STRUCTURAL DESIGN SOLVER DEVELOPMENT FOR OVERHEAD
INDUSTRIAL CRANES: EQUATIONS-OF-STATE SOLVER METHOD

by

JOEL CHRISTIAN WARREN

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Thesis Advisor: Dr. Roger D. Quinn

Department of Mechanical Engineering

CASE WESTERN RESERVE UNIVERSITY

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CASE WESTERN RESERVE UNIVERSITY

SCHOOL OF GRADUATE STUDIES

We hereby approve the thesis/dissertation of

Joel Christian Warren

candidate for the Master of Science degree *

(signed) Maurice Adams, Ph.D.

(chair of the committee)

Ken Loparo, Ph.D.

Malcolm Cooke, Ph.D.

Claire Rimanc, Ph.D.

(date) January 19th, 2012

* We also certify that written approval has been obtained for any proprietary material contained therein.
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Structural Design Solver Development for Overhead Industrial Cranes:  
Equations-of-State Method

Abstract

By

Joel Christian Warren

Structural engineering analysis for overhead industrial cranes is performed by a number of methods throughout industry: by the use of hand-calculations, behavioral simulation packages, or by different custom-made sizing programs. These methods are specific to a single aspect of crane design, and are not designed to focus on the analysis of the crane as a whole system. This requires re-iteration of engineering work for each subsequent stage, leading to unnecessary rework and waste of engineering resources.

To bring cohesion to the engineering analysis, a specialized analytical solver was developed. This solver combines the structural analysis for all aspects of the crane design. This analysis is performed using equations-of-state modeling in a two-stage process: the first stage addresses the crane structure and optimization, the second stage addresses the runway structure and optimization. Design checks against code and crane industry standards are performed constantly, to aid in the optimization process.
1. **Typical problems in structural analysis that requires solutions**

Structural analysis in overhead industrial cranes is a process subject to many revisions. Design parameters are often subjected to quick changes, based on requirements, and designs must remain flexible yet functional. An effective design is a functional design, and functionality requires design analysis and certification.

Quick and accurate certification is key to providing the best and most cost-effective designs. And often enough, design analysis tools are limited in their functionality and availability for many organizations. Common practice is the development of custom analysis tools, utilizing data common in industry and/or company practice.

Accurately estimating the most effective design (optimization) leads to more efficient use of resources, undercutting investment cost, increasing profitability. Design tools that capture most of the design parameters early in conceptual development allow for the best use of resources, while maintaining design versatility.
2. **Applications that apply directly to overhead crane systems**

Crane systems have to be designed under the consideration of four primary types of loading:

1. **Dead loads** - the weights of all structural, mechanical, and electrical systems of the crane assembly.
2. **Applied loads** - the weight to be lifted and conveyed during operation, generally a design-specific rated capacity.
3. **Fatigue loads** – cranes are utilized on a cyclical operation with repetitive loading and unloading of conveyed loads (or packages).
4. **Seismic loads** – cranes are typically operated on a suspended railway, which is anchored to a building structure or alternative support framework. Just as the building structure must be capable of handling excitation from seismic activity, so must the crane assembly framework.

These loads are typically modeled using static loading criteria (at least in reference to the dead loads and applied loads) upon the assembly. This is done to simplify the calculations and iterative process, as that dynamic loading requires time-step modeling, an extremely iterative calculation methodology. Because static-loading criteria are chiefly used in the design analysis process, design factors of safety are generally higher than what would normally be required. This serves as a safeguard to allot for what is unknown, in regards to the variable loading capacities and operation-lift time and traverse time.
The thesis of this paper is as follows: the development of equation-of-state solvers for structural (and mechanical) analysis leads to the optimized case for overhead industrial crane design - which is maximization of effectiveness whilst minimizing resource investment. In other words, the more which is known about the governing operations parameters and what can be accurately simulated, the more effective the designer can be with design development - and the more cost-effective the design becomes in the fabrication/construction of it.

Fig. 2.1-4: Sample images of different configurations of overhead industrial cranes
3. **C.M.A.A. Structural Design Specifications**

The Crane Manufacturers Association of America (C.M.A.A) serves as the self-established guiding body of industrial crane design and development throughout the United States of America. A subset of the Material Handling Industry of America (M.H.I.A.), the organization is composed of leading and established manufacturers of industrial cranes and hoisting equipment. This organization sets the standards for industry-wide design methodology, analysis, fabrication specifications, and acceptable manufacturing tolerances. A set of design specifications is provided to members, consisting of a governing guideline of codes, calculations and general rules of thumb.

The specifications draw heavily upon standard precedents of design, analysis, and fabrication set in place by the American Institute of Steel Construction (A.I.S.C.), as that the majority of industrial cranes are fabricated using plate and structural steel, and operate in conventional environment constructed according to A.I.S.C. standards. These specifications are written to ensure that all industrial crane designs maintain a high-enough margin of design safety to keep all allowable stresses within the elastic range of material behavior. C.M.A.A.-approved designs are expected to have a minimum factor-of-safety against yielding of 1.7 for structural members, a minimum factor-of-safety against ultimate tensile of 5.0 for mechanical load-bearing members, a column/elastic buckling minimum safety factor of 1.7, and a modal excitation minimum fundamental natural frequency of 2.4 Hz for structural and mechanical members.
Structural and mechanical analysis of the sections is a process that makes use of iterative design analysis. This serves as a means to determine which structural section and mechanical configuration provides the highest amount of design safety, whilst being the lightest (ergo, most cost-effective) and simplest to fabricate. The iterative process of design checking is a time-consuming one, as that many loading parameters and dynamic acceleration factors have to be considered in the calculations. In addition, multiple vectors of loading and dynamic excitation have to considered, modeling potential failures not only in the primary plane of loading, but also in the secondary planes of traverse loading as well.

To speed up this iterative process, many companies utilize standard design tables for quick-reference; these tables consists of design sections which have been carefully analyzed, and proven to be viable for the governing design parameters of loading capacity, span (distance between supports of crane), accelerations, operational and fatigue-life.

The development of the standards tables takes a great deal of time to complete, and often enough, the results are operationally-effective – but may not be cost-effective to the end-user to justify the purchase. As a result, many companies employ design-calculation software, written to quickly calculate the required properties of the structural section, to ensure an operationally-effective design and a more optimally cost-effective one too. This software is available in a number of formats: spreadsheets, mathematics packages, equations-of-state solvers – to name a few. Other companies have considered the use of finite-element packages
as well to speed up the analytical process, commercially-available and even some custom-developed packages, if feasible.
4. **A.I.S.C. Structural Design Specifications**

In the design of industrial steel structures, most designers and engineers turn to the guiding specifications of the American Institute Steel Construction (A.I.S.C.) organization. For over 90 years, the guidelines established by this body have been in place to ensure that all steel constructs have a high-degree of inherent safety and a long design life, ensuring functionality for many years.

In overhead industrial cranes, one of the foremost A.I.S.C. specifications is the Manual of Steel Construction, Allowable Stress Design, 9th Edition (ASD 9th Edition). This manual is especially important to the crane manufacturer in that the majority of overhead industrial specifications are directly inherited from it. ASD 9th Edition provides a foundation in the selection and analysis of structural members of the crane, including: design parameters, modeling specifications, tabulations of material strength properties and inertial properties for a majority of structural sections, reference diagrams based on principles of engineering mechanics and case studies, weldment and fabrication specifications, and acceptable criterion.
5. Proposal methods for structural analysis

The current proposal for improvement on this methodology is to use the template method, to develop geometric and analytical templates for the most-common full-system configurations. And to develop these system analyses modules as part of a global resource, by which all aspects of the crane and supporting structural system can be sized with limited preliminary information.

These analyses would require only the basic loading geometric and performance characteristics of the system in question. And from the choice in template, the governing equations-of-state are established and tailored to meet the dependent requirements of the system. This would then be solved by a multiple degree-of-freedom methodology (examples of numerical methods, linear regression, and homogenous matrix reduction) to further optimize the configuration and present preliminary findings for design/cost consideration.

Further refinements would be to consider loading cycles and variances, to determine if the general optimization (based on maximum rated capacity) can be further improved. By considering cases of actualization (distributing the actual lifted capacities according to theoretical lifts over a pre-specified period of time), one could better approximate the reactionary demands induced during operation. And one could design a system that is further optimized to specific and specialized needs.

In addition to static and dynamic design cycle analyses, consideration of fatigue must be addressed. Fatigue wear and degradation is induced in any system subjected to cyclical loading and operations. For many manufacturers, design of the crane
structure to infinite life fatigue criterion is simplest - as that the process of servicing fatigue-damaged cranes after years of use is very cost-intensive, and even dangerous if the failure occurs in the field before repairs are made. Designing the structure and mechanical systems to infinite life serves as a means of safeguarding the manufacturer against liability issues, and provide additional safety factor to account for degradation. Fatigue analysis which best incorporates the available design parameter information allows for the most effective and sustainable designs; and better fatigue sizing would allow manufacturers to better gauge their price-point commitment to warranty activities and repair services on the system aspects which are most likely to fail accordingly.

Since numerical methods are typically used for engineering analysis, there is really no need to develop in specialized engineering or analytical mathematical solver packages (examples of them are such as: Matlab, MAPLE, Mathematica) for solutions. In fact, all of the required calculations can be performed in a basic tabulated format (spreadsheet format). The versatility and ease of programming available in Microsoft Excel® allows for multiple sources of entry, data lookup, iteration, and calculation to be performed in one consolidated file. In addition, data protection enables the developer to tailor the information for source accordingly, without having to be concerned about erroneous reference entries or data manipulation. All in all, Microsoft Excel® serves as a very versatile program for the development of such a deliberated engineering solver, with excellent database development tools that are required for such a high-level project.
6. Aspects of analysis to be developed with new program

The development of this new calculations software would have an analysis setup, one that would enable quick and precise preliminary structural calculations for a number of overhead cranes and their supporting structural assemblies. It would enable users to start with preliminary templates for design and operation, and allow users to modify governing parameters to meet their needs. After which, precise and accurate reactions and stress/strain data would be available for review.

Approximately four different crane configurations are to be developed in this overall analysis program, with the top-running accounting for the first-stage of development:

1. Top-running double-box with top-running bridge end-trucks

![Fig. 6.1: Reference image for top-running double box crane with top-running bridge end trucks](image-url)
2. Under-running single-box with top-running bridge end-trucks

Fig. 6.2: Reference image for under-running single box crane with top-running bridge end trucks

3. Under-running single-box with under-running bridge end-trucks

Fig. 6.3: Reference image for under-running single box crane with under-running bridge end trucks

These calculations will cover a majority of the typical designs that would be commonly-used in most interior lifting applications. Capacities would range from fractional-tonnage to about 50 tons for safe free-standing applications. Larger
capacities would require greater in-depth analysis to be certain of design safety and developmental capability.

Results would consist of primary bending stresses (from loads) and secondary bending stresses (from traverse forces). These would be superimposed with primary shear stresses and secondary shear stresses, and using vector summation to get an accurate value of the full-stress range the system is exposed to. Stresses for the structural member would be calculated at the primary eight corners of the cross-section (ref. figure 6.4):

1. Top-left flange extrema
2. Top-right flange extrema
3. Top-left web/flange intersection
4. Top-right web/flange intersection
5. Bottom-left web/flange intersection
6. Bottom-right web/flange intersection
7. Bottom-left flange extrema
8. Bottom-right flange extrema
Fig. 6.4: Reference image girder cross-section with primary eight-points of cross-section stress concern

Stresses would also be calculated at the traverse rail for the hoist, using compression criteria. This would be repeated for the runway beam as well.
7. Programming Details

The development of the custom program required establishing the governing relationships between operational parameters, and acceptable/required design criterion, based on industry guidelines. The following listing serves as a master point of reference for what was considered important in the implementation of the custom solver package.

Starting with the basics of girder structural/mechanical analysis, we have the following procedure of development:

1. Identification of the driving dimensional and operational parameters
   a. Crane capacity
   b. Crane span
   c. Operational speeds
   d. Travel range (hook travel for lift, trolley travel for bridge, crane travel for runway)

2. Development of the geometric and structural requirements
   a. Length of the bridge (for trolley travel)
   b. Induced reactions and moments at bridge supports
   c. Required cross-sections for structural loading (inertial resistance)
   d. Required camber to meet structural deformation allowances

3. Selection of an initial cross-section to meet span and dimensional window requirements
   a. Optimization of the cross-section via iterative calculations
i. To maximize the inertial properties of the bridge section
   (structural resistance to stress and deformation)
ii. To minimize the weight of the structure (saving on material and
    labor investment)

4. Determining reactions based on applied loadings and weights
   a. Optimization of the crane end truck via iterative calculations
      i. To minimize the wheel-base to meet required runway travel
      ii. To maximize the wheel spacing (wheelbase) to reduce induced
          runway moments (and minimize stresses and deformation)

5. Selection of an initial runway supporting cross-section to meet crane loading
   and operational requirements
   a. Designed to handle structural/mechanical reactions based on applied
      loadings and weights from crane operation
   b. Optimization of the cross-section via iterative calculations
      i. To maximize the inertial properties of the runway section
         (structural resistance to stress and deformation)
      ii. To minimize the weight of the section (saving on material and
          labor investment)
   c. If pre-existing runway is in place, design check serves as a means of
      validating the use of the existing runway for the proposed crane design
      i. Verifying the structural integrity to be compliant to
         requirements as-is
ii. Or providing an optimized means to reinforce the runway as required for crane operation at proposed configuration

6. Identification of the first order hinge-points of yield throughout the supporting member (i.e. locations of the maximum bending moments and shears that lead to yield onset within the runway cross-members)
   a. Based upon the onset of the yield strength of the material under loading

![Stress-strain correlation curve](image-url)  

*Fig. 7.1: Stress-strain correlation curve for conventional ductile material response (i.e. low carbon steel)*
b. Superimposed for the case of beam bending

Fig. 7.2: Simplified stress diagram for material response to bending moments within the elastic range of response (before yield onset)

c. With the case of beam shear

Fig. 7.3: Simplified stress diagram for material response to bending shear within the elastic range of response (before yield onset)
For the girder weight calculation, the primary parameters to consider are as follows:

1. Dead weight of the crane, consists of:
   a. Bridge structure (beams, boxes, trusses, etc.)
   b. Hoist/trolley weights (frame, motors, drum and reeving – generally consolidated into a single unit)
   c. Bridge end trucks (for support of bridge along runway rail, allowing for traverse of crane up/down the runway)
   d. Additions (service platforms/walkways, electrification, magnets, lights, etc)

2. Lifted load the crane is to handle, consisting of:
   a. Maximum rated load for each hoist available on the crane
   b. Sum of each hoist’s maximum rated load is not to exceed the rated load of the bridge
3. Wheel loads of the crane, which is the reaction at the wheels of the end truck, during operation of the crane.

   a. This wheel load is calculated with:
      i. the hoist(s) at maximum rated load (sum not exceeding bridge rating, per safe operation definition)
      ii. at full traverse to one side of the bridge
      iii. at minimum spacing between wheels of the hoist/trolley units

   b. This load is then distributed amongst the wheels of the end truck via a moment balance, resultant being the highest wheel load reactions induced by crane operation into the runway

4. The moment balance to solve for the maximum wheel load for a single hoist/trolley system is as follows:

\[ R_{\text{max}} = \left( \frac{1/2 \text{ dead weight}}{\text{no. of wheels per end truck}} \right) + \left( \frac{\text{Span-a} \times P_1 + \text{Span-a-wheelbase} \times P_2}{\text{Span} \times \text{no. of wheels per end truck}} \right) \]  

Equation 7.1: End reaction calculation, to solve for maximum wheel load, for single hoist/trolley

As calculated for a two wheel end truck (symmetrical distribution of loads), where \( P_n \) is the applied load from each pair of wheels, equidistant from the center of the runway rail.
5. The moment balance to solve for the maximum wheel load for the two

hoist/trolley system is as follows:

\[
R_{\text{max}} = \left( \frac{1/2 \text{ dead weight}}{\text{no. of whts per end truck}} \right) + \left( \frac{\text{Span-dist to CL}}{\text{Span}} \right) \cdot P_1 + \left( \frac{\text{Span-dist to CL-whlbse-dist btw.inside whls}}{\text{Span}} \right) \cdot P_2 + \left( \frac{\text{Span-dist to CL}-\text{whlbse-dist btw.inside whls}}{\text{Span}} \right) \cdot P_3 + \left( \frac{1}{\text{(no.of whls per end truck)}} \right) \cdot P_4,
\]

Equation 7.2: End reaction calculation, to solve for maximum wheel load, for dual hoist/trolley

Multiple wheels are calculated using the same logic for iteration, where the

additional wheels of each remaining hoist is calculated using the following

generalized equation:

\[
R_{\text{max}} = \left( \frac{1/2 \text{ dead weight}}{\text{no. of wheels per end truck}} \right) + \left( \sum_{i=1}^{n} \left( \frac{\text{Span-spacing}_i}{\text{Span}} \right) \cdot P_i \right) \cdot \frac{1}{\text{(no.of wheels per end truck)}}.
\]

Equation 7.3: End reaction calculation, to solve for maximum wheel load, symbolic expression

for finite series of hoist/trolley systems

For the current programs, the calculation was developed to limit to a choice of either
two wheels or four wheels for the end truck, and to a single or dual hoist upon the
bridge. These choices approximate over 95% to 98% of most industrial overhead

crane configurations in existence.

To simplify the calculations in the program, a single equation was used, utilizing a (#
of hoists -1) multiplier to allow for a single or dual hoist application. This allows for
the nullification of computing the further-distance loads of the second hoist if it is not present in the design.

Fig 7.5: Runway End truck loading diagram, four wheel case

\[
R_{\text{max}} = \left( \frac{1/2 \text{ dead weight}}{\text{no. of whls per end truck}} \right) + \left( \frac{\text{Span-dist to CL}}{\text{Span}} \right) * P_1 + \left( \frac{\text{Span-dist to CL-whlbase}}{\text{Span}} \right) * P_2 + \\
(\# \text{ of hoists} - 1) * \left( \frac{\text{Span-dist to CL-whlbase-dist btw inside whls}}{\text{Span}} \right) * P_3 + (\# \text{ of hoists} - 1) * \left( \frac{\text{Span-dist to CL-2-whlbase-dist btw inside whls}}{\text{Span}} \right) * P_4 * \left( \frac{1}{\text{(no. of whls per end truck)}} \right), \hspace{1em} (7.4)
\]

Equation 7.4: End reaction calculation as utilized in program for adjustment between single & dual hoist/trolley systems

However, for the eight wheel bogie system, it is more traditional to utilize the equivalent center load (ECL) to determine the actual wheel loads

Fig 7.6: Runway End truck loading diagram, eight wheel case
\[ ECL = R_{max} \times (\text{no. of whls per end truck}) \]  
(7.5)

\[ P_{1\text{actual}} = \frac{1}{2} \times ECL \times (1 + \frac{\frac{1}{2} \times \text{dist btw inside whls + whlbase}}{\frac{1}{2} \times \text{dist btw inside whls}})^{-1} = P_{4\text{actual}} \]  
(7.6)

\[ P_{2\text{actual}} = \frac{1}{2} \times ECL - P_{1\text{actual}} \]  
(7.7)

\[ = \frac{1}{2} \times ECL \times (1 - (1 + \frac{\frac{1}{2} \times \text{dist btw inside whls + whlbase}}{\frac{1}{2} \times \text{dist btw inside whls}})^{-1}) = P_{3\text{actual}} \]  
(7.8)

Equation 7.5 - 8: End reaction calculation for four-wheel (bogie) end trucks, calculation of actual load for each wheel based on location

This is reflected in the advanced 8 wheel runway analysis.

For the girder section analysis, the method of built-up sections is used to develop optimized sectional shapes. The method of built-up sections is a means to calculate the inertial resistance of a body’s cross-section to deformation. Using this method, one can calculate the inertial properties of a body, and estimate the properties of a section that is not available in standard references.

The procedure starts with the division of a section into regions of simpler shapes (such as rectangles and squares). This is followed by establishing an axis of reference, typically at the extrema of the compound section to be calculated.

Each region of the body is then calculated using a tabulated format:

<table>
<thead>
<tr>
<th>Region No.</th>
<th>Area</th>
<th>Dist</th>
<th>Area*Dist</th>
<th>Area*Dist^2</th>
<th>Ixx</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A1</td>
<td>Dy1</td>
<td>A1*Dy1</td>
<td>A1*Dy1^2</td>
<td>Ixx1</td>
</tr>
<tr>
<td>2</td>
<td>A2</td>
<td>Dy2</td>
<td>A2*Dy2</td>
<td>A2*Dy2^2</td>
<td>Ixx2</td>
</tr>
<tr>
<td>Sum</td>
<td>A Sum</td>
<td>n/a</td>
<td>A Sum * Dy Sum</td>
<td>A Sum * Dy Sum^2</td>
<td>Ixx Sum</td>
</tr>
</tbody>
</table>

Fig 7.7: Symbolic tabulation for calculation of cross-section properties
1. The first column is the regional division of the cross-section to be analyzed
2. The second column is the region’s individual cross-sectional area
3. The third is the distance from each region’s centroid to the axis of reference
4. The fourth is the multiplication of the 1st two column values for each region
5. The fifth is the multiplication of the 1st column value by the 2nd column value squared
6. The sixth is the region’s own moment of inertia, with respect to its own centroidal axis of symmetry

The summation of each column (with exception to the distance column) is required to solve for the combined section’s properties. Using the following equations:

1. Total Cross-Sectional Area
   \[
   A_t = \sum_{i=1}^{n} A_i
   \]  
   (7.9)

2. Neutral Axis Location (as displaced from the reference axis)
   \[
   n_x = \frac{\sum_{i=1}^{n} (A_i * dy_i)}{A_t}
   \]  
   (7.10)

3. Cross-Section Moment of Inertia about the neutral axis
   \[
   I_x = \sum_{i=1}^{n} l_{xxi} + \sum_{i=1}^{n} (A_i * dy_i^2) - \sum_{i=1}^{n} A_i * n_x^2
   \]  
   (7.11)

4. Section modulus of cross-section
   \[
   S_{x(lower extrema)} = \frac{I_x}{n_x}  \\
   S_{x(upper extrema)} = \frac{I_x}{d - n_x}
   \]  
   (7.12, 13)
5. Radius of Gyration

\[ r_x = \sqrt{\frac{I_x}{A_t}} \]

Equation 7.9 - 14: Calculations for Cross-Sectional Area, Neutral axis location, Moment of Inertia, Section Modulus, and Radius of Gyration for Composite Cross-Sections using the Method of Built-Up Sections

![Diagram of cross-sections](Image)

**Tabulated Calculation of Moment of Inertia**

<table>
<thead>
<tr>
<th>Section</th>
<th>Height ( h )</th>
<th>Area ( b(h_1 - h) )</th>
<th>( b_1^2 )</th>
<th>Moment ( \frac{b(h_1^2 - h^3)}{2} )</th>
<th>Moment ( \frac{b(h_1^3 - h^4)}{3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.500</td>
<td>0.125</td>
<td>0.187</td>
<td>0.016</td>
<td>0.002</td>
</tr>
<tr>
<td>B</td>
<td>0.531</td>
<td>0.625</td>
<td>0.206</td>
<td>0.391</td>
<td>0.010</td>
</tr>
<tr>
<td>C</td>
<td>0.319</td>
<td>1.500</td>
<td>0.191</td>
<td>2.250</td>
<td>0.205</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d = \frac{M}{A} = \frac{0.315}{0.644} = 0.49)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The moment of inertia of the entire section with reference to the neutral axis \( xx \) is:

\[ I_{xx} = I_{DE} = Ad^3 \]

\[ = 0.272 - 0.644 \times 0.49^2 \]

\[ = 0.117 \]

Fig. 7.8: Sample calculation using the method of sections, from Machinery's Handbook, 26th Edition

Built-up (or box) sections are generally used for longer spans, due to the need to conserve weight while maximizing inertial resistance to stress and deformation.

Solving for the basic stresses and deflection:
1. The primary bending stress:

For single hoist/trolley:

\[ \sigma = \frac{M_x n_x}{I_{xx}}, \text{ with } M_x = \frac{1}{4} \times \text{dead load} \times \frac{1}{2} \times \text{span} + P_1 \times \frac{1}{2} \times (\text{span} - \text{whlbase}) \]  \hfill (7.15)

For dual hoist trolley:

\[ \sigma = \frac{M_x n_x}{I_{xx}}, \]

\[ M_x = \frac{1}{4} \times \text{dead load} \times \frac{1}{2} \times \text{span} + P_1 \times \frac{1}{2} \times (\text{span} - 2 \times \text{whlbase} - \text{dist btw. near whls}) + P_2 \times \frac{1}{2} \times (\text{span} - \text{whlbase} - \text{dist btw. near whls}), \text{ with } P_1 = P_2 = \frac{1}{4} \times (\text{trolley weight} + \text{live load}) \] \hfill (7.16)

Consolidated as a single equation:

\[ \sigma_{BM} = \frac{M_x n_x}{I_{xx}}, \]

\[ M_x = \frac{1}{4} \times \text{dead load} \times \frac{1}{2} \times \text{span} + P_1 \times \frac{1}{2} \times (\text{span} - \# \text{ of hoists} \times \text{whlbase} - \# \text{ of hoists} \times \text{dist}) + \# \text{ of hoists} \times P_2 \times \frac{1}{2} \times (\text{span} - \text{whlbase} - \text{dist}), \text{ with } P_1 = P_2 = 1/4 \times (\text{trolley weight} + \text{live load}) \] \hfill (7.17)

2. The secondary bending moment:

Consolidated as a single equation:

\[ \sigma_{BM} = \frac{M_x n_x}{I_{xx}}, \text{ with } M_x = P_1 \times \frac{1}{2} \times (\text{span} - \# \text{ of hoists} \times \text{whlbase} - \# \text{ of hoists} \times \text{dist}) + \# \text{ of hoists} \times P_2 \times \frac{1}{2} \times (\text{span} - \text{whlbase} - \text{dist}), \text{ with } P_1 = P_2 = 1/20 \times (\text{trolley weight} + \text{live load}) \] \hfill (7.18)

3. The primary shear:

\[ \tau_b = \frac{V_{max}}{Area_{cs}}, \text{ where } V_{max} = R_{max} \]  \hfill (7.19)
4. The torsional shear: 
\[ \tau_t = \frac{T \times r}{I_{zz}} = \frac{R_{max} \times ((d-n_x) + r_h)}{I_{xx} + I_{yy}}, \] (7.20)

5. Design Check: 
\[ \frac{\sigma_{bM}}{\sigma_{ys}} + \frac{\sigma_{bm}}{\sigma_{ys}} + \frac{\tau_b}{\tau_{ys}} + \frac{\tau_t}{\tau_{ys}} \leq 1 \] (7.21)

Equation 7.15 - 21: Bending Moment/Stress calculation for single & dual hoist/trolley systems, Primary Bending Shear, Torsional Shear and ASD Design Check calculations for girder analysis

Continuing with the basics of runway structural/mechanical analysis, the following procedure has been in-development; starting with the basic parameters for a four and eight-wheel system, followed by the advanced development for each respective case.

The loading calculations are, in essence, the same as those utilized on the girder sections. However, due to the runway generally consisting of simpler structures, there are a number of additional calculations and iterative checks to review.

Examples of the basic loading/parameter and the advanced loading/parameter cases (for both the four and eight wheel systems) are found in the appendix (Entries A through D).
8. **Solver Parameters**

Combination of Structural Calculations (girder, runway, column, base-plate) for different configurations:

1. Static loading (standard FOSy)
2. Dynamic loading (estimated FOSy)
   a. Using mean effective load factors to estimate cyclical loading variances
3. Seismic loading (estimated FOSy)
4. Fatigue failure (estimated life of operation until failure – girder only)

Considered Independent variables

1. Span of Bridge
2. Bridge Capacity
3. Lift Height Required
4. Speed of Bridge (@ maximum capacity)
5. Speed of Trolley (@ maximum capacity)
6. Lowest Overhead Obstruction Height
7. Columns Spacing (centerline spacing under runway)

Considered Dependent Variables

1. Maximum estimated wheel-load (@ maximum capacity @ maximum end-approach)
2. Allowable wheel-loads
   a. ASCE rail
      i. Permissible loads referenced in CMAA 70, Table 4.11.3 (2004 ed.)
b. Square-bar rail

3. Wheel-base of bridge end-truck
   a. Two wheel spacing as standard, with additional wheels added if wheel-loads exceed allowable rating

4. Maximum allowable end-approach
   a. Based on wheel-load and wheel-base only

5. Ixx & Iyy required for bending resistances

6. Estimated plate sizes & stiffener requirements
   a. Standard size diaphragm widths (manual selection)
   b. Provide selection chart for diaphragm selection

7. Estimated runway beam-size
   a. ASD criteria
   b. Provide selection chart for runway beam selection

8. Estimated deflection for runway beam

9. Column translational forces & moments
   a. Vertical compression
   b. Traverse Shear
   c. Bridge Shear
   d. Traverse Moment
   e. Bridge Moment

Column Size based on ASD design criteria (to be featured in future iterations of the solver)

1. Allowable compression (Euler buckling) versus Actual compression
a. Sigma y: ASTM A-36, A-992

b. Ratio of Slenderness-to-radius of gyration (L/r)

c. Charts for allowable compression

2. Allowable traverse bending moment
   a. Sigma y (% yield allowed, CMAA criteria, AISC criteria)
   b. Ixx required (@ minimum to meet stress criteria)

3. Allowable bridge bending moment
   a. Sigma y (% yield allowed, CMAA criteria, AISC criteria)
   b. Iyy required (@ minimum to meet stress criteria)

4. Determine if gussets are required
   a. Based on beam selection size
   b. Ixx beam versus Ixx required (both orientations)
   c. Selection charts to limit to “square” wide-flange beam (beam depth
      \(\sim=\) base flange)

5. ASD design check (Sigma, actual / Sigma, allowed)
   a. compression and bridge (simultaneously)
   b. compression and traverse (simultaneously)
   c. no bridge and traverse (two-tandem motions are restricted @
      maximum capacity)

Standard drawings for reference (responsibility of the designer to provide this data to
the customer/end-user)

1. Tabulated drawings using standard views for approximate sizing
   a. AutoCAD drawings superimposed into spreadsheet format
b. Values appear in dimension located as tabulated by program

2. One standard three-set view for girder
   a. Top
   b. Front
   c. Side

3. One standard three-view set for column cell
   a. Top
   b. Front
   c. Side

4. Two standard views for column base-plate
   a. Top (Plan) view with no gusset required
   b. Top (Plan) view with gussets in highest bending moment orientation
      (typical traverse by default)
      i. Estimated Hole sizing based on 4-bolt pattern & 8-bolt pattern
      ii. Consider tubing if values exceed allotted stresses for standard
          “square”
          wide-flanges and gusset combinations

For the development of the custom-based spreadsheets, the following preliminary logic was developed for brainstorming of the implementation procedure:

A listing of the Independent Parameters of the system:

1. Span (SP)
2. Capacity (LL)
3. Lift Height (LH)
4. Bridge Speed (VB)
5. Trolley Speed (VT)
6. Lowest Obstruction (LO)
7. Column Spacing (CS)
8. Girder Weight (GR)
9. Trolley/Hoist Weight (TR)
10. End-truck Weight (ET)
11. Beam Weight (BW)

The Dependent Variables, or Outputs, of the analysis:

1. Maximum Wheel-load (RM)
2. Allowable Wheel-load (RA)
3. Wheel-base (WB)
4. Minimum End-Approach (AP)
5. Ixx Girder (IxxG)
6. Iyy Girder (IyyG)
7. Ixx Runway (IxxR)
8. Iyy Runway (IyyR)
9. Calculated Deflection (DEL)
10. Runway End-Reaction (RE)
11. Vertical Compression (VC)
12. Trolley Shear (VT)
13. Bridge Shear (VB)
14. Trolley Bending Moment \((MT)\)
15. Bridge Bending Moment \((MB)\)
16. Runway Bending Moment \((MR)\)
17. Bending Moment Stress \((\sigma_{ys})\)

Figure 8.1: Early samples of runway calculation loading and reaction parameters, per AISC & CMAA design criterion

Calculations to determine basis design parameters:

- Determining Minimum End-Approach from Allowable Wheel-load:

\[
RA = \frac{1}{2} \times \left( \frac{1}{2} \times (ET + GR) + \frac{(SP - AP)}{SP} \times (LL + TR) \right);
\]

Solving for \(AP\) yields:
AP = SP – SP/(LL + TR) * [ 2*RA – ½ * (ET + GR) ]; with RM/RA <= 1.0

- For Beam Loading (Direction of Primary Deflection):

  \[ MR = \frac{1}{4} * BW * (CS – WB) + \frac{1}{2} * RM * (CS – WB) + \frac{1}{4} * BW * WB; \]

  with RM = \( \frac{1}{2} * \left[ \frac{1}{2} * (ET + GR) + (SP – AP)/SP * (LL + TR) \right] \);

- Bending Moment Stress (Direction of Primary Deflection):

  \[ \sigma_{BR} = \frac{MR*c/I_{xxR}}{c/I_{xxR}} = c/I_{xxR} * \left[ \frac{1}{4} * BW * (CS – WB) + \frac{1}{2} * RM * (CS – WB) + \frac{1}{4} * BW * WB \right]; \]

  Solving for I_{xxR} yields:

  \[ I_{xxR_{min}} = \frac{c}{\sigma_{BR}} * \left[ \frac{1}{4} * BW * (CS – WB) + \frac{1}{2} * RM * (CS – WB) + \frac{1}{4} * BW * WB \right]; \]

- Using Lightest Weight of Beam to optimize selection: (I_{xxR} vs. BW)

  (*) Use True/False Criteria for Selection (IF-ELSE loop, all three to be FALSE to stop iteration)

  \[ I_{xxR_{n+1}} - I_{xxR_{n}} \geq 0; \text{ next iteration of beam has higher } I_{xx} \text{ value} \]

  \[ WB_{n+1} - WB_{n} \leq 0; \text{ next iteration of beam has lower weight} \]

  \[ \% \sigma_{ys} \geq c/I_{xxR_{n}} * \left[ \frac{1}{4} * BW * (CS – WB) + \frac{1}{2} * RM * (CS – WB) + \frac{1}{4} * BW * WB \right]; \]

  bending resistance of beam does not let bending stress exceed allowable.

  (*) Use all three at FALSE to optimize selection for runway and/or girder.
To solve for beam optimization, nest IF-ELSE loops by:

1. $\% \sigma_{ys}$
2. $I_{xxR_{n+1}} - I_{xxR_{n}}$
3. $WB_{n+1} - WB_{n}$

For Beam Loading (Direction of Secondary Deflection):

$$MQ = 0.05 \times (TR + LL) \times (CS - WB);$$

Bending Moment Stress (Direction of Secondary Deflection):

$$\sigma_{bQ} = MQ \times c/I_{yyR} = c/I_{yyR} \times [0.05 \times (TR + LL) \times (CS - WB)];$$

Solving for $I_{yyR}$ yields:

$$I_{yyR_{min}} = c/\sigma_{bQ} \times [0.05 \times (TR + LL) \times (CS - WB)];$$

Using Lightest Weight of Beam to optimize selection: ($I_{yyR}$ vs. $BW$)

(*) Use True/False Criteria for Selection (IF-ELSE loop, all three to be FALSE to stop iteration)

$$I_{yyR_{n+1}} - I_{yyR_{n}} \geq 0; \text{ next iteration of beam has higher } I_{xx} \text{ value}$$

$$WB_{n+1} - WB_{n} \leq 0; \text{ next iteration of beam has lower weight}$$

$$\% \sigma_{ys} \geq c/I_{yyR_{n}} \times [0.05 \times (TR + LL) \times (CS - WB)]$$

bending resistance of beam does not let bending stress exceed allowable

(*) Use all three at FALSE to optimize selection for runway and/or girder.
• To solve for beam optimization, nest IF-ELSE loops by:

1. \( \% \sigma_{ys} \)
2. \( IyyR_{n+1} - IyyR_{n} \)
3. \( WB_{n+1} - WB_{n} \)

• Stresses of Concern:

1. \( \sigma_{Bx} \) (primary moment bending)
2. \( \tau_{Bx} \) (primary bending shear)
3. \( \sigma_{By} \) (secondary moment bending)
4. \( \tau_{By} \) (secondary bending shear)
5. \( \tau_z \) (torsional shear)
6. \( \sigma_{Rc} \) (rail compression, wheel-load)

• \( \sigma_{Bx} = MR * c_{x}/Ixx; \) MR = primary bending moment

• \( \tau_{Bx} = 3/2 * V_{x}/A_{cs}; \) \( V_{x} \) = primary bending shear, \( A_{cs} \) = area of structural cross-section

• \( \sigma_{By} = MQ * c_{y}/Iyy; \) MQ = secondary bending moment

• \( \tau_{By} = 3/2 * V_{y}/A_{cs}; \) \( V_{y} \) = secondary bending shear

• \( \tau_z = Mz * r/Izz = 1/Izz * [RT*nx*ny - QT*ny^2]; \) \( RT \) = trolley wheel-load, \( QT \) = trolley thrust-load, \( nx \) = distance to x-x neutral axis, \( ny \) = distance to y-y neutral axis, \( Izz = Ixx + Iyy \)

• \( \sigma_{Rc} = RT/A_{R} = RT/RB * 1/[2*(RH + TP_t) + 2]; \) \( RB \) = base-width of rail, \( RH \) = height of rail, \( TP_t \) = top-plate thickness
*This material will be added to future iterations of the solver, those which will be developed in higher-level programming languages (such as Visual BASIC®), to allow for multiple embedded “IF-ELSE” loops to properly sort and optimize to solution.
9. **Solver Development**

Updates to the programming led to the following development. These pages feature developed procedural examples of the calculation algorithm that has been developed for the crane analysis/optimization program. The datasheets are developed as individual calculations, utilizing Microsoft Excel® as a starting point for the basic development of the program, which serves as a common utility for many designers and engineers in the creation of accessible analytical software modules. From these models, a state of continuity is developed - where the system is modeled as a whole; and the variable geometric and loading/reaction parameters of each section is dependent on the others. This interweaving of the design analysis allows for a better development of global optimization, and a chance to create a more cost-effective structure for the industrial crane design.

For the 1st section of the calculation process, an indication of basic girder sizing and parameters must be made. This is done by the user entering the data for the plate sizes of the different sections of the girder, the required girder span, spacing between plates, and sizes of longitudinal stiffener angles, as shown in table (1):

![Girder section calculation worksheet](image)

**Fig 9.1: Table 1, Data entry of girder cross-section sizing and required span**

The input variables consist of the following (as noted, all sizing is in Imperial inches):

- **Fig 9.1:** Table 1, Data entry of girder cross-section sizing and required span
a. top plate width (tw)
b. top plate thickness (c1)
c. bottom plate width (bw)
d. bottom plate thickness (c2)
e. web plate 1 thickness (t1)
f. web plate 1 height (h)
g. rail width (rb)
h. rail height (rh)
i. stiffener elevation (s1)
  a. adjusted automatically if stiffeners are required to: 0.4 * h
j. stiffener leg 1 length (s4)
k. stiffener leg 2 length (s2)
l. stiffener leg thickness (s3)
m. distance between inside of web plates (dw)
n. web centerline offset (for adjustment for non-symmetrical boxes)
o. girder span (L, in inches – even with most spans being verified in foot-inch lengths)
p. distance between end of girder & centerline of span (dist), to adjust for actual box length
q. space btw diaphragm and bottom plate (to account for estimated diaphragm sizing for proper weight calculations)
r. required moment of inertia for stiffeners (if necessary) and current moment of inertia value for configured stiffener
a. Alerts are integrated to allow the user to know if the stiffener meets CMAA criterion established in Section 70.3.5.3.1.1

i. distance between diaphragm plates \((a) = \text{(average of) 60 inches}\)

ii. depth of girder \((h) = h\)

iii. thickness of web plates \((t) = t1\)

iv. Area of stiffener \((A_s) = (s4 \times s3) + (s3 \times (s2 - s3))\)

When one longitudinal stiffener is used it shall be placed so that its centerline is 0.4 times the distance from the inner surface of the compression flange plate to the neutral axis. It shall have a moment of inertia no less than:

\[
I_o = 1.2 \left[ 0.4 + 0.6 \left( \frac{a}{h} \right) + 0.0 \left( \frac{a}{h} \right)^2 + 0.6 \left( \frac{A_p}{A_f} \right) \right] h t^3 \quad \text{in.}^4
\]

Fig 9.2: Section 70.3.5.3.1.1, required sizing for Longitudinal Stiffeners for Girder Cross-Section

These variables are illustrated (for the most part) in the reference diagram shown below, indicative of the standard configuration for the majority of box girders fabricated by crane companies. For simplicity, it is assumed that the web plates are symmetrical and identical in most calculations, as that the finer control of cross-sectional properties comes from adjusting the top & bottom plates, since they contribute more to the cross-sectional inertial resistance.
The data from this point of entry is then processed within two tabulations, to compute the necessary values to calculate the moments of inertia, composite cross-sectional area, and other values of interest. These tables are composites of the method of built-up sections, as referenced in section 7 of this paper.
Fig 9.4: Table 2, calculations for the $I_x$ (strong-axis) inertial resistance of the cross-section

<table>
<thead>
<tr>
<th>Area number</th>
<th>calculations description</th>
<th>cross-section area</th>
<th>centroid dist from ref. axis</th>
<th>area&quot;dist&quot;</th>
<th>area&quot;dist&quot; dist</th>
<th>centroid 2nd moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>top pl.</td>
<td>10</td>
<td>31.75</td>
<td>0.25</td>
<td>7.950825</td>
<td>0.007950825</td>
</tr>
<tr>
<td>2</td>
<td>web pl.</td>
<td>10</td>
<td>20.375</td>
<td>1.63</td>
<td>545.40825</td>
<td>1545.40825</td>
</tr>
<tr>
<td>3</td>
<td>top pl.</td>
<td>10</td>
<td>50.375</td>
<td>2.55</td>
<td>257.350625</td>
<td>257.350625</td>
</tr>
<tr>
<td>4</td>
<td>web pl.</td>
<td>10</td>
<td>0.375</td>
<td>3.75</td>
<td>1.90625</td>
<td>1.90625</td>
</tr>
<tr>
<td>5</td>
<td>rail</td>
<td>10</td>
<td>0.75</td>
<td>4.5625</td>
<td>4.5625</td>
<td>4.5625</td>
</tr>
<tr>
<td>6</td>
<td>rail</td>
<td>10</td>
<td>40.5625</td>
<td>30.421875</td>
<td>1223.457305</td>
<td>0.009765063</td>
</tr>
<tr>
<td>7</td>
<td>rail</td>
<td>10</td>
<td>40.5625</td>
<td>30.421875</td>
<td>1223.457305</td>
<td>0.009765063</td>
</tr>
<tr>
<td>8</td>
<td>rail</td>
<td>10</td>
<td>40.5625</td>
<td>30.421875</td>
<td>1223.457305</td>
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<td>10</td>
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<td>30.421875</td>
<td>1223.457305</td>
<td>0.009765063</td>
</tr>
<tr>
<td>total</td>
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<td>48.181</td>
<td>28.1563</td>
<td>1189.904</td>
<td>4933.593</td>
<td>2692.240</td>
</tr>
</tbody>
</table>

Fig 9.5: Table 3, calculations for the $I_y$ (weak-axis) inertial resistance of the cross-section

All input values come directly from table 1, where the properties of the composite regions are calculated according to the following equations, for Table 2 ($I_x$):

a. Area of region ($A$) = plate width * plate thickness
   a. Bottom plate = $b_w * c_2$
b. Web plate 1 = $h * t_1$
c. Top plate = $t_w * c_1$
d. Web plate 2 = web plate 1 (symmetry)
e. Rail = $r_b * r_h$
f. Stiffener 1, leg 1 = $s_4 * s_3$
g. Stiffener 1, leg 2 = $s_2 * s_3$
h. Stiffener 2, leg 1 = Stiffener 1, leg 1 (symmetry)
i. Stiffener 2, leg 2 = Stiffener 1, leg 2 (symmetry)

b. The centroid distance from reference axis (Dy) = distance from bottom of girder (where extrema is located) to the regional center (centroid) of the area in calculation

a. Bottom plate = $\frac{1}{2} \times c_2$

b. Web Plate 1 = $c_2 + \frac{1}{2} \times h$

c. Top Plate = $c_2 + h + \frac{1}{2} \times c_1$

d. Web plate 2 = web plate 1 (symmetry)

e. Rail = $c_2 + h + c_1 + \frac{1}{2} \times rh$

f. Stiffener 1, leg 1 = $c_2 + h - s_1 + \frac{1}{2} \times s_3$

g. Stiffener 1, leg 2 = $c_2 + h - s_1 + s_3 + \frac{1}{2} \times (s_2-s_3)$

h. Stiffener 2, leg 1 = Stiffener 1, leg 1 (symmetry)

i. Stiffener 2, leg 2 = Stiffener 1, leg 2 (symmetry)

c. The regional moments of inertia are calculated according to the base formula for rectangular cross-sections (Ixx) = $\frac{1}{12} \times \text{base} \times \text{height}^3$

a. Bottom plate = $\frac{1}{12} \times bw \times (c_2)^3$

b. Web plate 1 = $\frac{1}{12} \times t_1 \times (h)^3$

c. Top Plate = $\frac{1}{12} \times tw \times (c_1)^3$

d. Web plate 2 = web plate 1 (symmetry)

e. Rail = $\frac{1}{12} \times rb \times (rh)^3$

f. Stiffener 1, leg 1 = $\frac{1}{12} \times s_4 \times (s_3)^3$

g. Stiffener 1, leg 2 = $\frac{1}{12} \times s_3 \times (s_2-s_3)^3$

h. Stiffener 2, leg 1 = Stiffener 1, leg 1 (symmetry)
i. Stiffener 2, leg 2 = Stiffener 1, leg 2 (symmetry)

d. All other values are calculated according to the formulas noted below (in index notation)

   a. Area * dist = A_i * Dy_i, i = 1..n
   b. Area * dist * dist = A_i * Dy_i^2, i = 1..n

e. The sums of each of these respective calculations are listed in the “total” region, as required for the calculation of the various cross-sectional properties

These calculations are repeated for Table 3 (Iy), with the extrema being located on the far-side of the girder (far right as drawn, away from the rail location):

   a. Area of region (A) = plate width * plate thickness

      a. All values same as for Table 2 (Ix) calculations
   b. The centroid distance from reference axis (Dx) = distance from right of girder (where extrema is located) to the regional center (centroid) of the area in calculation

      a. Bottom plate = ½ * bw
      b. Web Plate 1 = ½ * bw + ½ * dw + ½ * t1
      c. Top Plate = ½ * tw + (offset), if required
      d. Web plate 2 = ½ * bw - ½ * dw - ½ * t1
      e. Rail = web plate 1 (by simplicity of design, and for greatest stress resistance)

      f. Stiffener 1, leg 1 = ½ * bw + ½ * dw - ½ * s4
      g. Stiffener 1, leg 2 = ½ * bw + ½ * dw - s4 + ½ * s3
      h. Stiffener 2, leg 1 = ½ * bw - ½ * dw + ½ * s4
i.  Stiffener 2, leg 2 = \( \frac{1}{2} \times bw - \frac{1}{2} \times dw + s4 - \frac{1}{2} \times s3 \)

c.  The regional moments of inertia are calculated according to the base formula for rectangular cross-sections \((I_{yy}) = \frac{1}{12} \times \text{height} \times \text{base}^3\)

   a.  Bottom plate = \( \frac{1}{12} \times c2 \times (bw)^3 \)

   b.  Web plate 1 = \( \frac{1}{12} \times h \times (t1)^3 \)

   c.  Top Plate = \( \frac{1}{12} \times c1 \times (tw)^3 \)

   d.  Web plate 2 = web plate 1 (symmetry)

   e.  Rail = \( \frac{1}{12} \times rh \times (rb)^3 \)

   f.  Stiffener 1, leg 1 = \( \frac{1}{12} \times s3 \times (s4)^3 \)

   g.  Stiffener 1, leg 2 = \( \frac{1}{12} \times (s2-s3) \times (s3)^3 \)

   h.  Stiffener 2, leg 1 = Stiffener 1, leg 1 (symmetry)

f.  All other values are calculated according to the formulas noted below (in index notation)

   a.  Area \times \text{dist} = A_i \times D_{x_i}, \quad i = 1..n

   b.  Area \times \text{dist} \times \text{dist} = A_i \times D_{x_i}^2, \quad i = 1..n

D.  The sums of each of these respective calculations are listed in the “total” region, as required for the calculation of the various cross-sectional properties
From this table, the cross-sectional properties of the composite cross-section are calculated.

![Table 4](image)

Fig 9.6: Table 4, Sectional properties of composite cross-section, and design checks for minimum values, based on permissible loading/stresses/ratios.

a. The associated output variables of this table consists of:

a. The estimated linear weight of the girder (Wt) = Area of the cross-section (A) * density of steel (0.284 lbs/cubic in) * (1ft/ 12 in)

b. The area of the cross section (A) = \( \sum A_i \), \( i = 1..n \)

c. The depth of the girder (d) = c1 + h + c2 +rh

d. The moment of inertia for the strong axis (Ix) = \( \sum (A_i * D_{yi}^2) + \sum I_{xx_i} - \sum A_i * n_x^2 \)

e. The distance to the neutral axis, from the girder bottom (n_x1) = \( \frac{\sum (A_i * D_{yi})}{\sum A_i} \)

f. The modulus of the section, based on the girder bottom (S_x1) = I_x / n_x1

g. The distance to the neutral axis, from the girder top (n_x2) = d - n_x1
h. The modulus of the section, based on the girder top \( (S_{x2}) = \frac{I_x}{n_{x2}} \)

i. The radius of gyration, of the strong axis \( (r_x) = \frac{I_x}{\sum A_i} \)

j. The moment of inertia for the weak axis \( (I_y) = \sum(A_i \cdot Dx_i^2) + \sum I_{yy} - \sum A_i \cdot n_y^2 \)

k. The distance to the neutral axis, from the side opposite the rail \( (n_y) = \frac{\sum(A_i \cdot Dx_i)}{\sum A_i} \)

l. The modulus of the section, based on the side opposite the rail \( (S_y) = \frac{I_y}{n_y} \)

m. The radius of gyration, of the weak axis \( (r_y) = \frac{I_y}{\sum A_i} \)

The design check ratios are based on geometric allowances given in CMAA 70.

These ratios help to ensure that the girders have a good measure of inertial resistance in all directions, limiting undesired performance in the beams during operation, such as buckling, bouncing, and rolling. The allowances are as noted:

a. Span: Beam Depth Ratio = \( \frac{L \text{ (in inches)}}{d} \). Not to exceed 25, as best as possible

b. Span: Inside Webs Ratio = \( \frac{L \text{ (in inches)}}{dw} \). Not to exceed 65, as best as possible

c. Web Height: Web Thickness Ratio = \( \frac{h}{t_1} \). Not to exceed 200, else the use of longitudinal stiffeners is required

   a. Integrated alerts are present with the design checks to enable the user to know if the values are okay, or in need of adjustment accordingly
The minimum allowable values are based on the loading data for the crane, which is entered in the following table. The equations used to calculate these values will be provided later in the section.

In this table, the quantities, spacing, and weights of the crane components are calculated.

<table>
<thead>
<tr>
<th>ITEM NO.</th>
<th>QTY.</th>
<th>DESCRIPTION</th>
<th>DIMENSIONS (FOR STEEL SIZES - IN INCHES):</th>
<th>WEIGHT/CHS</th>
<th>WEIGHT/TOTAL</th>
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<td></td>
<td></td>
<td>HEIGHT</td>
<td>WIDTH</td>
<td>LENGTH</td>
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<td>20</td>
<td>732</td>
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<tr>
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<td>2</td>
<td>2 BOTTOM PLATE</td>
<td>0.275</td>
<td>20</td>
<td>732</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>4 WEB PLATE</td>
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<td>4</td>
<td>732</td>
</tr>
<tr>
<td>4</td>
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<td>GIRDER DIAPHRAGM</td>
<td>38.75</td>
<td>15.5</td>
<td>4</td>
</tr>
<tr>
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<td>12</td>
<td>FUEL DIAPHRAGM</td>
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<td>15.5</td>
<td>4</td>
</tr>
<tr>
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<td>4</td>
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<td>15.5</td>
<td>4</td>
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<tr>
<td>7</td>
<td>1</td>
<td>TOP/ST BM SPACER</td>
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<td>15.5</td>
<td>4</td>
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<tr>
<td>8</td>
<td>1</td>
<td>45 DEGREE ANGLE</td>
<td>4</td>
<td>732</td>
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<td>710</td>
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<tr>
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<td>2</td>
<td>END</td>
<td>20</td>
<td>0.275</td>
<td>710</td>
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<tr>
<td>12</td>
<td>4</td>
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<td>1</td>
<td>732</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>FESTOON CABLE, 1.25 uf/ft AVG.</td>
<td>831.6</td>
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<td>700</td>
</tr>
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<td>1</td>
<td>FESTOON C-TRACK</td>
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<td>1.5</td>
<td>700</td>
</tr>
<tr>
<td>16</td>
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<td>CONTROL PANEL, 5-BOY MOTION</td>
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<td>700</td>
</tr>
<tr>
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<td>1</td>
<td>END TRUCK (MN)</td>
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<td>1.5</td>
<td>700</td>
</tr>
<tr>
<td>18</td>
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<tr>
<td>19</td>
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<tr>
<td>20</td>
<td>1</td>
<td>WELD ADDER, 1.5 LBS/FT</td>
<td>1.5</td>
<td>1.5</td>
<td>700</td>
</tr>
</tbody>
</table>

**Fig 9.7: Table 5, Structural configuration of the crane’s bridge and the operational hoist/trolleys.**

This is currently configured for a traditional top-running double-box assembly, as that accounts for the majority of high-capacity top-running cranes in existence. This limits the quantities of plates to:

a. Two top plates: width and thickness as given, length of L – 2*dist
b. Two bottom plates: width and thickness as given, length of L – 2*dist
c. Four web plates: height and thickness as given, length of L – 2*dist
d. Girder Diaphragms: internal spacers placed at 5 foot intervals, quantity is calculated using, qty. = (L – 2*dist)/60, raised to the next whole number.
e. Two rails: width and height as given, length of \( L - 3 \times \text{dist} \)

f. Four end-stops: lengths required determined by the designer

g. Paint and Weld adder: based on linear distance of one full girder and one full end truck assembly, length = \((L - 2 \times \text{dist} + \text{end truck length}) / 12\)

h. Hoist/trolley assemblies, quantities and weights (TR)
i. All other quantities of structural components are as determined by the designer

The calculation also requires the spacing available between the wheels for the hoist/trolleys and the end trucks. This is done to calculate the maximum generated end truck wheel-load, in the worst loading case possible (maximum loading at minimum spacing between wheels and support). This is required for proper runway design, as will be explained later in the section. The spacing dimension required consists of:

a. Trolley wheelbase (wb1), each trolley is assumed to have the same wheelbase

b. End-approach (a) of 1st trolley’s outermost wheel to center of span/runway

c. Distance between near/inside wheels (di1), for two trolleys on single bridge

d. Lifted load of each trolley (LL), each trolley is assumed to have the same capacity

e. The end truck wheelbase (wb2), for bogie (or 8 wheel) end trucks only
f. The distance between near/inside wheels (di2), for bogie end trucks only

The results of the values entered/calculated in Table 5 are shown in the resultant table, Table 6.

| Sum of Weights for Bridge (W/Out Hoists) | 25638.56 lbs |
| Sum of Weights for Crane (W/ Hoists)     | 29038.56 lbs |
| Max Wheel Load (for A 4-Wheel Bridge System) | 25792.86 lbs |
| Inner Wheel Load (For the Inner Wheels of an 8-Wheel Bridge System) | 15475.72 lbs |
| Outer Wheel Load (For the Outer Wheels of an 8-Wheel Bridge System) | 10317.15 lbs |
| Drive Girder Weight (W/ Panel)           | 10220.41 lbs |
| Idler Girder Weight (W/ Festoon)         | 10518.16 lbs |
| End-Truck Weights                       | 2460.00 lbs  |
| Hoist Weights                           | 1700.00 lbs  |
| Girder Linear Weight (Steel Plate Only, No Adders) | 166.28 lbs/ft |

Fig 9.8: Table 6, the resultant weights and wheel-loads values, based on the girder cross-section sizing and component configuration

The resultant weight consists of:

a. The full-bridge weight (FBW), the crane assembly without hoist/trolleys

   a. FBW = \( \sum \) (Weight/Total of Items 1 to 19)

b. The full crane weight (FCW), the crane assembly with hoist/trolleys

   a. FCW = \( \sum \) (Weight/Total of Items 1 to 20)

c. The maximum wheel-load (MWL) for the traditional four-wheel end truck

   a. MWL = \( \frac{1}{4} \) * FBW + \( \frac{1}{4} \) * (TR + LL) * (L-a + L-a-wb1 + (# hoists -1) * L-a-wb1-di1 + (# hoists -1) * L-a-wb1-di1-wb1)

   b. As referenced in Equation 7.4

d. The outer/outside wheel-load (OWL) for a bogie (8 wheel) end truck

   a. OWL = MWL / (1 + \( \frac{1}{2} \) * \( \frac{1}{2} \) * di2 + wb2)/ \( \frac{1}{2} \) * di2)
b. As referenced in Equation 7.6

e. The inner/inside wheel-load (IW) for a bogie (8 wheel) end truck
   a. \( IW = MWL / (1 - (1 + (\frac{1}{2} \cdot di^2 + wb^2) / \frac{1}{2} \cdot di^2)) \)
   b. As referenced in Equation 7.8

f. The weight of the drive girder (DGW)
   a. \( DGW = \sum (\text{Weight}/\text{Total of Items 1 to 12, 18 to 19}) / 2 + \sum (\text{Weight}/\text{Total of Item 16}) \)

g. The weight of the idler girder (IGW)
   a. \( IGW = \sum (\text{Weight}/\text{Total of Items 1 to 12, 18 to 19}) / 2 + \sum (\text{Weight}/\text{Total of Items 13 to 15}) \)

h. All others weights (end truck, hoist/trolley, and linear weight) are provided, as given or calculated in earlier tables, shown only for references

Fig 9.9: Table 7, Stress Calculations, Girder Deflection, Design Check for Girder Cross-Section/Girder Loading

Stress calculations are compliant to required structural safety factors per CMAA Section 70. The table entries/resultants are as follows:
a. The primary moment (Mp) = \( \frac{1}{4} \cdot \text{FBW} \cdot \frac{1}{2} \cdot L + \frac{1}{4} \cdot (\text{TR} + \text{LL}) \cdot \frac{1}{2} \cdot (L - \# \text{hoists} \cdot \text{wb1} - \# \text{hoists} \cdot \text{di1}) + \# \text{hoists} \cdot \frac{1}{4} \cdot (\text{TR} + \text{LL}) \cdot \frac{1}{2} \cdot (L - \text{wb1} - \text{di1}) \)
   a. As referenced in Equation 7.18
b. The secondary moment (Ms) = \( \frac{1}{20} \cdot (\text{TR} + \text{LL}) \cdot \frac{1}{2} \cdot (L - \# \text{hoists} \cdot \text{wb1} - \# \text{hoists} \cdot \text{di1}) + \# \text{hoists} \cdot \frac{1}{10} \cdot (\text{TR} + \text{LL}) \cdot \frac{1}{2} \cdot (L - \text{wb1} - \text{di1}) \)
c. The primary shear (Vp) = MWL, as this load is seen at each end of girder in the maximum loading case
   a. As referenced in Equation 7.19
   b. Secondary bending shear is ignored, due to lower magnitude in comparison to other forces/moments
d. The rotational torsion (Tr) = \( \frac{1}{20} \cdot (\text{TR} + \text{LL}) \cdot n_x \cdot \frac{2}{\# \text{hoists}} \cdot 2 \)
   a. As referenced in Equation 7.20
e. The primary bending stress in tension (\( \sigma_p \)) = \( \frac{M_p \cdot n_x \cdot 1}{I_x} = \frac{M_p}{S_{x1}} \)
   a. As referenced in Equation 7.18
f. The primary bending stress in compression (\( \sigma_p \)) = \( \frac{M_p \cdot n_x \cdot 2}{I_x} = \frac{M_p}{S_{x2}} \)
   a. As referenced in Equation 7.18
g. The secondary bending stress (\( \sigma_s \)) = \( \frac{M_s \cdot n_y \cdot 1}{I_y} = \frac{M_s}{S_{y1}} \)
h. The bending shear stress (\( \tau_p \)) = \( \frac{V_p}{A} \)
   a. Secondary bending shear stress is ignored, due to lower magnitude in comparison to other stresses
i. The torsional shear stress (\( \tau_r \)) = \( \frac{\text{Tr} \cdot n_x \cdot 2}{(I_{xx} + I_{yy})} \)
j. The angle of twist under torsion (°) = \((180/\pi) \times (T_r \times L) / (2 \times G \times (I_{xx} + I_{yy}))\)
   a. \(L\) = span of girder (in inches)
   b. Where shear modulus of elasticity of steel (G) = 11,500,000 lb/in²

The girder deflection calculation comes from the ASD 9th edition, formulae for simple beam bending; symmetrical loading for a distributed load (runway dead load), plus symmetrical loading for a two-point loading on a single member. For the vertical loading (in direction of gravity and primary bending):
   a. For the dead load (runway beam members), a distributed-loading calculation is used:
      a. \(\Delta x = (5/384 \times W_t \times (L^4)) / (E \times I_x)\)
         i. Where modulus of elasticity of steel (E) = 30,000,000 lb/in²
      b. For the applied lifted loads, the trolley wheel loads as symmetrical-spaced point loads are used:
         a. \(\Delta x = ((1/24 \times 1/4 \times (T_R + L) \times a \times (3 \times L^2 + 4 \times a^2)) / (E \times I_x)) + (#\text{ hoists} - 1) \times ((1/24 \times 1/4 \times (T_R + L) \times b \times (3 \times L^2 + 4 \times a^2)) / (E \times I_x))\)
         b. Distance from girder support to 1st wheel load (a):
            i. \(a = L - wb1 - (#\text{ hoists} - 1) \times d_{i1} - (#\text{ hoists} - 1) \times wb1\)
         c. Distance from girder support to 2nd wheel load (b) – for 2 hoist/trolley case only:
            i. \(b = a + (#\text{ hoists} - 1) \times wb1\)
c. Permissible Deflection is calculated, based on a direct correlation to runway span:
   a. $\Delta x_{all} = \frac{L}{888}$, for cambered sections. These sections have curvature built into the structure to negate dead-loads of the supported structure
   b. Minimum camber required is based upon the deflection of the dead-load, plus 1/2 of the lifted-load rating

For the horizontal loading (perpendicular to gravity and in the direction of secondary bending):

   a. For the applied lifted loads, the trolley wheel loads as symmetrical-spaced point loads are used:
      a. $\Delta x = \frac{((1/24 \times 1/20 \times (TR + LL) \times a \times (3 \times L^2 + 4 \times a^2))}{(E \times I_x)} + (# \text{ hoists-1}) \times \frac{((1/24 \times 1/20 \times (TR + LL) \times b \times (3 \times L^2 + 4 \times a^2))}{(E \times I_x)}$
      
      i. The distances (a & b) are calculated the same as for the vertical loading

   b. Permissible Deflection is calculated, based on a direct correlation to runway span:
      a. $\Delta x_{all} = \frac{L}{400}$

The stress design check consists of checking the case of combined stresses - where the summation of the ratios of maximum stress-to-allowable stress, for all possible stresses of design failure for a particular loading, must be less than unity to be considered acceptable.
The following stress values are considered the maximum allowable (or permissible) per CMAA Section 70 for structural members:

a. Tensile Bending stresses (primarily due to loading, and all secondary) = 0.6 * $\sigma_{ys}$

b. Compressive Bending stresses (primarily due to loading, and all secondary) = 0.35 * $\sigma_{ys}$

   a. (mainly in top-flange, generally a concern for single web members only – as shown in the runway section of the calculation procedure)

   c. Shear stresses (due to bending, couples, or rotational moments) = 0.35 * $\sigma_{ys}$

For the design check: $\sigma_p / 0.6 * \sigma_{ys} + \sigma_s / 0.6 * \sigma_{ys} + \tau_p / 0.35 * \sigma_{ys} + \tau_r / 0.35 * \sigma_{ys} \leq 1$, for the girder cross-section to be considered acceptable for calculated loading case.

Stress calculations, based upon girder cross-section’s inertial resistance and crane configuration are calculated in Tables 7 & 8.
Fig 9.10: Table 8, Eight-Point Stress Calculation/Check for Strong & Weak Axis Bending Stresses, Shear Stresses & Combined Stresses for Selected Girder Cross-Section

The eight-point cross-section check is intended to serve as a means of checking stresses across the box seams, where the welds are most crucial to maintaining cross-sectional integrity. The primary points of checking are the four outer seams (where external arc welding takes place); and the outer flanges directly above and below the web plates. These spots are crucial, in that all loading and deformation is distributed from the web directly into the plates at these locations, and if failure takes place at one of these spots, catastrophic failure is imminent.
The locations are illustrated in Figure 9.11:

![Dimensional Reference Diagram for Eight-Point Stress Calculation for Selected Girder Cross-Section](image)

Fig 9.11: Dimensional Reference Diagram for Eight-Point Stress Calculation for Selected Girder Cross-Section

At each of these locations, a point calculation is made for the primary bending stress ($\sigma_p$), and the secondary bending stress ($\sigma_s$). In addition, a point calculation is made for the primary shear stress ($\tau_p$), and the secondary shear stress ($\tau_s$). For each location, the following relationships are used:

a. Primary bending stress ($\sigma_p$) = $M_x \times (D_i) / I_x$
   
   a. $M_x$ = maximum primary bending moment
   
   b. $D_i$ = distance from the reference neutral axis of the cross-section ($n_x$)
   
   c. $I_x$ = moment of inertia for strong axis
b. Secondary bending stress \( (\sigma_s) = My \times (d_i) / Iy \)
   
a. \( My = \) maximum secondary bending moment
   
b. \( d_i = \) distance from the reference neutral axis of the cross-section \( (n_y) \)
   
c. \( Iy = \) moment of inertia for weak axis

c. Primary shear stress \( (\tau_p) = Vx \times (A_i) \times \Delta_i / Ix \times b_i \)
   
a. \( Vx = \) primary shear force = MWL
   
b. \( A_i = \) cross-sectional area of the region of the location of shear (top/bottom flange)
   
c. \( \Delta_i = \) distance from the reference neutral axis of the cross-section \( (n_x) \)
   
d. \( Ix = \) moment of inertia for strong axis
   
e. \( b_i = \) thickness of cross-section region where point is located (along the reference neutral axis)

d. Secondary shear stress \( (\tau_s) = Vy \times (A_i) \times \delta_i / Iy \times b_i \)
   
a. \( Vy = \) secondary shear force = 0.2 \* MWL
   
b. \( A_i = \) cross-sectional area of the region of the location of shear (web of girder)
   
c. \( \delta_i = \) distance from the reference neutral axis of the cross-section \( (n_x) \)
   
d. \( Iy = \) moment of inertia for weak axis
   
e. \( b_i = \) thickness of cross-section region where point is located (along the reference neutral axis)

Stresses are summed according to type (bending & shear) and then are added vectorally to a final combined stress resultant.
a. Combined Stresses (\( \Sigma \)) = \((\sigma_p + \sigma_s)^2 + (\tau_p + \tau_s)^2)^{(1/2)}

A design check against maximum permissible combined stresses of 0.6 * \( \sigma_{ys} \) is calculated. As long as the values are not allowed to exceed 0.6 * \( \sigma_{ys} \) (recommended is 0.5 * \( \sigma_{ys} \), to account for amplification and impact factors), the cross-section is sufficient for the loading configuration.

The frequency check, listed in Table 9, is used to rate the fundamental frequency of the girder cross-section:

![Frequency Response](image)

**Fig 9.12: Table 9, Frequency Response of Girders to Vertical and Horizontal Excitation**

The fundamental frequency of the crane is based upon the corresponding spring stiffness of each member, which is a function of the cross-sectional properties of the girder. The frequency response considers only the dead weight of the crane (structural and mechanical members only); it does not account for the addition of lifted loads into the calculation.

The natural frequency of a simply supported member is denoted as:

a. \( \Omega = (k/m)^{(1/2)} \)
   a. Where \( k \) = the combined spring stiffness of the crane members
   b. Where \( m \) = the mass of the crane
b. Mass (m) = the FCW/\(g_c\)
   
a. Where \(g_c\) = gravitational constant (32.2 ft/sec/sec)
   
b. For the vertical excitation, the FBW is used
   
c. For the horizontal excitation, 20% of the FBW is used (in lieu of a more complicated dynamic analysis)

b. Spring stiffness \(k\) = \(4608^*E^*I_{xx}/(L^3)\), for the supported girder member
   
a. Where \(E\) = modulus of elasticity of steel (30,000,000 psi)
   
b. \(I_{xx}\) = moment of inertia for strong axis
   
c. \(L\) = span of the girder

d. The girders operate in parallel, so the resultant equivalent spring stiffness \(k_{eq}\) is:
   
a. \(k_{eq} = k_1^*k_2 / (k_1 + k_2) = 2304^* E^* I_{xx} / (L^3)\)

b. A minimum of 2.4 Hz is required by CMAA Section 70 for most members, but anything above 2 Hz is generally considered acceptable in most seismic loading zones (Zones 0 & 1)
The runway analysis is the second part of the calculation that is required for a proper structural crane analysis. Starting with either the parameter entry for Table 10:

![Bridge Properties Table](image)

**Table 10**: Traditional (four-wheel) end truck crane configuration parameters

**Table 11**: Bogie (eight-wheel) end truck crane configuration parameters

These two tables provide for data entry and carry-over from the previous girder analysis, where for the four-wheel case:

a. The girder weight = FCW

b. Wheel Load 1 & 2, Table 10= MWL

c. Wheelbase 1 = distance between end truck wheels on either side of the bridge
For the eight-wheel case:

a. The girder weight = FCW

b. Outside Wheel Load 1 & 2 = OWL

c. Insider Wheel Load 1 & 2 = IWL

d. Wheelbase 1 = distance between end truck wheels on either side of the bridge.

Most manufacturers use the same wheelbase on both trucks for simplicity, so Wheelbase 1 = Wheelbase 2 in the majority of cases

e. The impact factor is used to estimate dynamic impact upon the runway, due to snatching and dropping of loads. It is accurately calculated using an acceleration multiplier for hoist motion versus gravitational acceleration – but is easier to assume as a straight percentage of the lifted load for most cases. It defaults to three selections in this program:

   a. 10%, corresponding to light loadings, light shocks

   b. 15%, corresponding to moderate loadings, moderate shocks

   c. 25%, corresponding to heavy loadings, heavy shocks
For the selection of the runway members, Tables 12 through 14 are used to configure a desired cross-section.

**Fig 9.16: Table 12, selection of runway beam, cell length, and beam properties**

Selection of the runway beam comes from a table of the most-commonly used sections in the industry. Information for beam properties consists of:

a. Beam depth, the overall dimension of the beam from top of flange to bottom of flange
b. Base flange width, the dimension of the beam in the plane of the strong axis
c. The thickness of the base flange
d. The thickness of the web of the beam, the member connecting the two flanges together
e. The weak axis moment of inertia (Iyy)
f. The weak axis section modulus (Sy) = Iyy / (1/2 * d)
g. The weak axis radius of gyration (kyy) = (Iyy / A) ^ 1/2
h. The strong axis moment of inertia (Ixx)
i. The strong axis section modulus (Sx) = Ixx / (1/2 * d)
j. The strong axis radius of gyration ($k_{xx}$) = $\left( \frac{I_{xx}}{A} \right)^{1/2}$

k. The linear density of the beam (LDb), in lbs/foot of length

l. The cross-sectional area (A)

m. The weight of the beam = LDb * length of cell

Included in the selection is the yield strength, with the most commonly used section being rated at:

a. 36,000 psi  A-36 beams, commonly used before the year 2000

b. 50,000 psi  A-441 & A-992 alloy beams, commonly used after the year 2000

![Table 13: Channel Cap Properties](image)

Fig 9.17: Table 13, selection of runway channel cap, and cap properties

Selection of the channel cap comes from a table of the most-commonly used sections in the industry. Information for cap properties consists of:

a. Channel depth, the overall dimension of the channel from top of flange to bottom of flange

b. Channel flange width, the dimension of the channel in the plane of the strong axis
c. The thickness of the base flange

d. The thickness of the web of the channel, the member connecting the two flanges together

e. The weak axis moment of inertia (Iyy)

f. The weak axis section modulus (Sy) = Iyy / (1/2 * d)

g. The weak axis radius of gyration (kyy) = (Iyy / A) ^ 1/2

h. The strong axis moment of inertia (Ixx)

i. The strong axis section modulus (Sx) = Ixx / (1/2 * d)

j. The strong axis radius of gyration (kxx) = (Ixx / A) ^ 1/2

k. The linear density of the channel (LDc), in lbs/foot of length

l. The cross-sectional area (A)

m. The weight of the channel = LDc * length of cell

The alloy of the channel cap is generally assumed to be the same as that of the beam (or in many cases ignored), as that the runway beam is the true load-bearing member, and capping is often used to increase the sectional properties of a beam to offset deformation and stress. In many applications, a channel cap is not used, so a selection of “No Cap” is available to provide null values for those properties in the calculations.

Fig 9.18: Table 14, selection of runway rectangular rail, and rail properties
Configuration of the runway rail comes from a simple configurator used to size sections based on common bar sizes. Information for the rail consists of:

a. Rail height

b. Rail width

c. The strong axis moment of inertia (Ixx)
   a. Only the strong axis is considered, as that the rail contributes little to the weak axis moment of inertia for the composite section

d. The strong axis section modulus (Zxx) = Ixx/ (1/2 * d)
e. The strong axis radius of gyration (kxx) = (Ixx / A) ^ 1/2
f. The linear density of the rail (LDr), in lbs/foot of length
g. The cross-sectional area (A)
h. The weight of the rail = LDr * length of cell

As with the channel cap, the alloy of the rail is generally assumed to be the same as that of the beam (or in many cases ignored), as that the runway beam is the true load-bearing member, and the rail is mainly a means to confine the traverse of the bridge to the direction of propagation of the runway beam.

In many applications, runway rail consists of forged rail rather than bar. This allows for higher compressive and torsional resistance at a lower weight, and the ability to replace the rail as wear occurs. For simplicity, this rail can be modeled as a rectangular bar, in lieu of the high number of sections currently available on the market. Future additions of the program will incorporate tabulated selections of commonly-used rail sizes for improved performance.
The properties of the resultant runway cross-section are tabulated in Table 15, using the method of built-up sections, akin to what was used in the girder analysis:

<table>
<thead>
<tr>
<th>Region Name</th>
<th>Area</th>
<th>Distance from Ref. Axis</th>
<th>Area * Dist</th>
<th>Area* Dist*Dist</th>
<th>Ixx Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>W24 X 117</td>
<td>34.40</td>
<td>12.13</td>
<td>417.27</td>
<td>5061.51</td>
<td>35.40</td>
</tr>
<tr>
<td>CB X 11.5</td>
<td>3.38</td>
<td>13.82</td>
<td>46.11</td>
<td>645.46</td>
<td>1.32</td>
</tr>
<tr>
<td>RAIL</td>
<td>1.00</td>
<td>24.98</td>
<td>24.98</td>
<td>624.00</td>
<td>0.0033</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>38.78</td>
<td>50.93</td>
<td>488.860</td>
<td>6330.971</td>
<td>3541.403</td>
</tr>
</tbody>
</table>

Fig 9.19: Table 15, Sectional properties of composite cross-section, and design checks for minimum values, based on permissible loading/stresses.

a. The associated output variables of this table consists of:
   a. The estimated linear weight of the girder (Wt) = Area of the cross-section (A) * density of steel (0.284 lbs/cubic in) * (1ft / 12 in)
   b. The area of the cross section (A) = \( \sum A_i \), \( i = 1..n \)
   c. The depth of the runway beam (D) = c1 + h + c2 + rh
   d. The moment of inertia for the strong axis (Ix) = \( \sum (A_i * Dy_i^2) + \sum I_{xx} - \sum A_i * n_x^2 \)
   e. The distance to the neutral axis, from the runway bottom (n_x1) = \( \sum (A_i * Dy_i) / \sum A_i \)
   f. The modulus of the section, based on the runway bottom (S_x1) = Ix / n_x1
   g. The distance to the neutral axis, from the runway top (n_x2) = d - n_x1
   h. The modulus of the section, based on the runway top (S_x2) = Ix / n_x2
i. The radius of gyration, of the strong axis \((r_x) = \frac{I_x}{\sum A_i}\)

j. The moment of inertia for the weak axis \((I_y) = \sum (A_i \cdot D_{x_i}^2) + \sum I_{yy_i} - \sum A_i \cdot n_y^2\)

k. The distance to the neutral axis, from the edge of the beam flange \((n_y) = \frac{\sum (A_i \cdot D_{x_i})}{\sum A_i}\)

   a. Generally located at the center of the section, due to symmetrical configuration

l. The modulus of the section, based on the edge of the beam flange \((S_y) = \frac{I_y}{n_y}\)

   a. Calculated using the sum of the tabulated values for each cross-sectional region of the runway

m. The radius of gyration, of the weak axis \((r_y) = \frac{I_y}{\sum A_i}\)

Two of the primary design checks used in the analysis of the runway section are the single web buckling calculation and the top-flange compression stress allowance.

The single web buckling calculation is shown in Table 16:

![Table 16](image)

Fig 9.20: Table 16, Design Check for Buckling of Single-Web Members, as referenced from “Handbook of Structural Engineering”
Buckling is the phenomenon where compressive loading upon a member leads to a deformational shift of the member along the axis of compression. This shift results in the misalignment of the member, creating a buckle (or folding) of the web between the location of compression and the support of the member. This is a critical failure, in that the buckled member is plastically-deformed, and the loading capacity of a buckled member is severely-reduced, to the point where catastrophic failure (fracture) will most likely occur with continued use.

A case to be avoided at all cost, the primary means to safeguard against web buckling are:

a. intermittent diaphragms, providing additional members to transfer compressive stresses from the rail flange to the support flange without full-use of the web.
b. longitudinal stiffeners, to increase the cross-section’s inertial resistance, and lower the neutral axis closer to the support flange, reducing the likelihood of web buckling

c. load limiting/de-rating, limiting the permissible loads that the runway beam will have the carry, and safeguard against web buckling

This design check allows the user to determine the maximum permissible wheel load for the configured runway, without having to add diaphragms or stiffeners to improve the section. As referenced in the Load Resistance Factor Design Guide (LRFD), and based upon the ratio of:

a. Web Check Ratio \( R = \left( \frac{D_w}{t_w} \right) \left( \frac{L}{b_f} \right)^{-1} \)
   
   a. \( D_w = \) depth of the web \((D - 2*t_w)\)
   
   b. \( t_w = \) thickness of the web
   
   c. \( L = \) length of the cell (beam)
   
   d. \( b_f = \) width of the base flange

b. For values of the ratio < 2.3, and the beam is restrained against rotation (rigidly supported)

   a. Max. permissible wheel load = \( 0.85 \times (C_r \times (t_w^3) \times t_f \times (D_w)^{-2}) \times (1 + 0.4 \times (R^3)) \)

   i. \( C_r = \) adjustment factor (480,000 if \( Mu/My < 1; 960,000 \) if \( Mu/My \geq 1) \)

      1. \( Mu = \) highest bending moment in beam section
      2. \( My = \) the yield moment (where stresses cause onset plastic deformation in the beam)
c. For values of the ratio < 1.7, and the beam is not restrained against rotation (simply supported)

   a. Max. permissible wheel load = 0.85 * (Cr * (tw ^3) * tf * (Dw) ^ - 2) * (0.4 * (R^3))

d. For most runway beams, the runway is rigidly supported, so the main concern is the ratio (if R < 2.3). And if it is under the value for checking, whether the wheel load falls into the acceptable range. If not, the use of runway diaphragms or longitudinal stiffeners is required section is required.

The top flange compression check for single-web member is shown in Table 17:

![Table 17](image)

Fig 9.22: Table 17, Design Check for Top Flange Compression of Single-Web Members, as referenced from “A.I.S.C Allowable Stress Design, 9th Edition” & CMAA Section 70.3.5.7

Based upon the equation:

a. \( \sigma_{cc_all} = \frac{12,000}{(L*D) / (Af)} \), in units of kips/sq.in

   a. L = length of the cell (beam)

   b. D = depth of the beam

   c. Af = area of the base flange = bf * tf + Ac (Area of channel cap)

b. For modern sections, with the principal alloy being A992, the material properties of current beams is of a higher strength than past A36 beams. So in
consideration of the ratio of allowable –to-yield for compression being 1:3 in the A36 beam, an adjustment is used:

a. \( \sigma_{c_{all}} = \frac{1}{3} \times \sigma_{ys} / (L \times D) / (A_f) \), in units of kips/in\(^2\)

i. \( \sigma_{ys} \) = yield strength of the material, in units of lbs/in\(^2\)

c. The compressive stresses calculated are the vector sum of the primary bending stress & the secondary bending stress

a. \( \sigma_{c} = \left\{ \left( \frac{M_x}{S_x} \right)^2 + \left( \frac{M_y}{S_y} \right)^2 \right\}^{1/2} \)

d. If the bending compression exceeds this permissible value, then another section is recommended to safeguard against flange failure

For the traditional (4-wheel) single loading crane configuration, the resultant loading data is given in Table 18:

![Table 18](image)

Fig 9.23: Table 18, Single loading (4 wheel) resultant runway loads/stresses/design check/deflection
Stress calculations are compliant to required structural safety factors per CMAA Section 70. The table entries/resultants are as follows:

a. The location of the wheel loads are shown, as referenced from the left-hand support (per classic beam-bending theory)
   a. \( a = 1\textsuperscript{st} \) wheel location, absolute displacement, located at \( x = \frac{1}{2} \times (L - \text{wb}/2) \)
   b. \( b = 2\textsuperscript{nd} \) wheel location, absolute displacement = \( x + \text{wb} \)

b. The maximum vertical reactions, generated by the crane at the applied loading is shown
   a. \( R_{1c} = \text{MWL} \times (2 - \text{wb}/L) \), with the crane wheel at the location of maximum generated moment, due to motion of the bridge assembly

c. The maximum horizontal reactions, generated by the crane at the applied loading is shown
   a. \( r_{1c} = (n-1) \times \frac{1}{20} \times (\text{TR} + \text{LL}) \), with the crane wheel at the location of maximum generated moment, due to motion of the bridge assembly

d. The maximum primary bending moment (\( M_x \)) = \( \left( \text{MWL} / 2L \right) \times (L - \text{wb}/2)^2 \), for cases where the wheelbase is greater than 0.586 * cell length (L).
   If the wheelbase is less than 0.586* L, then \( M_x = \text{MWL} \times L/4 \)

e. The maximum secondary bending moment (\( M_y \)) = \( 2 \times r_{1c} \times x \times (L-x /L) \)

f. The primary shear (\( V_x \)) = \( R_{1c} \)
   a. Secondary bending shear is ignored, due to lower magnitude in comparison to other forces/moments

g. The rotational torsion (\( Tr \)) = \( r_{1c} \times n_c 2 \times a \)
h. The primary bending stress in tension ($\sigma_p$) = $M_x$ * $n_x$1 / $I_x$ = $M_x$ / $S_x$1
i. The primary bending stress in compression ($\sigma_p$) = $M_x$ * $n_x$2 / $I_x$ = $M_x$ / $S_x$2
j. The secondary bending stress ($\sigma_s$) = $M_y$ * $n_y$1 / $I_y$ = $M_y$ / $S_y$1
k. The bending shear stress ($\tau_p$) = $V_x$ / $A_{web}$ = $V_x$ / $(tw * (D - 2 * tf))$
   a. Secondary bending shear stress is ignored, due to lower magnitude in comparison to other stresses
l. The torsional shear stress ($\tau_r$) = $T_r$ * $n_x$2 / ($I_{xx}$ + $I_{yy}$)
m. The angle of twist under torsion ($\theta$) = $(180 / \pi) * (12 * T_r * L) / (2 * G * (I_{xx} + I_{yy}))$
   a. L = length of cell (in feet)
   b. Where shear modulus of elasticity of steel ($G$) = 11,500,000 lb/in$^2$

The runway deflection calculation comes from the ASD 9th edition, formulae for simple beam bending: symmetrical loading for a distributed load (runway dead load), plus symmetrical loading for a two-point loading on a single member. For the vertical loading (in direction of gravity and primary bending):

a. For the dead load (runway beam members), a distributed-loading calculation is used:
   a. $\Delta x = (5/384 * W_t * (L^4)) / (E * I_x)$
      i. Where modulus of elasticity of steel ($E$) = 30,000,000 lb/in$^2$
   b. For the applied lifted loads, the maximum wheel loads (MWL) as symmetrical-spaced point loads are used:
a.  \[ \Delta x = \frac{1}{24} \times MWL \times a \times (3L^2 + 4a^2)} {E Ix} \]

c.  Permissible Deflection is calculated, based on a direct correlation to runway span:

   a.  \[ \Delta x_{all} = \frac{L}{600} \], for uncambered sections.  These sections do not have curvature to negate dead-loads of the supported structure

For the horizontal loading (perpendicular to gravity, and in the direction of secondary bending):

   a.  For the applied lifted loads, the maximum wheel loads (MWL) as symmetrical-spaced point loads are used:

      a.  \[ \Delta x = \frac{1}{24} \times 0.2 \times MWL \times a \times (3L^2 + 4a^2)} {E Ix} \]

   b.  Permissible Deflection is calculated, based on a direct correlation to runway span:

      a.  \[ \Delta x_{all} = \frac{L}{400} \]

The stress design check consists of a checking the case of combined stresses - where the summation of the ratios of maximum stress-to-allowable stress, for all possible stresses of design failure for a particular loading, must be less than unity to be considered acceptable.

The following stress values are considered the maximum allowable (or permissible) per CMAA Section 70 for structural members:

   a.  Tensile Bending stresses (primarily due to loading, and all secondary) = 0.6 * \( \sigma_{ys} \)
b. Compressive Bending stresses (primarily due to loading, and all secondary) = $0.35 \times \sigma_{ys}$

a. (mainly in top-flange, generally a concern for single web members only
– as shown in the runway section of the calculation procedure)

c. Shear stresses (due to bending, couples, or rotational moments) = $0.35 \times \sigma_{ys}$

For the design check: $\sigma_p / 0.6 \times \sigma_{ys} + \sigma_s / 0.6 \times \sigma_{ys} + \tau_p / 0.35 \times \sigma_{ys} + \tau_r / 0.35 \times \sigma_{ys} \leq 1$, for the girder cross-section to be considered acceptable for calculated loading case.
For the bogie (8-wheel) dual loading crane configuration, the resultant loading data is given in Table 19:

<table>
<thead>
<tr>
<th>DOUBLE LOADING</th>
<th>RUNWAY WEB STRESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOR MAXIMUM RESULTANTS:</td>
<td>PRIMARY BENDING (TENSION)</td>
</tr>
<tr>
<td>FIRST LOCATION OF O.S. WHEEL (FROM SUPPORT)</td>
<td>11221.17 PSI</td>
</tr>
<tr>
<td>FIRST LOCATION OF I.S. WHEEL (FROM SUPPORT)</td>
<td>PRIMARY BENDING (COMPR)</td>
</tr>
<tr>
<td>SECOND LOCATION OF I.S. WHEEL (FROM SUPPORT)</td>
<td>10042.01 PSI</td>
</tr>
<tr>
<td>SECOND LOCATION OF O.S. WHEEL (FROM SUPPORT)</td>
<td>SECONDARY BENDING STRESS</td>
</tr>
<tr>
<td>MAX REACTION (INNER WHEEL AT SUPPORT)</td>
<td>5095.48 PSI</td>
</tr>
<tr>
<td>MAX HORIZ REACTION (INNER WHEEL AT SUPPORT)</td>
<td>BENDING SHEAR STRESS</td>
</tr>
<tr>
<td>MAX BENDING MOMENT (Mx)</td>
<td>3106.27 PSI</td>
</tr>
<tr>
<td>MAX TORSION REACTION (Tv)</td>
<td>TORSIONAL SHEAR STRESS</td>
</tr>
<tr>
<td>ANGLE OF TWIST</td>
<td>228.35 PSI</td>
</tr>
<tr>
<td>MAX BENDING MOMENT (Mx)</td>
<td>RUNWAY BEAM DEFLECTION</td>
</tr>
<tr>
<td>MAX BENDING MOMENT (Mx)</td>
<td>FOR VERTICAL LOADING</td>
</tr>
<tr>
<td></td>
<td>FULL LOAD DEFLECTION</td>
</tr>
<tr>
<td></td>
<td>ALLOWED DEFLECTION</td>
</tr>
<tr>
<td></td>
<td>0.15 IN</td>
</tr>
<tr>
<td></td>
<td>0.41 IN</td>
</tr>
<tr>
<td></td>
<td>FOR HORIZONTAL LOADING</td>
</tr>
<tr>
<td></td>
<td>LIVE LOAD DEFLECTION</td>
</tr>
<tr>
<td></td>
<td>ALLOWED DEFLECTION</td>
</tr>
<tr>
<td></td>
<td>0.23 IN</td>
</tr>
<tr>
<td></td>
<td>0.60 IN</td>
</tr>
</tbody>
</table>

**STRESS DESIGN CHECK:**

- **ACTUAL**
  - PRIMARY BENDING STRESS
  - SECONDARY BENDING STRESS
  - TORSIONAL SHEAR STRESS
  - MATERIAL YIELD STRESS

- **STRESS DESIGN CHECK:**
  - PRIMARY BENDING STRESS
  - SECONDARY BENDING STRESS
  - TORSIONAL SHEAR STRESS
  - MATERIAL YIELD STRESS

Fig. 9.24: Table 19, Dual loading (8 wheel) resultant runway loads/stresses/design check/deflection

Stress calculations are compliant to required structural safety factors per CMAA Section 70. The table entries/resultants are as follows:

a. The location of the wheel loads are shown, as referenced from the left-hand most support (per classic beam-bending theory)
   a. \( a = \) location of 1\(^{st}\) outside wheel, absolute displacement = \( x - \) Wheelbase 1
   b. \( b = \) location of 1\(^{st}\) inside wheel, absolute displacement = \( x - “b” = x - (wb / 2) * (x - 2)/(1 - x) \)
c. \( c = \text{location of 2}^{\text{nd}} \text{inside wheel, absolute displacement} = x + \text{distance between inside wheels (Dist)} \)

d. \( d = \text{location of 2}^{\text{nd}} \text{outside wheel, absolute displacement} = x + \text{Dist} + \text{Wheelbase 2} \)

b. The maximum vertical reactions, generated by the crane at the applied loading is shown

\[
a. \quad R_1c = MWL \times (2 - \frac{wb}{L}) \times (\frac{OWL}{IWL}), \text{ with the inner crane wheel at distance (b) from the location of maximum generated moment, due to motion of the bridge assembly}
\]

c. The maximum horizontal reactions, generated by the crane at the applied loading is shown

\[
a. \quad r_1c = (n - 1) \times \frac{1}{20} \times (TR + LL) \times (\frac{OWL}{IWL}), \text{ with the inner crane wheel at distance (b) from the location of maximum generated moment, due to motion of the bridge assembly}
\]

d. The maximum primary bending moment \( (Mx) = (MWL / 2*L) \times (L - \frac{wb}{2})^2 \), for cases where the wheelbase is greater than 0.586 * cell length \( (L) \).

If the wheelbase is less than 0.586 * \( L \), then \( Mx = MWL \times L/4 \)

e. The maximum secondary bending moment \( (My) = 2 \times r_1c \times x \times (L-x) /L) \)

f. The primary shear \( (Vx) = R1_c \)

\[
a. \quad \text{Secondary bending shear is ignored, due to lower magnitude in comparison to other forces/moments}
\]

g. The rotational torsion \( (Tr) = r_1c \times n_s2 \times a \)

h. The primary bending stress \( (\sigma_p) = Mx \times n_s1 / Ix = Mx / S_s1 \)
i. The secondary bending stress ($\sigma_s$) = $M_y \cdot n_y / I_y = M_y / S_y$

j. The bending shear stress ($\tau_p$) = $V_x / A_{web} = V_x / (t_w \cdot (D - 2 \cdot t_f))$

   a. Secondary bending shear stress is ignored, due to lower magnitude in comparison to other stresses

k. The torsional shear stress ($\tau_r$) = $T_r \cdot n_x / (I_{xx} + I_{yy})$

l. The angle of twist under torsion (°) = $(180 / \pi) \cdot (12 \cdot T_r \cdot L) / (2 \cdot G \cdot (I_{xx} + I_{yy}))$

   a. $L =$ length of cell (in feet)

   b. Where shear modulus of elasticity of steel ($G$) = 11,500,000 lb/in$^2$

The runway deflection calculation comes from the ASD 9th edition, formulae for simple beam bending: symmetrical loading for a distributed load (runway dead load), plus symmetrical loading for a two-point loading on a single member. For the vertical loading (in direction of gravity and primary bending):

   a. For the dead load (runway beam members), a distributed-loading calculation is used:

      a. $\Delta x = (5/384 \cdot W_t \cdot (L^4)) / (E \cdot I_x)$

      i. Where modulus of elasticity of steel ($E$) = 30,000,000 lb/in$^2$

   b. For the applied lifted loads, the outside and inside wheel loads (OWL & IWL) as symmetrical-spaced point loads are used:

      a. $\Delta x = ((1/24 \cdot OWL \cdot a \cdot (3 \cdot L^2 + 4 \cdot a^2)) / (E \cdot I_x)) + ((1/24 \cdot IWL \cdot b \cdot (3 \cdot L^2 + 4 \cdot b^2)) / (E \cdot I_x))$
c. Permissible Deflection is calculated, based on a direct correlation to runway span:
   a. \( \Delta x_{all} = \frac{L}{600} \), for uncambered sections. These sections do not have curvature to negate dead-loads of the supported structure

For the horizontal loading (perpendicular to gravity, and in the direction of secondary bending):

   a. For the applied lifted loads, the maximum wheel loads (MWL) as symmetrical-spaced point loads are used:
      a. \( \Delta x = \frac{(1/24 \times 0.2 \times OWL \times a \times (3L^2 + 4a^2))}{(E \times Ix)} + \frac{(1/24 \times 0.2 \times IWL \times b \times (3L^2 + 4b^2))}{(E \times Ix)} \)
   
   b. Permissible Deflection is calculated, based on a direct correlation to runway span:
      a. \( \Delta x_{all} = \frac{L}{400} \)

The stress design check consists of a checking the case of combined stresses - where the summation of the ratios of maximum stress-to-allowable stress, for all possible stresses of design failure for a particular loading, must be less than unity to be considered acceptable.

The following stress values are considered the maximum allowable (or permissible) per CMAAA Section 70 for structural members:

   a. Tensile Bending stresses (primarily due to loading, and all secondary) = 0.6 * \( \sigma_{ys} \)
b. Compressive Bending stresses (primarily due to loading, and all secondary) = 0.35 * σys

a. (mainly in top-flange, generally a concern for single web members only – as shown in the runway section of the calculation procedure)

c. Shear stresses (due to bending, couples, or rotational moments) = 0.35 * σys

For the design check: σp/ 0.6 * σys + σs/ 0.6 * σys + τp / 0.35 * σys + τr / 0.35 * σys <= 1, for the girder cross-section to be considered acceptable for calculated loading case.

For the fatigue check, the Soderberg fatigue criterion is used:

a. σamp / σfat + σmean / σys <= 1, where:

a. Mean stress (σmean) = k * σmax

i. k = mean effective load factor, per CMAA Section 70.5

b. Amplitude stress (σamp) = (1 – k) * σmax

c. Fatigue endurance stress (σfat) = Se * σuts

i. Material ultimate tensile strength (σuts) = endurance limit of material prior to extreme deformation and fracture

ii. Ratio of stress for infinite cycle life (Se), a variable based on S-N decomposition. Given as approx. 0.5* σuts for 1M cycles for most steels
d. Yield strength of material ($\sigma_y$s) = stress where plastic deformation onsets

b. All summary checks with values less than or equal to unity are considered acceptable for infinite life (1M or 1,000,000+ cycles)

As shown Tables 20 (for the girder fatigue check):

Fig. 9.25: Table 20, Fatigue check for girder loading/cross-section, per Soderberg fatigue criterion

As shown in Table 21 (for the runway fatigue check, single end truck loading):

Fig. 9.26: Tables 21, Fatigue check for runway single end truck loading, per Soderberg fatigue criterion
As shown in Table 22 (for the runway fatigue check, double end truck loading):

![Table 22](image)

Fig. 9.27: Tables 22, Fatigue check for runway single end truck loading, per Soderberg fatigue criterion

The mean effective load factor is the adjustment factor for the maximum crane rating, to determine what is the approximate statistical average payload actually handled by the crane. This is used to allow for more liberal adjustment in sizing critical load-bearing components, such as wheel/bearings, shafting and hoist reeving.

For each CMAA duty class of crane, there is a recommended mean effective load adjustment factor to utilize in these calculations:

1. Class A, $k = 0.75$,
   \[ \sigma_{\text{mean}} = 0.75 \times \sigma_{\text{max}} \]

2. Class B, $k = 0.75$,
   \[ \sigma_{\text{mean}} = 0.75 \times \sigma_{\text{max}} \]

3. Class C, $k = 0.80$,
   \[ \sigma_{\text{mean}} = 0.75 \times \sigma_{\text{max}} \]

4. Class D, $k = 0.85$,
   \[ \sigma_{\text{mean}} = 0.75 \times \sigma_{\text{max}} \]

5. Class E, $k = 0.90$,
   \[ \sigma_{\text{mean}} = 0.75 \times \sigma_{\text{max}} \]

6. Class F, $k = 0.95$,
   \[ \sigma_{\text{mean}} = 0.75 \times \sigma_{\text{max}} \]
The mean effective load factor is also used to calculate the amplitude stress for each case. Since the lifted load is not to exceed the maximum rated load of the bridge, the assumed amplitude stress is taken to be the variance between maximum and mean in both increment and decrement.

a. Class A, $k = 0.75$, Class A σ_{amp} = (1 - 0.75) * σ_{max}  
b. Class B, $k = 0.75$, Class A σ_{amp} = (1 - 0.75) * σ_{max}  
c. Class C, $k = 0.80$, Class A σ_{amp} = (1 - 0.80) * σ_{max}  
d. Class D, $k = 0.85$, Class A σ_{amp} = (1 - 0.85) * σ_{max}  
e. Class E, $k = 0.90$, Class A σ_{amp} = (1 - 0.90) * σ_{max}  
f. Class F, $k = 0.95$, Class A σ_{amp} = (1 - 0.95) * σ_{max}  

Each cycle class is considered to have an “expected” range of cyclical performance in handling payloads at the mean load rating. These CMAA cycle classes are broken down into four divisions, based on permissible operating range:

a. Class N1, 20k to 100k cycles  
b. Class N2, 100k to 500k cycles  
c. Class N3, 500k to 2M cycles  
d. Class N4, 2M+ cycles  

For each cycle class, based on a decomposition of a typical S-N chart for most mild steels, the following fatigue endurance factors (Se) were calculated in regression:

a. Class N1, Se = 0.38, σ_{fat} = 0.38 * σ_{uts}
b. Class N2, Se = 0.42 \[ \sigma_{fat} = 0.42 \times \sigma_{uts} \]

c. Class N3, Se = 0.47 \[ \sigma_{fat} = 0.47 \times \sigma_{uts} \]

d. Class N4, Se = 0.50 \[ \sigma_{fat} = 0.50 \times \sigma_{uts} \]

a. Where \( \sigma_{uts} \) is approximately equal to 1.666 \* \( \sigma_{ys} \) for most mild steels

All maximum stresses for primary bending and secondary bending are multiplied by the load adjustment factors for a particular duty class; and the material properties are multiplied by the fatigue adjustment factor for a particular cycle class. As long as the resultant values, as entered into the Soderberg relation is less than unity, the design is considered acceptable for fatigue – within that allotted duty and cycle class (or less).
10. **Strengths & Weaknesses of the comparative analysis methods**

The history of analysis for crane design and development is based on elastic material response & deformation theory. This theory, which borrows heavily from the Euler beam theory is used in most applications, since the preferred material of choice is low-content carbon steel, which is of high-strength, low cost and is readily available for industrial manufacture. The intent of the design is to maintain an elastic stress/strain response of the structure for all permissible loads and moments. This elastic response is where an approximate linear relationship is maintained between the material strain, and the induced stresses within the material.

Plastic deformation is not permissible for structural design, and is to be carefully guarded against. The plastic response is where a non-linear relationship (typically exponential) is maintained between the material strain, and the induced stresses within the material. As plastic deformation is non-recoverable, it can lead to a weakened structure which is incapable of handling rated loads, and eventually to structural failure (and possible collapse). This design scenario is to be guarded against at all costs.

To develop the most effective designs, the ones that will minimize material requirements and have desirable factors-of-safety, structural and mechanical analysis of the supporting framework must be done. There are several options that are commonly utilized by most companies, each with their own inherent strengths. They consist of the following:

1. Hand-Calculations
a. Can be performed easily with minimal resources (text, paper, writing tools)

b. Done using equation balance or dimensional verification

2. Spreadsheet Applications

a. Can be done quickly, with integrated real-time calculations

b. Can improve calculation time and accuracy with integrated equations/relationships of standard designs

3. Custom-software applications

a. Can be tailored for a specific company’s needs, outside of industry standard

b. Can integrate a large number of design standards more effectively

c. Can offer a greater degree of controlled access to analytical capability

4. Finite Element Analysis modules for CAD/CAM packages

a. Ability to simulate results in a virtual environment

b. Can get accurate data based on virtual deformations of the object mesh, rather than equation approximations

c. Can simulate and view virtual deformation under loading with results

Some of the inherent weaknesses of each method are as follows:

1. Hand-Calculations

   a. Is a slow process to iterate, can take many iterations to achieve a desired optimization of the design
b. Requires a generalized starting point, so most applications checked by hand will not vary far from industry standards
c. Required specialized knowledge to perform accurately, a general requirement of structural and/or mechanical scientific and engineering principles are necessary
   i. Restricted to (in most cases) – engineers and scientists
   ii. Personnel with many years of specialized industrial experience

2. Spreadsheet Applications
   a. Require setup and testing to properly develop and ensure accuracy
      i. Specialized knowledge is needed for effective development
   b. Can be altered without record of change (if not protected) and corrupted
   c. Is generally written to perform one type of analytical calculation alone
      i. But can be integrated to perform a multitude, if performed properly

3. Custom-software applications
   a. Requires setup, testing and de-bugging
      i. Specialized knowledge in engineering and software development is required
   b. Additional features can subtract from available development and re-work time
c. Many features can also be emulated through the use of custom spreadsheets and databases utilities

4. Finite Element Analysis modules for CAD/CAM packages
   a. Requires a great deal of computational power
      i. Systems will need to be tailor-made to properly handle the necessary calculations
   b. Often requires a fully (or nearly) realized design prior to testing
      i. More accessible during the finalization of a project, not the beginning
   c. Requires knowledge of determining boundary conditions to properly constrain model for simulation
   d. Extracting the most useful data can prove time-consuming
      i. Will offer comparable results quicker in most cases than actual prototype development, thereby reducing costs

The drive to develop analytical tools that are low in resource allocation, modular for optimization, easy to share and distribute is common-place in many areas of industry. The primary advisement for this particular area of application is a simple interface and programming platform, one which can allow for ease of development and execution of calculations using equations-of-behavioral state to perform calculations and iterations.

Custom development of analysis tools proves best for most organizations. They allow for the capture of data that is of commonality within their organization, can reference a standard(s) of working knowledge (theory, industrial standards, etc).
interface can be shared amongst a number of members, to improve the company-wide knowledge base and generate user support and critical feedback.

For a majority of the nominal analytical calculations performed by engineers and designers in crane manufacture, a standard spreadsheet proves effective for most singular applications. It’s where the need to merge resultant data from a number of different sources and arrays, is where the typical consideration is to develop custom-software applications. But the versatility of modern spreadsheet tools offers a number of options for custom development, and is the preferred choice for the initial phase of this consolidation of structural and mechanical analysis methodology for overhead industrial crane development.
11. **Path of design development in overhead crane systems**

In design development, a number of steps have to be considered in order to realize a fully-fledged and viable design.

1. Basic design intent – the general standards of design to be applied, such as: methodology, specifications, and validation criterion

2. Specification of design parameters – operational parameters that the crane is to achieve, such as: lift height, speeds and operating capacities. And this also includes restrictions on design, such as: clearance windows, load limits, power requirements, ambient environment conditions

3. Design development – progression of the design from conceptual (covering basic operating parameters and overall dimensions), to general (standard designs which covers the general requirements), to specialized (a project-specific design that covers the operational parameters and will accommodate the project-specific nature of the system)

4. Design tolerances & allowable deviations – specified requirements of dimensional and operational parameters that have to be met to guarantee reliability and repeatable function of the crane. Tolerances to ensure that components fit simply and smoothly into assemblies, specifications to ensure safety and durability, etc.

5. Design validation & methodology – the main methods of design validation that are referenced in crane design
6. Resources that are typically allotted to design development

a. Intent – the brainstorming aspect of design development, where the general limitations are of imagination and concise communication

b. Specification – the research into resources and allowable practices that will focus the scope of design, into a direction which will guarantee a useful product that has capability and cost-effectiveness

c. Development – the fabrication portion of design, where the conceptual aspect is transferred to form (papers, drawings, routings, bills of material). The work procedure is developed and partitioned according to the capabilities and capacities of the different groups/disciplines involved

d. Validation – the error and quality checkpoint of a design, to ensure it meets the design requirements applied to it. In addition, the validation
process is where the sustainability of a design is benchmarked, as to whether it is resource-efficient, meets safety standards, and is profitable in its implementation.

e. Iteration – repetition of the previous steps of the process, to allow for continuous improvement and refinement of the design
12. **Reasoning to bring cohesion to analysis methodology**

With so many choices for analytical methods to choose from, it becomes paramount to choose the right method for the right purpose. Reasons to consider in choice-methodology are as follows:

1. Availability of design information
   a. As always, the more information available at the initiation of design, the better. But often design parameters are still preliminary in nature during initial conceptual development, so information is often limited.

2. Time-constraints of design
   a. Demands to quickly bring about a realized product/project often limit the amount of time available for conceptual development. So quick analyses, which can allow for a variety of iterations is generally preferred.

3. Analytical resources available or on-demand
   a. In many cases, simple analyses are all that are required, as that design information for a project encompasses existing design standards, and only requires minor modulations to tailor them to the needs of the current design.
   b. For designs that have to be developed from scratch, greater amounts of design analysis will generally be required. And the greater the number of parameters, the more iteration required.

4. Optimization challenges
a. Designs often are in need of revision or significant changes to remain profitable as time passes. Fluctuations in costs of raw materials, machining, manufacture, actuation, and testing require that the design be developed to utilize these resources in the most optimal means possible. Often this means extracting components of the original design, or significant alterations which change the design, and require verification analysis to ensure functionality and compatibility.

The greater the variance allowed in a design, the more time and resources required to verify its feasibility. So what is generally best is to begin with a series of templates for each standard design variation, and adjust as them as much as possible to tailor-fit them to the needs of the custom variation.

And in addition to basic parameters, such as geometry and loading, generalized equations of state can be developed for each design template. Whereas alteration of the geometric parameters is done to match them accordingly to the custom variation, this can so be done with the equations of state as well. This would allow for more expedient updates to the analytical process, without having to re-create work or alter designs drastically.

Equations-of-state analysis uses the basis of static and dynamic equilibrium to develop the system reactions to applied forces and moments. From maintaining static equilibrium, one can determine the supporting reactions to the applied forces, along with the structural behavior (assuming known material properties and approximate linear behavior).
Dynamic equilibrium, using permissible loading factors and load-case modeling, leads to an understanding of motion of the structural system in response to applied forces and moments. And also motion simulation and analysis, which can be used to determine if design changes will prove effective in implementation, have a justifiable return of investment, and can lead to greater user versatility.

Equations-of-state analysis is the simplest methodology to computationally calculate and iterate. An extension of free-body analysis, it requires knowing governing parameters of the systems to provide constraints of motion and action. It is then principally driven by the Newtonian laws of motion, which dictate an equality balance of translational and rotational forces – that all forces which drive motion have equal and opposing counter-parts, and that the summation of these forces must equate to the final determined motion of the system.

It can become quite complex as the number of interactions increases. Each body is subjected to a number of degrees of translational and rotational freedom, known constraints of motion applied to the body eliminates degrees-of-freedom in motion. And these must be applied accurately to achieve feasible results. Under-constrained systems are not capable of yielding proper solution, which means that the reactions to applied forces and moments are not accurately related and unreliable. A strong foundational knowledge-base of free-body interaction is required to interpret results effectively and accurately.
13. Results and Conclusions

The development of equation of state solvers for structural engineering analysis for overhead industrial cranes (sizing of steel cross-sections and resistance to different types of loading and/or failure) is a cost-effective method to be considered for most companies. Due to the fact that most structural designs are within the elastic region of material response, applications that are linear in independent/dependant response can generally be utilized for most analytical requirements – and maintain a very high degree of analysis-to-actual correlation.

Continued improvement upon the progress made with this Microsoft Excel®-based linear solver, by incorporating the aspects into an updated solver. The optimization and analysis custom solver will be developed as a free-standing sub-routine, to be created in Microsoft Visual Studio 2008®. It will continue to be developed in a Visual BASIC® format to allow for macro development and integration with a number of design programs already available for the Windows® operating system platform. The datasheet results will be distributed amongst the input operational parameters, design criterion, embedded optimization, and analytical/optimization results of each iteration. This simplicity will allow designers to run multiple iterations, without having to re-compile the solver database each time.

The final goal is the development of a quick and simple self-standing structural analytical solver, able to work with basics of crane design parameters (critical dimensions and loading), integrated with a comprehensive structural database and iterative optimization sub-routines, to provide designers with a method of performing global analysis quickly and at minimum cost of set-up time and computing effort.
This will allow for crane manufacturers to produce the best product available for the lowest investment required.
14. Appendix

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A. Preliminary Runway Reactions, Moments, Stresses, and Design Checks:

For a 4-wheel bridge crane – sample calculation

*All calculations developed and executed in Maple 12 mathematical software package*

Case 1: Live Load

Independent Variables: P1, P2, L, a, wb
Dependent Variables: R1, R2, M1_LL, M2_LL

\[ \text{left-hand side reaction equation} \]
\[ R_1 := \frac{P_2 \cdot b}{L} + \frac{P_1 \cdot (b + \text{wb})}{L}, \]
\[ \text{right-hand side reaction equation} \]
\[ R_2 := \frac{P_1 \cdot a}{L} + \frac{P_2 \cdot (a + \text{wb})}{L}, \]

With \( b = L - (a + \text{wheelbase } \text{"wb"}) \), or \( b = L - (a + \text{wb}) \)
\( a = \) distance from left-hand support to P1; \( b = \) distance from right-hand support to P2

\[ b := L - (a + \text{wb}); \]

Maximum moment resides in region \( (a \leq x < b) \)

Case 2: Beam Load

Independent Variables: P1, P2, L, a, wb
Dependent Variables: R1, R2, Mmax_DL, W

Weight of Beam "W" is a factor of beam length and linear density. \( W = L \times \text{"ld"} \)
\( W := L I_d \)

\[ r_1 = \frac{W - L}{2}, \text{Self-weight reaction at left-hand side} \]

\[ r_1 = \frac{1}{2} L^2 I_d \]

\[ r_2 = \frac{W - L}{2}, \text{Self-weight reaction at right-hand side} \]

\[ r_2 = \frac{1}{2} L^2 I_d \]

\[ M_{\text{max,dl}} = \frac{W - L}{4}, \text{Maximum bending moment due to self-weight} \]

\[ M_{\text{max,dl}} = \frac{1}{4} L^2 I_d \]

Maximum reactions/moment is due to superposition of the beam load and the applied loads, in region \((a \leq x < b)\)

\[ R_{\text{lmax}} := R_1 + r_1; R_{2\text{max}} := R_2 + r_2; M_{\text{max}} := M_1 L_L + M_{\text{max,dl}}; \]

\[ R_{1\text{max}} := \frac{P_2 (L - a - wb)}{L} + \frac{P_1 (L - a)}{L} + r_1 \]

\[ R_{2\text{max}} := \frac{P_1 a}{L} + \frac{P_2 (a + wb)}{L} + r_2 \]

\[ M_{\text{max}} := \left( \frac{P_2 (L - a - wb)}{L} + \frac{P_1 (L - a)}{L} \right) a + \frac{1}{4} L^2 I_d \]

Case 4: Runway Horizontal Forces

Independent Variables: \(LL, Tr, L, a, wb\)

Dependent Variables: \(r_1_L, r_2_L, m_1_L, m_2_L\)

\[ \Sigma_{\text{Lateral}} := \frac{0.20}{4} (LL + Tr); \]

\[ \Sigma_{\text{Lateral}} := 0.0500 LL + 0.0500 Tr \]

\[ r_{L1} := \frac{P_2 a}{L} + \frac{P_1 (a + wb)}{L}, \text{Lateral reaction at left-hand side} \]

\[ r_{L1} := \frac{P_2 a}{L} + \frac{P_1 (a + wb)}{L} \]

\[ r_{L2} := \frac{P_1 b}{L} + \frac{P_2 (b + wb)}{L}, \text{Lateral reaction at right-hand side} \]

\[ r_{L2} := \frac{P_1 b}{L} + \frac{P_2 (b + wb)}{L} \]

With \(P_{1\_L}\) and \(P_{2\_L}\) both equal to one-quarter of the lifted load and trolley weight, or \(P_{1\_L} = P_{2\_L} = 0.25 \times (LL + Tr)\)

\[ P_{1\_L} := \frac{(LL + Tr)}{4}; \]

\[ P_{1\_L} := \frac{1}{4} LL + \frac{1}{4} Tr \]

\[ P_{2\_L} := P_{1\_L}; \]

\[ P_{2\_L} := \frac{1}{4} LL + \frac{1}{4} Tr \]

\[ r_{L2}; \]

\[ \left( \frac{1}{4} LL + \frac{1}{4} Tr \right) (L - a - wb) + \left( \frac{1}{4} LL + \frac{1}{4} Tr \right) (L - a) \]
Case 5: Longitudinal Forces (Horizontal)
Independent Variables: P1, P2, L, a, wb
Dependent Variables: RHorz, r1, r2

\[ R_{Horz} := 0.10 \cdot (R1 + R2); \]

Case 6: Torsion
Independent Variables: LL, Tr, L, a, wb, h
Dependent Variables: T1, T2, Ta, Tb, Tc, Tmax

\[ T1 := r1 \cdot h; \quad \text{torsional reaction at left-hand support due to lateral loading} \]
\[ T1 := \left( \frac{1}{4} LL + \frac{1}{4} Tr \right) (L - a - wb) + \left( \frac{1}{4} LL + \frac{1}{4} Tr \right) (L - a) \]

\[ T2 := r2 \cdot h; \quad \text{torsional reaction at right-hand support due to lateral loading} \]
\[ T2 := \left( \frac{1}{4} LL + \frac{1}{4} Tr \right) a + \left( \frac{1}{4} LL + \frac{1}{4} Tr \right) (a + wb) \]

With the following definitions:
\[ a1 = a \]
\[ b1 = wb \]
\[ c1 = L - (a + wb) \]

\[ Ta := T1; \quad Tb := 0; \quad Tc = T2; \quad \text{for 2 point runway loading (for a 4-wheel bridge)} \]
\[ Ta := \left( \frac{1}{4} LL + \frac{1}{4} Tr \right) (L - a - wb) + \left( \frac{1}{4} LL + \frac{1}{4} Tr \right) (L - a) \]
\[ Tb := 0 \]
\[ Tc = \left( \frac{1}{4} LL + \frac{1}{4} Tr \right) a + \left( \frac{1}{4} LL + \frac{1}{4} Tr \right) (a + wb) \]

Maximum torsion exists in regions a1 (0 ≤ x < a) & c1 (b ≤ x < L)
B. Preliminary Runway Reactions, Moments, Stresses, and Design Checks:

For an 8-wheel bridge crane – sample calculation

*All calculations developed and executed in Maple 12 mathematical software package*

Case 1: Live Load

Independent Variables: P1, P2, P3, P4, L, a, b, c, d, e, wb

Dependent Variables: R1, R2, M1_LL, M2_LL

> restart;

> Digits := 3;

> \( R1 := \frac{P4 \cdot (a + b + c + d)}{L} + \frac{P3 \cdot (a + b + c)}{L} + \frac{P2 \cdot (a + b)}{L} + \frac{P1 \cdot (a)}{L} \) left-hand side reaction equation

\[ R1 := \frac{P4 \cdot (a + b + c + d)}{L} + \frac{P3 \cdot (a + b + c)}{L} + \frac{P2 \cdot (a + b)}{L} + \frac{P1 \cdot (a)}{L} \]

> \( R2 := \frac{P1 \cdot (b + c + d + e)}{L} + \frac{P2 \cdot (c + d + e)}{L} + \frac{P3 \cdot (d + e)}{L} + \frac{P4 \cdot (e)}{L} \) right-hand side reaction equation

\[ R2 := \frac{P1 \cdot (b + c + d + e)}{L} + \frac{P2 \cdot (c + d + e)}{L} + \frac{P3 \cdot (d + e)}{L} + \frac{P4 \cdot (e)}{L} \]

With b & d = (wheelbase) "wb",

a = distance from left-hand support to P1; e= distance from right-hand support to P2;
c = distance between inside wheels on end truck

> \( R1, \)

\[ \frac{P4 \cdot (L - e)}{L} + \frac{P3 \cdot (-wb + L - e)}{L} + \frac{P2 \cdot (a + wb)}{L} + \frac{P1 \cdot a}{L} \]

> \( R2, \)

\[ \frac{P1 \cdot (L - a)}{L} + \frac{P2 \cdot (L - a - wb)}{L} + \frac{P3 \cdot (wb + e)}{L} + \frac{P4 \cdot e}{L} \]

\( M_{1_{\text{LLI}}} := R1 \cdot a \) Moment about left-hand support (R1) \((0 <= x < a)\)

\[ M_{1_{\text{LLI}}} = \left( \frac{P4 \cdot (L - e)}{L} + \frac{P3 \cdot (-wb + L - e)}{L} + \frac{P2 \cdot (a + wb)}{L} + \frac{P1 \cdot a}{L} \right) a \]

\( M_{1_{\text{LLL}}} := R1 \cdot a + (R1 - P1) \cdot b \) Moment about left-hand support (R1) \((a <= x < b)\)
Moment about right-hand support (R2) (d <= x < L)

\[ M_{2\text{LL2}} := \left( \frac{P_4 (L - e)}{L} + \frac{P_3 (-wb + L - e)}{L} + \frac{P_2 (a + wb)}{L} + \frac{P_1 a}{L} \right) a + \left( \frac{P_4 (L - e)}{L} + \frac{P_3 (-wb + L - e)}{L} + \frac{P_2 (a + wb)}{L} + \frac{P_1 a - p_1}{L} \right) wb \]

Moment about right-hand support (R2) (c <= x < d)

\[ M_{2\text{LL2}} := \left( \frac{P_1 (L - a)}{L} + \frac{P_2 (L - a - wb)}{L} + \frac{P_3 (wb + e)}{L} + \frac{P_4 e}{L} \right) e \]

Maximum moment resides in region (b <= x < c)

Case 2: Beam Load

Independent Variables: P1, P2, P3, P4, L, a, b, c, d, e, wb
Dependent Variables: R1, R2, Mmax_DL, W

Weight of Beam "W" is a factor of beam length and linear density. \( W = L \times \text{ld} \)

\[ W := L \cdot \text{ld} \]

Self-weight reaction at left-hand side

\[ r_1 = \frac{W \cdot L}{2} \]

Self-weight reaction at right-hand side

\[ r_2 = \frac{W \cdot L}{2} \]

Maximum bending moment due to self-weight

\[ M_{\text{max_DL}} := \frac{W \cdot L}{4} \]

Maximum reactions/moment is due to superposition of the beam load and the applied loads, in region (b <= x < c)

\[ R_{1\text{max}} := R_1 + r_1; R_{2\text{max}} := R_2 + r_2; M_{\text{max}} := M_{1\text{LL2}} + M_{\text{max_DL}}; \]

\[ R_{1\text{max}} := \frac{P_4 (L - e)}{L} + \frac{P_3 (-wb + L - e)}{L} + \frac{P_2 (a + wb)}{L} + \frac{P_1 a}{L} + r_1 \]
Case 4: Runway Horizontal Forces

Independent Variables: LL, Tr, L, a, wb

Dependent Variables: r1_L, r2_L, m1_L, m2_L

\[
\Sigma_{\text{Lateral}} = \frac{0.20}{4} \cdot (LL + Tr);
\]

\[
\Sigma_{\text{Lateral}} := 0.0500 \cdot LL + 0.0500 \cdot Tr
\]

\[
r_1_L := \frac{P4_L}{L} \cdot (a + b + c + d) + \frac{P3_L}{L} \cdot (a + b + c) + \frac{P2_L}{L} \cdot (a + b) + \frac{P1_L}{L} \cdot a; \text{ Lateral reaction at left-hand side}
\]

\[
r_2_L := \frac{P4_L}{L} \cdot (L - e) + \frac{P3_L}{L} \cdot (-wb + L - e) + \frac{P2_L}{L} \cdot (a + wb) + \frac{P1_L}{L} \cdot a; \text{ Lateral reaction at right-hand side}
\]

Distribution of loading for 8 point system is based on moment balance, where outside load (P1_L & P4_L) are:

\[
P1_L := \frac{L \cdot (b + 0.5c)}{8} \cdot \frac{(LL + Tr)}{L}; P4_L := \frac{L \cdot (b + 0.5c)}{8} \cdot \frac{(LL + Tr)}{L};
\]

\[
P1_L := \frac{(0.5L + 0.5a + 0.5e) \frac{1}{8} LL + \frac{1}{8} Tr}{L}
\]

\[
P4_L := \frac{(0.5L + 0.5a + 0.5e) \frac{1}{8} LL + \frac{1}{8} Tr}{L}
\]

Distribution of loading for 8 point system is based on moment balance, where inside load (P2_L & P3_L) are:

\[
P2_L := \frac{L \cdot (0.5c)}{8} \cdot \frac{(LL - Tr)}{L}; P3_L := \frac{L \cdot (0.5c)}{8} \cdot \frac{(LL + Tr)}{L};
\]

\[
P2_L := \frac{(0.5L + 0.5a + 0.5e + 1.0wb) \frac{1}{8} LL + \frac{1}{8} Tr}{L}
\]
\[ P_{3_L} := \frac{(0.5L + 0.5a + 0.5e + 1.0wb) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right)}{L} \]

> \( r_{1_L} \):

\[ \frac{(0.5L + 0.5a + 0.5e) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right)}{L^2} (L - e) \]

\[ + \frac{(0.5L + 0.5a + 0.5e + 1.0wb) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) (-wb + L - e)}{L^2} \]

\[ + \frac{(0.5L + 0.5a + 0.5e + 1.0wb) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) (a + wb)}{L^2} \]

\[ + \frac{(0.5L + 0.5a + 0.5e) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) a}{L^2} \]

> \( r_{2_L} \):

\[ \frac{(0.5L + 0.5a + 0.5e) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right)}{L^2} (L - a) \]

\[ + \frac{(0.5L + 0.5a + 0.5e + 1.0wb) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) (L - a - wb)}{L^2} \]

\[ + \frac{(0.5L + 0.5a + 0.5e + 1.0wb) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) (wb + e)}{L^2} \]

\[ + \frac{(0.5L + 0.5a + 0.5e) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) e}{L^2} \]

> \( m_{l_{L1}} := r_{1_L} \cdot a \); lateral moment about left-hand support (r1_L) \((0 \leq x < a)\)

\[ m_{l_{L1}} := \frac{(0.5L + 0.5a + 0.5e) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) (L - e)}{L^2} \]

\[ + \frac{(0.5L + 0.5a + 0.5e + 1.0wb) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) (-wb + L - e)}{L^2} \]

\[ + \frac{(0.5L + 0.5a + 0.5e + 1.0wb) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) (a + wb)}{L^2} \]

\[ + \frac{(0.5L + 0.5a + 0.5e) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) a}{L^2} \]

> \( m_{l_{L2}} := r_{1_L} \cdot a + (P_{3_L} \cdot b) \); lateral moment about right-hand support (r2_L) \((a \leq x < b)\)
\[ m_{1,l_1} := \left( \frac{0.5 \, L + 0.5 \, a + 0.5 \, e}{4} \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) (L - c) \right) \frac{L}{L^2} + \left( \frac{0.5 \, L + 0.5 \, a + 0.5 \, e + 1.0 \, wb}{4} \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) (-wb + L - e) \right) \frac{L}{L^2} + \left( \frac{0.5 \, L + 0.5 \, a + 0.5 \, e}{4} \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) (a + wb) \right) \frac{L}{L^2} + \left( \frac{0.5 \, L + 0.5 \, a + 0.5 \, e}{4} \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) a \right) \frac{L}{L^2} + \left( \frac{0.5 \, L + 0.5 \, a + 0.5 \, e}{4} \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) (L - e) \right) \frac{L}{L^2} + \left( \frac{0.5 \, L + 0.5 \, a + 0.5 \, e + 1.0 \, wb}{4} \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) (-wb + L - e) \right) \frac{L}{L^2} + \left( \frac{0.5 \, L + 0.5 \, a + 0.5 \, e}{4} \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) (a + wb) \right) \frac{L}{L^2} + \left( \frac{0.5 \, L + 0.5 \, a + 0.5 \, e}{4} \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) a \right) \frac{L}{L^2} - \left( \frac{0.5 \, L + 0.5 \, a + 0.5 \, e}{4} \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) wb \right) \frac{L}{L^2} \]

> \[ m_{2,l_1} := r_2 l_1 e, \text{ maximum lateral moment about right-hand support (r2_L) (d<= x <L)} \]

\[ m_{2,l_1} := \left( \frac{0.5 \, L + 0.5 \, a + 0.5 \, e}{4} \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) (L - a) \right) \frac{L}{L^2} + \left( \frac{0.5 \, L + 0.5 \, a + 0.5 \, e + 1.0 \, wb}{4} \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) (L - a - wb) \right) \frac{L}{L^2} + \left( \frac{0.5 \, L + 0.5 \, a + 0.5 \, e + 1.0 \, wb}{4} \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) (wb + e) \right) \frac{L}{L^2} + \left( \frac{0.5 \, L + 0.5 \, a + 0.5 \, e}{4} \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) e \right) \frac{L}{L^2} \]

> \[ m_{2,l_2} := r_2 l_2 e + (r_2 - p_{4_L}) e, \text{ maximum lateral moment about right-hand support (r2_L) (c<= x < d)} \]
Maximum moment resides in region \((b \leq x < c)\)

**Case 5: Longitudinal Forces (Horizontal)**

Independent Variables: \(P1, P2, L, a, wb\)

Dependent Variables: \(R_{Horz}, r1, r2\)

\[
R_{Horz} := 0.10 \cdot (R1 + R2);
\]

**Case 6: Torsion**

Independent Variables: \(LL, Tr, L, a, wb, h\)

Dependent Variables: \(T1, T2, Ta, Tb, Tc, Tmax\)

\[
T_l := r_{l_l}, h; \text{ torsional reaction at left-hand support due to lateral loading}
\]
\[ T_1 := \left\{ \frac{(0.5L + 0.5a + 0.5e) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) (L - e)}{L^2} \right. \]
\[ + \frac{(0.5L + 0.5a + 0.5e + 1.0wb) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) (wb + L - e)}{L^2} \]
\[ + \frac{(0.5L + 0.5a + 0.5e + 1.0wb) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) (a + wb)}{L^2} \]
\[ + \left. \frac{(0.5L + 0.5a + 0.5e) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) a}{L^2} \right\} h \]

> \( T_2 := r_2L \cdot h; \) torsional reaction at right-hand support due to lateral loading

\[ T_2 := \left\{ \frac{(0.5L + 0.5a + 0.5e) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) (L - a)}{L^2} \right. \]
\[ + \frac{(0.5L + 0.5a + 0.5e + 1.0wb) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) (L - a - wb)}{L^2} \]
\[ + \frac{(0.5L + 0.5a + 0.5e + 1.0wb) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) (wb + e)}{L^2} \]
\[ + \left. \frac{(0.5L + 0.5a + 0.5e) \left( \frac{1}{8} LL + \frac{1}{8} Tr \right) e}{L^2} \right\} h \]

With the following definitions:

a1 = a
b1 = wb
c1 = L - (a + e + 2*wb)
d1 = wb
e1 = e

> \( T_a := T1 - P1L \cdot h; T_b := T1; T_c := 0; T_d := T2; T_e := T2 - P1L \cdot h; \) for 4 point runway loading (for a 8-wheel bridge)
\begin{align*}
T_a & := \left( 0.5 \, L + 0.5 \, a + 0.5 \, e \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) \left( L - e \right) \frac{L^2}{L^2} \\
& + \left( 0.5 \, L + 0.5 \, a + 0.5 \, e + 1.0 \, wb \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) \left( -wb + L - e \right) \frac{L^2}{L^2} \\
& + \left( 0.5 \, L + 0.5 \, a + 0.5 \, e + 1.0 \, wb \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) \left( a + wb \right) \frac{L^2}{L^2} \\
& + \left( 0.5 \, L + 0.5 \, a + 0.5 \, e \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) a \frac{L^2}{L^2} h \\
& - \left( 0.5 \, L + 0.5 \, a + 0.5 \, e \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) h \frac{L^2}{L} \\
T_b & := \left( 0.5 \, L + 0.5 \, a + 0.5 \, e \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) \left( L - e \right) \frac{L^2}{L^2} \\
& + \left( 0.5 \, L + 0.5 \, a + 0.5 \, e + 1.0 \, wb \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) \left( -wb + L - e \right) \frac{L^2}{L^2} \\
& + \left( 0.5 \, L + 0.5 \, a + 0.5 \, e + 1.0 \, wb \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) \left( a + wb \right) \frac{L^2}{L^2} \\
& + \left( 0.5 \, L + 0.5 \, a + 0.5 \, e \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) a \frac{L^2}{L^2} h \\
T_c & := 0 \\
T_d & := \left( 0.5 \, L + 0.5 \, a + 0.5 \, e \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) \left( L - a \right) \frac{L^2}{L^2} \\
& + \left( 0.5 \, L + 0.5 \, a + 0.5 \, e + 1.0 \, wb \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) \left( L - a - wb \right) \frac{L^2}{L^2} \\
& + \left( 0.5 \, L + 0.5 \, a + 0.5 \, e + 1.0 \, wb \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) \left( wb + e \right) \frac{L^2}{L^2} \\
& + \left( 0.5 \, L + 0.5 \, a + 0.5 \, e \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) e \frac{L^2}{L^2} h }

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\[
    T_r := \left( 0.5 \, L + 0.5 \, a + 0.5 \, e \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) \left( L - a \right) \frac{1}{L^2} \\
    + \left( 0.5 \, L + 0.5 \, a + 0.5 \, e + 1.0 \, wb \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) \left( L - a - wb \right) \frac{1}{L^2} \\
    + \left( 0.5 \, L + 0.5 \, a + 0.5 \, e + 1.0 \, wb \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) \left( wb + e \right) \frac{1}{L^2} \\
    + \left( 0.5 \, L + 0.5 \, a + 0.5 \, e \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) \left( e \right) \frac{1}{h} \frac{1}{L^2} \\
    - \left( 0.5 \, L + 0.5 \, a + 0.5 \, e \right) \left( \frac{1}{8} \, LL + \frac{1}{8} \, Tr \right) \left( h \right) \frac{1}{L}
\]

Maximum torsion exists in regions b1 \( (a \leq x < b) \) & d1 \( (c \leq x < d) \)
C. Advanced Runway Reactions, Moments, Stresses, and Design Checks:

For a 4-wheel bridge crane – sample calculation

*All calculations developed and executed in Maple 12 mathematical software package*

\[
digits := 3 : \text{restart}:
\]

\[
\text{with(ExcelTools)}:
\]

\[
> \text{BeamData} \leftarrow \text{Import("BEAMDATA.xls")}
\]

Database for standard structural beam sections, imported into Maple 12 solver

\[
\text{BeamData} := \begin{array}{cccccccc}
1.462 \times 1.13 \text{ Array} \\
\text{Data Type: anything} \\
\text{Storage: rectangular} \\
\text{Order: Fortran_order}
\end{array}
\]

\[
> \text{RailData} \leftarrow \text{Import("RAILDATA.xls")}
\]

Database for standard square-bar rail sections, imported into Maple 12 solver

\[
\text{RailData} := \\
\begin{array}{cccccccc}
\text{"RAIL TYPE"} & \text{"AREA"} & \text{"D"} & \text{"BF"} & \text{"Ixx"} & \text{"Zxx"} & \text{"kxx"} & \text{"Ldensity"} & \text{0} & \text{0} \\
\text{"1.25 SQU"} & 1.56 & 1.25 & 1.25 & 0.2 & 0.32 & 0.36 & 5.38 & 0 & 0 \\
\text{"1.75 SQU"} & 3.06 & 1.75 & 1.75 & 0.78 & 0.89 & 0.51 & 10.55 & 0 & 0 \\
\text{"2.25 SQU"} & 5.06 & 2.25 & 2.25 & 2.14 & 1.9 & 0.65 & 17.44 & 0 & 0 \\
\text{"2.50 SQU"} & 6.25 & 2.5 & 2.5 & 3.26 & 2.61 & 0.72 & 21.53 & 0 & 0 \\
\text{"2.75 SQU"} & 7.56 & 2.75 & 2.75 & 4.77 & 3.47 & 0.79 & 26.05 & 0 & 0
\end{array}
\]

*Global Variables*

Beam Parameters:

0. Beam Name: 

1. Cell Length: (Length of Runway Cell, expressed in feet)

2. Beam Depth: 

   \[
   Dh := \text{BeamData}(\text{Row}_{1} + 1)_{3}:
   \]

3. Base Flange: 

   \[
   Bfb := \text{BeamData}(\text{Row}_{1} + 1)_{4}:
   \]
4. Thickness Flange: \( T_{fh} := BeamData_{(BDrow_{1} + 1)_{5}} \)

5. Thickness Web: \( T_{wb} := BeamData_{(BDrow_{1} + 1)_{6}} \)

6. Linear Density: \( \rho_{b} := BeamData_{(BDrow_{1} + 1)_{13}} \); weight per unit foot of beam length

7. Ixx: \( I_{xxb} := BeamData_{(BDrow_{1} + 1)_{7}} \)

8. Iyy: \( I_{yyb} := BeamData_{(BDrow_{1} + 1)_{10}} \)

9. Zxx: \( Z_{xxb} := BeamData_{(BDrow_{1} + 1)_{8}} \)

10. Zyy: \( Z_{yyb} := BeamData_{(BDrow_{1} + 1)_{11}} \)

11. Area
\[ Area := BeamData_{(BDrow_{1} + 1)_{2}} \]

Rail Parameters:

12. Rail Name: (Image)

13. Rail Length: (Length of Rail, expressed in feet)

14. Rail Depth: \( D_{r} := RailData_{(RDrow_{1} + 1)_{3}} \)

15. Rail Width: \( W_{r} := RailData_{(RDrow_{1} + 1)_{4}} \)

16. Linear Density: \( \rho_{r} := RailData_{(RDrow_{1} + 1)_{8}} \); weight per unit foot of rail length

17. Ixx: \( I_{xxr} := RailData_{(RDrow_{1} + 1)_{5}} \)

18. Iyy: \( I_{yyr} := RailData_{(RDrow_{1} + 1)_{5}} \)

Crane Parameters:

19. Trolley Weight \( w_{o} := 700 \): Weight of crane (hoist and trolley - in pounds)

20. Crane Live Load \( P_{L} := 2500 \): One-quarter of the live load (maximum capacity - in pounds)
\[ p_2 = 2500 : \text{One-quarter of the live load (maximum capacity - in pounds)} \]

21. Crane Wheelbase \[ w_b = 48 : \text{Wheel-base of crane trolley (in inches)} \]

22. Yield Strength of Material \[ y_{fr} = 30000 : (\text{psi}) \]

23. Distance from support (R1) to center of first wheel \[ a = 120 : (\text{in inches}) \]

Case of Single Crane Loading

Live Load Parameter:

\[
eq 1 := R_1 = \left( p_2 + \frac{W_c}{4} \right) \cdot \left( \frac{b}{12 \cdot L_b} \right) + \left( p_1 + \frac{W_c}{4} \right) \cdot \left( \frac{b + w_b}{12 \cdot L_b} \right) + \frac{1}{2} \cdot \left( \frac{p_r \cdot L_r}{b} \right)
\]

\[ R_1 = \frac{107}{6} b + 1271.000000 \]

\[
eq 2 := R_2 = \left( p_1 + \frac{W_c}{4} \right) \cdot \left( \frac{a}{12 \cdot L_b} \right) + \left( p_2 + \frac{W_c}{4} \right) \cdot \left( \frac{a + w_b}{12 \cdot L_b} \right) + \frac{1}{2} \cdot \left( \frac{p_r \cdot L_r}{b} \right)
\]

\[ R_2 = 3411.000000 \]

\[
eq 3 := M_{1_{LL}} = \text{rhs}(\text{eq1}) \cdot a
\]

\[ M_{1_{LL}} = 2140 b + 1.525200000 \times 10^5 \]

\[
eq 4 := M_{2_{LL}} = \text{rhs}(\text{eq2}) \cdot b
\]

\[ M_{2_{LL}} = 3411.000000 b \]

From given layout, \( b = L - (a + \text{wheelbase}) \) or \( b = L - (a + w_b) \). Substituting for \( b \) into reference equations gives:

\[
eq 5 := \text{lks}(\text{eq1}) = \text{subs}(\{ b = 12 \cdot L_r - (a + w_b) \}, \text{rhs}(\text{eq1}))
\]

\[ R_1 = 3625.000000 \]

\[
eq 6 := \text{lks}(\text{eq2}) = \text{subs}(\{ b = 12 \cdot L_r - (a + w_b) \}, \text{rhs}(\text{eq2}))
\]

\[ R_2 = 3411.000000 \]

\[
eq 7 := \text{lks}(\text{eq3}) = \text{subs}(\{ b = 12 \cdot L_r - (a + w_b) \}, \text{rhs}(\text{eq3}))
\]

\[ M_{1_{LL}} = 4.350000000 \times 10^5 \]

\[
eq 8 := \text{lks}(\text{eq4}) = \text{subs}(\{ b = 12 \cdot L_r - (a + w_b) \}, \text{rhs}(\text{eq4}))
\]

\[ M_{2_{LL}} = 4.502520000 \times 10^5 \]

Solving for the regional values of resultant bending shear force (V) yields the following: (units below are in lbf)
Between \((0 \leq x < a)\)

\[ eq11 := V = \text{rhs(eq5)} ; \]

\[ V = 3625.000000 \]

Between \((a \leq x < b)\)

\[ eq12 := V = \text{rhs(eq5)} - P1; \]

\[ V = 1125.000000 \]

Between \((b \leq x < c)\)

\[ eq13 := V = -\text{rhs(eq6)} - P2; \]

\[ V = -911.000000 \]

Between \((c \leq x < L)\)

\[ eq14 := V = -\text{rhs(eq6)}; \]

\[ V = -3411.000000 \]

Solving for the regional values of resultant primary bending moment \((M)\) yields the following: (units below are in lbf-in)

Between \((0 \leq x < a)\)

\[ eq15 := M = \text{rhs(eq5)} \cdot a ; \]

\[ M = 4.35000000 \times 10^5 \]

Between \((a \leq x < b)\)

\[ eq16 := M = \text{rhs(eq5)} \cdot a + \text{rhs(eq5)} \cdot (P1 \cdot (wb)); \]

\[ M = 4.89000000 \times 10^5 \]

Between \((b \leq x < c)\)

\[ eq17 := M = \text{rhs(eq6)} \cdot (12 \cdot Lr - (a + wb)) - \text{rhs(eq6)} \cdot (P2 \cdot (wb)); \]

\[ M = 4.06524000 \times 10^5 \]

Between \((c \leq x < L)\)

\[ eq18 := M = \text{rhs(eq6)} \cdot (12 \cdot Lr - (a + wb)); \]
Solving for the regional values of resultant secondary bending moment (m) yields the following: (units below are in lbf-in)

Between \((0 \leq x < a)\)

\[
eq 19 := m = 0.10 \cdot \text{rhs(eq5)} \cdot a ;
\]

\[m = 43500.00000\]

Between \((a \leq x < c)\)

\[
eq 20 := m = 0.5 \cdot (0.10 \cdot \text{rhs(eq5)} \cdot a + (0.10 \cdot \text{rhs(eq6)}) \cdot ((12 \cdot Lr - (a + wb)))) ;
\]

\[m = 44262.60000\]

Between \((c \leq x < L)\)

\[
eq 21 := m = (0.10 \cdot \text{rhs(eq6)}) \cdot ((12 \cdot Lr - (a + wb))) ;
\]

\[m = 45025.20000\]

Solving for the required values for regional minimum beam cross-section \((A_{\min})\) yields the following: \((\tau = V*A^2)\) (units below are in in^2)

Between \((0 \leq x < a)\)

\[
eq 22 := A_{\min} = \frac{\text{rhs(eq11)}}{0.35 \cdot y_s} ;
\]

\[A_{\min} = 0.3452380951\]

Between \((a \leq x < b)\)

\[
eq 23 := A_{\min} = \frac{\text{rhs(eq12)}}{0.35 \cdot y_s} ;
\]

\[A_{\min} = 0.1071428571\]

Between \((b \leq x < c)\)

\[
eq 24 := A_{\min} = \frac{\text{rhs(eq13)}}{0.35 \cdot y_s} ;
\]

\[A_{\min} = 0.08676190477\]

Between \((c \leq x < L)\)

\[
eq 25 := A_{\min} = \frac{\text{rhs(eq14)}}{0.35 \cdot y_s} ;
\]

\[A_{\min} = 0.3248571428\]

\[M = 4.502520000 \times 10^5\]
Solving for the required values for regional minimum beam second moment of primary cross-section \((I_{xx\_min})\) yields the following: \((\sigma = M*c*I_{xx}^{-1})\) (units below are in in^{4})

Between \((0<= x < a)\)

\[\text{eq26} := \quad I_{xx\_min} = \frac{\text{rhs(eq15)} \cdot Db}{2 \cdot 0.60\text{-ys}}; \quad I_{xx\_min} = 147.2958333\]

Between \((a<= x < b)\)

\[\text{eq27} := \quad I_{xx\_min} = \frac{\text{rhs(eq16)} \cdot Db}{2 \cdot 0.60\text{-ys}}; \quad I_{xx\_min} = 165.5808333\]

Between \((b<= x < c)\)

\[\text{eq28} := \quad I_{xx\_min} = \frac{\text{rhs(eq17)} \cdot Db}{2 \cdot 0.60\text{-ys}}; \quad I_{xx\_min} = 137.6535433\]

Between \((c<= x < L)\)

\[\text{eq29} := \quad I_{xx\_min} = \frac{\text{rhs(eq18)} \cdot Db}{2 \cdot 0.60\text{-ys}}; \quad I_{xx\_min} = 152.4603300\]

Solving for the required values for regional minimum beam second moment of torsional cross-section \((I_{zz\_min})\) yields the following: \((\tau = T*r*I_{zz}^{-1})\) (units below are in in^{4})

Between \((0<= x < a)\)

\[\text{eq30} := \quad I_{zz\_min} = \frac{\text{rhs(eq19)} \cdot (5 \cdot Db + Dr)}{0.35\text{-ys}}; \quad I_{zz\_min} = 34.57214286\]

Between \((a<= x < c)\)

\[\text{eq31} := \quad I_{zz\_min} = \frac{\text{rhs(eq20)} \cdot (5 \cdot Db + Dr)}{0.35\text{-ys}}; \quad I_{zz\_min} = 35.17822828\]

Between \((c<= x < L)\)

\[\text{eq32} := \quad I_{zz\_min} = \frac{\text{rhs(eq21)} \cdot (5 \cdot Db + Dr)}{0.35\text{-ys}}; \quad I_{zz\_min} = 35.78431371\]
Solving for the resultant values for regional primary bending stress ($\sigma$) yields the following: ($\sigma = M^*c^*I_{xx}^*-1$) (units below are in psi)

Between (0<= x < a)

$$eq33 := \sigma = \frac{rhs(eq15) \cdot Db}{2 \cdot I_{xxb}} ;$$

$\sigma = 6729.251270$

Between (a<= x < b)

$$eq34 := \sigma = \frac{rhs(eq16) \cdot Db}{2 \cdot I_{xxb}} ;$$

$\sigma = 7564.606600$

Between (b<= x < c)

$$eq35 := \sigma = \frac{rhs(eq17) \cdot Db}{2 \cdot I_{xxb}} ;$$

$\sigma = 6288.740560$

Between (c<= x < L)

$$eq36 := \sigma = \frac{rhs(eq18) \cdot Db}{2 \cdot I_{xxb}} ;$$

$\sigma = 6965.192740$

Solving for the resultant values for regional primary bending stress ($\tau$) yields the following: ($\tau = V^*A^*-1$) (units below are in psi)

Between (0<= x < a)

$$eq37 := \tau = \frac{rhs(eq11)}{Area} ;$$

$\tau = 246.5986395$

Between (a<= x < b)

$$eq38 := \tau = \frac{rhs(eq12)}{Area} ;$$

$\tau = 76.53061224$

Between (b<= x < c)

$$eq39 := \tau = -\frac{rhs(eq13)}{Area} ;$$

$\tau = 61.97278912$

Between (c<= x < L)
Solving for the required values for regional torsional stress ($\tau$) yields the following: ($\tau = T*\tau*Izz^{-1}$) (units below are in psi)

Between $0\leq x < a$

$$eq41 := \tau = \frac{rhs(eq19) \cdot (5 \cdot Db + Dr)}{(Izb + Iyb)};$$

$\tau = 806.1459027$

Between $a\leq x < c$

$$eq42 := \tau = \frac{rhs(eq20) \cdot (5 \cdot Db + Dr)}{(Izb + Iyb)};$$

$\tau = 820.2784744$

Between $c\leq x < L$

$$eq43 := \tau = \frac{rhs(eq21) \cdot (5 \cdot Db + Dr)}{(Izb + Iyb)};$$

$\tau = 834.4110460$

Design Stress Check for Combined Stressed: (if less than unity for each, then design choices are acceptable)

Verification in region $0\leq x < a$

$$eq44 := \frac{rhs(eq33)}{0.6 \cdot ys} + \frac{rhs(eq37)}{0.35 \cdot ys} + \frac{rhs(eq41)}{0.35 \cdot ys};$$

$0.4741086778$

Verification in region $a\leq x < b$

$$eq45 := \frac{rhs(eq34)}{0.6 \cdot ys} + \frac{rhs(eq38)}{0.35 \cdot ys} + \frac{rhs(eq42)}{0.35 \cdot ys};$$

$0.505663114$

Verification in region $b\leq x < c$

$$eq46 := \frac{rhs(eq35)}{0.6 \cdot ys} + \frac{rhs(eq39)}{0.35 \cdot ys} + \frac{rhs(eq42)}{0.35 \cdot ys};$$

$0.4333984054$

Verification in region $c\leq x < L$

$$eq47 := \frac{rhs(eq36)}{0.6 \cdot ys} + \frac{rhs(eq40)}{0.35 \cdot ys} + \frac{rhs(eq43)}{0.35 \cdot ys};$$
Design Check for Sideways Web Buckling: \((\text{ratio} = (h/\text{tw})/(L/\text{bf}))\), where \(h = \text{Db} - 2*\text{tf}\)

\[
\text{ratio} := \frac{\left(\frac{(\text{Db} - 2*\text{Tf})}{\text{Twb}}\right)}{\left(\frac{Lb}{Bfb}\right)}
\]

Use the following equation for the design check for sections for web sections with the ratio \(< 2.3\), and that are restrained against rotation. The maximum permissible wheel-load to prevent web buckling is:

\[
eq 48 := \text{evalb}(2.3 > \text{ratio})
\]

\(\text{false}\)

If the resultant is "true", use the design check below. If it is "false", buckling will not occur.

\[
eq 49 := \Phi Rn = 0.85 \cdot \left(\frac{\text{Cr} \cdot \text{Twb}^{3} \cdot \text{Tf}}{(\text{Db} - 2 \cdot \text{Tf})^{2}} \cdot \left[1 + 0.4 \cdot \left(\frac{\left(\frac{(\text{Db} - 2 \cdot \text{Tf})}{\text{Twb}}\right)}{\left(\frac{Lb}{Bfb}\right)}\right)^{\frac{3}{2}}\right]\right)
\]

\[
\Phi Rn = 0.08038042620 \cdot \text{Cr}
\]

If the moment at the load divided by yield moment \((\text{My} = \text{Ys} \cdot \text{Zxx})\) is less than one, use \(\text{Cr} = 960,000 \text{ psi}\); if the moment ratio is greater than or equal to one, use \(\text{Cr} = 480,000 \text{ psi}\)

\[
eq 50 := \text{My} = \text{ys} \cdot \text{Zxx}; \quad \text{eq}51 := r_m = \frac{\text{rhs(\text{eq}7)}}{\text{rhs(\text{eq}50)}};
\]

\[
\text{My} = 1.9410000 \times 10^8
\]

\[
r_m = 0.2241112828
\]

If \((\text{rhs(eq51)} < 1)\) then \(\text{subs}(\text{Cr} = 960000, \text{rhs(eq49)})\) else \(\text{subs}(\text{Cr} = 480000, \text{rhs(eq49)})\) end if

\[
77165.20915
\]

This is the maximum permissible wheel-load for sections restrained against rotation
Use the following equation for the design check for sections for web sections with the ratio < 1.7, and that are not restrained against rotation. The maximum permissible wheel-load to prevent web buckling is:

\[ eq52 := \text{evalb}(1.7 > \text{ratio}) \]

\[ \text{false} \]

If the resultant is "true", use the design check below. If it is "false", buckling will not occur.

\[ eq53 := \Phi r_n = 0.85 \cdot \left( \frac{Cr \cdot Twb}{(Db - 2 \cdot Tfb)^3} \right)^{0.4} \left( \left( \frac{Db - 2 \cdot Tfb}{Twb} \right) \left( \frac{Lb}{Bfb} \right) \right)^{1.5} \]

\[ \Phi r_n = 0.08014892431 \, Cr \]

If the moment at the load divided by yield moment (\( My = Ys \cdot Zxx \)) is less than one, use \( Cr = 960,000 \) psi; if the moment ratio is greater than or equal to one, use \( Cr = 480,000 \) psi

\[ eq54 := My = ys \cdot Zxx; eq55 := r_m = \frac{\text{rhs}(eq7)}{\text{rhs}(eq54)}; \]

\[ My = 1.9410000 \times 10^8 \]
\[ r_m = 0.2241112828 \]

\[ \text{if} \ (\text{rhs}(eq55) < 1) \ \text{then} \ (\text{subs}(Cr = 960000, \text{rhs}(eq53))) \ \text{else} \ (\text{subs}(Cr = 480000, \text{rhs}(eq53))) \ \text{end if} \]

\[ 76942.96734 \]

This is the maximum permissible wheel-load for sections not restrained against rotation.
D. Advanced Runway Reactions, Moments, Stresses, and Design Checks:

For an 8-wheel bridge crane – sample calculation

*All calculations developed and executed in Maple 12 mathematical software package*

\[
digits := 3 : \text{restart} : \\
\text{with(ExcelTools)} : \\
\]

Database for standard structural beam sections, imported into Maple 12 solver

\[
> \text{BeamData} := \text{Import("BEAMDATA.xls")} \\
\text{BeamData} := \begin{array}{c}
\text{Beam Data: anything} \\
\text{Storage: rectangular} \\
\text{Order: Fortran order} \\
\end{array} \\
\]

\[
> \text{RailData} := \text{Import("RAILDATA.xls")} \\
\text{RailData} := \\
\begin{array}{c}
\text{"RAIL TYPE" "AREA" "D" "BF" "lxx" "Zxx" "kxx" "density" } 0 \ 0. \\
\text{"1.25 SQU" 1.56 1.25 1.25 0.2 0.36 0.36 5.38 0 \ 0.} \\
\text{"1.75 SQU" 3.06 1.75 1.75 0.78 0.89 0.89 10.55 0 \ 0.} \\
\text{"2.25 SQU" 5.06 2.25 2.25 2.14 1.9 1.9 17.44 0 \ 0.} \\
\text{"2.50 SQU" 6.25 2.5 2.5 3.26 2.61 2.61 21.53 0 \ 0.} \\
\text{"2.75 SQU" 7.56 2.75 2.75 4.77 3.47 3.47 26.05 0 \ 0.} \\
\end{array} \\
\]

*Global Variables*

Beam Parameters:

0. Beam Name: \{111, W12x50\}

1. Cell Length: \[
(\text{Length of Runway Cell, expressed in feet})
\]

2. Beam Depth: \[
Db := \text{BeamData} \left( \text{BDrow} + 1 \right) .3 \\
\]

3. Base Flange: \[
Bfb := \text{BeamData} \left( \text{BDrow} + 1 \right) .4 \\
\]
4. Thickness Flange: \( T_{fh} := BeamData(BDrow_{1} + 1).5 \):

5. Thickness Web: \( T_{wb} := BeamData(BDrow_{1} + 1).6 \) :

6. Linear Density: \( \rho_{b} := BeamData(BDrow_{1} + 1).13 \) : weight per unit foot of beam length

7. Ixx: \( Ix_{xh} := BeamData(BDrow_{1} + 1).7 \):

8. Iyy: \( Iy_{yb} := BeamData(BDrow_{1} + 1).10 \):

9. Zxx: \( Zx_{xb} := BeamData(BDrow_{1} + 1).8 \):

10. Zyy: \( Zy_{yb} := BeamData(BDrow_{1} + 1).11 \):

11. Area \( Area := BeamData(BDrow_{1} + 1).2 \):

Rail Parameters:

12. Rail Name: 

13. Rail Length: (Length of Rail, expressed in feet)

14. Rail Depth: \( Dr := RailData(RDrow_{1} + 1).3 \):

15. Rail Width: \( Wr := RailData(RDrow_{1} + 1).4 \):

16. Linear Density: \( \rho_{r} := RailData(RDrow_{1} + 1).8 \) : weight per unit foot of rail length

17. Ixx: \( Ix_{xr} := RailData(RDrow_{1} + 1).5 \):

18. Iyy: \( Iy_{yr} := RailData(RDrow_{1} + 1).5 \):

Crane Parameters:

19. Trolley Weight \( w_{c} := 700 \) : Weight of crane (hoist and trolley - in pounds)

20. Crane Live Load \( P := 2500 \) : The equivalent center live-load (from 2 pt moment balance - in pounds)

21. Crane Wheelbase \( wb := 60 \) : Wheel-base of crane trolley (in inches)
22. Yield Strength of Material \( y_s := 30000 \) (psi)

23. Distance from support (R1) to center of first wheel \( a := 75 \) (in inches)

24. Distance between near wheels \( \text{dist} := 30 \):

Case of Single Crane Loading

Live Load Parameter:

\[
eq 1 \quad R_1 = \left( \frac{P_1 (b + c + d + e)}{12 \cdot L_b} \right) + \left( \frac{P_2 (c + d + e)}{12 \cdot L_b} \right) + \left( \frac{P_3 (d + e)}{12 \cdot L_b} \right) + \frac{1}{2} \left( \left( \text{pr} \cdot L_r \right) + \left( \text{pb} \cdot L_b \right) \right)
\]

\[
R_1 = \frac{1}{300} \cdot P_1 (b + c + d + e) + \frac{1}{300} \cdot P_2 (c + d + e) + \frac{1}{300} \cdot P_3 (d + e) + \frac{1}{300} \cdot P_4 e + 843.0000000
\]

\[
eq 2 \quad R_2 = \left( \frac{P_4 (a + b + c + d)}{12 \cdot L_b} \right) + \left( \frac{P_3 (a + b + c)}{12 \cdot L_b} \right) + \left( \frac{P_2 (a + b)}{12 \cdot L_b} \right) + \frac{1}{2} \left( \left( \text{pr} \cdot L_r \right) + \left( \text{pb} \cdot L_b \right) \right)
\]

\[
R_2 = \frac{1}{300} \cdot P_4 (75 + b + c + d) + \frac{1}{300} \cdot P_3 (75 + b + c) + \frac{1}{300} \cdot P_2 (75 + b) + \frac{1}{4} \cdot P_1 + 843.0000000
\]

\[
eq 3 \quad M_{1LL} = \text{rhs}(\eq 1) \cdot a
\]

\[
M_{1LL} = \frac{1}{4} \cdot P_1 (b + c + d + e) + \frac{1}{4} \cdot P_2 (c + d + e) + \frac{1}{4} \cdot P_3 (d + e) + \frac{1}{4} \cdot P_4 e + 63225.000000
\]

\[
eq 4 \quad M_{2LL} = \text{rhs}(\eq 2) \cdot b
\]

\[
M_{2LL} = \left( \frac{1}{300} \cdot P_4 (75 + b + c + d) + \frac{1}{300} \cdot P_3 (75 + b + c) + \frac{1}{300} \cdot P_2 (75 + b) + \frac{1}{4} \cdot P_1 + 843.0000000 \right) b
\]

From given layout, \( b = d = \text{wheelbase} \) or \( b = d = \text{wb} \); \( c = \text{distance between near wheels (dist)} \), \( e = 12 \cdot L_b \cdot (a+b+c+d) = 12 \cdot L_b \cdot (a+2 \cdot \text{wb} + \text{dist}) \). Substituting for \( b, c, d \) & \( e \) into reference equations gives:

\[
eq 5 \quad \text{lhs}(\eq 1) = \text{subs} \left( \left( b = \text{wb}, c = \text{dist}, d = \text{wb} \right) \right), \left( a = 12 \cdot L_r \right) \}
\]

\[
\text{rhs}(\eq 1))
\]

\[
R_1 = \frac{3}{4} \cdot P_1 + \frac{11}{20} \cdot P_2 + \frac{9}{20} \cdot P_3 + \frac{1}{4} \cdot P_4 + 843.000000000
\]

\[
eq 6 \quad \text{lhs}(\eq 2) = \text{subs} \left( \left( b = \text{wb}, c = \text{dist}, d = \text{wb} \right) \right), \left( a = 12 \cdot L_r \right) \}
\]

\[
\text{rhs}(\eq 2))
\]
Solving for the moment balance shows that the load on the outside wheels (P1 & P4) are as follows:

\[
R2 = \frac{3}{4} P4 + \frac{11}{20} P3 + \frac{9}{20} P2 + \frac{1}{4} P1 + 843.0000000
\]

\[
eq 8 \Rightarrow \text{lhs}(eq4) = \text{subs}(\{b = \text{wb}, c = \text{dist}, d = \text{wb}, e = 12 \cdot Lr - (a + 2 \cdot \text{wb} + \text{dist})\},
\]

\[
\text{rhs}(eq4))
\]

\[
M_{1LL} = \frac{225}{4} P1 + \frac{165}{4} P2 + \frac{135}{4} P3 + \frac{75}{4} P4 + 63225.00000
\]

\[
eq 8 \Rightarrow \text{lhs}(eq4) = \text{subs}(\{b = \text{wb}, c = \text{dist}, d = \text{wb}, e = 12 \cdot Lr - (a + 2 \cdot \text{wb} + \text{dist})\},
\]

\[
\text{rhs}(eq4))
\]

\[
M_{2LL} = 45 P4 + 33 P3 + 27 P2 + 15 P1 + 50580.00000
\]

Solving for the moment balance shows that the load on the inside wheels (P2 & P3) are as follows:

\[
eq 9 \Rightarrow P1 = \frac{1}{4} P \cdot \left( \frac{5 \cdot \text{dist}}{5 \cdot \text{dist} + \text{wb}} \right); \quad eq10 \Rightarrow P4 = \frac{1}{4} P \cdot \left( \frac{5 \cdot \text{dist}}{5 \cdot \text{dist} + \text{wb}} \right);
\]

\[
P1 = 125.0000000
\]

\[
P4 = 125.0000000
\]

Solving for the moment balance shows that the load on the inside wheels (P2 & P3) are as follows:

\[
eq 11 \Rightarrow P2 = \frac{1}{4} P \cdot \left( \frac{\text{wb}}{5 \cdot \text{dist} + \text{wb}} \right); \quad eq12 \Rightarrow P3 = \frac{1}{4} P \cdot \left( \frac{\text{wb}}{5 \cdot \text{dist} + \text{wb}} \right);
\]

\[
P2 = 500.0000000
\]

\[
P3 = 500.0000000
\]

Substituting the corrected wheel-loads into the reaction & moment equations yields the following:

\[
eq 13 \Rightarrow \text{lhs}(eq1) = \text{subs}(\{P1 = \text{rhs}(eq9), P2 = \text{rhs}(eq11), P3 = \text{rhs}(eq12), P4 = \text{rhs}(eq10)\}, \text{rhs}(eq5))
\]

\[
R1 = 1468.0000000
\]

\[
eq 14 \Rightarrow \text{lhs}(eq2) = \text{subs}(\{P1 = \text{rhs}(eq9), P2 = \text{rhs}(eq11), P3 = \text{rhs}(eq12), P4 = \text{rhs}(eq10)\}, \text{rhs}(eq6))
\]

\[
R2 = 1468.0000000
\]

\[
eq 15 \Rightarrow \text{lhs}(eq3) = \text{subs}(\{P1 = \text{rhs}(eq9), P2 = \text{rhs}(eq11), P3 = \text{rhs}(eq12), P4 = \text{rhs}(eq10)\}, \text{rhs}(eq7))
\]

\[
M_{1LL} = 1.101000000 \times 10^5
\]

\[
eq 16 \Rightarrow \text{lhs}(eq4) = \text{subs}(\{P1 = \text{rhs}(eq9), P2 = \text{rhs}(eq11), P3 = \text{rhs}(eq12), P4 = \text{rhs}(eq10)\}, \text{rhs}(eq8))
\]

\[
M_{2LL} = 88080.00000
\]
Solving for the regional values of resultant bending shear force ($V$) yields the following: (units below are in lbf)

Between $(0 \leq x < a)$

\[ V = rhs(eq13); \]

\[ V = 1468.00000 \]

Between $(a \leq x < b)$

\[ V = rhs(eq13) - rhs(eq9); \]

\[ V = 1343.00000 \]

Between $(b \leq x < c)$

\[ V = rhs(eq13) - rhs(eq9) - rhs(eq11); \]

\[ V = 843.00000 \]

Between $(c \leq x < d)$

\[ V = -(rhs(eq14) - rhs(eq10) - rhs(eq12)); \]

\[ V = -843.00000 \]

Between $(d \leq x < e)$

\[ V = -(rhs(eq14) - rhs(eq10)); \]

\[ V = -1343.00000 \]

Between $(e \leq x < L)$

\[ V = -(rhs(eq14)); \]

\[ V = -1468.00000 \]

Solving for the regional values of resultant primary bending moment ($M$) yields the following: (units below are in lbf-in)

Between $(0 \leq x < a)$

\[ M = rhs(eq13) \cdot a; \]

\[ M = 1.101000000 \times 10^5 \]

Between $(a \leq x < b)$

\[ M = (rhs(eq13) \cdot a) + ((rhs(eq13) - rhs(eq9)) \cdot (wh)); \]

\[ M = 1.906800000 \times 10^5 \]
Between \( b \leq x < c \)

\[
eq 25 := M = (\text{rhs}(eq13) \cdot a) + ((\text{rhs}(eq13) - \text{rhs}(eq9)) \cdot (wb)) + \left(\text{rhs}(eq13) - \text{rhs}(eq9) - \text{rhs}(eq11)\right) \cdot \left(\frac{\text{dist}}{2}\right);
\]

\[ M = 2.033250000 \times 10^5 \]

Between \( c \leq x < d \)

\[
eq 26 := M = (\text{rhs}(eq14) \cdot (12L - (a + 2 \cdot wb + \text{dist}))) + \left((\text{rhs}(eq14) - \text{rhs}(eq10)) \cdot (wb)\right) + \left((\text{rhs}(eq14) - \text{rhs}(eq10) - \text{rhs}(eq12))\right) \cdot \left(\frac{\text{dist}}{2}\right);
\]

\[ M = 2.033250000 \times 10^5 \]

Between \( d \leq x < e \)

\[
eq 27 := M = (\text{rhs}(eq14) \cdot (12L - (a + 2 \cdot wb + \text{dist}))) + \left((\text{rhs}(eq14) - \text{rhs}(eq10)) \cdot (wb)\right);
\]

\[ M = 1.906800000 \times 10^5 \]

Between \( e \leq x < L \)

\[
eq 28 := M = (\text{rhs}(eq14) \cdot (12L - (a + 2 \cdot wb + \text{dist})));
\]

\[ M = 1.101000000 \times 10^5 \]

Solving for the regional values of resultant secondary bending moment \( M \) yields the following: (units below are in lbf-in)

Between \( 0 \leq x < a \)

\[
eq 29 := m = 0.10 \cdot \text{rhs}(eq13) \cdot a;
\]

\[ m = 11010.00000 \]

Between \( a \leq x < b \)

\[
eq 30 := m = 0.10 \cdot \text{rhs}(eq13) \cdot a + 0.10 \cdot (\text{rhs}(eq13) - \text{rhs}(eq9)) \cdot (wb);
\]

\[ m = 19068.00000 \]

Between \( b \leq x < d \)

\[
eq 31 := m = 0.10 \cdot \text{rhs}(eq13) \cdot a + 0.10 \cdot (\text{rhs}(eq13) - \text{rhs}(eq9)) \cdot (wb) + 0.10 \cdot ((\text{rhs}(eq13) - \text{rhs}(eq9) - \text{rhs}(eq11)) \cdot (5 \cdot \text{dist}) + 0.10 \cdot (\text{rhs}(eq14) - \text{rhs}(eq10) - \text{rhs}(eq12)) \cdot (5 \cdot \text{dist});
\]

\[ m = 21597.00000 \]

Between \( d \leq x < e \)
Between \((e \leq x < L)\)

\[
eq 32 := m = 0.10 \cdot \text{rhs(eq14)} \cdot (12 \cdot Lr - (a + 2 \cdot wb + \text{dist})) + 0.10 \cdot \text{rhs(eq14)} - \text{rhs(eq10)} \cdot wb;
\]

\[m = 19068.00000\]

Solving for the required values for regional minimum beam cross-section \((A_{\text{min}})\) yields the following: \((\tau = V^*A^*-1)\) (units below are in \(\text{in}^2\))

Between \((0 \leq x < a)\)

\[
eq 34 := A_{\text{min}} = \frac{\text{rhs(eq17)}}{0.35 \cdot \text{ys}};
\]

\[A_{\text{min}} = 0.1398095238\]

Between \((a \leq x < b)\)

\[
eq 35 := A_{\text{min}} = \frac{\text{rhs(eq18)}}{0.35 \cdot \text{ys}};
\]

\[A_{\text{min}} = 0.1279047619\]

Between \((b \leq x < c)\)

\[
eq 36 := A_{\text{min}} = \left( \frac{\text{rhs(eq19)}}{0.35 \cdot \text{ys}} \right);
\]

\[A_{\text{min}} = 0.08028571428\]

Between \((c \leq x < d)\)

\[
eq 37 := A_{\text{min}} = -\left( \frac{\text{rhs(eq20)}}{0.35 \cdot \text{ys}} \right);
\]

\[A_{\text{min}} = 0.08028571428\]

Between \((d \leq x < e)\)

\[
eq 38 := A_{\text{min}} = -\left( \frac{\text{rhs(eq21)}}{0.35 \cdot \text{ys}} \right);
\]

\[A_{\text{min}} = 0.1279047619\]
Between \((e \leq x < L)\)

\[
eq 39 \quad A_{\text{min}} = - \left( \frac{\text{rhs}(eq22)}{0.35 \text{ in}^2} \right);
\]

\[A_{\text{min}} = 0.1398095238\]

Solving for the required values for regional minimum beam second moment of primary cross-section \((I_{xx_{\text{min}}})\) yields the following: \((\sigma = M^*c*I_{xx}^{-1})\) (units below are in \(\text{in}^4\))

Between \((0 \leq x < a)\)

\[
eq 40 \quad I_{xx_{\text{min}}} = \frac{\text{rhs}(eq23)\cdot Db}{2 \cdot 0.60 \text{ in}^2};
\]

\[I_{xx_{\text{min}}} = 37.28108333\]

Between \((a \leq x < b)\)

\[
eq 41 \quad I_{xx_{\text{min}}} = \frac{\text{rhs}(eq24)\cdot Db}{2 \cdot 0.60 \text{ in}^2};
\]

\[I_{xx_{\text{min}}} = 64.56636666\]

Between \((b \leq x < c)\)

\[
eq 42 \quad I_{xx_{\text{min}}} = \left( \frac{\text{rhs}(eq25)\cdot Db}{2 \cdot 0.60 \text{ in}^2} \right);
\]

\[I_{xx_{\text{min}}} = 68.84810416\]

Between \((c \leq x < d)\)

\[
eq 43 \quad I_{xx_{\text{min}}} = \left( \frac{\text{rhs}(eq26)\cdot Db}{2 \cdot 0.60 \text{ in}^2} \right);
\]

\[I_{xx_{\text{min}}} = 68.84810416\]

Between \((d \leq x < e)\)

\[
eq 44 \quad I_{xx_{\text{min}}} = \left( \frac{\text{rhs}(eq27)\cdot Db}{2 \cdot 0.60 \text{ in}^2} \right);
\]

\[I_{xx_{\text{min}}} = 64.56636666\]

Between \((e \leq x < L)\)

\[
eq 45 \quad I_{xx_{\text{min}}} = \left( \frac{\text{rhs}(eq28)\cdot Db}{2 \cdot 0.60 \text{ in}^2} \right);
\]
Solving for the required values for regional minimum beam second moment of torsional cross-section \((I_{zz_{\text{min}}})\) yields the following: \((\tau = T^*r^*I_{zz}^{-1})\) (units below are in \(\text{in}^4\))

Between \((0 <= x < a)\)

\[
\text{eq46} := I_{zz_{\text{min}}} = \frac{\text{rhs(eq29)} \cdot (5 \cdot Db + Dr)}{0.35 \cdot \text{ys}};
\]

\(I_{zz_{\text{min}}} = 37.28108333\)

Between \((a <= x < b)\)

\[
\text{eq47} := I_{zz_{\text{min}}} = \frac{\text{rhs(eq30)} \cdot (5 \cdot Db + Dr)}{0.35 \cdot \text{ys}};
\]

\(I_{zz_{\text{min}}} = 8.750328571\)

Between \((b <= x < d)\)

\[
\text{eq48} := I_{zz_{\text{min}}} = \left\{ \frac{\text{rhs(eq31)} \cdot (5 \cdot Db + Dr)}{0.35 \cdot \text{ys}} \right\};
\]

\(I_{zz_{\text{min}}} = 15.15452000\)

Between \((d <= x < e)\)

\[
\text{eq49} := I_{zz_{\text{min}}} = \left\{ \frac{\text{rhs(eq32)} \cdot (5 \cdot Db + Dr)}{0.35 \cdot \text{ys}} \right\};
\]

\(I_{zz_{\text{min}}} = 17.1647286\)

Between \((e <= x < L)\)

\[
\text{eq50} := I_{zz_{\text{min}}} = \left\{ \frac{\text{rhs(eq33)} \cdot (5 \cdot Db + Dr)}{0.35 \cdot \text{ys}} \right\};
\]

\(I_{zz_{\text{min}}} = 8.750328571\)

Solving for the resultant values for regional primary bending stress \((\sigma)\) yields the following: \((\sigma = M^*c^*I_{xx}^{-1})\) (units below are in \(\text{psi}\))

Between \((0 <= x < a)\)

\[
\text{eq51} := \sigma = \frac{\text{rhs(eq23)} \cdot Db}{2 \cdot I_{xxb}};
\]

\(\sigma = 1703.196700\)

Between \((a <= x < b)\)

\[
\text{eq52} := \sigma = \frac{\text{rhs(eq24)} \cdot Db}{2 \cdot I_{xxb}};
\]

\(\sigma = 2949.732488\)
Between \((b \leq x < c)\)

\[
\text{eq53} := \sigma = \frac{\text{rhs(eq25)} \cdot D_h}{2 \cdot I_{cxb}}.
\]

\(\sigma = 3145.344860\)

Between \((c \leq x < d)\)

\[
\text{eq54} := \sigma = \frac{\text{rhs(eq26)} \cdot D_h}{2 \cdot I_{cxb}}.
\]

\(\sigma = 3145.344860\)

Between \((d \leq x < e)\)

\[
\text{eq55} := \sigma = \frac{\text{rhs(eq27)} \cdot D_h}{2 \cdot I_{cxb}}.
\]

\(\sigma = 2949.732488\)

Between \((e \leq x < L)\)

\[
\text{eq56} := \sigma = \frac{\text{rhs(eq28)} \cdot D_h}{2 \cdot I_{cxb}}.
\]

\(\sigma = 1703.196700\)

Solving for the resultant values for regional primary bending stress \((\tau)\) yields the following: \((\tau = V \cdot A^{-1})\) (units below are in psi)

Between \((0 \leq x < a)\)

\[
\text{eq57} := \tau = \frac{\text{rhs(eq17)}}{\text{Area}};
\]

\(\tau = 99.86394558\)

Between \((a \leq x < b)\)

\[
\text{eq58} := \tau = \frac{\text{rhs(eq18)}}{\text{Area}};
\]

\(\tau = 91.36054422\)

Between \((b \leq x < c)\)

\[
\text{eq59} := \tau = \frac{\text{rhs(eq19)}}{\text{Area}};
\]

\(\tau = 57.34693878\)

Between \((c \leq x < d)\)

\[
\text{eq60} := \tau = \frac{\text{rhs(eq20)}}{\text{Area}};
\]

\(\tau = 57.34693878\)
Between \((d \leq x < e)\)

eq61 \(\Rightarrow \tau = -\left(\frac{\text{rhs(eq21)}}{\text{Area}}\right)\);

\(\tau = 91.36054422\)

Between \((e \leq x < L)\)

\(\tau = 99.86394558\)

Solving for the required values for regional torsional stress \((\tau)\) yields the following: \((\tau = T^\ast r^*Izz^{-1})\) (units below are in psi)

Between \((0 \leq x < a)\)

\(\tau = 204.0383078\)

Between \((a \leq x < b)\)

\(\tau = 353.3698867\)

Between \((b \leq x < d)\)

\(\tau = 400.2375416\)

Between \((d \leq x < e)\)

\(\tau = 353.3698867\)

Between \((e \leq x < L)\)

\(\tau = 204.0383078\)

Design Stress Check for Combined Stressed: (if less than unity for each, then design choices are acceptable)

Verification in region \((0 \leq x < a)\)
Verification in region \((a \leq x < b)\)

\[
eq 68 := \frac{\text{rhs(eq51)}}{0.6\cdot\text{ys}} + \frac{\text{rhs(eq57)}}{0.35\cdot\text{ys}} + \frac{\text{rhs(eq63)}}{0.35\cdot\text{ys}};
\]

0.1235651107

Verification in region \((b \leq x < c)\)

\[
eq 69 := \frac{\text{rhs(eq52)}}{0.6\cdot\text{ys}} + \frac{\text{rhs(eq58)}}{0.35\cdot\text{ys}} + \frac{\text{rhs(eq64)}}{0.35\cdot\text{ys}};
\]

0.2062293062

Verification in region \((c \leq x < d)\)

\[
eq 70 := \frac{\text{rhs(eq53)}}{0.6\cdot\text{ys}} + \frac{\text{rhs(eq59)}}{0.35\cdot\text{ys}} + \frac{\text{rhs(eq65)}}{0.35\cdot\text{ys}};
\]

0.2183208555

Verification in region \((d \leq x < e)\)

\[
eq 71 := \frac{\text{rhs(eq54)}}{0.6\cdot\text{ys}} + \frac{\text{rhs(eq60)}}{0.35\cdot\text{ys}} + \frac{\text{rhs(eq66)}}{0.35\cdot\text{ys}};
\]

0.2183208555

Verification in region \((e \leq x < L)\)

\[
eq 72 := \frac{\text{rhs(eq55)}}{0.6\cdot\text{ys}} + \frac{\text{rhs(eq61)}}{0.35\cdot\text{ys}} + \frac{\text{rhs(eq66)}}{0.35\cdot\text{ys}};
\]

0.2062293062

Verification in region \((e \leq x < L)\)

\[
eq 73 := \frac{\text{rhs(eq56)}}{0.6\cdot\text{ys}} + \frac{\text{rhs(eq62)}}{0.35\cdot\text{ys}} + \frac{\text{rhs(eq67)}}{0.35\cdot\text{ys}};
\]

0.1235651107

Design Check for Sideways Web Buckling: \((\text{ratio} = (h/tw)/(L/bf))\), where \(h = \text{Db} - 2*\text{tf}\)

\[
\text{ratio} := \frac{(\text{ Db} - 2\cdot \text{ tf} )}{\text{ Twb}} \left( \frac{\text{ Lf}}{\text{ Lbf} } \right)
\]

9.530032437

Use the following equation for the design check for sections for web sections with the ratio < 2.3, and that are restrained against rotation. The maximum permissible wheel-load to prevent web buckling is:

\[
eq 74 := \text{eval}(2.3 > \text{ ratio})
\]

false

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If the resultant is "true", use the design check below. If it is "false", buckling will not occur.

\[
\Phi_{Ra} = 0.85 \cdot \left( \frac{Cr \cdot Twb^3 \cdot Tfb}{(Db - 2 \cdot Tfb)^2} \right) \left( 1 + 0.4 \cdot \left( \frac{(Db - 2 \cdot Tfb)}{Twb} \right) \left( \frac{Lb}{Bfb} \right) \right) \]

\[
\Phi_{Ra} = 0.08038042620 \cdot Cr
\]

If the moment at the load divided by yield moment \((My = Ys \cdot Zxx)\) is less than one, use \(Cr = 960,000\) psi; if the moment ratio is greater than or equal to one, use \(Cr=480,000\) psi

\[
eq 76 := My = Ys \cdot Zxx; \quad eq77 := r_m = \frac{rhs(eq25)}{rhs(eq76)};
\]

\[
My = 1.9410000 \times 10^8
\]

\[
r_m = 0.1047527048
\]

if \((rhs(eq77) < 1)\) then \((subs(Cr = 960000, rhs(eq75)))\) else \((subs(Cr = 480000, rhs(eq75)))\) end if

77165.20915

This is the maximum permissible wheel-load for sections restrained against rotation

Use the following equation for the design check for sections for web sections with the ratio < 1.7, and that are not restrained against rotation. The maximum permissible wheel-load to prevent web buckling is:

\[
eq 78 := evalb(1.7 > ratio)
\]

false

If the resultant is "true", use the design check below. If it is "false", buckling will not occur.

\[
eq 79 := \Phi_{Ra} = 0.85 \cdot \left( \frac{Cr \cdot Twb^3 \cdot Tfb}{(Db - 2 \cdot Tfb)^2} \right) \left( 0.4 \cdot \left( \frac{(Db - 2 \cdot Tfb)}{Twb} \right) \left( \frac{Lb}{Bfb} \right) \right) \]

\[
\Phi_{Ra} = 0.08014892431 \cdot Cr
\]

If the moment at the load divided by yield moment \((My = Ys \cdot Zxx)\) is less than one, use \(Cr = 960,000\) psi; if the moment ratio is greater than or equal to one, use \(Cr=480,000\) psi
This is the maximum permissible wheel-load for sections not restrained against rotation

\[ eq80 := My = ys \cdot Zcxd; \quad eq81 := r_m = \frac{rhs(eq25)}{rhs(eq80)}; \]

\[ My = 1.9410000 \times 10^8 \]

\[ r_m = 0.1047527048 \]

\[
\text{if } (rhs(eq81) < 1) \text{ then } (\text{subs}(Cr = 960000, rhs(eq79))) \text{ else } (\text{subs}(Cr = 480000, rhs(eq79))) \text{ end if}
\]

76942.96734

This is the maximum permissible wheel-load for sections not restrained against rotation.
15. Bibliography

1. Crane Manufacturers Association of America (CMAA), “Specifications for Top-Running Bridge & Gantry-Type Multiple Girder Electric Overhead Traveling Crane”, Article #70, Revised 2004


