A STUDY ON THE PHYSICS OF ICE ACCRETION IN A TURBOFAN ENGINE ENVIRONMENT

by

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Nomenclature

\[ A = \text{unit area} \]
\[ \alpha/D_v = \text{Lewis Number} \]
\[ B = \text{ambient static pressure} \]
\[ b = \text{relative heating factor for super cooled liquid water icing} \]
\[ b' = \text{relative heating factor for ice crystal icing} \]
\[ c_{p,i} = \text{specific heat of ice} \]
\[ c_{p,w} = \text{specific heat of water} \]
\[ c_{p,a} = \text{specific heat of air} \]
\[ E_{xx} = \text{energy} \]
\[ E_m = \text{airfoil collection efficiency} \]
\[ \text{FAA} = \text{Federal Aviation Administration} \]
\[ \text{FOD} = \text{foreign object debris} \]
\[ h/c = \text{ratio of project height over chord length for an airfoil} \]
\[ h = \text{enthalpy of the moist air flow at ambient static temperature} \]
\[ h_c = \text{unit convective heat transfer coefficient (energy/time/area/temp)} \]
\[ h_m = \text{unit convective mass transfer coefficient (mass/time/area)} \]
\[ h_{s*} = \text{saturation enthalpy of the air at adiabatic saturation temperature} \]
\[ h_{w*} = \text{enthalpy of the water vapor at the adiabatic saturation temperature} \]
\[ \text{HPC} = \text{high pressure compressor} \]
\[ \text{HPT} = \text{high pressure turbine} \]
\[ \text{ICC} = \text{Ice Crystal Consortium} \]
\[ \text{IWC} = \text{ice water content (mass ice / volume of air)} \]
\[ L_{sub} = \text{latent heat of sublimation per mass} \]
\[ L_{evap} = \text{latent heat of evaporation per mass} \]
\[ L_{fus} = \text{latent heat of fusion per mass} \]
\[ \text{LPC} = \text{low pressure compressor} \]
\[ \text{LPT} = \text{low pressure turbine} \]
LWC = liquid water content (mass water / volume of air)
M_\text{xx} = mass
m’ = accumulated ice particle melt fraction
MTM = Modified Tribus-Messinger
MW_w = molecular weight of water
MW_a = molecular weight of air
n = impinging super cooled liquid water freeze fraction
n’ = impinging ice particle accumulation fraction
n” = surface water freeze fraction
NASA = National Aeronautics and Space Administration
NPRM = notice of proposed rulemaking
NRCC = National Research Council Canada
O = impinging super cooled liquid water accumulation fraction
O’ = impinging ice particle accumulation fraction
P_{AM} = logarithmic mean density factor
P_\infty = ambient partial pressure of water vapor
P_{\text{sur},i} = surface partial pressure of water vapor over ice
P_{\text{sur},w} = surface partial pressure of water vapor over water
P_w,s = saturation water vapor pressure at T
Q_{\text{surface}} = surface energy transfer
Q_{\text{conv}} = convective energy transfer
Q_{\text{evap}} = evaporative energy transfer
Q_{\text{fusion}} = fusion energy transfer
Q_{\text{sub}} = sublimation energy transfer
Q_{\text{sensible}} = sensible energy transfer
Q_{\text{viscous}} = viscous energy transfer
Q_{\text{kinetic}} = kinetic energy transfer
r = recovery factor
R_w = unit mass rate of water impingement (mass/time/area)
R_i = unit mass rate of ice impingement (mass/time/area)
R_{\text{sub}} = unit mass rate of sublimation (mass/time/area)
\( R_{\text{evap}} \) = unit mass rate of evaporation (mass/time/area)

RATFac = Research Altitude Test Facility

RH = percent relative humidity

t = static temperature

t_o = total temperature

t_{\text{sur}} = surface temperature

t_p = impinging particle temperature

t_{\text{se}} = equilibrium surface temperature

t_\infty = ambient static temperature

T = absolute temperature of air sample to calculate \( P_{w,s} \)

t/c = thermocouple

T-M = Tribus-Messinger

U_o = airspeed

V_\infty = free stream velocity

W = humidity ratio (mass water vapor / mass of dry air)

W_s = saturation humidity ratio (mass of water vapor / mass of dry air)

W_s^* = saturation humidity ratio at the adiabatic saturation temperature

W_{\text{surf}} = humidity ratio at the surface

W_\infty = humidity ratio in the free stream

**Subscripts**

\( \infty \) = ambient

1 = plane 1

2 = plane 2

a = air

c = convective

conv = convective

evap = evaporation

fus = fusion

i = ice

in = in
\( o \) = total
\( \text{out} \) = out
\( p \) = particle
\( p,a \) = constant pressure, air
\( p,i \) = constant pressure, ice
\( \text{psia} \) = pounds per square inch absolute
\( p,w \) = constant pressure, water
\( s^* \) = saturation of air at adiabatic saturation temperature
\( \text{sub} \) = sublimation
\( \text{sur} \) = surface
\( \text{sur},i \) = surface covered with ice
\( \text{sur},w \) = surface covered with water
\( w \) = water
\( w^* \) = water vapor at adiabatic saturation temperature

**Superscripts**

' = variable related to impinging ice particles
'' = variable related to accumulated surface water

**Overbar**

• = indicates variable is a rate per unit time
Numerous turbofan engine loss of thrust control events have led to a theory that ice is accreting on initially warmer than freezing internal engine hardware and affecting normal operation. The phenomenon is termed ice crystal icing. Previous ice crystal icing research on a simple airfoil empirically identified the effects of pressure, temperature and relative humidity on the equilibrium surface temperature and the characteristic ice that accretes. In these cases with freezing fraction and impinging particle temperature as experimental unknowns, the onset of ice accretion on an initially warmer than freezing surface was found to correlate with a below freezing thermodynamic wet-bulb temperature. In the present work, a novel ice accretion model, based on a modified Tribus-Messinger (MTM) surface energy balance, is developed. Implementing the model, the effect on the equilibrium surface temperature and freezing fraction near the leading edge stagnation region of an airfoil operating in icing conditions is investigated by varying the parameters of ambient static pressure, ambient static temperature, relative humidity and impinging particle temperature. The previously identified effects of pressure, temperature and relative humidity are identified as well as a new impinging particle temperature effect. A below freezing adiabatic saturation temperature, analogous to the thermodynamic wet-bulb temperature is calculated and shown to correlate with the predicted onset of ice accretion. Implementing the MTM model to investigate the physics of the modified surface energy balance, the pressure, temperature, relative humidity
and impinging particle temperature effects are shown to be coupled within the latent, convective and sensible energy terms of the modified balance. The model predicts the observed onset of accretion and characteristic ice that forms on the surface from two experimental test cases. Control of the onset of ice accretion for these cases is demonstrated by varying the inputs to the model. It is also shown that by changing the impinging particle temperature for a given test condition the onset and characteristic ice that accretes can be influenced and predicted. Since particle temperature influences the surface temperature the wet-bulb temperature alone may not always predict the onset of ice accretion.
Chapter 1: Introduction

Turbofan Engine Loss of Power Field Events

The Ice Particle Threat to Engines in Flight (1), was presented at the 44th AIAA Aerospace Sciences Meeting and Exhibit in Reno, Nevada in January 2006. It documented information the authors learned while investigating approximately forty-six turbofan engine high altitude field events characterized as a temporary loss of engine power during revenue service. The paper developed the hypothesis that the loss of power or loss of thrust control events were related to operation in high altitude ice crystal clouds at tropical, warmer than standard, day type temperatures. The majority of the field events occurred at altitudes outside of the FAA established engine icing certification envelope of 22,000 ft (2).

According to the authors, aircraft operating at high altitudes can ingest ice crystals into their engines. As a result, ice accretes inside the engine’s low pressure compression system along the air flow path leading to the engine core. In this region of a turbofan engine, the flow path hardware is typically warmer than the freezing temperature of water and, until this paper was published, was thought not to be susceptible to ice accretion involving ice crystal ingestion. The authors suggested that ice accretion in this area of the engine can affect the normal propulsion system operation and result in a loss of power or air flow path hardware damage. The paper termed this new phenomenon “Ice Crystal Icing”.

Locations of Field Events

Figure 1, later adapted from (1), demonstrates that the forty-six loss of power events occurred worldwide. When this chart was created many suspected events appeared to have occurred in the South Asian Pacific region of the world which the authors surmised to be more of a tropical environment, see map inset.

![World Map](image)

Figure 1: A world map adapted from (1) demonstrating approximately 46 loss of power events that have occurred suspected to be related to operation in ice crystal clouds at high altitude.

Common Symptoms Described for a Typical High Altitude Loss of Power Field Event

Several common observations were documented by the authors’ investigation as being associated with the loss of power field events (1):

1. Uncommanded, Loss of Thrust Control (LOTC)
2. Flameout
3. Roll back
4. Stall/surge
5. Total air temperature probe anomaly (rises to 0 C)
6. Tropical day type ambient temperatures at flight level
7. Operation outside of inclement weather certification envelope (>22K ft)
8. Operation in the vicinity of highly convective clouds/storms.
9. No noticeable airframe icing
10. No flight-radar weather echoes at flight level
11. Precipitation on cockpit wind screen (heated)
The authors also note that these symptoms are typically associated with operation in inclement weather but that the majority of pilots interviewed observed no notable inclement weather preceding the events and were operating outside of the icing envelope. Many revealed they were operating in the vicinity of convective storm clouds.

**Turbofan Engine Internal Environment**

In external airframe icing, the aerodynamic surface and the ambient air are both typically below freezing temperature. It is expected that impinging super cooled liquid water will readily accrete on an aerodynamic surface operating in this external to the engine environment. For the environment internal to a turbofan engine, both the ambient air and the aerodynamic surfaces are warmer than freezing temperature under normal operation. It is not expected that impinging super cooled liquid water will readily accrete on an aerodynamic surface in this internal to the engine environment.

A schematic of the cold modules, engine sections leading up to the combustor, for a typical turbofan engine is depicted in Figure 2. The environment shown is one that goes from an external to the engine sub-freezing ambient static temperature to an internal to the engine warmer than freezing environment. The internal to the engine static temperature can reach up to 250F, depending on engine operation, due only to the compression of the air as it passes through the low pressure compression stages. The relatively slow moving cloud particles at ambient static temperature external to the engine are simultaneously overtaken
by the aircraft, traveling at some airspeed, entrained into the engine airflow and ingested. Considering the high relative velocity with the aircraft, the ingested air and cloud particles typically travel from outside the engine to the ice crystal icing region of the low pressure compression system, shown as a red outline in Figure 2, in a time frame that is typically on the order of $1/100^{th}$ of second. Referring to Figure 2, the majority of air entering the red-outlined low pressure compression system (LPC) has the potential to flow into the engine core. The core is made up of the high pressure compressor (HPC), combustor (not shown) and high pressure turbine (HPT) systems (not shown). During off-design operation, the inter-compressor bleed ducts are open allowing various mass flow rates of air and foreign object debris (FOD) such as ingested or shed ice to bypass the HPC. During on design operation these ducts are closed and all air mass flow and FOD is ingested into the engine core. The air that passes through the core also passes through the low pressure turbine (LPT) system (not shown). The core generates high pressure, high temperature air flow that expands through the HPT to power the HPC, the high pressure system. The air then passes through the LPT to power the LPC and the fan, the low pressure system. The high pressure system is aerodynamically coupled to the low pressure system. The majority of thrust in a turbofan engine is provided by the fan component of the low pressure system. A small portion comes from the core flow. All core airflow entering the HPC must pass through the LPC which provides a boost to both the pressure and temperature of the airflow prior to entrance into the HPC. The AIAA paper
suggests that it is the normal operation of the core or the HPC operability that is affected by the ice crystal icing phenomenon that occurs in the LPC (1).

The high relative velocity between the particles and the engine hardware, translates to a very short residence time between initial ingestion and the particle reaching the hardware inside the red outlined area in Figure 2, where ice crystal icing is theorized to occur (1).

In comparison, the engine region where super cooled liquid water related or conventional engine icing occurs is bracketed in Figure 2. In this region the ambient air and engine surface temperature are at or below freezing. A key fundamental difference between ice crystal icing and conventional super cooled liquid water icing is that ice crystal icing is thought to only be able to occur on engine hardware that is normally at a warmer than freezing temperature prior to entering the ice particle cloud.

A question that arises from the ice crystal icing theory is if the ice particles are initially at sub-freezing ambient temperatures external to the engine, what temperature are they when they reach the ice crystal icing region of the engine? Are they melted, frozen or some combination of the two?

**Ice Crystal Icing Hypothesis**

The ice crystal icing hypothesis developed by the authors in the 2006 AIAA paper (1) is that while pilots perform a fly around to avoid operating the aircraft through convective storm clouds, the aircraft turbofan engines ingest ice crystals
outpouring from the convective storm cells. These ingested ice particles can
impinge upon initially warmer than freezing engine hardware inside the low
pressure compression system, break up, accumulate and melt. This creates a
scenario where initially warmer than freezing hardware can be cooled and ice
can begin to accrete upon it. The accreted ice can affect the normal operation of
the engine.

Figure 2: Schematic of typical high bypass turbofan engine cold modules, depicting hypothesized
potential regions where ice crystal icing could occur adapted from (3).
Chapter 2: Literature Review for Ice Crystal Icing in a Turbofan Engine Environment

Engine and aircraft manufacturers as well as the major certification agencies have been working together to address the ice crystal icing phenomena from both an administrative side, advising pilots how to recognize and avoid ice crystal icing environments and a science and engineering side, funding fundamental research to understand the physics of ice crystal icing in turbofan engines. An international industry sourced organization was formed between several engine and aircraft manufacturers. This group is named the Ice Crystal Consortium (ICC) and it has developed a technology roadmap towards understanding not only how the ice crystal issues will affect the certification of new products but also how to understand the fundamental physics of ice crystal icing in a turbofan engine environment. The ICC is a sub-group of the larger Engine Icing Working Group (EIWG) which works to improve engine inclement weather operation and certification. The ICC continues to track and investigate suspected ice crystal icing related events. Since Figure 1 was published in 2006 with forty-six field events, the ICC now has documented over two hundred and twenty field events that are suspected to be directly related to operation in a high altitude ice crystal environment.

Federal Aviation Administration Proposed Rulemaking

Based largely on the ICC efforts to track, investigate and understand these ice crystal icing related field events, on June 29, 2010, the Federal Aviation Administration (FAA) published a notice of proposed rulemaking (NPRM) in the
Federal Registrar (4). Among several additional proposed changes to the current inclement weather certification process, this document proposes to require aircraft and aircraft engine manufacturers to certify their products for operation in a high altitude ice crystal icing environment. The ICC effort and the proposed FAA rulemaking document have led directly to international engineering and scientific interest. This led to current ongoing efforts to investigate both the nature of high altitude ice crystal environments that aircraft and aircraft turbofan engines are or may be operating in and the fundamental physics of ice crystal ice accretion in an internal turbofan environment. This chapter will give a summary of the existing research on ice crystal icing in a turbofan engine environment.

**MacLeod, et al, 2007, NRC Canada Research, Ice Crystal testing on a warm flat plate with stagnation regions implemented**

The ICC funded initial research at the National Research Council Canada (NRCC). This work conducted by MacLeod et al (5), was based on the hypothesis that ice crystals can accumulate at stagnation regions, cool down the

![Figure 3: A heated flat plate with stagnation region implemented (5)](image)
warmer than freezing surface and result in the accretion of ice. The scope of the experiment was to operate a heated flat plate implemented with stagnation regions in an ice crystal cloud. Figure 3 depicts the test article. The ice accreted near the surface mounted blocks. It demonstrated that the rapid buildup of ice on a heated surface with a stagnation region was possible.


In 2010 Boeing and NRCC conducted joint research to investigate the s-duct transition region of a turbofan engine (3). This is typically the region in the low pressure compression system between the exit plane of the LPC and the entrance plane to the HPC, the inter-compressor bleed duct is also typically located in this region. The experiment modeled a heated airfoil protruding perpendicular to the flow path along the outer wall of the s-duct transition between the LPC and HPC and demonstrated that it was possible to form ice on the airfoil outer wall stagnation region as well as the outer wall surfaces surrounding the airfoil. Figure 4 depicts the test article before and after a simulated ice crystal ingestion event.

Figure 4: Model of a turbofan engine S-Duct region in the LPC prior to and after a simulated ice crystal ingestion event (6)
National Research Council Canada Ice Crystal Accretion Physics Research

The National Research Council Canada is conducting ongoing research under a collaborative research agreement with the ICC. Much of this work is proprietary and not publically available (5), (7).

NASA Turbofan Engine Icing Related Research

The National Aeronautics and Space Administration (NASA) is engaged in ongoing research efforts working to develop a physics based engine performance model incorporating the effects of ice crystal icing (8), (9). To validate this model and to perform full scale research on turbofan engines, NASA developed an existing altitude engine test facility, the Propulsion Systems Laboratory test cell three (PSL-3), with the capability to create an ice crystal cloud (8). This operational facility is capable of simulating a high altitude ice crystal icing environment on a full scale engine throughout the engines entire mission profile.

NASA is also working to incorporate ice crystal physics into the existing ice accretion modeling codes (10) and conducting joint research with NRCC to study the fundamental ice crystal accretion physics in a turbofan engine environment on a warmer than freezing surface operating in liquid water and mixed phase icing conditions. This joint NASA / NRCC research is reviewed in more detail in Chapter 3.
NASA High Ice Water Content (HIWC) Campaign

NASA is leading an international effort to scientifically explore and quantify high ice water content in the natural high altitude ice crystal environment. This research intends to develop measurement technologies and test methodologies to perform flight testing near convective storms while measuring the environment the aircraft is operating in. It is called the High Ice Water Content or (HIWC) campaign (11), (12).

European Research Efforts

Direction générale de l’armement Engine Icing Related Research

The French Government, General Directorate for Armament (DGA) have developed several ice crystal icing altitude tunnels used to conduct ice crystal icing related research on air data sensors and related test articles. They have also developed an ice crystal icing research and test capability in their existing altitude engine test facility.

High Altitude Ice Crystal (HAIC)

The European Aviation Safety Agency (EASA) has sponsored an international high altitude icing research program aimed at developing the understanding, measurement and modeling capacities of the phenomena to prepare the European aviation industry with acceptable means of compliance to future regulation changes (13).
Chapter 3: NASA and NRCC Joint Research Details and Overview

In 2010 NASA and NRCC began a joint research effort to study the fundamental physics of ice crystal ice accretion. A collaborative research agreement was established. The continuing research is being conducted in the NRCC Research Altitude Testing Facility (RATFac) on a simple airfoil design to simulate operation in a turbofan engine low pressure compressor (LPC) environment. To date the research has identified three key effects for ice accretion on warmer than freezing surfaces operating in icing conditions: ambient temperature, ambient pressure and ambient humidity. The joint research also correlated the combined effects of these parameters on the onset of ice accretion in this environment with a lower than freezing thermodynamic wet-bulb temperature (14), (15) and (16).

Research Altitude Test Facility (RATFac) Description

A schematic of the NRCC RATFac is shown in Figure 5. The simple airfoil test article used in the joint NASA/NRCC ice accretion studies is depicted in Figure 6. The wind tunnel section consists of an inlet duct with a bell mouth rectangular inlet and test section. The ice laden air-jet, liquid water spray plume and conditioned air entering the altitude chamber are drawn into the inlet bell mouth by a downstream compressor. These three flows mix together along the inlet duct before reaching the test cell. The cloud delivery system is complex and it is difficult to quantify the temperature and make-up of the cloud in the test section.
Joint Research Objective

The objective of the joint research conducted is to investigate the aerodynamic and heat transfer characteristics of the test article and to study the effect of key parameters on the onset and growth of ice accretion on the test article operating in the simulated LPC environment of a turbofan engine. The test cell used in the research is depicted in Figure 7. A 3-D Model of the test cell is shown in Figure 8.
Figure 7: Typical test cell set-up of the NASA test article in the NRCC experimental test facility (14).

Figure 8: A 3-D model of the Fundamental ice Accretion Test Cell showing the inlet bell mouth and the exhaust duct through which a downstream compressor pulls air through the test cell. The test article is shown in red (14).
Joint Research Findings

The ongoing joint NASA/NRCC research began in 2010 with initial testing that measured the aerodynamic and heat transfer characteristics of the test article and established the hypothesis that temperature, altitude and humidity play a role in the onset of ice accretion on its surface (14). Further joint NASA/NRCC research conducted in 2011 conducted on a test article in a mixed phase environment identified the connection between the thermodynamic wet-bulb temperature and the onset of ice accretion on its surface (15). Most recent, joint NASA/NRCC research conducted in 2012 at the RATFac on a heated test article, identified the thermodynamic wet-bulb temperature as a key indicator to the onset of ice accretion on a warmer than freezing temperature surface operating in both liquid water and a mixed phase (solid and liquid water) icing environment (16).

Joint NASA/NRCC Research Opportunity

One fundamental difficulty with analyzing the Joint NASA/NRCC research data is that it is not possible to know or measure the temperature of the ice and water particles that impinge onto the warm surface of the test article. It is a commonly accepted practice that, given enough residence time, clouds in nature and clouds generated in test facilities have particles that are the same temperature as the ambient static temperature in which they are entrained. For ingestion into an engine with very low residence times this may not be the case. This is a key missing parameter and presents an opportunity for improvement when
investigating the fundamental physics of ice crystal icing in these experiments. Knowledge or improved insight of the impinging particle temperatures is needed.

The joint NASA/NRCC research effort simulates an idealized steady state, LPC, turbofan, engine icing environment. The actual engine environment is not steady state nor is it feasibly measurable in the field. Further, referring to Figure 2, when an ice particle is ingested into an engine it must pass through multiple stages to arrive at the region of interest for ice crystal icing research. The residence time of the ice particle as it travels from an external to the engine cold and near saturated environment to the region of interest in the warm and relatively dry turbofan engine environment, is estimated to be on the order of 1/100th of a second. The realities of a turbofan engine internal environment, operating in icing conditions make it difficult to predict and model the temperature and make-up of an icing cloud that impinges onto the warmer than freezing engine hardware.

Researchers can estimate to some degree the size and make-up of the particles composing the icing cloud in the RATFac. However, researchers cannot yet measure the temperature of the particles making up the simulated icing cloud. A valuable result of these experiments is the observation of the characteristic or type of ice that forms on the surface of the test article.
Joint NASA /NRCC Research Need

Current Joint NASA NRCC Experimental testing has identified several key parameters that appear to effect ice accretion in a turbofan engine environment on a warmer than freezing surface:

1. Ambient Pressure
2. Ambient Temperature
3. Ambient Relative Humidity
4. Thermodynamic Wet-bulb temperature
5. The ice and liquid water content of the cloud

To help gain insight into physics of energy transfer on the surface from the experimental results and plan future experiments, a research tool is needed to better understand the impinging cloud particle temperatures.

Chapter 4: Dissertation Research Objectives and Approach

An opportunity exists to help researchers gain additional insight from the data generated by joint NASA/NRCC fundamental ice crystal accretion research. The research need is to use the results of the joint NASA/NRCC research data and gain a better understanding of the physics of surface energy transfer of ice crystal icing from the simulated turbofan engine conditions, impinging cloud particle temperature and the characteristic ice that accretes.
Research Question

When the ambient air temperature is above freezing, what effect do impinging droplet temperature, relative humidity, ambient temperature and ambient pressure have on the surface temperature and the potential for ice accretion formation, near the stagnation region of a simple airfoil operating in icing conditions?

Research Objectives

The objectives for this research are as follows:

1. Investigate icing problem using a computer model, developed implementing a modified Tribus-Messinger heat balance model, to investigate the effect of droplet temperature, ambient temperature, relative humidity and related parameters on the equilibrium surface temperature of a simple aerodynamic surface operating in icing conditions in the LPC environment of a turbofan engine.

2. Gain an understanding of the physics of ice accretion on warm surfaces by studying convection, latent, sensible, kinetic and viscous terms governing the heat balance in a turbofan engine environment.

3. Study the behavior of the thermodynamic wet-bulb temperature as a predictor for the onset of ice accretion in the NASA/NRC Canada Joint Research.

4. Implement the computer model and understanding of the physics gained to investigate the NASA and NRC Canada Joint Research experimental results to help provide insight and guidance for future experimental work.

Research Approach Overview

The research approach is as follows:

1. Understand and modify the Tribus-Messinger (T-M) heat balance model to separate the droplet temperature from ambient temperature.

2. Develop a computer program to implement the modified T-M model and compare the code for accuracy against experimental cases from the Messinger paper.

3. Implement the computer model to accomplish the following:
a. Study the effect of ambient temperature, ambient pressure, droplet temperature, relative humidity and related parameters on the equilibrium surface temperature in a turbofan engine icing environment.

b. Gain understanding of the physics by comparing the physical heat transfer terms

c. Study the behavior of the thermodynamic wet-bulb temperature as a predictor of warm surface ice accretion.

d. Investigate the results obtained by NASA and NRC Canada fundamental icing physics experimental research

4. Attempt to formulate the T-M heat balance model for the case of ice crystal only and mixed phase conditions—solid and liquid water particles.
Chapter 5: Development of the Modified Tribus Messinger Program

Understanding the Tribus-Messinger Heat Balance

For his 1949 dissertation at UCLA, Myron Tribus developed a method to determine the energy balance on the surface of an unheated aerodynamic surface operating in icing conditions (17). His main consideration was the de-icing of airplane propellers. Prior to Tribus’s thesis it was common practice to continuously heat aerodynamic surfaces operating in icing conditions to prevent them from icing up. With development of Tribus’s model the goal became to allow icing to occur but strategically de-ice the aerodynamic surfaces with intermittent heating. The magnitude of heat required could be determined by implementing Tribus’s model. His model considers only two equilibrium surface temperature regimes. The first is above freezing where if super cooled liquid water impinges upon the warmer than freezing temperature surface, no ice accretes. The second is below freezing temperature where if super cooled water impinges upon the colder than freezing temperature surface it freezes upon impact and ice accretes on the surface.

In 1951, Bernard Messinger recognized an important third temperature regime, if the equilibrium surface temperature is equal to freezing temperature the impinging super cooled water may only partially freeze upon impact. Messinger modified the Tribus heat balance model to include this third equilibrium temperature regime (18). To account for this partial freezing of impinging water, Messinger introduces the concept of the freezing fraction. He defines the
freezing fraction \( (n) \) as the ratio of mass of impinging super cooled liquid water that freezes upon impact to the mass of impinging super cooled liquid water. The dimensionless freezing fraction can have any value between 0 and 1 \( (0 < n < 1) \).

The combined work of Tribus and Messinger resulted in the Tribus-Messinger heat balance model (T-M). This heat balance model is the basis for the major ice accretion codes today.

**Mass Transfer Modes**

Figure 9 depicts the four mass transfer modes considered by Tribus in the derivation of his heat balance model (17): Water impingement, Evaporation/sublimation, Shedding and Runback. He neglected runback, where liquid water on the surface runs back under aerodynamic forces, because his thesis work was concerned only with the below freezing equilibrium temperature regime where all impinging water freezes upon impact.

![Figure 9: The four mass transfer modes considered by Tribus (17).](image-url)
Energy Transfer Modes

Figure 10 depicts the seven energy transfer modes considered by Tribus in the derivation of his heat balance model (17): Radiation, Viscous, Sensible, Sublimation/ Evaporation, Kinetic Energy of Droplet, Fusion of Ice and Convection. He neglects radiation. The viscous and kinetic terms are relatively small but are included for completeness as these terms become more significant at high airspeeds.

Surface Energy Balance

Tribus considered the modes of energy transfer for an aerodynamic surface operating in icing conditions and conducted a surface energy balance. A surface energy balance implies that there are no storage or generation terms. Figure 11 is adapted from (18) and depicts a schematic of an aerodynamic surface operating in super cooled liquid water icing conditions.
The Tribus-Messinger Energy Balance

\[
\dot{Q}_{\text{surface}} = \dot{Q}_{\text{convection}} + \dot{Q}_{\text{latent}} + \dot{Q}_{\text{sensible}} - \dot{Q}_{\text{kinetic}} - \dot{Q}_{\text{friction}}
\]

where

\[
\dot{Q}_{\text{latent}} = (\dot{Q}_{\text{evap}} + \dot{Q}_{\text{sub}} - \dot{Q}_{\text{fusion}})
\]

The strategy employed to use the T-M Model is that for some equilibrium surface temperature \(t_{se}\), equation 5.1 is equal to zero.

Figure 11: A schematic of the stagnation region of an aerodynamic surface operating in icing conditions adapted from (18).
Energy Transfer Term Equations

The T-M model was derived by considering the mass and energy transfer modes from Figures 9 and 10 that apply to a surface energy balance on an aerodynamic surface operating in icing conditions, represented by the schematic in Figure 11.

Convection

The convection term accounts for the heat transfer due to convective heat transfer driven by the difference between surface temperature of the aerodynamic surface and the ambient static temperature of the air.

\[ \dot{Q}_{\text{convection}} = h_c A (t_s - t_{\infty}) \]  

(5.2)

Latent

The latent heat term accounts for the latent energy associated with phase changes of water: evaporation, sublimation and fusion.

\[ \dot{Q}_{\text{latent}} = L_{\text{sub}} R_{\text{sub}} A + L_{\text{evap}} R_{\text{evap}} A - L_{\text{fus}} R_{w} A n \]  

(5.3)

Relation of the latent energy terms to convective energy transfer

In his dissertation, Tribus worked to convert the various energy transfer mode terms in the surface energy balance to be analogous to a convective energy transfer mode (17) between the surface and ambient temperature. This is treated in detail in the Tribus thesis (17) but is briefly discussed here for clarity to the reader. The evaporation term is addressed in this explanation but a similar procedure also treats the sublimation term.
Tribus equated the mass flux (mass/time) per unit area due to evaporation ($R_{\text{evap}}$) to the difference between the humidity ratios at the surface and the free stream multiplied by the convective mass transfer coefficient. The humidity ratio can be expressed as a ratio of the partial pressure of water vapor to the partial pressure of dry air factored by the ratio of molecular weights.

$$R_{\text{evap}} = h_m \left( W_{\text{surf}} - W_\infty \right) = h_m \left( \frac{M_{W_w}}{M_{W_d}} \right) \left( \frac{1}{B} \right) \left( P_{\text{surf, w}} - P_\infty \right) = h_m \left( \frac{18}{29} \right) \left( \frac{1}{B} \right) \left( P_{\text{surf, w}} - P_\infty \right)$$  \hspace{1cm} (5.4)

Implementing the Lewis relation, Tribus related the convective mass transfer coefficient ($h_m$) to the convective heat transfer coefficient ($h_c$) \hspace{1cm} (17).

$$\frac{h_c}{h_m c_p,a} = P_{AM} \left( \frac{\alpha}{D_\nu} \right)^{(2/3)}$$  \hspace{1cm} (5.5)

For air water systems the Lewis number ($\alpha/D_\nu$) is equal to 0.845 \hspace{1cm} (19), and for low diffusion rates where the heat-mass transfer analogy is valid, the logarithmic mean density factor ($P_{AM}$), is equal to unity $\rightarrow 18/29/(0.845^{(2/3)}) = 0.69 \approx 0.70$.

To relate the latent fusion term in equation 5.3 to convective heat transfer, Tribus introduced a dimensionless variable (b) that relates the sensible heat capacity of the impinging water on the surface to the convective heat transfer from the surface. In his work Messinger later called this dimensionless parameter the Relative Heat Factor \hspace{1cm} (18):

$$b = \frac{R_w}{h_c} \frac{c_{p,w}}{h_c}$$  \hspace{1cm} (5.6)
The $R_w$ term in equation 5.6 is a complex term and is a function of several key parameters. The accurate determination of this parameter is an important and critical step in aviation icing analysis. A detailed overview of the process to determine $R_w$ is contained in (20).

Considering the preceding conversion equations, equation 5.3 takes the following form that is implemented in the T-M model:

$$Q_{\text{latent}} = L_{\text{sub}} A \left( \frac{0.70 h_c}{c_{p,a}} \right) \left( \frac{P_{\text{surf}} - P_{\infty}}{B} \right) + L_{\text{evap}} A \left( \frac{0.70 h_c}{c_{p,a}} \right) \left( \frac{P_{\text{surf,w}} - P_{\infty}}{B} \right) - \frac{L_{\text{iw}} h_c}{c_{p,\text{w}}} A n b \quad (5.7)$$

Equation 5.7 is a function of the convective heat transfer coefficient, the surface and free stream water vapor saturation pressure, the free stream static pressure, the freezing fraction and the relative heating factor.

**Sensible**

The sensible heat term accounts for the heat required to warm or cool down the impinging liquid water droplets if they are different from the surface temperature they impinge and accumulate on.

$$\dot{Q}_{\text{sensible}} = R_w A c_{p,w} (t_s - t_{\infty}) = h_c A b (t_s - t_{\infty}) \quad (5.8)$$

**Kinetic**

The kinetic term represents the dissipation of the kinetic energy of the impinging liquid water droplets upon the aerodynamic surface. It is typically very small compared to the other terms in the energy balance. It was likely included by Tribus since he was working with aircraft speeds.
\[ \dot{Q}_{\text{kinetic}} = \left( R_{w} A V_{\infty}^{2}/2 \right) = \left( h_{c} b A V_{\infty}^{2}/2c_{p,w} \right) \]  
(5.9)

**Viscous**

The viscous term accounts for the dissipation of viscous forces as the air flows past the aerodynamic surface. This author’s observation is that this term was included for completeness by Tribus since he was dealing with aircraft speeds. This term is typically small for subsonic airspeeds compared to the other terms in the energy balance. The term for recovery factor \( r \) is assumed to be 0.875 for this work, a compromise between laminar and turbulent flow values of \( r \) (18).

\[ \dot{Q}_{\text{viscous}} = h_{c} A r \left( V_{\infty}^{2}/2c_{p,a} \right) \]  
(5.10)

**T-M Model Assumptions**

Tribus made the following assumptions when originally developing his Heat Balance Model for impinging liquid water that freezes upon impact (in the T-M model this means \( n=1 \)):

1. Steady State Model
2. Super cooled liquid water clouds only
3. Impinging water droplet is at ambient temperature
4. Ambient air is saturated
5. Surface water runback and shedding are neglected
6. Radiation heat transfer is neglected

The computer model developed in this dissertation and discussed in the following section, is conceived of as a research tool to help the NASA /NRCC researchers to analyze cases where \( n = 0 \) originally but that lead to \( n > 0 \) by varying the parameters of ambient pressure, ambient temperature, ambient relative humidity.
and impinging particle temperature. These are experimental cases that begin as equilibrium surface temperature \( t_{se} \) > freezing temperature, in a simulated turbofan LPC environment with \( n=0 \) that become \( t_{se} = \) freezing temperature with \( 0 < n < 1 \) due to operation in icing conditions. For these cases the T-M heat balance model assumptions are most valid near the stagnation region of an airfoil or other surface. For this reason the model will be used to study the stagnation region of an aerodynamic surface operating in icing conditions. This is also the area of interest for the NASA/NRCC work.

**Approach for Development of the Modified Tribus-Messinger Model**

The Modified Tribus-Messinger Model (MTM) is developed by taking the following approach:

1. Develop a computer model based on the T-M heat balance model
2. Compare the computer model for accuracy to the available Messinger Table of experimental data
3. Further develop the computer program implementing a modified T-M heat balance model
4. Compare the modified code showing that it also duplicates the Messinger Table of experimental data

**T-M Heat Balance Based Code**

Messinger included results of an experiment he conducted on Mt. Washington (18), showing good comparison between experimentally measured data and predicted data implementing the T-M heat balance for an aerodynamic surface operating in icing conditions. Excerpts from these results are presented in Table 1.
The approach to develop a code that will address the research question and objectives stated in Chapter 4 is to first write a computer code that implements the same assumptions as the T-M Heat Balance Model and show that it duplicates the predicted values of the Messinger experimental data from reference (18). The compared T-M based computer model will then be modified so that the impinging droplet temperature and ambient humidity can be varied as a function of ambient pressure and ambient temperature. Then experimental test points from Messenger’s table will be run in the MTM code and agreement with the Messinger experimental results will be demonstrated.

Table 1: Input’s and Measured values of Messinger’s Experiment (RH=100%, Impinging particle temperature is ambient temperature), Messinger and T-M code predicted values for Equilibrium Surface Temperature and Freezing Fraction (18).

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>Ambient Static Pressure (psi)</td>
</tr>
<tr>
<td>1</td>
<td>10.180</td>
</tr>
<tr>
<td>2-A</td>
<td>10.126</td>
</tr>
<tr>
<td>2-B</td>
<td>10.126</td>
</tr>
<tr>
<td>2-C</td>
<td>10.126</td>
</tr>
<tr>
<td>3</td>
<td>10.240</td>
</tr>
<tr>
<td>4</td>
<td>10.283</td>
</tr>
<tr>
<td>5</td>
<td>10.281</td>
</tr>
<tr>
<td>6-A</td>
<td>10.318</td>
</tr>
<tr>
<td>6-B</td>
<td>10.398</td>
</tr>
<tr>
<td>7</td>
<td>10.326</td>
</tr>
<tr>
<td>8</td>
<td>10.395</td>
</tr>
<tr>
<td>9</td>
<td>10.225</td>
</tr>
<tr>
<td>10</td>
<td>10.236</td>
</tr>
<tr>
<td>11</td>
<td>10.297</td>
</tr>
<tr>
<td>12-A</td>
<td>10.177</td>
</tr>
<tr>
<td>12-B</td>
<td>10.175</td>
</tr>
<tr>
<td>13</td>
<td>10.218</td>
</tr>
<tr>
<td>14</td>
<td>10.266</td>
</tr>
</tbody>
</table>

Table 1 shows the results of the T-M computer model implementing the T-M Heat Balance. It also shows the Messinger calculated and measured values along
with his experimental conditions. The Messinger calculated values for equilibrium surface temperature and freezing fraction are plotted against the T-M computer values in Figures 12 and 13. The line in each chart represents the point where the T-M computer model predicted value would match the Messinger calculated value. The standard deviation for this comparison (21) is calculated to be 0.5°F for the equilibrium surface temperature and 0.01 for the freezing fraction.

**MTM Model**

The T-M based code was further developed to include a modified T-M Heat Balance (MTM) model. The modifications included in the MTM model allow for varying the impinging particle temperature and calculation of the partial pressure of water vapor in the ambient air as a function of the program inputs in Table 2. A simple flow diagram demonstrating the logic of the program is shown in Figure 14. The full source code including sample input and output files are included in Appendix D. The computer model determines the outputs, key values of which are listed in Table 3, for a range of values of the impinging particle temperature. The particle temperature in the model ranges from the ambient static temperature, a T-M model assumption, to the theoretical cold limit for super cooled liquid water to exist in the ambient air, -40°F. The item listed as particle temperature increment in Table 2 defines the temperature change between each impinging particle temperature iteration.
Figure 12: Equilibrium Surface Temperature Messinger Calculated vs. T-M Code Predicted

Figure 13: Freezing Fraction Messinger Calculated vs. T-M Code Predicted
Impinging Droplet Temperature

The MTM code considers only a liquid water particle not an ice particle or mixed phase condition. The reason for this is that the liquid only consideration represents the simplest form of the heat balance model. In order to demonstrate the increased complexity of including ice crystals or mixed phase in the heat balance model, the ice crystal only and mixed phase form of the heat balance is presented in Appendices B and C.

Flow chart

Figure 14: A simple flow chart describing the logic of the MTM computer model.
In the Energy Transfer Terms section, equation 5.8 represents the sensible heat transfer term as an impinging water droplet warms to the surface temperature of the aerodynamic surface. According to the T-M Heat balance Model assumptions, the impinging water particle is considered to be at the ambient static temperature of the air flow. Considering the difficulty of knowing the internal turbofan engine environment, the nature of the experimental set-up in the RATFac for the joint NASA/NRCC fundamental Ice Crystal Accretion Studies and the low residence time inside the test chamber, it is conceivable that the impinging water droplets are not at the ambient static temperature. To account for an impinging water particle that is at a different temperature than the ambient

Inputs and Outputs

**Table 2: MTM computer program inputs**

**Inputs**
1. Ambient Static Pressure (psia)
2. Ambient Static Temperature (F)
3. Initial Surface Temperature (F)
4. Ambient Relative Humidity (%) 
5. Free Stream Velocity (knots)
6. Messinger b parameter (nd)
7. Particle temperature increment (F)

**Table 3: MTM computer program outputs**

**Outputs:**
1. Equilibrium Surface Temperature (F)
2. Freezing Fraction (nd)
3. Adiabatic saturation temperature (F)
4. The terms of the heat balance
5. Impinging particle temperature (F)
6. All additional equilibrium temperature parameters
static temperature, the MTM Model modifies equation 5.8 by introducing a program variable representing the temperature of the impinging water particle $t_p$. The resulting modified sensible heat equation for the MTM model is as follows:

$$Q_{\text{sensible}} = h_c A b (t_s - t_p) \rightarrow t_{se} > t_{fz}$$

$$= h_c A b \left[ (t_{fz} - t_p) + n \frac{c_{p,i}}{c_{p,w}} (t_{se} - t_{fz}) \right] \rightarrow t_{se} \leq t_{fz}$$

(5.11)

There are three temperature regimes addressed in the MTM model, greater than, less than and equal to freezing temperature. Just as in the sensible term equation 5.11, the sublimation component of the latent energy term, equation 5.7, is non-zero in the MTM model only for $t_{se} < t_{fz}$.

Figure 15: The reduction in ambient humidity as air is ingested into a turbofan engine and the static pressure and temperatures increase rapidly between engine and HPC inlet planes. Engine Schematic is adapted from (3).
Relative Humidity and Water Vapor Saturation Pressure

In the Energy transfer terms section equation 5.7 represents the latent energy transfer term as contained in the MTM model. The T-M Heat Balance model assumes that the ambient air is saturated at ambient conditions.

Outside of the turbofan engine, operating at high altitude, if solid or liquid water particles are present in the environment they are likely below freezing temperature. It is also plausible that the ambient air would be at or very close to a saturated condition. Considering that static temperature and pressure increase between the engine inlet and the inlet to the high pressure compressor for a typical turbofan engine, Figure 15, the ambient air obtains low relative humidity as it is ingested into the turbofan engine and approaches the HPC. To account for this reduction in relative humidity the MTM model implements a subroutine that determines the water vapor pressure as a function of the ambient static pressure and static temperature. From this the relative humidity of the airflow can be determined. The main equations implemented for the subroutine are taken from a standard ASHRAE Handbook adapted from the work of Hyland and Wexler (19).

To calculate the saturation partial pressure of water vapor over a surface covered by liquid water the following equation is used (19):

$$\ln\left(p_{ws}\right) = \frac{C_8}{T} + C_9 + C_{10} T + C_{11} T^2 + C_{12} T^3 + C_{13} \ln(T)$$  \hspace{1cm} (5.12)

Where the constants are as follows (19):

55
Note that this equation is good for temperature range 492 to 852R. For temperatures below this range, there is an equation for saturation pressure over ice (19).

**Thermodynamic Wet-bulb or Adiabatic Saturation Temperature**

The Joint NASA / NRCC Fundamental Ice Crystal Accretion studies correlated the on-set of ice accretion on a warmer than freezing surface operating in icing conditions with a below freezing calculated thermodynamic wet-bulb temperature.

The MTM model calculates the Adiabatic Saturation Temperature which is analogous to the Thermodynamic Wet-bulb temperature (19). The MTM program implements a subroutine which uses an iterative procedure taken from a Standard ASHRAE handbook (19). The adiabatic saturation temperature is the temperature that satisfies the following equation given initial enthalpy, humidity ratio and static pressure of the moist air flow (19):

\[ h + (W_w^* - W)h_w^* = h_s^* \]  
\[ (5.13) \]

Where:

- \( h \) = enthalpy of the original moist airflow at ambient static temperature
- \( h_w^* \) = the enthalpy of the water vapor added to the air at the adiabatic saturation temperature
- \( h_s^* \) = Saturation enthalpy of the airflow at the adiabatic saturation temperature
$W_s^* =$ Saturation Humidity Ratio at the adiabatic saturation temperature

$W =$ Humidity ratio of the airflow at the ambient static temperature and static pressure

The humidity ratio formula is taken from the standard ASHRAE handbook (19).  

$$ W = \frac{\text{MW}_w}{\text{MW}_a} \left( \frac{P_{s,w}}{B - P_{s,w}} \right) \tag{5.14} $$

Where:

$W =$ humidity ratio of the air at static temperature

$\text{MW}_w =$ molecular weight of water

$\text{MW}_a =$ molecular weight of air

$P_{s,w} =$ partial pressure of water vapor at static temperature

$B =$ ambient static pressure

To use the MTM Computer model an input file is created based on the desired test conditions. When the program is run, an output file is created containing several parameters. For each impinging particle temperature test run, there is a unique set of output parameters. Some of the key output parameters are listed in Table 3. A complete list is shown in appendix D. The main parameters analyzed in this work are the equilibrium surface temperature and the freezing fraction as the parameters of impinging particle temperature, relative humidity, ambient pressure and ambient temperature are varied.

The following chapter documents research conducted implementing the MTM computer program on an experimental case taken from Table 1 and two experimental cases taken from the joint NASA/NRCC Fundamental ice Crystal Accretion Studies.
Chapter 6: Results and Discussion of Research Conducted Implementing the MTM Program

The research question to be answered is as follows: When the ambient air temperature is above freezing, what effect do impinging droplet temperature, relative humidity, ambient temperature and ambient pressure have on the equilibrium surface temperature and the potential for ice accretion, near the stagnation region of a simple airfoil operating in icing conditions?

Existing research has identified the key effects of ambient pressure, ambient temperature, ambient relative humidity and wet-bulb temperature on the on-set and characteristic ice that forms on an aerodynamic surface operating in icing conditions (14), (15) and (16). The goal of the existing and ongoing NASA/NRCC experimental work is to gain an understanding of the physics of ice accretion in this difficult to understand, measure and duplicate turbofan engine LPC environment.

For the present work in this thesis, the following Messinger experimental case inputs and parameters are studied:

Table 4: Case 2A Inputs from Messinger experimental data

<table>
<thead>
<tr>
<th>The inputs to the Model are as follows:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Free Stream Velocity (knots)</td>
<td>219</td>
</tr>
<tr>
<td>2. Messinger b parameter (nd)</td>
<td>0.53</td>
</tr>
<tr>
<td>3. Impinging Particle Temperature (F)</td>
<td>varied</td>
</tr>
<tr>
<td>4. Ambient Static Pressure (psia)</td>
<td>varied</td>
</tr>
<tr>
<td>5. Ambient Static Temperature (F)</td>
<td>varied</td>
</tr>
<tr>
<td>6. Initial Surface Temperature (F)</td>
<td>varied</td>
</tr>
<tr>
<td>7. Ambient Relative Humidity (%)</td>
<td>varied</td>
</tr>
</tbody>
</table>
Table 5: Parameters Studied for Case 2A

<table>
<thead>
<tr>
<th>Parameters Studied</th>
<th>Range of Values Studied</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impinging Particle Temperature (F):</td>
<td>60 to -40</td>
</tr>
<tr>
<td>Ambient Pressure (psia):</td>
<td>14.7 to 4</td>
</tr>
<tr>
<td>Ambient Temperature (F):</td>
<td>60 to 4.6</td>
</tr>
<tr>
<td>Relative Humidity (%):</td>
<td>100 to 0</td>
</tr>
<tr>
<td>Initial Surface Temperature (F):</td>
<td>60 to 4.6</td>
</tr>
</tbody>
</table>

**Ambient Pressure Effects**

Figures 16 and 17 demonstrate the MTM model predicted effects on equilibrium surface temperature and freezing fraction vs. impinging particle temperature with the ambient pressure as a parameter for the Messinger experimental test point 2A in Table 1. The ambient temperature is 4.6F, the relative humidity is 100% and the ambient pressure varies from 4psia to 14.7psia.

For ambient static pressures equal to and higher than 9 psia, it is shown in Figure 16 that the model predicts the equilibrium surface temperature remains equal to the freezing temperature of water for all impinging particle temperatures. For ambient static pressures lower than 9 psia, the model predicts an equilibrium surface temperature lower than freezing for the colder impinging particle temperatures with transition to freezing at warmer impinging particle temperatures for the 8,7,6 and 5 psia test runs. The transition between freezing and colder than freezing is where the MTM model switches from an evaporative and fusion model of the latent term to one of sublimation and fusion. The 4 psi test run remains below freezing for all impinging particle temperatures. A below freezing equilibrium surface temperature is indicative that the freezing fraction is equal to one or n=1 and impinging water particles freeze immediately upon
Figure 16: The effect of impinging liquid water droplets on the equilibrium surface temperature at different ambient pressures for an ambient temperature of 4.6°F and 100% relative humidity.

Figure 17: The effect of impinging liquid water droplets on the equilibrium surface temperature at different ambient pressures for an ambient temperature of 4.6°F and 100% relative humidity.
impact. For the pressures lower than 9 psia the freezing fraction is between zero and one or 0<n<1 and impinging water particles partially freeze upon impact. This is shown in Figure 17. Both plots reveal that as the ambient static pressure decreases and the impinging particle temperature gets colder the equilibrium surface temperature decreases and the freezing fraction increases. This combined with the noted transition shows an ambient static pressure and impinging particle temperature effect on the equilibrium surface temperature and the characteristic ice that accretes.

**Ambient Temperature Effects**

Figures 18 and 19 demonstrate the MTM model predicted effects on equilibrium surface temperature and freezing fraction versus impinging particle temperature from varying the ambient static temperature for the Messinger experimental test point 2A in Table 1. The ambient pressure is 10.18 psia, the relative humidity is 100% and the ambient static temperature varies from 4.6F to 60F.

For ambient static temperatures greater than or equal to 35F, Figure 18 shows that the model predicts the equilibrium surface temperature is greater than the freezing temperature of water for the warmer impinging particle temperatures. This is indicative that no ice accretes and the freezing fraction is n=0. For ambient temperatures equal to or lower than freezing, the model predicts the equilibrium surface temperature is equal to the freezing temperature of water for all impinging particle temperatures. This is indicative that ice is accreting on the surface, the freezing fraction is 0<n<1 and that impinging liquid water particles
Figure 18: The effect of impinging liquid water droplets on the equilibrium surface temperature at different ambient temperatures for an ambient pressure of 10.2 psia and 100% relative humidity.

Figure 19: The effect of impinging liquid water droplets on the freezing fraction at different ambient temperatures for an ambient pressure of 10.2 psia and 100% relative humidity.
partially freeze upon impact. For ambient temperatures equal to and greater than 35F the equilibrium surface temperature decreases as impinging particle temperature decreases. A transition point is predicted for the 35F and 40F test runs where the equilibrium surface temperatures become equal to freezing for a given impinging particle temperature. This transition is more readily shown in Figure 19 and marks the onset of ice accretion on the surface. These plots demonstrate an ambient static temperature and impinging particle temperature effect on the equilibrium surface temperature and the characteristic ice that accretes.

**Relative Humidity Effects**

Figures 20 and 21 demonstrate the MTM model predicted effects on equilibrium surface temperature and freezing fraction versus impinging particle temperature from varying the ambient relative humidity for the Messinger experimental test point 2A in Table 1. The ambient pressure is 10.18 psia, the ambient temperature is 4.6F and the relative humidity varies from 0 to 100%.

For ambient relative humidity greater than or equal to 50%, Figure 20 shows that the model predicts the equilibrium surface temperature is less than the freezing temperature of water for the colder impinging particle temperatures. This is indicative that the freezing fraction is $n=1$ and that water particles freeze immediately upon impact with the surface. For ambient relative humidities greater than 50%, the model predicts the equilibrium surface temperature is equal to the freezing temperature of water for all impinging particle temperatures.
Figure 20: The effect of impinging liquid water droplets on the equilibrium surface temperature at varying relative humidity for an ambient temperature 4.6°F and an ambient pressure of 10.2 psia.

Figure 21: The effect of impinging liquid water droplets on the equilibrium surface temperature at varying relative humidity for an ambient temperature 4.6°F and an ambient pressure of 10.2 psia.
This is indicative that the freezing fraction is $0 < n < 1$ and that impinging water particles only partially freeze upon impact with the surface. For ambient relative humidities equal to and less than 50%, the equilibrium surface temperature decreases as impinging particle temperature decreases below a certain temperature. A transition point is predicted for these test runs where the equilibrium surface temperatures become equal to freezing for a given impinging particle temperature. This transition is more readily shown in Figure 21 and marks the onset of a different characteristic ice accretion on the surface.

These plots demonstrate ambient relative humidity and impinging particle temperature effects on the equilibrium surface temperature and the characteristic ice that accretes.

**Discussion of Ambient Static Pressure, Static Temperature and Relative Humidity Effects**

The analysis reveals that equilibrium surface temperature and freezing fraction are affected by variations in the ambient static pressure, static temperature and relative humidity. These effects were empirically identified in the Joint NASA/NRCC experiments. This analysis also identifies an impinging particle temperature effect that was not identified in the joint NASA/NRCC experiments.

The model predicts that the ambient pressure, temperature, relative humidity and impinging particle temperature can actually change the characteristic ice that accretes by affecting the predicted value of the freezing fraction. In several test runs a transition exists where the freezing fraction becomes greater than zero. In
these cases the on-set of ice accretion is affected by variation of these parameters. These can be interpreted physically as sensible, convective and latent energy term effects.

The latent energy accounts for the physical processes of sublimation, evaporation and fusion. From equation 5.7, the evaporation and sublimation terms are proportional to the difference between the vapor pressure on the surface at the equilibrium surface temperature and the ambient vapor pressure at the ambient static temperature, quantity divided by the ambient static pressure. As the ambient static pressure decreases the latent heat term becomes larger which lowers the equilibrium surface temperature. As the ambient static temperature and ambient relative humidity increase the vapor pressure term in the numerator of the evaporation and sublimation components of equation 5.7 increases. It was shown in Figures 16 to 21 that these three parameters all affect the equilibrium surface temperature which also affects the surface vapor pressure term in the evaporation and sublimation components of equation 5.7.

With decreasing equilibrium surface temperature, for a given impinging particle temperature, the sensible energy term in equation 5.11 becomes stronger as the difference between the equilibrium surface temperature and the impinging particle temperature increases. For a given ambient pressure, temperature and relative humidity, as the impinging particle temperature decreases, more sensible energy is required to warm the particle to the surface temperature. This decreases the equilibrium surface temperature and increases the value of the
freezing fraction if 0<n<1. The fusion term in equation 5.7 accounts for the latent energy when water freezes or melts. It is not taken into account until the freezing fraction becomes n>0 and is factored by n when freezing begins until n=1. The fusion term increases until n=1.

The convection term in equation 5.2 accounts for the energy transfer due to the difference between the ambient static temperature and the surface temperature. The latent and sensible terms affect the surface temperature and contribute to the variation in the convection term.

For this variation in ambient static pressure, static temperature, relative humidity and impinging particle temperature analysis, the model predicts that all the parameters have an effect on the equilibrium surface temperature and the characteristic ice that accretes. Analysis also suggests that the effects of these parameters are coupled in the latent, sensible and convection terms of the MTM heat balance. A closer look into the physics of the MTM heat balance terms follows in the next section.

**Investigating the Physical Heat/Energy Transfer Terms**

The T-M heat balance in Equation 5.1 is set up to model the balance of energy between the modes of energy transfer on the right hand side and the energy transfer from the surface on the left hand side. The strategy to solve the T-M model is that for some equilibrium surface temperature the right hand side of equation 5.1 is equal to zero and there is no energy transfer from the surface. In this sense each term on the right hand side is considered as transferring energy
to or from the surface. In this work the convention for energy transfer from the surface is positive.

**Figures 22 through 29** demonstrate the MTM physical predictions of unit energy transfer rate vs. impinging particle temperature for the energy transfer terms in equation 5.1. The data was obtained by running the Messinger Case 2A in the MTM computer program and varying the following parameters: ambient temperature, ambient pressure, relative humidity and impinging particle temperature. The sensible, convective, latent and viscous physical energy terms are analyzed. The kinetic energy term exists in the MTM model but it is small on the order of 0.2% of the summation of the other terms and is left out for clarity in the figures. The unit energy terms analyzed are calculated assuming a unit convective heat transfer coefficient and unit area. The Latent energy term is comprised of evaporation, sublimation and fusion components. To clarify insight into the physics, these individual components of the latent term are plotted along with the whole latent term in the figures. The figures include the equilibrium surface temperature (secondary y-axis) vs. impinging particle temperature as an aide to interpretation. The evap/sub. plotline represents evaporation when the equilibrium surface temperature is greater than or equal to freezing temperature and it represents sublimation below freezing. The parameters for each test run were picked to clearly contrast the behavior of the physical energy transfer terms in the MTM heat balance at different ambient conditions. The different
combinations of the input parameters analyzed, relative humidity, RH (%), static
temperature, t (F) and ambient static pressure, B (psia), are listed below:

1. Figure 22: RH = 100 t = 4.6 B = 10.1
2. Figure 23: RH = 100 t = 4.6 B = 5
3. Figure 24: RH = 50 t = 4.6 B = 10.1
4. Figure 25: RH = 50 t = 4.6 B = 5
5. Figure 26: RH = 100 t = 60 B = 10.1
6. Figure 27: RH = 100 t = 60 B = 5
7. Figure 28: RH = 50 t = 60 B = 10.1
8. Figure 29: RH = 50 t = 60 B = 5

The results shown in Figure 22 are from the original unmodified Messinger
experimental test point, Case 2A in Table 1, (18). The MTM test run parameters
are relative humidity=100%, ambient temperature=4.6F and ambient
pressure=10.1psia. The equilibrium surface temperature is predicted to be equal
to freezing for all impinging particle temperatures. This is indicative that the
freezing fraction is between zero and one, 0<\(n<1\). The MTM model predicts
there are two competing dominant cooling terms for this case. The convective
term dominates cooling for all impinging particle temperatures warmer than
approximately -20F and the sensible term dominates cooling for colder impinging
particle temperatures. The MTM model also predicts that the latent term, with a
strong negative fusion component, is the dominant heating term for all impinging
particle temperatures.

To compare and highlight the physical effect of decreasing the pressure, the test
run plotted in Figure 23 shows the results of running the original Messinger
experimental test point in the MTM model with a reduced ambient static
pressure: RH=100%, t=4.6F and B=5psia. A noticeable change is that the
Figure 22: Messinger original experimental test point 2A. The effect of impinging liquid water droplet temperature on the key physical heat transfer terms in the modified T-M Heat Balance Model. Relative Humidity 100%, Temperature 4.6F and Pressure 10.1 psia.

Figure 23: Modified Messinger test point 2A. The effect of impinging liquid water droplet temperature on the key physical heat transfer terms in the modified T-M Heat Balance Model. Relative Humidity 100%, Temperature 4.6F and Pressure 5 psia.
Figure 24: Modified Messinger test point 2A. The effect of impinging liquid water droplet temperature on the key physical heat transfer terms in the modified T-M Heat Balance Model. Relative Humidity 50%, Temperature 4.6F and Pressure 10.1 psia.

Figure 25: Modified Messinger test point 2A. The effect of impinging liquid water droplet temperature on the key physical heat transfer terms in the modified T-M Heat Balance Model. Relative Humidity 50%, Temperature 4.6F and Pressure 5 psia.
model now predicts the sole dominant cooling term to be the evaporation component of the latent term for all impinging particle temperatures > -30F. The fusion component of the latent term remains the dominant heating term for all impinging particle temperatures. The reduced pressure also results in a noticeable change in all the physical energy terms at an approximate -10F impinging particle temperature. This corresponds to a transition of the equilibrium surface temperature from freezing to below freezing that does not exist for the original Messinger test point. This is indicative of a freezing fraction transition from 0<n<1 to n=1 and marks a change in the characteristic ice accretion on the surface.

The MTM model predictions plotted in Figures 24 and 25 demonstrate the difference in the physical parameters from reducing the relative humidity from 100% to 50%, Figure 24, and by reducing both the relative humidity and ambient static pressure from the original Messinger point to RH=50% and B=5psia, Figure 25. Both figures have similar dominant term trends as the original Messinger point, Figure 22. Figure 24 shows the MTM model predicts the equilibrium surface temperature drops to below freezing at a -37F impinging particle temperature. The MTM model predicts that when both RH and B are reduced, the equilibrium surface temperature is less than freezing for all impinging particle temperatures, Figure 25.

Figures 26 to 29 present the MTM model results from test runs that changed the ambient temperature to a warmer condition, 60F, from the original Messinger
point, 4.6F. The test run matrix for reducing the pressure and relative humidity as presented in Figures 22-25, are repeated for the warmer ambient condition.

All of the warmer ambient temperature plots show similar dominant cooling term trends in that they predict the latent term to be the dominant term for the warmest impinging particle temperatures and transition to the sensible term as the dominant cooling term at some colder impinging particle temperature. The dominant cooling term is a bit more interesting.

Where the ambient temperature is the only parameter changed from the original Messinger test point, Figure 26, the dominant heating term is the viscous term. It is larger than all the cooling terms combined for the warmer impinging particle temperatures which show in the equilibrium surface temperature becoming warmer than ambient in both Figures 26 and 27. A transition occurs as the latent term becomes the dominant heating term at an impinging particle temperature of approximately 10F. The latent term is cooling due to evaporation from the surface when it is positive even though the ambient air is 100% RH. This is due to the warmer than ambient equilibrium surface temperature. This term transitions to heating due to condensation when it is negative. It is also the dominant cooling term at warmer impinging particle temperatures for this case. The dominant heating trends are similar for the MTM results presented in Figure 27. The reduced pressure for this test run results in a slightly warmer impinging particle temperature for the transition of the dominant heating term from viscous to latent at approximately 13F.
Figure 26: Modified Messinger test point 2A. The effect of impinging liquid water droplet temperature on the key physical heat transfer terms in the modified T-M Heat Balance Model. Relative Humidity 100%, Temperature 60F and Pressure 10.1 psia.

Figure 27: Modified Messinger test point 2A. The effect of impinging liquid water droplet temperature on the key physical heat transfer terms in the modified T-M Heat Balance Model. Relative Humidity 100%, Temperature 60F and Pressure 5psia.
Figure 28: Modified Messinger test point 2A. The effect of impinging liquid water droplet temperature on the key physical heat transfer terms in the modified T-M Heat Balance Model. Relative Humidity 50%, Temperature 60F and Pressure 10.1 psia.

Figure 29: Modified Messinger test point 2A. The effect of impinging liquid water droplet temperature on the key physical heat transfer terms in the modified T-M Heat Balance Model. Relative Humidity 50%, Temperature 60F and Pressure 5 psia.
Comparing Figure 28 to Figure 26, the relative humidity term is the only change; it is reduced to 50% in Figure 28. This change nearly eliminates the latent term from the competition for the dominant heating term. The viscous term is the dominant heating term at the warmest impinging particle temperature but a transition to the convective term occurs at an impinging particle temperature of approximately 38F. When both the pressure and relative humidity are reduced as presented in Figure 29, the convective term is the dominant heating term for all impinging particle temperatures. By inspection of the figures the MTM model predicts that for test cases with lower relative humidity at the warmer ambient static temperature, the equilibrium surface temperature decreases faster as the impinging particle temperature decreases.

**Modeling the Physical Energy Terms for a Turbofan Engine Internal Environment**

Consider a turbofan engine operating in a high altitude ice crystal cloud. According to the literature reviewed and summarized in Chapter 1, the theory is that ice crystal icing is occurring deep inside the low pressure compression system of a turbofan engine operating in a warmer than standard day type atmospheric condition. The theory also states that the phenomenon will only occur on flow path hardware that is initially warmer than freezing temperature (1). Impinging ice particles can somehow impact, accumulate and melt onto the warmer than freezing surfaces allowing for additional accumulation of ice particles. Eventually the surface cools to the point where ice accretion can occur. This suggests there is some transition where a freezing fraction of n=0
becomes $0 < n < 1$, due solely to the ingestion of ice crystals during normal operation.

In this section, the scenario is modeled for an aircraft flying at 35,000 ft which encounters an ice crystal cloud. Standard and tropical day type temperature and pressures for 35,000 ft are tropical day -45.6F/5psia and standard day -65.8F / 5psia, (22). The relative humidity of the environment external to the engine is assumed 100%. In general, a turbofan engine has several stages between the external ambient conditions and the area in the engine where ice crystal icing is thought to occur, between the LPC and HPC systems, Figure 2. This results in a temperature and pressure increase over that of ambient air external to the engine and a reduction in relative humidity, Figure 15. A temperature increase of 100F and a pressure increase of 5 psia over ambient external conditions are estimated for this scenario. The relative humidity is estimated to be 10% for the internal engine environment. Figure 30 and 31, present the MTM results comparing standard vs. tropical day for the predicted physical energy transfer terms in the turbofan engine environment due to impinging super cooled liquid water particles of varying temperature. The equilibrium surface temperature is also shown.

Figure 30 shows that for a standard day condition, the MTM model predicts that the internal environment equilibrium surface temperature is equal to freezing for all impinging particle temperatures. Figure 31 shows that for a tropical day condition, the MTM model predicts a transition from above freezing to equal to freezing for the equilibrium surface temperature where $0 < n < 1$. This marks the
Figure 30: MTM Predicted Physical Energy Transfer Terms for the internal environment of a Turbofan Engine operating in an ice crystal cloud during a standard day type; RH=10%, t=34.2F, B=10psia

Figure 31: MTM Predicted Physical Energy Transfer Terms for the internal environment of a Turbofan Engine operating in an ice crystal cloud during a tropical day type; RH=10%, t=54.4F, B=10psia
on-set of ice accretion. This analysis highlights the effect of operating in a tropical day vs. standard day environment and the impinging particle temperature. Both scenarios result in an ambient temperature at the engine hardware ice accretion location greater than freezing, 34F and 54F but the tropical day test run is initially 20F warmer. Both the tropical day type and the impinging particle temperature are significant in this result. Inspection of the figures shows that compared to the Standard day type, the Tropical day type latent and sensible energy terms are stronger cooling terms and the convective and viscous terms are stronger heating for all impinging particle temperatures. The evaporation and fusion components of the latent term exhibit similar trends in both cases but are predicted weaker in general for the tropical day type.

Discussion of Physical Heat Transfer Terms Analysis

When comparing the effect on the physical energy transfer terms of equation 5.1 by varying the stated input parameters, the results plotted in Figures 22 to 29 show definite trends observed for the various heating and cooling terms studied. The analysis also reveals that when varied together the parameters have stronger effects than the individually varied results, suggesting they are inter-related in the physics of the surface energy balance. The following sections discuss these observations in more detail.

Referring to Figure 22, as the impinging particle temperature becomes colder the equilibrium surface temperature is constant and equal to freezing. Intuitively, colder particles accumulating on the surface have potential to require more
energy from the surface in order to warm to surface temperature; this is shown by the sensible energy line as it increases with decreasing particle temperature. We know from the variation in pressure, temperature and relative humidity analysis that when the equilibrium surface temperature is equal to freezing, the freezing fraction becomes higher as the impinging particle temperature decreases. When a higher mass fraction of impinging water freezes upon impact it gives off the latent heat of fusion to the surface. This is shown by the decreasing (stronger heating effect) fusion line. In the chart the latent heat term is shown to become increasingly negative as the particle temperature decreases. This suggests that the fusion component is dominating the latent heat term and that the heat of fusion exceeds the heat of evaporation. The heat of evaporation is constant because it is driven by the difference between the surface water vapor pressure and the ambient water vapor pressure, equation 5.7. Since the equilibrium surface temperature is constant and the ambient static temperature is constant the driving force for evaporation is constant. The convective term is also a function of the surface temperature and ambient static temperature. It is shown to be constant as well. The latent and viscous terms compete with the sensible and convective terms to either cool the surface or heat the surface. The viscous term remains constant since it is a function of only the airspeed which is constant for this condition.

Referring to Figures 23-25, the same trends for the energy transfer terms are seen as discussed for Figure 22. One key difference is that in Figures 23-25...
there is a transition of equilibrium surface temperature from freezing to below freezing for a given impinging particle temperature. The transition occurs at approximately -10F, -38F and for all impinging particle temperatures respectively.

The test run presented in Figure 23 is for a reduction in the ambient static pressure as compared to the original Messinger test point in Figure 22. Equation 5.7 shows that the latent term is a function of the ambient static pressure, B. As B is in the denominator of the evaporation and sublimation components, the latent term should become more of a cooling term as the ambient static pressure decreases. Comparing the evaporation component in Figures 22 and 23 confirms it is larger (higher positive) in Figure 23 as expected.

The test run presented in Figure 24 is for a reduced relative humidity as compared to Figure 22. The relative humidity is a measure of the water vapor pressure in the ambient air. Equation 5.7 shows that the latent term is a function of the water vapor pressure in the evaporation and sublimation terms. The ambient water vapor pressure is in the numerator and is subtracted from the surface water vapor pressure. As the ambient water vapor pressure decreases the latent energy term should become more cooling as relative humidity decreases. Comparing the evaporation component in Figures 22 and 24 confirms it is larger (positive) in Figure 24 as expected.

The test run presented in Figure 25 is for both a reduced relative humidity and a reduced ambient static pressure as compared to Figure 22. Per the discussions on Figures 23 and 24, a larger evaporation component is expected with reduced
relative humidity and reduced ambient static pressure. Comparing the evaporation component in Figures 22 and 25 confirms it is larger (positive) in Figure 25 as expected.

The MTM test runs presented in Figures 26-29 are for a warmer ambient static temperature than in Figure 22. In general all equilibrium surface temperatures are warmer than freezing for all impinging particle temperatures. Since freezing fraction is equal to zero (equilibrium surface temperature is greater than freezing) there is no fusion component in the fusion term and the latent term is equal to the evaporation term; the plot lines coincide in the charts. The difference between the equilibrium surface temperature and the ambient static temperature determines both the magnitude and direction of the energy transferred due to convection, equation 5.2. Comparing the colder ambient air plots, Figures 22-25, against the warmer ambient air plots, Figures 26-29, confirms that the colder ambient air plots show convection term that is a cooling term and the warm air plots show it as a heating term. They also show that the equilibrium surface temperature is higher for the warmer air plots. The ambient air temperature is theoretically the temperature that the aerodynamic surface would equalize at if there were no water particles present. In an icing environment, the water vapor pressure in the ambient moist air and the impinging particle temperatures are influenced by the ambient static temperature.

In Figures 26-29, for colder impinging particle temperatures, there is a negative evaporation component of the latent energy term. This is indicative of water
vapor condensing onto the surface and giving off latent heat. As pressure and relative humidity are decreased individually (Figures 27, 28 respectively) and simultaneously (Figure 29) the impinging particle temperature at which evaporation transitions to condensation decreases.

In Figures 30 and 31, MTM results for the standard day versus tropical day energy transfer are compared. The difference in the two day types is basically that the equilibrium surface temperature is higher for a given impinging particle temperature due to the fact that the ambient static temperature is warmer for the tropical day condition. The biggest difference between the two charts is the predicted transition of the equilibrium surface temperature from warmer than freezing to freezing in the tropical day case. The latent term trends the same in both cases but the evaporation and fusion components are stronger for the standard day case. This is because the fusion component is given heat to the surface as ice forms upon impact since freezing fraction is between zero and one (0<n<1). The evaporation term is higher since the difference between the surface temperature and the ambient temperature drives more evaporation energy transfer. Though this analysis is for super cooled liquid water only, it is conceivable that the lower temperature impinging particles could account for the melting of impinging ice particles of an actual ice crystal encounter. A hypothesis formed from this comparison is that initially upon entrance into the ice crystal cloud during tropical day type ambient conditions, the ice crystals that are ingested, pass through the various engine stages along the flow path towards the
suspected ice crystal icing regions of the engine. Along the way they are broken up and warmed up or even melted. Initially only the smaller warmer particles are able to impinge, accumulate and melt on the 54F surface and as they do the surfaces become wetter. As the surfaces become wetter, the larger and colder particles are able to impinge, accumulate and melt. Evaporation occurs to cool the surface. The result is, as is shown for one estimated example in Figure 31, the equilibrium surface temperature becomes equal to freezing and the onset of ice accretion is reached. This assumes that the ice particles would not accumulate on the colder standard day surface.

The MTM model is implemented to study the physics of the surface energy balance that comprises the model. The analysis shows and discusses how and why the physical energy transfer terms behave with variations in the input parameters of relative humidity, ambient static temperature, ambient static pressure and impinging particle temperature. A change in one of the input parameters is shown to affect more than one of the surface energy balance terms, the equilibrium surface temperature and the characteristic ice that accretes (changes the freezing fraction value). This suggests that the input parameters are inter-dependent within the physics of the surface energy balance.

**NASA /NRC Canada Joint Research Cases**

This section presents the implementation of the MTM model to analyze the results of two experimental test cases conducted as part of the joint NASA/NRCC research to study the fundamental accretion physics of ice crystal icing in the
LPC of a turbofan engine. These cases were conducted in the National Research Council Canada RATFac on the simple airfoil test article discussed in Chapter 3. One goal of this dissertation research is to help the NASA/NRCC researchers gain insight and interpret the experimental observations of these tests by better understanding the environment they are studying which is both difficult to measure and to reproduce. In the previous sections it was demonstrated that the MTM model predicts ambient pressure, ambient temperature and relative humidity effects on the equilibrium surface temperature and freezing fraction of an aerodynamic surface operating in icing conditions. These effects were empirically identified in the joint NASA /NRCC research. In the previous sections it was demonstrated by implementing the MTM model that the model also predicts that the impinging particle temperature is a key influence on the equilibrium surface temperature and the freezing fraction or characteristic ice accretion.

Due to the short residence time for ice and water particles in the simulated engine LPC it is difficult to know the temperature of the impinging particles on the test article. It is likely that the particles are at some temperature that is warmer than the ambient temperature outside the simulated engine test cell but colder than the ambient temperature inside the test cell at the location of impingement. There is no effective way yet to measure the impinging water particle temperatures in the experiment. The MTM model is a tool that can be implemented to help overcome this experimental difficulty by allowing
researchers to gain valuable insight about the temperature of the impinging water particles for their experimental cases based on experimental observations of the characteristic ice accretion that forms on the test article.

The approach is to implement the MTM model to run several of the joint NASA/NRCC test cases to generate data relating the equilibrium surface temperature and freezing fraction to the impinging particle temperature. The predicted impinging particle temperature and freezing fraction are then correlated with the experimentally observed characteristic ice that accreted.

**Freezing Fraction and Characteristic Ice**

It is well established in the aviation icing community that qualitatively, the characteristic ice that forms on an aerodynamic surface operating in icing conditions is related to the value of the freezing fraction. For example a zero value of the freezing fraction or n=0 indicates no ice accretion on the surface. A value near to but greater than zero will tend to be more of an accumulation than an accretion and it will appear slushy in consistency. A value of freezing fraction in the middle ranges, 0.4<n<0.7 for example, will have a clear wet look, and be firmly accreted to the leading edge of the aerodynamic surface. It will also have a characteristic non-aerodynamic “horn" shaped profile which grows upstream into the direction of the flow. This is indicative that impinging water only partially freezes upon impact and a portion of it accumulates as surface water and can run back under aerodynamic forces. This type of ice is called glaze ice. Finally a freezing fraction near or equal to one (n=1) will be opaque and white in
appearance indicative of air being trapped in the ice formation as the impinging water freezes instantly upon impact. This ice is called rime ice. It is very hard and takes on the shape of the leading edge of the aerodynamic surface. The shape of accreted rime ice is aerodynamic compared to the glaze ice shape. Figure 32 shows some qualitative examples of the three different ice regimes.

One note is that the transitions to the different characteristic ice accretions are not well defined. It is not uncommon to see a characteristic ice that is made up of both rime and glaze ice for instance as the freezing fraction goes from a glaze ice value to a rime ice value.

Figure 32: Examples of ice formed on aerodynamic surfaces at different freezing fraction ranges n= near 1, n= near 0.5 and n=near 0.

Joint NASA / NRCC Fundamental Accretion Studies Case 539

Figure 33 depicts a drawing of a simple airfoil that is the test article in the Joint NASA /NRCC Fundamental Accretion studies discussed in Chapter 3. For these
experiments, various thermocouple sensors were embedded into the test article making up the surface of the air foil. The location labeled cover three t/c marks the location of the leading edge thermocouple which is embedded beneath the surface. Table 6 lists the experimental parameters for Case 539 considered here. Figure 34 shows the leading edge temperature vs. time as measured by the leading edge thermocouple. The experimental parameters were designed to simulate the conditions inside the LPC region of a turbofan engine. This region, where ice crystal icing is theorized to occur, is shown schematically by the red outlined area in Figure 2. For Case 539, the relative humidity is 8%, the ambient static temperature is 49F and the cloud is composed of only liquid water particles at LWC=1.54 grams of water per cubic meter of air. NASA and NRCC
researchers suggest that the tolerance for the LWC is +/-20% and the tolerance for the heat transfer coefficient listed in Table 6, calculated based on measured parameters 8mm from the leading edge is +/-30%. The initial airfoil leading edge measured temperature is approximately 55F (13C), Figure 34. Since the thermocouple is embedded below the surface the measured temperature may be slightly different, relative to the net energy transfer, than the actual surface temperature. For example it may be slightly warmer for a net cooling energy transfer and slightly cooler for a net heating energy transfer. The measured temperature does correspond to the cloud off total temperature in Table 6. One item to note is that the calculated wet bulb temperature is shown to be below freezing at 27F. Researchers in the joint NASA NRCC testing correlated a

Table 6: NASA /NRCC Joint Fundamental Ice Crystal Accretion Experimental Research Case 539 and an MTM implemented modified Case 539a (16).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>539 / 539a</td>
</tr>
<tr>
<td>Date</td>
<td>3/14/2001</td>
</tr>
<tr>
<td>Total Temperature (C, F)</td>
<td>12.9, 55.3</td>
</tr>
<tr>
<td>Total Pressure (psia)</td>
<td>6.567</td>
</tr>
<tr>
<td>B (psia)</td>
<td>6.32 / 10</td>
</tr>
<tr>
<td>Mach</td>
<td>0.249</td>
</tr>
<tr>
<td>Static Temperature (C,F)</td>
<td>9.4, 48.9</td>
</tr>
<tr>
<td>Dry bulb temperature (C,F)</td>
<td>72.9, 55.3</td>
</tr>
<tr>
<td>Relative humidity (%)</td>
<td>8.1</td>
</tr>
<tr>
<td>Wet bulb temperature (C, F)</td>
<td>-2.7, 27.1</td>
</tr>
<tr>
<td>Angle of Attack</td>
<td>0</td>
</tr>
<tr>
<td>IWC (g/m³)</td>
<td>0</td>
</tr>
<tr>
<td>LWC (g/m³)</td>
<td>1.54</td>
</tr>
<tr>
<td>Heat transfer coefficient, h_c (BTU/hr/ft²/F)</td>
<td>29.3</td>
</tr>
<tr>
<td>b</td>
<td>0.36</td>
</tr>
<tr>
<td>R_w (lb/hr/ft²)</td>
<td>10.4</td>
</tr>
</tbody>
</table>
Figure 34: Surface mounted thermocouple traces for case 539 of joint NASA/NRCC Fundamental Ice Crystal Accretion Experimental Research cover 3 is the leading edge surface temperature (16).

calculated, below freezing wet-bulb temperature with the on-set of ice accretion on initially warmer than freezing hardware. Ice accretion is expected for this condition. In Figure 34 the solid black line indicates when the cloud was turned on (1) and off (0). Note that at cloud on condition, the leading edge measured surface temperature drops to approximately 33F.

Figure 35 displays the MTM model outputs from running the Case 539 inputs in Table 6. It shows predictions for equilibrium surface temperature on the primary Y-axis and freezing fraction on the secondary Y-axis vs. impinging water particle temperature. The green data point with a plus sign is the average equilibrium surface temperature measured between 200 and 330 seconds by the leading
edge thermocouple, Figure 34. The data point marked with an orange color is the equilibrium surface temperature as predicted by the MTM model if the impinging particle temperature were the same as the ambient static temperature. This MTM model output point represents the assumption of the unmodified T-M Heat Balance Model.

The freezing fraction associated with this orange data point is approximately \( n = 0.15 \). From Figure 32 the characteristic ice for \( n \) close to 0 would be a slushy ice on the surface of the test article and more of an accumulation rather than an accretion. Figure 37 shows a before and after screen capture of the ice accreted for experimental run Case 539. It shows that for this condition the ice accretion observed was in the form of an accreted glaze ice. This suggests that the freezing fraction is not near zero but likely at some freezing fraction value between \( 0 < n < 0.3 \). Referring to Figure 35, for a freezing fraction in this range at the conditions of case 539, the impinging droplet temperature is predicted to be as cold as approximately 25F. Taking into account the uncertainties of the LWC and heat transfer coefficient which influence the calculated \( b \) parameter has a tolerance of 0.13 which propagates to a freezing fraction estimate between \( 0.2 < n < 0.45 \) at a particle temperature of 25F. The tolerances on the \( b \) parameter gives random error for the freezing fraction calculated in the MTM model. The range of freezing fraction is plotted in Figure 35. Note that these uncertainties have a significant effect on the value of the freezing fraction and the predicted vs. measured leading edge equilibrium surface temperature.
Figure 35: Equilibrium surface temperature and freezing fraction as a function of the impinging droplet temperature for case 539 of joint NASA/NRCC Fundamental ice Crystal Ice Accretion Experimental Research.

Figure 36: Equilibrium surface temperature and freezing fraction as a function of the impinging droplet temperature for case 539a of joint NASA/NRCC Fundamental ice Crystal Ice Accretion Experimental Research.
Figure 36 shows the predicted result of equilibrium surface temperature and freezing fraction vs. impinging particle temperature from running Case 539 in the MTM model and changing the ambient static pressure from 6.3 to 10psia. The new case is called 539a. The model predicts the equilibrium surface temperature is warmer than freezing (n=0) for impinging particle temperatures warmer than 34F. No ice accretion will occur. This was accomplished solely by changing the ambient static pressure.

The physical heat transfer terms for Cases 539 and 539a are shown in Figures 38 and 39. The behavior trends of the energy transfer terms are similar for both cases. In case 539a, Figure 39, there is a transition that occurs where the equilibrium surface temperature goes from warmer than freezing with no ice accretion (n=0) to freezing temperature (0<n<1). This marks the on-set of ice accretion on the surface and it highlights the pressure effect discussed in the previous paragraph. For the impinging particle temperature range close to 25F that fits the characteristic ice accretion observed for case 539, the dominating cooling term is the latent term and the dominating heating term is the convective term.

Discussion of Case 539 and 539a Results

Based on the experimental observation and analysis of the MTM model results, the temperature of the impinging particles on the test article was not at the ambient static temperature. The characteristic ice observed was an accreted glaze ice accretion. Taking into account uncertainties of heat transfer and
Figure 37: A screen capture of the simple air foil test article before and after the cloud on experimental run for Case 539 of the Joint NASA/NRCC Fundamental Ice Crystal Accretion Studies. A mixed rime/glaze ice accretion occurred (16).
Figure 38: The effect of impinging liquid water droplet temperature on the key physical heat transfer terms in the modified T-M Heat Balance Model. Case 539 of joint NASA/NRCC Fundamental Ice Crystal Ice Accretion Experimental Research.

Figure 39: The effect of impinging liquid water droplet temperature on the key physical heat transfer terms in the modified T-M Heat Balance Model. Case 539a of joint NASA/NRCC Fundamental Ice Crystal Ice Accretion Experimental Research.
total water content assuming the observed characteristic ice has a freezing fraction lower than \( n=0.3 \), the impinging particle temperature was between 20 and 40F. This is an example of how the MTM model can be implemented to help researchers gain insight into the existing experimental runs and to develop improved methods to measure the experimental parameters. The temperature of a liquid water droplet could become less than ambient static temperature due to evaporation from the particle surface as the droplets are entrained into 8% relative humidity air. Another hypothesis to consider is that an ingested ice crystal could remain cold and not warm up as it is ingested into the warmer turbofan engine environment. Consider an ambient ice crystal at 20,000ft. The ambient static temperature is on the order of -20F for standard day type. As the aircraft approaches, the ingestion process begins. The ice particle is entrained into the air that is being ingested into the engine and a high relative velocity with the air and with the aircraft ensues. Under normal aerodynamic considerations the micron sized particle would undergo convective heat transfer due to the high relative velocity, warm up and possibly melt. Literature has hypothesized that a surface melt layer on the order of molecules thick that can form on the surface of solid ice at below melting point temperatures and could act as a thermal resistance to the heating of the solid ice, (23). Considering this and that the ice particle travels in less than \( \frac{1}{100} \)th of a second to the ice crystal icing region of interest inside the engine, it is conceivable that the ice particle could be closer to the ambient static temperature external to the engine from which it was ingested. A better understanding of ice crystal heat transfer over the very short residence
times in a turbofan engine environment are important and are recommended for future work.

The MTM model predicts a pressure effect for Case 539a that affects the on-set of ice accretion. By changing only the pressure for this condition, the physics of the surface energy balance were changed significantly enough to delay the on-set of icing until a colder impinging particle temperature occurs. The higher ambient static pressure makes the evaporation component of the latent energy term less positive. This is an example of how the MTM model can be used to help researchers plan future experiments and better understand what measurements are required in the experiments. For instance in these cases it would be a justifiable need to develop impinging particle temperature and LWC measurement methodologies/technologies.

Referring to Figures 37 and 38, the physical energy transfer terms were plotted for case 539 and 539a respectively. As discussed previously these charts demonstrate that the input parameters of relative humidity, ambient static temperature and pressure and the impinging particle temperature are coupled in the physics of the surface energy balance.

**Joint NASA / NRCC Fundamental Accretion Studies Case 545**

*Figure 33* depicts the simple airfoil that is the test article in the Joint NASA /NRCC Fundamental Accretion studies for case 545. *Table 7* lists the experimental parameters for case 545 and *Figure 40* shows the measured temperature vs. time for the thermocouple embedded in the leading edge of the
test article. From Table 7 the relative humidity is 29%, the static temperature is 50F and the cloud is composed of only liquid water particles at 1.6 grams of water per cubic meter of air. The initial surface temperature of the leading edge is shown in Figure 40 to be approximately 59F (15C). This is slightly higher but close to the total temperature in Table 7 of 57F. The calculated wet bulb temperature is shown in Table 7 to be above freezing at 36F. No ice accretion is expected in this case. The same experimental considerations for tolerances/accuracies on LWC, heat transfer coefficient and leading edge measured temperature discussed in Case 539 apply to Case 545. Figure 41 shows that the MTM model predicts the equilibrium surface temperature is above freezing and freezing fraction is zero for impinging particle temperatures above approximately 17F. Below 17F the equilibrium surface temperature is at freezing temperature and the freezing fraction is 0<n<1.

It is shown in Figure 43, that no ice accretion was observed experimentally for the Case 545 condition. This indicates that the freezing fraction is n=0. The average measured equilibrium surface temperature between 125 and 250 seconds in Figure 40 is approximately 39F. If the impinging particle temperature is assumed to be equal to the ambient static temperature the MTM model predicts the equilibrium surface temperature would be 36F, Figure 41. This is 7.5% off of the experimentally measured value. Based on this observation, the MTM model predicts the impinging particle temperature was at or near to ambient static temperature.
Table 7: NASA /NRCC Joint Fundamental Ice Crystal Accretion Experimental Research Case 545 and an MTM implemented modified case 545a (16).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>545 / 545a</td>
</tr>
<tr>
<td>Date</td>
<td>3/14/2001</td>
</tr>
<tr>
<td>Total Temperature (C, F)</td>
<td>13.7, 56.6</td>
</tr>
<tr>
<td>Total Pressure (psia)</td>
<td>6.49</td>
</tr>
<tr>
<td>B (psia)</td>
<td>6.3</td>
</tr>
<tr>
<td>Mach</td>
<td>0.256</td>
</tr>
<tr>
<td>Static Temperature (C,F)</td>
<td>10.0, 50.0</td>
</tr>
<tr>
<td>Dry bulb temperature (C,F)</td>
<td>13.7, 56.6</td>
</tr>
<tr>
<td>Relative humidity (%)</td>
<td>29.4 / 8</td>
</tr>
<tr>
<td>Wet bulb temperature (C, F)</td>
<td>2.4, 36.2</td>
</tr>
<tr>
<td>Angle of Attack</td>
<td>0</td>
</tr>
<tr>
<td>IWC (g/m³)</td>
<td>0</td>
</tr>
<tr>
<td>LWC (g/m³)</td>
<td>1.6</td>
</tr>
<tr>
<td>Heat transfer coefficient, hacı (BTU/hr/ft²/R)</td>
<td>29.2</td>
</tr>
<tr>
<td>b</td>
<td>0.38</td>
</tr>
<tr>
<td>Rm (lbm/hr/ft²)</td>
<td>11.2</td>
</tr>
</tbody>
</table>

The results of running the case 545 condition inputs in the MTM model, changing the relative humidity to 8% are shown in Figure 42. The new case is called 545a. The model predicts that for all impinging particle temperatures the equilibrium surface temperature is equal to freezing at these new Case 545a conditions. The model predicts a relative humidity effect on the equilibrium surface temperature and the on-set of ice accretion. The physical heat transfer terms for Cases 545 and 545a are shown in Figures 44 and 45. The behavior trends of the energy transfer terms are similar for both cases. In case 545a the transition that occurs in case 545 where the equilibrium surface temperature goes from warmer than freezing with no ice accretion (n=0) to freezing temperature (0<n<1) no longer exists. This marks the on-set of ice accretion on the surface in case
545 and it highlights the relative humidity effect discussed in the previous paragraph. No ice is predicted to accrete for case 545 for impinging particle temperatures warmer than approximately 17°F. In the case of 545a ice accretion is predicted to occur for all impinging particle temperatures. Ice accretion is effectively turned on by reducing the ambient relative humidity.

Figure 40: Surface mounted thermocouple traces for case 545 of joint NASA/NRCC Fundamental Ice Crystal Accretion Experimental Research (16).
Figure 41: Equilibrium surface temperature and freezing fraction as a function of the impinging droplet temperature for case 545 of joint NASA/NRCC Fundamental ice Crystal Ice Accretion Experimental Research.

Figure 42: Equilibrium surface temperature and freezing fraction as a function of the impinging droplet temperature for case 545a of joint NASA/NRCC Fundamental ice Crystal Ice Accretion Experimental Research.
Discussion of Case 545 and 545a Results

Based on the experimental observation and analysis of the MTM model results, the temperature of the impinging particles on the test article are predicted to be near ambient static temperature of the test cell. This is an example of how the MTM model can be implemented to help researchers gain insight into the existing experimental runs.

The MTM model predicts a relative humidity effect for Case 545 that affects the on-set of ice accretion. By changing only the relative humidity for this condition, the physics of the surface energy balance were changed significantly enough to initiate the on-set of icing for all impinging particle temperatures. This is an

Figure 43: A screen capture of the simple air foil test article after the cloud on experimental run for Case 545 of the Joint NASA /NRCC Fundamental Ice Crystal Accretion Studies. No ice accretion occurred (16).
Figure 44: The effect of impinging liquid water droplet temperature on the key physical heat transfer terms in the modified T-M Heat Balance Model. Case 545 of joint NASA/NRCC Fundamental Ice Crystal Ice Accretion Experimental Research

Figure 45: The effect of impinging liquid water droplet temperature on the key physical heat transfer terms in the modified T-M Heat Balance Model. Case 545a of joint NASA/NRCC Fundamental Ice Crystal Ice Accretion Experimental Research

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example of how the MTM model can be used to help researchers plan future experiments.

When comparing the latent energy components between the RH= 29% (case 545) and the RH=8% (case 545a) runs, the lower relative humidity manifests itself as a significant increase in cooling effect of the evaporative component and a significant increase in heating effect of the fusion component of the latent term. The evaporative and fusion component for the RH=8% run are predicted to be 103% and 190% larger respectively than the RH=29.3% case. The MTM model predicts a significant relative humidity effect on evaporation and fusion components of the latent term.

Referring to Figures 44 and 45, the physical energy transfer terms were plotted for case 545 and 545a respectively. As discussed previously these charts demonstrate that the input parameters of relative humidity, ambient static temperature and pressure and the impinging particle temperature are all coupled in the physics of the surface energy balance.

**Thermodynamic Wet-bulb Temperature**

Table 8 is a compilation of the thermodynamic wet bulb temperatures predicted for selected cases studied in this dissertation using the MTM model. For Cases 539 and 545 the thermodynamic wet-bulb temperature was calculated by the NASA/NRCC researchers based on experimental observations. This is shown as the second value in the cell (predicted /observed). The Messinger Case 2A
Table 8: Adiabatic Saturation Temperature, Messinger Case 2A, NASA/NRCC Cases 539, 539a, 545 and 545a at impinging particle temperature equal to ambient static temperature.

<table>
<thead>
<tr>
<th>Case</th>
<th>Ambient Static Pressure (psia)</th>
<th>Ambient Static Temperature (F)</th>
<th>Ambient Relative Humidity (%)</th>
<th>Thermodynamic wet-bulb temperature (F)</th>
<th>Equilibrium Surface Temperature (F)</th>
<th>Ice Accretion</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A</td>
<td>10.1</td>
<td>4.6</td>
<td>100</td>
<td>4.6</td>
<td>34* / 32</td>
<td>Y*</td>
</tr>
<tr>
<td>2A.1</td>
<td>5.2</td>
<td>4.6</td>
<td>100</td>
<td>4.6</td>
<td>32</td>
<td>Y</td>
</tr>
<tr>
<td>2A.2</td>
<td>10.1</td>
<td>4.6</td>
<td>50</td>
<td>2.1</td>
<td>32</td>
<td>Y</td>
</tr>
<tr>
<td>2A.3</td>
<td>5.1</td>
<td>4.6</td>
<td>50</td>
<td>0.5</td>
<td>30.3</td>
<td>Y</td>
</tr>
<tr>
<td>2A.4</td>
<td>10.1</td>
<td>60</td>
<td>100</td>
<td>60</td>
<td>62.6</td>
<td>N</td>
</tr>
<tr>
<td>2A.5</td>
<td>5.1</td>
<td>60</td>
<td>100</td>
<td>60</td>
<td>61.6</td>
<td>N</td>
</tr>
<tr>
<td>2A.6</td>
<td>10.1</td>
<td>60</td>
<td>50</td>
<td>48.7</td>
<td>53.1</td>
<td>N</td>
</tr>
<tr>
<td>2A.7</td>
<td>5.1</td>
<td>60</td>
<td>50</td>
<td>45.9</td>
<td>49.3</td>
<td>N</td>
</tr>
<tr>
<td>539</td>
<td>6.3</td>
<td>49</td>
<td>8.1</td>
<td>27.1* / 25.7</td>
<td>32.5* / 32</td>
<td>Y*</td>
</tr>
<tr>
<td>539a</td>
<td>10</td>
<td>49</td>
<td>8.1</td>
<td>30.1</td>
<td>34.3</td>
<td>N</td>
</tr>
<tr>
<td>545</td>
<td>6.2</td>
<td>50</td>
<td>29.4</td>
<td>36.2* / 32.5</td>
<td>38.7* / 36.2</td>
<td>N*</td>
</tr>
<tr>
<td>545a</td>
<td>6.2</td>
<td>50</td>
<td>8</td>
<td>25.9</td>
<td>32</td>
<td>Y</td>
</tr>
</tbody>
</table>

* Determined/calculated from experimental observation — all other values are predicted using MTM program

Data is also presented for each MTM test run. The value of the adiabatic saturation temperature is shown to be affected by the different input parameters to the model. The column indicating whether there was ice formation or not shows that for wet-bulb temperatures below freezing there was ice accretion and for wet-bulb temperatures above freezing there was no ice accretion. The model prediction supports the experimental results that the on-set of ice accretion correlates with a thermodynamic wet-bulb temperature below freezing. MTM model also includes the impinging particle temperature which influences the equilibrium surface temperature. This suggests that the wet-bulb temperature alone may not always be the best indicator for the onset of ice accretion.
Chapter 7: Discussion of Results and Conclusions

Overview

The motivation for this research is turbofan engine loss of thrust control field events. These events are thought to be related to ice accretion inside the low pressure compressor system of the turbofan engines. A hypothesis was formed that turbofan engines operating at high altitudes near convective storms ingest ice crystals which are able to cool normally warmer than freezing engine hardware and form ice accretions leading to loss of engine control. There is ongoing research being conducted to understand the physics of this phenomenon which has been termed “Ice Crystal Icing”.

Recent NASA / NRCC research revealed ambient pressure, ambient temperature, and ambient relative humidity effects on the equilibrium surface temperature and the on-set of ice accretion on a simple aerodynamic surface operating in icing conditions in a simulated turbofan engine low pressure compressor environment. The NASA/NRCC researchers attempted to express the combined effects of these parameters by calculating the thermodynamic wet-bulb temperature based on experimental observations for the respective test cases. The surfaces inside a turbofan engine differ from typical super cooled liquid water ice accretion surfaces such as in air frame icing, in that they are initially warmer than freezing temperature.
The research conducted for this dissertation was performed to help provide insight into these joint experiments and study the physics of the surface energy balance. The question motivating this research is as follows: What is the effect of varying ambient pressure, temperature, relative humidity and impinging particle temperature on the equilibrium surface temperature, the physical terms of the energy balance and the on-set of ice accretion?

This present work is performed modeling super cooled liquid water impinging onto a simple airfoil, not ice crystals or mixed phase. The development of the simple liquid-only model is accomplished before attempting to develop the more complicated mixed phase or ice crystal only heat balance model. However the simple model is adequate to demonstrate the effect of particle temperature on the surface temperature. The appendix provides the derivation of both an ice crystal only (Appendix B) and a mixed phase (Appendix C) heat balance model. The complexities of each are presented and briefly discussed.

A large unknown exists for all existing experimental work for research in this area—the temperature of the ice particles themselves is unknown. Consider an ice particle initially at the altitude ambient static conditions that is entrained in air being ingested by an engine attached to the wing of an aircraft traveling at some typical flight Mach number. The ice particle would be exposed to very large relative velocities. It would enter a warmer, higher pressure environment, pass through several stages of the engine and impinge upon engine hardware in on the order of one hundredth of a second. What temperature is the ice particle? It
is conceivable that the ice particle is somewhere between the ambient static temperature at altitude and the ambient static temperature of the air flow in the low pressure compressor. Not enough is known about ice crystal or mixed phase impact physics or heat transfer over hundredths of a second. These would be good issues to study for future work. The work in this dissertation seeks to understand the liquid water particle only with a range of impinging temperatures prior to taking on the more complex mixed phase or ice crystal only development.

A research tool is developed implementing the well accepted Tribus-Messinger (T-M) Heat Balance model for aerodynamic surfaces operating in icing conditions. The T-M model is modified and adapted into a computer program. The T-M model strategy for a solution is to solve a surface energy balance by finding an equilibrium surface temperature and freezing fraction such that the energy transfer at the surface is zero. The physical terms that make up the surface energy balance are: convection, latent, sensible, viscous and kinetic energy. It is a steady state model.

The modified T-M (MTM) model is used to study the effect of impinging droplet temperature, ambient pressure, ambient temperature and ambient relative humidity on the equilibrium surface temperature of a simple airfoil operating in icing conditions. The inputs for the model runs are based on existing experimental data. Each model run is conducted using the original experimental inputs and then the original inputs are modified as needed to study the effect of varying the parameter of interest for its effect on equilibrium surface temperature.

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For each parameter of interest test run the pressure, temperature, and relative humidity of interest are all held constant and the impinging particle temperature is varied from the ambient static temperature to -40F. By this method the effect of the parameter of interest and the impinging particle temperature are studied for the various test runs. For each case, the model output includes the original unmodified inputs that are compared to the observations from the original experiment. The model output is found to compare within +/- 0.5F of the original Messinger experimental observations for equilibrium surface temperature and within +/- 0.01 of the Messinger calculated freezing fraction, based on his experimental observation.

**Messinger Experimental Case 2A**

The parameters of interest were studied using inputs from Messinger Case 2A shown in Table 1. The original ambient pressure (10.1psia), temperature (4.6F) and relative humidity (100%) were used as the original test run in the model.

In order to study the effect of the parameters of interest on the equilibrium surface temperature and the on-set of ice accretion, the following parameters were varied: ambient static pressure (B), ambient static temperature (t), relative humidity (RH) and impinging particle temperature. Two parameters were held constant while the third parameter of interest was varied through a range of values.
Pressure, Temperature, Relative Humidity and impinging Particle Temperature Effects

It was found that decreasing the input parameters of interest acts to decrease the equilibrium surface temperature and increase the value of the freezing fraction predicted by the MTM model when $0<n<1$. In general, changing the input parameters can effectively turn on and off the on-set of ice accretion and/or change the characteristic of the ice that accretes on the surface.

The input parameters of ambient temperature, pressure and relative humidity and impinging particle temperature are found to be coupled in the physics of the MTM surface energy balance in the latent, convective and sensible terms.

Physical Energy Transfer Terms

Understanding the effect on the physical energy transfer terms that make up the surface energy balance gives insight to the physics of what is happening on the surface for the various run conditions tested in the model. The terms are either heating terms (negative) or cooling terms (positive) according the convention used in the dissertation. The following physical terms are discussed:

Convection, Latent, Sensible and Viscous.

**Convection**

The convection term is governed by the difference between the surface temperature and the ambient static temperature factored by the heat transfer coefficient, equation 5.2. When the surface temperature is larger than the ambient temperature the convective term is positive or a cooling term. For many
of the test runs the equilibrium surface temperature is equal to freezing and it is constant. For these conditions the convection term is also constant since the ambient static temperature and pressure is constant for all test runs. For a given static ambient condition, the convective term trends as the equilibrium surface temperature trends. The surface temperature is influenced by impinging particle temperature through the sensible term and it is also related to the latent term.

**Latent**

The latent term, equation 5.3, is made up of three components: fusion, evaporation and sublimation. When the freezing fraction is less than 1, n<1, the sublimation term is zero in this model. If the freezing fraction is between zero and 1, 0<n<1, super cooled liquid water impinges and partially freezes upon impact. For this range of the freezing fraction both ice and water form on the surface and both the fusion term and the evaporation term combine to make up the latent term. If n=0, the fusion term is zero and the latent term is only accounting for the evaporation or condensation of surface water. In this model, the ambient pressure, ambient temperature and ambient relative humidity are components of the latent heat term, equation 5.7. The latent term is often predicted by the MTM model to be the dominant cooling or heating term and sometimes it transitions from dominant heating to dominate cooling as the impinging droplet temperature decreases. This highlights how the importance of the latent term is influenced by the impinging particle temperature through the sensible and convective terms of the surface energy balance.
**Sensible**

The sensible heat term in the MTM model, equation 5.11, is governed by the temperature difference between the incoming particle temperature and the surface temperature. The impinging particle temperature influences the other parameters through the sensible term. If the surface temperature is warmer than the incoming particle temperature, the sensible term is a cooling term. For most of the conditions studied it is a cooling term.

**Viscous**

The viscous term is a function of only airspeed. For the conditions studied the viscous term is constant and heating for all impinging particle temperatures. For the warm ambient static temperatures the viscous term can be the dominating heating term in the balance—especially for warmer impinging particle temperatures where the sensible energy contributions are small.

**Physical Energy Terms for Standard Day versus Tropical Day Ambient Conditions**

The theory of ice crystal icing suggests that the LOTC field events are occurring predominantly during tropical day type ambient conditions. In this work a comparison is made of the physical energy transfer terms predicted using the MTM model in the simulated LPC environment of a turbofan engine operating at standard day and tropical day ambient conditions. The result shows that for a standard day type the equilibrium surface temperature is equal to freezing for all impinging particle temperatures. The Ice Crystal icing theory implies that ice particles would likely not accumulate on a cold surface. For the tropical day type,
the initial surface temperature is 20F warmer than standard day conditions. The model predicts a transition from equilibrium surface temperature warmer than freezing to one equal to freezing for a given impinging particle temperature. The warmer initial surface temperature and the existence of the transition supports the hypothesis that wetted smaller ice particles can impinge, accumulate and melt on an initially warmer than freezing surface. The increase in surface water leads to potential accumulation and melting of larger particles that eventually cool the surface to freezing where ice accretion can begin.

**Case 539 and 539a**

Case 539 is an MTM model run of a joint NASA/NRCC ice accretion studies experimental case conducted on a simple airfoil operating in super cooled liquid water icing conditions in the simulated low pressure compressor environment of a turbofan engine. The NASA/NRCC researchers calculated the thermodynamic wet-bulb temperature, based on experimental observations, to account for the effects of the pressure, temperature and relative humidity on the equilibrium surface temperature and freezing fraction. They calculated the thermodynamic wet-bulb temperature to be 27F. Based on analysis from the joint NASA/NRCC fundamental accretion studies, ice accretion is expected to occur on the initially warmer than freezing surface for this condition when $t_{wb} < t_{fz}$. The experimental observation is that an accreted glaze ice formation occurred. This characteristic ice suggests a freezing fraction in the ranges between $0 < n < 0.3$. Researchers measured the equilibrium surface temperature of the leading edge of the airfoil to
be an average of 33°F. This was measured at steady state conditions using a thermocouple embedded beneath the surface.

Figure 35 presents these MTM model run results. The model predicts that the equilibrium surface temperature is equal to freezing temperature for all impinging particle temperatures. By inspection of these results the MTM model predicts that the impinging particle temperature, for this range value of freezing fraction that correlates with the experimentally observed characteristic ice accretion, is at least 25°F. If the impinging particle temperature is assumed to be the ambient static air temperature, a common assumption for air frame icing analysis, the model predicts the characteristic ice is a low n ice. This could correspond to an accreted glaze ice without horns. Based on the MTM model predictions, the impinging particle temperature is between 25°F and 49°F.

The temperature of a liquid water droplet could become less than ambient static temperature due to evaporation from the surface as the droplets are entrained into 8% relative humidity air. For ice crystals the development of a surface melt could allow cold ice crystals to survive the 1/100th of a second trip through the warmer turbofan engine environment to impingement. Super cooled liquid water and ice crystal heat transfer studies over the very short residence times in a turbofan engine environment are important and recommended for future work.

Figure 36 presents results of Case 539a. Case 539a inputs are developed by taking the case 539 inputs and increasing the ambient static pressure from 6.3 psia to 10psia. The pressure change is the only difference. Case 539a is run in
the MTM model. The results show that for an impinging particle temperature between 34F and 49F, the equilibrium surface temperature is predicted to be warmer than freezing. This suggests that freezing fraction is equal to zero (n=0) and that no ice will accrete. This demonstrates the MTM model prediction of a pressure effect that can turn on and off the on-set of ice accretion. It also demonstrates the MTM model prediction of a significant impinging particle temperature effect on equilibrium surface temperature and the characteristic ice that accretes on the surface.

**Physical Heat Transfer Terms, 539 and 539a**

Figures 38 and 39 show predicted results of the MTM model for energy transfer rate and equilibrium surface temperature vs. impinging particle temperature. The only difference between these two predicted results is that the ambient static pressure for case 539 is 6.3psia and it is 10psia for case 539a.

For case 539, ice accretes on the surface and the predicted impinging particle temperature is between 25F and 49F. For these conditions the dominant heating term is the convective term and the dominant cooling term is the latent term. The latent term has two components: fusion and evaporation. The fusion term is a heating term as super cooled liquid water that impacts the surface and releases heat in order to freeze. The evaporation term is a cooling term as super cooled liquid water requires heat to evaporate from the surface. The latent term competes directly with the sensible term for the dominant cooling role. As the impinging particles grow colder they require more heat in order to warm up to the
surface temperature. The evaporation term remains constant since it is driven by the difference in vapor pressure on the surface at the surface temperature and the ambient vapor pressure at the ambient static temperature in ratio with the ambient static pressure, equation 5.7. The latent energy term is relatively the same between case 539 and case 539a, but the magnitudes of the latent term components are significantly higher for the 6.3 psia case 539 compared to the case 539a. The evaporation component is 58% larger for case 539 and the fusion term is 500% higher at the particle impingement temperature of 25F. The fusion term has similar trend for both cases but the case 539a fusion term is zero until an impinging particle temperature of 33F. The evaporation term is more physical since the driving force for evaporation is a function of 1/B. The model predicts a significant pressure effect in the latent term, specifically the evaporation component.

**Case 545 and 545a**

Case 545 is a joint NASA/NRCC ice accretion studies experimental case conducted on a simple airfoil operating in super cooled liquid water icing conditions in the simulated low pressure compressor environment of a turbofan engine. The wet-bulb temperature was calculated to be above freezing, 36F, by the researchers which suggested to them based on past experience that ice accretion would not occur. The experimental observation is that no ice accreted or accumulated on the surface. This suggests a freezing fraction equal to zero (n=0). Researchers measured the equilibrium surface temperature of the leading edge of the airfoil to be an average of 39F.
The Case 545 inputs are run in the MTM model. Figure 41 presents these results. The model predicts that the equilibrium surface temperature is warmer than freezing temperature for impinging particle temperatures between 17F and 49F. Since no ice is experimentally observed, the model predicts that the impinging particle temperature is warmer than 17F. If the impinging particle temperature is assumed to be the ambient static air temperature the model predicts the equilibrium surface temperature to be 36F. This is within 7.5% of the experimentally measured value of 39F by the embedded leading edge thermocouple. The model predicts that the temperature of the impinging droplets is close to the ambient static temperature. It is worth noting that the relative humidity is 29.3%. This is over 260% higher than in Case 539 where ice accretion occurs and the particle temperature is predicted to be between 25F and ambient static temperature. The model predicts that the ambient relative humidity can have a significant effect on the temperature of the impinging particles. This is the result of less evaporation from the surface of the particles at the higher value of relative humidity. Considering an ice crystal instead of a super cooled liquid water droplet, this could be a limiting case of this model. The reason is that if a surface melt (23) forms on the ice crystal and is responsible for the thermal resistance of the ice particle warming up to the ambient static air, this effect would be possible at any relative humidity condition not just the lower relative humidity conditions. Future work should include a study of the ice crystal heat transfer physics and the mechanism if any that serves to thermally insulate
the ice from the warmer ambient static air temperature in the short residence
times and high relative velocity environment of the turbofan engine.

Figure 42 presents results of Case 545a. Case 545a inputs are developed by
taking the case 545 condition and reducing the relative humidity to 8% from
29.3%. Case 545a is run in the MTM model. The relative humidity change is the
only difference. The results show that for all impinging particle temperatures, the
equilibrium surface temperature is predicted to be equal to freezing temperature.
This suggests that freezing fraction is equal to a value between zero and one,
(0<n<1) and that ice will accrete on the surface. This demonstrates a significant
relative humidity effect. It also demonstrates a significant impinging particle
temperature effect on equilibrium surface temperature and the characteristic ice
that accretes on the surface.

Physical Heat Transfer Terms, 545 and 545a

Figures 44 and 45 show predicted results of the MTM model for energy transfer
rate and equilibrium surface temperature vs. impinging particle temperature. The
only difference between these two predicted results is that the ambient relative
humidity for case 545 is 29.3% and it is 8% for case 545a.

For case 545 no ice accretes on the surface for all impinging particle
temperatures warmer than 17F and the predicted impinging particle temperature
is the ambient static temperature of 49F. For these conditions the dominant
heating term is the convective term and the dominant cooling term is the latent
term. For this condition, the latent term has two components: fusion and
evaporation. The fusion term is a heating term as super cooled liquid water that impacts the surface releases heat in order to freeze. The evaporation term is a cooling term as super cooled liquid water requires heat to evaporate from the surface. The latent term competes directly with the sensible term for the dominant cooling role. As the impinging particles grow colder they require more heat in order to warm up to the surface temperature. The evaporation term is driven by the difference in vapor pressure on the surface at the surface temperature and the ambient vapor pressure at the ambient static temperature quantity in ratio with the ambient static pressure, equation 5.7. The surface vapor pressure is a function of the equilibrium surface temperature. The model predicts that ice accretion occurs for all impinging particle temperatures for the case 545a. The latent energy term is relatively the same between case 545 and case 545a, but the magnitudes of the latent term components are significantly higher for the 8% relative humidity case (545a) compared to the 29% relative humidity case (545). The evaporation component is 44% larger for case 545a and the fusion term is 100% higher at the estimated particle impingement temperature of 49F. The fusion term difference is explained by the fact that fusion is zero for the no ice accretion conditions of case 545. It does have a similar trend for both cases for all impinging particle temperature warmer than 17F. The evaporation term difference is that the driving force for the evaporation is a function of vapor pressure which is directly affected by relative humidity and ambient and surface temperature. The model predicts a significant relative humidity effect in the latent term, specifically the evaporation component.
Adiabatic Saturation / Thermodynamic Wet-bulb Temperature

The joint NASA/NRCC Fundamental Ice Accretion research identified a correlation between a colder than freezing calculated thermodynamic wet-bulb temperature and the on-set of ice accretion for an initially warmer than freezing surface operating in icing conditions. The joint experiments calculated the thermodynamic wet-bulb temperature for Case 539 and case 545. The result is that for warmer than freezing thermodynamic wet-bulb temperatures, no ice accretes and for colder than freezing temperature, ice accretion occurs. The MTM model outputs predict the adiabatic saturation temperature or thermodynamic wet-bulb temperature for all test conditions run in the model. For the test conditions run, the MTM model predicts that the adiabatic saturation temperature is below freezing for all cases where ice accretion is predicted to be able to occur and is warmer than freezing where ice accretion is predicted to not be able to occur. The MTM model includes the impinging particle temperature effect which influences the equilibrium surface temperature. This suggests that for some cases the wet-bulb temperature alone may not always predict the on-set of ice accretion.

Conclusions

A novel research tool, in the form of a computer model, is developed to study the physics of ice accretion in a turbofan engine environment. The model allows the user to investigate the effect of impinging particle temperature, ambient static pressure, ambient static temperature and relative humidity on the equilibrium surface temperature, the physical energy transfer terms and the thermodynamic
wet-bulb temperature of an aerodynamic surface operating in icing conditions in a simulated low pressure compressor turbofan engine environment.

The MTM program predicts that for the conditions studied, the following test parameters have a significant effect on the equilibrium surface temperature: impinging particle temperature, ambient static pressure, ambient static temperature and ambient relative humidity. The impinging particle temperature effect is newly identified in this research. The effects of ambient static temperature, pressure and relative humidity were empirically identified in the joint NASA/NRCC studies.

When equilibrium surface temperature is predicted to be warmer than freezing and no ice is predicted to accrete, a change in just one of the parameters is shown to result in a prediction that the equilibrium surface temperature is equal to or colder than freezing temperature and that ice accretes.

When equilibrium surface temperature is predicted to be equal to or colder than freezing and ice is predicted to accrete, a change in just one of the parameters is shown to result in a prediction that the equilibrium temperature is warmer than freezing and no ice accretes.

When equilibrium surface temperature is predicted to be equal to freezing temperature and ice is predicted to accrete with some freezing fraction between zero and one (0<n<1), changing the test parameters is shown to effect the freezing fraction and the characteristic ice that accretes.
Studying the individual parameters reveals relevant effects on the equilibrium surface temperature but studying the effect of changing two or more parameters is demonstrated to have a greater effect than the individual parameters. This suggests the parameters are coupled in the physics.

Investigating the effect on the physical terms from varying the test parameters reveals that they are related through the evaporation component of the latent term for warmer than freezing equilibrium surface temperatures. The evaporation component is a function of the vapor pressure at the surface temperature and the ambient vapor pressure at the ambient static temperature. It is also a function of the ambient static pressure. The sensible term also affects the surface energy balance which in turn affects the equilibrium surface temperature. The convective term is also affected by the equilibrium surface temperature and the ambient static temperature which affects the surface balance. Since the impinging particle temperature affects the sensible term, it influences all of the test parameters through the physics of the surface energy balance.

The model predicts the adiabatic saturation temperature is colder than freezing for predicted ice accretion and warmer than freezing for predicted ice accretion not to occur. This agrees with NASA/NRCC empirical findings. The MTM model shows that the impinging particle temperature affects the surface temperature which suggests that the wet-bulb temperature alone may not always predict the on-set of ice accretion.
Using the model to study the NASA/NRCC case 539 and case 545, insight as to the temperature of the impinging particles is gained. The model uses the observed characteristic ice and estimated freezing fraction associated with this type of ice accretion and arrives at the probable range of impinging particle temperatures. The model can be used for similar insight into existing experimental test runs and it can help plan future experiments by predicting pressure, relative humidities and temperatures that may turn on and off the on-set of ice accretion. Implementing the MTM model identified key technology required for improvement the experiment: the measurement of impinging particle temperature and liquid water content of the cloud.

The joint NASA/NRCC fundamental ice accretion studies identify three key effects on the equilibrium surface temperature: ambient static pressure, ambient static temperature and ambient relative humidity. They also correlate a below freezing thermodynamic wet-bulb temperature with the on-set of ice accretion on initially warmer than freezing surfaces. The research conducted in this dissertation predicts these effects and correlates the MTM predicted adiabatic saturation temperature with the on-set of ice accretion. It also identifies a new impinging particle temperature effect.

The research investigating the effect of these parameters on the physical energy transfer terms, identifies these parameters are coupled in the physics of the MTM energy balance.
Chapter 8: Summary and Future Work

Industry engineers investigating a mounting number of turbofan engine high altitude loss of power field events, presented a hypothesis in 2006 suggesting that the events are related to a phenomenon they termed ice crystal icing. They proposed that turbofan engines operating at high altitude are ingesting ice crystals that accrete inside the LPC on warm engine hardware, resulting in a temporary loss of power in some cases. A subgroup of the Engine Icing Working Group, the Ice Crystal Consortium (ICC) was formed from industry members to develop a technology roadmap to better understand this phenomenon. This effort led to several major ongoing research programs. The goal of the ICC is to better characterize the nature of the in-flight environment and to better understand the physics of ice crystal icing in a turbofan engine. The work of the ICC eventually led to an FAA newly proposed rulemaking (NPRM) entry into the Federal Registrar in June of 2010 (4).

Among several ongoing research efforts, NASA and NRCC entered into a collaborative research agreement to better understand fundamental ice crystal accretion in the internal environment of a turbofan engine. This research identified several key effects on the equilibrium surface temperature and the onset of ice accretion for a warm surface operating in icing conditions:

1. Ambient Static Pressure
2. Ambient Static Temperature
3. Ambient Relative Humidity
4. Thermodynamic Wet-bulb Temperature

5. The ice and liquid water content of the cloud.

The joint research produced super cooled liquid water, ice crystal and mixed phase clouds which impinged onto a warm surface. Observations were made on whether or not ice accreted on the surface. The joint research also calculated the thermodynamic wet-bulb temperature. The onset of ice accretion correlated with a below freezing thermodynamic wet-bulb temperature.

An opportunity for greater insight exists for the joint NASA/NRCC research. Because it is difficult to predict and produce the exact environment inside the turbofan engine, the true nature of the ingested ice crystals is not known. One important parameter is the temperature of the impinging cloud particles. This is not feasibly measured or predicted for the joint research testing due to a short residence time of particles at the warmer temperatures of the test cell. The ice particles start off cold, then are mixed with super cooled water spray and finally entrained into conditioned air being ingested into the test cell. The result is a mixed phase cloud of unknown particle temperatures.

This Dissertation documents the development and research conducted in order to help NASA/NRCC researchers gain insight into these empirical results. A computer code is developed based on a modified Tribus Messinger (T-M) heat balance model that allows for the variation of ambient pressure, ambient temperature, relative humidity and impinging particle temperatures. The model outputs the equilibrium surface temperature and freezing fraction on the surface.
being modeled as a function of the key effects identified in the joint NASA/NRCC research and a new effect, identified using the MTM model, the impinging cloud particle temperature. The MTM model allows the investigator to vary cloud particle temperatures and ambient vapor pressures.

The MTM code is compared against several experimental data points including several of the joint NASA/NRCC cases. It is demonstrated to predict the outcome of the NASA/NRCC experiments and to compare within 0.5°F and 0.01 of the equilibrium surface temperature and freezing fraction based calculated based on experimental observations by Messinger. It also shows that impinging particle temperature is a key effect on the equilibrium surface temperature and the onset of ice accretion.

Through analysis of the MTM model output and with knowledge of the characteristic ice formed in the joint NASA /NRCC experiments, the range of likely impinging particle temperatures was predicted. This demonstrates how researchers can better quantify the temperature of the impinging cloud particles and gain insight into existing and future experimental results. Research using the MTM model also shows that the model inputs are coupled in the latent, convective and sensible energy terms of the surface energy balance.

A novel computer model has been developed that can assist researchers in both analyzing existing experimental work and developing future experimental tests leading to increased fundamental understanding of the ice crystal icing phenomenon in an internal turbofan engine environment.
The following summary highlights the accomplishments of the research objectives documented in this dissertation and lists with suggested future work:

1. Provided the motivation for this and related research in this area:
   a. Turbofan Engine Ice Crystal Icing Field Events

2. Presented a current research problem that needs to be addressed:
   a. Difficult to simulate and measure internal turbofan engine environment. Need a tool to help analyze results to gain more insight into the physics of warm surface ice accretion and to help plan future experiments.

3. Linked research objectives to current research and a problem to be addressed:
   a. Developed a computer model based on a modified Tribus-Messinger heat balance model to study the effect of ambient pressure, ambient temperature and relative humidity on the equilibrium surface temperature in an internal turbofan engine environment.

   b. Implemented the computer model to gain insight to the physics of the ice accretion on warm surfaces by studying the convective, sensible, latent and viscous energy terms.

   c. Implemented the model to study the behavior of the adiabatic saturation temperature (analogous to thermodynamic wet-bulb temperature) in predicting the on-set of ice accretion on initially warm surfaces

4. The research conducted identified ambient pressure, ambient temperature and altitude effects on the equilibrium surface temperature, on-set of ice accretion and the characteristics of the ice that forms on the surface.

   a. The parameters identified by the model were also identified by the joint NASA/NRCC research in fundamental ice accretion physics in a simulated turbofan engine environment.

5. Research conducted identified adiabatic saturation temperature or thermodynamic wet-bulb temperature as a good indicator of ice accretion on an initially warm surface.
a. This was also identified by the NASA/NRC Canada work in fundamental ice accretion physics in a turbofan engine environment.

6. The research conducted made the following discoveries:

a. Impinging particle temperature affects the equilibrium surface temperature, the on-set of ice accretion and the characteristics of the ice that forms on the surface.

b. The test parameters are coupled in the physics contained in the latent, convective and sensible energy terms of the surface energy balance.

c. Developed insight to the physics which suggests the difference between standard day and tropical day conditions can lead to the on-set of ice accretion in a turbofan engine environment while operating in tropical day type ambient conditions.

Future work is suggested:

1. Use the MTM computer model to analyze additional super cooled liquid water cases from the joint NASA / NRCC studies on ice crystal accretion physics to gain insight as to the temperature of the impinging particles in the RATFac.

2. Use the MTM computer model to analyze ice crystal only and mixed phase cases from the joint NASA / NRCC studies. Develop an ice crystal only and mixed phase variation of this tool if needed.

3. Use the MTM model to help develop future experimental work.

4. In an engine, the rotors through which the cloud particles must pass to reach the outlined red regions in Figure 2, could easily centrifuge many of the ice/water particles to the outer casing of the LPC. Explore the nature of the internal engine ice crystal environment and adapt this tool for impingement of a different kind. For example it might be useful to adapt this tool for a mixed phase, aerodynamic driven flow running along the outer case surface of the engine that impacts an area of stagnation along the wall.

5. Explore simpler methods to analytically account for ice crystal or mixed phase clouds. For instance it might be possible to add in a term for latent heat to melt ice crystals contained in the flow rather than derive the heat balance to account for both the water catch and the ice catch. This might
avoid the complications of incorporating both streams in a derivation as shown in Appendices B, C

6. Conduct further research on the changes in the physical heat transfer terms as a function of the ambient pressure, temperature, relative humidity and impinging particle temperatures as it relates to understanding the limiting factors associated with how well the adiabatic saturation temperature can be used as an indication for the on-set of ice accretion on an initially warm surface operating in an icing environment.

7. Study the evaporation/sublimation of particles at the residence times relevant to ingestion into a turbofan engine environment

8. Study the heat transfer on cold particles entrained in a warm air stream at the residence times relevant to ingestion into a turbofan engine environment

9. Study the differences in the physical energy transfer terms between tropical day type and standard day type conditions.

10. Investigate surface melting on ice particles to see if it acts to insulate the ice from warming up when entrained in a warm air stream at the residence times relevant to ingestion into a turbofan engine environment.

11. Investigate Ice crystal and mixed phase impact physics

12. Investigate ways to uncouple the MTM model input parameters to facilitate studying their individual impact on the physics of the surface energy balance.
Appendix A: Control Volume Analysis Derivation of the T-M Heat Balance Model

The objective of providing this appendix is to perform a control Volume analysis for the Tribus case near the stagnation region of the schematic in Figure 11. Tribus derived the original heat balance based on a surface energy balance with no generation or storage terms. He ignored runback and shed ice (17). These will be included in the control volume derivation.

Consider a control volume drawn around the stagnation region of an aerodynamic surface operating in icing conditions, Figure 9.1. Moist air and super cooled liquid water are entering at plane 1. Moist air, super cooled liquid water (entrained in the air flow) and surface water driven by aerodynamic forces are leaving at plane 2.

Figure 9.1: A control volume drawn around the stagnation region of an aerodynamic surface operating in icing conditions. Moist air and super cooled liquid water are entering at plane 1. Moist air, super cooled liquid water (entrained in the air flow) and surface water driven by aerodynamic forces are leaving at plane 2.
forces are leaving at plane 2. Water and ice (super cooled water that freezes upon impact) accumulate inside the control volume.

Water and ice (super cooled water that freezes upon impact) accumulate inside the control volume. In this section a mass and energy balance is conducted for this control volume.

**Conservation of Mass**

\[
\frac{dM}{dt} = \sum M_{in} - \sum M_{out} \tag{9.1}
\]

\[
\sum M_{in} = M_{\text{dry air,1}} + M_{\text{water vapor,1}} + M_{\text{liquid water,1}} \tag{9.2}
\]

\[
\sum M_{out} = M_{\text{dry air,2}} + M_{\text{water vapor,2}} + M_{\text{liquid water,2}} \tag{9.3}
\]

\[
W_1 = \frac{\text{mass of water vapor}}{\text{mass of dry air}} = \text{humidity ratio} \tag{9.4}
\]

\[
M_{\text{dry air,1}} + M_{\text{water vapor,1}} = M_{\text{dry air,1}} + W_1 M_{\text{dry air,1}} \tag{9.5}
\]

\[
M_{\text{dry air,2}} + M_{\text{water vapor,2}} = M_{\text{dry air,2}} + W_2 M_{\text{dry air,2}} \tag{9.6}
\]

\[
M_{\text{dry air,1}} = M_{\text{dry air,2}} \tag{9.7}
\]

\[
M_{\text{liquid water,1}} = LWC_1 \times \text{Volume of Dry Air}
\]

\[
= LWC_1 \times \frac{M_{\text{dry air,1}}}{\rho_{\text{air,1}}} \tag{9.8}
\]

\[
M_{\text{liquid water,2}} = LWC_2 \times \text{Volume of Dry Air} + \text{surface runback}
\]

\[
= LWC_2 \times \frac{M_{\text{dry air,2}}}{\rho_{\text{air,2}}} + M_{\text{liquid water,runback}} \tag{9.9}
\]
O = water accumulation parameter
\[ O = \frac{\text{mass of water that impinges and accumulates}}{\text{mass of water that impinges}} \]  
Dimensionless \( 0 < O < 1 \)

\[ w_R = \text{Unit rate of water impingement} \]  

\[ LWC_2 = LWC_1 - \text{impinging water that accumulates + shedding water} \]  
\[ = LWC_1 - R_w O + M_{\text{liquid,shedding}} \]  

\[ \frac{dM}{dt} = \left\{ W_1 \frac{M_{\text{dry,1}}}{\rho_{\text{air,1}}} + LWC_1 \frac{M_{\text{dry,1}}}{\rho_{\text{air,1}}} \right\} 
- \left\{ W_2 \frac{M_{\text{dry,2}}}{\rho_{\text{air,1}}} + LWC_1 \frac{M_{\text{dry,1}}}{\rho_{\text{air,1}}} \right\} - R_w O + M_{\text{liquid,shedding}} \]  

\[ \frac{dM}{dt} = (W_1 - W_2)M_{\text{dry,1}} + R_w O - M_{\text{liquid,shedding}} \]  

Conservation of Energy
\[ \frac{dE}{dt} = \sum E_{\text{in}} - \sum E_{\text{out}} \]  

\[ E_{\text{in}} = \text{Enthalpy Moist Air} \]  
+ Kinetic Energy droplets  
+ Viscous Energy  
+ Conduction

\[ \text{Enthalpy Moist Air} = h_{\text{moist,air,1}} \]  

\[ \text{Kinetic Energy of droplets} = \frac{1}{2} R_w A V_{\infty}^2 \]
Viscous Energy = \( \frac{h_c \alpha V^2}{2c_{p,a}} \) \hspace{1cm} (9.19)

The recovery factor, \( r \), is equal to one near the stagnation region of interest.

\[
E_{\text{in}} = h_{\text{moist air,1}} + \frac{1}{2} R_w A V^2 + \frac{h_{\text{AV}}^2}{2c_{p,a}} + Q_{\text{conduction}}
\hspace{1cm} (9.20)
\]

\[
E_{\text{out}} = \text{enthalpy moist air} + \text{convection} + \text{sensible water ( warming impinging water )} + \text{sensible ice (cooling surface ice )} + \text{latent ( fusion)}
\hspace{1cm} (9.21)
\]

Enthalpy moist air = \( h_{\text{moist air,2}} \) \hspace{1cm} (9.22)

Convection

\[
Q_{\text{convection}} = h_c A (t_{\text{sur}} - t_x) \hspace{1cm} (9.23)
\]

Sensible heat as impinging liquid water at some particle temperature is warmed to surface temperature or to freezing temperature depending on the conditions.

\[
Q_{\text{sensible, impinging water}} = R_w A O (1-n) C_{p,w} (t_{\text{sur}} - t_p) \hspace{1cm} (9.24)
\]

Sensible heat as surface ice is cooled down to surface temperature

\[
Q_{\text{sensible, surface ice}} = n R_w A C_{p,i} (t_{\text{freeze}} - t_{\text{sur}}) \hspace{1cm} (9.25)
\]

Heat of fusion as super cooled water impinges upon surface and freezes upon impact.

\[
Q_{\text{fusion}} = n R_w A L_{\text{fusion}} \hspace{1cm} (9.26)
\]
Evaporation is due to convective mass transfer. Tribus implemented the Lewis law to relate convective mass transfer to convective heat transfer. This can be done with small error for a water air system (19). For a control volume analysis this term is handled by the conservation of mass for enthalpy of moist air.

\[
E_{out} = h_{moist,2} + h_c A (t_{sur} - t_\infty) + R_w A O (1-n) C_{p,w} (t_{sur} - t_p) \\
+ n R_w A C_{f,I} (t_{freeze} - t_{sur}) + n R_w A L_{fusion}
\]  

(9.27)

\[
\frac{dQ}{dt} = \left\{ h_{moist,1} + \frac{1}{2} R_w A V_\infty^2 + \frac{h_c AV_\infty^2}{2C_{p,a}} + Q_{conduction} \right\} \\
- \left\{ h_{moist,2} + h_c A (t_{sur} - t_\infty) + R_w A O (1-n) C_{p,w} (t_{sur} - t_p) \right\} \\
+ n R_w A C_{f,I} (t_{freeze} - t_{sur}) + n R_w A L_{fusion}
\]

(9.28)

For some quasi-steady state \( \frac{dQ}{dt} = 0 \) and energy in equals energy out. \( Q_{conduction} \) then becomes:

\[
Q_{conduction} = h_{moist,2} + h_c A (t_{sur} - t_\infty) \\
+ R_w A O (1-n) C_{p,w} (t_{sur} - t_p) \\
+ n R_w A C_{f,I} (t_{freeze} - t_{sur}) + n R_w A L_{fusion}
\]

(9.29)

The enthalpy of moist air can be related to the relative humidity using the following formula (19).

\[
h_{moist air} = h_{dry air} + \text{W h_{water vapor at t_{\infty}}}
\]

(9.30)

\[
h_{moist air,1} = h_{dry air,1} + \text{W1 h_{water vapor at t_{\infty}}}
\]

(9.31)
From conservation of mass:

\[ M_{\text{dry air},1} = M_{\text{dry air},2} \text{ and } t_{\infty,1} = t_{\infty,2} \rightarrow h_{\text{dry air},1} = h_{\text{dry air},2} \]  

(9.33)

Therefore in equation 9.29 the two enthalpy terms for moist air reduce to:

\[ h_{\text{moist air},2} - h_{\text{moist air},1} = (W_2 - W_1) h_{\text{water vapor at } t_{\infty}} \]  

(9.34)

If evaporation or sublimation occurs as the air passes over the water/ice contaminated surface then \( W_2 \) will be greater than \( W_1 \) and the effect is to take energy away from the surface. This is essentially the concept of evaporative cooling term.

Equation 9.29 becomes:

\[ Q_{\text{conduction}} = + (W_2 - W_1) h_{\text{water vapor at } T_{\infty}} + h_{c} A (t_{\text{sur}} - t_{e}) + R_{w} A O (1-n) C_{p,w} (t_{\text{sur}} - t_{p}) + n R_{w} A C_{p,i} (t_{\text{freeze}} - t_{\text{sur}}) + n R_{w} A L_{\text{fusion}} - \frac{1}{2} R_{w} A V_{\infty}^2 - \frac{h_{AV_{\infty}^2}}{2C_{\rho,a}} \]  

(9.35)
Grouping like terms,

\[
Q_{\text{cond}} = + \left( W_2 - W_1 \right) h_{\text{water vapor}, T_c} \\
+ h_c A \left( t_{\text{sur}} - t_{s,a} \right) + \frac{V_o^2}{2C_{p,a}} \\
+ h_c A \left\{ O \left( 1 - n \right) C_{p,w} \left( t_{\text{sur}} - t_p \right) \right\} \\
+ R_w A \left\{ n C_{p,i} \left( t_{\text{freeze}} - t_{s,a} \right) \right\} \\
+ n L_{\text{fusion}} - \frac{1}{2} V_o^2 \\
\]

Defining the relative heating factor (b),

\[
b = \frac{R_w C_{p,w}}{h_c} \quad (9.37)
\]

Implementing equation 9.37 into 9.36:

\[
Q_{\text{cond}} = + \left( W_2 - W_1 \right) h_{\text{water vapor}, T_c} \\
+ h_c A \left( t_{\text{sur}} - t_{s,a} \right) + \frac{V_o^2}{2C_{p,a}} \\
+ \frac{h_c A b}{C_{p,w}} \left\{ O \left( 1 - n \right) C_{p,w} \left( t_s - t_p \right) \right\} \\
+ n C_{p,i} \left( t_{\text{freeze}} - t_s \right) \\
+ n L_{\text{fusion}} - \frac{1}{2} V_o^2 \\
\]

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Continue to group like terms,

\[ Q_{\text{conduction}} = + (W2-W1) h_{\text{water, vapor, } T_s} \]

\[ + h_c A \left( t_{\text{sur}} - t_\infty \right) \frac{V_\infty^2}{2C_{p,a}} \]

\[ + \frac{O}{O_{\text{water}}} \left( 1-n \right) b \left( t_{\text{sur}} - t_p \right) \]

\[ + h_c A \left[ \frac{n c_p l}{c_p w} b \left( t_{\text{freeze}} - t_{\text{sur}} \right) \right] \]

\[ + \frac{nb c_{\text{fusion}}}{c_{p,w}} \left( 1 - \frac{V_\infty^2}{2C_{p,w}} b \right) \]

\[(9.39)\]

Continue to group like terms,

\[ Q_{\text{conduction}} = + (W2-W1) h_{\text{water, vapor, } T_s} + \ldots \]

\[ \left( t_{\text{sur}} - t_\infty \right) \frac{V_\infty^2}{2C_{p,a}} + O \left( 1-n \right) b \left( t_{\text{sur}} - t_p \right) \]

\[ + n \frac{c_p l}{c_p w} b \left( t_{\text{freeze}} - t_{\text{sur}} \right) \]

\[ + \frac{nb c_{\text{fusion}}}{c_{p,w}} \left( 1 - \frac{V_\infty^2}{2C_{p,w}} b \right) \]

\[(9.40)\]

Simplify further,

\[ Q_{\text{conduction}} = (W2-W1) h_{\text{water, vapor, } T_s} + \ldots \]

\[ \left( t_{\text{sur}} \left( 1+b \left( O(1-n) \frac{c_p l}{c_p w} \right) \right) - t_\infty + t_p \left( Ob(n-1) \right) \right) \]

\[ + t_{\text{freeze}} \frac{c_p l}{c_p w} b n + \frac{L_{\text{fusion}}}{c_{p,w}} n b - \frac{V_\infty^2}{2} \left( \frac{1}{c_{p,a}} + \frac{b}{c_{p,w}} \right) \]

\[(9.41)\]
The main difference between the surface energy balance and the control volume approach is how the latent energy of evaporation and sublimation is handled. The control volume derivation tracks the enthalpy of moist air through the conservation of mass. The surface energy balance derivation relates the convective mass transfer to an analogous convective heat transfer.
Appendix B: T-M Heat Balance Modified for Ice Crystal Only Impingement

This derivation is conducted to demonstrate the complexity of analyzing the cases of ice crystal only impingement. Consider the Chapter 5 derivation of the modified T-M Heat balance model replacing the liquid water particles with ice particle. Figure 9 and 10 and 11 still apply directly (substituting ice particles for the liquid water droplets). Equation 5.1 still applies directly. The T-M Model assumptions still apply.

Introduction of Three New Dimensionless Factors

*Relative Heat Factor for Ice Crystal Impingement and Accumulation (b’)*

The relative heat factor for ice crystal impingement and accumulation introduced here is analogous to the Messinger b parameter introduced for liquid water impingement. It is used to relate the sensible heating capacity to the convective heating capacity of the surface undergoing ice crystal impingement. In this work it the parameter is assigned the symbol (b’). A new accumulation fraction (O’), defined below, needs to be introduced and included to account for the fact that ice particles can simply impinge the surface and bounce off. Only the mass of ice particles that impinge and accumulate have an effect on the value of b’.

\[
b' = \frac{R_{\text{ice}} O' c_{p,i}}{h_c} = \frac{\text{surface ice particle sensible heat capacity}}{\text{surface convective heat capacity}} \tag{10.1}
\]

Where O’ is the ice particle accumulation fraction defined below.
Ice Particle Accumulation Fraction ($O'$)

\[ O' = \frac{\text{mass of accumulated impinging ice particles}}{\text{mass of impinging ice particles}} \rightarrow (0 < O' < 1) \]  \hspace{1cm} (10.2)

Ice Particle Melting Fraction ($n'$)

\[ n' = \frac{\text{mass of impinging ice particles that melt upon impact}}{\text{mass of impinging ice particles}} \rightarrow (0 < n' < 1) \]  \hspace{1cm} (10.3)

Convective and Friction Terms

The convective and frictional terms are combined into one equation. Since Tribus was dealing with aircraft speeds he included the viscous term in his heat balance for completeness. At aircraft speeds interesting to commercial transport aircraft these terms are relative small compared to the other terms. They are included for completeness.

Convection

\[ \dot{Q}_{\text{convection}} = h_c A \left( t_s - t_\infty \right) \]  \hspace{1cm} (10.4)

Viscous

\[ \dot{Q}_{\text{viscous}} = h_c A r \left( \frac{V_\infty^2}{2c_p} \right) \]  \hspace{1cm} (10.5)

Combined Convection and Viscous

\[ \dot{Q}_{\text{Convection+viscous}} = h_c A \left( t_s - t_\infty - \frac{rV_\infty^2}{2c_p} \right) \]  \hspace{1cm} (10.6)
Sensible Terms

Energy required to warm impinging ice particles that impinge and accumulate on the surface to freezing temperature. This also accounts for the liquid water (from melted surface ice) that warms to the surface temperature.

Warming to Freezing Temperature of Impinging and accumulating Ice

\[
\dot{Q}_{\text{sensible, ice}} = R_i A c_{p,i} \left( t_{\text{freeze}} - t_p \right) = h_c A b' \left( O' \right)^2 \left( t_{\text{freeze}} - t_p \right)
\]

using equation 10.1

Warming of impinging, accumulating and melting ice surface temperature

\[
Q_{\text{sensible, water}} = R_i A c_{p,w} O' n' \left( t_s - t_{\text{freeze}} \right) = h_c c_{p,w} \left( O' \right)^2 \left( t_s - t_{\text{freeze}} \right)
\]

using equation 10.1

Combined Sensible Terms

\[
\dot{Q}_{\text{sensible}} = h_c A b' \left( O' \right)^2 \left( t_{\text{freeze}} - t_p \right) + h_c c_{p,w} b' \left( O' \right)^2 \left( t_s - t_{\text{freeze}} \right)
\]

Latent Heat Terms

Sublimation

\[
\dot{Q}_{\text{sub}} = L_{\text{sub}} R_{\text{sub}} A = h_c L_{\text{sub}} b' O' \left( R_{\text{sub}} / R_{\text{ice}} \right) A / c_{p,i}
\]

using equation 10.1

Evaporation

\[
\dot{Q}_{\text{evap}} = L_{\text{evap}} R_{\text{evap}} A = h_c L_{\text{evap}} b' O' \left( R_{\text{evap}} / R_{\text{ice}} \right) A / c_{p,i}
\]

using equation 10.1
**Fusion**

\[ \dot{Q}_{\text{fus}} = L_{\text{fus}} R_{\text{ice}} \quad O' \; n' \; A = h_c \; L_{\text{fus}} \; b' \; (O')^2 \; n' \; A / c_{p,i} \]  \hspace{1cm} (10.12)

using equation 10.1

**Combined Latent Terms**

\[ \dot{Q}_{\text{latent}} = h_c \; L_{\text{sub}} \left( R_{\text{ice}} / R_{\text{sub}} \right) b' O' A / c_{p,i} + h_c \; L_{\text{evap}} \left( R_{\text{ice}} / R_{\text{evap}} \right) b' O' A / c_{p,i} - h_c \; L_{\text{fus}} b' (O')^2 n' A / c_{p,i} \]  \hspace{1cm} (10.13)

**Kinetic Term**

\[ \dot{Q}_{\text{kinetic}} = R_i A \left( V_{\infty}^2 / 2 \right) = h_c \; A \left( V_{\infty}^2 \over 2 c_{p,i} O' \right) \]  \hspace{1cm} (10.14)

**Governing Equation for Ice Crystal Only Impingement based on a Surface Energy Balance**

Combing the previous terms into one equation, equation 10.1 the surface energy balance for ice crystal only impingement becomes:

\[ \dot{Q}_{\text{surface}} = + \left\{ h_c A \left( t_s - t_{\infty} - rV_{\infty}^2 / \left( 2c_p \right) \right) \right\} + \left\{ h_c A b' \left( O' \right)^2 \left( t_{\text{freeze}} - t_p \right) \right\} + \left\{ h_c C_{p,w} b' \left( O' \right)^2 \left( t_s - t_{\text{freeze}} \right) \right\} + \left\{ h_c L_{\text{sub}} b' O' \left( R_{\text{sub}} / R_{\text{ice}} \right) A / c_{p,i} \right\} + \left\{ h_c L_{\text{evap}} b' O' \left( R_{\text{evap}} / R_{\text{ice}} \right) A / c_{p,i} \right\} - \left\{ h_c L_{\text{fus}} b' \left( O' \right)^2 n' A / c_{p,i} \right\} + \left\{ h_c A b' \left( V_{\infty}^2 / \left( 2c_{p,i} O' \right) \right) \right\} \]  \hspace{1cm} (10.15)
Where the terms in the brackets affect the surface energy transfer:

1. Convective and Viscous
2. Sensible
3. Latent
4. Kinetic

The complexity for solution to the ice crystal only, modified surface energy balance model is that multiple unique solutions exist for a given set of inputs. This is in contrast to the simpler super cooled liquid water only derivation which had one unique solution of equilibrium surface temperature and freezing fraction per given set of inputs.
Appendix C: T-M Heat Balance Modified for Mixed Phase with Accompanying Statistical Analysis

This derivation is conducted to demonstrate the complexity of analyzing the cases of ice crystal and liquid water impingement. This is also termed mixed phase. Consider the Chapter 5 derivation of the modified T-M Heat balance model with both liquid water and ice water impinging particles. Figure 9 and 10 and 11 still apply directly (substituting ice particles for the liquid water droplets). Equation 5.1 still applies directly. The T-M Model assumptions still apply.

\[ \dot{Q}_{\text{surface}} = \dot{Q}_{\text{convection}} + \dot{Q}_{\text{latent}} + \dot{Q}_{\text{sensible}} + \dot{Q}_{\text{kinetic}} + \dot{Q}_{\text{friction}} \]  

(11.1)

Introduce Two Dimensionless Parameters Developed by Tribus and Messinger

**Impinging Water Relative Heating Factor**

\[ b = \text{relative heating factor} = \frac{R_w C_{p,w} O}{h_c} \]  

(11.2)

where O is the impinging water particle accumulation fraction defined below

**Freezing Fraction (n)**

\[ n = \frac{\text{mass of impinging water particles that freeze upon impact}}{\text{mass of impinging water particles}} \rightarrow 0 < n < 1 \]  

(11.3)

Introduce Five New Dimensionless Parameters

**Impinging ice Relative Humidity Factor**

\[ b' = \text{relative heating factor} = \frac{R_i C_{p,i} O'}{h_c} \]  

(11.4)

where O' is the impinging ice particle accumulation fraction defined below
Impinging water particle accumulation fraction \((O)\)

\[
O = \frac{\text{mass of impinging water particles that accumulates}}{\text{mass of impinging water particles}} \rightarrow (0 < O < 1) \quad (11.5)
\]

Ice particle accumulation fraction \((O')\)

\[
O' = \frac{\text{mass of accumulated impinging ice particles}}{\text{mass of impinging ice particles}} \rightarrow (0 < O' < 1) \quad (11.6)
\]

Melting fraction \((n')\)

\[
n' = \frac{\text{mass of accumulated impinging ice particles that melt}}{\text{mass of accumulated, impinging ice particles}} \rightarrow (0 < n' < 1) \quad (11.7)
\]

Surface Water Freezing Fraction

\[
n'' = \frac{\text{mass of surface water that freezes}}{\text{mass of surface water}} \rightarrow (0 < n'' < 1) \quad (11.8)
\]

The following equations characterize the energy transfer to or from the aerodynamic surface operating in icing conditions via convection, latent, sensible, kinetic and viscous modes.

**Convective and Viscous Modes/Terms**

The convective and frictional terms are presented individually and then as one combined term.

**Convection**

\[
\dot{Q}_{\text{convection}} = h_c A (t_s - t_{\infty}) \quad (11.9)
\]
Viscous

\[ \dot{Q}_{\text{viscous}} = h_c A \left( \frac{rV_c^2}{2c_{p,a}} \right) \]  \hspace{1cm} (11.10)

Combined Convection and Viscous

\[ \dot{Q}_{\text{Convection+viscous}} = h_c A \left( t_s - t_{\infty} - \frac{rV_c^2}{2c_{p,a}} \right) \]  \hspace{1cm} (11.11)

Sensible Terms

Energy required to warm impinging water and ice particles that impinge and accumulate on the surface to surface temperature (liquid) or freezing temperature (ice particles). The water that results from ice melting and then warming to surface temperature is also accounted for.

Warming of Impinging and Accumulated Ice to Freezing Temperature

\[ \dot{Q}_{\text{sensible,ice}} = R_i O' c_{p,i} A \left( t_{\text{freeze}} - t_p \right) = h_c b_i (O')^2 A \left( t_{\text{freeze}} - t_p \right) \]  \hspace{1cm} (11.12)

Warming of impinging super cooled liquid water that impinges and does not freeze upon impact to surface temperature

\[ \dot{Q}_{\text{warming,liquid}} = R_w OAC_w \left( t_{\text{se}} - t_{\infty} \right) (1-n) = h_c b O^2 (1-n)A \left( t_{\text{se}} - t_{\infty} \right) \]  \hspace{1cm} (11.13)

Warming of Impinging, accumulated and melted Ice water to surface temperature

\[ \dot{Q}_{\text{sensible,water}} = R_i O' n' c_{p,w} A(t_s - t_{\text{freeze}}) = h_c b' (O')^2 c_{p,w} A(t_s - t_{\text{freeze}}) \]  \hspace{1cm} (11.14)
**Combined Sensible Terms**

\[
\dot{Q}_{sensible} = + h_c b_i (O')^2 \frac{R_i}{R_w} A (t_{freeze} - t_p) \\
+ h_c b O^2 (1-n) A \left( t_{se} - t_{\infty} \right) \\
+ h_c b' (O')^2 \frac{c_{p,w}}{c_{p,i}} A (t_s - t_{freeze})
\]  

(11.15)

**Latent Terms**

**Evaporation**

\[
\dot{Q}_{evap} = L_{evap} R_{evap} A = h_c L_{evap} \frac{R_{evap}}{R_w c_{p,w}} OA
\]  

(11.16)

**Ice Particles impinging accumulating melting**

\[
\dot{Q}_{fus} = L_{fus} R_{fus} n'A = h_c L_{fus} b'(O')^2 n' A / c_{p,i}
\]  

(11.17)

**Sublimation**

\[
\dot{Q}_{sub} = L_{sub} R_{sub} A = h_c L_{sub} b' O' \left( \frac{R_{sub}}{R_{ice}} \right) A / c_{p,i}
\]  

(11.18)

**Liquid water impinging and freezing on impact**

\[
\dot{Q}_{freezing, liquid on impact} = R_w OAL_{fusion} n = h_c L_{fus} b O^2 n A / c_{p,w}
\]  

(11.19)

**Liquid water on the surface that accumulates and then freezes**

\[
\dot{Q}_{freezing, liquid on surface} = (R_w O(1-n) + R_i O'n') A L_{fus} n'' = h_c L_{fus} n'' b \frac{O}{c_{p,w}} \left( O(1-n) + \frac{R_i}{R_w} O'n' \right)  
\]  

(11.20)
Combined Latent Terms

\[ \dot{Q}_{\text{Latent}} = \dot{h}_cL_{\text{evap}} \frac{R_{\text{evap}}}{R_w c_{p,w}} \text{OA} + \dot{h}_cL_{\text{fus},b'}(O')^2 n'A /c_{p,i} \]

\[ + \dot{h}_cL_{\text{sub},b'} \left( \frac{R_{\text{sub}}}{R_{\text{ice}}} \right) A / c_{p,i} \]

\[ + \dot{h}_cL_{\text{fus},b} O^2 n A / c_{p,w} \]

\[ + \dot{h}_cL_{\text{fus},n''} b O \left( \frac{O(1-n) + \frac{R_i}{R_w} O'n'}{c_{p,w}} \right) \]  

(11.21)

Kinetic Terms

Kinetic Heating from impinging water particles

\[ \dot{Q}_{k,w} = R_w A \left( \frac{V_{\infty}^2}{2} \right) = \frac{h_c b O A}{c_{p,w}} \left( \frac{V_{\infty}^2}{2} \right) \]  

(11.22)

Kinetic Heating from impinging ice particles

\[ \dot{Q}_{k,\text{ice}} = R_i A \left( \frac{V_{\infty}^2}{2} \right) = \frac{h_c b O A'}{c_{p,i}} \left( \frac{V_{\infty}^2}{2} \right) \]  

(11.23)

Combined Kinetic Terms

\[ \dot{Q}_{\text{kinetic}} = \frac{h_c b O A}{c_{p,w}} \left( \frac{V_{\infty}^2}{2} \right) + \frac{h_c b O A'}{c_{p,i}} \left( \frac{V_{\infty}^2}{2} \right) \]  

(11.24)
Combining the terms derived above Equation 11.1 becomes:

\[
\dot{Q}_{\text{surface}} = + \left\{ h_c \left( t_s - t_{\infty} - r V_{\infty}^2 \right) + \frac{c_s}{2} \right\} (1)
\]

\[
+ h_b (O')^2 \frac{R}{R_w} A (t_{\text{freeze}} - t_p) (2)
\]

\[
+ h_b O^2 (1-n) A \left( t_{\text{se}} - t_{\infty} \right) (3)
\]

\[
+ h_b' (O')^2 \frac{c_{p,w}}{c_{p,i}} A (t_s - t_{\text{freeze}}) (4)
\]

\[
+ \frac{h_{\text{evap}}}{R_w} \frac{\dot{R}_{\text{evap}}}{R_w} A_{\text{p}} (5)
\]

\[
+ h_{\text{sub}} b' (O')^2 n A \frac{1}{c_{p,i}} (6)
\]

\[
+ h_{\text{sub}} b' (O')^2 n A \frac{1}{c_{p,w}} (7)
\]

\[
+ h_{\text{sub}} b' (O')^2 n A \frac{1}{c_{p,i}} (8)
\]

Where the terms in the brackets represent the various energy transfer modes/terms affecting the surface energy transfer:

1. Convective and Viscous
2. Sensible
3. Latent
4. Kinetic

Similar to the ice crystal only derivation, the complexity of solution for the derived, mixed phase, modified, surface energy balance model is that multiple unique solutions exist for a given set of inputs.
Appendix D  Modified Tribus-Messinger Computer Program

Main Program

PROGRAM Mess_main
!
! Purpose:
! A computer model written to determine the equilibrium surface temperature of an aerodynamic surface operating
! in super cooled liquid water conditions based on Bernard Messinger heat balance model. The details of this derivation
! are available in the following reference [1]:
!
! 1. Messinger, Benard L., Equilibrium Surface Temperature of an Unheated Icing
! Surface as a Function of Air Speed, Journal of the Aeronautical Sciences, January 1953
!
! Messinger heat balance makes the following assumptions:
! 1. Super cooled liquid droplets are the same temperature as the ambient static air
! 2. The ambient air is saturated and therefore the saturation pressure (p_infinity) of water vapor in ambient
! atmospheric air is the saturation pressure of water vapor at the ambient air static temperature.
! 3. No mass leaves an implicit control volume that both Messinger [1] and Tribus [2] used to derive their heat balance
! models
!
! **************
!
! Record of Revision:
!
<table>
<thead>
<tr>
<th>Date</th>
<th>Programmer</th>
<th>Description of Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>13-MAR-2013</td>
<td>Michael J. Oliver</td>
<td>Original Code</td>
</tr>
</tbody>
</table>

IMPLICIT NONE
!

! Data dictionary: declare constants
REAL :: cp_air  ! specific heat of air
(BTU/lbm/R) REAL :: cp_h2o  ! specific heat of water (BTU/lbm/R)
REAL :: cp_ice  ! specific heat of ice (BTU/lbm/R)
REAL :: L_evap                                           ! latent heat of evaporation water (BTU/lbm)
REAL :: L_sublm                                          ! latent heat of sublimation water (BTU/lbm)
REAL :: L_fusn ! latent heat of fusion water (BTU/lbm)
REAL :: J  ! mechanical equivalent of heat (ft-lbf/BTU)
(lbm*slug)
REAL :: r  ! recovery factor applying to kinetic heating (dimensionless)
REAL :: t_freeze ! freezing temperature of water (F)
REAL :: convergence ! algorithm is converged if theta functions are less than this value
REAL :: n_increment ! increment value of n by this amount for each n do loop
REAL :: surface_temperature_increment ! increment value of surface temperature by this amount for each loop
REAL :: cold_limit ! algorithm cold limit boundary (R)
!

! Data dictionary: declare variable types, definitions and units
REAL :: ambient_pressure_static, ambient_temperature_static  ! from input file (psia), (F)
REAL :: x1 ! dummy variables for pressure and temperature passing to subroutine
REAL :: surface_temperature ! temperature of the aerodynamic surface operating in icing conditions (R)
REAL :: surface_temperature_M ! temperature of liquid water particles (F)
REAL :: surface_temperature_initial ! temperature of the surface initially (F)
REAL :: surface_temperature_equilibrium
REAL :: surface_temperature_equilibrium_M ! partial pressure of water vapor in ambient air (psia) over ice
REAL :: ambient_partial_pressure_water_vapor_w ! partial pressure of water vapor in ambient air (psia) over water
REAL :: surface_pressure_saturation_ice ! saturation pressure of water vapor over ice at equilibrium surface temperature
REAL :: surface_pressure_saturation_water ! saturation pressure of water vapor over water at equilibrium surface temperature
REAL :: surface_pressure_saturation_ice_M ! saturation pressure of water vapor over ice at equilibrium surface temperature
REAL :: surface_pressure_saturation_water_M ! saturation pressure of water vapor over water at equilibrium surface temperature
REAL :: relative_humidity_ice ! relative humidity over ice
REAL :: relative_humidity_water ! relative humidity over water
REAL :: FtoR ! conversion from Farenheight to Rankine +459.68
REAL :: particle_temperature_delta ! temperature delta from ambient air of the particle impinging on the aerodynamic surface(R)
REAL :: mess_n ! Messinger classic freezing fraction final
REAL :: mess_n_M ! Messinger classic freezing fraction final
REAL :: psiaTOinHg ! conversion from psia to inches of mercury *2.036
REAL :: airspeed ! Velocity of the air (knots)
REAL :: TWC ! Total Water Content (g/m^3)
REAL :: b_param ! relative heat factor
REAL :: Th1, Th2, Th3
REAL :: Th1_M, Th2_M, Th3_M
REAL :: tR, tR_M ! Thermodynamic wet bulb temperature (R)
REAL :: Qsen ! Sensible heat term (F)
REAL :: Qcon ! Convection heat term (F)
REAL :: Qlat ! Latent heat term (F)
REAL :: Qevap ! Latent heat of evaporation (F)
REAL :: Qfus ! Latent heat of fusion term (F)
REAL :: Qvis ! Viscous heat term (F)
REAL :: Qkin ! Kinetic heat term (F)
REAL :: Qsen_M ! Sensible heat term (F) modified
REAL :: Qcon_M ! Convection heat term (F) modified
REAL :: Qlat_M ! Latent heat term (F) modified
REAL :: Qevap_M ! Latent heat of evaporation (F) modified
REAL :: Qfus_M ! Latent heat of fusion term (F) modified
REAL :: Qvis_M ! Viscous heat term (F) modified
REAL :: Qkin_M ! Kinetic heat term (F) modified
INTEGER :: input_correct ! User specified variable to specify whether inputs where entered correctly
INTEGER :: np_increment ! accumulation fraction increment variable
INTEGER :: surface_temperature_loop_indicator ! output file integers
CHARACTER(len=70) :: filename
unit1 = 25
unit2 = 45
! Define Constants
FtoR = 459.68
cp_air = 0.240
cp_h2o = 1
cp_ice = 0.47
L_evap = 1077
L_sublm = 1221
L_fusn = 144
J = 778
g = 32.174
r = 0.875 ! This represents the stagnation line of the airfoil r = (1-(V^2/V_inf^2)(1-Pr^n)) n=1/2, 1/3 laminar, turbulent!
a compromise between laminar and turbulent flow
T_freeze = 32 + FtoR
convergence = 0.0005
cold_limit = -65 + FtoR
n_increment = 0.00001
surface_temperature_increment=0.00005
psiaTOinHg = 2.036
np_increment = 0.01
! Define some variables
surface_temperature_loop_indicator = 0
no=0
yes=1
input_correct=no
X1=-40
filename ="C:\Users\Mike\Documents\PhD\20130312 Big Back UP\Icephysics\OUTDAT" ! Change this to output to the desired direct
Open (UNIT=unit1, FILE=filename, STATUS='NEW', ACTION='WRITE', IOSTAT=ierror)
WRITE (25,*) 'Messinger Case 2A, ambient_pressure_static, ambient_temperature_static, RH, surface_temperature_initial,airspeed & TWC,b_param, surface_pressure_saturation_ice,surface_pressure_saturation_water, ambient_partial_pressure_water_vapor_w, & Th1,Th2,Th3,Mess_n,surface_temperature_equilibrium Qcon,Qlat,Qevap,Qfus,Qsen,Qvis,Qkin, tR, & & temperature_p,surface_pressure_saturation_water_M, surface_pressure_saturation_ice_M,Th1_M,Th2_M, Th3_M,Mess_n_M, & & surface_temperature_equilibrium_M, Qcon_M,Qlat_M,Qevap_M,Qfus_M,Qsen_M,Qvis_M,Qkin_M,tR_M'
! Get inputs to calculate the equilibrium surface Temperature
CALL input_file (ambient_pressure_static, & & ambient_temperature_static, & & surface_temperature_initial, & & airspeed, & & TWC, & & b_param & & particle_temperature_delta)
relative_humidity_ice = relative_humidity_water
temperature_p = ambient_temperature_static ! F
! Loop through several different particle temperatures
particle_loop:Do
! Sub Routine to run the Messinger and Tribus heat balance Model
CALL messinger (surface_temperature, & & ambient_temperature_static, & & relative_humidity_water, & & airspeed, & & TWC, & & b_param & & particle_temperature_delta)
relative_humidity_ice = relative_humidity_water
temperature_p = ambient_temperature_static ! F
! Loop through several different particle temperatures
particle_loop:Do
! Sub Routine to run the Messinger and Tribus Modified heat balance Model
CALL messinger_mod (surface_temperature_M, & & temperature_p, & & ambient_temperature_static, & & relative_humidity_water, & & TWC, & & b_param & & airspeed, & & ambient_partial_pressure_water_vapor_w, & & ambient_partial_pressure_water_vapor_i, & & surface_pressure_saturation_water, & & surface_pressure_saturation_ice, & & surface_temperature_equilibrium & & mess_n,Th1,Th2,Th3,tR,Qcon,Qlat,Qevap,Qfus,Qsen,Qvis,Qkin)
! Sub Routine to run the Messinger and Tribus Modified heat balance Model
CALL messinger_mod (surface_temperature_M, & & temperature_p, & & ambient_temperature_static, & & relative_humidity_water, & & TWC, & & b_param & & airspeed, & & ambient_partial_pressure_water_vapor_w, & & ambient_partial_pressure_water_vapor_i, & & surface_pressure_saturation_water, & & surface_pressure_saturation_ice, & & surface_temperature_equilibrium & & mess_n_M,Th1_M,Th2_M,Th3_M,tR_M & & Qcon_M,Qlat_M,Qevap_M,Qfus_M,Qsen_M,Qvis_M,Qkin_M)
Subroutine: Input_file.F95

SUBROUTINE input_file (ambient_pressure_static, &
& ambient_temperature_static, &
& surface_temp_initial, &
& relative_humidity_water, &
& airspeed, &
& TWC, &
& b_param &
& particle_temperature_delta)
!
! Purpose:
!
! To read in the parameters for the Messinger program
!
! Record of Revisions:
!
! DATE Programmer Description of change
! ====== ========= ===============
! 13-MAR-2013 Michael J. Oliver Original Code
!
IMPLICIT NONE

! Data dictionary: Declare constants
!
! Data Dictionary: Declare variable types, definitions, & units
CHARACTER(len=20) :: filename ! Name of file to open
CHARACTER(len=50) :: d1,d2,d3,d4,d5,d6,d12,d13 ! Identifies the field name
CHARACTER(len=80) :: list_name ! List name identifying the input list
INTEGER :: nvals = 0 ! Number of values read in
INTEGER :: status ! I/O status
REAL ::
ambient_pressure_static ic REAL ::
ambient_temperature_static REAL ::
surface_temp_initial REAL ::
relative_humidity_water REAL ::
airspeed REAL ::
TWC REAL ::
b_param REAL ::
particle_temperature_delta

!FORMAT SECTION
10 FORMAT(‘’,A20)
15 FORMAT( F12.4, A50 )
16 FORMAT(12, A50 )
20 FORMAT( ' ',A50 ,F12.4 )
21 FORMAT(' ',A50, I12 )
30 FORMAT( 'An error occurred reading line', I6)
40 FORMAT(0,'End of file reached. There were', I6, 'values in the file.' )
50 FORMAT( 'Error opening file: IOSTAT = ', I6)

inputloop : DO
! Get the file name, and echo it back to the user.
WRITE (*,*) 'Please enter the input file name including extension: ' READ(*,*) filename
WRITE(*,10) filename
! Open the file and check for errors on open.
OPEN ( UNIT=3, FILE=filename, STATUS='OLD', ACTION='READ', IOSTAT=status ) IF (status ==0) EXIT
WRITE(*,50) status
END DO inputloop
openif : IF ( status==0) THEN
! Open was ok. Read values.
READ(3, '(A80)') list_name
!Read values into program
! Ambient Pressure Static (psia)
READ ( 3, 15, IOSTAT=status ) ambient_pressure_static, d1 ! get next value
! Ambient Temperature Static ( F)
READ ( 3, 15, IOSTAT=status ) ambient_temperature_static, d2 ! get next value
! Initial surface temperature ( F)
READ ( 3, 15, IOSTAT=status ) surface_temp_initial, d3 ! get next value
! Relative humidity over water ( %)
READ ( 3, 15, IOSTAT=status ) relative_humidity_water, d4 ! get next value
! Airspeed ( knots)
READ ( 3, 15, IOSTAT=status ) airspeed, d5 ! get next value
! TWC g/m^3)
READ ( 3, 15, IOSTAT=status ) TWC, d6 ! get next value
! b_param ( non-dimensional)
READ ( 3, 15, IOSTAT=status ) b_param, d12 ! get next value
! particle temperature delta ( F)
READ ( 3, 15, IOSTAT=status ) particle_temperature_delta, d13 ! get next value
! The Read loop has terminated. Was it because of a READ error or because of the end of the input file?
readif: IF ( status > 0) THEN ! a Read error occurred. Tell user
WRITE(*, 30) nvals+1
ELSE ! the end of the data was reached. Tell user:
WRITE(*,40) nvals
END IF readif
ELSE openif
WRITE (*,50) status
END IF openif
! Close File
CLOSE (UNIT=3)
RETURN
END SUBROUTINE input_file

Subroutine: Saturation_pressure.F95

SUBROUTINE saturation_pressure ( &
 &t_static, &
 &B, &
 &t_surf, &
 &relative_humidity, &
 &ambient_partial_pressure_water_vapor_w, &
 &ambient_partial_pressure_water_vapor_i, &
 &surface_pressure_saturation_water, &
 &surface_pressure_saturation_ice)
Purpose:

Given: Ambient conditions T_static, P_Static, Relative humidity and a surface temperature

The purpose of this sub routine is to calculate the following values over water:
1. Ambient saturation pressure
2. Surface saturation pressure (at the current guess of the equilibrium surface temperature in the Messinger subroutine)

Formulas used in this subroutine were taken from the an ASHRAE handbook [1]

References:

Record of Revisions:

<table>
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<tr>
<td>13-MAR-2013</td>
<td>Michael J. Oliver</td>
<td>Original Code</td>
</tr>
</tbody>
</table>

IMPLICIT NONE

Data dictionary: Declare constants

REAL :: FtoR  !Temperature conversion from F to R = 459.68
REAL :: C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13  ! Constants for Thermodynamic properties of water saturation

Data dictionary: Declare variable types and definitions

REAL :: t_static  ! ambient static temperature (R)
REAL :: B  ! ambient static pressure (psia)
REAL :: t_surf   ! surface temperature (R)
REAL :: t_dp_w, t_dp_i  ! dew point temperature of ambient air ( R)
REAL :: ppwv_w  ! partial pressure of water vapor in ambient air (psia) over water
REAL :: ppwv_i  ! partial pressure of water vapor in ambient air (psia) over ice
REAL :: ps_t_surf_w  ! saturation pressure over water at surface temperature (psia)
REAL :: ps_t_surf_i  ! saturation pressure over ice at surface temperature (psia)
REAL :: surface_pressure_saturation_water
REAL :: surface_pressure_saturation_ice
REAL :: ambient_partial_pressure_water_vapor_w
REAL :: ambient_partial_pressure_water_vapor_i
REAL :: ambient_temperature_dewpoint_w
REAL :: ambient_temperature_dewpoint_i
REAL :: ps_t_static_w  ! saturation pressure of ambient air if it were saturated at t static and p static over water
REAL :: ps_t_static_i  ! saturation pressure of ambient air if it were saturated at t static and p static over ice
REAL :: RH_w  ! Relative humidity of the ambient air over water (dimensionless)
REAL :: RH_i  ! Relative humidity of the ambient air over ice (dimensionless)
REAL :: relative_humidity  ! Relative humidity of the ambient moist air (%)
REAL :: W_w,W_i  ! Humidity ratio of ambient air (lbm water vapor / lbm dry air)
REAL :: W_s_w, W_s_i  ! Saturation humidity ratio of ambient air (lbm water vapor / lbm dry air)
REAL :: mu_w, mu_i  ! degree of saturation (dimensionless)

Define Constants
FtoR = 459.68

Constants to calculate saturation pressure over ice from 1993 ASHRAE Handbook Fundamentals I-P Edition
C1 = -1.0214165E4
C2 = -4.8932428E0
C3 = -5.3765794E-3
C4 = 1.9202377E-7
C5 = 3.5575832E-10
C6 = -9.0344688E-14
C7 = 4.1635019E0
! Constants to calculate saturation pressure over water from 1993 ASHRAE Handbook Fundamentals I-P Edition
C8 = -1.0440397E4
C9 = -1.1294650E1
C10 = -2.7022355E-2
C11 = 1.2890360E-5
C12 = -2.4780681E-9
C13 = 6.5459673E0

!Calculate the saturation pressure of the ambient air saturated at ambient static pressure and temperature over water
ps_t_static_w = EXP( C8 / (t_static+FtoR) + C9 + C10 *(t_static+FtoR) + C11 *(t_static+FtoR)**2 &
& + C12 * (t_static+FtoR)**3 + C13 *LOG(t_static+FtoR) )

!Calculate the saturation pressure of the air over the ice covered surface if it were saturated at the current guess of
!the equilibrium surface temperature from the Messinger subroutine
ps_t_static_i= EXP( C1 / (t_surf+FtoR) + C2 + C3 * (t_surf+FtoR) + C4 * (t_surf+FtoR) **2&
& + C5 * (t_surf+FtoR)**3 + C6 * (t_surf+FtoR)**4 + C7 *LOG(t_surf+FtoR) )

!Calculate the saturation pressure of the ambient air saturated at ambient static pressure and temperature over ice
ambient_partial_pressure_water_vapor_i = EXP( C1 / (t_static+FtoR) + C2 + C3 * (t_static+FtoR) + C4 *
(t_static+FtoR)**2 &
& + C5 * (t_static+FtoR)**3 + C6 * (t_static+FtoR)**4 + C7 *LOG(t_static+FtoR) )

! Calculate the partial pressure of water vapor in ambient air given relative humidity and static temperature and
!pressure
RH_w=relative_humidity/100
RH_i =
relative_humidity/10
0 ppwv_w = RH_w
* ps_t_static_w
ppwv_i = RH_i
*ps_t_static_i

!Calculate the humidity ratio and saturation humidity ratio of the ambient air
W_w = 0.62198 * ppwv_w / ( B - ppwv_w)
W_s_w = 0.62198 * ps_t_static_w / ( B - ps_t_static_w)
W_i = 0.62198 * ppwv_i / ( B - ppwv_i)
W_s_i = 0.62198 * ps_t_static_i / ( B - ps_t_static_i)

! Calculate the degree of saturation mu
of ambient air mu_w = W_w / W_s_w
mu_i = W_i / W_s_i

!Calculate the dew point temperature of the ambient air for dew point temperatures below 32F
0 t_dp_w = 90.12 + 26.412 * log(ppwv_w) + 0.8927 * (log(ppwv_w))**2 t_dp_i = 90.12 + 26.412 * log(ppwv_i ) +
0.8927 * (log(ppwv_i ))**2

!Calculate the saturation pressure of the air over the water covered surface if it were saturated at the current guess of
the !equilibrium surface temperature from the Messinger subroutine
ps_t_surf_w = EXP( C8 / (t_surf+FtoR) + C9 + C10 *(t_surf+FtoR) + C11 *(t_surf+FtoR)**2 &
& + C12 * (t_surf+FtoR)**3 + C13 *LOG(t_surf+FtoR) )
ps_t_surf_i= EXP( C1 / (t_surf+FtoR) + C2 + C3 * (t_surf+FtoR) + C4 * (t_surf+FtoR) **2&
& + C5 * (t_surf+FtoR)**3 + C6 * (t_surf+FtoR)**4 + C7 *LOG(t_surf+FtoR) )
surface_pressure_saturation_water = ps_t_surf_w surface_pressure_saturation_ice = ps_t_surf_i

! Note the assumption that Messinger and Tribus make is the air is saturated at
ambient conditions ambient_partial_pressure_water_vapor_w = ppwv_w
ambient_partial_pressure_water_vapor_i = ppwv_i

! This is the dew point of the air prior to the liquid water being present given the
relative humidity ambient_temperature_dewpoint_w = t_dp_w-FtoR
ambient_temperature_dewpoint_i = t_dp_i-FtoR
RETURN
END SUBROUTINE saturation_pressure

Subroutine: Tribus_Messinger.F95

SUBROUTINE messinger (surface_temperature, &
& ambient_temperature_static, &
& ambient_pressure_static, &
& relative_humidity_water, &
& TWC, &
& b_param &
& airspeed, &
& ambient_partial_pressure_water_vapor_w, &
& ambient_partial_pressure_water_vapor_i, &
& surface_pressure_saturation_water, &
& surface_pressure_saturation_ice, &
& surface_temperature_equilibrium &
& mess_n, Th1, Th2, Th3, tr, Qcon, Qlat, Qevap, Qfus, Qsen, Qvis, Qkin )
!
! Purpose:
!
! To calculate the equilibrium surface temperature using the Tribus-Messinger defined heat balance
!
! Record of Revisions:
!
! DATE  Programmer  Description of change
! ======  =========  ===============
! 13-MAR-2013  Michael J. Oliver  Original Code
!
IMPLICIT NONE
! Data dictionary: Declare constants
REAL :: FtoR  !Temperature conversion from F to R = 459.68
REAL :: J, g, cp_h2o, cp_air, cp_ice, r
REAL :: psiaTOinHg
REAL :: L_evap, L_sublm, L_fusn
REAL :: convergence, cold_limit, n_increment
REAL :: t_freeze
REAL :: surface_temperature_increment
INTEGER :: surface_temperature_loop_indicator

! Data dictionary: Declare variable types and definitions
REAL :: surface_temperature ! user specified variable specifying pressure altitude brought in from main program
REAL :: ambient_partial_pressure_water_vapor_i
REAL :: ambient_partial_pressure_water_vapor_w
REAL :: ambient_pressure_static
REAL :: ambient_temperature_static
REAL :: surface_pressure_saturation_water REAL :: surface_pressure_saturation_ice
REAL :: airspeed
REAL :: n, mess_n REAL :: b_param REAL :: TWC
REAL :: surface_temperature_equilibrium
REAL :: tr
REAL :: relative_humidity_water ! RH_w1
REAL :: theta1, theta2, theta3 ! equilibrium surface temperature < 32F
REAL :: theta1p, theta2p, theta3p ! equilibrium surface temperature = 32F
REAL :: theta1pp, theta2pp, theta3pp ! equilibrium surface temperature > 32F
REAL :: Th1, Th2, Th3
REAL :: Qsen  !Sensible heat term (F)
REAL :: Qcon  ! Convection heat term (F)
REAL :: Qlat  ! Latent heat of fusion (F)
REAL :: Qevap  ! Latent heat of evaporation (F)
REAL :: Qfus ! Latent heat of fusion term (F)
REAL :: Qvis  ! Viscous heat term (F)
REAL :: Qkin  ! Kinetic heat term (F)

! Define Constants
FtoR = 459.68
surface_temperature_loop_indicator=0
n_increment = 0.001
convergence = 0.1
surface_temperature_increment =0.001
cold_limit= -65.0
t_freeze = 32.0
psiaTOinHg = 2.036
cp_air = 0.240
cp_h2o = 1.0
cp_ice = 0.47
L_evap = 1077.0
L_sublm = 1221.0
L_fusn = 144.0
j=778.0
g=32.174
r=0.875
n=1
surface_temperature=32
CALL saturation_pressure ( ambient_temperature_static, &
 & ambient_pressure_static, &
 & surface_temperature, &
 & relative_humidity_water, &
 & ambient_partial_pressure_water_vapor_w, &
 & ambient_partial_pressure_water_vapor_i, &
 & surface_pressure_saturation_water, &
 & surface_pressure_saturation_ice)
nloop : DO
!Theta Equations Tsurf,eq = t_freeze and 0<mess_n<1
theta1p = t_freeze*(1+ b_param)+ 2.9 * L_evap * surface_pressure_saturation_water / ambient_pressure_static
theta2p = -1*(ambient_temperature_static * (1 + b_param)+2.9*L_evap *ambient_partial_pressure_water_vapor_w &
 &/ ambient_pressure_static + 144 * b_param * n/cp_h2o)
theta3p = -1*((airspeed*1.688)**2/2/g/J*(r/cp_air+b_param*cp_h2o))
IF(ABS(theta1p + theta2p + theta3p)<convergence) THEN
surface_temperature_loop_indicator = 1
mess_n = n
Th1=theta1p
Th2=theta2p
Th3=theta3p
surface_temperature_loop_indicator=1
surface_temperature_equilibrium=surface_temperature
Qcon=t_freeze - ambient_temperature_static
Qlat=2.9*L_evap*(surface_pressure_saturation_water - ambient_partial_pressure_water_vapor_w) &
 &/ ambient_pressure_static - 144 * mess_n/cp_h2o*b_param
Qfus = - 144 * mess_n/cp_h2o*b_param
Qevap = Qlat-Qfus
Qsen =-(t_freeze - ambient_temperature_static)*b_param
Qvis = -r*(airspeed*1.688)**2/2/g/J/cp_air
Qkin = -(airspeed*1.688)**2/2/g/J/cp_h2o*b_param
EXIT nloop
ELSE IF (n<0) THEN
! Assign the initial equilibrium surface temperature for equilibrium loop for equilibrium temperature not equal to 32
surface_temperature = 212
n=1.0
EXIT nloop
ELSE
n = n - n_increment
END IF
ENDDO nloop
! Loop to determine equilibrium surface temperature where all the Theta functions add to zero--indicating
! no further heat transfer is occurring  
equilibrium_loop : DO  
If( surface_temperature_loop_indicator==1) THEN  
EXIT equilibrium_loop  
ENDIF  
CALL saturation_pressure ( ambient_temperature_static, &  
& ambient_pressure_static, &  
& surface_temperature, &  
& relative_humidity_water, &  
& ambient_partial_pressure_water_vapor_w, &  
& ambient_partial_pressure_water_vapor_i, &  
& surface_pressure_saturation_water, &  
& surface_pressure_saturation_ice) IF  
(surface_temperature > t_freeze) THEN  
!Theta Equations tsurf,eq > t_freeze  
theta1pp=surface_temperature*{(1+b_param)+2.9*L_evap*surface_pressure_saturation_water/ambient_pressure_static}  
theta2pp=-(ambient_temperature_static*{(1+b_param)+2.9*L_evap*ambient_partial_pressure_water_vapor_w}/ &  
& ambient_pressure_static)  
theta3pp=-(airspeed*1.688)**2/2/g/J*(r/cp_air+b_param/cp_h2o)  
IF(ABS(theta1pp + theta2pp + &  
& theta3pp)< convergence) THEN surface_temperature_loop_indicator = 1  
Th1=theta1pp  
Th2=theta2pp  
Th3=theta3pp  
n=0.0  
surface_temperature_equilibrium = surface_temperature  
Qcon= surface_temperature_equilibrium - ambient_temperature_static  
Qlat=2.9*L_evap*(surface_pressure_saturation_water - ambient_partial_pressure_water_vapor_w)/ &  
& ambient_pressure_static  
Qfus = 0  
Qevap = Qlat-Qfus  
Qsen = (surface_temperature_equilibrium - ambient_temperature_static)*b_param  
Qvis = -(airspeed*1.688)**2/2/g/J/(cp_h2o*b_param)  
Qkin = -(airspeed*1.688)**2/2/g/J/(cp_h2o*b_param)  
ENDIF  
ELSE  
! Theta Equations tsurf,eq < t_freeze  
theta1 = surface_temperature*{(1 + (cp_ice/cp_h2o)*b_param) + 2.9*L_sublm*surface_pressure_saturation_ice/ &  
& ambient_pressure_static}  
theta2 = -(ambient_temperature_static*{(1+b_param)+2.9*L_sublm*ambient_partial_pressure_water_vapor_w}/ &  
& ambient_pressure_static)  
theta3 = -(airspeed*1.688)**2/2/g/J/(cp_h2o*b_param)  
ENDIF END IF  
IF(ABS(theta1 + theta2 + theta3)< convergence) THEN  
surface_temperature_loop_indicator = 1  
Th1=theta1  
Th2=theta2  
Th3=theta3  
n=1.0  
surface_temperature_equilibrium = surface_temperature  
Qcon=surface_temperature_equilibrium - ambient_temperature_static  
Qlat=2.9*L_sublm*(surface_pressure_saturation_ice - ambient_partial_pressure_water_vapor_w)/ &  
& ambient_pressure_static - 144 / cp_h2o * b_param  
Qfus = - 144 / cp_h2o * b_param  
Qevap = Qlat-Qfus  
Qsen = b_param*( (t_freeze - ambient_temperature_static) - n * cp_ice/(t_freeze - &  
& surface_temperature_equilibrium))  
Qsen = (t_freeze - ambient_temperature_static)*b_param*cp_ice/cp_h2o  
Qvis = -(airspeed*1.688)**2/2/g/J/(cp_h2o)  
Qkin = -(airspeed*1.688)**2/2/g/J/(cp_h2o*b_param)  
IF (surface_temperature_loop_indicator==1) THEN  
EXIT equilibrium_loop  
ELSE IF (surface_temperature < cold_limit) THEN  
surface_temperature_equilibrium = 1000.00  
EXIT equilibrium_loop
Subroutine: Modified_Tribus_Messinger.F95

SUBROUTINE messinger_mod
    (surface_temperature_M, &
    & temperature_p, &
    & ambient_temperature_static, &
    & ambient_pressure_static, &
    & relative_humidity_water, &
    & TWC, &
    & b_param, &
    & airspeed, &
    & ambient_partial_pressure_water_vapor_w, &
    & ambient_partial_pressure_water_vapor_i, &
    & surface_pressure_saturation_water_M, &
    & surface_pressure_saturation_ice_M, &
    & surface_temperature_equilibrium_M, &
    & Qcon_M, Qlat_M, Qevap_M, Qfus_M, Qsen_M, Qvis_M, Qkin_M)

! Purpose:
! To calculate the equilibrium surface temperature using the Tribus-Messinger defined heat balance.

! Record of Revisions:

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IMPLICIT NONE

! Data dictionary: Declare constants
REAL :: FtoR !Temperature conversion from F to R = 459.68
REAL :: J, g, cp_h2o, cp_air, cp_ice, r
REAL :: psiaTOinHg
REAL :: L_evap, L_sublim, L_fusn
REAL :: convergence, cold_limit, n_increment
REAL :: t_freeze
REAL :: surface_temperature_increment
INTEGER :: surface_temperature_loop_indicator

! Data dictionary: Declare variable types and definitions
REAL :: surface_temperature_M ! user specified variable specifying pressure altitude brought in from main program
REAL :: temperature_p
REAL ::
ambient_partial_pressure_water_vap
or_w REAL ::
ambient_partial_pressure_water_vap
REAL ::
ambient_pressure_static
REAL ::
ambient_temperature_static
REAL ::
surface_pressure_saturation_water_M
REAL ::
surface_pressure_saturation_ice_M
REAL ::
airspeed

160
REAL ::
n
mess_n
M
REAL ::
b_param
m
REAL :: TWC
REAL :: surface_temperature_equilibrium
um_M REAL :: tR_M
REAL :: relative_humidity_water
REAL :: theta1_M, theta2_M, theta3_M
REAL :: theta1p_M, theta2p_M, theta3p_M
REAL :: theta1pp_M, theta2pp_M, theta3pp_M
REAL :: Th1_M, Th2_M, Th3_M
REAL :: Qsen_M
REAL :: Qcon_M
REAL :: Qlat_M
REAL :: Qevap_M
REAL :: Qfus_M
REAL :: Qvis_M
REAL :: Qkin_M

! Define Constants

FtoR = 459.68
surface_temperature_loop_indicator = 0
n_increment = 0.001
convergence = 0.1
surface_temperature_increment = 0.001
cold_limit = -65.0
t_freeze = 32.0
psiaTOinHg = 2.036
cp_air = 0.240
cp_h2o = 1.0
cp_ice = 0.47
L_evap = 1077.0
L_sublm = 1221.0
L_fusn = 144.0
J = 778.0
g = 32.174
r = 0.875
n = 1
surface_temperature_M = 32

! The Theta Equations

CALL saturation_pressure (ambient_temperature_static, &
& ambient_pressure_static, &
& surface_temperature_M, &
& relative_humidity_water, &
& ambient_partial_pressure_water_vapor_w, &
& ambient_partial_pressure_water_vapor_l, &
& surface_pressure_saturation_water_M, &
& surface_pressure_saturation_ice_M)

nloop = DO
! The Theta Equations

theta1p_M = t_freeze*(1 + b_param) + 2.9 * L_evap * surface_pressure_saturation_water_M / ambient_pressure_static + temperature_p * b_param + 2.9 * L_evap * &
& ambient_pressure_water_vapor_w / ambient_pressure_static + 144 * b_param / cp_h2o)
theta3p_M = -1*((airspeed*1.688)**2/2/gJ*(r/cp_air + b_param*cp_h2o))

IF(ABS(theta1p_M + theta2p_M + & &theta3p_M) < convergence) THEN

surface_temperature_loop_indicator = 1

mess_n_M = n

Th1_M = theta1p_M
Th2_M = theta2p_M
Th3_M = theta3p_M
surface_temperature_loop_indicator = 1
surface_temperature_equilibrium_M = surface_temperature_M
Qcon_M = t_freeze - ambient_temperature_static
Qlat_M = 2.9 * L_sublm * (surface_pressure_saturation_water_M - ambient_partial_pressure_water_vapor_w) &
ambient_pressure_static - 144 * mess_n_M / cp_h2o * b_param
Qfus_M = - 144 * mess_n_M / cp_h2o * b_param
Qevap_M = Qlat_M - Qfus_M
Qsen_M = b_param * (t_freeze - temperature_p) - mess_n_M * cp_ice / cp_h2o * (t_freeze - & surface_temperature_equilibrium_M) !modified by MJ 03/11/2013
Qvis_M = -r*(airspeed*1.688)**2/2/g/J/cp_h2o
Qkin_M = -r*(airspeed*1.688)**2/2/g/J/cp_h2o * b_param
EXIT nloop
ELSE IF (n<0) THEN
! Assign the initial equilibrium surface temperature and set n=0 for equilibrium loop for equilibrium temperature not equal to 32
surface_temperature_M = 212
n=1
EXIT nloop
ELSE
n = n - n_incremet
ENDIF
ENDIF
nloop
! Loop to determine equilibrium surface temperature where all the Theta functions add to zero--indicating
! no further heat transfer is occurring
 equilibrium_loop : DO
If( surface_temperature_loop_indicator==1 ) THEN
EXIT equilibrium_loop
END IF
CALL saturation_pressure ( ambient_temperature_static, &
ambient_pressure_static, &
surface_temperature_M, &
relative_humidity_water, &
ambient_partial_pressure_water_vapor_w, &
ambient_partial_pressure_water_vapor_i, &
surface_pressure_saturation_water_M, &
surface_pressure_saturation_ice_M)
IF(surface_temperature_M>t_freeze) THEN
!Theta Equations tsurf,eq > t_freeze
theta1pp_M = surface_temperature_M * (1+b_param) + 2.9 * L_sublm * surface_pressure_saturation_water_M /ambient_pressure_static + temperature_p * b_param + 2.9 * L_sublm * ambient_partial_pressure_water_vapor / ambient_pressure_static
theta3pp_M = -(airspeed*1.688)**2/2/g/J*(r/cp_air + b_param/cp_h2o)
IF(ABS(theta1pp_M + theta2pp_M + theta3pp_M)< convergence) THEN
surface_temperature_loop_indicator = 1
Th1_M=theta1pp_M
Th2_M=theta2pp_M
Th3_M=theta3pp_M
n=0
ELSE
!Theta Equations tsurf,eq < t_freeze
theta1_M = surface_temperature_M * (1+b_param) + 2.9 * L_sublm * surface_pressure_saturation_ice_M /ambient_pressure_static
theta3pp_M = -(airspeed*1.688)**2/2/g/J*(r/cp_air + b_param/cp_h2o)
IF(ABS(theta1pp_M + theta2pp_M + theta3pp_M)< convergence) THEN
surface_temperature_loop_indicator = 1
Th1_M=theta1pp_M
Th2_M=theta2pp_M
Th3_M=theta3pp_M
n=0
ENDIF
ENDIF
ELSE
!Theta Equations tsurf,eq > t_freeze
theta1pp_M = surface_temperature_M * (1+b_param) + 2.9 * L_sublm * surface_pressure_saturation_water_M /ambient_pressure_static + temperature_p * b_param
theta2pp_M = 2.9 * L_sublm * surface_pressure_saturation_water_M /ambient_pressure_static + temperature_p * & ambient_pressure_static
theta3pp_M = -(airspeed*1.688)**2/2/g/J*(r/cp_air + b_param/cp_h2o)
ENDIF
ENDIF
EXIT equilibrium_loop
ELSE
n=0
ENDIF
& ambient_pressure_static
theta2_M = -(ambient_temperature_static + 2.9 * L_sublim * ambient_partial_pressure_water_vapor_w/ ambient_pressure_static & b_param*(t_freeze - temperature_p) + 144*b_param + cp_ice/cp_h2o*b_param*t_freeze )
theta3_M = -(airspeed*1.688)**2/2/g/J*(r/cp_air+b_param*cp_h2o)

END IF
IF(ABS(theta1_M + theta2_M + theta3_M) < convergence) THEN
Th1_M = theta1_M
Th2_M = theta2_M
Th3_M = theta3_M
n = 1
mess_n_M = n
surface_temperature_equilibrium_M = surface_temperature_M
Qcon_M = surface_temperature_equilibrium_M - ambient_temperature_static
Qlat_M = 2.9 * L_sublim * (surface_pressure_saturation_ice_M - ambient_partial_pressure_water_vapor_w)/ & ambient_pressure_static - 144/cp_h2o*b_param
Qfus_M = -144/cp_h2o*b_param
Qevap_M = Qlat_M - Qfus_M
Qsen_M = b_param*(t_freeze - temperature_p) - mess_n_M*cp_ice/cp_h2o*(t_freeze - & surface_temperature_equilibrium_M) Qvis_M = -r*(airspeed*1.688)**2/2/g/J/cp_air
Qkin_M = -(airspeed*1.688)**2/2/g/J/cp_h2o*b_param

ENDIF
IF (surface_temperature_loop_indicator==1) THEN
EXIT equilibrium_loop
ELSE IF (surface_temperature_M < cold_limit) THEN
surface_temperature_equilibrium_M = 1000.00
EXIT equilibrium_loop
END IF
surface_temperature_M = surface_temperature_M - surface_temperature_increment
END DO equilibrium_loop
CALL wet_bulb( ambient_pressure_static, ambient_temperature_static, ambient_partial_pressure_water_vapor_w, tR_M)
ENDSUBROUTINE messinger_mod

Subroutine: wet_bulb_temperature.F95

SUBROUTINE wet_bulb (B, &
& Tinf, &
& Pinf_w, &
& tR )
!
! Purpose:
!
! The purpose of this sub routine is to determine the thermodynamic wet bulb temperature for a Messinger heat balance model
!
! the formulas reference are from an ASHRAE handbook [1]:
!
! References:
!
! Record of Revisions:
!
|
| DATE | Programmer | Description of change |
| ====== | ========= | =============== |
| 13-MAR-2013 | Michael J. Oliver | Original Code |

! Data Dictionary: Declare constants

! Data Dictionary: Declare variable types, definitions, & units
REAL :: B          ! ambient static pressure (psia)
REAL :: Tinf, Tinfw ! static temperature (F), static temperature working (R)
REAL :: Pinf_w    ! Pw if relative humidity < 100% else Pws at t static
REAL :: ps_tr_w, REAL :: ps_tr_i REAL :: WsR REAL :: W1
REAL :: tR        ! thermodynamic wet bulb temperature (R)
REAL :: temp_incr
REAL :: convergence
INTEGER :: trial
REAL :: C1,C2,C3,C4,C5,C6,C7
REAL :: C8,C9,C10,C11,C12,C13
REAL :: h, hwR, hsR
REAL :: A

! Define constants convergence = 0.01 temp_incr = .01

! Constants to calculate saturation pressure over ice from 1993 ASHRAE Handbook Fundamentals I-P Edition
C1 = -1.0214165E4
C2 = -4.8932428E0
C3 = -5.3765794E-3
C4 = 1.9202377E-7
C5 = 3.5575832E-10
C6 = -9.0344688E-14
C7 = 4.1635019E0

! Constants to calculate saturation pressure over water from 1993 ASHRAE Handbook Fundamentals I-P Edition
C8 = -1.0440397E4
C9 = -1.1294650E1
C10 = -2.7022355E-2
C11 = 1.2890360E-5
C12 = -2.4780681E-9
C13 = 6.5459673E0

tinfw=tinf+459.68 tR=672   !Degrees R
trial =1
wet_bulb_loop : DO

!Calculate the saturation pressure over water at web bulb temperature guess tR (R)
ps_tr_w = EXP( C8 / (tR) + C9 + C10 * (tR) + C11 * (tR)**2 &
& C12 * (tR)**3 + C13 * LOG(tR) )

!Calculate the saturation pressure over ice at web bulb temperature guess tR (R)
ps_tr_i= EXP( C1 / (tR) + C2 + C3 * (tR) + C4 * (tR)**2 &
& C5 * (tR)**3 + C6 * (tR)**4 + C7 * LOG(tR) )

! Bring in the humidity ratio from the messinger sub routine or at least the ambient_saturation_pressure_w
W1 = 0.62198 * Pinf_w/(B-Pinf_w)  ! at some relative humidity
WsR = 0.62198* ps_tr_w/(B-ps_tr_w)  ! at some temperature T = T* (R)
hsR = 0.240*(Tinf) + W1*(1061+0.444*(Tinf))

hwR = tR - 32   ! this is an estimate
hsR =0.240*(tR-459.68)+WsR*(1061+0.444*(tR-459.68))
A = h+(WsR-W1)*hwR  ! corrected typo changed hsR to hwR per ASHRAE handbook

3/10/2013 MJ O IF (abs(A-hsR)<=convergence) EXIT wet_bulb_loop
IF(trial ==0) EXIT
wet_bulb_loop tR =
   tR -temp_incr
IF (tR <=0) EXIT wet_bulb_loop
END DO wet_bulb_loop
RETURN

END SUBROUTINE wet_bulb
### Sample Input Text File

This input file is for Original Messinger Point Case 2A

| 10.18 | Ambient Pressure static (psia).................. |
| 4.6   | Ambient Temperature Static (F).................. |
| 4.6   | Initial Surface Temperature (F).................. |
| 100.  | Relative Humidity over water (%)................ |
| 219.  | Aircraft Airspeed (knots)........................ |
| 0.7   | Total Water Content (g/cubic meter)............. |
| 0.53  | messinger b parameter........................... |
| 1.0   | particle Temperature Delta (F)................... |

### Sample Output File

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Th1 Th2 Th3 Mess_n Tse Qcon Qlat Qevap Qfus Qsen

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Note these are the first twelve of a total of forty six rows of this sample output.
APPENDIX E   Relative Heat Factor Error Analysis

For the joint NASA/NRCC fundamental accretion studies, there were two experimental measurements made that are used to calculate the relative heat factor (b) used in the MTM model calculations. These are the liquid water content (LWC) and the convective heat transfer coefficient (h_c). The researchers making the measurements estimated the random error associated with each of these two independent measurements to be LWC +/- 20% and h_c +/- 30%.

The relative heat factor is calculated by the following equation (17), (18):

\[ b = \frac{R_w c_{p,w}}{h_c} \]  

(13.1)

The unit water catch rate can be calculated using the following equation (20):

\[ R_w = 0.379U_{o}LWC(h/C)E_m \left( \text{lbm/hr/ft}^2 \right) \]  

(13.2)

This gives an equation for the relative heat factor in the following form:

\[ b = \frac{0.379U_{o}LWC(h/C)E_m c_{p,w}}{h_c} \]  

(13.3)

Since these measurements with associated error are used as a product or quotient in the calculation of the relative heat factor, the propagation of the errors is determined as the additive some of the fractional uncertainties of the respective measurements (21):
\[
\frac{\delta b}{|b|} = \sqrt{\left(\frac{\delta LWC}{LWC}\right)^2 + \left(\frac{\delta h_c}{h_c}\right)^2}
\] (13.4)

Where for each quotient squared under the square root symbol, the absolute value term in the denominator is the best estimate of the measurement and the numerator represents the respective error associated with it.

For case 539 the estimate and error are as follows:

1. LWC
   a. Best estimate = 1.5 g/m\(^3\)
   b. Error is +/- 20% or 1.5*0.20 \(\rightarrow\) 0.308

2. h\(_c\)
   a. Best estimate = 29 g/m\(^3\)
   b. Error is +/- 30% or 29*0.30 \(\rightarrow\) 8.7

The fractional error associated with the relative heat factor, b is:

\[
\frac{\delta b}{|b|} = \sqrt{\left(\frac{0.308}{1.5}\right)^2 + \left(\frac{8.7}{29}\right)^2} = 0.36
\] (13.5)

In the case of Case 539 and 539a the calculated relative heat factor is 0.36. The error that propagates through to this is as follows:

\[
\delta b = 0.36 \times |b| = 0.36 \times 0.36 = 0.13
\] (13.6)

For case 539 the calculation of b has a tolerance of 0.13:

\[
b = 0.36 \pm 0.13
\] (13.7)

Similarly for case 545 and 545a the calculation of b has a tolerance of 0.14.
References


22. MIL-STD-210A.


