MODIFIED GRAVITY IN COSMOLOGY AND FUNDAMENTAL PARTICLE PHYSICS

by

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Submitted in partial fulfillment of the requirements
For the degree of Doctor of Philosophy

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Modified Gravity in Cosmology and Fundamental Particle Physics

Abstract

by

De-Chang Dai

The Standard Model of particle physics and General Relativity are very successful in describing present experimental results. Both of them, however, are assumed to be low-energy approximations of a more complete theory. There are several candidate theories that are proposed to be part of a new conceptual structure beyond the Standard Model. To solve the Hierarchy problem, the energy threshold of the new theories is expected to be the TeV scale. One paradigm, low energy quantum gravity, combines the Standard Model with General Relativity and assumes the existence of extra dimensions. This paradigm predicts that TeV black holes can be produced in the Large Hadron Collider (LHC). In this dissertation we describe a black-hole simulator, BlackMax, for coming accelerators. The generator is based on the Monte Carlo technique and predicts the signatures of black-hole production at the LHC.

The remainder of the dissertation is an example of the violation of Birkhoff’s law. Birkhoff’s law is analogous to a famous result of Newtonian theory, that the gravitational acceleration due to a spherical shell vanishes inside the shell. Since the universe is homogeneous and isotropic on large scales, local gravitational phenomena can be treated as local events only; and one can always ignore the influence from the distant mass distribution. This law is violated in modified gravity theories. It is shown that a spherical shell can affect the geometry in the extra dimensions. The change of geometry in the extra dimensions also changes the geometry inside the shell. The gravitational acceleration inside a spherical shell does not vanish.
Chapter 1

Introduction

1.1 The Standard Model of Particle Physics

The Standard Model is a quantum gauge field theory which was developed between about 1940 to 1973 [1]. This theory is consistent with quantum mechanics and special relativity. On a microscopic scale, it successfully describes interactions between fundamental particles through the strong force, the weak interaction and the electromagnetic force, but not gravity. The theory describes experimental measurements to very high accuracy. However the theory is not regarded as a complete theory of fundamental particles because it does not include gravity and because of the large number of parameters which must be fixed by experiments. It is a general belief that there is physics “Beyond the Standard Model” (BSM) and it may appear at the energy scale of several TeV. This energy scale is within the measurement range of the Large Hadron Collider (LHC) at CERN, which will collide protons with a center-of-mass energy at 14TeV. Many BSM models have been developed and will be tested by the LHC. The LHC may catch the first signature of physics Beyond the Standard Model. Of course, the other important aim of the LHC is to discover the Higgs boson, which is the only missing particle in the Standard Model. Although there is no direct evidence for the Higgs boson, in this thesis we assume the particle does exist.
1.1.1 Particles and Interaction of the Standard Model

There are two kinds of particles in the Standard Model. The first types carry “charges” and the other ones “mediate” interactions between particles by coupling directly to the charges. The former include both fundamental fermions and some gauge bosons; the latter are all gauge bosons. The charges include color, weak isospin, and weak hypercharge. The values of those charges are not predicted in the Standard Model, although their relative values are. The gauge boson fields allow the Lagrangian to be invariant under gauge transformations. This invariance implies the existence of conserved currents and charges in the model.

There are three main symmetries of the standard-model Lagrangian. The first is a $U(1)_Y$ hypercharge symmetry, which is an Abelian symmetry. The corresponding gauge boson is $B^\mu$ and the hypercharge is written as $Y$. The second is an $SU(2)$ weak isospin symmetry. This symmetry is also referred to as $SU(2)_L$ since the associated gauge fields only couple to left-handed fermions and right-handed anti-fermions. The corresponding gauge fields are $W^1_\mu$, $W^2_\mu$ and $W^3_\mu$. The conserved hypercharge is $I_3$, the third component of the weak isospin $I$. The last of the three gauge symmetries is $SU(3)$ color, the non-Abelian symmetry of Quantum Chromodynamics (QCD). The mediating gauge bosons are 8 gluons. The conserved charge is color.

The theory also introduces the Higgs field, a complex scalar field, to give masses to the particles and to cause the breaking of electroweak symmetry. As a result of the breaking and the Higgs mechanics, the $W^3_\mu$ from $SU(2)_L$ mixes with $B^\mu$ from $U(1)_Y$ and produces a massless photon and a massive $Z$ boson. $W^1_1$ mixes with $W^2_2$ to produce the $W^\pm$, which are also massive. All fermions acquire mass in the $SU(2)_L$ breaking except for neutrinos. The existence of off-diagonal neutrino masses is evidence of BSM physics that we will not discuss here. There is one symmetry left after the breaking, which is $U(1)_Q$-electromagnetic gauge symmetry, which is a combination of $U(1)_Y$ and $U(1)$ subgroup of $SU(2)$. The coupling between fermions and the photon is proportional to the electromagnetic charge $Q = Y + I_3$ of the fermions. The coupling
between fermions and the $Z$ boson is proportional to $I_3 - Q \sin^2 \theta_w$ where $\theta_w$ is the Weinberg angle, $\sin^3 \theta \sim 0.23$. Since $I_3$ depends on the fermion chirality, the coupling also depends on the fermion chirality.

Although there is no direct evidence for the existence of the Higgs, experiments still favor a low-mass Higgs, 219 to 251 GeV. The discovery of the Higgs will be one of the important goals of the LHC.

There are two extra global symmetries that have been found in the Lagrangian of the Standard Model- lepton number ($L$) and baryon number ($B$). These are not imposed on the model; they appear naturally. Although $B$ and $L$ symmetries are individually broken by non-perturbative processes in the Standard Model, the combination of $B - L$ is preserved in all process. Some of the theories beyond the standard model violate $B$ or $L$ perturbatively. Those violations cause various problems, one of them being too short a lifetime for the proton.

1.2 Theory Beyond the Standard Model

There are many new modified theories for physics beyond the Standard Model, eg. string theory, supersymmetry (SUSY). The work in the first part of this thesis is focused on models with “large” extra dimensions.

1.2.1 Hierarchy Problem

The Hierarchy problem is a common argument for the existence of Physics beyond the Standard Model. It can be expressed in two ways. The first one is the aesthetic hierarchy problem. The Standard Model does not include gravity. The mass scale in the Standard Model ($\sim 100$GeV) is so much smaller than the Planck scale, the mass scale of gravity ($\sim 10^{19}$ GeV). If there exists a “theory of everything”, the mass scales of interactions should be more similar. It is deemed unnatural to have a unified theory with two very different energy scales.
The second way to express the Hierarchy Problem is called the Technical Hierarchy Problem. The one-loop correction terms of the Higgs mass can be calculated from Figure 1.1. Since this diagram is quadratically divergent, a cutoff(\(\Lambda\)) must be introduced into the calculation. Here, we list one correction term that comes from fermions

\[
\delta M^2_{\text{Higgs}} = \frac{|\lambda_f^2|}{16\pi^2} \left(-2\Lambda^2 + 6m_f^2 \ln(\Lambda/m_f) + \ldots\right),
\]

(1.1)

where \(\lambda_f\) is the coupling constant and \(m_f\) is the mass of the fermion in the loop (Figure 1.1). In general, the value of the bare Higgs mass is not a problem, since the bare Higgs mass can be any value which provide the physical Higgs mass about Electroweak scale, \(\sim 100\text{GeV}\). However, the natural choice of \(\Lambda\) is the Planck mass. The bare Higgs mass will be about the order of the Planck mass, which is \(10^{16}\) times larger than the electroweak scale. A “fine-tuning” parameter is necessary in the theory. If one believes there is no a fine-tuning parameter, \(\Lambda\) should not be the Planck mass. Then, there must be a new physics around the TeV scale.

### 1.2.2 Supersymmetry (SUSY)

Supersymmetry is one of the most popular BSM theories. The basic idea of SUSY is that an extra symmetry is introduced so that the Lagrangian is invariant as fermions are exchanged with bosons and vice versa. Since none of the known particles are fermion-boson pairs, the number of particles approximately doubles. Those new particles provide new cancellations in the calculation of the Higgs mass. The quadratic term in equation 1.1 is cancelled and only the \(\ln \Lambda\) term is left. Fine-tuning is then not necessary in this model. However, supersymmetry must be broken at an energy
scales around a TeV and the mass of superparticles must be about 1 TeV so that the Higgs mass is about the electroweak scale.

The breaking of supersymmetry makes the theory become less beautiful. About 100 new parameters are needed to describe a particular model. A theory with so many parameters is unlikely to be ruled out or proved by a single experiment.

1.2.3 Extra-Dimensions

Extra-dimension models are a class of possible solutions to the hierarchy problem. These theories assume that there are some dimensions in which only gravitons propagate; the other standard-model particles are constrained to a (3+1)-dimensional hypersurface, called the “brane”. Since gravitons are perturbation of the geometry of space, they propagate in both the brane and the bulk. It is natural not to confine a graviton on the brane. In string theory, a Dirichlet brane is hypersurface on which open string ends. A standard model particle is the end of an open string. Therefore, it is constrained on a brane. A graviton is a closed-loop string. It does not end on a brane. It appears anywhere space in space.

From Gauss’ law the gravitational force decays as $\sim 1/r^{d+2}$ (d is the number of extra dimensions). The decay is much faster than any 4-dimensional force. The gravitational force, therefore, becomes small at macroscopic distances compared to the other forces. Today’s measurements, however, show that the gravitational force is not discrepant from Newtonian gravity for distances larger than 150$\mu$m [34]. If extra-dimensional space exists, the size of the extra dimensions must be less than 1mm. Even though the extra dimensions are small, this is enough to dilute the intensity of the gravitational force(If there are at least 3 extra dimensions or if the extra dimensions are negatively curved[131]).

Since only gravitons are able to leave the 4-dimensional regular brane, momentum in the extra dimensions is not conserved. The brane is taken to be infinitely heavy so that it absorbs any momentum in extra-dimensional directions that is carried by
Proton decay and $n\bar{n}$ oscillations are the other problems in the Standard Model. One solution to these problems is to put the fermions on different branes, as shown in Figure 1.2. The gauge bosons and Higgs are able to propagate in a “thick brane” which is still much thinner than the size of the extra dimensions. Since fermions are not so close to each other, the process of, say, a bound state of quarks decaying to a lepton becomes highly suppressed. The proton decay problem is thereby solved in the model. The same can be done for $n\bar{n}$ oscillations by splitting up and down-type quarks.¹

The (d+4)-dimensional Planck mass in these models is about a TeV; The Hierarchy problem is not a problem anymore. The lowness of the (4+d)-dimensional Planck mass provides a possibility of quantum black hole creation at the TeV scale. Once energy several times greater than the (4+d)-dimensional Planck mass is assembled within its own horizon, there is a chance for it to “shave its hair” and become a thermal black hole. Such objects, their successful or failed precursors, are potentially detectable at the LHC. The first part of this thesis focuses on the simulation of the expected signal from such black holes in models with extra dimensions at the LHC.

We will discuss the details in chapter 2.

¹There are other ways known to explain proton decay and $n\bar{n}$ oscillation. One is conserved gauge baryon number, or, alternately, one can introduce a local $U(1)$ symmetry, which is broken down to a discrete group, that prohibits worrisome dimension-4 trilinear quark superfield operators [134].
1.3 Modified Gravity in Cosmology

Like the Standard Model in particle physics, the ΛCDM model is the most successful current cosmological model. This model is based in General Relativity and introduces two forms of exotic matter, dark matter and dark energy, of which there is only indirect evidence of their existence. For this reason, scientists are increasingly asking whether General Relativity is an approximation of a more complete theory.

A first phenomenological intriguing attempt to replace dark matter with modified gravity was Milgrom and Bekenstein (1983) [127, 128], so called MOND (Modified Newtonian Dynamics). This theory introduces two parameters and one free function. It can explain galaxy-scale phenomena according to those new parameters. However, MOND is regarded as a phenomenological fit instead of a real theory. The most serious defect of the model is the lack of a covariant action. This defect has been solved recently by TeVeS [111] and by General Einstein Aether theory[123] (GEA).

In those theories dark matter is replaced by a dynamical scalar field or a vector field. They alter the response of spacetime to matter. There was no direct evidence to challenge those theories until the bullet cluster [129] was found. The bullet cluster is indeed two clusters passing through each other. The smaller subcluster has left a notable supersonic track in the sky. The mass distribution from gravitational lensing shows that the mass centers of the subclusters are not at the baryon mass centers. The deviation is treated as direct evidence of non-interacting dark matter. MOND itself is not able to explain this phenomenon, and neither is its relativistic version (e.g. TeVeS)\(^2\). The triumph, however, does not last too long. It has been discovered that Abell 520 [132] is a counterexample of the bullet cluster. If Abell 520 is real [132], it may overturn all contemporary gravity theories. Which means a new type of modified gravity theory is necessary.

There are several models that explain the expansion of the universe without introducing dark energy. The two most well known are DGP[113] and F(R) (for review,
see [130]). In these theories the expansion of the universe comes from dynamical reasons instead of matter with negative pressure. Although these modified gravity theories successfully replace dark energy or dark matter, the theories still have several unsolved problems. The most well known being the existence of many free parameters that can not be determined (e.g. extra vector field in GEA). These parameters lead to the difficulty of finding a suitable solution. The basic treatment of this problem is eliminating several degrees of freedom and making the solution become General Relativity at short scale. The solution is so close to General relativity that it is hard to rule out these modified theories. However, the modification also loses some of the quality of the General Relativity. One of them is Birkhoff’s law. Birkhoff’s law is a relativistic version of Gauss’s law. It states that the gravitational field outside a spherically symmetric body behaves as if the whole mass of the body were concentrated at the center; the gravitational field inside a spherical shell vanishes. This means that in an isotropic and homogeneous distribution matter does not contribute any gravitational field inside its own radius; gravitational phenomena are therefore governed by the local mass distribution. Once the theorem is violated, the gravitational acceleration can be due to matter far away. In chapter 3, we will use DGP as a toy model to produce an effect like dark matter. DGP surely is not a suitable replacement for dark matter. However, we are looking for a simple example of the ill-behavior of the result from violation of Birkhoff’s Law, before doing a long calculation in more realistic MOND model. From this point of view, DGP is a good model with which to begin. In future, we will study other models, like MOND and TeVeS, which are real models to replace dark matter.
Chapter 2

BlackMax: A Black-Hole Generator for the LHC

2.1 Introduction

Models with TeV-scale quantum gravity [3, 4, 5, 6] offer very rich collider phenomenology. Most of them assume the existence of a (3+1)-dimensional hypersurface, which is referred to as “the brane,” where Standard-Model particles are confined only gravity, and possibly other particles that carry no gauge quantum numbers, such as right handed neutrinos, can propagate in the full space the so called “bulk”. Under certain assumptions, this setup allows the fundamental quantum-gravity energy scale, $M_*$, to be close to the electroweak scale. The observed weakness of gravity compared to other forces on the brane (i.e. in the laboratory) is a consequence of the large volume of the bulk diluting the strength of gravity.

In the context of these models of TeV-scale quantum gravity, probably the most exciting new physics is the production of micro-black-holes in near-future accelerators like the Large Hadron Collider (LHC) [7]. According to the “hoop conjecture” [8], if the impact parameter of two colliding particles is less than two times the gravitational radius, $r_h$, corresponding to their center-of-mass energy ($E_{CM}$), a black-
hole with a mass of the order of $E_{CM}$ and horizon radius, $r_h$, will form. Typically, this gravitational radius is approximately $E_{CM}/M^2$.\(^1\) Thus, when particles collide at center-of-mass energies above $M_*$, the probability of black-hole formation is high.

Strictly speaking, there exist no complete calculation (including radiation during the process of formation and back-reaction) which proves that a black-hole really forms. It may happen that a true event horizon and singularity never forms, and that Hawking (or rather Hawking-like) radiation is never quite thermal. In [9] this question was analyzed in detail from the point of view of an asymptotic observer, who is, in the context of the LHC, the most relevant observer. It was shown that, though such observers never observe the formation of an event horizon even in the full quantum treatment, they do register pre-Hawking quantum radiation that takes away energy from a collapsing system. Pre-Hawking radiation is non-thermal and becomes thermal only in the limit when the horizon is formed. Since a collapsing system has only a finite amount of energy, it disappears before the horizon is seen to be formed. While these results have important implications for theoretical issues like the information loss paradox, in a practical sense very little will change. The characteristic time for gravitational collapse in the context of collisions of particles at the LHC is very short. This implies that pre-Hawking radiation will be experimentally indistinguishable from Hawking radiation calculated for a real black hole. Also, calculations in [9] indicate that the characteristic time in which a collapsing system loses all of its energy is very similar to the life time of a real black-hole. Thus, one may proceed with a standard theory of black-holes.

Once a black-hole is formed, it is believed to decay via Hawking radiation. This Hawking radiation will consist of two parts: radiation of Standard-Model particles into the brane and radiation of gravitons and any other bulk modes into the bulk. The relative probability for the emission of each particle type is given by the gray-body factor for that mode. This gray-body factor depends on the properties of the particle $^{1}$$h/c$ is ignored in the equation
(charge, spin, mass, momentum), of the black-hole (mass, spin, charge) and, in the context of TeV-scale quantum gravity, on environmental properties – the number of extra dimensions, the location of the black-hole relative to the brane (or branes), etc.

In order to properly describe the experimental signatures of black-hole production and decay one must therefore calculate the gray-body factors for all of the relevant degrees of freedom.

There are several black-hole event generators available in the literature [10] based on particular, simplified models of low-scale quantum gravity and incorporating limited aspects of micro-black-hole physics. Unfortunately, low-scale gravity is plagued with many phenomenological challenges like fast proton decay, large $n\bar{n}$ oscillations, flavor-changing neutral currents and large mixing between leptons [11, 12]. For a realistic understanding of the experimental signature of black hole production and decay, one needs calculations based on phenomenologically viable gravity models, and incorporating all necessary aspects of the production and evolution of the black-holes.

One low-scale gravity model in which the above mentioned phenomenological challenges can be addressed is the split-fermion model [13]. In this model, the Standard Model fields are confined to a “thick brane”, much thicker than $M_*^{-1}$. Within this thick brane, quarks and leptons are stuck on different three-dimensional slices (or on different branes), which are separated by much more than $M_*^{-1}$. This separation causes an exponential suppression of all direct couplings between quarks and leptons, because of exponentially small overlaps between their wave-functions. The proton decay rate will be safely suppressed if the spatial separation between quarks and leptons is greater by a factor of at least 10 than the widths of their wave functions. Since $\Delta B = 2$ processes, like $n\bar{n}$ oscillations, are mediated by operators of the type uddudd, suppressing them requires a further splitting between up-type and down-type quarks. Since the experimental limits on $\Delta B = 2$ operators are much less stringent than those on $\Delta B = 1$ operators, the u and d-type quarks need only be separated by a few times the width of their wave functions [13].
Current black-hole generators assume that the black-holes that are formed are Schwarzschild-like. However, most of the black-holes that would be formed at the LHC would be highly rotating, due to the non-zero impact parameter of the colliding partons. Due to the existence of an ergosphere (a region between the infinite redshift surface and the event horizon), a rotating black-hole exhibits super-radiance: some modes of radiation get amplified compared to others. The effect of super-radiance [14] is strongly spin-dependent, with emission of higher-spin particles strongly favored. In particular the emission of gravitons is enhanced over lower-spin Standard-Model particles. Since graviton emission appears in detectors as missing energy, the effects of black-hole rotation cannot be ignored. Similarly, black-holes may be formed with non-zero gauge charge, or acquire charge during their decay. This again may alter the decay properties of the black-hole and should be included.

Another effect neglected in other generators is the recoil of the black-hole. A small black-hole attached to a brane in a higher-dimensional space emitting quanta into the bulk could leave the brane as a result of a recoil. In this case, visible black-hole radiation would cease. Alternately, in a split-brane model, as a black-hole traverses the thick brane the Standard-Model particles that it is able to emit will change depending on which fermionic branes are nearby.

It is also the case that virtually all the work in this field has been done for the idealized case where the brane tension is negligible. However, one generically expects the brane tension to be of the order of the fundamental energy scale, being determined by the vacuum energy contributions of brane-localized matter fields[15]. As shown in [16], finite brane tension modifies the standard gray-body factors.

Finally, it has been suggested [17] that more common than the formation and evaporation of black-holes will be gravitational scattering of parton pairs into a two-body final state. We include this possibility.

Here we present a comprehensive black-hole event generator, BlackMax, that takes

\footnote{Although if the black-hole carries gauge charge it will be prevented from leaving the brane.}
into account practically all of the above mentioned issues\textsuperscript{3}, and includes almost all the necessary gray-body factors\textsuperscript{4}. Preliminary studies show how the signatures of black-hole production and decay change when one includes splitting between the fermions, black-hole rotation, positive brane tension and black-hole recoil. Future papers will explore the implications of these changes in greater detail.

In section 2.2 and 2.3 we discuss the production of black-holes and the gray-body factors respectively. The evaporation process and final burst of the black-holes is discussed in section 2.4 and 2.5. Sections 2.6 and describe the input and output of the generator. Section 2.7 shows some characteristic distributions of black-holes for different extra dimension scenarios. The reference list is extensive, reflecting the great interest in the topic \cite{7, 8, 10-12, 15-26, 28-32, 35-109}, and many more.

\section{2.2 Black-hole production}

We assume that the fundamental quantum-gravity energy scale $M_*$ is not too far above the electroweak scale. Consider two particles colliding with a center-of-mass energy $E_{CM}$. They will also have an angular momentum $J$ in their center-of-mass (CM) frame. By the hoop conjecture, if the impact parameter, $b$, between the two colliding particles is smaller than the diameter of the horizon of a $(d+1)$-dimensional black-hole (where $d$ is the total number of space-like dimensions) of mass $M = E_{CM}$ and angular momentum $J$,

$$ b < 2r_h(d, M, J), \quad \text{(2.1)}$$

then a black-hole with $r_h$ will form. The hoop conjecture is a classical estimate. A black hole with energy close to $M_*$ has wavelength roughly the same as $M_*$. Here, we choose the mass of a black hole to be larger than $5M_*$ to keep the estimation inside 10\% error. According to hoop conjection the Planck-black-hole cross section is

\textsuperscript{3}although not necessarily simultaneously

\textsuperscript{4}Except in the one case of the graviton gray-body factor for a rotating black-hole, where the calculation has yet to be achieved.
approximately equal to the interaction area $\pi(2r_h)^2$.

In Boyer-Lindquist coordinates, the metric for a $(d+1)$-dimensional rotating black-hole (with angular momentum parallel to the $\hat{\omega}$ in the rest frame of the black-hole) is[133]:

$$
    ds^2 = \left(1 - \frac{\mu r^{4-d}}{\Sigma(r, \theta)}\right) dt^2 \\
    - \sin^2 \theta \left(r^2 + a^2 \left(1 + \frac{\mu r^{4-d}}{\Sigma(r, \theta)}\right)\right) d\phi^2 \\
    + 2a \sin^2 \theta \frac{\mu r^{4-d}}{\Sigma(r, \theta)} dr d\phi - \frac{\Sigma(r, \theta)}{\Delta} dr^2 \\
    - \Sigma(r, \theta) d\theta^2 - r^2 \cos^2 \theta d\Omega. 
$$

(2.2)

where $\mu$ is a parameter related to mass of the black-hole, while

$$
    \Sigma = r^2 + a^2 \cos^2 \theta
$$

(2.3)

and

$$
    \Delta = r^2 + a^2 - \mu r^{4-d}. 
$$

(2.4)

The mass of the black-hole is

$$
    M = \frac{(d-1)A_{d-1}}{16\pi G_d} \mu, 
$$

(2.5)

and

$$
    J = \frac{2Ma}{d-1}
$$

(2.6)

is its angular momentum. Here,

$$
    A_{d-1} = \frac{2\pi^{d/2}}{\Gamma(d/2)} 
$$

(2.7)

is the hyper-surface area of a $(d-1)$-dimensional unit sphere. The higher-dimensional
gravitational constant $G_d$ is defined as

$$G_d = \frac{\pi^{d-4}}{4M_s^{d-1}}. \quad (2.8)$$

The horizon occurs when $\Delta = 0$. That is at a radius given implicitly by

$$r_h^{(d)} = \left[ \frac{\mu}{1 + (a/r_h^{(d)})^2} \right]^{1/2} = \frac{r_s^{(d)}}{\left[ 1 + (a/r_h^{(d)})^2 \right]^{1/2}}. \quad (2.9)$$

Here

$$r_s^{(d)} \equiv \mu^{1/(d-2)} \quad (2.10)$$

is the Schwarzschild radius of a $(d+1)$-dimensional black-hole, i.e. the horizon radius of a non-rotating black-hole. Equation 2.10 can be rewritten as:

$$r_s^{(d)}(E_{CM}, d, M_s) = k(d)M_s^{-1}[E_{CM}/M_s]^{1/(d-2)}, \quad (2.11)$$

where

$$k(d) \equiv \left[ 2^{d-3}\pi^{(d-6)/2}\frac{\Gamma[d/2]}{d-1} \right]^{1/(d-2)} \quad (2.12)$$

Figure 2.1 shows the horizon radius as a function of black-hole mass for $d$ from 4 to 10. We see that the horizon radius increases with mass; it also increases with $d$.

Figure 2.2 shows the Hawking temperature of a black-hole

$$T_H = \frac{d-2}{4\pi r_h} \quad (2.13)$$

as a function of the black-hole mass for $d$ from 4 to 10. The Hawking temperature is a measure of the characteristic energies of the particles emitted by the black-hole. $T_H$ decreases with increasing mass. However, the behaviour of $T_H$ with changing $d$ is complicated, reflecting the competing effect of an increasing horizon radius and an increasing $d - 2$ in equation 2.13.
Figure 2.1: Horizon radius (in GeV$^{-1}$) of a non-rotating black-hole as a function of mass for 4-10 spatial dimensions.

Figure 2.2: Hawking temperature (in GeV) of a non-rotating black-hole as a function of mass for 4-10 spatial dimensions.
Figure 2.3: Horizon radius (in GeV$^{-1}$) of a black-hole as a function of mass for different $B$ in $d=5$ spatial dimensions.

For the model with non-zero tension brane, the radius of the black-hole is defined as

$$r_h(t) = r_s B^{1/3},$$

(2.14)

with $B$ the deficit-angle parameter which is inverse by proportional to the tension of the brane.

Figure 2.3 shows the horizon radius as a function of black-hole mass for the model with non-zero tension brane. As the deficit-angle parameter increases, the size of the black-hole increases.

Figure 2.4 shows the Hawking temperature of a black-hole for the model with non-zero tension brane. The Hawking temperature decreases as the deficit angle decreases.

Figure 2.5 shows the horizon radius as a function of black-hole mass for a rotating black-hole. The angular momentum decreases the size of the horizon and increases the Hawking temperature (see figures 2.5 and 2.6).

If two highly relativistic particles collide with center-of-mass energy $E_{CM}$, and impact parameter $b$, then their angular momentum in the center-of-mass frame before the collision is $L_{in} = bE_{CM}/2$. Suppose for now that the black-hole that is formed retains all this energy and angular momentum. Then the mass and angular momentum
Figure 2.4: Hawking temperature (in GeV) of a black-hole as a function of mass for different $B$ in $d=5$ spatial dimensions.

Figure 2.5: Horizon radius (in GeV$^{-1}$) of a rotating black-hole as a function of mass for different angular momentum in $d=5$ spatial dimensions. Angular momentum $J$ is in units of $\hbar$. 

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Figure 2.6: Hawking temperature (in GeV) of a rotating black-hole as a function of mass for different angular momentum in d=5 spatial dimensions. Angular momentum J is in units of $\hbar$.

of the black-hole will be $M_{in} = E_{CM}$ and $J_{in} = L_{in}$. A black-hole will form if:

$$b < b_{max} \equiv 2r_h^{(d)}(E_{CM}, b_{max}E_{CM}/2). \quad (2.15)$$

We see that $b_{max}$ is a function of both $E_{CM}$ and the number of extra dimensions.

We can rewrite condition (2.15) as

$$b_{max}(E_{CM}; d) = 2r_s^{(d)}(E_{CM}) \left[ 1 + \left( \frac{d-1}{2} \right)^2 \right]^{\frac{1}{d-2}}. \quad (2.16)$$

There is one exception to this condition. In the case where we are including the effects of the brane tension, the metric (and hence gray-body factors) for a rotating black-hole are not known. In this case we consider only non-rotating black-holes. Therefore, for branes with tension

$$b_{tension}^{max}(E_{CM}, d) = 2r_s^{(d)}(E_{CM}). \quad (2.17)$$

Also, for branes with tension only the $d = 5$ metric is known.

At the LHC, each proton will have $E = 7$ TeV in the CM frame. Therefore, the total proton-proton center of mass energy will be $\sqrt{s} = 14$ TeV. However, it is not the
protons that collide to make the black-holes, but the partons of which the protons are made. If two partons have energy $vE$ and $uE$, much greater than their respective masses, then the parton-parton collision will have

$$s' = |p_i + p_j|^2 = |v(E,E) + u(E,-E)|^2 = 4uE^2 = us.$$  \hspace{1cm} (2.18)

We define a quantity $Q'$

$$Q' = E_{CM} = \sqrt{s'} = \sqrt{us}$$ \hspace{1cm} (2.19)

The center-of-mass energy for the two colliding partons will be $\sqrt{us}$, as will be the 4-momentum transfer $Q'^2$. The largest impact parameter between the two partons that can form a black-hole with this mass will therefore be $b_{max}(\sqrt{us};d)$, as given by equation 2.16.

The total proton-proton cross section for black-hole production is therefore

$$\sigma^{pp \rightarrow BH}(s; d, M_\star) = \int_{M_\star^2/s}^1 du \int_{u}^1 dv \frac{1}{v} \pi [b_{max}(\sqrt{us};d)]^2 \times \sum_{ij} f_i(v,Q') f_j(u/v,Q').$$ \hspace{1cm} (2.20)

Here $f_i(v,Q')$ is the $i$-th parton distribution function. Loosely this is the expected number of partons of type $i$ and momentum $vE$ to be found in the proton in a collision at momentum transfer $Q'$.

In [17] it is argued that strong gravity effects at energies close to the Planck scale will lead to an increase in the $2 \rightarrow 2$ cross section via the exchange of Planckian “black-holes” (by which any quantum gravity effect or resonance is meant). Final states with high multiplicities are predicted to be suppressed. Although the intermediate state is created in the strong gravity regime, it is not a conventional microscopic black hole. The state is not stable. Thermal Hawking radiation does not take place. Especially since inelastic collisions increase the energy loss, the threshold for creating stable black-holes shifts to even higher values. Thus $2 \rightarrow 2$ scattering may be the
most important signal in the LHC instead of black-holes evaporating via Hawking radiation. One should find that the cross section for two-body final states suddenly jumps to a larger value, as the energy reaches the quantum-gravity scale. We calculate the cross section of two-body final states by replacing \( \pi b_{\text{max}}^2 \) in equation (2.20) with

\[
\pi b_{\text{max}}(\sqrt{s'} > M_{\text{min}})^2 \approx \pi r_s^2 P_2
\]

where

\[
P_2 = e^{-<N>} \sum_{i=0}^{2} \frac{<N>^i}{i!} \tag{2.22}
\]

\[
< N > = \rho \left( \frac{4 \pi k (d) M_{BH}}{d-1} \right) \frac{(d-1)/(d-2)}{M_s} \tag{2.23}
\]

\[
\rho = \frac{\sum c_i g_i \Gamma_i \zeta(3) \Gamma(3)}{\sum c_i f_i \Phi_i \zeta(4) \Gamma(4)} \tag{2.24}
\]

\[
\Gamma_i = \frac{1}{4 \pi r^2} \int \frac{\sigma_i(\omega) \omega^2 d\omega}{e^{\omega/T} \pm 1} \left[ \int \frac{\omega^2 d\omega}{e^{\omega/T} \pm 1} \right]^{-1} \tag{2.25}
\]

\[
\Phi_i = \frac{1}{4 \pi r^2} \int \frac{\sigma_i(\omega) \omega^3 d\omega}{e^{\omega/T} \pm 1} \left[ \int \frac{\omega^3 d\omega}{e^{\omega/T} \pm 1} \right]^{-1} \tag{2.26}
\]

Figure 2.7 shows the cross section for non-rotating black-holes on a tensionless brane as a function of mass for different numbers of spatial dimensions, as generated by BlackMax. The cross section increases with the number of spatial dimensions.

Figure 2.8 shows the cross section for non-rotating black holes on a brane with positive tension as a function of mass for various deficit angle parameter, \( B \). The cross section increases as the tension increases (as \( B \) decreases).

Figure 2.9 shows the cross section for non-rotating black holes as function of the number of split-fermion space dimensions, \( n_s \). When a pair of partons are separated in the extra-dimensions they must approach more closely in the ordinary dimensions.
Figure 2.7: Cross section for production of a black-hole (rotating or non-rotating) as a function of the number of spatial dimensions, for a tensionless brane, with no fermion brane-splitting.

Figure 2.8: Cross section for production of a non-rotating black-hole as a function of the deficit angle parameter for $d = 5$ and $n_s = 2$.

in order to form a black-hole. Thus the effective cross-section for black-hole formation in collisions is decreased. This effect become more severe as $n_s$ increases because the partons are more likely to be more widely separated in the extra dimensions, therefore the cross section decreases with increasing $n_s$.

Figure 2.10 shows the cross section as a function of the chosen minimum black-hole mass. The parton distribution functions strongly suppress the events with high black-hole masses.

Figure 2.11 shows the cross section for the two-body final-state scenario as a function of the number of spatial dimensions, for $M_{min} = M_s = 1$ TeV, $M_{min} = M_s =$
Figure 2.9: Cross section for production of a non-rotating black-hole as a function of the number of fermion brane-splitting dimensions for $d = 10$.

Figure 2.10: Cross section for formation of a black-hole (rotating or non-rotating) as function of the minimum mass of black-hole, for a zero-tension brane, with no fermion brane-splitting. The vertical lines are the error bars.
3 TeV and $M_{\text{min}} = M_\ast = 5$ TeV. It increases with the number of spatial dimensions.

Figure 2.11: Cross section for the two-body final-state scenario as a function of number of spatial dimensions where $M_{\text{min}} = M_\ast = 1$ TeV, $M_{\text{min}} = M_\ast = 3$ TeV and $M_{\text{min}} = 5$ TeV.

2.2.1 Black-Hole Formation in BlackMax

Within BlackMax, the probability of creating a black-hole of center-of-mass energy $\sqrt{us}$, in the collision of two protons of center-of-mass-energy $\sqrt{s}$, is given by

$$P(Q') = \int_0^1 \frac{dv}{v} \sum_{ij} f_i(v,Q') f_j\left(\frac{u}{v},Q'\right).$$

(2.27)

According to the theory, there will be some minimum mass for a black-hole. We expect $M_{\text{min}} \sim M_\ast$, but leave $M_{\text{min}} \geq M_\ast$ as a free parameter. One can change $M_{\text{min}}$ according to the situations. Therefore, a black-hole will only form if $u > \left(\frac{M_{\text{min}}}{Q}\right)^2$.

The type of partons from which a black-hole is formed determines the gauge charges of the black-hole. Clearly, the probability to create a black-hole in the collision of any two particular partons $i$ and $j$ with energies (momenta) $vE$ and $\frac{uE}{v}$, is given by

$$P(vE,\frac{uE}{v},i,j) = f_i(v,Q')f_j\left(\frac{u}{v},Q'\right)$$

(2.28)

The energies and types of the two colliding partons determine their momenta and
affect their locations within the ordinary and extra dimensions. For protons moving in the $z$-direction, we arbitrarily put one of the partons at the origin and locate the second parton randomly within a disk in the xy-plane of radius $b_{\text{max}}(E_{\text{CM}} = \sqrt{us}; d)$.

We must, however, also take into account that the partons will be separated in the extra dimensions as well. Each parton type is given a wave function in the extra dimension. For fermions, these wave functions are parametrized by their centers and widths which are input parameters (cf. Fig 2.14). In the split-fermion case, the centers of these wave functions may be widely separated; but even in the non-split case, the wave functions have non-zero widths. The thickness of the brane is about $M_*$, which is about the size of the black holes. For gauge bosons, the wave functions are taken to be constant across the (thick) brane.

The output from the generator (described in greater detail below) includes the energies, momenta, and types of partons that yielded black-holes. The locations in time and space of the black-holes are also output.

The formation of the black-hole is a very non-linear and complicated process. We assume that, before settling down to a stationary phase, a black-hole loses some fraction of its energy, linear and angular momentum. We parameterize these losses by three parameters: $1 - f_E$, $1 - f_P$ and $1 - f_L$. Thus the black-hole initial state that we actually evolve is characterized by

$$
E = E_{\text{in}}f_E;
$$
$$
P_z = P_{z\text{in}}f_P;
$$
$$
J' = L_{\text{in}}f_L;
$$

(2.29)

where $E_{\text{in}}$, $P_{z\text{in}}$ and $L_{\text{in}}$ are initial energy, momentum and angular momentum of colliding partons, while $f_E$, $f_P$ and $f_L$ are the fractions of the initial energy, momentum and angular momentum that are retained by the stationary black-hole. We expect that most of the energy lost in the non-linear regime is radiated in the form
of gravitational waves and thus represents missing energy. Yoshino and Rychkov [48] have calculated the energy losses by numerical simulation of collisions. Their results will be incorporated in a future upgrade of BlackMax.

For a small black-hole, the numerical value for the angular momentum is of the order of several $\hbar$. In that range of values, angular momentum is quantized. Therefore a black-hole cannot have arbitrary values of angular momentum. We keep the actual angular momentum of the black-hole, $J$, to be the nearest half-integer, i.e. $2J = \left[2J' + \frac{1}{2}\right]$.

The loss of the initial angular momentum in the non-linear regime has as a consequence that the black-hole angular momentum is no longer in the transverse plane of the colliding protons. We therefore introduce a tilt in the angular-momentum

$$\theta \equiv \cos^{-1}\left(\frac{J}{\sqrt{J(J+1)}}\right).$$

(2.30)

Figure 2.12 illustrates this geometry.

In this version of the generator, we have assumed that the angular-momentum quantum numbers of the black-hole were $(J, J_m = J)^5$. We next randomly choose an angle $\phi$, and then reset the angular-momentum axis to $(\theta, \phi)$.

### 2.3 Gray-body Factors

Once the black-hole settles down to its stationary configuration, it is expected to emit semi-classical Hawking radiation. The emission spectra of different particles from a given black-hole depend in principle on the mass, spin and charge of the black-hole, on the “environment”\(^6\) and on the mass and spin of the particular particle. Wherever possible we have made use of the correct emission spectrum often phrased in terms of the gray-body factor for black-holes in 3+1-dimensional space-time. In most cases,\(^5\)

\(^5\)Future versions of the generator may randomize the choice of $J_m$.

\(^6\)Dimensionality and geometry of the bulk, brane tension, location of fermionic branes
these were extant in the literature, but we have calculated the spectra for the split-fermion model ourselves, and reproduced existing spectra independently. The sources of the gray-body factors are summarized in Table A.1.

- **Non-rotating black-hole on a tensionless brane:** For a non-rotating black-hole, we used previously known gray-body factors for spin 0, 1/2 and 1 fields in the brane, and for spin 2 fields (i.e. gravitons) in the bulk.

- **Rotating black-hole on a tensionless brane:** For rotating black-holes, we used known gray-body factors for spin 0, 1/2 and 1 fields on the brane. The correct emission spectrum for spin 2 bulk fields is not yet known for rotating black-holes, we currently do not allow for the emission of bulk gravitons from rotating black-holes. As discussed below, this remains a serious shortcoming of current micro-black-hole phenomenology, since super-radiance might be expected to significantly increase graviton emission from rotating black-holes, and thus increase the missing energy in a detector.

- **Non-rotating black-holes on a tensionless brane with fermion brane splitting:** In
the split-fermion models, gauge fields can propagate through the bulk as well as on the brane, so we have calculated gray-body factors for spin 0 and 1 fields propagating through the bulk, but only for a non-rotating black-hole for the split-fermion model. These are shown in Figures A.1-A.12.

• **Non-rotating black-holes on a non-zero tension brane:** The bulk gray-body factors for a brane with non-zero tension are affected by non-zero tension because of the modified bulk geometry (deficit angle). We have calculated gray-body factors for spin 0, 1 and 2 fields propagating through the bulk, again only for the non-rotating black-hole for a brane with non-zero tension and $d = 5$.

• **Two particle final states:** We use the same gray-body factors as a non-rotating black-hole to calculate the cross section of two-particle final states (excluding gravitons).

In all cases, the relevant emission spectra are loaded into a data base as described in appendix A.

### 2.4 Black-Hole Evolution

The Hawking radiation spectra are calculated for the black-hole at rest in the center-of-mass frame of the colliding partons. The spectra are then transformed to the laboratory frame as needed. In all cases we have not (yet) taken the charge of the black-hole into account in calculating the emission spectrum, but have included phenomenological factors to account for it as explained below.

The degrees of freedom of the Standard-Model particles are given in Table A.2. Using the calculated Hawking spectrum and the number of degrees of freedom per particle, we determine the expected radiated flux of each type of particle as a function of black-hole and environmental properties. For each particle type $i$ we assign to
it a specific energy, $h\omega_i$ with a probability determined by that particle’s emission spectrum. (The particle “types” are listed in Table A.2.)

Assume a black-hole with mass $M_{bh}$ emits a massless particle with energy $h\omega_i$. The remaining black-hole will have energy and momentum like

$$(M_{bh} - h\omega_i, -h\omega_i) \quad (2.31)$$

Here we ignore the other dimensions. We use a classical model to simulate the events. The mass of the remaining black-hole should remain positive. So from equation (2.31) one gets

$$h\omega_i < M_{bh}/2 \quad (2.32)$$

Combining this with the observation that energy of a particle is larger than its mass, leads us to require that

$$M_i < h\omega_i \quad (2.33)$$

here, $M_i$ is the mass of $i$’s particle type.

Figure 2.13: The emitted fermion intensity (normalized to one) as a function of the distance between the black-hole and the center of the gaussian distribution of a fermionic brane. As a black-hole increases the distance from a fermionic brane due to recoil the intensity of the emitted fermions of that type falls down quickly. The radius of black-hole is set to be $2M_s^{-1}$. The width of the fermionic brane is $M_s^{-1}$. The plot is for the case of one extra dimension.
We next need to determine whether that particle with that energy is actually emitted within one generator time-step $\Delta t$. The time-step itself is an input parameter (cf. figure 2.14). We choose a random number $N_r$ from the interval $[0, 1]$. Given $L_{F_i}$, the total number flux of particles of type $i$, and $N_i$, the number of degrees of freedom of that particle type, the particle will be emitted if

$$L_{F_i} N_i \Delta t > N_r.$$  

\hspace{2cm} (2.34)

In the single-brane model, $L_{F_i}$ is derived from the power spectrum of the Hawking radiation. In the split-fermion model, we include a suppression factor for fermions. The factor depends on the overlap between the particular fermion brane and the black-hole when the black-hole is not located on that fermion’s brane. Fig. 2.13 shows how the spectrum of emitted particles changes as the black-hole drifts away from the center of the fermion brane. As a black-hole increases its distance from a fermion brane due to recoil, the intensity of the emitted fermions of that type declines quickly.

If the particle is to be emitted, we choose its angular-momentum quantum numbers $(l, m)$ according to:

$$P_{em}(i, l, m, E) = \frac{L_{i,l,m}(E)}{\sum_{l',m'} L_{i,l',m'}(E)}.$$  

\hspace{2cm} (2.35)

Here $P_{em}(i, l, m, E)$ is the probability that a type $i$ particle with quantum numbers $(l, m)$ will be emitted. $L_{i,l,m}(E)$ is the emission spectrum of a particle of type $i$ with quantum numbers $(l, m)$. This step is omitted in the case of non-rotating black-holes since we do not follow the angular-momentum evolution of the black-hole.

Once the quantum numbers of the emitted particle are determined, we calculate the direction of emission according to the corresponding Spherical Harmonic wave function:

$$P_{em}(\theta, \phi) = |\Psi_{lm}(\theta, \phi)|^2 \Delta \theta \Delta \phi.$$  

\hspace{2cm} (2.36)
Here $P_{em}(\theta, \phi)$ is the probability of emission in the $(\theta, \phi)$ direction for the angular quantum numbers $(\ell, m)$. $\Psi_{lm}(\theta, \phi)$ is the (properly normalized) Spherical Harmonic wave function of the mode with those angular quantum numbers.

Once the energy and angular-momentum quantum numbers are determined for the $i$-th particle type, then, if that particle type carries SU(3) color we assign the color randomly. The color is treated as a three-dimensional vector $\vec{c}_i = (r, b, g)$, in which a quark’s color-vector is either $(1, 0, 0)$, $(0, 1, 0)$, or $(0, 0, 1)$. Similarly, an anti-quark has $-1$ entries in its color-vector. A gluon’s color-vector has one $+1$ entry, and one $-1$ entry.

The black-hole gray-body factors which quantify the relative emission probabilities of particles with different spin are calculated for a fixed background, i.e. assuming that the black-hole metric does not change during the emission process. However, as the black-hole emits particles, the spin and charge of the black-hole do change significantly at black-hole mass near $M_*$. 

### 2.4.1 Electric and Color Charge Suppression

A charged and highly rotating black-hole will tend to shed its charge and angular momentum. Thus, emission of particles with charges of the same sign as that of the black-hole and angular momentum parallel to the black-hole’s will be preferred. Emission of particles that increase the black-hole’s charge or angular momentum should be suppressed. The precise calculation of these effects has not as yet been accomplished. Therefore, to account for these effects we allow optional phenomenological suppression factors for both charge and angular momentum.

The following charge-suppression factors can currently be used by setting parameter 19 (cf. section 2.6) equal to 2.

\begin{align*}
F_Q &= \exp(\zeta_Q Q_{bh} Q_{em}) \quad \text{(2.37)} \\
F_3^a &= \exp(\zeta_3 c_{bh}^a c_{em}^a) \quad a = r, b, g. \quad \text{(2.38)}
\end{align*}
\(Q^{bh}\) is the electromagnetic charge of the black-hole, \(Q^{em}\) is the charge of the emitted particle; \(c^{bh}_{a}\), is the color value for the color \(a\), with \(a = r, b, g\), of the black-hole, and \(c^{em}_{a}\), is the color value for the color \(a\), with \(a = r, b, g\), of the emitted particle. \(\zeta_Q\) and \(\zeta_3\) are phenomenological suppression parameters that are set as input parameters of the generator.

We estimate \(\zeta_Q = \mathcal{O}(\alpha_{em})\) and \(\zeta_3 = \mathcal{O}(\alpha_s)\), where \(\alpha_{em}\) and \(\alpha_s\) are the values of the electromagnetic and strong couplings at the Hawking temperature of the black-hole. Note that we currently neglect the possible restoration of the electroweak symmetry in the vicinity of the black-hole when its Hawking temperature is above the electroweak scale. Clearly, since \(\alpha_{em} \simeq 10^{-2}\) we do not expect electromagnetic (or more correctly) electroweak charge suppression to be a significant effect. However, since \(\alpha_s(1\text{ TeV}) \simeq 0.1\), color suppression may well play a role in the evolution of the black-hole.

Once we have determined the type of particle to be emitted by the black-hole, we draw a random number \(N_r\) between 0 and 1 from a uniform distribution. If \(N_r > F^Q\) then the emission process is allowed to occur, if \(N_r < F^Q\) then the emission process is aborted. We repeat the same procedure for color suppression factor, \(F^3_a\). Thus, particle emission which decreases the magnitude of the charge or color of the black-hole is unsuppressed; this suppression prevents the black-hole from acquiring a large charge/color, and gives preference to particle emission which reduces the charge/color of the black-hole.

### 2.4.2 Movement of the Black-Hole during Evaporation

We choose the direction of the momentum of the emitted particle \((\hat{P}_e)\) according to equation (2.36) in the center-of-mass frame and then transform the energy and momentum to their laboratory frame values \(\hbar \omega'\) and \(\vec{P}_e'\). The black-hole properties (energy, momentum, mass, colors, and charge) are then accordingly updated for the
next time step:

\[
E(t + \Delta t) = E(t) - \hbar \omega' \\
\vec{P}(t + \Delta t) = \vec{P}(t) - \vec{P}'(t) \\
M(t + \Delta t) = \sqrt{E(t + \Delta t)^2 - \vec{P}(t + \Delta t)^2} \\
\vec{c}^{bh}(t + \Delta t) = \vec{c}^{bh}(t) - \vec{c}_i \\
Q^{bh}(t + \Delta t) = Q^{bh}(t) - Q_i
\]

Here $\vec{c}^{bh}$ is the color 3-vector of the black-hole and can have arbitrary integer entries.

Due to the recoil from the emitted particle, the black-hole will acquire a velocity $\vec{v}$ and move to a position $\vec{x}$:

\[
\vec{v}(t) = \vec{P}(t)/E \\
\vec{x}(t + \Delta t) = \vec{x}(t) + \vec{v}(t) \Delta t
\]

Since fermions are constrained to live on the 3+1-dimensional regular brane, the recoil from fermions is not important. Only the emission of vector fields, scalar fields and gravitons gives a black hole momentum in extradimension. Once a black hole gains momentum in extradimension, it is able to leave the regular brane if it carries no gauge charge. In the split fermion case, it can move within the mini-bulk even if it carries gauge charge. In the case of rotating black holes, because the gray-body factor for gravitons is not yet known, graviton emission is turned off in the generator and the black holes experience no bulk recoil.

Recoil can in principle change the radiation spectrum of the black-hole in two ways. First, the spectrum will not be perfectly thermal or spherically symmetric in the laboratory frame, but rather boosted due to the motion of the black-hole. However, as we shall see, the black-hole never becomes highly relativistic, so the recoil does not significantly affect the shape of the spectrum.
As the lifetime of a small black-hole is relatively short, and its recoil velocity non-relativistic, it does not move far from its point of creation. However, even a recoil of the order of one fundamental length $\sim M^{-1}$ in the bulk direction could dislocate the black-hole from the brane$^7$. In single-brane models this would result in apparent missing energy for an observer located on the brane able to detect only Standard Model particles. In the split-fermion model, as the black-hole moves off or on particular fermion branes, the decay channels open to it will change.

### 2.4.3 Rotation

Since two colliding particles always define a single plane of rotation, rotating black-holes are formed with a single rotational parameter. For two particles colliding along the $z$-axis, there should be only one rotation axis perpendicular to the $z$-axis. However, due to angular-momentum loss both in the formation process, and subsequently in the black-hole decay, three things can happen: i) the amount of rotation can change, ii) the rotation axis can be altered, and iii) more rotation axes can emerge, because there are more than three spatial dimensions. Also, if the colliding particles have a non-zero impact parameter in bulk directions$^8$ the plane of rotation will not lie entirely in the brane direction. Because solutions do not exist for rotating black-holes with more than one rotation axis, we forbid the emergence of secondary rotation axes. We do, on the other hand, allow the single rotation axis of the black-hole to evolve. However, no gray-body factors are known if the single rotation axis acquires components in the extra dimensions, therefore we limit the rotation axis to the brane dimensions. Relaxing these limitations is a subject for future research.

We next must determine the rotational axis of the black-hole. The rotation parameter of a black-hole with angular-momentum quantum numbers $(j, j)$ is taken to

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$^7$This is very unlikely because most of the black-holes have gauge charges.

$^8$Due to the finite thickness of the single brane or splitting between the quark branes.
be
\[ a = \frac{J \cdot d - 1}{M} \]
(2.46)
where \( J = \sqrt{j(j+1)}\hbar \). The direction of the black-hole angular momentum is taken to be
\[ \vec{J} = j\hbar \hat{\omega} + \sqrt{j\hbar} \hat{l}_\perp, \]
(2.47)
where \( \hat{l}_\perp \) is a unit vector in the plane perpendicular to \( \hat{\omega} \). We chose the direction of \( \hat{l}_\perp \) randomly.

When the black-hole emits a particle with angular-momentum quantum numbers \((l, m)\), there are several possible final states in which the black-hole can end up. We use Clebsch-Gordan coefficients to find the probability of each state.

\[ |j, j\rangle = \sum_{j' \leq j + l} \mathcal{C}(j, j; l, m, j', j - m) |j', j - m\rangle |l, m\rangle \]
(2.48)

We use \(|\mathcal{C}(j, j; l, m, j', j - m)|^2\) as the probability that the new angular-momentum quantum numbers of the black-hole will be \((j', j - m)\). From angular-momentum conservation \( \vec{J} = \vec{L} + \vec{J}' \), we can calculate the tilt angle of the black-hole rotation axis as:
\[ \cos \theta = \frac{j(j+1) + j'(j'+1) - l(l+1)}{2 \sqrt{j(j+1)j'(j'+1)}}. \]
(2.49)
We randomly choose a direction with the tilt angle \( \theta \) as a new rotation axis and change quantum numbers to \((j', j')\).

In calculating the gray-body factors, the black hole is always treated as a fixed unchanging background. The power spectrum of emitted particles can be calculated from
\[ \frac{dE}{dt} = \sum_{l, m} |A_{l, m}|^2 \frac{\omega}{exp((\omega - m\Omega)/T_H) \mp 1/2}\pi d\omega. \]
(2.50)
Here \( l \) and \( m \) are angular momentum quantum numbers. \( \omega \) is the energy of the
emitted particle. $\Omega$ is defined by

$$\Omega = \frac{a_s}{(1 + a_s^2)r_h}. \quad (2.51)$$

The exponential factor in the denominator of (2.50) causes the black hole to prefer to emit high angular momentum particles. However, since the TeV black holes are quantum black holes, the gray-body factors should really depend on both the initial and final black-hole parameters. The calculation of the gray-body spectra on a fixed background can cause some problems. In particular, in the current case, the angular momentum of the emitted particle (as indeed the energy) may well be comparable to that of the black hole itself. There should be a suppression of particle emission processes in which the black hole final state is very different from the initial state. We therefore introduce a new phenomenological suppression factor, parameter 17, to reduce the probability of emission events in which the angular momentum of the black hole changes by a large amount.

If parameter 17 is equal to 1 (cf. section 2.6), we do not take into account the suppression of decays which increase the angular momentum of the black-hole. If we are using $\Delta$Area suppression (parameter 17 equal to 2) then

$$F_L^L = \exp(\zeta_L r_{bh}^b(t + \Delta t)^2 / r_{bh}^b(t)^2 - 1)). \quad (2.52)$$

If we are using $J_{bh}$ suppression (parameter 17 equal to 3) then

$$F_L^L = \exp(-\zeta_L |J_{bh}^b(t + \Delta t)|). \quad (2.53)$$

If we are using $\Delta J_{bh}$ suppression (parameter 17 equal to 4) then

$$F_L^L = \exp(-\zeta_L |J_{bh}^b(t + \Delta t) - J_{bh}^b(t)|). \quad (2.54)$$
We might expect $\zeta_L \sim 1$, however there is no detailed theory to support this; as indeed there is no detailed theory to choose among these three phenomenological suppression factors. It is also worth noting that, while for $d = 3$ and $d = 4$ there is a maximum angular momentum that a black-hole of a given mass can carry, for $d \geq 5$ there is no such upper limit. We do impose that $a \leq R_s/2$ for $d = 3$ and $a \leq R_s$ for $d = 4$.

As for the charge and color suppression, we choose a random number $N_r$ between 0 and 1. If $N_r > F^L$ then the particle emission is aborted.

The procedure described in this section is then repeated at each time step with each particle type, and then successive time steps are taken until the mass of the black-hole falls below $M_*$. In practice, the time step should be set short enough that in a given time step the probability that particles of more than one type are emitted is small. We set the time step to $\Delta t = 10^{-5}$ GeV$^{-1}$.

In two-body final states, one expects no black-hole, and hence no black-hole decay by emission of Hawking radiation. The generator therefore proceeds directly to the final burst phase.
2.5 Final Burst

In the absence of a self-consistent theory of quantum gravity, the last stage of the evaporation cannot be described accurately. Once the mass of black-hole becomes close to the fundamental scale $M_*$, the classical black-hole solution can certainly not be used anymore. We adopt a scenario in which the final stage of evaporation is a burst of particles which conserves energy, momentum and all of the gauge quantum numbers. For definiteness, we assume the remaining black-hole will decay into the lowest number of Standard-Model particles that conserve all quantum number, momentum and energy.

A black-hole with electromagnetic charge $Q^{bh}$ and color-vector $\vec{c}^{bh} = (r^{bh}, b^{bh}, g^{bh})$ will be taken to emit $N_{-1/3}$ down-type quarks (i.e. d,s or b quarks), $N_{2/3}$ up-type quarks (u,c, or t), $N_{-1}$ charged leptons and W bosons, $N_{gl}$ gluons, and $N_{n}$ non-charge particles (ie. $\gamma$, Z an Higgs). We use the following procedure to determines $\vec{N}_{burst} \equiv (N_{-1/3}, N_{2/3}, N_{-1}, N_{gl}, N_{n})$.

Step 1: preliminary solution:

- Search all possible solutions with $N_n = 0$.
- Choose the minimum number of particles as preliminary solution.

Step 2: Actual charged/colored emitted particle count:

- The preliminary $\vec{N}_{burst} \equiv (N_{-1/3}, N_{2/3}, N_{-1}, N_{gl}, N_{n})$ having now been determined. If the minimum number of solution is less than 2, we then add $N_n$ to keep the total number equal to 2. Later we choose one of them randomly according to the degrees of freedom of each particle.

- After obtaining the number of emitted particles, we randomly assign their energies and momenta, subject to the constraint that the total energy and momentum equal that of the final black-hole state. We currently neglect any bulk components of the final black-hole momentum.
2.6 Input and Output

The input parameters for the generator are read from the file parameter.txt, see Fig.2.14:

1. Number_of_simulations: sets the total number of black-hole events to be simulated;

2. Center_of_mass_energy_of_protons: sets the center-of-mass energy of the colliding protons in GeV;

3. M_ph: sets the fundamental quantum-gravity scale ($M_*$) in GeV;

4. Choose_a_case: defines the extra dimension model to be simulated:
   
   (a) 1: non-rotating black-holes on a tensionless brane with possibility of fermion splitting,
   
   (b) 2: non-rotating black-holes on a brane with non-zero positive tension,
   
   (c) 3: rotating black-holes on a tensionless brane with $d=5$,
   
   (d) 4: two-particle final-state scenario;

5. number_of_extra_dimensions: sets the number of extra dimensions; this must equal 2 for brane with tension (Choose_a_case=2);

6. number_of_splitting_dimensions: sets the number of extra split-fermion dimensions (Choose_a_case=1);

7. extradimension_size: sets the size of the mini-bulk\(^9\) in units of 1/TeV (Choose_a_case=1)

8. tension: sets the deficit-angle parameter $B$ [15, 16] (Choose_a_case=2);

9. choose_a_pdf_file: defines which of the different CTEQ6 parton-distribution functions (PDF) to use;

\(^9\)This is the distance between fermion branes where only gauge bosons and Higgs field can propagate in split-fermion brane scenario.
10. **Minimum_mass**: sets the minimum mass $M_{\text{min}}$ in GeV of the initial black-holes;

11. **fix_time_step**: If equal to 1, then code uses the next parameter to determine the time interval between events; if equal to 2 then code tries to optimize the time step, keeping the probability of emitting a particle in any given time step below 10%.

12. **time_step**: defines the time interval $\Delta t$ in GeV$^{-1}$ which the generator will use for the black-hole evolution;

13. **Mass_loss_factor**: sets the loss factor $0$ for the energy of the initial black-hole, as defined in equation 2.29;

14. **momentum_loss_factor**: defines the loss factor $0 \leq f_p \leq 1$ for the momentum of initial black-holes as defined in equation 2.29;

15. **Angular_momentum_loss_factor**: sets the loss factor $0 \leq f_L \leq 1$ for the angular momentum of initial black-holes as defined in equation 2.29;

16. **Seed**: sets the seed for the random-number generator (9 digit positive integer);

17. **L_suppression**: chooses the model for suppressing the accumulation of large black-hole angular momenta during the evolution phase of the black-holes (cf. discussion surrounding equations 2.52-2.54);

   - 1: no suppression;
   - 2: $\Delta$ Area suppression;
   - 3: $J_{bh}$ suppression;
   - 4: $\Delta J$ suppression;

18. **angular_momentum_suppression_factor**: defines the phenomenological angular-momentum suppression factor, $\zeta_L$ (cf. discussion surrounding equation 2.52-2.54);
19. **charge** suppression turns the suppression of accumulation of large black-hole electromagnetic and color charge during the black-hole evolution process on or off (cf. discussion surrounding equation 2.37)

   - 0: charge suppression turned off;
   - 1: charge suppression turned on;

20. **charge** suppression factor sets the electromagnetic charge suppression factor, $\zeta_Q$, in 2.37;

21. **color** suppression factor sets the color charge suppression factor, $\zeta_3$ in 2.37;

21-94 (odd entries:) the widths of fermion wave functions (in $M^{-1}_e$ units); and (even entries:) centers of fermion wave functions (in $M^{-1}_e$ units) in split-brane models, represented as 9-dimensional vectors (for non-split models, set all entries to 0).

When the code terminates, the file output.txt with all the relevant information (i.e. input parameters, cross section) is output to the working directory. This file contains also different segments of information about the generation of black-holes which are labelled at the beginning of each line with an ID word (Parent, Pbh, trace, Pem, Pemc or Elast):

- **Parent**: identifies the partons whose collision resulted in the formation of the initial black-hole (see Fig. 2.16).

  - column 1: identifies the black-hole;
  - column 2: PDGID code of the parton;
  - column 3: energy of the parton;
  - columns 4-6: brane momenta of the parton.
• **Pbh**: contains the evolution of the charge, color, momentum and energy of the black-holes, and, for rotating black-holes, their angular momentum (cf. Fig. 2.17).
  
  – column 1: identifies the black-hole;
  
  – column 2: time at which the black-hole emitted a particle;
  
  – column 3: PDGID code of a black-hole;
  
  – column 4: three times the electromagnetic charge of the black-hole;
  
  – columns 5 to 7: color-charge vector components of the black-hole;
  
  – columns 8: energy of the black-hole in the laboratory frame;
  
  – columns 9 to 11: brane components of the black-hole momentum in the laboratory frame;
  
  – columns 12 to (8+d): bulk components of the black-hole momentum;
  
  – column (9+d): angular momentum of the black-hole, in the case of rotating black-holes; empty otherwise.
  
• **trace**: contains the evolution history of the black-holes’ positions (cf. Fig. 2.18):

  – column 1: identifies the black-hole;
  
  – column 2: the times at which the black-hole emitted a particle;
  
  – columns 3 to 5 are the brane components of the black-hole position vector when the black-hole emitted a particle;

  – columns 6 to (2+d): the bulk components of the black-hole position vector, when the black-hole emitted a particle.

• **Pem**: contains a list of the black-holes, with the history of their evolution (cf. Fig. 2.19):
- column 1: identifies the black-hole;
- column 2: the times at which the black-hole emitted a particle;
- column 3: PDGID code of the emitted particle;
- column 4: three times the charge of the emitted particle;
- columns 5 to 7: color-vector components of the emitted particle;
- columns 8: energy of the emitted particle in the laboratory frame;
- columns 9 to 11: brane components of the momentum of the emitted particle, in the laboratory frame;
- columns 12 to (8+d): bulk components of the momentum of the emitted particle.

- **Pemc**: contains the same information as Pem, but in the center-of-mass frame of the collision.

- **Elast**: contains the same information as Pem for the particles emitted in the final decay burst of the black-hole. Column 12 and onwards are omitted as these particles have no bulk momentum.

### 2.7 Results

We choose the following parameters for the distributions shown in this section and normalize them to an integrated luminosity of $10^{36} fb^{-1}$, unless otherwise stated. The values of the parameters are chosen to be the same as in figure 2.14, except where a parameter is varied to study its effect. We consider a black hole as a classical object in this simulation. To avoid quantum effects becoming significant we choose the initial black-hole mass to be larger than $5M_\odot$. Therefore, the wavelength of incoming partons is about 10 times smaller than the black hole’s radius. The lifetime is long enough that a black hole can thermalize.
• Number_of_simulations = 10000;

• Center_of_mass_energy_of_protons = 14000 GeV;

• M_ph = 1000 GeV;

• extradimension_size = 10 TeV^{-1};

• choose_a_pdf_file = 0;

• Minimum_mass = 5000 GeV;

• time_step = 10^{-5} GeV^{-1};

• Mass_loss_factor = 0.0;

• momentum_loss_factor = 0.0;

• Angular_momentum_loss_factor = 0.2;

• Seed = 123589341;

• L_suppression = 1;

• charge_suppression = 1;

• charge_suppression_factor irrelevant since charge_suppression = 1;

• color_suppression_factor irrelevant since charge_suppression = 1;

• widths of fermion wave functions = 1 M_\pi^{-1};

• centers of quark wave functions: \((10^{-2}/3, 0, 0, \ldots) \text{GeV}^{-1}\) (i.e. all quarks have Gaussian wave functions centered on a point displaced from the origin by \(10^{-2}/3 \text{GeV}^{-1}\) in the first splitting direction);

• distribution center of leptons: \((-10^{-2}/3, 0, 0, \ldots) \text{GeV}^{-1}\).
In this section, we present some distributions of properties of the initial and evolving black-holes and of the particles which are emitted by them during the Hawking radiation and final-burst phases.

2.7.1 Mass of the Initial Black-Holes

Figures 2.21, 2.22 and 2.23 show the the initial black-hole mass distribution for three different extra dimension scenarios: non-rotating black-holes on a tensionless brane, non-rotating black-holes on a non-zero tension brane and non-rotating black-holes with split fermion branes respectively. Because we chose 5 TeV as the minimum mass of the initial black-hole, the distributions have a cut off at 5 TeV.

2.7.2 Movement of Black-Holes in the Bulk

The generator includes recoil of black-holes due to Hawking radiation. The recoil modifies the spectrum due to the Doppler effect. Even if the effect is small, the high energy tail of the emitted particle’s energy spectrum is longer than for pure Hawking radiation. Fig. 2.24 shows the random motion in the mini-bulk for 10,000 black-holes as consequence of recoil. While most of the black-holes remain on the brane where they were formed, a significant number of them are capable of drifting all the way to the lepton brane. There are also a few events where the black holes leave the mini-bulk completely. Since Standard-Model charges are confined to the mini-bulk, a black-hole needs to carry zero charge in order to be able to leave the mini-bulk. Once out of the mini-bulk, a black-hole cannot emit Standard-Model particles anymore. Models with an additional bulk $Z_2$ symmetry (e.g. Randall Sundrum models) do not allow for a black-hole recoil from the brane [28]. Unfortunately, the number of black-holes which escape the mini-bulk is so small that experimentally we are unlikely to be able to distinguish between models on this basis.
2.7.3 Initial Black-Hole Charge Distribution

Most of the initial black-holes are created by u and d quarks. Denote by $N_{3Q}$ the number of black-holes that have electromagnetic charge $Q$. Fig. 2.25 is a histogram of $3Q$ for $n_s = 0$, $n_s = 4$, and $n_s = 7$ in $d = 10$ space. Since at these parton momenta, there are roughly twice as many u-quarks in a proton as d-quarks, we expect that most of the black-holes have $3Q = 4$. i.e. are made of two u-quarks ($f_{uu}$). One does indeed see a large peak at $3Q = 4$ in figure 2.25. A second peak at $3Q = 1$ corresponds to black-holes made of one d and one u-quark, or from one anti-d and one gluon. Since there are only a few gluons or anti-quarks at these momenta, $f_1 \simeq f_{ud}$.

Similarly, the small peak at $3Q = -2$ is predominantly $dd$ and not $\bar{u}$-gluon.

We expect that $f_{ud} \simeq 2\sqrt{f_{uu}f_{dd}}$, and thus $f_1 \simeq 2\sqrt{f_{f_{-2}}}$. This relation is roughly satisfied in Fig. 2.25. In the split-fermion case, since gluons can move in the mini-bulk, there is a further suppression of the gluon contribution due to the wave-function-overlap suppression between the gluons and fermions. In particular $3Q = 2$ and $3Q = -1$, which are dominated by gluon-quark collisions, are suppressed, as can be seen in Fig.2.25. For a large number of split dimensions, there are almost no gluon-gluon or gluon-quark black-holes. The decline in the gluon-quark configurations accounts for the simultaneous rise in the fraction of quark-only configurations (i.e. $3Q = 4, 1, -2$).

2.7.4 Initial Black-Hole Color Distribution

The colliding partons that form the black-hole carry gauge charges, in particular color and electromagnetic charge. From the PDFs [33] we see that, at the relevant parton momentum, most of the partons are u and d type quarks – essentially the valence quarks. Contributions from “the sea” – other quarks, antiquarks, gluons and other partons – are subdominant. We therefore estimate the distribution of the colors of
the initial black-holes to be

\[ N_i(0) : N_i(1) : N_i(2) = 4 : 4 : 1. \] (2.55)

Here \( N_i(p) \) is the number of black-holes whose \( i \)-th color-vector component (\( i=1 \) is red, \( i=2 \) blue, \( i=3 \) green) has the value \( p \). This agrees very well with the graph in Fig. 2.26. \( N_i(-1) \) and \( N_i(-2) \) refer to black-holes created from collisions involving gluons or anti-quarks. Their numbers are hard to estimate, but we expect that \( N_i(-2) \ll f_i(-1) \ll N_i(0 \leq p \leq 2) \), again consistent with Fig. 2.26.

At energy scales accessible at the LHC, the color distribution of black-holes in different brane world models does not differ from each other significantly. However, were \( M_{min} \) significantly lower (at or below 1 TeV), then black-hole production by gluon-gluon scattering would be more important, significantly altering the color distribution, and making it more sensitive to fermion brane-splitting (which lowers the gluon-gluon contribution).

### 2.7.5 Evolution of Black-Hole Color and Charge during the Hawking Radiation Phase

Figure 2.27 shows the color distribution of the black-holes which they accumulate during the evaporation phase. From equation 2.55, the expected average initial color of the black-holes is \( 2/3 \). Since the colors of emitted particles are assigned randomly, we expect the cumulative color distribution (CCD) to be symmetric around \( 2/3 \) and peaked at the value. This is indeed what we find.

The width of the CCD depends on the total number of particles emitted by the black-hole during its evaporation phase.

As discussed above, we allow for the possibility of suppressing particle emission which increases the charge, color or angular momentum of the black-hole excessively (cf. discussion around equations 2.37, and 2.52-2.54). Figure 2.27 shows also the
cumulative black-hole color distribution where we suppressed the accumulation of large color charges during the evaporation phase. In order to amplify the effect of color suppression, we have set $f_3 = 20$ instead of the expected $f_3 \simeq 0.1$. We see that the number of black-holes with a color charge larger than 1 is decreased.
Number of simulations
10000 (10000 and 100000)
Center of mass energy of protons
14000
M_ph(GeV)
1000.
Choose a case:(1:tensionless_nonrotating_2:tension_nonrotating_3:rotating_nonsplit_4:Lisa_two_particles_final_states)
1 (from 1 to 4)
number of extra dimensions
7 (from 1 to 7)
number of splitting dimensions
0 (from 0 to 7)
extradimension size(1/M_pl)
10.
tension(parameter of deficit angle:1_to_0)
1.0 (from 0.4 to 1.0)
choose a pdf file(0_to_41_cteq6)
0
Minimum mass(GeV)
5000. (1000. And 5000.)
Maximum mass(GeV)
14000,
fix time step(1:fix_2:no)
2 (1 and 2)
time step(1/GeV)
1.0e-5
Mass loss factor(0-1.0)
0.00
momentum loss factor(0-1.0)
0.0
Angular momentum loss factor(0-1.0)
0.0
Seed
123589341
L_suppression(1:none_2:delta_area_3:anular_momentum_4:delta_angular_momentum)
1
angular_momentum_suppression_factor
1
charge suppression(1:none_2:do)
1 (1 and 2)
charge suppression factor
1
color suppression factor
20
split fermion width(1/M_pl) and location(from-15to15)(up_to_9extradimensions)
u quark Right(Note:do not insert blank spaces)
1.0
1.0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
u quark Left(Note:do not insert blank spaces)
1.0
1.0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
u bar quark Right(Note:do not insert blank spaces)
1.0
1.0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
u bar quark Left(Note:do not insert blank spaces)
1.0
1.0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Figure 2.14: Parameter.txt is the input file containing the parameters that one can change. The words in parentheses are the parameters that are used in the paper.
**Parameter.txt**

<table>
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<tr>
<th>trace</th>
<th>PbH</th>
<th>PbH</th>
<th>Pem</th>
<th>Pemc</th>
<th>E_{lab}</th>
<th>E_{CM}</th>
<th>E_{lab}</th>
<th>E_{CM}</th>
<th>E_{lab}</th>
<th>E_{CM}</th>
<th>E_{lab}</th>
<th>E_{CM}</th>
<th>E_{lab}</th>
<th>E_{CM}</th>
</tr>
</thead>
<tbody>
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<td>5.20195e-04</td>
<td>-8.48948e-05</td>
<td>0.00000e+00</td>
<td>-1.56771e-04</td>
<td>5.74812e-03</td>
<td>0.00000e+00</td>
<td>0.00000e+00</td>
<td>1.89002e+03</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
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<td>0</td>
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<td>0.00000e+00</td>
<td>3.82207e+03</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parent</td>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1.22085e+03</td>
<td>0.00000e+00</td>
<td>0.00000e+00</td>
<td>1.22085e+03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Cross section**

The total cross section is $5.322505 \times 10^{-8}$ (GeV$^{-2}$) = $2.070454 \times 10^{-11}$ b. The cross section error is $1.161466 \times 10^{-8}$ (GeV$^{-2}$) = $4.518102 \times 10^{-12}$ b.

**Information about black holes and emitted particles**

Figure 2.15: Output.txt: There are three parts to this file. The first part is a copy of parameter.txt. The second part includes information about the black-hole and the emitted particles. The first column identifies the what type of information each row is supplying – parent is information about the two incoming partons; PbH is information on the energy and momenta of the produced black-holes; trace describes the location of the black-holes; Pem characterizes the emitted particles in the lab frame; Pemc characterizes the emitted particles in the center-of-mass frame; Elast describes the final burst. The third part of the file is the black-hole production cross-section as inferred from the events in this generator run.
Figure 2.16: Lines in the output file headed by the ID = Parent contain information about the initial partons which formed the black-hole.

Figure 2.17: Lines in the output file headed by the ID = Pbh contain the energies and momenta of the black-holes for each emission step. In case of rotating black-holes, the last column in the line is the angular momentum.

Figure 2.18: Lines in the output file headed by the ID = trace contain the location of the black-hole for each emission step.

Figure 2.19: Lines in the output file headed by the ID = Pem contain the types of the emitted particles, their energies and momenta in the lab frame and the times of their emission.
Figure 2.20: Lines in the output file headed by the ID = Elast contain the types, energies and momenta of particles of the final burst.

Figure 2.21: Mass distribution of initial black-holes (rotating and non-rotating) on a tensionless brane for various numbers of extra dimension.

Figure 2.22: Mass distribution of initial (non-rotating) black-holes on a non-zero tension brane for $B = 1.0$, $B = 0.8$ and $B = 0.6$. 
Figure 2.23: Mass distribution of initial (non-rotating) black-holes on a tensionless brane for $d = 10$ and different numbers of split-fermion branes.

Figure 2.24: black-hole movement in the mini-bulk due to recoil. $X_1$, $Y_1$ are coordinates in two extra dimensions. The red circle indicates the width of the quark brane. The blue circle indicates the width of the lepton brane. The black lines are black-holes traces. The size of the mini-bulk is $10 \text{TeV}^{-1} \times 10 \text{TeV}^{-1}$. black-holes with non-zero standard model gauge charges bounce back from the wall of the mini-bulk. black-holes with zero Standard Model gauge charges can leave the mini-bulk.
Figure 2.25: Electromagnetic charge distribution of the initial black-holes.

Figure 2.26: Initial color distribution of the created black-holes. The vertical lines are error bar.

Figure 2.27: Cumulative color distribution for non-rotating black-holes on a tensionless brane with $d = 4$ and no fermion brane splitting. Histogram with the black squares (open circles) is with (without) color suppression.
Figure 2.28: Number of particles emitted by a non-rotating black-hole on a tensionless brane prior to the “final burst” as function of number of extra dimension. Here $n_s = 0$. The error bars denote one standard deviation range.

Figure 2.29: Number of particles emitted by a non-rotating black-hole in the split-fermion model prior to the “final burst” as a function of the number of brane-splitting dimensions, with $d = 10$. Error bars denote one standard deviation range.
Figure 2.30: Number of particles emitted by a non-rotating black-hole on a non-zero tension brane prior to the “final burst” as function of deficit angle parameter $B$ with $d = 5$ and $n_s = 2$. Error bars denote one standard deviation range.

Figure 2.31: Number of particles emitted by a rotating black-hole prior to the “final burst” as function of number of extra dimension with $n_s = 0$. Error bars denote one standard deviation range.
2.7.6 Number of emitted particles

Figures 2.28 through 2.31 show the number of particles that are emitted by a microscopic black-hole during the decay process before its final burst for a variety of models.

In the single tensionless brane model (Fig. 2.28), the number of emitted particles first increases with the number of dimensions for a non-rotating black-hole, but then decreases. This behaviour is a result of the complicated interplay of a number of effects: the horizon size of a black-hole of a given mass as a function of $d$, and its effect on the particle emission spectra; the dependence of the Hawking temperature on $r_h^{(d)}$; the existence due to energy-momentum conservation of an upper limit of $M_{bh}^2/2$ on the energy of an emitted particle. The location of the peak will shift as a function of the input parameter $M_{\text{min}}$, the minimum initial mass of a black-hole.

In the split fermion model (Fig. 2.29), the number of emitted particles decreases with the number of extra dimensions. This is because, even for a fixed Hawking temperature, the average energies of emitted gauge bosons and scalar fields increase as $n_s$ increases.

In the model with a finite-tension brane (Fig. 2.30), the number of particles
decreases as the parameter $B$ increases, \textit{i.e.} with decreasing tension. As the tension increases, $B$ gets smaller but the horizon radius of the black-hole increases. The Hawking temperature therefore decreases, and, as a consequence, the average energy of emitted particles falls. More particles will therefore be emitted in the evolution of the black-hole.

For rotating black-holes (Fig. 2.31) (on a tensionless, unsplit brane), the number of emitted particles first increases, then decreases, and finally reaches a plateau. This is due to similar reasons as for non-rotating black-holes. Compared to non-rotating black-holes of the same mass, rotation shifts the energy of emitted particles to higher values because it decreases the horizon radius and increases the emission of higher angular-momentum modes. This decreases the total number of emitted particles. It also means that the effect of the upper kinematic limit of $M_{bh}/2$ on the emitted particle’s energy is magnified.

Figure 2.32 shows the number of particles that are emitted at the final burst stage for a non-rotating black-hole on a tensionless brane with $d = 5$ and $n_s = 0$. The average number of emitted particles is about 3. During the Hawking radiation phase a black-hole emits about 10 particles, so approximately 30% of the emitted particles will be from the final burst stage. In the case examined in Figure 2.32, we did not include suppression of large black-hole color or electric charge. Thus some black-holes acquire large color and electric charges by the end of the Hawking radiation phase. These black-holes then must decay into a large number of particles ($>5$) in the final burst.

2.7.7 Energy Distributions of the Emitted Particles

Once formed, the black-holes decay by emission of Hawking radiation, a process which continues until the mass of the black-hole falls to the fundamental quantum-gravity scale. At this stage we chose the black-holes to burst into a set of Standard-Model particles as described in section 2.5. The observable signatures of the decay will
depend on the distributions of energy, momentum and particle types of the emitted particles.

Figures 2.33 through 2.36 show the relation between the mass of the evolving black-hole and the average energy of emitted particles for different extra dimension models. The error bars denote $1/\sqrt{N}$ times the standard deviation of the mean energy. The minimum mass of the initial black-hole is taken to be $M_{\text{min}} = 5\, \text{TeV}$.

We see from figure 2.33 that, for a single (i.e. unsplit) brane, when $M_{\text{bh}} \gg M_*$, a black-hole in $d = 10$ emits higher energy particles than a black-hole in lower dimensions. For black-hole masses closer to $M_*$, the highest energy particles are emitted when the dimensionality of space is low, i.e. $d = 4$. This reversal can be understood from figure 2.2. In the LHC energy range, the curves of Hawking temperature as a function of black-hole mass for different dimensions cross. At high mass, high $d$ exhibits the highest Hawking temperature; at low mass, low $d$ exhibits the highest Hawking temperature. It is easy from this figure to estimate the number of emitted particles, and to roughly reproduce the results shown in figure 2.28.

The main difference between the curves for different $d$, comes from the changing size of the black-hole horizon. For low $d$, the horizon radius increases more quickly with the mass than for higher $d$, as seen in figure 2.1. The Hawking temperature of the black-hole is inversely proportional to $r_h$. So long as the Hawking temperature remains well below the black-hole mass (here for $M_{\text{bh}} \gtrsim 2\, \text{TeV}$), the energy of emitted particles decreases as the mass increases. However as equations (2.11-2.12) show, this change is slow for high $d$. By $d = 10$, the energy of emitted particles is almost constant from $2\, \text{TeV}$ to $5\, \text{TeV}$.

The increase of the energy of emitted particles stops at about $2\, \text{TeV}$ and then a decrease begins. The reason, as stated above (equation (2.32)), is that by energy-momentum conservation, a black-hole can only emit particles with less than half of its mass.

In the split-fermion model (Fig. 2.34), the average energy of the emitted particles
Figure 2.33: Average energy of the particles emitted by (non-rotating) black-holes on unsplit branes versus the mass of the black-hole at the time of emission. Here $d = 4$, $d = 7$ and $d = 10$. Note that the final burst is not included, because it occurs when the mass of the black-hole is less than 1 TeV.

... increases as the number of dimensions in the mini-bulk increases. The energy shift comes from the gauge bosons and scalar fields, which access the higher-dimensional phase space of the mini-bulk (cf. appendix A: figure A.1 through A.12).

For the brane with non-zero tension (Fig. 2.35), the radius of the black-hole increases with tension hence the energy of emitted particles decreases with tension.

For a rotating black-hole (Fig. 2.36), angular momentum decreases the size of the horizon. Thus since black-holes are typically formed with some initial angular momentum, they emit higher energy particles than non-rotating black-holes of the same mass. However, the black-hole tends to shed its angular momentum rapidly as it emits particles. This increases the horizon size, lowers the Hawking temperature, and lowers the average energy of the emitted particles. The rapid shedding of angular momentum thus leads to a drop in the average emitted particle energy around $M_{\text{min}}$.

If one compares rotating with non-rotating black-holes, one finds that the energy of the emitted particles is always larger for the rotating black-holes. By the time the mass of the black-hole has dropped well below $M_{\text{min}}$ (here to approximately $1 - 2$ TeV), almost all of the angular momentum has been lost and the difference between the rotating and non-rotating black-holes is small.
Figure 2.34: Average energy of the particles emitted by (non-rotating) black-holes on split branes versus the mass of the black-hole at the time of emission. Here $d = 10$ and $n_s = 0, n_s = 3, n_s = 7$.

As a Standard-Model particle is emitted from a black-hole, this particle may carry some momentum in the extra dimensional directions. As an observer in 3-space observes this particle, he/she will find the apparent mass of the particle is

$$M_{ob} = \sqrt{M_p^2 + \sum_e P_e^2}$$

(2.56)

Here $M_{ob}$ is the observed mass, $M_p$ is the true mass of the particle, and $P_e$ is the particles extra-dimensional momentum. Clearly $M_{ob} \geq M_p$. A Standard-Model particle, however, cannot leave the Standard Model brane. Its extra-dimensional momentum must therefore be absorbed by the brane or carried away by bulk particles (such as gravitons). We therefore calculate an emitted particle’s “energy on the brane” according to:

$$E_b = \sqrt{M_p^2 + \sum_{i=1}^{3} P_i^2}$$

(2.57)

where $P_i$ is the regular 3-momentum. We will assume that the shedding of extra-dimensional momentum is rapid, and henceforth we will refer to $E_b$ (rather than the initial emission energy) as the energy of the emitted particle.

Figure 2.37 shows the energy distribution of the particles from Hawking radiation.
Figure 2.35: Average energy of the particles emitted by (non-rotating) black-holes on a brane with tension, versus the mass of the black-hole at the time of emission. Here $d = 5$, $n_s = 2$ and $B = 1$, $B = 0.8$, $B = 0.6$.

in the single-brane model. The cross section for black-hole production increases with $d$ (figure 2.7). The area under the curves also increase with $d$. The peaks of the curves are around 200GeV to 400GeV. One can compare, for example, the energy distributions for $d = 9$ and $d = 10$. A black-hole in high $r$ $d$ tends to emit particles with higher energy, so the curve for $d = 10$ has a longer higher energy tail than for $d = 9$.

Figure 2.38 shows the energy distribution of the particles from the final burst in the single-brane model. The energy these particles share is much smaller than the energy in the earlier Hawking radiation phase. The peak in the energy distribution of these particle is around 200GeV to 300GeV. The tails extend just to 1TeV, which is the mass at which the black hole is taken to be unstable and undergo its final burst.

Figures 2.39 through 2.42 show the energy distribution of emitted particles (including final burst particles) in the various models that BlackMax can simulate.

Figure 2.39 shows the energy distribution in the single-brane model. The cross section of black-hole increases with $d$ (figure 2.7). The area under the curves also increases with $d$. The peaks of the curves are around 200 GeV to 400. Again comparing $d = 9$ and $d = 10$, a black-hole in higher $d$ tends to emit particles with higher
energy, so the curve for \( d = 10 \) has a longer high energy tail than for \( d = 9 \).

Figure 2.40 illustrates the split-fermion model. In this figure, we keep the total number of dimensions \( d \) fixed but change the dimensionality \( n_s \) of the mini-bulk. This affects the spectra of only the gauge boson and scalar fields, as only their propagation is affected by the the mini-bulk’s dimensionality. (Gravitons propagate in the full bulk; other Standard-Model particles propagate only on the brane.) As explained above, these spectra will shift to higher energies as the number of splitting dimensions is increased. One can see that the curve in \( n_s = 7 \) has the longest high energy tail.

Figure 2.41 illustrates the brane with tension model. The energies of the emitted particles shift to lower energy as \( B \) decreases.

The energy distribution of emitted particles for rotating black-holes (figure 2.42) has the same general characteristics as the distribution for non-rotating black-holes. Angular momentum causes a black-hole to tend to emit higher energy particles than a non-rotating black-hole. The curves have longer higher energy tails than the non-rotating black-holes.

Figure 2.43 shows the Energy distribution of emitted particles from two-body final-states scenario. The energy of the emitted particles is about the half of the
Incoming partons.

In figure 2.44 we show the energy distribution of different types of particles in the $d = 5$ single-brane model. The area of each curve is dependent on the degree of freedom of each particle and its power spectrum. One can compare the ratio of the same type of particles. For example, the area of gluons should be 8 times as large as the area of photons. It is roughly the same as what the figure shows.
2.7.8 Pseudorapidity Distributions of the Emitted Particles

Figures 2.45 to 2.48 show the pseudorapidity distributions of the emitted particles for different extra dimension scenarios.

Most of black-holes are made of two $u$ quarks and have charge $3/4$. For the shown figures we did not include charge suppression, because of that the black-holes tend to emit the same number of particles and antiparticles during Hawking radiation phase. This can be seen from the curves without final burst in figure 2.45 through figure 2.48. The majority of the black-holes are positive charged and will tend to emit positive particles in the final burst. That is why there are more positrons than electrons for the distributions which include the final burst particles.

Figure 2.49 shows the pseudorapidity distribution in the two-body final-state scenario. The distribution is much wider than the equivalent distribution from Hawking radiation. In this model we do not consider the angular momentum of the black hole. We therefore take the decay process in the two-body final state scenario to be isotropic in the ordinary spatial directions, just as in other models without angular momentum. In the center of mass frame, particles are therefore emitted in directions uncorrelated with the beam direction. Nevertheless, because the threshold energy of
Figure 2.40: Energy density distribution of emitted particles from non-rotating black-holes on a tensionless brane with fermion brane splitting. Shown are the distributions for \(d = 10\) and \(0 \leq n_s \leq 7\). The spectra include the final burst particles.

the two-body final-state model is much lower than that of other models, the intermediate state tends to have a higher velocity down the beam-pipe. The decay products are therefore emitted with larger pseudorapidity. In truth, one may expect that the intermediate state of the two-body final-state scenario has non-negligible angular momentum. However, since the intermediate state is not a real black hole, it is unclear exactly what role the angular momentum of the intermediate state plays.

In the final state scenario, the momenta of the two emitted particles are correlated with the initial parton momenta, and hence the pseudorapidity distribution of the emitted particles reflects that of the initial partons. The ratio of the number of events for pseudorapidity between 0 and 0.5 divided by that for pseudorapidity between 0.5 and 1 is about 1.1. This is much higher than the asymptotic QCD value of 0.6, as predicted by [17, 27]. If the ratio is found not to equal 0.6, then this would suggest new physics beyond the Standard Model.

### 2.7.9 Emitted Particle Types

Table A.3 shows, for a variety of representative extra-dimension scenarios the fraction of emitted particles which are of each possible type – quarks, gluons, (charged)
leptons, (weak) gauge bosons, neutrinos, gravitons, Higgs bosons and photons. One notable feature is that the intensity of gravitons relative to other particles increases with the number of extra dimensions. Note that the absence of gravitons in the case of a rotating black-hole is not physical, but rather reflects our ignorance of the correct gray-body factor.

2.8 Conclusion

Hitherto, black-hole generators for the large-extra-dimension searches at the LHC have made many simplifying assumptions regarding the model of both our three-dimensional space and the extra-dimensional space, and simplifying assumptions regarding the properties of the black-holes that are produced. In this paper we have discussed a new generator for black-holes at the LHC, BlackMax, which removes many of these assumptions. With regard to the extra-dimensional model it allows for brane tension, and brane splitting. With regard to the black-hole, it allows for black-hole rotation, charge (both electro-magnetic and color) and bulk recoil. It also introduces the possibility of a two-body final state that is not a black-hole.

Although BlackMax represents a major step forward, there remain important
Figure 2.42: Energy distribution of emitted particles for rotating black-holes for $4 \leq d \leq 10$.

Figure 2.43: Energy distribution of emitted particles for two-body final states.

deficiencies that will need to be addressed in the future. BlackMax continues to insist on a flat geometry for the bulk space, whereas there is considerable interest in a warped geometry [4] or in a compact hyperbolic geometry [5]. While BlackMax allows for black-hole rotation, the absence of either an analytic or a numerical gray-body factor for the graviton in more than three space dimensions for rotating black-holes is a serious shortcoming that can be expected to materially change the signature of black-hole decay for rotating black-holes. Other issues include how to properly account for the likely suppression of decays that cause a black-hole to acquire very “large” color, charge or angular momentum. (As opposed to the somewhat contrived
Figure 2.44: Energy distribution of each particle type in the $d = 5$ single brane model.

Figure 2.45: Pseudorapidity distribution of charge-leptons and anti-leptons for non-rotating black-holes on a tensionless branewith and without the final burst particles; $d = 5$.

phenomenological approach currently taken.) These are but a few of the fundamental issues that remain to be clarified.

Despite these (and no doubt other) shortcomings, we expect that BlackMax will allow for a much improved understanding of the signatures of black-holes at the LHC. Work in progress focuses on using BlackMax to explore the consequences for the ATLAS experiment of more realistic black-hole and extra-dimension models.
Figure 2.46: Pseudorapidity distribution of quarks and anti-quarks for non-rotating black-holes on a tensionless brane with and without the final burst particles; $d = 5$.

Figure 2.47: Pseudorapidity distribution of charged-leptons and anti-leptons for rotating black-holes, on a tensionless brane, with and without the final burst particles. Here $d = 5$.

Figure 2.48: Pseudorapidity distribution of quarks and anti-quarks for rotating black-holes, on a tensionless brane, with and without the final burst particles. Here $d = 5$. 
Figure 2.49: Pseudorapidity distribution for the two-body final-state scenario. The distribution in 2-body final states is much flatter near $\eta = 0$ than emission due to Hawking radiation.
Chapter 3

Birkhoff’s Theorem (BT)

3.1 Introduction

An important tool in astrophysics and cosmology is the ability to reconstruct an object’s mass from measurements of the gravitational force in its vicinity (say, by measuring velocities). Newtonian gravity’s inverse-square law guarantees that the gravitational flux through an enclosed surface is conserved, giving us a handle on the mass within that surface; this is Gauss’s law. Even though General Relativity (GR) is a highly non-linear theory, Birkhoff’s Theorem (BT) plays the same role and allows us to reconstruct the mass of spherically symmetric configurations. Two particularly important aspects of BT are that: (a) outside a spherically symmetric mass distribution, the gravitational potential (i.e. metric) depends on the distribution of the matter density only at second order in the potential, due to the role of binding energy as a source of gravity; and (b) a shell of mass has no effect on the metric in its interior. Without these properties we presumably could not calculate almost any gravitational fields without knowing details of the distribution of matter all over the Universe. Indeed many calculations in gravity would become undoable either in principle or practice, and many others would become enormously more difficult. In general, modifications to GR tend to violate BT [110].
Nevertheless, there has been considerable interest in theories that modify GR and break BT. The first of two primary phenomenological motivations for interest in such theories is the observation that at scales above stellar clusters there is too much gravity to be sourced by the observed matter. Thus either there is dark matter in such systems, or Newton’s Law is not valid in this domain. The latter possibility, known in a surprisingly successful, albeit phenomenological form as Modified Newtonian Dynamics (MOND), explains a range of such systems. The recent discovery of covariant theories [111, 112] which yield GR, Newton and MOND in appropriate limits has further fueled interest in modified gravity as an alternative to dark matter.

There is also interest in modified gravity theories as an alternative explanation to dark energy for the observed accelerated expansion of the universe. Again the existence of reasonable covariant extensions to the Einstein-Hilbert (EH) action, such as the DGP model of Dvali, Gabadadze and Porati [113] and \( f(R) \) theories [114], has been particularly important.

Theories that violate BT should also lose the benefits afforded by it. It would be expected that the geometry locally will depend on the detailed distribution of mass in the universe. If this dependence is sufficient to qualitatively affect the behavior of dynamical systems, then we could be in a most difficult situation of having alternative gravity theories in which we are fundamentally, or at least practically, unable to make firm predictions for a wide variety of important quantities.

We will not perform an exhaustive calculation of non-Birkhoffian behavior in all modified gravity theories, although that may well be a desirable program of research. Instead we will begin that endeavor by considering a particular example, DGP. We choose DGP both because it is a single theory, with only one free parameter of interest, and because the calculations are readily doable. Other modified theories of gravity in general will not respect BT, and a similar investigation for other theories of gravity is in progress [115], with preliminary results that are consistent with our expectations that DGP is a reasonable exemplar.
To cut precipitously to the bottom line, by solving the DGP equations for concentric spherical shells of dust in a background with accelerated expansion, we will demonstrate the violations of BT that can be expected in modified gravity theories. The violation in theories that seek to replace dark matter is likely to render the calculability of cosmology beyond the homogeneous background significantly more difficult or wholly impossible. Theories which replace dark energy may be safer, although even there we should still expect significant effects on observable scales and should not be complacent.

The paper is organized as follows. In section 3.2 we first review the background de-Sitter solution in DGP theory, and then solve the metric functions on the brane in the presence of a weak source on the brane. We show that the equations define the metric only up to a function, which we name \( g(r) \). We present our modeling for \( g(r) \) in section 3.3, and then consider and estimate the severity of the deviations from BT. We conclude in section 3.4.

### 3.2 Space time geometry

DGP introduces an infinite flat 5-dimensional space (bulk) in which gravitons move freely, while Standard Model fields are confined to the 4-dimensional brane world. The 5-dimensional gravity theory retains the EH action, while a 4-dimensional EH action is presumably induced by radiative corrections by the matter on the brane. The action of the theory (other than a possible Gibbons-Hawking term) is [113]

\[
S_{(5)} = -\frac{M_5^3}{16\pi} \int dz \, d^4x \sqrt{-g} R - \frac{M_p^2}{16\pi} \int d^4x \sqrt{-g^{(4)}} R^{(4)} + \int d^4x \sqrt{-\bar{g}^{(4)}} \mathcal{L}_m .
\]  

(3.1)

Here \( z \) is the extra dimension, \( M_5 \) is the fundamental five-dimensional Planck mass, \( M_p \) is the observed four-dimensional Plank mass, \( g \) and \( R \) are the 5-dimensional
metric and Ricci scalar, and $g^{(4)}$ and $R^{(4)}$ are the induced quantities on the brane. The gravity mass scale in the bulk is taken to be extremely low, $M_5 \sim 10^{-3} \text{eV}$. This introduces a new physical scale, the crossover scale, at which gravity becomes 5-dimensional, $r_0 = M_5^2/2M_5^3$. While Einstein gravity approximately holds on the brane on distances shorter than $r_0$, one expects modifications to gravity at larger distances due to the graviton’s free movement in the bulk. Actually, bulk effects leak into the brane on even shorter distances than $r_0$, due to non-linear interactions of the scalar degree of freedom of gravity [116, 117]. Taking this into account, the length scale at which gravity is modified, known as the Vainshtein radius, is $r_* = (r_0^2/r_s)^{1/3}$, where $r_s$ is Schwarzschild radius of the source. Since $r_* \ll r_0$, this may open an observational window to the existence of higher dimensions [118].

An attractive feature of DGP is that it has a branch of cosmological solutions in which cosmic acceleration occurs without dark energy [119] as the Hubble function approaches $r_0$ at late times. By choosing $r_0^{-1} \sim H_0$, DGP presents an alternative mechanism for the present acceleration. We follow [118] and solve in DGP for the metric of a spherically symmetric, static matter source in a de-Sitter background. We then check how the gravitational field changes when we keep the mass constant but change its distribution. We show that there may be a gravitational force within a spherical mass shell, depending on the form of an undetermined function, $g(r)$.

We start with the Einstein equations in the bulk:

$$\frac{G_{AB}}{2r_0} + \delta(z - z_0)G^{(4)}_{AB} = \frac{8\pi}{M_p^2} T_{AB}. \tag{3.2}$$

Here $G_{AB}$ is the bulk Einstein tensor, $G^{(4)}_{AB}$ is the intrinsic Einstein tensor on the brane, $T_{AB}$ is energy momentum tensor. We have explicitly specified that $T_{AB}$ vanishes except at the location $z_0$ of the brane in the extra dimension (see below, Eq.3.5). Without loss of generality, we can take $z_0 = 0$.

We wish to find the metric (at least on the brane) induced by a static, spherically
symmetric (3-d) weak matter source on the brane, in a background cosmology which, given observations, we take to be de Sitter-like. This problem was first addressed by Lue and Starkman [118]. Following them, we write the line element as

\[
\begin{align*}
\text{ds}^2 &= N^2(r, z)dt^2 - A^2(r, z)dr^2 - B^2(r, z) \\
&\quad \times [d\theta^2 + \sin^2\theta d\phi^2] - dz^2. \\
\end{align*}
\]

We assume a \(Z_2\) symmetry across the brane, and that there is no spatial curvature in the brane directions, \(B|_{z=0} = r\). The resulting bulk Einstein tensor and Einstein equations are presented in [118] (equations (2.7) and (2.8) respectively), and we do not reproduce them here.

### 3.2.1 Vacuum solution

The metric functions for the background de-Sitter solution [120]

\[
\begin{align*}
N(r, z) &= (1 \mp H|z|)(1 - H^2 r^2)^{1/2} \\
A(r, z) &= (1 \mp H|z|)(1 - H^2 r^2)^{-1/2} \\
B(r, z) &= (1 \mp H|z|)r, \\
\end{align*}
\]

with the vacuum energy-momentum tensor given by

\[
T^A_B = \delta(z)\text{diag}(\rho_H, -P_H, -P_H, -P_H, 0). \tag{3.5}
\]

The energy density and pressure due to the bulk are

\[
\rho_H \equiv -P_H \equiv \frac{3M_p^2H}{8\pi r_0}(r_0H \pm 1). \tag{3.6}
\]
3.2.2 Weak source on the brane

Adding a gravitational source on the brane, the energy momentum tensor remains of the form equation 3.5, with

\[ \rho = \rho_H + \rho_g(r) , \quad P = P_H + P_g(r) . \] (3.7)

Here \( \rho_g(r) \) and \( P_g(r) \) are the energy density and pressure due to the distribution of matter on the brane. It is convenient to redefine the metric functions,

\[ N(r, z) \equiv 1 + n(r, z) \]
\[ A(r, z) \equiv 1 + a(r, z) \] (3.8)
\[ B(r, z) \equiv (1 + b(r, z))r . \]

It can be seen from Eq. 3.4 that to first order in \( r \) and \( z \), we get \( n \approx H^2 r \mp H|z| \), and the solution for empty space has \( \dot{n}(r, 0) = \pm H \) (dots denote \( d/dz \)). Treating the mass distribution on the brane as a perturbation around the vacuum solution we can write

\[ \dot{n}(r, 0) = \mp H + g(r) , \] (3.9)

where \( g(r) \) is the correction due to the mass distribution. It is here that we deviate from [118], which took \( g(r) = 0 \). Even though \( g(r) \) is not strictly defined by the equations, [118] and [122] argue that \( g(r) = 0 \) is essential to recover a well-behaved bulk solution. This has been a matter of some dispute, with some authors [121] proposing alternative Schwarzschild solutions. Our own calculations suggest that the solution of [118] itself requires a small non-zero \( g(r) \) to recover the desired bulk solution for self-consistency.

We assume that the mass distribution is of negligible pressure, \( P_g(r) \approx 0 \), and we define \( R_g \) and \( G_g \) to be the 3-volume integrals over the mass density and \( g(r) \).
respectively,

\[
R_g(r) \equiv \frac{8\pi}{M_p^2} \int_0^r \tilde{r}^2 \rho_g(\tilde{r}) d\tilde{r},
\]

(3.10)

\[
G_g(r) \equiv \frac{3}{2r_0^2} \int_0^r \tilde{r}^2 g(\tilde{r}) d\tilde{r}.
\]

(3.11)

We now want to solve for the metric functions \(a\), \(b\), and \(n\). Except for \(G_{zz}\) and \(G_{zr}\), the bulk equations can be satisfied by choosing suitable quadratic terms. It has been shown that \(G_{zr}\) is identically zero when covariant conservation of the matter source and the brane boundary conditions are considered [118]. The equations we are left with (shown to first order) are the bulk equation \(G_{zz}\)

\[
n'' + \frac{2n'}{r} - 2(\frac{a}{r})' = (\pm H - g)(\dot{a} + 2\dot{b}) - (2\dot{a} + \dot{b})\ddot{b}
\]

(3.12)

(primes are derivatives with respect to \(r\) and dots are derivatives with respect to \(z\)), and the Einstein equations on the brane:

\[
\dot{a} + 2\dot{b} = r_0 \left( \frac{2a}{r^2} + \frac{2a'}{r} \right) - \frac{r_0}{r^2} R_g' - 3H(r_0H \pm 1)
\]

\[
2\dot{b} = r_0 \left( \frac{2a}{r^2} - \frac{2n'}{r} \right) - H(3Hr_0 \pm 2) - g
\]

(3.13)

\[
\dot{a} + \dot{b} = r_0 \left( -n'' - \frac{n'}{r} + \frac{a'}{r} \right) - H(3Hr_0 \pm 2) - g.
\]

Eliminating \(\dot{a}\) and \(\dot{b}\) and integrating \(dr\) yields

\[
n'r^2 + ra + \frac{1}{2}H^2r^3 + G_g - R_g = 0.
\]

(3.14)

We define a new quantity,

\[
f(r) \equiv rn' - \frac{1}{2r}(R_g - G_g) + (H^2 \pm \frac{H}{2r_0})r^2 + \frac{gr^2}{4r_0},
\]

(3.15)

and use it to replace \(\dot{b}\) and \(\dot{a} + 2\dot{b}\) in 3.12 and 3.13. Substituting into (3.12), multiplying
by \( r^2 \) and integrating with respect to \( r \) yields a quadratic equation for \( f(r) \). Imposing the boundary condition that asymptotically the solution approaches the de Sitter background yields

\[
f(r) = \frac{r}{8r_0^2} \left[ \sqrt{D} - r (3 - 2r_0 (g \mp H)) \right] ;
\]

\[
D(r) \equiv (3 - 2r_0 (g \mp H))^2 r^2 + 12gr^2r_0 \\
+ 16 \frac{r_0^2}{r} \left[ Q + \frac{1}{2} (R_g - G_g) + r^3 (2H^2 \pm \frac{3H}{2r_0}) \right]
\]

\[
Q(r) \equiv \mp H (r_0 G_g - \frac{1}{2} gr^3) - \frac{1}{2} \int_0^r g(\tilde{r}) g'(\tilde{r}) \tilde{r}^3 d\tilde{r} .
\]

With this solution for \( f(r) \) we get simple expressions for the metric functions on the brane:

\[
n(r, 0) = n_0 + \int_0^r d\tilde{r} \left[ \frac{f(\tilde{r})}{r} + \frac{1}{2r^2} (R_g(\tilde{r}) - G_g(\tilde{r})) \\
- \frac{g(\tilde{r})\tilde{r}}{4r_0} - (H^2 \pm \frac{H}{2r_0})\tilde{r} \right] \]

\[
a(r, 0) = \frac{1}{r} (R_g(r) - G_g(r)) - \frac{1}{2} H^2 r^2 - nr'(r, 0)
\]

\[
b(r, 0) = 0 .
\]

This solution reduces to that of [118] in the limit \( g(r) = 0 \). We have generalized [118] by allowing the influence of the bulk to be space-dependent, as one expects. While \( g(r) \) approaches zero far from the source, \( G_g(r) \) reaches a constant non-zero value. The effect of \( G_g \) is to modify \( R_g(r) \), and the sign of \( G_g(r) \) will determine whether the gravitational field will increase (suggesting a gravitational source of greater than actual mass) or decrease. The quantity relevant for the acceleration of a test particle is \( n'(r, 0) \), given by

\[
n'(r, 0) = \frac{f(r)}{r} + \frac{1}{2r^2} (R_g(r) - G_g(r)) \\
- \frac{g(r)r}{4r_0} - (H^2 \pm \frac{H}{2r_0})r .
\]
3.3 Gravitational effects of mass shells

We shall see that the effects of matter on the metric are not confined to the region exterior to it. To see this we study the gravitational force exerted by a spherical mass shell in both its interior and exterior with a reasonable model of $g(r)$. We next superpose several concentric shells, and construct a mock model of a galaxy, within a cluster and super-cluster for example.

3.3.1 Modeling

Based on dimensional analysis we expect a shell with mass $M_i$ and radius $r_i$ to modify $\dot{n}$ by roughly $\sqrt{2GM_i/r_i^3}$. By choosing this dimensional form we recognize $g(r)$ as a correction to the local Hubble scale due to the mass shell. We therefore model $g(r)$ as

$$
\begin{align*}
g(r) &= -\sqrt{2GM_i/r_i^3} \times \begin{cases} 
1 & \text{for } r \leq r_i \\
(r_i/r)^4 & \text{for } r > r_i 
\end{cases} 
\end{align*}
$$

Exterior to the shell, $g(r)$ needs to decay strongly enough to ensure that $Gg$ is finite in the limit $r \to \infty$, which is satisfied with our choice of $r^{-4}$. As can be seen later, the dominant contribution of $g(r)$ comes from the interior region, and the details of the decay external to the shell are less important. We have chosen the sign of $g(r)$ to augment the attractive force of gravity with an eye to our interest in modifications of gravity that mimic dark matter.

First consider a test particle external to the shell. For distances much larger than the shell radius, the only relevant contribution comes from the $G_g$ term of 3.17. Asymptotically $G_g \to 2\sqrt{2GM_i/r_i^3}/r_0^2$, which depends on the crossover scale $r_0$ as well as the shell mass and radius. Notice that the dependence of $G_g$ on the location of the shell is a positive power; hence for fixed mass the contribution grows for larger shells. This shows explicitly that the gravitational force of spherically symmetric systems
depends on the distribution of matter.

Now consider a test particle interior to the spherical shell. Newtonian gravity predicts that the particle feels no acceleration. For DGP, this depends on the bulk structure. Taking $g(r)$ non-zero for $r < r_i$ modifies the gravitational potential and gives a non-zero acceleration within the shell. A local deviation of $\dot{n}(r,0)$ from the DGP-de Sitter value in the presence of a matter concentration would seem to be expected if, looking at 3.9, we can think of $g(r)$ as a local change in the effective Hubble scale. Overdensities are expected to create such modifications. This result is extremely disturbing. It suggests, for example, that when calculating gravitational forces within a galaxy, one must consider the effects of the cluster and super-cluster in which it is embedded.

3.3.2 Multiple shells

Notice that $g(r)$ is non-linear in the mass, so one cannot superpose solutions. Concentric shells however, each contributing a $g(r)$ correction to the Hubble scale, is a solvable system. As a mock model of our extended neighborhood we model our galaxy, cluster and supercluster as concentric spherical dust shells, see Fig. 3.1. It is self evident that galaxies and clusters are not spherically symmetric and concentric shells, and we have not attempted any data fitting. Nonetheless, the spherically symmetric calculation does give an order of magnitude estimate as to how severe are the deviations between two calculations within the same theory. As we show, the deviations are not negligible. Indeed, another possible way to model the galactic neighborhood is with spheres of mass instead of shells. This will give yet another possible solution within the same theory. As we consider all of these solutions just an estimate to the order of magnitude of the problem, we chose to work with the simple shells model.

We first consider DGP as a dark energy alternative, and so take $r_0 = 1/H$. For the radii of the shells we take $10^5$pc, $3 \times 10^6$pc and $3 \times 10^7$pc, and $10^{12} M_\odot, 10^{15} M_\odot$.
Figure 3.1: Mock model of a galactic neighborhood as concentric spherical shells of mass.

and $10^{17}M_\odot$ for the masses. Fig. 3.2 shows the gravitational acceleration as a function of the distance. The plot compares the acceleration accounting only for the galaxy (solid), for the galaxy and cluster (dashed), and for the galaxy, cluster and super-cluster (dotted), to the Newtonian acceleration (dot-dashed). Clearly, the accumulating effect of external shells can be non-negligible. The implication that mass shells can alter the gravitational acceleration interior to them seems particularly ominous for modified gravity theories that seek to replace dark matter as an explanation for the anomalous dynamics of galaxies. Although DGP is not a candidate MOND theory, we wish to understand, in this calculable theory, the magnitude of the effects we might expect from BT violations. We now let our galaxy, cluster and supercluster shells have radii of $3 \times 10^4$pc, $3 \times 10^6$pc and $3 \times 10^7$pc, and masses of $10^{11}M_\odot$, $10^{13}M_\odot$ and $10^{15}M_\odot$, accounting only for the visible matter. To mimic dark matter, $r_0 \approx 2.6 \times 10^7$pc is chosen to boost the acceleration due to the gravity so that the circular velocity of a test particle in orbit at $r_g$ is 200km/s. As expected, smaller values of $r_0$ cause larger modifications to the gravitational force, as shown in Figure 3.3. The cluster and supercluster keep the local acceleration approximately constant.
Figure 3.2: Gravitational acceleration as a function of radius. $g$ (solid) includes only the galaxy as a source, $g+c$ (dashed) adds the cluster, and $g+c+sc$ (dotted) adds the super-cluster. The Newtonian acceleration is $g_n$ (dot-dashed). The crossover scale is $r_0 = 10^5 r_g = 10^{10}\text{pc} \approx 1/H_0$.

Figure 3.3: As in Fig. 3.2, but with $r_0 = r_g = 10^5\text{pc}$.
from the radius of the galaxy shell outward. In addition, the internal accelerations to the galaxy shell are substantial. Clearly “exterior” mass cannot be ignored.

3.4 Conclusions

In this work we have used DGP as an example of the sickness associated with violation of Birkhoff’s theorem. We have generalized the solution of [118], and shown that (a) even for spherically symmetric systems, the exact distribution of the matter affects the gravitational force external to the source, and (b) one cannot neglect distant gravitational sources, because even interior to a spherically symmetric mass shell the gravitational force is non-zero. These effects pose severe conceptual and calculational difficulties.

The problem for DGP may be due to our ignorance of the bulk. Conceivably a better understanding of how the 4-dimensional metric of the brane is induced and of bulk effects may change our grim conclusion and answer satisfactorily the issues we have raised. The full understanding of the gravi-scalar degree of freedom near a brane source should hopefully alleviate the ambiguity in the determination of \( g(r) \); as argued for example by [118] and [122], the gravi-scalar is frozen out, yielding \( g(r) = 0 \), effectively enforcing a Birkhoff-like vanishing of the gravitational acceleration inside a spherical shell. However, we may find ourselves with a well-determined solution which has a non zero \( g(r) \) and thus Birkhoff-violating.

This sickness is not unique to DGP theory: most theories of modified gravity are likely to suffer the same problem in various degrees. Since DGP is not a candidate MOND theory, it is not primarily in their application to DGP per se that our results should be viewed, rather as an example of the magnitude of the problem that is likely to apply to most modified gravity theories. These issues are the subject of further investigation. In future work we will also consider deviations from both spherical symmetry and smoothness and how they affect the gravity in modifications to GR.
possible conclusion of our work may be that we need Birkhoff’s theorem to hold for a theory to be calculationally tractable. Nature may, of course, refuse to cooperate.
Chapter 4

Conclusion

The individual chapters of the thesis contain their own conclusions. Here are a few comments about future work. In practice, the existing black hole simulators (TRUENOIR, CHARYBDIS and CATFISH) only incorporate Hawking radiation and the final explosion, and can deal with only one type of higher-dimension model, a single-zero-tension-brane model. However, since the improvement of the calculation of grey body factors, BlackMax is able to deal with the spin-down phase and several different kinds of higher dimension models. Since this major step forward, BlackMax allows for a much improved understanding of the signatures of black-holes at the LHC. However, it still includes many simplifying assumptions. The future work is doing a better theoretical analysis and focusing on using BlackMax to explore the consequences for the ATLAS and CMS experiments.

There are several modified gravity theories that may replace dark matter or dark energy. For example MOND, General Einstein Aether and TeVeS can explain dark matter; DGP and f(R) can explain dark energy. These theories become Newtonian gravity or GR at short distance scales, and Birkhoff’s s law is satisfied at this range, too. However, discrepancy happen at a galaxy scale (for dark matter) and at about a Hubble radius (for dark energy). Since these models mimic energy density at the scales, the gravitational acceleration does not come from the surrounding matter (in
DGP it can be from the higher dimensional space). Birkhoff’s law, therefore, does not necessarily hold. It is necessary to review the existant or incoming theories to see if the violation of Birkhoff’s law causes the calculation of the gravitational acceleration to become undoable. In future work we will also consider deviations from both spherical symmetry and smoothness and how they affect the gravity in modifications to GR.
Appendix A

Particle Emission Spectra

Nowadays gray-body factors can be found in a lot of papers. We collect the relative papers in table A.1. We follow these papers to calculate energy power spectra in our database except for the split-fermions model. We calculate spectra of gauge bosons and scalar fields in the split-fermions model by ourselves (figure A.1 to A.12). Anyone who has a better spectrum can upgrade our database. For example, we only calculate the spectra up to the \( l = 9 \) mode.

![Figure A.1: Spectra of scalar fields in d=5 space.](image-url)
Figure A.2: Spectra of scalar fields in d=6 space.

Figure A.3: Spectra of scalar fields in d=7 space.

Figure A.4: Spectra of scalar fields in d=8 space.
Figure A.5: Spectra of scalar fields in d=9 space.

Figure A.6: Spectra of scalar fields in d=10 space.

Figure A.7: Spectra of gauge bosons in d=5 space.
Figure A.8: Spectra of gauge bosons in d=6 space.

Figure A.9: Spectra of gauge bosons in d=7 space.

Figure A.10: Spectra of gauge bosons in d=8 space.
Figure A.11: Spectra of gauge bosons in $d=9$ space.

Figure A.12: Spectra of gauge bosons in $d=10$ space.
Table A.1: Literature sources for particle emission spectra

<table>
<thead>
<tr>
<th>Type of particle</th>
<th>Type of black hole</th>
<th>Brane model</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard-Model particles</td>
<td>non-rotating</td>
<td>unsplit; tensionless</td>
<td>[19][20]</td>
</tr>
<tr>
<td>gravitons</td>
<td>non-rotating</td>
<td>split/unsplit; tensionless</td>
<td>[18]</td>
</tr>
<tr>
<td>Standard-Model particles</td>
<td>non-rotating</td>
<td>split/unsplit; with tension</td>
<td>[16]</td>
</tr>
<tr>
<td>gravitons</td>
<td>non-rotating</td>
<td>split/unsplit; with tension</td>
<td>[16]</td>
</tr>
<tr>
<td>scalars and gauge bosons</td>
<td>non-rotating</td>
<td>split; tensionless</td>
<td>figures A.1-A.12</td>
</tr>
<tr>
<td>fermions</td>
<td>rotating</td>
<td>unsplit; tensionless</td>
<td>[24][25]</td>
</tr>
<tr>
<td>gauge bosons</td>
<td>rotating</td>
<td>unsplit; tensionless</td>
<td>[23][25]</td>
</tr>
<tr>
<td>scalar fields</td>
<td>rotating</td>
<td>unsplit; tensionless</td>
<td>[19][21][22][25]</td>
</tr>
</tbody>
</table>

Table A.2: Degrees of freedom of Standard-Model particles which are emitted from a black hole. For gravitons, the table shows 1, because the appropriate growth in the number of degrees of freedom is included explicitly in the graviton emission spectrum. $n_s$ is the number of extra dimensions in which vector and scalar fields can propagate.

<table>
<thead>
<tr>
<th>particle type</th>
<th>$d_0$</th>
<th>$d_{1/2}$</th>
<th>$d_1$</th>
<th>$d_2$</th>
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<tr>
<td>Quarks</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Charged leptons</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Neutrinos</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Photons or gluons</td>
<td>0</td>
<td>0</td>
<td>8(2 + $n_s$)</td>
<td>0</td>
</tr>
<tr>
<td>$Z^0$</td>
<td>1</td>
<td>0</td>
<td>2 + $n_s$</td>
<td>0</td>
</tr>
<tr>
<td>$W^+$ and $W^-$</td>
<td>2</td>
<td>0</td>
<td>2(2 + $n_s$)</td>
<td>0</td>
</tr>
<tr>
<td>Higgs boson</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Graviton</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Table A.3: The fraction of emitted particles of different types (including final burst particles) in a variety of extra dimension scenarios. *Note that the absence of gravitons in the case of a rotating black hole is due exclusively to our current ignorance of the correct gray-body factor.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>quarks</th>
<th>gluons</th>
<th>leptons</th>
<th>gauge bosons</th>
<th>neutrinos</th>
<th>gravitons</th>
<th>Higgs bosons</th>
<th>photons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d = 4$ $n_x = 0$ non-rotating BH</td>
<td>68.21</td>
<td>10.79</td>
<td>9.45</td>
<td>5.72</td>
<td>3.97</td>
<td>2.06e-01</td>
<td>4.98e-01</td>
<td>8.61e-01</td>
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<tr>
<td>$d = 5$ $n_x = 0$ non-rotating BH</td>
<td>65.37</td>
<td>13.29</td>
<td>9.04</td>
<td>6.12</td>
<td>3.76</td>
<td>4.60e-01</td>
<td>5.26e-01</td>
<td>1.13</td>
</tr>
<tr>
<td>$d = 6$ $n_x = 0$ non-rotating BH</td>
<td>63.68</td>
<td>14.61</td>
<td>8.93</td>
<td>6.58</td>
<td>3.51</td>
<td>7.18e-01</td>
<td>7.76e-01</td>
<td>1.30</td>
</tr>
<tr>
<td>$d = 7$ $n_x = 0$ non-rotating BH</td>
<td>61.25</td>
<td>15.94</td>
<td>8.75</td>
<td>7.17</td>
<td>3.45</td>
<td>1.28</td>
<td>8.04e-01</td>
<td>1.44</td>
</tr>
<tr>
<td>$d = 8$ $n_x = 0$ non-rotating BH</td>
<td>60.99</td>
<td>15.94</td>
<td>8.56</td>
<td>6.99</td>
<td>3.45</td>
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Bibliography


[38] M. Casals, P. Kanti, E. Winstanley, hep-th/0511163


[40] A.S. Cornell, W. Naylor, M. Sasaki; hep-th/0510009


[43] G. Duffy, C. Harris, P. Kanti, E. Winstanley, JHEP **0509** 049 (2005)


[52] V. P. Frolov, D. V. Fursaev, D. Stojkovic, Class. Quant. Grav. **21** 3483 (2004); JHEP **0406** 057 (2004);


[64] V. Frolov, M. Snajdr and D. Stojkovic, Phys. Rev. D68 044002 (2003);


[66] C. M. Harris, hep-ph/0502005


[68] V. Cardoso, O.J.C. Dias, J. P.S. Lemos, Phys.Rev.D67 064026 (2003); V. Car-
    doso, O. J.C. Dias, J. L. Hovdebo, R. C. Myers, hep-th/0512277


    D69 065010 (2004)


[73] M. Cavaglia, S. Das, Class.Quant.Grav. 21 4511 (2004); M. Cavaglia, S. Das, R.

[74] V. Cardoso, M. Cavaglia, L. Gualtieri, hep-th/0512116


[83] K.A. Bronnikov, S.A. Kononogov, V.N. Melnikov, gr-qc/0601114


