results from Run 3 of 4505.5 Model at Mach 3, Dataset 1
Figure 5.56: Results from Run 3 of 4505.5 Model at Mach 3, Dataset 2

Figure 5.57: Results from Run 1 of 7005 Model at Mach 3, Dataset 1
Figure 5.58: Results from Run 1 of 7005 Model at Mach 3, Dataset 2

Figure 5.59: Results from Run 2 of 7005 Model at Mach 3, Dataset 1
Figure 5.60: Results from Run 2 of 7005 Model at Mach 3, Dataset 2

Figure 5.61: Results from Run 3 of 7005 Model at Mach 3, Dataset 1
Figure 5.62: Results from Run 3 of 7005 Model at Mach 3, Dataset 2

Figure 5.63: Results from Run 1 of 4505.5 Model at Mach 2.5, Dataset 1, Reverse
Figure 5.64: Results from Run 1 of 4505.5 Model at Mach 2.5, Dataset 2, Reverse

Figure 5.65: Results from Run 2 of 4505.5 Model at Mach 2.5, Dataset 1, Reverse
Figure 5.66: Results from Run 2 of 4505.5 Model at Mach 2.5, Dataset 2, Reverse

Figure 5.67: Results from Run 3 of 4505.5 Model at Mach 2.5, Dataset 1, Reverse
Figure 5.68: Results from Run 3 of 4505.5 Model at Mach 2.5, Dataset 2, Reverse

Figure 5.69: Frequency for 1DOF Test and SHO Simulation for 7005
Figure 5.70: Frequency for 1DOF Test and SHO Simulation for 4505.5

Figure 5.71: Moment Slope Coefficient: 1DOF Test and Empirical Data for 7005
Figure 5.72: Moment Slope Coefficient: 1DOF Test and Empirical Data for 4505.5

Figure 5.73: Frequency for 7005 with Outliers Removed
Figure 5.74: Frequency for 4505.5 with Outliers Removed

Figure 5.75: Moment Slope Coefficient: 7005 with Outliers Removed
Figure 5.76: Moment Slope Coefficient: 4505.5 with Outliers Removed
APPENDIX A METHOD OF SOLUTION

APPENDIX A.1 MARKER TRACKING PROGRAM

Find Marker Pixel Location

Modified Code from Clara O Farrell by Abigail Sevier

Video Taken from Sony Cybershot IV

%2.15.17-FPSINE_2, 7.5 degree sine cam

for front facing camera

ii = 1;
picshift = 3; %syncs left and front camera from remote
pic = ii+picshift; %for ahead camera
%pic = ii %for behind camera
istart = 1;
 iend = 480-picshift; %2 seconds of 240 fps is 480 frames
positions = zeros((iend-istart),2);
corr = zeros((iend-istart),1);
threshold = single(0.93);
level = 0;
boxSize = 50;
initialGuess = int64([991, 533]); %initial location of marker center
box = [initialGuess(1)-boxSize/2 initialGuess(2)-boxSize/2 boxSize boxSize];
imName = sprintf('FPSINE_2 %03d.jpg',pic);
img = rgb2gray(imread(imName)); %converts to grayscale
Matlab System Objects
%Rotates Image
hRotate1 = vision.GeometricRotator('Angle', pi);
%Fast Fourier Transform (FFT) of target and image
hFFT2D1 = vision.FFT;
hFFT2D2 = vision.FFT;
%Inverse Fast Fourier Transform (IFFT) of convolution
hIFFFT2D = vision.IFFT;
%2D convolution
hConv2D = vision.Convolver('OutputSize','Valid');
TARGET DATA
%Read in images
imName = sprintf('FPSINE_2 %03d.jpg',pic);
imtar = rgb2gray(imread(imName)); %converts to grayscale
%Increase contrast:
se = strel('disk',30);
imgenh = imsubtract(imadd(imtar, imtophat(imtar,se)),
imbothat(imtar,se));
%Crops Image to Create Target
roi = [982, 524, 18, 18]; %[XMIN YMIN WIDTH HEIGHT]
target_img = imcrop(imgenh,roi);
imshow(target_img)
target_img = single(target_img);
SYSTEM OBJECT TO FIND TARGET IN SEQUENTIAL FRAMES

numberOfTargets = 1;
hFindMax = vision.LocalMaximaFinder( ...
'Threshold', single(-1), ...
'MaximumNumLocalMaxima', numberOfTargets, ...
'NeighborhoodSize', floor(size(target_img)/2)*2 - 1);

%%%%%%%%%%%%%%%%LOOPS THROUGH FRAMES TO FIND MARKER PIXEL LOCATION%%%%%%%%
count = 0;
for ii = istart:iend
count = count+1;
if rem(count,50) == 0
    count %displays progress of program
end
pic = ii+picshift; %for ahead camera
%pic = ii; %for behind camera
box = [initialGuess(1)-boxSize/2 initialGuess(2)-boxSize/2 boxSize boxSize]; %Box Size to Search For Target
%Crops Image
Icrop = imcrop(img,box);
Icrop = single(Icrop);
[ri, ci]= size(Icrop);
%Target Image Preparation for Convolution
target_dim_nopyramid = size(target_img);
target_image_gp = target_img;
target_energy = sqrt(sum(target_image_gp(:).^2)); % Magnitude will be used later for normalizing correlation
target_image_rot = step(hRotate1, target_image_gp); %Rotates Target Image
[rt, ct] = size(target_image_rot);
% For zero padding:
r_mod = 2^nextpow2(rt + ri); %ri is the initial row size for image
c_mod = 2^nextpow2(ct + ci);
target_image_p = [target_image_rot zeros(rt, c_mod-ct)];
target_image_p = [target_image_p; zeros(r_mod-rt, c_mod)];

% Compute the 2-D FFT of the target image
target_fft = step(hFFT2D1, target_image_p);

% Initialize constant variables used in the processing loop.
target_size = repmat(target_dim_nopyramid, [numberOfTargets, 1]);
gain = 2^(level);
Im_p = zeros(r_mod, c_mod, 'single'); % Used for zero padding
C_ones = ones(rt, ct, 'single');      % Used to calculate mean using conv

% Read in image to search for vent in:
imName = sprintf('FPSIANE_2 %03d.jpg',pic);
Im = rgb2gray(imread(imName)); % transform color to grayscale

% Increase contrast:
se = strel('disk',30);
Im = imsubtract(imadd(Im, imtophat(Im,se)), imbothat(Im,se));
% Crop to box size to speed up computation:
Im = imcrop(Im,box);
Im = single(Im);
Im_gp = Im;
Im_p(1:ri, 1:ci) = Im_gp;    % Zero-pad
%Computes 2D FFT of New Image
img_fft = step(hFFT2D2, Im_p);
% Frequency domain convolution.
corr_freq = img_fft .* target_fft;
% Inverse converts back to spatial domain
corrOutput_f = step(hIFFFT2D, corr_freq);
corrOutput_f = corrOutput_f(rt:ri, ct:ci);
% Calculate image energies and block run tiles that are size of target template.
IUT_energy = (Im_gp).^2;
IUT = step(hConv2D, IUT_energy, C_ones);
IUT = sqrt(IUT);
% Calculate normalized cross correlation.
norm_Corr_f = (corrOutput_f) ./ (IUT * target_energy);
% Find the location of the max correlation:
xyLocation = step(hFindMax, norm_Corr_f);
% Find the max value of the correlation:
maxCorr = max(norm_Corr_f(:));
positions(ii+1,:) = [double(box(1)) +
double(xyLocation(1))+floor(rt/2)-1,
double(box(2))+double(xyLocation(2))+floor(ct/2)-1];
corr(ii) = maxCorr;
initialGuess = positions(ii+1,:); % use this position as the initial guess for the next step
end
positions(1,:) = positions(2,:);
frontpix = positions;
APPENDIX A.2 VERIFICATION OF TRACKING PROGRAM PIXEL SELECTION

Check Program Marker Pixel Location

Abby Sevier 2.15.17

Video Taken from Sony Cybershot IV

FPSINE_2, 7.5 degree sine cam

For Front Facing Camera

ii = 250; %Frame number being consulted
imName = sprintf('FPSINE_2 %03d.jpg',ii);
Im = rgb2gray(imread(imName));
imshow(Im)
hold on
plot(frontpix(ii+1,1),frontpix(ii+1,2), '*')
hold off
APPENDIX A.3 ANGLE EXTRACTION PROGRAM FOR TWO CAMERA TEST

Position and Angle Transformation

Video Taken from Sony Cybershot IV

PSINE_2, 7.5 degree sine cam

stingf = [1014 579]; %location of point of rotation from front camera
stingl = [1205 732]; %location of point of rotation from left camera
stingloc = triangulate(stingf,stingl,stereoParams); %converts sting coordinates to physical coordinates
worldPoints = triangulate(frontpix,leftpix,stereoParams); %converts tracking coordinates to physical coordinates

a_corr = 55.27;
b_corr = 55.23;
xs = stingloc(1);
ys = stingloc(2);
zs = stingloc(3);
x = worldPoints(:,1);
y = worldPoints(:,2);
z = worldPoints(:,3);
[r c] = size(worldPoints);
dxi = x(1) - xs;
dyi = y(1) - ys;
dzi = z(1) - zs;
r_alpha = sqrt(dxi^2+dyi^2);
r_beta = sqrt(dxi^2+dzi^2);
for i=1:r
    dx(i) = x(i) - xs;
    dz(i) = z(i) - zs;
    dy(i) = y(i) - ys;
    a(i) = asind(dy(i)/r_alpha)+a_corr;
    b(i) = asind(dz(i)/r_beta)+b_corr;
end
APPENDIX A.4 ANGLE EXTRACTION PROGRAM FOR ONE CAMERA TEST

%%%%%%Video Taken from Sony Cybershot IV %%%%%%%

\[ t = [-206.698662846428, -36.2260121550246, 397.887743644043]; \]
%translation vector
\[ R = [0.9997 -0.0166 -0.0193; 0.0164 0.9998 -0.0138; 0.0195 0.0135 0.9997]; \]
%rotation matrix
\[ \text{stingf} = [1006 557]; \]
%pixel location of center of rotation
\[ \text{stingloc} = \text{pointsToWorld}(\text{cameraParams}, R, t, \text{stingf}); \]
%converts sting coordinates to physical coordinates
\[ \text{worldPoints} = \text{pointsToWorld}(\text{cameraParams}, R, t, \text{frontpix}); \]
%converts pixel tracking results to physical coordinates
\[ \text{a_corr} = 0; \]
\[ \text{x_s} = \text{stingloc}(1); \]
\[ \text{y_s} = \text{stingloc}(2); \]
\[ \text{x} = \text{worldPoints}(:,1); \]
\[ \text{y} = \text{worldPoints}(:,2); \]
\[ [\text{r c}] = \text{size(\text{worldPoints});} \]
\[ \text{dx} = \text{x}(1) - \text{x_s}; \]
\[ \text{dy} = \text{y}(1) - \text{y_s}; \]
\[ \text{r_alpha} = \sqrt{\text{dx}^2 + \text{dy}^2}; \]
\[ \text{for} \ i=1:\text{r} \]
\[ \hspace{1cm} \text{dx}(i) = \text{x}(i) - \text{x_s}; \]
\[ \hspace{1cm} \text{dy}(i) = -\text{y}(i) + \text{y_s}; \]
\[ \hspace{1cm} \text{a}(i) = \text{asin}(\text{dy}(i)/\text{r_alpha}) + \text{a_corr}; \]
\[ \text{end} \]
APPENDIX A.5 TRACKING PROGRAM FOR 1DOF TESTING

% Find Marker Pixel Location
% Abby Sevier with portions of code from Clara O Farrell
% Video Taken from Sony Cybershot IV
% 1 DOF Testing: A1 configuration

ii = 1;
f1 = 194; % frame at which model support retracts
f2 = 260; % frame where model movement has ceased
intensity = 5; % contrast level used by strel function
picshift = f1 - 1;

% for ahead camera
istart = f1 - picshift;

% for behind camera
iend = f2 - picshift;

positions = zeros((iend - istart), 2);
corr = zeros((iend - istart), 1);
threshold = single(0.93);
level = 0;

boxSize = 150;
initialGuess = int64([915, 531]); % initial location of marker center
box = [initialGuess(1) - boxSize/2 initialGuess(2) - boxSize/2 boxSize boxSize];
box = [max(box(1), 1) max(box(2), 1) min(box(3), 1080 - box(1)),
      min(box(4), 1920 - box(2))];
imName = sprintf('A1 %03d.jpg', pic);

% Matlab System Objects

hRotate1 = vision.GeometricRotator('Angle', pi);

% FFT of target and image
hFFT2D1 = vision.FFT;

% IFFT of conv
hIFFFT2D = vision.IFFT;

% 2D_conv
hConv2D = vision.Convolver('OutputSize', 'Valid');

% read in images
imName = sprintf('A1 %03d.jpg', pic);
imgtar = rgb2gray(imread(imName)); % converts to grayscale

% Increase contrast:
se = strel('disk', intensity);

imenh = imsubtract(imadd(imgtar, imtophat(imgtar, se)),
imbothat(imgtar, se));

roi = [907, 524, 15, 15]; % [XMIN YMIN WIDTH HEIGHT]
target_img = imcrop(imenh, roi);

% Matlab System Objects

ObjectNumber = 1;
hFindMax = vision.LocalMaximaFinder(...
'Threshold', single(-1), ...
'MaximumNumberOfLocalMaxima', numberOfTargets, ...
'NeighborhoodSize', floor(size(target_img)/2)*2 - 1);

%%%%%%%%%%%%%%%%%%%%%%%%%%LOOP DE
% LOOP%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
count = 0;
for ii = istart:iend
count = count+1;
    if rem(count,50) == 0
        count
    end
pic = ii+picshift; %for ahead camera
%pic = ii; %for behind camera
box = [initialGuess(1)-boxSize/2 initialGuess(2)-boxSize/2 boxSize]
    box = [max(box(1),1) max(box(2),1) min(box(3), 1080-box(1)),
    min(box(4), 1920-box(2))]
%cropimage
Icrop = imcrop(img,box);
imshow(Icrop)
[ri, ci] = size(Icrop);
%target image
target_dim_nopyramid = size(target_img);
target_image_gp = target_img;
target_energy = sqrt(sum(target_image_gp(:).^2));% for normalizing
%correlation
target_image_rot = step(hRotate1, target_image_gp);
    [rt, ct] = size(target_image_rot);
    % For zero padding:
    r_mod = 2^nextpow2(rt + ri); %ri is the initial row size for image
    c_mod = 2^nextpow2(ct + ci);
target_image_p = [target_image_rot zeros(rt, c_mod-ct)];
target_image_p = [target_image_p zeros(r_mod-rt, c_mod)];
% Compute the 2-D FFT of the target image
target_fft = step(hFFT2D1, target_image_p);
% Initialize constant variables used in the processing loop.
target_size = repmat(target_dim_nopyramid, [numberOfTargets, 1]);
gain = 2^(level);
Im_p = zeros(r_mod, c_mod, 'single'); % Used for zero padding
C_ones = ones(rt, ct, 'single'); % Used to calculate mean
using conv
% Read in image to search for vent in:
imName = sprintf('A1 %03d.jpg',pic);
Im = rgb2gray(imread(imName)); % transform color to grayscale
% Increase contrast:
se = strel('disk',intensity);
Im = imsubtract(imadd(Im, imtophat(Im,se)), imbothat(Im,se));
% Crop to ROI to speed up computation:
Im = imcrop(Im,box);
Im = single(Im);
Im_gp = Im;
% Frequency domain convolution.
Im_p(1:ri, 1:ci) = Im_gp; % Zero-pad
img_fft = step(hFFT2D2, Im_p);
corr_freq = img_fft .* target_fft;
corrOutput_f = step(hIFFFT2D, corr_freq);
corrOutput_f = corrOutput_f(rt:ri, ct:ci);
% Calculate image energies and block run tiles that are size of
% target template.
IUT_energy = (Im_gp).^2;
IUT = step(hConv2D, IUT_energy, C_ones);
IUT = sqrt(IUT);

%IUT = IUT(rt:xii,ct:cii); %ADDED IN

% Calculate normalized cross correlation.
norm_Corr_f = (corrOutput_f) ./ (IUT * target_energy);

% Find the location of the max correlation:
xyLocation = step(hFindMax, norm_Corr_f);

% Find the max value of the correlation:
maxCorr = max(norm_Corr_f(:));

positions(ii+1,:) = [double(box(1)) +
double(xyLocation(1))+floor(rt/2)-1,
double(box(2))+double(xyLocation(2))+floor(ct/2)-1];
corr(ii) = maxCorr;

initialGuess = positions(ii+1,:); % use this position as the
initial guess for the next step
end
[r c] = size(positions);
for i = 2:r
    frontpix(i-1,:) = positions(i,:);
end
APPENDIX A.6 ANGLE EXTRACTION CODE FOR 1DOF TEST

```
% Translation Vector from Extrinsic Calibration
R = [ 0.0189 0.9989 0.0440; -0.9940 0.0141 0.1087; 0.1080 -0.0458 0.9931]; % Rotation Matrix from Extrinsic Calibration

```

```matlab
sting = [874 553]; % location of pin
stingloc = pointsToWorld(cameraParams,R,t,sting); % converts pin location to physical coordinates
worldPoints = pointsToWorld(cameraParams,R,t,frontpix); % converts tracked pixel coordinates to physical coordinates

```

```matlab
a_corr = .4386; % offset to move final pitch angle to 0 degrees
fps = 480; % frames per second of camera
xs = stingloc(1); % x coordinate of sting
ys = stingloc(2); % y coordinate of sting
x = worldPoints(:,1); % marker x coordinates
y = worldPoints(:,2); % marker y coordinates
[r c] = size(worldPoints);
for i=1:r
    dx(i)= x(i)-xs;
    dy(i) = -y(i)+ys;
    a(i) = atand(dx(i)/dy(i))+a_corr;
end
```

```matlab
t = linspace(0,(1/fps)*r,r);
plot(t,a);
```
APPENDIX A.7 SHO FIT PROGRAM FOR 1DOF TEST

% SHO FIT CURVE

% 2.6.17 Abigail Sevier

% for 1DOF Testing

% VIDEO DATA INPUT

hold off

tsine = 0;
cosinefit = 0;
a_max = 0;
[r c] = size(a_o);
a = a_o(2:c);
[r c] = size(a);
fps = 480;
time = c/fps; %total video time
offset = mean(a); %moves x axis
offset = a(c);

% Initialize Counting Variables

count = 0;
count1 = 0;
count2 = 0;
cross = [0];
cycle_start = a(1);
cycle(1) = 2;

% Determine Frequency of Oscillation

for i = 2:c
    if a(i-1) < offset && a(i) >= offset % pitch angle negative to positive (.25 a cycle)
        count1 = count1+1;
cross(count1) = i; % determines frequency by crossing location
        interp(count1) = (-a(i-1)/(a(i)-a(i-1)))*(i-(i-1))+i-1;
    end
    if a(i-1) > offset && a(i) <= offset % pitch angle positive to negative (.75 a cycle)
        count2 = count2+1;
cross2(count2) = i; % determines frequency by crossing location
        interp2(count2) = (-a(i-1)/(a(i)-a(i-1)))*(i-(i-1))+i-1;
    end
end

[rf cf] = size(interp);
f1 = .25*(fps/(interp(1)-0)); % determines frequency of first 1/4 of cycle
f2 = .75*(fps/(interp2(1)-0)); % determines frequency of first 3/4 of cycle
f = (f1+f2)/2;
w = 2*pi*f; % Calculate angular velocity

% Break up data into cycles

d_cycle_dec = fps/f; % determines number of frames in one cycle
d_cycle = round(d_cycle_dec);
cycle = 0;
cycle(1) = 1;
i = 1;
while (cycle(i)+d_cycle) < fps*time
    i = i+1;
    cycle(i) = cycle(i-1)+d_cycle; %determine frame of start of new
cycle
end
cycle(i+1) = fps*time;
[rc cc] = size(cycle);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%DETERMINE MAXIMUM OF EACH
CYCLE%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 1:cc-1
    k = 0;
    for j = cycle(i):cycle(i+1)-1
        k = k+1;
        a_sep(k) = a(j); %separates one cycle into separate variable
    end
    [a_max(i) t_max(i)] = max(a_sep); %determines maximum positive
    amplitude of cycle
    [a_min(i) t_min(i)] = min(a_sep); %determines maximum negative
    amplitude of cycle
    t_max(i) = t_max(i) + cycle(i)-1; %computes time at which maximum
    occurred
    t_min(i) = t_min(i) + cycle(i)-1;
    a_sep = 0;
end
[ramax camax] = size(a_max);
[ramin camin] = size(a_min);
if camin > camax
    tc = camax;
else
    tc = camin;
end
x = 1;
for i = 1:tc
    if abs(a_min(1)) > abs(a_max(1)) %sorts maximum and minimum
        amp(x) = abs(a_min(i)); %takes absolute value of relative
        maximum amplitudes
        amp(x+1) = abs(a_max(i));
        t_amp(x) = t_min(i);
        t_amp(x+1) = t_max(i);
        x = x+2;
    elseif abs(a_min(1)) < abs(a_max(1))
        amp(x) = abs(a_max(i));
        amp(x+1) = abs(a_min(i));
        t_amp(x) = t_max(i);
        t_amp(x+1) = t_min(i);
        x = x+2;
    end
end
t = t_amp/fps;
a_max = amp_offset;
%IF VIBRATION IN STING
%a_max(tcc-3) = .00001;
a_max(tcc-2) = .00001; %flags to turn on if incorrect maximum is found
%a_max(tcc-1) = .00001;
a_max(tcc) = .00001;
[trr tcc] = size(a_max);
%%%%%%%%%%%%%%%%%%LEAST SQUARES FIT
EXPOENTIAL%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i=1:tcc
    an1(i) = (t(i)^2)*a_max(i);
    an2(i) = a_max(i)*log(a_max(i));
    an3(i) = t(i)*a_max(i);
    an4(i) = t(i)*a_max(i)*log(a_max(i));
    ad1(i) = a_max(i);
end
an1s = sum(an1);
an2s = sum(an2);
an3s = sum(an3);
an4s = sum(an4);
ad1s = sum(ad1);
d = ad1s*an1s-(an3s)^2;
aexp = (an1s*an2s-an3s*an4s)/d;
A = exp(aexp); %amplitude of exponential fit
el = real((ad1s*an4s-an3s*an2s)/d); %exponent found of exponential fit
phase = pi; %phase shift due to positive amplitude
%%%%%%%%%%%%%%%%%%%%%PLOT COSINE
FIT%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 1:round(fps*time)
    cosinefit(i) = A*exp(tsine(i)*el)*cos(w*tsine(i)+phase)+offset;
end
hold on
plot(tsine,a, '* k')
plot(tsine, cosinefit, 'k')
hold off
legend( 'data', 'Cosine fit', 'location', 'EastOutside')
xlabel( 'Pitch Angle (deg)' )
ylabel( 'Time (sec)' )
[rt ct] = size(tsine);
for i = 1:ct %separates variables
    M(i,1) = tsine(i);
    M(i,2) = a(i);
    M(i,3) = cosinefit(i);
    M(i,4) = frontpix(i,1);
    M(i,5) = frontpix(i,2);
end
fileID = fopen('B2.txt','wt');
[nrows, ncols] = size(M);
for row = 1:nrows %writes variables to text file to be exported to excel
    fprintf(fileID, '%.3f %.3f %.3f %.3f %.3f\n', M(row,:));
end
fclose(fileID);
hold off
APPENDIX B SOLIDWORKS DRAWINGS
Appendix B.1: Drawing of Wind Tunnel Bottom Panel in 1DOF Testing Apparatus
Appendix B.2 Drawing of Original Sting in 1DOF Testing Apparatus
Appendix B.3: Drawing of Ring in 1DOF Testing Apparatus
Appendix B.4: Drawing of Pin in 1DOF Testing Apparatus
Appendix B.5: Drawing of Diamond Strut for 1DOF Testing Apparatus
Appendix B.6 Drawing of 70 degree, 5 percent Test Model for 1 DOF Test Apparatus
Appendix B.7: Drawing of 70 degree, 5.5 percent Test Model for 1 DOF Test Apparatus
Appendix B.9: Drawing of 60 degree, 7.5 percent Test Model for 1 DOF Test Apparatus
Appendix B.10: Drawing of Lower Portion of 45 degree, 5.5 percent Test Model for 1 DOF Test Apparatus
Appendix B.11: Drawing of Upper Portion of 45 degree, 5.5 percent Test Model for 1 DOF Test Apparatus
Appendix B.12: Drawing of Assembly for 45 degree, 5.5 percent Test Model for 1 DOF Test Apparatus
Appendix B.13: Drawing of Lower Portion of 45 degree, 8.5 percent Test Model for 1 DOF Test Apparatus
Appendix B.14: Drawing of Lower Portion of 45° Degree 8.5 percent Test Model for 1 DOF Test Apparatus.
Appendix B.15: Drawing of Assembly of 45 degree, 8.5 percent Test Model for 1 DOF Test Apparatus
Appendix B.16: Drawing of Brass Retraction Sleeve
Appendix B.18: 15 Degree Model Support
Appendix B: 25 Degree Model Support
Appendix B.20: Drawing of Modified Sting for 1DOF Test Apparatus
Appendix B.21: Modified Sleeve for Retraction Mechanism
BIBLIOGRAPHY:


