ESSAYS ON SUPPLY CHAIN COMPETITION AND
COORDINATION OF OPERATIONS WITH FINANCE

By

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Essays on Supply Chain Competition and Coordination of Operations with Finance

Abstract

by

Qiaohai (Joice) Hu

This dissertation consists of four essays related to supply chain competition and coordination of operational and financial decisions.

The first essay models dynamic oligopoly supply chain games. The results include circumstances in which an echelon base-stock policy is not a best competitive response.

The second essay investigates how airlines might determine their network structures through a three-stage duopoly game. We find that at equilibrium either both airlines use hub-and-spoke networks or both use point-to-point networks. Furthermore, a hub-and-spoke network does not necessarily dominate a point-to-point network; and a high demand variance or a low mean demand generally favors a hub-and-spoke network.
The third essay examines a model with a financial criterion and investigates the interdependence of a firm’s capital structure and its short-term operating decisions concerning inventories, dividends, and liquidity. We conclude that the optimal inventory policy does not depend on the firm’s capital structure, but the optimal dividend and liquidity policy does depend on the inventory decisions. The optimal capital structure depends only on the marginal tax rate, the interest rate for long-term bonds, and the decision maker’s interest rate for intertemporal tradeoffs.

The fourth essay studies a serial multi-stage manufacturing system whose decision maker maximizes the expected present value of dividends via periodic decisions concerning production quantities at each echelon, dividends, and short-term borrowing. We show that the liquidity constraint and multiple stages cause echelon base-stock policies to be sub-optimal.
Chapter 1

Supply Chain Games

1.1 Introduction

The profitability of many firms depends on the competitive effectiveness of supply chains in which they are members. Glitches at a single supplier or a supplier’s supplier affect the chain’s overall ability to meet customer commitments. For example, Firestone tire distributors were hurt badly by the manufacturer’s pricing policy following the Ford Explorer-Firestone tire adverse publicity a few years ago. However, their competitors, such as Goodyear, benefited from the mishap (White, Aeppel and Arsberry (2001)).

Little is known about the operational dynamics of supply chain competitive behavior. In the economics of industrial organizations, an ample normative literature on competing supply chains has few operational details (Greenhut and Ohta
(1979), Grossman and Hart (1986), and Ziss (1995)). In contrast, most of the normative results in the operations literature is based either on perfect competition or monopoly. This literature during the past decade seems to have focused on the effects of echelon base-stock policies. That is, orders are placed at each stage in the supply chain in order to raise echelon inventory to a target level. However, it seems to us that imperfect competition characterizes the interactions among most supply chains. In other words, little is known about the normative operational dynamics of supply chains in their arguably most prevalent environment. This study investigates the extent to which echelon base-stock policies are appropriate for these environments.

The necessary attributes of a model of operational dynamics in supply chain competition include two (or more) supply chains, two (or more) echelons in each supply chain, and decisions made in numerous time periods. Several previous studies nearly meet these criteria. Boyaci and Gallego (2004) model competing two-echelon supply chains which attract Poisson demands that are proportional to their service rates. We say that a supply chain is coordinated or centralized if it has, in effect, one decision maker. The authors analyze the static strategic game in which each player (supply chain) has two alternatives, namely to coordinate or to remain uncoordinated. The payoffs in the resulting bimatrix game stem from the subsequent interactions of the firms. Coordination turns out to be a dominant strategy for both supply chains, but the aggregated expected profits of the chains
are smaller under the coordinated scenario than under the uncoordinated scenario. This paper differs from Boyaci and Gallego (2004) in two important ways. First, decisions are made every period, and second, the supply chains interact through product availability, i.e., each retailer’s demand depends both on her own and the competing retailer’s inventories. This inter-dependency of demand on inventory may be caused by a substitution effect or a demand stimulation effect of inventory. Netessine, Rudi and Wang (2004) review the literature in which customers substitute one product with another or switch from one retailer to another when their first-choice product or source is out of stock.

Several normative studies investigate interactions among the echelons in a single uncoordinated multi-echelon supply chain. These studies implicitly assume either monopoly or perfect competition, but they are relevant here due to their multiplicity of decision makers. Lee and Whang (1999) assume that the lower stage incurs backorder penalties, and the upper stage incurs holding costs. They design performance mechanisms to induce each member to choose a system-optimal base-stock level. In a similar model, Porteus (2000) analyzes responsibility tokens as the coordination scheme. Chen (1999) designs an accounting scheme which induces each member to minimize its own cost without compromising the system-wide performance. Cachon and Zipkin (1999) consider a static strategic game in which the wholesaler and retailer each make once-and-for-all choices of a base-stock level policy. A pair of payoffs, the long-run average profits per period, is
associated with each pair of choices. They show that the resulting bimatrix game has a unique equilibrium point that differs from the system-optimal solution. Hence, they develop a linear transfer scheme that coordinates the supply chain. In contrast to these references, the firms in this paper make decisions each period and they are not assumed \textit{ex ante} to use base-stock policies.

There are several game theoretic analysis of supply chain “front ends,” i.e., of models with only a single echelon (the “retailer”). Kirman and Sobel (1974) analyze a dynamic oligopoly model in which competing firms set prices and inventory levels each period. They characterize an equilibrium point when demand depends on prices but (unlike the present study) not on inventories. Netessine et al. (2004) analyze the same model except that demand depends on retailers’ inventories but not on prices. They investigate the effects of different representations of customer backlogging behavior and the impacts of retailers’ retention incentives on customers when stockouts occur. Bernstein and Federgruen (2004) model a dynamic inventory and pricing game for a distribution system with one wholesaler and two retailers engaged in price competition. They improve on Kirman and Sobel (1974) by obtaining sufficient conditions for the existence of a unique unrandomized equilibrium point.

The next section specifies a model with dynamic stochastic interactions between and within two supply chains who compete through product availability. Each chain consists of a wholesaler and a retailer with an established relationship.
If a customer at one retailer encounters a stockout, she might switch her patronage to the other retailer. As a result, each retailer’s realized demand depends on both retailers’ inventory levels which in turn depend on the wholesalers’ inventories. Hence, the supply chains interact through all firms’ inventory policies.

We analyze four scenarios: a single monopolistic decentralized supply chain whose retailer and wholesaler are separate decision-makers, two competing decentralized supply chains, two competing centralized supply chains, and a hybrid scenario in which one supply chain is centralized but the other is not. Mimicking the well-known result that an echelon base-stock policy is optimal for a centralized serial supply chain, we prove that the firms in a decentralized supply chain have equilibrium points in echelon base-stock policies. The result is valid for chains with more than two members. However, when two or more supply chains compete, centralized or not, we find that generally there is no equilibrium point in echelon base-stock policies.

Section 1.2 presents the model, and §1.3 shows that generally there is no equilibrium point in echelon base-stock policies when (a) centrally managed chains compete with each other, and (b) when a centrally managed chain competes with a decentralized chain. The interests of the firms in a decentralized chain are not congruent and, therefore, they are essentially competing with each other even if they all share the same information. In such a game §1.4 proves that there is an equilibrium point at which each firm uses an echelon base-stock policy. Then §1.5
analyses a dynamic game among the firms comprising two decentralized chains in which the wholesalers expedite shipments to the retailers whenever shortages occur. As a result, there is an equilibrium point at which all the firms employ echelon base-stock policies. Conclusions and unanswered questions are in §1.7.

1.2 Model

The model is a dynamic supply chain generalization of Cournot oligopoly. Each supply chain consists of a wholesaler and a retailer with complete information. Retail firms choose quantities of goods that are available to consumers and the market determines prices and quantities actually sold. At the beginning of each period \( t \) \((t = 1, 2, \cdots)\) the retailers and wholesalers review their inventories and make replenishment decisions. In supply chain \( i \) \((i = 1, 2)\), let \( x_{ri}^t \) and \( x_{wi}^t \) denote the retailer’s and wholesaler’s respective inventory levels at the beginning of period \( t \) and \( z_{ri}^t \) and \( z_{wi}^t \) their order quantities. We indicate a pair of variables for both chains by suppressing chain-identifying superscripts; for example, \( x_r^t = (x_{r1}^t, x_{r2}^t) \).

For expository simplicity we assume that there is a lag of one period to transfer goods from a supplier to a wholesaler, and that delivery is immediate from a wholesaler to a retailer. Therefore, the total supplies available to satisfy demand at the retailers are their order-up-to levels. However, with minor changes the results are valid for any integer-valued delivery lags between chain members. Let \( y_r^t \) and \( y_w^t \) denote the respective retailer and wholesaler inventory positions (i.e.
on-hand plus on-order) in supply chain $i$ after purchase orders are processed and transported in period $t$:

$$y_{ri}^t = x_{ri}^t + z_{ri}^t \quad \text{and} \quad y_{wi}^t = x_{wi}^t + z_{wi}^t - z_{ri}^t \quad (1.1)$$

We assume that each chain member’s storage is bounded above, namely that $y_{ri}^t \leq u_{ri}$ and $y_{wi}^t \leq u_{wi}$ with $u_{ri} < \infty$ and $u_{wi} < \infty$ for each $i$. Consistent with the complete information assumption, we assume that the retailers do not order more than the wholesalers’ on-hand inventories. Therefore, $0 \leq z_{ri}^t$ and $0 \leq z_{wi}^t \leq x_{wi}^t$. It can be shown that $z_{ri}^t \leq x_{wi}^t$ is a redundant constraint in a model with expediting and altered information conditions. When $a = (a_i)$ and $b = (b_i)$ are vectors of the same dimension, $\min\{a, b\}$ denotes the vector whose $i$th component is $\min\{a_i, b_i\}$. We preclude planned backlogging and, therefore, constrain $y_{ri}^t \geq 0$ and $y_{wi}^t \geq 0$. If the initial conditions satisfy $x_{ri}^1 \leq u_{ri}^t$ and $0 \leq x_{wi}^1 \leq u_{wi}^t$, the bounds correspond to

$$x_{wi}^t \leq y_{wi}^t \leq u_{wi}^t \quad x_{ri}^t \leq y_{ri}^t \leq \min\{x_{ri}^t + x_{wi}^t, u_{ri}^t\} \quad (1.2)$$

Let $D^i_t$ be the nonnegative random demand encountered by retailer $i$ in period $t$. Since each component of $D_t$ may depend on $y_{ti}^t$, we sometimes write $D_t(y_{ti}^t)$. This models an array of more specific customer behaviors (Kirman and Sobel (1974) and Netessine et al. (2004)). For any $\omega_1 \geq 0$ and $\omega_2 \geq 0$, we assume that $D_1(\omega_1, \omega_2), D_2(\omega_1, \omega_2), \cdots$ are independent and identically distributed random vectors; let $D(\omega_1, \omega_2) = (D^1(\omega_1, \omega_2), D^2(\omega_1, \omega_2))$ be a vector with the same
probability distribution as $D_1(\omega_1, \omega_2)$.

We let the retailers’ revenues and inventory-related costs be random variables whose probability distributions depend on the vector $y = (y^1, y^2)$ of retailer supply levels. Let $G_t^i$ be retailer $i$’s revenue net of inventory-related costs in period $t$ and let $G_t = (G_t^1, G_t^2)$. We assume that $G_t$ is conditionally independent of $G_1, ..., G_{t-1}$ given $y_t$ and that the conditional distribution of $G_t$ given $y_t = y$ is the same for all $t$. These assumptions are consistent with many specific models in operations and economics.

We assume that the wholesalers incur linear holding costs and that all firms incur proportional ordering costs and are risk-neutral profit-maximizers. However, the paper’s conclusions remain valid if decision-makers are risk-averse with exponential utility functions (Chung and Sobel (1987), Bouakiz and Sobel (1992)). Let $c^w_i$, $c^r_i$, and $h^w_i$ be wholesaler $i$’s respective unit purchasing cost, wholesale price, and unit holding cost; let $\rho_i$ be retailer $i$’s unit penalty cost of excess demand. For each unit of excess demand, if any, the wholesaler pays $\alpha_i \rho_i$ ($0 \leq \alpha_i \leq 1$) and the retailer pays $(1 - \alpha_i) \rho_i$.

We assume that excess demand $(D_t^i - y_t^r)^+$ is backlogged and that the following chronology of events occurs during each period $t$: inventory levels are observed, ordering decisions are made, retailer demands are realized, revenues and costs are credited and debited, and inventory balances are updated. Hence, the dynamics are
\begin{equation}
x_{t+1}^r = y_t^r - D_t(y_t^r) \quad x_{t+1}^w = y_t^w
\end{equation}

Let \( \mathcal{R}^+ \) denote nonnegative real numbers. If the backlogging assumption is replaced with \( x_{t+1}^r = y_t^r - \theta_t(y_t^r, D_t) \) where \( \theta_1(y^r, d), \theta_2(y^r, d), \cdots \) are independent and identically distributed nonnegative random vectors for each \( y^r \) and \( d \), and realized \( \theta_1(\cdot, d(\cdot, \alpha)) \) is concave on \( \mathcal{R}^+ \), the paper’s results do not significantly change. In particular, this includes excess demand being lost.

Let \( \beta_{ri} \) and \( \beta_{wi} \) be the single-period discount factors in supply chain \( i \), and define the following echelon variables:

\begin{align}
s_{t}^{ri} &= x_{t}^{ri} & a_{t}^{ri} &= y_{t}^{ri} & s_{t}^{wi} &= x_{t}^{ri} + x_{t}^{wi} \\
a_{t}^{wi} &= y_{t}^{ri} + y_{t}^{wi} & a_{t}^{r} &= (a_{t}^{r1}, a_{t}^{r2})
\end{align}

Let \( a^r = (a_{r}^{r1}, a_{r}^{r2}) \) denote a generic value of the pair \( a_{r}^{r} = (a_{r}^{r1}, a_{r}^{r2}) \) and define the following functions:

\begin{align}
L_i(a^r) &= E[\{G_i^{i} - (1 - \alpha_i)\rho_i(a_{i}^{ri} - D_{i}^{i})^{+} - [(1 - \alpha_i)\rho_i
\begin{align}
&+ \beta_{ri}c_{i}^{r}]D_{i}^{i}\} | a_{i}^{ri} = a^r] - [c_{i}^{r} (1 - \beta_{ri}) - (1 - \alpha_i)\rho_i] a_{i}^{ri} \\
H_{i}(a^r, a^{wi}) &= [\alpha_{i}\rho_i + h_{i}^{w} + \beta_{wi}^{r}(1 - \beta_{wi})] a_{i}^{r} - [h_{i}^{w} + c_{i}^{w} (1 - \beta_{wi})] a_{i}^{wi} \\
&- E\{\alpha_{i}\rho_i(a_{i}^{ri} - D_{i}^{i})^{+} - [\beta_{wi}(c_{i}^{r} - c_{i}^{w}) + \alpha_{i}\rho_i]D_{i}^{i} | a_{i}^{ri} = a^r, a_{i}^{wi} = a^{wi}\}
\end{align}

The constraints and dynamics in (1.2) and (1.3) correspond to

\begin{align}
s_{t}^{wi} &\leq a_{t}^{wi} & s_{t}^{ri} &\leq a_{t}^{ri} \leq s_{t}^{wi}
\end{align}
\[ s_{t+1}^{wi} = a_{t}^{wi} - D_{t}^{i} \quad s_{t+1}^{ri} = a_{t}^{ri} - D_{t}^{i} \]  

Let \( B_{ri} \) and \( B_{wi} \) denote the expected present values of the retailer’s and wholesaler’s profits, and let \( b_{ri} \) and \( b_{wi} \) denote the corresponding expected values. Then

\[
B_{ri} = \sum_{t=1}^{\infty} \beta_{ri}^{t-1} \left\{ G_{t}^{i} - c_{t}^{i} z_{t}^{r} - (1 - \alpha_{i}) \rho_{i}(D_{t}^{i} - y_{t}^{ri})^{+} \right\} \\
= \sum_{t=1}^{\infty} \beta_{ri}^{t-1} \left\{ G_{t}^{i} - c_{t}^{i}(1 - \beta_{ri})y_{t}^{ri} - \beta_{ri} c_{t}^{i} D_{t}^{i} - (1 - \alpha_{i}) \rho_{i}(D_{t}^{ri} - y_{t}^{ri})^{+} \right\} + c_{t}^{r} x_{t}^{ri} \\
So \]

\[ b_{ri} = E\left\{ \sum_{t=1}^{\infty} \beta_{ri}^{t-1} L_{i}(a_{t}^{r}) \right\} + c_{t}^{r} s_{1}^{r} \]  

(1.9)

The present value of the wholesaler’s profits is

\[
B_{wi} = \sum_{t=1}^{\infty} \beta_{wi}^{t-1} \left[ c_{t}^{w} z_{t}^{wi} - c_{t}^{w} z_{t}^{wi} - h_{t}^{w} y_{t}^{wi} - \alpha_{i} \rho_{i}(D_{t}^{i} - y_{t}^{ri})^{+} \right] \\
= \sum_{t=1}^{\infty} \beta_{wi}^{t-1} \left\{ (c_{t}^{r} - c_{t}^{w})(1 - \beta_{wi})y_{t}^{ri} + \beta_{wi}(c_{t}^{r} - c_{t}^{w})D_{t}^{i} - [h_{t}^{w} + c_{t}^{w}(1 - \beta_{wi})]y_{t}^{wi} \right. \\
- \alpha_{i} \rho_{i}(D_{t}^{i} - y_{t}^{ri})^{+} \right\} + c_{t}^{w} s_{t}^{wi} - c_{t}^{r} x_{t}^{ri} \\
So \]

\[ b_{wi} = E\left\{ \sum_{t=1}^{\infty} \beta_{wi}^{t-1} H_{i}(a_{t}^{r}, a_{t}^{wi}) \right\} + c_{t}^{w} s_{1}^{wi} - (c_{t}^{r} + c_{t}^{w}) s_{1}^{ri} \]  

(1.10)

An echelon base-stock policy, loosely speaking, selects order quantities to move each echelon’s inventory position as close as possible to a target level. Let \( a_{r}^{ri} \) and \( a_{w}^{wi} \), respectively, denote the retailer’s and wholesaler’s targets (decision variables)
in supply chain \( i \). In view of (1.7), an echelon base-stock policy in an infinite-horizon model stipulates \( a^{ri}_t = \min\{\max\{a^*_s, s^{ri}_t\}, s^{wi}_t\} \) and \( a^{wi}_t = \max\{a^{wi}_*, s^{wi}_t\} \). If \( s^{ri}_1 \leq a^*_s \) and \( s^{wi}_1 \leq a^*_s \) as we typically assume, then \( a^{ri}_t = \min\{a^*_s, s^{wi}_t\} \) and \( a^{wi}_t = a^*_s \) for all \( t \). The same is true in a finite-horizon model except that \( a^*_s \) and \( a^*_s \) acquire a time index.

Although the foregoing model is a sequential game in which the players’ time streams of rewards are discounted over an infinite horizon, for simplicity’s sake in the following sections we analyze the corresponding sequential game with a finite horizon. We introduce some notation to explain a solution concept (Heyman and Sobel (2004), p. 452) that slightly generalizes the standard notion of an equilibrium point of a sequential game.

In a sequential game let \( Q \) be the set of players; at various points in the following sections, the sequential game models have two to four players. For each \( q \in Q \), let \( \Pi_q \) be a strategy for player \( q \), that is a non-anticipative rule for deciding what action to take each period as a function of the elapsed history thus far. Let \( \Pi = (\Pi_q, q \in Q) \) be the tuple of all players’ strategies. It is common to write \( \Pi = (\Pi_q, \Pi_{-q}) \) where \( \Pi_{-q} \) consists of the strategies of all the players except player \( q \). Let \( v^q_s(\Pi, N) \) be the expected present value of the rewards to player \( q \) during an \( N \)-period horizon if the players employ strategies in \( \Pi \) and \( s \) is the state at the beginning of the first period. For each player \( q \) let \( \pi_q \) be a subset of player \( q \)’s policies, and let \( \pi = \times_{q \in Q} \pi_q \). Let \( S' \) be a subset of states. We say that
Π∗ = (Π∗∗q, q ∈ Q) is an N-period equilibrium point with respect to initial states s ∈ S′ and policies in π if

\[ v^q_s(Π^∗, N) ≥ v^q_s[ξ_q, (Π^∗ − q), N] \] for all s ∈ S′, ξ_q ∈ π_q, and q ∈ Q

Our interest is in supply chain games having echelon base-stock policies that are equilibrium points with respect to the set π of stationary policies and a non-empty set of initial states S'. In game-theoretic terminology, such a Π* would be Markov-perfect and each player would employ a time-invariant strategy.

1.3 Dynamic Competition Between Centralized Supply Chains

The most important finding in the paper, a negative result, concerns centrally managed supply chains. We explain why there is generally no equilibrium point at which each chain employs an echelon base-stock policy. As centralization of an industry’s competing supply chains becomes widespread, there is competitive advantage in managing supply levels with policies that are more complex than would be worthwhile if the chains were decentralized.

Competing centralized supply chains lack an equilibrium point among echelon base-stock policies

Let b_i be the sum of the retailer’s and wholesaler’s expected present values of
profits in supply chain \( i \) and define

\[
\gamma_{ri}(ar^i) = [c_i^w (1 - \beta_i) + h_i^w + \rho_i]a^ri + E[G_i^r - \rho_i D_i - \rho_i (a^ri - D^i)^+] \\
\gamma_{wi}(aw^i) = -[c_i^w (1 - \beta_i) + h_i^w]a^{wi}
\]

Summing (1.9) and (1.10) yields

\[
b_i = E\left\{ \sum_{t=1}^{\infty} \beta_i^{t-1} \left[ \gamma_{ri}(a_r^t) + \gamma_{wi}(a_w^t) \right] + c_i^w (s_i^w - s_i^r) \right\}
\]

As in Sinha and Sobel (1997), if supply chain two uses a base-stock policy with target echelon supply levels \( a^{r2}_* \) and \( a^{w2}_* \) at the retailer and wholesaler, respectively, and \( a^{w2}_* \geq s_i^{w2} \) and \( a^{r2}_* \geq s_i^{r2} \), then supply chain one’s best response corresponds to the following dynamic program:

\[
f^*_t(s^r1, s^w1, s^w2) = \sup_{a^r1, a^w1} \{ \gamma_{r1}(a^r_1, \min\{a^r2_*, s^w2\}) + \gamma_{w1}a^w1 + \beta_1 E[f^*_t(a^r1 - D_1, a^w1 - D_1, a^w2 - D_2)] : s^r1 \leq a^r1 \leq s^w1 \leq a^w1 \}
\]

That is, chain one’s best response depends on the inventory levels in chains one and two. An echelon base-stock policy at chain one would depend on inventory levels only in chain one and, therefore, it could not be a best response.

This conclusion is consistent with the following intuition. Suppose that both supply chains use echelon base-stock policies at an equilibrium point. When retailer two’s order is constrained by his wholesaler’s inventory while wholesaler one’s inventory is ample, because inventory information is common, retailer one can exploit its competitor’s shortage by raising his supply higher than its target level in that period due to demand substitutability. So supply chain two reaps
extra profit by deviating from echelon base-stock policies. Similarly, knowing supply chain two’s response, supply chain one should deviate from its echelon base-stock policy. Therefore, competing centralized supply chains generally lack an equilibrium point in echelon base-stock policies.

A similar argument shows that competition between centralized and decentralized supply chains lacks an equilibrium point among echelon base-stock policies.

1.4 Decentralized Monopolistic Supply Chain

1.4.1 The General Case

Here we suppress the chain label $i$ and consider the interactions between a wholesaler and a retailer in a monopolistic supply chain with an exogenous probability distribution of demand. If the wholesaler employs an echelon base-stock policy with base-stock level $a^w_1 \geq s^w_1$, then $a^w_t = a^w_1$ and $s^w_{t+1} = a^w_t - D_t$ for all $t$. So the retailer’s decision problem corresponds to the following dynamic program with $V_0(\cdot, \cdot) \equiv 0, s^r \leq s^w$ and $n = 1, 2, \cdots$:

$$V_n(s^r, s^w) = \max \left\{ W_n(a) : s^r \leq a \leq s^w \right\} \tag{1.11}$$

$$W_n(a) = L(a) + \beta_r E\{V_{n-1}(a-D, a^w - D)\}$$

It follows from (1.5) and the exogenous distribution of demand that $L(\cdot)$ is concave on its domain if $E(G_1|a^r_1 = a)$ is a concave function of $a$. Many specific examples yield concavity including $G_1 = p \cdot \min\{D_1, a^r_1\} - h^r \cdot (a^r_1 - d_1)^+$. The first term is
revenue that is proportional to the lesser of supply and demand, and the second term is an inventory cost that is proportional to the excess supply.

An induction starting with \( n = 0 \) establishes the following conclusion.

**Lemma 1.4.1.** If \( E(G_1|a'_1 = a) \) is a concave function of \( a \), then \( V_n(\cdot, \cdot) \) and \( W_n(\cdot) \) are concave functions on their domains \( (n = 1, 2, \ldots) \).

In a monopolistic decentralized chain, a base-stock policy is the retailer’s best response to the wholesaler’s use of an echelon base-stock policy.

**Theorem 1.4.2.** An echelon base-stock policy is optimal in (1.11). That is, there is a scalar \( a^*_{sn} \) such that \( a = \max\{s^r, a^*_{sn}\} \) is optimal in (1.11).

**Proof.** If the wholesaler employs an echelon base-stock policy, the retailer faces a Markov decision process in which the retailer’s best response must be an optimal policy. Select \( a^*_{sn} \in \arg\max W_n(\cdot) \); then \( a^* = \min\{s^w, \max\{a^*_{sn}, s^r\}\} \) is optimal in (1.11). There are three cases:

\[
a^* = \begin{cases} 
a^*_{sn} & \text{Case A: } s^r \leq a^*_{sn} \leq s^w; \\
s^r & \text{Case B: } a^*_{sn} < s^r \leq s^w \text{ (transient case);} \\
s^w & \text{Case C: } s^r \leq s^w \leq a^*_{sn}. 
\end{cases}
\]

Let \( \tilde{a} = \max\{a^*_{sn}, s^r\} \) and observe that \( L(\min\{\tilde{a}, s^w\}) = L(\min\{\max\{a^*_{sn}, s^r\}, s^w\}) = L(a^*) \). Therefore, \( \tilde{a} = a^*_{sn} = a^* \) and \( \tilde{a} = s^r = a^* \) in cases A and B, respectively. In case C, \( \tilde{a} = a^*_{sn} \) and \( a^* = s^w \), so \( L(a^*) = L(s^w) \) and \( L(\min\{\tilde{a}, s^w\}) = L(\min\{a^*_{sn}, s^w\}) = L(s^w) \). Therefore, \( \tilde{a} = \max\{s^r, a^*_{sn}\} \) is optimal. \( \square \)
Now consider the wholesaler’s problem. When the retailer uses a base-stock policy with target level $a^r_s \geq s^r_1$, then $a^r_t = \min\{s^w_t, a^r_s\}$ because the retailer cannot order more than the wholesaler’s on-hand inventory. So the wholesaler’s decision problem corresponds to the following dynamic program with $v_0(\cdot, \cdot) \equiv 0$, $s^r \leq s^w$, and $n = 1, 2, \cdots$:

$$v_n(s) = \max\{J^w_n(a, s) : a \geq s\} \quad (1.12)$$

$$J^w_n(a, s) = H(\min\{s, a^r_s\}, a) + \beta_w E[v_{n-1}[a - D_t(\min\{s, a^r_s\})]]$$

Lemma 1.4.3. For each $n$, $v_n(\cdot)$ and $J_n(\cdot, \cdot)$ are concave functions on their domains.

Proof. In order to begin a straightforward induction starting with $n = 0$, and $v_0 \equiv 0$ yield

$$J^w_1(a, s) = H(\min\{s, a^r_s\}, a) = \beta(c^r - c^w)d - \alpha\rho E[D_t - \min\{s, a^r_s\}]^+ \quad (1.13)$$

$$+ [c^r(1 - \beta) + h^w] \min\{s, a^r_s\} - [c^w(1 - \beta) + H^w]a$$

which is a sum of concave terms.

There is a pure strategy equilibrium point at which the firms in a monopolistic decentralized supply chain employ the same kind of policy as if they were in a monopolistic centralized supply chain.

Theorem 1.4.4. If $E(G_1|a^r_1 = a)$ is a concave function of $a$, then there are scalars $a^r_*$ and $a^w_*$ such that the decentralized monopoly supply chain game has an
Proof. Consider the one-period two-player strategic game $\Gamma$ in which the retailer and wholesaler, respectively, choose $a^r \in [0, u^r]$ and $a^w \in [0, u^w]$ and receive payoffs $L(a^r)$ and $H(a^r, a^w)$. Then $\Gamma$, termed the reduced game, has a pure strategy equilibrium point $(a^*_r, a^*_w)$ due to the concavity of $L(\cdot)$ and $H(\cdot, \cdot)$ and Debreu (1952). Because of the additive separability of (1.9) in $(a^r_t, s^r_t)$ and (1.10) in $(a^w_t, a^r_t; s^w_t, s^r_t)$, the nonnegativity of demands, (1.7), and (1.8), following Sobel (1981), an induction establishes that the dynamic game with payoffs (1.9) and (1.10) has an equilibrium point relative to $[0, a^*_r] \times [0, a^*_w]$ consisting of $(a^r_t, a^w_t) = (\min\{a^r_t, s^w_t\}, a^w_t)$ for all $t = 1, 2, \ldots$.

4.2 Decentralized Supply Chain with Expedited Shipment

This sub-section is related to Cachon and Zipkin (1999) in which each firm’s criterion is its long-run average profit per period (no discounting) and each firm chooses an echelon target inventory level. Thus the dynamic game becomes a static strategic game in which each firm’s decision is its target echelon supply level. Cachon and Zipkin (1999) analyze this strategic game and conclude that the wholesaler has little influence on the retailer’s strategy. We aim to garner additional insight into the wholesaler’s role in a decentralized supply chain.

We assume that whenever the wholesaler cannot completely fill the retailer’s
order, she expedites the shortfall from her supplier to the retailer at unit cost $\lambda c^w$ ($\lambda > 1$). Expediting may occur because the retailer has more power, or because the wholesaler wants to retain the retailer’s goodwill. In addition, if the retailer incurs large losses upon stockout, he may require the wholesaler to provide this premium service. We note that as $\lambda$ rises, the wholesaler should raise her base-stock level to reduce expediting cost.

The retailer’s expected profit per period remains the same as (1.9) but $s^r_t \leq a^r_t \leq s^u_t$ in (1.7) is replaced with $s^r_t \leq a^r_t$. For simplicity, let $\alpha = 0$, i.e., the wholesaler does not share the retailer’s backorder cost. This simplification affects the numerical values of the players’ stock levels but not the structure of equilibrium points. Moreover, since the wholesaler fulfills the retailer’s order in each period, it is reasonable to hold the retailer fully accountable for excess demand. Since the retailer’s ordering quantity is not constrained by the wholesaler’s inventory, an argument that is similar to the proof of Theorem 2 yields the optimality (relative to $[0, a^r_\ast]$) of the retailer’s use of a base-stock policy consisting of $a^r_t = a^r_\ast$ for all $t = 1, 2, \cdots$. Now we address the effect of this policy on the wholesaler.

The expediting feature of the model alters the expected present value of the wholesaler’s profit from (1.10). Under the expediting assumption, $a^w_t = (s^w_t - a^r_\ast)^+ + z^w_t + a^r_\ast$. The dynamics remain the same as in §2, i.e., $s^w_{t+1} = a^w_t - D_t$ and $s^r_{t+1} = a^r_\ast - D_t$, but the wholesaler’s ordering constraint $z^w_t \geq 0$ corresponds to $a^w_t \geq a^r_\ast$ for all $t = 1, 2, \cdots$, which is satisfied (as assumed in §1.2) because
planned backlogging is precluded. The resulting expected present value of the wholesaler’s profit is

\[ b_w = E \sum_{t=1}^{\infty} \beta_w^{t-1} \left\{ c^w (a^r_s - s^w_t) - c^w s^w_t - \lambda c^w (a^r_s - s^w_t)^+ - h^w (a^r_t - a^r_s) \right\} \]

\[ = E \sum_{t=1}^{\infty} \beta_w^{t-1} \left\{ \beta_w (c^r - c^w) D_t(a^r_s) + [h^w + c^r (1 - \beta_w)] a^r_s - [h^w + c^w (1 - \beta_w)] a^w_t - \beta_w c^w (\lambda - 1) [a^r_s - a^w_t + D_t(a^r_s)]^+ \right\} \]  

(1.14)

\[ = E \left[ \sum_{t=1}^{\infty} \beta_w^{t-1} M(a^r_s, a^w_t) \right] - (\lambda - 1) c^w (a^r_s - s^w_1)^+ - c^w s^w_1 + c^w s^w_1 \]

with the definition

\[ M(a^r, a^w) = \beta_w (c^r - c^w) E[D_1(a^r)] + [h^w + c^r (1 - \beta_w)] a^r - [h^w + c^w (1 - \beta_w)] a^w - \beta_w c^w (\lambda - 1) E[a^r - a^w + d(a^r) + \eta]^+ \]  

(1.15)

In the braces on the first line of (1.14), the first term is the wholesaler’s revenue in period \( t \), the second is the purchasing cost for a regular order, the third term is the purchasing cost of an expedited order, and the last term is the holding cost.

We observe that the wholesaler’s single-period measure of effectiveness, \( M(a, \cdot) \), has economies of scale, i.e., for any \( a \in \mathbb{R} \), \( M(a, \cdot) \) is concave on \( \mathbb{R}^+ \).

The wholesaler faces a Markov decision process with payoff (1.14) which is parameterized by the wholesaler’s target base-stock level. Let \( a^w_s \in \text{argmax} M(a^r_s, a^w) \).

If \( a^w_s \geq s^w_1 \), then \( a^w_s \geq s^w_t = a^w_s - D_{t-1} \) for \( t \geq 2 \) because \( D_t \) is nonnegative. So \( a^w_t = a^w_s \) is optimal for all \( t = 1, 2, \ldots \). In conclusion, the dynamic game with payoffs (1.9) and (1.14) has an equilibrium point relative to \([0, a^r_s] \times [0, a^w_s]\) consisting
of \((a^r_t, a^w_t) = (a^r, a^w)\) for all \(t = 1, 2, \ldots\).

More specific assumptions regarding the structures of revenues, costs, and demand lead to explicit solutions and comparative statics results. These assumptions correspond to an exogenous retail price, a linear retail revenue function, and concavity of the mean demand as a function of the retail supply level. In part (b) of the following result, \(d'\) denotes the right-hand derivative of \(d\), and setting \(dL(a^r)/da^r\) and \(\partial M(a^r, a^w)/\partial a^w\) to zero yields (1.16) and (1.17), respectively.

**Theorem 1.4.5.** (a) The decentralized monopolistic supply chain game under expedited shipment has an equilibrium point relative to \([0, a^r_\ast] \times [0, a^w_\ast]\) consisting of \((a^r_t, a^w_t) = (a^r_\ast, a^w_\ast)\) for all \(t = 1, 2, \ldots\).

(b) If

(i) \(G_1 = p \min\{a^r_1, D_1\} - h^r(a^r_1 - D_1)^+\)

(ii) \(D_1(a^r_1) = d(a^r_1) + \eta_t\) with \(\eta_1, \eta_2, \cdots\) independent and identically distributed random variables with mean zero and distribution function \(F(\cdot)\),

(iii) \(d(\cdot)\) concave and \(d(a) \leq d(a + \delta) < d(a) + \delta\) \((0 \leq a, 0 < \delta)\),

then

\[
a^r_\ast = d(a^r_\ast) + \frac{p - c^r + (\rho + \beta c^r)(1 - d')}{(\rho + p + \rho)(1 - d')} F^{-1}\left(\frac{h^w + c^w(1 - \beta_w)}{\beta_w c^w(\lambda - 1)}\right)
\]

(1.16)

\[
a^w_\ast = a^r_\ast + d(a^r_\ast) + \frac{h^w + c^w(1 - \beta_w)}{\beta_w c^w(\lambda - 1)} F^{-1}\left(\frac{h^w + c^w(1 - \beta_w)}{\beta_w c^w(\lambda - 1)}\right)
\]

(1.17)

The assumptions regarding expected demand as a function of supply level, \(d(\cdot)\), imply that the right-hand derivative satisfies \(0 \leq d'(a) < 1\). The hypothesis
of part (b) is presumed throughout the remainder of this section.

From (1.16) and (1.17), the chain members should set their target supply levels by considering the inventory effect on demand. However, the degree of this effect differs for the retailer and the wholesaler. The second term of (1.17) depends on several parameters but it does not depend on \(d(\cdot)\) because the wholesaler should set a supply level such that her expected overstock cost equals her expected shortage cost and these costs depend only on the parameters in the second term and \(F(\cdot)\). However, \(d'\) appears in the second term of (1.16) because the retailer should adjust his inventory both for the deterministic part of demand \((d)\) and for the random part of demand. His inventory level affects demand and, consequently, his profits.

The numerator of (1.17), \(c^w(\lambda \beta_w - 1) - h^w\), is the wholesaler’s unit shortage cost, while the denominator, \(\beta_w c^w(\lambda - 1)\), which can be rewritten as \([((\beta_w \lambda - 1)c^w - h^w] + [c^w(1 - \beta_w) + h^w]\), is the sum of the unit shortage cost and the unit overstocking cost. Hence, the wholesaler should set her echelon base-stock level as if she were an independent dynamic newsvendor.

We now characterize the dependence of echelon base-stock levels on cost and revenue parameters. We say “increasing” and “decreasing” for nondecreasing and nonincreasing, respectively. The results are summarized as follows.

**Proposition 1.4.6.** The retailer’s target supply level \(a^*_r\) increases as \(p\) or \(\alpha\) increase, it decreases as \(c^r\) increases, and if \(d(x + \delta) \leq d(x) + \delta (\delta > 0)\), then it also increases as \(h^r\) decreases or \(\beta^r\) increases. The wholesaler’s target echelon
level \( a_w^* \) increases as \( \lambda \), \( c^w \) or \( a^r \) increase or as \( h^w \) or \( c^r \) decrease. If \( \lambda < 2 \), then \( a_w^* \) increases as \( \beta_w \) increases.

**Proof.** Under the assumptions in Theorem 1.4.5(b), \( L(\cdot) \), \( d(\cdot) \) and \( M(\cdot, \cdot) \) are concave, so their derivatives from the right exist.

\[
\frac{\partial^2 L(a^r)}{\partial p \partial a^r} = 1 - [1 - d'(a^r)]F[a^r - d(a^r)] \geq 0
\]

\[
\frac{\partial^2 L(a^r)}{\partial c^r \partial a^r} = -\beta_r d'(a^r) - (1 - \beta_r) \leq 0
\]

\[
\frac{\partial^2 L(a^r)}{\partial a^w \partial a^r} = 0
\]

If \( d(x + \delta) \leq d(x) + \delta (\delta > 0) \), then \( d'(a^r) \leq 1 \) and

\[
\frac{\partial^2 L(a^r)}{\partial \alpha \partial a^r} = \rho [1 - d'(a^r)] + d'(a^r) \rho \geq 0
\]

\[
\frac{\partial^2 L(a^r)}{\partial h^r \partial a^r} = -[1 - d'(a)]F[a^r - d(a^r)] \leq 0
\]

\[
\frac{\partial^2 L(a^r)}{\partial \beta_r \partial a^r} = c^r [1 - d'(a^r)] \geq 0
\]

Also,

\[
\frac{\partial^2 M(a^r, a^w)}{\partial h^w \partial a^w} = -1 < 0
\]

\[
\frac{\partial^2 M(a^r, a^w)}{\partial \lambda \partial a^w} = \beta_w c^w [1 - F(a^w - a^r - d(a^r))] \geq 0
\]

\[
\frac{\partial^2 M(a^r, a^w)}{\partial \beta_w \partial a^w} = c^w (2 - \lambda) \{1 - F[a^w - a^r - d(a^r)]\} \geq 0
\]

\[
\frac{\partial^2 M(a^r, a^w)}{\partial a^r \partial a^w} = \beta_w c^w (\lambda - 1) \partial F[a^w - a^r - d(a^r)]/\partial a^r \geq 0
\]

\[
\frac{\partial^2 M(a^r, a^w)}{\partial c^r \partial a^w} = 0
\]

\[
\frac{\partial^2 M(a^r, a^w)}{\partial a^w \partial c^w} = -(1 - \beta_w) - \beta_w (\lambda - 1) F[a^w - a^r - d(a^r)] \leq 0
\]
The assumption $d(x + \delta) < d(x) + \delta(\delta > 0)$ means that a unit increase in inventory induces an increase of less than one unit of demand. When $p$, $\alpha$ or $\beta_r$ increase, the retailer’s expected backorder costs increase, so he should raise his base-stock level. Conversely, when $c^r$ or $h^r$ increase, his expected holding costs increase, so he should lower his base-stock level. Similarly, when $\lambda$ or $\alpha$ increase, the wholesaler should raise her echelon base-stock level to reduce the expected backorder cost. If expediting is not too expensive $\lambda < 2$, then the wholesaler should increases $a'^w$ as $\beta_w$ increases to reduced the expected backorder cost. Conversely, she should lower her echelon base-stock level to reduce the expected inventory cost when $c^w$ or $h^w$ increase. We note that because $a'^w$ increases as $a'^r$ increases, and $a'^r$ decreases as $c^r$ increases, $a'^w$ decreases as $c^r$ increases. This means that knowing that the retailer will lower his supply level if she raises the wholesale price, the wholesaler lowers her echelon supply level accordingly.

4.3 Pricing and Inventory Games

Since the wholesaler expedites any inventory shortfalls, the retailer always obtains the quantity that he orders. That is, the wholesaler’s inventory policy does not influence him directly $(\partial L/\partial a^r \partial a^w) = 0)$. However, the price that the wholesaler charges the retailer certainly does affect the retailer. Proposition 1.4.6 asserts that the retailer should lower his base-stock level when the wholesale price increases.
Since wholesale price is a vital element of contracts between wholesalers and retailers, in this subsection we enlarge the dynamic game to include the wholesale price decision by the wholesaler.

The enlarged model has two stages. At the first stage, the wholesaler fixes the wholesale price, \( c^r \), which remains constant thereafter. At the second stage, the retailer and wholesaler play the infinite-horizon game described in §4.2. However, expediting implies that the retailer optimizes without regard to the inventory choices made by the wholesaler. If the wholesaler chooses wholesale price \( c^r \) at the first stage, let \( a^*_r(c^r) \) make explicit the dependence of an optimal \( a^r \) on \( c^r \) in (1.5). Anticipating this choice by the retailer, the wholesaler’s second-stage decisions are selected to maximize (1.14), i.e., the wholesaler selects \( a^w \) to maximize

\[
M[a^*_r(c^r), a^w] \text{ subject to } a^w \geq a^*_r(c^r).
\]

Let \( a^w_r(c^r) \) be a maximizing value of \( a^w \). It follows from (1.14) that the wholesaler’s first-stage problem is to choose \( c^r \) to maximize

\[
M[a^*_r(c^r), a^w]/(1 - \beta_w) - (\lambda - 1)c^w(a^*_r - s^w_1)^+ c^r - s^*_r + c^w s^w_1
\]

The assumptions in Theorem 1.4.5(b) lead to a characterization of the resulting wholesale price.

**Theorem 1.4.7.** Under the hypothesis of Theorem 1.4.5(b), there is a pure strategy equilibrium point relative to \([0, a^*_r] \times [0, a^w_r] \) for the dynamic game preceded by the pricing decision in which the retailer and wholesaler employ echelon base-stock policies, and the wholesale price satisfies
Setting the derivative of (1.14) with respect to \( c^r \) to zero yields (1.18). From (1.18), the equilibrium wholesale price does not depend on the unit expediting cost but depends only on the wholesaler’s purchasing cost and holding cost, \( \beta_w \), demand’s marginal rate in inventory, the sensitivity of the retailer’s target supply level to the wholesaler price, and the retailer’s initial inventory!

We now characterize the impacts of \( c^w \), \( h^w \), \( s^r_i \) and \( \beta_w \) on \( c^*_r \). A proof similar to that of Proposition 1.4.6 establishes the following result.

**Proposition 1.4.8.** The equilibrium point wholesale price, \( c^*_r \), increases as \( c^w \) or \( h^w \) increases, and decreases as \( \beta_w \) increases.

Proposition 1.4.8 indicates that the wholesaler passes her higher purchasing and holding costs on to the retailer by raising the wholesale price. This transfer enhances “double marginalization”, the phenomenon that a decentralized retailer often sets a lower base-stock level than a centralized decision maker would select. A smaller discount factor \( \beta_w \) makes the present more valuable than the future; therefore, the wholesaler raises the wholesale price. Higher \( s^r_i \) means that the retailer buys less in the first period, so the wholesaler increases the wholesale price.
5. Dynamic Competition Between Decentralized Supply Chains

This section studies dynamic inventory competition between two decentralized supply chains. We continue to assume that the wholesalers expedite orders to the retailers at unit cost \( \lambda_i c^w_i (\lambda_i > 1) \) when they are out of stock.

It follows from (1.5) and the proof of Theorem 3 that the two retailers play an infinite horizon game having an embedded strategic game in which the payoffs to the retailers one and two, when retailers one and two select respective supply levels \( a \) and \( b \), is

\[
L_1(a, b) = E\{G^1_1 - \rho_1[a - D^1_1(a, b)]^+ - (\rho_1 + \beta_1 c^r_1)D^1_1(a, b)\} - c^r_1(1 - \beta_1)a
\]

\[
L_2(a, b) = E\{G^2_1 - \rho_2[b - D^2_1(a, b)]^+ - (\rho_2 + \beta_2 c^r_2)D^2_1(a, b)\} - c^r_2(1 - \beta_2)b
\]

Each retailer’s decision is affected directly by the other retailer because each one’s demand depends on both retailers’ supply levels. The concavity of \( L_1(\cdot, b) \) and \( L_2(a, \cdot) \) and the result in Debreu (1952) imply that the retailers’ imbedded game has an equilibrium point relative to \( x^2_{i=1} [0, a^*_i] \) consisting of \( (a^*_1, a^*_2) = (a^{r1}_t, a^{r2}_t) \) for all \( t = 1, 2, \ldots \).

Now we turn to the wholesalers. wholesaler one faces a Markov decision process with payoff

\[
M_i(a^{r1}_t, a^{r2}_t, a^{wi}_i)/(1 - \beta_i) + \lambda_i - 1)c^w_i(a^{ri}_t - s^{wi}_1)^+ - c^r_1 s^{ri}_1 + c^w_1 s^{wi}_1
\]

where
\begin{align}
M_i(a^{r_1}_i, a^{r_2}_i, a^{w}_i) &= \beta_{wi}(c^r_i - c^{w}_i)E[D_i^f(a^{r_1}_i, a^{r_2}_i)] + [h^w_i + c^r_i(1 - \beta_{wi})]a^{r_i}_i \\
& \quad - [h^w_i + c^w_i(1 - \beta_{wi})]a^{w}_i - \beta_{wi}c^w_i(\lambda_i - 1)E[a^{r_i}_i - a^{w}_i + d(a^{r_1}_i, a^{r_2}_i) + \eta_i]^+ \\
\end{align}

(1.19)

for \(i = 1, 2\). Above equation has the same form as (1.15) with \(a^r\) replaced by \((a^{r_1}_i, a^{r_2}_i)\). We observe that the retailers’ decisions directly affect the wholesalers, but the wholesalers do not interact with each other. The concavity of \(M_i(\cdot, \cdot, a^{w}_i)\) ensures the existence of \(a^{w}_i \in \arg\max M_i(a^{r_1}_i, a^{r_2}_i, a^{w}_i)\) for \(i = 1, 2\). In summary, the dynamic decentralized supply chain game with payoffs defined by (1.9) and (1.14) has an equilibrium point relative to \(\times_{i=1}^2[0, a^{r_i}_i] \times [0, a^{w}_i]\) consisting of \((a^{r_i}_i, a^{w}_i) = (a^{r_i}_t, a^{w}_i)\) for all \(t = 1, 2, \cdots\). More specific assumptions regarding the structures of revenues, costs and demand lead to an explicit solution and comparative results. Let \(d^i_t = \partial d_i(a^{r_1}, a^{r_2})/\partial a^{r_i} (i = 1, 2)\). The results are summarized as follows.

**Theorem 1.4.9.** (a) If \(E(G^i_1)\) is concave in \(a^{r_i}_i\) and \(E(D^i_1)\) is concave in \(a^{r_i}_i\), \(i = 1, 2\), then the decentralized supply chain game under expedited shipment has an equilibrium point relative to \(\times_{i=1}^2[0, a^{r_i}_i] \times [0, a^{w}_i]\) consisting of \((a^{r_i}_t, a^{w}_i) = (a^{r_i}_t, a^{w}_i)\) for all \(t = 1, 2, \cdots\).

(b) If

(i) \(G^i_1 = p_i \min\{D^*_i(a^*_1), a^{r_i}_1\} + h^r_i(a^{r_i}_i - D^*_i)^+\)

(ii) \(D^*_i(a^1, a^2) = d_i(a^1, a^2) + \eta_i\) with \(\eta_{i_1}, \eta_{i_2}, \cdots\) independent and identically distributed random variables with mean zero and distribution function \(F_i(\cdot)\),
(iii) $d_1(\cdot, a^2)$ and $d_2(a^1, \cdot)$ increasing and concave,
then,

$$a_{ri}^* = d_i(a_{ri}^1, a_{ri}^2) + F_i^{-1} \left[ \frac{\rho_i - c_i^r(1 - \beta_i)}{(\rho_i + h_i^r)(1 - d_i^r)} + \frac{(p_i - \rho_i - \beta_i c_i^r)d_i^r}{(\rho_i + h_i^r)(1 - d_i^r)} \right] \quad (1.20)$$

$$a_{wi}^* = a_{ri}^* + d_i(a_{ri}^1, a_{ri}^2) + F_i^{-1} \left[ \frac{h_i^w + c_i^w(1 - \beta_{wi})}{\beta_{wi} c_i^w (\lambda_i - 1)} \right] \quad (1.21)$$

(c) As $c_i^r, \beta_{ri}$, or $h_i^r$ increase, or as $c_j^r$ or $\rho_j$ decrease, $a_{ri}^*$ decreases ($i = 1, 2; i \neq j$).

The hypothesis in part (a) of Theorem 1.4.9 are sufficient for the concavity of $L_1(a, \cdot)$ and $L_2(\cdot, b)$ on their domains.

We note that expediting is essential for the existence of equilibrium points of the decentralized supply chain game. Without the expediting feature, the argument in §1.3 leads to the conclusion that competing decentralized multi-echelon supply chains generally lack equilibrium points in echelon base-stock policies.

### 1.5 Conclusions and Questions

Since Clark and Scarf (1960), sufficient conditions have been known for an echelon base-stock policy to be optimal in a monopolistic centralized supply chain. When these same conditions are applied to competing centralized chains, the class of echelon base-stock policies does not generally contain an equilibrium point. The negative result persists if a centralized chain competes with a decentralized one. Consequently, it is not clear whether a first-mover advantage accrues to the supply chain that invests first in coordination. In addition, it is unclear whether costless
coordination is profitable if the competitor chain is centralized. Since we assume that the chains compete through product availability, and that the firms’ revenues are functions of goods supply level, perhaps specific forms of the revenue functions $G^i$ could lead to answers to these questions.
Chapter 2

Are Hub-and-Spoke Networks Better than Point-to-Point Networks?

2.1 Introduction

During the first ten years of U.S. airline deregulation (in the 1980s), major airlines (e.g., Northwest) shifted dramatically from point-to-point networks to hub-and-spoke networks. Hub-and-spoke networks not only create economies of density by flying passengers from different cities to and from a few hub cities but also yield higher flight frequency and broader geographic coverage. The role of traffic density in the airline industry has been widely studied both theoretically (Bailey, Graham
and Kaplan (1985), Brueckner and Spiller (1991), Hendricks, Picciione and Tan (1995a)) and empirically (Caves, Christensen and Tretheway (1984), Brueckner, Dyer and Spiller (1992)).

However, some airlines moved back to point-to-point networks recently (e.g. Southwest). The growing level of congestion at major hub airports in the 1980s created opportunities for low-fare, no-frills, and point-to-point services exemplified by Southwest airline. Shunning congested airports and direct competition with major airlines, low-cost carriers carved out a thriving market niche by reviving point-to-point services. In response, several major airlines (Continental, Delta, United, and US Airways) created subsidiaries offering similar services using a single type of aircraft (to reduced aircraft maintenance costs) and lower-paid crews.

Major airlines are now experiencing financial difficulties: US Airways filed for bankruptcy for a second time, and Delta is near bankruptcy. Excess capacity, cut-throat competition, oil price surges, powerful labor unions, and terrorist threats contributed to this situation. Carey and McCartney (2004) suggest that major airlines should de-emphasize hub-and-spoke networks and add direct flights between non-hub cities following the low-cost carriers’ strategy.

This paper investigates how airlines might determine their network structures
through a three-stage duopoly game in which two airlines serve a three-city network under demand uncertainty. At the first stage, the airlines choose their network structures; at the second stage, while route demands are still uncertain, they construct capacities; at the third stage, after demands are known, they allocate seats and flights to each route given capacity constraints imposed by the second stage decision.

Besides economies of traffic density, a hub-and-spoke network provides an airline the flexibility of allocating capacities among markets after uncertainty is resolved. Hence, if a hub-and-spoke network does not incur excess extra investment costs in the hub, it might seem to be a better choice than a point-to-point network for a monopolistic airline. However, we show later in §2.4 that this intuition could be wrong.

We address the following questions:

- Is a hub-and-spoke network always better than a point-to-point network? If not, when is it dominated by a point-to-point network?

- Under what conditions are there subgame perfect equilibrium points for the three-stage duopoly game? What are the equilibrium network structures?

There is a stream of papers by economists studying network selection and strategic interactions between airlines. Oum, Zhang and Zhang (1995) study a duopoly model with three cities. Considering economies of density which affect
both costs and demand, they show that strategic interaction reinforces the tendency towards hubbing because hubbing reduces airlines’ marginal costs and increases product quality thus forcing their competitors to cut output. The authors further show that even if hubbing increases total costs, strategic considerations may lead airlines to adopt this network structure. However, hubbing causes the prisoner’s dilemma: both airlines adopt hub-and-spoke networks, but the competitive advantage cancels out, so both may be worse off. Hendricks, Picciione and Tan (1995b) identify conditions under which an equilibrium point with competing hub-and-spoke networks exists for an n-city network. Barla (1999) studies a three-stage duopoly game in which the airlines have different hubs but the same non-hub cities thus competing only on the non-hub route. He concludes that the airlines determine their network structures through balancing the flexibility value versus the committed advantage in the non-hub route. In contrast, we assume that the airlines compete across a collection of three cities of which one is a hub city.

The rest of the paper is organized as follows. §2.2 presents the model. §2.3 characterizes a subgame perfect equilibrium point in the quantity and capacity games for all possible cases: both airlines employ hub-and-spoke networks, both airlines employ point-to-point networks, and they employ different network structures. §2.4 studies the network game and identifies factors that determine the airlines’ network structures. §2.5 concludes the paper.
2.2 Model and Assumptions

Two risk-neutral airlines compete across a collection of three cities: a hub city $H$ and two non-hub cities, $A$ and $B$ (Figure 1). Markets $AH$ and $BH$ are addressed as the *local markets* and $AB$ as the *connecting market*. A passenger travelling from $A$ to $B$ has to take two flights, $AH$ and $HB$, if he or she flies with a hub-and-spoke network, but the same passenger take only one flight ($AB$) with a point-to-point network. We assume that the markets from and to the hub city are identical, i.e., markets $AH$ and $BH$ have a common demand function. We also assume that all three markets $AH$, $BH$, and $AB$ are bidirectionally symmetric, e.g., traffic from $A$ to $B$ has the same characteristics as the traffic from $B$ to $A$. As a result, the third-stage game is a two-product *Cournot* competition, that is, the airlines compete for the connecting and local passengers. Relaxing above two symmetric assumptions adds the number of markets but will not change the qualitative results.

If an airline has chosen a hub-and-spoke network at the first stage, then at the second stage while demands are still unknown, it decides on the aggregate capacity
(e.g., aircraft size and maximum number of flights to offer per week) for the local and connecting markets; if it has chosen a point-to-point network initially, then it chooses capacities for the local and connecting markets, respectively. At the third stage, after demands are known, the airline allocates seats between the local and connecting markets if it has chosen a hub-and-spoke network. If it has chosen a point-to-point network, it simply determines upon the number of seats and flights to offer in each market subject to the capacities. Each stage in the model is a simultaneous-move non-cooperative game with complete information.

Let \( c_h \) and \( c_p \) be the unit capacity costs, respectively, of a hub-and-spoke network and of a point-to-point network, \( q_{yi} \) be the number of seats that airline \( i \) offers in market \( y \), and \( p_y \) be the ticket price for market \( y \). We note that some of the results might change if an economy of scale is considered for the capacity cost. Demands for the origin-destination pairs (AH, BH, AB) are independent random variables. We assumed a linear inverse demand function \( p_y = M_y - (q_{y1} + q_{y2}) \) for \( y = l, c; l \) represents the local markets and \( c \) the connecting market). Random variable \( M_y \) is nonnegative and has a mean \( \mu_y \) and a variance \( \sigma^2_y \) with distribution function \( F_y(\cdot) \). The qualitative results would remain valid if a more general demand function were used.

We assume that a connecting passenger cannot be accommodated through the hub if he or she chooses to fly through a point-to-point network. Hence, a hub-and-spoke network provides an airline the flexibility of allocating capacities
between the local and connecting markets after demands are known, but a point-to-point network does not. In addition, we assume that connecting passengers are prevented from arbitrages, i.e., they have to buy a single ticket AB instead of two separate tickets AH and HB. The airlines can enforce such a policy simply by checking passengers’ destination cities on their boarding passes. Furthermore, for expository simplicity we examine only the cases in which the realization of markets satisfies $M_c ≤ M_l$, a sufficient condition for $p_c ≤ 2p_l$ thus eliminating arbitrage opportunities. This simplicity is justified since hub cities usually attract heavier traffic than non-hub cities. We assume $c_h ≤ μ_y$ and $c_p ≤ μ_y$ so that the airlines make profits. The variable costs are assumed to be zero because airlines’ operating costs are mostly associated with offering a seat rather than serving a passenger (Barla (1999)).

Network adjustment is infrequent in the airline industry especially when one of the endpoint airports is congested and gates and landing slots are hard to obtain. Capacity investment is also a long-term decision. However, quantity, number of seats to offer, is more flexible (e.g., airlines can adjust number of flights to offer depending on market realization).

Let airline one be the row-player and airline two be the column player, $H$ denote a hub-and-spoke network and $P$ a point-to-point network, and $Π_i$ represent airline i’s net profit. Three cases are possible: both airlines use hub-and-spoke
networks, both airlines use point-to-point networks, and they use different networks. Figure 2.2 represents the game by a $2 \times 2$ matrix. Let $\pi_i$ represent airline $i$’s profit in the *Cournot* competition. We assume that the capacities are finite. Let $K_i$ denote the capacity on either leg when airline $i$ uses a hub-and-spoke network, and $K_{yi}$ be airline $i$’s capacity in market $y$ when it uses a point-to-point network.

<table>
<thead>
<tr>
<th>point-to-point</th>
<th>hub-and-spoke</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\max_{q_{li},q_{ci}} \pi_i = E\pi_i - c_i(2K_i+K_{yi})$</td>
<td>$\max_{q_{li},q_{ci}} \pi_i = E\pi_i - 2c_iK_i$</td>
</tr>
<tr>
<td>$\max_{q_{li}} \pi_i = 2p_{li}q_{li} + q_{ci}$</td>
<td>$\max_{q_{li}} \pi_i = 2p_{li}q_{li} + p_{ci}$</td>
</tr>
<tr>
<td>s.t. $0 \leq q_{li} \leq K_{yi}$ ($y = l, c$)</td>
<td>s.t. $q_{li} + q_{ci} \leq K_i$</td>
</tr>
<tr>
<td>$0 \leq q_{li}, 0 \leq q_{ci}$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2.3: Capacity and Quantity Optimization

Working backward, we first obtain equilibrium quantities in the *Cournot* game, then the capacity competition, and finally the network structure game.

At the third stage, given their rival’s network structures and capacities, the airlines engage in the quantity game after demand realization. Specifically, the airlines choose numbers of seats $q_{li}$ and $q_{ci}$ to maximize their expected payoffs.
\((\pi_1, \pi_2)\) given capacity constraints imposed by the second stage decision and their network structures. For example, when airline one uses a hub-and-spoke network and airline two uses a point-to-point network, airline one’s problem is

\[
\max_{q_{l1}, q_{c1}} \pi_1 = 2pq_{l1} + pq_{c1} = 2[M_l - (q_{l1} + q_{l2})]q_{l1} + [M_c - (q_{c1} + q_{c2})]q_{c1}
\]

\[s.t. \quad q_{l1} + q_{c1} \leq K_1 \tag{2.1}\]

\[0 \leq q_{l1} \quad 0 \leq q_{c1}\]

Similarly, airline two’s problem is

\[
\max_{q_{l2}, q_{c2}} \pi_2 = 2pq_{l2} + pq_{c2} = 2[M_l - (q_{l1} + q_{l2})]q_{l2} + [M_c - (q_{c1} + q_{c2})]q_{c2} \tag{2.2}
\]

\[s.t. \quad 0 \leq q_{l2} \leq K_{l2} \quad 0 \leq q_{c2} \leq K_{c2}\]

At the second stage, in the capacity game the airlines determine their capacities with payoffs \((\Pi_1, \Pi_2)\), given their network structures. Continuing the example above, airline one’s problem is

\[
\max_{K_{l1}} \Pi_1 = E\pi_1 - 2c_hK_1 \tag{2.3}
\]

and airline two’s problem is

\[
\max_{K_{l2}, K_{c2}} \Pi_2 = E\pi_2 - c_p(2K_{l2} + K_{c2}) \tag{2.4}
\]

Figure 2.3 summarizes airline one’s capacity and quantity optimization problems given its network structure. Finally, at the first stage, given its rival’s network
structure, airline $i$ selects its network structure depending on the expected payoff $\Pi_i$.

\section*{2.3 Quantity and Capacity Games}

We now characterize a subgame perfect equilibrium point in the quantity and capacity games. In this section we assume that the airlines do not necessarily deplete their capacities, so the quantity and capacity games are different. Because the calculations are tedious and lengthy, we attach the details in Appendices A, B and C and outline the solutions and illustrate the calculation through the case in which both airlines adopt hub-and-spoke networks.

\subsection*{2.3.1 Hub-and-Spoke Networks}

Label the airlines so that $K_1 \leq K_2$. For the quantity competition, the airlines’ problems are defined as (2.1), a concave nonlinear maximization problem to which the Karush-Kuhn-Tucker (KKT) conditions apply. For example, airline one’s
KKT conditions are

\begin{align*}
2(M_l - 2q_l1 - q_l2) - u_1 + v_{l1} &= 0 \\
v_{l1}q_l1 &= 0 \\
M_c - 2q_c1 - q_c2 - u_1 + v_{c1} &= 0 \\
v_{c1}q_c1 &= 0 \\
u_1(q_c1 + q_l1 - K_1) &= 0 \quad (2.5) \\
q_c1 + q_l1 &\leq K_1 \\
u_1, v_{l1}, v_{c1} &\geq 0 \\
q_c1, q_l1 &\geq 0
\end{align*}

In (2.5), \(v_{l1}\) and \(v_{c1}\) are slack variables, and \(u_1\) is a Lagrange multiplier. The realization of demands defines binding conditions of the capacities and the values of the Lagrange multiplier and slack variables. There are six combinations of the variables that define the regions in Figure 2.4. The equilibrium quantities can be derived by solving (2.5) and its analogues for airline two for each region. In area 1, neither airline is capacity-constrained; in area 2, only airline one is capacity-constrained; in area 3, both airlines are capacity-constrained; in area 4, only airline one is capacity-constrained, and it serves only the local markets; in area 5, both airlines are capacity-constrained, and airline one serves only the local markets; in area 6, both airlines are capacity-constrained and serve only the local markets; the connecting market is not served. Areas 4, 5, or 6 occur when the
local markets are significantly larger than the connecting market.

We now illustrate the procedure to calculate equilibrium quantities for some areas and relegate the details to Appendix A. In area 3, since both airlines are capacity-constrained, \( u_i > 0 \) and \( v_{yi} = 0 \) \((i = 1, 2; y = c, l)\). Solving (2.5) and its analogues for airline two yields

\[
q_{li} = \frac{(2M_l - M_c + 3K_i) - 9}{9} \\
u_i = 2\left(M_c + M_l - 2K_i - K_j\right) > 0 \quad (i \neq j; i, j = 1, 2)
\]

Similarly, in area 4, airline two is not capacity-constrained, so \( u_2 = v_{c2} = v_{l2} = 0 \); airline one serves only the local markets.

\[
v_{c1} = M_l - M_c/2 - 3K_1 > 0 \quad q_{c1} = 0 \quad q_{c2} = M_c/2 \quad q_{l1} = K_1 \quad q_{l2} = (M_l - K_1)/2
\]

The other areas can be analyzed similarly, and the details are in Appendix A. In
summary, the areas can be defined as follows:

\[ \Delta_1 = \{(x_l, x_c) : x_l + x_c \leq 3K_1\} \]
\[ \Delta_2 = \{(x_l, x_c) : x_l + x_c \geq 3K_1, x_l + x_c \leq K_1 + 2K_2, 2x_l - x_c \leq 6K_1\} \]
\[ \Delta_3 = \{(x_l, x_c) : x_l + x_c \geq K_1 + 2K_2, 2x_l - x_c \leq 6K_1\} \]
\[ \Delta_4 = \{(x_l, x_c) : 2x_l - x_c \geq 6K_1, x_l + x_c \geq K_1 + 2K_2\} \]
\[ \Delta_5 = \{(x_l, x_c) : 2x_l - x_c \geq 6K_1, 2x_l - x_c \leq 2K_1 + 4K_2, x_l + x_c \geq K_1 + 2K_2\} \]
\[ \Delta_6 = \{(x_l, x_c) : 2x_l - x_c \geq 2K_1 + 4K_2\} \]

Notice that \((x_l, x_c)\) are restricted to the Northeastern quadrant. When the airlines have identical capacities, areas 2, 4, and 5 disappear as shown in Figure 2.4(b).

Note that \(E\pi_i\) in (2.3) or (2.4) is the sum of the product of the profit in each area and the probability of that area.

Having solved the quantity game, we now characterize an equilibrium point of the capacity game. The proof of the next result is in Appendix A.

**Proposition 2.3.1.** When both airlines employ hub-and-spoke networks, the capacity game has a pure strategy equilibrium point. Furthermore, if \(K_1 = K_2 = K\), the equilibrium point is unique, and \(K\) satisfies

\[
\frac{1}{3} \int_{\Delta_3} (x_l + x_c - 3K) dF_l dF_c + 2 \int_{\Delta_5} (x_l - 3K) dF_l dF_c = c_h \tag{2.6}
\]
2.3.2 Point-to-Point Networks

This section studies the quantity and capacity games when both airlines use point-to-point networks. We assume that one airline has larger capacities than the other in all routes. There are other cases (e.g., one airline has larger capacity in one market but smaller capacity in the other market than the other airline). However, as we focus on the symmetric case in the succeeding sections, this assumption is only for expository convenience. Label the airlines so $K_{y2} \geq K_{y1}$ ($y = l, c$).

Figure 2.5(a) shows nine areas that are defined by the binding conditions of the capacities.

The quantity game can be solved for each area as illustrated in §3.2. The details are in Appendix B. In area 1, neither airline is capacity-constrained; in area 2, airline one is capacity-constrained in the connecting market; in area 3, airline one is capacity-constrained in all markets; in area 4, airline one is capacity-constrained in the local markets; in area 5, both airlines are capacity-constrained in the local markets; in area 6, airline one is capacity-constrained in all markets, and airline two is capacity-constrained in the local markets; in area 7, both airlines are capacity-constrained in the connecting market; in area 8, airline one is capacity-constrained in all markets, and airline two is capacity-constrained in the connecting market; in area 9, both airlines are capacity-constrained in all markets. For the same reason as in the proof of Proposition 1, the capacity game has a pure strategy equilibrium point. The proof of the following result is relegated
to Appendix B.

**Proposition 2.3.2.** When both airlines adopt point-to-point networks, the capacity game has a pure strategy equilibrium point. If $K_{y1} = K_{y2} = K_y$ ($y = l, c$), the equilibrium point is unique and $K_c$ and $K_l$ satisfy

\[
\int \int_{\Delta_{7,9}} (x_c - 3K_c) dF_l dF_c = c_p \tag{2.7}
\]
\[
\int \int_{\Delta_{5,9}} (x_l - 3K_l) dF_l dF_c = c_p \tag{2.8}
\]

### 2.3.3 Different Network Structures

Let airline one employ a hub-and-spoke network and airline two a point-to-point network. Figure 6(a) represents the case in which $K_1 > K_{y2}(y = l, c)$. Note that there are other cases (e.g., $K_{l2} > K_1 > K_{c2}$), so the capacity game might have multiple equilibria. Figure 2.6(a) shows nine areas that are defined by the binding conditions of the capacities. The quantity game for each area can be solved explicitly. The details are in Appendix C. In area 1, neither airline is capacity-constrained; in area 2, airline two is capacity-constrained in the local
markets; in area 3, airline two is capacity-constrained in the connecting market; in area 4, airline one is capacity-constrained; in area 5, airline one is capacity-constrained, and airline two is capacity-constrained in the local markets; in area 6, airline one is capacity-constrained, and airline two is capacity-constrained in the connecting market; in area 7, airline one is capacity-constrained and serves only the local markets where airline two is capacity-constrained; in area 8, airline one is capacity-constrained and serves only the local markets and airline two is capacity-constrained in all markets; in area 9, both airlines are capacity-constrained in all markets.

A proof (attached in Appendix C) that is similar to those of Propositions 1 and 2 establishes the following result.
Proposition 2.3.3. When the airlines use different network structures, the capacity game has a pure strategy equilibrium point which satisfies

\[ c_h = \frac{1}{3} \int_{\Delta_4} (x_l + x_c - 3K_1)dF_l dF_c + \frac{2}{11} \int_{\Delta_5} (2x_c + 3x_l - 6K_1 - 3K_{12})dF_l dF_c \]

\[ + \frac{1}{5} \int_{\Delta_6} (3x_c + 2x_l - 6K_1 - 3K_{12})dF_l dF_c \]

\[ + 2 \int_{\Delta_7} (x_l - 2K_1 - K_{12})dF_l dF_c + \frac{2}{3} \int_{\Delta_8} (x_c + x_l) \]

\[ - 2K_1 - K_{c2} - K_{12})dF_l dF_c \]

\[ c_p = \int_{\Delta_2} (x_l - 3K_{12})dF_l dF_c + \frac{2}{11} \int_{\Delta_5} (x_c + 7x_l - 3K_1 - 18K_{12})dF_l dF_c \]

\[ + \frac{2}{5} \int_{\Delta_7} ((x_l - 2K_{12} - K_1)\int_{\Delta_5} (x_c + x_l - 2K_1 - K_{c2})dF_l dF_c \]

\[ - 10K_{12})dF_l dF_c \]

\[ c_p = \frac{1}{5} \int_{\Delta_6} (4x_c + x_l - 3K_1 - 9K_{c2})dF_l dF_c + \int_{\Delta_8} (x_c - 2K_{c2})dF_l dF_c \]

\[ + \frac{1}{6} \int_{\Delta_9} (5x_c + 2x_l - 4K_1 - 11K_{c2} - 2K_{12})dF_l dF_c \]

The equations in Propositions 1-3 provide little information about the subgame perfect equilibrium points. In order to investigate the network structure game further, we make two important assumptions in §4.

2.4 Network Structure Game

Having analyzed the quantity and the capacity games, we address the two questions raised in the introduction: what factors affect airlines’ network choices?
What are the airlines’ network structures at equilibrium? §2.4.1 states two additional assumptions. Following the logic of §2.3, §2.4.2 derives equilibrium capacities, prices, and profits for the three cases: both airlines use hub-and-spoke networks, both use point-to-point networks, and they use different network structures. §2.4.3 compares the cases. §2.4.4 investigates a monopolist’s network choice and a duopoly network game when the two network structures have the same unit capacity costs. Then §2.4.5 addresses the same questions when the unit capacity costs are different.

Henceforth, we assume that the airlines are identical, so the airline identity $i$ is suppressed.

2.4.1 Assumptions

As shown in §2.3, the quantity and capacity games have a subgame perfect equilibrium point. However, the equilibrium capacities lack closed-forms. In order to analyze the network game, we have to compare the profits. So we make the following assumptions.

Assumption 2.4.1. Both the local and connecting markets are always served by the airlines.

This assumption excludes the cases in which the local markets are so much larger than the connecting market that serving the connecting market is uneconomical. Specifically, when both airlines use hub-and-spoke networks, areas 4, 5,
and 6 disappear in Figure 2.4(a), and areas 7 and 8 disappear in Figure 2.6.

**Assumption 2.4.2.** If the airlines use a hub-and-spoke networks, \( q_{li} + q_{ci} = K_i \); if they use point-to-point networks, \( q_{yi} = K_{yi} \) \((y = l, c; i = 1, 2)\).

Reflecting characteristics of the U.S. airline industry, this assumption forces the airlines to deplete their capacities regardless of market realization. Because most expenses for a flight occur at departure and landing, and the marginal cost of an passenger is minuscule (Barla (1999)), it is beneficial for airlines to sell up to the maximum number of seats available per flight even at a sub-optimal price. For example, because demands decreased dramatically after September 11, U.S. airlines had to cut prices to fill more seats in the flights.

Assumptions 2.4.1 and 2.4.2 eliminate the complexity caused by different demand realization and binding conditions of the capacities. Furthermore, the quantity and capacity games are equivalent.

### 2.4.2 Equilibrium Capacities and Prices

Let superscripts \( h \), \( p \), and \( m \) represent, respectively: both airlines use hub-and-spoke networks, both airlines use point-to-point networks, and one airline uses a hub-and-spoke network and the other a point-to-point network. The proof of the following result is relegated to Appendix A.

**Proposition 2.4.3.** When both airlines employ hub-and-spoke networks, the capacity game has a subgame perfect equilibrium point at which
(a) the optimal capacity is $K^h = (\mu_l + \mu_c)/3 - c_h/2$,

(b) the expected prices are $p^h_c = (\mu_c + 2c_h)/3$ and $p^h_l = (\mu_l + c_h)/3$;

(c) the expected profit is

$$\Pi^h = \frac{(4\sigma_l^2 + \sigma_c^2)}{27} + \frac{(2\mu_l^2 + \mu_c^2)}{9} - \frac{5c_h(\mu_l + \mu_c)}{9} + \frac{2c_h^2}{3}$$

(2.12)

It can be verified that $K^h \geq 0$ and $\Pi^h \geq 0$ because $c_h \leq \mu_l$ and $c_h \leq \mu_c$ by assumption.

For the remainder of the paper, we use “increase” and “decrease” for “non-decreasing” and “nonincreasing”, respectively. From Proposition 2.4.3, it can be derived that the optimal capacity increases as mean demands increase or the unit capacity cost decreases and that the expected prices increase as the respective mean demand or the unit capacity cost increases. Moreover, (2.12) indicates that the airlines’ expected profits increase if demand variances increase, but they are not monotone in $c_h$, $\mu_l$, or $\mu_c$.

We now study the case in which both airlines adopt point-to-point networks. The proof of the following results is in Appendix B.

**Proposition 2.4.4.** When both airlines employ point-to-point networks, the capacity game has a subgame perfect equilibrium point at which

(a) the equilibrium capacities are $K^p_y = (\mu_y - c_p)/3$ ($y = l, c$);

(b) the expected prices are $p^p_y = (\mu_y + 2c_p)/3$;
(c) the expected profit is

\[
\Pi^p = \frac{(2\mu_l^2 + \mu_c^2)}{9} - 2c_p(2\mu_l + \mu_c)/9 + c_p^2/3
\]  

(2.13)

It can be verified similarly that \( K^p_y \geq 0 \) and \( \Pi^p \geq 0 \) since \( c_p \leq \mu_l \) and \( c_p \leq \mu_c \) by assumption. From Proposition 2.4.4, it can be verified that the monotonicity of capacities and prices with respective to \( \mu_y \) or \( c_h \) are the same as that from Proposition 4. However, demand variances no longer affect the expected profit. Furthermore, the expected profit increases as either demand mean increases, or as the unit capacity cost decreases, whereas this monotonicity does not exist when both airlines use hub-and-spoke networks.

Finally, we study the case in which the airlines employ different network structures. Let subscripts \( h \) and \( p \) represent the airline’s network structures, and superscript \( m \) represent the case. The results are summarized as follows, and its proof is attached in Appendix C.

**Proposition 2.4.5.** When the airlines employ different network structures, the capacity game has a subgame perfect equilibrium point at which

(a) the optimal capacities are \( K^m_h = \frac{(2\mu_c + 2\mu_l + 3c_p - 6c_h)}{6} \) and \( K^m_c = \frac{(\mu_c + c_h - 2c_p)}{3} \); and \( K^m_l = \frac{(2\mu_l + c_h - 2c_p)}{6}; \)

(b) the expected prices are \( p^m_l = \frac{(2\mu_l + c_p + c_h)}{6} \) and \( p^m_c = \frac{(\mu_c + c_h + c_p)}{3}; \)

(c) the expected profits are
\[ \Pi^m_h = \frac{(4\sigma_l^2 + \sigma_c^2)}{18} + \frac{[(2\mu_l^2 + \mu_c^2) - (\mu_l + \mu_c)(7c_h - 2c_p)]}{9} \]  
\[ + \frac{(10c_h^2 + c_p^2 - 7c_pc_h)}{6} \]  
\[ \Pi^m_p = \frac{(2\mu_l^2 + \mu_c^2)}{9} + \frac{\mu_l(2c_h - 7c_p)}{9} + \frac{2\mu_c(c_h - 2c_p)}{9} - \frac{5c_hc_p}{6} + \frac{c_h^2}{6} + \frac{c_p^2}{9} \]

From Proposition 2.4.5(a), it is straightforward to derive that the hubbing airline’s capacity increases as either mean market increases, and the equilibrium capacities for the local and connecting markets of the airline with a point-to-point network increase as the corresponding mean market increases. Furthermore, if hubbing becomes more expensive, the airline with a point-to-point network should raise its capacities, while the other airline should reduce its capacity and vice versa. The expected prices increase as the corresponding mean demands increase, or as either unit capacity cost increases. Demand variances positively affect the hubbing airline’s profit but has no impacts on the other airline’s profit. More interestingly, comparing (2.14) and (2.12), one observe that demand variances have larger impacts on the hubbing airline’s expected profit in this scenario than when both airlines adopt hub-and-spoke networks because the flexibility value is shared by the airlines in the latter case.

Notice the wording in Propositions 2.4.4 to 2.4.6, “equilibrium capacity” and “expected prices”. Because capacities are determined before demands are known,
so they are deterministic. However, prices are determined after uncertainty is resolved, so actual prices depend on demand realization. Therefore, “expected” is used to restrict “price”.

Table 1 summarizes each airline’s equilibrium capacities in the local and connecting markets at a subgame equilibrium point. Since the airlines deplete their
capacities, the capacity decision and pricing decision are equivalent, and the orderings of prices and capacities are opposite. The following results are immediate from Table 2 or Propositions 2.4.4 to 2.4.6.

**Corollary 2.4.6.** If $c_h > c_p$, then $p_i^h > p_i^m > p_i^p$. If $c_h > 3c_p$, then $p_i^h > p_i^m > p_i^p$. If $2c_p > c_h > c_p$, then $p_i^h > p_i^p > p_i^m$. If $3c_p > c_h > 2c_p$, then $p_i^m > p_i^p > p_i^h$.

Corollary 2.4.6 claims that if a hub-and-spoke network is more expensive than a point-to-point network, scenario $h$ provides the lowest capacity thus the highest price in the connecting market. However, if the unit capacity cost is the same regardless of the network structures, then all three cases offer the same price and capacity for the connecting market. In reality, low-cost carriers’ cost per mile seat could be 40% lower than that of a major hub-and-spoke airline (Borenstein (1992)), so $2c_p > c_h > c_p$ probably best reflects the reality. Hence, from Corollary 2.4.6 local passengers benefit most from the airlines’ network differentiation.

<table>
<thead>
<tr>
<th></th>
<th>p</th>
<th>h</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Local</strong></td>
<td>$\frac{1}{2}(\mu_l - c_p)$</td>
<td>$\frac{1}{2}(2\mu_l - c_h)$</td>
<td>$\frac{1}{2}\mu_l - \frac{1}{6}(c_p + c_h)$</td>
</tr>
<tr>
<td><strong>Connecting</strong></td>
<td>$\frac{1}{2}(\mu_c - c_p)$</td>
<td>$\frac{1}{2}(\mu_c - c_h)$</td>
<td>$\frac{1}{2}\mu_c - \frac{1}{3}(c_p + c_h)$</td>
</tr>
</tbody>
</table>

Table 2.1: Equilibrium Capacities
Furthermore, if $\mu_l = \mu_c$, the total capacities of the local markets is larger than that of the connecting market as long as at least one airline uses a hub-and-spoke network. Consequently, local flights are cheaper than connecting flights.

### 2.4.3 Network Choice with Uniform Capacity Cost

This subsection studies the airlines’ equilibrium network structures when $c_p = c_h = c$. This assumption is relaxed in §2.4.4.

**Monopoly**

As conjectured in §1, if fixed investment costs are ignored, a hub-and-spoke network might seem to dominate a point-to-point network for a monopolist because it possesses a flexibility value and economies of density. However, the following result shows that a monopolist might favor a point-to-point network over a hub-and-spoke network.

Let $\Pi^M_p$ and $\Pi^M_h$ be a monopolistic airline’s expected profit when it uses a point-to-point network and a hub-and-spoke network, respectively. Let $\Delta^M = \Pi^M_h - \Pi^M_p$. It can be verified that optimal capacities are $K_y = (\mu_y - c_p)/2$ $(y = l, c)$ if the monopolist uses a point-to-point network, and $K^h = (\mu_c + \mu_l - 3c_h)/2$ if it uses a
hub-and-spoke network. So

\[ \Pi_p^M = \left(2\mu_c^2 + \mu_t^2 - 4\mu_c c_p - 2\mu_c c_p + 3c_p^2\right)/4 \]

\[ \Pi_h^M = \left[(\sigma_c^2 + 2\sigma_t^2) + (\mu_c - 2\mu_t - 4(\mu_t - \mu_c)^2)/4 \right] \]

\[ \Delta^M = (\sigma_c^2 + 2\sigma_t^2 - \mu_c^2 + \mu_t^2 + 6c_p^2 - 3c_p^2 - 4\mu_c c_h + 2\mu_c c_p - 4\mu_t c_h + 4\mu_t c_p) / 4 \]  \hspace{1cm} (2.16)

For expository simplicity, let \( \sigma_t = \sigma_c = \sigma \) and \( \mu_t = \mu_c = \mu \), then \( \Delta^M = (3c^2 - 2\mu c + 3\sigma^2)/4 \). Let \( \underline{c} \) and \( \bar{c} \) are the respective smaller and larger roots of \( c \) by setting \( \Delta^M \) to zero. Then \( \bar{c} = (\mu + \sqrt{\mu^2 - 9\sigma^2})/3 \) and \( \underline{c} = (\mu - \sqrt{\mu^2 - 9\sigma^2})/3 \). So if \( \sigma/\mu \geq 3 \), \( \Delta^M > 0 \); otherwise, if \( c \in (\underline{c}, \bar{c}) \), then \( \Delta^M < 0 \), and if \( c \in (0, \underline{c}) \cup (\bar{c}, \mu) \), then \( \Delta^M > 0 \). The results are summarized as follows.

**Proposition 2.4.7.** (a) A monopolist uses a hub-and-spoke network if \( \sigma/\mu \geq 3 \). Otherwise, it uses a hub-and-spoke network if \( c \in (\bar{c}, \mu) \cup (0, \underline{c}) \) and a point-to-point network if \( c \in (\underline{c}, \bar{c}) \); (b) If \( \mu_c \) increases, or if \( c_p, \sigma_c \), or \( \sigma_t \) decreases, \( \Delta^M \) decreases.

Proposition 2.4.7 reveals that only if the demand variation coefficient is larger than 3, a monopolist favors a hub-and-spoke network. Otherwise, it might favor a point-to-point network depending on the parameters. For an extreme case, if \( \sigma = 0 \), then \( \Delta^M = (3c^2 - 2\mu c)/4 \). So if \( \mu/c < 3/2 \), the airline should use a hub-and-spoke network and otherwise a point-to-point network. Although a hub-and-spoke network has economies of density, it requires one seat in each leg.
to transport a connecting passenger, so a monopolist does not necessarily favor a hub-and-spoke network. When demand/cost ratio is greater than $3/2$, this capacity-costly disadvantage dominates economies of density. So a monopolist should use a point-to-point network.

In addition, Proposition 2.4.7 implies that under the same market condition, a low-cost airline is more likely to opt for a point-to-point network than a high-cost airline. This coincides with the fact that all the prosperous U.S. direct flight airlines are low-cost. In general a high demand variance or a low demand/cost ratio favors a hub-and-spoke network.

**Best Response**

This subsection examines an airline’s best response given its rival’s network structure. We first analyze the case in which the rival airline employs a hub-and-spoke network. Let $\Delta^H$ denote the profit gap of responding with a hub-and-spoke network versus a point-to-point network. From (2.12) and (2.15),

$$\Delta^H = \Pi^h - \Pi^m_p = (\sigma^2_c + 4\sigma^2_l)/27 + (c^2 - \mu_c c)/3$$  \hspace{1cm} (2.17)

Let $\bar{c}_1 = \{3\mu_c + [9\mu_c^2 - 4(\sigma^2_c + 4\sigma^2_l)]^{1/2}\}/6$ and $\underline{c}_1 = \{3\mu_c - [9\mu_c^2 - 4(\sigma^2_c + 4\sigma^2_l)]^{1/2}\}/6$. Following the reasoning as for Proposition 2.4.7, if (2.18) holds, $\Delta^H$ in (2.17) is nonnegative. So the best response is a hub-and-spoke network; otherwise, the sign of $\Delta^H$ depends on the value of $c$: if $c \in (\underline{c}_1, \bar{c}_1)$, $\Delta^H > 0$; if $c \in (\underline{c}_1, \mu) \cup (\bar{c}_1, \mu)$, $\Delta^H < 0$. The consequence results are summarized as follows.
**Proposition 2.4.8.** When its competitor employs a hub-and-spoke network, an airline’s best response is a hub-and-spoke network if

\[
\frac{\sqrt{\sigma_c^2 + 4\sigma_l^2}}{\mu_c} \geq 3/2
\]

(2.18)

Otherwise, the best response is a hub-and-spoke network if \(c \in (0, c_1) \cup (\bar{c}_1, \mu_c)\) and a point-to-point network if \(c \in (c_1, \bar{c}_1)\).

From (2.17), it can be derived that as \(\mu_c\) decreases, or as \(\sigma_l\) or \(\sigma_c\) increases, \(\Delta^H\) increases. A hub-and-spoke response enables the airline to share with the incumbent airline, a flexibility value, \((4\sigma_l^2 + \sigma_c^2)/27\). In contrast, a point-to-point response provides the airline a committed position in the connecting market whose value depends on the size of the mean of the connecting market. The airline chooses its network structure by weighing the flexibility value and the committed value. Specifically, when (2.18) holds, the flexibility value is larger, so it responds with a hub-and-spoke network. Otherwise, an intermediate capacity cost favors a point-to-point response, while a low or high capacity cost favors a hub-and-spoke response.

If \(\sigma_c = \sigma_l = \sigma\), then \(c > \bar{c}_1\) and \(c < c_1\), respectively, correspond to

\[
\mu_c/c - \sqrt{(\mu_c/c)^2 - 20/9(\sigma/c)^2} > 2
\]

(2.19)

\[
\mu_c/c + \sqrt{(\mu_c/c)^2 - 20/9(\sigma/c)^2} < 2
\]

(2.20)

Inequalities (2.19) and (2.20) imply that if \(\sigma/c\) is higher than \(\mu_c/c\) by a certain
increment, the best response is a hub-and-spoke network and otherwise a point-to-point network. So the airline determines its network structure through weighing $\mu_c/c$ versus $\sigma/c$, or roughly speaking, the commitment value and the flexibility value. One extreme case is $\sigma_c = \sigma_l = 0$. Then $\Delta^H = (c^2 - \mu_c c)/3 \leq 0$ because $\mu_c \geq c$ by assumption. So an airline should respond with a point-to-point network without uncertainty. The flexibility value diminishes without uncertainty, and economies of density cancel out if the airline responds with a hub-and-spoke network. Therefore, by responding with a point-to-point network, the airline improves its profit. In addition, if $\mu_c/c < (> 2)$, $\Delta^H$ increases (decreases) as $c$ increases because a hub-and-spoke’s economies of density dominate its cost disadvantage when demand/cost ratio is low. In summary, the demand/cost ratio affects the airline’s network choice similarly in a duopoly case as in a monopoly case.

We now consider the airline’s best response when its competitor uses a point-to-point network. Let $\Delta^P$ be the profit gap of responding with a hub-and-spoke network versus a point-to-point network. From (2.13) and (2.14),

$$\Delta^P = \Pi^m_h - \Pi^p = c^2/3 - c(\mu_l + 3\mu_c)/9 + (\sigma_c^2 + 4\sigma_l^2)/18 \quad (2.21)$$

Let $c_2 = \{\mu_l + 3\mu_c - [((\mu_l + 3\mu_c)^2 - 6(4\sigma_l^2 + \sigma_c^2)]^{1/2}\}/6$ and $\bar{c}_2 = \{\mu_l + 3\mu_c + [((\mu_l + 3\mu_c)^2 - 6(\sigma_c^2 + 4\sigma_l^2)]^{1/2}\}/6$. Similarly, following the same reasoning as for Propositions 7 and 8, the results below are consequences of (2.21).
Proposition 2.4.9. When its competitor employs a point-to-point network, an airline’s best response is a hub-and-spoke network if

\[ \frac{\sqrt{\sigma_c^2 + 4\sigma_l^2}}{\mu_l + 3\mu_c} > \sqrt{6}/6 \] (2.22)

Otherwise, the best response is a hub-and-spoke network if \( c \in (0, c_2) \cup (\bar{c}_2, \mu_l) \) and a point-to-point network if \( c \in (\bar{c}_2, \mu_l) \).

From (2.21), \( \Delta^P \) increases (decreases) as \( \sigma_c \) or \( \sigma_l \) (\( \mu_l \) or \( \mu_c \)) increases. In addition, if \( (\mu_l + 3\mu_c)/c \leq (>)3 \), \( \Delta^P \) increases (decreases) as \( c \) increases. Comparing (2.22) to (2.18), one observes that both \( \mu_l \) and \( \mu_c \) now affect the airline’s response when the incumbent uses a point-to-point network. Moreover, (2.22) indicates that \( \mu_c \) weights more than \( \mu_l \) in determining the airline’s response network.

Comparing Propositions 2.4.7 to 2.4.9, one observes that with or without competition, in general, a high demand variation coefficient or an extremely high or low unit capacity cost favors a hub-and-spoke network, but an intermediate capacity cost favors a point-to-point network.

Equilibrium Network Structures

Before analyzing the network game, we first compare the airline’s expected profits in the three cases. From Propositions 2.4.4 to 2.4.6,

\[ \Pi^h - \Pi^m_h = -(\sigma_c^2 + 4\sigma_l^2)/54 \leq 0 \quad \Pi^p - \Pi^m_p = c\mu_l/9 > 0 \]

So the following results are immediate.
Proposition 2.4.10. \( \Pi^p > \Pi^m_p, \Pi^m_h \geq \Pi^h \).

Proposition 2.4.10 reveals that a hubbing airline prefers its rival to use a point-to-point network, while an airline with a point-to-point network prefers its rival to use the same kind of network. These results are intuitive. When the airlines use different network structures, the hubbing airline enjoys not only the flexibility value but also the economies of density thus forcing the other airline to cut capacities. Consequently, the hubbing airline benefits if its rival uses a point-to-point network. For the same reason, if both airlines use point-to-point networks initially, as soon as one of them switches to a hub-and-spoke network, the other suffers. The ordering of the other pairs of expected profits depends on the parameters and demand characteristics.

We now analyze the network game. Let \( H \) and \( P \) denote a hub-and-spoke network and a point-to-point network, respectively, and let NEP stand for pure strategy Nash equilibrium point. The following result concerns the case in which \( \sigma_l = \sigma_c = \sigma \) and \( \mu_l = \mu_c = \mu \), and its proof is in Appendix D.

Proposition 2.4.11. The network game is characterized as follows:

(a) if \( \sigma/\mu > 2\sqrt{30}/15 \), \( (H,H) \) is the NEP,

(b) if \( 3\sqrt{5}/10 < \sigma/\mu < 2\sqrt{30}/15 \), and

(i) if \( c \in (0, c_2) \cup (\bar{c}_2, \mu) \), then \( (H,H) \) is the NEP,

(ii) if \( c \in (c_2, \bar{c}_2) \), then \( (H,H) \) and \( (P,P) \) are the NEPs;

(c) if \( \sigma/\mu < 3\sqrt{5}/10 \), then
(i) if \( c \in (0, c_2) \cup (c_2, \mu) \), then \((H, H)\) is the NEP,

(ii) if \( c \in (c_1, c_1) \), then \((P, P)\) is the NEP,

(iii) if \( c \in (c_2, c_1) \cup (c_1, c_2) \), \((H, H)\) and \((P, P)\) are the NEPs.

Proposition 2.4.11 concludes that the airlines adopt the same kind of networks at equilibrium. Specifically, for a high demand variation coefficient, both airlines choose hub-and-spoke networks; for an intermediate or low demand variation coefficient, two cases are possible: either both airlines choose hub-and-spoke networks, or both airlines choose hub-and-spoke networks. Note that in reality, airlines’ networks are much more complicated, so the symmetric equilibrium conclusion of this ideal three-city model does not exclude the coexistence of hub-and-spoke and point-to-point networks in reality.

2.4.4 Network Choice with Different Unit Capacity Costs

A hub-and-spoke network incurs extra cost such as gaining landing slots and gates, handling passengers’ packages in the hub, and coordinating flights. In addition, it contributes to airport congestion and lowers aircraft usage because aircrafts arrive and depart at about the same time for connecting passengers to take connecting flights. Moreover, in reality, low-cost carriers usually have a lower cost per mile seat than major airlines. So we assume \( c_p < c_h \).
Monopoly

The following results can be derived by computing the partial derivatives of $\Delta^M$ with respect to the corresponding parameters from (2.16).

**Proposition 2.4.12.** (a) As $\sigma_t$, $\sigma_c$, or $\mu_c$ decreases, or as $c_p$ increases, $\Delta^M$ increases; (b) If $(\mu_l + \mu_c)/c_h \geq (<)3$, $\Delta^M$ decreases (increases) as $c_h$ increases.

Comparing Propositions 2.4.12 and 2.4.7, one observes that the essence of the results do not change when $c_p < c_h$: larger demand variances favor a hub-and-spoke network, whereas larger market means favor a point-to-point network; if $(\mu_l + \mu_c)/c_h < 3$, as $c_h$ increases, the monopolist should tend to use a hub-and-spoke network because of its economies of density; otherwise, economies of density are dominated by the capacity-costly factor, so a monopolist should instead tend to choose a point-to-point network. In reality, there is an excess capacity in airline industry, namely, $(\mu_l + \mu_c)/c_h$ is relatively small. Thus, loosely speaking, airlines with dominant positions in their hubs should favor hub-and-spoke networks. This explains in part why major airlines, whose costs are high relative to low-cost carriers such as Southwest, favor hub-and-spoke networks.

In conclusion, the essence of the results does not change compared to the case with uniform capacity costs, so we here do not derive specific criterion for a monopolist’s network selection.
Best Response

We now study an airline’s best response to a hubbing competitor. From (2.12) and (2.15),

\[ \Delta^H = \Pi^h - \Pi^p = \left(4\sigma_t^2 + \sigma_c^2\right)/27 - 7\mu_l(c_h - c_p)/9 - \mu_c(7c_h - 4c_p)/9 \]

\[ + \frac{c_h^2}{2} + 5c_h c_p/6 - c_p^2 \]

The following results follow by computing the partial derivatives of \( \Delta^H \) with respect to the corresponding parameters.

**Proposition 2.4.13.** (a) As \( \sigma_l, \sigma_c, \text{ or } c_p \) increases, or as \( \mu_c \) or \( \mu_l \) decreases, \( \Delta^H \) increases; (b) \( \Delta^H \) is submodular in \( c_h \) and \( c_p \).

Proposition 2.4.13 claims that when there is a competing hubbing airline, the larger the demand variances, or the smaller the mean demands, the less an airline should tend to respond with a hub-and-spoke network. This monotonicity is not surprising because a hub-and-spoke network’s flexibility value increases when demand variances increase, and because economies of density are larger when mean demand is smaller. The submodularity of \( \Delta^H \) means that the marginal increasing rate of \( \Delta^H \) in \( c_p \) decreases as \( c_h \) decreases.

We now consider the airline’s best response when its rival employs a point-to-point network. From (2.13) and (2.14),

\[ \Delta^p = \Pi^m_h - \Pi^p = \left(4\sigma_t^2 + \sigma_c^2\right)/18 - \left[7c_h(\mu_l + \mu_c) - 2c_p(3\mu_l + 2\mu_c)\right]/9 \]

\[ + \frac{10c_h^2}{6} - c_p^2 - 7c_h c_p \]

(2.23)
The following results are obtained by deriving partial derivatives of $\Delta^P$ with respect to the corresponding parameters.

**Proposition 2.4.14.** (a) As $\sigma_l$ or $\sigma_c$ increases, or as $\mu_l$ or $\mu_c$ decreases, $\Delta^P$ increases; (b) $\Delta^P$ is submodular in $c_h$ and $c_p$.

Comparing (2.17) and (2.21) with (2.23) and (2.23), respectively, one observe that changing $c_h = c_p$ to $c_h > c_p$ does not alter the qualitative results. So we here do not derive specific conditions that determine the airlines’ network structures at equilibrium. Nevertheless, the submodularity of $\Delta^p$ or $\Delta^H$ in $c_h$ and $c_p$ captures the impacts of unit capacity costs on the airlines’ network selection. The submodularity of $\Delta^P$ means that the marginal decreasing rate of $\Delta^P$ in $c_p$ increases as $c_h$ decreases. Therefore, the lower its competing low-carrier’s cost, the greater a hubbing airline’s profit deteriorates if it does not follow suit and switch to a point-to-point network as well. So this partly explains major airlines’ response strategy, creating airlines with airlines to match low-cost carriers’ business model.

### 2.5 Conclusions

This paper examines how airlines might determine their network structures through a three-stage duopoly game in which two airlines serve a three-city network with uncertain demand. We show that at equilibrium the airlines employ the same
kind of networks. The monopoly case is also investigated to exclude competition’s impacts on the airline’s network selection. Even if fixed investment costs are ignored, we show that a hub-and-spoke network does not necessarily dominate a point-to-point network. A larger demand variation coefficient or an extreme high or low unit capacity cost generally favors hub-and-spoke networks whether there is competition or not.

This paper sheds some lights into how demand, cost, and competition affect the airlines network selection and capacity investment and why low-cost carriers exemplified by Southwest prospers. In reality, networks are much more complex, airlines use multiple types of aircrafts, passengers are sensitive to flight quality such as number of stops and flight frequency. Future research might consider these factors.
Chapter 3

Capital Structure and Inventory Management

3.1 Operations and Capital Structure

A large literature examines the effects of a firm’s mixture of debt and equity on its incentives to produce. For nearly 30 years, researchers have examined the consequences of splits between ownership and control (agency models) and the resulting diversity of claims on cash flow. A related stream of research examines the interactions of a firm’s capital structure with the product market behaviors of the firm’s competitors and suppliers and the firm’s reactions to those behaviors. Pioneered by (Brander and Lewis 1986), a stream of literature studies debt’s strategic commitment effect which leads to more aggressive behavior in the product
Another stream of papers studies agency issues of a financially constrained firm and their ramifications in product competition ((Jensen and Meckling 1976), (Povel and Raith 2004), (Jean-Baptiste and Riordan 2003)). The reviews by (Allen and Winton 1995), (Maksimovic 1995), and (Frydenberg 2004) are excellent portals to these large bodies of work. However, the firm’s production process is typically static in that research.

A parallel stream of research analyzes a plethora of models of inventory and production processes but until recently financial considerations have been conspicuously absent. (Buzacott and Zhang 2004) analyze a single period Stackelberg game between the bank and retailer. Anticipating the retailer’s response (order quantity and loan size), the bank determines the interest rate and loan limit. The authors assume that the retailer maximizes expected revenues while the bank maximizes expected profits. (Archibald, Thomas, Betts and Johnston 2002) assume that the objective of a start-up firm facing discretely distributed demands is maximizing long-term survival probability instead of average profit per period. The authors conclude that start-up firms should be more cautious in their component purchasing strategies than well-established firms and the purchasing strategies are not monotone in the capital available. (Li, Shubik and Sobel 2003) study a firm who makes production, borrowing and dividend decisions simultaneously in each period in an infinite horizon to maximize the expected present value of
dividends subject to nonnegative production, nonnegative short-term loan and liquidity constraints. The authors conclude that the firm should always use internally generated funds first which is consistent with “pecking order” theory, and a base stock policy is optimal both for the physical goods and cash reserves. (Sobel and Zhang 2003) extends the results in (Li et al. 2003) to models with nonlinear production costs. (Babich and Sobel 2004) examines the relationship between operational decisions and financial decisions to maximize a firm’s pre-IPO value. With the exception of (Xu and Birge 2004c), issues of capital structure are not addressed in the recent spate of research that infuses financial considerations in production-inventory models.

Here we analyze interactions between capital structure and short-term operating decisions concerning inventories, dividends, and liquidity in a production-inventory model that is simple, dynamic, and stochastic. However, we ignore agency issues and the possibility of conflicting claimants. (Xu and Birge 2004b) conclude that (i) the firm’s borrowing capacity is limited if bankruptcy cost is considered, and (ii) not coordinating operational and financial decisions may lead to suboptimal results. (Xu and Birge 2004c) investigate agency issues in a model that is similar to (Xu and Birge 2004b). (Xu and Birge 2004a) study a discrete-time, finite-horizon, partial equilibrium model in which the firm’s objective is to maximize the expected discounted value of net cash flow to the shareholders subject to period-by-period constraints that model resource evolution. The firm’s
financial resources are internal savings, current cash flow, single period debt, and external equity. The firm’s decisions in each period are to invest or to default, and how to finance production.

We employ the same framework as in (Li et al. 2003) and assume that the firm strives to maximize its market value, namely the expected present value of the time stream of dividends. Endowed with an initial equity and bonds, the firm determines in each period its production quantity and the amount of dividends to issue or subscribe. Also it pays a fixed coupon for its bond, and it pays taxes if it makes a positive profit.

For more than 40 years it has been known ((Miller and Modigliani 1961)) that a firm’s dividend policy does not affect its market value if the firm operates in capital markets that are perfect and complete. (Allen and Michaely 1995) review the literature on dividend policy. However, many firms, particularly if they are young and entrepreneurial, do not operate in perfect complete capital markets and their revenue is essential to fund operations as well as to invest in capacity expansion and R&D.

Bankruptcy laws encompass diverse consequences of insolvency and near-insolvency. See the review by (Senbet and Seward 1995). Until §3.5 we consider only reorganization bankruptcy which refers to a costly restructuring and continuation of operations. In §3.5 we briefly discuss wipeout bankruptcy which refers to a permanent shut-down of operations.
We address the following questions.

1. How should the firm’s inventory replenishment decisions depend on its mixture of debt and equity?

2. How should the firm’s dividend and short-term borrowing decisions depend on the interaction of its capital structure and its inventory replenishment decisions?

3. What are the determinants of the firm’s optimal mixture of debt and equity?

Section 3.2 specifies the model predicated on reorganization bankruptcy that is explained in greater detail by (Li et al. 2003). Then §3.3 finds the optimal policy for inventory replenishment, short-term borrowing, and dividends, given a mixture of debt and equity; §3.4 considers the debt-equity tradeoff in order to maximize the value of the firm. Finally §3.5 discusses extensions and variations including wipeout bankruptcy and multiple products.

### 3.2 Model

We assume that the firm’s debt consists of fixed coupon bonds with an infinite due date, and that the firm operates as a dynamic newsvendor with liquidity constraints. At the beginning of each period in this discrete-time model, the firm decides how much money to borrow, how much to produce (of a single good), and how much of a dividend to issue. Production quantities are nonnegative but dividends are not so constrained; a negative dividend corresponds to a capital
subscription which is common in entrepreneurial firms. Production quantities are available without delay, excess demand is backordered, and successive periods’ demands are independent and identically distributed nonnegative random variables. At the end of each period, a bond coupon payment and repayment of the short-term loan are due (if a loan was obtained at the beginning of the period). If there are insufficient funds for these payments, the firm is “reorganized” and incurs a bankruptcy penalty.

**Notation**

$b_n$: amount borrowed at the beginning of period $n$ and repaid at the end of period $n$;

$z_n$: quantity produced in period $n$ and available to meet demand in period $n$;

$v_n$: dividend issued in period $n$ if $v_n > 0$; capital subscription if $v_n < 0$.

$w_n$: amount of retained earnings at the beginning of period $n$;

$x_n$: inventory level at the beginning of period $n$;

$D_n$: demand in period $n$; $D, D_1, D_2, \cdots$ are independent and identically distributed random variables with distribution function $F(\cdot)$;

$Q$: coupon payment due at the end of each period;

$g(y_n, D_n)$: sales revenue net of inventory costs; until §3.5, $g(y, d) = r \min\{y, d\} - h(y - d)^+$ where $r$ and $h$ are the selling price and unit storage cost, respectively;

$c$: unit production cost;

$\rho$: short-term loan interest rate;
\(p(w_n)\): default penalty if \(w_n < 0\); until §3.5, \(p(w) = -\theta(-w)^+\);

1 – \(\tau\): the firm’s marginal income tax rate;

\(\beta\): a single-period discount factor;

\(B\): the firm’s expected present value of dividends.

The following chronology occurs each period.

At the beginning of the period, calculate retained earnings \(w_n\) and inventory level \(x_n\);

Pay the default penalty \(p(w_n)\) if \(w_n < 0\);

Choose the levels of short-term borrowing, production, and dividend, \((b_n, z_n, v_n)\), subject to the liquidity constraint that will be specified by (3.6);

Pay the dividend and loan interest, \(v_n\) and \(\rho b_n\), and implement the production decision \(z_n\) at a cost of \(cz_n\);

Observe demand \(D_n\) and receive sales revenue net of inventory-related costs \(g(y_n, D_n)\);

Pay the principal of the loan and coupon, \(b_n\) and \(Q\), and taxes, then update accounts.

It is convenient to define

\[
s_n = w_n - v_n - \tau[cz_n + \rho b_n + p(w_n)] \tag{3.1}
\]

\[
y_n = x_n + z_n \tag{3.2}
\]

Since produced goods are immediately available to satisfy demand, \(y_n\) is the amount of goods that is available to satisfy demand in period \(n\). We interpret \(s_n\) as the internal equity at the beginning of the period after the dividend is issued,
loan interest and production costs are paid, the bankruptcy penalty (if any) is incurred, and tax credits for these expenses are received. Thus the total working capital available in period $n$ is $b_n + s_n$, i.e. the sum of short-term loan and internal equity. This interpretation of (3.1) is consistent with accrual accounting.

The backlogging assumption and this notation imply

$$x_{n+1} = y_n - D_n \quad (3.3)$$
$$w_{n+1} = s_n + \tau[g(y_n, D_n) - Q] \quad (3.4)$$

These equations balance the flows of physical goods and cash, respectively. If $s_n + \tau g(y_n, D_n) > \tau Q$, then $b_n$ is repaid at the end of the period. Otherwise, the repayment occurs in period $n + 1$ and the default penalty $p(w_{n+1})$ is levied. Since $z_n \geq 0$ corresponds to $y_n \geq x_n$ and borrowed amounts and production quantities are constrained to be nonnegative,

$$b_n \geq 0 \quad \text{and} \quad y_n \geq x_n \quad (3.5)$$

The liquidity constraint is $b_n + w_n \geq v_n + \tau[p(w_n) + cz_n + \rho b_n]$ which obliges the firm to cover the expenditures in period $n$. This inequality corresponds to

$$b_n + s_n \geq 0 \quad (3.6)$$

Let $B$ denote the present value of dividends, and let $\beta$ be the single-period discount factor ($0 \leq \beta < 1$). Using (3.1), $v_n = w_n - s_n - \tau[cz_n + \rho b_n + p(w_n)]$;
thus (3.3) and (3.4) yield

\[
B = \sum_{n=1}^{\infty} \beta^{n-1} v_n = \sum_{n=1}^{\infty} \beta^{n-1} \{w_n - s_n - \tau [cz_n + \rho b_n + p(w_n)]\}
\]

\[
= \sum_{n=1}^{\infty} \beta^{n-1} \{-(1 - \beta)(s_n + \tau cy_n) + \tau \beta g(y_n, D_n) - \tau \beta p[s_n + \tau g(y_n, D_n)]
\]

\[
- \tau Q] - \tau \rho b_n\} + \tau cx_1 + w_1 - p(w_1) - \sum_{n=1}^{\infty} \beta^{n-1} c \tau \beta D_n - \frac{\tau \beta Q}{1 - \beta}
\]

(3.7)

Define

\[
K(b, s, y, Q) = -(1 - \beta)(s + \tau cy) - \tau \rho b + \tau \beta E\{g(y, D) - p[s + \tau g(y, D) - \tau Q]\}
\]

(3.8)

So the expected present value of dividends is

\[
E(B) = w_1 - p(w_1) + \tau cx_1 - \tau \beta Q/(1 - \beta) - \tau c \beta E(D)/(1 - \beta)
\]

\[
+ \tau \beta E[\sum_{n=1}^{\infty} K(b_n, s_n, y_n, Q)]
\]

(3.9)

### 3.3 Optimal Operating Decisions

In this section we fix the size of the coupon payment, \(Q\), and find optimal values for the production quantity, short-term borrowing, and liquidity (hence the dividend).

Given \(x_n\) and \(w_n\), the decision variables \(s_n, b_n,\) and \(y_n\) correspond to the decision variables \(v_n, z_n,\) and \(b_n\). So we define a policy to be a non-anticipative rule for choosing \(s_1, b_1, y_1, s_2, b_2, y_2, \cdots\) such that, for each \(n\), \((s_n, b_n, y_n)\) is a function of the elapsed history that satisfies (3.5) and (3.6). An optimal policy maximizes \(E(B|x_1 = x, w_1 = w)\) for each \((x, w)\). The goal is to characterize an optimal policy and its relation to the long-term debt level.
In this section and the next, we assume that the initial capitalization is \( w_1 = mQ + \eta \geq 0 \). That is, the periodic coupon payment of \( Q \) results from issuing bonds in the amount \( mQ \) and the firm has obtained equity in the amount \( \eta \). So \( p(w_1) = 0 \). The model here differs from that in Li et al. (1997) due to taxes but this difference is not essential and the following result is valid here. For \( Q \) fixed, let \((s, y) = (s^*, y^*)\) maximize \( K((-s)^+, s, y, Q)\). Then \((b_n, s_n, y_n) = ((-s^*)^+, s^*, y^*)\) for all \( n = 1, 2, \ldots \) maximizes \( E(B|x_1 = x, w_1 = w) \) for each \((x, w)\) such that \( x \leq y^* \). That is, (i) \( b_n^* = (-s_n^*)^+ \) without loss of optimality, and (ii) if the initial inventory is no higher than \( y^* \), then a myopic policy is optimal. Property (i) is consistent with pecking order theory.

Therefore, we assume that \((b_n, s_n, y_n) = ((-s)^+, s, y)\) for all \( n = 1, 2, \ldots \) and the remainder of this section seeks optimal values of \( s \) and \( y \). It follows from (3.9) that

\[
E(B) = mQ + \eta + \tau cx_1 - \tau \beta Q/(1 - \beta) - \tau cE(D)/(1 - \beta) \\
+ K((-s)^+, s, y, Q)/(1 - \beta)
\]

(3.10)

The assumption of linear functions for the default penalty and the revenue net of inventory costs,

\[
p(a) = -\theta(-a)^+ \\
g(y, d) = ry - (r + h)(y - d)^+
\]

(3.11) (3.12)

implies that \( p(\cdot) \) is nonincreasing convex, \( g(\cdot, d) \) is concave, and \( K \) is a concave function on its domain. Henceforth, we evaluate derivatives and partial derivatives
from the right. The following observations are useful in evaluating the partial
derivatives of $K$:

\[
dE[g(y, D)]/dy = r - (r + h)F(y) \tag{3.13}
\]

\[
\partial E\{p[s + \tau g(y, D) - \tau Q]\}/\partial s = \theta F
\]

where $F = F[hy + Q - s/\tau]$. From (3.8) and (3.13),

\[
\partial K(b, s, y, Q)/\partial y = -\tau c(1 - \beta) + \tau \beta \{r - (r + h)F(y) - \tau \theta h F\} \tag{3.14}
\]

Setting (3.14) to zero,

\[
(r + h)F(y) + \tau \theta h F = r - c(1 - \beta)/\beta \tag{3.15}
\]

Similarly, if $s \geq 0$ so $b = 0$,

\[
\partial K(0, s, y, Q)/\partial s = -(1 - \beta) + \tau \beta E\{p'[s + \tau g(y, D) - \tau Q]\} \tag{3.16}
\]

\[
= -(1 - \beta) + \tau \beta \theta F
\]

Setting (3.16) to zero,

\[
F = (1 - \beta)/(\tau \beta \theta)(s \geq 0) \tag{3.17}
\]

Combining (3.15) and (3.17),

\[
(r + h)F(y) + h(1 - \beta)/\beta = r - c(1 - \beta)/\beta
\]

which corresponds to

\[
F(y) = \frac{\beta r - (c + h)(1 - \beta)}{\beta(r + h)} \tag{3.18}
\]
Define $F^{-1}(u) = \sup\{v : F(v) \leq u\}$ for $0 \leq u < 1$. Then (3.18) corresponds to

$$y^* = F^{-1}\left[\frac{b - (c + h)(1 - \beta)}{\beta(r + h)}\right]$$

Substitute (3.20) for $y$ in (3.15) and rearrange terms to obtain

$$s^* = \tau\left\{Q - (r + h)F^{-1}\left[\frac{1 - \beta}{\beta \theta}\right] + hy^*\right\}$$

That is,

$$s^* = \tau\left\{Q - (r + h)F^{-1}\left[\frac{1 - \beta}{\beta \theta}\right] + hF^{-1}\left[\frac{b - (c + h)(1 - \beta)}{\beta(r + h)}\right]\right\}$$

Performing the same analysis under the assumption $s < 0$ so $b = (-s)^+$ yields

$$F = (1 - \beta - \tau \rho)/(\tau \beta \theta)(s < 0) \quad (3.19)$$

$$y^* = F^{-1}\left[\frac{b - (c + h)(1 - \beta) - h \tau \rho}{\beta(r + h)}\right] \quad (3.20)$$

$$s^* = \tau\left\{Q - (r + h)F^{-1}\left[\frac{1 - \beta - \tau \rho}{\beta \theta}\right] + hy^*\right\}$$

That is,

$$s^* = \tau\left\{Q - (r + h)F^{-1}\left[\frac{1 - \beta - \tau \rho}{\beta \theta}\right] + hF^{-1}\left[\frac{b - (c + h)(1 - \beta) - h \tau \rho}{\beta(r + h)}\right]\right\}$$

Regardless of whether $s^*$ is negative or not, we reach the following conclusion.

**Proposition 3.3.1.** The optimal base-stock inventory level, $y^*$, does not depend on the amount of debt, $mQ$, and does not depend on the tax rate, $\tau$, regardless of whether or not the optimal level of retained earnings, $s^*$, is nonnegative. The optimal level of retained earnings depends on the level of debt, the tax rate, and the parameters on which the optimal base-stock inventory level depends.
From (3.1) and (3.2), the optimal dividend in period $n$ is

$$v_n = w_n + \tau cx_n - \tau p(w_n) - s^* - \tau cy^* - \tau \rho(-s^*)^+$$

which depends on all the parameters.

### 3.4 Capital Structure

In this section we find the level of debt, namely $mQ$, that maximizes the expected present value of the dividends. We use (3.10) to evaluate $\partial E(B)/\partial Q$ and the differentiability of $y^*$ and $s^*$ with respect to $Q$ established in the previous section.

Let $K^{(i)}$ denote the partial derivative of function $K((-s)^+, s, y, Q)$ with respect to its $ith$ argument. Since $y$ and $s$ are unconstrained in the optimization of $K((-s)^+, s, y, Q)$, at an optimum $K^{(3)} = 0$ and $K^{(4)} = -\tau^2 \beta \theta \mathcal{F}$. Also, $K^{(2)} = 0$ if $s^* \geq 0$, and $-K^{(1)} + K^{(2)} = 0$ if $s^* < 0$. Therefore, from (3.10) regardless of the sign of $s^*$,

$$\frac{\partial E(B)}{\partial Q} = m + (K^{(4)} - \tau \beta)/(1 - \beta) = m - \tau \beta (\tau \theta \mathcal{F} + 1)/(1 - \beta) \quad (3.21)$$

We note that the value of $\mathcal{F}$ depends on the sign of $s^*$ via (3.18) and (3.19).

If $s^* \geq 0$ then (3.18) and (3.21) imply that financing entirely with debt is optimal if and only if

$$\frac{\partial E(B)}{\partial Q} \geq 0 \iff m(1 - \beta)/\tau \geq 1 \quad (3.22)$$

and all-equity financing is optimal if $m(1 - \beta) < \tau$. In perfect complete capital markets, $\beta = (1 + I)^{-1}$ for an appropriate interest rate $I$. Also, $m$ is the present
value of a $1 perpetual annuity, i.e. \( m = (1 - \beta)^{-1} = (1 + I)/I \). So 100% debt financing is optimal because

\[
m(1 - \beta)/\tau = 1/\tau > 1 \quad (\tau < 1)
\]

The time value of internal investment funds for an entrepreneur, i.e., \( I_{\beta} = (1 + I)^{-1} \), is likely to be greater than the implicit interest rate \( i \) at which the capital market is willing to lend long-term to the entrepreneur. In order to see that \( (1 - \beta)m \) grows with the disparity, let \( I = i + u \) where \( u > 0 \). Then

\[
(1 - \beta)m = \frac{(i + u)}{i} \frac{1 + i}{1 + i + u}
\]

so \( d(1 - \beta)m/du = \frac{1 + i}{i(1 + i + u)^2} > 0 \).

Therefore, the usual entrepreneurial situation makes the left side of (3.23) larger than \( 1/\tau \) and the risk-neutral entrepreneur should prefer debt to equity.

This argument considers neither risk-sensitivity nor the loss of control that accompanies debt financing. Of course the issues addressed in this paper arise only in markets where the Miller-Modigliani theorems do not apply.

We note that when \( s^* < 0 \), the optimality condition that corresponds to (3.22) for financing entirely with debt is \( m(1 - \beta)/[\tau(1 - \tau \rho)] \geq 1 \).

**Proposition 3.4.1.** The firm’s optimal capital structure is entirely debt or equity depending only on \( m, \beta, \tau, \) and \( \rho \), and not on the production-inventory system parameters \( r, c, h, \) and \( F \).
3.5 Extensions

The closed-form expressions in §3.4 exploit (3.11) and (3.12). However, similar results can be obtained for other representations of revenue, inventory-related costs, and default penalties.

The results in sections 3.3 and 3.4 refer to a model which includes reorganization bankruptcy. However, (Li et al. 2003) show that the model with wipeout bankruptcy has the same properties as the model with reorganization bankruptcy if one replaces the discount factor $\beta$ in the latter model with $\beta F$ in the former model (where $F$ is defined in §3.4).

Other generalizations mentioned by (Li et al. 2003) can be used here too at the expense of more cumbersome notation. In particular, we could obtain results similar to Propositions 3.3.1 and 3.4.1 for a nonstationary model and similar to Proposition 3.4.1 for a model with multiple products.
Chapter 4

Echelon Base-Stock Policies Are Financially Sub-Optimal

4.1 Introduction

Operational and financial policies are typically treated separately in research literatures. Yet many firms, particularly entrepreneurial growing firms, are often short of financial resources, and their operational decisions are often constrained by working capital and credit. Both operational and financial decisions are often made by the same small group of people (or a single person!) in small firms. Therefore, operational and financial decisions often interact forcefully.

This paper focuses on these interactions. The imposition of a financial criterion and a liquidity constraint on some operationally simple models has led to
the same form of optimal policy as in the corresponding cost minimization or profit maximization model. If the operational model corresponds to a dynamic newsvendor, the financial version has an optimal policy in which both inventory and liquidity levels are determined with base-stock level policies (Li et al. (2003)). If the dynamic newsvendor has a setup cost, then the financial version has an optimal $(s,S)$ inventory policy (Sobel and Zhang (2003)).

The present study is part of an effort to understand the degree of operational complexity for which it remains true that the financial version and the cost or profit maximization version both have the same form of optimal policy for a multi-echelon dynamic newsvendor. Therefore, this paper can be regarded as a multi-echelon generalization of Li et al. (2003). Since Clark and Scarf (1960), it has been known that an echelon base-stock policy minimizes the expected present value of purchasing, transshipment, and inventory-related costs in a multi-echelon model with linear purchasing costs and backlogging of excess demand. However, we present counterexamples which show that a liquidity constraint and multiple echelons cause this type of policy to be sub-optimal for the financial objective. For simplicity of exposition, the model and counterexamples contain two echelons. Nevertheless, the conclusions are valid for models with more than two echelons.

We briefly mention some recent contributions to the growing literature on the coordination of operational and financial decisions. There are further references in the cited contributions. Archibald et al. (2002) assume that a start-up firm faces
a discretely distributed demand and strives to maximize its long-term survival probability. One conclusion is that a start-up firm pursuing this objective should be more cautious in its component purchasing strategy than a well-established firm. Furthermore, the purchasing quantity is not a monotone function of the available capital.

Buzacott and Zhang (2004) study the relationship between a retailer’s operational policy and a bank’s lending policy. In their game model, the bank is a Stackelberg leader. Anticipating the retailer’s response (ordering quantity and loan size), the bank determines the interest rate and loan limit. The retailer is assumed to maximize its expected revenues, while the bank maximizes its expected profits.

Babich and Sobel (2004) study capacity expansion decisions that optimize the expected discounted proceeds from an initial public offering of stock.

Xu and Birge (2004b) address issues of capital structure in production-inventory models and conclude that (i) the firm’s borrowing capacity is limited if bankruptcy cost is considered, and (ii) not coordinating operational and financial decisions may lead to suboptimal results. Xu and Birge (2004c) address agency issues in a model that is similar to Xu and Birge (2004b). Xu and Birge (2004a) study a discrete-time, finite-horizon, partial equilibrium model in which the firm’s objective is to maximize the expected discounted value of net cash flow to the shareholders subject to period-by-period constraints that model resource evolution.
The firm’s financial resources are internal savings, current cash flow, single period debt, and external equity. The firm’s decisions in each period are to invest or to default, and how to finance production. Hu and Sobel (2004) study the effects of capital structure on a firm’s operational and financial decisions.

The rest of this paper is organized as follows: §4.2 states assumptions and presents the model; §4.3 analyzes the model and presents counterexamples; and §4.4 summarizes the results.

## 4.2 Model

We study a discrete-time infinite-horizon model where the firm decides how much money to borrow, how much of a dividend to issue, how much raw material to order, and how many finished goods to produce. Production quantities are non-negative, but dividends are not so constrained; a negative dividend corresponds to a capital subscription which is common in entrepreneurial firms. Then demand occurs, revenue is received, inventory-related costs are incurred, and repayment of the short-term loan is due. The firm repays the loan if it can, but reorganization bankruptcy occurs if the firm defaults. Upon bankruptcy, the firm is reorganized, incurs a reorganization cost, and resumes operation. An alternative treatment of bankruptcy, wipeout, would cause all operations to cease after the first default. It is clear from Li et al. (2003) that our conclusions would remain valid if we were to employ wipeout rather than reorganization bankruptcy in the model.
The following chronology occurs each period.

1) Pay the default penalty if retained earnings are negative,

2) Choose levels of borrowing, production at each stage, and dividends subject to the liquidity constraint,

3) Pay dividends and loan interest and implement the production plan,

4) Realize demand, receive sales revenues, and repay the short-term loan,

5) Update accounts by calculating the new levels of retained earnings and inventory at each stage.

Notation

\( i = 1, 2 \): echelon; goods move from stage \( i = 2 \) to stage \( i = 1 \);

\( n = 1, 2, \cdots \): time period;

\( b_n \): amount borrowed at the beginning of period \( n \) and due to be repaid at the end of period \( n \);

\( z_{in} \): quantity produced/purchased at stage \( i \) in period \( n \);

\( v_n \): dividend or capital subscription in period \( n \) according to \( v_n \geq 0 \) or \( v_n < 0 \);

\( w_n \): retained earnings at the beginning of period \( n \);

\( x_{in} \): inventory level at stage \( i \) at the beginning of period \( n \);

\( D_n \): demand in period \( n \); \( D, D_1, D_2, \cdots \) are nonnegative, independent, and identically distributed random variables;

\( c_i \): unit production cost at stage \( i \);

\( h_i \): unit holding cost at stage \( i \);
\( \rho \): single-period interest rate for short-term loans;

\( p(w_n) \): bankruptcy penalty if \( w_n < 0 \)

\( g(y_{1n}, D_n) \): sales revenue net of finished goods inventory-related costs where \( y_{1n} = x_{1n} + z_{1n} \) is the total supply of finished goods in period \( n \); for example,

\[
g(y_1, d) = r \min\{y_1, d\} - h_1(y_1 - d)^+ \quad \text{where} \quad r \text{ denotes unit sales price.}
\]

We assume that there is a one-period production lead time at the second stage but no lead time at the end stage. It is convenient to define

\[
y_{2n} = x_{2n} + z_{2n} - z_{1n} \quad (4.1)
\]

\[
y_{1n} = x_{1n} + z_{1n} \quad (4.2)
\]

\[
m_n = w_n - p(w_n) - v_n - (c_1 z_{1n} + c_2 z_{2n}) - \rho b_n \quad (4.3)
\]

Equations (4.1) and (4.2) reflect the lead time assumption. We interpret \( m_n \) as the internal equity at the beginning of the period after the dividend is issued, loan interest and production costs are paid, the bankruptcy penalty (if any) is incurred and before the loan is made and revenue and inventory costs are realized. Thus the total working capital available in period \( n \) is \( b_n + m_n \), i.e., the sum of short-term loan and internal equity. This interpretation of (4.3) is consistent with accrual accounting.

The backlogging assumption and one-period delay in stage two and this notation imply
\[ x_{1,n+1} = y_{1n} - D_n \]
\[ x_{2,n+1} = y_{2n} \quad (4.4) \]
\[ w_{n+1} = m_n + g(y_{1n}, D_n) - h_2 y_{2n} \]

These equations balance the flows of physical goods and cash, respectively. At the end of the period, the loan is repaid if \( m_n + g(y_{1n}, D_n) - h_2 y_{2n} \geq 0 \). Otherwise, the repayment occurs in period \( n + 1 \) and the default penalty \( p(w_{n+1}) \) is levied. Since \( z_{in} \geq 0 \) corresponds to \( y_{in} \geq x_{in} (i = 1, 2) \) and borrowed amount and production quantities are constrained to be nonnegative. Furthermore, stage one does not order more than stage two’s on-hand inventory. So

\[ b_n \geq 0 \quad x_{2n} \geq y_{1n} \geq x_{1n} \quad y_{2n} \geq x_{2n} \quad (4.5) \]

The liquidity constraint is \( b_n + w_n \geq v_n + p(w_n) + v_n + (c_1 z_{1n} + c_2 z_{2n}) + \rho b_n \) which obliges the firm to cover the expenditures in period \( n \). This inequality corresponds to

\[ b_n + m_n \geq 0 \quad (4.6) \]

Define the following echelon variables:

\[ s_1 = x_1 \quad s_2 = x_1 + x_2 \quad a_1 = y_1 \quad a_2 = y_1 + y_2 \quad (4.7) \]

So (4.4), (4.5), and (4.6) become
\[ s_{1,n+1} = a_{1,n} - D_n \]
\[ s_{2,n+1} = a_{2,n} - D_n \] \hspace{1cm} (4.8)

\[ w_{n+1} = m_n + g(a_{1n}, D_n) - h_2(a_{2n} - a_{1n}) \]

\[ s_1 \leq a_1 \leq s_2 \leq a_2 \quad 0 \leq b \quad 0 \leq b + m \] \hspace{1cm} (4.9)

Let \( B \) denote the present value of the time stream of dividends, \( \beta \) be the single-period discount factor \((0 \leq \beta < 1)\). Using (4.3), \( v_n = w_n - p(w_n) - m_n - c_1 z_1 - c_2 z_2 - \rho b_n \); thus (4.4) yield

\[
B = \sum_{n=1}^{\infty} \beta^{n-1} v_n = \sum_{n=1}^{\infty} \beta^{n-1} \left\{ w_n - p(w_n) - \sum_{i=1}^{2} c_i z_{i,n} - \rho b_n - m_n \right\}
= \sum_{n=1}^{\infty} \beta^{n-1} \left\{ -(1 - \beta)[m_n + c_2 y_{2n} + (c_1 + c_2) y_{1n}] + \beta[g(y_{1n}, D_n) - h_2 y_{2n}] - h_2 y_{2n} - \beta p[m_n + g(y_{1n}, D_n) - h_2 y_{2n}] \right\}
= \sum_{n=1}^{\infty} \beta^{n-1} D_n (c_1 + c_2) - w_1 + p(w_1) - (c_1 + c_2) x_{11} - c_2 x_{21} \] \hspace{1cm} (4.10)

Define

\[
\Gamma(m, b, a_1, a_2) = [\beta h_2 - c_1 (1 - \beta)] a_1 + \beta E[g(a_1, d)] - [c_2 (1 - \beta) + \beta h_2] a_2 - \beta Ep[m + g(a_1, d) - h_2 (a_2 - a_1)] - \rho b - (1 - \beta) m \] \hspace{1cm} (4.11)

So the expected present value of dividends is

\[
E(B) = E \sum_{n=1}^{\infty} \beta^{n-1} \Gamma(m, b, a_1, a_2) \]
\[ - E(D_1)(c_1 + c_2)/(1 - \beta) + w_1 - p(w_1) + c_1 s_{11} + c_2 s_{21} \] \hspace{1cm} (4.12)
4.3 Analysis

In this section we try to find the firm’s optimal policies governing its physical goods inventory and capital level. Given $s_n = (s_{1n}, s_{2n})$ and $w_n$, the decision variables $z_n = (z_{1n}, z_{2n})$, $v_n$, and $b_n$ correspond to $a_n = (a_{1n}, a_{2n})$, $m_n$, and $b_n$. So we define a policy to be a non-anticipative rule that chooses $a_1$, $m_1$, $b_1$, $a_2$, $m_2$, $b_2$, $\cdots$ such that, for each $n$, $(a_n, m_n, b_n)$ is a function of the elapsed history that satisfies (4.9). An optimal policy maximizes $E(B|s_1 = x, w_1 = w)$ for each $(s, w)$.

The goal is to characterize an optimal policy.

The term in the last square bracket of (4.10) does not depend on the decision variables, so a policy maximizes $E(B|H_1)$ if and only if it maximizes $E(B|H_1) + c_1s_{11} + c_2s_{21} - E(D_1)(c_1 + c_2)/(1 - \beta)$. Therefore, the firm’s problem is

$$\max_{b_n, m_n, a_{1n}, a_{2n}} E\left[ \sum_{n=1}^{\infty} \beta^{n-1} \Gamma(b_n, m_n, a_{1n}, a_{2n}) \right] \quad (4.13)$$

where the maximum is over the set of feasible policies that satisfy (4.9).

The dimensionality of the model can be reduced using an argument in Li et al. (2003). Although a straightforward dynamic program formulation of the the problem that corresponds to (4.9) and (4.13) appears to have three state variables $(s_{1n}, s_{2n}, w_n)$ and four decision variables $(b_n, m_n, a_{1n}, a_{2n})$, it can be reduced to the following dynamic program with two state variables $(s_{1n}, s_{2n})$ and three decision variables $(m_n, a_{1n}, a_{2n})$. The firm’s decision problem corresponds to the following dynamic program with $\Phi_0(\cdot, \cdot) \equiv 0$, $s_{1n} \leq s_{2n}$, and $n = 1, 2, \cdots$: 
\[ \Phi_n(s_1, s_2) = \sup_{b,m,a_1,a_2} \{ J_n(b,m,a_1,a_2) : 0 \leq b, 0 \leq b + m, s_1 \leq a_1 \leq s_2 \} \]

\[ J_n(b,m,a_1,a_2) = \Gamma(m,b,a_1,a_2) + \beta E[\Phi_{n-1}(a_1 - D, a_2 - D)] \]

The following proposition justifies the elimination of \( b_n \) as a decision variable.

**Proposition 4.3.1.** There is no loss of optimality in letting \( b_n(s_1, s_2) = [-m_n(s_1, s_2)]^+ \) for all \( n = 1, 2, \cdots \) and \( (s_1, s_2) \in \mathbb{R}^2 \).

**Proof.**

\begin{align*}
\Phi_n(s_1, s_2) &= \left\{ \begin{array}{l}
[\beta h_2 - c_1(1 - \beta)]a_1 + \beta E[g(a_1, D)] \\
- [\beta h_2 - c_2(1 - \beta)]a_2 + \beta E[\Phi_{n+1}(a_1 - D, a_2 - D)] \\
+ \sup_m \{ - (1 - \beta)m - Ep[m + g(a_1, d) - h_2(a_2 - a_1)] \} \\
+ \sup_b \{ - \rho(b) : b \geq 0, b \geq -m \} : m \in \mathbb{R} \} : s_1 \leq a_1 \leq s_2 \leq a_2 \right\} \\
&= \sup_{a_1,a_2} \left\{ \left[ \begin{array}{l}
[\beta h_2 - c_1(1 - \beta)]a_1 + \beta E[g(a_1, D)] \\
- [\beta h_2 - c_2(1 - \beta)]a_2 + \beta E[\Phi_{n+1}(a_1 - D, a_2 - D)] \\
+ \sup_m \{ - (1 - \beta)m - Ep[m + g(a_1, D)] \} \\
- h_2(a_2 - a_1) - \rho(-m)^+ : m \in \mathbb{R} \} : s_1 \leq a_1 \leq s_2 \leq a_2 \right\}
\end{align*}

(4.15)

This result is consistent with finance’s “pecking order” which advises a firm to resort to internal capital before borrowing externally. So “pecking order” remains valid for a multi-stage manufacturing firm.

A straightforward induction establishes the next result.
Proposition 4.3.2. If \( p(\cdot) \) is nonincreasing convex on \( R \) and \( g(\cdot,d) \) is concave on \( R \) for each \( d \geq 0 \), then the value function \( \Phi_n(\cdot, \cdot) \) is concave on \( R^2 \) and \( J_n(\cdot, \cdot, \cdot, \cdot) \) is concave on \( R^4 \).

If the inventory level vector is \( s = (s_1, s_2) \) and the level of retained earnings is \( w \), the dividend in period \( n \) is

\[
v_n(s, w) = w - p(w) + c_1 s_1 + c_2 s_2 - c_1 a_1 - c_2 a_2 - \rho b_n - m_n \tag{4.16}
\]

The intuitive consequence is that the amount of the dividend is a nondecreasing function of retained earnings if \( p(\cdot) \) is nonincreasing. The conclusion is summarized as follows.

Proposition 4.3.3. If \( p(\cdot) \) is nonincreasing on \( R \), then \( v_n(s, \cdot) \) is nondecreasing on \( R \) for each \( s \in R^2 \).

4.3.1 Counterexamples

We now address the fundamental question: Is there an optimal policy in which production decisions are consistent with an echelon base-stock policy? That is, does the financial criterion and the liquidity constraint alter the type of production policy that would maximize (minimize) the expected present value of profits (costs)? The primary differences between this model and the model in Clark and Scarf (1960) are the financial constraints, \( b \geq 0 \) and \( b + m \geq 0 \), and the inseparability of the one-period reward function \( \Gamma(m, b, a_1, a_2) \) in \( a_1 \) and \( a_2 \). The following
counterexamples show that echelon base-stock policies are generally not optimal for (4.14).

Let \( c_1 = c_2 = 0 \), \( s = (s_1, s_2) \) and \( a = (a_1, a_2) \), and assume that the default penalty and revenue net of inventory are piece-wise linear:

\[
p(x) = -\theta(-x)^+
\]

\[
g(a, d) = ra - (r + h_1)(a - d)^+
\]

**Counterexample 1**

Suppose that demand \( D \) takes the values \((0, 1, 2)\) with respective probabilities \((0.7, 0.2, 0.1)\). Let \( \beta = 0.9, \rho = 0.05, r = 1, \theta = 0.2, h_1 = 0.15 \), and \( h_2 = 0.10 \).

Table 4.2 displays part of the solution of (4.14) (obtained with policy iteration) when only integer values of \( s_1 \in [-3, 2] \), \( s_2 \in [-1, 4] \), \( a_1 \in [-1, 2] \) and \( a_2 \in [0, 4] \).

It is apparent in Table 4.2’s tabulation of optimal actions and cash levels for each state that a base stock policy is not optimal for stage one.

<table>
<thead>
<tr>
<th>s</th>
<th>(-3, -1)</th>
<th>(-2, -1)</th>
<th>(-2, 0)</th>
<th>(-2, 1)</th>
<th>(-2, 2)</th>
<th>(-2, 3)</th>
<th>(-2, 4)</th>
<th>(-1, -1)</th>
<th>(-1, 0)</th>
<th>(-1, 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>(-1, 2)</td>
<td>(-1, 2)</td>
<td>(0, 2)</td>
<td>(0, 2)</td>
<td>(1, 2)</td>
<td>(1, 3)</td>
<td>(2, 4)</td>
<td>(-1, 2)</td>
<td>(0, 2)</td>
<td>(0, 2)</td>
</tr>
<tr>
<td>(m^*)</td>
<td>1.3</td>
<td>1.3</td>
<td>0.2</td>
<td>0.2</td>
<td>-0.095</td>
<td>0.005</td>
<td>0.04</td>
<td>1.3</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>s</td>
<td>(-1, 2)</td>
<td>(-1, 3)</td>
<td>(-1, 4)</td>
<td>(0, 0)</td>
<td>(0, 1)</td>
<td>(0, 2)</td>
<td>(0, 3)</td>
<td>(0, 4)</td>
<td>(1, 1)</td>
<td>(1, 2)</td>
</tr>
<tr>
<td>a</td>
<td>(0, 2)</td>
<td>(1, 3)</td>
<td>(2, 4)</td>
<td>(0, 2)</td>
<td>(0, 2)</td>
<td>(1, 3)</td>
<td>(2, 4)</td>
<td>(1, 2)</td>
<td>(1, 2)</td>
<td>(1, 3)</td>
</tr>
<tr>
<td>(m^*)</td>
<td>0.2</td>
<td>0.005</td>
<td>0.04</td>
<td>0.2</td>
<td>0.2</td>
<td>0.005</td>
<td>0.04</td>
<td>-0.095</td>
<td>-0.095</td>
<td>0.005</td>
</tr>
<tr>
<td>s</td>
<td>(1, 3)</td>
<td>(1, 4)</td>
<td>(2, 2)</td>
<td>(2, 3)</td>
<td>(2, 4)</td>
<td>(3, 0)</td>
<td>(3, 1)</td>
<td>(-3, 1)</td>
<td>(-3, 2)</td>
<td>(-3, 3)</td>
</tr>
<tr>
<td>a</td>
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<td>(2, 4)</td>
<td>(2, 3)</td>
<td>(2, 3)</td>
<td>(2, 4)</td>
<td>(0, 2)</td>
<td>(0, 2)</td>
<td>(0, 2)</td>
<td>(0, 2)</td>
<td>(1, 3)</td>
</tr>
<tr>
<td>(m^*)</td>
<td>0.005</td>
<td>0.04</td>
<td>-0.06</td>
<td>-0.06</td>
<td>0.04</td>
<td>0.2</td>
<td>0.2</td>
<td>0.005</td>
<td>0.005</td>
<td>0.105</td>
</tr>
</tbody>
</table>

Table 4.2: An echelon base-stock policy is not optimal for stage one
Counterexample 2

Let demand, $D$, take the values $(0, 1)$ with respective probabilities $(0.7, 0.3)$. Let
\[ \beta = 0.9, \rho = 0.14, r = 2, \theta = 0.25, h_1 = 0.15, \text{ and } h_2 = 0.1. \]
Consider only integer values of $s_1 \in [-1, 1]$, $s_2 \in [0, 2]$, $a_1 \in [0, 1]$, and $a_2 \in [1, 2]$. Table 2 lists the optimal actions at each state. It is apparent that a base-stock policy is optimal neither for stage two production nor for the cash reserves.

<table>
<thead>
<tr>
<th>s</th>
<th>(-1, 0)</th>
<th>(-1, 1)</th>
<th>(-1, 2)</th>
<th>(0, 0)</th>
<th>(0, 1)</th>
<th>(0, 2)</th>
<th>(1, 1)</th>
<th>(1, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>(0, 2)</td>
<td>(1, 1)</td>
<td>(1, 2)</td>
<td>(0, 2)</td>
<td>(1, 1)</td>
<td>(1, 2)</td>
<td>(1, 1)</td>
<td>(1, 2)</td>
</tr>
<tr>
<td>$m^*$</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.3: An echelon base-stock policy is not optimal for stage two

4.4 Conclusions

The counterexamples show that base-stock policies are generally sub-optimal both for physical goods production and for the level of retained earnings. The firm should adjust its cash reserves and its inventory levels at various stages depending on recent realization of demand. We conclude that studying production and inventory decisions in isolation from the financial aspects of the firm might yield financially sub-optimal policies.
Appendix A

Hub-and-Spoke Networks

Airline one’s problem at the third stage is

\[ \pi_1 = \max_{q_{l1}, q_{c1}} 2[M_l - (q_{l1} + q_{l2})]q_{l1} + [M_c - (q_{c1} + q_{c2})]q_{c1} \quad (A.1) \]

\[ s.t. \quad q_{l1} + q_{c1} \leq K_1 \quad 0 \leq q_{l1} \quad 0 \leq q_{c1} \]

This is a concave nonlinear maximization problem which Karush-Kukh-Tucker (KKT) condition applies. Airline one’s KKT conditions are
\[2(M_l - 2q_{l1} - q_{l2}) - u_1 + v_{l1} = 0\]
\[v_{l1}q_{l1} = 0\]
\[M_c - 2q_{c1} - q_{c2} - u_1 + v_{c1} = 0\]
\[v_{c1}q_{c1} = 0\]
\[u_1(q_{c1} + q_{l1} - K_1) = 0\]
\[q_{c1} + q_{l1} \leq K_1\]
\[u_1, v_{l1}, v_{c1}, q_{c1}, q_{l1} \geq 0\]

where \(v_{c1}\) and \(v_{l1}\) are slack variables, and \(u_1\) is a Lagrange multiplier. There are six areas depending on the binding conditions of the capacities. The values of the slack variables or multipliers are specified only when they are positive because they define the various boundaries of the areas in Figure 4. The quantity game is solved for each area by taking appropriate values of the Lagrange multipliers and the slack variables from (A.2) and the analogues of airline two. *Neither airline is capacity-constrained* (\(\Delta_1\))

\[q_{y1} = q_{y2} = A_y/3 \quad y = l, c\]

*Airline one is capacity-constrained; airline two is not* (\(\Delta_2\))

\[q_{li} = (2M_l-M_c-3K_i)/9 (i = 1, 2) \quad q_{ci} = (4M_c+M_l-3K_i)/9 \quad u_1 = (M_l+M_c)/3-K_1 > 0\]

*Both airlines are capacity-constrained* (\(\Delta_3\))
\[ q_{li} = \frac{(2M_l - M_c + 3K_i)}{9} \quad q_{ci} = \frac{(M_c - 2M_l + 6K_j)}{9} \]

\[ \mu_i = 2\left(M_c + M_l - 2K_i - K_j\right)/3 > 0 \quad (i \neq j; i, j = 1, 2) \]

**Airline one is capacity-constrained and serves only the local markets (\(\Delta_4\))**

\[ q_{c1} = 0 \quad q_{c2} = M_c/2 \quad q_{l1} = K_1 \quad q_{l2} = (M_l - K_1)/2 \]

\[ u_1 = M_l - 3K_1 > 0 \quad v_{c1} = M_l - M_c/2 - 3K_1 > 0 \]

**Both airlines are capacity-constrained; airline one serves only the local markets (\(\Delta_5\))**

\[ q_{l1} = K_1 \quad q_{c1} = 0 \quad q_{l2} = \frac{(2M_l - M_c - 2K_1 + 2K_2)}{6} \]

\[ q_{c2} = \frac{(M_c - 2M_l + 2K_1 + 4K_2)}{6} \quad v_{c1} = \frac{(2M_l - M_c - 6K_1)}{3} > 0 \]

\[ u_1 = \frac{(M_c + 4M_l - 10K_1 - 2K_2)}{3} > 0 \quad u_2 = \frac{2(M_c + M_l - K_1 - 2K_2)}{3} > 0 \]

**Both airlines are capacity-constrained and serve only the local markets (\(\Delta_6\))**

\[ q_{li} = K_i \quad q_{ci} = 0 \quad u_i = 2(M_l - 2K_i - K_j) > 0 \]

\[ v_{ci} = 2M_l - M_c - 4K_i - 2K_j > 0 \quad (i, j = 1, 2; i \neq j) \]

**Proof of Proposition 2.3.1**

**Proof.** It can be verified that the airlines’ payoff functions \(\Pi_i\) \((i = 1, 2)\) are concave in their own decision variable regardless of their rival’s decision. It follows from
Debreu (1952) that the capacity game has a pure strategy NEP. From (2.3), the first-order condition (FOC) of $\Pi_i$ with respect to $K_i$ in the capacity game is

$$E \frac{\partial \pi_i}{\partial K_i} = 2c_h \quad (A.3)$$

Using Leibnitz’s rule to derive $\partial \pi_i / \partial K_i$ for each area in Figure 2.4, for airline one, (A.3) becomes

$$\frac{1}{3} \iint_{\Delta_2} (x_l + x_c - 3K_1)dF_l dF_c + \frac{2}{3} \iint_{\Delta_3} (x_l + x_c - 2K_1 - K_2)dF_l dF_c$$
$$+ \iint_{\Delta_4} (x_l - 3K_1)dF_l dF_c + \frac{1}{3} \iint_{\Delta_5} (x_c + 4x_l - 10K_1 - 2K_2)dF_l dF_c \quad (A.4)$$
$$+ 2 \iint_{\Delta_6} (x_l - 2K_1 - K_2)dF_l dF_c = c_h$$

for airline two, (A.3) becomes

$$\frac{2}{3} \iint_{\Delta_{3,5}} (x_l + x_c - K_1 - 2K_2)dF_l dF_c + 2 \iint_{\Delta_6} (x_l - K_1 - 2K_2)dF_l dF_c = c_h \quad (A.5)$$

When the airlines have the same capacities, areas 4, 5, and 6 disappear, and (A.4) and (A.5) can be simplified to (2.6).

The following condition is sufficient for uniqueness via contraction (Vives (1999)).

$$|\partial^2 \Pi_j / \partial K_i \partial K_j| < |\partial^2 \Pi_j / \partial (K_i)^2| \quad (i, j = 1, 2; i \neq j) \quad (A.6)$$

The second-order derivatives are

$$\left| \frac{\partial^2 \Pi_2}{\partial (K_2)^2} \right| = \frac{4}{3} \iint_{\Delta_3} dF_l dF_c + 4 \iint_{\Delta_6} dF_l dF_c$$
$$\left| \frac{\partial^2 \Pi_2}{\partial K_1 \partial K_2} \right| = \frac{2}{3} \iint_{\Delta_3} dF_l dF_c + 2 \iint_{\Delta_6} dF_l dF_c$$
So (A.6) holds.

Proof of Proposition 2.4.3

Proof. Under Assumptions 1 and 2, area 3 takes the whole space in Figure 2.4, so \( q_l = (2M_l - M_c + 3K^h)/9 \), \( q_c = (M_c - 2M_l + 6K^h)/9 \), and (2.6) becomes

\[
c_h = 2(\mu_l + \mu_c - 3K^h)/3
\]

(A.7)

So \( K^h = (\mu_l + \mu_c)/3 - c_h/2 \), \( p_c^h = M_c - 2q_c \), \( p_l^h = M_l - 2q_l \) and

\[
\Pi^h = 2p_l^h q_l + p_c^h q_c - 2c_h K^h
\]

\[
= \frac{1}{81} \iint_{\Delta} \left[ (7x_c + 4x_l - 12K^h)(x_c - 2x_l + 6K^h) \\
+ 2(5x_l + 2x_c - 6K^h)(2x_l - x_c + 3K^h) \right] dF_l dF_c - 2c_h K^h
\]

(A.8)

The result follows after some algebraic manipulations.
Appendix B

Point-to-Point Networks

Airline one’s problem is

\[
\max_{q_{l1},q_{c1}} \pi_1 = 2[M_l - (q_{l1} + q_{l2})q_{l1} + [M_c - (q_{c1} + q_{c2})]q_{c1} \quad (B.1)
\]

\[
\text{s.t.} \quad 0 \leq q_{c1} \leq K_{c1} \quad 0 \leq q_{l1} \leq K_{l1}
\]

The KKT conditions of (B.1) are
The quantity game is solved for each area by taking appropriate values of the Lagrange multipliers and the slack variables from (A.2) and its analogues for airline two.

**Neither airline is capacity-constrained (Δ₁)**

\[ q_{yi} = A_y/3 \]

**Airline one is capacity-constrained in the local markets (Δ₄)**

\[ q_{l1} = K_{l1} \quad q_{c1} = q_{c2} = M_c/3 \quad q_{l2} = (M_l - K_{l1})/2 \quad u_{l1} = M_l - 3K_{l1} > 0 \]

**Airline one is capacity-constrained in the connecting market (Δ₂)**

\[ q_{c1} = K_{c1} \quad q_{c2} = (M_c - K_{c1})/2 \quad q_{l1} = q_{l2} = M_l/3 \quad u_{c1} = (M_c - 3K_{c1})/2 > 0 \]
Airline one is capacity-constrained in all markets ($\Delta_3$)

\[ q_{y1} = K_{y1} \quad q_{y2} = (A_y - K_{y1})/2 \quad u_{y1} = (A_y - 3K_{y1})/2 > 0 \]

Airline one is capacity-constrained in all markets and airline two is capacity-constrained in local markets ($\Delta_6$)

\[ q_{l1} = K_{l1} \quad q_{c1} = K_{c1} \quad q_{l2} = K_{l2} \quad u_{l1} = M_l - 2K_{l1} - K_{l2} > 0 \]
\[ u_{c1} = (M_c - 3K_{c1})/2 > 0 \quad u_{l2} = M_l - 2K_{l2} - K_{l1} > 0 \]

Airline one is capacity-constrained in all markets and airline two is capacity-constrained in the connecting market ($\Delta_8$)

\[ u_{c2} = (M_c - 3K_{c1})/2 > 0 \quad u_{l1} = (M_l - 3K_{l1})/2 > 0 \]
\[ u_{c1} = M_c - 2K_{c1} - K_{c2} \quad q_{l2} = (M_l - K_{l1})/2 \]

Both airlines are capacity-constrained in all markets ($\Delta_9$)

\[ q_{yi} = K_{yi}^p \quad u_{yi} = A_y - 2K_{yi} - K_{yj} > 0 \quad i, j = 1, 2; i \neq j; y = l, c \]

Both airlines are capacity-constrained in the local markets ($\Delta_5$)

\[ q_{l1} = K_{l1}^p \quad q_{l2} = K_{l2} \quad q_{c1} = q_{c2} = M_c/3 \quad u_{li} = M_l - 2K_{li} - K_{lj} > 0 \]

Both airlines are capacity-constrained in the connecting market ($\Delta_7$)

\[ q_{c1} = K_{c1} \quad q_{c2} = K_{c2} \quad q_{l1} = q_{l2} = M_c/3 \quad u_{ci} = A_y - 2K_{ci} - K_{cj} > 0 \]

Proof of Proposition 2.3.2
Proof. Following the same reasoning as in the proof of Proposition 1, for airline one, \( \partial \Pi_1 / \partial K_{c1} = \partial E \pi_1 / \partial K_{c1} - c_p = 0 \) which translates into

\[
c_p = \frac{1}{2} \int \int_{\Delta_{2,3,6}} (x_c - 3K_{c1}) dF_c dF_c + \int \int_{\Delta_{7,8,9}} (x_c - 2K_{c1} - K_{c2}) dF_c dF_l \tag{B.3}
\]

Similarly, \( \partial \Pi_1 / \partial K_{l1} = \partial E \pi_1 / \partial K_{l1} - 2c_p = 0 \) which translates into

\[
c_p = \int \int_{\Delta_{4}} (x_l - K_{l1}) dF_l dF_c + \frac{1}{2} \int \int_{\Delta_{3,8}} (x_l - 3K_{l1}) dF_l dF_c + \int \int_{\Delta_{5,6,9}} (x_l - 2K_{l1} - K_{l2}) dF_l dF_c \tag{B.4}
\]

Similarly for airline two, the FOC of \( \Pi_2 \) w.r.t \( K_{c2} \) and \( K_{l2} \) yields, respectively.

\[
\frac{1}{2} \int \int_{\Delta_{7}} (x_c - 3K_{c1}) dF_c dF_c + \int \int_{\Delta_{8,9}} (x_c - 2K_{c2} - K_{c1}) dF_c dF_l = c_p \tag{B.5}
\]

\[
\int \int_{\Delta_{5,6,9}} (x_l - 2K_{l2} - K_{l1}) dF_l dF_c = c_p \tag{B.6}
\]

When the airlines have the same capacities, areas 2, 3, 4, 6 and 8 disappear, Figure 2.5(b) shows the situation. As a result, (B.5) and (B.6) are simplified to (2.7) and (2.8), respectively. Similar to the proof of Proposition 1, a sufficient condition for uniqueness is

\[
\left| \frac{\partial^2 \Pi_i}{\partial (K_{yi})^2} \right| > \left| \frac{\partial^2 \Pi_i}{\partial K_{yi} \partial K_{yj}} \right| \quad (y = c, l; i, j = 1, 2; i \neq j) \tag{B.7}
\]

The second-order derivatives are

\[
\left| \frac{\partial^2 \Pi_1}{\partial (K_{l1})^2} \right| = 2 \int \int_{\Delta_{5,9}} dF_l dF_c \tag{B.8}
\]

\[
\left| \frac{\partial^2 \Pi_1}{\partial K_{l1} \partial K_{l2}} \right| = \int \int_{\Delta_{5,9}} dF_l dF_c \tag{B.9}
\]
So (B.7) holds.

**Proof of Proposition 2.4.4**

*Proof.* Under Assumptions 1 and 2, area 9 takes the whole space in Figure 2.5, so equations (2.7) and (2.8) translate into $\mu_c - 3K_c^p = c_p$ and $\mu_l - 3K_l^p = c_p$ respectively. Thus $K_y^p = (\mu_y - c_p)/3$ for $y = l, c$. So $p_y^p = (A_y + 2c_p)/3$

\[
\Pi^p = 2p_l^p K_l^p + p_c^p K_c^p - c_p(2K_l^p + K_c^p)
\]

\[
= 2(\mu_l - 2K_l^p)K_l^p + (\mu_c - 2K_c^p)K_c^p - c_p(2K_l^p + K_c^p) \quad \text{(B.10)}
\]

\[
= (2\mu_l^2 + \mu_c^2)/9 - 2c_p(2\mu_l + \mu_c)/9 + c_p^2/3
\]

\[\square\]
Appendix C

Different Networks

Let airline one use a hub-and-spoke network and the other airline a point-to-point network. Airline one’s problem is defined by (A.1) and the corresponding KKT conditions are defined by (A.2). Airline two’s problem is the analogues of (B.1) and the corresponding KKT conditions are the analogues of (B.2). The quantity game is solved for each area by taking appropriate values of the Lagrange multipliers and slack variables.

Neither airline is capacity-constrained ($\Delta_1$)

\[ q_{l1} = q_{l2} = M_l/3 \quad q_{c1} = q_{c2} = M_c/3 \]

Airline two is capacity-constrained in the local markets ($\Delta_2$)

\[ q_{l1} = (M_l - K_{l2})/2 \quad q_{l2} = K_{l2} \quad q_{c1} = q_{c2} = M_c/3 \quad \mu_{l2} = M_l - 3K_{l2} > 0 \]

Airline two is capacity-constrained in the connecting market ($\Delta_3$)

\[ q_{c1} = (M_c - K_{c2})/2 \quad q_{c2} = K_{c2} \quad q_{l1} = q_{l2} = M_c/3 \quad u_{c2} = (M_c - 3K_{c2})/2 > 0 \]
Only airline one is capacity-constrained \((\Delta_4)\)

\[
q_{c1} = (M_c - 2M_l + 6K_1)/9 \quad q_{c2} = (4M_c + M_l - 3K_1)/9
\]

\[
q_{l1} = (2M_l - M_c + 3K_1)/9 \quad q_{l2} = (M_c + 7M_l - 3K_1)/18
\]

\[
u_1 = (M_c + M_l - 3K_1)/3 > 0
\]

Airline one is capacity-constrained; airline two is capacity-constrained in the local markets \((\Delta_5)\)

\[
q_{l1} = (4M_l - M_c + 3K_1 - 4K_{l2})/11 \quad q^m_{l2} = K_{l2}
\]

\[
q^m_{c1} = (M_c - 4M_l + 8K_1 + 4K_{l2})/11 \quad q_{c2} = (5M_c + 2M_l + 4K_1 + 2K_{l2})/11
\]

\[
u_1 = 2(2M_c + 3M_l - 6K_1 - 3K_{l2})/11 > 0 \quad u_{l2} = 2(M_c + 7M_l - 3K_1 - 18K_{l2})/11 > 0
\]

Airline one is capacity-constrained; airline two is capacity-constrained in the connecting market \((\Delta_6)\)

\[
q_{c1} = (M_c - M_l + 3K_1 - K_{c2})/5 \quad q_{c2} = K_{c2}
\]

\[
q_{l1} = (M_l - M_c + 2K_1 + K_{c2})/5 \quad q_{l2} = (M_c + 4M_l - 2K_1 - K_{c2})/10
\]

\[
u_1 = (3M_c + 2M_l - 6K_1 - 3K_{c2})/5 > 0 \quad u_{c2} = (4M_c + M_l - 3K_1 - 9K_{c2})/5 > 0
\]

Airline one is capacity-constrained and serves only the local market where airline two is capacity-constrained \((\Delta_7)\)

\[
q_{l1} = K_1 \quad q_{c1} = 0 \quad q_{c2} = M_c/2 \quad u_{l2} = 2M_l - 4K_{l2} - 2K_1 > 0
\]

\[
u_1 = 2M_l - 4K_1 - 2K_{l2} > 0 \quad v_{c1} = (4M_l - M_c - 8K_1 - 4K_{l2})/2 > 0
\]
Airline one is capacity-constrained and serves only the local markets; airline two is capacity-constrained in all markets (Δ₈)

\[ q_{l1} = K_1 \quad q_{c1} = 0 \quad q_{c2} = K_{c2} \quad q_{l2} = K_{l2} \]

\[ u_{c2} = M_c - 2K_{c2} > 0 \quad u_{l2} = 2M_l - 4K_{l2} - 2K_1 > 0 \]

\[ u_1 = 2M_l - 4K_1 - 2K_{l2} > 0 \quad v_{c1} = 2M_l - M_c - 4K_1 - 2K_{l2} + K_{c2} > 0 \]

Both airlines are capacity-constrained in all markets (Δ₉)

\[ q_{l2} = K_{l2} \quad q_{l1} = (2M_l - M_c + 2K_1 + K_{c2} - 2K_{l2})/6 \]

\[ q_{c2} = K_{c2} \quad q_{c1} = (M_c - 2M_l + 4K_1 - K_{c2} + 2K_{l2})/6 \]

\[ u_1 = 2(M_c + M_l - 2K_1 - K_{c2} - K_{l2})/3 > 0 \]

\[ u_{c2} = (5M_c + 2M_l - 4K_1 - 11K_{c2} - 2K_{l2})/6 > 0 \]

\[ u_{l2} = (4M_l + M_c - 2K_1 - K_{c2} - 10K_{l2})/3 > 0 \]

**Proof of Proposition 2.3.3**

*Proof.* For airline one, \( \partial \Pi_1/\partial K_1 = E(\partial \pi_1/\partial K_1) - 2c_h = 0 \), and \( E(\partial \pi_1/\partial K_1) \) can be derived by summing the product of the first-order derivative of profit w.r.t. \( K_1 \) and the probability of each area in Figure 2.6 using the Leibnitz rule. So the FOC of \( \Pi_1 \) w.r.t. \( K_1 \) is \( E\partial \pi_1/\partial K_1 = 2c_h \), which translates into

\[
c_h = \frac{1}{3} \int \int_{\Delta_4} (x_l + x_c - 3K_1) dF_l dF_c + \frac{2}{11} \int \int_{\Delta_5} (2x_c + 3x_l - 6K_1 - 3K_{l2}) dF_l dF_c
\]

\[
+ \frac{1}{5} \int \int_{\Delta_6} (3x_c + 2x_l - 6K_1 - 3K_{l2}) dF_l dF_c + 2 \int \int_{\Delta_7,8} (x_l - 2K_1 - K_{l2}) dF_l dF_c
\]

\[
+ \frac{2}{3} \int \int_{\Delta_9} (x_c + x_l - 2K_1 - K_{c2} - K_{l2}) dF_l dF_c
\]
For airline two, $\partial \Pi_2 / \partial K_{i2} = \partial E \pi_2 / \partial K_{i2} - 2c_p = 0$, which can be changed into

$$c_p = \int \int_{\Delta_2} (x_i - 3K_{i2})dF_idF_c + \frac{2}{11} \int \int_{\Delta_5} (x_c + 7x_i - 3K_1 - 18K_{i2})dF_idF_c$$

$$+ 2 \int \int_{\Delta_7,8} ((x_i - 2K_{i2} - K_1)dF_idF_c + \frac{1}{3} \int \int_{\Delta_9} (4x_i + x_c)$$

$$- 2K_1 - K_{c2} - 10K_{i2})dF_idF_c$$

similarly, $\partial \Pi_2 / \partial K_{c2} = \partial E \pi_2 / \partial K_{c2} - c_p = 0$, which is equivalent to

$$c_h = \frac{1}{5} \int \int_{\Delta_6} (4x_c + x_i - 3K_1 - 9K_{c2})dF_idG + \int \int_{\Delta_8} (x_c - 2K_{c2})dF_idF_c$$

$$+ \frac{1}{6} \int \int_{\Delta_9} (5x_c + 2x_i - 4K_1 - 11K_{c2} - 2K_{i2})dF_idF_c$$

Proof of Proposition 2.4.5

Proof. Under Assumptions 1 and 2, area 9 takes the whole space in Figure 6. So (2.9), (2.10), and (2.11), respectively, become

$$c_h = 2(\mu_l + \mu_c - 2K^m_h - K^m_c - K^m_l)/3$$

$$c_p = (4\mu_l + \mu_c - 2K^m_h - K^m_c - 10K^m_l)/3$$

$$c_p = (2\mu_l + 5\mu_c - 4K^m_h - 11K^m_c - 2K^m_l)$$

Solving above equations yields expected equilibrium capacities. The expected equilibrium quantities for area 9 is
\[ q_{l2} = K_l^m \quad q_{l1} = (2x_l - x_c + 2K_h^m + K_c^m - 2K_l^m)/6 \]
\[ q_{c2} = K_c^m \quad q_{c1} = (x_c - 2x_l + 4K_h^m - K_c^m + 2K_l^m)/6 \]
\[ E p_l^m = E[x_l - (q_{l1} + q_{l2})] = (2\mu_l + c_p + c_h)/6 \]
\[ E p_c^m = E[x_c - (q_{c1} + q_{c2})] = (\mu_c + c_p + c_h)/3 \]

The airlines’ expected profits are

\[ \Pi_h^m = 2p_l^m q_{l1} + p_c^m q_{c1} - 2c_h K_h^m = (p_c^m - 2p_l^m)q_{c1} + (2p_l^m - 2c_h)K_h^m \]
\[ = \frac{1}{18} \int \int_{\Delta} (x_c - 2x_l)(x_c - 2x_l + 4K_h^m - K_c^m + 2K_l^m)dF_l dF_c \]
\[ + K_l^m(2\mu_l + c_p - 5c_h)/3 \]
\[ \Pi_p^m = 2(p_l^m - c_p)K_l^m + (p_c^m - c_p)K_c^m \]
\[ = [K_l^m(2\mu_l - 5c_p + c_h) + K_c^m(\mu_c + c_h - 2c_p)]/3 \] (C.1)

The results follow after a few algebraic manipulations.
Appendix D

The Proof of Proposition 2.4.11

Proof. By definition, at equilibrium each player’s strategy is a best response to its rival’s strategy. In case a, from Propositions 2.4.8 and 2.4.9, a hub-and-spoke network is the best response regardless of its rival’s network structure. The same is true for case b(i). In case b(ii), from Propositions 2.4.8 and 2.4.9, each airline follows a tit-for-tat strategy, i.e., using the same network structure as its rival. It can be verified that \( c < \bar{c}_2 \) is equivalent to \( 6c^2 - 8\mu_c + 5\sigma^2 < 0 \). So

\[
\Pi^h - \Pi^p = (6c^2 - 8\mu c + 5\sigma^2 + 3c^2 - 4\mu_c)/27 < 0 \quad (D.1)
\]

\[
\Pi^p - \Pi^m_h = -(6c^2 - 8\mu c + 5\sigma^2)/18 > 0 \quad (D.2)
\]

Therefore, (D.1), (D.2) and Proposition 10 yield \( \Pi^m_p < \Pi^h < \Pi^m_h < \Pi^p \) in case b(ii). This confirms that (P,P) and (H,H) are the pure strategy NEPs. Figure D.1 is an numerical example of the network game for case b(ii).

It can be verified that \( \bar{c}_1 < \bar{c}_2 \) and \( \underline{c}_1 > \underline{c}_2 \). Case c can be described by Figure
Figure D.1: Case b(ii)

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>(3,3)</td>
<td>(3.5, 1.5)</td>
</tr>
<tr>
<td>P</td>
<td>(1.5, 3.5)</td>
<td>(4, 4)</td>
</tr>
</tbody>
</table>

Airline Two

Airline One

Figure D.2: Case c

D.2 where $\rightarrow$ denotes responses, e.g., $P \rightarrow H$ indicates that a hub-and-spoke network is the best response to a point-to-point network. The results for part c are established following the same reasoning for part b.
Bibliography


Barla, P. 1999. Demand uncertainty and airline network morphology with strategic interactions. working paper, Department of Economics, University of Laval.


