OPTIMIZATION OF FIRE BLANKET PERFORMANCE

BY

VARYING RADIATIVE PROPERTIES

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August 11, 2011

*We also certify that written approval has been obtained for any proprietary material contained therein.
This work is dedicated
to the memory of my grandmother,

Maryflor Blanco Horn

and

*ad majorem Dei gloriam.*
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Finally, I would like to thank the members of my family for their unending support.
## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_B$</td>
<td>Total heat blocking efficiency, see Equation 5-1</td>
</tr>
<tr>
<td>$T_B$</td>
<td>Blanket back-side surface temperature (K)</td>
</tr>
<tr>
<td>$T_{B,EM}$</td>
<td>Blanket back-side surface temperature in emissive scheme ($\varepsilon_F = 1$) (K)</td>
</tr>
<tr>
<td>$T_{B,REF}$</td>
<td>Blanket back-side surface temperature in reflective scheme ($\varepsilon_F = 0$ and $\varepsilon_B = 1$) (K)</td>
</tr>
<tr>
<td>$T_{CH}$</td>
<td>Blanket front-side surface temperature at which the front surface emissivity is changed from $\varepsilon_F$ to $\varepsilon_{F,CH}$</td>
</tr>
<tr>
<td>$T_F$</td>
<td>Blanket front-side surface temperature (K)</td>
</tr>
<tr>
<td>$T_{F,EM}$</td>
<td>Blanket front-side surface temperature in emissive scheme ($\varepsilon_F = 1$) (K)</td>
</tr>
<tr>
<td>$T_{F,REF}$</td>
<td>Blanket front-side surface temperature in reflective scheme ($\varepsilon_F = 0$ and $\varepsilon_B = 1$) (K)</td>
</tr>
<tr>
<td>$T_H$</td>
<td>Temperature of the structure (K)</td>
</tr>
<tr>
<td>$T_{SWITCH}$</td>
<td>“Switch temperature,” see §3.4.2 (K)</td>
</tr>
<tr>
<td>$T_\infty$</td>
<td>Ambient air temperature (K)</td>
</tr>
<tr>
<td>$\dot{Q}_{CONV}$</td>
<td>Incident convective heat flux from fire (W/m$^2$)</td>
</tr>
<tr>
<td>$\dot{Q}_{RAD}$</td>
<td>Incident radiative heat flux from fire (W/m$^2$)</td>
</tr>
<tr>
<td>$\dot{Q}_{TOT}$</td>
<td>Total incident heat flux from fire (W/m$^2$)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$h$</td>
<td>Convective coefficient on back surface (W/m$^2$-K)</td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal conductivity of the blanket (W/m-K)</td>
</tr>
<tr>
<td>$k_{\text{AIR}}$</td>
<td>Thermal conductivity of air (W/m-K)</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness of the blanket (m)</td>
</tr>
<tr>
<td>$t_{\text{GAP}}$</td>
<td>Thickness of the air gap between the back surface of the blanket and the structure (m)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Absorptivity of the blanket</td>
</tr>
<tr>
<td>$\varepsilon_B$</td>
<td>Back surface emissivity of the blanket</td>
</tr>
<tr>
<td>$\varepsilon_F$</td>
<td>Front surface emissivity of the blanket</td>
</tr>
<tr>
<td>$\varepsilon_{F,CH}$</td>
<td>Front surface emissivity after $T_F$ has exceeded $T_{CH}$</td>
</tr>
<tr>
<td>$\eta_B$</td>
<td>Quasi-steady heat blocking efficiency, see §2.3</td>
</tr>
<tr>
<td>$\eta_{B,\text{REF}}$</td>
<td>Quasi-steady heat efficiency by a reflective scheme</td>
</tr>
<tr>
<td>$\eta_{B,\text{EM}}$</td>
<td>Quasi-steady heat efficiency by an emissive scheme</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Mean value for a normal distribution</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Reflectivity of the blanket</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan-Boltzman constant (W/m$^2$-K$^4$)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation for a normal distribution</td>
</tr>
</tbody>
</table>
Optimization of Fire Blanket Performance by
Varying Radiative Properties

Abstract
By
KEVIN M. BRENT

There is an effort to develop a thin fire blanket which can be deployed to protect structures in wildland-urban interfaces from exterior fires. The surface radiative properties of fire blankets for the protection of structures from wildland fires were variables in three optimization studies. A quasi-steady heat transfer model was used. In steady-state, there were two modes of blocking heat from reaching the structure: reflection and re-radiation of heat back toward the surroundings. The first study considered only the effectiveness of the blanket. The second considered both the effectiveness and the surface temperature facing the structure. The third used time-variant incident heat conditions. In the first study, a method to predict which mode of heat blocking was most effective, based on the incident heat conditions, was developed. In the second study, the optimal fronts provided by these two modes of blocking heat were described explicitly. In the third study, it was found that the same optimal fronts existed even when the incident heat conditions were time-variant. Finally, it was found that allowing the emissivity of the surface facing the fire to change at some temperature could improve heat blocking performance.
1. Introduction

1.1 Fire blankets for the protection of structures from wildland fires

1.1.1 Description

At present, the use of fire blankets is generally limited to smothering small laboratory or kitchen fires. A more recent application, on a larger scale, has been the protection of structures from exterior fires, particularly wildland-urban interface fire hazards. Figure 1-1 below shows the use of such a fire blanket for the protection of a shed during a controlled burn [1].

![Thin fire blanket being used in a test for the protection of a structure from an exterior fire](image)

**Figure 1-1.** Thin fire blanket being used in a test for the protection of a structure from an exterior fire [1].

This use is more similar to the protective clothing of a fireman than it is to the fire blankets found in laboratories and kitchens, in that the goal is the protection of some subject rather than smothering the fire. However, protective clothing for fire-fighters is heavy and thick, and it is designed to keep the wearer from excessive temperature for a
period of time as the outside surface and material of the clothing heat up. For ease of deployment, fire blankets for the protection of structures must be lightweight and thin. This means that, unlike protective fire-fighting gear, they must be effective in quasi-steady-state conditions, since the material must be too thin for protection by a transient absorption of incident heat.

In an effort to alter the way in which the fire blankets interact with incident heat, coatings and thin layers of aluminum are often added to the surface facing the fire. In experimental studies [2,13-17], these modifications have been added to either surface. By means of these coatings, the radiative properties of both surfaces may be controlled independently.

1.1.2 Assumptions

The requirements that the fire blanket material be lightweight, thin, and resistant to high temperature and large temperature gradients limits the material selection to woven, fibrous materials, such as fiberglass. Thin fabrics of this type achieve steady-state on the order of seconds when exposed to intense heat fluxes. However, the heat flux conditions in a wildland fire change on the order of minutes [2]. Therefore, a quasi-steady-state assumption was made. At each instance in time, it was assumed that the fire blanket would be operating at steady-state with its surroundings.

Because the materials used in these fire blankets are both fibrous and thin, the transfer of incident radiation through them is a complex phenomenon combining absorption, scattering, emission, and conduction. It was assumed that this interaction could be summarized as reflection, absorption, and emission in very thin surface layers.
For the purpose of using a simple model which would be suitable for optimization, these were treated as surface phenomena.

It was further assumed that the material of the blanket had zero transmissivity, so that all incident radiation was either reflected or absorbed as heat.

Gray media and surfaces were assumed. With thermal equilibrium, Kirchhoff’s law was assumed to be valid. Kirchhoff’s law states that the emissivity ($\varepsilon$) and absorptivity ($\alpha$) of a surface are equal. This, combined with the zero transmissivity assumption, meant that one minus the reflectivity ($1-\rho$) was equal to the emissivity ($\varepsilon$). The surface absorptivity and reflectivity could then be expressed as a function of the surface emissivity only.

Finally, it was assumed that the front and back surface emissivities could be controlled independently by adding surface coatings or layers to the blanket.

1.2 Literature Review

1.2.1 Patents

The use of a fire blanket to protect an entire structure from an exterior fire is not a new idea. In 1942, Paul Rhoads Wagner filed a patent [3] for a “Conflagration-Retardative Curtain.” This was a system of “a plurality of fire-resistant curtains” which was draped over the roof and sides of a building near a fire. However, Wagner set the trend for all future patents related to this use of fire blankets by noting that “no particular incombustible materials are claimed as part of this invention.” Wagner then went on to suggest a variety of heavy, wire reinforced materials, but never considered that this application might require a type of fire blanket different from the fire resistant materials encountered in fire fighting and in kitchens and laboratories.
A series of similar patents follow. Virgil R. Ballinger filed a patent [4] for a “Fire Protection Apparatus for a Building” in 1971, in which “fire-resistant or fire-retardant material” is used to cover a building. Also in 1971, Norman D. Mitchell filed a patent [5] for a “Fire Protection Device for Building Structure,” which deployed a “fire resistant curtain” over the roof of a structure. In 1988, Kyle McQuirk filed a patent [6] for “Fire Protection for Structures,” which was a deployment method for a “fire resistant sheet material.” In 1993, David J. Hitchcock filed a patent [7] for an “Automated Exterior Fire Protection System for Building Structures,” which was yet another deployment method for a “fire resistant blanket.” Hitchcock’s idea even appeared in the March, 1997 issue of *Popular Mechanics* as an “Automatic Fire Shield for Homes” [8]. Common to every one of these claims is the invention of some deployment method which uses some unspecified “fire resistant,” “fire retardant,” or even “fire retardative” material. None of the inventors considered that this different application of a fire blanket would require a different type of material, other than those already in existence for fire fighting and smothering small fires.

### 1.2.2 Heat transfer models for other applications

There have been a multitude of studies attempting to model heat transfer through fire resistant materials, but the vast majority of these are focused on materials for fire fighting gear or other heavy materials which operate in a transient manner. A few of these models, however, are worthy of mention as being applicable to a thin, fibrous material operating in steady state. In 1980, T. W. Tong and C. L. Tien published analytical models for radiative heat transfer in fibrous materials for building insulation [9]. In 1983, they published a two part study “Radiative Heat Transfer in Fibrous
Insulations.” The first study was an analytical one [10], while Q. S. Yang joined for the second, experimental study [11]. In 1999, D. A. Torvi and J. D. Dale published the study “Heat Transfer in Thin Fibrous Materials Under High Heat Flux” in *Fire Technology* [12]. This study developed a heat transfer model for thin, fibrous materials, and applied it using finite element analysis. This study, however, focused on the use of such material in an inherently transient application for the protection of people.

### 1.2.3 Experimental studies and heat transfer model

During 2009 and 2010, a group of researchers led by F. Takahashi published a series of papers which specifically looked at the use of thin fire blankets for the protection of structures [2,13-17]. These papers developed a means of quantifying the performance of a material in such an application (the heat blocking efficiency, $\eta_B$). Two types of laboratory tests were conducted: one using a Meker burner so that the heat input to the samples was predominantly convection, and the other using a radiant cone so that the heat input was predominantly radiation. A wide range of blanket materials have been tested, both single- and multi-layered, with and without an aluminum coating on either surface. A 2011 poster at the 5th International Wildland Fire Conference (WILDFIRE 2011) also presented the results of a live, prescribed burn which tested four thin fire blankets in the protection of a structure [1]. In 2011, S.-Y. Hsu was the first author of a paper from the same group of researchers, entitled “Modeling Heat Transfer in Thin Fire Blanket Materials under High External Heat Fluxes” [2]. This heat transfer model solved one-dimensional, radiative heat transfer equations including in-depth absorption, emission, and scattering within the thin, fibrous materials. The model focused on
predicting steady- and quasi-steady-state performance, and comparisons were made with experimental data.

1.3 Optimization

1.3.1 The case for optimization

To the knowledge of the author, there has been no study seeking to find the optimal properties of a material for use as a thin fire blanket for the protection of structures from exterior fires. Because such fire blankets operate in quasi-steady-state conditions, their use is inherently different from that of protective fire-fighting gear or fire blankets found in kitchens and laboratories. Therefore, it cannot be assumed that the ideal properties of a material for these other applications are also ideal for a large-scale, thin fire blanket for structure protection. An optimization study must be performed in order to build the most effective fire blankets possible for the protection of structures from exterior fires.

1.3.2 Nature of the study

The object of this study was to find the optimal front and back surface radiative properties for a large-scale, thin fire blanket. Due to the set of assumptions made, the surface absorptivity ($\alpha$) and reflectivity ($\rho$) could both be expressed as a function of emissivity ($\varepsilon$) only. Therefore, the two variables for optimization were the front and back surface emissivities, $\varepsilon_F$ and $\varepsilon_B$, respectively.

The requirements that the fire blanket be lightweight, thin, and be able to withstand high temperature were restrictions on the choice of material. For this reason, the thermal conductivity and thickness of the fire blanket were left as input parameters. The surface emissivities are not similarly restricted by the choice of material, since
surface coatings are commonly applied to change the surface radiative properties. Often, the surface facing the fire is aluminized to give it a high reflectivity and low emissivity. However, this is not done based on any study or optimization, but based on the experiences of the manufacturers and users of fire-resistant materials.

**Three parts of the study.** This study was divided into three parts. The first part of the study was a single-objective optimization in which a single variable, the heat blocking efficiency, was maximized. (The heat blocking efficiency was a value used to quantify the effectiveness of a fire blanket, described in §2.3.) At various incident heat conditions, the pair of surface emissivities, front and back, which offered the highest heat blocking efficiency for those specific conditions was found. The sensitivity to various other input parameters (e.g. blanket thickness) were also investigated.

The second part of the study was a double-objective optimization, in which the back surface temperature was added as a second objective function. A high heat blocking efficiency and low back surface temperature represented objectives contrary to each other. Like the first part of the study, the surface emissivities were optimized for one specific set of incident heat conditions, but various conditions were examined.

These first two optimization studies were for steady-state conditions (i.e. each optimization case is for one constant set of incident heat conditions). In the third optimization study, a transient heat input was specified to represent the heat load to a structure from a passing wildland fire. This required the objective functions to be redefined. The heat blocking efficiency became the total heat blocking efficiency, and the back surface temperature was replaced with the peak back surface temperature. This third part of the study was itself divided into two cases. The first considered the front and
back surface emissivities to be independent of temperature, so that they would not vary over the course of the transient heat input. In the second case, the front surface emissivity was allowed to vary with temperature in a specific manner. This was motivated by the experimental observations that, when exposed to large heat fluxes, the front surface of the fire blanket can change color and emissivity, and aluminum coatings can melt or otherwise disappear at elevated temperatures [2,13-17]. In this second case, the temperature at which this change occurred and the front surface emissivity after the change were two additional variables to be optimized.

1.3.3 Methods for optimization

1.3.3.1 Enumerative search

For a simple, single-objective optimization, the entire search space may be sampled to find the set of inputs which give the best objective function. The result of such an enumerative search is a single optimized result.

1.3.3.2 Multi-Objective Genetic Algorithm (MOGA)

The case for an evolutionary algorithm. For multi-objective optimization, in which two or more objectives might be contrary to one another, an evolutionary algorithm has the capacity to find a population of non-dominated solutions. Such an algorithm does not require the use of a user-designed weighting function which determines the relative importance of the various objectives, as classical methods of optimization often do. The population of non-dominated solutions, which is the goal of using an evolutionary algorithm, is a set of trade-off solutions between the various contradictory objectives.
**Description of evolutionary algorithms [18]**. Evolutionary algorithms generally share the same sequence of operations, which is depicted on the following page in Figure 1-2. First, a random population of potential solutions is initialized. Each member of this population is evaluated based on some fitness function. The fitness assigned to each member of the population is the input for the selection operator, which favors solutions with a high fitness and discourages weaker solutions from being passed onto the next generation. The crossover operator then takes the selected members of the population and switches some sets of their properties between pairs of them. This is an attempt to mimic biological reproduction, in which sections of genetic code between two “parents” are interchanged to produce “children.” After crossover, each member of the population is subjected to random mutation. At this point the entire population is judged to be optimized or not, often by the number of iterations, and the process is repeated until the population satisfies this test.
A Multi-Objective Genetic Algorithm (MOGA) is a specific example of an evolutionary algorithm in which a specific method for assigning fitness is used.

The fitness function [18]. The fitness which is first assigned to a potential solution in a MOGA is based on the normalized number of times that that potential solution is dominated by other members of the population. Thus, non-dominated members of the population have a fitness of one, and those dominated the most number of times have a fitness of zero. That fitness is then modified based on a niching function, which encourages a diverse set of solutions by punishing those whose properties are too similar.

Stochastic Universal Selection operator [19]. A Stochastic Universal Selection (SUS) operator is a standard feature of a MOGA. The SUS first constructs a “roulette-wheel” based on the fitness of each member of the population. Rather than randomly
selecting a single solution from that roulette wheel $N$ times, however, the SUS selects $N$
solutions at evenly-spaced intervals across the wheel. In this manner, only one random
number is used for the entire process.

**Simulated Binary Crossover operator [20].** Typically, a MOGA uses a Single-
Point Crossover operator, in which a random point in a string of variables is chosen. All
of the variables on one side of that randomly chosen point are switched between the two
“parents” to produce the “children” string of variables. However, in this case, there were
only two real-valued variables: the front and back surface emissivities. In an effort to
preserve the behavior of a Single-Point Crossover for this two-element, real-valued
string, a Simulated Binary Crossover (SBX) operator was used.

**Non-uniform operator [18].** A non-uniform mutation operator was used, in
which each variable in each member of the population was subjected to mutation within a
range of values. As the algorithm progressed from the first to the last iteration, the range
over which each variable could mutate was reduced, until there was no mutation at the
last iteration. In this manner, much “searching” across the search space was done early
on in the execution of the algorithm. In later iterations, when it could be assumed that the
population had been somewhat optimized, less “searching” was done in order to preserve
the improved population.
2. The Model

2.1 Review of assumptions

Due to the requirement that the fire blanket material be thin, it was assumed that the blanket would operate in quasi-steady-state [2]. That is, as the incident heat conditions changed over time, the blanket was assumed to be operating at steady-state in each instant. For the simplicity of the model, it was assumed that reflection, absorption, and emission were surface phenomena. Gray media and surfaces were assumed. It was further assumed that there was no transmissivity, and that Kirchhoff’s Law was valid.

2.2 Derivation of the governing equations

Figure 2-1, which follows, shows the setup of the heat transfer model. The incident heat is described by a quantity of incoming radiative and convective heat flux. A portion of the radiative flux is reflected, and the rest of the heat is absorbed at the front surface. Of the absorbed heat, some is re-emitted off of the front surface, and some is conducted through to the back surface. From the back surface, heat is transmitted to the house (or structure, otherwise) either by convection or radiation. Already, it is seen that heat can be blocked from transmission to the structure by the reflection of incident radiant heat, or the re-emission of absorbed heat.
Figure 2-1. Setup for modeling heat transfer through a fire blanket.

When the first law of thermodynamics is applied to a small control volume which contains the front surface but terminates in the interior of the blanket, the following equation is obtained:

\[
\dot{Q}_{\text{RAD}} - \rho_F \dot{Q}_{\text{RAD}} + \dot{Q}_{\text{CONV}} = \varepsilon_F \sigma (T_F^4 - T_{\infty}^4) + \frac{k}{t} (T_F - T_B).
\]  

(2-1)

Since the transmissivity of the material was assumed to be zero and Kirchhoff’s Law was assumed to be valid,

\[
\varepsilon = 1 - \rho.
\]  

(2-2)

When this is substituted into Equation 2-1, the following is obtained:

\[
\varepsilon_F \dot{Q}_{\text{RAD}} + \dot{Q}_{\text{CONV}} = \varepsilon_F \sigma (T_F^4 - T_{\infty}^4) + \frac{k}{t} (T_F - T_B).
\]  

(2-3)
Doing the same for a control volume which extends from the interior of the blanket beyond the back surface, the following equation is obtained:

\[
\frac{k}{t} (T_F - T_B) = \varepsilon_B \sigma (T_B^4 - T_H^4) + h(T_B - T_H).
\]  

Equations 2-3 and 2-4 are the two governing equations for this one-dimensional quasi-steady-state model of heat transfer through a fire blanket. These two equations can be solved iteratively for the front surface temperature, \(T_F\), and the back surface temperature, \(T_B\). With these two temperatures determined, all quantities relating to the system can be found.

### 2.3 Description of the heat blocking efficiency, \(\eta_B\)

As mentioned in §1.3.2, the heat blocking efficiency is a value which quantifies the effectiveness of a fire blanket. The heat blocking efficiency is defined as the ratio of incident heat blocked from reaching the structure to the total amount of incident heat [17]. This can be expressed as:

\[
\eta_B = 1 - \frac{\dot{Q}_{B,\text{OUT}}}{\dot{Q}_{F,\text{IN}}}. \tag{2-5}
\]

Using the values relevant to the heat transfer model being used in this study, the heat blocking efficiency may be expressed in several ways:

\[
\eta_B = 1 - \frac{\varepsilon_B \sigma (T_B^4 - T_H^4) + h(T_B - T_H)}{\dot{Q}_{\text{RAD}} + \dot{Q}_{\text{CONF}}}, \tag{2-6a}
\]

\[
= 1 - \frac{k}{t} (T_F - T_B) \frac{\dot{Q}_{\text{RAD}} + \dot{Q}_{\text{CONF}}}{\dot{Q}_{\text{RAD}} + \dot{Q}_{\text{CONF}}}, \tag{2-6b}
\]
Equation 2-6d is an especially convenient definition of the heat blocking efficiency, as it shows clearly the two modes of blocking heat in the numerator: reflection of radiation and re-emission of absorbed heat.

### 2.4 Comparison with experimental results and a more advanced model

#### 2.4.1 Results and model in Hsu et al. [2]

Before doing further work with this proposed model, it should be verified as a representation of reality. In a prior study, Hsu et al. [2] constructed a heat transfer model based on the absorbing, emitting, and scattering of radiation across the fibers of the material, and conducted experiments to verify their predictions. There were two cases:

- Case #1 involved a single layer of fiberglass with a thickness of 0.91mm, and
- Case #2 involved the same fiberglass with the addition of 0.025mm of aluminum foil to the front surface.

Both of these were exposed to 83kW/m² of incident radiative heat flux. For the application to the proposed model, the thermal conductivity of the fabric was assumed to be constant across the material, and equal to that given by the average of the two surface temperatures. It was further assumed that the gray surface emissivity of the fiberglass was 0.5, and that of the aluminum was 0.07. This assumed emissivity of aluminum is a widely reported value. The emissivity of fiberglass was chosen to give results closest to
those in Hsu et al. [2]. The value for the back surface convective coefficient used for comparison was 6.9 W/m\(^2\)-K. Again, this value gave the set of results closest to those in Hsu et al. [2]. Table 2-1, which follows, shows the results of comparison.

<table>
<thead>
<tr>
<th>Case #1</th>
<th>(\eta_B)</th>
<th>(T_F) (K)</th>
<th>(T_B) (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hsu et al. experiment [2]</td>
<td>0.752</td>
<td>881</td>
<td>656</td>
</tr>
<tr>
<td>Hsu et al. model [2]</td>
<td>0.774</td>
<td>932</td>
<td>815</td>
</tr>
<tr>
<td>Proposed model</td>
<td>0.807</td>
<td>976</td>
<td>818</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case #2</th>
<th>(\eta_B)</th>
<th>(T_F) (K)</th>
<th>(T_B) (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hsu et al. experiment [2]</td>
<td>0.933</td>
<td>798</td>
<td>517</td>
</tr>
<tr>
<td>Hsu et al. model [2]</td>
<td>0.937</td>
<td>630</td>
<td>579</td>
</tr>
<tr>
<td>Proposed model</td>
<td>0.938</td>
<td>645</td>
<td>588</td>
</tr>
</tbody>
</table>

**Table 2-1.** Comparison of experimental data and model in Hsu et al. [2] to the proposed model.

It was noted in Hsu et al. [2] that of the three types of data recorded in the experiment, the heat flux through the blanket (and, by extension, the heat blocking efficiency) was the most reliable. The two surface temperatures were recorded by the use of thermocouples, which, in large temperature gradients, become highly dependent upon contact pressure and position [2]. The small discrepancies in surface temperature between the proposed model and the more advanced model presented in Hsu et al. may find cause in the rough estimation for the emissivity of fiberglass. The reasonable agreement of the proposed model with both experimental heat blocking efficiencies and the surface temperatures predicted by a more advanced, published model justify its use for this optimization study. The simplicity of the present model reduces the number of blanket properties to be optimized while highlighting those most important for an optimization study.
2.4.2 Results in Takahashi et al. [17]

Since it was suspected that the large temperature gradients present in the experimental data used in §2.4.1 rendered the thermocouple data unreliable [2], results given by the proposed model were compared to lower radiant heat flux data collected by Takahashi et al. [17]. In that study, a sample of 0.91mm-thick fiberglass was exposed to 21kW/m², 42 kW/m², and 63 kW/m² of incident radiant heat flux, and the surface temperatures and heat blocking efficiencies were found. For the purposes of comparison, it was assumed that the average emissivity of fiberglass was 0.541, the average thermal conductivity of fiberglass was 0.136W/m-K, and the back surface convective coefficient was 23.5W/m²-K. These values were chosen to give predicted results which were closest to those presented in Takahashi et al. [17]. Table 2-2 below shows the comparison of the experimental data in Takahashi et al. [17] and the results of the proposed model.

<table>
<thead>
<tr>
<th>Incident Radiation</th>
<th>ηB</th>
<th>TF (K)</th>
<th>TB (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 kW/m²</td>
<td>0.631</td>
<td>606</td>
<td>546</td>
</tr>
<tr>
<td>Takahashi et al. [17] experiment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proposed model</td>
<td>0.625</td>
<td>590</td>
<td>537</td>
</tr>
<tr>
<td>42 kW/m²</td>
<td>0.690</td>
<td>723</td>
<td>653</td>
</tr>
<tr>
<td>Takahashi et al. [17] experiment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proposed model</td>
<td>0.677</td>
<td>744</td>
<td>652</td>
</tr>
<tr>
<td>63 kW/m²</td>
<td>0.698</td>
<td>858</td>
<td>713</td>
</tr>
<tr>
<td>Takahashi et al. [17] experiment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proposed model</td>
<td>0.709</td>
<td>850</td>
<td>727</td>
</tr>
</tbody>
</table>

Table 2-2. Comparison of experimental data Takahashi et al. [17] to the proposed model.

It is seen that the temperatures predicted by the model and those observed in Takahashi et al. [17] agree more closely at these, low radiant heat fluxes, than in the previous higher heat flux. The average emissivity of fiberglass used in these comparisons, 0.541, is admittedly lower than the range of values typically reported for that value. For this reason, the table below shows the same data as Table 2-2, in addition
to the predicted values of the heat blocking efficiency and front and back surface
temperatures using a range of different values for the average emissivity of fiberglass.

<table>
<thead>
<tr>
<th></th>
<th>$\eta_B$</th>
<th>$T_F$ (K)</th>
<th>$T_B$ (K)</th>
<th>% error in $\eta_B$</th>
<th>% error in $T_B$</th>
<th>% error in $T_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>21 kW/m²</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Takahashi et al. [17] exp.</td>
<td>0.631</td>
<td>606</td>
<td>546</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$\varepsilon = 0.541$</td>
<td>0.625</td>
<td>590</td>
<td>537</td>
<td>0.010</td>
<td>0.026</td>
<td>0.016</td>
</tr>
<tr>
<td>$\varepsilon = 0.55$</td>
<td>0.620</td>
<td>592</td>
<td>539</td>
<td>0.017</td>
<td>0.023</td>
<td>0.013</td>
</tr>
<tr>
<td>$\varepsilon = 0.6$</td>
<td>0.598</td>
<td>601</td>
<td>544</td>
<td>0.052</td>
<td>0.008</td>
<td>0.004</td>
</tr>
<tr>
<td>$\varepsilon = 0.65$</td>
<td>0.577</td>
<td>608</td>
<td>549</td>
<td>0.086</td>
<td>0.003</td>
<td>0.005</td>
</tr>
<tr>
<td>$\varepsilon = 0.7$</td>
<td>0.556</td>
<td>615</td>
<td>553</td>
<td>0.119</td>
<td>0.015</td>
<td>0.013</td>
</tr>
<tr>
<td>$\varepsilon = 0.75$</td>
<td>0.536</td>
<td>621</td>
<td>556</td>
<td>0.151</td>
<td>0.025</td>
<td>0.018</td>
</tr>
<tr>
<td>$\varepsilon = 0.8$</td>
<td>0.516</td>
<td>627</td>
<td>559</td>
<td>0.182</td>
<td>0.035</td>
<td>0.024</td>
</tr>
<tr>
<td><strong>42 kW/m²</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Takahashi et al. [17] exp.</td>
<td>0.690</td>
<td>723</td>
<td>653</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$\varepsilon = 0.541$</td>
<td>0.677</td>
<td>744</td>
<td>652</td>
<td>0.019</td>
<td>0.029</td>
<td>0.002</td>
</tr>
<tr>
<td>$\varepsilon = 0.55$</td>
<td>0.673</td>
<td>745</td>
<td>653</td>
<td>0.025</td>
<td>0.030</td>
<td>0.000</td>
</tr>
<tr>
<td>$\varepsilon = 0.6$</td>
<td>0.655</td>
<td>754</td>
<td>657</td>
<td>0.051</td>
<td>0.043</td>
<td>0.006</td>
</tr>
<tr>
<td>$\varepsilon = 0.65$</td>
<td>0.639</td>
<td>762</td>
<td>660</td>
<td>0.074</td>
<td>0.054</td>
<td>0.011</td>
</tr>
<tr>
<td>$\varepsilon = 0.7$</td>
<td>0.622</td>
<td>769</td>
<td>663</td>
<td>0.099</td>
<td>0.064</td>
<td>0.015</td>
</tr>
<tr>
<td>$\varepsilon = 0.75$</td>
<td>0.607</td>
<td>775</td>
<td>664</td>
<td>0.120</td>
<td>0.072</td>
<td>0.017</td>
</tr>
<tr>
<td>$\varepsilon = 0.8$</td>
<td>0.592</td>
<td>780</td>
<td>666</td>
<td>0.142</td>
<td>0.079</td>
<td>0.020</td>
</tr>
<tr>
<td><strong>63 kW/m²</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Takahashi et al. [17] exp.</td>
<td>0.698</td>
<td>858</td>
<td>713</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$\varepsilon = 0.541$</td>
<td>0.709</td>
<td>850</td>
<td>727</td>
<td>0.016</td>
<td>0.009</td>
<td>0.020</td>
</tr>
<tr>
<td>$\varepsilon = 0.55$</td>
<td>0.706</td>
<td>851</td>
<td>727</td>
<td>0.011</td>
<td>0.008</td>
<td>0.020</td>
</tr>
<tr>
<td>$\varepsilon = 0.6$</td>
<td>0.691</td>
<td>860</td>
<td>730</td>
<td>0.010</td>
<td>0.002</td>
<td>0.024</td>
</tr>
<tr>
<td>$\varepsilon = 0.65$</td>
<td>0.677</td>
<td>868</td>
<td>731</td>
<td>0.030</td>
<td>0.012</td>
<td>0.025</td>
</tr>
<tr>
<td>$\varepsilon = 0.7$</td>
<td>0.663</td>
<td>874</td>
<td>732</td>
<td>0.050</td>
<td>0.019</td>
<td>0.027</td>
</tr>
<tr>
<td>$\varepsilon = 0.75$</td>
<td>0.650</td>
<td>880</td>
<td>733</td>
<td>0.069</td>
<td>0.026</td>
<td>0.028</td>
</tr>
<tr>
<td>$\varepsilon = 0.8$</td>
<td>0.637</td>
<td>886</td>
<td>733</td>
<td>0.087</td>
<td>0.033</td>
<td>0.028</td>
</tr>
</tbody>
</table>

Table 2-3. Comparison of experimental data Takahashi et al. [17] to the proposed model

...for a range of values representing the emissivity of fiberglass.

It can be seen that the predicted surface temperatures remain accurate even with
much higher values for the emissivity of fiberglass. The predicted values for the heat
blocking efficiency, however, can suffer when the assumed emissivity of fiberglass is
high. Nevertheless, the proposed model predicted the heat blocking efficiency and
surface temperatures with sufficient accuracy, when compared with experimental data, for the model to be justified for use in optimization.

$\eta_B$

3.1 Introduction

In this first part of the optimization study, the surface radiative properties were optimized to produce the highest heat blocking efficiency possible.

3.2 Method

3.2.1 Objective function

The only objective function was the heat blocking efficiency. As discussed in §2.3, the heat blocking efficiency is defined as the ratio of the incident heat blocked from reaching the structure to the total amount of incident heat. Reflection of radiative heat serves the purpose of blocking it from reaching the structure. The re-emission of absorbed radiative and convective heat at the front surface also prevents it from being transmitted through to the structure. It should be noted that if the incident heat is completely convective, the re-radiation of heat off of the front surface is the only mechanism to block heat transmission to the structure in the quasi-steady regime.

3.2.2 Optimization algorithm

Because there was only one objective function, an enumerative search scheme was used. The entire search space was studied in a sequential fashion to find the highest heat blocking efficiency with the current set of input parameters. The search space in this study was represented by a zero to one range for both the front and back surface emissivities.
3.3 Results

3.3.1 Two optimal schemes

It was found, as could be expected, that when the incident heat was dominated by radiation, a perfectly reflective front surface gives the highest heat blocking efficiency. When the incident heat was not completely dominated by radiation (~90% radiation), a perfectly emissive front surface combined with a perfectly reflective back surface gave the highest heat blocking efficiency. For ease of reference, these two schemes were called a “reflective scheme” and an “emissive scheme,” respectively.

For the reflective scheme, the emissivity of the back surface had no effect on the heat blocking efficiency. It did, however, effect the surface temperatures. The more emissive the back surface, the lower the temperatures, as the heat which was entering the front of the blanket was more easily emitted toward the structure. Therefore, the optimal reflective scheme consisted of a reflective front surface ($\varepsilon_F = 0$) and an emissive back surface ($\varepsilon_B = 1$). The heat blocking efficiency was then equal to the radiative fraction of the incident heat:

$$\eta_{B,REF} = \frac{\dot{Q}_{RAD}}{\dot{Q}_{RAD} + \dot{Q}_{CONV}}.$$  \smallskip (3-1)

It is quickly seen that the heat blocking efficiency for a reflective scheme is independent of the surface temperatures, and by extension, the back surface emissivity.

For the emissive scheme, the amount of heat that was emitted off of the front surface was temperature-dependent, and therefore the back emissivity had an effect on the heat-blocking efficiency of the blanket. A lower back surface emissivity drove the temperatures up, allowing the front surface to emit more of the absorbed incident heat.
back toward the fire. Therefore, the optimal emissive scheme consisted of an emissive surface \((\varepsilon_F = 1)\) and a reflective back surface \((\varepsilon_B = 0)\). The heat blocking efficiency was then given by the fraction of incident heat which was re-emitted toward the surroundings:

\[
\eta_{B, EM} = \frac{\sigma(T_F^4 - T_{\infty}^4)}{\dot{Q}_{RAD} + \dot{Q}_{CONV}}. \tag{3-2}
\]

### 3.3.2 Influence of various input parameters

#### 3.3.2.1 Incident heat conditions

It was seen from Equation 3-1 that the heat blocking efficiency of a reflective blanket only depended on the fraction of incident heat that was radiation. The total amount of incident heat did not affect the heat blocking efficiency of such a fire blanket, so long as the ratio of radiative to total heat remained the same. The front and back temperatures of a reflective fire blanket depended on the amount of heat which was conducted through the blanket. This meant that the two surface temperatures depended on both the total amount of incident heat and its composition.

It was seen from Equation 3-2 that the heat blocking efficiency of an emissive blanket depended on the total amount of incident heat and the temperature of the front surface. The surface temperatures of the blanket were dependent upon the amount of heat absorbed by the front surface. Since, in this case, the front surface was completely emissive and absorbed all of the incoming heat, the surface temperatures depended only on the total amount of incident heat regardless of its radiative or convective composition. By extension, the heat blocking efficiency, too, depended only on the total amount of incident heat regardless of its composition.
Which scheme was actually the best for some set of incident heat conditions is illustrated in the following charts. For these charts, $k/t = 100 \text{ W/m}^2\text{-K}$, $h = 5 \text{ W/m}^2\text{-K}$, $T_H = 300 \text{ K}$, and $T_\infty = 300 \text{K}$.

**Figure 3-1.** Reflective and emissive temperatures and heat blocking efficiencies for 10 kW/m$^2$ of incident heat.
Optimum Reflection and Emission Efficiencies and Temperatures, $\dot{Q}_{\text{TOT}} = 20\text{kW/m}^2$

Figure 3-2. Reflective and emissive temperatures and heat blocking efficiencies for 20 kW/m$^2$ of incident heat.
Figure 3-3. Reflective and emissive temperatures and heat blocking efficiencies for 50 kW/m² of incident heat.
It is seen from Figures 3-1 through 3-4 that a reflective scheme offers the highest heat blocking efficiency over a range of radiation-dominated heat inputs which diminishes as the total amount of incident heat increases. This means that for larger fluxes of incoming heat, an emissive scheme offers the best heat blocking efficiency for all cases except for those which are completely dominated by radiation. It is also seen from the above charts that the temperatures of these front-emissive fire blankets can easily exceed the melting point of aluminum (~933K), which is commonly used to form a reflective surface, and even the softening point of heat-resistant fiberglass (~1000-1300K). Aside from being inconsistent with the nature of a feasible fire blanket, these high temperatures also threaten the ignition of the structure.

Figure 3-4. Reflective and emissive temperatures and heat blocking efficiencies for 100 kW/m² of incident heat.
3.3.2.2 Back surface emissivity

Allowing heat to be emitted off of the back surface brings down the back surface temperature to more feasible values, but at the cost of heat blocking efficiency. The front surface temperature was not observed to lower significantly by increasing the back surface emissivity for large fluxes of incident heat. The table below was compiled for the same scenario as that in the charts: $k/t = 100 \text{ W/m}^2\text{-K}$, $h = 5 \text{ W/m}^2\text{-K}$, $T_H = 300 \text{ K}$, and $T_\infty = 300 \text{K}$. This table shows the effect of raising the back surface emissivity from the optimal $\varepsilon_B = 0$ to $\varepsilon_B = 1$.

<table>
<thead>
<tr>
<th>$\dot{Q}_{\text{TOT}}$ (kW/m$^2$)</th>
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<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
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<td>573.0</td>
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<td>561.0</td>
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<td>0.608</td>
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<td>632.2</td>
<td>613.5</td>
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<td>$\Delta T_{\text{EM}}$ (K)</td>
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<td>135.5</td>
<td>151.6</td>
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<td>$\eta_{B,\text{EM}}$</td>
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<td>834.1</td>
<td>807.7</td>
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<td>0.815</td>
<td>0.782</td>
<td>0.757</td>
<td>0.738</td>
</tr>
</tbody>
</table>

Table 3-1. Front-emissive temperatures and heat blocking efficiencies with varied back surface emissivities.

3.3.2.3 Back surface convective coefficient

The convective coefficient between the back surface of the fire blanket and the structure, $h$, affected the temperature and efficiency of the blanket in a manner similar to
the back surface emissivity. A higher convective coefficient allowed the absorbed heat to be transferred from the blanket to the structure at a lower temperature.

For the emissive scheme, this meant that the heat blocking efficiency suffered as the surface temperatures dropped with a higher value for $h$. For the reflective scheme, the convective coefficient did not affect the heat blocking efficiency, as it was independent of surface temperatures.

The table below shows some representative values which demonstrate the effect of the convective coefficient on the surface temperatures and heat blocking efficiency for both the emissive and reflective schemes. These data are for the case in which $Q_{\text{TOT}} = 20$ kW/m², $Q_{\text{CONV}}/Q_{\text{TOT}} = 0.2$, $k/t = 100$ W/m²-K, $T_H = 300$K, and $T_\infty = 300$K. It should be noted that the coefficient $h$ was somewhat more encompassing than just a convective coefficient, and represented all non-radiative heat transfer between the back surface of the fire blanket and the structure. This means that the special case in which $h = 0$ for the emissive scheme (in which $\varepsilon_B = 0$) represents a situation in which no heat can be transferred to the structure from the blanket. This drove up the front surface temperature so that all incident heat was re-radiated off of the front surface toward the surroundings.

<table>
<thead>
<tr>
<th>$h$ (W/m²-K)</th>
<th>0</th>
<th>1</th>
<th>2.5</th>
<th>5</th>
<th>10</th>
<th>25</th>
<th>50</th>
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</thead>
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<td><strong>Emissive Scheme</strong></td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>$T_{F,EM}$ (K)</td>
<td>775.0</td>
<td>770.6</td>
<td>764.2</td>
<td>753.7</td>
<td>734.6</td>
<td>687.9</td>
<td>635.8</td>
</tr>
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<td>$T_{B,EM}$ (K)</td>
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<td>695.1</td>
<td>610.4</td>
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<td>39.5</td>
<td>77.5</td>
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<td>0.977</td>
<td>0.943</td>
<td>0.892</td>
<td>0.803</td>
<td>0.612</td>
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<td><strong>Reflective Scheme</strong></td>
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<td></td>
</tr>
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<td>$T_{F,REF}$ (K)</td>
<td>569.6</td>
<td>562.8</td>
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<td>537.5</td>
<td>509.8</td>
<td>452.6</td>
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<td>$T_{B,REF}$ (K)</td>
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<td>513.0</td>
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<td>469.8</td>
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<td>40.0</td>
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<td>$\eta_{B,REF}$</td>
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<td>0.800</td>
<td>0.800</td>
<td>0.800</td>
<td>0.800</td>
</tr>
</tbody>
</table>

**Table 3-2.** Temperatures and heat blocking efficiencies with varied convective coefficients, $h$. 

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Note on the range of realistic values for the convective coefficient. Very little can be said about the convective coefficient, $h$, without knowing the specific geometry of the space between the blanket and the structure. One can imagine that the blanket might be hung loosely over the top of the structure and draping down the sides, or wrapped tightly around it as in Figure 1-1. Despite this, some guesses as to its order of magnitude can be ventured by examining some hypothetical cases.

The first case is a blanket draped over the side of a structure. The structure is assumed to be at 300K and have a large thermal mass. The value for $h$ should then be somewhere between the rate of heat transfer by conduction through still air, given by $k_{\text{AIR}}/t_{\text{GAP}}$, and the value for $h$ given by natural convection across the gap. Consider some hypothetical structure with a height of 8m and a blanket draped 0.5m from its exterior walls. The back surface temperature of the blanket is 600K. The thermal conductivity of the air between the blanket and the structure is assumed to be about 0.03W/m-K. The value for $k_{\text{AIR}}/t_{\text{GAP}}$ then comes out to about 0.06W/m$^2$-K. The Rayleigh number for natural convection in this scenario is about $4.5 \times 10^{12}$, which gives a value for $h$ of about 5.5W/m$^2$-K. For this hypothetical application, then, the actual value of $h$ is likely between 0.06 W/m$^2$-K and 5.5W/m$^2$-K.

The second case is a blanket wrapped tightly around the sides of the structure. Assuming the same temperatures and a one-millimeter gap between the back surface and the structure, it is quickly seen that $k_{\text{AIR}}/t_{\text{GAP}} \approx 30$ W/m$^2$-K. Between the two cases, the value for $h$ can be grossly estimated as being between $10^{-2}$ W/m$^2$-K and $10^{2}$ W/m$^2$-K. Beyond this, nothing more specific can be said without knowing how the blanket is used.
3.3.2.4 Thermal conductivity and thickness

For a constant thermal conductivity, \( k \), in a quasi-steady analysis, the thermal conductivity of the material of the blanket and its thickness only appear together as the constant value \( k/t \). For ease of reference, this value will be called the “conductive coefficient.” As this conductive coefficient increases, heat travels through the blanket across a weaker temperature gradient. This brings the front and back surface temperatures closer together.

In an emissive scheme, a higher conductive coefficient lowers the front surface temperature. This in turn lowers the heat blocking efficiency, meaning that more heat is conducted through the blanket. For this heat to be convected off of the back surface, a higher back surface temperature is required. Therefore, a higher conductive coefficient causes the front and back surface temperatures to become closer to some middle value and lowers the heat blocking efficiency.

In a reflective scheme, the heat blocking efficiency, and therefore the flux of heat conducted through the blanket, is independent of the front surface temperature. This means that the same amount of heat is absorbed regardless of the conductive coefficient. Therefore, the back temperature remains mostly insensitive to the conductive coefficient, as there is no extra heat that must be shed. As the conductive coefficient increases, the front surface temperature lowers and the heat blocking efficiency is unaffected.

Table 3-3, which follows, demonstrates these effects with some representative values. These data are for the case in which \( Q_{TOT} = 20 \text{ kW/m}^2 \), \( \dot{Q}_{CONV}/\dot{Q}_{TOT} = 0.2 \), \( h = 5 \text{ W/m}^2\text{-K} \), \( T_H = 300\text{K} \), and \( T_\infty = 300\text{K} \).
Table 3-3. Temperatures and heat blocking efficiencies with varied conductive coefficients, $k/t$.

Note on the range of realistic values for the conductive coefficient. The materials that are used for thin fire blankets are generally woven fibrous materials (e.g. fiberglass). In an attempt to get some feel for the range of the conductive coefficient, it was assumed that the thermal conductivity of some such material would be between that for air (~0.025 W/m-K) and that for glass (~1 W/m-K). The thickness of these thin fire blankets is limited to a few millimeters. This gives a range of 10 to $10^3$ W/m$^2$-K for the conductive coefficient.

3.4 Discussion

3.4.1 Discussion of findings

For every case examined, it was found that for incident heat conditions which were dominated by radiation, a reflective front surface offered the best heat blocking efficiency. This is, of course, not surprising. In this case, the back surface emissivity had no effect on the heat blocking efficiency, but an emissive back surface served to keep the temperatures as low as possible. Once the convective fraction of incident heat exceeded some value, an emissive front surface and a reflective back surface offered the best heat
blocking efficiency. The convective fraction of incident heat beyond which an emissive front surface was more efficient lowered as the total amount of incident heat increased.

The back surface emissivity directly affected the two surface temperatures of the fire blanket. A higher back surface emissivity resulted in lower temperatures. As noted, when the front surface was reflective, the heat blocking efficiency was independent of the surface temperatures, and therefore independent of the back surface emissivity. When the front surface was emissive, however, the heat blocking efficiency was dependent upon the front surface temperature of the blanket. That front surface temperature was in turn dependent upon the back surface emissivity. Therefore, a reflective, or non-emissive, back surface offered the highest front surface temperature and the highest heat blocking efficiency. Increasing the back surface emissivity brought about lower temperatures, but lower heat blocking efficiencies as well. For large fluxes of incident heat, the effect became less dramatic.

The convective coefficient between the back surface and the structure had a similar effect on the temperatures and performance of the fire blanket as the back surface emissivity. This makes sense, as an increase in either allows the same amount of heat to be transferred away from the back surface at a lower temperature. This convective coefficient was not a target for optimization, but is the result of how the blanket is employed. Depending on the geometry of the space between the blanket and the structure it is protecting, it was estimated that the convective coefficient would be between $10^{-2}$ W/m$^2$-K and $10^2$ W/m$^2$-K.

The value of $k/t$, which was called the “conductive coefficient” for ease of reference, was also not a target for optimization, but a result of the fire blanket design. It
was assumed that the material of the blanket would be fibrous, in the manner of
fiberglass. Therefore, the thermal conductivity of the material would be between that of
air and that of the solid material comprising the fibers. The thermal conductivity for air
was estimated to be 0.025 W/m·K. It was assumed that the fiber material would be some
sort of glass, with a thermal conductivity estimated to be 1 W/m·K. If the thickness of
the blanket was limited to a few millimeters at most, these estimates give the value for
the conductive coefficient as being between 10 W/m²·K and 10³ W/m²·K.

3.4.2 “Switch Temperature” concept

It would be useful to be able to predict whether some fire blanket exposed to
some incident heat conditions would be most efficient with a reflective or emissive
scheme. Thus far, the only guides toward this end are the charts presented earlier and the
qualitative statement concerning whether or not the incident heat is “dominated by
radiation.” In order to make a more quantitative prediction, the concept of a switch
temperature is introduced. The switch temperature is that temperature for a given fire
blanket at which it is possible to emit heat off of the front surface at the same rate at
which heat could be reflected:

\[ Q_{RAD} = \sigma T_{SWITCH}^4 \]  \hspace{1cm} (3-3)

Equations 2-3 and 2-4 can be solved for the front and back surface temperatures
with the emissive scheme:

\[ \dot{Q}_{RAD} + \dot{Q}_{CONV} = \sigma(T_{F,EM}^4 - T_{\infty}^4) + \frac{k}{t}(T_{F,EM} - T_{B,EM}), \text{ and} \]

\[ \frac{k}{t}(T_F - T_B) = h(T_{B,EM} - T_H). \]  \hspace{1cm} (3-5)
If the front temperature with the emissive scheme, $T_{F, EM}$, is less than the switch temperature, $T_{SWITCH}$, then the blanket is unable to reach a high enough temperature to make the emission of heat off of the front surface more efficient than reflection. In this case, the reflective scheme offers the highest heat blocking efficiency. On the other hand, if $T_{F, EM}$ is greater than $T_{SWITCH}$, then the emissive scheme offers the highest heat blocking efficiency. To demonstrate this, Figure 4 is reproduced below with the addition of the switch temperature. It can be seen that at the point where $T_{F, EM}$ exceeds $T_{SWITCH}$, the emissive scheme becomes more efficient than the reflective scheme.

**Optimum Reflection and Emission Efficiencies and Temperatures, $\dot{Q}_{TOT} = 10$ kW/m²**

![Graph showing the comparison of efficiencies for different schemes at varying temperatures.]

**Figure 3-5.** The previous Figure 3-1 with the addition of the switch temperature, $T_{SWITCH}$. 
4. Double-Objective Optimization: Heat Blocking Efficiency ($\eta_B$) and Back Surface Temperature ($T_B$)

4.1 Introduction

The previous part of the optimization study was a single-objective optimization with high heat blocking efficiency as the only objective function. It was seen that this sometimes resulted in optimized solutions that operated at unrealistic temperatures. In that part of the study, it was noted that raising the back surface emissivity in a front-emissive scheme served to lower the surface temperatures of the fire blanket, but also lowered the heat blocking efficiency. This tradeoff was interpreted as evidence of an optimal front in high heat blocking efficiency and low back surface temperature. The goal of this second part of the study was to find that front.

4.2 Method

4.2.1 Objective functions

In response to the unrealistic temperatures found by the single-objective optimization, in which the only objective function was a high heat blocking efficiency, a second objective function was added: low back surface temperature. It was seen in the first part of the study that these can be contrary objectives.

4.2.2 Optimization algorithm

Because this optimization sought two objectives which were contrary to each other, a Multiple-Objective Genetic Algorithm (MOGA) was used. The MOGA used is described in detail in §1.3.3.2, and is attached as Appendix B.
4.3 Results

4.3.1 Outputs from the algorithm

4.3.1.1 10 kW/m² incident heat flux

Figures 4-1 through 4-5 show the results of the MOGA with 10 kW/m² of incident heat and various fractions of the total heat being convection. For these results, 

\[ k/t = 100 \text{ W/m²-K}, \ h = 5 \text{ W/m²-K}, \ T_H = 300 \text{ K}, \ \text{and} \ T_\infty = 300\text{K}. \]

In Figure 4-1, with \( \dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT.}} = 0.1 \), it is seen that there is a single solution which dominates all others. It is a reflective scheme, in which the front surface is reflective and the back surface is emissive. This solution had both a higher heat blocking efficiency and a lower back surface temperature than all other possible solutions. In Figure 4-2, with \( \dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT.}} = 0.2 \), it is seen that the same reflective scheme offers 80% heat blocking efficiency. There is also a set of emissive schemes which offer higher efficiencies at significantly higher temperatures. In Figure 4-3, with \( \dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT.}} = 0.3 \), the trend continues: The reflective scheme offers the lowest back surface temperature, and there is a front of higher efficiency and higher temperature emissive schemes. Figure 4-4, with \( \dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT.}} = 0.4 \), shows the same behavior. In Figure 4-5, with \( \dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT.}} = 0.5 \), the reflective scheme is dominated by the least efficient and lowest temperature emissive scheme. Therefore, it no longer shows up in the set of non-dominated solutions, and only the emissive front remains.
Non-Dominated Heat Blocking Efficiencies and Back Surface Temperatures, $\dot{Q}_{TOT} = 10 \text{kW/m}^2$, $\dot{Q}_{CONV}/\dot{Q}_{TOT} = 0.1$.

Figure 4-1. Optimized objective functions, $\dot{Q}_{TOT} = 10 \text{kW/m}^2$ and $\dot{Q}_{CONV}/\dot{Q}_{TOT} = 0.1$. 
Non-Dominated Heat Blocking Efficiencies and Back Surface Temperatures, $\dot{Q}_{\text{TOT.}} = 10$ kW/m$^2$, $\dot{Q}_{\text{CONV.}}/\dot{Q}_{\text{TOT.}} = 0.2$.

**Figure 4-2.** Optimized objective functions, $\dot{Q}_{\text{TOT.}} = 10$ kW/m$^2$ and $\dot{Q}_{\text{CONV.}}/\dot{Q}_{\text{TOT.}} = 0.2$. 
Non-Dominated Heat Blocking Efficiencies and Back Surface Temperatures, $\dot{Q}_{\text{TOT.}} = 10 \text{ kW/m}^2$, $\dot{Q}_{\text{CONV.}}/\dot{Q}_{\text{TOT.}} = 0.3$.

Figure 4-3. Optimized objective functions, $\dot{Q}_{\text{TOT.}} = 10\text{ kW/m}^2$ and $\dot{Q}_{\text{CONV.}}/\dot{Q}_{\text{TOT.}} = 0.3$. 
Non-Dominated Heat Blocking Efficiencies and Back
Surface Temperatures, $\dot{Q}_{\text{TOT.}} = 10$ kW/m$^2$, $\dot{Q}_{\text{CONV.}}/\dot{Q}_{\text{TOT.}} = 0.4$

Figure 4-4. Optimized objective functions, $\dot{Q}_{\text{TOT.}} = 10$ kW/m$^2$ and $\dot{Q}_{\text{CONV.}}/\dot{Q}_{\text{TOT.}} = 0.4$. 
Figure 4-5. Optimized objective functions, $\dot{Q}_{TOT} = 10\text{ kW/m}^2$ and $\dot{Q}_{CONV}/\dot{Q}_{TOT} = 0.5$.

4.3.1.2 50 kW/m$^2$ incident heat flux

Running the algorithm with 50 kW/m$^2$ of total incident heat revealed behavior that was qualitatively very similar. Figures 4-6 through 4-9 show the results of running the MOGA with 50 kW/m$^2$ of incident heat and various fractions of the total heat being convection. For these results, $k/t = 100\text{ W/m}^2\text{-K}$, $h = 5\text{ W/m}^2\text{-K}$, $T_H = 300\text{ K}$, and $T_\infty = 300\text{K}$. The qualitative behavior is the same as that for 10 kW/m$^2$ of incident heat, except that shifts in behavior occur at lower values of $\dot{Q}_{CONV}/\dot{Q}_{TOT}$. For example, the reflective scheme is already dominated by at least some emissive schemes when $\dot{Q}_{CONV}/\dot{Q}_{TOT} = 0.1$ and is dominated by the entire emissive front when $\dot{Q}_{CONV}/\dot{Q}_{TOT} = 0.4$. 

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Non-Dominated Heat Blocking Efficiencies and Back Surface Temperatures, $\dot{Q}_{TOT} = 50 \text{ kW/m}^2$, $\dot{Q}_{CONV}/\dot{Q}_{TOT} = 0.1$.

**Figure 4-6.** Optimized objective functions, $\dot{Q}_{TOT} = 50\text{ kW/m}^2$ and $\dot{Q}_{CONV}/\dot{Q}_{TOT} = 0.1$. 
Non-Dominated Heat Blocking Efficiencies and Back Surface Temperatures, $\dot{Q}_{TOT.} = 50$ kW/m$^2$, $\dot{Q}_{CONV.}/\dot{Q}_{TOT.} = 0.2$

Figure 4-7. Optimized objective functions, $\dot{Q}_{TOT.} = 50$ kW/m$^2$ and $\dot{Q}_{CONV.}/\dot{Q}_{TOT.} = 0.2$. 
Figure 4-8. Optimized objective functions, $\dot{Q}_{TOT} = 50 \text{kW/m}^2$ and $\dot{Q}_{\text{CONV}}/\dot{Q}_{TOT} = 0.3$. 
Figure 4-9. Optimized objective functions, $\dot{Q}_{TOT} = 50 \text{ kW/m}^2$ and $\dot{Q}_{CONV}/\dot{Q}_{TOT} = 0.4$.

4.3.2 Superposition of outputs

In Figures 4-1 through 4-5, the front of emissive schemes appears in the same place. The only dependence of this front on the type of heat input is its lower bound, which is given by the heat blocking efficiency of the reflective scheme. Therefore, these images can be superimposed, to show the entire emissive front and the various reflective schemes. The performance of the reflective scheme depends on both the total flux of incident heat and the fraction of that heat which is convection or radiation. The emissive front, however, only depends upon the total flux of incident heat. The entire optimal front is then given by this emissive front and the reflective scheme corresponding to the particular value of $\dot{Q}_{CONV}/\dot{Q}_{TOT}$. If there are solutions on the emissive front which provide a lower heat blocking efficiency and higher back surface temperature than the
reflective scheme at that particular set of incident heat conditions, they are obviously dominated and not part of the Pareto-optimal front. This superposition is shown below as Figure 4-10.

![Graph](image-url)

**Figure 4-10.** Superposition of Figures 4-1 through 4-5.

As with the 10kW/m² case, Figures 4-6 through 4-9 can be superimposed to show the range of emissive schemes for 50kW/m² incident heat flux, which are valid for all values of $\dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT}}$, along with a sample of the reflective schemes, only one of which is valid for each value of $\dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT}}$. Figure 4-11, which follows, is such a superposition of Figures 4-6 through 4-9.
Non-Dominated Heat Blocking Efficiencies and Back Surface Temperatures, $\dot{Q}_{TOT.} = 50 \text{ kW/m}^2$, $\dot{Q}_{CONV.}/\dot{Q}_{TOT.} = 0.1$ to 1

**Figure 4-11.** Superposition of Figures 4-6 through 4-9.

### 4.3.3 Optimal front

Since the front and back emissivities of all of the non-dominated solutions were known after optimization, Figure 4-10 could be reproduced based on the governing equations. The two surface temperatures of the single reflective scheme are given by the following equations:

$$
\dot{Q}_{CONV} = \frac{k}{t} (T_{F,REF} - T_{B,REF}) \quad \text{and} \quad (4-1)
$$

$$
\frac{k}{t} (T_{F,REF} - T_{B,REF}) = \sigma (T_{B,REF}^4 - T_H^4) + h (T_{B,REF} - T_H). \quad (4-2)
$$

The heat blocking efficiency of the reflective scheme is given by

$$
\eta_{B,REF} = \frac{\dot{Q}_{RAD}}{\dot{Q}_{RAD} + \dot{Q}_{CONV}}. \quad (4-3)
$$
Similarly, the two surface temperatures of the set of emissive tradeoff solutions are given by the following equations:

\[
\dot{Q}_{RAD} + \dot{Q}_{CONV} = \sigma \left( T_{F,EM}^4 - T_\infty^4 \right) + \frac{k}{t} \left( T_{F,EM} - T_{B,EM} \right) \quad \text{and} \quad (4-4)
\]

\[
\frac{k}{t} \left( T_{F,EM} - T_{B,EM} \right) = \varepsilon B \sigma \left( T_{B,EM}^4 - T_H^4 \right) + h \left( T_{B,EM} - T_H \right). \quad (4-5)
\]

The heat blocking efficiency of these emissive schemes is given by

\[
\eta_{B,EM} = \frac{\sigma T_{F,EM}^4}{\dot{Q}_{RAD} + \dot{Q}_{CONV}}. \quad (4-6)
\]

Using these equations, Figure 4-12 was constructed as a continuous version of Figure 4-10. In this plot, the front of emissive tradeoff solutions, the behavior of which is the same regardless of the type of incident heat, is shown as a solid line. The behavior of the reflective scheme, which does change with the type of incident heat, is shown as a dotted line. At each set of incident heat conditions, there is some reflective scheme which may dominate all, some, or none of the emissive tradeoff solutions. Similar to Figure 4-12, Figure 4-13 is a continuous version of Figure 4-11.
Figure 4-12. Reflective (dotted line) and emissive (solid line) fronts, 10kW/m² total incident heat flux.
Optimized Heat Blocking Efficiencies and Back Surface Temperatures, $\dot{Q}_{\text{TOT}} = 50$ kW/m$^2$

**Figure 4-13.** Reflective (dotted line) and emissive (solid line) fronts, 50kW/m$^2$ total incident heat flux.
Figure 4-14 shows the same information as Figures 4-12 and 4-13, along with the same curves for $\dot{Q}_{\text{TOT}} = 20\text{kW/m}^2$.

![Optimized Heat Blocking Efficiencies and Back Surface Temperatures, $\dot{Q}_{\text{TOT}} = 10, 20, \text{and } 50\text{ kW/m}^2$](image)

**Figure 4-14.** Reflective and emissive fronts for various amounts of incident heat flux.

### 4.4 Discussion

#### 4.4.1 Discussion of findings

The most important contribution of this double-objective optimization to the entire study is the description of the optimal front contained in §4.3.3.

It was found that, for each set of incident heat conditions, there exists a single front-reflective scheme and a continuous set of front-emissive schemes. The front-reflective scheme consists of a reflective front surface and an emissive back surface. The emissive schemes consist of an emissive front surface, and a back surface emissivity ranging from unity (with the highest heat blocking efficiency and highest back surface
temperature) to zero (with the lowest heat blocking efficiency and lowest back surface temperature). While the performance of the reflective scheme depends on both the total amount of incident heat and what fraction of that heat is radiation or convection, the set of emissive schemes depends only on the total flux of incident heat.

If the incident heat is almost entirely radiation, the reflective scheme dominates the entire set of emissive tradeoff solutions. The threshold of “almost entirely radiation” is dependent upon the total flux of incident heat. As can be seen in Figure 4-13, the reflective scheme dominates all of the emissive tradeoff solutions at $\dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT}} < \sim 15\%$ for 10 kw/m$^2$ of incident heat, at $\dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT}} < \sim 10\%$ for 20 kW/m$^2$ of incident heat, and at $\dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT}} < \sim 5\%$ for 50 kW/m$^2$ of incident heat.

For incident heat conditions in which $\dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT}}$ is greater than these values, but in which the incident heat is still mostly radiation, the reflective scheme dominates only a portion of the set of emissive schemes. Here again, the threshold of “mostly radiation” depends on the total flux of incident heat. Figure 4-13 shows that the reflective scheme dominates only some of the emissive tradeoff solutions at $\dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT}} < \sim 47\%$ for 10 kw/m$^2$ of incident heat, at $\dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT}} < \sim 42\%$ for 20 kW/m$^2$ of incident heat, and at $\dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT}} < \sim 33\%$ for 50 kW/m$^2$ of incident heat.

For incident heat conditions in which $\dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT}}$ is greater than these values, the set of emissive schemes forms the entirety of the optimal front, and the reflective scheme is dominated.

4.4.2 Comparison with experimental results in Hsu et al. [2]

At this point it was deemed prudent to pause and compare the results found in this study with available experimental results. To the knowledge of the author, the only
published experimental data recording heat blocking efficiencies and surface temperatures of fire blankets with a variety of configurations are contained in a study by Hsu et al. [2]. In that study, fiber glass fabric in one or two layers with aluminized surfaces in different configurations were studied. The table below describes the six cases studied.

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Single layer, no aluminized surfaces</td>
</tr>
<tr>
<td>2</td>
<td>Single layer, front surface aluminized</td>
</tr>
<tr>
<td>1’</td>
<td>Double layer, no aluminized surfaces</td>
</tr>
<tr>
<td>2’</td>
<td>Double layer, front surface of front layer aluminized</td>
</tr>
<tr>
<td>3’</td>
<td>Double layer, back surface of back layer aluminized</td>
</tr>
<tr>
<td>4’</td>
<td>Double layer, front surface of front layer and back surface of back layer aluminized.</td>
</tr>
</tbody>
</table>

**Table 4-1.** Description of the configurations tested by Hsu et al. [2].

In order to compare the results of exposing the materials in these cases to incident heat in a laboratory with the trends found in the present work, each case must be described in terms of the present model. It was assumed that a plain fiberglass surface had a moderate to high emissivity (between 0.5 and 0.9). It was further assumed that the aluminized surfaces had a low emissivity (less than 0.1). Hsu et al. noted that at high temperature the aluminum would melt or otherwise disappear, leaving a darkened surface [2]. Therefore, the aluminized surfaces were assumed to have an emissivity close to the plain fiberglass surfaces if the surface temperature was very high (more than 900K). Based on these estimates for emissivity, the various cases can be described in terms of reflective and emissive schemes. The table below shows these descriptions.
Table 4-2. Description of the configurations tested by Hsu et al. [2] in terms of reflective and emissive schemes.

In Hsu et al. [2], these materials were each subjected to two types of incident heat flux: 83 kW/m$^2$ of convection (using a Meker burner) and 83 kW/m$^2$ of radiation (using a radiant cone). The resulting steady-state heat blocking efficiencies and surface temperatures were recorded. These are presented in the tables below, along with the respective optimal fronts. For comparison with the optimal fronts, it was assumed that $h = 6.9$ W/m$^2$-K, $T_H = 300$ K, and $T_\infty = 300$K. It was further assumed that $k/t = 100$ W/m$^2$-K for the single layers, and that $k/t = 30$ W/m$^2$-K for the double layers. The actual value of the average thermal conductivity of the material was not determined in Hsu et al. [2], as the width of the air gap between the two blankets was not known to a sufficient degree of accuracy. The authors of that paper assumed that this gap of air was approximately 0.5 mm [2].

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Description in terms of present model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mid-range emissive scheme ($\varepsilon_B \sim 0.5-0.8$)</td>
</tr>
<tr>
<td>2</td>
<td>Reflective scheme (Mid-range emissive scheme for high $T_F$)</td>
</tr>
<tr>
<td>1'</td>
<td>Mid-range emissive scheme ($\varepsilon_B \sim 0.5-0.8$), lower $k/t$</td>
</tr>
<tr>
<td>2'</td>
<td>Reflective scheme, lower $k/t$ (Mid-range emissive scheme for high $T_F$)</td>
</tr>
<tr>
<td>3'</td>
<td>Upper-range emissive scheme ($\varepsilon_B &lt; 0.1$)</td>
</tr>
<tr>
<td>4'</td>
<td>Neither scheme, both surfaces reflective (Upper-range emissive scheme for high $T_F$)</td>
</tr>
</tbody>
</table>
Figure 4-15. Experimental results of exposing the configurations in Tables 4-1 and 4-2 to 83 kW/m² of radiant heat flux, compared with optimal front.

When exposed to radiative incident heat flux, the three cases with aluminized front surfaces, cases 2, 2’, and 4’, had the highest heat blocking efficiencies. Cases 2 and 2’ were non-ideal but realistic approximations of the reflective scheme. Thus, they had the lowest back surface temperatures and very high heat blocking efficiencies. Case 4’ was neither a reflective nor an emissive scheme. Its aluminized front surface gave it a heat blocking efficiency in the range of the reflective scheme and of cases 2 and 2’, but its aluminized back surface drove up the back surface temperature into the range of cases 1, 1’, and 3’. Cases 1, 1’, and 3’ represented non-ideal but realistic approximations of different emissive schemes. No emissive solutions appeared on the optimal front because the incident heat was entirely radiative. Therefore, it was not surprising that cases 1, 1’,
and 3’ were dominated by the solutions approximating the reflective scheme (cases 2 and 2’).

**Figure 4-16.** Experimental results of exposing the configurations in Tables 4-1 and 4-2 to 83 kW/m² of convective heat flux, compared with optimal front.

When exposed to 83 kW/m² of convective incident heat flux from the Meker burner, the front surface temperatures were high enough to cause the aluminized front surfaces in cases 2, 2’, and 4’ to become emissive. Because of these high temperatures, both cases 1 and 2 were non-ideal emissive schemes which fell behind the optimal front comprised of emissive trade-off solutions. A different optimal front was used for comparison with the double layer configurations, since the value of $k/t$ was significantly lower for those configurations. It was seen that the two double layer configurations with an aluminized back surface (cases 3’ and 4’) had higher heat blocking efficiencies and
higher back surface temperatures than those without an aluminized back surface (cases 1’ and 2’). This agrees with the results found in the double-objective optimization study, in which emissive schemes with various back surface emissivities represented trade-off solutions in high heat blocking efficiency and low back surface temperature.

As in §2.4.1, it should be noted here that Hsu et al. [2] noted in their study that of the three types of data collected (heat blocking efficiency, front surface temperature, and back surface temperature), the heat blocking efficiency data was most reliable. With large heat fluxes and temperature gradients, the temperature data collected by thermocouples became dependent upon contact pressure and position. Nonetheless, the experimental data presented in Hsu et al. [2] support the conclusions drawn in the present study.
5. Multi-Objective Optimization for Time-Varying Heat Input

5.1 Introduction

The object of this last part of the study was to demonstrate that the quasi-steady-state model developed in §2 could be optimized for a time-varying heat input which represented an approaching and passing exterior fire. Rather than solving the two governing equations (Equations 2-3 and 2-4) for one set of incident heat conditions, a whole sequence of incident heat conditions representing the fire were used, and the resulting performance characteristics were recorded at short time intervals.

Fires in wildland-urban interface areas which behave differently will produce different sequences of incident heat conditions. Rather than attempting to model a real wildfire approaching a building, a simpler approach was taken. The normal distribution was used to create a ten-minute long sequence of values for the total incident heat flux, by setting the mean, \( \mu \), equal to 300 seconds, and the standard deviation, \( \sigma \), equal to 100 seconds. This could be easily normalized to produce whatever peak heat flux was to be studied.

Four different sequences of incident heat conditions were chosen for analysis. These sequences were sampled at ten-second intervals, so that the entire ten-minute sequence consisted of analysis at 61 incident heat conditions. Each of these four sequences had a peak total incident heat flux, \( \dot{Q}_{TOT} \), of 100 kW/m². “Sequence 1” maintained a constant value for \( \dot{Q}_{CONV}/\dot{Q}_{TOT} \) of 0.1. “Sequence 2” maintained a constant value for \( \dot{Q}_{CONV}/\dot{Q}_{TOT} \) of 0.5. “Sequence 3” consisted of completely radiative heat flux (\( \dot{Q}_{CONV}/\dot{Q}_{TOT} = 0 \)) from time zero up to 280 seconds, or the first 29 steps in the sequence. From 290 seconds until 310 seconds, the incident heat flux was entirely convective.
\( \dot{Q}_{\text{conv}}/\dot{Q}_{\text{tot.}} = 1 \). From 320 seconds until the end of the sequence, the incident heat flux was again entirely radiative \( \dot{Q}_{\text{conv}}/\dot{Q}_{\text{tot.}} = 0 \). This gave an overall total value for \( Q_{\text{conv}}/Q_{\text{tot.}} \) of about 0.119 (note that these are \( Q \) and not \( \dot{Q} \) – total amounts of heat over the sequence and not instantaneous quasi-steady heat fluxes). “Sequence 4” consisted of completely radiative heat flux \( \dot{Q}_{\text{conv}}/\dot{Q}_{\text{tot.}} = 0 \) from time zero up to 230 seconds, or the first 24 steps in the sequence. From 240 seconds until 360 seconds, the incident heat flux was entirely convective \( \dot{Q}_{\text{conv}}/\dot{Q}_{\text{tot.}} = 1 \). From 370 seconds until the end of the sequence, the incident heat flux was again entirely radiative \( \dot{Q}_{\text{conv}}/\dot{Q}_{\text{tot.}} = 0 \). This gave an overall total value for \( Q_{\text{conv}}/Q_{\text{tot.}} \) of about 0.484. Figures 5-1 through 5-4, which follow, are depictions of these four input sequences. The lines in these figures were constructed from the 61 evenly-spaced points used for analysis in the algorithm, without any smoothing added by the graphing software. It is seen that this number of samples was sufficient to produce smooth curves.
Figure 5-1. Input “Sequence 1,” constant $\dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT}} = 0.1$. 
Figure 5-2. Input “Sequence 2,” constant $\dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT}} = 0.5$. 
Figure 5-3. Input “Sequence 3,” overall total $Q_{\text{CONV}}/Q_{\text{TOT}} = 0.119$. 
Figure 5-4. Input “Sequence 4,” overall total $\frac{Q_{\text{CONV}}}{Q_{\text{TOT.}}} = 0.484$.

For these sequences of incident heat conditions, the governing equations (Equations 2-3 and 2-4) were solved every ten seconds across the entire ten-minute sequence. It is recognized by the author that these input sequences are rather arbitrary and are not derived from actual fire data. It is important to note here that the object of this study was not to optimize a fire blanket for application to some observed, real fire, but rather to demonstrate the ability to do so, given data corresponding to the type of wildfire common in the area of proposed application.

This study was divided into two parts. Both parts of the study sought a population of non-dominated solutions for each of the four input sequences just discussed. Part 1 of the study assumed that no properties of the fire blanket would change over the course of the input sequence. Thus, the two variables were still the front and back surface
emissivities, as before. It was found in experiment, however, that the front surface of the fire blanket material often changes in appearance at very high temperature, when either the front surface of the blanket material or some coating changes color or some added layer of aluminum melts and falls away [2,13-17]. Part 2 of this study was an attempt to consider this phenomenon. Once the front surface temperature exceeded some value, $T_{CH}$, the front surface emissivity was changed from $\varepsilon_F$ to $\varepsilon_{F,CH}$ for the remainder of the input sequence. Both $T_{CH}$ and $\varepsilon_{F,CH}$ were left as variables to be optimized.

For all of the optimizations performed in this study, as for all previous optimizations, $k/t = 100 \text{ W/m}^2\text{-K}$, $h = 5 \text{ W/m}^2\text{-K}$, $T_H = 300 \text{ K}$, and $T_\infty = 300 \text{ K}$.

5.2 Method, Part 1: Constant properties

5.2.1 Objective functions

In the previous optimization study, the objective functions were steady-state values: the heat blocking efficiency, $\eta_B$, and the back surface temperature, $T_B$. In this study, however, the incident heat conditions were time dependent. Thus, some new objective functions, analogous to the heat blocking efficiency and back surface temperature, were needed.

Instead of using the previous heat blocking efficiency, $\eta_B$, which was a ratio of quasi-steady heat fluxes, the total heat blocking efficiency was used. This was a ratio of the total amount of heat blocked from reaching the structure to the total amount of incident heat over the course of the entire input sequence:

$$H_B = \frac{\int_{\tau_i}^{\tau_f} \dot{Q}_{\text{BLOCKED}} \, dt}{\int_{\tau_i}^{\tau_f} \dot{Q}_{\text{TOT.}} \, dt} = \frac{\int_{\tau_i}^{\tau_f} \eta_B \dot{Q}_{\text{TOT.}} \, dt}{\int_{\tau_i}^{\tau_f} \dot{Q}_{\text{TOT.}} \, dt}. \quad (5-1)$$
For the discrete sampling used in the algorithm, the integrals in Equation 5-1 were replaced with left-sided Riemann sums.

Similarly, the steady-state back surface temperature, $T_B$, which was used as an objective function in the previous study, was replaced with the peak back surface temperature over the course of the entire input sequence, $T_{B,\text{PEAK}}$.

### 5.2.2 Optimization algorithm

The same MOGA used in the previous study (and described in §1.3.3.2) was used again, with the new objective functions. Only slight modifications were needed to calculate to total heat blocking efficiency, $H_B$, and the peak back surface temperature, $T_{B,\text{PEAK}}$, from the quasi-steady heat blocking efficiency, $\eta_B$, and the quasi-steady back surface temperature, $T_B$.

### 5.3 Results, Part 1: Constant properties

Figure 5-5 shows the total heat blocking efficiencies, $H_B$, and peak back surface temperatures, $T_{B,\text{PEAK}}$, for the non-dominated solutions given by the MOGA with input “Sequence 1” (described in §5.1 and depicted in Figure 5-1). Table 5-1 is provided to show the values for the front and back surface emissivities which produce these non-dominated solutions. It was seen that the same reflective scheme found in the previous optimization study (§4) again appeared. The high-temperature end of an emissive front is also visible. It was noted that Figure 5-5 is qualitatively very similar to Figure 4-6, in which a reflective scheme dominated most, but not all, of the emissive front.
Figure 5-5. Optimized objective functions, “Sequence 1,” constant $\dot{Q}_{\text{CONF}}/\dot{Q}_{\text{TOT}} = 0.1$, constant front surface emissivity.

<table>
<thead>
<tr>
<th>$H_B$</th>
<th>$T_{B,\text{PEAK}}$ (K)</th>
<th>$\varepsilon_F$</th>
<th>$\varepsilon_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.940</td>
<td>1104</td>
<td>0.908</td>
<td>0.000</td>
</tr>
<tr>
<td>0.940</td>
<td>1102</td>
<td>0.928</td>
<td>0.001</td>
</tr>
<tr>
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<td>1102</td>
<td>0.930</td>
<td>0.001</td>
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<td>0.924</td>
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<td>0.931</td>
<td>0.022</td>
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<td>0.900</td>
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<tr>
<td>0.900</td>
<td>628.1</td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 5-1. Non-dominated solutions in Figure 5-5.

Figure 5-6 shows the total heat blocking efficiencies, $H_B$, and peak back surface temperatures, $T_{B,\text{PEAK}}$, for the non-dominated solutions given by the MOGA with input “Sequence 2” (described in §5.1 and depicted in Figure 5-2). Table 5-2 is provided to show the values for the front and back surface emissivities which produce these non-dominated solutions. It is seen in the plot and in the table that the full range of emissive schemes, representing an emissive front, is present. This is qualitatively very similar to
the results of the double-objective optimization for a constant set of incident heat conditions presented in §4.

![Non-Dominated Peak Back Surface Temperatures and Total Heat Blocking Efficiencies, Input Sequence No. 2, Constant Properties](image)

**Figure 5-6.** Optimized objective functions, “Sequence 2,” constant $\dot{Q}_{\text{CONV.}}/\dot{Q}_{\text{TOT.}} = 0.5$, constant front surface emissivity.
Table 5-2. Non-dominated solutions in Figure 5-6.

<table>
<thead>
<tr>
<th>$H_B$</th>
<th>$T_{B,PEAK}$ (K)</th>
<th>$\epsilon_F$</th>
<th>$\epsilon_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.940</td>
<td>1107</td>
<td>0.962</td>
<td>0.000</td>
</tr>
<tr>
<td>0.940</td>
<td>1109</td>
<td>0.948</td>
<td>0.000</td>
</tr>
<tr>
<td>0.940</td>
<td>1107</td>
<td>0.962</td>
<td>0.000</td>
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<td>0.894</td>
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<td>0.810</td>
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<td>0.800</td>
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<tr>
<td>0.798</td>
<td>928.3</td>
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<td>0.773</td>
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<td>0.768</td>
<td>901.8</td>
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<td>0.758</td>
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<td>0.727</td>
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<td>0.930</td>
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<tr>
<td>0.702</td>
<td>849.3</td>
<td>0.710</td>
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<tr>
<td>0.691</td>
<td>809.1</td>
<td>0.956</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Figure 5-7 shows the total heat blocking efficiencies, $H_B$, and peak back surface temperatures, $T_{B,PEAK}$, for the non-dominated solutions given by the MOGA with input “Sequence 3” (described in §5.1 and depicted in Figure 5-3). Table 5-3 is provided to show the values for the front and back surface emissivities which produce these non-dominated solutions. Here again, the entire emissive front is seen, with the addition of two very high-temperature solutions. These two solutions had very low front and back surface emissivities, and fit into neither the reflective nor emissive optimal schemes described in §3.3.1. Because these two solutions offered such a small gain in total heat blocking efficiency yet operated at such great temperatures, they were disregarded as unfeasible and unimportant solutions. Figure 5-8 is a detail of Figure 5-7 which excludes these high-temperature solutions, and presents the entire non-dominated emissive front in a better scale.
Figure 5-7. Optimized objective functions, “Sequence 3,” overall average 
\[ \dot{Q}_{\text{CONF}}/\dot{Q}_{\text{TOT}} = 0.119, \] constant front surface emissivity.
Figure 5-8. Detail of Figure 5-7, showing the emissive front.
Table 5-3. Non-dominated solutions in Figures 5-7 and 5-8.

Figure 5-9 shows the total heat blocking efficiencies, $H_B$, and peak back surface temperatures, $T_{B,PEAK}$, for the non-dominated solutions given by the MOGA with input “Sequence 4” (described in §5.1 and depicted in Figure 5-4). Table 5-4 is provided to show the values for the front and back surface emissivities which produce these non-dominated solutions. Yet again, the entire emissive front was non-dominated, and the results are qualitatively similar to those presented in the previous optimization study (§4).
Figure 5-9. Optimized objective functions, “Sequence 4,” overall average

\[ \frac{Q_{\text{CONF}}}{Q_{\text{TOT}}} = 0.484, \text{ constant front surface emissivity.} \]

<table>
<thead>
<tr>
<th>( H_B )</th>
<th>( T_{B,\text{PEAK}} (\text{K}) )</th>
<th>( \varepsilon_F )</th>
<th>( \varepsilon_B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.940</td>
<td>1103</td>
<td>0.995</td>
<td>0.000</td>
</tr>
<tr>
<td>0.939</td>
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<td>0.001</td>
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<td>0.369</td>
</tr>
<tr>
<td>0.781</td>
<td>908.1</td>
<td>1.000</td>
<td>0.412</td>
</tr>
<tr>
<td>0.745</td>
<td>876.9</td>
<td>0.932</td>
<td>0.593</td>
</tr>
<tr>
<td>0.743</td>
<td>866.7</td>
<td>0.984</td>
<td>0.613</td>
</tr>
<tr>
<td>0.740</td>
<td>864.6</td>
<td>0.980</td>
<td>0.627</td>
</tr>
<tr>
<td>0.736</td>
<td>860.3</td>
<td>0.978</td>
<td>0.652</td>
</tr>
<tr>
<td>0.729</td>
<td>858.3</td>
<td>0.934</td>
<td>0.694</td>
</tr>
<tr>
<td>0.708</td>
<td>838.8</td>
<td>0.896</td>
<td>0.848</td>
</tr>
</tbody>
</table>

Table 5-4. Non-dominated solutions in Figure 5-9.
5.4 Method, Part 2: Temperature-dependent front surface emissivity

5.4.1 Objective functions

The objective functions were the same as those in Part 1 of this study: the total heat blocking efficiency, $H_B$, and the peak back surface temperature, $T_{B,PEAK}$.

5.4.2 Optimization algorithm

The same MOGA which was used in the previous study and in Part 1 of this study was used again. Only slight modifications were needed to incorporate the additional variables $T_{CH}$ and $\varepsilon_{F,CH}$, which are the temperature at which the front surface emissivity is altered, and the front surface emissivity after that change.

5.5 Results, Part 2: Temperature-dependent front surface emissivity

Figure 5-10, which follows, shows the total heat blocking efficiencies, $H_B$, and peak back surface temperatures, $T_{B,PEAK}$, for the non-dominated solutions given by the MOGA with input “Sequence 1” (described in §5.1 and depicted in Figure 5-1). Table 5-5 is provided to show the values for the front and back surface emissivities which produce these non-dominated solutions. These results are nearly identical to those given in Figure 5-5 for the same input sequence using a constant front surface emissivity.

There is a single reflective scheme solution, which dominates most, but not all, of an emissive front. The upper end of the emissive front presented here is found to be less than perfectly emissive. However, the location of the non-reflective scheme solutions, and the fact that their front surface emissivity is greater than 0.65 in all cases, justify their classification as part of a front of emissive schemes. As with Figure 5-5, note the qualitative similarity between Figures 5-10 and 4-6.
Figure 5-10. Optimized objective functions, “Sequence 1,” constant

\[ \dot{Q}_{\text{CONV}} / \dot{Q}_{\text{TOT}} = 0.1 \], non-constant front surface emissivity.

<table>
<thead>
<tr>
<th>( H_B )</th>
<th>( T_{B,\text{PEAK}} ) (K)</th>
<th>( T_{CH} )</th>
<th>( \varepsilon_F )</th>
<th>( \varepsilon_B )</th>
<th>( \varepsilon_{F,CH} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.940</td>
<td>1110</td>
<td>337.5</td>
<td>0.423</td>
<td>0.000</td>
<td>0.669</td>
</tr>
<tr>
<td>0.940</td>
<td>1109</td>
<td>337.5</td>
<td>0.400</td>
<td>0.000</td>
<td>0.671</td>
</tr>
<tr>
<td>0.939</td>
<td>1109</td>
<td>337.5</td>
<td>0.399</td>
<td>0.001</td>
<td>0.670</td>
</tr>
<tr>
<td>0.937</td>
<td>1105</td>
<td>337.5</td>
<td>0.399</td>
<td>0.004</td>
<td>0.687</td>
</tr>
<tr>
<td>0.930</td>
<td>1093</td>
<td>337.5</td>
<td>0.405</td>
<td>0.014</td>
<td>0.740</td>
</tr>
<tr>
<td>0.926</td>
<td>1085</td>
<td>337.5</td>
<td>0.395</td>
<td>0.021</td>
<td>0.839</td>
</tr>
<tr>
<td>0.900</td>
<td>628.1</td>
<td>337.5</td>
<td>0.254</td>
<td>1.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 5-5. Non-dominated solutions in Figure 5-10.

Figure 5-11, which follows, shows the total heat blocking efficiencies, \( H_B \), and peak back surface temperatures, \( T_{B,\text{PEAK}} \), for the non-dominated solutions given by the MOGA with input “Sequence 2” (described in §5.1 and depicted in Figure 5-2).

Table 5-6 is provided to show the values for the front and back surface emissivities which produce these non-dominated solutions. These results are nearly identical to those given
in Figure 5-6 for the same input sequence using a constant front surface emissivity. As in Figure 5-6, a fully developed emissive front is visible. Not all of the non-dominated solutions have initial front surface emissivity values close to one. However, those non-dominated solutions with lower initial front emissivity ($\varepsilon_F$) values also have a lower value for $T_{CH}$ (“lower” meaning close to the ambient 300K). This means that $T_{CH}$ was quickly exceeded, and the front surface emissivity switched from the $\varepsilon_F$ value to the $\varepsilon_{F,CH}$ value early in the sequence. The values for $\varepsilon_{F,CH}$ are invariably close to one.

**Figure 5-11.** Optimized objective functions, “Sequence 2,” constant $\dot{Q}_{CONV}/\dot{Q}_{TOT} = 0.5$, non-constant front surface emissivity.
<table>
<thead>
<tr>
<th>$H_B$</th>
<th>$T_{B,PEAK}$ (K)</th>
<th>$T_{CH}$</th>
<th>$\varepsilon_F$</th>
<th>$\varepsilon_B$</th>
<th>$\varepsilon_{F,CH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.940</td>
<td>1111</td>
<td>302.8</td>
<td>0.317</td>
<td>0.000</td>
<td>0.931</td>
</tr>
<tr>
<td>0.937</td>
<td>1108</td>
<td>303.0</td>
<td>0.328</td>
<td>0.003</td>
<td>0.932</td>
</tr>
<tr>
<td>0.937</td>
<td>1108</td>
<td>303.0</td>
<td>0.325</td>
<td>0.003</td>
<td>0.933</td>
</tr>
<tr>
<td>0.937</td>
<td>1104</td>
<td>1070</td>
<td>0.826</td>
<td>0.003</td>
<td>0.959</td>
</tr>
<tr>
<td>0.926</td>
<td>1085</td>
<td>345.1</td>
<td>0.306</td>
<td>0.019</td>
<td>0.986</td>
</tr>
<tr>
<td>0.921</td>
<td>1078</td>
<td>1046</td>
<td>0.799</td>
<td>0.026</td>
<td>0.997</td>
</tr>
<tr>
<td>0.904</td>
<td>1067</td>
<td>311.0</td>
<td>0.467</td>
<td>0.051</td>
<td>0.919</td>
</tr>
<tr>
<td>0.870</td>
<td>1031</td>
<td>1141</td>
<td>0.827</td>
<td>0.113</td>
<td>0.935</td>
</tr>
<tr>
<td>0.867</td>
<td>1023</td>
<td>316.5</td>
<td>0.472</td>
<td>0.121</td>
<td>0.899</td>
</tr>
<tr>
<td>0.864</td>
<td>1017</td>
<td>300.0</td>
<td>0.449</td>
<td>0.130</td>
<td>0.914</td>
</tr>
<tr>
<td>0.859</td>
<td>1011</td>
<td>302.9</td>
<td>0.466</td>
<td>0.142</td>
<td>0.912</td>
</tr>
<tr>
<td>0.852</td>
<td>1003</td>
<td>303.3</td>
<td>0.468</td>
<td>0.158</td>
<td>0.906</td>
</tr>
<tr>
<td>0.815</td>
<td>957.9</td>
<td>1060</td>
<td>0.757</td>
<td>0.260</td>
<td>0.934</td>
</tr>
<tr>
<td>0.815</td>
<td>951.9</td>
<td>1053</td>
<td>0.756</td>
<td>0.267</td>
<td>0.989</td>
</tr>
<tr>
<td>0.806</td>
<td>946.8</td>
<td>1060</td>
<td>0.763</td>
<td>0.293</td>
<td>0.933</td>
</tr>
<tr>
<td>0.801</td>
<td>941.0</td>
<td>1060</td>
<td>0.748</td>
<td>0.309</td>
<td>0.940</td>
</tr>
<tr>
<td>0.791</td>
<td>929.1</td>
<td>1057</td>
<td>0.766</td>
<td>0.347</td>
<td>0.946</td>
</tr>
<tr>
<td>0.774</td>
<td>909.9</td>
<td>1070</td>
<td>0.737</td>
<td>0.418</td>
<td>0.939</td>
</tr>
<tr>
<td>0.747</td>
<td>872.2</td>
<td>511.8</td>
<td>0.563</td>
<td>0.581</td>
<td>0.968</td>
</tr>
<tr>
<td>0.697</td>
<td>812.7</td>
<td>597.7</td>
<td>0.401</td>
<td>0.958</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 5-6. Non-dominated solutions in Figure 5-11.

Figure 5-12, which follows, shows the total heat blocking efficiencies, $H_B$, and peak back surface temperatures, $T_{B,PEAK}$, for the non-dominated solutions given by the MOGA with input “Sequence 3” (described in §5.1 and depicted in Figure 5-3).

Table 5-7 is provided to show the values for the front and back surface emissivities which produce these non-dominated solutions. As in the previous Figure 5-7, some very high-temperature solutions appear in the set of non-dominated solutions. These fall into neither a reflective scheme nor an emissive scheme, and they offer only a very slight advantage in performance over the other solutions in the non-dominated set. On the basis of this slight advantage and the gain of at least 2000K in peak back surface temperature over the other solutions, these are disregarded as unfeasible and not useful. Figure 5-13
is a detail of Figure 5-12 which excludes these high-temperature solutions, and shows the optimal front in a better scale.

Figure 5-13 shows an optimal front of solutions that cannot be easily classified as either reflective or emissive schemes. All of these solutions begin the sequence with very low front surface emissivities ($\varepsilon_F$), indicating reflective front surfaces. Most of them, however, do not have the emissive back surface of the reflective scheme as defined in §3.3.1. Since “Sequence 3” begins with a period of exposure to purely radiative incident heat flux, however, almost all of this heat was blocked by the initially reflective front surfaces, leaving the back surface emissivities with little effect on the surface temperatures. Once the incident heat flux switched to complete convection, all of the nearly 100kW/m$^2$ incident heat flux was absorbed at the front surface for an instant, driving the front surface temperatures above $T_{CH}$. At this point the $\varepsilon_F$ values for the front surface emissivities were replaced with the $\varepsilon_{F,CH}$ values, which are nearly one for all of the solutions present in Figure 5-13. From this point in the input sequence until its completion, these solutions were emissive schemes. These results show that the MOGA took advantage of the input sequence and the ability to change the front surface emissivity by initially reflecting all of the incident radiant heat flux, and then switching to an emissive scheme when the incident heat flux became convective.
Figure 5-12. Optimized objective functions, “Sequence 3,” overall average $\dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT}} = 0.119$, non-constant front surface emissivity.
Figure 5-13. Detail of Figure 5-7, disregarding the four unfeasible non-dominated solutions.

Table 5-7. Non-dominated solutions in Figures 5-12 and 5-13.

<table>
<thead>
<tr>
<th>$H_B$</th>
<th>$T_{B,PEAK}$ (K)</th>
<th>$T_{CH}$</th>
<th>$\varepsilon_F$</th>
<th>$\varepsilon_B$</th>
<th>$\varepsilon_{F,CH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.974</td>
<td>3666</td>
<td>453.9</td>
<td>0.000</td>
<td>0.000</td>
<td>0.006</td>
</tr>
<tr>
<td>0.973</td>
<td>3638</td>
<td>444.8</td>
<td>0.573</td>
<td>0.000</td>
<td>0.007</td>
</tr>
<tr>
<td>0.972</td>
<td>3384</td>
<td>454.3</td>
<td>0.465</td>
<td>0.000</td>
<td>0.009</td>
</tr>
<tr>
<td>0.968</td>
<td>2912</td>
<td>451.7</td>
<td>0.443</td>
<td>0.000</td>
<td>0.018</td>
</tr>
<tr>
<td>0.965</td>
<td>1133</td>
<td>524.9</td>
<td>0.006</td>
<td>0.000</td>
<td>0.893</td>
</tr>
<tr>
<td>0.965</td>
<td>1133</td>
<td>524.9</td>
<td>0.006</td>
<td>0.000</td>
<td>0.893</td>
</tr>
<tr>
<td>0.965</td>
<td>1132</td>
<td>526.9</td>
<td>0.006</td>
<td>0.000</td>
<td>0.896</td>
</tr>
<tr>
<td>0.962</td>
<td>1126</td>
<td>1188</td>
<td>0.010</td>
<td>0.004</td>
<td>0.902</td>
</tr>
<tr>
<td>0.959</td>
<td>1114</td>
<td>1200</td>
<td>0.010</td>
<td>0.012</td>
<td>0.913</td>
</tr>
<tr>
<td>0.951</td>
<td>1079</td>
<td>1222</td>
<td>0.000</td>
<td>0.043</td>
<td>0.941</td>
</tr>
<tr>
<td>0.947</td>
<td>1071</td>
<td>1276</td>
<td>0.001</td>
<td>0.053</td>
<td>0.936</td>
</tr>
<tr>
<td>0.919</td>
<td>1032</td>
<td>1061</td>
<td>0.041</td>
<td>0.100</td>
<td>0.970</td>
</tr>
<tr>
<td>0.908</td>
<td>997.3</td>
<td>1055</td>
<td>0.019</td>
<td>0.176</td>
<td>0.941</td>
</tr>
<tr>
<td>0.882</td>
<td>930.8</td>
<td>1300</td>
<td>0.001</td>
<td>0.389</td>
<td>0.887</td>
</tr>
<tr>
<td>0.840</td>
<td>831.0</td>
<td>1300</td>
<td>0.003</td>
<td>0.823</td>
<td>0.998</td>
</tr>
<tr>
<td>0.838</td>
<td>830.7</td>
<td>1220</td>
<td>0.008</td>
<td>0.823</td>
<td>1.000</td>
</tr>
<tr>
<td>0.832</td>
<td>810.5</td>
<td>1193</td>
<td>0.000</td>
<td>0.977</td>
<td>1.000</td>
</tr>
<tr>
<td>0.827</td>
<td>807.7</td>
<td>1217</td>
<td>0.008</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Figure 5-14, which follows, shows the total heat blocking efficiencies, $H_B$, and peak back surface temperatures, $T_{B,\text{peak}}$, for the non-dominated solutions given by the MOGA with input “Sequence 4” (described in §5.1 and depicted in Figure 5-4). Table 5-8 is provided to show the values for the front and back surface emissivities which produce these non-dominated solutions. An optimal front very similar to that found in the previous Figure 5-13 is visible. Once again, all of these solutions begin the sequence with very low front surface emissivities ($\varepsilon_F$), indicating reflective front surfaces. A range of back surface emissivities is present. All of the values for the front surface emissivity after $T_{CH}$ has been exceeded on the front surface ($\varepsilon_{F,CH}$) are nearly one. As before, the MOGA took advantage of the input sequence and the ability to change the front surface emissivity by initially reflecting all of the incident radiant heat flux, and then switching to an emissive scheme when the incident heat flux became convective.
Figure 5-14. Optimized objective functions, “Sequence 4,” overall average

\[ \frac{\dot{Q}_{\text{CONV}}}{\dot{Q}_{\text{TOT}}} = 0.484 \], non-constant front surface emissivity.
<table>
<thead>
<tr>
<th>$H_B$</th>
<th>$T_{B,PEAK}$ (K)</th>
<th>$T_{CH}$</th>
<th>$\varepsilon_F$</th>
<th>$\varepsilon_B$</th>
<th>$\varepsilon_{F,CH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.963</td>
<td>1103</td>
<td>596.1</td>
<td>0.001</td>
<td>0.000</td>
<td>0.995</td>
</tr>
<tr>
<td>0.962</td>
<td>1102</td>
<td>1284</td>
<td>0.004</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.961</td>
<td>1100</td>
<td>1286</td>
<td>0.004</td>
<td>0.002</td>
<td>1.000</td>
</tr>
<tr>
<td>0.955</td>
<td>1093</td>
<td>1300</td>
<td>0.002</td>
<td>0.016</td>
<td>0.974</td>
</tr>
<tr>
<td>0.951</td>
<td>1089</td>
<td>1289</td>
<td>0.018</td>
<td>0.014</td>
<td>1.000</td>
</tr>
<tr>
<td>0.946</td>
<td>1069</td>
<td>662.7</td>
<td>0.000</td>
<td>0.037</td>
<td>0.988</td>
</tr>
<tr>
<td>0.943</td>
<td>1064</td>
<td>666.5</td>
<td>0.000</td>
<td>0.045</td>
<td>0.994</td>
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<tr>
<td>0.897</td>
<td>992</td>
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<td>0.167</td>
<td>0.991</td>
</tr>
<tr>
<td>0.894</td>
<td>1038</td>
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<td>0.287</td>
<td>0.081</td>
<td>0.998</td>
</tr>
<tr>
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<td>1169</td>
<td>0.014</td>
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<td>1.000</td>
</tr>
<tr>
<td>0.878</td>
<td>954.8</td>
<td>1147</td>
<td>0.020</td>
<td>0.256</td>
<td>1.000</td>
</tr>
<tr>
<td>0.877</td>
<td>950.9</td>
<td>1033</td>
<td>0.016</td>
<td>0.268</td>
<td>1.000</td>
</tr>
<tr>
<td>0.872</td>
<td>940.3</td>
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<td>0.012</td>
<td>0.299</td>
<td>1.000</td>
</tr>
<tr>
<td>0.843</td>
<td>884.3</td>
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<td>0.001</td>
<td>0.518</td>
<td>0.995</td>
</tr>
<tr>
<td>0.841</td>
<td>920.2</td>
<td>953.0</td>
<td>0.047</td>
<td>0.413</td>
<td>0.914</td>
</tr>
<tr>
<td>0.833</td>
<td>867.1</td>
<td>1252</td>
<td>0.002</td>
<td>0.604</td>
<td>0.994</td>
</tr>
<tr>
<td>0.828</td>
<td>863.6</td>
<td>1238</td>
<td>0.007</td>
<td>0.634</td>
<td>0.977</td>
</tr>
<tr>
<td>0.808</td>
<td>858.3</td>
<td>929.0</td>
<td>0.048</td>
<td>0.696</td>
<td>0.931</td>
</tr>
<tr>
<td>0.800</td>
<td>843.2</td>
<td>1074</td>
<td>0.052</td>
<td>0.775</td>
<td>0.951</td>
</tr>
<tr>
<td>0.796</td>
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<td>1042</td>
<td>0.000</td>
<td>0.819</td>
<td>1.000</td>
</tr>
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<td>1148</td>
<td>0.050</td>
<td>0.910</td>
<td>0.955</td>
</tr>
<tr>
<td>0.784</td>
<td>818.1</td>
<td>998.5</td>
<td>0.016</td>
<td>0.916</td>
<td>1.000</td>
</tr>
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<td>0.784</td>
<td>814.4</td>
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<td>0.000</td>
<td>0.959</td>
<td>0.984</td>
</tr>
<tr>
<td>0.780</td>
<td>813.2</td>
<td>1289</td>
<td>0.061</td>
<td>1.000</td>
<td>0.945</td>
</tr>
</tbody>
</table>

**Table 5-8.** Non-dominated solutions in Figure 5-14.

**Note on the relative values of $T_{B,PEAK}$ and $T_{CH}$.** It can be seen many times in Tables 5-5 through 5-8 that the value of $T_{CH}$ exceeds that of $T_{B,PEAK}$. This does not mean that the front surface temperature did not exceed $T_{CH}$. Once a set of incident heat conditions applied to a fire blanket with a front surface emissivity of $\varepsilon_F$ resulted in a front surface temperature in excess of $T_{CH}$, the front surface emissivity was changed to $\varepsilon_{F,CH}$ and the temperatures and heat blocking efficiency were re-evaluated at that time-step. This means that the front surface temperature which first exceeded $T_{CH}$ was not recorded, and thus the peak back surface temperature might be significantly lower than $T_{CH}$. A record of whether or not $T_{CH}$ was exceeded was necessarily kept, however, and in every
non-dominated solution presented in this section, the front surface temperature did exceed $T_{CH}$ during the course of the input sequence.

5.6 Discussion

5.6.1 Comparison of Parts 1 and 2

Figure 5-15 below is a superposition of the previous Figures 5-5 and 5-10. It shows the results of the MOGA using a constant front surface emissivity along with those from the MOGA allowed to change the front surface emissivity for the input sequence called “Sequence 1” (described in §5.1 and depicted in Figure 5-1). The superposition of these two sets of non-dominated solutions shows that they lie on the same optimal front. This meant that there was no advantage or disadvantage to switching the front surface emissivity once the front surface temperature exceeded some $T_{CH}$.

![Graph showing superposition of non-dominated solutions](image)

**Figure 5-15.** Superposition of Figures 5-5 and 5-10.
Figure 5-16 below is a superposition of the previous Figures 5-6 and 5-11. It shows the results of the MOGA using a constant front surface emissivity along with those from the MOGA allowed to change the front surface emissivity for the input sequence called “Sequence 2” (described in §5.1 and depicted in Figure 5-2). Much like the previous Figure 5-15, this plot shows that there is no advantage or disadvantage to switching the front surface emissivity once the front surface temperature exceeded some value $T_{CH}$. 

![Non-Dominated Peak Back Surface Temperatures and Total Heat Blocking Efficiencies, Input Sequence No. 2, Superposition of Results](image)

**Figure 5-16.** Superposition of Figures 5-6 and 5-11.
Figure 5-17 below is a superposition of the previous Figures 5-8 and 5-13. It shows the results of the MOGA using a constant front surface emissivity along with those from the MOGA allowed to change the front surface emissivity for the input sequence called “Sequence 3” (described in §5.1 and depicted in Figure 5-3). Unlike the two previous Figures 5-15 and 5-16, this plot shows that there is a significant advantage to switching the front surface emissivity once the front surface temperature exceeded some value $T_{CH}$.

Figure 5-17. Superposition of Figures 5-8 and 5-13.
Figure 5-18 below is a superposition of the previous Figures 5-9 and 5-14. It shows the results of the MOGA using a constant front surface emissivity along with those from the MOGA allowed to change the front surface emissivity for the input sequence called “Sequence 4” (described in §5.1 and depicted in Figure 5-4). As with the previous Figure 5-17, this plot shows that there is a significant advantage to switching the front surface emissivity once the front surface temperature exceeded some $T_{CH}$.

![Figure 5-18](image)

**Figure 5-18.** Superposition of Figures 5-9 and 5-14.

For the two input sequences which maintained a constant value for $\dot{Q}_{\text{conv}}/\dot{Q}_{\text{tot}}$ throughout the duration of the sequence (“Sequence 1” and “Sequence 2”), the non-dominated solutions from both the constant front surface emissivity and the non-constant front surface emissivity algorithms lied on the same optimal front. Because $\dot{Q}_{\text{conv}}/\dot{Q}_{\text{tot}}$ was the same throughout the entire sequence, the incident heat conditions always favored
the same scheme, reflective or emissive. Therefore, there was no advantage to be found in allowing the scheme to change mid-sequence by allowing the substitution of a new front surface emissivity once some thermal condition was met.

For the two input sequences which were composed of purely radiative incident heat flux interrupted by a stretch of purely convective incident heat flux (“Sequence 3” and “Sequence 4”), the non-dominated solutions from the non-constant front surface emissivity algorithm out-performed those from the algorithm restricted to a constant front surface emissivity. The change from radiative to convective incident heat flux meant that a reflective front surface was no longer optimal, and an emissive scheme was best. The algorithm restricted to a constant front surface emissivity for the duration of the sequence was unable to respond to these changing conditions mid-sequence, and was forced to converge upon a population of emissive schemes as the optimal front. The algorithm which was allowed to change the front surface emissivity was able to reflect the first exposure to radiative incident heat flux, and then switch to an emissive scheme once exposed to the convective incident heat flux.

Figures 5-19 and 5-20 were created to demonstrate the difference in performance between blankets with a constant and non-constant front surface emissivity when exposed to input “Sequence 3.” Figure 5-19 was created using the last non-dominated solution presented in Table 5-3. Figure 5-20 was created using the last non-dominated solution presented in Table 5-7. In Figure 5-19, the front and back surface emissivities are both near one for the duration of the sequence. This is a low-temperature, low-efficiency emissive scheme, which is on the bottom end of the emissive front presented in Figures 5-7 and 5-8. In Figure 5-20, the front surface emissivity is almost zero initially,
and almost all of the incident heat is reflected. Once the incident heat flux switches from being radiative to convective, the front surface emissivity becomes nearly one, and the same emissive scheme seen in Figure 5-19 is produced for the remainder of the sequence. By comparing the two plots, it becomes obvious how a blanket with a front surface emissivity that changes outperforms one with a constant front surface emissivity. Note that the heat blocking efficiency presented in these plots is not the total heat blocking efficiency ($H_B$), but instantaneous heat blocking efficiency ($\eta_B$) as a function of time.

Figure 5-19. Incident heat fluxes, surface temperatures, and heat blocking efficiency as functions of time, “Sequence 3,” constant front surface emissivity.
Figure 5-20. Incident heat fluxes, surface temperatures, and heat blocking efficiency as functions of time, “Sequence 3,” non-constant front surface emissivity.

5.6.2 Discussion of findings

It was found that for a time-variant sequence of incident heat conditions, the optimal front and back surface emissivities can be classified as reflective or emissive schemes when all properties of the blanket are constant and independent of temperature. Occasionally, some solutions which cannot be classified as reflective or emissive appear in the optimal front, but these operate at exceedingly high temperatures and offer only a slight advantage in total heat blocking efficiency over the most efficient emissive scheme. Further, the optimal fronts created by these reflective and emissive schemes are qualitatively very similar to those found in §4.
Solutions on those optimal fronts may be dominated when the front surface emissivity is allowed to change once the front surface temperature exceeds some value, $T_{CH}$, depending on the nature of the sequence of incident heat conditions. If the ratio of convective to total incident heat flux, $Q_{\text{CONV.}}/Q_{\text{TOT.}}$, is constant for the duration of the input sequence, then allowing this change in the front surface emissivity affords no gain in performance. If the incident heat flux is first dominated by radiation and later switches to being dominated by convection, then allowing the front surface emissivity to change once the front surface temperature exceeds some value results in higher total heat blocking efficiencies for the same peak back surface temperature.

The sequences of incident heat conditions used in this optimization study were devised for the purpose of demonstrating the ability to optimize fire blanket performance for a time-variant sequence of heat inputs. They were meant to represent incident heat fluxes found in a wildland fire, but they were not collected from experiment or observation. Actual observed data regarding convective and radiative heat fluxes as functions of time in a wildfire would be unique to each wildfire. Further, the author is not aware of any such data having been collected prior to this study. It can be hypothesized that seasonal fires in wildland-urban interface areas are roughly similar over time in each location, depending on the surroundings. Therefore, if the convective and radiative heat fluxes provided by such wildfires were recorded as functions of time, an optimization such as those presented in this study could determine the optimal properties of a fire blanket for use in that location.
6. Summary and Conclusions

6.1 Nature of the study

Three optimization studies have been conducted on a one-dimensional, quasi-steady-state heat transfer model. The goal of these studies was to optimize the performance of a fire blanket for use in protecting a structure from an exterior fire. Each of these studies took the front and back surface emissivities of the blanket as variables. The incident heat conditions, expressed as a total incident heat flux and a ratio of convective to total incident heat flux, were input parameters. The temperature of the air surrounding the blanket-covered-structure, the temperature of the structure, the thermal conductivity and thickness of the blanket, and the convective coefficient between the back surface of the blanket and the structure were all left as parameters.

The effectiveness of the blanket was quantified by the heat blocking efficiency. This was the ratio of incident heat prevented from reaching the structure to the total amount of incident heat. Due to the quasi-steady nature of the model, incident heat could be prevented from reaching the structure by two means. First, incident radiative heat flux could be reflected by a front surface with a very low emissivity. Second, any incident heat flux could be absorbed, raising the front surface temperature. This elevated temperature could then be used to re-radiate the absorbed heat back toward the fire by a front surface with a very high emissivity. Thus, reflection and the re-emission of absorbed heat were the two means of blocking incident heat from being transmitted through to the structure.
6.2 Single-objective optimization

In the first optimization study, the only objective was a high heat blocking efficiency. Two optimal schemes were found, which operated on the two modes of blocking heat just discussed. A reflective scheme had a reflective front surface and an emissive back surface. The reflective front surface reflected all incident radiative heat flux. The emissive back surface had no effect on the heat blocking efficiency, but allowed any absorbed heat to be transferred to the structure at the lowest possible temperature. The emissive scheme had an emissive front surface and a reflective back surface. The emissive front surface absorbed all incident heat flux. The reflective back surface prevented this heat from being radiated toward the structure, and drove the front and back surface temperatures up to their highest possible values. The hot, emissive front surface then radiated much of the absorbed heat back toward the fire, greatly increasing the heat blocking efficiency.

It was found that the reflective scheme offered the highest heat blocking efficiency only when the incident heat flux was dominated by radiation. When there was an appreciable amount of convective heat flux present in the incident heat conditions, the emissive scheme was most effective. A method was developed to predict whether the reflective or emissive scheme would be most efficient. This method compared the front surface temperature given by the emissive scheme to the minimum temperature required to achieve the reflective heat blocking efficiency by means of the emissive scheme (called the “switch temperature”).

Finally, in this first study, the sensitivity of the performance of the blanket to the various input parameters was tested. It was found that there was a set of trade-off
emissive schemes which operated at lower temperatures but offered lower heat blocking efficiencies. This was done by increasing the back surface emissivity from zero (reflective) across the range to one (emissive).

6.3 Double-objective optimization

Many of the optimal solutions given by the first optimization study operated at very high temperatures, well in excess of the melting point of aluminum (a commonly used coating) and beyond the softening point of fiberglass (a fairly common material for this application). Further, a very high back surface temperature could cause the structure to ignite. In response, the second optimization study had two objective functions: high heat blocking efficiency and low back surface temperature. A Multi-Objective Genetic Algorithm (MOGA) was used to find a population of optimal trade-off solutions for these two objectives. Sets of non-dominated solutions were found for various incident heat conditions. These non-dominated solutions were then used to characterize the optimal fronts based on the incident heat conditions. A method was developed to explicitly define those optimal fronts.

6.4 Time-variant heat input

The third optimization study applied a time-varying sequence of incident heat conditions to the MOGA used in the previous study. The total heat blocking efficiency replaced the previous heat blocking efficiency as an objective function. The total heat blocking efficiency was a ratio of the total amount of heat per unit area blocked from reaching the structure to the total amount of heat per unit area incident upon the front surface of the blanket for the duration of the sequence. The previous heat blocking efficiency was a ratio of fluxes. The previous objective of low back surface temperature
was replaced with a low peak back surface temperature. It was found that the same optimal fronts described in the previous double-objective optimization study appeared here as well.

In a second part of this last study, the front surface emissivity was allowed to change once the front surface temperature exceeded some value. This was intended to mimic the charring, melting, or vaporization of the front surface or front surface coating of the blanket, which has been observed in experimental studies [2,13-17]. Both the temperature at which the front surface emissivity changes and the front surface emissivity after the change were variables. For sequences of incident heat conditions which maintained a constant ratio of convective to total heat flux, the same optimal fronts found before appeared again. For sequences which were at first dominated by radiative heat flux, and then switched to being dominated by convective heat flux, allowing the front surface emissivity to change resulted in a new set of non-dominated solutions. These solutions out-performed those found when the front surface emissivity was constant throughout the sequence.

6.5 Future work

Future work based on this research could take three directions:

- The results could be applied toward the development of better fire blankets for the protection of structures from wildland fires,

- Closer approximations of the optimal solutions found in this research could be developed in a laboratory and tested in an attempt to both confirm the results of this study and develop methods of producing a more ideal fire blanket, and
• The same three optimization studies could be repeated with a more advanced heat transfer model, such as that contained in Hsu et al. [2] in an attempt to repeat the results.

Instantaneous radiative and total heat fluxes could be recorded as functions of time in both wildland fires and prescribed burns. These data could be used as time-varying heat inputs for an optimization in the manner of that contained in §5. Solutions which fall on the resulting optimal fronts could serve as guidelines for manufacturers. At present, it is difficult to control the emissivity of a surface beyond adding a layer of aluminum to make it reflective. Further variables, such as \( T_{CH} \), are further beyond the control of manufacturers. Nonetheless, these guidelines could serve as goals for manufacturers to attain to with their present methods.

In a laboratory setting, where the method of production does not need to be nearly as economical as in manufacturing, fire blankets could be produced which are much closer approximations of the optimal solutions found in this study through the use of various coatings beyond aluminum. These could be tested by the same methods used in previous experimental studies [2,13-17] in an attempt to experimentally produce the results of the optimization study contained in §4. In the process of making fire blankets which are closer to the ideal schemes, it could happen that an economical method of controlling the surface emissivities of a fibrous material.

Finally, the same three studies reported in this paper could be repeated with a more advanced heat transfer model. The model used in this research was intentionally simple, both for ease of optimization and so that the results could be more easily applied in manufacturing. For the purposes of reproducing these results, however, a heat transfer
model such as that presented in Hsu et al. [2], which incorporates several variables
describing the scattering of thermal radiation within a fibrous material, could be used.
Appendix A: solver.m

This is the MATLAB function which was used to find the quasi-steady front and back surface temperatures, along with the quasi-steady heat blocking efficiency, of a fire blanket exposed to a set of incident heat conditions. The incident heat conditions, ambient temperature, temperature of the structure, thermal conductivity and thickness of the blanket, front and back surface emissivities of the blanket, and the convective coefficient between the back surface of the blanket and the structure were all input parameters. The function began with an initial guess for the surface temperatures. The first governing equation (Equation 2-3) was then solved explicitly for the front surface temperature, using the initial guess for the back surface temperature. The second governing equation (Equation 2-4) was solved explicitly for the back surface temperature, using the new value for the front surface temperature. This process was iterated until the sum of the change in both temperatures between iterations was less than 0.001K. The quasi-steady heat blocking efficiency was calculated once the function converged on a set of surface temperatures.

```matlab
function [HBE T] = solver(Qtot,QconvQtot,Tinf,Th,h,e,kt)
%SOLVER takes the various inputs and finds the heat blocking efficiency and %surface temperatures of the fire blanket described by e and kt when %exposed to the incident heat conditions described by Qtot, QconvQtot, %Tinf, and h. -- K. Brent, Cleveland, June 3rd, 2011

%Note the Stefan-Boltzmann Constant
s = 5.6704e-008;
%Write the incident heat conditions in terms of radiation and convection
Qconv = Qtot*QconvQtot;
Qrad = Qtot-Qconv;
%Establish some initial values
T = [1000 1000];
delta = inf;
iter = 0;
%Iterate!
while delta > 0.001 && iter < 600
```

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\[ P_1(1) = e(1) s; \]
\[ P_1(4) = kt; \]
\[ P_1(5) = -(e(1) s T_{inf}^4 + kt T(2) + e(1) Q_{rad} + Q_{conv}); \]
\[ R_1 = \text{roots}(P_1); \]
\[ \text{for } i = 1:\text{length}(R_1) \]
\[ \quad \text{if } R_1(i) > 0 \]
\[ \quad \quad \text{if real}(R_1(i)) == R_1(i) \]
\[ \quad \quad \quad T_{new} = R_1(i); \]
\[ \quad \end{if} \]
\[ \end{for} \]
\[ \text{delta} = \text{abs}(T(1) - T_{new}); \]
\[ T(1) = T_{new}; \]

\[ P_2(1) = e(2) s; \]
\[ P_2(4) = kt + h; \]
\[ P_2(5) = -(kt T(1) + e(2) s T_h^4 + h T_h); \]
\[ R_2 = \text{roots}(P_2); \]
\[ \text{for } i = 1:\text{length}(R_2) \]
\[ \quad \text{if } R_2(i) > 0 \]
\[ \quad \quad \text{if real}(R_2(i)) == R_2(i) \]
\[ \quad \quad \quad T_{new} = R_2(i); \]
\[ \quad \end{if} \]
\[ \end{for} \]
\[ \text{delta} = \text{delta} + \text{abs}(T(2) - T_{new}); \]
\[ T(2) = T_{new}; \]

\[ \text{iter} = \text{iter} + 1; \]
\[ \end{end} \]

\[ HBE = 1 - (kt * (T(1) - T(2))) / (Q_{rad} + Q_{conv}); \]
Appendix B: MOGA2.m

This is a MATLAB program which used a MOGA to find the optimal front in terms of heat blocking efficiency and back surface temperature for a given set of incident heat conditions. The function solver.m was used to evaluate the fitness of each potential solution. The aggressiveness of the simulated binary crossover (SBX) operator was controlled by the variable “hc.” The lower this number, the more aggressively the SBX operator searched for new solutions. Similarly, the aggressiveness of the non-uniform mutation operator was controlled by the variable “b.” The lower this number, the more slowly the range of possible mutations converged upon a delta function.

clear all
c1c

%MOGA2 uses a Multiple Objective Genetic Algorithm to optimize a population
%of fire blankets. The objective functions are (1) high heat blocking efficiency and (2) low back surface temperature. The variables are the front and back surface emissivities. The parameters describing the thermal conductivity, thickness of the fire blanket, incident heat conditions, and convection to the structure are left as inputs.
% -- K. Brent, Cleveland, June 3rd, 2011
tic

%Set some constants describing the blanket and incident conditions
kt = 100;           %k/t in W/m^2*K
Qtot = 50000;       %Qtot in W/m^2
QconvQtot = 0.4;    %Qconv/Qtot
Th = 300;           %in K
Tinf = 100;           %in K
h = 5;              %in W/m^2*K

%Initialize a random population
%set population size
pop = 100;  %***MUST BE AN EVEN NUMBER***
%choose some surface emissivities
e = rand(2,pop);

%Set number of iterations
itermax = 1500;

%Initialize a record of heat blocking efficiencies
HBE = zeros(1,pop);
%Initialize a record of temperatures (K)
T = zeros(2,pop);
%Initialize a record of dominations
dominated = zeros(1,pop);

%Initialize a record of fitness
fitness = zeros(1,pop);

%Set some parameters for the shared fitness
alpha = 1;
sigShare = 0.01;
%Initialize some values for the shared fitness
distance = 0;
nc = 0;
thmax = 0;
thmin = 0;

%Initialize some vectors for the Stochastic Universal Selection Operator
Fsum = 0;
R = zeros(1,pop);
P = zeros(1,pop);

%Choose a parameter for the Simulated Binary Crossover Operator
hc = 5;

%Choose a parameter for the Non-Uniform Mutation Operator
b = 2;

%Initialize the next generation
newe = zeros(2,pop);

%Iterate!
for iter = 1:itermax

%Evaluate the performance of each solution
for i = 1:pop
    [HBE(i) T(:,i)] = solver(Qtot,QconvQtot,Tinf,Th,h,e(:,i),kt);
end

%Find the number of times each solution is dominated
for i = 1:pop
    dominated(i) = 0;
    for j = 1:pop
        if HBE(j)>HBE(i) && T(2,j)<T(2,i)
            dominated(i) = dominated(i) + 1;
        end
        if HBE(j)==HBE(i) && T(2,j)<T(2,i)
            dominated(i) = dominated(i) + 1;
        end
        if HBE(j)>HBE(i) && T(2,j)==T(2,i)
            dominated(i) = dominated(i) + 1;
        end
    end
end
end

% Assign a fitness based on the number of times each solution is dominated and enforce some constraints
for i = 1:pop
    fitness(i) = (pop-dominated(i))/pop;
    if HBE(i) < 0
        fitness(i) = 0;
    end
end

% Reevaluate the fitness based on a niching function
HBEmax = max(HBE);
TBmax = max(T(2,:));
HBEmin = min(HBE);
TBmin = min(T(2,:));
for i = 1:pop
    nc = 0;
    for j = 1:pop
        distance = (((HBE(i)-HBE(j))/(HBEmax-HBEmin))^2+(T(2,i)-T(2,j))/(TBmax-TBmin))^2)^0.5;
        if distance <= sigShare
            nc = nc + (1-(distance/sigShare)^alpha);
        end
    end
    fitness(i) = fitness(i)/nc;
end

% Stochastic Universal Selection Operator
% determine the probability table
Fsum = sum(fitness);
P(1) = fitness(1)/Fsum;
% establish a random string
R(1) = rand();
for i = 2:pop
    P(i) = P(i-1) + fitness(i)/Fsum;
    R(i) = mod(R(i-1) + 1/pop,1);
end
% choose solutions for the new generation
for i = 1:pop
    if R(i) >= 0 && R(i) <= P(1)
        newe(:,i) = e(:,1);
    end
    for j = 2:pop
        if R(i) > P(j-1) && R(i) <= P(j)
            newe(:,i) = e(:,j);
        end
    end
end
% establish the new generation
e = newe;

% Simulated Binary Crossover Operator
for i = 1:2:pop
    % perform the crossover for the front surface emissivity
    u = rand();
if u<=0.5
    bq = (2*u)^(1/(hc+1));
else
    bq = (1/(2*(1-u)))^(1/(hc+1));
end

temp(1,1) = 0.5*(((1+bq)*e(1,i)+(1-bq)*e(1,i+1));
temp(1,2) = 0.5*(((1-bq)*e(1,i)+(1+bq)*e(1,i+1));

%perform the crossover for the back surface emissivity
u = rand();
if u<=0.5
    bq = (2*u)^(1/(hc+1));
else
    bq = (1/(2*(1-u)))^(1/(hc+1));
end

temp(2,1) = 0.5*(((1+bq)*e(2,i)+(1-bq)*e(2,i+1));
temp(2,2) = 0.5*(((1-bq)*e(2,i)+(1+bq)*e(2,i+1));

%record the crossover
e(1,i) = temp(1,1);
e(2,i) = temp(2,1);
e(1,i+1) = temp(1,2);
e(2,i+1) = temp(2,2);

% Be sure to keep the emissivities between 0 and 1.
for i = 1:pop
    for j = [1 2]
        if e(j,i)<0
            e(j,i) = 0;
        end
        if e(j,i)>1
            e(j,i) = 1;
        end
    end
end

%Non-Uniform Mutation Operator
for i = 1:pop
    for j = [1 2]
        tau = 2*round(rand())-1;
        e(j,i) = e(j,i)+tau*(1-rand()^((1-iter/itermax)^b));
        if e(j,i)<0
            e(j,i) = 0;
        end
        if e(j,i)>1
            e(j,i) = 1;
        end
    end
end

% Recalculate the fitness after the last iteration
% Evaluate the performance of each solution
for i = 1:pop
    [HBE(i) T(:,i)] = solver(Qtot, QconvQtot, Tinf, Th, h, e(:,i), k);
end

% Find the number of times each solution is dominated
for i = 1:pop
    dominated(i) = 0;
    for j = 1:pop
        if HBE(j)>HBE(i) && T(2,j)<T(2,i)
            dominated(i) = dominated(i) + 1;
        end
        if HBE(j)==HBE(i) && T(2,j)<T(2,i)
            dominated(i) = dominated(i) + 1;
        end
        if HBE(j)>HBE(i) && T(2,j)==T(2,i)
            dominated(i) = dominated(i) + 1;
        end
    end
end

%Assign a fitness based on the number of times each solution is dominated and enforce some constraints
for i = 1:pop
    fitness(i) = (pop-dominated(i))/pop;
    if HBE(i) < 0
        fitness(i) = 0;
    end
end

%Reevaluate the fitness based on a niching function
HBEmax = max(HBE);
TBMmax = max(T(2,:));
HBEmin = min(HBE);
TBmin = min(T(2,:));
for i = 1:pop
    nc = 0;
    for j = 1:pop
        distance = (((HBE(i)-HBE(j))/(HBEmax-HBEmin))^2+((T(2,i)-T(2,j))/(TBMmax-TBmin))^2)^0.5;
        if distance <= sigShare
            nc = nc + (1-(distance/sigShare)^alpha);
        end
    end
    fitness(i) = fitness(i)/nc;
end

frontNum = length(find(dominated==0));
onDe = zeros(frontNum,2);
onDT = zeros(frontNum,2);
onDHBE = zeros(frontNum,1);

j = 1;
for i = 1:pop
    if dominated(i)==0
        nonDe(j,1) = e(1,i);
        nonDe(j,2) = e(2,i);
        nonDT(j,1) = T(1,i);
        nonDT(j,2) = T(2,i);
        nonDHBE(j) = HBE(i);
        j = j+1;
    end
end

plot(nonDT(:,2),nonDHBE,'bx');
xlabel('Back surface temperature, K');
ylabel('Heat blocking efficiency');
toc
Appendix C: MOGA3trans.m

This is a MATLAB program which used the same MOGA as MOGA2.m to find the optimal front in terms of total heat blocking efficiency and peak back surface temperature of a fire blanket when exposed to a sequence of time-varying incident heat conditions. In addition to the front and back surface emissivities, this algorithm took the front surface temperature at which the front surface emissivity changes, $T_{CH}$, and the front surface emissivity after that temperature is exceeded, $\varepsilon_{F}$, as two additional variables.

clear all
%clc

%MOGA3trans is a multi-objective genetic algorithm. The objective functions are: low back surface temperature and high $Q_{blocked}/Q_{total}$. The variables are: front and back surface emissivity, temperature at which the front surface emissivity changes ($T_{ch}$), and the front surface emissivity after the change ($\varepsilon_{Fch}$). All properties are assumed to be constant, except for the front surface emissivity. Once $T_{ch}$ is exceeded, $\varepsilon_{F}$ is replaced with $\varepsilon_{Fch}$. The blanket is exposed to a sequence of incident heat conditions which represent a fire.
% -- K. Brent, CWRU, Cleveland, July 22, 2011

tic

%Create input sequence based on normal distribution
t = 0:10:600;
Qtot = (1000000/normpdf(300,300,100)).*normpdf(t,300,100);
QconvQtot = zeros(1,length(t));
QconvQtot(t>=240) = 1;
QconvQtot(t>360) = 0;
%Set some constants describing the blanket and incident conditions
kt = 100; % k/t in W/m^2*K
Th = 300; %in K
Tinf = 300; %in K
h = 5; %in W/m^2*K

%Initialize a random population
%set population size
pop = 100; %***MUST BE AN EVEN NUMBER***
%choose some surface emissivities
e = rand(2,pop);
% e(1,:) = zeros(1,pop);
%choose some temperatures for the front emissivity to change
Tch = 1000*rand(1,pop)+300;
%choose the front emissivity after Tch is exceeded
% Set number of iterations
itermax = 500;

% Initialize a record of dominations
dominated = zeros(1,pop);

% Set some parameters for the shared fitness
alpha = 1;
sigShare = 0.015;

% Initialize some values for the shared fitness
distance = 0;
nc = 0;

% Initialize some vectors for the Stochastic Universal Selection Operator
Fsum = 0;
R = zeros(1,pop);
P = zeros(1,pop);

% Choose a parameter for the Simulated Binary Crossover Operator
hc = 5;

% Choose a parameter for the Non-Uniform Mutation Operator
b = 2;

% Initialize the next generation
newe = zeros(2,pop);
newTch = zeros(1,pop);
neweFch = zeros(1,pop);

% Iterate!
for iter = 1:itermax
    % record whether Tch has been exceeded
    ch = zeros(1,pop);

    % Evaluate the performance of each solution
    QbQt = zeros(1,pop);
    TBpeak = zeros(1,pop);
    for i = 1:pop
        for j = 1:length(tt)
            if ch(i)==0
                [HBE T] = solver(Qtot(j),QconvQtot(j),Tinf,Th,h,e(:,i),kt);
            end
            if ch(i)==1
                [HBE T] = solver(Qtot(j),QconvQtot(j),Tinf,Th,h,[eFch(i);e(2,i)],kt);
            end
            if ch(i)==0 && T(1)>Tch(i)
                ch(i) = 1;
                [HBE T] = solver(Qtot(j),QconvQtot(j),Tinf,Th,h,[eFch(i);e(2,i)],kt);
            end
        end
    end
end
QbQt(i) = QbQt(i)+(5*Qtot(j)*HBE);
if T(2)>TBpeak(i)
    TBpeak(i) = T(2);
end
end
QbQt(i) = QbQt(i)/(5*sum(Qtot));
end

%Find the number of times each solution is dominated
for i = 1:pop
    dominated(i) = 0;
    for j = 1:pop
        if QbQt(j)>=QbQt(i) && TBpeak(j)<=TBpeak(i)
            dominated(i) = dominated(i) + 1;
        end
        if QbQt(j)==QbQt(i) && TBpeak(j)==TBpeak(i)
            dominated(i) = dominated(i) - 1;
        end
    end
end

%Assign a fitness based on the number of times each solution is dominated and enforce some constraints
fitness = (pop-dominated)./pop;

%Reevaluate the fitness based on a niching function
QbQtmax = max(QbQt);
TBpeakmax = max(TBpeak);
QbQtmin = min(QbQt);
TBpeakmin = min(TBpeak);
for i = 1:pop
    nc = 0;
    for j = 1:pop
        d(1) = ((QbQt(i)-QbQt(j))/(QbQtmax-QbQtmin))^2;
        d(2) = ((TBpeak(i)-TBpeak(j))/(TBpeakmax-TBpeakmin))^2;
        distance = sum(d);
        if distance <= sigShare
            nc = nc + (1-(distance/sigShare)^alpha);
        end
    end
    fitness(i) = fitness(i)/nc;
end

%Filter out solutions which fail to utilize eFch
for i = 1:pop
    if ch(i)==0;
        fitness(i) = 0;
    end
end

%Stochastic Universal Selection Operator
%determine the probability table
Fsum = sum(fitness);
P(1) = fitness(1)/Fsum;
%establish a random string
R(1) = rand();
for i = 2:pop
    P(i) = P(i-1) + fitness(i)/Fsum;
    R(i) = mod(R(i-1) + 1/pop,1);
end
%choose solutions for the new generation
for i = 1:pop
    if R(i) >= 0 && R(i) <= P(1)
        newe(:,i) = e(:,1);
        newTch(i) = Tch(1);
        neweFch(i) = eFch(1);
    end
    for j = 2:pop
        if R(i) > P(j-1) && R(i) <= P(j)
            newe(:,i) = e(:,j);
            newTch(i) = Tch(j);
            neweFch(i) = eFch(j);
        end
    end
end
%establish the new generation
e = newe;
Tch = newTch;
eFch = neweFch;

%Simulated Binary Crossover Operator
for i = 1:2:pop
    %perform the crossover for the front surface emissivity
    u = rand();
    if u<0.5
        bq = (2*u)^(1/(hc+1));
    else
        bq = (1/(2*(1-u)))^(1/(hc+1));
    end
    temp(1,1) = 0.5*((1+bq)*e(1,i)+(1-bq)*e(1,i+1));
    temp(1,2) = 0.5*((1-bq)*e(1,i)+(1+bq)*e(1,i+1));
    %perform the crossover for the back surface emissivity
    u = rand();
    if u<0.5
        bq = (2*u)^(1/(hc+1));
    else
        bq = (1/(2*(1-u)))^(1/(hc+1));
    end
    temp(2,1) = 0.5*((1+bq)*e(2,i)+(1-bq)*e(2,i+1));
    temp(2,2) = 0.5*((1-bq)*e(2,i)+(1+bq)*e(2,i+1));
    %perform the crossover for the temperature at which eF changes
    u = rand();
    if u<0.5
        bq = (2*u)^(1/(hc+1));
    else
        bq = (1/(2*(1-u)))^(1/(hc+1));
    end
    temp(3,1) = 0.5*((1+bq)*Tch(i)+(1-bq)*Tch(i+1));
    temp(3,2) = 0.5*((1-bq)*Tch(i)+(1+bq)*Tch(i+1));
    %perform the crossover for the front emissivity after Tch is exceeded
    u = rand();
    if u<=0.5
        newe(:,i) = e(:,1);
        newTch(i) = Tch(1);
        neweFch(i) = eFch(1);
    end
end
\[ bq = (2u)^{(1/(hc+1))}; \]

\textbf{else}
\[ bq = (1/(2*(1-u)))^{(1/(hc+1))}; \]
\textbf{end}

\[ \text{temp}(4,1) = 0.5 \times ((1+bq) \times eFch(i) + (1-bq) \times eFch(i+1)); \]
\[ \text{temp}(4,2) = 0.5 \times ((1-bq) \times eFch(i) + (1+bq) \times eFch(i+1)); \]

\% record the crossover
\[ e(1,i) = \text{temp}(1,1); \]
\[ e(2,i) = \text{temp}(2,1); \]
\[ Tch(i) = \text{temp}(3,1); \]
\[ eFch(i) = \text{temp}(4,1); \]
\[ e(1,i+1) = \text{temp}(1,2); \]
\[ e(2,i+1) = \text{temp}(2,2); \]
\[ Tch(i+1) = \text{temp}(3,2); \]
\[ eFch(i+1) = \text{temp}(4,2); \]
\textbf{end}

\% Be sure to keep the emissivities between 0 and 1
\[ e(e<0) = 0; \]
\[ e(e>1) = 1; \]
\[ eFch(eFch<0) = 0; \]
\[ eFch(eFch>1) = 1; \]

\% Be sure to keep Tch between 500K and 1100K
\[ Tch(Tch<300) = 300; \]
\[ Tch(Tch>1300) = 1300; \]

\% Non-Uniform Mutation Operator
\textbf{for} \ i = 1:pop
\textbf{for} \ j = [1 2]
\[ \tau = 2 \times \text{round(rand())-1}; \]
\[ e(j,i) = e(j,i) + \tau \times (1-\text{rand()}^{((1-\text{iter}/\text{itermax})^b)}); \]
\textbf{if} \ e(j,i)<0
\[ e(j,i) = 0; \]
\textbf{end}
\textbf{if} \ e(j,i)>1
\[ e(j,i) = 1; \]
\textbf{end}
\textbf{end}
\[ \tau = 2 \times \text{round(rand())-1}; \]
\[ Tch(i) = Tch(i) + \tau \times (1-(350 \times \text{rand()}^{((1-\text{iter}/\text{itermax})^b)}); \]
\textbf{if} \ Tch(i)<300
\[ Tch(i) = 300; \]
\textbf{end}
\textbf{if} \ Tch(i)>1300
\[ Tch(i) = 1300; \]
\textbf{end}
\[ \tau = 2 \times \text{round(rand())-1}; \]
\[ eFch(i) = eFch(i) + \tau \times (1-\text{rand()}^{((1-\text{iter}/\text{itermax})^b)}); \]
\textbf{if} \ eFch(i)<0
\[ eFch(i) = 0; \]
\textbf{end}
\textbf{if} \ eFch(i)>1
\[ eFch(i) = 1; \]
\textbf{end}
\textbf{end}
Recalculate the fitness after the last iteration
Evaluate the performance of each solution
record whether \( Tch \) has been exceeded

\[
\begin{align*}
ch &= \text{zeros}(1, \text{pop}); \\
QbQt &= \text{zeros}(1, \text{pop}); \\
TBpeak &= \text{zeros}(1, \text{pop}); \\
\text{for } i &= 1: \text{pop} \\
&\text{for } j = 1: \text{length}(tt) \\
&\quad \text{if } ch(i) == 0 \\
&\quad \quad [\text{HBE } T] = \text{solver}(Qtot(j), QconvQtot(j), Tinf, Th, h, e(:, i), kt); \\
&\quad \text{end} \\
&\quad \text{if } ch(i) == 1 \\
&\quad \quad [\text{HBE } T] = \text{solver}(Qtot(j), QconvQtot(j), Tinf, Th, h, [eFch(i); e(2, i)], kt); \\
&\quad \text{end} \\
&\quad \text{if } ch(i) == 0 \&\& T(1) > Tch(i) \\
&\quad \quad ch(i) = 1; \\
&\quad \text{end} \\
QbQt(i) &= QbQt(i) + (5 \cdot Qtot(j) \cdot \text{HBE}); \\
&\text{if } T(2) > TBpeak(i) \\
&\quad TBpeak(i) = T(2); \\
&\text{end} \\
QbQt(i) &= QbQt(i) / (5 \cdot \text{sum}(Qtot)); \\
\end{align*}
\]

Find the number of times each solution is dominated

\[
\begin{align*}
\text{for } i &= 1: \text{pop} \\
&\text{dominated}(i) = 0; \\
&\text{for } j = 1: \text{pop} \\
&\quad \text{if } QbQt(j) >= QbQt(i) \&\& TBpeak(j) <= TBpeak(i) \\
&\quad \quad \text{dominated}(i) = \text{dominated}(i) + 1; \\
&\quad \text{end} \\
&\quad \text{if } QbQt(j) == QbQt(i) \&\& TBpeak(j) == TBpeak(i) \\
&\quad \quad \text{dominated}(i) = \text{dominated}(i) - 1; \\
&\quad \text{end} \\
&\text{end} \\
&\text{end} \\
\end{align*}
\]

Assign a fitness based on the number of times each solution is dominated and enforce some constraints

\[
\text{fitness} = (\text{pop} - \text{dominated}) / \text{pop};
\]

Reevaluate the fitness based on a niching function

\[
\begin{align*}
QbQtmax &= \text{max}(QbQt); \\
TBpeakmax &= \text{max}(TBpeak); \\
QbQtmin &= \text{min}(QbQt); \\
TBpeakmin &= \text{min}(TBpeak); \\
\text{for } i &= 1: \text{pop} \\
&\text{nc} = 0; \\
&\text{for } j = 1: \text{pop} \\
&\quad d(1) = ((QbQt(i) - QbQt(j)) / (QbQtmax - QbQtmin))^2; \\
&\text{end} \\
&\text{end} \\
&\text{fitness} = \text{fitness} \cdot (1 + d(1)); \\
&\text{end} \\
\end{align*}
\]
d(2) = ((TBpeak(i)-TBpeak(j))/(TBpeakmax-TBpeakmin))^2;
distance = sum(d);
if distance <= sigShare
    nc = nc + (1-(distance/sigShare)^alpha);
end
end
fitness(i) = fitness(i)/nc;
end

frontNum = length(find(dominated==0));
nonDe = zeros(frontNum,2);
nonDTch = zeros(frontNum,1);
nonDeFch = zeros(frontNum,1);
nonDch = zeros(frontNum,1);
nonDTBpeak = zeros(frontNum,1);
nonDQbQt = zeros(frontNum,1);
j = 1;
for i = 1:pop
    if dominated(i)==0
        nonDe(j,1) = e(1,i);
        nonDe(j,2) = e(2,i);
        nonDTch(j) = Tch(i);
        nonDeFch(j) = eFch(i);
        nonDch(j) = ch(i);
        nonDQbQt(j) = QbQt(i);
        nonDTBpeak(j) = TBpeak(i);
        j = j+1;
    end
end
toc
Appendix D: The Development of an Optimal Front by a MOGA

The following series of plots show the convergence of an initially random population of potential solutions onto an optimal front as MOGA2.m (presented as Appendix B) progresses from the first to the thousandth iteration. The total incident heat flux was $\dot{Q}_{\text{TOT}} = 10\text{kW/m}^2$, the ratio of convective to total heat flux was $\dot{Q}_{\text{CONV}}/\dot{Q}_{\text{TOT}} = 0.3$. As for all previous optimizations, $k/t = 100\text{ W/m}^2\text{-K}$, $h = 5\text{ W/m}^2\text{-K}$, $T_H = 300\text{ K}$, and $T_\infty = 300\text{ K}$. The population size was 100.

Figure D-1 below shows the initial random population of front and back surface emissivities. This is called the decision variable space, because it defines the bounds on all of the variables which define a solution.
Figure D-2 below shows the same population which is shown in Figure D-1, except it is now shown in the objective variable space. The objective variable space defines the bounds on all of the objective functions corresponding to the solutions which lie in the decision variable space. Note that the shape of the objective variable space is very different from that of the decision variable space.

**Random Initial Population of Solutions, Objective Space**

D-2. Random initial population, objective space.

Figures D-3 through D-6 show the population of solutions converging upon the optimal front in the objective variable space as the MOGA progresses from the first to the thousandth iteration.
Population of Solutions After 200 Iterations

At this point, the non-dominated solutions can be selected from the population and plotted. The result, Figure D-7 below, is a near-duplicate of Figure 4-3.

**Non-Dominated Solutions After 1000 Iterations**

![Graph showing non-dominated solutions after 1000 iterations.](image)

References


