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MODEL-BASED FAILURE DETECTION IN INDUCTION MOTORS
USING NONLINEAR FILTERING

by

KUN-CHU LIU

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy

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August, 1995
CASE WESTERN RESERVE UNIVERSITY

GRADUATE STUDIES

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MODEL-BASED FAILURE DETECTION IN INDUCTION MOTORS
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Abstract

by

KUN-CHU LIU

In this dissertation, we present a model-based failure detection method for induction motors. Our method addresses two failure modes of the squirrel-cage induction motors; namely, rotor bar failures and stator winding insulation problems. Under the assumption that the stator voltages, the stator currents and the rotor speed are measurable, the induction motor can be modeled as a linear time-varying system. We apply a nonlinear filtering algorithm for failure detection using a discrete-time representation for the system. Failure events are treated as jumps in the parameters of the linear model. These events are random processes and are modeled as a finite state Markov chain. The conditional probability for each operating condition given the observation sequence is obtained by a nonlinear recursive algorithm. Then, the failure of the induction motor can be isolated by the record of conditional probabilities.

To compensate for the variation of the rotor resistance due to thermal effect,
we use the extended Kalman filter (EKF) to estimate the real-time value of the 
rotor resistance. Because speed sensor is a high-cost device, and may not be 
installed in some compact systems, we use a model reference adaptive system 
design technique to identify the rotor speed of the induction motor. thereby 
eliminating the need for a rotor speed measurement sensor in the failure detec-
tion scheme.
Dedicated to my parents and to my wife.
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Chapter 1
Introduction

1.1 Motivation

The detection and diagnosis of faults in dynamic systems can help avoid costly catastrophic failures, which can result in more severe damage and even human fatalities. Fault detection, diagnosis, and recovery are essential ingredients in the design of advanced systems where higher demands on reliability and safety are necessary. Typical examples include aeronautical and nuclear power plant equipment. The development of systems capable of early fault detection of critical plant components and subsystems can improve the overall operational reliability of the plant.

Electrical machines play an important role in many types of industrial processes. In practice, we can use a variety of measurement sensors to collect signals from various parts of the electrical machine. This sensor data can be used for failure detection and monitoring. Typical sensors might measure stator voltage and current, air-gap and magnetic flux density, rotor position and speed, and motor vibration. The job then, of the fault monitoring and detection systems is to isolate conductor shorts and open circuits, demagnetized magnets, rotor unbalance, and bearing wear. It is apparent that if it is necessary to diagnose other faults of the electrical machine, additional sensors, some very costly, would
be required.

Fig. 1. The general structure of model-based failure detection.

One approach to fault detection is to make use of a mathematical model of the plant. These techniques are referred to model-based methods, and often include analytical redundancy to accommodate hardware failures. The general structure of a model-based approach is shown in Fig. 1 ([1], [2]). This method requires knowledge of the physical characteristics of the plant, and this information is used to build a mathematical model of the plant and develop the decision analysis algorithms for each failure mode. The advantages of model-based failure detection methods are as follows:

1. They are software intensive methods, thus, expensive equipment is not necessary.
2. They can reliably detect incipient faults if the model is accurate.

3. We can easily upgrade the algorithm without additional hardware so that the system can detect more failure events.

The traditional methods of incipient fault detection as applied to large electric machines are generally not applicable for induction motors due to the limitations of sensing device size and the cost considerations. From an application perspective, the field-oriented control of induction motors, which is a major high performance control technique, includes a state estimation scheme in its implementation. Using this existing observer/state estimator, the implementation of a model-based fault detection scheme for induction motors can become more efficient and economical than other methods.

1.2 Literature Survey of Fault Detection in Electrical Machines

Although electrical machines are usually well constructed and robust, the possibility of failure arises from the stresses that come from the conversion of electrical energy to mechanical energy and vice versa. As mentioned in the previous section, there are many different fault detection methods in electrical machines that have been proposed in the literature. Basically, all can be divided into two classes of approaches: invasive and noninvasive. Both of these approaches have been successfully used in industrial applications.

In [3], Timperley provides an incipient fault identification method using electrical neutral RF monitoring. This technique has been installed and tested in large hydro-electric machines, as well as steam turbine generators. This invasive approach takes advantage of radio frequency measurement of noise at
the machine's neutral point. Accurate, wide-band spectrum analysis can be used on the measured signal to detect various types of stator deterioration as well as design defects.

Keyhani and Miri [4] use linear discrete dynamic equations to model the dynamic behavior of a round rotor synchronous machine connected to an infinite bus. This model is used to develop an observer whose inputs are the operating data of the synchronous generator and whose outputs are the estimates of unmeasurable damper winding currents. Once the unmeasurable states have been estimated, the trajectories for estimated machine parameters are computed using a recursive least squares method. The trajectory for a specified parameter is compared to a base-line trajectory for that parameter from a healthy machine, and any deterioration which might cause changes in this parameter can be detected.

In [5], a signal-based fault diagnosis scheme was proposed for low-power dc motors. Information about the condition of the rotor is contained mainly in the periodic components of the current signal. Four fault modes were investigated: a disconnected armature winding, commutator bars with different heights, a short circuit between adjacent commutator bars, and an eccentric collector. The current signal is sampled synchronously with the rotor speed, and residuals (the difference between actual and predicted values) associated with each failure mode are generated using a bank of linear filters which include the normal mode and all the fault modes. The filter bank incorporates linear prediction models of the system in "all" modes of operation. The serial correlation coefficients $\rho_m$ of the residuals, where $m$ represents the lag between the samples, are used to
detect and localize faults automatically. This correlation coefficient is estimated using the following equations:

\[
\hat{\rho}_m = \frac{\sum_{l=m}^{N} (e(l) - \bar{e})(e(l - m) - \bar{e})}{\sum_{l=m}^{N} (e(l) - \bar{e})^2}
\]  

(1.1)

where

\[
\bar{e} = \frac{1}{N} \sum_{l=1}^{N} e(l)
\]

and \(N\) is the number of samples.

Cho et al. [6] studied the detection of broken rotor bars for induction motors by using a least squares estimator to identify rotor resistance, which is the parameter that directly relates to a rotor fault. In this paper, a single-phase mathematical model was chosen to describe balanced steady-state electrical operation of an induction motor. In order to avoid an ill-posed estimation problem, it is assumed that the stator and the rotor have the same self-inductance. The compensation for the thermal variation of rotor resistance and stator resistance is based on a thermal model of the induction motor. Different load torques are applied to obtain sets of measurement data from at least two different operating points. This helps to avoid the adverse consequences of measurement disturbances. The experimental results showed that the induction motor with a broken rotor bar, which correspond to 2.4% increase in rotor resistance, could be identified.

Chow et al. ([7], [8]) investigated two failure modes of an induction motor, a stator winding short and bearing failure, by using an artificial neural net-
work. This was accomplished by defining a relationship map between \((I, \omega_r)\) and \((N, b)\), where \(I, \omega_r\) denote the measurable variables of stator rms current and rotor speed, and \(N, b\) denote the equivalent stator winding turns and the equivalent damping coefficient of the bearing, respectively. For failure detection, the values of \(N\) and \(b\) are divided into three condition levels (good, fair, bad) to yield \(\hat{N}_c\) and \(\hat{b}_c\), respectively, which qualitatively describe the conditions of the motor. At the beginning of a neural network training session, the detection and diagnosis of the motor's condition, the estimated values of which are \(\hat{N}_c, \hat{b}_c\) will not be correct. Based on the difference between the quantized motor condition values \((N_c, b_c)\) and the estimated values \((\hat{N}_c, \hat{b}_c)\), an error quantity \(e = (\hat{N}_c - N_c)^2 + (\hat{b}_c - b_c)^2\) is generated and used to adjust the network weights to improve the estimated values \((\hat{N}_c, \hat{b}_c)\). A successful result is presented by using multilayer feedforward net and backpropagation training algorithm on fault detection for a split-phase squirrel-cage induction motor.

### 1.3 Problem Formulation

The induction motor is a rugged, reliable low-cost ac machine. It has been used for both high-performance and low-performance drive applications. Incipient fault detection of an induction motor can enhance the performance of the overall drive system by identifying a potential problem before a major failure occurs. Early fault detection could allow preventive maintenance to be scheduled for the complete system, thereby reducing maintenance costs, and possibly preventing extended down time of the system caused by a forced outage of the electrical machine. Fault detection and monitoring the operational status and health
of an induction motor can reduce maintenance cost and improve the overall reliability of the system.

The methodology of our work is to study a physical model of incipient faults in the squirrel-cage induction motor. Then, we present an adaptive failure detection scheme for the induction motor. To test the performance of the failure detection scheme developed in this work, a simulation study using a three-phase mathematical model of the induction motor is used to generate the measurable variables which are stator voltages, stator currents and the time varying parameter rotor speed, $\omega_r$, in the plant.

1.3.1 Three-phase Mathematical Model of an Induction Motor

In general, a three-phase induction motor has three stator windings and three rotor windings for each pole pair. The stator windings are identical, sinusoidally distributed, and displaced 120° with equivalent turns. It is assumed that the rotor windings have the same arrangement. Let $\theta_r$ denote the mechanical angular position of the rotor and $\theta_e$ denote the angle measured in electrical units. The relationship between $\theta_r$ and $\theta_e$ is given by:

$$\theta_e = n_p \left( \int_0^t \omega_r(\tau) d\tau + \theta_e(0) \right)$$  \hspace{1cm} (1.2)

where $\omega_r$, $n_p$ represent the rotor speed and number of pole pairs of the induction motor, respectively. The three-phase voltage equations for the stator and rotor
of an induction motor can be expressed in an explicit matrix form, given by:

\[
R_s I_s + L_{ss} \frac{dI_s}{dt} + L_{sr} \frac{dI_r}{dt} + n_p \omega_r G_{sr} I_r = V_s \tag{1.3}
\]

\[
R_r I_r + L_{rr} \frac{dI_r}{dt} + L_{rs} \frac{dI_s}{dt} + n_p \omega_r G_{rs} I_s = V_r \tag{1.4}
\]

where the vectors of stator and rotor voltages and currents in each phase are

\[V_s = [v_{sa}, v_{sb}, v_{sc}]^T, \quad V_r = [v_{ra}, v_{rb}, v_{rc}]^T, \quad I_s = [i_{sa}, i_{sb}, i_{sc}]^T, \quad I_r = [i_{ra}, i_{rb}, i_{rc}]^T,\]

respectively. The parameter matrices are defined as follows:

\[
R_s = \begin{bmatrix}
R_s & 0 & 0 \\
0 & R_s & 0 \\
0 & 0 & R_s
\end{bmatrix},
\]

\[
R_r = \begin{bmatrix}
R_r & 0 & 0 \\
0 & R_r & 0 \\
0 & 0 & R_r
\end{bmatrix},
\]

\[
L_{ss} = \begin{bmatrix}
L_{ls} + L_{ms} & -0.5L_{ms} & -0.5L_{ms} \\
-0.5L_{ms} & L_{ls} + L_{ms} & -0.5L_{ms} \\
-0.5L_{ms} & -0.5L_{ms} & L_{ls} + L_{ms}
\end{bmatrix},
\]

\[
L_{rr} = \begin{bmatrix}
L_{lr} + L_{ms} & -0.5L_{ms} & -0.5L_{ms} \\
-0.5L_{ms} & L_{lr} + L_{ms} & -0.5L_{ms} \\
-0.5L_{ms} & -0.5L_{ms} & L_{lr} + L_{ms}
\end{bmatrix},
\]
\[ L_{sr} = L_{ms} \begin{bmatrix} \cos \theta_e & \cos (\theta_e + 120^\circ) & \cos (\theta_e - 120^\circ) \\ \cos (\theta_e - 120^\circ) & \cos \theta_e & \cos (\theta_e + 120^\circ) \\ \cos (\theta_e + 120^\circ) & \cos (\theta_e - 120^\circ) & \cos \theta_e \end{bmatrix} \cdot \]

\[ G_x = \frac{\partial L_{sr}}{\partial \theta_e} \]

\[ = -L_{ms} \begin{bmatrix} \sin (\theta_e) & \sin (\theta_e + 120^\circ) & \sin (\theta_e - 120^\circ) \\ \sin (\theta_e - 120^\circ) & \sin (\theta_e) & \sin (\theta_e + 120^\circ) \\ \sin (\theta_e + 120^\circ) & \sin (\theta_e - 120^\circ) & \sin (\theta_e) \end{bmatrix} \cdot \]

\[ L_{rs} = L_{sr}^T, \]

\[ G_{rs} = G_x^T. \]

The subscripts \( s \) and \( r \) stand for stator and rotor, respectively. The definitions of all parameters are as follows:

\( R_s \): the stator resistance of each stator phase winding.

\( R_r \): the rotor resistance of each rotor phase winding.

\( L_{ls} \): the stator leakage inductance of each stator phase winding.

\( L_{lr} \): the rotor leakage inductance of each rotor phase winding.

\( L_{ms} \): the magnetizing inductance of each stator phase winding.

Most induction motors are not equipped with coil-wound rotor windings, instead, the rotor current flows in copper or aluminum bars which are uniformly distributed and which are embedded in a ferromagnetic material with all bars terminating in a common ring at each end of the rotor. This type of rotor
configuration is referred to as a squirrel-cage rotor. The uniformly distributed winding is properly described by its fundamental sinusoidal components and is represented by an equivalent three-phase winding. In our research work, we consider squirrel-cage induction motors for which the rotor voltage $V_r$ equals 0. Then, the equation (1.4) becomes:

$$\mathbf{R}_r I_r + \mathbf{L}_{rr} \frac{dI_r}{dt} + \mathbf{L}_{rs} \frac{dI_s}{dt} + n_p \omega_r \mathbf{G}_{rs} I_s = 0. \quad (1.5)$$

Rearranging equation (1.3) and equation (1.5), the electrical dynamics of an induction motor can be described by:

$$\frac{dI_s}{dt} = (\mathbf{I}_3 - \mathbf{D} \mathbf{L}_{rs})^{-1} \left[ \mathbf{L}_{ss}^{-1} V_s + \left( n_p \omega_r \mathbf{D} \mathbf{G}_{rs} - \mathbf{L}_{ss}^{-1} \mathbf{R}_s \right) I_s + \left( \mathbf{D} \mathbf{R}_r - n_p \omega_r \mathbf{L}_{ss}^{-1} \mathbf{G}_{sr} \right) I_r \right] \quad (1.6)$$

$$\frac{dI_r}{dt} = (\mathbf{I}_3 - \mathbf{E} \mathbf{L}_{sr})^{-1} \left[ -\mathbf{E} V_s + \left( \mathbf{E} \mathbf{R}_s - n_p \omega_r \mathbf{L}_{rt}^{-1} \mathbf{G}_{rs} \right) I_s + \left( n_p \omega_r \mathbf{E} \mathbf{G}_{sr} - \mathbf{L}_{rr}^{-1} \mathbf{R}_r \right) I_r \right] \quad (1.7)$$

where the matrices $\mathbf{D} = \mathbf{L}_{ss}^{-1} \mathbf{L}_{sr} \mathbf{L}_{rt}^{-1}$, $\mathbf{E} = \mathbf{L}_{rr}^{-1} \mathbf{L}_{rs} \mathbf{L}_{ss}^{-1}$ and the matrix

$$\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$  

The mechanical dynamics of the rotor are given by the following equations:

$$\frac{d\omega_r}{dt} = \frac{1}{J} (T_e - T_i) - \frac{b}{J} \omega_r \quad (1.8)$$
\[ T_e = n_p l_s^2 \frac{\partial L_{sr}}{\partial \theta_e} I_r \]  

(1.9)

where \( J, b \) are the inertia of the rotor and the equivalent viscous damping coefficient of the bearings, respectively. \( T_e \) denotes the torque generated by the motor, and \( T_l \) denotes the load torque.

Therefore, the electrical characteristics of an induction motor are given by a linear time varying system, when the rotor speed is measurable. Define the state vector \( x(t) = [i_{sa}(t), i_{sb}(t), i_{sc}(t), i_{ra}(t), i_{rb}(t), i_{rc}(t)]^T \) and the known input vector is \( u(t) = [v_{sa}(t), v_{sb}(t), v_{sc}(t)]^T \). The state space model of the induction machine is given by:

\[ \dot{x}(t) = A(t)x(t) + B(t)u(t) \]  

(1.10)

where

\[
A(t) = \\
\left[ \begin{array}{c}
(I_3 - DL_{rs})^{-1}(n_p \omega_r D_{rs} - L_{rs}^{-1} R_s) \\
(I_3 - DL_{rs})^{-1}(n_p \omega_r D_{sr} - L_{sr}^{-1} G_{sr})
\end{array} \right] - \\
\left[ \begin{array}{c}
(I_3 - EL_{sr})^{-1}(E_{sr} - n_p \omega_r L_{rs}^{-1} G_{sr}) \\
(I_3 - EL_{sr})^{-1}(n_p \omega_r E_{sr} - L_{rr}^{-1} R_r)
\end{array} \right],
\]

\[
B(t) = \\
\left[ \begin{array}{c}
(I_3 - DL_{rs})^{-1} L_{rs}^{-1} \\
-(I_3 - EL_{sr})^{-1} E
\end{array} \right].
\]

A four-pole, 3hp, 220V, 60Hz, delta-connected type squirrel-cage induction motor used in [12] is chosen for our study. In this model, \( R_s = 0.687 \Omega, L_s = 83.97 mH, R_r = 0.842 \Omega, L_r = 85.28 mH, M = 81.36 mH, J = 0.03 km^2, b = 0.01 km^2 \text{sec}^{-1} \). The parameters of the three-phase model are given by:

\( L_{ms} = \frac{2}{3} M = 54.24 mH, L_{ls} = L_s - M = 2.61 mH, L_{lr} = L_r - M = 3.69 mH. \) We
use MATLAB\textsuperscript{TM} to simulate the model (1.10) of the induction motor, which is driving a constant load $T_i = 25 \, Nm$. The three-phase stator and rotor currents, the torque generated by the motor and the rotor speed are shown in Fig. 2 through Fig. 5. Note that the frequency of rotor currents corresponds to the difference in the speed of the rotating MMF (magnetomotive force) induced by the stator currents and the rotor speed.

\begin{center}
\begin{itemize}
\item[-] $i_{sa}$
\item[-] $i_{sb}$
\item[-] $i_{sc}$
\end{itemize}
\end{center}

Fig. 2. The three-phase stator currents of the induction motor.
Fig. 3. The three-phase rotor currents of the induction motor.

Fig. 4. The torque generated by the induction motor.
1.3.2 Two-phase Mathematical Model of an Induction Motor

A four winding primitive representation of an electrical machine was suggested by G. Kron in [9]. In this representation three-phase ac machines can be described using a two-phase model. A change of variables which defines a transformation of the three-phase variables of stationary circuit elements to an arbitrary reference frame may be expressed as follows:

\[ f_{sdq0} = K_s f_{abc} \]  \hspace{1cm} (1.11)
where $f_{sq0} = [f_{sd}, f_{sq}, f_{so}]^T$ and $f_{abc} = [f_{sa}, f_{sb}, f_{sc}]^T$ and

$$K_s = \frac{2}{3} \begin{bmatrix}
\sin \theta & \sin \left(\theta - \frac{2\pi}{3}\right) & \sin \left(\theta + \frac{2\pi}{3}\right) \\
\cos \theta & \cos \left(\theta - \frac{2\pi}{3}\right) & \cos \left(\theta + \frac{2\pi}{3}\right) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}$$

$$\theta = \int_0^t \omega(\tau) \, d\tau + \theta(0).$$

In the above equation, $f$ can represent either voltage, current, flux, or electrical charge. The $\omega$ denotes the angular velocity of the rotating reference frame.

---

![Diagram](image)

Fig. 6. The complete four-winding d-q primitive machine.

As shown in [10], a lumped parameter model of an induction motor can be derived from the dynamic relationships between flux linkage, current, and voltage of an ac machine. Using various coordinate transformations, the three-phase induction motor can be represented as the well-known d-q model (shown in Fig. 6).
If we choose the reference frame for variables fixed at stator, usually referred to as an a-b model, the dynamics of an induction motor is described by the following set of equations:

\[
\begin{align*}
R_s i_{sa} + \frac{d\Psi_{sa}}{dt} &= u_{sa} \\
R_s i_{sb} + \frac{d\Psi_{sb}}{dt} &= u_{sa}
\end{align*}
\]

(1.12)

\[
\begin{align*}
R_r i_{rd} + \frac{d\Psi_{rd}}{dt} &= 0 \\
R_r i_{rq} + \frac{d\Psi_{rq}}{dt} &= 0
\end{align*}
\]

(1.13)

where \( R, i, \Psi, v \) denote resistance, current, flux linkage and voltage; subscripts \( s \) and \( r \) stand for stator and rotor. \((a,b)\) denotes the components of a vector with respect to the reference frame fixed at the stator. Similarly, \((d,q)\) will denote the components of a vector with respect to a rotating reference frame at speed \( n_p \times \omega_r \), where \( n_p \) is the number of pole pairs and \( \omega_r \) is the rotor speed of the induction motor.

Let \( \theta_r \) denote the angular position of the rotor, and \( \theta_e \) denote the angle in electrical units such that \( \frac{d\theta_e}{dt} = n_p \times \frac{d\theta_r}{dt} \) with \( \theta_r(0) = 0 \) (see Fig. 7). Then, the vectors \([i_{rd}, i_{rq}]^T\) and \([\Psi_{rd}, \Psi_{rq}]^T\) in the rotating frame can be transformed into the vectors \([i_{ra}, i_{rb}]^T\) and \([\Psi_{ra}, \Psi_{rb}]^T\) which are related to the stationary frame \((a,b)\) by

\[
\begin{bmatrix}
i_{ra} \\
i_{rb}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_e & -\sin \theta_e \\
\sin \theta_e & \cos \theta_e
\end{bmatrix}
\begin{bmatrix}
i_{rd} \\
i_{rq}
\end{bmatrix}
\]

(1.14)
\[
\begin{bmatrix}
\Psi_{ra} \\
\Psi_{rb}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_e & -\sin \theta_e \\
\sin \theta_e & \cos \theta_e
\end{bmatrix}
\begin{bmatrix}
\Psi_{rd} \\
\Psi_{rq}
\end{bmatrix}.
\]  

(1.15)

Using this transformation, (1.12) and (1.13) become

\[
\begin{align*}
R_s i_{sa} + \frac{d \Psi_{sa}}{dt} &= v_{sa} \\
R_s i_{sb} + \frac{d \Psi_{sb}}{dt} &= v_{sb} \\
R_r i_{ra} + \frac{d \Psi_{ra}}{dt} + n_p \omega_r \Psi_{rb} &= 0 \\
R_r i_{rb} + \frac{d \Psi_{rb}}{dt} - n_p \omega_r \Psi_{ra} &= 0.
\end{align*}
\]  

(1.16)

Fig. 7. The angular position of the rotor refers to the a-b reference frame.
Under the assumption of linearity of the magnetic circuits and symmetry of the inductance as well as neglecting iron losses, the flux equations are:

$$
\Psi_{sa} = L_s i_{sa} + M i_{ra} \\
\Psi_{sb} = L_s i_{sb} + M i_{rb} \\
\Psi_{ra} = L_r i_{ra} + M i_{sa} \\
\Psi_{rb} = L_r i_{rb} + M i_{sb}
$$

(1.17)

where $L_s, L_r$ are self-inductances for the stator and rotor windings, respectively, and $M$ is the mutual inductance. Eliminating $i_{ra}, i_{rb}, \Psi_{sa}$ and $\Psi_{sb}$ in equation (1.16) by using (1.17), we obtain

$$
R_s i_{sa} + \frac{M}{L_r} \frac{d\Psi_{ra}}{dt} + (L_s - \frac{M^2}{L_r}) \frac{di_{sa}}{dt} = v_{sa} \\
R_s i_{sb} + \frac{M}{L_r} \frac{d\Psi_{rb}}{dt} + (L_s - \frac{M^2}{L_r}) \frac{di_{sb}}{dt} = v_{sb} \\
\frac{R_r}{L_r} \Psi_{ra} - \frac{R_r}{L_r} M i_{sa} + \frac{d\Psi_{ra}}{dt} + n_p \omega_r \Psi_{rb} = 0 \\
\frac{R_r}{L_r} \Psi_{rb} - \frac{R_r}{L_r} M i_{sb} + \frac{d\Psi_{rb}}{dt} - n_p \omega_r \Psi_{ra} = 0.
$$

(1.18)

The mechanical dynamics of the rotor are governed by the equations

$$
\frac{d\omega_r}{dt} = \frac{1}{J}(T_e - T_l) - \frac{b}{J} \omega_r
$$

(1.19)

$$
T_e = \frac{3n_p M}{2L_r} (\Psi_{ra} i_{sb} - \Psi_{rb} i_{sa})
$$

where $J$ and $b$ are the inertia of the rotor and the equivalent viscous damping coefficient of the bearings which support the rotor of the induction motor, $T_e$ denotes the torque generated by the motor and $T_l$ denotes the load torque.
Rearranging equations (1.18) and (1.19), the overall dynamics of an induction motor are given by the following set of first order nonlinear differential equations:

\[
\begin{align*}
\frac{di_{sa}}{dt} &= -\left(\frac{M^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2}\right) i_{sa} + \frac{M R_r}{\sigma L_s L_r^2} \Psi_{ra} + \frac{n_p M}{\sigma L_s L_r} \omega_r \Psi_{rb} + \frac{1}{\sigma L_s} v_{sa} \\
\frac{di_{sb}}{dt} &= -\left(\frac{M^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2}\right) i_{sb} - \frac{n_p M}{\sigma L_s L_r} \omega_r \Psi_{ra} + \frac{M R_r}{\sigma L_s L_r^2} \Psi_{rb} + \frac{1}{\sigma L_s} v_{sb} \\
\frac{d\Psi_{ra}}{dt} &= \frac{R_r M}{L_r} i_{sa} - \frac{R_r}{L_r} \Psi_{ra} - n_p \omega_r \Psi_{rb} \\
\frac{d\Psi_{rb}}{dt} &= \frac{R_r M}{L_r} i_{sb} + n_p \omega_r \Psi_{ra} - \frac{R_r}{L_r} \Psi_{rb} \\
\frac{d\omega_r}{dt} &= \frac{3 n_p M}{2 L_r J} (\Psi_{ra} i_{sb} - \Psi_{rb} i_{sa}) - \frac{T_i}{J} - b \omega_r
\end{align*}
\] (1.20)

where \( \sigma = 1 - \left(\frac{M^2}{L_s L_r}\right) \).

Define the state vector \( x(t) = [i_{sa}(t), i_{sb}(t), \Psi_{ra}(t), \Psi_{rb}(t)]^T \) and the known input vector \( u(t) = [v_{sa}(t), v_{sb}(t)]^T \). The dynamics of the electrical variables of the induction machines are governed by the following linear time-varying state equation:

\[
\dot{x}(t) = A(t)x(t) + B(t)u(t)
\] (1.21)

where

\[
A(t) = \begin{bmatrix}
-\left(\frac{M^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2}\right) & 0 & \frac{M R_r}{\sigma L_s L_r^2} & \frac{n_p M}{\sigma L_s L_r} \omega_r \\
0 & -\left(\frac{M^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2}\right) & -\left(\frac{n_p M}{\sigma L_s L_r} \omega_r\right) & M R_r \\
\frac{R_r M}{L_r} & 0 & -\left(\frac{R_r}{L_r}\right) & -\left(n_p \omega_r\right) \\
0 & \frac{R_r M}{L_r} & \frac{n_p \omega_r}{L_r} & -\left(\frac{R_r}{L_r}\right)
\end{bmatrix}
\]
Because the rotor speed is a time dependent variable, \( A(t) \) is the time-varying matrix. We use MATLAB\textsuperscript{TM} to simulate an a-b (two-phase) model of the induction motor considered in the previous section. The results are shown in Fig. 8 through Fig. 11.

![Graph showing two-phase stator currents](image)

Fig. 8. The two-phase stator currents of the induction motor.
Fig. 9. The two-phase rotor flux linkages of the induction motor.

Fig. 10. The torque generated by the induction motor.
1.3.3 Failure Modes for an Induction Motor

According to previous studies which have examined the reliability of induction motors and their failure modes, the three most probable failures are: failures in rotor shaft bearings, short circuits in the stator, and rotor bar failures. Table 1 from [17] summarizes the information from three surveys: The report of the Motor Reliability Working Group (MRWG), the investigation carried out by the Electric Power Research Institute (EPRI), and the data collected by Krauß of Allianz, a technical insurance company. Rotor and stator failures will cause electrical parameters change, and the bearing failure will increase the rotational friction coefficient of the rotor.
<table>
<thead>
<tr>
<th>Component of failures</th>
<th>MRWG</th>
<th>EPRI</th>
<th>Allianz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing</td>
<td>50%</td>
<td>41%</td>
<td>8%</td>
</tr>
<tr>
<td>Stator</td>
<td>25%</td>
<td>37%</td>
<td>60%</td>
</tr>
<tr>
<td>Rotor</td>
<td>2%</td>
<td>19%</td>
<td>22%</td>
</tr>
<tr>
<td>Other</td>
<td>22%</td>
<td>12%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Table 1. The failure rates of induction motor components.

The model-based fault detection method can take advantage of easily accessible measurements to predict motor condition and isolate failures. Our diagnostic methodology relies on the three-phase stator current measurements and mathematical modeling of the failure modes. Taking into consideration the physical characteristics of an induction machine, the diagnostic algorithm is aimed at detecting and isolating two of the most common electrical failure modes, which are rotor bar failures and stator winding insulation problems.

The detection of a broken rotor bar in an induction motor is based on the fact that the rotor resistance of the induction motor apparently increases when a rotor bar fails. The change of rotor resistance will cause the operating characteristics of the induction motor to change, which in turn causes the dynamic behavior of the states of the system to change. An on-line parameter or state estimation scheme will then be able to detect the occurrence of a fault.

From fundamental electromagnetic theory and basic conductor characteristics, the flux linkage of the windings is a function of the number of equivalent turns and the resistance of the winding is proportional to the length of the winding. The stator windings for each phase are identical with equivalent turns.
$N_s$ and resistance $R_s$. The self-inductance of the stator windings and the mutual inductance between stator windings are proportional to $N_s^2$ and $N_s$, respectively, and the mutual inductance between stator and rotor windings are proportional to $N_s$. Using these relationships, all elements in the inductance parameter matrices, including $L_{ss}$, $L_{sr}$ and $G_{sr}$, for a stator winding insulation failure can be computed from the nominal values of a healthy motor. Because the stator resistance is not a dominant parameter of induction motors, we assume that the percentage of turns shorted in the stator winding will cause the same percentage of stator resistance decrease.

1.4 Dissertation Outline

In this chapter, we introduced two different mathematical models for the induction motor; a three-phase model and a two-phase model. We also investigated the physical properties of two of the dominant electrical failure modes: broken rotor bar and shorted circuit in the stator windings.

In chapter 2 a failure detection scheme based on nonlinear filtering theory for squirrel-cage induction motors is developed. First, the well-known statistical observer, the Kalman filter is used to estimate rotor flux linkages of the induction motor under the assumption that stator voltages, stator currents and rotor speed are measurable. The failure events are modeled as a random jump process taking values in a finite set with known a priori probability, then a nonlinear recursive equation is derived to compute the conditional (a posteriori) probabilities for each failure mode given the observation sequence. Faulty components of the induction motor can be detected and isolated by analyzing of the conditional
probabilities.

In Chapter 3 we propose a parameter estimation algorithm for the induction motor which is based on the discrete-time extended Kalman filter (EKF) theory, given that stator voltages and currents, and rotor speed are measurable. The EKF algorithm is used to estimate the rotor resistance by treating the rotor resistance and its rate of change at each integration step as extra states to form an augmented nonlinear system. Then, on-line tracking of rotor resistance changes can be obtained through the evolution of the states of the extended Kalman filter. To compensate for the variation of rotor resistance caused by thermal effects, we use the EKF to estimate rotor resistance, and this information is used to update the nominal parameter values that are used in the failure detection scheme.

In Chapter 4, we investigate rotor speed estimation by using a model reference adaptive system design technique. The circuit equations of an induction motor can be decoupled into two independent flux observers which are referred to as the stator equation and the rotor equation. The stator equation is regarded as the reference model and the rotor equation, which involves the rotor speed, is regarded as the adjustable model. The rotor speed is obtained as the output of the adaptive mechanism which is driven by the difference of the flux values between these two flux simulators. We also investigate the dynamic response for the overall system related to the gain of the adjustment mechanism. The estimated speed value is then used in place of an actual rotor speed measurement in the failure detection scheme. Computer simulations are used to evaluate the performance of the failure detection system.
Chapter 2

An Application of Nonlinear Filtering to Failure Detection of Induction Motors

2.1 Introduction

The main motivation behind the application of nonlinear filtering to failure detection comes from the fact that the mean time between failures of components is generally much greater than the settling time of the system or the nonlinear filtering algorithm. Thus, it is reasonable to assume that the filter output will stabilize to a particular parameter set before another change would occur. This approach is based on the modeling of a failure event as a sudden shift in model parameters, possibly governed by a random jump in the process with known a priori distribution. To use this approach, one must determine a set of system model parameters for each operating mode, including the normal and each possible failure mode. These models are used to construct a nonlinear filter whose output is the conditional probability of the jump parameter process.

A discrete-time stochastic model of an induction motor is used in this study and we assume that the three-phase stator voltages and currents, and the rotor speed are measurable. The nonlinear filter uses a bank of Kalman filters as state observers, each tuned to a particular operating mode of the induction motor. The one-step prediction and covariance matrix of the Kalman filter
bank are used to compute the conditional probability of each operating mode. The configuration of this failure detection scheme is shown in Fig. 12.

![Diagram](image)

Fig. 12. The configuration of the failure detection scheme.

### 2.2 Kalman Filter for Induction Motor State Estimation

In the previous chapter, we introduced a two-phase equivalent induction motor representation. In this model, the electrical characteristics of the induction motor can be expressed as a linear time-varying system when rotor speed is treated as a measured parameter. Due to the presence of modeling error, input disturbances and measurement noise, a stochastic model is the most ap-
appropriate. A linear discrete-time stochastic model with additive plant and measurement noises is used. Define the state vector \( \mathbf{x}(k) = [i_{sa}(k), i_{sb}(k), \Psi_{ra}(k), \Psi_{rb}(k)]^T \), the input vector \( \mathbf{u}(k) = [v_{sa}(k), v_{sb}(k)]^T \) and the output vector \( \mathbf{y}(k) = [i_{sa}(k), i_{sb}(k)]^T \), the discrete-time state space model of an induction motor is given by:

\[
\begin{align*}
\mathbf{x}(k+1) &= A(k)\mathbf{x}(k) + B\mathbf{u}(k) + \mathbf{w}(k) \\
y(k) &= H\mathbf{x}(k) + \mathbf{v}(k)
\end{align*}
\]  

(2.1)

where

\[
A(k) = \begin{bmatrix}
1 + a_{11} & 0 & a_{13} & a_{14}\omega_r(k) \\
0 & 1 + a_{22} & a_{23}\omega_r(k) & a_{24} \\
a_{31} & 0 & 1 + a_{33} & a_{34}\omega_r(k) \\
0 & a_{42} & a_{43}\omega_r(k) & 1 + a_{44}
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
b_{11} & 0 \\
0 & b_{22} \\
0 & 0 \\
0 & 0
\end{bmatrix},
\]

\[
H = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
\]
with
\[
\begin{align*}
    a_{11} &= -\left( \frac{M^2R_rT + L_r^2R_rT}{\sigma L_rL_r^2} \right), \\
    a_{13} &= \frac{MR_rT}{\sigma L_rL_r^2}, \\
    a_{14} &= \frac{n_pMT}{\sigma L_rL_r}, \\
    b_{11} &= \frac{T}{\sigma L_r}, \\
    a_{22} &= a_{11}, \\
    a_{23} &= -a_{14}, \\
    a_{24} &= a_{13}, \\
    b_{22} &= b_{11}, \\
    a_{31} &= \frac{R_rMT}{L_r}, \\
    a_{33} &= -\frac{R_rT}{L_r}, \\
    a_{34} &= -n_pT, \\
    a_{42} &= a_{31}, \\
    a_{43} &= -a_{34}, \\
    a_{44} &= a_{33}.
\end{align*}
\]

and $T$ is the sampling period in the discretizing procedure.

The initial state $x(0)$, process noise $w(k)$ and measurement noise $v(k)$ are independent Gaussian white noise processes with the statistics:

\[
\begin{align*}
    E\{x(0)\} &= x_0 \\
    E\{[x(0) - \hat{x}(0)][x(0) - \hat{x}(0)]^T\} &= P_0, \quad P_0 \geq 0 \\
    E\{w(k)\} &= 0 \\
    E\{[w(k)][w(j)]^T\} &= Q\delta_{kj}, \quad Q \geq 0 \\
    E\{v(k)\} &= 0 \\
    E\{[v(k)][v(j)]^T\} &= R\delta_{kj}, \quad R > 0. \\
\end{align*}
\]  

(2.2)

Here $\delta_{kj}$ is the Kronecker delta function.

For a linear stochastic system with Gaussian noise, the well-known Kalman filter is an optimal least-squares state estimator. The discrete-time Kalman filter equations are as follows: [13]

Time update:

\[
P(k + 1 \mid k) = A(k)P(k \mid k)A^T(k) + Q
\]
\[ \hat{x}(k+1 \mid k) = A(k)\hat{x}(k \mid k) + Bu(k) \quad (2.3) \]

initialized by \( P(1 \mid 0) = P_0 \) and \( \hat{x}(1 \mid 0) = x_0 \).

**Measurement update:**

\[
K(k+1) = P(k+1 \mid k)H^THP(k+1 \mid k)H^T + R_*^{-1}
\]

\[
P(k+1 \mid k + 1) = [I - K(k+1)H]P(k+1 \mid k)
\]

\[
\hat{x}(k+1 \mid k + 1) = \hat{x}(k+1 \mid k) + K(k+1)[y(k+1) - H\hat{x}(k+1 \mid k)]
\quad (2.4)
\]

where \( K(\cdot) \) denotes the Kalman gain matrix, \( \hat{x}(\cdot) \) denotes the estimated state and the estimation error covariance matrix \( P(\cdot \mid k) = E\{[x(k) - \hat{x}(k \mid k)][x(k) - \hat{x}(k \mid k)]^T\} \).

### 2.3 Nonlinear Filtering for Fault Detection

The application of nonlinear filtering to fault detection is based on the modeling of failure events as a jump in the model parameters, the probabilistic description of which is modeled as a finite state Markov chain. This approach requires that a set of system model parameters for the normal mode and for all possible failure modes are available. The conditional probability given the observation record for each mode will be obtained by a nonlinear recursive algorithm. [14]

In our work, the stochastic system behavior can be described by a set of jump linear difference equations which are given as:

\[
x(k+1) = A(\xi_k)x(k) + B(\xi_k)u(k) + w(k)
\]
\[ y(k) = H(\xi_k)x(k) + v(k) \]  \hspace{1cm} (2.5)

where the state vector \( x(k) \in \mathbb{R}^n \), the input vector \( u(k) \in \mathbb{R}^m \) is assumed to be known and the observation vector \( y(k) \in \mathbb{R}^r \). The matrices \( A(\xi_k), B(\xi_k), H(\xi_k) \), which depend on \( \xi_k \), are of dimensions \( n \times n, n \times m \) and \( r \times n \) respectively.

\( \{w(k)\} \) is an \( n \)-dimensional random noise sequence.

\( \{v(k)\} \) is an \( r \)-dimensional random noise sequence.

\( \{\xi_k\} \) is a stationary finite state Markov process where \( \xi_k \in \{1, 2, 3, \ldots, N\} \).

\( N \) denotes the number of possible states in the chain. The one-step transition probabilities of \( \{\xi_k\} \) are:

\[ p_{ij}(k) = p(\xi_{k+1} = j \mid \xi_k = i) \]  \hspace{1cm} (2.6)

and are assumed to be known for all \( i, j = 1, 2, 3, \ldots, N \).

The initial state of the system \( x(0) \) is a Gaussian random variable with mean \( x_0 \), covariance \( P_0 \), and the process and measurement noises are independent white Gaussian random sequences with zero mean and covariance matrices \( Q \) and \( R \) respectively. Then,

\[ x(0) \sim N(x_0, P_0), \quad P_0 \geq 0 \]

\[ w(k) \sim N(0, Q), \quad Q \geq 0 \]

\[ v(k) \sim N(0, R), \quad R > 0. \]  \hspace{1cm} (2.7)

Define the observation sequence \( Y_k = \{y(1), y(2), \ldots, y(k)\} \), and the parameter sequence \( \Xi_k = \{\xi_1, \xi_2 = c_2, \ldots, \xi_k = c_k\}, \quad c_i \in \{1, 2, \ldots, N\} \)
for \( i = 1, 2, \ldots, k \). Applying Bayes’ rule and the theorem of Total Probability, the probability distribution of the jump parameter sequence for the system conditioned on the observation sequence can be obtained as the output of the following nonlinear post processor (a detailed proof is given in Appendix A):

\[
p(\Xi_k \mid Y_k) = \frac{p(y(k) \mid \Xi_k, Y_{k-1})p(\xi_k = c_k \mid \xi_{k-1} = c_{k-1})p(\Xi_{k-1} \mid Y_{k-1})}{\sum_{s_1=1}^{N} \cdots \sum_{s_k=1}^{N} p(y(k) \mid \Xi_{s_k}, Y_{k-1})p(\xi_k = s_k \mid \xi_{k-1} = s_{k-1})p(\Xi_{s_k-1} \mid Y_{k-1})}
\]

(2.8)

where

\[
p(y(k) \mid \Xi_k, Y_{k-1}) = \left(\frac{\det[H^T(\xi_k)R^{-1}H(\xi_k) + P_{\Xi_{k-1}}^{-1}(k \mid k-1)]}{(2\pi)^r \det(R) \det(P_{\Xi_{k-1}}(k \mid k-1))}\right)^{\frac{1}{2}} \times \exp\left[-\frac{1}{2}g(\Xi_k, y(k))\right]
\]

(2.9)

with

\[
g(\Xi_k, y(k)) = y(k)^T R^{-1} y(k) + \hat{x}_{\Xi_{k-1}}^T(k \mid k-1) P_{\Xi_{k-1}}^{-1}(k \mid k-1) \hat{x}_{\Xi_{k-1}}(k \mid k-1) - D^T(\Xi_k)[H^T(\xi_k)R^{-1}H(\xi_k) + P_{\Xi_{k-1}}^{-1}(k \mid k-1)]^{-1}D(\Xi_k)
\]

\[
D(\Xi_k) = H^T(\xi_k)R^{-1}y(k) + P_{\Xi_{k-1}}^{-1}(k \mid k-1) \hat{x}_{\Xi_{k-1}}(k \mid k-1)
\]

\[
\Xi_k = \{\xi_1 = c_1, \xi_2 = c_2, \ldots, \xi_k = c_k\}
\]

\[
\Xi_{s_k} = \{\xi_1 = s_1, \xi_2 = s_2, \ldots, \xi_k = s_k\}
\]

and \( \hat{x}_{\Xi_{k-1}}(k \mid k-1) \), \( P_{\Xi_{k-1}}(k \mid k-1) \) are the mean of the one step prediction distribution and the covariance of the Kalman filter tuned to the sequence of
parameters $\Xi_{k-1} = \{\xi_1 = c_1, \xi_2 = c_2, \ldots, \xi_{k-1} = c_{k-1}\}$.

Note that equation (2.8) is a nonlinear recursive equation. This nonlinear processor uses an N-element array of parallel Kalman filters, where at the k-th step, $N^k$ sets of sufficient statistics are required. It is also important to note that the realization of this nonlinear filtering algorithm will require an increasing amount of memory, since the set of sufficient statistics is increasing exponentially at each stage. Fortunately, for many fault detection applications, the Markov process that models the failure process, i.e., the jump parameter sequence, has a one step transition probability matrix that is $I_N$ (an N by N identity matrix). This models a random failure process where the operating mode of the system is uncertain, but not changing in time. This simplified model can also be used to model a random failure process where the evolution on the failure parameter space is such that recovery from a failure is not possible and only single point failures are to be identified. This is of course a reasonable situation for our application if we assume that the system always begins in the normal operating model and the time between failures is much longer than the convergence time of the detection algorithm. In this case, the detection filter is reset after each detection to accommodate the random characteristic of the failure process. For this simple case, the amount of memory needed at each stage for this nonlinear processor implementation will increase linearly and the conditional probability for each mode can be computed by:

$$p_j(\Xi_k \mid Y_k) = \frac{p_j(y(k) \mid \Xi_k, Y_{k-1})p_j(\Xi_{k-1}, Y_{k-1})}{\sum_{j=1}^{N} p_s(y(k) \mid \Xi_{s_k}, Y_{k-1})p_s(\Xi_{s_{k-1}}, Y_{k-1})}$$

(2.10)
with

\[ \Xi_k = \{ \xi_0 = j, \xi_1 = j, \ldots, \xi_k = j \}, \quad j \in \{1, 2, \ldots, N\} \]
\[ \Xi_n = \{ \xi_0 = s, \xi_1 = s, \ldots, \xi_k = s \}, \quad s \in \{1, 2, \ldots, N\}. \] (2.11)

The initial condition for equation (2-10) is given by

\[ p_j(\xi_0 = j), \quad j = 1, 2, \ldots, N. \] (2.12)

### 2.4 Computer Simulation

A classical control technique for induction motors in the industry is the field oriented control. This scheme partially linearizes the system by controlling it in a reference frame which rotates with the rotor flux vector of the motor. Since the rotor flux is usually not measurable, this scheme includes the observer design which uses the two-phase mathematical model as shown in many recent publications (see [15], [16]). When we apply the coordinate transformation to obtain the two-phase \( a-b \) model, the zero variables, including \( i_{s0} \) and \( \Psi_{r0} \), which equal zero require the assumption that the motor operates under balanced condition. To reduce the complexity of the filter in the algorithm, we use a unsymmetrical \( a-b \) model to approximate the mathematical model of the faulty three-phase induction motor which is operating in a slightly unbalanced condition. This is reasonable because the value of the zero variables as well as the parameters corresponding to the zero variables are small in the transformation procedure, when the motor is in the early fault mode.

Because the stator windings of the three-phase induction motor with an
insulation problem are not identical, it is necessary to transform approximately all motor variables to the stationary reference frame in order to derive faulty model equations with constant parameters. We know that the three parameters, \( R_s \), \( L_s \), and \( M \) in the a-b model change when the stator windings are shorted, these are included in the stator failure mode investigated in this work. Using the same procedures as in section 1.3.2, we obtain the following unsymmetrical a-b model equations used for the implementation of the Kalman filter bank in the nonlinear filtering failure detection scheme:

\[
\dot{x}(t) = A_s(t)x(t) + B_s(t)u(t) \tag{2.13}
\]

where

\[
A_s(t) = \begin{bmatrix}
-\left( \frac{M^2 R_r + L^2 r_{1a}}{\sigma_a L_{1a} L_r^2} \right) & 0 & \frac{n_p M_a}{\sigma_a L_{1a} L_r} & \frac{n_p M_a}{\sigma_a L_{1a} L_r} \\
0 & -\left( \frac{M^2 R_r + L^2 r_{1b}}{\sigma_b L_{1b} L_r^2} \right) & \frac{n_p M_b}{\sigma_b L_{1b} L_r} & \frac{n_p M_b}{\sigma_b L_{1b} L_r} \\
\frac{R_s M_a}{L_r} & 0 & -\left( \frac{R_s}{L_r} \right) & -\left( n_p \omega_r \right) \\
0 & \frac{R_s M_b}{L_r} & n_p \omega_r & -\left( \frac{R_s}{L_r} \right)
\end{bmatrix}
\]

\[
B_s = \begin{bmatrix}
\frac{1}{\sigma_a L_{1a}} & 0 \\
0 & \frac{1}{\sigma_b L_{1b}} \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

with \( \sigma_a = 1 - \left( \frac{M^2}{L_{1a} L_r} \right) \), \( \sigma_b = 1 - \left( \frac{M^2}{L_{1b} L_r} \right) \). The subscripts \( a \) and \( b \) denote the unsymmetrical parameters referred to the a-axis and b-axis of the a-b model,
respectively. Note that the values of unsymmetrical parameters depend on which phase has the winding insulation problem.

Because the Kalman filter is fundamentally a stochastic estimator, it requires the necessary statistical information about the modeling uncertainties and measurement noise. The precision of the estimation result will depend on the accuracy of the model and the noise level of the environment. Process noise of the discrete-time stochastic induction motor model used in this work is a result of the errors in the modeling process, for example, input disturbances and parameter measurement error such as \( \omega_r, L_r, L_s, M \) etc.; the measurement noise comes from stator current measurement errors. In the simulation studies which follow, we include two cases of the noise environment: a low noise level and a high noise level. The low noise level is defined as follows: the random noise added to the system will cause 1% variation of peak values in the states and current measurements also have a 1% of peak values error. The high noise level is defined as: the random noise added to the system cause 5% variation of peak values in the states and current measurements also have a 5% of peak values error.

In this chapter, we illustrate the use of the multiple-model identification method for fault detection in an induction motor. In this method, a bank of Kalman filters corresponding to the possible modes of operation, the normal mode (N), the rotor bar failure mode (R) and a stator winding insulation problem (S), are designed and implemented. For the low noise environment, a 2% increase of \( R_r \) is used for the R mode and 1% of the stator windings are shorted in phase A is used for S mode. For the high noise environment, an 8% increase
of $R_r$ is used for the R mode and 2% of the stator windings are shorted in phase A is used for the S mode. The conditional probability for each mode is computed by the nonlinear post processor algorithm which is described in section 2-4. Fig. 13 through Fig. 15 and Fig. 16 through Fig. 18 show the computer simulation results for low level and high level noise environments, respectively. These results illustrate that the induction motor with one rotor bar broken or 1% of the stator windings shorted can be detected by the nonlinear filtering method when the system is at low noise environment. Moreover, we obtained the following results from the simulation studies:

1. This method has good robustness properties. In particular, the algorithm performs satisfactorily with a 1% modelling error in stator self-inductance $L_s$, rotor self-inductance $L_r$ or mutual inductance $M$ and also a ±25% variation in the stator resistance or a ±0.2 rad/sec speed measurement error.

2. Since the initial probability for each operating mode is not the main factor affecting the convergence rate for each computing stage, we choose almost equal initial probabilities for all modes in the algorithm.
Fig. 13. The conditional probability of each mode when IM is at N mode.

Fig. 14. The conditional probability of each mode when IM is at R mode.
Fig. 15. The conditional probability of each mode when IM is at S mode.

Fig. 16. The conditional probability of each mode when IM is at N mode.
Fig. 17. The conditional probability of each mode when IM is at R mode.

Fig. 18. The conditional probability of each mode when IM is at S mode.
Chapter 3
Extended Kalman Filter for Induction Motor State and Parameter Estimation

3.1 Introduction

In the previous chapter, we presented a robust failure detection scheme for detecting rotor bar and stator winding short circuits in induction motors. One difficulty in the implementation of this scheme is that the variation of motor temperature can cause a significant change in the rotor and stator resistance up to ±50% [18]. This will result in increasing the false alarm rate, and even corrupt the results of the detection and isolation scheme. Thus, these variations of resistance should be tracked as they occur. Compensation for thermal variations in resistance could use the thermal model proposed in [6]. It would be necessary to install appropriate temperature sensors in the rotor and stator. The temperature-sensing devices and associated components are relatively expensive and may not be practically applicable in smaller motors. They could also reduce the reliability of the machine.

In recent years, many papers have considered the on-line identification of the rotor time constant and other parameters for the development of adaptive monitoring and control of induction motors. Standard methods for the estimation of induction motor parameters include the locked rotor test, the no-load test,
and the standstill frequency response test (see [19], [20]). In [19], an automatic procedure is described in which a sequence of such tests is performed, each designed to isolate and measure a specific parameter. This method is applied to the automatic tuning of an induction motor drive. In [20], the authors proposed the real-time estimation of the induction motor parameters based on a simplified standard model expressed in the rotor coordinate frame. It is assumed that the stator currents and the rotor position (or velocity) measurements are available. The recursive least squares algorithm is then used to estimate the electrical and mechanical parameters.

In this chapter, we use the extended Kalman filter (EKF) for joint state and parameter estimation of the induction motor. For this purpose, we treat selected parameters as extra states and form an augmented state vector. One consequence is that the new augmented model is nonlinear, even if the original state space model is linear. To use the Kalman Filter with a nonlinear stochastic system, we linearize the model around the nominal estimation state trajectory, this yields a linear perturbation model. The standard Kalman filter can then be used for state estimation and the on-line estimated parameter value can be obtained from the specified state value. The emphasis here is to use this method to estimate rotor resistance, and the estimated value is used in conjunction with the failure detection algorithm developed in Chapter 2.
3.2 EKF for Joint State and Parameter Estimation

Consider the following nonlinear discrete-time state space model:

\[
\begin{align*}
x(k+1) &= f(x(k), u(k)) + w(k) \\
y(k) &= h(x(k)) + v(k).
\end{align*}
\] 

(3.1)

The statistics of \(x(0), w(k), v(k)\) are defined by equation (2.2), the associated EKF equations are:

\[
\begin{align*}
P(k+1 | k) &= F(k)P(k | k)F^T(k) + Q \\
K(k+1) &= P(k+1 | k)G^T(k)[G(k)P(k+1 | k)G^T(k) + R]^{-1} \\
P(k+1 | k+1) &= [I - K(k+1)G(k)]P(k+1 | k) \\
\hat{\hat{x}}(k+1 | k+1) &= f(\hat{\hat{x}}(k | k), u(k)) + \\
&\quad K(k+1)[y(k+1) - h(\hat{\hat{x}}(k+1 | k))]
\end{align*}
\] 

(3.2)

where

\[
\begin{align*}
F(k) &= \left(\frac{\partial f(x, u, k)}{\partial x}\right)_{x=\hat{\hat{x}}(k | k)} \\
G(k) &= \left(\frac{\partial h(x, u, k)}{\partial x}\right)_{x=\hat{\hat{x}}(k | k)}.
\end{align*}
\] 

(3.3)

Equation (3.2) is initialized by \(P(1 | 0) = P_0\) and \(\hat{\hat{x}}(1 | 0) = x_0\).

For our application, the EKF is applied to the simultaneous estimation of stator currents, rotor flux linkages and rotor resistance. Define the state vector
\( x(k) \), the output vector \( y(k) \) and the known input \( u(k) \) as follows:

\[
x(k) = [x_1(k), x_2(k), x_3(k), x_4(k), x_5(k), x_6(k)]^T \\
= [i_{sa}(k), i_{sb}(k), \Psi_{sa}(k), \Psi_{sb}(k), R_r(k), \Delta R_r(k)]^T \\
y(k) = [y_1(k), y_2(k)]^T \\
= [i_{sa}(k), i_{sb}(k)]^T \\
u(k) = [u_1(k), u_2(k)]^T \\
= [v_{sa}(k), v_{sb}(k)]^T.
\]

This yields a sixth-order augmented state space model, with a nonlinear state equation and a linear observation equation, for the induction motor:

\[
x_1(k+1) = x_1(k) + f_{11}x_1(k) + f_{15}x_1(k)x_5(k) + f_{13}x_3(k)x_5(k) + f_{14}\omega_r(k)x_4(k) + b_{11}u_1(k) + w_1(k) \\
x_2(k+1) = x_2(k) + f_{22}x_2(k) + f_{25}x_2(k)x_5(k) + f_{23}\omega_r(k)x_3(k) + f_{24}x_4(k)x_5(k) + b_{22}u_2(k) + w_2(k) \\
x_3(k+1) = x_3(k) + f_{31}x_1(k)x_3(k) + f_{33}x_3(k)x_5(k) + f_{34}\omega_r(k)x_4(k) + w_3(k) \\
x_4(k+1) = x_4(k) + f_{42}x_2(k)x_5(k) + f_{43}\omega_r(k)x_3(k) + f_{44}x_4(k)x_5(k) + w_4(k) \\
x_5(k+1) = x_5(k) + x_6(k) \\
x_6(k+1) = x_6(k)
\]

\[
y(k) = Hx(k)
\]
where
\[
\begin{align*}
f_{11} &= -\left(\frac{L_2 R_T}{\sigma L_L L_T^2}\right), \quad f_{15} = -\left(\frac{MT}{\sigma L_L L_T^2}\right), \quad f_{13} = \frac{MT}{\sigma L_L L_T^2}, \quad f_{14} = \frac{\eta_{MT}}{\sigma L_L L_T}, \quad b_{11} = \frac{T}{\sigma L_T}, \\
f_{22} &= f_{11}, \quad f_{25} = f_{15}, \quad f_{23} = -f_{14}, \quad f_{24} = f_{13}, \quad b_{22} = b_{11}, \\
f_{31} &= \frac{MT}{L_r}, \quad f_{33} = -\frac{T}{L_r}, \quad f_{34} = -n_p T, \\
f_{42} &= f_{31}, \quad f_{43} = -f_{34}, \quad f_{44} = f_{33}
\end{align*}
\]

with \( T \) is sampling period.

\[
H = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

Then, the Jacobian matrices for the implementation of the EKF are:

\[
F(k) = 
\begin{pmatrix}
1 + f_{11} + f_{15} x_5 & 0 & f_{13} x_5 & f_{14} \omega_r(k) & f_{13} x_3 + f_{15} x_1 & 0 \\
0 & 1 + f_{22} + f_{25} x_5 & f_{23} \omega_r(k) & f_{24} x_5 & f_{24} x_4 + f_{25} x_2 & 0 \\
f_{31} x_5 & 0 & 1 + f_{33} x_5 & f_{34} \omega_r(k) & f_{31} x_1 + f_{33} x_3 & 0 \\
0 & f_{42} x_5 & f_{43} \omega_r(k) & 1 + f_{44} x_5 & f_{42} x_2 + f_{44} x_4 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\]

(3.6)

\[
G(k) = H.
\]

(3.7)

The extended Kalman filter is an approximate filter for nonlinear systems and consequently, the estimated parameter values may be biased or divergent.
Ljung (see [21]) proposed a modified EKF algorithm using the innovation representation of the filter. This algorithm has excellent global convergence properties. If necessary, this modified EKF can be applied to rotor resistance estimation for induction motors.

3.3 Computer Simulation

Our simulation study showed that the induction motor system is not that sensitive to changes in stator resistance, i.e. large changes do not cause significant changes in the states. This result is in agreement with that obtained in [22]. Therefore, it is difficult for the extended Kalman filter (EKF) algorithm to estimate the stator resistance of the induction motor with acceptable accuracy when the stator resistance is changing, but this can be an advantage for model-based failure detection schemes. In other words, this class of failure detection algorithms is robust to variations of stator resistance but identification of small changes in stator resistance will be difficult. From the simulation results shown in Chapter 2, we showed that the nonlinear filtering failure detection scheme applied to the induction motor, which is a model-based method, is robust to the variation of the stator resistance.

The EKF algorithm developed in the previous section can be used for online tracking of variations in rotor resistance and this parameter data can then be used to adapt the failure detection algorithm of the induction motor. In order to verify this approach, we use the three-phase motor model to generate the measurable data in which the value of rotor resistance increases suddenly by 50%. The computer simulation results of the EKF used to estimate rotor
resistance is shown in Fig. 19 and Fig. 20. The definitions of noise environments are the same as in Chapter 2.

Secondly, we consider using the estimated value from the EKF to modify the nominal value of the rotor resistance in the nonlinear filtering failure detection scheme. In practice, changes in the rotor resistance are caused not only by changes in rotor temperature, but also by broken rotor bars. Since the EKF algorithm can not distinguish between these two factors, the failure detection algorithm will produce false rotor bar failure reports when the rotor temperature increases. Fortunately, we can use the fact that the change in $R_\tau$ caused by the thermal effect is very slow, and that caused by the rotor bar broken is sudden. The nonlinear filtering algorithm is implemented to operate in sequential time windows. Because the settling time for the filter is only three milliseconds, it is reasonable to use the previous value of $\hat{R}_\tau$ estimated from the EKF as the nominal value of rotor resistance in the failure detection algorithm.

In the failure detection algorithm, we simulate that rotor resistance increase slowly as the rotor temperature rises. The EKF tracks the true value of the rotor resistance with about 1% error. Because of the estimated error of $\hat{R}_\tau$, the failure detection scheme is less successful. By increasing the $R_\tau$ value 1% in the $R$ mode Kalman filter algorithm for each experimental simulation, we found that the adaptive failure detection scheme can detect the $R$ mode which is defined as a 5% increase in the rotor resistance. Fig. 21 through Fig. 23 show the simulation results of the adaptive failure detection scheme, which includes the nonlinear filter for failure detection and the EKF for rotor resistance estimation in the low noise environment.
Fig. 19. The estimation of rotor resistance in the low noise environment.

Fig. 20. The estimation of rotor resistance in the high noise environment.
Fig. 21. The conditional probability of each mode when IM is at N mode.

Fig. 22. The conditional probability of each mode when IM is at R mode.
Fig. 23. The conditional probability of each mode when IM is at S mode.
Chapter 4

Speed Estimation of An Induction Motor

4.1 Introduction

The installation of a speed sensor could diminish the robustness and simplicity of an induction motor. This lowers the system reliability, especially in hostile environments, and also requires careful cabling arrangement with special attention to reducing electrical noise. Because the speed sensor cannot be mounted in some cases, for example in the compact equipments or high speed motor drives, speed estimation has been one of the important requirements for induction motors.

There are many schemes based on simplified motor models that have been devised to estimate the speed of the induction motor from measured terminal voltages and currents. [23] In this chapter, we apply a model reference adaptive system (MRAS) to speed estimation in a squirrel-cage induction motor. This technique is less complex and more stable than previous results. It is based on the observation that the two-phase mathematical model of an induction motor can be decoupled into two flux observers. One, that does not involve rotor speed, \( \omega_r \), will be regarded as the reference model; the other includes the parameter \( \omega_r \) and may be regarded as the adjustable model. The error between the states of the two models is used to derive the adjusting scheme which generates the
estimated parameter \( \dot{\omega}_r \) for the adjustable model.

We use the speed estimation method proposed by Schauder [24] to design the adaptation mechanism based on the idea of hyperstability, and to investigate the dynamic response of the system. Using the pole allocation equation, we can choose an adequate gain of the PI controller in the adaptation mechanism to achieve excellent performance. We also show that the estimated rotor speed is accurate enough for the failure detection scheme developed in Chapter 2.

### 4.2 Speed Identification Using MRAS Technique

There are three basic approaches to the design of model reference adaptive systems: (1.) Local parametric optimization theory, (2.) Lyapunov functions. (3.) Hyperstability and positivity concepts. In [25], Landau described a practical synthesis technique for MRAS structures based on hyperstability. The stability approach, in particular the use of hyperstability and positivity concepts, is the most successful method for designing the adaptation mechanism of MRAS.

In order to obtain an accurate dynamic equation of motor speed, we treat it as an adjustable parameter in the coupled circuit equations which includes both stator and rotor equations. Since the motor supply voltages and stator currents are measured in a stationary reference frame, it is convenient to express these equations in the stationary frame. Rearranging the electrical part of equation (1.20) in section 1.3.2, we can obtain the stator equation, regarded as the reference model, and the rotor equation, regarded as the adjustable model as follows:
Stator equation
\[
\frac{d\psi_{ra}}{dt} = \frac{L_r}{M} \left( v_{sa} - R_s i_{sa} - \sigma L_s \frac{di_{sa}}{dt} \right) \\
\frac{d\psi_{rb}}{dt} = \frac{L_r}{M} \left( v_{sb} - R_s i_{sb} - \sigma L_s \frac{di_{sb}}{dt} \right).
\]  
(4.1)

Rotor equation
\[
\frac{d\psi_{ra}}{dt} = -\frac{R_r}{L_r} \psi_{ra} - n_p \omega_r \psi_{rb} + M \frac{R_r}{L_r} i_{sa} \\
\frac{d\psi_{rb}}{dt} = n_p \omega_r \psi_{ra} - \frac{R_r}{L_r} \psi_{rb} + M \frac{R_r}{L_r} i_{sb}.
\]  
(4.2)

The configuration of the MRAS for rotor speed estimation of an induction motor is shown in Fig. 24.

Fig. 24. The structure of MRAS for rotor speed estimation.

In general, the rotor speed, \( \omega_r \), is a variable and the model then is a linear
time-varying system. However, it is valid to initially treat $\omega_r$ as a constant parameter of the reference model. Define $\epsilon_a = \Psi_{ra} - \hat{\Psi}_{ra}$ and $\epsilon_b = \Psi_{rb} - \hat{\Psi}_{rb}$, where $[\Psi_{ra}, \Psi_{rb}]^T$ and $[\hat{\Psi}_{ra}, \hat{\Psi}_{rb}]^T$ are vector outputs of the reference model and the adjustable model, respectively. Subtracting equation (4-2) from (4-1), we obtain the following state error equations:

$$
\begin{bmatrix}
\frac{d\epsilon_a}{dt} \\
\frac{d\epsilon_b}{dt}
\end{bmatrix} =
\begin{bmatrix}
-\frac{R_c}{L_r} & -n_p \omega_r \\
n_p \omega_r & -\frac{R_c}{L_r}
\end{bmatrix}
\begin{bmatrix}
\epsilon_a \\
\epsilon_b
\end{bmatrix} - n_p (\omega_r - \hat{\omega}_r)
\begin{bmatrix}
\hat{\Psi}_{rb} \\
\hat{\Psi}_{ra}
\end{bmatrix}.
\tag{4.3}
$$

Let the state error vector $\epsilon = [\epsilon_a, \epsilon_b]^T$, then we obtain:

$$
\frac{d\epsilon}{dt} = A\epsilon - W
\tag{4.4}
$$

where

$$
A = \begin{bmatrix}
-\frac{R_c}{L_r} & -n_p \omega_r \\
n_p \omega_r & -\frac{R_c}{L_r}
\end{bmatrix},
$$

$$
W = \begin{bmatrix}
n_p (\omega_r - \hat{\omega}_r) \hat{\Psi}_{rb} \\
n_p (\omega_r - \hat{\omega}_r) \hat{\Psi}_{ra}
\end{bmatrix}.
$$

Since $\hat{\omega}_r$ is a function of the state error vector $\epsilon$, these equations define a non-linear feedback system illustrated in Fig. 25.
Fig. 25. The representation of MRAS as a nonlinear feedback system.

Following Landau’s results, hyperstability is assured provided that the linear time-invariant forward path transfer matrix is strictly positive real (SPR), and the nonlinear feedback loop, which includes the adaptation mechanism, satisfies Popov’s criterion for hyperstability. In the present system, the forward transfer matrix $G(s) = (sI - A)^{-1}$ is given by:
\[ G(s) = \frac{1}{(s + \frac{R_r}{L_r})^2 + (n_{\phi r})^2} \begin{bmatrix} s + \frac{R_r}{L_r} & -n_{\phi r} \\ n_{\phi r} & s + \frac{R_r}{L_r} \end{bmatrix}. \quad (4.5) \]

Then,

\[ G(j\omega) + G^T(-j\omega) = k \begin{bmatrix} \frac{R_r}{L_r} \left( \frac{R_r}{L_r} \right)^2 + (n_{\phi r})^2 + \omega^2 \\ -\frac{4R_r n_{\phi r} \omega j}{L_r} \frac{R_r}{L_r} \left( \frac{R_r}{L_r} \right)^2 + (n_{\phi r})^2 + \omega^2 \end{bmatrix} \]

with

\[ k = \frac{2}{\left[ (\frac{R_r}{L_r})^2 + (n_{\phi r})^2 + \omega^2 \right]^2 + \left( \frac{4R_r n_{\phi r}}{L_r} \right)^2} \]

\[ j = \sqrt{-1}. \]

Note that \( k \) is a real constant and its denominator is not identically equal to 0 for all frequency values \( \omega \). We obtain \( G(j\omega) + G^T(-j\omega) \) is a positive definite Hermitian Matrix for all \( \omega \), which implies that \( G(s) \) is indeed SPR.

Popov's criterion requires that

\[ \int_0^t e^T W d\tau \geq -\gamma_0^2, \text{ for all } t \geq 0 \quad (4.6) \]

where \( \gamma_0^2 \) is a positive constant. Referring to [24], satisfying Popov's criterion leads to a candidate adaptation mechanism given as follows:

\[ \dot{\phi}_r = \phi_2(\epsilon) + \int_0^t \phi_1(\epsilon) d\tau \quad (4.7) \]
where

\[
\phi_1(\epsilon) = k_i \left( \epsilon_b \dot{\psi}_{ra} - \epsilon_a \dot{\psi}_{rb} \right) \\
= k_i \left( \psi_{rb} \dot{\psi}_{ra} - \psi_{ra} \dot{\psi}_{rb} \right)
\]

\[
\phi_2(\epsilon) = k_p \left( \epsilon_b \dot{\psi}_{ra} - \epsilon_a \dot{\psi}_{rb} \right) \\
= k_p \left( \psi_{rb} \dot{\psi}_{ra} - \psi_{ra} \dot{\psi}_{rb} \right).
\]

Define the state error function \( e = \psi_{rb} \dot{\psi}_{ra} - \psi_{ra} \dot{\psi}_{rb} \), the estimated rotor speed \( \dot{\omega}_r \) can be obtained by:

\[
\dot{\omega}_r = \left( k_p + \frac{k_i}{s} \right) e. \tag{4.8}
\]

Fig. 26 shows the block diagram of the rotor speed identification scheme using (4.8) for the adjustment mechanism.
Fig. 26. The block diagram of MRAS speed estimation system.
4.3 Dynamic Response of MRAS Speed Identification

In designing the adaptation law for an MRAS, we have to take into account the overall stability of the system, and ensure that the estimated parameter quantity will converge to the true value with suitable dynamic characteristics. In order to investigate the dynamic response of the MRAS for rotor speed identification, it is necessary to linearize the adjustable model equations for small deviations about a particular steady state solution. If this is derived in the motor model referred to the stationary reference frame, the result will still be a system of linear time-varying equations. It is useful to first transform the model equations to the reference frame rotating synchronously with the stator current frequency. Then, through linearization with respect to a certain operating point, we obtain the transfer function relating the estimation error $\Delta (\omega_r - \hat{\omega}_r)$ to $\Delta e$ as follows:

$$G_1(s) = \frac{\Delta e}{\Delta (\omega_r - \hat{\omega}_r)} = \frac{(s + \frac{R_e}{L_r}) |\Psi_r|^2}{(s + \frac{R_e}{L_r})^2 + \omega_s^2}$$

(4.9)

where $\omega_s$ denotes the slip frequency of the induction motor and $|\Psi_r|^2 = \Psi_{rs}^2 + \Psi_{rb}^2$. Equation (4.9) can be represented using the block diagram shown in Fig. 27.
Fig. 27. The dynamic response of the MRAS speed estimator.

Assume $\omega_* = 0$ in equation (4.9) for simplicity, we can obtain the closed-loop speed estimation system transfer function from $\Delta \omega_r$ to $\Delta \hat{\omega}_r$ as follows: [26]

$$\frac{\Delta \hat{\omega}_r}{\Delta \omega_r} = \frac{(sk_p + k_i)|\Psi_r|^2}{s^2 + s(k_p|\Psi_r|^2 + \frac{B_e}{L_r}) + k_i|\Psi_r|^2}$$

$$= \frac{(2\zeta \omega_n - \frac{B_e}{L_r})s + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$$  \hspace{1cm} (4.10)

where $\zeta$ and $\omega_n$ stand for the damping ratio and the undamped natural frequency of the system, respectively. Then, we can specify $\zeta$ and $\omega_n$ by using $k_p$ and $k_i$ of the adjustment mechanism as follows:

$$k_p = \frac{(2\zeta \omega_n - \frac{B_e}{L_r})}{|\Psi_r|^2}$$

$$k_i = \frac{\omega_n^2}{|\Psi_r|^2}$$  \hspace{1cm} (4.11)
4.4 Simulation and Application

The characteristics of the MRAS speed estimation are verified by computer simulation. We use MATLAB to simulate the same model as in Chapter 2 and Chapter 3. In practice, the original models of (4-1) and (4-2) are difficult to implement because of the pure integrator which requires an initial value and has drift problems. In order to avoid these problems, we replace the pure integrator with a low-pass filter. The MRAS speed estimator does not work for arbitrary values of \( k_p \) and \( k_i \). In the case of \( \zeta = 0.3 \) and \( \omega_n = 100 \), we obtain the gain of the adjust mechanism \( k_p \approx 75 \) and \( k_i \approx 15600 \) from equation (4-11). Fig. 28 shows the simulation result of the speed estimation by MRAS. The estimated speed is used in the Kalman filter bank of the failure detection algorithm developed in Chapter 2. Because the estimated speed error is around 0.5 rad/sec, the performance of the failure detection scheme will degenerate. Experiments which included different random noise sequences with an amplitude around 0.5 rad/sec on the measured rotor speed were run. and \( R_e \) value was increased by 2% in the R mode Kalman filter for each experimental simulation. By using similar procedures to adjust the parameter values in the S mode Kalman filter, we found that the failure detection scheme incorporating rotor speed estimation can successfully detect the following failures: the R mode defined as an 8% increase of \( R_e \) and the S mode defined as a 2% of the stator windings shorted in phase A. The simulation results are shown in Fig. 29 through Fig. 31.
Fig. 28. The estimation of rotor speed by MRAS.

--- : Measured value  -- -- : Estimation value

--- : N mode  -- -- : R mode(8%)  ······ : S mode(2%)

Fig. 29. The conditional probability of each mode when IM is at N mode
Fig. 30. The conditional probability of each mode when IM is at R mode.

Fig. 31. The conditional probability of each mode when IM is at S mode.
Chapter 5
Conclusions

5.1 Summary

In this thesis, we developed a model-based failure detection method for squirrel-cage induction motors based on nonlinear filtering theory. The scheme provides for the detection and isolation of two failure modes, broken rotor bars and short circuits in the stator windings. The algorithm was developed by combining a Kalman filter bank and a nonlinear post processor derived in Chapter 2. Finally, the performance of the method was verified by computer simulation. The results show that early faults of the rotor and the stator in squirrel-cage induction motors can be detected reliably and the method is robust to motor parameter variations.

It is important to know that rotor resistance variations due to changes in rotor temperature are much greater than that due to a broken rotor bar. This causes the failure detection scheme to produce incorrect results. In order to compensate for the thermal effect, a novel EKF algorithm was proposed for real-time estimation of the rotor resistance. The estimated value from the EKF can be used in the failure detection algorithm to compensate for slowly-varying changes in rotor resistance not identified with a failure. The influence of the parameter variation can be removed in the parameter adaptive failure detection scheme which combines nonlinear filtering and EKF. The proposed scheme was
verified through computer simulation.

In order to eliminate the speed sensor, we applied the MRAS technique to estimate the speed of a squirrel-cage induction motor. The rotor speed is estimated based on the difference between the outputs of two flux simulators. By investigating the dynamic response of the overall system, it was possible to choose adequate parameters of the adjustment mechanism to achieve fast and reliable speed estimation. Simulation results demonstrated that the error of rotor speed estimation is less than 1%. Using this rotor speed estimator we demonstrated how the nonlinear filtering failure detection scheme for the squirrel-cage induction motor could be implemented without speed sensor.

5.2 Future work

There are several aspects related to this work which deserve further study.

1. Bearing failures account for a large number of motor problems. We believe that a motor with defective bearings has a larger equivalent damping coefficient than that of a healthy motor in the mechanical dynamic equation. However, an accurate failure model is difficult to derive. Artificial neural networks may be used to model bearing failure and to isolate motor failures which include bearing failure.

2. In [21], Ljung proposed a modified EKF algorithm. He had shown that the global convergence results for this algorithm can be obtained in the general case. It is worthwhile to apply the modified EKF algorithm to the estimation of rotor resistance as discussed in this work.

3. As briefly discussed in section 4-1, there are many methods for computa-
tion or estimation of rotor speed. The robustness properties of the estimators need to be studied. Perhaps, a better robust estimator can be derived using the sliding mode observer design technique.
Bibliography


Appendix A

Theorem:

The stochastic system behavior is described by a set of jump difference equations:

\[ x(k + 1) = A(\xi_k) x(k) + B(\xi_k) u(k) + w(k) \]
\[ y(k) = H(\xi_k) x(k) + v(k) \]

(A.1)

where the state vector \( x(k) \in R^n \), the input vector \( u(k) \in R^m \) and the output vector \( y(k) \in R^r \). The matrices \( A(\xi_k), B(\xi_k), H(\xi_k) \), which depend on \( \xi_k \), are of consistent dimensions.

\( \{w(k)\} \) is an \( n \)-dimensional random noise sequence.

\( \{v(k)\} \) is an \( r \)-dimensional random noise sequence.

\( \{\xi_k\} \) is a stationary finite state Markov process with \( \xi_k \in \{1, 2, 3, \cdots, N\} \), \( N \) denotes the number of possible states in the chain. The transition probabilities of \( \{\xi_k\} \) is assumed to be known.

The initial state of the system \( x(0) \), process noises \( w(k) \) and measurement noises \( v(k) \) are independent white Gaussian random variables associated with
the following statistics properties:

\[ x(0) \sim N(x_0, P_0), \ P_0 \geq 0 \]
\[ w(k) \sim N(0, Q), \ Q \geq 0 \]
\[ v(k) \sim N(0, R), \ R > 0. \]

Define the observation sequence \( Y_k = \{y(1), y(2), \ldots, y(k)\} \), and the parameter sequence \( \Xi_k = \{\xi_1 = c_1, \xi_2 = c_2, \ldots, \xi_k = c_k\} \), \( c_i \in \{1, 2, \ldots, N\} \) for \( i = 1, 2, \ldots, k \). Then, the probability distribution of jump parameter sequence for the system conditioned on the observation sequence can be obtained by:

\[
p(\Xi_k \mid Y_k) = \frac{\prod_{s_k=1}^{N} p(y(k) \mid \Xi_k, Y_{k-1}) p(\xi_k = c_k \mid \xi_{k-1} = c_{k-1}) p(\Xi_{k-1} \mid Y_{k-1})}{\sum_{s_1=1}^{N} \sum_{s_2=1}^{N} \cdots \sum_{s_k=1}^{N} p(y(k) \mid \Xi_{s_k}, Y_{k-1}) p(\xi_k = s_k \mid \xi_{k-1} = s_{k-1}) p(\Xi_{s_{k-1}} \mid Y_{k-1})}
\]

(A.2)

where

\[
p(y(k) \mid \Xi_k, Y_{k-1}) = \left( \frac{\det[H^T(\xi_k)R^{-1}H(\xi_k) + P_{\Xi_{k-1}}^{-1}(k \mid k - 1)]}{(2\pi)^r \det(R) \det(P_{\Xi_{k-1}}(k \mid k - 1))} \right)^{\frac{1}{2}} \times \exp\left[ -\frac{1}{2} g(\Xi_k, y(k)) \right]
\]

with

\[
g(\Xi_k, y(k)) = y(k)^T R^{-1} y(k) + \hat{\xi}_k^T \hat{\xi}_k \approx \hat{\xi}_{k-1}^T \hat{\xi}_{k-1} P_{\Xi_{k-1}}^{-1}(k \mid k - 1) \hat{\xi}_k \hat{\xi}_{k-1} (k \mid k - 1) - D(\Xi_k) [H^T(\xi_k)R^{-1}H(\xi_k) + P_{\Xi_{k-1}}^{-1}(k \mid k - 1)]^{-1} D(\Xi_k)
\]
\[ D(\Xi_k) = H^T(\xi_k)R^{-1}y(k) + P_{\Xi_{k-1}}^{-1}(k \mid k - 1)\hat{x}_{\Xi_{k-1}}(k \mid k - 1) \]

\[ \Xi_k = \{ \xi_1 = c_1, \xi_2 = c_2, \cdots, \xi_k = c_k \} \]

\[ \Xi_{s_k} = \{ \xi_1 = s_1, \xi_2 = s_2, \cdots, \xi_k = s_k \} \]

and \( \hat{x}_{\Xi_{k-1}}(k \mid k - 1) \), \( P_{\Xi_{k-1}}(k \mid k - 1) \) are the mean of one-step prediction distribution and the covariance of Kalman filter tuned to the sequence of parameters \( \Xi_{k-1} = \{ \xi_1 = c_1, \xi_2 = c_2, \cdots, \xi_{k-1} = c_{k-1} \} \).

**Proof:**

Applying Bayes' formula, we obtain:

\[
p(\Xi_k \mid Y_k) \\
= p(\xi_k = c_k, \Xi_{k-1} \mid Y_k) \\
= p(\xi_k = c_k \mid \Xi_{k-1}, Y_k)p(\Xi_{k-1} \mid Y_k) \\
= p(\xi_k = c_k \mid \Xi_{k-1}, y(k), Y_{k-1})p(\Xi_{k-1} \mid y(k), Y_{k-1}) \\
= \frac{p(y(k) \mid \Xi_k, Y_{k-1})p(\xi_k = c_k \mid \Xi_{k-1}, Y_{k-1})p(\Xi_{k-1} \mid Y_{k-1})}{p(y(k) \mid Y_{k-1})} \tag{A.3}
\]

Using the fact that \( \{ \xi_k \} \) is a Markov process which is independent of \( x_0 \), \( \{ w(k) \} \) and \( \{ v(k) \} \). Then,

\[
p(\xi_k = c_k \mid \Xi_{k-1}, Y_{k-1}) = p(\xi_k = c_k \mid \xi_{k-1} = c_{k-1}).
\]

By the Theorem of Total Probability and the above equation, the denomi-
nator of equation (A-3) becomes:

\[
p(y(k) | Y_{k-1}) = \sum_{s_1=1}^{N} \left[ p(y(k) | Y_{k-1}, \xi_1 = s_1)p(\xi_1 = s_1 | Y_{k-1}) \right]
\]

\[
= \sum_{s_1=1}^{N} \left\{ \sum_{s_2=1}^{N} \left[ p(y(k) | Y_{k-1}, \xi_1 = s_1, \xi_2 = s_2)p(\xi_2 = s_2 | Y_{k-1}, \xi_1 = s_1) \right] \times p(\xi_1 = s_1 | Y_{k-1}) \right\}
\]

\[
= \sum_{s_1=1}^{N} \sum_{s_2=1}^{N} p(y(k) | Y_{k-1}, \xi_1 = s_1, \xi_2 = s_2)p(\xi_1 = s_1, \xi_2 = s_2 | Y_{k-1})
\]

\[
\vdots
\]

\[
= \sum_{s_1=1}^{N} \sum_{s_2=1}^{N} \cdots \sum_{s_{k-1}=1}^{N} p(y(k) | Y_{k-1}, \xi_1 = s_1, \xi_2 = s_2, \ldots, \xi_{k-1} = s_{k-1}) \times p(\xi_1 = s_1, \xi_2 = s_2, \ldots, \xi_{k-1} = s_{k-1} | Y_{k-1})
\]

\[
= \sum_{s_1=1}^{N} \sum_{s_2=1}^{N} \cdots \sum_{s_{k-1}=1}^{N} \sum_{s_k=1}^{N} \left[ \sum_{s_k=1}^{N} p(y(k) | Y_{k-1}, \Xi_{s_k})p(\xi_k = s_k | Y_{k-1}, \Xi_{s_k}) \right] \times p(\Xi_{s_k} | Y_{k-1})
\]

\[
= \sum_{s_1=1}^{N} \sum_{s_2=1}^{N} \cdots \sum_{s_k=1}^{N} p(y(k) | Y_{k-1}, \Xi_{s_k})p(\xi_k = s_k | \xi_{k-1} = s_{k-1})p(\Xi_{s_k} | Y_{k-1})
\]

(A.4)

with

\[
\Xi_{s_k} = \{ \xi_1 = s_1, \xi_2 = s_2, \ldots, \xi_{k-1} = s_{k-1}, \xi_k = s_k \}.
\]

Therefore,

\[
p(\Xi_k | Y_k) =
\]
\[
p(y(k) \mid \Xi_k, Y_{k-1}) p(\xi_k = c_k \mid \xi_{k-1} = c_{k-1}) p(\Xi_{k-1} \mid Y_{k-1})
\]

\[
\sum_{s_1=1}^N \cdots \sum_{s_k=1}^N p(y(k) \mid \Xi_{s_k}, Y_{k-1}) p(\xi_k = s_k \mid \xi_{k-1} = s_{k-1}) p(\Xi_{s_k} \mid Y_{k-1})
\]

(A.5)

Claim:

\[
p(y(k) \mid \Xi_k, Y_{k-1}) = \left(\frac{\text{det}[H^T(\xi_k)R^{-1}H(\xi_k) + P^{-1}_{\Xi_{k-1}}(k \mid k-1)]}{(2\pi)^r \text{det}(R) \text{det}(P^{-1}_{\Xi_{k-1}}(k \mid k-1))}\right)^{\frac{1}{2}}
\times \exp\left[-\frac{1}{2} g(\Xi_k, y(k))\right]
\]

(A.6)

where

\[
g(\Xi_k, y(k)) = y(k)^T R^{-1} y(k) + \hat{x}^T_{\Xi_{k-1}}(k \mid k-1) P^{-1}_{\Xi_{k-1}}(k \mid k-1) \hat{x}_{\Xi_{k-1}}(k \mid k-1)
\]

\[-D^T(\Xi_k)[H^T(\xi_k)R^{-1}H(\xi_k) + P^{-1}_{\Xi_{k-1}}(k \mid k-1)]^{-1} D(\Xi_k)\]

\[
D(\Xi_k) = H^T(\xi_k) R^{-1} y(k) + P^{-1}_{\Xi_{k-1}}(k \mid k-1) \hat{x}_{\Xi_{k-1}}(k \mid k-1)
\]

and \(\hat{x}_{\Xi_{k-1}}(k \mid k-1)\), \(P_{\Xi_{k-1}}(k \mid k-1)\) are the mean of one-step prediction distribution and the covariance matrix of Kalman filter tuned to the sequence of parameters \(\Xi_{k-1} = \{\xi_1 = c_1, \xi_2 = c_2, \cdots, \xi_{k-1} = c_{k-1}\}\).

Note that:

1. \(p(y(k) \mid \Xi_k, Y_{k-1}, x(k) = x) = p(y(k) \mid \xi_k = c_k, x(k) = x)\).

   Because output equation \(y(k) = H(\xi_k)x(k) + v(k)\), \(y(k)\) is Gaussian random vector with mean \(H(\xi_k)x\) and covariance \(R\), i.e.

\[
p(y(k) \mid \xi_k = c_k, x(k) = x) \sim N(H(\xi_k)x, R).
\]

(A.7)
(2.) \( p(x(k) = x \mid \Xi_k, Y_{k-1}) = p(x(k) = x \mid \Xi_{k-1}, Y_{k-1}) \).

Since \( x(k) \) is independent of \( \xi_k \) and is obtained as one-step prediction from Kalman filter tuned to the parameter set \( \xi_{k-1} = c_{k-1} \). Consider the parameter set jump sequence in the Markov chain has been \( \Xi_{k-2} \), we obtain:

\[
p(x(k) = x \mid \Xi_{k-1}, Y_{k-1}) \sim \mathcal{N}(A(\xi_{k-1})\hat{x}_{\Xi_{k-1}}(k-1 \mid k-1) + B(\xi_{k-1})u(k-1),
A^T(\xi_{k-1})P_{\Xi_{k-1}}(k-1 \mid k-1)A(\xi_{k-1}) + Q).
\]

(A.8)

Applying the Theorem of Total Probability,

\[
p(y(k) \mid \Xi_k, Y_{k-1}) = \int_{\mathbb{R}^n} p(y(k) \mid \Xi_k, Y_{k-1}, x(k) = x) \times p(x(k) = x \mid \Xi_k, Y_{k-1}) \, dx.
\]

(A.9)

Then, by rearranging the argument of the exponential in the Gaussian distribution function of equation (A-9) and solving the integration, we obtain equation (A-6).