3C-SIC MULTIMODE MICRODISK RESONATORS AND SELF-SUSTAINED OSCILLATORS WITH OPTICAL TRANSDUCTION

by

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DEDICATION

To My Parents

and

To My Fiancée
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3C-SiC Multimode Microdisk

Resonators and Self-Sustained

Oscillators with Optical Transduction

Abstract

by

HAMIDREZA ZAMANI

These days, sensor chips in very small form factors are ubiquitous. In this dissertation, I explain my efforts and achievements in proposing the first center-clamped 3C-SiC RF microdisk resonators with out-of-plane vibration as a candidate for future resonant sensing applications. First, nanomachining of 3C-SiC material using Focused Ion Beam (FIB) to find sputtering yield and lateral deformations for many different FIB conditions as well different patterns (and scales), is investigated accompanied by SRIM simulations.

Then, I report theoretical analysis, fabrication and measurement of (center-clamped) RF multi-mode micromechanical resonators based upon vibrating circular disks made of a ~500nm thin SiC epilayer grown on single crystal Si. The fabrication method - composed of FIB sputtering and HNA etching - has been used for the first time and the vibration resonance
peaks (in frequency spectrum) are detected through laser-interferometry measurements. A ~40µm-diameter SiC disk with a slender (~800nm) anchor exhibits more than a dozen flexural modes between ~2MHz–20MHz with quality factors (Q’s) of ~1000–4000. Two other disks with diameters of ~40µm and ~30µm, and wide anchors (~20µm and ~10.3µm, respectively) have their set of major flexural peaks (associated with modes of zero-circular nodes) between 15–19MHz with Q’s of ~500–2500.

This dissertation then describes the efforts to achieve an improved self-sustained oscillator based on such resonators. First, a theory-inspired method is developed for design of electrostatically-transduced MEMS/NEMS referenced Pierce oscillators to improve phase noise (by increasing motional inductance and thus motional resistance) while also addressing oscillation start-up and other specifications e.g. frequency detuning and power consumption.

Finally, the laser-based optical transduction scheme is used to make self-sustained oscillators from microdisk resonators. Laser actuation is very compatible with 3C-SiC material and the targeted application of sensing in harsh-environments and very immune to the undesired circuit loading effects and power handling inefficiencies of other transductions. The oscillator is analyzed by behavioral simulations and also measured with a home-built setup. The transduction with the microdisk resonator is via interferometry-based sensing and laser-based optical actuation. The measured phase noise of such oscillator shows a noticeable improvement of the effective $Q$ (~2.3) over that of the open-loop microdisk resonators.
Chapter 1: Introduction

Late nineteenth and early twentieth century are marked in the international and especially US public minds with the pictures of skyscrapers. For years, the high rises in New York and Chicago reminded each and every individual walking daily in the neighboring streets that these are the state-of-the-art in technology. The people felt the wild planet is more harnessed than ever with the massive airplanes and gigantic liners. More than the temporary feeling of surprise and pride, those creatures made the human feel safe. They were the masterworks of the world those days: World of giant constructions.

However, the world is very different in the twenty first century. World wars have passed long ago, and today, at least the regular individuals don’t see any prominent large-scale terrestrial or cosmic dangers to worry about. Feeling even safer since end of Cold War, human has always worked hard to improve his quality of life. They have looked for tools to help have a better memory, think faster and sense better. And these are exactly what the new advances in semiconductor and MEMS technologies provide. If used responsibly, these technologies have even provided a safer environment for us to live. That’s the reason the new world is no longer driven by the desire to make the big. This is the era of very tiny things.

MEMS technology has passed a long journey to blossom. The reportedly first pressure sensor was made by Kulite Semiconductor in 1961 and the first MEMS resonators in late sixties. [1,2] However, it took years until these products improved in reliability and manufacturability. Although, the first high-volume pressure sensors were fabricated by
National Semiconductor in 1974, MEMS technology experienced the real jump in 1980s and early 1990s thanks to effective fabrication technologies developed and matured by then. The popular Analog Devices ADXL50 accelerometer which came to market in 1993 is an example of this technology flourish.

Roughly two decades ago, MEMS fabrication technology was already mature enough at least for silicon (and some other materials). As said, some of the research efforts since then were directed towards making more complex systems including the resonators and the electronic readout/actuation circuits. The other aspect which researchers worked on was test and characterization of new materials (e.g. SiC that will be discussed more in later sections) for the MEMS resonators.

All these years, there was also a constant move towards mechanical resonators with sub-micrometer sizes, called NEMS (Nano Electro Mechanical Systems). MEMS (and NEMS) resonators can be categorized based on different perspectives. From the application perspective, they can be grouped as either microfluidic MEMS, optical MEMS (MOEMS), BioMEMS, RF MEMS and other types. Solid-state MEMS resonators can also be grouped based on their transduction scheme, i.e. the way the mechanical energy is transferred to the other forms of energy. The amplitude of mechanical vibrations of a resonator can be sensed by electrostatic (capacitive), electromagnetic, piezoelectric, optical and other transduction methods while they can be actuated by electrostatic, piezoelectric and thermal mechanisms. From the geometry and vibration-mode point of view; they can be further categorized which is explained in Section 1.3.
A mechanical resonator can be seen as a very high quality factor \((Q\)-factor\) resonator. From this viewpoint, the resonator can be simply modeled with a single degree-of-freedom spring-mass damper system where the resonance frequency is determined from Eq. (1.1) below in which \(\omega_{res,peak}\), \(f_{res,peak}\), \(k\) and \(m\) stand for angular natural frequency, ordinary natural frequency, elastic constant and the effective mass of the resonator respectively.

\[
\omega_{res,peak} = 2\pi f_{res,peak} = \sqrt{\frac{k}{m}}
\]  

(1.1)

Equation (1.1) helps to understand better the main two applications for the solid-state MEMS resonators: resonant sensing (defined better in Section 1.1) and reference clock generation. While in the latter, the resonator has a non-changing high-\(Q\) resonance frequency; in the former, the changes in the resonance frequency due to changes in either \(k\) or \(m\) will be detected. In both of these applications, the resonator is actuated by an input signal, and the output signal is an RF signal resonating either at the resonator’s intrinsic frequency or at a (slightly) different frequency due to loading. Another application, RF filtering, can be seen as a special case of open-loop clock generation application in which the stimulation signal is an RF signal with frequency components also around the MEMS/NEMS resonance frequency. Once that loop is closed, and the Barkhausen criteria [3] are met, we will have a self-sustained oscillator based on MEMS/NEMS resonator (Fig. 1.1). In either of the resonant sensing and clock generation applications, the high-\(Q\) of the resonance peak in frequency spectrum helps as in the former, it improves the sensing precision and in the latter, it helps to have a high quality, low phase noise clock. This will be revisited in later chapters.
In this dissertation, the resonant sensing application has been more of interest when designing the microdisk MEMS resonators (Chapter 3).

Figure 1.1. Three different applications of solid-state MEMS/NEMS resonators.

In the following sections of this chapter, we will first see the motivations and drivers for MEMS/NEMS resonators and oscillators. This includes a comparison of the state-of-the-art MEMS/NEMS resonators with the competing technology of quartz resonators. We will then see examples of the state-of-the-art MEMS/NEMS resonators and oscillators in the market. Then, resonator geometries and vibration modes as well as resonators’ transduction mechanisms are briefly analyzed. Different loss mechanisms in the resonators are discussed after that and the quality factor is accurately defined. Silicon carbide (SiC) as an advanced material for making MEMS/NEMS resonators is defined and its advantages are discussed. A summary of the works on SiC MEMS/NEMS resonators follows this section.

After that, the sections on challenges and motivations for making MEMS/NEMS resonators come which explains the reasons to selection of microdisk geometry and SiC as the material
for the resonators in this dissertation. The importance of self-sustained oscillators for making resonators was also discussed in that section. Based on the motivations derived from this analysis, and the fact that there are not many works done on the MEMS/NEMS disk resonators so far, a literature review on the MEMS/NEMS referenced oscillators is done. This literature review is done in several subsections for electrostatically-transduced 1D (i.e. beam) and 2D (planar) resonators as well as piezoelectrically-transduced flexural-mode and FBAR (Thin Film Bulk Acoustic Resonator) ones. At the ending section of this chapter, we see an overview of what will come in later chapters of this dissertation.

1.1 Drivers for MEMS/NEMS Resonators and Oscillators

In the previous section, three main motivations for using solid-state MEMS/NEMS were discussed: for RF signal processing, for resonant sensing and for clock generation applications. In RF front-end transmitters, the signals generated in base band and up-converted to RF band need to be filtered before the gain stages; otherwise, added noise will be also amplified. A similar (but reverse in order) procedure should be done in the RF receivers. The filtering should be done with very high-$Q$ filters that have been usually off-chip (e.g. SAW or ceramic filters), because on-chip components usually do not have large enough quality factors. Even before filtering, the mixer which upconverts the baseband signal to RF signal also needs a very clean (in frequency spectrum) clock signal which means the clock generation should have been done using high-$Q$ passive components – which was historically done with crystal resonators. In better words, in both clock-generation and filtering applications, passive resonating components with high $Q$ factors are needed for RF signal processing. However, the problem is that the high-$Q$ off-chip components
(ceramic/SAW filters and crystal resonators) do not easily shrink in form-factor with the rate integrated chips (ICs) do. Hence, MEMS and NEMS at micro- and nanoscale sizes respectively can easily do the job and even be good candidates for integration with electronic circuits (i.e. fabricated in the CMOS process) in future at a comparative price (to CMOS).

There is a strong motivation for using MEMS/NEMS in sensing applications. The fact that a sensing resonator can provide a high-$Q$ resonance peak in frequency domain, easily means the power of the sensed signal at a tiny frequency interval around the resonance frequency $f_{res}$ is boosted in amplitude and all signal power in other frequencies are filtered out. However, in contrast to filtering application, the resonance frequency is not constant anymore. Any particle which is absorbed or adsorbed to the MEMS/NEMS resonator, implies a loading which changes the effective mass, and therefore changes the resonance frequency. In other MEMS/NEMS sensors, some particle implantations can also change the effective $k$ in Eq. (1.1) and thus change the resonance frequency. The larger the quality factor is, the smaller each discernable step change in resonance frequency of the resonator and thus the finer the resolution of the sensor would be. That’s why MEMS/NEMS resonators which provide resonant sensing (amplifying sensor response at a specific frequency (or several frequencies) and filtering out the response at other frequencies) - while being realized at small form-factors - are very good candidates for a large range of future sensing applications. In this dissertation the second application (resonant sensing) was more in mind when designing the SiC microdisk resonators (Chapter 3).
1.2 Examples of Commercial MEMS/NEMS Resonator Products

MEMS/NEMS resonators are currently being used in many high-tech products in automobile, biomedical and communications industry. Although there are unproved claims by different MEMS oscillator fabrication companies about their MEMS-based timing products (e.g. oscillators) having better phase noise performance compared to the crystal counterparts (refer to different articles by experts from Maxim and Micro Oscillator, as well as SiTime, Discera and Silicon Clocks during the last 8 years), however the long-term frequency-temperature stability for crystal oscillators are still better (<35ppm over 0 to 70°C). However, there are novel sensing applications that for which of course, crystal-based oscillators cannot be used. Portable communication device industry has been seriously using MEMS/NEMS resonators for timing and filtering purposes. It is several years the accelerometer and gyroscope sensor chips made from MEMS/NEMS have been successfully used in hand-held smart phones. L3G4200DH 3-axis MEMS gyroscope [4] and LIS331DLH 3-axis MEMS accelerometer chips [5] made by STMicroelectronics Co. used in Apple iPhone 5 and MPU-65006 integrated gyroscope and accelerometer chip [6] made by InvenSense Co. used in Samsung Galaxy S5 are only a few examples of many.

Figure 1.2. Two MEMS resonators used in SiTime oscillators and timing products.
There are also a couple of companies working on MEMS/NEMS based products for timing applications. SiTime, a MEMS and analog semiconductor company announced formally on October 2014 that will be acquired by MegaChips, has different oscillator products based on two all-Silicon MEMS resonators. The two resonators, shown in Figure 1.2, have kHz (left) and MHz (right) resonance frequencies which are used in the company’s VCXO and TCXO products. Using these resonators, sub-1ps jitter oscillators are produced with the frequency range 1-80MHz (SiT8208) [7] and 80-220MHz (SiT8209) [8].

Discera (acquired by Micrel on Aug. 2013) is another company which has MEMS-based as well as crystal-based oscillator products. The oscillator series DSC11XX [9] by the company have jitter of ~1.7ps and work at 2.3-170MHz or 2.3-460MHz frequency ranges.

There are also other companies (with MEMS sections) emerged in the last few years like Sand 9, Toshiba, Silicon Labs, Mobius Microsystems, Synergy and Vectron and research labs like HRL laboratories and Sandia National Laboratories as well as academic laboratories which are exploring MEMS/NEMS research frontiers and/or producing high-performance or low-power MEMS-based oscillators.

### 1.3 Resonator Geometries and Vibration Modes

Solid state mechanical resonators can have different shapes. They can be one-dimensional (1D) in the form of strings and beams (or cantilevers) or higher dimensions. The reason they are called 1D is that they are much longer in one dimension than the others. Beams (cantilevers) are grouped into clamped-free, clamped-clamped and free-free. The MEMS/NEMS resonators can be parallel, serial or parallel-serial combination of the above configurations.
These resonators can either resonate along the longest direction (extensional-mode vibration), perpendicular to that dimension (flexural-mode vibration) or in shear-mode. In extensional-mode, longitudinal standing-waves and in flexural-mode resonators, transverse standing waves are created. In bulk acoustic wave (BAW) resonators, the displacement is in-plane and longitudinal acoustic waves are created. One of the advantages of these planar configurations is that they have larger surface (to mass ratio) which means lower damping for the acoustic waves. Thus, they usually have higher $Q$ factor which comes from higher potential for storing energy (due to lower loss). However, the fact that they have lower cross-section area makes them worse in terms of power handling and also sensing compared to the flexural-mode resonators. Below, we will look at them from a more analytical view.

As mentioned in the description for Eq. (1.1), the resonators can be modeled as single degree-of-freedom spring-mass damper system. In extensional mode cantilevers, the spring constant would be $k_e = \frac{EA}{L}$, while in bending (flexural-mode) resonators, $k_e = \frac{3EI}{L^3}$; where $I = \frac{ab^3}{12}$. In these equations, $E$, $A$ and $L$, are Young’s modulus, cantilever’s cross section area and static length of the cantilever along which it extends respectively, and also $I$, $b$ and $a$ are the cantilever’s second moment of inertia, depth dimension along which the cantilever bends and the cantilever’s third dimension (other than the length $L$ and depth $b$) respectively. As the resonator’s resonance frequency is proportional to square root of the spring constant (Eq. (1.1)), we can expect that for the beams (cantilevers) of the same dimension, extensional-mode resonators have a higher resonance frequency than the flexural-mode ones. However, the flexural-mode resonators usually have a larger vibration amplitude and also can be more easily actuated/sensed which makes them a better choice for sensing applications. The
planar-geometry flexural-mode resonators also have an extra good feature: The sensing area is larger which improves the power handling of the resonator.

Planar geometries can have different geometries, rectangular (or square), ring, disk and other shapes. The count and location of the clamps in these resonators can be chosen based on the resonator’s vibration mode of interest, or by considerations e.g. decreasing clamping loss. Similar to the vibration modes of the 1D resonators, planar resonators can vibrate in-plane (i.e. bulk/extensional, and torsional) and out-of-plane (i.e. flexural mode). Disk resonators which resonate in extensional-mode usually vibrate in Lame’ mode, while non-radially symmetric (e.g. rectangular) resonators usually vibrate in other modes (e.g. wine-glass mode). It is important to note that while Lame mode and wine-glass modes of vibration are similar, they are actually different [10]. Similar to the beam/cantilever resonators, extensional mode planar resonators usually have a larger resonance frequency than flexural-mode resonators for a given geometry and dimensions. Flexural planar resonators though have a wide area which lets them lend themselves easily to actuation and sensing with high power handling. This latter one is a very good property which candidates the flexural-mode resonators for high-performance sensing applications (with and without feedback loop). The microdisk resonators in this dissertation (Chapter 3) are flexural-mode disk resonators anchored at their center.

1.4 Sensors Transduction Schemes

As discussed before, the mechanical resonator vibrates in a specific frequency that is determined by the exciting signal (if the resonator is assumed a linear system). This resonance frequency, thus may or may not be the natural frequency of the resonator and this
fact determines the amplitude of the mechanical vibrations. Irrespective of how large the vibration amplitude is, the mechanical kinetic energy of the resonators need to be converted to the electrical energy so that the read-out circuits can read (i.e. sense) and process the mechanical vibrations. In electrostatic transduction, the mechanical (vibration) energy of the resonator is transformed to the electrical energy without any need to a medium in between the mechanical resonator and the electrode. In this mechanism, first an electrostatic field is created between the resonator (or a conductive layer on that) and the sensing/actuation electrode by applying a DC voltage difference between them. This configuration can be modeled by a capacitor, where the mechanical vibrations modulate the electrical charge on the electrodes of this capacitor by periodic change of the gap size. Eq. (1.2) below models change of electrode current with periodic changes of the capacitor gap size.

\[
\frac{di}{dt} = \frac{d}{dt}(C(V_{dc} + v_{ac})) = C_0 \frac{d}{dt}(v_{ac}) + V_{dc} \frac{d}{dt}(C) + V_{dc} \frac{d}{dt}(C) \frac{dv_{ac}}{dt} = C_0 \frac{d}{dt}(v_{ac}) + V_{dc} \frac{d}{dt}(C) \frac{dv_{ac}}{dt}
\]

The good thing about the capacitive transduction is that there is no medium needed for energy transduction, thus \( Q \) factor is expected to be high. Although this transduction mechanism can be a good choice in some applications due to rather lower noise or easy configuration for that specific application, in some sensing applications other transduction mechanisms are preferred for higher power handling. In other words, the fact that there is no medium in between the resonator and the electrode usually leads to high losses.

Piezoelectric transduction is one of the energy conversion mechanisms in which the resonator is in contact with an intermediary piezoelectric layer which converts mechanical energy to
electrical energy. Some materials create mechanical charge (or voltage) when they are deformed – i.e. under mechanical stress. The inverse conversion is also done in these materials. These two physical phenomena called direct and inverse effects of piezoelectricity and can be utilized for sensing and actuation respectively. In these sensors, the piezoelectric layer is usually sandwiched in between two electrodes and piezoelectric vibrations generates an electric field between the two electrodes. Piezoelectric materials are anisotropic and in the sandwiched configuration described above, the mechanical deformation can be either along the electric field or perpendicular to that. This fact determines the final sensor configuration, i.e. the way the piezoelectric material should connected to the mechanical resonator.

PZT (lead zirconium titanium oxide) is a good piezoelectric material with very high coupling coefficients (above 1000). However, like most piezoelectric materials, its $Q$ factor is not high which seriously degrades the loaded $Q$ factor of the resonators if used at the sensing end. For actuation though, high $Q$ factors are less of a concern as the resonator will make a high-$Q$ signal out of the poor-$Q$ transduced actuation force. That is the reason, a thin layer of PZT was used in Chapter 3 for actuation of my microdisk resonators in vibration measurement experiments. ZnO and AlN are two recently widely used piezoelectric-based resonators thanks to their high $Q$ factors, although their coupling coefficient is about two orders of magnitude lower than PZT. Literature review on BAW resonators with AlN and ZnO used in their configurations is done in later sections of this chapter).

There are also other transduction mechanisms. In piezoresistive transduction, electrical resistance changes by mechanical force. Though, this effect (piezoresistivity) occurs in all materials, it is much stronger in some specific materials e.g. Si (single- and poly-crystalline).
Piezoresitive transduction is easy to implement for electrical interfaces. That is why it was one of the favorite transduction schemes in early MEMS sensors. However, it has some major drawbacks: DC power consumption in this transduction scheme is high. Also, it is very sensitive to temperature changes because electrical resistance changes a lot by changes in temperature.

Optical (laser) transduction mechanism is a non-intrusive and accurate sensing mechanism. This reading scheme which lends itself easily to different resonator geometries, can provide a method for reading out vibrations (and determine resonance frequency) while it only needs a small area –equal to laser spot size - of the resonator surface for sensing/readout. This property is very crucial when the dimensions of the resonators decrease (e.g. in NEMS resonators). Also, for larger (i.e. micro-size) resonators, it plays an important role, especially when the resonator has several resonance modes and by wise choice of size and position of the laser spot on the resonator surface, specific harmonics of the fundamental resonance frequency will be more strongly sensed. In addition to the above, this sensing mechanism does not degrade the quality factor of the resonator (e.g. compared to the piezoelectric or piezoresistive schemes). These were the reasons, this sensing scheme was used in Chapter 3 and 4 for the microdisk resonators and also for the microdisk-based oscillators. Details on the laser interferometry-based sensing configuration is given in Chapter 3.

Laser-based optical actuation was the transduction mechanism used in the feedback loop of the microdisk-based oscillators in Chapter 4. In this scheme, a high-energy laser (i.e blue laser as opposed to red laser used for sensing) was used to increase the local temperature of the spot area on the resonator surface. This temperature increase in turn leads to mechanical
displacement, thus resonator’s mechanical vibrations. The good thing about this mechanism is that it is also non-intrusive similar to the laser-interferometry-based sensing mechanism used. Also for SiC material that resists even high temperatures - i.e. does not make inelastic deformations by laser-based temperature increase – such transduction mechanism seems to be a good choice. More details are given in Chapter 4.

1.5 Loss Mechanisms and Quality Factor

Similar to any other physical device, damping also exists in mechanical resonators. In addition to the extrinsic (i.e. non-mechanical) losses in the resonators, there are several well-known intrinsic loss mechanisms (i.e. not pertinent to the read-out/actuation circuitry and the transduction interface); which exist more or less in different mechanical resonators. It should be noted that these losses have different effects e.g. decrease in mechanical power of the resonator and increase in the thermal noise generated. A good physical parameter which well shows the effect of loss is quality factor ($Q$ factor). Quality factor for a one degree-of-freedom DOF system can be defined in different ways for a damped spring-mass system. The first definition for quality factor is as below

$$Q = \frac{f_{\text{res,peak}}}{\Delta f} = \frac{1}{2\zeta}$$  \hspace{2cm} (1.3)

where $f_{\text{res,peak}}$ is the peak resonance frequency of the resonator which is correspondent to the frequency at which the vibration amplitude is maximum, $\Delta f$ is half-power bandwidth of the resonator and $\zeta$ is the damping factor. It is important to note that if $Q$ factor is very high – i.e. $\zeta$ is very small - $f_{\text{res}}$ will be relatively very close to $f_{\text{res,peak}}$; thus, Eq. (1.3) changes to
Figure 1.3. Frequency spectrum (magnitude at top and phase at bottom) of three decaying sinusoidal signals with different \( Q \)-factors all with resonance frequency of 1 GHz.

Fig. 1.3 shows the effect of increase in \( Q \)-factor on the shape of the resonance circuit frequency spectrum (both magnitude and phase). Another definition for \( Q \)-factor can be derived from the ratio of the resonator’s total mechanical energy (\( U_{\text{diss}} \)) to the dissipated mechanical energy in the resonator (\( U_{\text{tot}} \)) as below

\[
\frac{U_{\text{tot}}}{U_{\text{diss}}} = \frac{1}{\frac{1}{2} k x_{\text{res, peak}}^2} = \frac{1}{\frac{1}{2} \pi c_d x_{\text{res, peak}}^2} = \frac{\omega_{\text{res, peak}} Q}{2\pi} (1.5)
\]

where \( x_{\text{res, peak}} \) and \( c_d \) are the maximum vibration amplitude in case of sinusoidal vibration and the damping coefficient – defined as the ratio of damping force over speed - respectively. It is due to mention that in Eq. (1.5), \( U_{\text{diss}} \) is found as below
\[ U_{\text{diss}} = \int F_{\text{diss}} dx = \int F_{\text{diss}} v dt = \int c_d v^2 dt = \pi c_d x_{\text{res, peak}}^2 \omega \] (1.6)

where \( F_{\text{diss}} \) and \( v \) are the dissipation force and speed respectively. From Eq. (1.5), it is obvious that quality factor \( (Q) \) equals \( 2\pi \) times the total mechanical energy (per-cycle) over the dissipated energy (per-cycle) of a mechanical resonator only when the resonance frequency is equal to the frequency at which peak vibration amplitude happens. In high-\( Q \) factor resonators, the definition also holds for the resonance frequencies other than the peak resonance frequency (i.e. the natural frequency of the resonator). It should be noted that in the above equation, the single-sideband resonance spectrum is assumed i.e. always \( \omega \leq \omega_n \).

As a conclusion from this equation, to have the highest \( Q \) factor in making oscillators from the mechanical resonators, the best strategy is to reduce the difference between the oscillation frequency of the oscillator and the resonance frequency of the resonator; in other words reducing the pulling factor as introduced in Chapter 4.

In the following, some major loss mechanisms in the resonators are briefly discussed: The motivation for this brief study is to achieve some conclusions on why SiC was chosen in this dissertation as the material for making the resonators and also getting familiar with different losses to be expected before fabricating the resonators and measuring their \( Q \) factors.

1.5.1 Material Intrinsic Losses

There are several intrinsic material losses and Akhiezer loss due to Akhiezer Effect (AKE) is one of them. Akhiezer damping occurs as a result of heat flow among different phonon modes which leads to phonon scattering. As a consequence of this damping, a temperature difference between different modes builds up which leads to some entropy and hence
dissipation. It is a damping process which only depends on the material properties and sets an upper limit on the highest $Q$-factor to be expected from a material irrespective of the resonator design and vibration modes. Authors in the recent work [11] rank different MEMS/NEMS material based on the maximum $Qf$ product to be achieved due to AKE dissipation as given in Table I.

Table I. Some of the Common MEMS/NEMS materials with Maximum $Qf$ According to AKE

<table>
<thead>
<tr>
<th>Material</th>
<th>$Qxf$ ($10^{-13}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>2.3</td>
</tr>
<tr>
<td>Quartz</td>
<td>3.2</td>
</tr>
<tr>
<td>AlN</td>
<td>2.5</td>
</tr>
<tr>
<td>Diamond</td>
<td>3.7</td>
</tr>
<tr>
<td>Sapphire</td>
<td>11.3</td>
</tr>
<tr>
<td>SiC</td>
<td>64</td>
</tr>
</tbody>
</table>

This table shows how SiC is an exceptional good material in terms of material-dependent intrinsic losses. Although the table suggests that SiC promises to give exceptionally high $Qf$ product for any given resonator geometry, transduction mechanism and vibration mode, we should note that this loss mechanism is only one of several loss mechanisms and should not be taken as the only important loss among others. From non-quantum viewpoints, material losses are losses inherent in the material structure, which only depend on the material properties and frequency but not on the geometry. Assuming that intrinsic material losses are the only losses of the resonator, Kaajakari [12] gives a formulation for the $Q$ factor to be expected due to this loss assuming that the stress $T$ in the system is only due to elastic and viscous forces, thus

$$T = ES + \mu \frac{\partial S}{\partial t},$$  \hspace{1cm} (1.7)
where $E$, $S$ and $\mu_y$ are Young’s modulus, strain and viscosity of the material. Therefore, the quality factor due to material loss can be found as below

$$Q = \frac{E}{\mu_y \omega},$$  \hspace{1cm} (1.8)

In other words, the higher the Young’s modulus, the higher the $Q$-factor will be. Another conclusion from Eq. (1.8) would be that if the material intrinsic loss is the dominant loss, then $Q$-$f$ product due to this specific loss would be constant for the resonators made out of the same material. That is why this kind of loss is only dependent on the material properties and not on the resonator geometry and design.

1.5.2 Thermoelectric dissipation (TED)

TED is another quantum mechanical loss due to transport (not scattering as for AKE) of phonons. These strains induce temperature flow gradient which in turn causes energy dissipation. Increase in strain gradient leads to increases in entropy of the system and thus to this damping. This loss is one of the most dominant losses when the period of the resonator is of the same order as the thermal time constant across the beam. [13] In flexural-mode vibration resonators, the highest frequency in which this loss shows up is inversely proportional to the thickness of the resonator; thus with increase in the resonance frequency of the resonator by scaling down the physical sizes, effect of this loss will not be constrained to very low frequencies (relative to the center frequency). This loss process was explained (and modeled) by Clarence Zener in 1937. [14] It is important to know that TED losses also depend on the resonator design, as based on the design, loss modes might be coupled to some
or all of the resonator vibration modes. So, this loss is not an exclusively material dependent loss.

### 1.5.3 Surface Losses

Surface related losses are more important when the size of the resonator becomes very small e.g. for NEMS resonators. Roughness of the surface and any surface contamination or oxide layer on the surface can cause this kind of loss. The fact that SiC is very chemical inert, as will be discussed later, is a reason that this loss would be lower in SiC resonators (for a given surface roughness).

### 1.5.4 Air Damping

We saw that the surface losses becomes noticeable only at nano-scales. However, for larger sizes and especially at lower frequencies, air damping becomes the dominant loss mechanism for the mechanical resonator. Air damping is loss of mechanical energy to the air molecules surrounding the resonating structure. In other words, this loss is a viscosity loss mechanism because the resonator is vibrating inside air (or any other fluid). The amount of this loss is dependent on the mean-free-path of the fluid, i.e. the statistical average distance a single molecule of the fluid moves before hitting another molecule of the fluid. That is why decreasing the pressure of the liquid which increases mean-free-path, decreases this loss. Temperature, pressure and composition of the environment all affect this loss. In both the SiC microdisk resonators (Chapter 3) and also for the resonator-based self-sustained oscillator (Chapter 4) developed in this dissertation, the resonators were placed inside a
vacuum chamber (with a few mPa pressure) to almost eliminate effect of this loss on the resonator’s $Q$ factor.

### 1.5.5 Clamping Loss

Clamping (or anchor) loss is the energy coupled from the resonator to the anchors (at the clamps). In other words, a large portion of this coupled energy can convert to the mechanical kinetic energy in the anchors - vibrate the resonator anchors. This kind of loss can be minimized by good design approaches which might include decreasing the anchoring area - so the coupling factor will be smaller - or designing the anchors to be at the *nodal* points (and/or nodal lines for planar resonators) of the resonator. We will see in Chapter 3 that higher-$Q$ micordisk resonators are achieved with lower-area center clamping anchors.

### 1.6 Attractions of SiC As an Advanced Materials for MEMS/NEMS

Silicon carbide is an attractive material for making reliable and high performance NEMS resonators due to its large Young’s modulus and robust surface (Mohs scale of hardness of 9 similar to diamond with hardness of 10) [15]. SiC also has wear resistance value of 9.15 comparable with diamond (10.0). Also NEMS switches which are now at the focus of researcher’s attention due to their low power consumption, are made from SiC [16] partly due to the above good physical properties. High temperature stability of silicon carbide is another property which candidates it for special applications demanding high temperature resilience [17].

Chemical inertness is another good characteristics of silicon carbide which along with its temperature stability candidates it for pattern transfer applications. Nanostencil masks for fast printing of VLSI interconnects and antennas can be made from SiC [18]. (Thermal)
nanoimprint lithography method is another fast fabrication method for which SiC mold seems to be a good choice. In these two printing applications, high-resolution and clean (without contaminants) patterning of nanostructures on SiC is necessary.

In addition to these application-demanded material selection criteria, there are some other considerations to choose a material for higher performance in MEMS/NEMS. In particular, (1) SiC resonators can resonate at a relatively higher frequency for a given effective geometric factor. Eq. (1.9) shows the equation for resonance frequency of a double-clamped flexural-mode vibrating beam

$$f = 1.03 \sqrt{\frac{E}{\rho}} \frac{t}{L^2}$$

(1.9)

where \(E, \rho, t\) and \(L\) are Young’s modulus, mass density, thickness and length of the resonator. The value \(\sqrt{\frac{E}{\rho}}\) which is similar but not equal to the velocity of sound in the material for SiC is \(1.5 \times 10^4\) m/s, while for other MEMS materials e.g. Si and GaAs is \(8.4 \times 10^3\) m/s and \(4.4 \times 10^3\) m/s respectively which are considerably lower than SiC. [19] This simply translates (according to Eq. (1.9)) to higher resonance frequency for SiC resonators. A similar reasoning can be done for resonators of geometries other than doubly-clamped.

(2) Some major loss factors were discussed in the previous sections. AKE quantum-effect considerations place an upper limit on the maximum achievable frequency-\(Q\) factor product, and are only dependent on the resonator’s material. According to Table I, Akhiezer Effect (AKE) limit on \(f-Q\) factor product— which is the maximum limit based on quantum mechanical analysis of scattering effects - is exceptionally high for SiC \((64 \times 10^{13})\)
compared to other materials usually used for making MEMS/NEMS resonators e.g. Si ($2.3 \times 10^{13}$), quartz ($3.2 \times 10^{13}$), AlN ($2.5 \times 10^{13}$), diamond ($3.7 \times 10^{13}$) and sapphire ($11.3 \times 10^{13}$). Although, the goal in this dissertation was not attempting to achieve such high $fQ$ products by our current resonator geometries, but the choice of material for research should always be based on some long-term considerations and goals. Also, from Eq. (1.8) (i.e. if we assume material-dependent losses in the resonators are the dominant ones), $Qf$ product of the MEMS-resonators will be proportional to Young’s modulus ($E$), where 3C-SiC has Young’s modulus of ~360GPa [20] which is higher than Si ~130-185GPa [21], AlN ~345GPa [22]. In addition to all of these, SiC is very chemically inert which simply means less chance of oxidation layer grown on it, potentially leading to lower surface losses.

These were the reasons and motivations towards using SiC for the microdisk resonators in this PhD research project.

1.7 Summary of Works on MEMS/NEMS Resonators Made of SiC

Although there are very few works of oscillators referenced on SiC MEMS/NEMS resonators in the literature, there are many good works on SiC resonators. Some of them are explained in Table III below. The silicon carbide used in these works had been mostly poly-SiC. 3C-SiC was the mostly used polytype and less frequently amorphous SiC ($a$-SiC) was used. The geometries reported for the SiC resonators has been mostly beams, with the exception of two LOBARs (lateral overtone bulk acoustic resonators) and a planar resonator vibrating in Lame mode.
### Table II. SiC MEMS Resonators in the Literature

<table>
<thead>
<tr>
<th>Ref</th>
<th>Material – transduction mechanism</th>
<th>Geometry - Vibration Mode</th>
<th>Motional Resistance</th>
<th>Resonance Frequency</th>
<th>Sizes</th>
<th>Bias Volt.</th>
<th>Resonator gap/ PZE thickness/ Magnetic field</th>
<th>Q-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>[23]</td>
<td>Poly-SiC-PZE transduction</td>
<td>Rectangular Lateral overtone bulk (LOBAR)</td>
<td>NA</td>
<td>2.9 GHz</td>
<td>Diffent</td>
<td>NA</td>
<td>NA/100 nm &amp; 200 nm/ NA</td>
<td>&gt;70,000 (air)</td>
</tr>
<tr>
<td>[24]</td>
<td>Poly-SiC-Electrostatic transduction</td>
<td>Rectangular Lame mode</td>
<td>220kΩ (single-ended)</td>
<td>173MHz</td>
<td>NA</td>
<td>NA</td>
<td>195 nm/ NA/ NA</td>
<td>1050 (single-ended)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>18kΩ (fully diff.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9200 (fully differentia l)</td>
</tr>
<tr>
<td>[25]</td>
<td>3C-SiC-magnetomotive transduction</td>
<td>Doubly clamped and free-free beam-in-plane resonances</td>
<td>NA</td>
<td>178MHz (clamped-clamped)</td>
<td>174MHz (free-free)</td>
<td>l: 3μm, w: 0.15μm</td>
<td>NA/NA/1T to 8T</td>
<td>4500 (clamped-clamped)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11000 (free-free)</td>
</tr>
<tr>
<td>[26]</td>
<td>3C-SiC-PZE transduction</td>
<td>Rectangular Lateral overtone bulk (LOBAR)</td>
<td>1kΩ to 2.9kΩ</td>
<td>1.75 GHz</td>
<td>l:84 μm x w:453 μm SiC plate w/ 4 IDT with 2.5μm period</td>
<td>NA</td>
<td>NA/250nm/ NA</td>
<td>4250</td>
</tr>
<tr>
<td>[27]</td>
<td>3C-SiC-electromagnetic transduction for detection</td>
<td>Clamped-clamped beam-In-plane and out-of-plane modes</td>
<td>NA</td>
<td>6.8MHz - 134 MHz</td>
<td>h: 250nm</td>
<td>250V</td>
<td>NA/NA/0-5T</td>
<td>$10^3 &lt; Q &lt; 10^4$</td>
</tr>
<tr>
<td>[28]</td>
<td>3C-SiC-electrostatic transduction</td>
<td>Folded beam (array)-flexural mode</td>
<td>25MΩ</td>
<td>30.2kHz</td>
<td>l:154μm w:2.4μm (each beam)</td>
<td>10V</td>
<td>2μm/ NA/ NA</td>
<td>10300</td>
</tr>
<tr>
<td>[39, 30]</td>
<td>Nitrogen-doped poly-SiC-electrostatic transduction</td>
<td>Double-ended tuning fork (DETF)-flexural mode</td>
<td>NA</td>
<td>207.9kHz and 210.8kHz</td>
<td>l: 150 μm h: 7μm</td>
<td>NA</td>
<td>1.5-4.5 μm</td>
<td>560</td>
</tr>
<tr>
<td>[31]</td>
<td>α-SiC-electrostatic transduction</td>
<td>Clamped-clamped beam-flexural mode</td>
<td>26 kΩ</td>
<td>8.3MHz</td>
<td>l:25 μm x w:45μm g: 200nm</td>
<td>NA</td>
<td>NA</td>
<td>1040</td>
</tr>
</tbody>
</table>

1.8 Challenges and Motivations

1.8.1 Multiple Mode Operation

One-dimensional MEMS/NEMS resonators (e.g. single-clamped and doubly-clamped) with different geometries and vibration modes, each resonate at several resonance frequencies. It is important to note that in this dissertation, resonance frequency or resonance mode of a resonator is a term different from vibration mode in that vibration mode refers to the geometrical plane in which the resonator resonates e.g. in-plane (shear or bulk/contour mode) or out-of-plane (flexural mode); while there are several resonance frequencies for vibration in each of those planes. Two dimensional 2D (i.e. planar) MEMS/NEMS resonators, either rectangular (or square), disk, ring etc., provide a larger number of such resonance frequencies with large vibration amplitudes. This gives even more flexibility to the oscillator designer.

Mechanical resonators with several vibration modes have advantages over single-mode vibration resonators for sensing applications. Assuming that the desired parameter in the sensed signal of the sensor would be frequency, the stimulus (either applied force, small particles absorbed on the resonator surface or any other stimuli) changes the resonance frequency. Now, if there are more than one resonance mode - thus, more than one resonance
peaks seen in the frequency spectrum of the sensed signal - the sensed signal upon processing give us more information e.g. about the position (on the resonator surface) that stimulus is applied at. This fact becomes more obvious when we know that several resonance frequencies are associated with each vibration mode in the resonator. Each vibration mode has different spatial period (though their frequencies might not be necessarily an integer multiple of the fundamental mode). This means, the spatial position of the applied stimulus (force, or absorbed particle which adds mass) determines the resonance frequencies.

Also, it is important to note that the best way for MEMS/NEMS resonators to operate as sensors is when they are part of oscillators (and even PLLs) as discussed in Section 8.3. A programmable sensing oscillator which can programatically oscillate at (or very close to) different resonance frequencies definitely gives more degrees of freedom to the sensor designer. This feature can be easily provided using a multiple-(resonance) mode MEMS/NEMS resonator. There are also motivations for multiple-mode resonators in the clock-generation applications. Modern technology has enabled multi-mode communication using the same transceiver on portable devices. A very low-power solution to this is again an oscillator with multiple-(resonance) mode resonator. That’s why, planar MEMS resonators are good choices for MEMS/NEMS resonators for sensing and clock-generation applications.

1.8.2 Challenges with Fabrication and Signal Transduction

As a conclusion from the previous section, SiC MEMS/NEMS resonators with multiple resonance frequencies should be fabricated. For SiC resonators, both signal transduction
and fabrication are challenging. Silicon carbide is a wide bandgap material, about 3 times that of Si. This means poor electrical conductivity unless it is doped. And doping is not always desirable in vibrating NEMS as it changes the vibration characteristics of the device. That’s why SiC (without a conducting layer on top of that) does not lend itself easily to electrostatic transduction compared to Si and some other materials. From this respect, optical and piezoelectric sensing methods are better choices for SiC. Piezoelectric transduction is not a good choice for sensing as discussed in Section 4 of this chapter. That is why optical/laser interferometry-based method is used as the sensing transduction method in this dissertation for the SiC microdisk resonators in this chapter.

In terms of high-quality and efficient fabrication of MEMS/NEMS structures from SiC, there are challenges with the traditional patterning methods what will be discussed in Chapter 2 (Section 1). That’s why, FIB milling is used in this dissertation as precise and high-throughput machining method.

1.8.3 Need to Self-Sustained Oscillators

No resonator by itself is a good choice for sensing applications. The reason is that there are different loss mechanisms in MEMS/NEMS resonators that not only degrade the vibration amplitude over time, but also provides a frequency response (resonant peak) with lower Q factor. Self-sustained oscillators provide a feedback loop to compensate for the resonator loss and provide sharper resonance peaks (Fig. 1.4). Phased locked loops (PLLs) provide another degree of freedom by being able to lock to reference signals of arbitrary frequency (within the locking range). This way, a PLL can provide a low-phase noise duplicate to the output signal of a MEMS-based oscillator which may work at different resonance
frequencies.

![Graph showing frequency vs. Q factor](image)

Figure 1.4. Larger line width in resonator (left) versus sustaining oscillator (right).

1.9 Little Works So Far on Design of MEMS-Based Oscillators

Although it is not a short time since the first work on MEMS-based oscillators published [19], only a small research had been done on design of MEMS oscillators with optimum performance thereafter. Instead, most of the MEMS community attention on the next years have been focused on fabricating resonators with higher performance (e.g. higher $f-Q$ factor), smaller feature size or investigating properties of MEMS/NEMS from different materials. In the majority of MEMS-referenced oscillators in these years, the academic researchers have either used the same oscillator architecture as [19], or simply tried to copy
and use the famous architectures used for LC oscillators without considering any specific attention to the considerations for optimum design for that architecture when the resonator replaces LC-oscillator. However, the fact is that those complex architectures did not easily lend them to MEMS resonators with usually high motional resistance $R_m$. In other words, even some widely known oscillator circuits like Pierce oscillator, need some design procedure to oscillate with high $R_m$ MEMS/NEMS. This was the reasons part of this dissertation research was focused on optimized design of MEMS.NEMS-based oscillators (Chapter 4).

1.10 MEMS/NEMS Referenced Oscillators in The Literature

In this section, only works which include oscillators based on MEMS/NEMS resonators in the literature are reviewed. In some categories like beam resonators with piezoelectric transduction; however, that no major oscillator work has been found, the open-loop MEMS/NEMS resonators were listed. The resonators in them are not necessarily made from SiC and in fact mostly from non-SiC material.

1.10.1 MEMS Oscillators with Electrostatic Transduction

**MEMS Oscillators Based on Beam Resonators:**

There are many works in the literature based on MEMS resonators with electrostatic transduction. Table III below shows a summary of some of these oscillators.
Table III. Electrostatic Beam-Resonator Based Oscillators

<table>
<thead>
<tr>
<th>Ref</th>
<th>Material &amp; Geometr y</th>
<th>Motional Resistanc e</th>
<th>Vibratio n Mode</th>
<th>Vibration Frequency</th>
<th>Sizes</th>
<th>Amplifier Type</th>
<th>Measured Phase Noise (or jitter)</th>
<th>$P_{out}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[34]</td>
<td>Poly-Si Folded beam</td>
<td>621kΩ and 2483kΩ</td>
<td>flexural</td>
<td>16.5kHz</td>
<td>l:185.3μm, w:1.9μm, h:2μm</td>
<td>Trans-impedance</td>
<td>NA</td>
<td>58pW , 0.28n W and 0.52n W (Vbias = 30, 40, 50V)</td>
</tr>
<tr>
<td>[35]</td>
<td>Poly-Si Clamped-clamped</td>
<td>8790Ω</td>
<td>flexural</td>
<td>9.34MHz</td>
<td>l:40μm, w:8μm, h:2μm</td>
<td>Trans-impedance - TSMC 0.35μm</td>
<td>-82 dBc/Hz at 1 kHz</td>
<td>-42.2 dBm</td>
</tr>
<tr>
<td></td>
<td>340Ω</td>
<td>flexural</td>
<td>8.614MHz</td>
<td>l:40μm, w:40μm, h:2μm</td>
<td>Trans-impedance</td>
<td>-80 dBc/Hz at 1 kHz</td>
<td>-33.9 dBm</td>
<td></td>
</tr>
<tr>
<td>[36]</td>
<td>Si-Comb drive</td>
<td>600 kΩ, 250kΩ, 150 kΩ (at $V_{bias}=1.5$ V, 3V and 5V) (Q= 60000, $V_{bias}=2.5$ V)</td>
<td>flexural</td>
<td>32kHz</td>
<td>l:10μm,</td>
<td>Ring oscillator</td>
<td>NA</td>
<td>&lt;1μW</td>
</tr>
<tr>
<td>[37]</td>
<td>Poly Si-Comb drive</td>
<td>NA</td>
<td>flexural</td>
<td>175.5kHz</td>
<td>NA</td>
<td>Pierce</td>
<td>-60 dBc/Hz (at 10Hz)</td>
<td>NA</td>
</tr>
<tr>
<td>[38]</td>
<td>Poly-Si free-free</td>
<td>125kΩ for single FF-beam and 15kΩ for 20 FF-beam resonator (at $V_{bias}=14$ V and 30V)</td>
<td>flexural</td>
<td>15.4 MHz</td>
<td>l: 40μm, w: 10μm, h:2μm</td>
<td>Trans-impedance</td>
<td>-109 dBc/Hz (at 1kHz)</td>
<td>NA</td>
</tr>
<tr>
<td>[39]</td>
<td>Poly Si Clamped-clamped</td>
<td>340Ω (at $V_{bias}=13$ V)</td>
<td>flexural</td>
<td>8.614 MHz</td>
<td>l: 40μm, w: 40μm, h:32μm</td>
<td>Trans-impedance</td>
<td>-80 dBc/Hz (at 1kHz)</td>
<td>400μW</td>
</tr>
<tr>
<td></td>
<td>340Ω</td>
<td>flexural</td>
<td>8.614MHz</td>
<td>l: 40μm, w: 40μm, h:32μm</td>
<td>Trans-impedance</td>
<td>-95 dBc/Hz (at 1kHz) With ALC</td>
<td>830μW With ALC</td>
<td></td>
</tr>
<tr>
<td>[40]</td>
<td>Si Clamped-clamped</td>
<td>1MΩ</td>
<td>flexural</td>
<td>14 MHz</td>
<td>l: 44μm, w: 4μm h: 8μm</td>
<td>Trans-impedance</td>
<td>-105 dBc/Hz (at 1kHz)</td>
<td>NA</td>
</tr>
</tbody>
</table>
These works begin with first prominent efforts on design of oscillators based on MEMS resonators in 1998 and 1999 ([37] and [34] respectively). The oscillator in the first work was a Pierce oscillator and in the second work was a trans-impedance one. The architecture of most works in the literature thereafter is closely related to the configurations of these two oscillators especially the trans-impedance one.

Lee et. al. [46] in 2001 designed a Pierce oscillator based on clamped-clamped beam resonators. One of the important results of this work was that the measured close-in phase noise of the oscillator was $\sim$-80dBc/Hz at 1kHz which is much higher than the predicted (from simulation) value of $\sim$125dBc/Hz at that offset frequency. This observation at this work had been a
motivation for some works in later years to model this observed phenomenon (increase in $1/f^3$ noise) and also attempt to decrease it.

Lin et. al. [39] use a voltage limiting amplifier (ALC) to limit the voltage amplitude of the oscillator. The authors observe that because ALC prevents the resonator to work in the nonlinear regime, the $1/f^3$ region phase noise decreases. Although in this work, the authors suggest that using ALC is the best solution to lower phase noise of the oscillators, we will see in later works by Prof. Nguyens’s group in Berkeley and also other academic groups, that the best approach for minimizing close-in phase noise would be making the oscillator not operate in the nonlinear regime (i.e. output voltage amplitude of the oscillator does not exceed the maximum voltage amplitude in the oscillator’s current-limited regime), without using an ALC. The reason is ALC introduces some new noise because due to the additional active circuitry.

Lee and Nguyen [38] proposed use of mechanically-coupled (free-free beam) resonators for the oscillators. The authors claim that using a parallel array of resonators leads to lower phase noise because the motional resistance of the array is smaller than that of a single resonator. They also make note that this decrease in motional resistance $R_x$ is achieved at the expense of some decrease in effective total $Q$. The analysis of the authors is correct in the sense that arraying resonators definitely lowers the effective noise due to mechanical system; however, the fact that if it would necessarily decrease phase noise in an oscillator – which is a closed loop system - seems to need more detailed analyses and also experiments.

The authors also have another important observation: The best phase noise is achieved when the oscillation amplitude is just below the critical Duffing value (i.e. right before the oscillator enters the nonlinear region). This approves the hypothesis that you don’t need an ALC to achieve a low phase noise. But one only needs some oscillator topology that can work at a voltage level in which the resonator works in linear regime. This is very similar to what the LC-oscillator
designers had found out years ago that the lowest phase noise is achieved when the oscillator is biased in the current-limited regime at closest point to the voltage-limited regime [47].

**MEMS Oscillators Based on Planar Resonators:**

In addition to the oscillators with beam structures, there are also electrostatically-transduced MEMS/NEMS based oscillators with planar resonator structures reported in the literature. The electrostatically-transduced planar resonators are rectangular (or square)-shaped, circular disk shaped, ring-shaped or in other geometries. Table IV below summarizes some of these important works.

Table IV. Electrostatic Micro-Disk (or -Rectangular)-Resonator Based Oscillators

<table>
<thead>
<tr>
<th>Ref</th>
<th>Material &amp; Geometry</th>
<th>Motional Resistance</th>
<th>Vibration Mode</th>
<th>Vibration Freq.</th>
<th>Sizes</th>
<th>Amplifier Type</th>
<th>Measured Phase Noise</th>
<th>$P_{con}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[35]</td>
<td>Poly-Si Single wine-glass disk</td>
<td>1500Ω</td>
<td>wine-glass</td>
<td>61.2 MHz</td>
<td>R: 32μm, h: 3μm</td>
<td>Trans-impedance -TSMC 0.35μm</td>
<td>-110 dBc/Hz (at 1 kHz)</td>
<td>-24.6 dBm</td>
</tr>
<tr>
<td>[48]</td>
<td>Poly-Si-A series of wine-glass disks</td>
<td>5.75 kΩ, 3.11 kΩ, 1.98 kΩ, 1.25 kΩ (for 1, 3, 5 and 9 disks)</td>
<td>Wine-glass</td>
<td>60 MHz</td>
<td>R: 32μm, h: 3μm</td>
<td>Trans-impedance TSMC 0.35μm</td>
<td>-123 dBc/Hz (at 1 kHz)</td>
<td>NA</td>
</tr>
<tr>
<td>[49]</td>
<td>Single-crystal Si-Square extensional disk ($Q=130000$)</td>
<td>4.47 kΩ ($V_{bias}=75$V)</td>
<td>Extensional (bulk)</td>
<td>13.1 MHz</td>
<td>l: 320μm, w: 290μm, h: 10μm</td>
<td>Trans-impedance</td>
<td>-138 dBc/Hz (at 1 kHz)</td>
<td>NA</td>
</tr>
<tr>
<td>[50]</td>
<td>Si-BAR</td>
<td>2.4kΩ ($Q=51,000$ at $V_{bias}=14$V)</td>
<td>Extensional (bulk)</td>
<td>145 MHz</td>
<td>l: 270μm, w: 27μm</td>
<td>Trans-impedance (0.18μm CMOS)</td>
<td>-111 dBc/Hz (at 1kHz)</td>
<td>3.6m W</td>
</tr>
<tr>
<td>[51]</td>
<td>Si-BAR</td>
<td>50kΩ, 5kΩ (at $V_{bias}=14$V and 18V)</td>
<td>Extensional (bulk)</td>
<td>103 MHz</td>
<td>Trans-impedance (0.18μm CMOS)</td>
<td>-108dBc/Hz (at 1kHz)</td>
<td>2.6m W</td>
<td></td>
</tr>
<tr>
<td>[52]</td>
<td>Si Square</td>
<td>76kΩ (at $V_{bias}=60$V)</td>
<td>Extensional (bulk)</td>
<td>2.18 MHz</td>
<td>l: 2mm, w: 2mm</td>
<td>Trans-impedance</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>[53]</td>
<td>Single-crystal Si square</td>
<td>50kΩ (at Vbias&lt;5V, Q=58,000)</td>
<td>Lame mode</td>
<td>100 MHz</td>
<td>l: 41.25 μm</td>
<td>Trans-impedance</td>
<td>-110 dBc/Hz (at 1kHz)</td>
<td>NA</td>
</tr>
<tr>
<td>[54]</td>
<td>Ring</td>
<td>750kΩ (Q=1826)</td>
<td>In-plane (contour-mode)</td>
<td>5.2 MHz</td>
<td>Rin/Rout=0.73</td>
<td>Trans-impedance</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>[55]</td>
<td>Ring and beam</td>
<td>65kΩ (Q=160,000)</td>
<td>Extensional (and in-plane)</td>
<td>20 MHz</td>
<td>l: 202 μm w: 12 μm h: 59 μm Rin: 20 μm</td>
<td>Trans-impedance</td>
<td>far phase noise floor of -131 dBc/Hz</td>
<td>6.9mW</td>
</tr>
<tr>
<td>[56]</td>
<td>Si square</td>
<td>76.9kΩ (Q=8,000)</td>
<td>Lame mode</td>
<td>17.6 MHz</td>
<td>h: 10 μm</td>
<td>Trans-impedance (0.18μm CMOS)</td>
<td>-121 dBc/Hz (at 1kHz)</td>
<td>5.9mW</td>
</tr>
<tr>
<td>[57]</td>
<td>Si ring</td>
<td>NA (Q=70,000, Qmeasured=2,300)</td>
<td>Extensional (contour-mode)</td>
<td>2.05 GHz</td>
<td>NA</td>
<td>Trans-impedance</td>
<td>-80 dBc/Hz (at 10kHz)</td>
<td>NA</td>
</tr>
<tr>
<td>[58]</td>
<td>Si square</td>
<td>6.7kΩ (Vbias<del>3V) 18.5kΩ (Vbias</del>1.8V)</td>
<td>Lame mode</td>
<td>17.6 MHz</td>
<td>NA</td>
<td>Trans-impedance</td>
<td>DIDO: -127 dBc/Hz (at 1kHz)</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>SIDO: -100 dBc/Hz (at 1kHz)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

One of the earliest good works in this list is done by Kaajakari et. al in 2004 [49]. They designed a square resonator-based oscillator in which the resonator resonates in square-extensional mode. The authors achieve very good phase noise of -138dBc/Hz at 1kHz offset from the center frequency which was record low for MEMS-based oscillators. It is due to mention that the resonance mode of their resonator is square-extensional mode which is different from Lame’-mode. The next good works in this category belongs to the Berkeley group [35, 48]. In [35] a single wine-glass disk resonator serves as the reference for the oscillator, while in [48] an array of wine-glass disk resonators is used in order to decrease phase noise.

Works [50], [51] discuss oscillators based on Si bulk acoustic resonators. Oscillator configuration used for both of these works is trans-impedance amplifier (TIA) architecture. The goal of the authors in these works have been using resonators with minimum motional resistance so that they can make oscillators based on the resonators which work at high frequencies. The resonators
work in their fundamental width-extensional mode. The phase noises achieved are not as good as the ones reported in [49] but still meet the GSM phase noise specifications.

There are also some oscillator works in the literature (e.g. [54], [55] and [57]) which are based on ring-shaped resonators. The resonators of the latter two have high intrinsic $Q$-factors which helps to have low phase noise. The oscillator in [57] operates at high frequency of ~2GHz. Fig. 1. 9 and 10 depict the ring resonators in [54] and [57] respectively. The work in [58] is a recent Lame'-mode square resonator-based oscillator reported in 2013. In that work two different oscillator architectures are tested: SIDO (single-in, differential-out) and DIDO (differential-in, differential-out). The DIDO oscillator has a good reported close-in phase noise of -127 dBc/Hz (at 1kHz).

### 1.10.2 MEMS Oscillators with Piezoelectric (PZE) Transduction

#### PZE MEMS Oscillators Based on Flexural-Mode Beam Resonators:

Piezoelectricity is another energy transduction mechanism between the mechanical resonator and the oscillator circuit. The oscillators of this group can be divided in two groups of flexural beam and FBAR resonators in which piezoelectric layer can be either a beam (or membrane) or FBAR. The fabrication of these structures usually involves deposition of a piezoelectric film on a supporting substrate followed by removal of a portion of the substrate to define the suspended resonator. Silicon and gallium arsenide have been used with success in such resonators [59]. Recently, ZnO have also been successfully deployed as the piezoelectric material [60].

Table V shows three recent works in the literature with beam-shaped resonators that have piezoelectric transduction. Although they can be used as part of self-sustained oscillators, the works reported here by themselves do not contain any self-sustained oscillators.

| Table V. Piezoelectric Flexural-Mode Beam-Resonators (with Sustaining Amplifier) |
The authors in [61] and [62] claim that electrostatically-coupled resonators suffer from high motional resistance and high biasing voltages needed to decrease their motional resistance; while, piezoelectric transduced resonators can be designed with low-motional resistance. In [61], based on the fact that in piezoelectric transduction, motional resistance is inversely proportional to the coupling coefficient, the authors try to maximize this factor – to minimize motional resistance - by carefully sizing and shaping the AlN and the SOI.

In [63], an IBAR resonator structure is deployed for piezoelectric transduction which used to be employed in electrostatic transduction resonators. This structure includes a single rod and two flanges. The structure has two anchors tethered with a tiny size the rod. This work in contrast to the works in [61] and [62] also includes the sustaining amplifier circuit for the resonator. The
oscillator circuitry is not fabricated, but only the simulation results for the designed oscillator with the equivalent circuit of the resonator inside are given.

**PZE MEMS Oscillators Based on FBAR Resonators:**

Table VI shows a couple of the piezoelectrically transduced oscillators with FBAR resonators.

<table>
<thead>
<tr>
<th>Ref</th>
<th>Material &amp; Geometry</th>
<th>Motional Resistance</th>
<th>Vibration Mode</th>
<th>Vibration Frequency</th>
<th>Amplifier Type</th>
<th>Measured Phase Noise</th>
<th>$P_{\text{con}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[64]</td>
<td>Lateral-field-excited (LFE) piezoelectric AlN FBAR</td>
<td>82Ω ($Q=1450 &lt;2200$)</td>
<td>Contour mode</td>
<td>1.05 GHz</td>
<td>Pierce</td>
<td>-81 dBc/Hz (at 1 kHz)</td>
<td>3.5mW</td>
</tr>
<tr>
<td>[65]</td>
<td>Lateral-field-excited (LFE) piezoelectric AlN FBAR</td>
<td>50 Ω ($Q&lt;2500$)</td>
<td>Contour mode</td>
<td>1.5 GHz</td>
<td>Pierce</td>
<td>-85 dBc/Hz (at 10 kHz)</td>
<td></td>
</tr>
<tr>
<td>[66]</td>
<td>High-order laterally-excited (LFE) bulk acoustic resonators =LBAR</td>
<td>150Ω ($Q_{\text{unloaded}}=7100$)</td>
<td>Laterally vibrating</td>
<td>724 MHz</td>
<td>Trans-impedance -- 0.18μm CMOS</td>
<td>-87 dBc/Hz (at 1 kHz)</td>
<td>7.2mW</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.006 GHz</td>
<td></td>
<td>26.75 MHz</td>
<td></td>
</tr>
<tr>
<td>[67]</td>
<td>SiO$_2$ pillars in Si resonators. AlN film sandwiched between Mo electrodes for excitation and sensing.</td>
<td>(Q=7600)</td>
<td>1st longitudinal extensional mode</td>
<td>26.75 MHz</td>
<td>Trans-impedance</td>
<td>-101 dBc/Hz (at 1 kHz)</td>
<td>NA</td>
</tr>
<tr>
<td>[68]</td>
<td>Lateral-field-excited (LFE) piezoelectric AlN FBAR</td>
<td>25Ω ($Q=2050$)</td>
<td>Contour-mode</td>
<td>204MHz</td>
<td>Pierce</td>
<td>-77 dBc/Hz (at 1 kHz)</td>
<td>47μW</td>
</tr>
<tr>
<td>[69]</td>
<td>Lateral-field-excited (LFE) piezoelectric AlN FBAR</td>
<td>Tens of ohms</td>
<td>Contour-mode</td>
<td>1.16 GHz</td>
<td>Colpitts</td>
<td>-143 dBc/Hz (at 100 kHz)</td>
<td>4.2mW</td>
</tr>
</tbody>
</table>

One of the favorite resonator structures in the listed papers is contour-mode AlN piezoelectric Lateral-field-excited (LFE) FBAR resonator used as the reference resonator in the oscillators. One of the advantages of the LFE structures is its compatibility with CMOS technology. A similar piezoelectric LFE technology in other works reported is called LBAR by the authors in

Table VI. Oscillators with Piezoelectric FBAR-Mode Resonators
the Georgia Tech. University. These structures have still lower $Q$-factor compared to FBAR but their $Q$-factor have increased through the years.

### 1.11 Objectives and Organization of the Next Chapters

This chapter served as an introductory chapter in which motivations for choosing SiC microdisk resonator (from material, mechanics and interface/transduction aspects) were explained. Through the sections of this chapter, use of this resonator with optical transduction scheme for sensing was justified. In addition to that, some key terms and technical background on the subject were also introduced/discussed and review of the relevant literature was done.

In Chapter 2, first choice of an appropriate nanomachining method for fabrication of SiC disk resonators will be described and this nanomachining method will be fully characterized (for 3C-SiC) before being applied. Then in Chapter 3, theoretical analysis of the 3C-SiC microdisk resonators in conjunction with simulation results will be presented. Fabrication of the microdisk resonators with an innovative fabrication method will be then discussed and the RF vibration measurement results of this resonator which was the first of its kind in the world– i.e. the first reported MHz-range SiC disk resonator – will be presented. Based on such measurements and post-processing of the data for several SiC microdisks, these resonators will be fully characterized.

Chapter 4, first provides a brief overview of noise and in particular phase noise theoretical background and then discusses an experiment to make an oscillator based on the microdisk resonators. This oscillator which works based on optical transduction mechanism is then modeled by behavioral simulators and the results are compared and discussed with each
other. Chapter 5 talks about conclusions from the research in this dissertation and the opportunities for future works.
Chapter 2: SiC Nanomachining Using FIB Milling

In the previous chapter, an introduction to the micro/nano mechanical resonators and their classification in terms of geometry and mode of vibration as well as mechanical properties and some other relevant concepts was presented. Also, properties of Silicon Carbide (SiC) which makes it a strong candidate for making MEMS/NEMS resonators were explained. Choosing the correct material is the first step in terms of making mechanical resonators. The very next step is analysis of the good possible methods of nanomachining – the term means cutting out at nano scale here - and from which selecting a good one as our nanofabrication method of choice. This chapter begins with this topic and then proceeds with characterization of this technique. It is noteworthy that the whole resonator fabrication process is not discussed here, but the selected fabrication tool for SiC resonators is characterized in detail, both experimentally and theoretically.

2.1 Nanomachining of MEM/NEM Resonators

One of the outstanding challenges for SiC in these novel applications is efficient and high-quality nanomachining of the material. The conventional machining methods for the materials can be categorized as photolithography and non-photolithography-based ones. A study of different nanomachining methods is presented here, and advantages and disadvantages of each are explained in detail. Although some of these methods can be used
for SiC nanomachining, only one is selected based on its advantages for making the prototype MEMS/NEMS resonators of this PhD dissertation out of SiC.

### 2.1.1 Analysis of Nanomachining Tools

Conventional photolithography is poorly suited for patterning non-planar surfaces. In addition, it can hardly be used to generate structures in sub-100 nm scale due to limitation of optical resolution (because of diffraction of light, backscattering from the substrate, and difficulty in developing the patterns) as the critical device dimensions shrink [69, 70]. Isotropic etchants produce undercuts with large bias, distance of undercutting, which is not desirable in many applications. Also their etching process is hard to repeat precisely due to sensitivities to change of temperature and concentration changes. Development by wet anisotropic etchant is a better method to generate 3D patterns due to sharper and better-controlled edges. However, the final geometry of micromachined structures are restricted by the crystal orientation. Also, as another disadvantage, it is very difficult - if not impossible - to find a fast anisotropic wet etchant for SiC (maybe one of the only choices is etching by phosphoric acid, an isotropic etchant, at 200 °C) [71]. It is important to note that the last reason is not a determining factor for not selecting wet anisotropic etchants for SiC nanomachining as throughput is not a main factor for making the dissertation prototypes.

Plasma (e.g. fluorine-based plasma of SF₆/O₂) can be used [72] has been already successfully used for etching SiC since 1997 [73]; however, it should be noted that it is also a rather slow etchant for SiC. Comparative studies of the F₂ plasmas for deep etching
of SiC is available in the literature [74]. SF₆/O₂ is recently used to etch SiC using mask patterns as small as 115 nm [75], and also for making MEMS resonators of SiC. [76].

Nevertheless, a disadvantage of using fluorine-based plasma etching for prototyping of the MEMS/NEMS resonators is that the etching profile (shape) strongly depends on composition of the gasses. In special case of SF6 plasma, density of oxygen strongly affects the profile of SiC i.e. changes anisotropy and etch rate in a nonlinear way. This is considered a bad characteristic for fast and repeatable production of SiC micro/nanoresonators in a university lab. In other words, in the characterization Experiments I and II (explained later in this chapter), good control of the etching size (and pattern) were demanded to be able to quantify the milling (nanomachining) effects for different milled patterns and/or under different milling conditions. As the densities of the gasses were not exactly controllable, plasma etching method was not used for etching SiC in this project.

(Pulsed) laser radiation is a direct-write process and does not need any masks which is good for making SiC MEMS/NEMS resonators. There have been several published studies on use of nano and pico second [77] as well as femtosecond [78] (pulse-duration) lasers to improve the lateral resolution of (nano)machining. As the duration of pulse decreases, theoretically absorption depth becomes greater than heat diffusion length in most materials and causes better spatial resolution. However, in femto-second lasers which could be used for submicron machining, the prevalence of energy transfer through photon-atom interaction over heat generation is a nonlinear (nonmonotonic) function of pulse duration and also dependent on target material and environment conditions [79]. This does not mean
that pulsed laser radiation is not a good method for SiC nanomachining. However, for the characterization process in this dissertation (specifically the concept discussed in this chapter), it would be very difficult to use pulsed laser for nanomachining of SiC, as control of the above-mentioned laser parameters to make several prototypes at the same fabrication conditions for characterization purposes is not easy.

Electron-beam lithography (EBL) is advantageous to photolithography in the respect that it can beat the diffraction limit of light and make features in the nanometer regime. Although, the spot size of SEM electrons can be less than $5\,\text{Å}$, the sputtering lateral resolution is limited by forward scattering of electrons in the resist layer and secondary electrons from the underlying substrate [80]. Here, sub-100 nm features are routinely achieved using EBL technique; e.g. 50 nm dimensions have been reported in patterning nanochannels etched into substrate using RIE [81]. A disadvantage to EBL to make nanostencil mask and nanoimprint molds, is that it is mask-based and the residues of the mask (after removal) can have adverse effects. Overall, EBL could also be a choice for SiC nanomachining, and the only reason it was not finally chosen as the nanomachining method in this project, was the long time it takes to sputter arrays of SiC rings – to make suspended microdisks from – where each ring has $\sim20-40\,\mu\text{m}$ diameter (i.e. the difference between the ring inner and outer diameters) and $\sim0.5\,\mu\text{m}$ depth.

Focused Ion Beam (FIB) is an advanced powerful alternative method to EBL lithographic methods for nano-milling. Though photons are intrinsically larger than electrons, FIB principally does not have much lower lateral resolution (i.e. linewidth) compared to EBL since secondary electrons in FIB milling have lower relative energies than primary
particles, while in the case of electron beam it is not the case. Also advantageously, FIB provides a direct (i.e. mask-less) machining method that enables it to do away with the photoresist and its contaminations. The heavy mass and at the same time electrical reactivity of incident charged ions on the other hand set the way for a fast (high yield) patterning technique. Recently, structures with 30 nm feature sizes at surface (sub-5 nm at deepest position of the trench) have been realized with mere rastering of the beam (i.e. no advanced additional technique) [82]. These recent works (and also the ones specifically on SiC [83]) usually only push for fabrication of the smallest trenches (e.g. as small as several 10 nm long at the surface). However, to make 3D structures like mechanical resonators or other structures like SiC molds for nanoimprinting micro-/nano-sized planar antennas and interconnects, we also need to know the exact depth (and sputtering yield) of different geometries. In addition, the precise shape of the milled structures at depths (and not only at the surface) for different feature sizes should be investigated. Therefore, the smallest lateral (surface) feature size chosen in the experiments in this chapter is ~0.25 μm so that the characterization quantities will be applicable to the microdisks to be machined in the next chapter.

2.1.2 Review of Fabrication Methods for SiC Resonators

Several resonator works have already been reviewed in terms of their performance and operational characteristics in Section 1.7. Talking about the fabrication methods in these works, SiC is deposited using LPCVD in [23], [24] and [25]; while, The resonators are patterned using RIE in [23], dry etching and FIB [24], and using NF₃/O₃/Ar plasma etching in [25]. The patterning of non-SiC parts in [26] is done via KOH wet etching, followed by
Cl₂-based RIE to define the SiC cavities. In [27], SiC is grown using APCVD process and the SiC resonator fabrication is done by anisotropic electron cyclotron resonance (ECR) NF₃/O₃/Ar plasma etching. In [29], poly-SiC is deposited by LPCVD and the poly-SiC layer is patterned using low-temperature oxide as the etch mask. Finally, the deposition of SiC in [30] is via LPCVD and the poly-SiC patterning through HBr and Cl₂-plasma etching (instead of fluorinated-gas based etching which is claimed to increase surface roughness).

2.2 Basic Terminology of the FIB Milling

Focused ion beam (FIB) is a beam of ionized atoms incident from a FIB column towards a surface (called target surface) at a specific angle and with specific beam spot dimensions. Before the ionized particles (called primary particles, usually Ga⁺ ions) are accelerated by applying an electrostatic force of energy \( E \), the target material is mounted on the FIB machine stage in a chamber. Then, the pressure of this chamber that FIB column exists inside which is lowered using gasses and motors so that the incident Ga⁺ ions can accordingly fly at the lowest air viscosity – so that they will not be deviated (much) from the incidence angle once they are projected.

Upon hitting the target surface, the momentum transfers from the primary particles to the target atoms. During this ion-atom collision which can be either elastic (also called nuclear) or inelastic (also called electronic), part of the momentum transfers from the primary particles to the target atoms or third particles. The primary particles – if not reflected when hitting the target surface - will finally lose their kinetic energy after (several) collisions, thus become stand still and rest (i.e. are embedded) at some depth in the target material. Further effects of the embedded Ga⁺ ions on the future incident ions will be
discussed in Experiment 1 (Section 6 in this chapter). Target atoms, on the other hand, either only recoil (i.e. move from their original position a limited distance) due to the collision and then stand still or get enough energy from the collision to leave the target surface. The latter phenomenon is called sputtering and the minimum energy the target atoms need to get from the primary particles to sputter is called surface binding energy (SBE). Third particles can be generated during this collision which are secondary electrons or ions generated, which would either reside in the target material or exit it some way.

![Figure 2.1](image.png)

*Figure 2.1. A graph of some physical interactions (recoils, ion implantation and atoms sputtering) occurring between the incident Ga⁺ ion and the 3C-SiC (Si and C) atoms (adapted from [84]).*

### 2.3 Polytype 3C-SiC and Its Growth

Cubic 3C-SiC is one of the most common polytypes of the SiC, which in conjunction with hexagonal 4H-SiC and 6H-SiC has been deployed for making high-temperature resilient electronic components (e.g. PN junction diodes and MOSFETs) and mechanical transducers. The latter application as discussed in the previous chapter is due to many good mechanical (e.g. large Young’s Modulus even at high temperatures) and chemical (inertness) properties of SiC. Fig. 2.2 shows a sketch of crystalline cubic-lattice of 3C-SiC in the form of 1, 2, 3, 1, 2, 3 (double layers in unit cell). [85] As seen, it has a zinc-blende lattice structure with larger Si atoms and smaller C atoms (created with VESTA software). [86]
The 3C-SiC has single- and poly-crystalline structures. The 3C-SiC used for the characterization experiments in this chapter is single-crystalline. Single-crystalline 3C-SiC thin films can be grown on Si wafers by atmospheric pressure chemical vapor deposition (APCVD). The process of growth is done in several steps: The first step involves heating up Si wafer to 1000°C with H₂ present to remove silicon native oxide. The wafer is then cooled to less than 500°C. When the wafer temperature is brought down, a very thin (few nanometers deep) layer of SiC film is already grown on the wafer. However, the thick continuous single-crystalline SiC layer is formed in a step called carbonization: In this step, the heated Si substrate is exposed to a gas-shape hydrocarbon (e.g. propane C₃H₈). The gaseous mix of H₂ and the hydrocarbon is passed over the wafer surface when the wafer is heated to 1300°C. [87] The single-crystalline SiC is grown at about 10-20nm/min, thus SiC films of different thickness can be grown by different lengths of the carbonization step.
2.4 SRIM (and TRIM) Simulator

TRIM simulator is core of a set of simulators called SRIM originally developed by J.F. Ziegler and J.P. Biersack three decades ago [88] to study the phenomena associated with the collision of ions with atoms. These phenomena which were briefly discussed earlier in this chapter (Fig. 2.1) were covered in the software simulations. The software does 2D Monte Carlo simulation of the collisions which might occur for the case a single accelerated ion hits the material surface with known material (properties). The simulator then gives the distribution of ions and atoms, as well as the average penetration depth of the ion and its spread along (straggler) and orthogonal to the ion beam. Sputtering rate is another parameter that the simulation provides.

Although the software is very popular and widely-used in the radiation and FIB communities, it also has some several shortcomings that deserves attention for a wise interpretation of the simulation results. First, at each single simulation which is in fact a Monte Carlo set of simulations, a single atom is incident towards the material surface. This simply means the effect of ion current $I$ is not considered in the simulations e.g. in deriving parameters like average sputtering yield. Also, the crystalline structure of the target material is not taken into account in the simulations, consequently there would be no difference in simulation results for different polytypes of SiC as the target material. In other words, the target material is considered absolutely amorphous, i.e. the ion channeling phenomena is not covered in the simulation model.

In real-life milling of a trench in any target material by accelerated ions, the recoiled ions next to the edges of the trenches do not leave the trench area, but accumulate (redeposit)
along the trench edges (still inside the trench area). These redeposited atoms make up a
tsloped (called *V-Shaped*) accumulation of atoms. However, in TRIM software, the different
simulations each associated with *independent* consecutive collisions of an incident ion with
the target material atoms, are not correlated to each other, i.e. collisions in the new Mont
Carlo runs of the simulations are totally independent of the previous ones. In other words,
this simulator is not designed to consider the accumulated damage (and redeposition of the
sputtered atoms). As a consequence, the effect of implanted charge-inducing Ga ions on the
future incident ions is also not modeled by the software. We will see the latter effect in
experiment measurements from Experiment I and Experiment II.

### 2.5 Theoretical Analysis of SiC Using TRIM Simulator

Despite of the above few shortcomings, TRIM simulator results are usually considered a
standard metric in that the results are accurate enough for many applications. The software
gives accurate results when the beam-milling is governed by binary and immediate
displacements in collision cascades. It only does not take into account dynamic properties of
the target material (e.g. mobility, annealing) [89] It was the motivation in this dissertation for
doing TRIM simulations before the real experiments are done.

After the TRIM simulations, Experiments I and II are performed to 1) see the effect of the
phenomena that TRIM simulator falls short of accounting for (e.g. redeposition and effect of
change of current), 2) measure some lateral and vertical factors for different target trench
area and depth sizes.

The first parameter of interest in this dissertation to be calculated using TRIM simulator is
sputtering yield. The other parameters are trajectories of Ga\(^+\) ions inside SiC target material,
the Ga⁺ implantation sites at different FIB energies, the collision points between primary ions and the target atoms and maybe the most important of these four, the peak-density range (i.e. depth) as well as lateral spread of the Ga ions implanted inside SiC target material.

2.5.1 Sputtering Yield

The simulation in this section is aimed to provide some theory-based data to compare with the measured parameters, on the sputtering yield of SiC (and its constituent atoms C and Si) at the given FIB milling conditions. Sputtering yield is defined as the average number of atoms leaving the surface of a solid per incident primary particle. TRIM simulator can calculate the sputtering yield based on a Monte Carlo simulation of hitting the SiC target surface with the incident Ga ions. It is important to note that if the target material is made of several elements, TRIM gives sputtering yield in terms of number of sputtered atoms not number of sputtered molecules. [90] The Monte Carlo simulations in my simulations were done for 1000 ions (which typically is enough for the software to give results with less than %10 accuracy). [91]

As said before, the sputtering yield derived is a function of mass of both primary and target atoms and the primary particle energy (thus voltage) but not its current.
Fig. 2.3 depicts SRIM modeling of the sputtering yield for the case SiC target material is hit by accelerated Ga ions (incident at different energies) normally (i.e. at 90° angle) onto SiC target surface, similar to what can be done in a focused ion beam (FIB) machine. It is also noticeable that the sputtering yield versus incident ion energies experiences different slopes; sharper at the beginning and then more flat as energy increases with a maximum at some specific energy in the range. In nanomachining applications, if highest sputtering yield is of interest, this maximum FIB energy should be theoretically chosen for milling. Sputtering yield for Si atoms at the same FIB energy is higher than that for the constituent C atoms at the same FIB energy. It is important to note that sputtering occurs when the energy transferred from the incident ions to the target atoms (constituent C and Si) is higher than their surface bounding energies (SBE). The surface binding energy (SBE) for SiC used for sputtering yield calculations in TRIM is 3.4 eV. The minimum energy transferred is proportional to

\[ \frac{1}{(1 + \frac{M_{\text{ion}}}{M_{\text{target}}})(1 + \frac{M_{\text{target}}}{M_{\text{ion}}})} \]  \hspace{1cm} (2.1) 

This equation implies that the more the atomic masses of the target material and the primary incident material is similar in quantity, the higher portion of the energy of the incident material will transfer to the target material. That is why the portion of the energy transferred from Ga ion (mass: 69.7u) to the Si (mass: 28.08u) is higher than that to the carbon (mass: 12u). This higher kinetic energy – in fact that part of which in excess to SBE – contributes
to the sputtering yield, that is why the sputtering yield of Si atoms is higher than that of the C atoms.

Later in this chapter (Experiment I in Section 6), the above results are compared to those derived from the experiment measurements. We will see that the sputtering yield is also a function of the Ga ions current.

### 2.5.2 Charge-Contributing Ga Atoms Implementation Profile

This section discusses FIB milling simulations of SiC using TRIM simulator to model the ion-atom collisions and derive the implantation profile of charge-contributing Ga ions inside the target material (SiC). This simulation study is aimed to give rather accurate insights to the behavior of FIB-milling parameters that have not been covered by the two experiments in Section 6 of this chapter. It is due to mention that the Ga ions when implanted might have already lost their charge and tuned into neutral Ga atoms; however, the charging effect exists at spots very close to the implantations spots because SiC is not a conducting material.

Fig. 2.4 depicts damage of the target material atoms as well as the implanted ions distribution profile for 30keV incident Ga. Fig. 2.4a and 2.4b show profile of charge-contributing Ga atoms inside SiC colliding with Si and C atoms respectively, while Fig. 2.4c shows distribution of (still) implanted ions inside SiC. Fig. 2.4d shows all those collision trajectories and implantation sites of Ga ions in one plot. The peak density of implanted ions is also shown in Fig. 2.4c to be 188 Å (18.8 nm) from the surface.
Figure 2.4. TRIM Simulation for trajectory of \( \text{Ga}^+ \) ions inside SiC atoms as well as the \( \text{Ga}^+ \) implantation sites for \( E \sim 30 \) keV FIB. (a,b) demonstrate the collision points between primary ions and the target atoms (respectively for Si and C) along the \( \text{Ga}^+ \) trajectory inside SiC. (c) The black nodes show a 2D profile of implanted \( \text{Ga}^+ \) ions resting under the surface of SiC; while gray line curve shows ion density versus depth. Peak density ion range of 18.8nm is also designated in the figure. (d) Collision points and implantation sites versus depth all in one graph. Simulations done for 1000 incident ions (to derive distance of peak implanted ions) but shown here for 150 ions for the sake of clarity of graphs.

Fig. 2.5 depicts the plot in Fig. 2.4d for different incident ion energies. This figure clearly shows that as FIB energy increases, the depth for peak density range of the implanted ions as well as their lateral spread increases. Fig. 2.5d is exactly a duplicate of Fig. 2.4d (i.e. at 30keV FIB energy); while, Fig. 2.5a, 2.5b, and 5c show the plot for other FIB energies. Fig. 2.5b and 5d are specifically important as they are done at two FIB energies 10 keV and 30 keV used in Experiment I (Section 6 of this chapter) which respectively have peak implanted ions density at depths at 9.2nm and 18.8nm. As we will see in later sections, this explains why the repelling effect of implanted ions - the ions inside and outside the slot area (see Section 6.1, Experiment 1) - is stronger for the set of rectangles milled at 10 keV compared to the one at 30 keV, as the implanted ions are nearer to the surface.
Figure 2.5. TRIM Simulation of Figure 4d done at different FIB incident ion energies: 5 keV (a), 10 keV (b), 20 keV (c) and 30 keV (d). Lateral spread as well as the depth at which peak density of ions are implanted are shown for different FIB energies. It is seen that higher FIB energies result in more lateral and in-depth spread of damage and larger distribution of implanted ions. Simulations done for 1000 incident ions (to derive distance of peak implanted ions) but shown here for 150 ones for the sake of clarity of graphs.

2.6 Experiments

2.6.1 FIB/SEM Instruments

The experiments are done with FEI Helios NanoLab 650, which is an advanced dual-beam FIB-SEM machine for high resolution characterization in 2D and 3D with advanced field emission SEM (FESEM) and focused ion beam (FIB) technologies all in one machine. The energy range for e-beam is 20 eV-30 keV and for ion beam is 500 V-30 kV. Probe current range for e-beam is 0.8 pA to 26 nA and for ion-beam in the range of 1 pA to 65 nA. [92] Both SEM imaging and FIB milling were done with their associated beam column perpendicular to the target surface; i.e. SEM was done when the SEM column had a tilt
angle of 0°; while, FIB was done when FIB column had a tilt angle of 52°. The setting for SEM imaging of Experiment 1 (slots) and Experiment 2 (set of scaled-sized shapes) in Section 6 of this dissertation (both at zero tilt angle) which were used for lateral size measurements are reflected in Table I.

<table>
<thead>
<tr>
<th>Variables &amp; Parameters</th>
<th>HFW* (μm)</th>
<th>VFW* (μm)</th>
<th>WD* (mm)</th>
<th>energy (keV)</th>
<th>current (pA)</th>
<th>pixel size (nm)</th>
<th>dwelling time(ns)</th>
<th>detector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment 1</td>
<td>53.7</td>
<td>46.3</td>
<td>3.9</td>
<td>5</td>
<td>800</td>
<td>52.4</td>
<td>6000</td>
<td>ETD*</td>
</tr>
<tr>
<td>Experiment 2</td>
<td>17.1</td>
<td>14.7</td>
<td>4.1</td>
<td>5</td>
<td>200</td>
<td>8.33</td>
<td>400</td>
<td>ETD</td>
</tr>
</tbody>
</table>

*HFW, VFW, WD and ETD respectively stand for horizontal field width, vertical field width, working distance and Everhart-Thornley detector (ETD). ETD is a secondary electron detector.

2.6.2 Experiment I (Rectangular Slots)

In the first experiment, two-sets of 2 μm x 5 μm rectangular slots were milled on the same SiC chip and sputtering yield [atoms/ions] as defined by average number of total SiC atoms (either Si or C atoms) removed per incident Ga⁺ particles is calculated. The focused Ga⁺ ions of the machine's FIB column were irradiated at two FIB energies (10 keV and 30 keV) and different currents reported. The voltages and currents are reported in Table II in conjunction with the associated milling time, t and depth, d measured using Agilent 5500 AFM (contact mode, dwelling time ~ 300 ns, normal incidence). Fig. 2.6 depicts the top-view SEM images (taken at zero tilt angle of the stage with respect to SEM column) of the rectangular slots which had been previously milled normal to the FIB column respectively at 10 keV and 30 keV energies with milling time in seconds shown above each slot in the figure. It is seen that the trenches milled in 10 keV have more lateral deformation compared to the ones milled in 30 keV. This is more due to the lower energy of 10 keV FIB which
implants Ga\(^+\) ions nearer to the surface compared with 30 keV FIB (see simulation results in Section 5 of this chapter); thus the implanted ions repel and deviate the ones irradiated afterwards more strongly. An important consideration for the experiment was that for lower currents, higher milling times had been used to ideally achieve similar depth ranges in the slots (except for one slot in each set). Fig. 2.6c-2.6e as well as 6f-6h present the AFM image as well as depth profiles (along with slot width and length) for the right-end slots (shown inside boxes) among SEM images of Fig. 2.6a and 2.6b. These two slots are chosen for case show as they were milled at nearly the same FIB current and milling time. These curves show how much the average V-shape slope of the trench walls (due to redeposition) in the higher energies (e.g. 30 keV) is steeper than that in lower energies (e.g. 10 keV).

To quantitatively compare the effect of change of FIB current in vertical profile of the milled slots, *sputtering yield*, \(Y\), can be derived experimentally (Eq. (2.2)). [93]

\[
Y = \frac{dN_T}{(It/Aq)} \tag{2.2}
\]
with \(d, N_T, I, t, A\) and \(q\) respectively represent milled depth, target (SiC) atomic density, FIB current, milling time, scan (milling) area and ion electrical charge. All parameters have SI units and \(N_T\) is assumed \(9.66 \times 10^{28} \text{ m}^{-3}\) for 3C-SiC [94]. Fig. 2.7a shows the sputtering yield versus energies found from TRIM simulator which works based on theory [91], shown in nodes connected with solid lines as well as those from the experiment at two energies of 10 keV and 30 keV (shown by stars). TRIM does not account for the effect of different FIB currents on sputtering yield nor does it account for the phenomena like redeposition and change of yield during milling time (see Section 5). We argue that these phenomena serve as two reasons for difference between the simulation and measurement results. The other reason for the discrepancy can be the difference between real density of the 3C-SiC and what has been considered here. Other possible reasons are discussed in Section 5.

For different slots in this experiment, \(N_T, A\) and \(q\) are the same, so sputtering yield from Eq. (2.2) can be considered a weighted \(d\), i.e. averaged over \(It\). Fig. 2.7b demonstrates measurement results of sputtering yield versus current for two sets of milled slots. This curve shows an interesting phenomenon: sputtering yield for 30 keV is rather constant versus current; while at 10 keV FIB energy, it decreases (rather linearly) with current. Sputtering yield, as a rate of milling ideally should not depend on duration of milling, but it is proportional to rate of charge accumulation underneath surface of the milled area; thus on current \(I\). In other words, the higher the current, the higher the rate of charge accumulation; thus, the more repelling of ions originally irradiated towards the target material. In the higher FIB energy of 30 keV, the Ga atoms are implanted deeper and have negligible effect on the \(Ga^+\) ions irradiated thereafter. It is due to mention that the Ga ions when implanted might have already lost their charge and tuned into neutral Ga atoms;
however, the charging effect exists at spots very close to the implantations spots because SiC is not a conducting material.

Table II: Focused ion beam milling specifications in Experiment 1 (Slot Milling Experiment).

<table>
<thead>
<tr>
<th>Variables &amp; Parameters</th>
<th>*E, I, t, d (Fig. 2.6a)</th>
<th>*E, I, t, d (Fig. 2.6b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slot 1</td>
<td>10,0.576,26,150</td>
<td>30,1.120,25,340</td>
</tr>
<tr>
<td>Slot 2</td>
<td>10,0.365,77,300</td>
<td>30,0.603,48,345</td>
</tr>
<tr>
<td>Slot 3</td>
<td>10,0.202,144,320</td>
<td>30,0.275,87,290</td>
</tr>
<tr>
<td>Slot 4</td>
<td>10,0.104,235,275</td>
<td>30,0.155,254,472</td>
</tr>
</tbody>
</table>

*E, I, t and d respectively represent energy [keV], current [pA], milling time [s] and milled depth [nm] of focused ion beam.

Figure 2.7. (a) Sputtering yield versus FIB energy from TRIM simulations (nodes connected with solid line) as well as those from Experiment I measurements (star nodes). (b) Sputtering yield for the slots in Experiment I versus current. (c,d) quantitatively analyze the slope of V-shape walls milled at the two FIB energies by demonstrating $\frac{\Delta W}{W}$, defined in the research, when normalized by milling time (c) and by sputtering depth (d). The inset in (c) defines the parameters used to define $\Delta W$ and the inset in (d) shows the ions implanted inside and out of slot area as well as their repelling effects on forthcoming ions. The numbers above the 10 kV FIB data in (d) represent the value of $It$ (i.e. FIB current multiplied by milling time) in nanoCoulomb.
Fig. 2.7c and 2.7d show characteristics of V-shape walls as a lateral benchmark for FIB milling of 3C-SiC. Fig. 2.7c shows average width of the V-shape slope (per second) versus current for the two sets of rectangular slots (see Fig. 2.6a and 2.6b). The average V-shape wall width is defined as \( \Delta W = \frac{1}{2}(W_{\text{top}} - W_{\text{bottom}}) \) with \( W_{\text{top}} \) and \( W_{\text{bottom}} \) parameters defined graphically in inset of Fig. 2.7c. Figure 2.7d shows \( \frac{\Delta W}{2} \), i.e. average slope of the V-shape walls. The value of \( Q = It \) for each milled slot is written next to the corresponding node in the curve.

Three of the rectangular slots (in each set) are milled at nearly equal \( It \) bit in all four slots in each set, for higher the FIB current \( I \), the lower the milling time \( t \) is chosen. As \( \Delta W \) is expected to be directly proportional to the number of charge-contributing Ga ions hence to \( It, \frac{\Delta W / 2}{t} \) (Fig. 2.7c) which can be written as \( \frac{\Delta W / 2}{It} \); is expected to be almost a linear function of \( I \) in the slots with nearly equal \( It \) as can be seen for 30keV curve.

But this does not explain the behaviors of 10keV curves in Fig. 2.7c and 7d. In Region II of Fig. 2.7c for 10 keV FIB, we see a nonlinear, i.e. an order of larger-than-one functionality with \( I \). Also, both \( \Delta W \) and \( d \) are ideally linearly proportional to the number of charge-contributing Ga ions, hence proportional to \( It \). Therefore, we don’t expect to see a nearly constant \( \frac{\Delta W / 2}{d} \) versus \( I \) for three slots in Fig. 2.7d while for 10 keV FIB curve it does not perform according to expectations. To explain these anomalies seen in the 10keV curves in Fig. 2.7c and 2.7d, the following background and the explaining theory is
suggested. There are basically two major phenomena affecting the sputtering depth, \( d \) and V-shape wall width \( \Delta W / 2 \) of 3C-SiC slots:

(1) The first phenomenon is about the effect of the Charge-contributing Ga ions implanted inside the rectangular slot area. The charged ions implanted underneath 3C-SiC surface inside the slot area repel and deviate the future Ga\(^+\) radiated ions, off the slot area (inset of Fig. 2.7d). The Ga\(^+\) ions originally targeted towards center of the slots are then deviated to the slot corners and even off the slot area, this phenomenon decreases \( d \) and also increases \( \Delta W / 2 \) - the latter by increasing \( W_{\text{top}} \). This explains why in Region II of Fig. 2.8c for 10keV FIB, i.e. for higher currents in contrast to lower currents in Region I, \( \frac{\Delta W / 2}{t} \) seems to be higher-than-one function of \( I \) and not a linear function as seen in Region I. This phenomenon is also responsible for higher-than-one increasing functionality with \( I \) of \( \frac{\Delta W / 2}{d} \) in Region II of Fig. 2.7d that \( I \) is large. It is important to note that this phenomenon is only a function of \( I \) and not \( t \), because the charge-contributing Ga ions implanted inside the slot area do not accumulate over time.

(2) The second phenomenon is related to the effect of the charge-contributing Ga ions which were originally targeted inside the slot area but were embedded outside of (but close to the slot area). In Region I, which corresponds to longer milling times (thus lower FIB currents as seen in Table II), the first phenomenon is weak and this second phenomenon with an accumulative-over-time effect dominates. The reason the effect of these implanted ions is accumulative over time in contrast to those of the first phenomenon is that the ions implanted outside of the slot area are not sputtered by the Ga\(^+\) ions irradiated later, while
the ions of the first phenomenon which were implanted inside the slot area, were sputtered out by the ions irradiated later toward the slot area; thus don’t accumulate over time. In fact, the Ga\(^+\) ions which were targeted inside the slot area but hit the target surface outside of the slot area, have lower energies compared to the ones hit the surface inside the slot area, and this energy is not enough for sputtering either 3C-SiC atoms or previously implanted Charge-contributing Ga ions; hence these ions are only implanted under the surface without sputtering anything (inset of Fig. 2.7d).

Therefore, their repelling effect on forthcoming Ga\(^+\) ions adds up gradually. That is the reason for larger \(t\) (i.e. smaller \(I\)) more forthcoming Ga\(^+\) ions originally targeted outside of the slot area, are deviated by these implanted ions inward, i.e. inside the slot area. This, in turn, decreases \(\Delta W / 2\) by increasing \(W_{\text{bottom}}\) and also increases \(d\). Therefore, a higher-than-one functionality of \(\frac{\Delta W / 2}{d}\) with \(I\) in Region I of Fig. 2.7d is seen.

### 2.6.3 Experiment II

In Experiment I (Rectangular Slots), I investigated vertical and lateral characteristics of FIB milling of fixed-dimension 3C-SiC rectangular trenches for different FIB \(I\) and \(E\). However, MEMS 3D fabrication and also other applications e.g. nanoimprint demands characterizing the material for different trench shapes and dimensions. FIB instruments these days enable milling ready-to-use circular and polygonal shapes. In Experiment 2, trenches with several popular shapes (that might serve as building blocks for the more complex shapes for future applications) are milled out. These shapes include: (1) circle, (2) square, (3) cross (composed of two crossing rectangles with aspect ratio 4:1 each, with the
vertical one milled after the horizontal one), and (4) triangle. The feature sizes of the of the shapes are 2 $\mu$m, 1 $\mu$m, 0.5 $\mu$m and 0.2 $\mu$m, while for cross the width of each of its two rectangles is $\frac{1}{4}$ of the feature size. Four array, each including 4 x 4 trenches – with each of the above four shapes at four dimensions - have been milled out (e.g. Fig. 2.8a). The whole patterns on each array were milled out in a single run; i.e. all the 16 shapes of each pattern were milled at the same aimed depth ($d_{aimed}$) and also with the same FIB column settings, i.e. the same FIB current and voltage. The arrays are milled at aimed depths of ~100 nm and ~150 nm and each, at two currents of ~80 pA and ~800 pA, totally four arrays.

**Figure 2.8.** Milling of different shapes and study of the effect of lateral-size scaling, at FIB $E$~30 keV and $I$~80 pA in Experiment 2. (a) SEM image of the array milled with shapes of feature sizes 2 $\mu$m, 1 $\mu$m, 500 nm and 200 nm. The cross is composed of two overlapping 4:1 rectangles (i.e. width of each rectangle is $\frac{1}{4}$ of the feature size at each scale); with horizontal one milled before the vertical one. (b,c) demonstrate the AFM image of the triangle with the corresponding depth profiles along the red lines respectively in vertical and horizontal traces of the AFM tip.

![Diagram](image.png)
Figure 2.9. Two lateral deformation figure of merits, Ratio of diagonals (a) and Normalized average Diagonals (b), as well as sputtering yield (c) to compare scaling effect on different geometries of different dimensions in the array. For triangles, altitude and for circles, diameter is used instead of diagonal.

Figure 2.10. Milling of different shapes and study of the effect of lateral-size scaling, at FIB E~30 keV and I~800 pA in Experiment 2. (a) SEM image of the array milled with shapes of feature sizes 2 µm, 1 µm, 500 nm and 200 nm. The cross is composed of two overlapping 4:1 rectangles (i.e. width of each rectangle is ¼ of the feature size at each scale); with horizontal one milled before the vertical one. (b,c) demonstrate the AFM image of the triangle with the corresponding depth profiles along the red lines respectively in vertical and horizontal traces of the AFM tip.

Figure 2.11. Two lateral deformation figure of merits, Ratio of diagonals (a) and Normalized average Diagonals (b), as well as sputtering yield (c). For triangles, altitude and for circles, diameter is used instead of diagonal.

It is worth mentioning that to keep the same milling conditions for all the trenches in one array, so that the milling results can be compared together, the FIB machine settings (e.g. focus and astigmation) were set once for all shapes in one array, and then all trenches in each array were milled out in a single milling run (thus for the same aimed milling depth).
Based on the lateral effects study in Experiment 1, 30 keV FIB energy was used throughout Experiment 2 to minimize the lateral damage. The AFM depth measurement (used later to calculate $Y$) was only done for two arrays: the arrays milled at $I \sim 80$ pA and $\sim 800$ pA, both with the larger aimed depth of $d_{aimed} \sim 150$ nm. However, to analyze lateral damage by change of milling current, SEM images of the other two sets with the same aimed depth of $d_{aimed} \sim 100$ nm ($I \sim 80$ pA and $\sim 800$ pA) were used, shown in Fig. 2.10a and 8a respectively. From the latter two arrays, the one milled at lower current of $\sim 80$ pA has shapes with sharper edges; while in the one milled at $\sim 800$ pA rounded edges can be clearly seen which in turn lead to change in length of diagonals. Therefore, the size of diagonals can serve as a quantitative benchmark for lateral damage study.

Fig. 2.8b and 2.8c as well as 2.10b and 2.10c depict the AFM 2D depth profiles of two triangles (as examples of the array shapes) in arrays milled at $I \sim 80$ pA and $\sim 800$ pA (both $d_{aimed} \sim 150$ nm) respectively. The triangles whose AFM profiles are shown in these figures have 0.5 $\mu$m feature size (marked in SEM images). Fig. 2.9a and 9b as well as 11a and 11b show lateral size study of the arrays milled respectively at $\sim 80$ pA and $\sim 800$ pA both for the same $d_{aimed} \sim 100$nm. In this dissertation, two lateral figure of merits (FOMs) are proposed and used for these two arrays: FOM 1, ratio of diagonals, as a measure of change-of-aspect-ratio from the aimed aspect ratio; and FOM 2, (normalized) average of diagonals, as a measure of rounding of the shapes (at the edges). Fig. 2.9a and 11a depict FOM 1, the ratio of diagonals (with altitudes in case of triangles and diameters in case of circles instead of diagonals). For $I \sim 80$ pA, we observe that while at larger feature sizes (here $>1 \mu$m), the change-of-aspect-ratio deformation vanishes; for smaller feature sizes,
FOM 1 is larger and the shapes in descending order of tolerance against this deformation are square, circle, triangle and cross. For \( I \approx 800 \text{ pA} \), the order is rather unchanged except that cross suspiciously seems to be the most immune one against this deformation which is due to the excessive rounding (of edges) in the 800 pA case which made exact measurement of diagonals impossible.

Neglecting the specific case of cross at \( \approx 800 \text{ pA} \) in which the measurement was not reliable, the order of tolerance against change-of-aspect-ratio during FIB milling for arrays of \( I \approx 80 \text{ pA} \), and \( \approx 800 \text{ pA} \) can be explained in terms of two geometric parameters of the ideal shapes which give a sense of how thin the shapes are: (1) aspect ratio and (2) shape inner angles. The higher the aspect ratio (e.g. in cross) and the smaller the inner angles of the shape (e.g. in triangle), the larger the shape would be vulnerable to change-of-aspect ratio (deformation) due to effects during FIB milling e.g. charge-induced drift or imperfect focus/astigmatism. At small feature sizes e.g. 250 nm though, we observe exceptions to the above rule e.g. circle seems to have a FOM 1 slightly larger than that of triangle. In fact, most undesired phenomena during FIB milling are independent of the shape size; and reasonable affect smaller feature-size shapes more. This is the reason for larger FOM 1 at lower feature sizes especially if the phenomena mostly affect along one of the shape diagonals.

Graphs in Fig. 2.11b (for 80 pA) and 9b (for 800 pA) depict FOM 2, the normalized average diagonals, defined by average of two diagonals, divided by the ideal (i.e. aimed) size of the diagonal. In contrast to FOM 1 which was a directional figure-of-merit, this FOM quantifies effect of rounding at all edges in average which are represented in terms of
average diagonal elongation from all sides. We see that in ~80 pA case (Fig. 2.9b), triangle (and sometimes square) are the least vulnerable to rounding effect. For ~800 pA (Fig. 2.11b), this FOM is comparatively larger compared to ~80 pA case as higher relative current in ~800 pA cause higher charge-induced drift and elongate the diagonals more. Order of shapes in 800pA case is rather unchanged compared to 80pA (Fig. 2.11b).

The depth measurement and sputtering yield calculation (Eq. 2.1) were done at the deepest points of the milled shapes. The measurement was done after leveling using Agilent AFM post-processing software and also the free software Gwyddion [95] for each shape and the results are reflected in Fig. 2.10c and 12c. For the array milled at ~80 pA, square and circle have the highest yields; with expected (comparable to Experiment 1) yields at feature sizes of 1 μm or higher. However, at smaller feature sizes, their yield drops more than 2.5 times because of size scaling. Triangle has a sputtering yield relatively lower than them for all feature sizes with scaling effect on sputtering even obvious at 1 μm feature size which probably draws on the fact that triangles have smaller angles (thus they are thinner from all sides) compared to circle and square shapes.

Cross is the worst one in this regard because of its higher aspect ratio (thus very thin at least along each of its widths): not only has the lowest sputtering yield compared to other shapes at all feature sizes, but also we see the scaling effect on sputtering yield is not negligible even at 2μm feature size (i.e. when the width size is 0.5μm). For the array milled at ~800pA, two major differences exist: (1) Nearly all shapes seem to have less scaling effect on sputtering yield value for sizes above 0.5μm (compared to their corresponding ones in ~80pA array); (2) Cross and triangle have yields comparable to the other two shapes
at all feature sizes, and for the largest feature size of 2 μm, we observe a sudden increase in cross sputtering yield to nearly 1.8 times of square and circle at the same feature size. The jump in sputtering yield of cross at 2 μm feature size draws on the fact that milling of a cross has been performed by milling two overlapping rectangles one after the other, and it means that ideally, the depth at the overlapping region where depth was measured should be twice that of the one for other shapes at the same feature size. That explains for the jump in cross sputtering yield to 1.8 times that of square and circle at 2 μm feature size in 800 pA; while for 80pA it doesn't even happen.

2.7 Conclusions
In the previous sections of this chapter, incentives for using SiC to make MEM/NEM resonators was reviewed. Following that, 3C-SiC polytype of SiC was introduced and its properties and fabrication method were discussed. In order to use the material for fabrication of mechanical resonators, focused ion beam (FIB) milling was chosen as a method to incarnate the microdisk mechanical resonators discussed in next chapter. However, to that, there is a need to do a thorough characterization of SiC FIB milling which has not been done in the literature. To fill this gap, first a theoretical simulation of the collisions between the accelerated Ga ions – the ions usually used in FIB machines – and the Si and C atoms in SiC target material was done. Although these simulations cannot model the physical phenomena for the case of crystalline SiC polytypes as target material – TRIM simulator assumes an amorphous structure for the target material – however, the simulations results are still fairly accurate. Using these simulations, good intuition in the sputtering yield and ion implantation profile as well as the ion-atom collision locates for different FIB energies is gained.
Experimental measurements were done in the framework of two experiments. In the first experiment, rectangular arrays of few micron lateral size were milled at different FIB energies and currents and the effect of change of FIB current on the sputtering yield was quantitatively calculated. Then, in the second experiment, different geometries of about 250nm (in some geometries even as small as 125nm) to 2μm were milled at different FIB conditions and the effect of change of geometry and size of the pattern on the sputtering yield as well as the lateral deformations were quantitatively measured. The 3C-SiC FIB milling characterization using these two experiments has provided a good basis for fabrication of micro and nano 3C-SiC resonators of different geometries and scales. The results of this research also be used to make SiC nanostencil masks for fast printing of VLSI interconnects and antennas. A separate research on FIB milling of single-crystalline, poly-crystalline and amorphous SiC towards making nanostencil masks is presented in Appendix A.

In the next chapter, fabrication of 3C-SiC microdisks which also includes a FIB milling step will be described in detail. Theoretical analysis and computer simulation results of the microdisk resonator will be presented and then measurement results followed by a thorough characterization of the resonators are given.
Chapter 3: 3C-SiC Microdisk Resonators

3.1 Motivation For the 3C-SiC Microdisk Resonators

Recent advances in microelectromechanical systems (MEMS) have led to various high-$Q$ microdisk resonators [96,97] which are advantageous towards RF/microwave frequency control and timing applications. Also, nanoelectromechanical systems (NEMS) capitalize on many emerging nanostructures such as nanowires/tubes and nanobeams/cantilevers, flexural motions (bending, deflection); and high-$Q$ flexural resonances are often best used for dynamical sensing of ultra-small quantities and effects [98], and for making low voltage and ultra-low power RF/microwave NEMS [99]. SiC is an attractive material and proven to be excellent for NEMS towards these goals [100,101], thanks to its exceptional electromechanical properties and nanomanufacturing availability in very thin layers on various sacrificial materials.

As discussed in Chapter 1, beam MEM/NEM resonators (either single-clamped or doubly-clamped) with different geometries and vibration modes, each resonate at several resonance frequencies called resonance modes. Mechanical disk structures offer versatile resonance modes due to their many interesting vibrations ranging from flexural (transverse, bending) to bulk (extensional, contour), that are dependent upon their geometries and boundary conditions. Such resonances have hence been widely used for signal processing [102]. Literature review of different MEM/NEMS beam and planar resonators (including disk resonators) is presented in Chapter 1.
The circular disk in this work can be imagined as extrusion (i.e., rotation in plane around its clamp) of a very thin singly-clamped SiC NEMS cantilever. This new disk is intriguing from both NEMS and MEMS viewpoints as it is much thinner than mainstream bulk-mode MEMS disk resonators; and it has an extremely large surface area in contrast to its original cantilever template (hence a tremendous enhancement on available active surface for capturing, critical for sensing applications) compared to wine-glass [103,104] and contour-mode [105] disk resonators.

3.2 Chapter Overview

Based on the characterization of FIB milling of 3C-SiC material done in the previous chapter, Chapter 3 reports on a set of experiments aimed to make 3C-SiC microdisk resonators with a novel FIB-based approach. The fabrication method is fully described and its advantages as well as side effects are discussed. Then, the dynamics (mostly resonance frequency) of microdisk resonators are investigated using the accurate as well as approximate analytical formulas from the literature. This theoretical analysis then proceeds with COMSOL simulations to analyze the vibration modes and the associated resonance frequencies calculated. Finally, the frequency spectrum of the vibrating resonators are derived (measured) using a home-built vibration measurement setup based on laser interferometry detection method. The microdisks were in turn actuated by piezoelectric transduction. Using the above experiments and also post-processing of the measurement results, a full-characterization of such microdisk resonators of different sizes was done and design perspectives achieved.
3.3 Multi-Mode Analytical Model

Vibrating disk structures clamped at outer circumference and/or center, have been previously studied [106-108]. In an original work, Southwell [106] did a mathematical analysis on thin circular turbine disks clamped at the center and free at the outer circumference, for flexural (out-of-plane) vibrations which is a structure very similar in geometry to our microdisks.

Fig. 3.1a shows side view of our fabricated structure which is similarly a disk anchored at the center, with anchoring and circumference circles of radii $b$ and $a$, respectively. This structure is slightly different from the simple disk model studied in [106]: (1) It has a parabolic-cone pillar with J-curve sides (due to our specific method of fabrication) anchored to the disk at its center. (2) The disk edges are smoothed due to FIB milling effects (more details in Section III) which slightly contradicts the even thickness assumption of the disk made in Southwell’s equations. We will observe in Section IV that the spectrum of vibration frequencies of this structure is rather similar to that predicted in [106]. Fig. 3.1b shows scanning electron microscope (SEM) photo of a realization to such structure.
Figure 3.1. Schematic and SEM of the microdisk resonators and the flexural modes of resonance. (a) Side-view of our fabricated microdisk resonator structures. The smoothing of disk edges due to FIB-milling is also shown at an exaggerative scale. (b) Tilted view scanning electron microscopy (SEM) image of such a disk demonstrating lateral dimensions of the microdisk and the underneath anchor. (c) The first flexural modes in disk resonators, \( n \) is the number of node diameters and \( m \) is the number of node circles.

The flexural vibration modes of a symmetrical disk structure as discussed in [106] can be expressed by the number of nodal circles \( m \) and nodal diameters \( n \) (Fig. 3.1c). Any deviation from symmetry in shape of the disk or in shape or relative position of the anchor changes the resonance frequencies of the modes in Fig. 3.1c.

Under assumptions of small out-of-plane deflections for the disk and negligible shear vibrations, the theory [106-108] suggests the following equation for radial frequency \( \omega \)

\[
\omega = \sqrt{\frac{3D(1-v^2)}{\rho h}} \frac{\lambda}{a^2} = \lambda \sqrt{\frac{E}{4\rho}} \frac{h}{a^2}
\]  

(3.1)

where \( \rho \) is volumetric mass density, \( h \) refers to \( h_{\text{disk}} \) in Fig. 3.1a and \( D \) is the flexural rigidity defined as \( Eh^3/12(1-v^2) \) with \( E \) and \( v \) denoting the Young modulus and Poisson’s ratio of the disk material. Also, \( \lambda \) is a non-dimensional coefficient [108] strongly dependent on both \( b/a \) and the resonance mode, and nearly independent of \( v \) changes (for \( v \) values
between 0 and 0.5). This coefficient is derived and tabulated in [108] for \((b/a)\) values of 0.1, 0.3, 0.5 and 0.7 where \(\nu = 0.3\). We will use these tabulated values as an approximate to the resonance frequencies for our SiC disks with \(\nu \sim 0.23\).

### 3.3.1 Disk Resonance Modes

Equation (1) does not show explicit relation of \(\omega\) with the anchor radius \(b\) (or the ratio \((b/a)\)) but through the coefficient \(\lambda\). In other words, the effect of a change of physical dimensions on the resonance frequency is explicit for the ratio \((h/a^2)\) and implicit for the ratio \((b/a)\) (expressed through parameter \(\lambda\)).

Fig. 3.2a shows the parameter \(\lambda\) (in log-domain) for the first resonance modes versus \((b/a)\). The legend shows the mode numbers in the form of \(f_{m,n}\). We observe that the resonance frequencies increase as \((b/a)\) increases (\(a\) assumed constant). This is intuitive since increase in \(b\) means the radius of the suspended freely-vibrating disk decreases. Also we clearly observe that modes with higher \(n\) and/or \(m\) have larger resonance frequencies except for modes \(f_{0,0}\) and \(f_{0,1}\) roughly at \((b/a) \leq 0.3\). The latter fact is better shown in Fig. 3.2b with a linear view of \(\lambda\) which compares \(\lambda\) of those two modes at different values of \((b/a)\). This figure also shows a polynomially fit to the four values given in [108].

### 3.3.2 An Approximate Equation for Vibrating Disk

The MEMS design engineers need some non-complex intuitional (though inaccurate) equations to help them with design of the electromechanical structures. These non-complex intuitional equations for multi-mode resonators are usually only expected to predict the lowest mode frequencies (with some error). Here, we analyze their applicability
also for other modes. The simplest and most intuitive equation of this form is

$$\omega = \frac{k_1}{(a-b)^2}$$

(3.2)

where $k_1$ does not depend on disk physical dimensions. In order to observe how closely (2) can follow (1), $k_1$ has been found by equating (1) and (2) at $(b/a) = 0.1$ and then the two had been compared for other values of $(b/a)$. Fig. 3.2c shows the normalized error between $\lambda$ predicted from (1) and its equivalent term from Eq. (3.2).

It is seen that for two modes with no nodal diameters (i.e. $n = 0$), this error is not higher than 5% for $(b/a) \leq 0.7$. It is higher though in other modes. In other words, we can use this approximate equation for modes with $n = 0$ to get a good sense of $f_{0,0}$ or $f_{0,1}$ resonance frequency.

### 3.3.3 Simulation Results for the Disk with Pillar Anchor

In this subsection, we compare the simulation results with the theory from (1). To this end, the structure is modeled in COMSOL software with parabolic cone-shaped pillars (with J-shape sides) from Si anchored from below to the center of the SiC disk with homogenous thickness. The simulation is done at three different values at which measurement is performed (Section IV): (1) $a = 20 \mu m$ and $(b/a) = 0.02$; (2) $a = 20 \mu m$ and $(b/a) = 0.5$; (3) $a = 15 \mu m$ and $(b/a) = 0.34$, all with anchor heights $\approx (a-b)$. The anchor heights are chosen so because silicon material is etched out by the isotropic etchant HNA.

Fig. 3.2d shows the error between the first five resonance frequencies from simulation, and frequencies of the corresponding modes from (1) drawn versus resonance modes
(degenerate-modes for the same $m$ and $n$ are assumed one mode). It should be mentioned that as $\lambda$ in (1) was not given for $(b/a) \sim 0.02$ in the references, linear fitting was used to estimate it.

The error in case (1) with very small $(b/a)$ is not larger than 4%. In cases (2) and (3) with higher ratios of $(b/a)$, the difference is respectively 12.6% and 20.6%. These simply demonstrates that at lower $(b/a)$ values, theoretical equations can be used to roughly predict the resonance frequencies with acceptable error. For larger $(b/a)$, using FEM simulators is inevitable.

There is another interesting observation to Fig. 3.2d: Though the error (i.e. normalized difference) is not negligible for large values of $(b/a)$, it is rather constant for all resonance modes in each structure. It means that the $\lambda$ values (derived from the theory) for different modes of the same physical structure need to be all scaled by a constant proportion (this can be done through calibration of all modes at a single resonance frequency), and then the $\lambda$ values can be successfully used as a good approximation to our structure of anchored SiC disk with parabolic cone-shaped pillar. Fabrication and test (characterization) results for our device are given in the next sections.

3.4 Fabrication

3.4.1 Fabrication Steps

The 500nm thin SiC epilayer is heteroepitaxially grown on single crystal Si (100) by an atmospheric pressure chemical vapor deposition (APCVD) process detailed elsewhere [109]. The device nanomachining process (Fig. 3.3) features a simple mask-less design
and a resist-less protocol. Following epitaxial growth (Fig. 3.3a), circular *rings* are directly patterned and nanomachined on the SiC epilayer, even milling slightly past the SiC/Si interface to make sure SiC has been totally milled out (Fig. 3.3b). The milling is done using focused ion beam (FIB) with FEI Helios NanoLab 650. This milling step is performed to make arrays of circular rings on four SiC/Si chips. The array dimensions are reflected in Table I (discussed more in Section IV).

The exposed Si is then etched in HNA (HF, nitric acid, DI water 1:2:1), each chip with a different duration (Fig. 3.3c). These different etching times ($t_{HNA}$) as well as different ring dimensions in each array were designed to produce suspended disks with center anchors of different diameters. Chips were held in the HNA tank for the duration of wet etching. Then, the chip is immediately taken out of the HNA container and held (and also moved back and forth to enhance removal of acid) in a container of DI water.

As shown in Fig. 3.3c, the wet etching not only builds suspended disks anchored at their centers, but also produces suspended rims at the outer circumference of the rings. This additional rim also vibrates and generates undesired resonance frequency components which may weakly couple to the vibrating disks. In Chip 2, we cut out the rims using FIB milling to avoid this unwanted effect (Fig. 3.3d).

Chip 1 has two rows of rings, with smaller and larger *intra-ring radius*, which is defined in terms of the ring inner and outer diameters as $\sqrt{D_{\text{ring, out}}^2 - D_{\text{ring, in}}^2}$. The chip was etched for ~35s which made all disks *decapped* (i.e. the anchoring pillar totally etched out.) except for the ring with the largest inner diameter of 40$\mu$m in the second row in Table I, which turned into a disk with a slim anchor. Chips 2 and 3 were etched for ~11s to leave two
largest rings alive (i.e., not decapped), turning them into disks with thick anchors. We will see later that measurement results are provided for Chips 1 and 2 but not for Chip 3 as no resonance peaks were observed for disks on that chip. To have a good estimate of the etching times needed in each case, a formula based on mass consumption was developed (next subsection) to better predict the required etching time at each case.

3.4.2 Etch Rate

HNA is an isotropic etchant (for Si) [110]. Theoretically, the instantaneous etch rate for all rings (with inner and outer diameters of $D_{\text{ring,in}}$ and $D_{\text{ring,out}}$ in Table I) is proportional to the area of the Si surface to be etched [111].

**Figure 3.2.** Theoretical (from precise (3) and approximate (4) equations) as well as simulation results: (a) shows relative frequencies of the first resonant modes at different $\left(\frac{b}{a}\right)$ values from (3), (b) shows an enlarged (zoomed) comparison of modes $f_{0,0}$ and $f_{0,1}$ at small and large $\left(\frac{b}{a}\right)$, (c) depicts the normalized difference between the precise and approximate equations versus $\left(\frac{b}{a}\right)$. It shows that this error (normalized difference) is not higher than 5% for $\left(\frac{b}{a}\right) \leq 0.7$ at the two modes with no nodal diameter ($n = 0$); while higher for other modes, (d) demonstrates the difference between simulation and theory (3) for the device dimensions fabricated and measured in next sections. It shows that this error is small when the anchor is slim. Also for non-slim anchors, it is nearly constant for all $\left(\frac{b}{a}\right)$ values. Thus if the results from theory will be calibrated with simulation results at a single $\left(\frac{b}{a}\right)$, they can serve as a good approximation to simulation results.
As Fig. 3.3c shows, the etching of the ring is done both inward, making the suspended disks; and outward, making the neighboring suspended rims. Therefore, we will have inner and outer anchors associated to the disk and the neighboring rim, respectively. The etching time - based on the assumption of uniform mass consumption - can be formulated in terms of the diameters of these anchors. The time which takes to etch out a ring to have rim anchor diameter $D_{anch,rin}$ is

$$t_{HNA} \propto \frac{\pi}{2} \int_{0}^{\frac{\pi}{2}} 2k\pi(R_2 - R_2 \sin \varphi)(R_1 + \frac{R_2 - R_2 \sin \varphi}{2}) d(R_2 \sin \varphi) \propto R_2^2(R_1 + \frac{R_2}{3})$$

(3.3a)

And the time it takes to have a disk anchor diameter of $D_{anch}$ is

$$t_{HNA} \propto R_2^3(R_1 - \frac{R_2}{3})$$

(3.3b)

where $R_1$ equals $D_{ring, out}$ and $D_{ring, in}$ respectively in (3.a) and (3.b); and $R_2$ equals $\sqrt[3]{(D_{ring, out} - D_{ring, in})}$ and

Table I: FIB Milling of Rings

<table>
<thead>
<tr>
<th>Chip</th>
<th>Ring Sizes [µm]</th>
<th>FIB Voltage [keV]</th>
<th>FIB Current [$10^{-12}$A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(10,30),(20,40),(40,60)</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>(10,20),(20,30),(30,40),(40,50)</td>
<td>30</td>
<td>67</td>
</tr>
<tr>
<td>3</td>
<td>(10,30),(20,40),(30,50),(40,60)</td>
<td>30</td>
<td>47</td>
</tr>
</tbody>
</table>

Figure 3.3. Process flow for fabrication and prototyping of SiC microdisks, (a) 500nm 3C-SiC epilayer on Si (100) substrate, (b) Mask-less patterning and nanomachining of SiC using FIB, (c) Etching of Si substrate by HNA to yield suspended SiC microdisks, and (d) Cutting out the neighboring rims.
\[ \sqrt{2}(D_{\text{ring,in}} - D_{\text{anch}}). \] It is noteworthy that \( R_2 \) is positive in both (3.3a) and (3.3b).

These equations explain why in Chip 1 which was aimed to produce slim-anchored disks (\( R_1 \sim 20\mu m \) and \( R_2 \sim 19.6\mu m \)), the etching time \( t_{\text{HNA}} \) is \( \sim 35s \), but e.g. in Chip 2 which was aimed to produce disks with thick-anchors (e.g. \( R_1 \sim 20\mu m \) and \( R_2 \sim 10\mu m \)) etching time is much smaller, \( \sim 10s-11s \) which agrees with (3.b). In practice, the etching of Chip 2 was done in three steps with the total time of \( \sim 11s \) (discussed more in Section IV).

Equations (3.a) and (3.b) also show why for the same etching time, the radius of the suspended rim is always smaller than radius of the suspended disk (i.e. the two \( R_2 \) in (3.a) and (3.b)). This difference in the radius of the disk and the rim are more pronounced for slim-anchored disk structures (also see Fig. 3.4.h)). We also remember from (2) that the flexural vibration frequencies of the suspended disks (and similarly their neighboring rims) are approximately inversely proportional to the square of their suspended radii (also see Fig. 3.3c). These two facts mean that the neighboring suspended rim of each disk always makes up flexural frequencies larger than those of the suspended disk. Thus, even if the fabrication step of Fig. 3.3d was not performed, and the resonances have been coupled from the rims to the disks, they would not affect the fundamental resonance modes of the disk.

For the same HNA recipe and lab temperature, there are also other factors affecting etch rate which are not as effective as the parameters discussed in (3). The first factor is difference in the ring intra-radius which is the reason the largest ring on row 1 of Chip 1 (Table I) was decapped; while the largest one on row 2 was not. The second factor is the
level of Ga ions implanted during FIB milling. These different Ga-concentration levels are caused by different FIB currents which have been used for chips (Table I), and cause different vertical etch rates in the three chips but rather the same horizontal etch rates for given physical parameters $D_{\text{ring, in}}$ and $D_{\text{ring, out}}$.

Dependence of Si etch rates on different Ga concentration levels for anisotropic etchants (e.g. KOH) has been studied in [112], where etch rates decrease with increase in concentration level. The effect of p- and n-type doping on Si etch rates for isotropic etchant (e.g. HNA) is very different: Increase in doping/concentration level, increases the etch rate [113]. This latter fact causes higher (vertical) etched depth for rings of the same dimension in Chip 2 compared to those in Chip 3. This different etched depth under SiC disk (for the same initial ring sizes) may also change the amplitude of interferometry-based detected signal at the laser detector and can even cause the amplitude of augmented travelling laser beams at the detector nearly vanish (Section 4). This phenomenon happened in Chip 3 which made the detection of vibration peaks through laser interferometry impossible.

3.5 Vibration Measurement and Characterization

In this section, we present the experimental results for different chips fabricated in Section III. For each chip, the microscopy images (optical and/or scanning electron) are shown and discussed. For Chips 1 and 2, the RF measurement results based on laser interferometry are also presented.
3.5.1 Chip 1: Slim-Anchored Disk

3.5.1.1 Microscopy

Fig. 3.4a shows the chip before HNA etching with dimensions of the rings milled using the FIB machine given in Table I. The pattern is composed of two rows of rings with intra-ring radii \( \sqrt{\left( D_{\text{ring,out}} - D_{\text{ring,in}} \right)} \) of \( \sim 10\,\mu m \) and \( \sim 5\,\mu m \). At etching time of 35s, only the largest ring in the second row survives, turning into a disk of slim anchor \( \sim 0.8\,\mu m \) diameter.

Fig. 3.4b through 3.4g show SEM images of each ring before HNA etching for better comparison. This etching experiment also demonstrates that the size of intra-ring radius may also slightly change etch rate (not predicted by (3.a) and (3.b) in Section 3 of this chapter) otherwise the largest disk in row 1 of Table I would also survive. In other words, larger intra-ring radius increases HNA etch rate.
Figure 3.4. SEM and optical microscope images of the FIB-milled rings in Chip 1, (a) Whole pattern of rings before HNA etching, (b)-(g) Individual rings of the pattern before HNA etching, (h) & (i) Optical microscope images of the slim-anchored disk (after wet etching).
Fig. 3.4h and 4i show optical microscope images of the slim-anchored disk (after etching) in which the slim anchor under SiC disk is clearly visible. The fabrication steps for the device in this chip do not include the final step of rim-cutting (Fig. 3.3d).

3.5.1.2 Vibration Measurement

The suspended microdisk is tested at room temperature in a vacuum of ~12 mTorr to examine their resonance response, by using a home-built apparatus [114] which incorporates a two-port RF network analysis measurement with a sensitive laser interferometry for displacement readout. SiC microdisk samples are driven by a piezoelectric thin disk actuator (Fig. 3.5).

Every observed micromechanical resonance is examined and verified to be from vibration modes of the SiC microdisk. To ensure this, we have employed a combination of techniques and tests, which generally include examinations of amplitude dependency, linear and nonlinear behaviors at varying drive levels, precise response dependency of laser spot locations on the device surface, control tests of response and dependency when the same laser spots focused upon areas with no devices (e.g. Fig. 3.4c, regions between microdisks and far from the suspended rims), careful calibration and tuning of the spot size (and other specs) of the laser interferometry, dependency on varying laser intensity, examination of thermomechanical fluctuations (Brownian motion) of micromechanical devices, etc. For each device, after verifying the observed micromechanical modes, optical detection is further optimized to measure the overall response. The detection is then tuned and optimized for characterizing individual peaks and modes.
Fig. 3.6 shows the main results of measured flexural response from a 500nm-thick, ~40µm-diameter SiC disk with an anchoring stem of only ~800nm in diameter. All the verified resonance peaks are from out-of-plane vibrations of the thin disk *loaded with* the slender stem. Analytical and finite element (COMSOL) modeling are combined to analyze and determine the mode shapes, as illustrated in Fig. 3.6a.

There are several observations onto this spectrum: As Fig. 2b shows, the first (set of) resonance peak corresponds to \( f_{0,1} \) and not \( f_{0,0} \), for this disk with \( b/a = 0.02 \). Also, the step-like pattern of the frequency versus mode number (blue triangles) in Fig. 3.6b clearly indicates the predicted degenerate modes – frequencies very close to each other - due to the disk’s many asymmetry possibilities (with abundant theoretical nodal diameters and nodal circles). Though the designed geometry was asymmetric, asymmetries due to fabrication process break degeneracy and cause mode splitting; hence many of the observed peaks, each in fact is a superposition of two split degenerate modes (Fig. 3.6b). The dash-dotted lines in Fig. 3.6b separate peaks belonging to the same theoretical resonance mode (same \( m \) and \( n \)) from the other resonance peaks. Fig. 3.6b and 6c show a non-monotonic \( Q \)-frequency dependency and an overall trend of possibly overcoming the conventional \( Q-f \) trade-off.

Fig. 3.7a shows the first mode \( f_{0,1} \) flexural resonance at ~2.81MHz, with its simulated mode shape shown in Fig. 6a. The resonance shape is very pure, consisting of just one peak (not a degenerate pair). The inset shows the peak voltage amplitudes for different drive voltages. The resonance peak shows a very nonlinear behavior which draws on the fact that the amplitude is very high (compared to other resonance peaks). Fig. 3.7b and 3.7c
demonstrate selected examples of observed mode-splitting, and transition from linear to nonlinear characteristics, respectively. Fig. 3.7b demonstrates a pair of modes at ~6.6MHz which correspond to the mode \( f_{0,2} \) mode. Fig. 3.7c shows one peak of the pair of resonance peaks corresponding to the mode \( f_{0,3} \).

3.5.1.3 Summary

We saw that etching out a ring to produce disks of slim anchor needs careful and precise estimation of the etching time; otherwise, the anchoring pillar would be totally etched out. This adds a challenge to the precise fabrication of slim-anchored disks. Meanwhile, there are some characteristics specific to the slim-anchored disks: 1) The frequencies of the modes are kept small, which might be desirable for some applications; 2) The detected amplitude of vibration can be very high (depends also on the position of laser spot), which makes the read-out signal of the potential sensor higher at the sensing circuit. The latter fact might also have a disadvantage: production of usually undesirable harmonics (due to nonlinearity). We could not measure any harmonic components due to resonator nonlinearity because the measurements were done by a network analyzer and not using a spectrum analyzer.
Figure 3.5. Schematic of the vibration resonance measurement setup with optical apparatus, photo detector, the specimen in vacuum and the electrical measurement instrument (network analyzer). This home-built measurement setup works based on laser interferometry to measure amplitude of the resonance peaks.

Figure 3.6. Resonance response and multi-mode characteristics measured from a ~40µm-diameter SiC microdisk with a very slender anchoring stem (diameter ~800nm): (a) Wide range RF multi-mode response measured by using two-port RF network analysis with laser interferometric detection techniques. A resonance mode is designated to each observed and carefully verified micromechanical resonance peak, in the order of increasing frequency. Insets: simulated mode shapes (in COMSOL), (b) Measured resonance frequency (triangles) and Q’s (circles) versus Resonance ID #. The dash-dotted lines separate frequencies of the modes from each other, and (c) Measured Q versus resonance frequency with a trend defying the conventional Q-frequency trade-off.

Figure 3.7. Characteristics of individual resonance modes measured from the SiC microdisk with a very slender supporting stem: (a) The fundamental out-of-plane flexural resonance at ~2.81MHz, with Q≈1400, (b) A pair of modes at ~6.6MHz involving flexural vibrations of the slender stem, and (c) A high frequency flexural mode at ~16.3MHz, with Q≈1130, with nonlinear behavior at high amplitude. Insets: measured peak amplitude of the resonance (or the dominant resonance, in panel (b)) versus drive voltage, along with linear fitting.

3.5.2 Chip 2: Thick-Anchored Disks

3.5.2.1 Microscopy

The pattern which is milled out by FIB machine is a series of rings with intra-ring radius
of $\sim 10 \mu m$ as reflected in Table I (the disks in the two replicate rows have the dimensions given in Table I). Fig. 3.8a depicts two replicate rows of such pattern after FIB-milling. The enlarged (zoomed) version of each half are shown in Fig. 3.8b and 3.8c.

HNA wet etching for this chip is done in two steps (not just one step as for Chip 1) to have better observation and control on what occurs to the chip before the total etching duration $t_{\text{HNA}} \sim 11 s$. At each step, the chip was held by a pair of tweezers in HNA tank and then held in the DI water. Then, the chip was dried out, and SEM (and optical) images were taken. Fig. 3.8d shows SEM image of one of the rings ($D_{\text{ring, in}} \sim 20 \mu m$) after the first step of wet etching, taken by a Hitachi S4500 machine at 2keV electron field (all other SEM images are taken using an FEI Nova Nanolab 200 machine). Fig. 3.8d shows the optical microscope image of the same (etched) ring with the measurement of the outward etching radius (red line) done by the microscope machine.
Figure 3.8. SEM and optical microscope images of the ring pattern in Chip 2 discussed in Table I. (a) depicts two rows of the pattern (after FIB-milling), (b) & (c) The enlarged (zoomed) version of each half, (d) shows SEM image of one of the rings ($D_{\text{ring},in} \approx 20 \mu m$) after the first level of wet etching, (e) Optical microscope image of the same etched ring, and (f)-(k) show SEM images taken at 15keV after the last (second) level of etching.

Fig. 3.8f through 3.8k show SEM images taken at 15keV after the last (second) level of etching. Fig. 3.8f shows the suspended disks (formerly rings of $D_{\text{ring},in} \approx 30 \mu m$ and $40 \mu m$) as well as decapped disk from the original ring of $D_{\text{ring},in} \approx 20 \mu m$) at tilted angle. The sharp-edge pillar of the decapped disk is visible in the images). Fig. 3.8g shows a closer tilted view (smaller WD, working distance) to the survived suspended disks where the Si anchoring area under the SiC disk is visible. More enlarged SEM images of the two disks (perpendicular and tilted views) are shown in Fig. 3.8h through 3.8k. The images at perpendicular view help to have a measure of the anchor diameter which would help to do
more precise COMSOL simulations. The anchors in Fig. 3.8h and 3.8j do not have a fully-circular shape. This might have happened as a result of chips inside HNA container not being held absolutely horizontally during the etch causing *azimuthal asymmetry*. The back and forth movement of the disk in DI water (to enhance acid removal) while there is still acid left in contact with the disk pillar might also contribute to such asymmetries. The asymmetry is more pronounced in the smaller disk of $D_{\text{ring, in}} \sim 30\mu\text{m}$.

### 3.5.2.2 Vibration Measurement

**40mm-Diameter Thick Anchor Disk**

Fig. 3.9a shows frequency spectrum of measured flexural response from a 500nm-thick, ~40µm-diameter SiC disk with an anchoring pillar of ~20µm in diameter on Chip 2.

Finite element (COMSOL) simulation results are used closely to analyze and determine the mode shapes, as illustrated in Fig. 3.9a. The first resonance peak predicted by COMSOL and analytical formulas is located at ~15.5MHz. The enlarged peak is depicted in Fig. 3.10a. The peak in Fig. 3.10a is decomposed to two major Lorentzian peaks and a minor one with low amplitude. The two larger peaks suggest a pair of peaks corresponding to mode $f_{0,0}$. It is important to mention that in contrast to the slim-anchored disk in Chip 1, here $(b/a)$ is large and $f_{0,0}$ peaks precede those from $f_{0,1}$ mode (see Fig. 3.2a and 3.2b). Modes $f_{0,1}$ and $f_{0,2}$ and $f_{0,3}$ also have a pair of resonance peaks, shown in Fig. 3.9a as well as 10b, 10c and 10d.

Fig. 3.9b shows the resonance frequencies and $Q$ versus Resonance ID# in the order of increasing frequency. The individual resonance frequencies and $Q$ factors are derived after
examination of resonance peaks as done in Fig. 3.10. The Lorentzian peaks are fitted to each resonance peak in Fig. 3.10 and the associated $Q$ is derived based on the definition of 3-dB (power) bandwidth. Except for the first resonance frequency which belongs to a resonance peak of relatively very low amplitude and is a negligible peak in our analysis (see also Fig. 3.10a), the other resonance frequencies all show up in mode-split pairs. An important observation about Fig. 3.9b and 9c is that the pair of frequencies of $f_{0,0}$ has a smaller $Q$ compared to other major peaks, which is due to the deviation of the anchor shape from a circular cross section (Fig. 3.8j).

This reduction of $Q$ is more obvious for 30μm-diameter thick-anchored disk, since deviation of the anchor from circular shape is larger in that case (Fig. 3.8i).

Fig. 3.9b shows the resonance frequencies and $Q$ versus Resonance ID# in the order of increasing frequency. The individual resonance frequencies and $Q$ factors are derived after examination of resonance peaks as done in Fig. 3.10. The Lorentzian peaks are fitted to each resonance peak in Fig. 3.10 and the associated $Q$ is derived based on the definition of 3-dB (power) bandwidth. Except for the first resonance frequency which belongs to a resonance peak of relatively very low amplitude and is a negligible peak in our analysis (see also Fig. 3.10a), the other resonance frequencies all show up in mode-split pairs. An important observation about Fig. 3.9b and 9c is that the pair of frequencies of $f_{0,0}$ has a smaller $Q$ compared to other major peaks, which is due to the deviation of the anchor shape from a circular cross section (Fig. 3.8j). This reduction of $Q$ is more obvious for 30μm-diameter thick-anchored disk, since deviation of the anchor from circular shape is larger in
that case (Fig. 3.8i).

Figure 3.9. Resonance response and multi-mode characteristics measured from a ~40µm-diameter SiC microdisk in Chip 2 with a thick anchoring stem (diameter ~20µm), (a) Wide range RF multi-mode response measured by using two-port RF network analysis with laser interferometric detection techniques. A resonance mode is designated to each observed and carefully verified micromechanical resonance peak, in the order of increasing frequency. Insets: simulated mode shapes (COMSOL), (b) Measured resonance frequency (triangles) and Q’s (circles) versus Resonance ID #. The dash-dotted lines separate frequencies of each mode from others resonance peaks, and (c) Measured Q vs. resonance frequency.

Figure 3.10. Characteristics of individual resonance modes measured from a ~40µm-diameter SiC microdisk in Chip 2 with a thick supporting stem of 20µm in diameter, (a) The fundamental out-of-plane flexural resonance with a pair of
frequencies at ~15.5MHz, with Q~990 and Q~840. There is also another peak of very smaller amplitude at ~15.5MHz which is minor compared to other two major peaks, (b) A pair of modes at ~16.21MHz and 16.24MHz with Q~1160 and Q~1610, (c) A pair of high frequency flexural mode peaks at ~17.8MHz, with Q~520 and Q~1510, (d) A pair of high frequency flexural mode at ~18.55MHz, with Q~1700 and Q~1150. Insets: measured peak amplitude of the resonance versus drive voltage, along with linear fitting of Lorentzian peaks to the measured resonance peaks.

Another observation in Fig. 3.9b and 9c is that the pair of resonance frequencies for all other (larger) modes, which are analyzed in detail in Fig. 3.10b through 3.10d, have different Q factors. However, the highest Q of each pair of peaks of the resonance modes are at the same range.

30MM-DIAMETER THICK ANCHOR DISK

Fig. 3.11a shows frequency spectrum of measured flexural response from a ~30µm-diameter SiC disk with an anchoring stem of ~10.3µm in diameter ((b / a) ≈ 0.34) on Chip 2. Finite element (COMSOL) modeling is also done and mode shapes added to the figure. Fig. 3.11b shows the resonance frequencies and their Q versus Resonance ID# in the order of increasing frequency. Fig. 3.11c depicts measured Q versus frequency for the resonance modes shown in Fig. 3.9b.

The first resonance peak predicted by COMSOL and analytical formulas is $f_{0,1}$ as predicted in Fig. 3.2b (in contrast to the 40µm-thick anchor disk with larger ratio $(b / a) = 0.5$). This resonance mode has a pair of measurement peaks (Resonance ID# 1 and 3 in Fig. 3.11b). Resonance peaks with ID# 1 and 2 at ~15.2MHz are shown in more detail in Fig. 3.12a. As shown, Resonance ID# 2 has a relatively lower amplitude; hence, not assumed a major peak. The resonance mode of $f_{0,0}$ found in COMSOL simulations corresponds to the measured peak with Resonance ID# 4, $f_{0,2}$ to Resonance ID# 6 and $f_{0,3}$ to Resonance ID# 7 and 8.
An important observation about Fig. 3.11b and 11c is that the resonance peak of $f_{0,0}$ (Resonance ID# 1, 3 and 4) has a smaller $Q$ compared to (most of) other major peaks, which is due to the deviation of the anchor from a circular cross section (Fig. 3.8j). We also observe that compared to the slim-anchored disk from Chip1, the $Q$ in both thick-anchored disks are smaller.

### 3.5.3 Chip 3

Chip 3 has a FIB-milled pattern of rings very similar to those in Chip 2 (see Table I). In fact, four sizes of rings were FIB milled on the chip, with $D_{\text{ring,in}}$ the same as those in Chip 2 but with larger $D_{\text{ring,out}}$. The other difference between the patterns in the two chips was the FIB current used for milling the rings.

Chip 3 was originally fabricated in order to make devices for vibration measurement purposes. However, lower FIB currents used for milling the rings (before HNA etching) resulted in less Ga implantation in Si. This fact changed the HNA vertical etching rate, resulting in thus pillars of smaller heights formed. This changes the path length of the laser which passes through SiC, reflected by Si body surface at the bottom of the pillar and reaching the SiC disk upon return. This returned laser beam (with changed phase) makes the augmented laser beam received at the laser detector vanish.

This happens because the laser detected at the laser detector diode is augmentation of at least two return laser beams: one beam reflected back from SiC surface and the other beam which has passed the semi-transparent SiC disk, has been reflected from Si surface after that and finally has passed through SiC again and reached the laser detector. These two
beams both experience *frequency modulations* equal to the disk vibration frequencies; thus, they would have the same frequency but different phase and amplitude at the laser detector diode.

Now, if the vertical depth of Si pillar which is half the total distance the second laser beam traverses between SiC and Si surfaces, induces a phase difference between the two beams which makes the two laser beams have the same amplitude but ~180° phase difference at the laser detector, these two laser beams cancel out each other and no laser beam with a frequency component equal to disk vibration frequency will be detected at the detector. This is exactly what happened in Chip 3 which resulted in no detectable peaks through interferometry-based measurement by the apparatus shown in Fig. 3.5.

![Figure 3.11. Resonance response and multi-mode characteristics measured from a ~30µm-diameter SiC microdisk in Chip 2 with a thick anchoring stem (diameter ~10.3µm), (a) Wide range RF multi-mode response measured using two-port RF network analysis with laser interferometric detection techniques. A resonance mode is designated to each observed and carefully verified micromechanical resonance peak, in the order of increasing frequency. Insets: simulated mode shapes (in COMSOL), (b) Measured resonance frequency (triangles) and Q’s (circles) versus Resonance ID #. The dash-dotted lines separate frequencies of the modes from each other, and (c) Measured Q versus resonance frequency.](image-url)
Figure 3.12. Characteristics of resonance modes measured from a SiC microdisk of diameter ~30μm in Chip 2 with a thick supporting anchor of diameter 10.3μm, (a) The fundamental out-of-plane flexural resonance with a pair of frequencies at ~15.2MHz, with Q=1634 and Q=394, (b) A single high frequency resonance mode at ~15.75MHz with Q=1228, (c) A single flexural mode at ~17.65MHz, with Q=1347. The slightly nonlinear behavior can also be clearly seen. (d) A pair of high frequency flexural mode at ~18.24MHz and ~18.28MHz, with Q=960 and Q=1287. Inset: Linear fitting of Lorentzian peaks to the measured resonance peaks.

Fig. 3.13 shows the set of SEM and optical microscope images taken from different stages of fabrication of disks on Chip 3. Fig. 3.13a depicts SEM images taken of Chip 3 by an FEI Nova Nanolab 200 machine at 5keV electron field. This image is taken of the two rows of the rings before any HNA etching. Fig. 3.13b and 3.13c show optical microscope and SEM images of the smallest ring (D_{ring, in}~10μm) after first stage of HNA etching. The etched disk is obviously decapped, i.e. the anchor totally etched out. The SEM image is taken using a Hitachi S4500 machine at 2keV electron field. Similarly, Fig. 3.13d through 13(i) show SEM images (perpendicular and tilted views) taken with the same machine for the other (larger-sized) rings after the first level of HNA etching. Fig. 3.13j through 3.13o
show SEM images (FEI Nova machine) after the final (i.e. second) stage of HNA etching. At the end of this stage of wet etching, the second-smallest disk is also decapped (Fig. 3.13j), while the other rings (the two larger-sized disks) have survived. Fig. 3.13l through 3.13o show zoomed versions of these suspended disks at perpendicular and tilted angles.

Figure 3.13. SEM and optical microscope images of the pattern in Chip 3, (a) depicts SEM images (taken at 5keV) of two rows of the pattern (after FIB-milling) discussed in Table I, (b) Optical microscope image of the smallest ring of inner diameter 10μm which is decapped after first stage of HNA etching, (c) SEM image of the picture in figure b, (d)-(i) SEM images (perpendicular and tilted views) of the three larger rings after the first level of etching which have turned into suspended disks (not decapped), (j) SEM image of the suspended disks after second (i.e. final) stage of HNA etching (the two smallest-sized disks were decapped) taken at 15keV electron field, (k)-(o) depict enlarged (zoomed) version of the two larger-sized suspended disks (D_{ring in} ~30μm and ~40μm) still alive. Figures (n) and (o) are respectively tilted views to Figures (l) and (m).
3.6 Comparison with Other Works In The Literature

In this section, a brief graphical performance comparison with some of the best works in the literature is provided. These works are reflected in references [115-128] and were selected in a table in [115].

![Graphical comparison of f-Q products](image)

*Figure 3.14. Comparison of my microdisk resonator with some of the best resonators in the literature with highest reported f-Q product.*

It is important to note that the microdisk resonators designed in this PhD work were not aimed at the highest quality factor or $f$-$Q$ product and the resonator material, size and geometry was chosen based on the needs of sensing applications in harsh environments and our optical transduction method. Otherwise, the flexural-mode resonator would not be used at all because compared to the extensional (aka contour-mode) resonators, the flexural-mode offer lower quality factor. The reason for the latter fact is that the ratio of vibrating area-to-resonator volume is higher in flexural-mode resonators which translates to higher losses. Contour-mode disk resonators, while provide higher $Q$ factor and resonance frequency, are less efficient in terms of power handling.

3.7 Conclusions

For the first time, flexural-mode SiC microdisks anchored on Si center parabolic-cone
pillars (with J-sides) have been fabricated. These suspended disks were studied theoretically, both with precise and approximate equations, to provide a good intuition into their properties. FEM simulations were also done to show how much analytical relations of the suspended disk are close in frequencies to the simulation results for SiC disks with cone-shaped Si pillars. Finally four chips with different arrays of microdisks at various sizes and HNA etching durations were fabricated to provide devices for measurement. An analytical equation (based on mass consumption) to calculate the required HNA etching time for our microdisk structures was also proposed. Results from these measurements set a stage to explore all major resonance peaks seen in practice. The peaks were studied in detail to understand mode shape of each resonance peak. Based on these theoretical and measurements studies, the advantages and disadvantages as well as practical fabrication challenges of both slim-anchor and thick-anchor microdisk resonators were discussed.

There were a couple of important observations to these measurements for thick-anchored disks, which were not seen in the slim-anchored disk of Chip 1:

1. Azimuthal asymmetries due to non-circular cross section anchors exist which are quite noticeable in SEM images. These deviations from circular form is larger than just to cause mode-splitting near the ideal degenerate frequencies. In other words, not only most of resonance modes e.g. $f_{0,0}$, $f_{0,1}$ and $f_{0,2}$ turn into a pair of split modes, but also it affects the $Q$ factors. This asymmetry can be decreased through use of improved etching mechanisms.

2. The thick-anchored disks discussed here did not have such large vibration amplitudes seen from slender-anchored disk in Chip 1. This draws on the fact that the suspended radii of both disks in Chip 2 are much smaller, nearly half of that for the slim-anchored disks.
(3) Thick-anchored disks provide a method to increase resonance frequencies of the disk (by keeping $a$ constant and increasing $(b/a)$). It is different from the other method to increase frequency that keeps $(b/a)$ constant and decreases $a$ in slim-anchored disks. In the former, frequencies of the resonance modes with zero nodal circle are relatively very close to each other; while in the latter they are not (also see Fig. 3.2a). This can give options to the MEMS resonator designer.

The precise analyses in this paper have provided a framework to fabrication and use of these resonators for future clock generation and sensing applications. Use of multi-mode resonators such as our microdisks for wireless communication and/or sensing applications demands precise characterization of the resonance modes based on which the read-out electrodes (and circuitry) can be designed. The analyses in this research pursued this goal and provided a knowledge base to candidate these resonators for such applications. Ongoing research is focusing on building oscillators based on these microdisk resonators with programmable read-out electrodes and circuitry.

Described in this chapter is a class of several-MHz RF multi-mode micromechanical resonators which were fabricated out of 3C-SiC as the first microdisk resonator of this kind from SiC prototypes and they were dynamically fully characterized (from resonance frequency point of view).

In this respect, they were first theoretically analyzed and then fabricated. Several microdisks with different ratio of anchor diameter to disk diameter were fabricated. These vibrating circular disks all made of a ~500nm thin SiC epilayer grown on single crystal Si are then tested and measurement results are analyzed. The resonators had a novel
fabrication method composed of FIB sputtering and HNA etching and the resonance peaks (in frequency spectrum) are detected through laser-interferometry measurements. The resonant peaks detected were expected from theory to be in the few MHz range. Also we see that in practice, a ~40µm-diameter SiC disk with a slender (~800nm) anchor exhibits more than a dozen flexural modes between ~2MHz–20MHz with quality factors (Q’s) of ~1000–4000. Two other disks with diameters of ~40µm and ~30µm, and wide anchors (~20µm and ~10.3µm, respectively) have their set of major flexural peaks (associated with modes of zero-circular nodes) between 15–19MHz with Q’s of ~500–2500.  

Chapter 4: New Design Method for M/NEMS-Referenced Pierce Oscillators

4.1 Introduction on Oscillators

4.1.1 Self-Sustained Oscillators

In Chapter 1, it was discussed how self-sustained oscillators are superior to resonators with decaying-amplitude output signals. However, the theoretical concept of self-sustained oscillation was not explained. Oscillators with sustained amplitude (i.e. not damped) signals are feedback systems in which there is no positive phase margin, in contrast to the amplifiers. In fact, in designing self-sustained oscillators, the designers design in a way to provide zero (or negative) phase margin. The zero phase margin in the complex domain simply means a constant amplitude; while a negative phase margin means a growing signal amplitude. To make that, an open-loop physical resonator which has decaying amplitude (i.e. poles and zeros in the left-half plane) immediately gets right-half-plane poles and/or zeros upon closure of positive feedback loop, which leads to grow in its signal amplitude before the amplitude increase stops.

Grow in the signal amplitude simply means increase in system power consumption. Physical systems with limited source of energy usually have some mechanism to limit the increase in amplitude (otherwise their consumed energy will be higher than their total
available energy), which in turn leads to decrease in the absolute value of their negative phase margin. Fig. 4.1 shows the root-locus of the closed loop two-pole system, and based on the loop gain, the system poles can be anywhere on this root-locus. However, to have an oscillator the poles should rest on the imaginary \( j\omega \) axis.

Migration of the system poles/zeros from right-hand side to the \( j\omega \)-axis requires a saturation mechanism which exists in some way in the system. When the closed-loop system poles/zeros rest on \( j\omega \)-axis, the system becomes a steady-state self-sustained oscillator in the sense that the output signal is a constant-amplitude sinusoidal signal. This steady-state condition is what Barkhausen criteria for self-sustained oscillation states

\[
|T(j\omega)| = 1\\
\angle T(j\omega) = 2k\pi
\]

where \( T(j\omega) \) is the loop gain in frequency domain and \( k \) is an integer number. Loop gain \( T(j\omega) = H(j\omega)G(j\omega) \) can be best defined in terms of the closed-loop oscillator transfer function

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{G(j\omega)}{1 + G(j\omega)H(j\omega)}
\]
where \( G(j\omega) \) and \( H(j\omega) \) are defined graphically in Fig. 4.2.

![Feedback amplifier model of the oscillator](image)

*Figure 4.2 Feedback amplifier model of the oscillator*

Self-sustained oscillators - defined in terms of either of the negative-resistance or transfer function models which will be discussed later – are simply called oscillators from now on in this dissertation.

### 4.1.2 Resonator-Based Oscillators

Oscillators can be grouped in terms of the shape of their output signal in the steady-state conditions. In this regard, most oscillators fall in either of the groups harmonic oscillators, square wave oscillator or quadrature oscillator. Harmonic oscillators are the oscillators which generate a continuous-time sinusoidal signal (in time domain) while in square-wave oscillators, the signal has a square-wave shape. Quadrature oscillators are very similar to the harmonic oscillators in which two sinusoidal signals – in-phase and out-of-phase – are generated. While the frequency spectrum of an ideal square-wave oscillator has many harmonics, the ideal frequency spectrum of a harmonic oscillator is a single impulse function at the oscillation frequency. The nonideal frequency spectrum of a harmonic oscillator will be discussed in Section 4.2.1.
From the oscillator architecture point of view, the oscillators today are (mostly) either ring oscillators or resonator-based oscillators. We remember that according to the second equation in Barkhausen criteria (Eq. (4.1)), the total phase shift in a feedback loop should be an integer multiple of $2\pi$. Ring oscillators are usually composed of $N$ inverters where each has a single (dominant) pole thus, introducing maximum frequency-dependent phase shift of $270^\circ$ ($180^\circ$ dc phase shift and $90^\circ$ frequency-dependent phase shift). That is why with an odd number of stages, $(2m+1)180^\circ$ dc phase shift (where $m$ is an integer) is generated, and another $180^\circ$ frequency-dependent phase shift is generated by all the inverters in the loop together at a frequency $\omega < \infty$. That is how the phase condition of Barkhausen criteria for oscillation is met. The gain condition can also simply be satisfied by increasing the gain of each inverter above a threshold.

Resonator-based oscillators are the resonators with a passive circuit which provide some inductive phase shift to compensate for the capacitive phase shifts due to passive or active stray capacitors or the capacitive loads. This passive circuit can be either an integrated (or off-chip) inductor as in LC-oscillators or a crystal resonator in crystal oscillators. Recently, MEMS/NEMS reference oscillators have emerged as a strong competitor for crystals oscillators. The passive circuit (including the whole circuit stray capacitances and losses) of the resonator-based oscillators can be modeled as a tank circuit separate from the active circuit. Negative-resistance model is a method of modelling oscillators, which is even more intuitive than the loop transfer function model (aka feedback amplifier model), especially for design of the resonator-based oscillators. Fig. 4.3 depicts the passive resonator with a
loss component – the latter modeled with a positive resistance – on the left side, while the active circuit on the right side is modeled by a negative resistance (and an active impedance) which compensates for the loss attributed to the passive and active components all migrated to the left side.

Figure 4.3 Negative-resistance model of the oscillator

Negative-resistance model can be also used to explain transient behavior of the oscillation start-up. However, the most implication from this modeling approach is the steady-state conditions for oscillation as below

\[
\text{Re}\{Z_{\text{active}}(j\omega)\} = -1 \times \text{Re}\{Z_{\text{passive}}(j\omega)\} \quad (4.3)
\]

4.1.3 Pierce Family Oscillators

Pierce family oscillators primarily include three architectures Colpitts, Pierce and Santos oscillators [127]. These oscillators all have only one transistor in their architecture and can be implemented in either bipolar or CMOS processes. Figure 4.4 shows these three oscillators when the resonators are MEMS/NEMS resonators. From the 3 nodes of each transistor (e.g. drain, gate and source for MOS), one is ac grounded and the resonator is connected between the other two. In Pierce oscillators which will be discussed more in this chapter, source is ac grounded. In order to satisfy the Barkhausen phase condition (Eq. (4.1)),
at least two shunt capacitors are needed to be connected to the nodes to which the resonator tank is connected. The good thing about the Pierce family oscillators compared to LC cross coupled and ring oscillators is that the number of active components is less which potentially leads to less noise generation and better phase noise performance. There are also disadvantages for these oscillators. Razavi [128] analytically shows that at least for Colpitts oscillator, a larger transistor voltage gain is needed for the oscillator to begin oscillation compared to the LC cross-coupled which might potentially lead to higher power consumption. The MEMS/NEMS reference oscillator based on Pierce architecture will be discussed in Section 4.4.

![Figure 4.4 (From the left): Colpitts, Pierce and Santose oscillators](image)

4.2 Phase Noise

4.2.1 Definition

It was mentioned in the previous section that the ideal harmonic oscillator generates a sinusoidal signal

\[ V(t) = V_0 \sin(\omega_0 t) \]  \hspace{1cm} (4.4)

The actual output signal of the harmonic oscillators, the amplitude and phase fluctuations can also be seen

\[ V(t) = (V_0 + v_0(t)) \sin(\omega_0 t + \phi(t)) \]  \hspace{1cm} (4.5)
Amplitude fluctuations are usually removed due to the saturation mechanism (intrinsic or through an extrinsic automatic gain control AGC) in the oscillator circuit. However, the phase fluctuation which according to the equipartition theorem of thermodynamics, in equilibrium constitutes half of the total noise power [129], leads to inaccuracy in frequency and time measurements. The random phase fluctuation has two parts: (1) the part which in average amounts to a periodic phase rotation, (2) the part that in average does not amount to any periodic phase rotation. The mathematical expression for these two kinds of phase variation is as below

\[ \phi(t) = A_d \sin(\omega_m t + \phi_n(t)) \]  

(4.6)

where \( A_d \) is the amplitude of the periodic random phase shifts and \( \phi_n(t) \) is the non-periodic random phase noise. The phase variations due to frequency modulations simply make other tones in addition to the main tone of the signal, at offset frequency of \( \omega_o \) from the main frequency \( \omega_0 \). These are not deterministic non-idealities, as the random nature of the phase noise causes the offset frequency change by changes in the time, noise was integrated. For very small random phase variations \( \phi(t) \approx \sin(\phi(t)) \); thus Eq. (4.5) (without the amplitude modulation part) can be written as

\[ V(t) = V_0 [\sin(\omega_0 t) \cos(A_d \sin(\omega_m t + \phi_n(t))) + \cos(\omega_0 t) \sin(A_d \sin(\omega_m t + \phi_n(t)))] \]  

(4.7)

which for very small \( A_d \) takes the following form

\[ V(t) \approx V_0 [A_d \cos(\omega_0 t) \cos(\omega_m t) + \sin(\omega_0 t)] \]

\[ = V_0 [\sin(\omega_0 t) + \frac{A_d}{2} \sin((\omega_0 + \omega_m)t) - \frac{A_d}{2} \sin((\omega_0 - \omega_m)t)] \]  

(4.8)

Eq. (4.8) exactly depicts the frequency spectrum for the signal with phase variation: Similar to the ideal oscillator in Eq. (4.4), there will be a large peak at the main frequency \( \omega_0 \) accompanied by a
pair of peaks at offset frequency $\omega_m$ from the main frequency with an amplitude modulated by a (less than unity) factor $A_d$. This mathematical expression shows how each pair of the offset peaks is generated, which in other words explain for the skirt of the main peak in the frequency spectrum of the practical (=non-ideal) harmonic oscillator (Fig. 4.5). The higher $\omega_m$, the smaller $A_d$ and so as the amplitude of the frequency spectrum at the associated offset frequency.

![Figure 4.5 Oscillator phase noise power spectrum](image)

Phase noise is calculated from the power spectral density (PSD) of the phase signal, which is simply the auto covariance of the random phase fluctuations in Eq. (4.6). The single-sideband phase noise used to be defined as the PSD at frequency offset $\omega_m$ i.e. the power in the frequency duration $\Delta\omega=1$ Hz - to the carrier power at the main frequency $\omega_0$ (where there is a peak in the frequency spectrum).

$$L(\omega_m) = 10 \log \frac{PSD(\omega_0 + \omega_m)}{P_{carrier}}$$ (4.9a)

However, for some reasons e.g. the fact that this definition does not discriminate between phase and amplitude noise, while in practice phase noise is measured by phase detectors, the following definition in IEEE standard 1139 [130]

$$L(\omega_m) = 10 \log \left( \frac{1}{2} S_p(\omega_0 + \omega_m) \right)$$ (4.9b)
where $S_{\phi}(\omega)$ is a single-sided power spectral density of the phase noise.

In this regard, phase noise is a measure of the short term random frequency instability of the oscillator output. There are also long term instabilities in the frequency of the oscillator output signal (e.g. due to aging) and environmental instabilities (e.g. due to variations in temperature or power supply). The noise which is generated from the white noise, flicker noise and shot noise in different parts of the oscillator circuit (passive or active) is then amplified and shaped in the loop to make the skirt around the peak component in frequency $\omega_0$. The details of the shape of this skirt and some phase noise theories which have been proposed to explain that are discussed in the next section.

### 4.2.2 Overview of the Oscillator Phase Noise Models (with Parallel Tanks)

In this section, the phase noise models proposed for oscillators with parallel tanks will be discussed. The first major work in this group was made by D. Leeson [131]. Although this concept was previously discussed in physics under the topic of Schawlow-Townes linewidth for the laser oscillator, For years Leeson was known to be the first person to publish a good linear time-invariant (stationary) model for the effect of oscillator noise sources on the single side band (SSB) phase noise in 1966.

$$L(\omega_m) = 10 \log \left( \frac{FK_BT}{P_{\text{carrier}}} \frac{1}{8Q_{\text{loaded}}^2} \left( \frac{\omega_0}{\omega_m} \right)^2 \right)$$  \hspace{1cm} (4.10)

where $P_{\text{carrier}}$, $F$, $K_B$, $T$ and $Q_{\text{loaded}}$ are the oscillator power, noise factor at the operating power, Boltzman’s constant ($1.38 \times 10^{-23}$ [J/K]), temperature and loaded quality factor of the resonator.

Though this equation has been widely used through the years to have a coarse prediction of phase noise performance of the oscillators, Leeson’s formula only has one term and accounts only for the
thermal noise of the oscillator (and for example not even the $1/f$ flicker noise). Thus, the predicted phase noise is a single-slope line (20dB/decade slope). According to Eq. (4.10) the phase noise at some offset frequency decreases if any of the carrier power or the resonator loaded $Q$ factor increases.

However, real measurements with instruments with a phase noise performance better than the oscillator under the test, reveal there are in fact several regions in the phase noise spectrum of the oscillators as shown in Fig. 4.5. In fact, even in the 20 dB/dec region which is seen in the measurement results, the measured amplitude is much larger than what predicted by Leeson, which might be due to the noise of the active circuits not completely covered by the formula. Also, the phase noise spectrum flattens out at large offset frequencies. To address the latter issue, a modified version of Leeson formula is suggested as below

$$L(\omega_m) = 10 \log \left( \frac{FK_b T}{P_{carrier}} \frac{1}{8Q_{\text{loaded}}} \left(1 + \left(\frac{\omega_m}{\omega_0}\right)^2\right) \right)$$

(4.11)

Razavi [132] presents a phase noise model for inductor-less voltage-controlled oscillators (e.g. ring oscillators) based on two-port LTO feedback system. The system is based on calculating quality factor and then loop transfer function based on loop phase noise due to noise sources in the circuit. Then phase noise is calculated based on the loop gain and phase. The higher this loaded $Q$ factor is, the larger this calculated instantaneous frequency change is.

Hajimiri and Lee [130] proposed a linear time-variant model. The authors give examples to explain why the noise is time-variant and based on that they define an Impulse Sensitivity Function (ISF) $\Gamma(x)$ to characterize the vulnerability of the signal at each time to additive noise. This function is maximum at signal zero crossings – as even a small noise can change the zero crossing and thus leads to phase noise – while it is minimum at the single time-domain peaks. ISF function closely
depends on the shape of the signal wave in time-domain. Based on this assumption, the authors find the phase noise in the $1/f^2$ and $1/f^3$ regions respectively as below

$$L(\omega_n) = 10 \log \left( \frac{PSD(i_n^2) \Gamma_{rmu}^2}{2q_{max}^2 \omega_n^2} \right)$$

(4.12)

$$L(\omega_m) = 10 \log \left( \frac{PSD(i_m^2)c_0^2 \omega_{1/f}^2}{8q_{max}^2 \omega_m^2 \omega_m} \right)$$

(4.13)

Where $q_{max}$ and $\omega_{1/f}$ are maximum charge density of the parallel tank capacitor and the $1/f$ noise corner frequency; while $c_0$ is equal to $2\Gamma_{(dc)}$. An important implication of the above formulation is that 1) similar to Leeson formula, higher $Q$ helps to lower phase noise if other factors in the formula are kept constant. Also the formula says that there both sensitive and insensitive moments in each oscillation period. This latter conclusion can give insights to the designer to design the oscillators which can be periodically turned off at the sensitive moments. Also a direct result of Eq. (4.13) is that the more symmetrical an oscillator is, the lower $\Gamma_{(dc)}$ and thus $c_0$ is, and therefore the lower the phase noise will be.

One of the most comprehensive and theoretic famous phase noise models was proposed by Demir [133]. In contrast to the Hajimiri-Lee model, this model is not useful for designers and does not give any intuition on the design of low-phase noise oscillator circuits. In this model, limit cycle represents the oscillator without any perturbations while deterministic and random perturbations make the trajectory deviate from the limit cycle. The perturbations are either orbit perturbations (corresponding to amplitude noise in other models) or phase perturbations (corresponding to phase noise in other models). Demir’s model provides the same results as Hajimiri’s model for stationary
noise sources. In case of injection locked oscillators, Demir’s model is believed to provide more accurate results. Especially, Demir’s model is more accurate in close-in offset frequencies.

D. Ham [134] developed a physics-based intuitional model which was done mostly when he was a PhD student in Caltech. He looks at the phase noise from the view that oscillation peak in oscillators always is wider than an impulse response that he refers to that as linewidth. This linewidth is caused by phase noise and is a good measure of the spectral purity of the signal. He discusses an experiment in which \( N \) oscillators with the same oscillation frequency begin from the initial phase of zero and get different phases after a sufficiently long time. From this experiment, he discusses time-dependent probability distribution of phase \( P(\phi,t) \) where \( P(\phi,t).d\phi \) represents the probability of phase to be in \((\phi,\phi+d\phi)\) at a given time, \( t \). If the only noise in the oscillator is white noise, the width of the probability distribution is derived from Eq. (4.14) which is a measure of the phase diffusion

\[
\langle \phi^2(t) \rangle = 2Dt \quad (4.14)
\]

where \( D \) is called phase diffusion constant and shows how fast the phase diffuses. Using Eq. (4.14), Eq. (4.5) in the absence of amplitude perturbations, power spectral density will have the Lorentzian shape as below

\[
S_f(\omega) = V_0^2 \frac{D}{(\omega_m)^2 + D^2} \quad (4.15)
\]

where \( \omega_m \) is the offset frequency. For smaller \( D \), the Lorenzian will experience a similar effect as smaller quality factor (in LTI models) would have on the phase noise, but change of \( D \) does not change the total oscillation energy of \( V_0^2/2 \). The phase noise is then derived as

\[
L(\omega_m) = 10 \log\left( \frac{S_f(\omega)}{\omega_m^2 + D^2} \right) = 10 \log\left( \frac{2D}{(\omega_m)^2 + D^2} \right) \quad (4.16)
\]
If $\omega_m^2 \gg D$, Eq. (4.16) reduces to the 20dB/dec behavior seen in Leeson formula and Fig. (4.5) as below

$$L(\omega_m) = 10 \log \left( \frac{2D}{\omega_m^2} \right)$$

(4.17)

He then finds an expression for $D$ based on the oscillator circuit topology and in terms of the oscillator circuit components. However, the diffusion constant $D$ can be also described in terms of the physical parameters demonstrating friction via the Einstein relation

$$D = \frac{k_B T}{m} \frac{1}{\gamma}$$

(4.18)

where $\frac{k_B T}{m}$ is a factor showing sensitivity of the Brownian particle to perturbations and is larger for lighter particles, and $\frac{1}{\gamma}$ demonstrates friction loss. If ignoring the time-varying effects, to write $D$ in terms of the LTI circuit models, it can be written as below for the parallel resonance circuit

$$D \approx \frac{1}{V_0^2} \frac{k_B T}{C} \frac{\alpha_b}{Q_{eff}}$$

(4.19)

### 4.3 Lowering Phase Noise in MEMS/NEMS Referenced Pierce Oscillators

Pierce family oscillators were discussed in Section 4.1.3. Following review of the phase noise theories in the Section 4.2.2 for oscillator circuits with parallel RL C tanks, this section discusses methods to design low-phase noise Pierce oscillators with MEMS/NEMS resonators which can begin oscillation. However before that, the general trends in lowering phase noise of all oscillators are discussed.
4.3.1 Phase Noise Theories of Non-MEMS/NEMS Referenced Oscillators

Oscillator phase noise models were discussed in Section 4.2.2. These theories also included nonlinear theories (e.g. Demir’s model) and LTV models (e.g. Hajimiri’s and Ham’s models) which suggested some architecture selection aspects to design low-phase noise oscillators. For example linear time-variant models imply using charge-injected oscillators as well as symmetrical-architecture oscillators can contribute to lower close-in phase noise in oscillators. Nonlinear models have also been used recently [135,136] especially in the MEMS/NEMS resonator based oscillators to lower phase noise by driving the oscillator to work in the nonlinear operation regime. This way they change the oscillator behavior to deviate from the usually-seen -20db/dec and -30dB/dec phase noise regions. In better words, using nonlinearity they make the close-in phase noise spectrum at one side to become steeper and in some way they increase what in linear systems it would be called the effective quality factor. Although these methods can be used to tweak the oscillator phase noise better; however, they also have some unexpected effects (e.g. increasing the nonlinearity and oscillator power diffusion to the harmonic components) that may not be desirable anytime. Thus, the best approach to design a low-phase noise oscillator might be designing it based on the intuitions form the LTI models and then tweaking it further based on the more complex theories if applicable.

According to the modified Leeson LTI model,

\[
L(\omega_m) = 10 \log\left(\frac{FK_mT}{P_{carrier}} \frac{1}{8Q^2_{loaded}} \left(1 + \left(\frac{\omega_h}{\omega_m}\right)^2\right)\left(1 + \frac{\omega_h}{\omega_m}\right)\right) \tag{4.20}
\]

This formula which can be also used for oscillators with series RLC circuits (similar to the equivalent electronic circuit model for the mechanical resonators), clearly suggests that to decrease phase noise loaded quality factor or carrier signal power should increase in a way that the denominator of the equation increases. Loaded quality factor has a stronger effect –parameter
squared exists in the denominator in contrast to the carrier power. From this view point, the approaches to either 1) decrease the *noise-to-carrier (power) ratio (NCR)*, or 2) increase the quality factor can help.

### 4.3.2 Increasing Loaded Quality Factor

Increasing the loaded quality factor was actually one of the main motivations for emerge of crystal oscillators and then MEMS/NEMS based oscillators. In case of electrostatic (=capacitive) transduction, these components both can be modeled by a series RLC circuit – called motional impedance and shown as $Z_m$ in Fig. 4.6 - in parallel with a static ac feed-through electrical capacitance ($C_{static}$) and two parasitic capacitances ($C_{p1}$ and $C_{p2}$).

It is due to mention that for other transduction schemes, the equivalent electronic circuit of the MEMS/NEMS resonator can be different. For thermal actuation, the equivalent circuit will be discussed in the next chapter.

![Figure 4.6 Equivalent circuit for MEM/NEM resonator](image)

In case of equivalent circuit in Fig. 4.6, the unloaded quality factor of the series resonance circuit can be found from

$$Q_{unloaded} = \frac{1}{\omega R_m C_m} = \frac{\omega L_m}{R_m}$$  \hspace{1cm} (4.21)
MEMS/NEMS resonators have a very high quality factor thanks to a large ratio of $L_m$ over $R_m$. The three other capacitors in Fig. 4.6 not only make up a parallel resonance frequency (in addition to the series one) but also degrade the quality factor a little. At large $C_{\text{static}}$ that $\frac{Q}{C_{\text{static}}}$ is very low, the oscillator even ceases to oscillate [137]. Therefore, the loaded quality factor is lower than what expressed in Eq. (4.21) and one of the oscillator circuit design criteria should be to minimize this degradation of quality factor due to loading effects. This is addressed in the proposed method in Section 4.4.

4.3.3 Lowering NCR in MEMS/NEMS Referenced Oscillators

**General Ideas**

In order to increase the signal power at the denominator of Eq. (4.20) one approach is to increase the signal power. In order to avoid the power consumption exceed so much, for the same bias current of the (MOS) transistors, the drain-source voltage should be maximized. As [138] suggests, the best operation region for the oscillator is in the *current-limited* regime (but very close to the *voltage-limited* regime). The reason is that further in the voltage-limited regime, though the current (and thus the power consumption) increase, the oscillator voltage does not increase that much; thus $P_{\text{carrier}}$ in Eq. (4.20) will not increase more. This approach provides larger NCR for the same power consumption.

**Parallel Arraying of the Mechanical Resonators**

Another approach which is used to increase NCR successfully in open-loop MEMS/NEMS resonator systems is *parallel arraying*. As the resonator is usually modeled with a series RLC circuit (with some parallel shunt capacitors), the resonator vibrations are converted to electrical
current. Thus, it seems quite reasonable to use a parallel array of the mechanical resonators so that the electrical currents coming out of the resonators (entering the electronic circuits) add up. There are some works in the MEMS/NEMS literature which try to incorporate this property to decrease NCR as discussed in Section 1.10.1. Lee and Nguyen [139] claim that using a parallel array of resonators helps to lower phase noise since the motional resistance of the array is smaller than that of a single resonator. They also make note that this decrease in motional resistance $R_x$ is achieved at the expense of some decrease in effective total $Q$.

The analysis of the authors is correct in the sense that arraying resonators definitely lowers the effective noise due to mechanical system; however, the fact that if it would necessarily decrease phase noise in an oscillator – which is a closed loop system - seems to need more detailed analyses and also experiments. In fact, parallel series-RLC branches provide a lower phase noise and also a higher current; however, in turn it also decreases the input voltage amplitude that drives the resonators (i.e. the voltage across the resonators). This fact leads us to this conclusion: Parallel arraying of the resonator has no effect on the loop gain of the oscillator. It has a good effect which reduces the total noise generated by the resonators; however, it also has a bad effect that decrease the total effective quality factor of the resonators – due to slight variation of their oscillation frequencies. So, the answer to the question that if parallel arraying improves phase noise of the MEMS/NEMS referenced oscillators lies in the fact that which of these two effects win. It is important to note that even if parallel arraying leads to lower phase noise by decreasing the generated mechanical noise, there is a limit to this approach for decreasing phase noise in oscillators. In better words, when total mechanical noise becomes very low, the noise due to active (electrical) components of the oscillator dominates; thus NCR will no longer change considerably; while total effective $Q$-factor goes on lowering by arraying.
**Tank Energy Maximization Approaches**

In addition to the above methods, there are also some approaches in the literature for maximizing the energy of the tank circuit. D. Ham and A. Hajimiri [134] prove analytically for a simplified MOS LC-oscillator with parallel tank that NCR can be decreased with increasing parallel tank capacitance. Using this concept for MEMS/NEMS referenced Pierce oscillators is much more involved, as their resonators with equivalent series RLC components in parallel with a static capacitance. Also, the Pierce family oscillators have an asymmetrical structure which leads to more complex loop transfer functions (compared to the simplified LC oscillator model presented in [134]). In addition to all these, any component values cannot be selected for the MEMS/NEMS oscillators as not only the choice of their $R_m$, $L_m$ and $C_m$ values are interrelated, large motional resistances make the oscillator not begin oscillation. Section 4.4 proposes a design methodology for the low phase noise MEMS/NEMS referenced Pierce oscillators with low-power consumption and low phase noise.

### 4.4 Proposed Design Method for Low-Phase Noise Pierce Oscillators

The main goal of this section is to provide some fundamentals for design of 3C-SiC microdisk resonator referenced Pierce oscillators. Here, I discuss my proposed design methodology for 3C-SiC MEMS/NEMS resonator-based Pierce oscillators which can begin to oscillate and has low phase noise at a limited power consumption. In this regard, first a theoretical modeling of such oscillators is done and the difficulties to design such resonators which have high motional resistance. Based on the theory discussed, the proposed steps to design the oscillator with these specifications are explained. These steps are applied toward design of a several-MHz 3C-SiC 1D (nanowire/beam) resonator based Pierce oscillator, and the simulation results are presented. This analysis can later be extended to the design of Pierce oscillators based on multi-mode microdisk-
resonators. These resonators have several resonance frequencies instead of a single one in case of the 1D resonators used in Pierce oscillator designed in this chapter. However, the design methodology will not be very different from what discussed here for oscillators based on single-resonance frequency resonator.

4.4.1 Improving Phase Noise in MEMS/NEMS Based Pierce Oscillators

As discussed, a MEMS/NEMS resonator of single resonance frequency can be displayed as shown in Fig. 4.6. In that figure, the parasitic capacitors $C_p$ were added to not lose the generality of the model. However, when modeling a MEMS/NEMS resonator in a Pierce oscillator configuration, not only those parasitic capacitances find a clear meaning, but also the electronic capacitors in parallel to the motional RLC circuit are completely defined and are not just limited to the static capacitor of a stand-alone MEMS/NEMS resonator. Fig. 4.7a shows RF view of a Pierce oscillator as well as the equivalent electronic circuit of a NEMS (MEMS) resonator with electrostatic transduction. This is composed of a series $RLC$ circuit ($motional$ branch) in parallel with a static parallel capacitor $C_g (=C_{static}$ in Fig. 4.6). The oscillator consists of three non-resonator-based capacitors $C_1$, $C_2$ and $C_3'$, (the parallel combination of the latter with $C_g$ is called $C_3$). Below, I derive signal-to- noise ratio (SNR) and phase noise equations to achieve insights for design of low phase noise oscillators. In deriving these, the equations in [137] have been effectively used.
To find the oscillation frequency $f_{osc}$ of a Pierce oscillator with mechanical resonator, we need to analyze the small-signal schematic (Fig. 4.7c), which is derived by approximations to Fig. 4.7b if the condition $C_3 << C_1, C_2$ holds. In general, Change of $G_m$ affects the intersection point of the radius ($Z_{gd,M}$) and the straight line representing the negate of the motional branch impedance. However, according to Fig. (4.7d), if $C_{12} = \frac{C_1 C_2}{C_1 + C_2}$ is large, $f_{osc}$ will be dependent of the series of the $C_m$ and $C_3$.

$$\omega_{osc} = 2\pi f_{osc} = \frac{1}{\sqrt{L_m \left( \frac{C_m C_3}{C_m + C_3} \right)}} \quad (4.22)$$

However, if $C_3 << C_m$, oscillation frequency will be nearly independent of $C_m$, and will be derived as
The following equations are derived in case of $C_1 << C_{1,2}$; i.e. when Eq. (4.23) holds. In such configuration, $f_{osc}$ will not be much different from the NEMS (MEMS) unloaded resonance frequency. Now, the equations for SNR and phase noise are derived.

In order to find the phase noise of the oscillator, first noise current power spectral density is found. Loss of the mechanical parts modeled by $R_m$ generates a noise voltage source of $4k_BT R_m$ power density; whereas the MOS generates a noise voltage source of $4k_BT \eta R_m$ where $\eta$ is the ratio of the MOS noise to the mechanical noise. The mechanical noise voltage source sees an impedance of

$$Z_L = j\omega L_m + \frac{1}{j\omega C_{tot}} = j\omega L_m \frac{(\omega + \omega_{osc})(\omega - \omega_{osc})}{\omega^2}$$  \hspace{1cm} (4.24)$$

where $C_{tot}$ is the total capacitance equal to the series of $C_m$ and $C_3$. Assuming that the offset frequency $(\omega - \omega_{osc})$ is much smaller than the oscillation frequency, the impedance can be written as

$$Z_L = 2jL_m(\omega - \omega_{osc})$$  \hspace{1cm} (4.25)$$

Thus, the power spectral density of the noise current would be

$$\omega_{osc} = 2\pi f_{osc} \equiv \frac{1}{\sqrt{L_m C_3}}$$ \hspace{1cm} (4.23)$$
Note that offset frequency is not shown here by \( \omega_m \) but by \( \Delta \omega \), so as not to be mistaken with frequency of the motional branch. Phase noise PSD can be derived from

\[
S_{I_i} = \frac{4k_bT R_m(1+\eta)}{2L_m(\omega - \omega_{osc})^2} = \frac{k_bT R_m(1+\eta)}{L_m^2(\Delta \omega)^2} = \frac{k_bT R_m(1+\eta)\omega^2}{Q^2 R_m^2(\Delta \omega)^2} \tag{4.26}
\]

Eq. (4.26) clearly shows that keeping the ratio of the oscillation frequency to quality factor (\( \omega/Q \)) constant, and meanwhile increasing \( L_m \) and \( R_m \) proportionally, current noise PSD (Eq. (4.26)) will decrease at the same rate. One of the scenarios to realize this is that \( R_m \) and \( L_m \) become \( n \) times larger, and \( C_m \) becomes \( n \) times smaller. This change preserves series resonance frequency and also series branch quality factor but not the loaded (by \( C_3 \)) resonance frequency and quality factor. However, still the ratio of the oscillation frequency

\[
\omega_{osc} = \frac{1}{\sqrt{L_m C_{tot}}} \quad \text{and} \quad Q_{loaded} = \frac{1}{\omega_{osc} R_m C_{tot}} = \frac{\omega_{osc} L_m}{R_m} \tag{4.26}
\]

in Eq. (4.26). Below, the effect on phase noise PSD (Eq. (4.27)) will be discussed.

In order to study the effect of \( n \) times increase in \( L_m, R_m \) and \( n \) times decrease in \( C_m \) on phase noise (Eq. (4.27)), we need to also know how \( I_m \) changes with such changes. An equation that helps in this regard, is that from [137] which gives the MOS transcondutance \( G_m \) at the oscillation time or better to say \( G_{m,min} \) in Fig. (4.7d)
\[ G_{m,\text{min}} = \omega^2 R_m C_1 C_2 (1 + \frac{C_3}{(C_1 C_2 / (C_1 + C_2))})^2 \] (4.28)

This simply means if keeping \( C_1, C_2 \) and \( C_3 \) constant, by making \( R_m \) larger \((n \text{ times})\), the ac transconductance (and thus the amplitude of the motional current \( I_m \)) needed at the oscillation frequency becomes \( n \) times. This is a result which also is compatible with the intuition that higher \( R_m \) needs a higher \( G_m \) and thus a higher power consumption to make the oscillator begin to oscillate. Therefore, even if we use different \( C_1, C_2 \) and \( C_3 \) values to keep the MOS bias current and thus the MOS power consumption constant for all values of \( R_m \), the phase noise in Eq. (4.27) increases \( n \) times by increasing \( R_m \) and \( L_m \) and decreasing \( C_m \), \( n \) times. This discussion will continue in Section 4.4.2.

We saw in Eq. (4.26) and (4.27) that to lower noise and phase current spectral densities, \( L_m \) and \( R_m \) should increase \( n \) times and \( C_m \) should decrease \( n \) times. There are limited methods to apply such change in MEMS/NEMS resonators. C.T.-C. Nguyen [139, 140] gives the formulation by which \( L_m \) and \( R_m \) scale inversely- while \( C_m \) directly-proportional to square of parameter \( \kappa = V_p \frac{\partial C}{\partial x} \) where \( V_p \) is the polarization (DC) voltage and \( \frac{\partial C}{\partial x} \) is the change in capacitance between the mechanical resonator and the electrode per unit displacement (\( x \)) across the resonator-electrode gap. The parameter \( \kappa \) shows how \( L_m \) can be increased by decreasing either the bias voltage across resonator-electrode or by decreasing the area between the 1D resonator and the electrode and/or increasing the gap between the electrode and the resonator. In all of these cases \( R_m \) experiences the same proportional increase and \( C_m \)
decreases at the same rate. This clearly shows the necessity of dealing with the so called challenge of high-$R_m$ resonators in design of low-phase noise Pierce oscillators. Oscillation Start-Up and Reducing Power Consumption

Impedance of only the motional series branch can be restated as below

$$Z_m(\omega) = R_m + j \frac{2p}{\omega C_m}$$

(4.29)

where $p$ represents frequency pulling defined as $p = \frac{\omega_{osc} - \omega_{series}}{\omega_m}$ with $\omega_{osc}$ and $\omega_{series}$ denoting the oscillation frequency and motional-branch resonance frequency. Negate Impedance of the motional branch for different frequencies makes up a line trajectory with constant real part ($-R_m$) and variable imaginary part is a line in Laplace plane (Fig. (4.7d)). All impedance seen by the motional branch includes a) the MOS impedance between gate and drain nodes, and 2) the parallel static capacitor of the NEMS (MEMS) resonator $C_g$ ($=C_{static}$), all modeled by the parameter $Z_c$. This impedance changes by change of the transconductance $G_m$ of the driver MOS and makes up a circular trajectory (in case of linear circuit operation) in the Laplace plane in Fig. (4.7d). Fig. (4.7d) shows that oscillation happens at intersection of the line (impedance $-R_m$) and the circle (impedance $Z_c$) assuming both resonator and circuit have linear behavior. This is in fact a pictorial expression for Barkhausen criterion [138]. Oscillation begins for $G_m$ between $G_{m,min}$ and $G_{m,max}$ and the stable oscillation eventually happens at $G_{m,min}$. 
It is clear from Fig. (4.7d) that for resonators with high-$R_m$, we need to lower $C_3$. This way the factor $QC_m/C_3$ would not drop so much that the oscillator would cease to oscillate. Need to lower $C_3$ also implies that there is a limit on increase in MOS $C_{gd}$ and thus on the MOS dimensions. This clearly shows that increasing $R_m$ and $L_m$ as concluded from Eq. (4.26) and (4.27) should not lead to increase in $G_m$ and thus increasing MOS $W/L$. That is why while $C_3$ decreases to make the oscillation happen, using Eq. (4.28) we understand that to keep $G_m$ (and thus power consumption) constant with increase in $R_m$, both $C_1$ and $C_2$ should decrease. As a challenge, we see in Fig. 4.7d that decreasing $C_1$ and $C_2$ has a bad effect on pulling factor (i.e. increases $p$). So, there is a trade-off between power consumption and pulling factor by changing $p$.

### 4.4.2 Design Steps for MEMS/NEMS Based Pierce Oscillators

We saw that to improve phase noise, $L_m$ and $R_m$ need to increase and $C_m$ decrease. However, if $R_m$ increases, not only $C_3$ should be chosen small enough, but a trade-off between power consumption and pulling factor should be made; i.e. ether power consumption increases by increasing $I_m$ and $G_m$ to achieve oscillation or at the expense of losing good pulling factor, we keep the $G_m$ consntat by decreasing $C_1$ and $C_2$.

If $C_1$ and $C_2$ cannot be decreased enough due to parasitic capacitances, two inductors in parallel with these two capacitors can be used to cancel out (most) of their capacitive admittance at the oscillation frequency. The inductors should have a very high self-
resonance frequency which is higher than the oscillator oscillation frequency so that adding them would help decrease the effective \( C_1 \) and \( C_2 \) and not increase them.

Thus, to bring a Pierce oscillator with high-\( R_m \) and high-\( \omega_{\text{series}} \) resonator into oscillation, two steps should be taken. First, the circle should shift up to cross the line of maximum acceptable \( p \) by increasing both \( C_1 \) and \( C_2 \) (Fig. 4.8b). Then, the circuit diameter should increase by decreasing the effective \( C_3 \) (Fig. 4.8c). This way, the circle and the line will intersect at frequency \( \omega_{\text{osc}} \).

Using the above design guidelines, Fig. 4.9 and Fig. 4.10 show the Pierce oscillator architecture for minimum power consumption and for minimum deviation from the resonator’s resonance frequency respectively. In the first architecture, there are two inductors in parallel with \( C_1 \) and \( C_2 \) that cancel out most of their capacitive admittance at the oscillation frequency. In Fig. 4.10 though, not only those inductors do not exist, but also \( C_1 \) and \( C_2 \) are
also increased as much as possible to make the oscillator begin oscillation. It is due to mention that it is quite possible that $R_m$ is not chosen that high to need very low $C_1$ and $C_2$. Therefore, in these cases, although it is not a goal to keep the minimum pulling factor $p$, but to have a very low power consumption. However, because needed $R_m$ is not so high, $L_1$ and $L_2$ are not needed, thus the architecture in Fig. 4.10 can be used.

**Figure 4.9.** The Pierce oscillator architecture when the inductors $L_1$ and $L_2$ are needed to nearly resonate out the capacitors $C_1$ and $C_2$. Two bias voltage sources $V_{bias1}$ and $V_{bias2}$ exist in this circuit.

**Figure 4.10.** The Pierce oscillator architecture without resonating-out inductors. The bias voltage source is connected to the $L_m$ to determine the bias voltage of the MOS gate.
4.4.3 Simulation Example for 3C-SiC MEMS Based Pierce Oscillator

In accordance with the 3C-SiC resonators designed in Chapter 3, the goal in this section was to design Pierce oscillators working with 3C-SiC resonators. We saw in that chapter that 2D center-clamped microdisk resonators have multiple resonance frequencies that depend on the resonance mode and the ratio of the disk radius to the center anchor radius \((a/b)\). The linear model for a multi-mode resonator is actually composed of several parallel series RLC motional branches, with a single parallel static electronic capacitor. That parasitic capacitor in parallel with the input impedance of the rest of the oscillator circuit is called \(Z_c\) in Fig. 4.11.

The important fact that reduces design of the 2D or any multi-mode resonance-frequency resonator to a single-resonance frequency resonator is that at each resonance frequency, the impedance of only one motional branch drops to a minimum of \(R_m\) in that branch, while the impedance of the other series RLC branches is very high (can be assumed as infinity in magnitude). The higher the \(L_m\)’s and the smaller the \(C_m\)’s are, the more the other motional...
branches impedance tend to that of an open circuit at the oscillation frequency of the other ones.

The designed double-clamped mechanical resonator designed has the parameters given in Table I.

<table>
<thead>
<tr>
<th>$E$ [Pa]</th>
<th>Density ($\rho$)</th>
<th>$f_{res}, f_{osc}$ [MHz]</th>
<th>Width ($w$) [m]</th>
<th>Depth ($d$) [m]</th>
<th>Length ($l$) [m]</th>
<th>Gap ($g$) [m]</th>
<th>DC Voltage [V]</th>
<th>$Q$</th>
<th>$R_m$ [Ω]</th>
<th>$L_m$ [H]</th>
<th>$C_m$ [F]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.33E+11</td>
<td>3.21E+03</td>
<td>20, 8.0E-08</td>
<td>2.00E-08</td>
<td>3.46E-06</td>
<td>1.98E-08</td>
<td>3.3</td>
<td>1.00E+3</td>
<td>5.224E+03</td>
<td>4.16E-2</td>
<td>1.522E-15</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$E$ [Pa]</th>
<th>Density ($\rho$)</th>
<th>$f_{res}, f_{osc}$ [MHz]</th>
<th>Width ($w$) [m]</th>
<th>Depth ($d$) [m]</th>
<th>Length ($l$) [m]</th>
<th>Gap ($g$) [m]</th>
<th>DC Voltage [V]</th>
<th>$Q$</th>
<th>$R_m$ [Ω]</th>
<th>$L_m$ [H]</th>
<th>$C_m$ [F]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.33E+11</td>
<td>3.21E+03</td>
<td>20, 1.333</td>
<td>2.00E-08</td>
<td>3.46E-06</td>
<td>4E-08</td>
<td>3.3</td>
<td>1.00E+3</td>
<td>8.74E+04</td>
<td>6.96E-1</td>
<td>9.10E-17</td>
<td></td>
</tr>
</tbody>
</table>

The Pierce oscillator architecture is that of Fig. 4.10 with $I_{BIAS}$ of 1μA, $V_{BIAS}$ of 0.82V. $V_{DD1}$ connected to the source of MP1 is 3.3 and $V_{DD1}$ connected to the source of MP2 is 1.5V. The MOS sizes are $\frac{W}{L}_{MP1} = \frac{10\mu m}{1\mu m}$, $\frac{W}{L}_{MP2} = \frac{10\mu m}{1\mu m}$ and $\frac{W}{L}_{MN1} = \frac{5\mu m}{1\mu m}$. The oscillator parameters are designed based on the equations provided in the previous sections in this chapter so that it can begin oscillation at this frequency ($f_{osc}=20MHz$). This oscillator was simulated by Spice Simulator and with the CMOS 0.35μm technology (including also flicker noise). Fig. 4.12 shows the time-domain voltage amplitude of the NMOS drain node. From
the measurement labels on the figure, it is obvious that $C_{tot}$ (4.762 MHz) is very different from the resonance frequency of the motional branch (20 MHz) due to the loading effect of the $C_3$ capacitor (which stands for the NMOS $C_{gd}$ and the static capacitor of the resonator) on the motional branch.

Figure 4.12 Transient voltage signal of the Pierce oscillator with the resonator in Table I.

Having known the oscillation frequency of the oscillator from the time-domain (transient) analysis, the harmonic analysis is done first to derive the oscillation frequency considering 11 harmonics. Then, the phase noise values were derived at the offset frequency range of 100Hz to 0.3MHz, considering both frequency dependent and bias dependent noise sources. The values derived were redrawn by Matlab shown below
In order to verify the theoretical claims supporting use of higher value $L_m$ and $R_m$ and lower value $C_m$ to lower phase noise, the same oscillator active circuit (with exactly the same sizes for MN1, MP1 and MP2) is used with the same bias voltage and currents. The resonator parameter values are reported in Table II (two pages ahead in the dissertation).

As seen, the $L_m$ and $R_m$ are 16.7308 larger and $C_m$ is 16.7308 than the values in Table I. The other parameter which is different between the two tables is the gap size which is 2.0225 times –equal to $\sqrt{16.7308}$ larger than the value in Table I. The transient domain voltage signal is shown in Fig. 4.14.
It is noteworthy that the oscillation frequency calculated from measurements in Fig. 4.14 is 1.3333 MHz which is $3.5716 \times \frac{1}{\sqrt{16.7308}}$ smaller than the oscillation frequency in Experiment 1. This again demonstrates that $C_3$ is much smaller than $C_m$ in both cases and change of oscillation frequency from the first Experiment to the second one mostly draws on change of $L_m$ and not $C_m$.

Having known the oscillation frequency of the oscillator (1.3333 MHz) from the time-domain (transient) analysis, the harmonic analysis is done first to derive the oscillation frequency considering again 11 harmonics. Then, the phase noise values were derived at the same offset frequency range as Experiment I, considering both frequency dependent and bias dependent noise sources. The values derived from the simulator were redrawn by Matlab (Fig. 4.15).
Figure 4.15 Phase noise for the Pierce oscillator based on the resonator in Table II.

As seen above, the phase noise in Fig. 4.15 is 7.2982dB smaller at offset frequency of 100Hz and 7.3974dB smaller than that in Fig. 4.13. This clearly shows the effect of increasing $L_m$ (and thus increasing $R_m$ and decreasing $C_m$ with the same proportion) on the phase noise performance of the MEMS/NEMS based oscillators. According to Eq. (4.26) and (4.27), the new $L_m$, $R_m$ and $C_m$ would suggest a decrease of ~12.2dB in case $Q_{\text{loaded}}$ would not change. However, due to loading which also has changed the oscillation frequency, ~7.4dB improve in phase noise is achieved.

Therefore, these two experiments with oscillators that can oscillate with the same active circuit but different mechanical resonators, approve the design methodology suggested in the previous sections of this chapter.
4.5 Summary and Conclusions

As seen in Chapter 1, any sensor is best to be used in self-sustained architectures which preserve a sustained (un-damped) signal amplitude. Following that major goal, this chapter attempts to provide a new methodology for design of MEMS-based Pierce oscillators with electrostatic transduction. The main intuition from this analysis is increasing motional inductance (in the equivalent circuit for MEMS/NEMS resonator) is a very effective method to improve the phase noise. I also analyzed how increasing motional inductance might lead to increase in motional resistance, as a potentially bad effect which might make the oscillator not to begin oscillation. However, this chapter suggested methods based on theory to make sure that the oscillation can begin and at the same time phase noise of the oscillator would decrease. These both good things can be achieved at either higher pulling factor (detuning) or higher power consumption (they can be traded off with each other).

Following the above analysis and the simulation results for electrostatically-transduced 3C-SiC microdisk-referenced Pierce oscillators which demonstrated effectiveness of the method, the next chapter will focus on laser-based oscillators with 3C-SiC microdisk reference resonators. We will see not only those oscillators provide spot sensing (good for multi-mode resonators) and low-power optical actuation, but also they potentially cause less detuning due to impedance loading effects compared to the oscillators with electrostatically-coupled resonators. Other benefits and the detailed procedure is explained in next chapter.
Chapter 5: Microdisk-Based Opto-Acoustic Oscillators

5.1 Introduction

In Chapter 3, 3C-SiC microdisk resonators were fabricated with a novel fabrication method (based on FIB milling and HNA etching) and theoretical analysis as well as finite-element simulation of the structure were done to find the vibration amplitudes and frequencies of different vibration modes. Following that, the RF vibration of the different-size microdisks were measured using a home-built measurement setup based on optical-laser sensing, interferometry and using spectrum-analyzer. The spectrum-analyzer assisted vibration measurements were then proceeded with processing of the measurement results to fully characterize the resonator vibration mode in terms of the resonance frequency and quality factor.

Chapter 4 discussed a novel process based on theory for design of Pierce oscillators with electrostatically-transduced MEMS/NEMS-based resonators. The design method relied on increasing the mechanical motional inductance to improve phase noise and proceeded with strategies to address/tackle the oscillation start-up, power consumption and pulling factor (i.e. detuning). These theory-based design intuitions also clearly demonstrated the trade-offs between these design goals.

Based on the background from the previous two chapters, in this chapter one of the 3C-SiC microdisk resonators fabricated in Chapter 3 is employed in a feedback oscillator loop with optical (laser)-based sensing and actuation to realize a self-sustained feedback oscillator. A step-by-step
modeling framework is proposed and the results are presented and several key observations are made. Following that, the measurement results of the phase noise are presented with comparison of the observed results with the simulation results. This practical and theoretic analysis of such oscillators helps to candidate them as novel oscillators of their kind for future sensing applications.

5.2 The Elements of Microdisk-Referenced Oscillators

5.2.1 Laser-Based Actuation

The reason for using laser-based sensing technique was fully discussed in Section 1.4 and Section 3.5.1 where accurate displacement sensing, non-destructiveness of the method, ease of setup and also localized (spot) sensing which enabled its application in sensing of specific vibration modes (in multi-mode resonators) and also for future nanoscale mechanical resonators were discussed. A setup similar to Fig. 5.1 can provide such advantages as well as accurate control of the laser power when the MEMS resonator is placed in vacuum chamber to avoid mechanical losses due to air damping (see Section 1.5.4).

![Figure 5.1. Schematic of the vibration resonance measurement setup with optical apparatus, photo detector, the specimen in vacuum and the electrical measurement instrument (network analyzer). This home-built measurement setup works based on laser interferometry to measure amplitude of the resonance peaks.](image)

As we know, all (self-sustained) resonator-based oscillators have a feedback loop that includes both a sensing and an actuation mechanism to couple to their mechanical resonators. The laser-based
has a comparatively less complex structure, consists of only a variable-power laser source with one or a few objective lenses. The motivations for using laser-based actuation as an excitation method for vibration of the microdisk resonators in this dissertation is very similar to those for optical (laser) sensing. The laser setup can provide a localized spatial access to all spots on the resonator surface. Also, the ease of implementation (i.e. simplicity of the setup) was another incentive for using laser for actuation technique. This actuation method does not suffer from the infringing electrostatic fields as capacitive transduction does, and is immune to the loss of power due to non-directional mechanical energy flow as for example piezoelectric methods suffer from.

In addition to these, in non-optical transduction methods there are usually components that impose loading on the resonator, which therefore would make the oscillation frequency different from resonance frequency of the resonator. However, in laser-based transductions, not only the effect of non-mechanical loading (on the mechanical resonator) due to the laser generation/propagation components is lower but also since laser propagates in air, the parasitics seen especially in electrostatic and piezoelectric-based transduced resonators are not seen here. That is why in the oscillator explained in this chapter which is based on resonators with laser-based optical transduction, the oscillation frequency is very close to the mechanical motional resonance frequency. This was not the case in the 3C-SiC electrostatically-transduced resonator-referenced Pierce oscillators simulated in the previous chapter. In addition to all these, laser-based actuation is very compatible with SiC as laser power in the laser-based actuation can generate very high heating which leads to very large local temperatures on the mechanical resonator surface. Because SiC is very immune to high temperatures, this approach for actuation of the MEMS resonator can be easily implemented in 3C-SiC microdisk resonators, while resonators made from other materials might melt at such temperatures.
It is very important that in contrast to thermal actuation, *optothermal (i.e. optical) actuation*, i.e. thermal actuation through laser, does not suffer from huge power consumption for heat generation by passing current through the resonator. In fact, optical actuation through laser saves on the consumed power by orders of magnitude. This is another advantage of using optical actuation for the 3C-SiC microdisk resonator-based oscillators.

### 5.2.2 Laser Sources and Amplifiers

In this section, laser generation (and amplification) is concisely explained. To generate lasers, it needs to have a laser generation oscillator. We know that the light is generated (i.e. *photons are emitted*) either by *spontaneous* or *stimulated* emissions. In spontaneous emissions, an electron transfers from higher energy state to lower energy band and in doing that, the atom would emit photons. The reverse process can also hold; i.e. electron moves to lower energy band once the atom receives photon. This former procedure is called spontaneous photon emission. Needless to say, the photons emitted from atoms of the same material through spontaneous emission, do not necessarily have the same phase i.e. they are incoherent. To make them coherent, stimulated emission process is used in which photons emitted from each atom enter another atom to help electrons move from an energy state to another state. Each photon emitted in this way has the same phase as the photon initially entered that atom, which make this process a coherent photons emission scheme. *Optical amplifiers* and *laser sources with tunable power* work based on the concept of stimulated emission – the latter is utilized in my oscillator setup to be discussed in Section 5.3.1. Using such optical amplifiers as well as optical resonant cavity structures for the emitted photons, the two Barkhausen conditions (Eq. 4.1) for the laser generation oscillator can now hold. As a result, the laser with a very tiny linewidth is generated.
5.2.3 Laser Phase and Frequency Modulators

In the previous section, the theory behind laser sources and amplifiers were discussed. In this section, another relevant property of the lasers are discussed concisely. Lasers are usually made in direct-bandgap materials and their frequency are determined by the material they are made in. However, upon propagation through other materials, the laser frequency also changes based on the medium’s refractive index. That is exactly why laser is frequency-modulated with the resonance frequency of the resonator when it passes through transparent MEMS/NEMS resonators. This property is used by electro-optical modulators (EOMs) in which the refractive index of the material is changed using electrical signal. Using such devices, frequency and phase of the lasers can be changes. The phase modulators are used in making the oscillator explained in this chapter to meet the Barkhausen phase criteria (Eq. 4.1).

In the oscillator circuits in which optical energy can be converted to electrical energy and then converted back to optical (laser), phase can be alternatively modulated using electrical phase modulators. This latter approach was taken in the micro-disk resonator referenced oscillator in this chapter.

5.3 Modeling The Microdisk-Based Oscillators

5.3.1 Schematic

A schematic of my microdisk-based oscillator with optical laser sensing and optical laser actuation is shown in Figure 5.2. The oscillator includes a feedback loop with sensing and actuation components. The sensing mechanism includes a rather low-power HeNe laser source of 632.8nm wavelength, different lenses (including objective lenses) to focus laser on the pot on the microdisk surface, and finally the photodetector to convert the laser power to electrical current. The loop
feedback path actually begins with the photodetector, and also includes LNAs, bandpass filters (BPF) around the signal center frequency, and also the phase modulator.

![Figure 5.2. Laser-based transduced Microdisk-Based Oscillator](image)

LNAs are actually low-noise transimpedance amplifiers which provide a large enough electrical voltage signal from the input current signal without adding much noise. Bandpass filter have a bandwidth of nearly 3MHz around the center frequency of 20MHz and are included to cancel out the harmonics of the 20MHz signal generated due to possibly nonlinear behavior of the LNAs as well as cancelling the white noise in all other frequencies. The phase modulator is where the phase of the signal is compensated to provide $360^\circ$ phase shift in the loop. It can be an automated or manual procedure. In my setup is was done manually using an analog dc control. The RF signal from the output of the phase modulator is then split using a splitter (directional coupler) to provide signal for two paths. One path goes to the measurement instrument (e.g. network analyzer or oscilloscope) and the second is in fact the oscillator feedforward path. This path includes the actuation mechanism devices including the amplitude modulator and the laser source. The
amplitude modulator modulates the intensity of the laser by changing the signal which controls the laser source output power. The choice of blue laser for the actuation (feedforward) path and modeling the optical generation and heat diffusion effects in microdisk resonators will be discussed more in the later sections.

### 5.3.2 Model Step 1: Feedback Loop with Constant Resonance Frequency

In order to get insights about the operation of the microdisk-based acousto-optic oscillator with optical actuation, a behavioral simulation of the system in Matlab Simulink is done. 

![Simulink model](image)

*Figure 5.3 The Simulink model (Step 1) for the oscillator in Fig. 5.2.*

In this model, the microdisk resonator is modeled as a linear system of order 2 with center frequency of ~20MHz (=19,859,282 Hz) and a quality factor of 1000. The LNAs are modeled simply as gain stages because their poles and zeros are much larger than 20MHz. The phase modulator is modeled by a delay stage which generates the same delay time for all frequencies. In reality, the phase modulator components can only provide desired phase modulation for a certain frequency band. BPF (filters) are modeled using a linear order-two system with BW of 4.4MHz which is much
larger than the mechanical resonator bandwidth. The noise in the system is assumed to be Gaussian noise with zero mean and variance of $1\times10^{-10}$.

The oscillation startup (in time domain) of this system is shown in Figure 5.4

![Figure 5.4 Oscillation start-up in the oscillator model (Step 1).](image)

It is due to mention' that the laser source generation takes some time – i.e. causes some delay - which is really negligible in our operational frequency ($\sim20\text{MHz}$) and also laser amplitude modulator responds to the variation of the control signal with some delay which is still very small due to the high bandwidth of the amplitude modulators ($200\text{MHz}$ in case of Newport 4102NF and 4104NF); otherwise they need to be included in the model as series low-pass filters.

Before deriving phase noise of the oscillator and comparing it to that of the model in Step2, we see the frequency response of the oscillator in Fig. 5.5.
5.3.3 Model Step 2: Feedback Loop With Varying Oscillation Frequency

As we know, part of the laser (photon) absorption in materials leads to heat generation. As heat transfers from the laser spot on 3C-SiC disks to the other locations on the microdisk, the material expands. Expansion of the material due to thermal energy changes the physical size of the mechanical resonator as well as its mass density, thus changes the microdisk resonance frequency. This change in the frequency has a mean value and a variance if modeled as a Gaussian process. We observe that though the mean value change of the resonance frequency causes a shift in the oscillation frequency, the variance of resonance frequency change does not seem to have any prominent effect on the phase noise. To model the variance frequency change, the “Mechanical Resonator” block in Fig. 5.3 was replaced with the subsystem in Fig. 5.6.
Figure 5.6 Dynamically changing of the resonance frequency with a variance of deltaF to account for the laser-induced temperature changes in 3C-SiC microdisk resonators (Step2).

Using the above block to model a microdisk resonator whose resonance frequency changes with a random normal continuous source (with arbitrary mean value of 1.5kHz and variance of 1.5kHz) over the simulation time, the effect of changes in the phase noise due to frequency instability can be derived. Fig. 5.7 shows the frequency spectrum of the oscillator which shows ~1.5kHz shift of oscillation frequency compared to that in Fig. 5.5 (Model Step1). Before deriving the phase noise from the oscillator’s time-domain signal, it is intuitive to know qualitatively how frequency changes with changes in temperature.

Mechanical strain changes with temperature changes as below

\[ S_x = \alpha \Delta T \quad (5.1) \]

where \( S_x \) is the mechanical strain, \( \alpha \) is the linear thermal expansion of 3C-SiC and \( \Delta T \) is the change in temperature. This equation simply means changes in the physical sizes of the resonator are proportional to both temperature changes and the initial physical dimension. So, any temperature
increase by $\Delta T$, simply leads to increase in radius $a$ (and the suspended radius $a-b$), thickness $h$, and density $\rho$ by $\alpha \Delta T$, $\alpha \Delta T$ and $\alpha^{-3} \Delta T^{-3}$ . Assuming that the frequency mode constant $\lambda$ in the resonance frequency of the oscillator (Eq. (5.2)) does not change considerably with such small changes in the ratio $a/b$ (which is a valid assumption), the frequency will increase by the factor $\alpha^{1/2} \Delta T^{1/2}$

$$\omega = \sqrt{\frac{3D(1-v^2)}{\rho h}} \frac{\lambda}{a^2} = \lambda \sqrt{\frac{E}{4\rho}} \frac{h}{a^2} \quad (5.2)$$

(Refer to Eq. (3.1) in Chapter 3 for definition of the terms). This means with any increase in 3C-SiC microdisk resonator temperature due to laser heating, the out-of-plane resonance frequency of the resonator also increases though at a slower pace. Knowing the thermal expansion coefficient of 3C-SiC $42e-6 ^\circ K$ [141], each 2.381$^\circ$K increase in 3C-SiC temperature, leads to $1e-4$ times increase in the physical dimensions and 1% increase in the microdisk out-of-plane resonance frequency. This can give us a hint to reverse engineer the variance of $\Delta T$ from the variance of frequency shift observed in the measurements.
Figure 5.7 Frequency spectrum to measure oscillation frequency in the oscillator model (Step 2). About 1.5kHz oscillation frequency change compared to what seen in Fig. 5.5 is observed.

To derive the oscillator phase noise, a Matlab (M-file) code is needed to derive the power of the signal at each frequency offset from the center frequency and find the ratio of that power to the signal power at the resonance frequency. It is due to mention that according to the new definition for phase noise, half of that power is defined as the phase noise. Please refer to Appendix A for the phase noise calculation M-file.
Figure 5.8 demonstrates the single sided phase noise from the model developed so far in Step 1 and Step 2. The dc value of the phase noise is only a function of the assumed signal amplitude which has been assumed arbitrarily, thus does not convey much information (that is, signal amplitude can change by change of the loop gain). However, an important observation from this graph is that, the phase noise (averaged over time) falls with the slope of $-20\text{dB/dec}$. This latter phenomenon is what we expected from the simulation, as the only noise source in the oscillator system is Gaussian noise which generates $20\text{dB/dec}$ phase noise slope (see review of Leeson and other phase noise models in Section 4.2.2). Now, I go forward to make the model more realistic by making the frequency changes time-variant (i.e. evolving over time with temperature variations).
5.3.4 Model Step 3: Loop with Time-Evolving Frequency Variations

As mentioned in the previous section, temperature variations lead to linear thermal expansion and thus resonance frequency changes in the (microdisk) mechanical resonators. However, assuming the heat generation at the laser spot on 3C-SiC microdisk happens fast enough to be assumed instantaneous, the heat transfer through the material takes some time and is not a very fast process. In other words, the process of heat transfer and thus frequency change should be modeled as a transient process which evolves over time.

The accurate analysis of heat transfer needs solving of 3D heat transfer equation for the microdisk geometry which is a boundary-value problem in which the laser spot on the disk determines the initial condition. The lumped response for simplified cases (only available for rather simple initial conditions) can be found in mathematical books. For disk geometries, assuming the heat source is at the center of the disk, the solution turns into Bessel differential equation. However, for the case of our experiment in which the laser spot was on the edge of the microdisk, the analytical solution is much more complex (if exists). However, the problem can be simplified by assuming that the heat only transfers along the thickness and also along the radius toward the center anchor. In reality because the heat also transfers in the tangent direction to the microdisk perimeter (φ direction), the realistic heat transfer time constant will be even higher. So, assuming that the heat only transfers in the direction of radius (i.e. radial direction) and thickness independent of each other, the thermal depth of penetration over time $t$ in any direction is given approximately by

$$x = (3\kappa t)^{1/2} \quad (5.3)$$
where \( x \) is the depth of penetration in each of the radius or thickness directions, \( \kappa \) is the heat diffusivity and \( t \) is time. This simply means the thermal time constant is equal to \( \frac{x^2}{3\kappa} \). In other words, the equation for elongation in the direction of microdisk radius and thickness can be derived from the equation below \[142\]

\[
\Delta x(t) = \Delta x(1 - e^{\frac{-3\kappa}{x^2} t})
\] (5.4)

Thus, according to the discussion in Section 5.3.3, the resonance frequency of the microdisk changes as

\[
\Delta f_{res}(t) = \Delta f_{res} \left(1 - e^{\frac{-3\kappa}{(a-b)^2} t}\right)^{3/2} \left(1 - e^{\frac{-3\kappa}{x^2} t}\right) (5.5)
\]

Thus, the heat transfer time constants along radius and thickness are not equal. Heat diffusivity of 3C-SiC is 1.6e-4 m²s⁻¹ \[143\], knowing which we can find the heat transfer time constants and calculate Eq. (5.5). The new model is based on this new derivation. To implement the above time-evolving random frequency change instead of the instantaneous random frequency changes as was the case in the Step 2 of the model, the single block (“random source”) in Fig. 5.6. which made the frequency change with given mean and variance \(( = N_{\Delta f_{res}}(\mu, \sigma) )\) change to the subsystem shown in Fig. 5.9 which realizes Eq. (5.6)

\[
\Delta f_{res}(t) = N_{\Delta f_{res}}(\mu, \sigma). \frac{(1 - e^{\frac{-3\kappa}{(a-b)^2} t})^{3/2}}{(1 - e^{\frac{-3\kappa}{x^2} t})} (5.6)
\]
As Fig. 5.9 shows, the frequency still undergoes random changes as a normal distribution with the same variance. However, here each random frequency change happens every 10 system sample time. During these 10 sample durations, the resonance frequency of the microdisk has a constant random value \( N_{f_{\text{res}}} (\mu, \sigma) \) but evolves over time with the model suggested by Eq. (5.6). This provides a more realistic case compared to the one in Step 2 of the model (Section 5.3.3).
Phase noise of the oscillator with the new changes is shown in Fig. 5.10. The -20dB/dec region of the phase noise can be seen which is again due to Gaussian noise sources (see Fig. 5.3). The phase noise seems very similar to the red curve in Fig. 5.8. However, there are some minor variations especially at lower offset frequencies. It should be mentioned that the phase noise profile can change more if the time difference between temperature changes is larger in the system (as opposed to the model in Step 2 which basically modeled continuous variations to the resonance frequency).

During these three subsections, the oscillator system was behaviorally modeled and step-by-step the details were added to the model to make it more similar to the realistic case. It shows that if changes in the oscillation frequency have a normal distribution with zero mean, the phase noise does not change much. However, in reality not only there is a nonzero mean for resonance frequency change in the oscillator, but also the frequency change distribution is also not a complete...
normal distribution. In the next section, the real experiment of making the acousto-optic oscillator in the lab using off-chip components will be explained and the measurement results will be presented.

5.4 Measurement Results for Microdisk-Based Opto-Acoustic Oscillator

The acousto-optic oscillator with optical actuation was modeled in the previous section and the phase noise was derived. In this section, the experiment measurement results will be discussed. Fig. 5.11 shows the microdisk chip to be placed in the vacuum chamber for the experiment.

![Figure 5.11 The microdisk resonator chip placed in the vacuum chamber.](image)

Fig. 5.12 shows the experiment setup for the oscillator. As seen, both blue and red (HeNe) laser sources as well as the mirrors and objective lenses to direct and focus them on the microdisk surface can be easily seen in this picture. Before the closed loop system in Fig. 5.2 is set up and tested, the
open loop system was tested using oscilloscope to make sure that the sensing system and also the HeNe laser source, photodetector, LNAs, and the vacuum chamber all work well (Fig. 5.12).

![Image](image_url)

*Figure 5.12 Oscillator setup. The HeNe (red) and blue laser sources as well as mirrors and lenses can be seen in the figure.*

The main goal of the open loop analysis was to measure the resonance frequency of the resonance mode that is dominant on the HeNe laser spot (sensing path). For complete characterization of all different resonance peaks in the vibration spectrum of the microdisk please refer to Chapter 3. The off-chip components seen in Fig. 5.13 are Mini-Circuits ZFSCJ-2-1-S 1-500MHz splitter, and the measurement instruments are Agilent E4440A 3Hz-26.5GHz PSA series spectrum analyzer, Mini-Circuits SBP-21.4+ 50Ω, 19.2-23.6MHz bandpass filter, Mini-Circuits ZFL-1000LN+, 0.1-1000MHz amplifier (LNA) – which is usually used for amplification of fast signals e.g. in photon
counting applications, optical devices from Thorlabs and HP 3577A network analyzer 5Hz-200Mhz (not shown in the figure).

Figure 5.13 The open-loop oscillator circuit. LNAs as well as splitter, BPF, dc power supply and measurement instruments to measure resonance frequency can be seen in the figure.
Fig. 5.14 shows one of the vibration peaks of the microdisk resonator on spectrum analyzer in the open-loop measurements. After finding different resonance peaks of the resonator and observing the dominant one (which strongly depends on the location of the laser spot on the surface of the microdisk resonator), the closed-loop oscillator is set up and the phase noise measurements for the oscillator is done. Fig. 5.13 shows the oscillation peak in closed loop (left) as well as the measured phase noise (right).

Figure 5.15 Wide spectrum (Left) and phase noise (Right) measurement results from closed-loop oscillator setup.
As seen in Fig. 5.15 (left and right), there are also two other non-major resonance peaks shown in both pictures with red circles. The correct phase noise slope should be measured after suppressing the effect of non-major peaks (and interferences) from the measurement results as a data post-processing step.

Also, it is important to note that in the model developed in the previous section, the temperature variations had a normal (Gaussian) distribution with zero mean and known variance where any nonzero mean frequency would mean a constant shift in the resonance frequency (and thus oscillation frequency). However, in real-world experiments as in this measurement, the temperature variations are not necessarily a normal distribution, thus the phase noise profile might undergo noticeable changes even with nonzero-mean frequency changes.

The main observation, however, is that the flat region which can be attributed to phase diffusion has a width ~5kHz. This simply means that the effective quality factor of the oscillator is

\[
Q = \frac{f_{\text{osc}}}{\Delta f} = \frac{f_{\text{osc}}}{D_\phi} = \frac{20 \text{MHz}}{5 \text{kHz}} = 4000
\]

(5.7)

where \(D_\phi\) is the oscillator diffusion constant. [144] This latter observation (~2.3 times increase of maximum effective \(Q\) factor from ~1700 for the thick-anchored resonators in Chapter 3 to ~4000 for the oscillator in this chapter) is exactly what we were seeking by developing a self-sustained oscillator.

5.5 Summary and Conclusions

In this chapter, a ~40μm diameter (~20μm suspended diameter), ~0.5μm 3C-SiC microdisk resonator fabricated and analyzed in Chapter 3 with different resonance modes was utilized in a sustained feedback loop with a laser-based sensing and actuation (the latter known as optical...
actuation). The microdisk resonators provide a large surface with numerous RF resonance frequencies and high $Q$ factor which is very proper for accurate sensing applications. The laser-based sensing and actuation mechanisms employed in this setup provide accurate spot sensing and low-power optical actuation with minimum impedance loading effect on the microdisk resonator – which otherwise might have affected the resonance frequency strongly. 3C-SiC material lends itself very well to laser-based optical actuation as the material does not melt or ablate at operational temperatures of the (blue) laser.

Prior to practical measurements, a theory-based model was developed in three steps and simulated behaviorally in Simulink to give insights about the process and the measurement results to be done. Then, the oscillator was tested first in an open-loop and then in closed-loop and after tuning the components, the oscillation started. The wide spectrum of the oscillator output signal as well as phase noise were measured and the measurement results were discussed. The main observation for the experiments was that the effective quality factor of the self-sustained oscillators developed in this dissertation is larger by a factor of 4 than the quality factor of the microdisk resonators seen in Chapter 3.

This dissertation ends here. The goal from the first chapter was to design and test a (novel) oscillator for 3C-SiC microdisk resonators. The novel center-clamped 3C-SiC RF microdisk mechanical resonators fabricated and characterized in Chapter 3, are alone good candidates for sensing applications and using laser-based optical read-out. optical ly actuation mechanisms enables them to be used in self-sustained oscillator architectures where we see strongly improved effective quality factors. This even more promotes the application of these microdisk resonators in future accurate sensing applications.
Chapter 6: Conclusions and Future Works

In this dissertation, I explained my efforts and achievements in proposing the first open-loop 3C-SiC RF microdisk resonators with out-of-plane vibration as well as optically-transduced self-sustained (closed-loop) oscillator as a candidate for future sensing applications. This effort first begins with material-level analyses which includes investigation of 3C-SiC material and its nanomachining using Focused Ion Beam (FIB). Sputtering yield and lateral deformations were measured (or calculated based on measurements) for many different FIB conditions as well different pattern geometries (and scales) to-be-milled. SRIM Monte Carlo simulations also helped to have a better understanding of the Ga implantations.

In the device and system (i.e. self-sustained oscillator) level investigations of this thesis, I have managed to propose a new micro-scale (with sub-micron thickness) mechanical resonator for RF applications from 3C-SiC which has high temperature-resistance and chemical inertness (with also many other good physical properties) for sensing applications. I also fully characterized these resonators via many measurements and data post-processing. Also, in pursuit of a MEMS-based resonant sensing system with self-sustained (non-damping) amplitude and improved performance, an analytical investigation of design method for better performance was done. This analytical investigation began with electrostatically-transduced MEMS-referenced Pierce oscillators in which design guidelines
for having improved phase noise performance were eventually proposed. This linear analysis mostly based on reducing the thermal (white) noise generated by the mechanical sections (of MEMS resonator), also includes its limits (when active circuitry induced noise would dominate) and trade-offs (with either of power consumption or frequency detuning).

Having known the merits and limits of electrostatically-transduced MEMS.NEMS-referenced oscillators based on the previous investigation, the dissertation then describes efforts on utilization of optical actuation for making self-sustained microdisk-based oscillators. The laser-based optical actuation and laser interferometry-based sensing not only enables spot sensing and actuation to potentially access specific resonance modes, it also has the advantage of low-power consumption for actuation (in contrast to piezoresistive actuation) as well as circuit loading effects (in contrast to electrostatic transduction) and maybe better power handling in some transduction schemes. Also, laser-based sensing and actuation is very well compatible with the sensing applications in the scope of this dissertation. The 3C-SiC microdisk resonator can be used in a harsh environment to which there is no access except laser-based read-out and actuation.

Behavioral modeling as well as experimental tests and measurements were done for these optically-actuated microdisk-referenced oscillators. The measurement results of phase noise showed a noticeable (>2.3) improvement of the effective $Q$ factor over that of the open-loop microdisk resonators. These oscillators are expected to be very good candidates as multiple-resonance sensors in future adaptive sensing applications especially in harsh environments.
Also, in addition to all good properties, these microdisk-based oscillators are very compatible with future fast-progressing optical systems to provide precision resonant sensing.

There are several directions for future works and probable improvements to this dissertation. In (open-loop) microdisk resonator characterizations reported in Chapter 3, degenerate modes can affect use of this (and any other MEMS/NEMS) resonator for resonant sensing applications. The degenerate modes are created due to unwanted geometry asymmetries (e.g. due to edge deshaping of disk edges in FIB milling, radially-asymmetric HNA etching of the anchor or nonuniform thickness of the microdisk) and material imperfections. Good effort has been done to avoid these unwanted asymmetries as by polishing 3C-SiC microdisk surface before FIB milling and holding the 3C-SiC chip as even as possible when doing HNA etching. However, it is expected that the degenerate modes can be weakened tremendously by applying better lab measures and facilities.

In Chapter 4, the design method/guideline had only considered thermal noise sources in mechanical resonator and also thermal MOS channel noise. As a future work, effect of other thermal (e.g. bias noise) and non-thermal (e.g. MOS 1/f noise) noise sources can be taken into account to better identify the limits of the analysis and design method proposed in this chapter. As an ongoing sequel to the work in this chapter, the theoretical and simulation results are also being measured practically to find out the limits of the method in practice. In Chapter 5 on optically-transduced 3C-SiC microdisk based oscillators, reasonably the best location for maximum vibration actuation and measurement is the very sensing/actuation laser spot anywhere near the microdisk edge. However, the effect of the interference of the
two lasers on each other is not well known and can be investigated in future works. Also, the current oscillator setup utilized was not an all opto-mechanical setup. In order to use electrical LNAs and band-pass filters as well as spectrum-analyzer-based vibration peak measurements, optical-to-electrical (and vice-versa) signal conversions were done. However, with development of all-optical components, all these operations can be done optically and at very small form-factors. It is very much anticipated that the proposed 3C-SiC referenced optically-actuated self-sustained oscillator can be implemented in near future using micron-sized all-optical devices which can inspire minimum undesired impedance-loading on the reference resonator; therefore, having higher loaded quality factor, thus better resonance sensing resolution.
Appendix A

This appendix includes the M-file code to calculate phase noise from the time-domain oscillator signals derived from the Simulink behavioral simulations in Chapter 5.

%% Code to calculate PSD from the oscillator time-domain

%% derived from Simulink and saved as parameter 'osc_timeData'.

Fs =1/(osc_timeData.time(2)-osc_timeData.time(1));

x=(osc_timeData.signals.values);

%%The following two lines are supposed to make the signals smoother.

% % x1=upsample(x,4);
% % x2=downsample(x1,4);
t = 0:1/Fs:length(x)/Fs;

nfft = 2^nextpow2(length(x));

%%% Pxx = abs(fft(x,nfft)).^2/length(x)/Fs;

Pxx = abs(fft(x,32*nfft)).^2/length(x)/Fs;  %%%%NOTE: For minimum choose nfft instead of 32*nfft.

% Create a single-sided spectrum

figure(1)

Hpsd_ssb = dspdata.psd(Pxx(1:length(Pxx)/2),'Fs',Fs);

plot(Hpsd_ssb);

% Create a double-sided spectrum

Hpsd_dsb = dspdata.psd(Pxx,'Fs',Fs,'SpectrumType','twosided');

% plot(Hpsd_dsb)

%%%%%% Phase Noise Measurement %%%% %%%%%

freq1=Hpsd_ssb.Frequencies;

deltaFreq1=freq1(2)-freq1(1);

maxssboffset=0.3e6;

Noffsetssb=ceil(maxssboffset/deltaFreq1);

[M,index]=max(Hpsd_ssb.Data);
f_pn=[freq1(index)-Noffsetssb*deltaFreq1:deltaFreq1:freq1(index)+Noffsetssb*deltaFreq1];

f_pn0=freq1(index)-Noffsetssb*deltaFreq1;

f_pn=f_pn';

PSD_pn=Hpsd_ssb.Data(index-Noffsetssb:index+Noffsetssb)/Hpsd_ssb.Data(index)/2;  
L=(1/2)*S_phi
figure(2)

lenf=length(f_pn); lenpn=length(PSD_pn);

semilogx(f_pn(-2+ceil(lenf/2):end)-f_pn(-2+ceil(lenf/2)),10*log10(PSD_pn(-
2+ceil(lenf/2):end)),'r');

xlabel('Offset Frequencies (Log)');

ylabel('Phase Noise (dBc/Hz)');
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