IDENTIFICATION AND CANCELLATION OF HARMONIC DISTURBANCES IN RADIO TELESCOPES

by

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Identification and Cancellation of

Harmonic Disturbances in Radio Telescopes

Abstract

by

TIMOTHY J. FRANKE

A new class of algorithms for the identification and cancellation of harmonic disturbances on rotary dynamic systems is proposed and demonstrated with applications on the Green Bank Telescope (GBT). The approach is a model-based iterative algorithm that exploits the structure of the problem to significantly reduce the number of tests needed to perform identification. During each such test, the system is in steady-state periodic operation. The crucial trick involves constructing a correspondence between the coefficients of disturbance terms and their time-periodic harmonics. This transformation enables the modeling and calibration of the system in a compact, harmonic representation.

This approach has numerous advantages. The harmonic model fully captures the behavior of arbitrarily complex linear systems. It is not a feedback approach and therefore will not destabilize existing controllers. The algorithm displays rapid convergence and requires a minimal number of tests to construct a model. Finally, a previously identified model may be reused to quickly update a calibration. All of these properties make it ideal for the calibration of feedforward compensators on a wide range of systems.

The nature of cogging on the GBT motors is rigorously studied. Various system identification tests are performed to characterize the behavior of the cogging with
Abstract

respect to operating conditions. A single motor calibration routine is developed and deployed on telescope hardware. The performance of the GBT with individually calibrated motors is tested. Additionally, the algorithm is extended to handle multiple interacting motors. A solution method is presented that yields reliable, physically reasonable solutions to the multiple motor problem. The calibration method is updated to compensate for the GBT encoder measurement error. The behavior of interpolation error was studied on two different encoders. Global variation of encoder calibrations is studied over the range of the telescope’s elevation axis. Finally, generalization of the algorithm to non-constant rate, but still periodic trajectories, is explored. These tests probe the limits of the algorithm’s underlying ideas.
Chapter 1

Introduction

The National Radio Astronomy Observatory (NRAO) in Green Bank, West Virginia operates several radio telescopes, including one of the world’s largest and most precise—the Green Bank Telescope (GBT). One of the primary responsibilities of their technical staff is to extend the scientific capabilities of the observatory. To this end, they are perpetually analyzing their telescopes and upgrading their systems to further improve performance.

During their investigation into the GBT’s tracking performance the dominant component of the tracking error was discovered to be a periodic disturbance whose frequency varied with telescope tracking speed [14]. It is clear that the origin of these tracking errors is in the servo system. Subsequent analysis of the problem revealed that the motors exhibited noticeable motor cogging. This cogging disturbance manifests itself at low speeds, such as when the telescope is observing, and was believed to be the cause of the observed errors [21]. Subsequent analysis revealed that systematic measurement error in the encoders is also a major source of the tracking error. These errors are output measurement errors and are non-physical, but the position loop introduces pointing error in an attempt to cancel the falsely measured position deviations.
This dissertation describes work that took place over a three year collaboration centering on improving the tracking capabilities of the GBT. A new method of model-based identification for harmonic disturbances was developed and applied to the problems of motor cogging cancellation and encoder measurement linearization. The particular design was chosen since it could operate within the unique requirements of the GBT as well as provide opportunities for generalization.

1.1 Motor Cogging

The first recognized cause of periodic tracking error on the GBT was motor cogging. Cogging is a periodic torque disturbance on an electric motor due to non-uniform magnetic attraction between rotor and stator [5]. DC motors contain rotors that consist of regions of magnetically susceptible iron laminations and less magnetic armature windings. Additionally, eccentricity in the rotor mounting results in similar
magnetic variations. Due to these sources of heterogeneity, there are angles that are magnetically more favorable for the motor to align. The net effect of this is modeled as an angle-dependent torque on the rotor. This torque is zero mean and consists of multiple equilibrium points around the rotor.

During operation, the cogging torque is overpowered by the motor’s driving electromechanical torque. However, a periodic ripple still remains and manifests itself as vibration on the rotor. At high speeds, this disturbance is attenuated by the inertia of the rotor. However, at low speeds this vibration can be problematic. It can be especially evident when the motor is attached to a structure with resonance modes in the range of the low speed motor rotational frequency. On the GBT, precision is most needed during low speed astronomical tracking. Elimination of the induced vibrations, and the tracking inaccuracies caused by them, is one promising way to reduce tracking error and thereby increase the GBT’s maximum astronomical observation frequency.
1.2 Encoder Interpolation

When the telescope is given a tracking command for a fixed rate target, a periodic position error is observed. Originally, this was believed to be either wind induced structural vibration or motor cogging. However, this error’s frequency changes proportionally with the rate of the telescope. This fact rules out the vibrational theory because structural modes appear at fixed frequencies, regardless of tracking speed. Cogging seemed to be the likely cause until it was observed that this effect only appeared when the position loop controller was active. When the telescope was controlled by the rate loop only, the effect disappeared entirely; the effects of cogging should have been more pronounced without the additional control loop to attenuate the disturbance.

The error is caused by nonlinearity in the encoder measurement of the telescope’s position. The shortcoming is due to imperfect interpolation in the encoder. The effects of this and precise details of how to measure it are discussed in Section 3.1. However, first an introduction of the mechanism that underlies this problem.

The orientation of each axis of the GBT is measured by an absolute position encoder. The operation of this type of sensor is at the heart of the observed effect
and provides a hint of how to mitigate it. An absolute position encoder has a series of encoded tracks, one per bit, that uniquely code for the current rotational angle. Though these position encoders measure an absolute position, not all bits can be directly inscribed onto the disk. Instead, only the most significant bits are marked. Additional bits are obtained by interpolation. Various methods for generating the analog interpolation signals exist. In a common setup, two tracks, offset by 90 degrees, of fine gradation are read as two analog intensities. These two tracks yield sinusoids as the encoder is turned. By approximating the inverse tangent of these two offset sinusoids, a much finer angle can be measured by disk markings alone. If these were read in a binary sense, then one period of these two would yield four angles. With interpolation, the resolution is limited by the resolution of the analog measurements and the quantization of the digital system.

The issue is that, in order to achieve additional precision, additional position information is extracted by analyzing the finest gradations as an analog signal and this is not a perfect process. By measuring the intensity of the signal, a fractional position can be extracted. This permits the encoder to measure position beyond what would be possible due to physical limitations in the gradation size.

Interpolation involves several non-ideal processes, which all contribute to the overall error. The analog to digital converter must be calibrated to generate a true value. The sensors themselves are not entirely linear. The optical measurement may be non-uniform in intensity to begin with. So while a good job at mapping backwards to obtain the true underlying angle is possible, it will invariably involve a certain degree of error. And since all of the sources of this error are systematic, the error will be systematically repeated over the interpolation interval. The repeatability is what will permit this error to be identified and counteracted.
1.3 The Green Bank Telescope

The GBT is the world’s largest fully steerable, single dish radio telescope. The primary dish is a 100 meter by 110 meter off-axis paraboloid. This asymmetric design permits the feed arm to be located out of the path of the observation beam. The unblocked aperture design eliminates a major source of signal interference due to scattering, providing an extremely sensitive device with astounding dynamic range. The GBT is unique among large radio telescopes in this regard.

The GBT has a range of receivers, providing nearly continuous coverage from 100 MHz to 90 GHz. These receivers are primarily located on a revolving turret on the feed arm, where the desired receiver can be rotated into the focus position at any time. Recently, NRAO has been developing receiver arrays to operate at the higher end of the frequency range.

The telescope is one of several operated by NRAO at Green Bank. NRAO is a division of the National Science Foundation and is charged with maintaining, operating, and expanding the United State government’s fleet of radio telescopes. The observatory is situated deep in the Appalachian mountains near the town of Green Bank, West Virginia. It’s remote location and natural shelter provide an environment naturally free of radio interference for their many telescopes. The organization is headquartered in Charlottesville, Virginia and also operates the Very Large Array in New Mexico and is a member of the Atacama Large Millimeter Array in Chile.

1.3.1 Telescope Control System

The GBT is an azimuth-elevation mounted telescope. The control system consists of a position loop and a rate loop for each axis. The position loop is responsible for regulating the rate setpoints to ensure that the telescope follows a given trajectory. The rate loop acts upon that reference and drives all motors to track that speed.
Additionally, the rate loop measures differences in motor speed and trims each motor’s commanded current to maintain uniform rotation across all motors on the axis.

The GBT control system was designed as a mostly analog system during the 1990’s. Since commissioning, subsystems have been upgraded in a piecemeal fashion so that few of the original components remain fully intact. During a recent system upgrade, each of the twenty-four drives was outfitted with a Motor Controller Interface (MCI). Each MCI is an embedded computer that is responsible for interfacing each motor’s encoder with the control system. Additionally, each MCI has the ability to monitor and alter the commanded armature current for its motor. The MCIs are the ideal place to add compensation for motor cogging.

In addition to the motors, the same type of networked embedded computer interface was provided for each of the two axis encoders. These Position Encoder Interfaces (PEI) have been modified to be able to add an angle dependent correction to the encoder reading. These PEIs will be used to correct for the encoder interpolation errors.

Ultimately, the MCIs and PEIs were meant to be networked together using a special real-time network protocol and hardware. A central computer on the telescope would subsume all control responsibilities for the entire telescope. This project has not yet been completed and that network was inoperable at the time this work was performed. Instead, these devices have been interfaced to the telescope’s dedicated ethernet system during the experiments described here. All communication with these
The azimuth axis is a wheel on track system controlled by sixteen 30 Hp motors while the elevation axis has a fully geared drive train and is powered by eight 40 Hp motors. All motors are field-wound DC motors and have their armature currents controlled by individual drives.

The azimuth axis presents two problems, due to its wheel on track configuration, that makes it non-ideal for testing. The wheels slip slightly and sporadically when they are driving the telescope. This can be measured and accounted for, but it is another layer of complexity. The other problem occurs where the track segments are spliced together. When a wheel crosses a track splice, the resistance is non-uniform and the controller ramps up to compensate for it. As the methods of this work focus on systems in periodic operation, this poses a major problem. The solution would be to map out where these occur and to only collect data when the system is away from these points. This is doable, but again introduces unnecessary complications. For these reasons, all experimental work focuses on the elevation axis.

On the elevation axis, each motor is connected with another motor, through gearboxes, and together they turn a single pinion gear. These pinion gears, in turn, drive a bull gear which is attached to the elevation structure. There are four pairs of these motors on the axis. The equivalent gear ratio from the motors to the elevation structure is 31640 to one. All motors are preloaded with a baseline torque to take up backlash in the gearboxes as well as between the pinion gears and bull gear. A schematic of the arrangement of these motors, along with their preloading biases, appears in Figure 1.5.
Figure 1.5: Elevation axis drive train. The eight 40 Hp motors are shown in their configuration on the telescope. Each is paired with another, through gearboxes. They share pinion gears that turn the bull gear (dashed). Preloading torque is indicated for each motor.

1.4 Prior Work with Motor Cogging

There are two strategies for dealing with motor cogging: either cancel it with input feedforward compensation or design a motor not to have it in the first place. Designing a motor to minimize cogging typically involves in-depth use of finite element analysis of Maxwell’s equations, material properties, and geometric representation of the motor under design. All of these facets are coupled together into an optimization problem. The field of literature on motor design methods to reduce or eliminate cogging through electro-mechanical design is vast and will not be surveyed here.

The class of approaches that is relevant to this dissertation is the use of feedforward cancellation of cogging. The idea is to superimpose an equal-but-opposite electrical torque to the cogging torque onto the desired input. The net effect of this is to remove cogging, but otherwise leaving the motor’s behavior unaffected.

Fundamentally, the feedforward torque needs to be computed based upon the current angle of the motor. To make this process tractable, this signal is most often parameterized by a truncated Fourier series. Each harmonic has two coefficients—a Cosine term and a Sine term, or equivalently a magnitude and phase. The task of the identification process is to identify values for these coefficients to minimize
the remaining harmonic ripple torque on the rotor. Just how these coefficients are
determined is what differentiates the approaches.

Before continuing, it should be noted that other representations are possible, but
not particularly common. Interpolation among a discretization of angles can be used,
with an anti-cogging estimate fixed to each point. In the case of linear motors some
form of spline representation, whether by itself or hybridized with trigonometric func-
tions, is often used since the edge behavior in these motors varies significantly from
the center. Unless explicitly noted, the anti-cogging signal parameterization is as a
Fourier series.

1.4.1 Static Parameterization

To cancel cogging vibrations, an inverse cogging signal must be parameterized and
injected into the telescope motors, driving the net torque to zero. The purpose of the
identification method is to deduce the harmonic coefficients of the truncated Fourier
series which define that signal. The most straightforward way to parameterize the
cogging torque is to directly measure it. If the motor under test can be instrumented
with a torque transducer and attached to a dynamometer, a profile of the cogging
torque can be attained. From this point, it is straightforward to extract the parameter-
erization of a best fit anti-cogging solution.

The need to isolate the motor, in addition to the need for special test equipment,
has limited interest in the above approach. What is more desirable is a method that
requires no additional instrumentation to what is already on a servo system.

The identification methodology developed in this dissertation is iterative—the
parameterization only changes at discrete moments and the time interval between
these moments is sufficiently longer than the transient response of the system. Since
the compensation is quasi-static, it avoids many of the stability issues which plague
continuously updating adaptive controllers.
1.4.2 Continuously Updating Parameterization

At the other end of the spectrum are those methods which continuously vary the feedforward parameters. With an appropriately chosen update rule, the controller will modify the anti-cogging parameters until they converge on the correct solution.

Most commonly, engineers have relied upon the construction of an adaptive cogging compensators to parameterize the signal that cancels these harmonics. The classical approach to motor cogging cancellation with adaptive control follows from a Lyapunov based approach [3, 4, 17, 13, 16]. These yield accurate results but the adaptive methods that are employed do not account for the possibility of structural resonances. This can lead to instability if such a method is attempted to be used on a system with significant structural dynamics. Additionally, these methods place explicit assumptions about the feedback controller to be used.

Recursive Least Squares (RLS) based approach has been employed by Zhao and Tan [22]. In this approach, a least squares fit is maintained to produce the optimal model that explains the observations of the system. While it is straight-forward to account for the controller command in the model, no provisions are made for the effects of structural dynamics. Further, implementing the RLS algorithm for a large number of harmonics can be computationally expensive, growing quadratically in cost. In our limited systems, this proved to be challenging to do in real-time.

A final approach is based on learning from the control signal [9, 12, 15]. These learning controllers build up a feedforward signal by averaging past feedback signals that were applied at the same rotor angle. Since these methods use feedback from an existing controller, they can be used as drop-in solutions. However, the dynamics of the system still play a role in convergence of the solution and the resulting stability of the system.
Beyond Perfect Cancellation

While adaptive algorithms strive to exactly cancel the modeled harmonics, Bodson et al. [2] explore a way in which continuous adaptation can outperform an optimal, but static, solution. The primary idea is that with careful adaptation gain calibration, the adaptive solution can attenuate harmonics that are not directly parameterized. In such a controller, the parameters do not converge to fixed values, but reach a limit cycle. These dynamically changing parameter values, when used for feedforward cogging cancellation, result in cancellation of their specified harmonics and attenuation of those not directly modeled, but present.

1.4.3 Multiple Motors

One of the primary feats of this dissertation is to develop an identification method that can identify and cancel cogging of a multiple motor system. In particular, the elevation axis has eight motors that are connected through a large structure as well as through a feedback controller. Despite the abundance of prior work with cogging cancellation on a single motor, there is an absence of prior attempts at addressing multiple, interacting motors. Perhaps this is due to the inherent difficulty of extending adaptive control methods to this problem. The approach taken in this dissertation is a departure from the adaptive control methods typically employed in cogging cancellation. However, this departure is justified by the fact that the iterative method can address the multiple motor problem.

1.5 Prior Work in Encoder Interpolation

Much of the literature on encoder interpolation is focused on efficiently computing finer angles [6]. In general, an encoder must be able to make these computations very rapidly and accuracy is traded off for simplicity of implementation. This work
is interested with going beyond the accuracy of the traditional methods.

The work of Mayer [10] attempts to negotiate this tradeoff. They propose efficient interpolation hardware that permits relative scaling and offsets between the two quadrature signals. The characterization of the error is relatively simple. Others have explored more complex models of error.

In Tan and Tang [18], a neural network is used to compute the interpolated angle from the analog waveforms. They consider a set of radial gaussian basis functions to account for non-sinusoidal analog measurements. By using the nonlinear fitting ability of a neural network, they construct a correction stage that maps from the non-ideal analog measurements, to clean encoder counts.

While not concerned with improving accuracy (correctness), another class of work focuses on improving encoder precision (number of digits) by accounting for the servo system’s dynamic behavior. Merry et al. [11] fit polynomials to the digital measurements to extract finer angle measurements. This is making the assumption that the encoder moves with little acceleration—hence can be well modeled by a low order polynomial. Kovudhikulrunsri and Koseki [8] use a state observer to similar effect. These dynamic methods yield measurements that are higher precision than the nominal encoder, though not necessarily more accurate.

The majority of the literature focuses on compensating for non-ideal encoder measurements by dealing directly with the analog signals. This is because the majority of these works are meant to be applied internally within the encoders. The work in the dissertation focuses on correcting the measurement after it has been reported by the encoder. This has been forced by necessity.
Chapter 2

Single Motor Calibration

The proposed identification method proceeds by analyzing a motor’s steady-state behavior while subjected to different perturbations of anti-cogging parameters. Analysis of the model exposes the input-output relationship between the injected signal parameters and the resulting rotor ripple harmonics. Identification consists of two stages: model identification and solution iteration. To build the harmonic model, a range of test input harmonic perturbations are injected to the system and their effects on the output harmonics are recorded. A linear input-output model, consisting of a matrix and offset, is fit to the aggregated data using a least squares method. From this model, an initial solution is computed. To iterate the solution, the current solution is applied and harmonic residuals are measured. The model matrix is used to compute an update from this residual. Each time, the input harmonic coefficient update is computed to reduce the measured residual ripple. Convergence is rapid and only requires running a single hardware experiment per iteration. Importantly, by re-using the model, a previously identified solution may be cheaply recalibrated with a single step.

In addition to the speed of convergence of this method, there are other reasons that make it ideal for mitigating motor cogging on telescopes. Since parameters
are updated discretely, the method avoids closed loop stability issues inherent in traditional, continuously updating adaptive controllers. This enables the algorithm to be integrated without modification of a system’s existing control loops. Further, since the approach builds an *ad hoc* harmonic model, unmodeled structural dynamics and resonances are not a problem and there is no need to carry out system identification as a separate step.

Portions of the work described in this chapter were originally published in [7].

### 2.1 Single Motor Modeling

#### 2.1.1 Dynamics of a Motor with Cogging

The telescope’s servo system consists of field wound DC motors. During operation, the field windings are held at a constant current and torque modulation is achieved by varying the armature currents. Those currents are controlled by drives whose electrical dynamics are substantially faster than the mechanical dynamics, and can be disregarded.

For a given motor with rotor angle $\theta$, the dynamics are

$$J\ddot{\theta} = ku(t) + \tau(t) + \varphi(\theta),$$

(2.1)

where $J$ is the rotor inertia, $k$ is the motor torque coefficient, $u$ is the armature current, $\varphi$ is the cogging torque, and $\tau$ accounts for the load dynamics and any friction torques.

Motor cogging is an angle dependent torque that acts on the rotor. Since it is a function of rotor angle, it is inherently periodic on the interval between $0$ and $2\pi$.
2.1. MODELING

radians and can be represented as the truncated Fourier series

$$\varphi(\theta) = \sum_{n \in H} (\varphi_{c,n} \cos(n\theta) + \varphi_{s,n} \sin(n\theta)),$$

where $H$ is a finite set of positive integers, denoting the harmonics of interest. Each harmonic has two harmonic coefficients, $\varphi_{c,n}$ and $\varphi_{s,n}$ which scale the cosine and sine terms. The initial task is to identify an appropriate set, $H$, that encompasses the most active cogging harmonics.

A motor does not operate independently, but is part of the telescope servo system. During operation, the motor is supplied with a current set-point, $u$, from an external feedback control system. Additionally, a reaction torque, $\tau$, is caused by the dynamics of the motor’s load. The modeling of these external signals will be deferred for the time being, but at this point note that they are governed by linear dynamical systems of the rotor angle.

2.1.2 Operation of the Motor System

The identification of the anti-cogging model and solution will be accomplished by systematically probing the system with different input coefficient perturbations. The test setup is rather straightforward to implement in practice. A linear feedback controller will be used to regulate the rotor to operate at a nearly fixed speed. Additionally, that controller will inject the anti-cogging test signal into its output, which is sent to the motor’s drive. The motor transients will be permitted to settle and, due to the cogging dynamics, will reach periodic operation. During this periodic operation state, the speed will not be entirely constant due to the non-zero cogging torque. However, these fluctuations will repeat every time the rotor turns through a complete rotation. These residual fluctuations will be measured and used to characterize rotor ripple.

This experimental protocol was selected as it is both simple to analyze and simple
to implement. No requirements are imposed on the form of the feedback controller other than it be linear and able to asymptotically track the speed reference. Further, the actual feedback gains of the controller do not need to be known. This makes it possible to use whatever pre-existing feedback controller is on the servo system without modification or identification. While the injected anti-cogging hardware would not previously exist on the servo-system, it would be necessary to implement for the final system, anyway.

Let the external feedback armature current command be denoted as \( v \). The anti-cogging signal is added to that, yielding the applied command, \( u \). The resulting command is fed into the motor drive is

\[
    u = v + \sum_{n \in H} (p_{c,n} \cos (n\theta) + p_{s,n} \sin (n\theta)),
\]

(2.3)

where the coefficients \( p_{*,n} \) are the estimated values of the anti-cogging harmonic coefficients. When \( u \) is substituted into the rotor dynamics, the resulting rotor dynamics become

\[
    J\ddot{\theta} = ku(t) + \tau(t) + \sum_{n \in H} ((\varphi_{c,n} + kp_{c,n}) \cos (n\theta) + (\varphi_{s,n} + kp_{s,n}) \sin (n\theta)).
\]

(2.4)

where the \( \varphi_{*,n} + kp_{*,n} \) terms denote the residual harmonic components of the cogging torque after the cancellation signal is applied. It’s important to note that injecting an anti-cogging signal does not change the structure of the dynamics, only the effective cogging torque.

The periodicity of the resulting motor operation will prove to be the key to identification. It will be required that the motor speed must be non-zero at all times. The reason for this restriction is two-fold. The first reason is a practical concern; if the rotor were to stop at any time, static friction on the rotor would enter the dynamics and invalidate the modeling. Secondly, a one-to-one relationship between
angle and time will be sought. A change in the sign of velocity would result in certain angles being visited multiple times within a single cycle. Such behavior complicates analysis without introducing any benefit. Relaxation of this constraint is explored in Chapter 5.

Along a trajectory that satisfies the above criterion, there exists a one-to-one mapping between time and angle. For a given trajectory denote the fundamental rotational frequency with $\omega_{\text{ref}}$ and the period time of the motor’s operation as $T_{\text{ref}}$. The mapping is only one-to-one for $t \in [0, T_{\text{ref}})$ and $\theta \in [0, 2\pi)$. On this domain, angle can be expressed uniquely as a function of time, $\theta(t)$, as well as time can be expressed as a function of angle, $t(\theta)$. Again, these functions are only valid for a given trajectory of the motor system. If one were to change the input harmonic coefficients or the speed setpoint, the motor would converge to a different periodic trajectory with its own version of these functions. The importance of this time-angle relation is that it permits quantities that are easily modeled as time periodic to be modeled as such while treating those that are angle based in their own way. Whenever transformation is needed, the unification between time periodic and angle periodic quantities makes this possible. This enables analysis by permitting both sets of tools to be used—freely choosing the most appropriate representation for the task at hand.

### 2.2 Single Motor Calibration

The anti-cogging harmonic solution will be identified by experimentally varying the input harmonics and measuring the residual rotor ripple. In order to accomplish this identification, a model of the motor response to such input is needed. Such a model describes how the parameters of the anti-cogging signal impact the rotor ripple harmonics of the system operating at nearly constant speed. The motor model is nonlinear and does not permit simple analysis. By virtue of the carefully selected
experimental conditions, the time-angle correspondence enables the input-output behavior to be modeled.

### 2.2.1 Analyzing Experiments

Due to the above described one-to-one relationship between angle and time over a single rotation, all periodic quantities can be parameterized equivalently as functions of time or of angle. To take advantage of this in the system dynamics, we will denote the Fourier series coefficients of some arbitrary angle-dependent periodic signal, \( z \), whose expansion is

\[
z(\theta) = \sum_{n \in H} \left( z_{c,n}^\theta \cos(n\theta) + z_{s,n}^\theta \sin(n\theta) \right)
\]

(2.5)

as

\[
z^\theta = \begin{bmatrix} z_{c,1}^\theta & z_{s,1}^\theta & z_{c,2}^\theta & z_{s,2}^\theta & \cdots \end{bmatrix}^T.
\]

(2.6)

The superscripted \( \theta \) denotes that these coefficients are computed with respect to an angle-dependent expansion. Similarly, the coefficients for the same signal, expressed as a time-dependent series, are denoted by \( z^t \). The superscripted \( \theta \) or \( t \) may be omitted when the domain of the expansion is clear from the context.

A small signal harmonic model of the dynamics can be constructed using this new harmonic notation. For a given trajectory, consider the quantity \( \tilde{\theta}(t) = \theta(t) - \omega_{\text{ref}}t \) and by choose \( \theta(0) = 0 \). The quantity \( \tilde{\theta} \) can be interpreted as the deviation of the rotor angle from its ideal value, \( \omega_{\text{ref}}t \), at a given time. Likewise define \( \tilde{u} \) and \( \tilde{\tau} \) to denote the external command signal and torque with their DC components subtracted out. Then rewriting (2.2) using the ripple harmonic coefficients as

\[
JS^2 \tilde{\theta}^t = kv^t + \tau^t + \varphi^t,
\]

(2.7)
where $S$ is a matrix consisting of $2 \times 2$ blocks, $S_n$, on the diagonal. The block that corresponds to the harmonic $n$ is

$$
S_n = \begin{bmatrix}
0 & n\omega_{\text{ref}} \\
-n\omega_{\text{ref}} & 0
\end{bmatrix}.
$$

(2.8)

The matrix $S$ should be easily recognizable as a linear system that takes the derivative of a signal.

Since both the structural reaction torque and the external feedback command are governed by linear dynamical systems, their time harmonic behavior is governed by their transfer functions and obey the relationships

$$
\tilde{v}^t = G\tilde{\theta}^t \quad \text{and} \quad \tilde{\tau}^t = P\tilde{\theta}^t,
$$

(2.9)

where the block diagonal matrices $G$ and $P$ implement the transfer functions $G(s)$ and $P(s)$ with respect to the coefficients, for a fixed fundamental frequency. Specifically, for the $n^{th}$ harmonic, the $2 \times 2$ block for those coefficients is

$$
G_n = \begin{bmatrix}
g_1 & -g_2 \\
g_2 & g_1
\end{bmatrix}
$$

(2.10)

where

$$
|G(jn\omega)| = \sqrt{g_1^2 + g_2^2} \quad \text{and} \quad \angle G(jn\omega) = \arctan \left( \frac{g_2}{g_1} \right).
$$

(2.11)

Substituting the relationships from (2.9) into (2.7) and solving yields

$$
\tilde{\theta}^t = (JS^2 - kG - P)^{-1} [\varphi^t + kp^t].
$$

(2.12)

Note that $(JS^2 - kG - P)^{-1}$ is a linear combination of linear dynamics. For a fixed rotation base rotation speed, this is a matrix of $2 \times 2$ blocks on the diagonal. Since
all experiments will be performed with the same speed setpoint, that matrix will be constant across all tests. Thus, the input-output relationship between the anti-cogging coefficients and the ripple harmonics is given by a fixed linear equation.

2.2.2 Simplifying Approximations

Equation (2.12) expresses the input-output relationship time-domain harmonics. The anti-cogging signal is most readily specified with angle-domain harmonics which need to be converted. In general, the transformation between domains involves inverting the time-angle correspondence. However, in certain operating regimes, the difference between angle and time domain signals can be minimized. When the ripple magnitude is small compared to the base speed, then $\theta(t) \approx \omega_{\text{ref}} t$, and the coefficients coincide. In Figure 2.1 both time and angle representations of a fictitious motor’s rotor speed ripple are shown at various speeds. The angle-dependent decompositions nearly match with the time dependent ones when the rotor speed ripple is small when compared to the baseline rotor speed. Chapter 5 explores how to proceed when this approximation cannot be made.

Under this operating regime, the input-output mapping can be approximated as

$$\tilde{\theta} = (JS^2 - kG - P)^{-1} [\varphi^\theta + kp^\theta].$$

(2.13)

In this equation, the relationship between input and output harmonics is linear and the harmonics are completely decoupled. Due to the approximation of a trivial time-angle correspondence, linearity of the input-output relationship is not exact, but holds well in practice. Therefore, an initial estimate of the linear relationship retains fidelity as the base-point of the linearization changes. Aggregating the terms in (2.13), and
2.2. CALIBRATION

CHAPTER 2. SINGLE MOTOR

Figure 2.1: Simulation of a rotor at various speeds, subjected to a non-zero second cogging harmonic. The rotor speed is periodic in both angle and time, but the Fourier decomposition of those two representations differ in general. This is most clearly apparent at low rotor speeds. When the rotor speed is large compared to the magnitude of the ripple, the representations begin to coincide and their analysis can be simplified. However, at these higher speeds the magnitude of the ripple is reduced and thus harder to measure. Selection of proper identification speed involves negotiating this trade off.
using the time-angle approximation, the input-output model is

\[ \tilde{\theta}^t = Mp^\theta + b. \] (2.14)

This form permits the identification of \( M \) to be performed at the beginning and to reuse that map in each subsequent iteration.

The independence of harmonics permits a significant reduction in the cost of model identification, by allowing all harmonics to be simultaneously perturbed. Complexity of implementation. As they are independent, the perturbation of a given input harmonic only affects its corresponding output harmonic. This means that the number of hardware experiments that need to be run in order to tune the system is independent of the number of harmonics of interest. All of these facts together permit identification to be performed far more quickly than would be possible by using a naïve minimization approach.

2.3 Identification Algorithm

Having analyzed the dynamics of a motor with cogging, the identification algorithm can now be formulated. The purpose of this algorithm is to build an input-output harmonic model of the motor and to iteratively use that to cancel any residual motor ripple. As the harmonics are independent, each one will be modeled individually. In order to discuss sequential tests, an extension of notation is needed. For a given harmonic, denote the input coefficients to the \( k^{th} \) experiment as \( p_c[k] \) and \( p_s[k] \), or collectively as a \( 2 \times 1 \) vector, \( p[k] \). Likewise, the output ripple coefficients for the harmonic of interest are denoted by \( y_c[k] \) and \( y_s[k] \). The input-output model for the
harmonic of interest is

\[
\begin{bmatrix}
y_c \\
y_s \\
\end{bmatrix}
= \begin{bmatrix}
m_c & -m_s \\
m_s & m_c \\
\end{bmatrix}
\begin{bmatrix}
p_c \\
p_s \\
\end{bmatrix}
+ \begin{bmatrix}
b_c \\
bs \\
\end{bmatrix},
\] (2.15)

where the matrix elements \(m\) and the vector \(b\) are the parameters of the affine harmonic model. Matrices in the particular form of \(M\) correspond to a magnitude scaling and a phase shift and therefore \(M\) can represent the transfer function dynamics expressed in (2.14).

### 2.3.1 Building the Harmonic Model

The model is parameterized by collecting data and then extracting the model parameters with a least squares fit. Data is collected from a number of perturbed experiments. Denote the total number of experiments as \(N_{\text{pert}}\). To achieve this, the model equation needs to be recast into a form with the parameters external to the data.

\[
\begin{bmatrix}
y_c[1] \\
y_s[1] \\
y_c[2] \\
y_s[2] \\
\vdots \\
\end{bmatrix}
= \begin{bmatrix}
p_c[1] & -p_s[1] & 1 & 0 \\
p_s[1] & p_c[1] & 0 & 1 \\
\vdots & \vdots & \vdots & \vdots \\
\end{bmatrix}
\begin{bmatrix}
m_c \\
m_s \\
b_c \\
bs \\
\end{bmatrix}
\] (2.16)

The two rows in the harmonic vector and the regression matrix are repeated, for each experiment, but with the appropriate values of \(y[k]\) and \(x[k]\) substituted. This forms an overconstrained least squares problem. Solving this problem yields the four harmonic model parameters. Since the harmonics are treated independently, this fitting procedure will need to be repeated for each harmonic. Note however that each experiment has all harmonics perturbed, so the total number of tests is unaffected by
the number of harmonics.

### 2.3.2 Iterating the Solution

From the model, the initial solution is computed for each harmonic as

\[ p[N_{\text{pert}} + 1] = M^{-1}b. \]  \hfill (2.17)

As these values were obtained from fitting the model to experimental data, they are often quite effective at canceling the cogging. However, since the linearity of the model is an approximation, this initial solution is not perfect.

The solution can be iterated to improve cancellation. To start this process, a rotor experiment is performed with the current anti-cogging solution. Having measured the residual rotor ripple coefficients, \( y[k] \), the solution is updated as

\[ p[k + 1] = p[k] - M^{-1}y[k] \quad \text{for} \quad k \geq N_{\text{pert}}. \]  \hfill (2.18)

Repeated application of this update will drive the residual acceleration to zero. To save experimental time, the matrix \( M \) is not updated at every iteration. The initial \( M \) matrix is often close enough to permit quick convergence despite uncertainty.

### 2.4 Calibration on the Testbed

The algorithm was tested on a motor testbed in the laboratory of NRAO Green Bank. The testbed consists of an elevation axis motor and an azimuth axis motor directly coupled together. The stages of an identification experiment, carried out on the testbed, appear in Figure 2.2. The anti-cogging coefficients resulting from testbed identification appear in Table 2.2.
2.4. CALIBRATION ON THE TESTBED  

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{40}$</td>
<td>82.9 rad sec$^{-2}$ V$^{-1}$</td>
</tr>
<tr>
<td>$k_{30}$</td>
<td>34.0 rad sec$^{-2}$ V$^{-1}$</td>
</tr>
<tr>
<td>$b$</td>
<td>0.466 sec$^{-1}$</td>
</tr>
<tr>
<td>$\mu_k$</td>
<td>11.8 rad sec$^{-2}$</td>
</tr>
</tbody>
</table>

Table 2.1: Identified parameters for the testbed dynamics given in (2.19).

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>1$^\text{st}$</th>
<th>2$^\text{nd}$</th>
<th>3$^\text{rd}$</th>
<th>4$^\text{th}$</th>
<th>6$^\text{th}$</th>
<th>12$^\text{th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cosine</td>
<td>-0.00671</td>
<td>0.03409</td>
<td>0.00508</td>
<td>-0.01112</td>
<td>0.00294</td>
<td>-0.00772</td>
</tr>
<tr>
<td>Sine</td>
<td>-0.00454</td>
<td>0.01645</td>
<td>-0.00263</td>
<td>-0.00367</td>
<td>0.00190</td>
<td>0.00228</td>
</tr>
</tbody>
</table>

Table 2.2: Identified testbed anti-cogging signal coefficients. The cogging coefficients can be computed from this set by negating them and scaling by the torque gain, $k_{40}$. These coefficients are valid in the positive direction of rotation.

System identification was carried out on the motor testbed. The model

$$\ddot{\theta} = k_{40}u_1 + k_{30}u_2 - b\omega - \mu_k \text{signum}(\omega) + \varphi(\theta), \quad (2.19)$$

where $k_{40}$ and $k_{30}$ are the 40 Hp and 30 Hp motor gains, $b$ is the damping coefficient, and $\mu_k$ is the coefficient of kinetic friction. Cogging terms are lumped into $\varphi$. Values for these coefficients appear in Table 2.1.

Once anti-cogging solutions have been identified and applied, it was informative to test the operation domain in which these solutions held. With a working anti-cogging solution loaded into the controller, the speed reference was changed. No variation of the magnitude of the speed affected the goodness of fit—the speed that the identification is performed at does not affect the solution. Having two motors on the testbed, the second motor was used to load the driving motor. Changes in the levels of loading did not alter the efficacy of the anti-cogging solution, either. What did prove to be critical is the direction of operation. Changing direction changes the cogging coefficients. A solution obtained in one direction will not work when
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the motor changes direction. To handle this, the identification is run twice—in the forward and reverse directions—and two sets of cogging coefficients are identified. On the MCI, two coefficient tables are stored and the appropriate one is used to compute the anti-cogging signal, based upon direction of rotation. Details of the MCI anti-cogging compensation appear in Appendix Section A.1.

To test long term solution stability, a series of identifications were performed on the testbed over the course of a week. The coefficients from the identification are plotted in Figure 2.3. The coefficients form clear clusters. Further, the drift from identification to identification occurs in random directions—there is no evident, systematic drift. All of this indicates that the anti-cogging solution is either fixed, or varies on a time scale much slower than a week. Either way, occasional calibration for the motors will be a sufficient solution for the needs of the GBT.

2.5 Single Motor Calibration on the Telescope

Identification experiments were performed on each of the eight 40 Hp motors on the elevation axis on the Green Bank Telescope. To perform identification, each motor was decoupled from the telescope and calibrated individually. It was necessary to implement an ad hoc speed controller for the motor under test since the telescope’s control system was disabled during this time. Due to the time consuming nature of physically decoupling motors, they were calibrated at various times over the course of two weeks. A listing of these calibrations appear in Appendix Section C.1.

Once all motors had been calibrated, the elevation axis was commanded to a constant speed setpoint. Plots of the speed fluctuations appear in Figure 2.4. Cogging compensation reduced the root mean square velocity fluctuation from 0.0904 to 0.0438 radians per second as well as reducing the peak-to-peak from 0.463 to 0.266 radians per second. While the cancellation of cogging made an impact on the velocity error,
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![Graphs showing rotor speed over time for different experiments](image)

(a) Rotor without compensation  
(b) First modeling experiment  
(c) Second modeling experiment  
(d) Third modeling experiment  
(e) Fourth modeling experiment  
(f) Initial solution  
(g) After one iteration  
(h) After two iterations

Figure 2.2: Snapshots of the rotor speed history during the identification process where the 1st, 2nd, 3rd, 4th, 6th, and 12th harmonics are canceled. The initial harmonic coefficients perform well and no improvement is seen after a single iteration. Error is reduced to the encoder resolution.
Figure 2.3: Calibration coefficients taken from successive testbed cogging identification experiments over the course of a week. Though there is variation from identification to identification, the relative proximity to their starting points hints at long term anti-cogging solution stability.

less of an impact was seen on the position error, where encoder measurement error dominates.
Elevation Speed without Compensation

Elevation Speed with Compensation

Figure 2.4: Elevation velocity fluctuations on the GBT, both normally and with cogging compensation. Cogging compensation reduced the root mean square velocity fluctuation from 0.0904 to 0.0438 radians per second as well as reducing the peak-to-peak from 0.463 to 0.266 radians per second.
Chapter 3

Calibrating Encoders

Initially, it was believed that the primary cause of the tracking error was due to motor cogging. Upon further investigation of the system it was discovered that a systematic encoder nonlinearity was responsible for the lion’s share of the tracking error. This measurement error is repeatable and therefore can be corrected for. The methodology developed to calibrate the motor cogging is adapted to solve this new class of identification problems.

3.1 Diagnosing Encoder Nonlinearity

The earliest attempt at canceling motor cogging on the elevation axis was unsuccessful, but ultimately revealed another source of tracking error. The strategy was simple: use the single motor cogging cancellation algorithm to cancel the net cogging torque on the elevation axis by using a single motor. If the drive train was sufficiently stiff, only the net cogging torque would appear to perturb the structure. This could be canceled by injecting a signal at a single motor to compensate it. This strategy was non-ideal in that it transferred compensation torque across the drive train components, but for the elevation axis it had a chance to work.

What happened wasn't an issue of algorithmic convergence, instead it was not
even possible to consistently measure cogging ripple. Repeated measurements of the same configuration yielded different output ripple harmonic fits. Further, the ripple data was dominated by the first harmonic, whereas in the lab tests, it was found that the second and fourth were the biggest contributors. Repeated measurement of the first harmonic displayed a systematic drift in phase—it appeared to be slowly moving which made identification impossible.

This systematic drift turned out not to be a problem measuring cogging, but was due to another error source, altogether. What appeared to be the first harmonic at the motors was in fact the second harmonic in the encoder interpolation error. The gear train ratio of a motor to the elevation structure is 31640 to one. The second harmonic of the encoder interpolation repeats 32768 (a power of two) times per rotation of the elevation structure. The proximity of these ratios was the cause of the difficulty: these two effects manifest at almost the same frequency when measured at the motors.

A plot of the tracking error appears in Figure 3.1a. To test the encoder, the original elevation axis encoder was replaced with one of a higher accuracy. The new encoder reduced the tracking error, a plot of which appears in Figure 3.1b. It also changed the pattern of the error, and, despite being smaller in magnitude, the error still occurs at the same period. Had the cause of the error been anything other than a measurement error, switching to a more accurate encoder would not change the error. Figure 3.2 shows an extended period of GBT tracking error with the new encoder. Successive passes over the interpolation interval of both encoders is shown in Figure 3.3. Note that, though the new encoder is more accurate, an interpolation error pattern remains. Much of the accuracy of the new encoder derives from the smaller magnitude of the interpolation error trend. The residual noise visible on both trends in Figure 3.3 is roughly the same magnitude. As this residual indicates the lower bound for tracking accuracy, it is fascinating to note that it is of the same magnitude for both encoders. This indicates that both encoders may be correctable
Figure 3.1: Plots of tracking error under the two tested elevation axis encoders. Since the change of the sensor to one of higher accuracy drastically changed the measured error profile, it is clear that the measurement nonlinearity is the cause of the observed tracking error.
3.2 Quantifying the Error

The similarity of this phenomenon to the motor cogging problem is uncanny. Both are periodic disturbances that are efficiently modeled by a few harmonics in a Fourier series, as can be seen from the error profile plots in Figure 3.3. Additionally, both deal with active feedback dynamics that alter the effect of the disturbance—the disturbance cannot be directly measured.

Encoder interpolation nonlinearity differs from cogging for the fundamental reason that it is an output disturbance rather than an input disturbance. This manifests itself in many ways, but the most immediate is how tracking performance degrades with speed. Cogging is a disturbance torque that acts on the rotor. From a torque to an angle, the rotor inertia behaves as a double integrator. At high speeds the dynamics of the system attenuate the disturbance. However, with an output disturbance, the opposite is true. The feedback controller will servo the system so as to eliminate any position error that it reads on the output. When the disturbance is within the controller’s bandwidth, the controller can effectively attenuate the output error. As speed increases, the ability to do this decreases and the uncorrected error increases accordingly. This arrangement has the peculiar quality that better physical tracking occurs at higher speeds than at lower ones, despite the fact that the measured tracking error is larger at high speeds than it is at low speeds. This is precisely because the error is not really there, but the controller still attempts to cancel it through compensating motion—thereby introducing real tracking error into the system.
Figure 3.2: Elevation axis position error for an extended observation period plotted over time in seconds. The data is marked every $2^{-14}$ of a rotation. The lock-step stability of the error pattern with respect to the markings indicates that this phenomenon is indeed occurring at that periodicity.
Figure 3.3: A plot of superimposed traces of the GBT position error over the interpolation interval for both encoders. The underlying trend can be closely modeled by a Fourier series, consisting of the first four harmonics. The interpolation interval is approximately 80 Arc-seconds long.

### 3.3 Modeling the Encoder Nonlinearity

A reasonable model must be chosen by making assumptions about the underlying error source. The strongest hint of the model’s structure is the quality of fit with a harmonic model to the averaged encoder error. A model consisting of the first four harmonics was parametrized to these data sets and achieved excellent matching. Further, each harmonic seemed to contribute significantly in magnitude to the overall fit. Consequently, the signal is entirely described by the first four harmonics.

Continuing the above reasoning, the starting point for analysis will be to view the encoder as implementing the following function

\[
\theta_{\text{meas}} = \theta_{\text{act}} - \sum_{n=1}^{4} \left[ p_{c,n} \cos \left( 2\pi 14^n \theta_{\text{act}} \right) + p_{s,n} \sin \left( 2\pi 14^n \theta_{\text{act}} \right) \right].
\]  

(3.1)
3.3.1 Selection of the Correction

The inversion of Equation (3.1) is non-trivial. However, an approximate inverse can be derived. In the cases of the encoder, the magnitude of the distortion is much smaller than the angle range. The approximation \( \cos(\theta_{\text{act}}) \approx \cos(\theta_{\text{meas}}) \) is used for the small trigonometric terms. Therefore

\[
\theta_{\text{act}} \approx \theta_{\text{meas}} + \sum_{n=1}^{4} \left[ p_{c,n} \cos \left( 2 \pi 2^{14} n \theta_{\text{meas}} \right) + p_{s,n} \sin \left( 2 \pi 2^{14} n \theta_{\text{meas}} \right) \right] \tag{3.2}
\]

provides an excellent approximation of the form of the inverse to be used for identification and cancellation.

3.3.2 Implementation and Identification

Mathematically, the identification problem is identical to the cogging problem—aside from the difference in period. There is a small harmonic disturbance in a rotational system operating at nearly constant speed. The harmonic coefficients of the cancellation signal need to be identified through a series of tests. Therefore, the identification algorithm itself will be used without modification.

The primary system modification necessary is that the injected signal must be added into the measurement given to the position loop controller. So, unlike cogging which is added to the output of a controller, this correction is added to the input of one. Though this distinction is extremely important, it does not alter the mathematics of the identification problem in any other way.

3.4 Elevation Axis Results

A series of experiments were carried out on the elevation axis of the GBT to calibrate and compensate for the encoder nonlinearity. These were performed while tracking a
constant rate target. Snapshots from each stage of identification of a particular test appear in Figure 3.4. These results demonstrate that the algorithm works extremely well for encoder calibration.

### 3.4.1 Calibration Details

It was observed that the cancellation residual grew after iterating the initial solution. This was caused by measurement noise. Initially, all tests were performed for the same duration. But as five tests were used to derive the initial solution, the effects of noise on this solution was effectively averaged down. By doubling the duration of the subsequent iteration tests, the residual error no longer increased after iteration.

The notion of signal to noise is important here. Perturbations have large magnitudes relative to fluctuations due to random errors and can be identified with a shorter duration of measurement. When iterating on the residual, the harmonic residual component is much smaller than the noise due to the rest of the system. This is evident in the plots in Figure 3.4.

### 3.4.2 Calibration and Angle

While reinterpolating the measured angle works well locally, how these calibrations hold over large changes in angles is another matter entirely. To answer the question of how the calibration varies across the range of the encoder, a series of calibrations were performed at various angles. The range of angles tested was limited by the safe operating range of the GBT elevation structure. Plots of the calibration coefficients performed at each angle appear in Figure 3.5. As can readily be seen from those plots, the first harmonic’s coefficients vary drastically across the range of the encoder while the other harmonics are relatively consistent.
Figure 3.4: Profile plots from all stages of identification of the elevation axis encoder calibration, based around 45 Degrees and moving at +15 Arc Minutes per Minute. All plots show the ripple profile (Arc-Seconds) plotted over the reinterpolation period (Arc-Seconds). The final solution has an RMS position error of 0.099 Arc-Seconds compared to the non-compensated RMS error of 0.207 Arc-Seconds.
Figure 3.5: These plots display the calibrations obtained from eight different identifications across the operational range of the GBT elevation axis. In order, 1\textsuperscript{st}, 2\textsuperscript{nd}, 3\textsuperscript{rd}, and 4\textsuperscript{th} harmonic coefficients (in 24 bit corrections) identified over the range of operation of the GBT elevation axis (in degrees). Circles denote Cosine coefficients and Xs mark the Sine coefficients.
3.4.3 Independent Corroboration of Results

Unlike motor cogging, where a torque was injected into the servo system, integrated twice by the dynamics, and measured by an encoder, there is no layer of physics between the encoder correction and the performance figure of merit. Measured output ripple is canceled by changing the measurement itself. Up to this point, success has been defined as an output that has been corrected to be ripple-free. To verify that the compensation is indeed improving tracking, and not just fooling itself, some form of independent measurement of error is needed.

The accepted standard at Green Bank for measuring tracking accuracy is the half-power point tracking test. In such a test, the edge (half-intensity point) of a known astronomical source is tracked. Any deviation from true pointing will be reflected in the measured source intensity. From this data, astronomical pointing accuracy can be measured. Unfortunately, as of the time of writing this dissertation, this experiment has not been performed with the compensation active. Eventually it will need to be done, but in the meanwhile residual structural vibration may be used as a substitute.

The GBT has several accelerometers on its structure. One of these, at the base of the feedarm, near the edge of the dish, is used to measure structural vibration during the series of identification tests. Thirty seconds of data is analyzed without compensation and another thirty with compensation. This is repeated at every angle of identification. Since acceleration measurements greatly amplify high frequency data, while attenuating low frequency content, a reconstructed position measurement will be extracted from the acceleration data. Acceleration data is detrended—with a first-order vectorized model—to subtract gravity and any linear trend. The detrended acceleration is integrated twice. As it has been linearly detrended as acceleration data, the reconstructed displacement will have zero mean. The root-mean-square (RMS) magnitude of the reconstructed position error will be used as the figure of merit. However, this encoder has not been calibrated. So, distance measurements are scaled
Figure 3.6: Reconstructed position error for an uncalibrated accelerometer on the base of the feedarm at various angles, with and without encoder linearization applied. As a general rule, the corrected case functions with less disturbance. However, the magnitude of other disturbances is larger than the magnitude of the improvements; caution should be employed interpreting these results.

by an unknown factor. Still, relative differences can be compared.

A plot of the reconstructed position error magnitudes appears in Figure 3.6. While the trend is clear that the linearized encoder readings perform better than the uncorrected encoder, reconstructed pointing error achieved by this method is not necessarily very accurate. The accelerometer is measuring all manner of disturbances on the telescope and attributing everything it sees to a single cause can be tricky. Instead, this result should be considered promising, but not definitive, proof of improvement. Half-power point tracking performance will be the final arbiter.
Chapter 4

Calibrating Multiple Motors

The calibration method devised in the previous chapter will be generalized to the multiple motor case. Ostensibly, this is as simple as extending the model to a vector of parameters and a matrix of coupling sub-matrices. However, numerical challenges preclude the use of a naïve approach in solving for a parameter update.

4.1 Generalizing the Calibration Method

The single motor calibration method developed previously can be extended to the multiple motor system. The general idea is to arrange the system model as a matrix of motor models. The aggregate matrix of models consists both of the individual motor models as well as models of the coupling between motors.

In this setting, the control system is still linear, though it involves multiple inputs and outputs with internal coupling. Likewise, the assumption that the perturbation magnitude is small compared to the system velocity also holds—justifying continued use of the independent harmonics approximation. The new element of complexity arises from the mechanical and feedback coupling between the motors. Fortunately, the structural and controller responses to small harmonic quantities are governed by linear dynamics. Consequently, this form of coupling can also be modeled by a linear
4.2 ILL-CONDITIONING

system model, just as the dynamics of the individual motors were.

To simultaneously model multiple motors, the notation used in the single motor problem is extended by adding additional subscripts. For example, \( p_{s,3,4} \) denotes the input sine coefficient of the third harmonic of the fourth motor. We gather all coefficients of the \( n^{th} \) motor into the vector of harmonic coefficients \( p_n \). The model of that motor from input to output harmonics, as measured on that same motor, is \( M_{n,n} \). Likewise, the offset, as measured at motor \( n \), is \( b_n \). What is new is the additive effects that all the other motors have on the observed harmonics at the \( n^{th} \) motor.

Since the structural and controller coupling of the motors is linear, the effects of the injected signals at all motors superimpose to appear on any given motor. These are subject to scaling and phase shifting, just as in the single motor case. Therefore, the effect of the injected cogging harmonics from motor \( m \), as they appear on motor \( n \), is modeled by the matrix \( M_{n,m} \). Aggregating these components into a single model yields

\[
\begin{bmatrix}
  y_1 \\
  y_2 \\
  \vdots
\end{bmatrix} = \begin{bmatrix}
  M_{1,1} & M_{1,2} & \cdots \\
  M_{2,1} & M_{2,2} & \cdots \\
  \vdots & \vdots & \ddots
\end{bmatrix}
\begin{bmatrix}
  p_1 \\
  p_2 \\
  \vdots
\end{bmatrix} + \begin{bmatrix}
  b_1 \\
  b_2 \\
  \vdots
\end{bmatrix}.
\] (4.1)

Just as was true in the single motor case, the sub-matrices \( M_{n,m} \) consist of \( 2 \times 2 \) blocks on the diagonals and zeros elsewhere. Further these block matrices are scaled orthogonal matrices and can be thought of as representing a phase shift and a scaling.

4.2 Ill-Conditioning of Multiple Motor Systems

It is well known that linear equations involving a singular matrix do not have a unique solutions, while non-singular matrices will have unique solutions. However, for invertible matrices that are close to singular, there can be considerable difficulty obtaining those solutions. The measure of singularity of a matrix is called the conditioning
The condition number of a matrix is equivalent to the fraction of the largest singular value to the smallest. Consequently, this number captures how different components in the solution are scaled relative to each other. When the condition number is large, the problem is said to be ill-conditioned. When solving equations of this type, those components associated with small singular values are amplified relative to the others, resulting in a solution dominated by the nearly degenerate subspace. The disproportionate scaling of these components results in a solution completely dominated by noise.

The condition number yields information about the solvability of the problem. While sufficiently ill-conditioned problems are unsolvable due to the limited precision of a computer’s floating point representation, the real limitation for cogging calibration is the measurement error due to random system disturbances. In this way, it serves as a measure of noise sensitivity.

In the multiple motor case, the tightness of the inter-motor coupling is the root of numerical problems. In these motors, harmonics injected into one motor show up almost unchanged on all other motors. Such a system has a nearly singular harmonic model. As will be seen, this results in the amplification of harmonics injected in one motor and canceled by some other. Since such a solution puts great loading across the interconnections, it would be potentially damaging to apply to the system. What is needed is a way to find good solutions to the problem that have a minimal amount of this type of counter-productive loading.

### 4.2.1 Simulation of a Two Motor System

In order to study how the degree of coupling affects the numeric solvability of the multiple motor calibration problem, a simplified two motor system is studied. A schematic representation of the dynamics appears in Figure 4.1. Each motor is mod-
4.2 ILL-CONDITIONING

CHAPTER 4. MULTIPLE MOTORS

Figure 4.1: Schematic of the two motor setup. The dynamics of this system are modeled in equation (4.2).

eled as an inertia with damping and subject to an externally applied torque. The two identical motors are connected through a flexible coupling with a finite stiffness. As a variation of the typical linear damping, the damping is applied based upon the difference in speed between the rotor and the reference speed. This setup corresponds to a situation where the motors are driven by a constant torque that exactly cancels the affects of the damping at the reference speed. This gives a system that responds similar to a speed-controlled motor system but without the additional dynamics that a feedback controller would introduce.

The simulation is meant to illustrate the general behavior, not that of a particular motor system. Consequently, gross simplifications are made both in the modeling as well as the choice of parameters. The dynamics of the two motor system are modeled as

\[
\begin{aligned}
\dot{\theta}_1 &= \omega_1 \\
J \dot{\omega}_1 &= \tau_1 + k(\theta_2 - \theta_1) - b(\omega_1 - \omega_{ref}) \\
\dot{\theta}_2 &= \omega_2 \\
J \dot{\omega}_2 &= \tau_2 - k(\theta_2 - \theta_1) - b(\omega_2 - \omega_{ref})
\end{aligned}
\]

(4.2)

where \( J = 1 \) is the inertia, the damping, \( b \), is 0.25. The link stiffness, \( k \), is to be studied over a range of values. During simulation the reference speed is chosen to be \( \omega_{ref} = 1 \) and the initial conditions start at zero angle moving at the reference speed.

As the model does not include an intrinsic cogging torque, the system is equivalent to one where the cogging is canceled. Small perturbations added to the external
torques of the system, $\tau$, can identify the small signal model—the offset term is zero. Therefore, a series of four tests were performed for the first harmonic cosine and sine coefficients were perturbed and the result measured. Aggregating these together yields the model matrix. This process of model fitting was repeated for a range of link stiffnesses and the resulting model conditioning numbers are plotted in Figure 4.2.

### 4.2.2 Behaviors Observed in the System

There are two distinct domains of behavior in the two motor system: loosely coupled and tightly coupled. In Figure 4.2, these regions correspond to how the conditioning of the system changes with stiffness. Figure 4.3 shows motor vibration caricatures with three different representative stiffnesses. When the system is loosely coupled, the affects of a perturbation from one motor show up elsewhere, but the numerical
Figure 4.3: Normalized perturbed behaviors of the two motor system with various link stiffness. On the left is the loosely coupled case ($k = 0.1$). The critical transition case ($k = 1$) is in the center and a tightly coupled case ($k = 10$) is on the right. Both the relative magnitudes and phasing contribute to the solvability of the problem.

The problem can largely be solved in the naïve manner since the perturbation appears distinctively on each motor. When the motors are tightly coupled, there is little observable difference between how the perturbation appears on each motor. This corresponds to an inability to discern which motor is causing the perturbation. The numeric problem is nearly singular and attempting to solve it will result in a solution that cancels the cogging, but does so in a way that may put significant torque across the link.

It is informative to look at the decomposition of one of the tightly coupled calibration models. The model matrix for $k = 10$ has singular value decomposition with four singular values, two are 0.9713 and the other two are 0.0526. These values correspond to a matrix that has condition number 18.468. When solving the calibration problem, the components that

\[
V_{k=10} = \begin{bmatrix}
-0.577 & 0.409 & -0.293 & 0.643 \\
0.409 & 0.577 & -0.643 & -0.293 \\
-0.577 & 0.409 & 0.293 & -0.643 \\
0.409 & 0.577 & 0.643 & 0.293
\end{bmatrix}
\]

and

\[
\sigma_{k=10} = \begin{bmatrix}
0.9713 \\
0.9713 \\
0.0526 \\
0.0526
\end{bmatrix}
\]
where the first two columns are associated with the singular value 0.9713 and the last two with 0.0526.

Those solution components associated with the larger singular value correspond to cases where the load is shared between the two motors—the cosine coefficients are the same as are the sine coefficients. Conversely, the sub-space associated with opposing torques has a small singular value. Ill-conditioned systems arise when the process under inspection attenuates some, but not all, components of the input. In the case of the coupled motors, the components that work together appear on the output while those that oppose each other are largely attenuated. Any measurement error will, when the equations are solved, amplify the attenuated components and yield a solution that is dominated by the oppositional components. Such a solution, when fed into the system, will fit the measured behavior very closely. However, that type of solution carries significant torque across the motor shaft.

Contrast this behavior with that of the loosely coupled case

\[ V_{k=0.1} = \begin{bmatrix}
-0.630 & 0.320 & -0.617 & -0.346 \\
0.320 & 0.630 & 0.346 & -0.617 \\
0.630 & -0.320 & -0.617 & -0.346 \\
-0.320 & -0.630 & 0.346 & -0.617
\end{bmatrix}, \quad \sigma_{k=0.1} = \begin{bmatrix}
1.196 \\
1.196 \\
0.971 \\
0.971
\end{bmatrix}.\]

Though the component vectors of the $V$ matrix are similar in shape (both cooperative and opposing terms), the singular values are very close in magnitude. This means that all four components have similarly scaled effects on the output. Therefore, error affects the obtained solution uniformly and there is not the concern that there was in the tightly coupled case about numeric instability.

In general, these approximate null space solutions should not appear in the solution since they lead to mechanical stress while contributing very little to the cancellation of cogging. Some action must be taken to mitigate the problems associated
4.3 Computing Calibrations

4.3 Calibrating the Multiple Motor System

Fortunately, reliable methods of obtaining solutions to ill-conditioned problems has been an active research area in the field of applied mathematics for some time now. The discipline is known as inverse problem theory [19, 1]. The key observation is that inversion of nearly singular problems is difficult, but by solving a modified problem, reliable solutions can be obtained. The new problem agrees with the original in the components with large singular values, but deviates from it for the components of small singular values. Those nearly singular subspaces are often not physically relevant to the solution, and their absence is not missed. Therefore, some form of regularization is employed whereby the solution is taken, not form the entire possible solution space, but a physically likely subset of it.

4.3.1 Introduction to Regularization Methods

There are two major schools of thoughts in inverse problems theory: deterministic and probabilistic. The deterministic approach achieves regularization through penalizing solutions that do not fit the preconceived notion of what the solution should look like. These are often formulated and solved as optimization problems. When probabilistic methods are used, the model forms a conditional probability of an event being observed for a particular solution. Regularization is achieved through the choice of the \textit{a priori} distribution. Monte Carlo methods are often used to estimate the solution's distribution.

In this work, deterministic optimization based method will be exclusively considered. Regularizers will impose our physical intuition for what properties the solutions ought to have. The solution update is then computed by solving an optimization
4.3. COMPUTING CALIBRATIONS

4.3.2 Proposed Regularization Methods

Several regularization methods are proposed based upon physical understanding of the system. Before introducing these regularizers, it is informative to see how the solutions obtained with no regularization performs. The problem of solving a system of linear equations can easily be recast as the optimization problem

\[
\min_x ||Ax - b||^2,
\]

where \( x = A^{-1}b \) uniquely solves this problem in the case that \( A \) is nonsingular. In the case that \( A \) is singular, the most common way to find a solution is by computing the pseudo-inverse of the matrix. The solution obtained in those cases is a solution to the optimization problem. In particular, it is the solution with minimum norm.

**Conflict Based Regularization**

while the solutions must be kept small to avoid issues with actuator saturation the primary concern has always been the minimization of mechanical stress across the structure. Instead of penalizing the magnitude of the coefficients, the magnitude of the coefficients projected onto the null space of the rigidly coupled system is penalized.

\[
\min_x \ ||Ax - b||^2 + \alpha||Nx||^2
\]

Where the matrix \( N \) projects the harmonic vector onto the nullspace of the perfectly rigidly coupled problem.

While a straightforward weighting like this will be used for the elevation axis, the azimuth axis is more complex. That axis is more non-uniformly coupled. Due to the structure, the motors are hierarchically organized. Some motors are grouped...
closely and tightly interacting, others are positioned far apart on the flexible structure. Deviations between motors on the same truck will need to be penalized more than differences between trucks. Hierarchical weighting of the nullspaces would be employed.

4.3.3 Selection of the Regularization Parameter

In (4.4), the parameter $\alpha$ controls the trade off between weighting model fit with the cost of antagonistic cross-motor torque. Selection of this parameter must be done in a way that seeks a good fit, while being physical realistic. There is no way to do this automatically—its value must be hand-tuned. Practically speaking, a range of values are used and the hypothetical solutions are computed. For each of these values, the cost for both terms of the objective function in (4.4) are evaluated. The best $\alpha$ is one that sits at the point of diminishing return on the goodness-of-fit in light of the observation noise of the system.

4.4 A Simulated Calibration

The work presented in this section is a simulation of how a multiple motor cogging solution could be calculated for the GBT elevation axis motors. All model data is derived from a single identification experiment, carried out on Motor 1, where it was perturbed and the resulting ripples on all motors were measured. Symmetry of the drive train arrangement will be used to extend this single test into an approximation of the full model. While exploitation of such symmetry can drastically reduce the amount of calibration time needed, a full modeling experiment will need to be performed at some point in the future to validated model symmetry is a good assumption.

By consulting Figure 1.5, similarities between motors can be constructed. The
basic idea is that the models should reflect across axes of symmetry: motors on the left affect those on the right as those on the right affect those on the left. However, there are two symmetric groups, an inner four motors and the outer four. Since the model data was only performed for one motor, one of these sets of relations needs to be approximated. The similarity structure is given by

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 7 & 8 & 7 & 8 \\
2 & 1 & 4 & 3 & 8 & 7 & 8 & 7 \\
3 & 4 & 1 & 2 & 5 & 6 & 5 & 6 \\
4 & 3 & 2 & 1 & 6 & 5 & 6 & 5 \\
5 & 6 & 5 & 6 & 1 & 2 & 3 & 4 \\
6 & 5 & 6 & 5 & 2 & 1 & 4 & 3 \\
7 & 8 & 7 & 8 & 3 & 4 & 1 & 2 \\
8 & 7 & 8 & 7 & 4 & 3 & 2 & 1 \\
\end{array}
\] (4.5)

where a number in the matrix, \( n \), corresponds to \( M_{n,1} \) being placed there in the system-wide matrix \( M \). On the telescope, all of the individual motor encoders have been configured to turn in the same direction when the elevation structure is moving. This permits the symmetry to be constructed without the need to reflect individual model matrices to account for differing orientation conventions.

### 4.4.1 Solving for Updates

While the two motor system demonstrated the potential of the multiple motor system for ill-conditioning, the condition number of the full model matrix, \( M \), is only 136. While this is high enough to cause problems with noisy data, it is not so unreasonably large to prohibit direct solution. Denote this direct solution as \( x_{\text{direct}} \). The norm of this solution is 0.526, which is would be a large step, but not dangerously large by any means.
Instead of using the direct solution approach, the regularization can be applied to the problem. Figure 4.4 shows the error components when solving (4.4) for a range of $\alpha$ parameters. Once a tradeoff point has been selected, in this case $\alpha = 0.001$ is chosen, the regularized solution can be computed. The norm of $x_{\text{regularized}}$ is 0.115. The residual solution error is 0.004 and is the cost paid by reigning in the step size. As the model offset has norm 0.015, this remaining solution error is not large, but may require additional solution iteration steps.

Another major concern for negotiating this tradeoff is the effect measurement noise has on the solutions. In general, over-fitting of a solution should be avoided. Consequently, one way to choose the tradeoff point would be to pick the parameter $\alpha$ from Figure 4.4 so that the residual equation error is not smaller than the expected measurement noise. This would require repeating a measurement several times to characterize the noise.
4.5 Reflections on Regularization

It is system noise that exacerbates the ill-conditioning of the numeric problem rather than the limited precision of floating point representations that plagues this method. This is likely true in most engineering problems. Consequently, the threshold between well and ill conditioned is much lower than the limits imposed by computation.

At opposite ends of the spectrum lie the uncoupled and the totally coupled cases. The uncoupled case requires each motor calibration be determined by itself and applied locally. If the motors were rigidly coupled, all motors behave as one and the best solution is to look at the total cogging and divide the compensation evenly among all motors. The fascinating thing is that the method of regularization provides a continuous bridge between these two special cases. When the motor system is not coupled strongly, the numeric problem is determined by the robust matrix solution and mostly ignores the regularization. When the system is loosely coupled, the dominant singular values are unchanged and the weaker, opposing solutions are attenuated by the regularization factor.

But there is a much deeper issue involved here. Regularization methods provide a critical method for injecting a designer’s intuition about what constitutes an acceptable solution into a complex machinery of a numeric problem. Controlling the form of the solution while staying close to a solution to the problem can help reign in the sometimes unpredictability of an overly analytic approach to engineering.
Chapter 5

Exploring Questions in the Underlying Theory

Up to this point, the calibration methodologies presented in this work, though surprisingly general, were only applicable to problems of a specific kind. All of these problems consisted of a rotational system moving at a nearly constant velocity, subjected to angle dependent disturbances. The final branch of this dissertation will explore further generalization of the identification methodology.

At the heart of this method is the exploitation of periodicity of operation. Quantities that depend upon state variables become periodic in time. But what can be said for systems in periodic operation of non-constant rate? A motivating example is a mechatronic system where the motor is restricted to a limited range of angles. Can a periodic back and forth motion be used to carry out a cogging identification?

In order to generalize the algorithm, the nonlinear conversion from state to time representations of harmonic quantities will require explicit treatment. When the rate is near constant and the perturbations are small, state periodic and time periodic representations of harmonic quantities coincide. The justification for this approximation was given in Section 2.2.2. In this chapter, approximations will be relaxed
and an identification method using arbitrary reference trajectories is developed. In this general setting, the theoretical strengths and limitations of the underlying theory become evident. Efficacy of this approach will be demonstrated on a simulated motor cogging problem.

**Demonstration Problem**

As a motivating example, the single motor with cogging will again be considered. However, unlike before, the periodic operating point will not be at a nearly constant speed, but following the sinusoidal angular reference trajectory

\[
r(t) = \pi + \frac{3\pi}{4} \sin(\omega_{\text{ref}} t).
\]

Note that, in addition to the continuously changing direction, this trajectory does not pass through all angles of the rotor; it is confined to stay between 45° and 315°. This limitation occurs in practice when the range of motion of a motor system is restricted to a range of angles, such as might occur in a robotic actuator.

Using the a motor model, the rotor dynamics that will be used for this problem are

\[
\begin{align*}
\ddot{\theta} &= 82.9 u - 0.466 \dot{\theta} + \varphi \\
\varphi &= -2.82 \cos(2 \theta) - 1.36 \sin(2 \theta) \\
e &= \theta - r \\
u &= -0.1 e - 0.1 \dot{e} + \frac{1}{82.9} \dot{\theta}_{\text{ref}}.
\end{align*}
\]

The parameters for these dynamics were simplified from the 40 Hp motor testbed. This motor system has been controlled by a simple proportional and derivative (PD) controller with acceleration feedforward. For simplicity, all direction dependent terms have been omitted in these dynamics, and therefore a unidirectional cogging solution will be sought.
5.1. Generalizing the Algorithm

In the previous chapters, the algorithm used was derived with the particular assumption that the mapping from angle domain to time domain harmonic coefficients could be approximated by the identity transformation. This was a consequence of the particular periodic trajectory of those systems—operating with near constant speed. As it turns out, the concept of the profile does not generalize nicely to the non-uniform motion case, so analysis must be carried out on the coefficients directly.

The generalization of rotor profiles appear in Figure 5.1. In these profile plots of tracking error, the differences between the uniform speed and non-uniform speed cases become readily apparent. Since the motor changes direction, angles are doubly covered on this interval. Further, since the time history at a doubly covered angle for either direction varies, the tracking error takes on multiple values for a given angle. When more complex reference signals are employed, angles may be covered more than twice.

Another issue is the non-uniform covering of the interval of angles. While some angles are not covered at all, of those that are visited the sampling of data points

Figure 5.1: Profiles of the non-uniformly excited motor with cogging system for several input amplitudes. The effect of nonlinearity is evident in 5.1a while the response of 5.1b displays the characteristic magnitude scaling of a linear system.

5.1 Generalizing the Algorithm
is not uniform. The system slows down as it changes direction, consequently the endpoints of the interval are more densely sampled. Any attempt to use this data as-is will result in a biased least squares problems when the model is fit. Ideally, the data must either be resampled or a weighting must be applied to the sample points to extract the model. Though this complication is not addressed in this work.

Finally, in order to solve for the anti-cogging solution, the response of the ideal system to the reference must be known. In the case of constant speed setpoints, the ideal response is trivial—all residual harmonic content should be zero. However, when the reference is a sinusoid, as it is here, the output of the system will be a phase shifted and scaled sinusoid of the same frequency. The frequency response of the system from the reference input to tracking error dictates the precise steady state values. The problem is that this response is not precisely known and must be identified. Only the frequency response of the motor input needed to be modeled in the earlier methods. Now, both the linear responses of the cogging and the reference must be modeled.

### 5.1.1 Analyzing the Periodic Quantities

The complexity introduced by the linear system dynamics prevents the direct treatment of the angle-dependent terms. Instead, all signals will be converted into time-domain periodic signals. Since the coefficients of the Fourier series are the quantities of interest, discovering the method for converting coefficients of the angle-domain cogging basis functions into equivalent time-domain coefficients must be discovered. Additionally, a way to convert back into angle-domain quantities must also be developed so that anti-cogging coefficient updates can be computed.

To start, consider the Fourier series expansion of the reference trajectory

\[
r(t) = r_0 + \sum_n r_{c,n} \cos(\omega_{\text{ref}}t) + r_{s,n} \sin(\omega_{\text{ref}}t).
\]  

(5.3)
5.1. GENERALIZATION

CHAPTER 5. UNDERLYING THEORY

Figure 5.2: Time domain representation of the cogging basis functions for the first two harmonics. The complexity of these shapes results from the change of basis matrix, \( \Gamma \), about this particular trajectory.

Since the system is driven into a steady-state periodic operation, the driving frequency \( \omega_{\text{ref}} \) dictates the periodicity of all time signals. All time-domain harmonic representations will be expressed as linear combinations of \( \cos(n\omega_{\text{ref}}t) \) and \( \sin(n\omega_{\text{ref}}t) \), similar to (5.3).

Fundamentally, the transformation between the two representations is a change of basis. And though the cogging basis functions are all zero mean when parameterized by angle, their time-domain signals have non-zero offsets. The coefficients of angle-domain quantities, \( y^\theta \), can be transformed into their equivalent time domain representation, \( y^t \), by

\[
y^t = \begin{bmatrix}
\gamma_{0,c,1} & \gamma_{0,s,1} & \gamma_{0,c,2} & \gamma_{0,s,2} & \cdots \\
\gamma_{c,1,c,1} & \gamma_{c,1,s,1} & \gamma_{c,1,c,2} & \gamma_{c,1,s,2} \\
\gamma_{s,1,c,1} & \gamma_{s,1,s,1} & \gamma_{s,1,c,2} & \gamma_{s,1,s,2} \\
\vdots & & \ddots & \ddots
\end{bmatrix}
y^\theta.
\] (5.4)
where $\gamma_{c,n,c,m}$ is the component of $\cos(n\omega_{ref}t)$ in $\cos(m\theta(t))$. More explicitly, the $\gamma_s$ elements are obtained by the Fourier series analysis equation and are such that

$$\cos(m\theta(t)) = \gamma_{0,c,m} + \sum_n \gamma_{c,n,c,m} \cos(n\omega_{ref}t) + \gamma_{s,n,c,m} \sin(n\omega_{ref}t)$$

holds. Each such decomposition yields a column in the matrix, $\Gamma$.

Figure 5.2 depicts the transformation of some simple angle-domain cogging basis functions into the time-domain. This transformation is performed around the operating point of (5.2), but without the effect of cogging. The complex time shape of these relatively simple angle quantities emphasizes an important point: harmonics are no longer independent—an isolated cosine harmonic has components in many different harmonics when transformed. Consequently, a wide range of time-domain harmonics should be indentified in order to accurately identify a few cogging harmonics. The matrix $\Gamma$ will not be square and conversion from time-domain coefficients to a angle-domain ones will be an information destroying projection.

### 5.1.2 Modeling System Response

As before, all quantities are transformed to the time-domain for analysis. The effect of these quantities on the measured ripple is dictated by the linear system’s treatment of harmonics at the base frequency. However, this time there are two inputs to the linear system dynamics: cogging torque and reference signal. Both input-output dynamical behaviors must be accounted for. The measured ripple is given by

$$y^t = M_c (p^t - p_0^t) + M_r r^t,$$

(5.5)

where $M_c$ and $M_r$ are block diagonal matrices that scale and phase-shift each harmonic. The matrix $M_c$ models the systems response to input cogging correction time harmonics and $M_r$ models the system’s response to the reference signal. Here, $p_0$ de-
notes the unknown zero-point for the anti-cogging coefficients, transformed to apply to the motor’s input. Through the use of the change of basis transformation, the model can be rewritten as

\[ y^t = M_c \Gamma_{\theta(t)} (p^\theta - p^\theta_0) + M_r r^t. \]  

(5.6)

Finally, since the trajectory \( \theta(t) \) is uniquely determined by the position error ripple harmonics, \( y \), the equations are

\[ y^t = M_c \Gamma_y (p^\theta - p^\theta_0) + M_r r^t. \]  

(5.7)

In the past, the ripple components due to the natural cogging torque were lumped into an offset vector, \( b \). However, the nonlinearity of the transformation is important and their effect cannot be separated so easily: their contribution to the disturbance harmonics depend on the particular trajectory of the system.

5.1.3 Parameterizing the Model

As before, the ripple equation will be used to fit model parameters to measured experimental data to identify the model. This formulation’s ripple equation, (5.5), must be solved for \( M_c, p^\theta_0 \) and \( M_r \). Since \( M_c \) and \( p^\theta_0 \) appear in the same term, the model identification problem is nonlinear and is handled by a general nonlinear solver.

In order to accurately fit a model, a full range of test data must be compiled. By inspecting (5.5), it becomes clear that the input reference must be varied to determine \( M_r \). Also, the inputs to \( M_c \) must be varied. However, there is the nonlinear matrix \( \Gamma \) to contend with. The cogging perturbation input, \( p \), must be chosen on the basis of how will \( \Gamma p \) will perturb the system.
5.1.4 Solving for Updates

Given a model and the harmonic decomposition of the residual $y_{\text{res}}^t$, then (5.7) can be solved for an update of $p$. This can either be handled by a nonlinear solver or by an iterative approach. In the iterative approach, $\Gamma$ is held constant about the current trajectory and a tentative update $p$ is computed. Then the trajectory is recomputed, in simulation not experiment, and $\Gamma$ is updated. The process repeats by solving for $p$ with $\Gamma$ fixed. When satisfied, $p$ is applied to the actual system as a solution update.

5.2 Anti-Cogging Simulation

Cogging identification was carried out for the system (5.2), subject to the input (5.1). For the cogging perturbations,

\[ p_{\text{pert-1}} = 0.05 \cos(2\theta) \]
\[ p_{\text{pert-2}} = 0.05 \sin(2\theta) \]
\[ p_{\text{pert-3}} = 0.05 \cos(2\theta) + 0.05 \sin(2\theta) \]

were used. To perturb the reference,

\[ r_{\text{ref-pert}} = \frac{3}{4} \left( \pi + \frac{3\pi}{4} \sin(\omega_{\text{ref}}t) \right) \]

was used. Profiles of the simulated motor system appear in Figure 5.3.

5.3 Improvements and Simplifications

5.3.1 Working without Knowing the Reference Signal

It may be possible to avoid modeling the response of the system to the reference signal. The sinusoidal input given by (5.1) contains only the fundamental harmonic
Figure 5.3: Error profiles during the simulated identification process. In the general setting, profiles are not used directly by the algorithm. However, they are instructive for the progress and behavior of identification.
and an offset term. If, instead of treating all output harmonics as relevant, the first harmonic were to be ignored then the remaining residuals should be driven to zero. The coupling of harmonics through the change of basis would all but guarantee that the significant portions of each cogging harmonic would appear in the other time harmonics. This approach would simplify model identification because no perturbations to the reference signal would be needed, but it is only applicable to references that have limited frequency content. If there is significant overlap between the reference and cogging harmonics, this strategy will yield an underdetermined identification problem.

Approximating Around a Trajectory

The chief complication to solving (5.7) for a solution update is the nonlinear dependency that the change of the trajectory has on the transformation matrix, $\Gamma$. One approach to solving these equations would be to use a fixed $\Gamma$, say based around the previous solution trajectory, and compute the update as if $\Gamma$ were constant.

Since the step is computed based on an assumption of linearity where none exists, no update will be truly correct, and it will become necessary to iterate the solution to achieve better cancellation. The differences between the functions in Figure 5.4 is small, but still present. These small errors arising from a fixed transformation matrix requires iteration. In the uniform speed cases treated in this dissertation, it was common for the algorithm to converge rapidly—often not requiring subsequent iteration of the solution.

While much simple to implement, the problem with this approach is that the iterations involve actual experiments on the system. These tend to carry with them a large cost in the amount of time required to perform. This is a cost that the approach of solving for an update using the nonlinear equations largely circumvents.
5.3. IMPROVEMENTS

5.3.2 Improved Model Solver

The major complication to handling a large number of cogging harmonics with this generalized method is the difficulty of solving for the model parameters from experimental data. In the earlier methods, the modeling equations were linear in the parameters to be identified. These were easily and accurately solved by a least squares linear solver. However, the model equation (5.5) is nonlinear in the $M_c$ and $p_0$ variables, as these are multiplied by each other. As the dimensionality tends to be high for the modeling problem, problems with general nonlinear solvers failing to converge appear in problems with many harmonics.

One approach to circumvent this difficulty is to solve for the model parameters in two steps. First, using a constant reference signal and small cogging perturbations,
5.4. INSIGHTS

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the matrix $M_c$ can be solved from the equation

$$y^t = M_c p + b,$$

(5.8)

where $b$ lumps the terms $M_c \Gamma p_0^t$ and $M_r r^t$. Since it has been shown that $\Gamma$ is nearly constant for small cogging perturbations, the nonlinear term $M_c \Gamma p_0^t$ should be constant across these experiments. Once $M_c$ has been obtained, $p_0$ and $M_r$ are computed from the full set of experimental data, holding $M_c$ constant, using (5.5).

5.4 Insights on the Underlying Theory

Having at last arrived at this general algorithm, it is appropriate to reflect on what are the fundamental properties that underly the algorithms researched in this dissertation. Periodicity permits the conversion of nonlinear terms to ones that are periodic in time. What makes this useful is that the nonlinearities that have been addressed are ones that are a linear combination of nonlinear basis functions. In all of the problems considered, the goal had been to manipulate their coefficients to null those terms.

Use of explicit, harmonic modeling of linear components permits the coefficients of nonlinearities to be laid bare for modeling. The steady state time-domain model of the system permits a compact representation that can be used to predict response when parameters are modified, despite the presence of nonlinearities in the system dynamics.

For a fixed set of nonlinear basis functions, a wide range of $\Gamma$ matrices exist, depending upon the particular system trajectory. Some of these matrices have poor numerical properties while others are easily solvable. The solvability of the inverse $\Gamma$ computation pertains directly to how well the basis functions are differentiated on the chosen trajectory. Therefore, the design of the periodic operating point is critical to the calibration method. In this chapter, the reference was chosen without regard to
these issues. The question of just how to tailor the operating point to the quantities that need to be identified remains an open question for the time being.
Chapter 6

Conclusions and Future Work

6.1 Major Contributions of this Work

In this dissertation, an iterative approach to canceling harmonic disturbances for systems in periodic operation was proposed. In particular, the algorithm is applied to the problems of motor cogging cancellation and encoder interpolation correction and demonstrated on the GBT. At its root, the algorithm is an optimization problem—it seeks to minimize residual ripple by identifying the parameters of a cancellation feedforward. Numerous tricks and observations permit the algorithm to be made extremely efficient with regards to the number of tests needed, which is critical when the objective function is evaluated by running the system for some period of time and then analyzing the residual error.

The approach developed in this dissertation contrasts with other approaches in several significant respects. First, most approaches taken by others are continuously varying adaptive controllers. These close feedback loops on the parameters, and consequently have very non-trivial stability issues. By virtue of being iterative, the proposed method avoids all of these issues. Further, since the method explicitly models the system dynamics during identification, it can function in a system where
the load and controller dynamics are unknown.

The algorithm was developed and specialized for the single motor cogging cancellation problem. Performance was tested on a motor testbed and the nature of the cogging was characterized. Cogging is discovered to be independent of speed and loading but it varies based upon direction. Later, this calibration algorithm was ported to the individual motors on the GBT elevation axis. Each motor was disconnected and individually calibrated. When all motors had been calibrated, the system was tested with and without compensation. A reduction in velocity error was observed.

The single motor algorithm was used to treat the problem of encoder linearization. Though the source of the encoder interpolation error is non-physical, it is a measurement artifact, it can be treated with the same approach as motor cogging. A significant part of the GBT’s position error is traced to the encoders. A measurement correcting term is added in software to the measurement used for feedback. The existing identification algorithm is used to identify the ideal correction, thereby linearizing the measurement. The success of this approach, for the particular problem, was demonstrated on the GBT. Finally, how the behavior of these calibrations changes over the range of the encoder was studied.

The methodology was extended to calibrate a system consisting of multiple motors, coupled across a structure. To the best of our knowledge, no other researcher has attempted this feat and instead has only dealt with one motor at a time. The core difficulty of this problem is that the motors are tightly coupled, both through the structure and through the action of feedback. The challenge is to generalize the method in a way that it is able to decouple the interactions and correctly attribute the individual contributions of the disturbance to their respective motors. This has been proposed to be handled using regularization. Solution updates computed using this method were simulated.

Finally, the extend of generalization of the underlying idea was explored. All three
previous problems have considered a rotational system operating at nearly constant speed. How the algorithm generalizes to other periodic trajectories was studied and an extended method was derived. In particular, the problem of single motor cogging identification of a motor with limited angular range was investigated.

6.2 Future Work

The calibration of the encoders appears to have worked extraordinarily well. Convergence was rapid and the accelerometer data backed up the hypothesis of improvement. However, conclusive proof will only be obtained by performing the half-power point tracking experiments. Finally, deviation across large angles impacts the calibration. A protocol must be developed for global pointing correction. Perhaps either by performing an identification before observing or by collecting and storing calibrations throughout the encoder’s range. Either way, this will need to be incorporated into the observing system.

The results presented for the multiple motor problem were simulation only. Further, assumptions of model symmetry were made to approximate a full model from a test performed at a single motor. A full harmonic model identification procedure needs to be performed—if for no other reason than to verify the correctness of the symmetric model. Further, different computed solutions should be tested on the telescope itself. Only then can the relative costs and benefits of different regularization methods be explored.

Finally, the generalization of the method has exposed many new questions that were not apparent in the uniform motion case. The choice of trajectory affects the transformation from state-based signals to time-domain harmonic representations. How best to construct a reference excitation signal to ensure a solvable identification problem remains an open question. Additionally, there is no requirement that har-
monic nonlinear basis functions need to be considered. The periodicity of the time operation of the system guarantees that any nonlinear function will be expressible as a Fourier series in the time domain. One possibility would be to use this fact to identify and cancel static friction that affects motors.
Appendix A

MCI Software

While the identification algorithm provides the theoretical framework to cancel motor cogging, the implementation of such a method on a resource constrained embedded system is another matter entirely. This chapter presents implementation details for the MCI based algorithms. The next chapter discusses implementation of the networked solution. With some derivations and careful selection of harmonic perturbations, the computational cost of identification can be made cheap. We will start by presenting the methods for injecting the anti-cogging signal as well as quantifying rotor ripple before getting into the details for the identification algorithm itself.

A.1 Anti-Cogging Signal Injection

The anti-cogging signal is generated by a Fourier series evaluated at the current rotor angle. A finite number of harmonics are summed with each consisting of a Cosine and Sine term. More explicitly, the estimated cogging torque, $\hat{\varphi}$, is given by

$$\hat{\varphi} = \sum_{n \in H} (\hat{\varphi}_{c,n} \cos(n\theta) + \hat{\varphi}_{s,n} \sin(n\theta)).$$  \hspace{1cm} (A.1)
However, the cogging torque estimates are never directly estimated. Instead, the anti-cogging signal obtained during estimation is injected. That anti-cogging signal is parameterized by two identified parameters for every harmonic and is injected as during identification.

While it is straightforward to specify this signal, robustly incorporating it into the controller posed several difficulties. The embedded computer in the MCI had inefficient floating point arithmetic. The sine and cosine routines in the standard C library were too slow to run in real-time. Instead, a lookup table based approach was used. Further, it is not desirable to inject the cancellation signal into the motor at high speeds. Cogging is not a problem in this operating regime and attempting to cancel for it may run into drive slew-rate limitations. The harmonics are rolled-off at high speed to solve this problem. This is done on a harmonic by harmonic basis. When the frequency of a harmonic (rotor speed times harmonic number) exceeds a threshold, it is attenuated. This ensures a smoother transition than if the entire anti-cogging system were simply shut-off.

A.1.1 Fast Trigonometric Evaluation

The embedded computer in the MCIs did not have hardware support for floating point operations. Consequently, the evaluation of Cosine and Sine using the standard C math library procedures was extremely slow and caused a loss of real-time with even a few evaluations. To avoid this difficulty, a lookup table was generated and used to evaluate Cosine between 0 and 90 degrees. Trigonometric identities were used to generate Cosine and Sine everywhere from this prototype function.

The lookup table used linear interpolation to generate values. To ensure that the run-time computational burden was as light as possible, a first order approximation was fit to each interval offline and the real-time system has a table of slope and offset values. Consequently, to evaluate the function prototype, a single multiplication and
addition are all that is needed. This is much cheaper than naively doing interpolation between the values at the interval endpoints.

### A.1.2 Rolloff at High Speeds

Cogging cancellation is needed at low speed only. At high rotational speeds, it is desirable to switch it off. Instead of simply turning off all compensation above a certain speed, each harmonic is rolled-off individually. This way, the transition is smooth and does not interfere with motor operation.

During identification, the speed of the rotor must be sufficiently slow such that all harmonics are faithfully injected into the system. Care must be taken to ensure this when selecting the rolloff frequency. If this is violated, identification of the higher harmonics will fail, a singularity in the input-output map will result, and incorrect and potentially dangerous values for the harmonic coefficients may be found.

### A.1.3 Bi-Directional Cogging Signal

Through experimentation it was discovered that the necessary cogging signal varied with rotor direction. For a given direction, the coefficients did not vary with speed or load. However, when the direction of rotation was reversed and identification was repeated, different results were obtained. To ensure cancellation of cogging in all operational regimes, the anti-cogging signal injector stores two sets of coefficients and computes the signal based upon those as well as the current rotor speed.

When the rotor is stopped, or nearly so, the act of switching between the two sets of coefficients can cause chattering problems. The problem arises due to discontinuities of the anti-cogging signals in either direction at a given angle. The control system can rapidly switch between the two directions, and in certain cases, the jump in torque can amplify this into a rapid vibration on the motor. To circumvent issues with chattering, the two sets of coefficients are linearly interpolated when the rotor
speed is sufficiently small. The transition speed region is chosen to be smaller than the nominal tracking speeds, but large enough to encompass the speed regions where rotor stiction can occur. In our experience, selecting this can be something of an art more than a science. The general advice would be to start small and increase it if chattering is observed.

This is an important implementation detail but it is not relevant to the methodology of identification. That is because identification speeds are kept out of this region. To calibrate this system, identification was performed twice, once in each direction. Since the complication of directionally dependent harmonic coefficients does not alter the identification methodology, it will not be discussed further and identification is only discussed in a single direction.

A.2 Measurement of Rotor Ripple Harmonics

While it is possible to identify the output ripple harmonics as either time-domain or angle-domain signals, angle-domain analysis lends itself to much simpler on-line identification. The fundamental reason for this is that the rotor angle will always be known, but the rotational frequency will need to be approximated.

The general strategy is to build a profile of rotor acceleration as a function of rotor angle. The acceleration will be the average acceleration seen at each pre-selected angle. The angle-domain harmonic coefficients are obtained by taking a Fourier decomposition of the profile. All of these steps are done incrementally and rely upon certain mathematical observations to keep them feasible in real-time.

A.2.1 Computing Accelerations

Perhaps the most difficult part of constructing the profile is estimating the acceleration. That is because computing a second derivative both requires future knowledge
of the signal and can greatly amplify any noise. The first challenge is not a major problem. That is because the rotor profile is an analysis tool and is not used for real-time feedback. This means that it can operate on a delayed signal without consequence. The issue of noise could prove problematic, but if the signal is already delayed, robust, band-limited acceleration estimator can be implemented.

To estimate the acceleration, a finite impulse response (FIR) filter is used. The FIR filter combines an estimate of the second derivative combined with a low-pass filter. This type of filter was chosen because it is possible to systematically design these filters to not distort the phase of a signal. It is possible to choose arbitrary magnitude and phase response and approximate it. This comes at the cost of a processing delay—the filters need to access future samples and therefore it is necessary to delay the output. The typical delay is half of the filter’s order worth of time steps. While such a delay would be problematic for feedback control, this delay does not hinder identification. That is because the identification algorithm is off-line and does not close any control loops in real-time. However, when building the rotor profile, it is important to use the correct angle. The rotor angle must be time delayed by the same amount as the acceleration is.

As a final note, the profile could also have been generated from rotor velocity instead of acceleration. Acceleration will more distinctly show the ripple. However, in a noisy environment this benefit may be completely negated by the increase in noise. It will be an engineering decision which works best in a particular case.

A.2.2 Averaging Methodology

Being fed a continuous stream of angles and accelerations, the job of the averaging subsystem is to build the rotor profile. This subsystem should perform well with two respects: it should acquire a rotor profile quickly and it should track long-term drifts in behavior. To achieve this, the averaging method we used starts as the arithmetic
A.2. RIPPLE

average. Then, after \( N \) values are averaged, the method switches to a geometric weighted average. For a hypothetical signal \( a_n \), the average \( \bar{a}_n \) is incrementally constructed by

\[
\bar{a}_{n+1} = \begin{cases} 
\frac{n}{n+1} \bar{a}_n + \frac{1}{n+1} a_{n+1} & n < N \\
\frac{N}{N+1} \bar{a}_n + \frac{1}{N+1} a_{n+1} & n \geq N
\end{cases}.
\]

(A.2)

To build the rotor profile, consider a discretization of the angles between 0 and \( 2\pi \). Denote these angles by the set \( \Theta \). Each angle corresponds to a bin where the averaged acceleration will be computed. Every time the rotor passes one of these angles, compute the acceleration of the rotor while it was at that angle and average it into that bin using the method of (A.2). Note that this method requires storing the visitation count as well as the average signal at each one of these angles.

A.2.3 Updating Harmonic Coefficients

Traditionally, a Fourier decomposition will be performed on the entire signal at once. However, in this application we cannot stop and run an expensive operation such as that. Instead, it is recognized that each change to the profile can easily be used to compute the corresponding change in the Fourier coefficients.

The Fourier decomposition of the angle-domain rotor profile is

\[
\alpha_{c,n}^\theta = \frac{1}{\text{count}(\Theta)} \sum_{\theta \in \Theta} \alpha(\theta) \cos(n\theta),
\]

(A.3)

with similar expressions for the sine coefficients. Each time the rotor profile is changed these coefficients need to be updated. Consider the case where the rotor profile average has just changed at the angle \( \theta' \) from \( \alpha_0 \) to \( \alpha_1 \). Then the change in the Fourier coefficients is

\[
\Delta \alpha_{c,n}^\theta = \frac{(\alpha_1 - \alpha_0) \cos(n\theta')}{\text{count}(\Theta)}.
\]

(A.4)
Using this incremental formula, all Sine and Cosine coefficients for each harmonic must be updated.

This method for maintaining the harmonic decomposition of the rotor ripple profile is numerically unstable. Over time, round-off error will accumulate and eventually render the values unusable. Therefore, this method cannot be used for extended periods of time. Since the duration of the experiments is short, perhaps 10 or fewer revolutions, numerical instability did not cause significant problems. If a long term decomposition is desired, it can be directly computed from the rotor profile. Obtaining the decomposition this way will work since the rotor profile is numerically stable since it relies upon averaging.

### A.3 Putting It All Together

Identification proceeds as a series of experiments. Each harmonic of has its anti-cogging input coefficients perturbed and the change in each harmonic of the output ripple is measured. The results of those tests are aggregated into the over-constrained linear system of equations (2.16). The least squares solution is found to extract the model parameters $M$ and $b$ for each harmonic. The iteration phase uses that model to improve the cancellation. Start with (2.17) and update using the update rule (2.18). Since each harmonic is independent, the identification for all harmonics can be carried out simultaneously. Consequently, identifying a dozen harmonics takes as many experiments as canceling one.
Appendix B

Telescope Interface Manager

The online analysis programs that were necessary in the single MCI case have been rendered obsolete by the real-time communications network described here. Those older functions permitted a single MCI to be calibrated in an extremely band-width limited environment. When all communication was moved to ethernet, the needs to do local analysis of data was no longer needed, and in fact it was detrimental to analysis since it returned summary statistics while forgetting the actual time history of data.

At the core of the new system is the Telescope Interface Manager (TIM). The TIM is a software program that serves as a central clearing house of real-time telescope information. It is run as a server on a rack-mounted computer in the GBT servo room and sits on the same network as the MCIs and PEIs. It handles the network configuration of attached devices and facilitates the synchronization and sharing of information across the system. The central abstraction that TIM provides is to make the telescope control system, though distributed across 24 MCIs and 2 PEIs, appear as a single system.

Each connected device is given a device ID by the TIM. Messages sent to specific device IDs will be forwarded to the correct device by the TIM. In order for a device
to be managed by the system, the TIM will need to be programmed to poll for it at a given IP address. Since all devices on the telescope sub-network have static IPs, this system is sufficient to uniquely identify devices and does not require that each device be individually configured with a unique identity. To start the connection, the TIM sends out a Request Logging packet that specifies the device’s ID. This causes the device to configure itself and begin sending state messages to the TIM.

The TIM is responsible for maintaining the set of the most recent states for each connected device. Then, other processes can request the TIM state, and a message with the latest states will be sent to the requester. Through the mechanisms of aggregation of state and dispatch to devices, the TIM provides an abstraction of the telescope system to make it appear as a single entity.

B.1 Telescope Devices

In addition to the TIM, there are two classes of network devices: those that correspond to telescope hardware and those that are software services. The MCI and PEI interface the telescope hardware into the system. The TIM aggregates all individual device state and routes commands to the appropriate devices. The most notable software service is the telescope monitor program that displays information on each connected hardware device. Calibration programs for the encoders and motors also interact with the TIM as software services. In those cases, a calibration script coded in Octave calls low-level C programs to perform actions. Each of those C programs is built off the network manager and connects to the TIM and reads or sends the data needed to perform a particular task, then terminates.
B.1.1 MCI Software

The software that controls each MCI has been modified to accommodate the network interface. The MCI software consists of several internal state variables that control the device behavior. Those that were deemed necessary to control for cogging cancellation and calibration were given messages that modify them. The cogging coefficients, the cogging compensation enabling flag, and the command source flag were all given means to be set with appropriate telescope protocol configuration messages. An MCI’s anti-cogging coefficient table can be loaded remotely over the network. This is an essential feature for calibrating the motors. There is a special message to this effect that contains a table of cogging coefficients for both directions of rotation. Further, there is another message that enables or disables the compensation of cogging without affecting the stored anti-cogging table.

Similarly to configuring the device, the real-time state information, consisting of the encoder count, the requested motor torque, and the applied motor torque, are all sent to the TIM server when the device is in logging mode. A device can either be actively logging or in a dormant state. The device waits, not sending any messages, until it receives a device configuration message from the TIM. That message configures its device ID and starts logging. When the device is actively logging, it will send state update packets to the TIM address that sent the configuration message until an internal timer has expired. To prevent timing out, the TIM periodically re-sends configuration packets to all connected devices, resetting their timers and ensuring they stay active. This architecture allows the communication activity to be cut out when no TIM program is running, causing the system to revert to its un-modified behavior. This allows us to keep the network code on the telescope when we are not experimenting; and to start an experiment by simply starting a TIM process.

Since the TIM system was originally intended to function as a logging system that facilitated calibration, each device has its own time domain. All state messages
transmitted to the TIM are time-stamped with the devices local time. When the TIM receives such a message, it is stored with the device time and the TIM time as well. It is up to the analysis program to make sense of the 27 different clocks on the system.

**Control**

The MCI software has a flag that controls the source of the motor torque command. During normal operation, it is configured to read it as an analog voltage sent to it by the rate-loop board. Also, as discussed, the motor has an internal rate feedback controller for performing calibration when the motor is disconnected from the system. Finally, a networked control mode was added. When the MCI is configured in this mode, it applies a torque as given to it by a telescope protocol MCI command message. There is an internal timeout that limits how long the MCI can go without receiving one of these command packets before triggering a shutdown. On the telescope’s version of the MCI software, there is a compile-time declaration that prohibits the system from using any command source but the telescope’s rate-loop board. This declaration prohibits the system from acting on these configuration or control messages and they are ignored by the MCI.

As an experiment, the ability to specify an armature current command over the network was added. This permitted an MCI to be controlled over the existing ethernet connection. This was attempted in the lab and was quite successful. The system was operated with a 100 Hz sampling rate and implemented a Proportional-Integral (PI) controller. Additionally, cogging compensation was computed on the remote computer added to the normal feedback. This worked to cancel cogging and suggests that local compensation is not strictly necessary. An additional network command message was added to the protocol to accommodate control over the network.

The way the system was developed leaves much wanting from the aspect of control.
A large body of pre-existing code had been written for the MCI both by myself and by NRAO software developers. In order to interface these devices to the TIM as quickly as possible, it was necessary to re-use most of this code. Consequently, the basic sampled operation was kept—too many sub-systems depended upon a constant sampling rate. In a proper networked control system, all devices should be under a unified clock domain. To get every device to sample and apply their new control value simultaneously is a job for a phase-locked loop. Such a system reads from a central clock and tweaks the sampling rate, slower or faster, to force its local clock into lock-step with the master clock. Unfortunately, this could not be implemented due to the above software constraints. Should NRAO choose to implement networked control over ethernet, the software would need to be re-worked to build in the assumption of a variable sampling rate.

B.1.2 PEI Software

Similar to the MCI, the PEI software logs relevant state data, loads calibration tables, and accepts remote enable/disable commands for the compensation. The timestamped state packet consists of the most recent encoder reading and the computed compensation value. Reinterpolation calibration tables can consist of up to 10 harmonics, each with a harmonic number, a cosine coefficient, and a sine coefficient. Additionally, there is a bit shift count to control the bit at which the interpolation interval is computed.

While the PEI has a network-enabled CPU, the translation of the encoder reading to the CCU is handled by an attached Field Programmable Gate Array (FPGA) board. The CPU of the networking card communicates to the FPGA by means of a shared memory bus. To compute the compensation, the CPU reads the encoder angle from the correct register, computes the reinterpolation correction, and writes that correction to the appropriate register. Internally, the FPGA shifts this correction 4-
bits to the right before adding it to the encoder reading. Consequently, the correction
given to the FPGA must be 16 times larger than what is to appear on the output.
In this document, all PEI corrections have been normalized to their equivalent 24-bit
values. Whenever the FPGA reports an angle to the CCU, it first adds the last given
trim to it. Since these two systems operate at different sampling rates, this was seen
as the easiest work-around for the situation.

B.1.3 Monitoring and Control

With the above described configuration, any program on the telescope’s restricted
sub-network can interact with the TIM and the devices, provided it implements
the telescope network protocol. Such a program is called a software service. Several
such programs were created; some only for lab use and others meant for the GBT.

One such GBT service is a monitoring program. This system can be run anywhere
on the NRAO network and connects to an active TIM process. Once connected, the
program allows the user to view a summary page of connected telescope devices as
well as individual pages for each device. Both the current state variables as well as
the device’s configuration variables are displayed in a manner appropriate for the type
of device. This program has proved invaluable for debugging network problems and
verifying that configuration variables are indeed being changed as requested.

The entire purpose of developing this system was to perform calibration on the
encoders and the motors. To accomplish this, a slew of small, single purpose telescope
services were written in C and use the Network Manager code. These appear as
command line utilities that perform a certain interaction with the TIM server and
then quit. For example, they may load an anti-cogging calibration into a particular
MCI or may modify the compensation enabling flag. Another one such program
dumps TIM state data to a file at a specified sampling rate for a requested duration of
time. The calibration code is programmed as a collection of Octave scripts. Function
wrappers have been coded for all these interactions with the telescope, so in the Octave code, telescope actions appear as function calls using the native data types. This setup has worked well for the task it was intended.

In the laboratory setup, network based feedback of a single motor was developed as a telescope service. The client program permits the user to set a rate setpoint, proportional and integral feedback gains, and to display the system state variables in an animated scrolling graph. The client program queries the TIM for the motor’s state, computes the feedback, and sends the command to TIM, which in turn forwards it to the motor.

Even though this network control arrangement was never used on the GBT, it was a sufficiently difficult task that by doing it, several bugs were exposed. To successfully perform feedback control, tight timing requirements must be met. Analysis of the round trip delay helped to detect a network manager bug that caused significant delay in handling events. Further, the ability to command the motor to move and to change its calibration permitted debugging of the communication network and greatly eased the transition to the telescope. Implementing real-time control in the lab exposed several bugs that would have been very difficult to uncover on the GBT and may have been impossible to fix otherwise.

B.2 Software and Communication

All devices on the network speak the same language. A standardized set of messages has been settled upon and comprises the telescope protocol. The standard feature of all messages is that their first byte is a code for the type of message it is. Following that is a predetermined length of data. All messages of a given type have the same data values in the same format.
B.2. SOFTWARE AND COMMUNICATION

APPENDIX B. TIM SOFTWARE

B.2.1 Network Manager Software

At the heart of each device on the developed network is an event-driven process that responds to received packets and transmits state updates. This is accomplished by using a system we have dubbed the Network Manager. The network manager code is shared between all devices on the communication network. It provides functionality to serialize and send outgoing messages and to receive incoming ones. The modularity of this design has facilitated the no fewer than six programs that communicate over the network protocol with minimal difficulty.

Each message type can have a callback function that can be bound to it and every different program has implemented callback functions for some or all message types. These callbacks are passed the data from the message and take specific actions in the program that they are part of. They may change global variables, respond with one or more messages, or do nothing. If no callback is associated with a message tag type, then such messages are ignored. So, when coding a network service, all that is necessary is to start a network manager and to code the appropriate message handlers.

In addition to message events, the Network Manager provides for a programmable sampling rate which triggers a timeout event with the prescribed regularity. This permits synchronous behaviors, such as sampling a value, computing a feedback command, or updating a display, to be handled concurrently with the event based message system. By using the timeout event, the network manager guarantees that no other event handler will be run at the same time. Thus one could program a system as if it were single-threaded. This removes the need for inter-process synchronization and communication and makes correct programs far easier to code.

There are two copies of the network manager code—one for Linux PCs and one for the MCI/PEI embedded systems. This split was necessary because the way networking is handled at an operating system level varies between these two architectures.
All of the top level interfaces have been preserved.

The MCI/PEI architecture necessitated that the timeout event be ignored. To keep with the fixed sampling time, messages are read on fixed intervals after the real-time computations are made, but before going back to sleep. This keeps this system single-threaded, but introduces small delays due to the sampling rate granularity.

### B.2.2 Communication Protocol

Below is a paraphrased listing of the major Telescope Protocol messages. Each carries with it certain specified data that is used by the recipient. Most messages that are about a device include a Device ID. In general, when a device specific message is sent to the TIM, it forwards it to the appropriate device.

**Logging Request** Sent by TIM to hardware devices. Specifies the Device ID for the device and signals the device to start sending logging packets to TIM.

**Request Telescope State** When sent to the TIM, it responds with a Telescope State message.

**Telescope State** A large packet that consists of all state information from all 26 telescope devices. To keep bandwidth requirements low, it does not include calibration data.

**Request Configuration** When sent to TIM, returns the corresponding device calibration message for the requested Device ID.

**MCI State** Sent by MCIs to TIM when logging. Contains a local timestamp, the current encoder reading, the requested command, and the applied motor command.

**MCI Configuration** Contains the cogging coefficients, the compensation flag, and the command source.
B.2. SOFTWARE AND COMMUNICATION  APPENDIX B. TIM SOFTWARE

**MCI Set Table**  When sent to an MCI, specifies the anti-cogging coefficients to use.

**MCI Compensation Enable**  Contains a flag that specifies whether the cogging compensation is to be enabled or disabled.

**MCI Set Source**  Specifies where the MCI should look for the requested motor command. Ignored on the GBT—lab use only.

**MCI Command**  When the MCI’s command source is the network, this packet sets the requested motor torque. When sent to TIM, it is forwarded to the appropriate MCI named by the Device ID. Ignored on the GBT—lab use only.

**PEI State**  Sent by the PEI when logging. It contains a timestamped reading of the axis encoder and the value of the correction applied to it.

**PEI Configuration**  Contains the current reinterpolation table coefficients and the compensation flag.

**PEI Set Table**  When sent to the PEI, causes it to use the specified reinterpolation coefficients.

**PEI Compensation Enable**  Either enables or disables the reinterpolation correction, depending upon the flag.
Appendix C

Telescope Calibrations

C.1 Individual Motor Tuning

The calibration was performed by Tim Weadon over the course of two weeks in March 2014. Each motor was disconnected from the telescope and an automated calibration routine was run. The resulting anti-cogging configurations were saved into MCI memory and were used for testing. Those calibrations appear in the table below.
Table C.1: Configuration obtained by individual motor tuning.

<table>
<thead>
<tr>
<th>Motor</th>
<th>Harmonic</th>
<th>Cosine</th>
<th>Sine</th>
<th>Cosine</th>
<th>Sine</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>Forward</td>
<td></td>
<td>Reverse</td>
<td></td>
</tr>
<tr>
<td>EL-1</td>
<td>1</td>
<td>2.1</td>
<td>0.4</td>
<td>−1.9</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>−0.3</td>
<td>−0.6</td>
<td>0.1</td>
<td>−0.8</td>
</tr>
<tr>
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<td>3</td>
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<td>−0.7</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
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</tr>
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Appendix D

Encoder Calibration Results

In this appendix, results from a series of encoder calibration experiments appear. Calibration was performed over a range of angles of the elevation structure.
### Table D.1: Encoder calibration results obtained at various angles. All coefficients are 24 bit equivalent corrections, to be added to 24 bit position measurements. All tests were performed with the elevation structure moving at 15 arcseconds per second in the positive direction.

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Table D.2: Derived model parameters for the elevation axis encoder at various angles of identification. Quantities are expressed in polar format, with angles being in radians, offset magnitudes in equivalent 24 bit values, and matrix magnitudes being unitless gains. All tests were performed at 15 arcseconds per second in the positive direction. The matrix parameters are governed by the system’s dynamic response and are relatively position invariant. Meanwhile the offset changes drastically over the range. This indicates that the tracking error does indeed vary over the range of operation of the elevation structure.
Bibliography


