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Inclusion of "value" concepts in evaluation of demand-side management in electric utility planning

Nelson, Sushil Kumar, Ph.D.
Case Western Reserve University, 1992

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INCLUSION OF "VALUE" CONCEPTS IN EVALUATION OF DEMAND-SIDE MANAGEMENT IN ELECTRIC UTILITY PLANNING

BY
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Submitted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy

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INCLUSION OF "VALUE" CONCEPTS IN EVALUATION OF DEMAND-SIDE MANAGEMENT IN ELECTRIC UTILITY PLANNING

Abstract

by

SUSHIL KUMAR NELSON

This thesis presents a theoretically correct and comprehensive criterion to screen electric utility demand-side management (DSM) programs. This criterion, the most value/generalized total resource cost test (MV/GTRC) test corrects several shortcomings of existing tests. Extensions of this test are presented that have never before been considered. But the MV/GTRC test is still a static test in that the various inputs are pre-specified and the program is screened based on these empirical inputs. A nonlinear bilevel model is formulated in this thesis to analyze the impact of various DSM issues on utility planning by modeling the interaction between the utility and its customers. The utility at the upper level seeks to optimize its objective by fixing electric rates and its investment in conservation and the lower level customer classes seek to maximize their net benefits by fixing electricity consumption and their investment in conservation. Efficient branch and bound algorithms are presented to overcome the nonconvexity problems caused by imbedding the Kuhn Tucker conditions of the lower level problems as constraints in the upper level model. This model is the first application of a nonlinear nonquadratic bilevel formulation solved in the literature.
To my mother and father
who are my greatest blessings
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CHAPTER 1

INTRODUCTION

1.1 Contributions

The electric utility industry in the United States is the most capital intensive among all industries. The generation, transmission and distribution aspects of power systems have been exhaustively researched and extensive literature exists on the supply-side issues of electric utility planning. But billions of dollars are sought to be spent on electric utility demand-side management (DSM) programs selected on the basis of faulty criteria and nebulous assumptions. It is vital that these programs are selected on the basis of economically efficient criteria and that the demand-side inputs to existing electric utility planning and DSM screening models analyzed rigorously on a consistent framework.

The contributions made by this thesis are as follows:

* Derivation of a new screening tool, which is an improvement over existing criteria, to evaluate economic viability of DSM programs.

* This test has been implemented as a model that is used by several utilities to screen DSM programs.

* Several extensions of this test are presented in this thesis which have never
before been considered.

* A nonlinear bilevel problem has been formulated in this thesis to model the interactions between the utility and its customers.

* This model is the first example of a nonlinear bilevel to be solved in the literature.

* Efficient branch and bound techniques are presented in this thesis to solve these bilevel models.

1.2 Background

Before 1973, the main objective of the electric utility was to supply reliable electric power to its customers at the "least cost." Economies of scale led utilities to construct huge power plants to meet demand which, spurred by the decrease in electric rates in real dollars, was growing at 7%/yr. But the oil embargo of 1973 raised electric rates and cut demand growth by nearly half. This led to an emphasis on the "proper and efficient" use of energy. Lovins (1977) and Sant (1979) further advanced the concept of "a least-cost strategy which provides energy services with the least-costly mix of energy supplies and energy efficiency improvements." In the early 1980's, "least-cost planning" underwent a transformation from the above inclusive view of all forms of energy to a closer focus on supply side and demand side options in electric utility planning. This focus of least-cost planning changed
further from an emphasis on energy conservation’s technical potential to minimization of society’s energy costs to an integrated evaluation of both supply-side and demand-side measures. This concept is now known as "integrated resource planning" (IRP). A recent survey conducted by Oak Ridge National Laboratory (Schweitzer and Hirst, 1990) shows that utility commissions in a majority of the states require this process to be undertaken by utilities in their states.

IRP can be defined as a process which evaluates both supply-side and demand-side options simultaneously and selects an optimal mix of these alternatives. This process has the following three broad areas of technological/methodological issues that need to be resolved before implementing least-cost plans:

- Quantification, comparison and integration of supply-side options;
- Quantification, comparison and integration of demand-side options;
- Comparison and integration of demand-side with supply-side options;

Supply-side options include coal-fired plants, nuclear plants, hydroelectric plants, power purchases from independent power producers etc., the output of which meet customer demands for electric power. On the supply side, some of the complicating issues are the economies and diseconomies of scale, lead times, unit sizes, reliability, and fuel price escalation. The environmental impacts of generating options have made the costs of supply-side resources even more uncertain. There are large IRP models in existence which evaluate these supply-side issues in considerable
detail. Examples include the Electric Power Research Institute's MIDAS, Energy Management Associates' PROMOD, Stone and Webster's EGEAS, and Gerber and Associates' MIDAS.

Demand-side management (DSM) programs are the measures taken by utilities to alter the load shape of customers. These measures include: conservation, peak clipping, valley filling, load shifting, strategic load growth, and load curtailment measures. There is also a very broad list of complicating factors for demand-side planning. These include:

- Customers demand energy services (e.g. light, motor power etc.) and not electricity or natural gas. A major failing of the IRP process, as practiced today, is that it assumes that quantity of energy services consumed by participants remains constant with and without the DSM program.

- Customers may underutilize capital in their decisions on how to produce energy services. This can occur due to several reasons:

  * Energy users may not own the building that they live in, and thus have no incentive to make the required capital investments. Even homeowners may be unwilling to make such investments if they are unsure about how long they will live in a building.
  * Customers may have a limited access to capital and may not want to make investments with a payback period of several years.
  * Customers may be ignorant about energy saving advantages of such investments.
  * Energy users may not follow economic rationality in their demand-side investments and may be influenced by intangibles
such as appearance, fashion etc.

* Manufacturers may not be willing to market energy service equipment with a long payback period.

The above reasons form the rationale for utility investment in conservation programs.

Electric rates have to be adjusted so that all utility costs (including DSM costs) are recovered from ratepayers. If the rates of all customers are changed due to the program, nonparticipants subsidize participants. This raises the questions of "equity" among ratepayers and "inefficiency" of prices if price is less than marginal costs.

What objective should the utility pursue in designing demand-side management programs? The following are the three common economic objectives:

- minimize the cost of providing energy services
- minimize electric rates to consumers
- maximize the net value to consumers of consuming energy services

There is a strong disagreement among utility planners about the right objective (Lovins, 1985, Ruff, 1988, Hobbs and Nelson, 1989 and Foley, 1989).

The above are some of the conceptual issues complicating the IRP process. The bilevel model presented in this thesis seeks to analyze the impact of these and other issues on the utility and the customer.

An irreplaceable element of the IRP process is the evaluation of the cost-
effectiveness evaluation of supply-side and demand-side programs. The supply-side measures have been evaluated for several decades now and the choices are decided entirely on costs (fixed and variable), and demand projections. In recent years, environmental factors are also explicitly considered before selecting the optimal supply-side resources.

The selection of demand-side measures in the integrated resource planning approach is often complicated by one or many of the factors listed below:

- Actual energy and capacity savings. There are many reasons why the actual savings are less than the projected savings. This may be due to lower participation rates, the presence of free riders (consumers who would have made these investments even without the utility program) and rebound (utility investments leading to consumers consuming more energy services because of the price of energy services has fallen due to the subsidy). During the last two years, there have been many extensive pre- and post-program evaluation studies undertaken to bridge this gap between actual and estimated savings.

- Impacts on transmission and distribution costs. These costs include the possible deferral of substation erection, and transformer purchases. There have been several recent efforts in evaluating the impacts of DSM programs on T&D resources. In a recent study undertaken by Lawrence Berkeley Laboratory for Pacific Gas & Electric, they estimate the avoided transmission and distribution costs to be 6 to 8% of total avoided energy and capacity costs (LBL, 1989).

- Reliability impacts. In most cases, conservation programs result in a decrease in loss of load probability when there are no avoided capacity
costs (i.e., in periods of adequate reserve margins.) But an important question that utility planners have to ponder over is whether one KW of avoided capacity due to a DSM program is equivalent to a KW of built capacity in years of inadequate reserve margins.

- Optimal size and timing of the utility DSM program. A few typical questions that most utility planners ask are: What is the size of the program that yields maximum benefits and when is the best time to introduce a DSM program to maximize net benefits?

- Optimal cost sharing arrangement between program participants and utility.

- Impacts on utility shareholders. Some regulatory commissions have adopted shareholder incentive schemes to make conservation programs attractive to investor owned utilities. The Electric Rate Adjustment Mechanism (ERAM) in existence in California is one such scheme which allows electric utilities to recover program costs between rate cases.

- Equity among participants and nonparticipants. In the absence of price elasticity of demand, participants face lower bills due to the DSM program whereas nonparticipants face higher bills due to increased rates when prices exceeds marginal costs. Thus other ratepayers subsidize program participants in this case.

- Efficiency impacts of rates. When price is greater than marginal costs, less electricity than what is economically efficient is being used. Introduction of DSM programs increase rates leading to even lesser electricity usage.

- Environmental impacts of the DSM programs. Generation of electricity leads to various types of water and air pollution problems. Energy conservation measures are "benign" resources that avoid such environmental costs. For example, regulators in New York allocate
14.05 mills per KWh saved as avoided environmental costs.

There have been several criteria which have been proposed to screen demand-side management programs (White, 1981, Lovins and Gilliam, 1986, Ruff, 1988, and Hobbs, 1991). Most of the complicating factors listed above are sought to be considered by most of the criteria, but there exists some fundamental differences among these criteria. The various other cost-benefit tests proposed in the literature are presented and analyzed in Chapter 2. A new criterion, the most value/general total resource cost (MV/GTRC) test, presented in this thesis in Chapters 2-4, considers certain important factors ignored by all other criteria. It is the only applicable criterion that explicitly accounts for the fact that the quantity and quality of energy services consumed by participants and nonparticipants change due to the adoption of the DSM program.

1.3 Problem Definition

In this section, we present the need for the analysis performed in this thesis. In sub-section 1.3.1 is presented the rationale for the derivation of the MV/GTRC test and why it is the most comprehensive economic criterion to screen demand-side management programs. Subsection 1.3.2 reiterates the need for the bilevel model to analyze the interactions between the utility and its customers. The screening model treats certain factors, such as free riders, participation rates and rebound etc., as
exogenous while evaluating DSM programs. The effects of these factors and their impacts on customer-utility interactions are analyzed by the bilevel model. Thus the bilevel model provides a framework to analyze customer behavior given such factors.

1.3.1 Need for a New Screening Criterion

Electric utilities in the United States currently spend about two billion dollars on conservation and load management options, with several billion dollars to be spent in the next twenty years. The use of wrong tests to screen these programs could lead to large wastage of resources. Current screening criteria ignore the effects of program expenditures on nonparticipants and the effects of factors like rebound and free riders on customer "value". "Rebound" occurs when electric utility subsidies lower the effective price of energy services for participants leading to increased consumption of energy services. "Free riders" are those program participants who would have invested in the conservation measures even without utility subsidies. "Value" can be defined as the benefits obtained by customers by the consumption of energy. The California Standard Practice (California Public Utilities Commission and California Energy Commission, 1983 and 1987), the benchmark for evaluating demand-side programs, focuses just on resource costs omitting the above stated program effects.

The increase in nonparticipant rates due to program expenditures could lead
to certain nonparticipants leaving the utility system. Factors such as cogeneration, competition among electric utilities, and competition with natural gas could make this possibility a reality. As a simple example, let the price of electricity be 0.06 $/kWh and the marginal cost of electricity be 0.03 $/kWh. If the DSM program raises the price of electricity to 0.07 $/kWh, some customers might opt to cogenerate at a cost of 0.065 $/kWh. There is a net loss in value to society of (0.065-0.03)$ per kWh. Thus if increase in cogenerated power due to such rate increases is 50,000 kWh/yr, society loses $1750/yr on uneconomic generation of power.

Most utility DSM planners tend to feel that the rebound effect is bad since it reduces the anticipated kWh savings. They fail to recognize that rebound has value to participants, since the latter would not have increased their consumption of energy services unless they derive some benefit from it. The MV/GTRC test is the only criterion that recognizes the value of rebound to participants.

Most societal cost tests ignore the effects of free riders since they feel it is just a matter of income distribution between the utility and its customers (Lovins and Gilliam, 1986, and Costello and Galen, 1984). This is not true because the program costs spent on free riders have to be recovered from all ratepayers which affects nonparticipant energy demands and, thus, value. The MV/GTRC test is the only test that takes this factor into account.
Thus the MV/GTRC test derived in this thesis would be a significant contribution to electric utility planning in that it corrects certain shortcomings of existing screening criteria which could otherwise lead to erroneous program selection.

The MV/GTRC test presents the concept of "maximization of customer value" as a criterion for evaluation of demand-side management programs. The "maximization of value" is in this case the same as "consumer surplus" maximization which is the basis of much of applied welfare economics and benefit-cost analysis (Mishan, 1973). Costello and Galen (1984) analyze the impacts of DSM programs on the basis of consumer surplus but do not provide a single criterion to screen potential DSM programs.

Consumer surplus can be defined as the difference between the customer’s willingness to pay for a product minus the cost of the product. The net benefits or consumer surplus can be approximated by the area under the demand curve for the product minus the total cost of consumption. This is exactly true if we ignore the income effects of product consumption. We ignore income effects, in this thesis, since we assume that the power expenditures of customers are a small fraction of their income (Willig, 1976). Thus the net benefits to customers due to consumption of energy services is the area under the demand curve for energy services minus their power bills.
The MV/GTRC criterion, in its simplest form, is derived by maximizing the gross benefits to participants in a program (area under the demand curve) plus the gross benefits to nonparticipants minus the total power bills and the societal costs of the program. Due to regulatory intervention, the power bills in each year are set to equal the total cost of generation plus the utility costs of the program. This thesis presents more general versions of the MV/GTRC test where power bills need not be recovered in the year of expenditure. Taking the total derivatives of the discounted sum of those benefits leads to the MV/GTRC criterion.

In this thesis, we present the formulation that leads to the MV/GTRC test and its consideration of various benefits that are ignored by all other screening criteria. This thesis would look at various economic aspects of demand-side utility planning and explore the incorporation of various issues that affect day-to-day electric utility demand-side planning.

1.3.2 Effects of DSM on Utility and Customers

The MV/GTRC test, discussed in the previous subsection, is a static test in that it screens demand-side management programs given a certain scenario of prices, electricity savings, free rider fractions, marginal costs, customer costs of the program, utility costs etc. It accepts the above data assumptions as given in evaluating DSM
programs. Such information can be obtained from empirical studies. But what we lack in utility planning is a framework for interpreting the results of such empirical analysis. Thus there is a need to model customer-utility interactions to understand the results of surveys and other data collection efforts. For instance, if the objective is to maximize net benefits to customers, at what level of customer capital market distortion should the utility invest in DSM programs? What if the objective was to minimize total costs? What level of free riders or rebound occur at what levels of distortions in the capital market? Distortions in the capital markets occur when customers in the market place face a higher discount rates for their purchases of consumer durables than the electric utility. The answers to such "what if" questions can help electric utilities decide what objectives are important, what parameters are important, and what data is needed for program evaluation. Such questions are addressed by using bilevel models in this thesis.

The bilevel models have the electric utility at the upper level and the various customer sectors (residential, commercial etc.) or end users (air-conditioners, refrigerators etc.) at the lower level. The utility fixes its price for electric power and its investment in conservation subject to the customer's consumption of electricity and the customers investment in conservation measures. The utility either seeks to maximize the net societal benefits (maximize value) or minimize the total costs of operation whereas the customer at the lower level seeks to maximize his net benefits
(i.e., area under the demand curve for energy services minus his power bills and perceived DSM costs).

We present two formulations of the above problem in this thesis. In one formulation, the utility has the option of investing in conservation measures directly, whereas in the other, the utility offers an incentive to the various customers or end-users to invest in conservation measures. The first formulation is appropriate for programs like low income weatherization in which the utility directly, or through energy service companies, weatherizes residences. On the other hand, the second formulation is germane for programs like air-conditioner subsidies in which the utility offers rebates to customers who install energy efficient air-conditioners.

Bilevel programming is a relatively new branch of mathematical programming in which the upper and lower levels are fundamentally in conflict with each other. It differs from traditional decomposition methods in that the latter methods essentially solve a single objective function over a fixed feasible region (Bialas and Karwan, 1982). Multiobjective programming, on the other hand, finds a compromise among the various goals of planners. But they still do not account for possible independent actions taken by individual units or the order in which these actions are taken. The basic difference between bilevel programming and multiobjective programming is that the former prohibits cooperation between players.
Bilevel programming is essentially a non-cooperative Stackelberg game in which the follower (lower level) seeks to maximize his objective naively assuming that the leader (upper level) will not change the values of the decision variables he controls. However, the Stackelberg leader makes his decision with full knowledge of how the follower will react (Simaan and Cruz, 1973).

The bilevel utility model presented here is solved by incorporating the Kuhn-Tucker (K-T) conditions of the lower level model as constraints in the upper level model. This is the rational reaction set of the follower and the leader optimizes his objective knowing how the follower will react. The complementary slackness conditions of the K-T equations make the constraint set nonconvex. A branch and bound method will be utilized to eliminate these conditions and search for the optimal solution among a set of solutions for convex reduced problems. Chapter 5 provides a detailed literature survey of the existing bilevel solution methodologies and the rationale for using a bilevel model to analyze demand-side planning. Chapter 6 presents the formulation of this bilevel model while Chapter 7 discusses solution techniques for this problem. A four customer example is solved in Chapter 8 to illustrate the solution procedures.
1.4 **Scope**

The next chapter provides a discussion of the existing DSM screening criteria and a rationale for the MV/GTRC criterion. A table is also presented in that chapter listing the advantages it has over existing screening criteria. A comprehensive version of this test has already been implemented and utilized by an Ohio utility in filing their IRP plans with the Public Utilities Commission of Ohio (Centerior Energy Corporation, 1990).

Chapter 3 contains a formulation of the value problem from which the basic form of the MV/GTRC criterion is obtained. This formulation accounts for multiple customer classes in the case in which programs which affects only electricity use in multiple periods. Four sample DSM programs are also evaluated in that chapter to illustrate the advantages of the MV/GTRC test over all other screening criteria.

The basic form of the MV/GTRC test, too, has some shortcomings. In that derivation, we assume that the effects of rate changes on nonparticipant loads occur instantaneously in the year in which the program costs are recovered through rates. This is not necessarily true, since change in energy usage, especially change in energy using appliances occurs over a longer period of time. Section 4.2 of Chapter 4 formulates a "value" problem and derives a version of the MV/GTRC test that
accounts for time lags in customer response to rate changes. The basic version of the MV/GTRC test assumes that there are no distortions in the capital market for alternate energy sources. But such distortions do exist and need to be accounted for in programs that involve fuel switching. Section 4.3 derives the multi-fuel version of the MV?GTRC test and illustrates the importance of this derivation with an example.

Economic efficiency alone may not be the sole basis by which DSM programs are screened by the MV/GTRC tests. For example, low income weatherization programs may not pass the screening tests, but if "benefits to the poor" is an equally important attribute, then the program might be selected based on a weighted multiattribute score. Such an analysis and the framework for it are discussed in Section 4.4.

Chapter 5 provides a comprehensive survey of the literature on bilevel programming. This survey provides the rationale for the imbedded K.T. conditions approach used to solve the bilevel model in this thesis. Most of the literature on bilevel programming are for linear problems and all of the applications are linear or quadratic problems. This thesis would be the first nonquadratic nonlinear bilevel application solved in the literature. Chapter 5 also presents the need for the bilevel model and the type of questions that can be addressed by the model.
The formulation of the two bilevel models discussed earlier in this chapter is presented in Chapter 6. The structure of both bilevel models is examined in that chapter for all three objectives. The problems, in all cases except one, were found to be a concave maximization or a linear minimization problem with convex equality and inequality constraints.

Techniques to solve the problems, formulated in Chapter 6, are discussed in Chapter 7. Examples are also provided in Chapter 8 to show the advantages of using these solution techniques. The final chapter provides some closing comments on what has been achieved in this thesis and what remains to be done.
CHAPTER 2

BENEFIT-COST ANALYSIS OF DSM PROGRAMS

2.1 Introduction

This chapter presents a comprehensive account of the existing literature in integrated resource planning (IRP). IRP can be defined as a process which evaluates both supply-side and demand-side options simultaneously and attempts to optimize the mix of these alternatives. This process is also known as "least cost planning" (LCP). The concept of integrated resource planning has been widely accepted by the U.S. electric utility industry in recent times. The importance of this process is underlined by the fact that most regulated electric utilities follow the IRP process when they file their resource plans before their regulatory commissions. Section 2.2 gives a brief introduction to the concept of demand-side management. Different screening criteria to evaluate such DSM programs can be found in Section 2.3. It is to be noted that these tests assume that the quantity and quality of energy services remain fixed. It is explained in section 2.4 how ignoring the value of the change in energy services could lead to erroneous demand-side management (DSM) program selection. Chapters 3 and 4 of this thesis derive some important results which improve the IRP process by including these value changes in the planning process.
2.2. Demand-Side Management Programs

Demand-side management programs can be defined as those utility activities that alter the load shape of the customer. Types of DSM programs include: peak clipping which lower peak consumption, strategic conservation which lowers the energy demands during all periods, load shifting which shifts the load from periods of higher demands to periods of lower demands, valley filling which builds off-peak loads, strategic load growth which stimulates energy usage other than valley filling discussed earlier, and flexible load shaping which reward customers who accept reduced reliability with lower rates. Gellings (1987) provides a comprehensive list of the various types of DSM programs identified in the literature.

These DSM programs affect supply costs and the value the customers obtain from consumption of energy services. Energy services can be defined as the ultimate services such as cooling, heating, lighting, drive power etc. which customers derive from consumption of energy sources such as electricity or natural gas.

2.3. DSM Screening Criteria

Screening of DSM programs is an integral aspect of the IRP process. There are several hundred DSM programs that can be identified as promising for any
particular utility. Southern California Edison, for example, had a portfolio of about 800 potential DSM programs in their 1991 plan. There are several evaluation criteria presented in the literature for screening potential DSM programs. White (1981) evaluated DSM program cost-effectiveness from the following four perspectives: participant, ratepayer, utility, and society. The California Standard Practice manual, the most widely used method for evaluating DSM programs includes the four basic tests of White. Given below is a short account of the cost-effectiveness of DSM programs from the four perspectives presented by White (1981):

**Participant Perspective**

A program is recommended from this perspective if the following condition is satisfied:

\[
NET \text{ BENEFIT} = INCENTIVES + BILL \text{ SAVINGS} - DIRECT \text{ COSTS} > 0
\]

Incentives include subsidies from the utility and other sources (e.g., federal tax credits). Bill savings are changes in the participants’ electric bill because of energy savings and finally, direct costs include all costs that are associated with installing and operating a particular demand-side measure borne by the participant.

**Nonparticipant Perspective**

The nonparticipant perspective is to measure the distribution equity impacts
of demand-side measure programs on nonparticipating utility ratepayers. The issue in this case is to estimate to what degree nonparticipants must pay or benefits from a DSM program. This test is also called the "no losers test." This test is incorporated as the "ratepayer impact measure" (RIM) in the California Standard Practice. The nonparticipant test" or the "no losers test" can be formally expressed as follows:

\[
NET\ BENEFITS = AVOIDED\ SUPPLY\ COSTS - REVENUE\ LOSSES - PROGRAM\ COSTS > 0
\]

Avoided supply costs are the decrease in generation costs (marginal cost times energy savings) due to the DSM program. Revenue losses can be defined as the loss in revenue (price times energy savings) to the utility due to the DSM program. Program costs define the total amount spent by the utility on the DSM program. These include utility DSM equipment costs, marketing and administrative costs, and utility subsidies. Thus the above test is cost-effective under this perspective if the cost of conservation per kWh saved is less than the difference between marginal cost and the average price of electricity per kWh (Ruff, 1988).

Utility Perspective

The utility perspective can be defined in one of two ways: as an accounting of utility costs as in the nonparticipant perspective (the assumption of White (1981) in his paper), or as a stockholder perspective focused on opportunities to earn a return.
The latter viewpoint is complicated and is not incorporated in any existing screening methods. However, with shareholder incentives for DSM programs now in vogue, the shareholder perspective is the correct index. The utility perspective can be formally stated as follows:

\[ \text{NET BENEFITS} = \text{AVOID. SUPPLY COSTS} - \text{UTIL. PROG. COSTS} > 0 \]

Supply-side alternatives are also usually screened by this version of the utility perspective. That is, there are a range of supply-side alternatives available to meet future projected power demands and the utility chooses the one having the lowest cost.

**Societal Perspective**

This perspective is known as the "total resource cost" (TRC) test in the California Standard Practice. This perspective eliminates the distinction between participants and nonparticipants. The objective is to determine whether a program is cost-effective to society based on the total costs and benefits of a program, independent of its allocation among shareholders, ratepayers, and participants. The societal costs can also incorporate so-called externalities (e.g. acid rain damage costs) in the cost/benefit analysis. In this case, the TRC test is the sum of the participant and the nonparticipant net benefits since the bill savings and the revenue losses cancel each other. We define the TRC test formally, as follows:
$NET\ BENEFITS = AVOID.\ SUPPLY\ COSTS + AVOID.\ ENV.\ COSTS$
- $CUST.\ PROG.\ COSTS - UTIL.\ PROG.\ COSTS > 0$

The TRC test is also known as the "least cost" (LC) test. This test will be henceforth referred to as the LC/TRC test.

The above four perspectives together form the basis for the California Standard Practice manual, the most widely used method for evaluating DSM programs. The Standard Practice recommends the LC/TRC test as a basis for screening conservation programs and the RIM test (nonparticipant perspective) for screening load building programs. There are a few utility economists who recommend that the "nonparticipant test" should be used for all programs (Ruff, 1988). But RIM is primarily used as a test of equity rather than economic efficiency in the Standard Practice.

The LC/TRC test could lead to erroneous program selection under any of the following conditions:

- DSM program participants have a non-zero price elasticity of demand for energy services, i.e., they alter the amount of heat, light, etc., they consume in response to changes in the effective price of providing these energy services.

- Program participants have a non-zero price of elasticity of demand for
electricity and the price they pay for electricity does not equal the marginal cost of electricity supply.

The program is a load building program. The LC/TRC test would incorrectly reject these programs in all cases, even when participants and nonparticipants alike receive substantial benefits.

In each of the above cases, DSM programs affect the value received by utility customers. The above conditions often occur necessitating a more comprehensive evaluation criterion for program selection. The need for a new test cannot be overemphasized considering the magnitude of DSM program investments and its effects on national energy self-sufficiency. The State of California alone spends more than $250 million annually on DSM programs. The type of value effects that are ignored by the LC/TRC, due to the assumption that the amount of energy services are fixed, are discussed in section 2.4.

This thesis derives a criterion, the "most value/ generalized total resource cost" (MV/GTRC) as a more appropriate efficiency test, as it accounts for changes in customer value. It is an enhancement of the LC/TRC test of the California Standard Practice in that it considers the benefits of the changes in the amount and value of energy services provided by electricity in addition to the various cost categories considered by the TRC test. Unlike the LC/TRC test which is recommended by the Standard Practice only for load management and conservation programs. The MV/GTRC test can be used to evaluate both load reduction and load building
programs on a consistent basis. The differences between the Standard Practice tests and the MV/GTRC tests are as found in Table 2.1.

The LC/TRC test is an appropriate measure of economic efficiency under the following basic condition: that the DSM does not alter, directly or indirectly, the amount or quality of energy services provided to participants in the program or the amount of electricity bought by nonparticipants. But DSM programs can and do often have such effects as described in the next subsection.

2.4. Value of Energy Services

In this subsection is presented a few reasons why the quantity or quality of energy services could change following the implementation of the DSM program.

Takeback/Rebound

Participants in a DSM program may take advantage of the utility subsidy to consume more energy services rather than to save energy. Thus, part of the anticipated energy savings does not occur -- the so-called rebound or takeback effect. This happens because this subsidy lowers the effective price of energy services to the customer. If the participant has a non-zero price elasticity of demand for energy services, then the customer will consume more energy services (Frey and
Table 2.1
Categories of Benefits/Costs
of the Various Tests

<table>
<thead>
<tr>
<th>TESTS</th>
<th>BENEFITS</th>
<th>COSTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOST VALUE/GENERAL TOTAL RESOURCE COST TEST (MV/GTRC)</td>
<td>AVOIDED SUPPLY COSTS; AVOIDED ENV. COSTS; DIRECT BENEFITS TO PARTICIPANTS; BENEFITS OF RATE CHANGES;</td>
<td>NET SOCIETAL COSTS EXCLUDING FREE RIDERS;</td>
</tr>
<tr>
<td>LEAST COST/TOTAL RESOURCE COST TEST (LC/TRC)</td>
<td>AVOIDED SUPPLY COSTS; AVOIDED ENV. COSTS;</td>
<td>NET SOCIETAL COSTS EXCLUDING FREE RIDERS;</td>
</tr>
<tr>
<td>RATEPAYER IMPACT MEASURE (RIM)</td>
<td>AVOIDED SUPPLY COSTS;</td>
<td>LOST REVENUE; TOTAL DSM PROGRAM COSTS;</td>
</tr>
<tr>
<td>GENERAL RATEPAYER IMPACT MEASURE (GRIM)</td>
<td>AVOIDED SUPPLY COSTS; BENEFITS OF RATE CHANGES;</td>
<td>LOST REVENUE; TOTAL DSM PROGRAM COSTS;</td>
</tr>
</tbody>
</table>

LaBay, 1988). For example, participants in a weatherization program may decide to set their thermostats to more comfortable levels because the cost of doing so has fallen. Although the actual savings fall short of the expected (technical or engineering) savings, participants are better off because they could have chosen to
keep their thermostats at the same levels, but did not do so.

Changes in Quality of Energy Services

Improvements in convenience, frequency, voltage, and other attributes are of concern to utilities who are operating in an increasingly competitive environment. They essentially seek to enhance the attractiveness of their product by altering its various attributes and tailoring them to each customer's needs (i.e. product differentiation). DSM programs provide benefits in this regard. For example, a weatherization program could eliminate "cold spots" in a house leading to increased comforts. Such improvements are the thrust of the Electric Power Research Institute's concept of "Integrated Value-Based Planning" (Gelling and Chamberlin, 1987).

Load Building/Fuel Substitution Programs

These DSM programs increase the amount of energy services provided by electricity either by creating new services (load building; e.g., security lighting or thermal storage programs) or by switching the fuel used to provide existing energy services (fuel substitution; e.g., heat pump programs for customers now using natural gas for heating). In the case of new services, the Standard Practice states that the RIM test rather than the LC/TRC test should be used as the main criterion for cost-effectiveness. If the quantity of energy services remain unaffected, then it is a valid test. Using RIM in this manner, assumes that the value of energy services that
receive equal their increase in electric bills. Moreover if the quality or quantity of energy services change, then the change in benefits should, in theory, be accounted for.

**Electric Rate Impacts**

In general, implementation of DSM programs will result in changes in electric rates due to the interaction of avoided costs, revenue losses, and the program’s costs to the utility. If the demand for electricity of the affected ratepayer classes has a nonzero price elasticity, these rate changes will affect the amount of electricity purchased. For instance, if electric rates fall, then nonparticipants will buy more power increasing their consumption of energy services, decrease their conservation efforts, and switch from competing fuels. But if electric rates increase, nonparticipants could increase conservation efforts, or switch to other fuels, or "bypass." It is to be noted that if the price of electricity equals the marginal cost of electricity, these changes are not of concern since the marginal benefit of energy services (which equal price) equal the marginal cost of providing them.

Thus these effects could be significant and are ignored by the Standard Practice. This assumption, as is clear from the previous paragraph, is valid only if the nonparticipants have a price elasticity of demand for electricity equal to zero or the marginal cost of electric power equals the price of power.
Reliability Benefits

If the expected loss of load falls as the result of DSM, supply costs increase, but so too do the benefits received from electricity (Sanghvi, 1990). These benefits should be credited to DSM programs.

Although the Standard Practice acknowledges the existence of many of these above value effects of DSM upon energy services, the versions of its tests that have been implemented exclude benefits of energy services and focus just on resource costs. Until recently, there have been good reasons for doing so, including an absence of the data and methodologies needed to estimate those benefits.

However, a practical version of a societal test that includes several of these categories of benefits has been developed in this thesis and is presented in the next chapter. Hobbs (1991) provides a derivation of this criterion for a simple case in which the program expenditures and impacts all occur in a single year and there is just one customer class. This test is, in a sense, a reduced gradient (the gradient of the value function w.r.t implementation of the DSM program while satisfying revenue requirements constraints). The MV/GTRC test can include the benefits of takeback, load building/fuel substitution programs, and electric rate changes. The measurement of these benefits is firmly grounded in the widely recognized and accepted concept of "consumers surplus", which is the basis of much of applied welfare economics and
benefit-cost analysis (Mishan, 1973). Quality and reliability benefits can also be included in the test in an ad hoc manner. The step-by-step derivation of the general version of the MV/GTRC criterion would be presented in Chapter 3. MOSTVALUE, a spreadsheet model which was developed for the Centerior Energy Corporation to screen demand-side management programs, is described in section 3.5 of chapter 3. Several DSM programs will also be evaluated in section 3.6, using MOSTVALUE, to illustrate the differences between the various screening criteria.

A more general version of the "ratepayer impact measure" (RIM) or the "no losers" test which includes the effects of rate changes caused by the adoption of DSM programs will also be presented in this thesis in Section 3.4 of Chapter 3. This test basically measures the change in consumer surplus or net benefits to nonparticipants when the price elasticity of demand for electricity is not equal to zero. This test is called the "general ratepayer impact measure" (GRIM). This test is recommended primarily as a measure of equity, since the MV/GTRC test can be used to evaluate all types of DSM programs on a consistent basis. Nelson and Hobbs (1992) show that GRIM reduces to the MV/GTRC test if there are no externalities or free riders, and participants are indifferent to the program.

Chapter 4 corrects certain shortcomings of the basic form of the MV/GTRC
test presented in Chapter 3. Section 4.2 discusses the version of the MV/GTRC test with time lags in customer response to DSM program induced rate changes. This thesis will also present the derivation of a version of the MV/GTRC test which corrects a shortcoming of the earlier versions with regard to fuel switching programs. It accounts for changes in "value" in the alternative fuel market as well during fuel switching programs. This thesis would be the first attempt at incorporating such value changes which result from fuel switching. Finally, a multiattribute framework to analyze the impacts of DSM programs is also discussed in Chapter 4.
CHAPTER 3

DERIVATION AND APPLICATION OF THE BASIC MV/GTRC TEST

3.1 Introduction

This chapter provides the derivation and equations for the basic form of the MV/GTRC test described in the previous section. The MV/GTRC test is intended to serve as a tool for electric utilities to screen their demand-side management (DSM) programs. This test should be satisfied if the objective of the regulated utility is to maximize net societal benefits subject to the utility recovering its costs. The Standard Practice tests (California Public Utilities Commission and California Energy Commission, 1983 and 1987), which are currently the norm for screening DSM programs, are valid only if the programs do not alter the amount or value of energy services provided to customers. But, as shown earlier, DSM programs do often have such effects, and, as a result, the value customers receive is changed. The MV/GTRC test presented in this thesis is the only theoretically correct and practical test available for considering the effects of DSM on customer value.
Section 3.2 discusses the basis for the formulation of the "value" problem and its resultant derivation, the MV/GTRC test. This section also contains a detailed discussion about assumptions made in deriving the test. Section 3.3 presents the derivation and formulation of the MV/GTRC criterion for the case where there are many customer classes, several blocks in the load duration curve, and a multi-year time horizon. Section 3.4 presents the derivation of the generalized ratepayer impact measure (GRIM) test discussed in the previous chapter. This measure is intended to be a measure of equity rather than a measure of economic efficiency.

Section 3.5 discusses the MOSTVALUE spreadsheet model, implemented by Nelson and Hobbs (1990a), based on the criteria derived in Sections 3.3 and 3.4. This spreadsheet model was used by a large midwestern utility for screening demand-side management programs while formulating their long-term forecast plan. Several sample DSM programs are evaluated in Section 3.6 to illustrate the differences between the MV/GTRC test and the LC/TRC test.

This chapter deals with only the basic form of the MV/GTRC test. The criterion presented in this chapter is valid for DSM programs if we assume:

- rate changes impact demands only in the year in which the utility program expenses are rate based, i.e., there is no lagged effects of rates on customer demand for electricity.
the DSM programs do not involve fuel switching.

- economic efficiency is the sole criterion for the selection of a DSM program.

Chapter 4 discusses versions of the MV/GTRC test that incorporates the above three major extensions of the basic form of the MV.

3.2 Assumptions of the MV/GTRC Test

A regulated investor owned utility's goal when designing a demand-side management program is assumed to be to maximize the net economic benefits to participants and nonparticipants, as measured by their "consumer surplus," minus external environmental costs.

The essence of the Marshallian concept of consumer's surplus is that a consumer derives extra satisfaction or utility from the consumption of goods than the actual price they pay for them (Marshall, 1920). This concept is derived from the law of diminishing utility. As we purchase more units of a good, its marginal utility goes on diminishing. The consumer is in equilibrium when marginal utility becomes equal to the given price. In other words, consumers purchase the number of units of a commodity at which marginal utility equals price. Hicks (1946) provides three ways
of quantifying the consumer's surplus.

Let the distance OA in Fig. 3-1 represent the consumer's income. He achieves a tangency solution at point D on indifference curve I₂. If he were unable to consume Q, he would be at point A on a lower indifference curve I₁. He would have to be given an income increment of AB dollars to restore him to the previous indifference curve I₂. This increment, called compensating income variation, is denoted by c, and provides a measure of the change in consumer's welfare.

At the given prices, the consumer would be willing to forego AC dollars of income rather than lose his opportunity to consume good Q. With income OC, his consumption is at E, which is on the same indifference curve as A. The amount corresponding to AC is called equivalent income variation, is denoted by e, and provides an alternate measure of consumer surplus. A third measure is provided by the demand curve in Fig. 3.2. It equals the area ABC, which is the difference between the area lying under the demand curve (OABD) and the consumer's expenditure (ODBC) and is denoted by s. Willig (1976) shows that $c \geq s \geq e$. Strict inequalities hold in Fig. 3-1 because of the income effect. If the consumer were forced to pay more to consume the good, his demand would decline because of a lower effective income, and the area under the curve would exceed the amount that he would pay rather than forego consumption of the good. Fig. 3-3 shows the case
Figure 3.1: Theory of Consumer Surplus
Figure 3.2: Theory of Consumer Surplus
Figure 3.3: Theory of Consumer Surplus
where the income effect is zero throughout. In this case AB=AC, and the three measures of consumer's surplus are the same. Now if the amount expended on good Q is a small portion of the consumer's income, then we can ignore the income effect. Therefore Fig. 3-3 applies to such a case. In this thesis, we assume that the average customer's expenditure on electricity is a small portion of his income and that the area under the demand curve provides an accurate measure of consumer's surplus. Willig (1976) shows that a good would have to be more than 10% of overall income for this effect to matter.

Consumer surplus has previously been proposed as an appropriate objective for IRP and DSM evaluation, (e.g., Costello and Galen, 1984, Barrager and Buckley, 1987, and Caves et al., 1983). The net benefit to the utility (the so-called producer surplus) can be omitted since it equals zero under the assumption that all utility costs are recovered from ratepayers. The current trend of regulatory incentives to utility shareholders based on achieved savings implies that the producer surplus is not zero in these cases. From society's viewpoint, it is an income transfer from the ratepayers to the utility shareholders if rate effects are ignored.

The MV/GTRC test differentiates between participants and nonparticipants. The term "participants" refer to the directly impacted uses of power customers who participate in a utility DSM program, excluding customers who would have made an
investment in the demand-side measure even in the absence of the utility program ("free riders"). "Nonparticipants" include: consumers not participating in the program; free riders; and participants’ electricity uses that are unaffected by the program.

The participants’ net benefits are defined as the integral of their demand curve for the energy service affected by the program minus (1) any direct costs they incur as part of the program and (2) their electricity bills. Nonparticipants’ net benefits equal their consumer surplus, defined as the integral of their demand curve for electricity minus their power bills. It is proved elsewhere that using the consumer surplus measured using the demand curve for electricity is the same as that calculated using the demand for energy services for nonparticipants (Hobbs, 1991 and Nelson and Hobbs, 1991a). Thus the problem can be defined as:

**MAX: PW OF**

\[\text{BENEFITS OF CONSUMPTION OF ENERGY SERVICES BY PARTICIPANTS IN PERIOD } i \text{ OF YEAR } t + \text{BENEFITS OF ENERGY SERVICES CONSUMPTION BY NON-PARTICIPANTS IN PERIOD } i \text{ OF YEAR } t - \text{COSTS OF POWER GENERATION IN PERIOD } i \text{ OF YEAR } t - \text{DSM PROGRAM COSTS IN YEAR } t - \text{EXTERNAL ENVIRONMENTAL COSTS IN PERIOD } i \text{ OF YEAR } t\]

\[\text{s.t: REVENUE RECOVERY CONSTRAINT IN EACH YEAR (3.1)}\]
Let us now derive the MV/GTRC test after defining a few variables. Let $\theta$ be a variable which designates whether or not a program is implemented. If $\theta=0$, then no DSM expenditures are made. If $\theta=1$, then the program is fully implemented. Values of $\theta$ between 0 and 1 represent partial implementation. The problem at hand is to determine the change in net social benefits or "value" resulting from the implementation of the DSM program. Value is defined in this case to be equal to the sum of consumer surplus for the participants and the nonparticipants, minus the external costs. The variables used in our derivation are defined below:

$$P_t = \{P_{ct}\},$$ the vector of prices [$/kWh] for customer classes $c$ in year $t$. In general, only those classes whose prices would be affected by DSM are considered.

$$Q_i = \{Q_{ci}\},$$ the vector of nonparticipant energy demands [kWh] for the classes from which DSM costs will be recovered in period ‘i’ of year $t$. $Q_{ci}$ is a function of price $P_{ci}$.

$$QP_i = \text{Participants’ energy demands [kWh] in period i of year t. It is assumed below that this is a small fraction of the total system load. For the participant customer class p, total energy demand } = QP_i + \Delta QP_i. \text{ } \Delta QP_i \text{ is the change in electricity consumption in period ‘i’ of year ‘t’ if the program is implemented.}$$

$$B_p(P_{pt}, \theta) = \text{Gross benefits [$]} \text{ in year t of the consumption of energy services by participants, equal to the integral of the ordinary demand curve:}$$
\[
\sum_{i} \int_{0}^{Q_{S_{pi}}(P_{pi}, \theta)} p_{S_{pi}}(q) \, dq 
\]  

(3.2)

where \(Q_{S_{pi}}\) is the quantity of energy services consumed by participants and \(p_{S_{pi}}\) is the demand curve (price) of energy services to participants in period \(i\) of year \(t\). \(Q_{S_{pi}}\) is a function of \(P_{pi}\) and \(\theta\).

\[B_{m}(P_{i}) = \text{Gross benefits [\$] in } t \text{ of electricity usage by nonparticipants as a function of the price of electricity.}\]

\[
\sum_{i} \sum_{c} \int_{0}^{q_{ci}(P_{ci})} p_{ct}(q) \, dq 
\]  

(3.3)

where \(p_{ct}(q)\) is the demand curve for electricity by nonparticipants in class \(c\) in year \(t\). \(B_{m}\) is a function of price because the upper limit of the integral \(q_{ci}\) depends upon \(P_{ct}\).

\[C_{i}(P_{i}, \theta) = \text{Cost of electricity production in year } t. \text{ It is a function of } Q_{i} \text{ and } QP_{i}, \text{ but since } QP_{i} \text{ is a function of } P_{pi} \text{ and } \theta, \text{ and } Q_{i} \text{ is a function of } P_{i}, \text{ the cost of electricity production can be expressed as a function of } P_{i} \text{ and } \theta.\]

\[= C_{c} \left( \sum_{i} \left[ \int_{0}^{Q_{S_{pi}}(P_{pi}, \theta)} p_{S_{pi}}(q) \, dq + QP_{ct}(P_{ct}) + \theta QP_{ct}^{\theta} \right] \right) \]

where \(\Delta QP_{i}^{\theta}\) is the change in electricity consumption by participants in period \('i'\) of year \('t'\) if the program is implemented.

\[CE_{i} = \text{Utility's expenditure on DSM equipment for participants in } t\]
For example, purchases of light bulbs by the utility that are distributed to participants.

\[ CM_t = \text{Fixed marketing and administrative costs of the program in } t \text{ [$.]} \]

\[ CP_t = \text{Transfer payments to participants in } t \text{ [$.]. Examples include appliance rebates and discounts on electric bills.} \]

\[ DB_{pt} = \text{Gross direct benefits to participants in year ‘} t \text{’ if the program is a load building program. At the minimum, this equals the incentive price times the increased consumption of electricity.} \]

\[ EQ_t = \text{Participant’s direct equipment costs in } t \text{, plus any other participant expenses such as installation time [$.].} \]

\[ EX_t(P, \theta) = \text{External costs of electricity production, such as air pollution damages [$.]. It is a function of } Q_t \text{ and } QP_t \text{ and, thus, } P_t \text{ and } \theta \text{ for the same reason as } C_t(P, \theta) \text{ above.} \]

\[ Fr = \text{Fraction of program participants who are free riders.} \]

\[ NE_t(\theta) = \text{Net DSM program cost [$.] from a societal perspective, which excludes transfer payments and free rider costs. Free rider costs are excluded because utility expenditure just replaces participant costs that would have taken place even without the program. This equals:} \]

\[ = \theta[(1-Fr)(CE_t+EQ_t) + CM_t] \]

\[ UE_t(\theta) = \text{Utility’s expenditure on the program in year } t \text{ [$.], which is assumed to be recovered from ratepayers in that year. (Other assumptions concerning cost recovery, such as rate-basing of} \]
DSM program costs, can also be made by appropriately defining CE, CM, and CP.)

\[ \theta(\text{CE}_t + \text{CM}_t + \text{CP}_t) \]

3.3 Derivation of the MV/GTRC Test

In terms of the variables defined above, the present worth of the value to society of power consumption \( V(\mathbf{P}, \theta) \) equals:

\[
V(\mathbf{P}, \theta) = \sum_{t} \frac{1}{(1+i)^t} \left[ B_{pt}(\mathbf{P}_{pt}, \theta) + B_{nt}(\mathbf{P}_n) \right. \\
- \left. C_{t}(\mathbf{P}_t, \theta) - NE_{t}(\theta) - EX_{t}(\mathbf{P}_x, \theta) \right]
\]  

(3.4)

where \( i \) is the interest rate and \( \mathbf{P} \) is the vector of all energy prices for all classes and years. This is equivalent to the sum of the consumer surplus for participants and nonparticipants, minus any external costs.

The net benefits of the DSM program can be approximated by the total derivative of the value function (3.4) with respect to \( \theta \), if the DSM program is small. The expression \( dV/d\theta \) includes \( dP_{ed}/d\theta \), which describes how rates change if the program is implemented. The latter derivative is obtained by assuming that in each year, the utility’s revenues just cover its supply and DSM costs:
\[ P_{pt} \cdot QP_{\epsilon} (P_{pt}, \theta) + \sum_{c} P_{ct} Q_{ct} (P_{ct}) = C_{\epsilon} (P_{\epsilon}, \theta) + UE_{\epsilon} (\theta) \] (3.5)

where: \( QP_{\epsilon} = QP_{\epsilon}^0 + \Delta QP_{\epsilon} \)

There is one such revenue recovery constraint for each year \( t \). By (1) taking the total derivative of the above expression with respect to \( \theta \), and (2) assuming a relationship among the \( P_{ct} \) in a given \( t \) (here, we assume that \( dP_{ct} / d\theta \) is the same for all \( c \)), \( dP_{ct} / d\theta \) can be obtained. The MV/GTRC test is derived by inserting the various expressions for \( dP_{ct} / d\theta \) in \( dV/d\theta \).

The derivation of the MV/GTRC test for a simple case, in which the program expenditures and impacts all occur in a single year and costs are recovered from just one customer class (e.g., residential, commercial, or industrial), is presented by Hobbs (1991).

In the discussion below, we derive the MV/GTRC test in two parts. Initially, in subsection 3.3.1, we assume that there are no benefits due to rebound or takeback, i.e., the DSM program does not alter the amount of energy services consumed by the participants. If the program is a load building one, the benefits to participants equal the direct benefits \( DB_{pt} \) which, at a minimum, equals the amount participants pay for their additional power consumption. Load building programs alter the amount of
energy services consumed by creating new demand for energy services. We then calculate, in subsection 3.3.2, the change in benefits to participants in a conservation program due to takeback/rebound separately and combine the two parts to form the MV/GTRC test. Subsection 3.3.3 provides an interpretation of the various components of the MV/GTRC test.

3.3.1 MV/GTRC Test Without Rebound or Takeback

Now, the net benefits of the program approximately equal the derivative of the value function (3.4) as long as the program is small\(^1\). This derivative evaluated at \(\theta=0\), is:

---

\(^1\)
The actual net benefits due to the program is the difference in the value function when \(\theta=1\) and \(\theta=0\). Costello and Galen (1984) use this approach to determine the value due to a DSM program. This leads to a complicated expression for the change in value that is not easy to interpret in a concise form. The derivation in this thesis takes a simplifying approach in that we assume that the slope of this function at the reference point (\(\theta=0\)) represents the value change due to the DSM program. This is done for the following reasons: (1) the programs screened by this model are not too large, and (2) we need a criterion that is as easy to interpret as the existing tests while offering enhancements.
\[
\frac{dv}{d\theta} = \\
\sum \frac{1}{(1+I)^2} \left[ \sum \frac{dp_{ct}}{d\theta} \sum \frac{\partial b_{ct}}{\partial q_{ct}^i} \frac{\partial q_{ct}^i}{\partial P_{ct}} + DB_{pt} (1-Fr) \right] \\
- \sum \frac{\partial c_{ct}}{\partial Q_{ct}^i} \left[ \sum \frac{dp_{ct}}{d\theta} \frac{\partial q_{ct}^i}{\partial P_{ct}} + \Delta QP_{ct}^i (1-Fr) \right] \\
- \sum \frac{\partial E_{ct}}{\partial Q_{ct}^i} \left[ \sum \frac{dp_{ct}}{d\theta} \frac{\partial q_{ct}^i}{\partial P_{ct}} + \Delta QP_{ct}^i (1-Fr) \right] \\
- \left[ (CE_{ct} + EQ_{ct}) (1-Fr) + CM_{ct} \right] \]

Note that the total derivative is taken rather than the partial derivative, since \( P \) is an implicit function of \( \theta \).

By definition, the following substitutions can be made:

\[
\frac{\partial b_{ct}}{\partial q_{ct}^i} = p_{ct} \quad (3.7)
\]

The above definition follows directly from the theory of consumer surplus, where at the optimal level of consumption of a good, the marginal benefit of consumption of a good equals the price of a good.
\[ \frac{\partial C_t}{\partial Q^t} \Delta MC^t_{1t} \] (3.8)

The above definition defines the marginal cost of power generation \(MC^t_{1t}\) for each period 'i'. We assume, in this derivation, that the cost of power generation in any period is independent of power consumption in any other period. In Nelson (1989), a situation was analyzed where cost of power generation depends on power consumption in all other periods.

\[ \frac{\partial EX^t}{\partial Q^t} \Delta ME^t_{1t} \] (3.9)

The above definition gives the marginal external cost of power generation for each period 'i'. This could, for example, be the marginal damage cost per KWh of power generated.

\[ \frac{\partial Q^t_{ci}}{\partial P^t_{ct}} = -E^t_{ct} \frac{P^t_{ct}}{Q^t_{ci}} \quad \forall i \] (3.10)

The expression (3.10) above is obtained from the classic definition of price elasticity of demand \(E_{ct}\) for a small change in price. We also assume that there is no variation in price elasticity of demand over different periods in a year.

Substituting (3.7)-(3.10) into (3.6) yields:
\[
\frac{dV}{d\theta} = \sum \frac{1}{(1+F)^t} \left[ -\sum E_{ct} Q_{ct} \frac{dP_{ct}}{d\theta} + DB_{pt}(1-Fr) \right] - \sum MC_{st} \left[ -\sum E_{ct} \frac{Q_{ct}}{P_{ct}} \frac{dP_{ct}}{d\theta} + \Delta Q_{ct} \right] (1-Fr) \]

(3.11)

\[
- \sum ME_{st} \left[ -\sum E_{ct} \frac{Q_{ct}}{P_{ct}} \frac{dP_{ct}}{d\theta} + \Delta Q_{ct} \right] (1-Fr) - \left[ (CE_t + EQ_t)(1-Fr) + CM_t \right]
\]

It might be argued that, for a small program, the change in rates \(dP_{ct}/d\theta\) is so small that it can be ignored. The above expression (3.11) shows that it would be wrong to disregard \(dP_{ct}/d\theta\), since the net benefit of the program \(dV/d\theta\) is linearly related to it. If the program is small, all the terms on the right side of (3.11) are small, and the contribution of the \(dP_{ct}/d\theta\) terms can still be relatively important.

For the derivation to be complete, we need to calculate the values of the derivative of the price with respect to \(\theta\) in (3.11). The value of this derivative depends upon the pricing policy of the utility and its regulating authority. If the DSM costs including lost revenues are not recovered through rates, then this derivative is zero and the benefits of rate changes are zero as well. Lost revenues equal the difference between price and marginal cost times the energy savings. If price exceeds marginal cost, lost revenues are positive or vice versa. In this derivation, we assume
that all program costs are recovered through rates, i.e. the utility has zero economic profit. Thus revenue minus costs equals zero and in each year 't':

\[ \sum_c P_{ct} \left[ \sum_i Q_{ci}^t \left( P_{ct} \right) \right] + P_{pc} \left[ \sum_t QP_i^0 + \theta \cdot \Delta QP_i^t (1 - Fr) \right] \]
\[ -C_t \left( \sum_c Q_{ct}^t \left( P_{ct} \right) + QP_i^0 + \theta \cdot \Delta QP_i^t (1 - Fr), \forall i \right) \]
\[ -UE_t(\theta) = 0 \]

(3.12)

where \( UE_t(\theta) \) is the total expenditure by the utility including those for free riders.

Let us now take the total derivative of (3.12) with respect to \( \theta \) and evaluate the expression at \( \theta = 0 \). Now an assumption must also be made regarding the distribution of price changes across classes c. In our derivation, we assume that the value \( dP_c / d\theta \) is the same over all classes. We also assume that the participant load is a small fraction of the total customer load.

Therefore, rearranging terms and, taking the total derivative of (3.12) yields:

\[ \frac{dP_{ct}}{d\theta} = \frac{(1 - Fr) \sum_i (P_{ct} - MC_{ct}) \Delta QP_i^t - UE_t}{[Q_{nt} - \sum_c E_{ct} Q_{ct}^t + \sum_t MC_{ct} \sum_c \frac{E_{ct} Q_{ct}^t}{P_{ct}}]} \]

(3.13)

Now the numerator of the expression on the right hand side can be defined as the RIM test of the California Standard Practice for year 't'. RIM, as explained earlier in Chapter 2, equals the lost revenues minus total utility costs. Thus
substituting (3.13) into (3.11) and dividing the numerator and the denominator of (3.13) by $Q_{nt}$ leads to the MV/GTRC test:

$$\frac{dV}{d\theta} = \sum_{c} \frac{1}{(1+I)^{c}} \left[ \sum_{i} (MC_{it} + ME_{it}) \Delta QP_{i}^{c} (1-Fr) \right.$$

[Avoided Costs]

$$+ DB_{pt} (1-Fr)$$

[Direct Participant Benefits]

$$- [(CE_{c} + EQ_{c}) (1-Fr) + CM_{c}]$$

[Net Societal Program Costs]

$$\left. - \frac{E_{nt}}{E_{nt}} - \sum_{i} (MC_{it} + ME_{it}) \sum_{c} \frac{E_{ct} f_{ci}^{c}}{P_{ct}} \right]$$

[Benefits of Rate Changes]

$$-RIM_{t}$$

$$1 - E_{nt} - \sum_{i} MC_{it} \sum_{c} \frac{E_{ct} f_{ci}^{c}}{P_{ct}}$$

where:

$$RIM_{t} = (1-Fr) \sum_{i} (P_{pt} - MC_{it}) \Delta QP_{i}^{c} - [CE_{c} + CM_{c} + CP_{c}] .$$

$P_{pt}$ = Price in year ‘t’ for the participant class.

$E_{nt}$ = Average price elasticity of electricity for the nonparticipant class in year ‘t’

$$= \sum_{c} E_{ct} f_{ci}^{c}$$

$f_{ct}^{i}$ = Fraction of electricity consumed in period ‘i’ of year ‘t’ by customer class
'c'. \[ \sum_c \sum_l f_{ct}^c = 1. \]

Equation (3.14) above may overestimate the net benefits of large DSM programs in which the energy savings are several percent of the load. This is because of the diminishing returns effect. For example, the largest supply cost savings result from the first few MW that are saved. This bias is a potential problem with both the LC/TRC and the MV/GTRC tests. As another example, if P>M, then the greater the energy savings, the larger will be the deviation between P and MC because a conservation program will simultaneously cause P to increase and MC to decrease\(^2\). For many conservation programs, this will worsen the benefits of rate changes term in (3.14). However this inaccuracy is unimportant for typically sized programs. As an example, consider the case in which P = $0.08/kWh, marginal cost MC = $0.05/kWh, price elasticity for nonparticipants \(E_a\) is 0.5, and the program costs $0.04/kWh saved. Even for a large program which would lower the utility’s load by 3.3%, the benefits of rate changes term in (3.14) would still be no more than 10% smaller in magnitude than the true value.

Derivation of expression (3.14) completes the first part of the derivation.

\(^2\) If the DSM is a load shifting program, decrease in peak consumption and increase in off-peak consumption could actually cause price and marginal cost to decrease.
Herein, we assumed that the cost of capacity additions are included in the term \( C_c(P_n, \theta) \). In the derivation which led to the development of the MOSTVALUE model, we assumed a cost structure with explicit separation of capacity and energy costs. The resulting criterion is presented in Nelson and Hobbs (1990a). In (3.14) above, we assume that there is no rebound/takeback for the DSM programs i.e., the conditional elasticity of demand for electricity is zero. We use the term "conditional" because this elasticity is conditional on a fixed set of conservation investments. This is not necessarily true for conservation programs. Costello and Galen (1984) suggest several ways that the conditional price elasticity can be calculated. Given below is the derivation of the change in benefits to participants in a conservation program due to takeback/rebound should \( E_p \), the conditional demand for electricity, be non-zero.

3.3.2 MV/GTRC Test with Rebound/Takeback

Let \( QP_b \) be the participant's electricity demand as a function of price \( P_{pi} \) and \( \theta \), the implementation of the program.

\[
QS_{pi}^x = \text{Participant’s demand for energy services (heat, light etc.) in period i of year t.}
\]
\[= K_\theta QP_{it}^\alpha, \text{ a production function}^3, \text{ where:} \]

\[K_\theta = \text{a constant which depends on whether the DSM program is implemented (i=0) or not (i=1).} \]

\[\alpha = \text{a production coefficient.} \]

\[= \beta_{s\tau}^\varepsilon P_{st}^{-\varepsilon}, \text{ a demand function where:} \]

\[\beta_{s\tau}^\varepsilon = \text{a constant} \]

\[P_{st} = \text{effective price of energy services [$/unit]} \]

\[\varepsilon = \text{price elasticity of demand for } QS_{pi}^i \]

From the theory of consumer surplus, we can say that the participant spends money on consuming energy services (i.e., his electric bill) till the marginal cost of energy services equals the marginal benefit (price) of energy services. Since \( QS_{pi}^i \) is

\[3^3 \]

This is equivalent to the familiar Cobb-Douglas production function. For the two input case, it takes the following mathematical form:

\[Q = K^*Q_1^\alpha Q_2^\beta \]

where \(Q\) is the output (energy services), \(Q_1\) is one input (electricity), \(Q_2\) is the other input (capital), and \(K, \alpha\) and \(\beta\) are constants. In our derivation, we assume that \(Q_2\) can take on two values \(Q_{2B}, \theta=0.1\). Therefore,

\[QS_{pi}^i = K_\theta QP_{it}^\alpha \]

where: \(K_\theta \triangle Q_{2B}^{(1-\alpha)}\)
a function of $Q_{P_{II}}$, we have:

$$P_{pt} Q_{P_{II}} = P_{pt} \left[ \frac{Q_{S_{P_{I}}^c}}{K_0} \right]^{1/\alpha}$$  \hspace{1cm} (3.15)

Differentiating (3.15) above with respect to $Q_{S_{P_{I}}^c}$ gives the marginal cost of consumption of energy services. Therefore:

$$\frac{P_{pt}}{\alpha K_0^{1/\alpha}} (Q_{S_{P_{I}}^c})^{\frac{1}{\alpha}-1} = P_{s_{II}}^c$$

$$= \beta_{s_{II}}^{c_{1/\epsilon_c}} Q_{S_{P_{I}}^c}^{-\frac{1}{\epsilon_c}}$$  \hspace{1cm} (3.16)

Rearranging the above expression yields to the following expression:

$$Q_{S_{P_{I}}^c} = \left[ \frac{P_{pt}}{K_0^{1/\alpha} \alpha \beta_{s_{II}}^{c_{1/\epsilon_c}}} \right]^{1-\frac{1}{\epsilon_c}-\frac{1}{\alpha}}$$  \hspace{1cm} (3.17)

Since $Q_{S_{P_{I}}^c}$ is a function of $Q_{P_{II}}$, the above expression can be translated to a demand curve for $Q_{P_{II}}$. 
\[ \mathcal{Q}_{P_{1t}} = \frac{1}{K_0^{1/\alpha}} \left( K_0^{1/\alpha} \alpha \beta_s^{\varepsilon/\varepsilon} \left[ \frac{1}{1 - \alpha \cdot \frac{Q}{P_{pt}}} \right] \right) \mathcal{P}_{P_{s\varepsilon}} \left[ \frac{1}{1 - \alpha \cdot \frac{Q}{P_{pt}}} \right] (3.18) \]

Now the gross benefit of consuming \( Q_{S_{pl}} \) is defined as the integral of the inverse of (3.17) from 0 to \( Q_{S_{pl}} \). It is shown later in this section that this also equals the integral of the inverse of (3.18) from 0 to \( Q_{P_{1t}} \).

The benefit of consuming \( Q_{P_{1t}} \), \( B_{\theta,1}(Q_{P_{1t}}) \) is as given below:

\[ B_{\theta,1}(Q_{P_{1t}}) = \int_0^{Q_{P_{1t}}} \frac{Q}{P_{\theta,1}}^{-1/\varepsilon} dQ (3.19) \]

where \( \varepsilon_p \) is the conditional price elasticity of demand for electricity.

Now if \( \varepsilon_p < 1 \), then the above integral is undefined. In that case, let the lower limit be \( 0^* \), and a constant \( C \) is added to represent the value of consumption from 0 to \( 0^* \). Since it is the change in benefits which matter, this constant does not affect the results of this section.

Therefore the gross benefits to program participants are:
\[ B_{\theta, t}^c (QP_{t,t}) = C + \beta_{\theta, t}^{1/\varepsilon_p} \frac{QP_{t,t}^{(1-\varepsilon_p^{(1-\varepsilon_p)}}}{(1-1/\varepsilon_p)} \]

\[ = C + \frac{\beta_{\theta, t}^{1/\varepsilon_p}R_{\theta, t}^{1-1/\varepsilon_p}}{(1-1/\varepsilon_p)} QP_{t,t}^{(\alpha-\xi_t)} \]

\[ = C + H_{\theta, t} \frac{QP_{t,t}^{(1-\varepsilon_p)}}{(1-1/\varepsilon_p)} \]

\[ = C + J_{\theta, t} P_{p,t}^{1-\varepsilon_p} \]

where: \( J_{\theta, t} = \frac{\beta_{\theta, t}}{(1-1/\varepsilon_p)} \)

The increase \( DB_{p,t} \) in participant's gross benefits resulting from implementing the conservation program is the difference between (3.20) with and without the program:

\[ DB_{p,t} = \sum_i \left[ (C + J_{1, t}) P_{p,t}^{1-\varepsilon_p} - (C + J_{0, t}) P_{p,t}^{1-\varepsilon_p} \right] \]

\[ = \frac{P_{p,t}}{(1-1/\varepsilon_p)} \sum_i \left[ \beta_{1, t} - \beta_{0, t} \right] P_{p,t}^{1-\varepsilon_p} \]

We will assume that the change in price \( P_{p,t} \) due to the implementation of the program is negligible to the change in the value \( J \). Now inserting (3.18) in (3.20) we get:

\[ DB_{p,t} = \frac{P_{p,t}}{1-1/\varepsilon_p} \sum_i \left[ QP_{t,t} (P_{p,t}, 1) - QP_{t,t} (P_{p,t}, 0) \right] \]

\[ = -P_{p,t} \frac{\varepsilon_p}{(1-\varepsilon_p)} \sum_i \Delta QP_i^t \]

where \( \Delta QP_i^t \) is the change in participant kWh demanded in period \( i \) of year \( t \) due to the program, including rebound effects.
Now if $QS_{pl}$ is unchanged (i.e. no rebound/takeback), the ratio of energy consumption by participants after the program/before the program can be obtained from the definition of $QS_{pl}$ expressed as a function of $QP_{it}$. Thus:

$$\left. \frac{QP_{it}(P_{pt}, 1)}{QP_{it}(P_{pt}, 0)} \right|_{qs\ constant} = \left( \frac{K_0}{K_1} \right)^{\alpha} \Delta G \hspace{1cm} (3.23)$$

$(1-G)$ can be interpreted as the fraction of the original use $QP_{it}(P_{pt}, 0)$ which is saved (conserved by the program, if there is no takeback). The value of $G$ can be estimated from engineering studies.

Using (3.18), we can also estimate the ratio of energy consumption after the program to before the program including rebound. This equals:

$$\left. \frac{QP_{it}(P_{pt}, 1)}{QP_{it}(P_{pt}, 0)} \right|_{qs\ variable} = \left[ \frac{K_1^{1/\alpha}}{K_0^{1/\alpha}} \right]^{\tau-1} = G^{(1-\varepsilon)} \hspace{1cm} (3.24)$$

The quantity $(1-G^{1-\varepsilon})$ can be interpreted as the fraction of the original use $QP_{it}(P_{pt}, 0)$ which is saved, including takeback. Thus the change in participant load, i.e. the net savings including rebound/takeback, is:

$$\Delta QP_{it}^f = QP_{it}(P_{pt}, 1) - QP_{it}(P_{pt}, 0) = (G^{(1-\varepsilon)} - 1) QP_{it}(P_{pt}, 0) \hspace{1cm} (3.25)$$

Therefore knowing $G$ from engineering studies (e.g., typically 75-85% for weatherization programs in the Pacific Northwest), $\varepsilon_p$ from econometric studies (e.g.,
Costello and Galen, 1984, Parti and Parti, 1980), $QP_t$ (the present use), and $P_{pt}$ allows us to calculate the change in gross benefits to participants due to changes in energy services $QS_{pi}$ given by (3.22).

Thus the direct benefits to participants $DB_{pt}$ in expression (3.14) can be defined as follows:

$$DB_{pt} = \text{Direct benefits of changes in energy services provided each year to participants as a result of the DSM program, assuming that none of the participants are free riders.}$$

$$= - \varepsilon_p \frac{P_{pt}}{1 - \varepsilon_p} \Delta QP_t$$  
[if a conservation program]

$$= GB_t$$  
[if a load building program]

where:

$$GB_t = \text{Gross direct benefits of program to participants if the program is a load building program. At a minimum, it equals } P_{pt} \text{ times } \Delta QP, \text{ the amount participants pay for their additional power consumption.}$$

It is now shown that the benefits to participants are the same regardless of whether the demand curve for energy services given by (3.17), or the demand curve for electricity given by (3.18), is used. This is important because it allows us to calculate the benefits to nonparticipants of consuming electricity using the integral of
their electricity demand curve.

Now the benefits of consuming \( Q_{sp}^1 \) is:

\[
B_{sl}(Q_{sp}^1) = C + \frac{\beta_{sp}^{\epsilon_0/\epsilon_s}}{1-1/\epsilon_s} Q_{sp}^{1/\epsilon_s} \tag{3.26}
\]

where \( C \), as defined earlier, is the integral from 0 to \( 0^+ \).

Since \( Q_{sp}^1 \) is a function of \( Q_{P_{lt}} \), we have:

\[
B_{sl}(Q_{sp}^1) = C + \frac{(\beta_{sl}^{\epsilon_0/\epsilon_s})^{1/\epsilon_s} K_0^{(1-1/\epsilon_s)} Q_{P_{lt}}^{(1-1/\epsilon_s)}}{1-1/\epsilon_s} \tag{3.27}
\]

\[
= B_{sl}(Q_{P_{lt}}, \theta)
\]

The equivalence of the changes in \( B_{sl}(Q_{P_{lt}}, \theta) \) with the changes in \( B_{\theta, i}(Q_{P_{lt}}) \) can be shown by calculating the derivatives of both and showing them to be equal.

From (3.19), we have:

\[
\frac{dB_{\theta, i}}{dQ_{P_{lt}}} = \frac{Q_{P_{lt}}^{-1/\epsilon_s}}{\beta_{\theta, i}} \tag{3.28}
\]

The derivative of \( B_{sl}(Q_{P_{lt}}, \theta) \) follows directly from (3.26) and is as given below:
\[
\frac{dB_{st}(QP_{it}, \theta)}{dQP_{it}} = (\beta_{st}^{it})^{1/\epsilon_s} K_{\theta}^{(1-1/\epsilon_s)} \alpha Q_{P_{it}}^{(\alpha - \frac{\alpha - 1}{\epsilon_s})} \\
= (\beta_{st}^{it})^{1/\epsilon_s} K_{\theta}^{(1-1/\epsilon_s)} \alpha Q_{P_{it}}^{-1/\epsilon_s}.
\] (3.29)

Substituting the expression for \( \beta_{st} \) from (3.18) into (3.28) shows that

\[
\frac{dB_{st}(QP_{it}, \theta)}{dQP_{it}} = \frac{dB_{s,t}(QP_{it})}{dQP_{it}}.
\]

3.3.3 Categories of Benefits and Costs of the MV/GTRC Test

In this subsection, we interpret the different components of (3.14), the MV/GTRC test.

Avoided Supply Costs

The utility’s avoided costs include energy, capacity, and external costs, all of which are considered by the TRC test of the California Standard Practice. As defined earlier in Chapter 2, TRC test is defined as the difference between avoided costs and the costs of the program to society.

Direct Participant Benefits

These include the value of new energy services (e.g. security lighting), avoided
fuel costs (fuel substitution programs), the benefits of takeback, and value losses due to curtailment programs. Of these, only avoided fuel costs are included in the California Standard Practice.

In the case of load building, a lower bound for the value of energy services is the price of electricity times the increase in consumption because, presumably, participants would not buy the energy if it was worth less than what they pay for it. However, they might be willing to pay more than that price. It can be shown that if (1) this value just equals the electric bill of participants plus their direct equipment costs and (2) there are no external costs or free riders, then the MV/GTRC test reduces to the "no losers" or GRIM test (presented as 3.32 below); i.e., a load building program will have positive net benefits if and only if benefits nonparticipants (Nelson and Hobbs, 1991a). This is because the net benefits to participants under this assumption are zero.

The benefits of rebound/takeback are calculated in a manner consistent with that of Costello and Galen (1984). The Standard Practice does not mention these benefits, although some analysts argue that they can be significant (e.g., Foley, 1989). If the conditional elasticity of demand for electricity can be estimated, then the benefits of takeback can be calculated. There are several econometric estimates of conditional elasticities for residential customers (Dubin and McFadden, 1982, Parti and Parti,
1980, and Hausmann, 1979). For instance, values on the order of 0.25 have been obtained for electric space heating, electric water heating, and air conditioning. For example, Seattle City Light estimated a value of 0.27 for electric heating in their service territory (Wilson and Gamponia, 1990). Further discussion of the estimation of these elasticities can be found in Hobbs (1991).

In the case of load curtailment programs, direct participant benefits are negative due to the decrease in energy services. Their value is at least as large as the price of electricity, assuming that the participant forgoes the service entirely or uses another fuel to provide it. But if the participant instead shifts the load to some other time, then there may be minimal losses in value.

**Net Societal Program Costs**

These include marketing and administrative costs (other than payments to participants) and net payments by the utility and participants for equipment. Transfer payments from the utility to participants are excluded. In addition, net program costs exclude expenditures by or for free riders, since these would have occurred even without the program.
Benefits of Rate Changes

These benefits equal the change in nonparticipant consumer surplus due to rate changes caused by the program. This category of benefits is not considered by the California Standard Practice (CPUC and CEC, 1983, 1987). As Hobbs (1991) and Nelson and Hobbs (1992) show, these benefits are proportional to the product of:

1. the difference between price and marginal social cost; and

2. the increase in nonparticipants’ electricity demands that result from rate changes. This is, in turn, proportional to the price elasticity of demand for nonparticipants and the increase in net revenues resulting from the DSM program, as calculated by the RIM test.

The RIM test reflects in part the utility’s expenditures on the program. The utility’s expenditures include not just marketing and equipment costs but also all transfer payments to participants, including any free riders. Because the benefits of rate changes depend upon the total expenditures by the utility $UE$, rather than the net resource costs $NE$, transfer payments $CP$, and free riders $FR$ do affect the net social benefits of DSM programs. Estimates of free rider fractions sometimes exceed 70% (Fang and Lui, 1989) and can have a significant effect on rates. Hence, transfer payments and free riders are not merely a matter of income distribution, contrary to assertions elsewhere (Lovins and Giliam, 1986, and Costello and Galen, 1984).
The benefits of rate changes can be positive for conservation programs if the program causes rates to move closer to marginal social cost and price elasticity is nonzero. This can occur, for example, with load building programs if they lower rates and if marginal social cost is less than price. The benefits of rate changes can be negative in the same circumstance for a conservation program if it raises rates. This increases the divergence between the price and marginal cost. As a result, if the nonparticipant price elasticity of demand is non-zero, economically efficient uses of electricity are discouraged. As a simple example, let the price of electricity be 0.08 $/kWh, and the marginal cost of electricity be 0.05 $/kWh. If the DSM program raises the price of electricity to 0.09 $/kWh, some nonparticipants might opt to cogenerate using natural gas at a cost of 0.085 $/kWh. The result is a net social loss of (0.085-0.05) $/kWh. As illustrated by examples in section 3.6, this loss in benefits can be significant and could make the difference between accepting a DSM program and rejecting it.

If instead price is less than marginal social cost, then the benefits of rates term will be negative if $\Delta Q_{ds}$ is positive. That is, if the DSM program causes rates to go down, thus stimulating more demand, the net social benefit of this additional demand will be negative, if $P<MC+ME$. However, the total value of the program is still likely to be positive, since both participants and nonparticipants are better off. In contrast, the benefits of rate terms can be positive if external costs $EX$ are high enough so that
P>MC but P<MC+ME. In this case, a conservation program will raise rates, bringing them closer to the social cost of power and as a consequence, uneconomic demands will be discouraged (Hobbs and Nelson, 1989). The benefits of rate changes for different types of programs are illustrated with examples in section 3.6.

3.4 Derivation of the GRIM Test

The RIM test, as defined earlier, is the difference between the benefits to (nonparticipant) customer bills and the costs to (nonparticipant) customer bills. The benefits include avoided supply costs and revenue gains due to the program, and costs include increased utility supply costs, utility DSM expenses, and revenue losses.

This assumption is valid if: 1) program is small relative to the total load of participants and 2) the price elasticity of demand for nonparticipants is zero. If, however, their price elasticity is nonzero, the net benefits to nonparticipants will be incorrectly calculated by the RIM test. This is because the price increase to nonparticipants will reflect not only the direct cost and revenue impacts of the program, but also the change in nonparticipant demands induced by rate changes. Therefore, we need a more general form of the RIM test which includes the impact of program induced rate changes. This test, which we call the "general ratepayer impact measure" (GRIM), is the change in nonparticipants' consumer surplus due to
the DSM program.

For a small program, the change in consumer surplus is adequately approximated by the negative of the change in price times the original quantity demanded by nonparticipants (See Fig. 3.4). Now this change in price is calculated by \( dP_c/d\theta \) in the derivation in subsection 3.3.1. If \( Q_n \) represents the nonparticipant load, then the GRIM test equals \( -Q_n \frac{dP_c}{d\theta} \) if \( dP_c/d\theta \) is the same for all classes in year \( t \). Therefore, from (3.13), we have:

\[
GRIM_c = \frac{(1-Fr) \sum (P_{pt} - MC_{it}) \Delta QP^*_i - [CE_t + CM_t + CP_t]}{[1 - E_{nt} + \sum \frac{MC_{it} \sum E_{ct} f_{cl}^t}{P_{ct}}]} \tag{3.30}
\]

\[
= \frac{RIM_c}{[1 - E_{nt} + \sum \frac{MC_{it} \sum E_{ct} f_{cl}^t}{P_{ct}}]}
\]

Now (3.30) be written as follows:
Figure 3.4: Change in Consumers Surplus of Nonparticipants
\[ GRIM_t = RIM_t \left[ 1 - \frac{E_{nt} + \sum_i MC_{nt} \sum_c \frac{E_{ct} \xi_{ct}}{P_{ct}}}{1 - E_{nt} - \sum_i MC_{nt} \sum_c \frac{E_{ct} \xi_{ct}}{P_{ct}}} \right] \] (3.31)

Therefore, from (3.31), GRIM in year \( t \) equals the RIM in year \( t \) plus the benefits of rate changes term for year \( t \) in (3.14). Finally, to get the net benefits under the GRIM test, we use:

\[ GRIM = \sum_t \frac{GRIM_t}{(1 + I)^t} \] (3.32)

If the price elasticity of electricity for nonparticipants is zero, then the RIM and GRIM tests are the same. Thus the GRIM test enhances the RIM test in that GRIM also considers the effects of rate changes upon nonparticipant demands.

The California Standard Practice recommends that the RIM test be used as the criterion to evaluate load building programs. But this is unnecessary, as the MV/GTRC test is a single economic criterion that can be used to evaluate both conservation and load building programs on a consistent basis. In the spreadsheet model (presented in section 3.5), the GRIM test is used instead as a measure of equity.
3.5. MOSTVALUE - A Spreadsheet Model

MOSTVALUE is a spreadsheet model developed by Nelson and Hobbs (1990a) to screen DSM programs on the basis of the four tests discussed so far in this thesis. These screening criteria are the MV/GTRC (derived in this thesis in section 3.3), the LC/TRC (presented in section 2.3), the RIM (presented also in section 2.3), and the GRIM (derived in this thesis in section 3.4) tests. Let us discuss briefly the features of MOSTVALUE before we present the examples solved using this model in section 3.6.

MOSTVALUE incorporates the full version of the MV/GTRC test presented as (3.14). MOSTVALUE calculates the various components of avoided costs (energy, capacity and environmental costs) separately. This model was utilized by an Ohio utility in screening their demand-side management programs. MOSTVALUE has a maximum planning horizon of 17 years. It can also accommodate up to three customer classes and a nine block load duration curve. Automatic sensitivity analysis can be performed using this model for various program specific data such as participation rates, program timings, free rider fractions, conditional price elasticities, program energy savings, and interest rates. Sensitivity analysis can also be performed on various supply-side scenarios (demand, price elasticities of demand, rates, marginal costs, marginal capacity costs, marginal external costs). It takes less
than 30 seconds to screen a demand-side program using MOSTVALUE on an IBM PC-AT.

MOSTVALUE assumes that the DSM program is small enough so that electricity prices and marginal costs of supply do not change enough to affect the evaluation. If a large program is being evaluated, then the estimates of net benefits may be overstated. The main reason for this phenomenon is the diminishing returns in the benefits of avoided supply costs and electric rate changes. Diminishing returns occur one block of a large program results in reduction in marginal costs and increase in electric rates for other blocks of the program. Thus, in actuality, there may be a lower total avoided costs and lower net benefits due to rate changes. An approximate way of considering these benefits is to test the program twice, with and without the program. The with program values will, most probably, understate the net benefits; the actual net benefits will lie between that estimate and the upper bound resulting from the without program values.

3.6 Example Applications of the MV/GTRC Test

This section presents several illustrations of the application of the MV/GTRC test to specific programs and compares the results with those obtained from the LC/TRC test of the California Standard Practice. The results of the RIM and the
GRIM tests are presented too. We consider a conservation program (residential lighting program), a load management program (direct load control of residential air conditioners), a valley fill program (industrial process heating), and a combination conservation and valley fill program (add-on heat pumps).

**Input Data**

Table 3.1 summarizes the price, load, and supply cost assumptions. These assumptions are similar to those used by a midwestern US utility. The base case assumptions of each program are presented below. In the present worth calculations, all cash flows are assumed to occur at the beginning of the year and an 11%/yr interest rate is used.

**Residential Energy Efficient Lighting.** This conservation program is directed at residential lighting. The utility pays $22,000 for administrative costs and $90,000 in rebates to participants, all in the first year. The incremental equipment cost to participants is $180,000. These expenditures are assumed to have the following annual impacts for 10 years: a decrease in the needle peak of 700 kW, a decrease in peak energy of 504,000 kWh/yr, and a decrease in off-peak energy of 1,008,000 kWh/yr. In the base case, it is also assumed that there is no takeback ($c_p=0$) or free riders.
Table 3.1
Avoided Cost, Demand, and Price Assumptions

Number of Seasons: three (summer, winter, spring/fall)
Number of hours in each period:
Needle peak: 1 hour/yr
Peak: 1393 hours/season (each season)
Off-peak: 1527 hours/season (each season)

First year marginal energy costs:
0.043 $/kWh (needle peak and peak; escalates @ 5%/yr)
0.020 $/kWh (off-peak; escalates @ 3.8%/yr)

Marginal Capacity Costs:
0 $/peak kW/yr (years 1-6)
130 $/peak kW/yr (year 7)
200 $/peak kW/yr (years 8-12)
270 $/peak kW/yr (years 13-15)

First year average power demands (MW) (Growth Rate = 1.6%)

<table>
<thead>
<tr>
<th>Class</th>
<th>Period</th>
<th>Winter</th>
<th>Summer</th>
<th>Spring/Fall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>Needle</td>
<td>-</td>
<td>1461</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Peak</td>
<td>1082</td>
<td>1061</td>
<td>826</td>
</tr>
<tr>
<td></td>
<td>Off-peak</td>
<td>880</td>
<td>863</td>
<td>660</td>
</tr>
<tr>
<td>Commercial</td>
<td>Needle</td>
<td>-</td>
<td>1894</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Peak</td>
<td>991</td>
<td>1210</td>
<td>908</td>
</tr>
<tr>
<td></td>
<td>Off-peak</td>
<td>751</td>
<td>843</td>
<td>632</td>
</tr>
<tr>
<td>Industrial</td>
<td>Needle</td>
<td>-</td>
<td>2056</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Peak</td>
<td>1931</td>
<td>1933</td>
<td>1932</td>
</tr>
<tr>
<td></td>
<td>Off-peak</td>
<td>1498</td>
<td>1498</td>
<td>1498</td>
</tr>
</tbody>
</table>

Price Elasticity: Mean Electric Rate, first year:
0.2 (residential) 0.105 $/kWh (residential)
0.5 (commercial) 0.090 $/kWh (commercial)
0.7 (industrial) 0.063 $/kWh (industrial)
(based on econometric analysis) (each escalates at 3.65%/yr)
Direct Air Conditioning Load Control. This is a load management program for residential air conditioners. In return for a lump sum payment (via a decrease in the monthly bill), participants agree to allow the utility to turn off their air conditioners for a limited time during system peaks. It is assumed that the decrease in the value of the energy services they receive is negligible. The utility's costs are $66,000 for administration and $26,000 for equipment in the first year, and $40,000 in payments in each of the ten years of the program. The resulting decrease in the needle peak is 1250 kW.

Industrial Process Heating. This valley fill program involves utility expenditures of $11,000 for administration and $45,000 for equipment. It yields an increase of 1,238,400 kWh in the demand for off-peak electricity for ten years. Because this is a new load which was not previously served by the utility, the value to the participants of this load must be assessed in order to use the MV/GTRC test correctly. It is assumed that this value is just equal to the price that the participants pay for the extra power they consume. A special off-peak rate which is 1.5¢/kWh less than the average industrial rate applies to this load. As a result, the value of new energy services is $59,443 the first year ($\left((0.063/\text{kWh} - 0.015/\text{kWh})\right)\times 1,238,400 \text{kWh}$), and increases by 3.65%/yr afterwards, which is the assumed increase in the price of electricity. Because this load pays a lower price than other loads in that customer
class, the value of this rate concession must be inserted in the spreadsheet. It is included for the purpose of calculating the lost revenues in the RIM and GRIM tests correctly according to the price paid by participants. It equals $18,576 (=0.015*1,238,400) in each year.

Add-On Heat Pump. The add-on heat pump can be considered to be a conservation as well as a valley fill program. The utility spends $180,000 on administration and $725,000 on equipment in the first year. The annual load impact, which is assumed to last for 15 years, is:

280 kW decrease in needle peak
168,000 kWh/yr decrease in summer peak energy
100,800 kWh/yr decrease in summer off-peak energy
806,400 kWh/yr increase in spring/fall peak energy
1,344,000 kWh/yr increase in spring/fall off-peak energy

These loads result from assuming that the heat pump replaces a traditional air conditioner in the summer and a natural gas furnace in the spring/fall season. During the winter, it is assumed that natural gas carries all the heating load. This program has rate distortion effects on the natural gas market which are ignored in this example, but can be calculated by using the method derived in Chapter 4.

Because the heating supplied in the spring/fall is an energy service which was not
previously provided by the utility, its value must be estimated. In the base case, it is assumed that its value is 2¢/kWh greater than the payments for electricity, and is based on the expense of natural gas. The participant pays a special heat pump rate for power during the spring/fall which is $0.05/kWh less than the average residential rate. This results in an electric bill of $118,272 for spring/fall power in the first year; adding the 2¢/kWh premium yields a total value of services in that year of $161,282. The value of services in subsequent years is higher because of the assumed escalation in electricity prices. The total rate concession must also be estimated and put into the spreadsheet; in the base case, this is $0.05/kWh times the spring/fall energy consumption, or $107,520. In the base case, it is assumed that there is no rate concession for summer energy use by the heat pump.

Results

In this subsection, we analyze the results of the MV/GTRC and GRIM test when applied to the four programs. These programs are also evaluated using the LC/TRC and the RIM tests of the California Standard Practice. Table 3.2 summarizes the base case results, along with various sensitivity analyses. Table 3.3 breaks down the benefits and costs of each program for each case.

Residential Energy Efficient Lighting. The residential lighting program is
Table 3.2
Benefits of Analyses of the DSM Programs
(present worth, in $1000)

<table>
<thead>
<tr>
<th>Program</th>
<th>Assumptions</th>
<th>MV/GTRC</th>
<th>LC/TRC</th>
<th>GRIM</th>
<th>RIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>RL</td>
<td>Base</td>
<td>76</td>
<td>353</td>
<td>-1024</td>
<td>-747</td>
</tr>
<tr>
<td>RL</td>
<td>70% Fr, 10 yr life</td>
<td>-7</td>
<td>353</td>
<td>-1317</td>
<td>-957</td>
</tr>
<tr>
<td>RL</td>
<td>Savings 7 yr life</td>
<td>-183</td>
<td>93</td>
<td>-1007</td>
<td>-731</td>
</tr>
<tr>
<td>RL</td>
<td>Takeback, 7 yr life</td>
<td>62</td>
<td>37</td>
<td>-836</td>
<td>-614</td>
</tr>
<tr>
<td>AC</td>
<td>Base</td>
<td>250</td>
<td>322</td>
<td>-13</td>
<td>59</td>
</tr>
<tr>
<td>AC</td>
<td>15 yr lifetime</td>
<td>705</td>
<td>751</td>
<td>385</td>
<td>431</td>
</tr>
<tr>
<td>AC</td>
<td>10 yr, no payments</td>
<td>330</td>
<td>322</td>
<td>329</td>
<td>320</td>
</tr>
<tr>
<td>AC</td>
<td>15 yr, no payments</td>
<td>790</td>
<td>751</td>
<td>789</td>
<td>750</td>
</tr>
<tr>
<td>IPH</td>
<td>Base</td>
<td>279</td>
<td>-243</td>
<td>279</td>
<td>217</td>
</tr>
<tr>
<td>IPH</td>
<td>15 year lifetime</td>
<td>376</td>
<td>-298</td>
<td>374</td>
<td>309</td>
</tr>
<tr>
<td>HP</td>
<td>Base</td>
<td>114</td>
<td>-1337</td>
<td>-413</td>
<td>-189</td>
</tr>
<tr>
<td>HP</td>
<td>30% Fr</td>
<td>-150</td>
<td>-990</td>
<td>-993</td>
<td>-660</td>
</tr>
<tr>
<td>HP</td>
<td>Summer discount</td>
<td>63</td>
<td>-1337</td>
<td>-658</td>
<td>-382</td>
</tr>
</tbody>
</table>

Note:  
RL = Residential Energy Efficiency Lighting  
IPH = Industrial Process Heating  
HP = Add-on Heat Pump  
AC = Direct Load Air-Conditioning Load Control

marginally attractive under the MV/GTRC test and base case assumptions. It is
Table 3.3
Breakdown of Benefits and Costs
(present worth, in $1000)

<table>
<thead>
<tr>
<th>Program</th>
<th>Assumptions</th>
<th>AVC</th>
<th>DPB</th>
<th>BRC</th>
<th>NPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>RL</td>
<td>Base</td>
<td>555</td>
<td>0</td>
<td>-277</td>
<td>202</td>
</tr>
<tr>
<td>RL</td>
<td>70% Fr, 10 yr life</td>
<td>555</td>
<td>0</td>
<td>-360</td>
<td>202</td>
</tr>
<tr>
<td>RL</td>
<td>7 yr life</td>
<td>295</td>
<td>0</td>
<td>-276</td>
<td>202</td>
</tr>
<tr>
<td>RL</td>
<td>Takeback 7 yr life</td>
<td>238</td>
<td>247</td>
<td>-222</td>
<td>202</td>
</tr>
<tr>
<td>AC</td>
<td>Base</td>
<td>414</td>
<td>0</td>
<td>-72</td>
<td>92</td>
</tr>
<tr>
<td>AC</td>
<td>15 yr lifetime</td>
<td>844</td>
<td>0</td>
<td>-46</td>
<td>92</td>
</tr>
<tr>
<td>AC</td>
<td>10 yr, no payment</td>
<td>414</td>
<td>0</td>
<td>9</td>
<td>92</td>
</tr>
<tr>
<td>AC</td>
<td>15 yr, no payment</td>
<td>844</td>
<td>0</td>
<td>39</td>
<td>92</td>
</tr>
<tr>
<td>IPH</td>
<td>Base</td>
<td>-187</td>
<td>463</td>
<td>59</td>
<td>56</td>
</tr>
<tr>
<td>IPH</td>
<td>15 year lifetime</td>
<td>-242</td>
<td>608</td>
<td>65</td>
<td>56</td>
</tr>
<tr>
<td>HP</td>
<td>Base</td>
<td>-341</td>
<td>1675</td>
<td>-224</td>
<td>995</td>
</tr>
<tr>
<td>HP</td>
<td>30% Fr</td>
<td>-239</td>
<td>1173</td>
<td>-333</td>
<td>751</td>
</tr>
<tr>
<td>HP</td>
<td>Summer discount</td>
<td>-341</td>
<td>1675</td>
<td>-276</td>
<td>995</td>
</tr>
</tbody>
</table>

Note:
AVC = Avoided Supply Costs
DPB = Direct Participant Benefits
BRC = Benefits of rate changes
NPC = Net societal program costs

heavily penalized by the cost of rate changes ($277,000), which even exceeds the net
expense of the program to the utility and participants ($202,000). This is a good example of a conservation program which saves energy, but inflates rates because avoided costs are smaller than lost revenues.

Sensitivity analyses in the tables show the effect of different assumptions concerning free riders, light bulb lifetimes, and takeback.

Free riders significantly affect the conservation program's net benefits, since they inflate utility expenditures and can, as a result, lower the benefits of rate changes. In the "70% Fr, 10 yr life" case, it is assumed that participation is 1/(1-0.7) times that in the base case, but that 70% of all participants are free riders (Fr=0.7). As a result, the net energy savings are the same as in the the base case but the utility's expenditures on equipment are 333% of the base case value. The net benefits in this case under the Standard Practice TRC test are the same as in the "base" case because avoided costs are unaffected and the real program cost (net of payments to free riders and their equipment purchases) is still $202,000. But the MV/GTRC test benefits worsen by $83,000 and the program fails that test. This shows that free riders can affect economic efficiency through their impacts on rates; contrary to assertions elsewhere (Lovins and Giliam, 1985 and Costello and Galen, 1984), free riders are not merely a matter of income distribution.
Alternatively, if a 7 year lightbulb life is assumed instead of a 10 year life, the program fails the MV/GTRC by a wide margin ($183,000), although it is still found beneficial by the California Standard Practice’s TRC test. The tests disagree because the latter criterion assumes that price elasticities of electricity are zero.

Takeback, too, can make an important difference in the lighting program, but in the opposite direction. In the "Takeback, 7 yr life" case, it is assumed that the program doubles the efficiency of lighting and that the conditional elasticity of demand for lighting is 0.25. Comparing this case with the 7 year life/no takeback case shows that 19% of the savings would be "taken back." The avoided cost savings are 19% smaller, yet the net benefits of the program under MV/GTRC actually increase (from -$183,000 to +$62,000). The LC/TRC test reflects the conventional (and incorrect) wisdom that takeback lowers benefits. The reason that wisdom is wrong is that the takeback has value; the participants are receiving more energy services. Table 3.3 shows this value to be $247,000, which is more than the cost of the program. This value is perhaps overstated, since the assumed elasticity (0.25) for lighting may be too high. Unfortunately, no estimates are available of the conditional price elasticity for lighting, although they exist for other residential uses (Dubin and McFadden, 1984, and Parti and Parti, 1980). The point, however, remains; takeback may decrease savings, but it enhances the value of a program.
Direct Air Conditioning Load Control. This program is attractive under both the MV/GTRC and LC/TRC tests for all sets of assumptions because of the size of the avoided capacity costs. Extending the lifetime of the equipment from 10 to 15 years doubles the benefits because capacity costs greatly increase during the later years. Two sensitivity analyses show the effect of the $40,000/yr payments to participants. If somehow participants could be enticed to join the program without those payments, then the net benefits of the program under MV/GTRC would increase by about $80,000, although, by definition, LC/TRC does not change. The reason for this is the rate benefits term; fewer expenditures mean lower rates, which are beneficial because price is greater than marginal cost.

This program also illustrates the effect of the divergence between price and avoided cost upon the benefits of rate changes. Because of rapidly mounting capacity costs, price and avoided cost are close to each other between years 8 and 15. As a result, there is little distortion in electricity consumption decisions in these years and changes in electric rates have relatively little effect upon net benefits then. Thus, the rate impacts in earlier years dominate the benefits of rate changes term. This explains why it is possible for the program to pass the RIM test, yet have a negative benefit of rate changes. The decreases in rates occur in later years when such decreases do not greatly affect net social benefits. However, rates increase in early years when price is much greater than avoided cost, causing the benefits of rate changes to be
negative.

**Industrial Process Heating.** In this valley fill program, benefits to participants are assumed to equal the price they pay for electricity. Under this assumption, the GRIM and MV/GTRC tests will be consistent. As a result, the program is either accepted by both criteria or rejected by both. Table 3.2 shows that this is indeed the case; the MV/GTRC net benefits are positive and the GRIM and RIM net benefits are also positive. But, by definition, the program fails the LC/TRC test, since that test ignores direct benefits to participants.

**Add-On Heat Pump.** The add-on heat pump program is unique in that it is both a conservation program (in summer) and a load building program (in spring/fall). As a result, it increases the utility's energy costs, yet it decreases its capacity expenses. The Standard Practice [CPUC and CEC (1983, 1987)] is ill-suited to evaluate such a program, since it says that the TRC test should be used for conservation programs, but that the RIM test is more appropriate for load building. The advantage of the MV/GTRC test is that it is a general economic efficiency criterion which encompasses the Standard Practice tests as special cases. Under the base case assumptions, the heat pump program's net benefits are positive under the MV/GTRC test.
But if 30% of the heat pump program participants are free riders, the MV/GTRC test becomes negative. This is in part because of the greater rate increases that result from the program expenditures remaining constant while the net gain in revenue shrinks. Therefore, if a significant fraction of the participants are anticipated to be free riders, heat pump economics can be adversely affected.

The performance of the heat pump program is also worsened if the participants are also given a rate discount of 1¢/kWh in the summer. Such a discount means that even more revenues would have to be made up by nonparticipants, which exacerbates the rate increase problem. Consequently, the net benefits under the MV/GTRC test fall by about $50,000, which is the amount by which the rate benefit term decreases. The LC/TRC test is of course unaffected, since it ignores the benefits of rate changes.
CHAPTER 4
EXTENSIONS OF THE MOST VALUE/GENERALIZED
TOTAL RESOURCE COST TEST

4.1 Introduction

In this chapter, we discuss extensions of the MV/GTRC test which are more
general than the basic test presented in Chapter 3. The version in Chapter 3 is
appropriate if we assume that: 1) effects on demands due to rate changes are
immediate and temporary; that is, electricity demanded in year t depends only on
prices in that year; 2) the DSM programs do not require switching of fuels; and 3)
economic efficiency is the sole basis upon which DSM programs are selected; and 4)
economic profits by the utility are zero (revenue requirements equal costs). But the
above assumptions are not true for all DSM programs. So the basic MV/GTRC
criterion, which corrects several shortcomings of existing screening tests, needs to be
generalized further.

Section 4.2 considers a version of the MV/GTRC test in which rate changes affect
demands not only in the year in which it occurs but also in subsequent years. In the
short run, most of the changes in energy usage by customers from changes in the
amount of energy services (for example, by altering thermostat settings). But in the
long run, most of the change in electricity demanded is due to changes in energy-
using equipment. Thus, because a rate change in any year ‘t’ will motivate changes
in equipment, the effects of such a rate change are felt for many years afterwards.
The derivation of this version of the MV/GTRC criterion is found in subsection 4.2.2
and is illustrated with an example in subsection 4.2.3.

The basic version of the MV/GTRC test assumes that the price of natural gas
equals the marginal cost of natural gas supply, where natural gas is an alternative fuel.
Market failures\(^1\) in the electricity sector were accounted for, but not those in the
natural gas market. Since natural gas markets are also regulated, the same failures
that plague the electricity market also occur in gas markets. Section 4.3 presents an
improved MV/GTRC criterion which accounts for market failures in both the
electricity and natural gas markets. This revision is useful for evaluating "fuel
switching" programs in which consumers are encouraged to use natural gas instead
of power for particular end-uses. Such programs are being considered in several
jurisdictions and are controversial.

\(^1\) There are two main types of market failures, 1) the customer's underutilization of
capital and 2) price set equal to average cost rather than marginal cost. The former
is the most commonly stated rationale for utility investment in conservation even
when price is greater than marginal cost, while the latter type of market failure leads
to inefficient consumption of electricity - including too little conservation if price is
less than marginal cost.
The version of the MV/GTRC test derived in Section 4.3 accounts for rate distortions and takeback/rebound in the natural gas as well as the electricity markets for fuel switching programs. A typical example of a fuel switching program would be a gas chiller program which replaces a traditional electric air conditioner during summer when the electricity rates are much higher than natural gas rates. A major contribution of Section 4.3 is the presentation of the benefits of takeback/rebound in the case of a fuel switching program. Rebound, as defined in Chapter 2, occurs when customers facing a lower price of energy services (due to utility investment in conservation) and a non-zero price elasticity respond by increasing their consumption of energy services. There is no previous work on calculating the effects of rebound with fuel switching. Hobbs (1991) and Nelson and Hobbs (1992) calculate the effect of rebound for conservation programs which do not involve fuel switching. Section 4.3 presents a way of calculating the effect of rebound and natural gas rate distortion benefits on the viability of fuel switching programs. A summary of the results of that section is presented in Nelson and Hobbs (1990b).

Economic efficiency is the sole basis on which DSM programs are evaluated in all the screening tests discussed in Chapter 3 including the MV/GTRC test. This perspective may be considered to be too narrow because they exclude many other important but nonmonetizable impacts of DSM programs. For example, a "low income weatherization" program may not pass the MV/GTRC test, but if "benefits to
the poor" is an important attribute, this program may be adopted. We discuss a multiattribute model in Section 4.4 that can include nonmonetizable impacts of DSM programs. This model uses an weighted additive utility function to rank candidate DSM programs. Nelson and Hobbs (1991) discuss the model in detail together with a few of the important attributes of DSM programs. Section 4.4 also discusses the inclusion of one of the important attributes, shareholder benefits, in the MV/GTRC test. These benefits affect economic efficiency in two ways: (1) it changes social welfare since producer surplus is not zero in this case; and (2) it changes the benefits of rate changes since it increases the amount of expenditure to be recovered from ratepayers. An example is provided in that section to illustrate the applicability of this version of the MV/GTRC tests.

4.2 Lagged Effects of DSM Programs on Nonparticipant Loads

In evaluating the benefits and costs of changes in electric rates, one critical parameter is the price elasticity of electricity demand. The price elasticity shows the responsiveness of loads to changes in electricity prices. If it is low, then nonparticipants will hardly alter the amount of electricity they demand and the benefits/costs of rate changes in the MV/GTRC test will be very small. But if the price elasticity is high, then changes in nonparticipant demands can be as large in magnitude as energy savings directly resulting from the DSM program, and the
benefits/costs of rate changes can be large. Thus, a defensible model of the response of load to price, and reliable estimates of price elasticities of demand are of paramount importance.

The magnitude of elasticity is subject to considerable uncertainty. There is little agreement on 1) the best approach to evaluate elasticities, 2) accuracy of elasticities obtained, 3) whether elasticities are constant or change as energy prices or other variables change, and 4) whether the short term or the long term price elasticity should be considered in screening DSM programs (see Bohl (1981) or Betancourt (1981) for a discussion on uncertainties of price elasticities).

One shortcoming of the MV/GTRC test in equation (3.14) is its failure to incorporate time lags in demand response to price (elasticity). The MV/GTRC test, as formulated then, assumes that the response of nonparticipants to a price change is instantaneous and that such a change does not affect demands in later years. This might be a good approximation if the amount of electricity demand shifts only because there is a change in the amount of energy services demanded. But in the long run, much or most of the change in the quantity of electricity demanded is due to changes in energy using equipment (stock adjustment). For example, due to a change in price, customers might buy more efficient appliances than they would have otherwise, or invest in equipment that could use alternate fuels. Thus a more accurate
model of nonparticipant response to price changes would consider how a price change affects demands not just in the year it occurs, but also in subsequent years.

We develop an improved version of the MV/GTRC test in this section which incorporates time lags in demand response to price changes. In most practical situations, customers do not react instantaneously to price or income changes, but do so in a more gradual manner. For example, if a person receives a salary bonus of $1000 this year, this may induce him to raise his expenditures by $500 this year, by another $200 the next year, and by another $100 the third year, so that the ultimate effect is an increase in expenditure of $800. Thus we have a lag of expenditures distributed over three years.

Subsections 4.2.1 discusses a specific form of a lag model that has been widely used in estimating electricity demands. This lag structure is geometric and greatly simplifies the estimation of cross-price elasticity of demand (defined as the change in the quantity of a commodity consumed to the change in price of another commodity, or the same commodity in a different period). Subsection 4.2.2 presents the derivation of the MV/GTRC criterion of the lagged form which makes use of the cross-price elasticities of demand for electricity. Finally, Subsection 4.2.3 presents an example to illustrate the difference between the lagged MV/GTRC criterion and the version with no lag.
4.2.1 Concept of Distributed Lags

In this subsection, we assume a very simple form of the lag equation discussed by Koyck (1954). In this expression, there is only one explanatory variable which occurs in distributed lag form:

\[ X_t = \beta_0 + \beta_1 Y_t + \beta_2 Y_{t-1} + \ldots \] (4.1)

where the beta values are coefficients that relate the consumption of one good to explanatory variables over different time periods.

In most cases it is plausible to assume that the beta values converge toward zero. Let us suppose, for convenience, that the decline is of the geometric type and that it starts at the first lag: \( \beta_2 = \lambda \beta_1, \beta_3 = \lambda^2 \beta_1, \ldots \) where lambda lies between zero and 1. Thus the equation takes the form:

\[ X_t = \beta_0 + \beta_1 Y_t + \lambda \beta_1 Y_{t-1} + \lambda^2 \beta_1 Y_{t-2} + \ldots \] (4.2)

Geometric distributed lags are frequently used to describe electric demands in the literature (see Taylor, 1975 and Thiel, 1976). Now, if \( Y_t \) equals the natural logarithm of price in year \( t \) and \( X_t \) equals natural log of electricity consumption in year \( t \), we get a distributed geometric lag logarithmic model for electricity consumption. Therefore, (4.2) can be modified as follows:
\[ \ln Q_t = \beta_0 + \beta_1 \ln P_t + \lambda \beta_1 \ln P_{t-1} + \lambda^2 \beta_1 \ln P_{t-2} + \ldots \] (4.3)

This implies that:

\[ Q_t = e^{\beta_0} P_t^{\beta_1} P_{t-1}^{\lambda \beta_1} P_{t-2}^{\lambda^2 \beta_1} \ldots \]
\[ = e^{\beta_0} (P_t^{\beta_1} P_{t-1}^{\lambda \beta_1} P_{t-2}^{\lambda^2} \ldots)^{\beta_1} \] (4.4)

From the definition of price elasticity of demand, we can define the short term elasticity of demand \( \varepsilon \) as equal to \( \beta_1 = -\frac{d\ln Q_t}{d\ln P_t} \). Now from (4.3) above, it is apparent that the short term price elasticity of demand \( \varepsilon \) equals \( -\beta_1 \). The cross-elasticity of demand, defined as the change in quantity consumed in year \( t \) with respect to the change in price in year \( 't-j' \), equals \( -\lambda \varepsilon \beta_1 \).

Now, the long term elasticity of demand \( E \) can be assumed as the summation of the short term price elasticities and the cross-price elasticities of demand from years 1 through infinity. Therefore:

\[ E = \beta_1 + \lambda \beta_1 + \lambda^2 \beta_1 + \ldots \]
\[ = \beta_1 \sum_{j=0}^{\infty} \lambda^j = \frac{\varepsilon}{1-\lambda} \] (4.5)

Thus, given estimates of \( \varepsilon \) and \( E \), the value of \( \lambda \) can be inferred. For example, if \( \varepsilon_t=0.12 \) and \( E_t=0.6 \), the cross-price elasticity of demand for price changes two years
earlier equals 0.0768. Electric utility econometricians usually calculate the short and long term price elasticity of demand (e.g., Erickson et al., 1973). From these numbers, the cross price elasticities for different years of lag can be easily calculated by estimating the value of lambda.

4.2.2 Formulation of the Lagged MV/GTRC Test

In this subsection, we derive a version of the MV/GTRC test for the lagged demand case. Thus, in this formulation, we assume that the nonparticipant demand $Q_{a}$ depends not only on $P_{t}$ but also $P_{t-1}, P_{t-2}$ etc. In this derivation, we assume there is a single period in each year and that there is just one customer class. We also assume that all customer and utility expenditures in the DSM program occur in year 0. Derivations for the more detailed versions would follow a similar approach. The benefits to society of power consumption can be defined as follows:

$$V(P, \theta) = \sum_{\tau} \frac{1}{(1+\lambda)^{\tau}} [B_{nt}(P) + \theta DB_{pe} (1-F_{x}) - C(P, \theta) - NE_{e}(\theta)]$$

(4.6)

where:

$B_{nt}(P) = \text{Gross benefits [\$] to nonparticipants in year 't' as a function of prices P (P_{\tau}, \tau=0,...,T-1) which in turn is dependent on \theta. As defined earlier, \theta denotes the level of implementation of the program. \theta=0 indicates no DSM expenditure and \theta=1 denotes}$
full implementation of the program.

\[ C_{(P, \theta)} = \text{Cost of electricity production [\$]. It is a function of quantity consumed by nonparticipants, participants, and the program implementation.} \]

\[ = C_t(Q_{at} + QP_0 + \theta \Delta QP_t(1-Fr)) \]

\[ DB_{Pt} = \text{Direct benefits to participants in year 't' [\$]. We assume that there is no rebound or takeback in this derivation for a conservation program.} \]

\[ NE_t(\theta) = \text{Net societal cost of the DSM investment in year 0 [\$].} \]

\[ = \theta^*[(CE_t+EQ_d)(1-Fr)+CM_t] \]

where:

\[ CE_t = \text{Utility equipment costs [\$].} \]

\[ CM_t = \text{Utility marketing and admin. costs [\$].} \]

\[ EQ_d = \text{Customer equipment costs [\$]} \]

\[ QP_t^0 = \text{Participant energy demands in year t [kWh]. It is assumed that the participant load is a small fraction of the overall load.} \]

\[ \Delta QP_t = \text{Change in energy usage in year 't' due to the DSM program [KWh].} \]

Now, as assumed earlier, the net benefits of the DSM program can be approximated by the derivative of this value function with respect to \( \theta \) as long as the program is small. This equals:
\[
\frac{dV(P, \theta)}{d\theta} = \sum_t \frac{1}{(1+i)^t} \left[ \frac{\partial B_{nt}}{\partial Q_{nt}} \sum_{s=0}^{t} \frac{\partial Q_{nt}}{\partial P_s} dP_s \right. \\
- \frac{\partial C_t}{\partial Q_{nt}} \sum_{s=0}^{t} \frac{\partial Q_{nt}}{\partial P_s} dP_s + \Delta Q P_e (1-Fr) \\
\left. - ((CE_e + CM_e) (1-Fr) + EQ_e) \right]
\] (4.7)

The above derivative differs from the derivation of the basic form in (3.6) in that the quantity of electricity consumed in any particular year 't' is a lagged effect of the price of electricity in years 0 to t. Let us now redefine a few terms:

\[
\frac{\partial B_{nt}}{\partial Q_{nt}} = P_e \\
\frac{\partial C_t}{\partial Q_{nt}} = MC_e \\
E_{ts} = -\frac{\partial Q_{nt}}{\partial P_s} \frac{P_s}{Q_{nt}}
\] (4.8)

The first of these definitions in (4.8) indicates that the customer in any year consumes electricity till his marginal benefit of consuming electricity equals the price of electricity in that year. The second definition is the classic definition of the marginal cost of electricity. The last definition in (4.8) defines the cross price elasticity between quantities in year 't' and prices in earlier years. We can now simplify (4.7) to yield:

---

2 In this formulation, we assume that the ratepayers are rational and minimize the present worth of their costs or maximize their net benefits and that the customers correctly forecast future prices other than the price effect of DSM programs
\[
\frac{dV}{d\theta} = \sum_t \frac{1}{(1+I)^t} \left[ -P_e \sum_{s=0}^t \frac{E_{ts} Q_{nt}}{P_{nt}} \frac{dP_s}{d\theta} \right.
\]
\[
-\frac{MC_e}{1+I} \left[ \sum_{t=0}^t \frac{E_{ts} Q_{nt}}{P_{nt}} \frac{dP_s}{d\theta} + \Delta Q P_e (1-Fr) \right] + [ (CE_e + CM_e) (1-Fr) + EQ_e ] \]

Simplifying (4.9) further yields:

\[
\frac{dV}{d\theta} = \sum_t \frac{1}{(1+I)^t} \left[ - \left( P_e - MC_e \right) \sum_{s=0}^t \frac{E_{ts} Q_{nt}}{P_{nt}} \frac{dP_s}{d\theta} \right.
\]
\[
-\left( P_e - MC_e \right) \Delta Q P_e (1-Fr)
\]
\[
- [ (CE_e + CM_e) (1-Fr) + EQ_e ] \]

In the above equation, the only variable not calculated as yet is the value of the derivative \(dP/d\theta\). This value can be obtained from the revenue recovery constraint for that year. This corresponds to the regulatory constraint that utility revenues must equal the cost of electricity production and the utility costs of DSM programs to be recovered through rates in year ‘t’. The constraint is:

\[
\sum_t P_t \left[ Q_{nt} (P_e) + QP^0 + \theta \Delta Q P_e (1-Fr) \right] =
\]
\[
C_t(Q_{nt} (P_e) + QP^0 + \theta \Delta Q P_e (1-Fr)) + \theta UE_t
\]

where \(UE_t\) is the portion of the utility DSM costs to be recovered in year ‘t’.
Taking the total differential of (4.11) with respect to \( \theta \) leads to the following expression:

\[
\frac{dP_t}{d\theta} = Q_{nt} + P_t \sum_{z=0}^{\tau} \frac{\partial Q_{nt}}{\partial P_s} \frac{dP_s}{d\theta} + \Delta QP_t (1-Fr) + \Delta t
\]

\[\ldots\]  

(4.12)

Simplifying (4.12), using the definition of cross-price elasticity of demand in (4.8), and rearranging yields the following expression:

\[
\frac{dP_t}{d\theta} = \left(UE_t - \sum_{z=0}^{\tau} \frac{E_{ts}}{P_s} \frac{dP_s}{d\theta} \right) \left[ \sum_{z=0}^{\tau} \frac{\partial Q_{nt}}{\partial P_s} \frac{dP_s}{d\theta} + \Delta QP_t (1-Fr) \right] \left( \frac{Q_{nt}}{P_t} \right)
\]

(4.13)

Note that, if there are \( T \) years in the planning horizon, there are \( T \) equations like (4.13) above with \( T \) \( dP/d\theta \) variables to be solved\(^3\). The easiest way to solve for these variables is to first calculate \( dP/d\theta \) since it does not depend on any other value.

\(^3\) Note that although energy savings due to a program may last only a few years, the rate impacts may extend beyond the end of the planning horizon. So, in theory, is \( S \) is the largest \( s \) such that \( E_{st}>0 \), then \( T+S \) equations of the form (4.13) need to be solved.
of \(dp/d\theta\), then \(dp/d\theta\) since it only depends on \(dp/d\theta\), and so on until \(dp/d\theta\) is calculated. Substituting the expressions for \(dp/d\theta\) into expression (4.10) would yield the MV/GTRC test with time lags.

Now, (4.13) can be rewritten as follows:

\[
\frac{dp_t}{d\theta} = \frac{UE_t - [P_t - MC_t] \left[ \Delta QP_t (1 - \epsilon_r) + \Delta Q_{nt} \right]}{Q_{nt}}
\]

(4.14)

where: \(\Delta Q_{nt} = Q_{nt} \sum_{s=0}^{s} \frac{E_{ts}}{P_{ts}} \frac{dp_s}{d\theta}\) is the change in nonparticipant demands.

The above expression (4.14) can be interpreted as the change in utility costs over the total consumption of electricity. If price elasticities are ignored, the numerator of the right hand side of the expression is the negative of the ratepayer impact measure (RIM) for year ‘t’. If cross-price elasticities are ignored (\(E_u = 0\) except, if \(t=s\), when it equals \(e_s\), the expression above reduces to the following form:

\[
\frac{dp_t}{d\theta} = \frac{P_t \left[ UE_t - (P_t - MC_t) \Delta QP_t \right]}{[P_t - E_s (P_t - MC_t)]}
\]

(4.15)

where \(e_t\) is the price elasticity of demand in year \(t\) with respect to price in that same year.
Expression (4.15) is the value for $\frac{dP}{d\theta}$ derived in section 3.3 of Chapter 3 for a single period, single customer class problem. Thus the time lag version can be reduced to the version of the MV/GTRC test derived earlier if the cross price elasticities are ignored. As explained in subsection 4.2.1, knowing the short term and the long term price elasticities of demand facilitates calculation of these cross price elasticities of demand.

The version of the MV/GTRC test presented in this subsection is illustrated with an example below in subsection 4.2.3.

4.2.3 An Example of the MV/GTRC Criterion With and Without Lag

The example is for a program which lasts for 9 years. However, as noted earlier, the price impacts last through the planning horizon. Let us assume that the planning horizon is 15 years. This DSM program is a conservation program which saves 40,000 kWh each year. We assume that there is just one customer class and that there is just one period each year. Prices are 0.1 $/kWh escalating at an annual rate of 2%/year over the planning horizon whereas marginal costs are initially 0.05 $/kWh escalating at an annual rate of 5%. The total load for the utility is 100,000 MWh escalating at an annual rate of 2%. 
We also assume that the utility incurs all its DSM expenses ($10,000) in the first year and are recovered from revenues in year 0. We also assume that there are no participant expenses. The short term price elasticity of demand is equal to 0.2 and the long term price elasticity of demand equals 0.8. Thus, if we assume a geometric lag structure, the lag ratio \( \lambda \) equals 0.75. This implies that the price elasticity of demand in year ‘t’ due to a price increase three years ago equals \( 0.75^3 \) times 0.1. Table 3.5 below gives the results of screening according to the LC/TRC test and the MV/GTRC test with and without demand lags. We use both short term and the long term price elasticity of demand in the case of the MV/GTRC test with no lagged effects. \( E_{SR} \) is the short term price elasticity of demand and \( E_{LR} \) id the long term price elasticity.

To illustrate the differences between the MV/GTRC with lag and without lag, let us first consider the benefits of rate changes calculated with (1) the no lag models using the short term price elasticity of demand and (2) the lag model. As we can see from columns (10) and (14) of the table, the benefits of rate changes in both these cases are the same in year 0. This is so because the lag effect affects demand only in later years. We can also note that the MV/GTRC test with no lag faces significantly higher rate effects in the first year as compared to all other years. This is because, in the formulation of the MV/GTRC in Section 3.3 of Chapter 3, we assume that the rate effects due to program costs all occur in the same year in which
it is expensed and the rate effects in later years are just the revenue losses (gains) due to the program. These effects last only until the last year in which energy savings occur (year 9). In this case, we ignore the effect that price changes in any particular year could have on customer decisions in later years. Thus, considering only the short term effects could lead to underestimation of the effect of rate changes in earlier years on demands in later years.

On the other hand, the lag model shows a more gradual reduction of rate effects in later years. This can be observed by comparing columns (10) and (14) in the table with and without lag effects. The main reason for the lagged model's higher rate effect in later years is our accounting for demand changes in later years due to price changes in earlier years through the lag effect. From the summary table 4.2, we note that the benefits of rate changes in the lag case (-$9151) are much greater than the case with no lag (-$2303) using $E_{sr}$. In this case, we note that this makes the difference between selecting and rejecting the DSM program. That is, the program passes the MV/GTRC test with no lag, whereas it fails the MV/GTRC test with lag effects (Table 4.2). Now, as observed earlier, most customers make decisions about energy using equipment in the long term and energy usage in the short term. Thus, the MV/GTRC test with no lag ignores changes in capital expenditure and underestimates the effects of rate changes, if the short term price elasticity is used.
In the examples given in Section 3.6 of the previous chapter, the long term price elasticity of demand was used to analyze the sample DSM programs. Now, in Table 4.1, we can observe from the table that the effects of rate changes are significantly higher when the long term price elasticity $E_{LR}$ was used. The benefits of rate changes in the first year alone is six times the magnitude of the other two cases. Thus, using the long term price elasticity of demand, significantly increases the magnitude of effects of rate changes and the benefits (costs in this case) may be overestimated. But the present worth of benefits for the lag model will be closer to the no lag model with $E_{LR}$ than the model with $E_{SR}$.

Thus, calculation of the benefits of rate changes with the short term price elasticity of demand underestimates the effects of rate changes by ignoring long term effects of price changes (for example, possible purchases by consumers of more energy efficient equipment). In contrast, calculation of the effects of rate changes with the long term elasticities overestimates the effects of rate changes by accounting presently for changes that actually occur only in later years. But the lag model adopts a more defensible approach and provides a method of looking at both short and long-term effects in a theoretically correct framework.
Table 4.1: Lagged Effects of DSM Programs: An Example

Short term price elasticity of demand = 0.2
Long term price elasticity of demand = 0.8
Lag Ratio = 0.75
Interest Rate = 0.1

<table>
<thead>
<tr>
<th>(1) Year</th>
<th>(2) Price $/kWh</th>
<th>(3) MC $/kWh</th>
<th>(4) Total Load MWh</th>
<th>(5) Energy Savings kWh</th>
<th>(6) Avoided Costs PW $</th>
<th>(7) Utility DSM Costs</th>
<th>(8) RIM $</th>
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<tr>
<td>0</td>
<td>0.1</td>
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<td>40000</td>
<td>2000</td>
<td>10000</td>
<td>-12000</td>
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<td>10200</td>
<td>40000</td>
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<td>0</td>
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<td>2</td>
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<td>0.055</td>
<td>10404</td>
<td>40000</td>
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<td>0</td>
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<tr>
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<td>0.058</td>
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<td>40000</td>
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<tr>
<td>4</td>
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<td>0.061</td>
<td>10824</td>
<td>40000</td>
<td>1660</td>
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## Table 4.1: Lagged Effects of DSM Programs: An Example (Continued)

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<tr>
<th>Year</th>
<th>(1) dP/dθ&lt;sup&gt;4&lt;/sup&gt;</th>
<th>(9) Benefits, Rate Changes PW $ no lag&lt;sup&gt;4&lt;/sup&gt;</th>
<th>(10) dP/dθ&lt;sup&gt;5&lt;/sup&gt;</th>
<th>(11) Benefits, Rate Changes PW $ no lag&lt;sup&gt;5&lt;/sup&gt;</th>
<th>(12) dP/dθ with lag</th>
<th>(13) Benefits, Rate Changes PW $ with lag</th>
</tr>
</thead>
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<sup>4</sup> calculated with short-run price elasticity of demand

<sup>5</sup> calculated with long-term price elasticity of demand
Table 4.2
Results of the Lagged Effects of DSM Programs: An Example

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<th>Results</th>
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<td>PW of Rate Benefits (with lag)</td>
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<tr>
<td>PW of Rate Benefits (no lag^2)</td>
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<td>= - $2303</td>
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<tr>
<td>PW of Rate Benefits (no lag^3)</td>
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<td>= - $13,480</td>
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<tr>
<td>= $15,052 - $10,000</td>
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<td>= $5,052</td>
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<tr>
<td>GTRC Benefits (No lag, E_{SR})</td>
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<td>= $15,052 - $10,000 - $2303</td>
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<tr>
<td>= $2,749</td>
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<tr>
<td>GTRC Benefits (No lag, E_{LR})</td>
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<td>= - $8,428</td>
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<td>Lagged GTRC Benefits</td>
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<tr>
<td>= $15,052 - $10,000 - $9,151</td>
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<td>= - $4,100</td>
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4.3 Extension of the MV/GTRC Test to Consider Fuel Switching Programs

The version of the test presented here assumes that program participants are switching from electricity to natural gas. The MV/GTRC test for the reverse case can be derived analogously. It is assumed that there are C different customer classes whose rates are affected by the conservation program. Let us also assume that there are T years in the planning period and I different time periods in each year. As defined in the previous chapter, the term "participants" refer to the directly impacted uses of customers who participate in a utility DSM program, including "free riders." "Free riders" denote program participants who would have invested in the DSM option even without the utility subsidy. "Nonparticipants" include: consumers not participating in the program; and participant energy uses not directly impacted by the program. An example of the latter would be gas heating loads for the case of a lighting rebate program, although there could be secondary effects. An example of a secondary effect would be if more efficient lighting means less heat needed in winter but more air-conditioning in summer. The participant's energy demand is assumed to be a small fraction of the overall load. However, a more general version of the criterion is possible if the DSM program affects a significant portion of the utility’s total load.
4.3.1 Participant Net Benefits

Let $Q_{sp}^{it}$ be the amount of energy services in units demanded by the participants in period ‘i’, year ‘t’. Letting $e_{sp}$ be the price elasticity of energy services, and $P_{sp}^t$ the price or the marginal willingness to pay for energy services in $/unit. The demand function for $Q_{sp}^{it}$ is assumed to be of the constant elasticity form:

$$Q_{sp}^{it} = \beta_{sp} e_{sp} P_{sp}^{t-\infty}$$  \hspace{1cm} (4.16)

Inverting (4.1) yields:

$$P_{sp}^t (Q_{sp}^{it}) = \left( \frac{Q_{sp}^{it}}{\beta_{sp}} \right)^{1/e_{sp}}$$  \hspace{1cm} (4.17)

Thus, the benefits or total willingness to pay for the consumption of energy services, $B (Q_{sp}^{it})$ equals:
\[ B(Q_{sp}^{1\tau}) = \int_0^{\infty} P_{sp}^z(q) \, dq \]
\[ = K + \left( P_{sp}^z \right)^{1/\varepsilon_{sp}} \left( Q_{sp}^{1\tau} \right)^{1-1/\varepsilon_{sp}} \]

where \( K \) denotes the value of the integral from 0 to \( 0^+ \).

Let \( \theta \) be a variable which designates whether or not a program is implemented. If \( \theta = 0 \), then there is no program and electricity is used. If \( \theta = 1 \), then the fuel switching program is fully implemented and natural gas is used. Values between 0 and 1 represent partial implementation of the program. It is assumed in this section that only one of the two energy forms is used; electricity without the program (\( \theta = 0 \)) and natural gas with the program (\( \theta = 1 \)).

The production function for \( Q_{sp}^{1\tau} \) is of the form given below:

\[ Q_{sp}^{1\tau} = K_\theta \left[ Q_{sp,0}^{1\tau} \right]^\alpha \]  

where \( \alpha \) is a parameter, \( K_\theta \) is a constant reflecting the amount of capital investment in conservation, and \( Q_{sp,0}^{1\tau} = (Q_{sp,0}^{1\tau}, Q_{sp,1}^{1\tau}) \) is the amount of electricity or natural gas used to provide the energy services required by the participants in period \( i \) of year \( t \).

Let \( P_{sp,\tau} \) be the prevailing price of electricity in year 't' in the participant's customer class. We assume that the participant derives benefits from consumption of
electricity until the marginal benefit equals the price of electricity. Thus we can derive an expression for the derived demand for electricity. It is obtained by substituting (4.17) into (4.18), and differentiating the resulting expression with respect to \( Q_{E_{p,0}} \), setting it equal to \( P_{esp} \), and solving for \( Q_{E_{p,0}} \):

\[
Q_{E_{p,0}} = \beta_{p,0}^t P_{esp}^\gamma
\]  

(4.20)

where:

\[
\beta_{p,0}^t = \frac{1}{K_0^{1/\alpha}} \left[ K_0^{1/\alpha} \alpha (\beta_{sp}^t)^{1/\varepsilon_p} \right]^{1/\gamma}
\]

\[
\gamma = \left[ \frac{1}{\alpha / \varepsilon_p + 1 - \alpha} \right]
\]

(Alternatively expression (4.20) can be obtained by maximizing (4.18) minus electricity bills subject to (4.19)). The gamma term defined above is the conditional price elasticity of demand for energy services conditioned on a fixed set of conservation investments.

Similarly, if the energy services were instead provided by natural gas, we obtain the derived demand function for natural gas by setting the marginal benefit of gas use equal to the price of natural gas. Thus the derived demand function for natural gas is of the form given below:
where:

\[ \beta_{p,1} = \frac{1}{K_1} \left[ K_1^{1/\alpha} \cdot (\beta_{sp})^{1/\alpha} \right]^\alpha \]

The benefits of the consumption of electricity and natural gas are obtained from (4.22) and (4.23) below:

\[ B(\xi_{p,0}) = \int_0^{\xi_{p,0}} P_{eqt}(q) \, dq \]

\[ = C + (\beta_{p,0})^{-\gamma} \left( \frac{\xi_{p,0}}{1-\gamma} \right) \]

\[ B(\xi_{p,1}) = \int_0^{\xi_{p,1}} P_{eqt}(q) \, dq \]

\[ = C + (\beta_{p,1})^{-\gamma} \left( \frac{\xi_{p,1}}{1-\gamma} \right) \]

Now the gross benefits of the consumption of energy services with electricity with no DSM and natural gas with DSM equals:
\[ E_p(Q_{p,0}^{t_c}, \theta) = (1-\theta) \ast B(QE_{p,0}^{t_c}) + \theta \ast B(QG_{p,1}^{t_c}) \]  \hspace{1cm} (4.24)

Let us omit the constants C from future consideration because they do not affect the change in benefits due to the DSM program. Now substituting (4.20) and (4.21) in (4.22) and (4.23) respectively, yields:

\[ B(QE_{p,0}^{t_c}) = \frac{\beta_{p,0}}{1-\gamma} P_{ep}^{1-\gamma} \]  \hspace{1cm} (4.25)

\[ B(QG_{p,0}^{t_c}) = \frac{\beta_{p,1}}{1-\gamma} P_{gp}^{1-\gamma} \]  \hspace{1cm} (4.26)

Let \( \tau \) be the ratio of energy use by participants after the program to energy use before the DSM program. It is important to note that the natural gas usage QG and electricity QE should be expressed in the same units (kWh or mmBtu). Let us assume that the units are in mmBtu. Using equation 4.19, we can show that for a fixed level of energy services (i.e. \( Q_{tp}^{t_c} \) is constant) consumed by participants:

\[ \tau = \left( \frac{K_0}{K_1} \right)^{\gamma/\sigma} \]  \hspace{1cm} (4.27)

Thus \( (1-\tau) QE_{p,0}^{t_c} \) would be the amount of energy savings by the participants in the absence of rebound/takeback. The savings would of course be negative if more natural gas is used than electricity. These savings are often called "technical" or "engineering" savings.
Dividing (4.21) by (4.20) yields the ratio of consumption of natural gas after the DSM program to the consumption of electricity before the program, accounting for rebound. This value \( \mu_c \) equals:

\[
\mu_c = \frac{\beta_{p,1}^{\text{iz}}}{\beta_{p,0}^{\text{iz}}} \left[ \frac{P_{\text{gpt}}}{P_{\text{ept}}} \right]^{\gamma}
= \tau^{(1-\gamma)} \left[ \frac{P_{\text{gpt}}}{P_{\text{ept}}} \right]^{\gamma}
\]

(4.28)

The subscript \( t \) is maintained because there could be a change in the ratio between price of natural gas and electricity. It should be emphasized that the price of electricity and natural gas should both be expressed in compatible units (e.g., $/\text{mmBtu}$).

Now \( (1-\mu_c) \mathcal{Q}_{E_{p,0}}^{\text{iz}} \) is the net savings including rebound. If \( Fr \) denotes the fraction of program participants who are actually free riders, then the net savings due to the program actually equal \( (1-Fr) (1-\mu_c) \mathcal{Q}_{E_{p,0}}^{\text{iz}} \). Thus, expressing the ratio of net savings to technical savings as the correct savings fraction (CSF), we have:
\[ CSF = \frac{1 - \tau(1 - \gamma) \frac{P_{\text{ext}}}{P_{\text{ext}}}^{1 - \gamma}}{1 - \tau} (1 - Fr) \]  

\[ = \frac{\text{Actual Savings}}{\text{Savings ignoring rebound, free riders}} \]  

Now the gross benefits to participants of the program in year \( t \) are obtained by calculating the summation of the expression (4.24) over all periods and is as given below:

\[ BP_c(P_{\text{ext}}, P_{\text{gpc}}, \theta) = \sum_i (1 - \theta) \frac{\beta^{t}_{p,0}}{1 - 1/\gamma} P_{\text{ext}}^{1 - \gamma} + \theta \frac{\beta^{t}_{p,1}}{1 - 1/\gamma} P_{\text{gpc}}^{1 - \gamma} \]  

(4.30)

4.3.2 Nonparticipant Benefits

As stated earlier, we assume that there are C different customer classes for both elasticity and natural gas and the participants form a small portion of the load of the customer class they belong to. It is also assumed that the customer demand follows a constant elasticity demand, i.e.,

\[ QE^{t}_{c} = \beta^{t}_{c} P^{\gamma_{c}} \]  

(4.31)

where \( \beta^{t}_{c} \) is a constant and:

\[ QE^{t}_{c} = \text{Quantity of electricity consumed by class } c \text{ in period } i \text{ of year } t \]

excluding uses by program participants that are directly affected by the DSM program.
\( P_{\text{ect}} \) = Average price of electricity for class c in year t.

\( \gamma_{\text{ect}} \) = Price elasticity of demand for electricity for class c in year t.

\[
= -\frac{dQ_{E_{c}}^{t \epsilon}}{dP_{\text{ect}}} \frac{P_{\text{ect}}}{Q_{E_{c}}^{t \epsilon}}
\]

It is not necessary for the demand curve to be of the constant elasticity form if the program is small. Any downward sloping curve with continuous first derivatives can be used.

Now natural gas consumption in class c is also assumed to be of the same form:

\[
Q_{G_{c}}^{t \epsilon} = \beta_{g_{c}c}^{t \epsilon} P_{g_{c}c}^{t \epsilon} - \gamma_{g_{c}c}^{t \epsilon} \tag{4.32}
\]

where \( \beta_{g_{c}c}^{t \epsilon} \) is a constant and:

\( Q_{G_{c}}^{t \epsilon} \) = Quantity of natural gas consumed by class c excluding program participant loads directly affected by the DSM program in period i of year t.

\( P_{g_{c}c} \) = Average price of natural gas for class c in year t.

\( \gamma_{g_{c}c} \) = Price elasticity of demand for natural gas for class c in year t.

Moreover, let us define \( Q_{E_{c}}^{t} \) and \( Q_{G_{c}}^{t} \) as the total consumption of electricity and natural gas respectively, by customer class c in year t.
The gross benefits to nonparticipants in year \( t \) due to the consumption of electricity and natural gas equal:

\[
BNP_c(P) = \sum_c \sum_t \left[ \frac{\beta^t_{ec}}{1-1/\gamma_{ec}} P_{act}^{1-\gamma_{ec}} + \sum_t \frac{\beta^t_{gc}}{1-1/\gamma_{gc}} P_{act}^{1-\gamma_{gc}} \right] \quad (4.33)
\]

The terms inside the brackets are obtained by integrating the demand curves (4.31) and (4.32), i.e., the area under their demand curves. The total gross benefits to participants and non-participants can be got by taking the sum of the present values of expressions (4.30) and (4.33).

4.3.3 Net Benefits of the DSM Program

Thus the gross benefits to participants and non-participants in year \( t \) are defined by equations (4.30) and (4.33) respectively. Let us now consider the costs associated with generation of electricity and natural gas, the program costs, and finally the external costs due to generation of electricity and natural gas.

Let the cost of electricity production in any period ‘i’ be \( CE^t_i (QE_{p_i,0} + QE_{c_i}^t) \).

This cost function is dependent on \( P_{act} \) and \( \theta \), since \( QE_{p_i,0} \) is dependent on \( P_{act} \) and \( \theta \) and \( QE_{c_i} \) on \( P_{act} \). Let the external costs in each period due to electricity generation be \( EXE^t_i (QE_{p_i,0} + QE_{c_i}^t) \). These costs are also dependent on \( P_{act} \) and \( \theta \). There are
similar costs $CG^*_t (QG_{p,1}^{t*} + QG_{c}^{d*})$ and $EXG^*_t (QG_{p,1}^{d*} + QG_{c}^{d*})$ for natural gas production.

It is to be noted that these that all the above costs exclude the costs of the DSM program.

Utility costs due to the program is a function of $\theta$ and equal:

$$C_{cost}^E (\theta) = \theta \cdot [CM_t + CE_t + CP_t] \quad (4.34)$$

where:

$CM_t =$ Marketing and administrative costs to the utility for implementing the DSM program in year $t$.

$CE_t =$ Utility equipment costs for implementing the DSM program in year $t$.

$CP_t =$ Transfer payments from the utility to the participants in the form of incentive payments.

Of the amount in expression (4.34), only $\theta \cdot (1 - FR) CE_t + CM_t$ are net expenditures from the society’s perspective. This is because the portion of the $Ce_{eq}$ expended on behalf of free riders would have been invested anyway even without the utility DSM program. Now if program participants also bear direct costs as a result of the program, these too must be considered. Thus the net societal costs due to the program equals:
\[ C_{soc}^s (\theta) = \theta (1 - Fr) \left[ CE_t + EQ_t \right] + CM_t \]  

(4.35)

where:

\[ EQ_t = \text{Customer equipment costs in year } t. \]

Now the net benefits of the program equal the sum of the present worth of the 
participant and the nonparticipant gross benefits minus the costs of electricity and 
natural gas production, societal costs of the program, and external costs of electricity 
and natural gas production. Therefore the net benefits of the DSM program equal:

\[
NB \left( P_{ect}, P_{gct}, \theta \right) = \sum_{t} \frac{1}{(1+i)^t} \left[ 
BP_t \left( P_{ect}, P_{gct}, \theta \right) + BNP_t \left( P_{ect}, P_{gct}, \theta \right) 
- \sum_{t} \left[ CE \left( P_{ect}, \theta \right) + CG \left( P_{gct}, \theta \right) \right] 
- \sum_{t} \left[ EXE \left( P_{ect}, \theta \right) + EXG \left( P_{gct}, \theta \right) \right] 
- C_{soc}^s (\theta) \right] 
\]

(4.36)

In the version of the MV/GTRC criterion presented in this section, we do not 
explicitly consider capacity and environmental control costs which we assume are 
included in the cost of energy supply. As shown in a derivation in Nelson and Hobbs 
(1990a), their inclusion is a simple matter.
The problem at hand is to determine the change in net social benefits resulting from implementation of the DSM program. If the program is not too large, this approximately equals the sum of the total derivatives of expression (4.36) taken one term at a time, and is as given below:

\[
\frac{dNB}{d\theta} = \sum \frac{1}{(1+i)^t} \left[ \frac{dB_P}{d\theta} \varepsilon + \frac{dBNP}{d\theta} \varepsilon + \frac{dC_{soc}}{d\theta} \varepsilon \right. \\
- \left[ \frac{dCE}{d\theta} \varepsilon + \frac{dCG}{d\theta} \varepsilon \right] \\
- \left[ \frac{dEXE}{d\theta} \varepsilon + \frac{dEXG}{d\theta} \varepsilon \right] \right]
\]

(4.37)

Taking the total derivative of (4.30), we get:

\[
\frac{dB_P}{d\theta} = \sum \left[ -\frac{\beta_{P_0}^{it}}{(1-\theta)} \frac{1-\gamma}{\gamma} P_{ept}^{1-\gamma} \right. \\
+ \left. \frac{\beta_{P_1}^{it}}{(1-\theta)} \frac{1-\gamma}{\gamma} P_{p}^{1-\gamma} \right] \\
- \gamma \frac{dP_{ept}}{d\theta} (1-\theta) \beta_{P_0}^{it} P_{ept}^{1-\gamma} \right] \left. - \gamma \frac{dP_{ept}}{d\theta} \theta \beta_{P_1}^{it} P_{ept}^{1-\gamma} \right]
\]

(4.38)

We can ignore the last two terms inside the square brackets because the derivatives of price w.r.t \( \theta \) is much smaller than the prices themselves. Simplifying further by substituting (4.20)-(4.21) in (4.38) yields:
\[
\frac{dB_P^c}{d\theta} = \sum_i \frac{\gamma}{1-\gamma} \left[ P_{cpt} Q_{E_P}^{t_c} - P_{gpt} Q_{C_P}^{t_c} \right]
\]  \hspace{1cm} (4.39)

(Change in Bill)

Multiplying and dividing (4.39) by \( Q_{E_P}^{t_c} \) and substituting (4.28) in the first term on the right hand side of (4.39) would then yield:

\[
\frac{dB_P^c}{d\theta} = \sum_i \frac{\gamma P_{cpt}}{1-\gamma} \left[ 1 - \left( \frac{P_{cpt}}{P_{gpt}} \right)^{\gamma} \right] Q_{E_P}^{t_c} \hspace{1cm} (4.40)
\]

The above expression gives the benefits due to rebound/takeback for participants. This expression would reduce to the benefits due to rebound derived in the previous chapter if we assume that there is just one fuel (\( P_{cpt} = P_{gpt} \)).

Now taking the total derivative of (4.34) w.r.t. \( \theta \) yields:

\[
\frac{dB_{NP}}{d\theta} = - \sum_c \sum_i \left( \gamma_{act} \frac{dP_{act}}{d\theta} P_{act}^{t_c} \gamma_{em} + \gamma_{gct} \frac{dP_{gct}}{d\theta} P_{gct}^{t_c} \gamma_{em} \right) \hspace{1cm} (4.41)
\]

Substituting (4.32)-(4.33) into (4.41) yields:
\[
\frac{dBNP_t}{d\theta} = -\sum_c \left( \gamma_{ect} \sum_i Q_{Ect}^{it} \frac{dP_{ect}}{d\theta} + \gamma_{gct} \sum_i Q_{Gct}^{it} \frac{dP_{gct}}{d\theta} \right) \quad (4.42)
\]

Now, by definition of price elasticity of demand, \( \gamma_{act} = -\frac{P_{act}}{Q_{act}} \frac{dQ_{act}}{dP_{act}} \). For small programs, the change in nonparticipant loads in customer class \( c \) \( \Delta Q_c^{it} \) can, by using (4.32) above, be approximated as (4.43) below:

\[
\Delta Q_{Ect}^{it} = \frac{dQ_{Ect}^{it}}{d\theta} = \frac{dQ_{Ect}^{it}}{dP_{ect}} \frac{dP_{ect}}{d\theta} = -\frac{\gamma_{ect} Q_{Ect}^{it}}{P_{ect}} \frac{dP_{ect}}{d\theta} \quad (4.43)
\]

The above description applies to natural gas usage as well. Inserting this result in (4.42) yields:

\[
\frac{dBNP_t}{d\theta} = \sum_c \left[ P_{ect} \sum_i \Delta Q_{Ect}^{it} + P_{gct} \sum_i \Delta Q_{Gct}^{it} \right] \quad (4.44)
\]

Now \( CE_t^c \) \( (P_{act}, \theta) \) is the cost of generating electric power in period \( i \) of year \( t \) as a function of prices \( P_{act} \) for all \( c \), and the implementation of the DSM program \( (\theta=0 \) indicates no implementation, while \( \theta=1 \) indicates full implementation) in any period 'i'. Similar reasoning applies to natural gas as well. Therefore the cost of electricity or natural gas production over all periods of a year equal:
\[ CE^t(P_{ect}, \theta) = \sum_i CE^t_i \left( \sum_c QE^t_{c} + (1-\theta) QP_{p,0} \right) \quad (4.45) \]

\[ CG^t(P_{qct}, \theta) = \sum_i CG^t_i \left( \sum_c QG^t_{c} + \theta QP_{p,1} \right) \quad (4.46) \]

Now differentiating (4.45)-(4.46) w.r.t. to \( \theta \) yields (4.47)-(4.48) respectively:

\[ \frac{d CE^t}{d \theta} = \sum_i \frac{\partial CE^t_i}{\partial QE^t_i} \left[ \sum_c \frac{\partial QE^t_{c}}{\partial P_{ect}} \frac{d P_{ect}}{d \theta} - QP_{p,0} \right] \quad (4.47) \]

\[ \frac{d CG^t}{d \theta} = \sum_i \frac{\partial CG^t_i}{\partial QG^t_i} \left[ \sum_c \frac{\partial QG^t_{c}}{\partial P_{qct}} \frac{d P_{qct}}{d \theta} + QP_{p,1} \right] \quad (4.48) \]

Now we can define \( MC_{e,i}^t \) and \( MC_{q,t}^t \) as the marginal cost of electricity and natural gas production respectively, in period 'i' of year 't', i.e., \( \frac{\partial CE^t_i}{\partial QE^t_i} = MC_{e,i}^t \) and

\[ \frac{\partial CG^t_i}{\partial QG^t_i} = MC_{q,t}^t. \] Simplifying (4.47)-(4.48) and using (4.43), and setting \( \theta \) equal to zero, yields (4.49)-(4.50) below:
\[
\frac{dCE^t}{d\theta} = \sum_i MC_{ei}^t \left[ \sum_c \Delta QE_c^i - Q_{E,0}^i \right] \quad (4.49)
\]

\[
\frac{dCG^t}{d\theta} = \sum_i MC_{qi}^t \left[ \sum_c \Delta QG_c^i + Q_{G,1}^i \right] \quad (4.50)
\]

By using a procedure similar to (4.45)-(4.50) above and defining \( ME_{ei}^t \) and \( ME_{qi}^t \)
as the marginal external costs of electricity and natural gas, respectively, we get:

\[
\frac{dEXE^t}{d\theta} = \sum_i ME_{ei}^t \left[ \sum_c \Delta QE_c^i - Q_{E,0}^i \right] \quad (4.51)
\]

\[
\frac{dEXG^t}{d\theta} = \sum_i ME_{qi}^t \left[ \sum_c \Delta QG_c^i + Q_{G,1}^i \right] \quad (4.52)
\]

Finally, the societal program costs as a function of \( \theta \) given by (4.36) can be differentiated w.r.t to \( \theta \) to yield:

\[
\frac{dC_{soc}^t}{d\theta} = (1-F_r) \left[ C_{eq}^t + C_{i}^t \right] + C_{maa}^t \quad (4.53)
\]

Thus the net benefits of a DSM program is approximated by the total derivative of NB w.r.t to \( \theta \) given by expression (4.37) and is equal to the sum of equations (4.40), (4.44), and (4.49)-(4.53) derived above:
Net Benefits=
\[
\sum_{t} \frac{1}{(1+r)^t} \left[ \sum_{i} \gamma P_{pct} \left[ 1 - \left( \frac{\gamma P_{pct}}{P_{pct}} \right)^{1-\gamma} \right] QE_{p,t} \right] + \sum_{i} [MC_{a1}^t QE_{p,t} - MC_{q1}^t QG_{p,t}] \\
+ \sum_{i} [ME_{a1}^t QE_{p,t} - ME_{q1}^t QG_{p,t}] \\
+ \sum_{c} \left[ P_{ect} - \sum_{i} (MC_{a1}^t + ME_{a1}^t) \Delta QE_{c,t} \right] \\
+ \sum_{c} \left[ P_{gct} - \sum_{i} (MC_{q1}^t + ME_{q1}^t) \Delta QG_{c,t} \right] \\
- \left[ (C_{c}^t + C_{eq}^t) (1-Fx) + C_{maa}^t \right] \\
\text{[Societal Costs]}
\]

(4.54)

The expression above gives the most general version of the most value/generalized total resource cost test. For the derivative to be complete, we need to calculate the derivatives of \( P_{ext} \) and \( P_{gext} \) with respect to \( \theta \), in order to obtain \( \Delta QE_{c,t} \) and \( \Delta QE_{c,t}^t \).

The value of these derivatives depends upon the pricing policies of the utility and
its regulatory commission. If the program costs and lost (gained) revenues are not recovered through rates, then the derivatives equal zero and the benefits of rate changes are zero as well. Here, we assume that all program costs are recovered through rates, i.e. each utility has zero economic profits. Thus revenue minus costs equal zero. It is also assumed that the gas and electric utility each have a separate revenue recovery constraint as given below by equations (4.40) and (4.41) respectively. The revenue recovery constraints for each year t is as given below:

\[
\sum_c P_{act}QE_{c}^{lt} + (1 - \theta) P_{opt}QE_{p,0} = \sum_c CE_i (\sum_c QE_{c}^{lt} + (1 - \theta) QE_{p,0}) \quad (4.55)
\]

\[
\sum_c P_{gct}QG_{c}^{lt} + \theta P_{opt}QE_{p,t} = \sum_i CG_i (\sum_c QG_{c}^{lt} + \theta QG_{p,1}) + \theta[CE_t + CP_t + CM_t] \quad (4.56)
\]

where:

\[
C_{tot} = (CE_t + CM_t + CP_t)
\]

\[
CP_t = \text{Transfer payments to participants by the gas utility (rebates, incentive payments etc.)}
\]

Taking the total derivative of the two revenue recovery constraints would yield the values of the derivatives of electricity and natural gas prices with respect to \( \theta \).

The total derivative of equation (4.55) w.r.t. \( \theta \) is given below, evaluated at \( \theta=0 \):
\[ \sum_c Q_{E_c}^{it} \frac{dP_{e ct}}{d\theta} + \sum_c P_{e ct} \frac{\partial Q_{E_c}^{it}}{\partial P_{e ct}} \frac{dP_{e ct}}{d\theta} = \sum_i M_{c e i} \left( \sum_c \frac{\partial Q_{E_c}^{it}}{\partial P_{e ct}} \frac{dP_{e ct}}{d\theta} - Q_{E_p}^{it} \right) \] (4.57)

Similarly, the total derivative of the gas revenue recovery constraint w.r.t. \( \theta \) is given below:

\[ \sum_c Q_{G_c}^{it} \frac{dP_{g ct}}{d\theta} + \sum_c P_{g ct} \frac{\partial Q_{G_c}^{it}}{\partial P_{g ct}} \frac{dP_{g ct}}{d\theta} + P_{g pt} Q_{G_p}^{it} = \sum_i M_{c g i} \left( \sum_c \frac{\partial Q_{G_c}^{it}}{\partial P_{g ct}} \frac{dP_{g ct}}{d\theta} + Q_{G_p}^{it} \right) + C_{e ot} \] (4.58)

Now if we assume that \( dP_{e ct}/d\theta \) and \( dP_{g ct}/d\theta \) is the same for all classes, and using the definition of own price elasticity of demand, (4.57) and (4.58) can be simplified to yield:
\[
\frac{dP_{\text{act}}}{d\theta} = \frac{\sum_i (P_{\text{act}} - MC^t_{\text{act}}) QE^t_{p,0}}{\sum_i (1 - \gamma_{\text{act}}) QE^t_c + \sum_i MC^t_{\text{act}} \sum_c \gamma_{\text{act}} QE^t_{c,0}} \tag{4.59}
\]

\[
\frac{dP_{\text{gct}}}{d\theta} = \frac{\sum_i (MC^t_{\text{gct}} - P_{\text{gct}}) QG^t_{p,1} + C^t_{\text{tot}}}{\sum_i (1 - \gamma_{\text{gct}}) QG^t_c + \sum_i MC^t_{\text{gct}} \sum_c \gamma_{\text{gct}} QG^t_{c,0}} \tag{4.60}
\]

It can be noted that the numerator of the R.H.S expression of (4.59)-(4.60) is the "ratepayer impact measure" test of the California Standard Practice for that year t. Substituting equation 4.59 and 4.60 in equation 4.54 would yield the total derivative of the net benefits w.r.t. to the DSM program, i.e., the most general form of the MV/GTRC criterion given below:
Net Benefits =

\[ \sum_{t} \frac{1}{(1+i)^t} \left( \sum_{i} \gamma \frac{P_{apt}}{1-\gamma} \left[ 1 - \left( \frac{P_{apt}}{P_{apt}} \right)^{1-\gamma} \right] Q_{E,0}^{t,1} \right) \]

[Rebound Benefits]

\[ + \sum_{t} [MC_{e,t} Q_{E,0}^{t,1} - MC_{g,t} Q_{G,1}^{t,1}] \]

[Avoided Energy Costs]

\[ + \sum_{t} [ME_{e,t} Q_{E,0}^{t,1} - ME_{g,t} Q_{G,1}^{t,1}] \]

[Avoided External Costs]

\[-\left( (EQ_{t} + CE_{t}) (1-Fr) + CM_{t} \right) \]

[Societal Program Costs]

\[ + \sum_{t} \gamma_{act} Q_{E}^{t,1} - \sum_{t} (MC_{e,t} + ME_{e,t}) \sum_{c} \gamma_{act} \frac{Q_{E}^{t,1}}{P_{act}} RIM_{e}^{t} \]

\[ + \sum_{c} (1-\gamma_{act}) Q_{E}^{t,1} + \sum_{t} MC_{e,t} \sum_{c} \gamma_{act} \frac{Q_{E}^{t,1}}{P_{act}} \]

[Benefits of Electric Rate Changes]

\[ + \sum_{c} \gamma_{gct} Q_{G}^{t,1} - \sum_{t} (MC_{g,t} + ME_{g,t}) \sum_{c} \gamma_{gct} \frac{Q_{G}^{t,1}}{P_{gct}} RIM_{g}^{t} \]

\[ + \sum_{c} (1-\gamma_{gct}) Q_{G}^{t,1} + \sum_{t} MC_{g,t} \sum_{c} \gamma_{gct} \frac{Q_{G}^{t,1}}{P_{gct}} \]

[Benefits of Gas Rate Changes]

where:

\[ RIM_{e}^{t} = - \sum_{t} (MC_{e,t} - P_{apt}) Q_{E}^{t,1} \]

\[ RIM_{g}^{t} = \sum_{t} (P_{apt} - MC_{g,t}) Q_{G}^{t,1} - G_{tot}^{t} \]
The MV/GTRC criterion derived above can be further simplified and presented as below:

\[
\text{Net Benefits} = \sum_t \frac{1}{(1+i)^t} \left( \sum_i \gamma P_{i,t} \left[ \frac{\tau P_{p,t}}{1-\gamma} \left( \frac{P_{q,t}}{P_{s,t}} \right)^{1-\gamma} \right] QE_{p,t} \right)
\]

[Rebound Benefits]
+ \sum_i [MC^{c,t}_i QE_{p,t}^{c,t} - MC^{q,t}_i QG_{p,t}^{c,t}]

[Avoided Energy Costs]
+ \sum_i [ME^{c,t}_i QE_{p,t}^{c,t} - ME^{q,t}_i QG_{p,t}^{c,t}]

[Avoided External Costs]
- \left[ (EQ_c + CE_c) (1-F_r) + CM_c \right]

[Societal Program Costs]
+ \sum_c \gamma_{ct} QE_{c,t}^{c,t} - \sum_i (MC^{c,t}_i + ME^{c,t}_i) \sum_c \gamma_{ct} \frac{f_{ct}^{c,t}}{P_{ct}} RIM_{c,t}
\]

[Benefits of Electric Rate Changes]
+ \sum_c \gamma_{qct} QG_{c,t}^{c,t} - \sum_i (MC^{q,t}_i + ME^{q,t}_i) \sum_c \gamma_{qct} \frac{f_{qct}^{c,t}}{P_{qct}} RIM_{q,t}

[Benefits of Gas Rate Changes]

where \( f_{ct}^{c,t} \) and \( f_{qct}^{c,t} \) are fractions of total electricity and natural gas usage respectively, consumed during period i of year t by customer class c.
The above derivation is the most general form of the MV/GTRC test as applied to the multi-fuel case. In the next subsection, an example is provided to illustrate the differences between the MV/GTRC and the LC/TRC tests for a fuel switching program.

4.3.4 A Gas Chiller Example

In this subsection, we present a simple example to illustrate the use of the general form of the MV/GTRC criterion presented in the previous subsection (equation 43.) In this example, electricity is the fuel being switched and natural gas is the new fuel. This program is based on information provided in a paper by Broderick and Patel (1988).

The installed air-conditioning capacity in the United States is estimated at over 10 million tons of which 95% is electrically driven. At present, electric utilities in many parts of the country are starved of capacity to meet electric demands at the summer peak. The operation of air-conditioning equipment is seen as the major reason for this peak demand. In this subsection, we analyze the potential benefits of a gas engine-driven water chiller for a commercial building. By the installation of this equipment, the electric utility will gain immediate relief from the summer peak load and, in the longer term, reduce the need to add electric generation capacity. Meanwhile, the
natural gas system, whose peak demand is in the winter, is utilized at a higher average capacity. This helps spread the capital costs for the transmission and distribution facilities over a larger base, thus lowering rates.

The lifetime period of the chiller is 10 years. For simplicity, all the dollar figures in this example are annualized and so we ignore the superscript 't' in all our calculations below. It is assumed, in this example, that marginal costs and prices do not change in real terms. It is also assumed here that there are two periods in a year (summer and winter) and there are three customer classes (residential, commercial and industrial).

It is assumed in this example that the building owners must choose between a electric chiller and a gas-fired chiller. It is also assumed that the owners would not invest in the gas chiller without the subsidy (Fr=0). It is also assumed that there are no external costs.

The data for the gas chiller which replaces a traditional central electric air conditioner for a building of 100,000 square foot area and 12 floors high is given below. Each year of operation in the office building, the gas chiller consumes over three times as much energy as the traditional electric chiller. The gas chiller uses 10,000 mmbtu/year at a total yearly operating cost of about $50,000/year whereas the
electric chiller uses 3000 mmBtu/year of electricity for a total energy cost of $48,000/year. A key operational characteristic of the electric chiller is a peak demand of 853 KW. This leads to demand charges of about $156,000/year. Thus the electric chiller has yearly operating costs exceeding $210,000 for the building owners.

The gas utility spends $0/yr on marketing and advertising expenses. It also spends $4000/yr on equipment purchase per participant. The participant spends $8000/yr on equipment as well. Therefore:

\[
\begin{array}{ll}
C_{\text{max}} & = \quad $ 0 \\
C_{\text{eq}} & = \quad $ 4000 \\
C_{c} & = \quad $ 8000 \\
C_{\text{reb}} & = \quad $ 0 \\
Fr & = \quad 0
\end{array}
\]

Thus the total program costs and the net societal costs are (in the absence of free riders) as given below:

\[
\begin{align*}
C_{\text{soc}} & = $(8000+4000) = $12,000 = \text{Societal Program Costs} \\
C_{\text{tot}} & = $(0+4000) = $ 4,000 = \text{Total Utility Costs}
\end{align*}
\]

Now the prices of natural gas in the various sectors are 3.5 $/mmBtu for the residential class (c=r), 3.0 $/mmBtu for the commercial (c=c) and industrial classes (c=i). The average price of electricity in the three customer classes are as follows: 14.0 $/mmBtu, 12.0 $/mmBtu, and 12.0 $/mmBtu for the residential, commercial and
industrial classes respectively. If we assume an average heat rate of 3416 Btu/kWh, these rates correspond to 4.78 cents/kWh, 4.10 cents/kWh, and 4.10 cents/kWh respectively.

\[
P_{ex} = \$ 14.0/mmBtu \quad \quad P_{eg} = \$ 2.5/mmBtu \\
P_{es} = \$ 12.0/mmBtu \quad \quad P_{eg} = \$ 3.0/mmBtu \\
P_{ei} = \$ 12.0/mmBtu \quad \quad P_{gi} = \$ 3.0/mmBtu
\]

The marginal cost of natural gas supply during the summer (i=s) and the winter (i=w) seasons are 2.4 $/mmBtu and 3.2 $/mmBtu respectively. The marginal costs of electricity are 20 $/mmBtu during summer and 1.8 $/mmBtu during winter. The prices and marginal costs of electricity are generally expressed in $/kWh but, for consistency with natural gas units, these are expressed in $/mmBtu. For example, $20/mmBtu summer peak marginal cost of electricity is consistent with a peak marginal cost of 0.068 $/kWh for a utility with an average heat rate of 3416 Btu/kWh.

\[
MC_{es}^s = \$ 1.8/mmBtu \quad \quad MC_{eg}^s = \$ 3.2/mmBtu \\
MC_{es}^w = \$ 20.0/mmBtu \quad \quad MC_{eg}^w = \$ 2.4/mmBtu
\]

The technical savings potential of this program is as given below. Note that all the savings occur during the summer season. Thus:

\[
Q_{E_{p,0}}^s = -10,000 \text{ mmBtu} \quad \quad Q_{G_{p,1}}^s = 3000 \text{ mmBtu} \\
Q_{E_{p,0}}^w = 0 \text{ mmBtu} \quad \quad Q_{G_{p,1}}^w = 0 \text{ mmBtu}
\]

We also assume that 30% of both the electrical as well as the natural gas load is
residential, 35% commercial, and that the other 35% industrial. This service territory has 10% of the natural gas load and 20% of the electricity load during summer. Thus:

\[ f_{er} = 0.06 \quad f_{g_{r}} = 0.03 \]
\[ f_{ei} = 0.07 \quad f_{g_{i}} = 0.035 \]
\[ f_{ec} = 0.07 \quad f_{g_{c}} = 0.035 \]
\[ f_{ew} = 0.24 \quad f_{g_{w}} = 0.27 \]
\[ f_{eiw} = 0.28 \quad f_{g_{iw}} = 0.315 \]
\[ f_{ecw} = 0.28 \quad f_{g_{cw}} = 0.315 \]
\[ f_{er} = 0.3 \quad f_{ei} = 0.35 \quad f_{ec} = 0.35 \]
\[ f_{g_{r}} = 0.3 \quad f_{g_{i}} = 0.35 \quad f_{g_{c}} = 0.35 \]

It is assumed that the conditional price elasticity of demand for cooling equals 0.25. It is to be noted that if the production function exponent \( \alpha \) in (4.18) is different for gas and electricity, the conditional price elasticity of demand for electricity and natural gas will differ. But, in this section, we assume that the \( \alpha \) values are the same for both sectors. Moreover, the price elasticity of demand for electricity is assume to be 0.3 in the residential class, 0.5 for the commercial (participant) class, and 0.7 in the industrial class. The price elasticities for natural gas are 0.5, 0.7, and 0.7 in the residential, commercial and industrial classes, respectively:

\[ \gamma = 0.25 \]
\[ \gamma_{er} = 0.3 \quad \gamma_{ec} = 0.5 \quad \gamma_{ei} = 0.7 \]
\[ \gamma_{gr} = 0.5 \quad \gamma_{gc} = 0.7 \quad \gamma_{gi} = 0.7 \]
Using the values given above in this subsection, we can calculate the following values:

\[ \tau = 3.33 \]

\[ QE_{p,0}^s = 3000 \text{ mmBtu} \]

\[ QG_{p,1}^s = [3.333^{(1.0.25)}][3.0/12.0]^{0.25}]^*3000 \text{ mmBtu} = 10466.3 \text{ mmBtu} \]

[from equation (4.29)]

\[ RIM_e = -12*(3000) + (20*3000) - (1.8*0) = $24,000 \]

[from definition of RIM below (4.61)]

\[ RIM_g = 3*10,466.3-2.4*10466.3-3.2*0-4000 = $2279.81 \]

It should be noted that the actual natural gas consumption is larger than the technical estimate of 10,000 mmBtu. This is because of the rebound effect, defined earlier. Thus, the customer uses more air-conditioning because the effective price of air conditioning has decreased. The RIM benefits for both the gas and the electric utility are positive indicating that both electricity and gas nonparticipants are better off.

Let us now calculate the benefits of each of the categories of benefits identified in equation 4.62.

I. Rebound Benefits
Benefits due to rebound =

\[
\frac{0.25 \times 12}{(1 - 0.25)} (3000) \left[ 1 - (3.333 \frac{3}{12})^{1 - 0.25} \right] = \$ 1533.65
\]

2. **Marginal Cost Savings**

Avoided energy costs =

\[
\$ -10,466.3 \times 2.4 + 3000 \times 20 = - \$ 34880.88
\]

3. **Net Societal Program Costs**

Net societal costs =

\[
\$ 12,000
\]

4. **Benefits of Rate Changes**

Average price elasticity of electricity =

\[
0.3 \times 0.3 + 0.5 \times 0.35 + 0.7 \times 0.35 = 0.51
\]

Average price elasticity of natural gas =

\[
0.5 \times 0.3 + 0.7 \times 0.35 + 0.7 \times 0.35 = 0.64
\]

\[
\sum_{i} MC_{ei} \sum_{c} \gamma_{ec} \frac{f_{ei}}{P_{ec}} =
\]

\[
20 \left[ \frac{0.06 \times 0.3}{14} + \frac{0.07 \times 0.5}{12} + \frac{0.07 \times 0.7}{12} \right] +
1.8 \left[ \frac{0.24 \times 0.3}{14} + \frac{0.28 \times 0.5}{12} + \frac{0.28 \times 0.7}{12} \right] = 0.2254
\]
\[ \sum_{i} MC_{gi} \sum_{c} \gamma_{gc} \frac{P^i_{gc}}{P_{gc}} = \]

\[ 2.4 \left[ \frac{0.03 \times 0.5}{3} + \frac{0.035 \times 0.7}{3} + \frac{0.07 \times 0.7}{3} \right] + 3.2 \left[ \frac{0.27 \times 0.5}{3} + \frac{0.28 \times 0.7}{3} + \frac{0.28 \times 0.7}{3} \right] = 0.6433 \]

Benefits of electric rate changes =

\[ \frac{0.51 - 0.2254}{1 - 0.51 + 0.2254} \times 24,000 = 9,547.67 \]

Benefits of natural gas rate changes =

\[ \frac{0.64 - 0.6433}{1 - 0.64 + 0.6433} \times 2279.81 = -7.50 \]

Thus the net benefits of the program under MV/GTRC equal:

\[ (1533.65 + 34880.88 - 12,000 + 9547.67 - 7.50) = 33954.70 \]

Therefore we find that the gas chiller program is attractive under the MV/GTRC criterion. The benefits under the LC/TRC criterion are much smaller = avoided costs - program costs ($34,880.88 - 12,000 = $22,880.88) because it ignores 1) the positive
rate benefits due to electric rate changes (nearly a quarter of the avoided cost savings) and 2) the rebound benefits. The electric rate benefits are positive because electric rates go down due to the saving of electric power whose marginal cost is much higher than the average price of electricity. The gas rate benefits are slightly negative even though the RIM benefits are positive. This is primarily due to the fact that the average marginal cost of supply is slightly more than the price of natural gas.

It is shown in Nelson and Hobbs (1990b) that the presence of 10% free riders (Fr=0.1) would lead to the program failing the LC/TRC test while still remaining attractive under the MV/GTRC test. This is primarily because the avoided costs due to the program are less than what is anticipated but the rebound benefits and the benefits of rate changes, though smaller, still make the program attractive under the MV/GTRC criterion.

The example presented in this subsection thus clearly shows the advantage that the new criterion has over the ubiquitous LC/TRC test. This is because the MV/GTRC test considers the benefits of rate changes and rebound/takeback that are ignored by the LC/TRC tests. The example thus illustrates the importance of considering the value changes that occur when the quantity of energy services consumed by participants and energy by nonparticipants do not remain the same after the program.
4.4 Alternative Criteria for DSM Program Screening

4.4.1 Incorporation of Noneconomic Criteria

It is argued that the versions of the MV/GTRC criterion derived earlier in this chapter and Chapter 3, like all other efficiency tests, may be too narrow because they exclude many other important, but, nonmonetizable impacts of DSM programs. Presented below is a list of the various criteria that are often considered in the evaluation of DSM programs (NARUC, 1988).

DSM Program Impact Issues

* energy savings
* capacity savings
* impacts on transmission and distribution costs
* system reliability impacts
* predictability of program timing and savings
* possibility of free riders and takeback
* societal (utility + customer) costs of programs
* customer share of program costs
* consumer satisfaction and comfort

DSM Program Technology Issues

* maturity of program technology
* incidence of DSM equipment failure
* market saturation
* fuel type
Administrative Issues

* planning and operating flexibility
* measurability of program impacts
* management quality and DSM experience

Regulatory Issues

* regulatory risks and impacts on utility shareholders
* equity among participants and nonparticipants
* shareholder returns

External Issues

* security of fuel supply (e.g., oil import dependence)
* environmental impacts
* public health and safety
* employment effects
* impacts on the poor

The above factors are presented in no particular order of importance. Utility DSM planners would have to decide which of these factors are important to their utility. Many of these factors can be combined into one criterion. For example, the new MV/GTRC criterion considers the following issues: energy savings, capacity savings, possibility of free riders and takeback, societal costs of programs, customer share of program costs, and shareholder benefits. But there exists no economic rationale for including others (e.g., environmental effects, administrative issues, or impacts on the poor). A multi-criterion decision making (MCDM) approach offers
a way of considering the latter factors together with economic factors in a single planning framework. There are several source in the literature which discuss various MCDM methods for considering multiple attributes in project screening (see, e.g., Chankong and Haimes, 1983, Cohon, 1978, or Hobbs, Chankong, and Hamadeh, 1990).

The best approach for screening is one whose validity is accepted, yet is simple and transparent. It should be a method which does not require decision makers to specify arbitrary parameters (such as goals) which complicate the analysis. Additive value functions are clearly the best method from the above perspective. The only parameters required are weights and the scaled attributes. Moreover, experiments show that there is little evidence to suggest that any particular nonlinear method (say, multiplicative utility functions) is both 1) more valid than additive utility functions and 2) results in significantly different decisions (Hobbs, 1986).

A model, developed by Nelson and Hobbs (1991), provides a framework to analyze the multiattribute aspects of DSM programs. It utilizes an additive utility function method. The objective of this model is to select the most attractive portfolio of candidate DSM programs subject to a budget constraint. The DSM programs would thus be ranked according to the value of the weighted sum of attributes. The overall value of a DSM program ‘j’ is given as:
\[ V(X_j) = \sum_{i} w_i V_i(X_{ij}) \]  

where:

\[ X_{ij} \] = Level of attribute \( i \) for plan \( j \);
\[ X_j \] = Values of attributes \( X_{ij} \) for option \( j \);
\[ W_i \] = Weights assigned to attribute \( i \);
\[ V_i(\cdot) \] = Value function of attribute \( i \), translating the attribute into a measure of worth.

Thus, for example, low income weatherization programs may pass the multiattribute test even if it fails the MV/GTRC test, if "benefits to the poor" is an important attribute.

Two of the most important attributes (environmental externalities and shareholder incentives) are extensively discussed in Nelson and Hobbs (1991). A lot of attention has been given of late to incorporating the environmental issues in integrated resource planning (Ottinger, 1990, and Cohen, 1990). These issues include air emissions, land use, water pollution, solid wastes etc. There has been an effort in recent years to include the environmental costs of various resource options in the planning process. Through monetization, these externalities are expressed as a cost per unit of externality, such as a $/lb of emissions or $/gal. of water consumed (Chernick and Caverhill, 1990). Such costs are avoided by the adoption of DSM options and the
benefits of DSM programs should include avoided external costs. This is the basis for the term \( E_X \), in the value function (3.3) in Chapter 3.

4.4.2 Shareholder Benefits

One attribute that was not previously considered in the MV/GTRC criterion is shareholder benefits. We had previously assumed, in the MV/GTRC formulation, that utility has zero economic profit, i.e., the producer surplus is zero. But many states have implemented or are considering DSM incentive mechanisms (Chamberlin, 1990). Cole and Cummings (1990) present a discussion of the various type of incentives for electric utilities to implement DSM programs. Presented below is a discussion of how MV/GTRC and other economic efficiency tests are modified to include shareholder benefits.

It is important to consider the effect of these shareholder benefits on the four tests of the MOSTVALUE model (MV/GTRC, LC/TRC, RIM, GRIM). Shareholder benefits have no impact on the TRC test since they are considered as a transfer payment from utility customers to the utility shareholders. But they do affect the MV/GTRC benefits because of its effects on nonparticipant rates (the benefits of rate changes term). If social marginal cost (utility marginal cost [MC] and external cost [ME]) is less than the price (\( P \)), and the change in nonparticipant demands due to rate
changes ($\Delta Q_{mp}$) is negative, then shareholder incentives will worsen the benefits of rate changes term. This is because such incentives will increase $P$ even more, further discouraging consumption of energy whose value ($P$) exceeds its cost ($MC+ME$). But if $MC+ME>P$, then shareholder benefits actually increase the benefits of rate changes, since higher rates discourage consumption of energy whose value is less than its cost. RIM benefits are lowered because shareholder benefits have to be paid for by nonparticipants. GRIM is affected similarly because it is a summation of the RIM benefits and the benefits due to rate changes.

Thus, from the above discussion, we note that the only modification that has to made to the screening tests is the term $RIM_\epsilon$ in expression (3.14) of Chapter 3. This term should now include shareholder benefits ($SB_\epsilon$) as utility costs that need to be recovered through rates. Thus the definition of $RIM_\epsilon$ in equation 3.14 of Chapter 3 now becomes:

$$RIM_\epsilon =
(1-F_I) \sum \left(P_{pt} - MC_{I\epsilon} \right) \Delta Q P^e_I - \left[ CE_\epsilon + CM_\epsilon + C^e_\epsilon + SB_\epsilon \right]$$  \hspace{1cm} (4.64)

A simple example is provided below to illustrate the impact of shareholder benefits on the benefits of the various screening tests. This example is for a single year DSM program with no free riders.
AN EXAMPLE OF THE IMPACT OF SHAREHOLDER BENEFITS
ON THE SCREENING TESTS

A. Price of electricity = 0.10 $/kWh
B. Marginal cost of electricity = 0.05 $/kWh
C. Elasticity of electricity consumption = 0.5
D. DSM Program Savings = 100,000 kWh
E. DSM Program Costs = 5000 $

NO SHAREHOLDER BENEFITS

F. Total Avoided Costs (B*D) = 5000 $
G. Net TRC Benefits (F-E) = 0 $
H. Net RIM Benefits ([B-A]*D-E) = -10,000 $
I. Benefits of rate changes \( \frac{(C*H*[A-B])}{(A-C*[A-B])} \) = -3333.33 $
J. MV/GTRC Benefits (F+I-E) = -3333.33 $
K. GRIM Benefits (H+I) = -13,333.33 $

WITH SHAREHOLDER BENEFITS

Assume that shareholder benefits are 15% of TRC Benefits PLUS 5% of total avoided (energy + capacity + environmental) costs.

L. Shareholder Benefits \( (0.15*G+0.05*F) \) = 1250 $
M. Net TRC Benefits (F-E) = 0 $
N. Net RIM Benefits (H-L) = -11,250 $
O. Benefits of rate changes \( \frac{(C*N*[A-B])}{(A-C*[A-B])} \) = -3750 $
P. MV/GTRC Benefits (F+O-E) = -3750 $
Q. GRIM Benefits (N+O) = -15,000 $
CHAPTER 5

BILEVEL PROGRAMMING

5.1 Introduction

Chapter 2 provided a discussion of existing screening criteria and the rationale for developing a new criteria (MV/GTRC) based on maximizing value. Chapter 3 contained the derivation of the basic form of the MV/GTRC test and provided examples to illustrate the advantages of the formulation. The MV/GTRC presented in Chapter 3 is applicable only for: (1) programs that do not involve fuel switching, (2) rate changes affect demands only in the year in which it occurs, and (3) economic efficiency is the sole basis for evaluating DSM programs. Chapter 4 presents extensions of the MV/GTRC test to screen all types of DSM programs and illustrated each modification of the test with an example.

Screening tests, such as the LC/TRC and the MV/GTRC test, are static in that the various DSM program and supply-side inputs are both specified and the program is screened based on these empirical inputs. Thus, collection of empirical data is an important task in the inclusion of demand-side management programs in the integrated
resource planning process. They could document, for example, the extent of free riders that could be expected in an add-on heat pump program of a certain size. But there is often no theoretical basis for the empirical data used in the screening process. There is an important need for a theoretical model to analyze the results of empirical data collection efforts and to estimate the impacts of various assumptions.

Chapters 5-7 will present one such theoretical model to analyze various issues affecting electric utility planning. This model is bilevel and has the electric utility at the higher level and its various customer classes at the lower level. The bilevel model is intended to provide insights into interactions between the utility and the customer classes and the impacts of certain assumptions on decisions taken by the utility and its customers. The customer at the lower level maximizes his net benefits of consuming energy services whereas the utility at the upper level either maximizes the net societal benefits (VBP), or minimize the total revenue requirements (LCP1) subject to the utility recovering all its costs. There is also another objective that could be considered: the minimization of net societal costs (LCP2).

This bilevel model has an upper level (utility) and lower level (customers) with conflicting objectives. Section 5.2 provides the rationale for the development of such a model and an overview of the type of issues that can be addressed by this model. Section 5.3 describes the various solution techniques considered by others in the field.
of bilevel programming and the reason why the procedure to be used in this thesis was chosen.

5.2 Bilevel Planning Model

5.2.1 Need for a Bilevel Model

There are several hundred DSM programs that might be appropriate for any utility's service territory. It is virtually impossible to consider all of them in an IRP model. Screening tests play a role of "weeding" out bad programs and greatly reducing the number of programs to be considered by the IRP model. One such test, the MV/GTRC test, is derived in Chapters 3 and 4 to correct various shortcomings of the LC/TRC test which is the industry benchmark for screening demand-side programs. As illustrated in section 3.6., the difference between the results of these two tests could be significant and could lead to more correct DSM program selection. But the MV/GTRC test screens DSM programs based on empirical data that is given. There is a need for a model to interpret the results of such empirical data collection efforts, that are often flawed, and to examine the effects of various assumptions on electric utility and customer decisions. Any such model would enhance the results of the MV/GTRC test which by itself is an enhancement of current techniques for DSM program selection.
As described in Chapter 2, "least cost planning" (LCP) can be described as a process which considers both supply-side and demand-side resources and identifies the combination which minimizes costs to both utility and consumer. During the last few years, "value-based planning" (VBP) has begun to be viewed as an alternative to LCP (Chamberlin and Hanser, 1987). The main objective of VBP is to maximize net customer value while meeting the customers' needs, as opposed to LCP, which aims to minimize the cost of providing energy services.

Though LCP and VBP would appear to involve conceptually simple maximization (or minimization) problems, the societal issues involved are complex, affecting all aspects of utility planning. There exist many questions over which debate rages among utilities, regulators and public interest groups.

Examples of these questions include:

- Should DSM be an administrative or market-driven process?

  In an administrative-based process, planners would decide what DSM programs are justified and all ratepayers would subsidize the chosen programs. In a more market-driven approach, the focus would be upon giving appropriate price signals so that both utilities and consumers are motivated to make efficient decisions. With limited exceptions, DSM programs would be paid for by the participants without
subsidies from nonparticipants. The market would then determine what programs are economic (Ruff, 1988). The bilevel model can be used to analyze, for instance, the possible effect of administrative vs. market-driven approaches upon participation.

To what extent do market failures exist which justify DSM programs?

DSM has been justified on the basis that it can correct market failures in the electricity market (Lovins, 1985). Market failures which could lead to underinvestment in conservation by customers include marginal costs of supply which exceed the price of electricity, environmental externalities, ignorance of the potential energy savings from conservation investments, lack of access to credit, and split incentives, in which consumers of energy services do not control the capital investments affecting energy use (e.g., apartment tenants). The extent and even existence of these failures has been debated. The bilevel model can show the implications of such failures for the evaluation of DSM programs.

Does the objective matter?

What should the primary objective of the utility be? Some advocate "least cost" (Lovins, 1985) whereas others, in effect, argue that "least rates" to the customers should be the utility's goal (Ruff, 1988). Hobbs and Nelson (1989) have advocated yet another criterion, i.e., the maximization of consumer surplus or "value." The bilevel model explicitly analyzes the effects of each of these objectives upon utility
and customer investments in conservation.

Do free riders matter in the evaluation of the DSM programs?

Free riders are customers who take advantage of DSM programs to subsidize conservation investment they would have made anyway. There is controversy as to the extent of free riders in DSMs and their implications for their evaluations [Fang and Lui, 1988]. The bilevel model can estimate the portion of program participants who are free riders, their impacts on DSM economics, and how they are affected by distortions in the capital market and electric rate structures.

Is rebound likely to be significant?

As stated previously, the rebound effect occurs when the customer responds to a DSM by increasing his consumption of energy services (e.g., by increasing room temperature, rather than by conserving energy). Although rebound can decrease the effectiveness of conservation programs, it does enhance the welfare of consumers. Some feel that this effect could be significant [Foley, 1989], whereas others disagree [Lovins, 1985].

Both empirical and theoretical research is needed to answer the above questions. Empirical studies could document, for instance, the extent of market failure or rebound in particular circumstances. Theoretical studies, such as this thesis,
provide a framework for interpreting the results of empirical investigations and for exploring the implications of different assumptions. The purpose of our model is to provide a theoretically rigorous framework for "what if" analyses. For instance, if there exist capital market imperfections and customers buy electricity and invest in conservation to maximize their net benefits, would the rebound effect or free riders become important? Would economically efficient levels of participation take place in market-oriented DSM programs? Would choice of planning objective matter? The answers to such "what if" questions can help utility planners decide what objective is appropriate, what parameters are relevant, and what data is needed in DSM planning.

5.2.2 Summary of Model Structure

The reason why this bilevel model is effective in addressing such "what if" questions is that it makes explicit the assumptions about consumer behavior that most other models leave implicit. This explicitness permits a rigorous analysis of these questions, the conclusions of which contradict those of previous studies whose logical flaws are hidden behind qualitative, descriptive arguments. Furthermore, our model is comprehensive as it simultaneously accounts for many factors ignored in other analyses. These include:

- consumer reactions to conservation subsidies including free riders and
rebound,

- sub-optimal consumer investment in conservation due to capital market distortions, and
- electric rates which are based on average rather than marginal cost.

A bilevel model is needed because the instruments available to utilities for motivating consumers to make efficient decisions are imperfect. These instruments include electric rates and DSM programs. Electric rates in the U.S. are generally based on average cost, rather than marginal cost, which distorts incentives and can cause the consumer’s objective to be inconsistent with the utility’s objective (e.g., maximizing net social benefits). Another distortion results from the "double payments" problem, in which DSM payments by utilities to consumers might provide too strong a motivation for conservation. A "double payment" occurs when the consumer is paid twice to save the same kWh: once directly by the utility (e.g., as an appliance rebate) and a second time in the form of reduced utility bills. These possible distortions in incentives are most effectively represented by a bilevel approach which explicitly models the linkages between utility and its customers. The model presented here is not intended to replace traditional utility planning models. Data and algorithmic difficulties preclude the development of a version of this model which would be sufficiently detailed for making specific decisions on supply and conservation measures. Rather, the model is to be a tool for "what if" analyses of certain economic issues that are ignored by large-scale models.
This model is intended to capture the interactions between the utility and its ratepayers in making decisions about their respective optimal DSM investments, the rates (fixed by the utility), and energy consumption (fixed by the customers). Such a framework provides an ideal setup to investigate the impacts of various assumptions on optimal decisions chosen by the two players. The lower level model is that of consumer choice regarding energy consumption and investments in conservation measures. There are ‘m’ such models, one for each customer class or particular end-use. The upper level model optimizes some objective such as net benefits or utility costs. The customer makes electricity use and conservation investment decisions naively assuming that the utility’s decisions regarding electric rates and conservation subsidies are immutable. It is assumed that there is no interconnection among the lower level models except through the utility model at the upper level.

The bilevel model captures the following essential aspects of the utility planning process:

- Consumers demand energy services such as heat or light; electricity or natural gas is just one input used to produce those services. Other inputs, such as capital investment in conservation are partial substitutes for electricity.

- Consumer demands for energy services are not fixed requirements; rather, they are economic demands which are responsive to price.

- Because of the market failures mentioned above, consumers may underutilize capital in their decisions on how to produce
energy services. Distortions in the capital market are modeled in this thesis by attaching a premium to the consumer's cost of capital.

Electric rates are set at a level such that all utility costs (including that of DSM) are recovered. These costs - fixed and variable - cause electric rates to deviate from the marginal cost of production.

The above model is a bilevel nonlinear model which is solved by substituting the lower level model Kuhn-Tucker conditions at constraints for the upper level model. The K.T. complementary slackness conditions make this model nonconvex and several simplified models have to be solved to obtain the optimal solution to the overall model. A review of the existing literature on bilevel programming to solve such problems is presented in the next section and would provide a justification for the methodologies used in this thesis.

5.3. A Survey of Bilevel Programming Solution Techniques

Many planning situations involve multiple decision makers with different and sometimes conflicting objectives. Often these groups are arranged in a hierarchical structure. Multilevel decision making has always been an integral aspect of the planning problem. Except in a few cases, methods for solving these multilevel problems are based on the decomposition methods of Dantzig and Wolfe (Dantzig and Wolfe, 1960, and Kornai and Liptak, 1965). This methodology can be viewed as an
adjustment phase with the upper level planner sending information to the lower level sub-units, observing their reactions and then updating the overall objective. Although these methods are intended to represent the behavior of a multilevel organization, they essentially solve a single objective function over a fixed feasible region. The decomposition methods imply that the single corporate objective decomposes into the objectives of the sub-units or that the upper level planner is willing to accept the aggregated objectives of the sub-units as his own (Dirickx and Jennergren, 1979). The basic fallacy of using decomposition models to solve problems in which the policy planner does not have control over all the variables is brought out by the following statement (McCari and Spreen, 1980):

"A classic example of misuse is one where a research team convinces itself that the country wishes to optimize nutritional production and then the research team replaces the objective function to do so. One needs only to wonder how many farmers will forgo income to produce calories for the aggregate population to see the fault of this modelling strategy."

Multi-objective analysis has been offered as an elixir for the above stated dilemma (Keeney and Raiffa, 1976). This class of methods purports to find a simultaneous compromises among the various goals of the planners. But these techniques still assume the objectives are those of a "harmonious bevy of planners" (Bialas and Karwan, 1982) and these techniques fail to account for possible independent actions taken by individual units or the order in which these decisions are
taken. The basic difference between bilevel programming and multiobjective programming is that the former prohibits cooperation between players. This precludes the possibility of using any of the approaches of multiobjective and goal programming. In fact, solutions of bilevel problems may not even be Pareto optimal (Bard, 1984).

The method proposed in this thesis is based on the concept of a Stackelberg game which is a noncooperative two person game (Simaan and Cruz, 1973). A major difference between bilevel programming and the standard noncooperative game is that in the former, the order in which strategies are selected is established at the outset. In the latter, all players are required to move at the same time. Bard (1984) proves that the order in which the player act would insure that the person who moves first will always achieve at least as good a result as he would if the order were reversed.

In the standard formulation of the Stackelberg game, the upper level decision maker is called the "leader" and the lower level decision maker is called the "follower". The leader has control over the decision vector $x \in X$ while the $P$ ‘followers’ individually control the decision vectors $y_p \in Y_p$, $p = 1, \ldots, P$. It is also assumed that the leader is given the first choice and selects an $x \in X$ to minimize his objective function $F$. Based on this information, the followers select a $y_p \in Y_p \cap \Psi_p (x)$ to minimize their individual objective functions $f_p$, where $\Psi_p (x)$

places additional conditions on the feasible regions of the followers due to the leader’s decisions, and $X$ is the feasible region for the leader. For the static case, it leads to the bilevel programming problem (BLPP) (Bard and Falk, 1982, Bialas and Karwan, 1984, Fortuny-Amat and McCarl, 1981) given below:

$$\begin{align*}
\min_{x} & \quad F(x, y(x)) \\
\text{s.t.} & \quad x \in X = \{x : H(x) \geq 0\} \\
\min_{y_p} & \quad f_p(x, y_p) \\
\text{s.t.} & \quad g_p(x, y_p) \geq 0 \\
& \quad y_p \in Y_p = \{y_p : G_p(y_p) \geq 0\} \\
& \quad p = 1, \ldots, P
\end{align*}$$

(5.1)

where $H$, $G_p$, and $g_p$ are vector valued functions.

Applications of bilevel programming have been made in many areas: these include cogeneration of power (Haurie et al., 1990), agricultural economics (Candler et al., 1981), economic development policy (Anandalingam, 1985), road network design (LeBlanc and Boyce, 1986), defense allocation problems (Bracken and McGill, 1973), and electric utility demand-side planning (Hobbs and Nelson, 1992).

The properties of the bilevel program are more complex than familiar mathematical programming problems. Consider the simplest nontrivial two level problem, given below (Bialas and Karwan, 1984):
\[
\begin{align*}
\max \ c_{11}x_1 + c_{12}x_2 & \quad \text{where } x_2 \text{ solves} \\
& \quad x_1 \\
\max \ c_{21}x_1 + c_{22}x_2 & \quad x_2 \\
\text{st: } A_1x_1 + A_2x_2 & \leq b \\\n& \quad x_1, x_2 \geq 0
\end{align*}
\] (5.2)

Although the feasible region for the lower level problem is a convex polyhedron, the actual problem is a nonconvex problem because the set of rational reactions which is the feasible region for the upper level problem is nonconvex (the dark shaded region in Figure 5.1). In this simple example, \( x_1 \) and \( x_2 \) are single component vectors. For any fixed choice of \( x_1 \), the lower level model will choose the value of \( x_2 \) which maximizes the lower level problem. This results in the rational reaction set which is the hatched region in the figure. The obvious choice of \( x_1 \) for the upper level problem is that which yields the maximum value of the upper level objective with respect to the hatched region. Thus, we are maximizing a linear objective function over a nonconvex set.

Most solution methodologies offered in the field of bilevel programming are for linear problems. Algorithms for solving linear bilevel problems can be classified into the following three general types (Kolstad, 1985):
Figure 5.1: Rational Reaction Set for a Two-Level Problem
Extreme Point Search Methods
Descent Methods
Kuhn-Tucker-Karush Methods

The first set of methods are concerned with moving from one extreme point to the another until an optimum is found. The second set of methods are based on various descent approaches for the upper level problem with gradient information from the subproblems. The final set of these methods utilizes the KKT conditions of the subproblem as constraints on the overall problem, thus converting the bilevel problem into a single level nonconvex mathematical problem. Provided below is an overview of each of these methods and their applicability to our bilevel model.

5.3.1. Extreme Point Ranking Methods

These methods are based on an important result proved by Bialas and Karwan (1982), i.e., any solution to the overall problem occurs at one of the extreme points of the constraint set of the lower level problem. Candler and Townsley (1982) have an algorithm which focuses on searching the bases of the lower level linear problem until a solution to the overall problem is obtained. They show that if there exists an optimal solution to the overall problem, then there exists a basis of the lower level problem with nonnegative reduced costs with respect to it such that the lower level solution solves the overall problem.
The Kth best algorithm of Bialas and Karwan (1982) adopts a slightly different approach by ignoring the lower level objective and including the constraints of the lower level problem as constraints in the upper level problem. This approach is possible because we know that the solution occurs at one of the extreme points of the lower level constraint set. Suppose the entire set of M extreme points of the new problem are identified and ordered according to ascending values of the objective, we know that one of these will solve the overall problem. The algorithm terminates when the Kth extreme point solves the lower level problem. Global optimality is assured since the solution is approached from a region of infeasibility and the Kth best extreme point satisfies the optimality conditions for the lower level model.

Papavassilopoulos (1981) adopts a similar approach, in that he generates a sequence of extreme points, each of which will be feasible to the overall problem. An adjacent extreme point is searched only if it improves the upper level objective while remaining optimal with respect to the lower level problem. This algorithm does not assure global optimality since we approach the solution from a region of feasibility.

The extreme point ranking methods are applicable only to bilinear problems and are of no importance to our nonlinear bilevel model. Thus they will not be discussed further.
5.3.2. Descent Methods

There are not many bilevel methods which involve using derivative information to proceed towards the optimum. The main reason is the potential for multiple local optima and the difficulty in calculating the derivatives associated with the subproblem. Referring to equation (5.1), the scheme is to apply any one of the many gradient-based methods to the upper level problem. In that formulation, y is viewed as a function of x, defined by the lower level problem. Gradients of $F(x,y)$ and $g(x,y)$ can be obtained if the derivative information from the lower level problem can be obtained. But, in reality, $y(x)$ may not be uniquely defined and may not be differentiable for all x.

Shimizu and Aiyoshi (1981) re-write the subproblem as an unconstrained minimization problem through the use of the penalty function. Assuming that the unconstrained function is strictly convex in y, they use the stationary conditions to express y as a function of x and solve the overall problem with respect to x. This method is generally applicable only to very small problems because y may not be uniquely defined or differentiable in x. Aiyoshi and Shimizu (1984) take this approach one step further and include the unconstrained lower level function discussed above into the leader’s objective leading to an overall unconstrained optimization problem. The authors prove that the simultaneous increase in the values of the
penalty parameters lead towards a local optimum. The global optimality of the overall problem is not assured unless we start with a convex problem. Thus, it is suggested that many different initial points be searched for the global optimum. Extremely slow convergence is the bane of this method because of the ill-conditioning caused by the updating of the penalty parameters.

Kolstad (1985) mentions several other descent methods but none of them offer any promise to the problem at hand. This is mainly because of the fact that lower level variables may not be uniquely defined or differentiable at all values of the upper level variables.

5.3.3. **Kuhn-Tucker-Karush Methods**

Let us consider the following bilevel problem:

\[
\begin{align*}
\min_{x} & \quad f(x, y) \\
\text{s.t.} & \quad g(x, y) \leq 0 \\
\min_{y} & \quad f(x, y) \\
\text{s.t.} & \quad h(x, y) \leq 0
\end{align*}
\]

These class of methods involve replacing the lower level model by its Kuhn-Tucker-Karush (KKT) conditions for optimality and then rewriting (5.3) as:
\[
\begin{align*}
\min_{x, y, \mu} & \quad F(x, y) \\
\text{s.t.} & \quad g(x, y) \leq 0 \\
& \quad \nabla_y f(x, y) + \mu \nabla_y h(x, y) = 0 \\
& \quad \mu h(x, y) = 0 \\
& \quad h(x, y) \leq 0 \\
& \quad \mu \geq 0
\end{align*}
\] (5.4)

This problem is equivalent to the overall problem since any solution to the overall problem satisfies the KKT conditions for the lower level model and thus solves it (provided \(f(x, y)\) is strictly quasi-convex and \(g(x, y)\) is quasi-convex with respect to \(y\)).

Now this problem is nonconvex because of the complementary slackness KKT conditions. Therefore, most gradient-based methods cannot be used to solve the above problem.

Bard and Falk (1982) consider the linear version of the problem in which the only problematic constraints are the complementary slackness conditions. They present the linear bilevel problem as a separable convex program by transforming \(\mu h(x, y)\) into \(\sum_i [\min (0, \lambda_i) + \mu_i]\) and \(\lambda_i - h_i + \mu_i = 0\) where \(\lambda_i \geq 0\) is a new variable. They then use an existing branch and bound technique which involves a partition of the feasible region to obtain a solution.

Later, Bard (1983a and 1983b) proposed a new algorithm to solve the KKT
problem. This method involves a grid search between estimated upper and lower bounds on the main objective function. This method does not guarantee a global solution and the computation time depends entirely on the interval between the lower and upper bounds. The smaller the interval, the fewer the grid points are needed to achieve a preselected spacing. Clark and Westerberg (1988) and Hauric et al. (1990) prove that this method yields a global optimum only if the solutions are Pareto-optimal. But, in general, solutions are not necessarily Pareto-optimal. Unlu (1987) attempts a similar approach based on finding efficient (i.e. weighted) bases (linear problem) for both the upper and the lower level problem and finding the optimal value among the efficient solutions. But Wen and Hsu (1989) show that this formulation does not necessarily yield the optimal solution. They illustrate that if the solution to the bilevel problem is not Pareto-optimal, there may exist solutions that do not decrease certain objective functions even while increasing certain other objective functions. This shows that in general, there is no relationship between multilevel and multi-objective formulations and this precludes the use of this linkage for all multilevel formulations.

Bialas and Karwan (1982) developed a parametric complementary pivoting procedure to solve the KKT problem for a linear case. Thus, they reformulate (3) as that of finding a feasible $x,t$, and $\mu$ such that the objective (minimization) is less than some upper bound $\epsilon$. By solving the model with successively smaller upper bounds
until no feasible solution can be found, an optimum will be obtained. This method should be considered as a heuristic since no convergence is guaranteed.

Fortuny-Amat and McCarl (1981) handle the complementary slackness conditions by transforming the problem into a larger mixed integer problem. They rewrite the complementary slackness condition \( \mu h(x, y) \) into \( h_i \leq M \eta \) and \( \mu \leq (1-\eta) M \) where \( \eta \in \{0, 1\} \) and \( M \) is a large positive constant. Nelson (1989) and Hobbs and Nelson (1991a) used a similar approach in solving their bilevel electric utility model. There exists considerable computational problems when the number of customer classes (lower level subproblems) is large. This thesis presents a branch and bound algorithm to limit the number of subproblems to be solved before the optimum is found. This method closely resembles approaches discussed in Bard (1988), Bard and Moore (1990), and Edmunds and Bard (1990).

The general idea of all the above methods is to relax the complementary slackness conditions and solve the corresponding problem. At each node, one of the complementary slackness conditions is enforced (by either setting \( h_i \) or \( \mu_i = 0 \)) and the problem is solved. The selection of the constraint to be enforced is arbitrary and in this thesis is based on the highest value of \( |\mu_i h_i| \). If there are \( n \) complementary slackness conditions, there are a total of \( 2^{n+1} - 1 \) nodes that might need to be solved before the optimal point is obtained. Efficient algorithms reduce significantly the
number of nodes that might need to be solved.

Bard (1988) uses such an algorithm to solve a convex quadratic problem. Bard and Moore (1990) employ such an approach for a linear integer problem whereas Edmunds and Bard (1990) generalize the above approach for nonlinear problems. Though the algorithm was presented for a general nonlinear problem, the problems solved had quadratic objective functions and linear constraints. Thus it is difficult to check the efficiency of their algorithm for a general nonlinear problem.

The problems presented in Chapter 6 are highly nonlinear, though the objectives are proved to be convex (concave) for minimization (maximization) problems. The constraint set for the different nodes in a branch and bound tree are also proved to be either convex equality or inequality constraints. The branch and bound algorithm presented in Chapter 7 solves this nonlinear bilevel model and the branching is dependent on the problem structure. So far, there has been no bilevel nonlinear nonquadratic application solved in the literature. We use our branch and bound algorithm to solve one such case in Chapter 8.
CHAPTER 6
FORMULATION OF THE BILEVEL PROBLEM

6.1 Introduction

An application of bilevel programming to resource planning in the electric utility industry is formulated in this chapter. This model is nonlinear and can be used to analyze the various issues that affect electric utility integrated resource planning. The electric utility at the upper level of the model seeks to minimize costs or maximize benefits while controlling electric rates and subsidizing energy conservation programs. The ‘n’ customer classes at the lower level attempt to maximize their net benefits from energy services by consuming electricity and investing in conservation. The purpose of this model is to provide a theoretically rigorous framework for "what if" analysis. The answers to the various questions raised in Section 5.2 can help utility planners to decide what objective is appropriate, what parameters are relevant, and what data is needed in demand-side planning.

Section 6.2 presents the assumptions and development of the bilevel model for the case where utilities consider their investment in energy efficiency improvements as variables. Let us call this model BLP1. The utility at the upper level optimizes
his objective by deciding on electric rates and his investments in conservation measures for different customer classes. The customer classes at the lower level, on the other hand, seek to maximize their net benefits of consumption of energy services by choosing their consumption of electricity or their investment in conservation measures in response to utility decisions. A single level model is obtained by substituting the Kuhn-Tucker conditions of the lower level model as constraints in the upper level model. This approach was previously discussed in section 5.3. This model is highly nonconvex and requires some simplification before a solution technique is applied. This simplification involves the reduction of the overall model into several reduced problems.

Section 6.3 analyzes the structure of such a reduced model. Convexity of the solution set can be proven for the intersection of all the constraints except certain convex equality constraints (constraints of the form \( f(x) = 0 \) where \( f(x) \) is a convex function). The reduced model, which is still nonconvex, is solved by formulating it as a augmented Lagrangian (AL) problem. Rockafellar (1973) and Powell (1972) prove that a global optimum can be found for such a model with convex equality and inequality constraints by using the augmented Lagrangian approach.

Section 6.4 considers the bilevel model which is formulated in such a way that the price subsidies for conservation investments are decision variables, instead of the
amount of investment. Let us call this model BLP2. BLP2 does not need to be solved with reduced models because the complementary slackness conditions of the lower level model in the Kuhn Tucker approach (discussed in section 5.3) are automatically satisfied for this problem, unlike BLP1. This implies that only one model needs to be solved to obtain the solution of the overall bilevel model BLP2.

The appropriateness of the bilevel model to be used for analysis depends on the type of DSM programs. For example, for programs where the utility gives away light bulbs to commercial customers, it is more appropriate to use BLP1. In contrast, if a program gives a rebate of X $ for every Y $ of investment in efficient air-conditioners by customers, BLP2 would be more appropriate.

6.2 Formulation of the BLP1 Model

(Direct Utility Expenditures on Conservation Measures)

In this section, we formulate one form of the bilevel model (BLP1) described earlier in section 5.2. The structure of the model is as shown in Figure 6.1. This is a bilevel nonlinear model. At the upper level, the utility optimizes some objective (minimize costs or maximize net benefits) subject to, if there are average cost pricing constraints, a revenue recovery constraint. The customers at the lower level seek to maximize their own benefits of consumption of energy services. Energy services are, in turn, obtained by the consumption of electricity and investment in DSM programs.
Figure 6.1: Structure of the Bilevel Model
There are ‘m’ such models in the lower level representing, for example, different end uses (heating, refrigeration, lighting, air conditioning etc.) or customer classes. An example is provided in Chapter 8 which looks at four different end-uses within the same customer class.

We formulate this model as a Stackelberg model with the utility at the upper level as the leader and the customer classes at the lower level as the followers. Under Stackelberg behavior, the leader makes his decision first and the followers react by optimizing their objective functions conditioned on the leader’s decisions. Thus, the follower optimizes his consumption of electricity and additional investment in conservation in response to the utility fixed prices and subsidy levels. It is assumed that there is no direct interaction between the customer classes except through utility fixed prices and DSM program subsidies.

6.2.1 Consumer Choice Model

The lower level consumer choice model seeks to represent the actions of a rational consumer. It is assumed that consumers at the lower level desire to maximize their net benefits by optimizing their consumption of electricity and their investment in DSM equipment in response to utility fixed prices and subsidies. The use of consumer surplus to represent the gross benefits to program participants was
previously explained in Section 3.2. It was explained in that section that, ignoring income effects, the consumer surplus could be defined as the area under the ordinary demand curve.

We assume in this model that the customer class optimizes his consumption of energy services such that his consumer surplus is maximized. This approach to modeling consumer choice has been criticized. Komor and Wiggins (1989) argue that the microeconomic theory of consumer choice used in this thesis is not an accurate descriptive model of consumer behavior. This is because the model excludes nonfinancial goals, assumes consumers are rational, and does not differentiate between actual and perceived costs. They present alternative models of consumer behavior based on duplicating actual customer behavior. But, as the authors acknowledge, these behavioral models have too poor a predictive record to be considered viable for conservation analysis.

Thus, though the net benefit maximizing model may not describe the process by which consumers reach their decisions nor explain how they reach their decisions, it appears to be the best predictor of consumer behavior. For the above reasons, we adopt the consumer-surplus maximizing model in this thesis.

Let us assume that the demand curve for energy services in any class 'i'
follows a constant elasticity form given below:

\[ q_{st} = C_i p_{st}^{-\varepsilon_{st}} \tag{6.1} \]

where \( \varepsilon_{st} \) is the price elasticity of demand for energy services, \( p_{st} \) the price of energy services in $/unit, and \( q_{st} \) the quantity of energy services in units consumed by customers in class ‘i’.

Let us now discuss the concept of "production function". This expression is used to refer to the physical relationship between a firm’s input of productive resources (raw materials, capital, labor etc.) and its output of goods or services per unit of time (Thompson, 1983). This relationship can be expressed symbolically as:

\[ Q = F(X_1, X_2, X_3, \ldots, X_n) \tag{6.2} \]

where \( Q \) is a output vector of goods or services and \( X = \{X_1, X_2, \ldots, X_n\} \) is the input vector.

In the lower level consumer choice model, it is assumed that energy services are produced by capital investments on the customer side of the meter by both the customer as well as the utility and the power consumed by the customer class. Thus the quantity of energy service consumed by customer class ‘i’ equals:
where \( q_{e1} \) equals the amount of electricity [kWh/yr] sold by the utility to the customer class, \( d_{ui} \) the investments by the utility on the demand side of the customer, \( K_i \) the minimum capital investment made by the customers, and \( d_{ai} \) the additional capital investments made by the customer class (for example, utility sponsored low income weatherization programs). It is assumed that the customer in class 'i' invests \( K_i \) dollars in initial capital equipment in any case.

The Cobb-Douglas production function is perhaps the most ubiquitous form of production functions in economic literature. It owes its popularity to the exceptional ease with which it can be manipulated. It is often estimated econometrically. In its unrestricted form, the Cobb-Douglas function (C-D) can be written as:

\[
\ell(x) = A \prod_k x_k^{\alpha_k}
\]

where \( A \) is a technological efficiency parameter, and \( \alpha_k \) is the elasticity of \( f(x) \) with respect to a particular input \( x_k \). An attractive property of the C-D form is that \( p \), the sum of the \( \alpha_k \), reflects the degree of homogeneity, i.e. \( m^p f(x) = f(mx) \). Thus if the sum of \( \alpha_k \) equals one, it reflects constant returns to scale. If the sum is greater (less) than 1, this indicates increasing (decreasing) returns to scale.
The drawbacks of the C-D function include:

1. elasticity of substitution is unity\(^1\). This implies that, given fixed prices, proportions between the factors that will be used for production will always remain the same.

2. no factor of production is completely substitutable for the other.

3. any technological advance that changes the ease of substitution among factors of production \(x\) cannot be represented. This means that the ratio of \(\alpha_i/\alpha_j\) cannot be changed and remains constant.

In the following model, energy services \(q_{s1}\) are produced by capital investments \((K_i+d_{ui}+d_{ci})\) and electricity usage according to the C-D form given below:

\[
q_{s1} = \beta_i (K_i + d_{ui} + d_{ci})^{\alpha_i} q_{e1}^{1-\alpha_i}
\]  \hspace{1cm} (6.5)

where \(\beta_i\) and \(\alpha_i\) are constants. Because the sum of the exponents equal 1, a doubling of each input would result in a doubling of energy services provided. It is also assumed that \(d_{ci}\) and \(d_{ui}\) are completely substitutable. A summary of the definitions of variables and parameters can be found in Table 6-1.

---

\(^1\) Elasticity of substitution \(e_{su}\) can be defined as the proportionate change in the ratio of the inputs divided by the proportionate rate of change of the rate of technical substitution:

\[
e_{su} = -\frac{d \ln (x_s/x_1)}{d \ln (f_1/f_2)}
\]

where \(f_1\) and \(f_2\) are the partial derivatives of the production function with respect to \(x_1\) and \(x_2\) respectively.
Now let us consider the gross benefits obtained by consumption of energy services. This is equal to the area under the demand curve for energy services. It can be calculated by integrating $p_t(q_{si})$, given by expression (6.1), from $0^+$ to $q_{si}$. As explained earlier in Section 4.3, the lower limit is set so that for extremely high value of prices, the quantity demanded falls to zero. This is done to insure that the consumer surplus calculated is finite for all values of elasticities of energy services less than 1. As long as prices of energy services remain below that high threshold value, this assumption does not affect the analysis. Thus the gross benefits of energy services $B_t(q_{si})$ for customer class 'i' can be calculated according to:

$$B_t(q_{si}) = K + \int_{0}^{q_{si}} p_{zt}(q) \, dq$$  \hspace{1cm} (6.6)

Inverting (6.1) and substituting it into the above and integrating yields:

$$B_t(q_{si}) = K + \frac{c_t^{1/\epsilon_{st}}}{1-1/\epsilon_{st}} q_{st}^{1-1/\epsilon_{st}}$$  \hspace{1cm} (6.7)

The constant $K$ in (6.6) and (6.7) can be dropped since it would not affect the calculations (the consumer and utility objectives are linear in $B_t(q_{si})$).

The model of consumer behavior can now be constructed. The consumer is assumed to maximize the net benefits of energy service minus his or her electricity bills $p_tq_{si}$ and the perceived cost of capital $L_0q_{si}$, subject to the production function (6.5). The parameter $L_0$, which is dimensionless and generally exceeds 1.0, accounts
Table 6.1

DEFINITIONS

(the normalized variables are in lower case and parameters are in upper case or greek symbols)

\[ q_{si} = \text{Quantity of energy services consumed by customers in class or by end use 'i'.} \]
\[ q_{ei} = \text{Quantity of electricity consumed by customers in class or end-use 'i'.} \]
\[ d_{ui} = \text{Amount invested by the utility in DSM programs in class or end-use 'i'.} \]
\[ d_{ci} = \text{Additional amount spent by customers in class or end-use 'i' on DSM programs.} \]
\[ A = \text{Fixed costs of electricity production per year} \, [\$/yr]. \]
\[ B = \text{Variable costs of electricity production in} \, \$/MWh. \]
\[ C_i = \text{Demand function coefficient for the demand for energy services.} \]
\[ K_i = \text{Minimum amount invested by customers in class or end-use 'i' on DSM equipment.} \]
\[ L_i = \text{A factor reflecting the distortions in the capital market for customer class 'i'. A value of} \, L_i=1 \text{ indicates no distortion in the capital market whereas a value of 8 reflects a high distortion in the market and that the customer requires a very short payback period. Also refer to equation (6.8).} \]
\[ \alpha_i = \text{Production function exponent for the capital investment in DSM programs and} \, (1-\alpha_i) \text{ is the exponent for the quantity of electricity consumed.} \]
\[ \beta_i = \text{Production function coefficient for the Cobb-Douglas production function.} \]
\[ \epsilon_{ri} = \text{Elasticity of demand for energy services for the constant elasticity demand function which is assumed.} \]

for the bias of consumers against energy saving investments. It is modeled by assuming that consumers make energy saving investments as if the cost of capital is much higher than it truly is. Several studies, for example, have found that such implied interest rates are 30%/yr, 40%/yr, or even higher (Hausmann, 1979 and Dubin
and McFadden, 1984). This, in fact, is the strongest argument made by pro-
conservationists for utility sponsored DSM programs. For example, if \( L_c = 1 \), it can be
shown that \( d_w \) will remain zero if the average price is greater than the marginal cost
of power generation. This is so because the customer class has the same cost of
capital as the utility and would purchase the amount of capital equipment even
without utility subsidies. Wirl (1988) and Ruff (1987) show in this case, the
maximum amount of subsidies to be offered by the utility should be no more than the
difference between marginal cost and price. Thus if price is greater than marginal
cost and there are no distortions in the customer’s capital market, no conservation
subsidies are necessary -- indeed, the customer may invest too much in conservation.

Using the notation of engineering economics, \( L_c \) can be defined as follows:

\[
L_c = \frac{(A \mid P, I_c, N)}{(A \mid P, I_m, N)} \quad (6.8)
\]

where:

\[
(A \mid P, I, N) = \text{annual worth or capital recovery factor (CRF), given interest rate } I \text{ and number of years } N = \frac{I (1 + I)^N}{(1 + I)^N - 1}
\]

\( L_c \) = effective interest rate for consumers

\( L_m \) = market interest rate

\( N \) = investment life (which might be perceived to be lower by consumers than the true life, in which case, a separate \( N_c \) could be defined)

This is the ratio of the apparent annual cost of capital to the actual cost.
The consumer model at the lower level can thus be defined as follows:

\[
\begin{align*}
\text{MAX} & \quad B_l(q_{st}) - P_{el} q_{e1} - L_1 d_{c1} \\
( q_{st}, q_{e1}, d_{c1} )
\end{align*}
\]

s.t.

\[
\begin{align*}
q_{st} &= \beta_1 (K_t + d_{c1} + d_{ul}) \alpha_1 q_{e1}^{1-\alpha}, \\
q_{st}, q_{e1}, d_{c1} &\geq 0
\end{align*}
\]

where \( p_{el} \) is the price of electricity charged by the utility to the customer class in \$/kWh. The utility subsidies (\( d_{ul} \)) and the price of electricity (\( p_{el} \)) are fixed by the utility and are assumed by the consumer to be unchangeable. Based on the values of \( p_{el} \) and \( d_{ul} \), the customer class chooses its investment (\( d_{c1} \)) and consumption of electricity (\( q_{el} \)) while maximizing the net benefits accruing from energy services. Refer to Figure 6.1, presented earlier, for the structure of this model.

Now since \( q_{el} \) follows the constant elasticity demand function and \( q_{ul} \) is positive for all values of \( p_{ul} \), it automatically results in \( q_{ul} \) and \( q_{el} \) being greater than or equal to zero. Thus the problem can be simplified, substituting the C-D form for \( q_{el} \) in the objective and defining \( d_{l} = K_t + d_{ul} + d_{c1} \), to the following form:

\[
\begin{align*}
\text{max} & \quad B_l(q_{st}) - P_{el} q_{e1} - L_1 d_{c1} \\
( q_{e1}, d_{c1} )
\end{align*}
\]

s.t.

\[ d_{c1} \geq 0 \]

(6.10)

Applying the KKT conditions for optimality, we obtain the following set of equations:
\[
\frac{\partial B_i(d_t, q_{el})}{\partial q_{el}} - p_{el} = 0 \quad (6.11)
\]
\[
d_{ct} \left[ \frac{\partial B_i(d_t, q_{el})}{\partial d_{ct}} - L_t \right] = 0 \quad (6.12)
\]
\[
\frac{\partial B_i(d_t, q_{el})}{\partial d_{ct}} - L_t \leq 0 \quad (6.13)
\]
\[
d_{ct} \geq 0 \quad (6.14)
\]

Note that there are \( m \) different customer classes or end-uses. There is one set of such constraints for each. After substituting the C-D form for \( q_{ct} \) in (6.7) and differentiating the expression with respect to \( q_{ct} \), we get:

\[
C_i^{1/\epsilon_i} \left[ \beta_t d_t^{\alpha_i} \right] \left[ (1-1/\epsilon_i) \right] (1-\alpha_i) q_{ct}^{\alpha_i (1-1/\epsilon_i) - 1} - p_{ct} = 0 \quad (6.15)
\]
\[
d_{ct} \left[ C_i^{1/\epsilon_i} \left[ \beta_t q_{ct}^{1-\alpha_i} \right] \left[ (1-1/\epsilon_i) \right] (\alpha_i) d_t^{\alpha_i (1-1/\epsilon_i) - 1} - L_t \right] = 0 \quad (6.16)
\]
\[
C_i^{1/\epsilon_i} \left[ \beta_t q_{ct}^{1-\alpha_i} \right] \left[ (1-1/\epsilon_i) \right] (\alpha_i) d_t^{\alpha_i (1-1/\epsilon_i) - 1} - L_t \leq 0 \quad (6.17)
\]
\[
d_{ct} \geq 0 \quad (6.18)
\]

The KKT conditions can be interpreted as follows: (6.15) says that the marginal benefit of consuming electricity must equal the price. Equations (6.16) to (6.18) state that if the customer invests in conservation measures, the marginal benefit of that investment must equal the marginal cost of capital \( L_t \). However, if the consumer does not invest in conservation, the marginal benefits must not exceed \( L_t \). It is interesting
to note that if \( d_{ui} \) and \( d_{ei} \) equal zero (i.e., capital is fixed), the optimal electricity demanded \( q_{ei} \) from 6.15 above as a function of price \( p_{ei} \) would follow a constant elasticity demand curve, just like \((6.1)^2\). However the price exponent will be 
\[-1/(\alpha_i + (1-\alpha_i)/\varepsilon_{ei})\] rather than \( \varepsilon_{ei} \). The resulting elasticity exceeds \( \varepsilon_{ei} \), if \( \alpha_i > 0 \) and \( \varepsilon_{ei} < 1 \). This implies that the demand for power is more elastic than the demand for energy services.

6.2.2 Utility Planning Model

This is the upper level model which optimizes some objective with respect to system constraints and the rational reaction set of the lower level consumers. We consider three such objectives:

Least Cost Planning Objective: Minimize Revenue Requirements (LCP1)

\[
\begin{align*}
\text{MIN } F_i &= A + \sum_i (B \sigma_{a_i} + d_{ui}) \\
&= (6.19)
\end{align*}
\]

Value Based Planning Objective: Maximize Net Customer Benefits (VBP)

If \( d_{ui} = 0 \), we can eliminate \( K_i + d_{ci} \) from equations (6.15)-(6.16). This leads to an unconditional demand curve for power that is more elastic than the conditional demand curve.
\[
\text{MAX } F_2 = \sum_i [B_i (q_{atl} - d_{sl})] - F_1
\]  
\[(6.20)\]

Least Cost Planning Objective 2: Minimize Societal Costs (LCP2)

\[
\text{MIN } F_3 = A + \sum_{i} (B q_{atl} + d_{atl} + d_{ctl})
\]  
\[(6.21)\]

All the objectives are in $/yr.

The first objective assumes that generation cost can be approximated as being linear in \(q_{atl}\). \(A\) represents the yearly fixed costs of the system and \(B\) is the variable cost of generation including operation and maintenance costs, though some of it may be fixed. We also assume that these costs include a fair return to shareholders.

Nelson (1989) models the cost portion of the objective in more detail for a two customer class model by using the multi-block load duration curve approach of Turvey and Anderson (1977). To accomplish this, it is necessary to define demand variables for each class and time period, generation variables for each generating unit and period, and constraints relating generation to demand and capacity. Since the objective of development of the model is to gain qualitative insights rather than quantitative decisions, we will use the linear cost function in this thesis. This objective is similar to the Utility Cost (UC) test of the California Standard Practice for screening DSM programs and is the traditional objective of electric utilities in the United States.
The second objective equals the net benefits to consumers of energy services, i.e. the gross benefits minus the cost of electricity generation and cost of utility and customer investment in conservation. According to this objective, it is interesting to note that the social cost of $d_{ei}$ is 1, not $L_i$. This objective is similar to the MV/GTC test derived in Chapters 3 and 4.

The third objective considered (6.21) is the minimization of societal costs. This objective seeks to minimize the total costs to the society (utility + customer) of power generation and DSM expenses. This objective is similar to the total resource cost (TRC) test of the California Standard Practice. Though minimization of revenue requirements (LCP1) is the objective of most utility integrated resource planning models, minimization of societal costs (LCP2) is the nominal objective of DSM screening models of the Standard Practice. The bilevel model provides a forum for understanding the difference between these two objectives in terms of their impacts on DSM resource selection.

Besides the objective, there are the following constraints:

- consumer reaction set (6.15-6.18)
- revenue recovery (price equals average cost)
\[ \sum_i p_{et}q_{et} = A + \sum_i (Bq_{et} + d_{ul}) \quad (6.22) \]

- non-negativity constraints

\[ d_{ul} \geq 0 \quad (6.23) \]

As discussed earlier, the optimality conditions of the lower level model are incorporated as constraints in the upper level model leading to a single model to be solved. This single level model is as given below:

\[
\text{OPTIMIZE} \quad F_k \quad (k=1, 2, 3) \quad (6.24)
\]

\[ \text{s.t.} \]

\[ \sum_i p_{et}q_{et} = A + \sum_i (Bq_{et} + d_{ul}) \quad (6.25) \]

For i=1, 2, ..., n

\[ C_i^{1/\varepsilon} \left[ \beta_i \alpha_i \right] (1-\alpha_i) \quad q_{et}^{(1-\alpha_i)} \quad \left( \alpha_i \right) q_{et}^{(1-\alpha_i)} - p_{et} = 0 \quad (6.26) \]

\[ d_{et} \left[ C_i^{1/\varepsilon} \left[ \beta_i \alpha_i \right] (1-\alpha_i) \quad \left( \alpha_i \right) q_{et}^{(1-\alpha_i)} - L_i \right] = 0 \quad (6.27) \]

\[ C_i^{1/\varepsilon} \left[ \beta_i \alpha_i \right] (1-\alpha_i) \quad \left( \alpha_i \right) q_{et}^{(1-\alpha_i)} - L_i \leq 0 \quad (6.28) \]

\[ d_{et} \geq 0 \quad (6.29) \]

Another objective that was the regulators' mandate was the minimization of
electric rates. In the absence of utility sponsored DSM programs, minimization of revenue requirements yields the same solution as the objective of minimization of electric rates. But, if the utility can invest in conservation measures, the latter objective does not lead to any such investment as long as price exceeds marginal cost. This is because investment in conservation reduces the consumption of electricity while increasing the costs to the utility. Thus higher costs have to be recovered from lower sales leading to higher rates. On the other hand, LCP1 can still lead to investment in conservation if there are high distortions in the capital: (L>1). But, if price is less than marginal cost, this objective might advocate investment in conservation measures as long as these costs are less than the difference between avoided costs and revenues (Ruff, 1988, and Cicchetti and Hogan, 1988).

6.2.3 Model Normalization

The model, as presented above, has 4 parameters for each customer class together with the utility’s fixed cost and the marginal cost of power generation. Normalization of the model would help in reducing the number of independent parameters in the model. This helps us to derive conclusions from the model that are as general as possible. In the discussion below, we restrict ourselves to the case where average price is greater than marginal cost. Nelson (1989) considers problems
with price less than marginal costs$^3$.

Let us define the following parameters for each customer class, for the case where the average cost (price) exceeds marginal costs:

$$M_i = \frac{C_i^{1/\varepsilon_i} \beta_i^{1-1/\varepsilon_i} B^{1-1/\varepsilon_i} (\alpha_i-1)}{(1-1/\varepsilon_i)} A^{-1/\varepsilon_i}$$

$$U_i = (1-1/\varepsilon_i) \alpha_i$$

$$V_i = (1-1/\varepsilon_i) (1-\alpha_i)$$

(6.30)

Using the above transformation of variables in equations (6.24)-(6.29), the following normalized model is obtained:

---

$^3$ In cases where price is less than marginal cost, the DSM program costs push the rates towards the marginal cost of power. This increases the net benefits to consumers by leading to more efficient energy choices (VBP). But it is conceivable that average cost (price) will then exceed marginal costs if the program costs are high enough. In this case, all three objectives would lead to investment in conservation measures for smaller levels of distortions in the capital market.
\[
\begin{align*}
\text{MIN } F_1 &= 1 + \sum_i q_{ei} + \sum_i d_{ui} \\
\text{MAX } F_2 &= \sum_i \left( M_i (K_i + d_{ci} + d_{ci})^{\theta_i q_{ei}} - q_{ei} - d_{ui} - d_{ci} \right) \\
\text{MIN } F_3 &= 1 + \sum_i q_{ei} + \sum_i (d_{ui} + d_{ci}) \\
\end{align*}
\]

subject to:

\[
\sum_i P_{ei} q_{ei} = 1 + \sum_i q_{ei} + \sum_i d_{ui} 
\]

For each customer class 'i', we have the following constraints:

\[
\begin{align*}
V_i M_i (K_i + d_{ui} + d_{ci})^{\theta_i q_{ei}^{\gamma_i-1}} - P_{ei} &= 0 \\
\gamma_i^2 [U_i M_i (K_i + d_{ui} + d_{ci})^{\theta_i q_{ei}^{\gamma_i-1}} - L_i] &= 0 \\
U_i M_i (K_i + d_{ui} + d_{ci})^{\theta_i q_{ei}^{\gamma_i-1}} - L_i &\leq 0 \\
d_{ci}, d_{ui}, P_{ei} &\geq 0 
\end{align*}
\]

The new variables are related to the variables of the original model as follows:

new \( q_{ei} \) = (B/A)* old \( q_{ei} \)
new \( d_{ui} \) = old \( d_{ui} / A \)
new \( d_{ci} \) = old \( d_{ci} / A \)
new \( p_{el} \) = old \( p_{el}/B \)
new \( F_1 \) = old \( F_1/A \)
new \( F_2 \) = old \( F_2/A \)
new \( F_3 \) = old \( F_3/A \)

We can further simplify the optimization problem (6.31-6.38) by substituting the equality constraint (6.33) into (6.34), yielding:

\[
\sum_i V_i M_i (K_i + d_{ul} + d_{el}) q_{e1} q_{e1}^{'} = 1 + \sum_i q_{e1} + \sum_i d_{ul} \quad (6.39)
\]

The complementary slackness conditions (6.36)-(6.37) and the revenue recovery constraint (6.39) above make this problem non-convex. Before we discuss solution techniques, let us examine the properties of each of these objectives and constraints in the next subsection.

6.3 Model Structure

In this thesis, we only consider the case where price is greater than marginal cost, i.e., the fixed costs \( A \) is greater than zero. The solution technique and the analysis of the model structure can be presented analogously for the case where price is less than marginal cost.
The multilevel problem represented in Section 6.2.3 by equations (6.31-6.38) is nonconvex. The presence of the lower level KKT conditions is one of the reasons for the nonconvexity of the problem. The other constraint which leads to nonconvexity is the revenue recovery constraint represented by (6.34). We solve the problem by making use of algorithms explained in Chapter 7, which search through a set of feasible reduced problems before arriving at an overall optimum. We show that the general form of the reduced problem, under some assumptions, is a convex problem yielding the global optimum. The best solution among the set of these global optima for the reduced problems provides the overall optimum.

6.3.1 Objective Functions

Least cost planning objectives (6.31 and 6.33) are linear, so if the constraint set is convex, the local optimum would be the global optimum for the reduced problem. We now have to prove that the value-based planning (VBP) objective presented as (6.32) is a concave function.

We can easily prove that a twice differentiable function is convex (concave) if and only if the Hessian of the function $f(x)$ is negative semi-definite (positive semi-definite (nsd) for all $x \in \mathbb{R}^n$ (Luenberger, 1984 and Chankong, 1989). We therefore check the convexity (concavity) of the functions of the bilevel problem by examining
the Hessian of these functions. The variables in this function are $d_i = (K_i + d_{ui} + d_{ei})$ and $q_{ei}$. If there are a total of $m$ customer classes, then there are a total of $2m$ variables in the VBP objective.

Differentiating with respect to $d_i$ and $q_{ei}$ would yield the following expression for each customer class $i$. We can check the concavity of the function class by class, since there is no interaction in (6.32) between variables of different classes, i.e., the expressions are classwise separable. Therefore:

$$
\nabla VBP_i = (U_i M_i (d_i)^{q_{ei}} q_{ei}^{v_i-1}, V_i M_i (d_i)^{q_{ei}} q_{ei}^{v_i-1})
$$

(6.40)

The above is the gradient of the VBP objective w.r.t. to $d_i$ and $q_{ei}$. There is a row vector with the gradients for each customer class $i$ as presented below:

$$
\nabla VBP = (\nabla VBP_1, \nabla VBP_2, \ldots, \nabla VBP_c)
$$

(6.41)

Differentiating the above expression w.r.t. $d_i$ and $q_{ei}$ yields the Hessian of the VBP objective which is block diagonal with a $2 \times 2$ block per customer class or end use 'i'. Let us now prove that the Hessian of the VBP objective is nsd. The Hessian for the $2 \times 2$ block is given below:
\[ \nabla^2 V_{BP_1} = \begin{pmatrix} U_i (U_i - 1) M_i d_i^{u_i - 2} q_{el}^{v_i} & U_i V_i d_i^{u_i - 1} q_{el}^{v_i - 1} \\ U_i V_i M_i d_i^{u_i - 1} q_{el}^{v_i - 1} & (V_i - 1) V_i M_i d_i^{u_i} q_{el}^{v_i - 2} \end{pmatrix} \] (6.42)

Let \( \alpha(k) \) be the determinant of the leading principal submatrix of order \( k \) of a matrix. According to the Sylvester’s Theorem, a matrix is positive definite if and only if \( \alpha(k) > 0 \) for all \( k = 1, \ldots, n \). On the other hand, a matrix is negative definite if and only if \( \alpha(k) < 0 \) for odd \( k \) and \( \alpha(k) > 0 \) for even \( k, k = 1, \ldots, n \) (Chankong, 1989).

We can easily note that the leading principal submatrix of order 1 of the matrix defined by (6.42) is less than or equal to zero because \( U_i, M_i \) are negative and \( D_i, q_{el} \) are positive (equation 6.30). This is so because the elasticity of energy services \( \varepsilon_{si} \) and the production function coefficients \( \alpha_i \) are assumed to be less than or equal to 1.

The determinant of the 2×2 matrix is as follows:

\[
U_i V_i (U_i - 1) (V_i - 1) M_i^2 d_i^{2u_i - 2} q_{el}^{2v_i - 2} - U_i^2 V_i^2 M_i^2 d_i^{2u_i - 2} q_{el}^{2v_i - 2} \geq 0 \] (6.43)

Canceling out common terms (collectively positive causing no change in signs), we have:
\[ (U_i - 1)(V_i - 1) - U_i V_i \geq 0 \]  \hspace{1cm} (6.44)

Simplifying further, we should show that:

\[ U_i + V_i \leq 1 \]  \hspace{1cm} (6.45)

We know that this relationship is always satisfied as long as \( \epsilon_{ui} \) is less than or equal to one (See definitions of \( U_i \) and \( V_i \) in equation 6.30). Now all the bilevel problems defined in this chapter are assumed to have the price elasticity of energy services \( \epsilon_{ui} \) less than 1. Thus we can conclude that the VBP function (6.32) is strictly concave. The significance of this result is that if any function \( f(x) \) is concave, the maximization of such a function would always lead to the global maximum. This is subject to the constraint set being convex.

6.3.2. Reduced Constraint Set

This section looks at the constraint set of the K-T problem obtained by substituting the Kuhn-Tucker conditions of the lower level model as constraints for the upper level model (6.31-6.38). The reduced constraint set is obtained by relaxing the complementary slackness conditions (6.36) and (6.37) for each customer class. This is important because if the reduced constraint set is convex then, if a solution exists, the local optimum becomes the global optimal for that particular reduced
problem. The best solution among the solutions for the various reduced problems provides the global optimal optimum for the bilevel problem.

Now let us substitute (6.35) into (6.34), and assume that J customer classes have non-zero capital investment in conservation measures. This implies that K (=m-J) customer classes make no additional investment in conservation measures, i.e., \( d_{ck} = 0 \).

Thus, in this case, the constraints (6.34-6.38) can be reduced to the following set:

\[
\sum_j V_j M_j d_j^{\alpha_j} q_{ej}^{\nu_j} + \sum_k V_k M_k d_k^{\alpha_k} q_{ek}^{\nu_k} - \sum_j (d_{uj} + q_{ej}) - \sum_k (d_{uk} + q_{ek}) - 1 = 0
\]

(6.46)

\[
U_j M_j (d_{uj} + d_{cj})^{\nu_j-1} q_{ej}^{\nu_j} = L_j \quad \forall j
\]

(6.47)

\[
U_k M_k d_{uk}^{\nu_k-1} q_{ek}^{\nu_k} \leq L_k \quad \forall k
\]

(6.48)

\[
d_{cj}, d_{uj} \geq 0 \quad \forall j
\]

(6.49)

\[
d_{ek} = 0, \quad d_{uk} \geq 0 \quad \forall k
\]

(6.50)

Let us look at the structure of each of these constraints. We can notice that there are J+1 equality constraints, K inequality constraints, and (2J+K) non-negativity constraints. Let us now prove that each of the K constraints presented as (6.48) are convex.
We have just two variables for each of these constraints (a total of \( k \) such constraints) and each of these functions are easily twice differentiable. It can be easily shown that if a function \( f(x) \) is convex, then \( f(x) \leq 0 \) is a convex set (Luenberger, 1984). Therefore let us prove that the function \( f(d_{ak}, q_{ak}) \) defined by (6.48) is convex. Let us now calculate the gradient of the function:

\[
\nabla F = (U_k (U_k - 1) M_k d_k^{u_k - 2} q_{ak}^{v_k}, U_k V_k M_k d_k^{u_k - 1} q_{ak}^{v_k - 1})
\]

(6.51)

Now, the Hessian of the function \( f(d_{ak}, q_{ak}) \) is given by the following expression:

\[
\nabla^2 F (d_k, q_{ak}) = \begin{pmatrix}
U_k (U_k - 1) (U_k - 2) M_k d_k^{u_k - 2} q_{ak}^{v_k} & U_k V_k (U_k - 1) M_k d_k^{u_k - 2} q_{ak}^{v_k - 1} \\
U_k V_k (U_k - 1) M_k d_k^{u_k - 2} q_{ak}^{v_k - 1} & U_k (V_k - 1) V_k M_k d_k^{u_k - 1} q_{ak}^{v_k - 2}
\end{pmatrix}
\]

(6.52)

According to Sylvester's Theorem, for the matrix to be pd, the determinant of the leading principal submatrix of order 1 and 2 should be greater than zero. We can easily observe that the order 1 leading principal submatrix is greater than zero, because \( U_k, V_k, M_k \) are negative and \( d_k, q_{ak} \) are positive. Taking the determinant of the Hessian (6.52) and canceling out common positive terms leads to the expression below:

Simplifying further, we get:
\[ U_k + V_k \leq 2 \] (6.54)

From the definitions of \( U_i \) and \( V_i \) in (6.30), we know that this is always true. Therefore the function \( f(d_k, q_{ek}) \) is strictly convex. Therefore the space defined by \( f(d_k, q_{ek}) \leq 0 \) is convex.

But we still have \( J \) equality constraints where \( f(d_j, q_{ej}) \) is a convex function but they do not define a convex set because \( f(x) = 0 \) need not form a convex set even when \( f(x) \) is convex.

Let us now consider the revenue recovery constraint given by (6.46). We can substitute expression (6.47) into (6.46) yielding a function with \((m-J=K)\) nonlinear functions and \((2m+J)\) linear functions with positive constants as given below:

\[
\sum_k \left( V_i M_k d_{uk} q_{ek}^{V_i} - q_{ek} - d_{uk} \right) + \sum_j \frac{V_j L_j}{D_j} d_{uj} \\
+ \sum_j \left( \frac{V_j L_j}{D_j} - 1 \right) d_{uj} - 1 = 0
\] (6.55)

So, to prove that the overall function is convex, all we need to show is that the function \( h(d_{uj}, q_{ej}) = V_j M_j d_{uj} q_{ej}^{V_j} \) is convex, as the other terms are zero. The gradient and the Hessian of the function are as follows:
\[ \nabla h(d_{uj}, q_{ej}) = (U_j V_j M_j d_{uj}^{\gamma_j-1} q_{ej}^{\gamma_j-1} v_j^{\gamma_j-1} -1, V_j^2 M_j d_{uj}^{\gamma_j-1} q_{ej}^{\gamma_j-1} v_j^{\gamma_j-1} -1) \] (6.56)

\[ \nabla^2 h = \begin{pmatrix} U_j (U_j-1) V_j M_j d_{uj}^{\gamma_j-2} q_{ej}^\gamma v_j & U_j V_j^2 M_j d_{uj}^{\gamma_j-1} q_{ej}^{\gamma_j-1} v_j^{\gamma_j-1} \\ U_j V_j^2 M_j d_{uj}^{\gamma_j-1} q_{ej}^{\gamma_j-1} v_j^{\gamma_j-1} & V_j^2 (V_j-1) M_j d_{uj}^{\gamma_j} q_{ej}^{\gamma_j-2} v_j^{\gamma_j-2} \end{pmatrix} \] (6.57)

We see that the leading principal submatrix of order 1 is greater than zero, because of the reasons stated earlier. Now the determinant of the Hessian which is the same as the determinant of the leading principal submatrix of order 2, is given by:

\[ (U_j - 1) (V_j - 1) - U_j V_j \geq 0 \] (6.58)

The above relationship is strictly true since \( U_k + V_k \) is strictly less than one as long as the elasticity of energy services is less than 1. Thus the function is convex and the sum of \( K \) such functions and \( (2m+1) \) linear functions is also convex. But the set defined by \( f(.)=1 \) is not necessarily convex.

Let us now present the problem to be solved.
\[
\text{MIN } 1 + \sum_{i} q_{el} + \sum_{i} d_{ui} \tag{6.59}
\]
\[
\text{MIN } 1 + \sum_{i} q_{el} + \sum_{i} d_{ui} + \sum_{i} d_{ci} \tag{6.60}
\]
\[
\text{MAX } \sum_{i} \left( M_i (K_i + c_{ci} + d_{ui}) u_i q_{el} - q_{el} - d_{ui} - d_{ci} \right) - 1 \tag{6.61}
\]
subject to:
\[
\sum_{k} \left( V_k M_k d_{uk} q_{ek} - q_{ek} - d_{uk} \right) + \sum_{j} \frac{V_j L_j}{U_j} d_{cj} + \sum_{j} \left( \frac{V_j L_j}{U_j} - 1 \right) d_{uj} - 1 = 0 \tag{6.62}
\]
\[
U_j M_A (d_{uj} + c_{cj})^{\gamma_j - 1} q_{ej}^{\gamma_j} = L_j \quad \forall j \tag{6.63}
\]
\[
U_k M_k d_{uk}^{\gamma_k - 1} q_{ek}^{\gamma_k} \leq L_k \quad \forall k \tag{6.64}
\]
\[
d_{cj}, d_{uj} \geq 0 \quad \forall j \tag{6.65}
\]
\[
d_{ck} = 0, \quad d_{uk} \geq 0 \quad \forall k \tag{6.66}
\]

Therefore the problem above is a linear minimization objective or a concave maximization objective together with \(K+1\) equality constraints (defined by convex functions), \(J\) inequality constraints (convex sets), and \(2K+J\) nonnegativity constraints.
The solution of this reduced problem in its present form would not guarantee the global optimum due to the presence of the convex equality constraints (6.62) and (6.63). Thus the original problem was nonconvex but so too is the reduced problem. In this thesis, we use the augmented Lagrangian method to solve reduced problems with convex equality constraints. This method is explained in Section 7.2 in the next chapter along with a discussion of various other methods to solve such problems.

6.4. Formulation of the BLP2 Model

(Utility Sponsored Price Subsidies for Purchase of DSM Resources)

The version of the bilevel model, presented in Section 6.2 and 6.3 above, considers investment in conservation measures (\(d_{ui}\) and \(d_{u0}\)) as variables. This formulation is ideally suitable for DSM programs where the utility invests directly in conservation measures. An example of such a program would be direct utility investment in low income weatherization programs where the utility installs a predetermined amount of insulation and energy efficient windows. In this section, we instead present a version of the bilevel model which is ideal for DSM programs where the utility offers price subsidies or rebates for customers to invest in conservation measures. A program where the utility offers rebates to customers to purchase energy-efficient air-conditioners is an example of this type of program. One feature of BLP2 is that it automatically ensures that the complementary slackness conditions
for the lower level problems are satisfied. Wirl (1989) adopts an approach similar to the one considered in this section for analysis of DSM measures. Though he does not consider this problem in a bilevel framework, he implicitly assumes that customers' choice of energy purchases are based on their net benefit maximization.

Now, as assumed earlier, we posit that consumers are concerned about energy services and not about commodities from which they are derived. These energy services (comfortable heating, bright lights etc.) are derived from two types of input, K (capital) and energy q_e. Thus, the energy services q_s are obtained from the two inputs and can be defined as follows:

$$q_{s1} = \beta_{s1} \cdot K_1^{\alpha_1} q_{e1}^{1-\alpha_1}$$

(6.67)

Let us also assume that the demand for energy services is of the constant elasticity demand form, i.e., $q_{s1} = C_1 P_{s1}^{-\varepsilon_s}$. Therefore, the benefits of consumption of energy services is the area under the demand curve for energy services. This equals:

$$B_1(q_{s1}) = \frac{q_{s1}}{\varepsilon_s} P_{s1}(q) dq$$

(6.68)

$$= \gamma_1 K_1^{\alpha_1} q_{e1}^{\nu_1}$$

where:
\[
\gamma_l = \frac{C^1_{\delta}}{1-1/\varepsilon_{s_l}} \beta_{s_l}^{1-1/\varepsilon_{s_l}}
\]

\[
U_i = \alpha_i (1 - 1/\varepsilon_{s_l})
\]

\[
V_i = (1 - \alpha_i) (1 - 1/\varepsilon_{s_l})
\]

Now the lower model can be defined as follows:

\[
\begin{align*}
\max \quad & B_i(q_{s_l}) - P_{e_l}q_{e_l} - L_i (1 - I_i) K_i \\
\text{subject to} \quad & q_{s_l} = \beta_{s_i} K_i^\alpha q_{e_l}^{1-\alpha} \\
& I_i, K_i, q_{e_l} \geq 0
\end{align*}
\]

where:

\(L_i\) = Ratio of the perceived cost of capital (the purchase price of energy efficiency) to the actual cost of capital. \(L_i K_i\) represents the cost of energy efficiency [$/unit of capital], actual or perceived, to the consumer. This could, for example, be the cost to the customer of buying an energy-efficient air-conditioner in the market place.

\(I_i\) = Fraction of the total investment in conservation (\(K_i\)) paid as utility incentives, in $/unit of capital, for the purchase of energy efficiency in sector ‘i’. This could, for example, be the rebate the utility offers for purchasing the efficient air-conditioner.

Note that \(K_i\) is measured in $ in the model.

Let us now discuss the lower level model. The objective of the customers at
the lower level is to maximize their gross benefits minus their electricity bills and their cost of purchasing energy efficient equipment. The term \( L_i(1-L_i)K_i \) in the objective function defines the costs to the customer of purchasing energy efficiency. The actual or perceived costs that he faces \( L_iK_i \) is lowered by the utility subsidies \( L_iI_iK_i \). Though society (utility) spends only \( I_iK_i \$ \), the consumer acts as if \( L_i \) times that amount has been spent on conservation.

An important feature of this model is that the Cobb-Douglas (C-D) form for the energy services (6.67) ensures that the two inputs \( K \) and \( q_e \) are not completely substitutable for each other. Therefore, unless the quantity of energy services consumed by that class is zero, \( K \) and \( q_e \) can never be zero. Moreover, the C-D form (6.67) also ensures that \( K \) and \( q_e \) are greater than zero. Therefore, the non-negativity constraints for \( K \) and \( q_e \) can be eliminated.

Now the upper level model can be either the LCP1 problem given below as (6.70), or the VBP problem given as (6.71), or the LCP2 problem given as (6.72). Below:
\[
\min \ C(\sum I_i q_{et}) + \sum I_i K_i 
\]

\[
\max \ \sum B_i (q_{et}) - C(\sum I_i q_{et}) - \sum K_i 
\]

\[
\min \ C(\sum I_i q_{et}) + \sum K_i 
\]

where:

\[ C(q_{et}) = \text{Cost of electricity production which, for simplicity, we assume is a linear function.} \]

In this formulation, we have assumed that the distortions in the capital market \( L_i \) occurs, truly, because consumers are short-sighted in their energy usage decisions. One of the main reasons could be lack of information.

Now the revenue recovery constraint can be given as follows:

\[
\sum P_{et} q_{et} = C(\sum I_i q_{et}) + \sum I_i K_i 
\]

The bilevel model is converted into a single level model by replacing the lower level models by their Kuhn-Tucker conditions. These conditions can still be easily obtained since the net benefits term is still differentiable with two variables \( K_i \) and \( q_{et} \). Thus the bilevel model becomes:
\[
\text{min } C(\sum q_{el}) + \sum I_i K_i \tag{6.74}
\]

\[
\text{max } \sum B_i(q_{el}) - C(\sum q_{el}) - \sum K_i \tag{6.75}
\]

\[
\text{min } C(\sum q_{el}) + \sum K_i \tag{6.76}
\]

subject to:

\[
\sum P_{el} q_{el} = C(\sum q_{el}) + \sum I_i K_i \tag{6.77}
\]

\[
\frac{\partial B_i(K_i, q_{el})}{\partial q_{el}} - P_{el} = 0 \quad \forall i \tag{6.78}
\]

\[
\frac{\partial B_i(K_i, q_{el})}{\partial K_i} - L_i (1 - I_i) = 0 \quad \forall i \tag{6.79}
\]

\[
0 \leq I_i \leq 1 \quad \forall i \tag{6.80}
\]

Note that the complementary slackness conditions for both the variables are removed because \( K_i \) and \( q_{el} \) are always greater than zero under the C-D assumption.

Rewriting the above model (6.74-6.80) in terms of \( K_i \) and \( q_{el} \), and using the linear cost function \( C(\sum q_{el}) = A + MC \sum q_{el} \), where \( A \) is the fixed cost per year and \( MC \) is the marginal cost of energy production, we get the following bilevel model:
\[
\begin{align*}
\text{min} & \quad A + MC \sum_i q_{e1} + I_i K_i \\
\text{max} & \quad \sum_i [B_i (q_{e1}) - K_i] - MC \sum_i q_{e1} - A \\
\text{min} & \quad A + MC \sum_i q_{e1} + \sum_i K_i
\end{align*}
\] (6.81)

\[
\begin{align*}
\sum_i P_{e1} q_{e1} = A + MC \sum_i q_{e1} + \sum_i I_i K_i \\
\gamma_1 V_i K_i U_i q_{e1}^{v_i-1} - P_{e1} = 0 \quad \forall i \\
\gamma_1 U_i K_i U_i q_{e1}^{v_i} - L_i (1 - I_i) = 0 \quad \forall i \\
I_i \geq 0 \quad \forall i
\end{align*}
\] (6.84)

(6.85)

(6.86)

Equation (6.85) says that the marginal benefit of consumption of an additional unit of electricity equals the price of electricity. Equation (6.86) indicates that the amount of incentive per unit of capital perceived by the customer should equal the customer’s cost of the durable minus the marginal benefit of adding a unit of capital.

6.4.1 Normalization and Analysis of the Model Structure

Now the above model (6.81)-(6.87) can be further simplified by replacing
(6.85) in (6.84) to yield the following constraint for (6.84) and eliminating the (6.85) constraints. Thus, (6.84) becomes:

\[ \sum_i \gamma_i V_i K_i^{\alpha_i} q_{ei} \nu_i = A + MC \sum_i q_{ei} + \sum_i I_i K_i \]  \hspace{1cm} (6.88)

It is important to note that if \( i \) represents uses in the same class, the prices are the same for all \( i \) in (6.85) and these equations are to be retained separately to enforce price equality.

Now the model can be further normalized by defining:

\[ M_i = \gamma_i A^{-1/\varepsilon_i} MC^{(1-1/\varepsilon_i) \alpha_i} \]  \hspace{1cm} (6.89)

and redefining the variables as follows:

- new \( K_i = \) old \( K_i/A \)
- new \( q_{ei} = (MC/A) \) old \( q_{ei} \)
- new \( I_i = \) old \( I_i \)
- new \( p_{ei} = \) old \( p_{ei}/MC \)
- new \( F_i = \) old \( F_i/A \) for all 3 objectives

The normalized version of the model represented by equations (6.81)-(6.88) can now be written as follows:

\[ \min 1 + \sum_i q_{ei} + \sum_i I_i K_i \]  \hspace{1cm} (6.90)
\[
\max \sum_i M_i q_{el}^{v_i} K_i^{q_i} - \sum_i K_i - \sum_i q_{el} - 1 \quad (6.91)
\]

\[
\min 1 + \sum_i q_{el} + \sum_i K_i \quad (6.92)
\]

subject to:

\[
\sum_i V_i M_i K_i^{q_i} q_{el}^{v_i} = 1 + \sum_i q_{el} + \sum_i I_i K_i \quad (6.93)
\]

\[
L_i (1 - I_i) - M_i U_i K_i^{q_i - 1} q_{el}^{v_i} = 0 \quad \forall i \quad (6.94)
\]

\[
0 \leq I_i \leq 1 \quad \forall i \quad (6.95)
\]

For the purpose of analyzing the structure of the model, we will eliminate \( I_i \) wherever it occurs using equation (6.94) yielding the following model:

\[
\min 1 + \sum_i q_{el} + \sum_i (K_i - \frac{V_i M_i}{L_i} K_i^{q_i} q_{el}^{v_i}) \quad (6.96)
\]

\[
\max \sum_i M_i q_{el}^{v_i} K_i^{q_i} - \sum_i K_i - \sum_i q_{el} - 1 \quad (6.97)
\]

\[
\min 1 + \sum_i q_{el} + \sum_i K_i \quad (6.98)
\]

subject to:
\[
\sum_i \left( \frac{U_i}{L_i} + V_i \right) M_i K_i q_{el}^i - \sum_i q_{el} - \sum_i K_i = 1 \quad (6.99)
\]
\[
M_i U_i K_i q_{el}^{i-1} q_{el}^i \leq L_i \quad \forall i \quad (6.100)
\]

Thus, there are two variables \( K_i \) and \( q_{el} \) for each customer class. We can easily see that the objective defined by (6.98) is linear and that (6.97) is of the same form as the VBP objective (6.32) for BLP1. We can show that (6.97) is concave in the same manner as it was proved for (6.32) by examining the Hessian of the 2x2 matrix.

Moreover, the constraint (6.100) is of the same form as equation (6.48) in Section 6.3 and it can be easily proved that (6.100) too defines a convex set. The only two equations that remain to be analyzed are (6.96) and (6.99).

Equation (6.96) defines \( 2m \) linear terms minus \( m \) non-linear terms. If we can prove that each of these nonlinear terms are concave, then the objective function defined by (6.96) is necessarily convex. So all we need to show is that the function
\[
h(K_1, q_{el}) = H_1 K_1 q_{el}^1 \quad \text{where } H_1 = U_1 M_1 L_1 \text{, is concave, as the other terms are linear.}
\]

The gradient and the Hessian of the function are as follows:
\[ \nabla h(K_1, q_{el}) = (U_1 H_1 K_1^{\alpha_{el}} q_{el}^{\alpha_{el}}, V_1 H_1 K_1^{\alpha_{el}} q_{el}^{\alpha_{el}}) \] (6.101)

\[ \nabla^2 h = \begin{pmatrix} U_1 (U_1 - 1) H_1 K_1^{\alpha_{el} - 2} q_{el}^{\alpha_{el}} & U_1 V_1 H_1 K_1^{\alpha_{el} - 1} q_{el}^{\alpha_{el}} \\ U_1 V_1 H_1 K_1^{\alpha_{el} - 1} q_{el}^{\alpha_{el} - 1} & V_1 (V_1 - 1) H_1 K_1^{\alpha_{el}} q_{el}^{\alpha_{el}} V_1^{-2} \end{pmatrix} \] (6.102)

Now the leading principal submatrix of order 1 is greater than zero, because \( U_1 \) is negative and \( H_1, K_1, \) and \( q_{el} \) are all positive. This implies that the Hessian is not negative, i.e., the function is not concave. In fact, we can prove that this function is convex. The determinant of the Hessian which is the same as the determinant of the leading principal submatrix of order 2, is given by:

\[ (U_1 - 1) (V_1 - 1) - U_1 V_1 \geq 0 \] (6.103)

The above relationship is strictly true since \( U_1 + V_1 \) is strictly less than one as long as the elasticity of energy services is less than 1. Thus the function \( h(.) \) is convex. This implies that (6.96) is non-convex, since \( h(.) \) would have to be concave for it to be so. Thus a global optimum is not guaranteed for the LCP1 problem.

Let us finally consider the constraint defined by (6.99) and note that it is of the same form as the function \( h(K_1, q_{el}) \) defined above. We proved by steps (6.101).
(6.103) that the function is convex. Thus the sum of these functions for \( m \) customer classes is also convex. But the set defined by \( f(.) = 1 \) is not necessarily a convex set. Therefore (6.99) defines a convex equality constraint which is not a convex set.

Thus the constraint set of the bilevel model BLP2 has convex equality \( f(x) = 0 \) where \( f(x) \) is a convex function) and inequality constraints. If the objective is convex (concave) or linear for minimization (maximization) problems, we can use the augmented Lagrangian approach discussed in the next section to obtain the global optimum. This is true for (6.97) which is a concave maximization function, and for (6.98) which is a linear function. But the method cannot guarantee a globally optimal solution for the LCP1 problem since the objective (6.96) is nonconvex.
CHAPTER 7
ALGORITHMS FOR SOLVING THE BILEVEL MODELS

7.1 Introduction

Chapter 6 presented the formulation of bilevel models to analyze the impacts of
demand-side management programs on electric utility resource planning. These
models can be useful for analyzing the effects of different objectives and various
program parameters on DSM resource selection while explicitly modeling the linkage
between the utility and its customers.

Section 6.2 considered a bilevel formulation where the utility can invest directly
in conservation measures (BLP1). BLP1 was reduced to a single level model by
incorporating the Kuhn-Tucker conditions of the lower level model as constraints in
the upper level model. This model becomes nonconvex even for the linear case and
complementary slackness conditions have to be removed before solving the model.
As explained in Section 7.3, the optimal solution for the bilevel model is obtained by
solving a series of trial problems each of which represents one possible resolution of

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the complementary slackness conditions. The optimal values of these trial problems are searched before the optimal solution of the original problem is obtained.

Section 6.4 formulated a bilevel model where the utility offers incentives to customers to invest in DSM programs (BLP2) rather than invest directly in conservation measures. This formulation ensures that all the complementary slackness conditions are automatically satisfied. Thus BLP2 does not require a special solution technique and is not discussed in this chapter.

We analyzed the structure of a reduced problem of BLP1 in Section 6.3 and found that this problem has a convex objective function with convex equality and inequality constraints. The structure of BLP2 was analyzed in Section 6.4, and all objectives, except for the minimization of revenue requirements, were linear or convex objectives with convex equality and inequality constraints. The significance of convexity is that if a feasible solution can be obtained for a convex problem, it is the global optimum for that problem.

Section 7.2 provides a discussion of various solution techniques to solve these reduced models of the BLP1 and BLP2 formulations. A rationale for the use of the augmented Lagrangian (AL) approach is presented.
The rest of the chapter is devoted to the discussion of solution algorithms to solve the overall BLP1 problem by solving the fewest number of reduced problems. We employ a branch and bound algorithm in Section 7.3 to solve the "maximization of value (VBP)" and the "minimization of societal cost (LCP2)" problems. But the "minimization of utility revenue requirements (LCP1)" objective cannot be solved by the branch and bound algorithm identified in Section 7.3. This is due to the fact that customer investments are pushed towards infinity for any problem for which the complementary slackness conditions are not imposed. Reasons for this problem and a heuristic to solve the LCP1 problem are presented in Section 7.4.

7.2 Algorithms to Solve the Reduced Problem

In this section, we consider various types of algorithms to solve the single level problem obtained by replacing the lower level problems by their Kuhn-Tucker conditions. Let us define the following problem as the general form of the reduced problem:

\[
\begin{align*}
\text{minimize } & f(x) \\
\text{s.t. } & g(x) \leq 0 \\
& h(x) = 0
\end{align*}
\]

(7.1)

where \( x \) is the set of upper and lower level variables, while \( f, g, \) and \( h \) define convex
functions. We assume that these functions are twice differentiable.

There are several candidate techniques for solving such constrained optimization problems (Luenberger, 1984, and Rao, 1984). These methods include:

- Heuristic Methods
- Cutting Plane Methods
- Feasible Direction Methods
- Penalty and Barrier Function Methods
- Dual Methods
- Multiplier Methods

Heuristic search methods are based on intuition and have little theoretical basis. An example of such an approach would be the one devised by Box (1965) to solve problems with only inequality constraints. This method is similar to the simplex method as a sequence of geometric figures each having $k \geq n+1$ vertices is formed to find the constrained minimum. But this method becomes rapidly inefficient when the number of variables increase. Moreover, if the feasible region is nonconvex, it is possible that the process will get stuck at a local optimum.

*Cutting plane* methods were originally described by Cheney and Goldstein (1959) and Kelley (1960) for convex programming problems. In this method, the non-linear constraints are linearized (Taylor series approximation) to approximate the feasible region by linearized envelopes. Since the constraint set is convex, the linearized envelopes always lie outside the feasible region. The objective has to be a linear
function or converted into one (this could be easily done by making the objective to minimize \( x_{n+1} \) and by adding a constraint \( f(x) - x_{n+1} \leq 0 \)). The resulting linear program can be easily solved. If the solution is not satisfactory, we relinearize the binding constraints about the current point, formulate a new approximating LP problem, and solve it again using the simplex method. This approach is repeated until a sufficiently accurate solution is obtained. This method is applicable only for convex programming problems and since all the optima of the linear program are likely to lie outside the feasible region, we need to determine a suitable tolerance level.

*Successive linear programming*, also known as the method of approximation problems, follows a similar approach by solving a sequence of linear problems (Palacios-Gomez et al., 1982). It is generally suitable for large sparse non-linear programs. In such problems, usually only some variables appear non-linearly in the objective and constraints and in addition, there is often a large subset of linear constraints. Boddington et al. (1979) discuss such an approach for nonlinear blending at Chevron Oil Co. This approach is not suitable for the problems of this thesis because of the high degree of nonlinearity of the functions and because any linearization could lead to non-optimal solutions.

The idea of *feasible direction methods* is to take steps through the feasible region of the form \( x_{k+1} = x_k + \alpha_k d_k \), where \( d_k \) is a direction vector and \( \alpha_k \) is a nonnegative
scalar. This method can be considered as a natural extension of unconstrained descent techniques. Each step is composed of selecting a feasible direction and a constrained line search. But, in the case of constrained nonlinear problems, this method is susceptible to zigzagging where the sequence of points generated by the process converges to a point that is not even a constrained local minimum.

It may be possible to solve problems with inequality constraints using an active set strategy. In this approach, certain constraints are active and others are treated as inactive. By systematically adding and dropping constraints from the working set, the optimal set of active constraints can be determined. But this may require a number of constrained problems to be solved exactly. Two of the most popular methods of this type are Rosen’s gradient projection method and Wolfe’s reduced gradient approaches. Both of these can be considered as the method of steepest descent applied to the surface defined by the active constraints. Of these two methods, the reduced gradient approach seems to the better one. It can be easily modified to avoid zigzagging and requires fewer computations per iterative step. Due to these reasons, for most problems, this method converges more rapidly than gradient projection methods. But for highly nonlinear problems, generalized reduced gradient (GRG) algorithms may fail to return to the feasible set. The popular GRG1, GRG2, and

\footnote{Beergrehn (1991) presents a problem, resembling the one in this thesis, which could not be solved by reduced gradient methods. That model too has variables raised to negative fractional powers.}
GINO, developed by Waren and Lasdon (1979), use the generalized reduced gradient method for nonlinear problems. This method works very well for problems with nearly linear constraints. But attempts to solve the single level problems presented in the previous sections using GRG2 failed to yield optimal solutions.

Another approach that seems to work for problems with a few nonlinear constraints is to adopt a type of a barrier function approach by penalizing a function of nonlinear constraints and leaving the linear constraints in the constraint set. Thus the problem becomes a series of optimization problems with nonlinear objective functions and a linear constraint set. It differs from normal barrier function methods in that the linear constraints are left as constraints unpenalized and not included in the objective

*Penalty* and barrier function methods are procedures for approximating constrained optimization problems by unconstrained problems. This is accomplished in the case of the penalty function methods by adding to the objective function a term that prescribes a high cost for violating constraints. In the case of barrier function methods, it is done by adding a term that favors points interior in the feasible region to ones near the boundary. There is a parameter $r_k$ associated with these terms and as $r_k$ tends to infinity, the approximated problem becomes increasingly accurate. There are two important factors to be considered. One is to check whether, as
parameter $r_k$ tends to infinity, the solution of the unconstrained problem converges to
the solution of the constrained problem. The second factor is that as $r_k$ increases, the
structure of the problem becomes increasingly unfavorable, slowing the convergence
rate of many of the algorithms. Most successful implementations of penalty or barrier
methods employ Newton’s method to solve the unconstrained problems and have
partly avoided the effects of ill-conditioned Hessians.

Let us define the barrier function problem for (7.1) with no equality constraints
as:

$$
\Phi(X, r_k) = f(X) - r_k \sum_{j=1}^{n} \frac{1}{g_j(X)}
$$

(7.2)

where $g_j(x)$ is the $j$th inequality constraint.

Let us also define the penalty function problem for (7.1) with no equality
constraints as:

$$
\Phi(X, r_k) = f(X) + r_k \sum_{j=1}^{n} (g_j(X))^q
$$

where:

$$
g_j(X) = \max(g_j(X), 0)
$$

where $r_k$ is a positive penalty parameter and $q$ is a nonnegative constant.
It can be easily proved that if one of the constraint functions is strictly convex and the objective and the other constraints are convex, then the unconstrained problem for both the barrier and the penalty function methods will be a strictly convex problem in $X$ (Rao, 1984). Each time, the problem is solved, it will provide the global optimum for (7.2) and (7.3). As $r_k$ increases, the global solution to the overall problem can be guaranteed.

The algorithms described in the above paragraph is valid only for problems with inequality constraints. Considering (7.1) above, the unconstrained minimization problem in which there are equality constraints $h_j(X)$ can be constructed as follows:

$$
\Phi_k = f(X) + r_k \sum_{j=1}^{K} G_j\left(g_j(X)\right) + H\left(r_k\right) \sum_{j=1}^{J} h_j^2(X) \quad (7.4)
$$

where $G_j$ is some function of $g_j$ tending towards infinity as $r_k$ tends to zero and as $H(r_k)$ tends to infinity, the summation of quantity $h_j(x)^2$ tends towards zero. Rao (1984) discusses such a solution procedure for solving a problem with both equality and inequality constraints.

Fiacco and McCormick (1966, 1988) present the following form of (7.4) for mixed problems with equality and inequality constraints:
\[ \Phi_k = f(X) - \frac{x_k}{\sum_{j=1}^{K} \frac{1}{g_j(X)}} + \frac{1}{\sqrt{r_k}} \sum_{j=1}^{J_k} h_j^2(X) \] (7.5)

It is well known and proved by Rao (1984) that the above unconstrained problem (7.5) tends towards the optimal solution as \( r_k \) increases and is guaranteed to be the optimal solution, if the following conditions are satisfied:

- \( f(X) \) is convex,
- \( g_j(X), j=1,2,\ldots,m \) are convex,
- \( \sum_{j=1}^{n} h_j^2(X) \) is convex in the interior feasible domain defined by the inequality constraints,
- one of the functions in the unconstrained approximation problems is strictly convex.

This approach will solve the reduced problems defined in Sections 6.2 (equations 6.59-6.66) and 6.4 (equations 6.81-6.87) and lead to global optimality. But the scale disparities that arise between the penalty parameters leads to extremely slow convergence of the algorithms and acceleration steps are necessary.

*Dual methods* seek to determine the Lagrange multipliers which are the fundamental unknowns in a constrained problem (Luenberger, 1984). Therefore, once these values are known, determination of the solution is relatively simple in most
cases. Dual methods tackle the dual problems whose variables are the Lagrange multipliers rather than the original problem.

Let us consider nonlinear programming problems of the form (7.1) with no inequality constraints. The corresponding dual function can be written as:

$$\Phi(\lambda) = \min_{\{\lambda\}} \left[ f(X) + \lambda^T h(X) \right]$$  \hspace{1cm} (7.6)

According to the local duality theorem (Luenberger, 1984, pp. 399), if the original unconstrained problem has a local solution at $x^*$ (regular point) with a solution $r^*$ and Lagrange multiplier $\lambda^*$ and if the Hessian of the Lagrangian $L(x^*)$ is positive definite, then the dual problem

$$\text{maximize } \Phi(\lambda)$$

has a local solution at $\lambda^*$ with corresponding solution $r^*$ and $x^*$ as the point corresponding to $\lambda^*$ in the definition of $\Phi$. This result can be easily extended for inequality constraints as well. Now if we also assume that $f$ and $g$ are convex and $h$ is convex in the feasible domain of the inequality constraints, we can then show that we can obtain a global optimum using the dual function. But, without some special structure (e.g., separable problems), this method is extremely costly to execute because every evaluation of $\Phi$ requires the solution of an unconstrained problem in the unknown $X$. 
Augmented Lagrangian or multiplier methods can be viewed as a combination of penalty functions and local duality methods (Hestenes, 1969). By using both methods together, it becomes possible to remove some of the undesirable features associated with each method alone. The augmented Lagrangian, for the equality constrained problem

\[
\begin{align*}
\text{minimize } & f(X) \\
\text{subject to: } & h(X) = 0
\end{align*}
\]  

is the function \( L_\lambda (X, \lambda) = f(X) + \lambda^T h(X) + \frac{c}{2} |h(X)|^2 \) for some positive constant \( c \).

For a fixed value of the vector \( \lambda \), the augmented Lagrangian \( L_\lambda \) is simply the standard quadratic penalty function and if the proper value of \( \lambda^* \) (correct value of the Lagrange multiplier) is used, the augmented Lagrangian is now the exact penalty function. It is exact in the sense that the solution of the penalty problem yields the exact solution for the original problem for that value of the penalty parameter. A typical step of an augmented Lagrangian commences with a vector \( \lambda_k \). Then \( X_k \) is found as a minimum point of
Next \( \lambda_k \) is updated to \( \lambda_{k+1} \). A standard method for this update is 
\[
\lambda_{k+1} = \lambda_k + c h(X_k) \quad \text{(Luenberger, 1984)}.
\]

The main iteration in augmented Lagrangian methods is with respect to \( \lambda \), but the penalty parameter may also be adjusted during the process. The sequence of \( c \)'s is usually either fixed or increased slowly towards a constant value. This remedies to a great extent the ill-conditioning problems that occur when \( c \) is increased towards infinity. From the viewpoint of duality, the problem (7.7) above is equivalent to:

\[
\text{minimize } f(X) + \frac{C}{2} | h(X) |^2 \\
\text{s.t. } h(X) = 0
\]

Hestenes (1959) and Powell (1972) were the first to propose this approach where squares of the equality constraint functions are added as penalties to the Lagrangian, and a certain simple rule is used for updating the Lagrange multipliers after each cycle. Powell (1972) shows that the rate of convergence is linear if one starts with a sufficiently high penalty and near to a local solution, if the usual second order
conditions for optimality are satisfied. Rockafellar (1973) generalizes this approach for inequality constrained problems. He shows that for convex problems (convex minimization problem with convex inequality constraints), global convergence to an optimal solution can be established for an arbitrary penalty factor.

Thus this method holds promise for the reduced problem defined in the previous subsections. The main reasons for adopting the augmented Lagrangian approaches in this thesis is that, for a finite penalty parameter with a convex objective and a constraint set comprised of convex equality and inequality constraints, global optimality can be obtained. This is because, at each iteration of the solution procedure, a convex unconstrained problem is solved and for a finite penalty parameter, the global optimum should be achieved. Moreover, the method is computationally efficient for problems like the ones discussed in this chapter where variables are raised to negative fractional powers. Beergrehn (1991) presents problems of this type solved by using his augmented Lagrangian (AL) model that could not be solved by existing constrained optimization models. Thus the AL model will be used in this thesis to solve the multiple customer class problem presented in the next chapter.

7.3 Solution Algorithms for VBP and LCP2 Objectives in the BLP1 Formulation

The bilevel models to be solved using the branch and bound technique, as
presented in Section 6.2 as equations (6.32)-(6.38), are defined below:

\[
\text{VBP} \quad \max \sum_i \left( M_i (K_i + d_{c1} + d_{s1}) u_i q_{el} v_i - q_{el} - d_{s1} - d_{c1} \right) - 1 \quad (7.10)
\]

or

\[
\text{LCP2} \quad \min 1 + \sum_i q_{el} + \sum_i (d_{s1} + d_{c1}) \quad (7.11)
\]

subject to:

\[
\sum_i V_i M_i (d_{s1} + d_{c1}) u_i q_{el} v_i = 1 + \sum_i q_{el} + \sum_i d_{s1} \quad (7.12)
\]

For each customer class ‘i’, we have the following constraints:

\[
d_{c1} \left[ U_i M_i (d_{s1} + d_{c1}) u_i^{-1} q_{el} v_i - L_i \right] = 0 \quad (7.13)
\]

\[
U_i M_i (d_{s1} + d_{c1}) u_i^{-1} q_{el} v_i - L_i \leq 0 \quad (7.14)
\]

\[
d_{c1}, d_{s1} \geq 0 \quad (7.15)
\]

For simplicity, we assume that \( K_i = 0 \) for all customer classes. This implies that \( d_{si} \) reflects the investment in conservation made by the customer class rather than the additional conservation investment as defined earlier. The same solution procedure applies if instead \( K_i \geq 0 \).
Bilevel problems have been solved with a variety of techniques in literature (Fortuny-Amat and McCarl, 1981, Bard and Falk, 1982, and Bard and Moore, 1990). But all of these approaches were applied to problems in which all the functions are linear, or quadratic in the case of Bard (1988).

There is, to date, no procedure for solving the problem with KKT constraints when the underlying functions are of arbitrary form. So far, only a few specialized versions have been solved with any degree of success. One such specialized version is the one used by Aiyoshi and Shimizu (1981). They use sequential unconstrained minimization technique (SUMT) type penalty functions to convert the problem into an unconstrained problem and solve the problem using a Fletcher-Reeves' conjugate gradient method. Although this approach solved small problems, it is hampered by extremely slow convergence and an inability to verify global optimality regardless of the structure of the model.

Cruz (1982) presents an efficient branch and bound scheme for a convex case where the objectives of the upper and the lower level models were quadratic and the upper and lower level constraints were linear. Edmunds and Bard (1990) present two approaches to solving nonlinear problems: a branch and bound technique and a objective function cut technique. The branch and bound technique is similar to the one used in this thesis, although the branching criteria are different. The test
problems analyzed in that paper had only linear constraints and were solved with a successive quadratic programming (SQP) algorithm. Personal communication with one of the authors (Bard) indicated that they have had very little success in reducing the number of branches to be fathomed in the branch and bound technique when the constraints were nonlinear. The objective function cut algorithm solves the problem by appending a constraint

\[ F(x, y) \leq F \]

to the KKT constraint set, where F is the upper level objective, x and y are the lower and upper level controlled variables respectively, and F is initially set at \( \infty \). Once a feasible point is found, the bounds are tightened eliminating current feasible points until an \( \epsilon \)-optimal solution is found. But this method fails to find solutions for many test problems (the same ones tested using the branch and bound technique); its success depended on the user allocated maximum number of quadratic programs the SQP algorithm had to solve in order to obtain a local solution. If the number is too small, it may lead to the conclusion that the problem is infeasible even though a local solution may exist whereas, if the number was too high, CPU time is wasted. Thus considerable time may be wasted in solving subproblems which may be infeasible and the authors indicate that numerical difficulty may be encountered when the lower level variables are unrestricted.
The branch and bound procedure considered in this thesis may be viewed as a rooted binary tree. The nodes correspond to subproblems in which various combinations of \( d_{ai} = 0 \) and \( g_i = 0 \) are enforced in (7.13), where \( g_i \) is defined as:

\[
    g_i (c_{ei}, d_{u1}, q_{e1}) = U_i M_i (d_{u1} + d_{ei})^{q_i - 1} q_{e1} q_i - L_i \quad (7.16)
\]

Note that when either \( d_{ai} \) or \( g_i(.) = 0 \) in equations (7.13) and (7.14), the complementary slackness condition for a particular customer class is satisfied. Arcs connect the nodes and lead to the linkage between different subproblems. It is necessary to come up with an efficient algorithm to search for the optimal node without considering all the \( 2^{m+1} - 1 \) possible nodes, where \( m \) is the number of customer classes. Thus, if there are 2 customer classes, there is a total of 7 reduced problems to be solved or fathomed (Figure 7.1). We adopt a branch and bound method to determine which reduced problems are to be solved.

Let us define a few parameters before we discuss the solution procedure.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_{\text{max}} )</td>
<td>Maximum number of nodes to be considered ((=2^{m+1} - 1)).</td>
</tr>
<tr>
<td>( k )</td>
<td>Counter to update the nodes backtracked.</td>
</tr>
<tr>
<td>( z )</td>
<td>Counter to update the number of reduced problems solved.</td>
</tr>
</tbody>
</table>
Figure 7.1: A Branch and Bound Tree For a Two Customer Class Problem
an array of dimension \( m \times n_{\text{max}} \) holding path vectors of backtrackable nodes.

\[ S^+ = \{ i : g_i = 0 \} \]. This is the row vector of the index of constraints with \( g_i \) set to 0 in any particular path.

\[ S^- = \{ i : d_{ei} = 0 \} \]. This is the row vector of the index of constraints with \( d_{ei} \) set to 0.

\[ \Gamma = \{ i : i \in S^+, i \notin S^- \} \]. This is the set of the indices of the complementary slackness conditions not considered in the path.

\[ J^- = \{ i : i \in S^+, g_i = 0 \} \]. This is the set of the index of the CS conditions which satisfies \( g_i = 0 \) even though they are not set equal to zero.

\[ F_{\text{opt}} \] Maximum value of the model (LCP2) or (VBP) to be solved.

We convert the LCP2 problem to a maximization problem by multiplying its objective by -1.

\[ F_{\text{max}} \] Maximum possible value of the objective. This value is the best solution that can be expected.

\[ \eta \] Branching parameter. If the capital inefficiency parameter \( L_i \) is less than or equal to this value, \( g_i \) is set equal to zero, whereas if \( L_i \) is greater than \( \eta \), \( d_{ei} \) is set equal to zero. This is a useful parameter and is employed to make use of our prior knowledge that at high levels of consumer distortions \( L_i \), utility
subsidies are likely to replace customer investment in conservation.

A flowchart of the algorithm to be discussed below is displayed as Figure 7.2 and the above notation will be used throughout the discussion.

Step-1 (Initialization)
Assign value to \( n_{\text{max}} = 2^{m+1} - 1 \). Set \( k = 0 \), \( z = 0 \), \( S^* = \{0\} \), \( S' = \{0\} \), \( J = \{0\} \), and set \( \Gamma = \{1,2, \ldots m\} \).

Step 2 (Upper Bound)
Formulate and solve the reduced problem with the complementary slackness conditions \( d_{ci}g_i = 0 \) removed. Set \( z \leftarrow z + 1 \) and let the optimal solution be \( \{d_{ci}^0, d_{ui}^0, q_{st}^0\} \). Update \( J' \). If this solution satisfies all \( d_{ci}g_i = 0 \), set \( F_{\text{opt}} = F_z \) and go to step 11; otherwise, fix \( F_z = F \) and set \( F_{\text{max}} = F_z \) and proceed to step 3.

Step 3 (Lower Bound)
Set the upper level variables \( d_{ui} = 0 \) and set \( g_i = 0 \) and solve the VBP model. Set \( z \leftarrow z + 1 \). Label the solution \( \{d_{ci}^*, d_{ui}^*, q_{st}^*\} \) and set \( F_{\text{opt}} = F_z \), the current solution.
Now if \( F_z = F_{\text{max}} \) go to step 11; else proceed to step 4.  

Step 4 (Accounting for Binding Constraints) 
Set the current node such that \( S^+ \leftarrow S^+ \cup i \in J^*, S^* \leftarrow S^* \), and \( \Gamma \leftarrow \Gamma \). If \( J = \{0\} \), go to step 5. Else, let the index of a customer class in \( J \) be \( s \). Set \( k \leftarrow k + 1 \) and update the array of backtrackable nodes by setting \( S^+ \leftarrow S^+ \), \( S^* \leftarrow S^* \cup s \), \( \Gamma \leftarrow \Gamma \), \( J \leftarrow J / s \) and the current node by setting \( S^+ \leftarrow S^* \cup s \), \( S^* \leftarrow S^* \), and \( \Gamma \leftarrow \Gamma \). Keep updating the set of backtrackable nodes and the current node till \( J = \{0\} \). Go to step 5. 

Step 5 (Branching) 
If \( \Gamma = \{0\} \), go to step 8; else consider the customer class 'i' with the highest value of \(| \alpha_{ei} g_i | \) and set the index of that customer class equals \( r \). If \( L_i \) for that class, is greater than \( \eta \), go to step 6. Otherwise, update \( z \leftarrow z + 1 \), and solve the reduced problem by setting \( g_i = 0 \) for all \( i \in S^* \cup r \) and \( d_{ui} = 0 \) for all \( i \in S^* \). Let \( F_z = F \). Set \( k \leftarrow k + 1 \) and update the array of backtrackable nodes \( L[\cdot,k] \) by setting \( S^+ \leftarrow S^+ \), \( S^* \leftarrow S^* \cup r \), \( \Gamma \leftarrow \Gamma / r \). The current node contains \( S^+ \leftarrow S^+ \cup r \), \( S^* \leftarrow S^* / r \), and \( \Gamma \leftarrow \Gamma \). Go to step 7. 

---

\(^2\) We set \( g_i \) also equal to zero because if \( d_{ui} = 0 \), then \( d_{ui} \) has to necessarily be greater than zero. Thus the only way for the CS conditions to be satisfied is for \( g_i = 0 \). This step provides a feasible point for the overall model. If no other solution is found, this point would be the optimal point.
Figure 7.2: Branch and Bound Algorithm for the VBP and LCP2 Problems
Figure 7.2: Branch and Bound algorithm for the VBP and LCP2 Problem
Step 6 (Alternate Branch)

Set $d_q = 0$, and then update $S^r \leftarrow S^r \cup r$, $S^+ \leftarrow S^+$, and $I^r \leftarrow I^r / r$. Now solve the reduced problem by setting $g_i = 0$ for all $i \in S^+$ and $d_q$ for all $j \in S^-$. Update $z \leftarrow z + 1$. Set $k \leftarrow k + 1$ and update the array of backtrackable nodes by setting $S^+ \leftarrow S^+ \cup r$, $S^- \leftarrow S^- \setminus r$, $I^r \leftarrow I^r$. Let $F_z = F$. Go to step 7.

Step 7 (Feasibility Check and Fathoming)

If no solution exists, fathom node and proceed to step 8. If feasible, proceed to step 9.

Step 8 (Backtracking)

If $k = 0$, go to step 11; else, solve the reduced problem obtained by setting $g_i = 0$ or $d_q = 0$ according to the indices of $S^+$, $S^-$, and $I$ contained in the path vector $B[-,k]$. Set $k \leftarrow k - 1$, and set $B[-,k+1] = 0$. Go to step 7.

Step 9 (Update objective values)

Check complementary slackness conditions of $i \in I^-$. If all conditions are satisfied, update $F_{op}$, fathom node and go to step 8. If any condition is not satisfied, go to step 10.
Step 10 (Bounding)

Check if $F_m < F_{opt}$. If so, fathom node and go to step 8. If not, go to step 4.

Step 11 (Termination)

Declare the feasible point in the inducible region associated with $F_{opt}$ as the optimal solution for the bilevel model.

At step 1, all the counters are set to zero, the branching value $\eta$ for L set, and the backtracking array $B$ is filled with zeros. At step 2, the upper level model is solved with no complementary slackness conditions. This solution would provide the most optimistic solution for the upper level problem. At step 3, a feasible point is obtained by setting all $d_u=0$. This provides a point in the inducible region and this can be set as the lower bound of the maximization objective. Now if this solution is same as the solution obtained without the complementary slackness conditions, the optimal solution is obtained and the algorithm can be terminated (step 11). The solution in step 3 is the status quo solution under the VBP and the LCP2 objective.

At step 4, we add all CS constraints which were binding, when the problem was solved with no complementary slackness conditions, to the set of active constraints to be fixed. This is equivalent to skipping nodes along a branch in the tree. The nodes which are skipped are stored in the backtracking array. At step 5, we choose
the customer class (r) with the highest value of $|d_{cr}g_r|$ and set $g_r=0$ for that particular customer class, if $L\leq\bar{n}$. This can be termed as the "entering constraint". This method of selection of entering constraint offered the best promise for this bilevel problem. This is because of our knowledge about the problem that as the distortions in the capital market (L) becomes higher, the customer investment $d_{cr}$ in DSM programs drop. This is the main reason for investment in DSM programs when price is greater than marginal cost. Edmunds and Bard (1990) also use $|d_{cr}g_r|$ as a branching criterion for their general problem and this criterion seemed to offer the most promise. Bard and Moore (1990) try out several entering criteria for their linear integer bilevel problems. They do not recommend any particular criteria, and concede that it is problem specific. For the bilevel model considered in this thesis, selection according to the value of $|d_{cr}g_r|$ seems to be the best alternative. There was no significant advantage to using any one branching rule except that we discovered that branching on the value $g_r$ led to greater number of nodes being solved than any of the other rules. Once the constraint to be made active is decided, the other branch at the same node is saved in the backtracking array. The reduced problem obtained is then solved.

Now if $L>\bar{n}$, $d_{cr}$ is set equal to zero and the reduced problem is solved and the backtracking array is updated with the other branch of the node (Step 6). The solutions of the branching steps 5 and 6 are checked in step 7.
If no solution exists (step 7), the node is fathomed and backtracking is performed (step 8). If the problem has a solution (Step 7), it is checked whether the solution satisfies all the complementary slackness conditions (step 9). If the solution is feasible for the overall bilevel model, the node is fathomed, and backtracking is performed and a new node is selected for advancing (step 8). On the other hand, if all the complementary slackness conditions are not satisfied, the upper bound of the objective is updated (step 10). If the solution value is lower than the present feasible solution, the node is fathomed and backtracking is performed. If the value is greater, we desire to go lower down the tree (step 4) to obtain a feasible point for the overall problem. Once all the nodes have been backtracked, the best feasible solution provides the overall optimum for the problem (step 11).

This is the algorithm used in the example, to be provided in Section 7.4, for solution of the VBP and LCP2 problem. This algorithm was tested and solved using the augmented Lagrangian optimization technique implemented by Beergrehn (1991).

7.4 Algorithm for the LCP1 Model

Let us now consider the LCP1 bilevel model, presented as equation (6.31) subject to (6.34)-(6.38) in Chapter 6, given below:
\[ \text{MIN } 1 + \sum q_{el} + \sum d_{ul} \] (7.17)

subject to:
\[ \sum V_i M_i (d_{ul} + d_{ci}) q_{el}^{V_i} = 1 + \sum q_{el} + \sum d_{ul} \] (7.18)

For i=1,2, ..., m:
\[ d_{cl} [U_i M_i (d_{ul} + d_{ci}) q_{el}^{V_i} - L_i] = 0 \] (7.19)
\[ U_i M_i (d_{ul} + d_{ci}) q_{el}^{V_i} - L_i \leq 0 \] (7.20)
\[ d_{cl}, d_{ul} \geq 0 \] (7.21)

The branch and bound algorithm presented in Section 7.3 does not require all the complementary slackness conditions to be satisfied at each node. Thus (7.11) does not apply at each node for each customer class in that case. The branch and bound approach does not work for the LCP1 problem above because, if the complementary slackness conditions are not satisfied for any particular class, the objective would push the \( d_{ui} \) for that particular class tends towards infinity. This occurs because increasing \( d_{el} \) (which does not appear in the objective) arbitrarily to satisfy (7.13) reduces the
need for $d_{ui}$ and $q_{si}$ (which do appear in the objective) to increase\(^3\). On the other hand, if the CS conditions are satisfied at each customer class, $d_{ci}$ is either set to zero or $g_i$ is binding for all classes. This limits the value of $d_{ci}$ to a finite value for all classes.

Thus we need to use a new algorithm to solve the LCP1 model which ensures that the complementary slackness conditions are satisfied for all classes in all iterations. We use a heuristic solution procedure to solve this model. This procedure, unlike the branch and bound algorithm in the previous section, does not guarantee global optimality under all circumstances. This is because the heuristic does not examine or eliminate all feasible nodes, as in the other case.

One major assumption of this algorithm is that, if utility investment in conservation $d_{ai}$ in any class alone does not improve the base case solution, utility investment in this class is unlikely to be in the optimal solution. The base case solution is the solution obtained when there are no electric utility DSM programs in all classes. This assumption is similar to the argument made when identifying DSM options to be included in the optimal mix of resources. That argument is that if a DSM program fails the screening test, it should not be considered further in the

\(^3\) $U_i$ and $V_i$ are negative fractional powers as long as $e_{ni}<1$. 
integrated resource planning process.

This assumption is reasonable because any program which fails the test individually, barring technical interactions\textsuperscript{4}, is unlikely to be a part of an optimal DSM portfolio. As noted in Chapter 3, there are "diminishing returns" when packages of DSM programs are included. Consider two peak shaving programs both of which pass the screening tests of Chapters 2-4. Introduction of the first programs clips the most expensive MW demand of the utility load whereas the second program would lead to lesser savings (clipping of a less expensive MW load) because of the effect of the first program. So it is possible that even a combination of two programs that individually pass screening tests may not be recommended as a package. Therefore it is unlikely a combination of programs, one of which fails the screening test, would be recommended as a package.

The other assumption that I make in the following algorithm is that, when the constraint (7.12) is binding, no utility subsidy is recommended. When price is greater than marginal cost, the utility invests in conservation only at high levels of distortions in the capital market, i.e., high values of $L_t$. Now the marginal benefit to the

\textsuperscript{4} Technical interactions occur when one DSM program affects the benefits of some other program. For example, investment in energy-efficient light bulbs could lead to reduction of the heating load in commercial buildings.
consumer of investing in conservation is, most often, less than the marginal cost of capital investment (equation 7.13) for high values of \( L_4 \). The complementary slackness conditions are satisfied, if (7.12) is not active, only if \( d_{ci} = 0 \). Therefore, we assume that both utility and customer investment in conservation cannot be greater than zero. If we do not make this assumption, all \( 2^m \) reduced problems need to be solved if the optimal point is to be obtained. This assumption is reasonable due to the fact that utility investments in conservation rather than customer investment is recommended only at high values of \( L_4 \). Nelson (1989) discovered a few cases where both \( d_{ci} \) and \( d_{ui} \) were both greater than zero, but none of them occurred with the LCP1 objective.

The heuristic presented below could significantly reduce the number of reduced problems solved if a smaller fraction of customer classes need utility DSM programs. For example, if there are \( m \) customer classes and only \( n \) (\( n < < m \)) classes recommend DSM programs, only \( (2^n + m - n) \) problems need to be solved before an optimal point is chosen.

Let us define a few variables below:

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>Counter updating the customer classes or end-uses</td>
</tr>
</tbody>
</table>
screened.

l Counter updating the number of customer classes or end-uses passing the screening step.

m Number of customer classes or end-uses.

n Counter updating the number of customer classes or end-uses considered at a time.

o Number of customer classes or end-uses passing the screening step.

z Counter updating the number of problems solved.

B[-] A row vector containing symbols representing the combination of classes considered.

C[-,-] An array containing the solutions of different reduced problems. The row representing the number of classes considered at a time and the column representing the combination of classes considered for program implementation.

F_{opt} Current optimal value for the problem.

F_{z} Value of the zth reduced problem solved.

P[-] A row vector containing the indices of all customer classes 'i', i=1,2,...,m.

Q[-] A row vector containing the indices of customer classes
that pass the screening test.

The algorithm is as given below:

1. (Initialization)
   Set \( k=0, \ l=0, \ z=0, \ n=1, \ o=0, \ B[-]=\{0\}, \ C[-,-]=\{0\}, \ P[-]=\{1,2, ..., m\}, \) and \( Q[-]=\{0\}. \)

2. (Initial Feasible Point)
   Set \( z\leftarrow z+1 \) and solve the traditional LCP model by setting \( g_i=0 \) and \( d_u=0 \) for all \( i \). Let the solution \( F_{\text{init}} \leftarrow F_z \). Order the indices of all the customer classes in vector \( P \) according to descending values of \( L_i \).

3. (Screening Individual Classes)
   Set \( z\leftarrow z+1 \) and \( k\leftarrow k+1 \). Solve a problem replacing the complementary slackness conditions of the customer class with index corresponding to \( P(k) \) by setting \( d_{ik}=0 \) and \( g_i>0 \). For all \( i \) not equal to \( k \), set \( g_i=0 \) and \( d_{ik}>0 \). If \( F_n>F_{\text{init}} \), go to step 4. Else set \( l=l+1, B[l]=\{k\} \), and \( C[n,l]=F_m \). Loop back to step 3.

4. (Checking)
   If \( k=m \), \( F_{\text{opt}} = \min \{ C[n,l] \} \) and go to step 5. Else go to step 3.

5. (Updating the Optimal Point)
   If \( l=0 \), go to step 8. Else, set \( n\leftarrow n+1 \), and go to step 6.
Figure 7.3: Heuristic for Solving the LCP1 Model
Figure 7.3: Heuristic for Solving the LCP1 Model
6. (Checking $2^{i+1}$ Combinations)
   
   Set $k=0$, $o=1$, $R=2^i$ and $Q[-] = \{ i : i \in B[-] \}$.

7. (Solving different combinations)
   
   Set $m\leftarrow m+1$, $k\leftarrow k+1$ and solve the problem by setting all classes 'i' $\in Q[-]$ such that $g_i=0$, and set $d_i=0$ for a combination of $j \in Q[-]$ taken $n$ at a time.

   Go to step 9.

8. (Stopping Criterion)
   
   $F_{opt}=F_{init}$ STOP.

9. (OptimalityUpdating)
   
   If $k=R$, go to step 10. If $F_m < F_{opt}$, set $l\leftarrow l+1$, $C[n, l]=F_m$ and set $B[l]$ with a symbol to represent a particular combination of 'n' $d_i=0$. Go to step 7. If $F_m > F_{opt}$, go to step 7.

10. (Updating the Optimal Point)
    
    If $l=0$, go to step 11. Else, set $n=n+1$, and go to step 6.

11. (Stop)
    
    The optimal solution for the problem is $F_{opt}$. STOP.

The algorithm is as shown in Figure 7.3 and is very easy to implement. It is a heuristic and cannot guarantee global optimality. But considering the likely nature of the optimal solution, as described above, it will lead to an optimal or near optimal point.
In step 1, the initialization process commences and the counters and arrays are filled with zeros except \( P \) which is an array of the indices of the various customer classes. In step 2, an initial feasible point is obtained \( (F_{\text{init}}) \). This point is the base solution for the LCP model in which the utility does not invest in any conservation subsidies. Steps 3, 4 and 5 are similar to the screening procedures (MV/GTRC and LC/TRC tests) discussed in Chapters 3 and 4. Thus, \( m \) different problems are solved in this loop by setting \( d_{ci} = 0 \) for each customer class. For those classes that yield a solution better than \( F_{\text{init}} \), the indices are stored in the \( B \) array and the solutions are stored in the \( C \) array. The minimum value among the solutions in the \( C \) array is designated as \( F_{\text{opt}} \). At Step 5, if utility investments in no class leads to better solutions than \( F_1 \), the optimal value for the problem equals \( F_1 \) (Step 8).

At step 6, the search starts to determine the optimal combination of utility investments in different classes that lead to an optimal value. So, at step 7, \( d_{ci} \) values are set to equal zero for customer classes selected \( 'n' \) \( (n=2,...,O) \) at a time. At step 9, the solutions for each combination of program is compared to \( F_{\text{opt}} \) determined with \( d_{ci} = 0 \) for \( 'n-1' \) customer classes. Any combination of programs leading to better results is stored in the \( B \) array and the values stored in the \( C \) array. At step 10, if no such combination exists, the optimal value for the problem equals \( F_{\text{opt}} \) obtained at the \( 'n-1'^{th} \) stage of the problem. If such a solution exists, a series of problems are solved again by setting \( d_{ci} = 0 \) for \( 'n+1' \) classes at a time. This procedure continues till all \( O \)
levels of the problem has been solved. The example solved in Section 8.2 of the next chapter provides an illustration of this solution technique.
CHAPTER 8

EXAMPLES FOR THE BILEVEL MODELS

8.1 Introduction

In this chapter, we present two examples to illustrate the solution techniques discussed in Chapter 7. Both these examples are for a four customer class model. It is assumed here that these are four different end-uses within the same customer class. Therefore, the four end-use sectors face the same price. For example, if the overall customer class is residential, the four end-use sectors could be water heating, space heating, cooling, and lighting. These are four end-uses within the same class (residential) facing the same price of electricity.

Section 8.2 presents an example for the BLP1 model. Two examples are discussed in this section. The first one is for a utility whose end-users in the base case face a price of electricity ($34/MWh) not much greater than the marginal cost of power generation ($30/MWh) while the second example is for the case where price ($80/MWh) is much greater than marginal cost ($30/MWh). The solution of the first example is discussed in detail by tracing the steps recommended by the algorithm presented in Chapter 7 for the LCP1, VBP and LCP2 models in Subsections 8.2.1,
8.2.2, and 8.2.3 respectively. There are a total of 31 \(2^{4+1}-1\) nodes in the branch and bound tree for the VBP and LCP2 models. This example yields the solution for the VBP model in nine steps, LCP1 in six, and the LCP2 in one step. A step refers to the number of solutions of the single level model (nodes) that need to be solved. In general, I find that fewer conservation subsidies are recommended as the divergence between price and marginal cost increase. Moreover, higher the distortion in the customer's capital market, more DSM programs are recommended. I discuss the results of this case where price is not much greater than marginal cost in Subsection 8.2.4. The assumptions and results of the case where price is much greater than marginal cost are presented in Section 8.2.5.

As noted earlier, BLP1 models are recommended for DSM programs in which the utility or its subcontractors invests directly in conservation measures. Section 8.3 provides an example for the BLP2 model in which the utility offers subsidies (or rebates) to customers to invest in conservation measures. As noted in Chapter 6, solutions for this model are found by solving only one model. This is because the complementary slackness conditions are automatically satisfied for this model.

Finally, it should be noted that the models solved in this chapter are devised to illustrate the applicability of the algorithms discussed in Chapter 7 for bilevel models. Though certain insights can be drawn from the solution of these models, these
examples are not exhaustive and are not intended to provide definitive conclusions. Section 8.4 offers some closing comments.

8.2 Example for the BLP1 Model

The example presented in this subsection is for a bilevel model with four end-uses within the same customer class. This model is solved for all three objectives (LCP1, VBP, LCP2) discussed in Chapters 6 and 7. As stated earlier, it is assumed that all customers in a class face the same electric rates and that price is greater than marginal cost (i.e., the parameter $A$, in Section 6.2, is greater than zero)\(^1\). Table 8.1 presents the parameter list for the four end-uses for this case. It can be seen that the end-uses 3 and 4 are similar except that end-use 4 is smaller than end-use 3 and faces a higher distortion in the capital market. For example, if the end-use is space heating, those belonging to 3 may belong to a wealthier class or may know more about the benefits of weatherization (reflected in their low value of $L$) than consumers in class 4. End-use 1 is efficient and faces no distortion in the capital market as customers using end-use 1 do not perceive a higher cost of capital relative to energy usage. Thus we would expect that the end-users of group 1 will invest in an adequate amount

\(^1\) If prices are to the same in all sectors we have equations (6.34)-(6.35) as constraints rather than (6.39) in the normalized model. This is done to ensure that all sectors have the same prices in this example.
of conservation even without utility subsidies.

| Table 8.1 |
| Parameter List for the 4 customer class model |

<table>
<thead>
<tr>
<th>Customer class ‘i’</th>
<th>Elasticity of energy services $e_i$</th>
<th>Production function exponent $\alpha_i$</th>
<th>Production function constant $\beta_i$</th>
<th>Demand function Constant $C_i$</th>
<th>Distortions in capital market $L_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15</td>
<td>0.5</td>
<td>1.0</td>
<td>1900</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
<td>0.55</td>
<td>1.1</td>
<td>2314</td>
<td>6.2</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>0.6</td>
<td>1.2</td>
<td>3200</td>
<td>2.1</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.6</td>
<td>1.2</td>
<td>2500</td>
<td>7.4</td>
</tr>
</tbody>
</table>

Some of the typical questions that could be addressed include:

1. Does the objective followed by the electric utility make a difference in DSM program selection?
2. Which of the four end-uses require utility subsidies under the three objectives?
3. What level of distortions in the end-use capital market make utility programs desirable?
4. What happens to the electric rates of customers if the utility invests in conservation measures?

5. What is the effect of conservation subsidies on energy consumption in end-use sectors that do not require utility investment?

6. What is the extent of free ridership among end-users when the utility invests in DSM programs?

7. What effect does the divergence between price and marginal cost have on DSM program selection?

These questions are answered in Subsection 8.2.4 of this section on the basis of the example problems solved in this section. Subsections 8.2.1-8.2.3 discuss the solution procedures for the LCP1, VBP, and LCP2 models respectively.

The bilevel model corresponding to the parameter list in Table 8.1 is given in Table 8.2. The VBP and LCP2 models are solved using the branch and bound algorithm discussed in Section 7.3 and the LCP1 model is solved with the heuristic presented in Section 7.4. The tree for the VBP and LCP2 algorithm is similar to the one shown as Figure 7.1. Each reduced problem is solved with the augmented Lagrangian model implemented by Beergrehn (1991). There is a maximum of 31 \( (2^{2+1}-1) \) nodes and \( 2^{4} \) feasible solutions that might need to be solved before the optimal point is obtained.
Table 8.2
4 End-Use Sector Example Problem

<table>
<thead>
<tr>
<th>Constants:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1 = -19.4894$</td>
</tr>
<tr>
<td>$c_2 = -2.2775$</td>
</tr>
<tr>
<td>$c_3 = -5.6448$</td>
</tr>
<tr>
<td>$c_4 = -0.4781$</td>
</tr>
</tbody>
</table>

| Objective(s): |
| LCP1 $\min$ $1 + q_{a1} + d_{a1} + q_{e1} + d_{a2} + q_{e2} + d_{a3} + q_{e3} + d_{a4} + q_{e4}$; |
| or |

| VBP $\max$ |
| $c_1^*(d_{a1} + d_{c1})^{2.83}q_{a1}^{2.83} - q_{a1} - d_{a1} - d_{c1}$ |
| $c_2^*(d_{a2} + d_{c2})^{3.11}q_{a2}^{2.55} - q_{a2} - d_{a2} - d_{c2}$ |
| $c_3^*(d_{a3} + d_{c3})^{5.4}q_{a3}^{3.6} - q_{a3} - d_{a3} - d_{c3}$ |
| $c_4^*(d_{a4} + d_{c4})^{5.4}q_{a4}^{3.6} - q_{a4} - d_{a4} - d_{c4}$; |

| LCP2 $\min$ $1 + q_{a1} + d_{a1} + q_{c1} + q_{a2} + d_{a2} + d_{c2}$ |
| $+ q_{a3} + d_{a3} + q_{c3} + d_{a4} + d_{c4}$; |

| Constraints: |
| $p_{a1}q_{a1} + p_{a2}q_{a2} + p_{a3}q_{a3} + p_{a4}q_{a4} - d_{a1} - d_{a2}$ |
| $d_{a3} - d_{a4} - q_{e1} - q_{e2} - q_{e3} - q_{e4} = 1$; |
| $55.2199*(d_{a1} + d_{c1})^{2.83}q_{a1}^{2.83} = p_a$; |
| $5.8076*(d_{a2} + d_{c2})^{3.11}q_{a2}^{2.55} = p_a$; |
| $20.3213*(d_{a3} + d_{c3})^{5.4}q_{a3}^{3.6} = p_a$; |
| $1.7213*(d_{a4} + d_{c4})^{5.4}q_{a4}^{3.6} = p_a$; |
| $d_{c1}*(55.2199*(d_{a1} + d_{c1})^{2.83}q_{a1}^{2.83} - 1.0) = 0$; |
| $d_{c2}*(7.0981*(d_{a2} + d_{c2})^{4.11}q_{a2}^{2.55} - 6.2) = 0$; |
| $d_{c3}*(30.482*(d_{a3} + d_{c3})^{5.4}q_{a3}^{3.6} - 2.1) = 0$; |
| $d_{c4}*(2.5819*(d_{a4} + d_{c4})^{5.4}q_{a4}^{3.6} - 7.4) = 0$; |
| $d_{a1}, d_{a2}, d_{a3}, d_{a4} \geq 0$; |
| $d_{c1}, d_{c2}, d_{c3}, d_{c4} \geq 0$; |
8.2.1 Solution of the LCP1 Model

The heuristic solution methodology for the LCP1 model was presented in Figure 7.3 and discussed in Section 7.4. As discussed there, the initial feasible point is defined as the solution of the traditional utility planning model in which the utility does not make any investment in DSM programs. We also set the constraints (7.12) to be binding because the complementary slackness conditions will not be satisfied when the constraints are not binding. This is because if all \(d_{ui} = 0\), then all \(d_{ui}\) have to be greater than zero. In this case, the only way that the CS conditions of (6.11) are satisfied is if all 4 constraints of the type (6.12) are satisfied. The results of this run is found under the heading "Iteration 1" and is shown in Table 7.2 with \(F_{\min} = 9.12\).

The next step of the solution procedure is to determine whether utility DSM investments are justified for any customer class. This is done by making \(d_{ci} = 0\) for each class, leaving \(g_{c} < 0\). Iterations 2, 3, 4, and 5 provide the solutions for these runs. It can be found from the solutions that only classes 2 (\(F_2 = 8.28\)) and 4 (\(F_4 = 8.11\)) justify utility subsidies (i.e., the solution has a lower value than \(F_{\min} = 9.12\)). So we determine that the only step to be completed is to evaluate whether utility subsidies are justified for both classes. Note that this step can be taken only if we assume that \(d_{ci}\) and \(d_{ui}\) cannot both be greater than zero for a given \(i\).
### Table 8.3: Results of the LCP1 Model (contd.)

<table>
<thead>
<tr>
<th>Class ‘i’</th>
<th>$d_c$</th>
<th>$d_u$</th>
<th>$q_e$</th>
<th>SLACK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.92</td>
<td>0</td>
<td>1.71</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.57</td>
<td>0</td>
<td>2.59</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.21</td>
<td>0</td>
<td>1.50</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.53</td>
<td>0</td>
<td>2.32</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Class ‘i’</th>
<th>$d_c$</th>
<th>$d_u$</th>
<th>$q_e$</th>
<th>SLACK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.15</td>
<td>0</td>
<td>1.46</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.64</td>
<td>0</td>
<td>2.19</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.33</td>
<td>0</td>
<td>1.26</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1.61</td>
<td>0.59</td>
<td>6.59</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Class ‘i’</th>
<th>$d_c$</th>
<th>$d_u$</th>
<th>$q_e$</th>
<th>SLACK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.17</td>
<td>0</td>
<td>1.44</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1.78</td>
<td>0.88</td>
<td>5.29</td>
</tr>
<tr>
<td>3</td>
<td>1.34</td>
<td>0</td>
<td>1.25</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.59</td>
<td>0</td>
<td>1.92</td>
<td>0</td>
</tr>
</tbody>
</table>

**Iteration 1:** "No DSM" Scenario

$F_i = 9.12$  
$F_{min} = 9.12$  
$F_{opt} = 9.12$  
$p_e = 1.12$

**Iteration 2:** $g_1, g_2, g_3, d_{ct} = 0$

$F_i = 8.11$  
$F_{min} = 9.12$  
$F_{opt} = 9.12$  
$p_e = 1.47$

**Iteration 3:** $g_1, d_{c2}, g_3, g_4 = 0$

$F_i = 8.28$  
$F_{min} = 9.12$  
$F_{opt} = 9.12$  
$p_e = 1.51$
| Iteration 4: \( g_1, g_2, d_{c3}, g_4 = 0 \) |
|---|---|---|---|---|
| \( F_d = 9.39 \) | \( F_{\text{min}} = 9.12 \) | \( F_{\text{opt}} = 9.12 \) | \( p_\varepsilon = 1.48 \) |
| Class 'i' | \( d_c \) | \( d_u \) | \( q_\varepsilon \) | SLACK |
| 1 | 2.16 | 0 | 1.46 | 0 |
| 2 | 0.64 | 0 | 2.18 | 0 |
| 3 | 0 | 2.06 | 0.76 | 1.28 |
| 4 | 0.58 | 0 | 1.94 | 0 |

| Iteration 5: \( d_{c1}, g_2, g_3, g_4 = 0 \) |
|---|---|---|---|---|
| \( F_5 = 10.06 \) | \( F_{\text{min}} = 9.12 \) | \( F_{\text{opt}} = 9.12 \) | \( p_\varepsilon = 1.49 \) |
| Class 'i' | \( d_c \) | \( d_u \) | \( q_\varepsilon \) | SLACK |
| 1 | 0 | 2.30 | 1.39 | 0.01 |
| 2 | 0.64 | 0 | 2.17 | 0 |
| 3 | 1.34 | 0 | 1.26 | 0 |
| 4 | 0.58 | 0 | 1.94 | 0 |

| Iteration 6: \( g_1, d_{c2}, g_3, d_{c4} = 0 \) |
|---|---|---|---|---|
| \( F_6 = 7.75 \) | \( F_{\text{min}} = 9.12 \) | \( F_{\text{opt}} = 8.11 \) | \( p_\varepsilon = 2.02 \) |
| Class 'i' | \( d_c \) | \( d_u \) | \( q_\varepsilon \) | SLACK |
| 1 | 2.46 | 0 | 1.22 | 0 |
| 2 | 0 | 1.52 | 0.93 | 4.68 |
| 3 | 1.49 | 0 | 1.03 | 0 |
| 4 | 0 | 1.41 | 0.65 | 6.00 |

The optimal point is found at "Iteration 6"
\( F_{\text{opt}} = 7.75 \)

Table 8.3: Results of the LCP Model
Thus the final step of the procedure is performed by setting $d_{c2}$ and $d_{c4}$ equal to zero and solving the problem. This provides an optimal point for the problem ($F_{\text{opt}}=7.75$). Thus a solution is obtained for this bilevel problem by solving six subproblems. The results of the reduced problems solved are summarized later in this chapter (Table 8.6, and discussed in Subsection 8.2.4).

The global optimality of this algorithm cannot be assured considering the assumptions made in Section 7.4. But, as observed in Nelson (1989), for sectors with higher values of distortion in the capital market, utility investment in conservation completely replaces the initially low customer investment (i.e. $d_{c1}=0$). This phenomenon lends credence to our belief that the LCP1 solution is optimal or very close to optimal.

### 8.2.2 Solution of the VBP Model

At every node, for the class 'i' with the highest value of $|\sigma_{ci}g_i|$, customer investment $d_{ci}$ is set equal to zero if $L_i>\eta$ or $g_i$ equals zero if $L_i<\eta$ (Figure 8.1). Taking the above step satisfies the complementary slackness condition for this class. In choosing $\eta$ (branching parameter), we make use of prior knowledge about the fact that the higher the distortion in the capital market, more the attractiveness of utility expenditures in DSM programs. This factor is also dependent on the divergence
between price and marginal cost. In this example, we use $\eta=4$ to choose the branch 
($d_{ai}=0$ or $g_i=0$ for the sector chosen). It is possible that other values might lead to 
fewer iterations; however, we did not test values other than $\eta=4$.

The model is initialized by setting all counters to zero, and filling all arrays with 
zeros. In the first iteration, the problem with no complementary slackness conditions 
is solved. This solution provides the best possible outcome for the upper level model 
and $F_{\text{max}}$ is set to this value (-12.80). The results of this run is given under the 
heading "Iteration 1" in Table 8.4. It should be noted that if either the slack variable 
or $d_{ai}$ corresponding to any particular class equals zero, the complementary slackness 
condition for that particular class is satisfied (equation 7.12). Figure 8.1 displays the 
branch and bound tree to be solved. It shows that the problem with no 
complementary slackness conditions is the originating node and is marked as (1).

In Iteration 2, we find a feasible point for the VBP model. This point 
corresponds to the status quo condition where the utility does not invest in any 
conservation measures ($d_{ai}=0$, $\forall i$) and is marked as (2) in Figure 8.1. We then set $F_{\text{opt}}$ 
to this value (-15.35). If we do not find a feasible value for any other node, we can 
conclude that the best solution is the current solution.

In Iteration 3, the problem is solved for the first main branch of the solution
Figure 8.1: Tree for the VBP Problem of the BLPI Formulation
### Iteration 1: No Complementary Slackness Conditions

\[ F_2 = -12.80 \quad F_{\text{max}} = -12.80 \quad F_{\text{opt}} = -\infty \quad p_e = 1.22 \]

<table>
<thead>
<tr>
<th>Class 'i'</th>
<th>( d_e )</th>
<th>( d_u )</th>
<th>( q_e )</th>
<th>SLACK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.98</td>
<td>0</td>
<td>1.63</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.44</td>
<td>0</td>
<td>1.13</td>
<td>5.04</td>
</tr>
<tr>
<td>3</td>
<td>1.62</td>
<td>0</td>
<td>1.05</td>
<td>0.92</td>
</tr>
<tr>
<td>4</td>
<td>1.27</td>
<td>0</td>
<td>0.81</td>
<td>6.23</td>
</tr>
</tbody>
</table>

### Iteration 2: No Utility DSM Investments \((d_u = 0, \forall i)\)

\[ F_2 = -15.35 \quad F_{\text{max}} = -12.80 \quad F_{\text{opt}} = -15.35 \quad p_e = 1.12 \]

<table>
<thead>
<tr>
<th>Class 'i'</th>
<th>( d_e )</th>
<th>( d_u )</th>
<th>( q_e )</th>
<th>SLACK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.92</td>
<td>0</td>
<td>1.71</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.57</td>
<td>0</td>
<td>2.59</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.21</td>
<td>0</td>
<td>1.50</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.53</td>
<td>0</td>
<td>2.32</td>
<td>0</td>
</tr>
</tbody>
</table>

### Iteration 3: \( d_{ct} = 0 \)

\[ F_3 = -12.93 \quad F_{\text{max}} = -12.80 \quad F_{\text{opt}} = -15.35 \quad p_e = 1.50 \]

<table>
<thead>
<tr>
<th>Class 'i'</th>
<th>( d_e )</th>
<th>( d_u )</th>
<th>( q_e )</th>
<th>SLACK</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>2.17</td>
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<td>1.45</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.43</td>
<td>0</td>
<td>1.07</td>
<td>4.83</td>
</tr>
<tr>
<td>3</td>
<td>1.60</td>
<td>0</td>
<td>1.01</td>
<td>0.68</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1.18</td>
<td>0.85</td>
<td>5.79</td>
</tr>
</tbody>
</table>

Table 8.4: Results of the VBP Model
Iteration 4: $g_1, \; d_{c2}, \; d_{c4} = 0$

<table>
<thead>
<tr>
<th>Class ‘i’</th>
<th>$d_c$</th>
<th>$d_u$</th>
<th>$q_x$</th>
<th>SLACK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.34</td>
<td>0</td>
<td>1.30</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1.29</td>
<td>1.11</td>
<td>4.30</td>
</tr>
<tr>
<td>3</td>
<td>1.58</td>
<td>0</td>
<td>0.99</td>
<td>0.40</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1.14</td>
<td>0.85</td>
<td>5.39</td>
</tr>
</tbody>
</table>

Iteration 5: $g_1, \; d_{c2}, \; g_3, \; d_{c4} = 0$

<table>
<thead>
<tr>
<th>Class ‘i’</th>
<th>$d_c$</th>
<th>$d_u$</th>
<th>$q_x$</th>
<th>SLACK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.34</td>
<td>0</td>
<td>1.30</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1.31</td>
<td>1.10</td>
<td>4.36</td>
</tr>
<tr>
<td>3</td>
<td>1.43</td>
<td>0</td>
<td>1.11</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1.16</td>
<td>0.83</td>
<td>5.46</td>
</tr>
</tbody>
</table>

Iteration 6: $g_1, \; d_{c2}, \; d_{c3}, \; d_{c4} = 0$

<table>
<thead>
<tr>
<th>Class ‘i’</th>
<th>$d_c$</th>
<th>$d_u$</th>
<th>$q_x$</th>
<th>SLACK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.54</td>
<td>0</td>
<td>1.17</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1.23</td>
<td>1.10</td>
<td>3.81</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1.53</td>
<td>0.99</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1.08</td>
<td>0.86</td>
<td>4.80</td>
</tr>
</tbody>
</table>

Table 8.4: Results of the VBP Model (contd.)
### Iteration 7: $g_1$, $g_2$, $d_{st} = 0$

<table>
<thead>
<tr>
<th>Class 'i'</th>
<th>$d_e$</th>
<th>$d_u$</th>
<th>$q_e$</th>
<th>SLACK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.18</td>
<td>0</td>
<td>1.44</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.64</td>
<td>0</td>
<td>2.15</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.14</td>
<td>0.50</td>
<td>0.99</td>
<td>0.73</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1.28</td>
<td>0.77</td>
<td>6.02</td>
</tr>
</tbody>
</table>

### Iteration 8: $d_{st}$, $d_{st} = 0$

<table>
<thead>
<tr>
<th>Class 'i'</th>
<th>$d_e$</th>
<th>$d_u$</th>
<th>$q_e$</th>
<th>SLACK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>2.52</td>
<td>1.18</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.37</td>
<td>0</td>
<td>1.01</td>
<td>4.28</td>
</tr>
<tr>
<td>3</td>
<td>1.53</td>
<td>0</td>
<td>0.99</td>
<td>0.02</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1.07</td>
<td>0.88</td>
<td>4.78</td>
</tr>
</tbody>
</table>

### Iteration 9: $g_4 = 0$

<table>
<thead>
<tr>
<th>Class 'i'</th>
<th>$d_e$</th>
<th>$d_u$</th>
<th>$q_e$</th>
<th>SLACK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.88</td>
<td>1.32</td>
<td>1.42</td>
<td>0</td>
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<td>2</td>
<td>1.46</td>
<td>0</td>
<td>1.04</td>
<td>4.86</td>
</tr>
<tr>
<td>3</td>
<td>1.64</td>
<td>0</td>
<td>0.98</td>
<td>0.72</td>
</tr>
<tr>
<td>4</td>
<td>0.02</td>
<td>0.57</td>
<td>1.90</td>
<td>0</td>
</tr>
</tbody>
</table>

All nodes have been fathomed and the optimal solution is found in Iteration 5 and $F_{opt} = -13.16$

Table 8.4: Results of the VBP Model.
algorithm corresponding to the scenario with \( d_{ct} = 0 \) (marked as (3) in Figure 8.1). This corresponds to the customer class with the highest value of \( |d_{ct} \sigma_1| \), with \( \eta \) less than \( L_1 \). The optimal value for this problem in this case equals -12.93 but the complementary slackness conditions are not satisfied for classes 2 and 3. Notice that this point does satisfy the CS condition for class 1. In Iteration 4, the problem is solved by setting \( g_1 = 0 \) and \( d_{ct} = 0 \) together with setting \( d_{c2} = 0 \) (the customer class with the next highest value of the complementarity condition and is marked as (4) in Figure 8.1). This value equals -13.15. But this value still does not satisfy the CS conditions for class 3. At Iteration 5, this problem is further updated by setting \( g_3 = 0 \) rather than \( d_{c3} = 0 \), as \( L_2 < \eta \) (marked as (5) in Figure 8.1). This yields a feasible solution for the entire problem and equals -13.16. Since this solution is better than that for the status quo model of Iteration 2, \( F_{opt} \) is updated.

At Iteration 6, the other fork of the branch at Iteration 4 (marked as (6) in Figure 8.1) with \( g_1, d_{c2}, d_{c3}, d_{ct} = 0 \) is solved. Since this solution (-13.46) is worse than \( F_{opt} \) (-13.16), this point is fathomed. Iteration 7 looks at a backtrackable node with \( g_1, d_{ct}, \) and \( g_2 = 0 \) (7 in Figure 8.1). This leads to a solution which is infeasible for the entire problem (feasible for the relaxed problem) and equals -13.87. Since this solution is worse than \( F_{opt} \), this node is fathomed. Iteration 8 further backtracks and solves a problem with \( d_{c1}, d_{ct} = 0 \) (8 in Figure 8.1). The optimal solution at this node (-13.39) is worse than \( F_{opt} \) (-13.16) and so the node is fathomed.
This leaves one backtracking node with $g_x=0$ (9 in Figure 8.1). This solution yields an optimal value of -13.87 which is worse than $F_{opt}$ (-13.16). Thus this node is fathomed and all possible nodes are either solved or fathomed. Thus, out of a total of 31 possible nodes, the optimal value is verified in iteration 9. This value was obtained at "Iteration 5" and equals -13.16. The results are discussed in subsection 8.2.4.

8.2.3 Solution of the LCP2 Model

This model uses the same approach used for solving the VBP model in section 7.3. The branch and bound method was employed for the model and the results were obtained when just one reduced model is solved.

InIteration 1, we solve the reduced problem with no complementary slackness conditions. Table 8.5 gives the results for the LCP2 model and the results are given under the heading "Optimal Policy." We notice the complementary slackness conditions are satisfied for all end-uses and utility investment in conservation measures are justified for all end-uses. Since $F_{opt}=F_{min}$, the algorithm stops in just one iteration and all complementary slackness conditions are satisfied automatically. Thus, we find that this objective recommends more conservation measures than the other two objectives. If the divergence between price and marginal costs were higher,
<table>
<thead>
<tr>
<th>Results</th>
<th>LCP2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Base Case</td>
<td>Optimal Policy</td>
</tr>
<tr>
<td>Societal Costs</td>
<td>13.35</td>
<td>11.47</td>
</tr>
<tr>
<td>d_c1</td>
<td>1.92</td>
<td>1.25</td>
</tr>
<tr>
<td>d_c2</td>
<td>0.57</td>
<td>0</td>
</tr>
<tr>
<td>d_c3</td>
<td>1.21</td>
<td>0</td>
</tr>
<tr>
<td>d_c4</td>
<td>0.53</td>
<td>0</td>
</tr>
<tr>
<td>d_a1</td>
<td>0</td>
<td>1.46</td>
</tr>
<tr>
<td>d_a2</td>
<td>0</td>
<td>1.05</td>
</tr>
<tr>
<td>d_a3</td>
<td>0</td>
<td>1.61</td>
</tr>
<tr>
<td>d_a4</td>
<td>0</td>
<td>1.04</td>
</tr>
<tr>
<td>p_e</td>
<td>1.12</td>
<td>2.52</td>
</tr>
<tr>
<td>q_e1</td>
<td>1.71</td>
<td>1.07</td>
</tr>
<tr>
<td>q_e2</td>
<td>2.59</td>
<td>1.21</td>
</tr>
<tr>
<td>q_e3</td>
<td>1.50</td>
<td>0.90</td>
</tr>
<tr>
<td>q_e4</td>
<td>2.32</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Table 8.5: Results of the LCP2 Model
then utility investment in conservation measures may not be recommended in all end-
uses as in this case. This is verified by the example in Section 8.2.5 where price is
much greater than marginal cost. Conservation expenditures are recommended for
only one of the sectors in that example for this objective.

8.2.4 Discussion of Results

Table 8.6 combines the results of all the models solved in Subsections 8.2.1-
8.2.3. The second column gives the solution for the case where there is no investment
in conservation programs for all end-uses. Columns 3, 4, and 5 tabulate the optimal
solutions of the LCP1, VBP, and LCP2 objectives respectively as compared to the
base case. We find that all three objectives lead to greater net benefits, lower revenue
requirements (except LCP2), as well as lower societal costs as compared to the base
case solution. So this bilevel model recommends some DSM investment for all three
objectives. Let us now discuss the solutions of the bilevel models.

The customer investment in conservation is maximum in the base case. As
utility investment in conservation increases, the customer investment in conservation
decreases in the end-use sector where the utility spends on conservation under all
three objectives. This result suggests a significant number of free riders. Free riders
are those consumers who would have invested in the program even without utility
## RESULTS

<table>
<thead>
<tr>
<th></th>
<th>BASE CASE</th>
<th>LCP1</th>
<th>VBP</th>
<th>LCP2</th>
</tr>
</thead>
<tbody>
<tr>
<td>REV. REQUIREMENTS</td>
<td>9.12</td>
<td>7.75</td>
<td>7.82</td>
<td>10.22</td>
</tr>
<tr>
<td>SOCIETAL COSTS</td>
<td>13.35</td>
<td>11.71</td>
<td>11.59</td>
<td>11.47</td>
</tr>
<tr>
<td>$d_{c1}$</td>
<td>1.92</td>
<td>2.46</td>
<td>2.34</td>
<td>1.25</td>
</tr>
<tr>
<td>$d_{c2}$</td>
<td>0.57</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$d_{c3}$</td>
<td>1.21</td>
<td>1.49</td>
<td>1.43</td>
<td>0</td>
</tr>
<tr>
<td>$d_{c4}$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$d_{e1}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.46</td>
</tr>
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<td>0</td>
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<td>1.31</td>
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<td>0</td>
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<td>1.61</td>
</tr>
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<td>1.41</td>
<td>1.16</td>
<td>1.04</td>
</tr>
<tr>
<td>$P_e$</td>
<td>1.12</td>
<td>2.02</td>
<td>1.80</td>
<td>2.52</td>
</tr>
<tr>
<td>$q_{e1}$</td>
<td>1.71</td>
<td>1.22</td>
<td>1.30</td>
<td>1.07</td>
</tr>
<tr>
<td>$q_{e2}$</td>
<td>2.59</td>
<td>0.93</td>
<td>1.10</td>
<td>1.21</td>
</tr>
<tr>
<td>$q_{e3}$</td>
<td>1.50</td>
<td>1.03</td>
<td>1.11</td>
<td>0.90</td>
</tr>
<tr>
<td>$q_{e4}$</td>
<td>2.32</td>
<td>0.65</td>
<td>0.83</td>
<td>0.88</td>
</tr>
<tr>
<td>TOTAL ENERGY USE</td>
<td>8.12</td>
<td>3.83</td>
<td>4.34</td>
<td>4.06</td>
</tr>
</tbody>
</table>

Note: $L_1 = 1$, $L_2 = 6.2$, $L_3 = 2.1$, and $L_4 = 7.4$

Table 8.6: Comparison of Results of the LCP1, VBP, LCP2 Models
investment. Figures 8.2-8.4 show the extent of free riders in all four end-use sectors for the LCP1, VBP and LCP2 objectives. The figures illustrate the maximum number of free riders occur when LCP2 (minimization of societal costs) is the objective selected by the utility. For example, the free rider fraction for end-use sector 3 under LCP2 is nearly 75%. This means that most of the utility investment merely displaces investment by consumers. This leads to unnecessarily high rate increases. The free rider fractions are calculated by the following formula:

\[
\left(\frac{d_{e0} - d_e}{d_u}\right) \times 100\%
\]

where \(d_{e0}\) is the investment made by end-users in the base case run. We also notice that the free rider fractions are lowest for LCP1.

Figure 8.5 shows the total investment in conservation (customers + utility) for all end-uses under the three objectives. We notice that the total investment in conservation is highest under LCP1, although this is not true for individual sectors. The customer investment in sectors 1 and 3, where the utility does not invest in conservation measures, increases under the LCP1 and VBP objectives. This is entirely due to the increase in rates caused by utility investment in conservation in sectors 2 and 4. This causes the nonparticipating classes (1 and 3) to reduce their consumption of electricity by conserving more. Besides the equity question of "why nonparticipating uses should subsidize participating uses", such income transfers can
Figure 8.2
FREE RIDER FRACTIONS FOR BLP1

UNDER THE VBP OBJECTIVE

Figure 8.3
Figure 8.4

FREE RIDER FRACTIONS FOR BLP1
UNDER THE LCP2 OBJECTIVE

END-USE SECTOR

0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0

FRACTION OF FREE RIDERS
Figure 8.5
also worsen the net benefits to society. When price is greater than marginal cost, efficient customers \((L=1)\) already consume less electricity than what is the optimal quantity. When there is a further price increase caused by utility investments in conservation, they decrease their energy consumption even further. This creates a loss in benefits. This is the effect which we observe in the results of the MV/GTRC test as "benefits of rate changes"; we noticed from the examples in Section 3.6 of Chapter 3 that this effect often makes the difference between selecting or rejecting a program. Because VBP is sensitive to these loss of benefits, it does not invest as much in sectors 2 and 4 (and thus does not raise rates as much) as LCP1 or LCP2.

VBP rates are the lowest among all the objectives. This is because of lower investment in utility measures in sectors 2 and 4 than the other two objectives. So sectors 1 and 3 suffer a lower loss of benefits under VBP due to the rate effect. Thus, VBP appears to recommend the lowest investment in conservation measures by the utility among the three objectives, as long as price is greater than marginal cost, because it is sensitive to the loss of benefits caused by rates being too high. If rates are too high, less energy services are consumed and, in sectors 1 and 3, too much capital is used.

Utility investment in conservation is the maximum under the LCP2 objective (Figure 8.6). Utility investment is recommended even for the sector with \(L_1=1\). In
INVESTMENT IN CONSERVATION
BY THE ELECTRIC UTILITY

Figure 8.6
Figure 8.7
contrast, VBP and LCP1 recommend utility investments only for end-uses with higher levels of distortion in the capital market \((L_2=6.2\) and \(L_4=7.4\)). This causes the rates to increase much more for LCP2 than the other two objectives (Table 8.6). LCP2 leads to investment in conservation by the utility under low levels of distortions in the capital market. This implies a higher number of free riders among the end-users who, at low values of \(L\), would have invested in the program anyway (Figure 8.4).

This also implies that "social costs" are lowered under the LCP2 objective by increasing the rates so high that the demand for energy services are lowered more in more efficient sectors 1 and 3. This in turn lowers the electricity demand and capital. At high levels of distortion \((L>6)\), where utility programs are also recommended under VBP and LCP2, less utility investment in conservation is recommended under LCP2. This is because the higher rates due to utility investment in the other sectors as well, mean that lower utility investment is needed to lower societal costs in the more inefficient sectors. Thus, if minimization of societal costs is the objective, there are more transfer payments from more efficient to less efficient end-uses. Under the California Standard Practice, the TRC screening test (discussed in Chapters 2, 3, and 4) follows the same objective as LCP2. But the latter assumes that there is no change in energy services consumed by the end-user due to the program. The LCP2 model, in contrast, takes advantage of the price elasticity of energy services by raising rates so high that the consumer needs fewer services, and thus less capital and electricity.
Figure 8.7 compares the solutions obtained under each of the models (LCP1, VBP, and LCP2) according to the revenue requirements, societal costs, and net benefits.

Let us now summarize this discussion as answers to the questions that were presented in the introduction to this section:

1. Does the objective followed by the electric utility make a difference in DSM program selection?

   Yes, it definitely does. As discussed above, each of the objective leads to different levels of investments in conservation by the utility and its end-users. LCP2 leads to excessive investment in conservation because driving up rates is good under that objective. VBP leads to the least investment in conservation because of the loss of benefits caused by increasing rates (due to utility investment).

2. What end-uses in this customer class require utility subsidies under the three objectives?

   All end-uses require utility investment under the LCP2 objective, whereas, utility investment is recommended for sectors 2 and 4 under the VBP and LCP1 objectives. Utility investment is necessary in the
latter enduses under all objectives because, at higher levels of distortion, there is inefficient usage of energy that require utility capital expenditures to bolster or replace low customer investment in conservation. Sectors 2 and 4 are the end-uses that face the highest levels of distortion in the capital market.

3. What level of distortions in the end-use capital market justify utility programs?

Given a divergence between price and marginal cost, the level of distortion in the capital market is the single most important determinant of whether utility investment is required or not. The higher the distortion in the capital market, the more attractive are utility investments. This answer assumes that price is greater than marginal costs. The greater the divergence between price and marginal cost (due to a higher value of the fixed cost A or a lower value of the marginal cost of power generation B), less utility investment would be recommended. This is because of the fact that distortions in end-users’ decisions due to incorrect rate signals becomes an important factor too when price is much greater than marginal cost. This is illustrated in Sub-section 8.2.5 for the example where price is much greater than marginal cost.
4. What happens to electric rates if the utility invests in conservation measures?

Electric rates increase for all three objectives when the utility invests in conservation measures. For example, the base case rates are $34/MWh (30*1.12) whereas the electric rates under the LCP2 objective is about $75/MWh (30*2.52). This is because higher costs are sought to be recovered from lower electricity sales. The rates are highest for the LCP2 objective since it results in the maximum investment by the utility in conservation. These rates for LCP2 are more than double the base case prices. This is because discouraging consumption of energy services results in lower costs.

5. What is the effect of utility conservation investments on energy consumption in end-use sectors in which utility investment is not made?

We note that the customer investment in conservation increases in these sectors under VBP and LCP2 objectives. This is because the increase in rates due to utility investment in conservation leads to lower energy consumption in the nonparticipant sectors. There is a net loss in benefits since it lowers consumption of energy which is under-consumed when price exceeds marginal cost, assuming efficient usage
of electricity (L=1).

6. What is the extent of free ridership among end-users when the utility invests in DSM programs?

The free rider effect is most significant under the LCP2 objective and is lowest under the LCP1 objective. Refer to Figures 8.2-8.4 for a graphical representation. In sector 3, under the LCP2 objective, we find nearly 75% of the end-users are free riders. Free riders are bad under the VBP objective because the investment by the utility to free riders increases rates and lowers the benefits to nonparticipating uses. Free riders are bad under LCP1 too because it increases the utility revenue requirements without lowering additional consumption of electricity. But LCP2 objective find free riders useful because they increase rates which lowers the end-users’ energy service demands while leading to lower capital and energy requirements.

8.2.5 Example for the P>>MC Case

Table 8.7 gives the parameters for the four end-use model with the same price for all end-uses. In this example, the fixed costs ($500 million) are much higher than the example discussed earlier in this section ($75 million). As a result, the base case
case price is $80/MWh. The rest of the parameters are the same as the previous example except that the price elasticities of energy services are lower in sectors 1 and 2.

<table>
<thead>
<tr>
<th>Customer class ‘i’</th>
<th>Elasticity of energy services $\varepsilon_i$</th>
<th>Production function exponent $\alpha_i$</th>
<th>Production function constant $\beta_i$</th>
<th>Demand function Constant $C_i$</th>
<th>Distortions in capital market $L_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.12</td>
<td>0.5</td>
<td>1.0</td>
<td>1900</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>0.12</td>
<td>0.55</td>
<td>1.1</td>
<td>2314</td>
<td>6.2</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>0.6</td>
<td>1.2</td>
<td>3200</td>
<td>2.1</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.6</td>
<td>1.2</td>
<td>2500</td>
<td>7.4</td>
</tr>
</tbody>
</table>

Due to the high fixed costs, price is much higher than marginal cost in this example. Table 8.8 compiles the results of the solution of the LCP1, VBP and LCP2 models. VBP and LCP2 models were solved using the branch and bound method of Section 7.3 and the LCP1 model was solved using the heuristic method of Section 7.4.
The VBP solution was obtained in seven steps while the LCP2 solution was obtained in six steps of the branch and bound method. We discovered the base case solution was the optimal solution for LCP1 after solving four problems. The results in Table 8.8 are not normalized as those in Table 8.6 and are discussed below.

We note that utility programs are justified only for the sector with the highest distortion in the capital market (sector 4, $L=7.4$) under the VBP and LCP2 objective. LCP1 does not justify any DSM program for all four sectors. This is because the price is so high that much conservation is taking place anyway. Thus the base case solution remains the best solution under this objective. The total consumption of electricity remains at 10 TWh and prices at $80/MWh.

VBP increases the revenue requirements by about $7 million and improves the net benefits by about $11 million. Under this objective, utility investment in conservation in sector 4 leads to higher rates ($80/MWh to $105/MWh) and reduced energy consumption (10 TWh to 7.67 TWh). The increase in rates leads to reduced consumption by all other sectors lowering their benefits. But the increase in benefits for sector 4, especially rebound, outweigh the loss in benefits in all other sectors.

LCP2 too justifies utility programs in sector 4. The results of the VBP and LCP2 runs are almost similar, though revenue requirements are increased by over $9.2
## RESULTS

<table>
<thead>
<tr>
<th></th>
<th>BASE CASE</th>
<th>LCP1</th>
<th>VBP</th>
<th>LCP2</th>
</tr>
</thead>
<tbody>
<tr>
<td>REV. REQUIREMENTS</td>
<td>800.0</td>
<td>800.0</td>
<td>806.6</td>
<td>809.3</td>
</tr>
<tr>
<td>NET BENEFITS</td>
<td>-854.2</td>
<td>-854.2</td>
<td>-843.1</td>
<td>-843.5</td>
</tr>
<tr>
<td>SOCIETAL COSTS</td>
<td>1125.4</td>
<td>1125.4</td>
<td>1111.5</td>
<td>1111.2</td>
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<tr>
<td>(d_{c1})</td>
<td>104.0</td>
<td>104.0</td>
<td>117.3</td>
<td>116.1</td>
</tr>
<tr>
<td>(d_{c2})</td>
<td>55.2</td>
<td>55.2</td>
<td>61.5</td>
<td>60.9</td>
</tr>
<tr>
<td>(d_{c3})</td>
<td>114.3</td>
<td>114.3</td>
<td>126.1</td>
<td>125</td>
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<tr>
<td>(d_{c4})</td>
<td>51.9</td>
<td>51.9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(d_{r1})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(d_{r2})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(d_{r3})</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(d_{r4})</td>
<td>0</td>
<td>0</td>
<td>76.6</td>
<td>72.8</td>
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<tr>
<td>(p_e)</td>
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<td>80.0</td>
<td>105.2</td>
<td>102.7</td>
</tr>
<tr>
<td>(q_{e1})</td>
<td>1.3</td>
<td>1.3</td>
<td>1.12</td>
<td>1.13</td>
</tr>
<tr>
<td>(q_{e2})</td>
<td>3.5</td>
<td>3.5</td>
<td>2.97</td>
<td>3.01</td>
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<tr>
<td>(q_{e3})</td>
<td>2</td>
<td>2</td>
<td>1.68</td>
<td>1.71</td>
</tr>
<tr>
<td>(q_{e4})</td>
<td>3.2</td>
<td>3.2</td>
<td>1.91</td>
<td>2.04</td>
</tr>
<tr>
<td>TOTAL ENERGY USE</td>
<td>10</td>
<td>10</td>
<td>7.67</td>
<td>7.88</td>
</tr>
</tbody>
</table>

Note: All prices are in $/MWh, electricity consumption in TWh (10^6) MWh, and all other variables are in 10^6 $.

Table 8.8: Comparison of Results of the LCP1, VBP, LCP2 Models
million in the latter case over the base case solution. The utility investment in this case is lower than the utility investment under the VBP solution. This is inconsistent with the results of the example presented in Subsection 8.2.4 where price is closer to marginal cost. This is primarily due to the rebound effect which is ignored by the LCP models but is included in the VBP benefits. In this case, utility subsidies in sector 4 lower the effective price of energy services in that sector and increase the consumption of energy services by decreasing the energy savings more than predicted. This increases the societal costs and therefore LCP2 recommends lesser investment in conservation than VBP. On the other hand, we find that a higher investment in sector 4 increases the net benefits due to the rebound effect. This effect is more important in this example, as compared to Table 8.6, because utility investment in conservation leads to a more dramatic lowering of the price of energy services for the inefficient sector.

Therefore, from the example presented in this subsection, we find that much less investment in conservation is required for the case where price is greater than marginal cost and VBP requires more investment in conservation than any other objective. In contrast, we discovered that LCP2 recommends more DSM programs for the case where price is closer to marginal cost. In conclusion, the effects of the various objectives depend strongly on the relationship between the price and marginal cost.
8.3 Example for the BLP2 Model

The example for the BLP2 model is the same as the example problem for BLP1 where price is much greater than marginal cost (Table 8.7). But, in this case, the utility offers a subsidy to the customer, rather than direct investment, to promote energy efficiency. Table 8.9 shows the BLP2 model to be solved. As explained earlier in Section 6.4, only one model needs to be solved to obtain the optimal solution for the models. The LCP1 model, which is nonconvex (as shown in Section 6.4), was solved for several starting points and all of them led to the LCP1 solution provided in Table 8.10. The results of the VBP and LCP2 models are also tabulated in Table 8.10.

An important fact to be noted about this formulation is that though the utility spends only $\hat{I}_K$ dollars on any sector, the customer responds to it as if $L_1$ times that amount has been spent on conservation. Thus more subsidies can be expected at higher level of distortions to bridge the gap between perceived cost and the marginal benefit of conservation investment.
Table 8.9
4 End-Use Sector Example for the BLP2 Model

<table>
<thead>
<tr>
<th>Constants:</th>
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</thead>
<tbody>
<tr>
<td>( c_1 = -0.8571 )</td>
<td></td>
</tr>
<tr>
<td>( c_2 = -4.8209 )</td>
<td></td>
</tr>
<tr>
<td>( c_3 = -11.0836 )</td>
<td></td>
</tr>
<tr>
<td>( c_4 = -1.3552 )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Objective(s):</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LCP1 ( \text{min} )</td>
<td>( 5 + 0.3*(q_{e1} + q_{e2} + q_{e3} + q_{e4}) + I_1K_1 + I_2K_2 + I_3K_3 + I_4K_4; )</td>
</tr>
<tr>
<td>OR</td>
<td></td>
</tr>
<tr>
<td>VBP ( \text{max} )</td>
<td>( \frac{c_1*(K_1)^{3.67}q_{e1}^{3.67} - q_{e1} - K_1}{c_2*(K_2)^{4.03}q_{e2}^{3.3} - q_{e2} - K_2} )</td>
</tr>
<tr>
<td>OR</td>
<td></td>
</tr>
<tr>
<td>LCP2 ( \text{min} )</td>
<td>( 5 + 0.3*(q_{e1} + q_{e2} + q_{e3} + q_{e4}) + K_1 + K_2 + K_3 + K_4; )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraints:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_eq_{e1} + p_eq_{e2} + p_eq_{e3} + p_eq_{e4} - I_1K_1 - I_2K_2 - I_3K_3 - I_4K_4 - 0.3*(q_{e1} + q_{e2} + q_{e3} + q_{e4}) = 5; )</td>
<td></td>
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<tr>
<td>( 3.1425*(K_1)^{3.67}q_{e1}^{4.67} = p_e; )</td>
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<td>( 15.9092*(K_2)^{4.03}q_{e2}^{4.3} = p_e; )</td>
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</tr>
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<td>( 39.9011*(K_3)^{5.4}q_{e3}^{4.6} = p_e; )</td>
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<tr>
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<td>( 19.4445*(K_2)^{5.03}q_{e2}^{3.3} + 6.2i_2 = 6.2; )</td>
<td></td>
</tr>
<tr>
<td>( 59.8516*(K_3)^{6.4}q_{e3}^{3.6} + 2.1i_3 = 2.1; )</td>
<td></td>
</tr>
<tr>
<td>( 7.3183*(K_4)^{6.4}q_{e4}^{3.6} + 7.4i_4 = 7.4; )</td>
<td></td>
</tr>
<tr>
<td>( I_1, I_2, I_3, I_4 \geq 0; )</td>
<td></td>
</tr>
<tr>
<td>RESULTS</td>
<td>BASE CASE</td>
</tr>
<tr>
<td>---------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>REV. REQUIREMENTS</td>
<td>800.0</td>
</tr>
<tr>
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</tr>
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</tr>
<tr>
<td>$K_1$</td>
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<td>$K_2$</td>
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<tr>
<td>$I_2$</td>
<td>0</td>
</tr>
<tr>
<td>$I_3$</td>
<td>0</td>
</tr>
<tr>
<td>$I_4$</td>
<td>0</td>
</tr>
<tr>
<td>$p_e$</td>
<td>80.0</td>
</tr>
<tr>
<td>$q_{e1}$</td>
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<td>$q_{e2}$</td>
<td>3.5</td>
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<tr>
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<tr>
<td>$q_{e4}$</td>
<td>3.2</td>
</tr>
<tr>
<td>TOTAL ENERGY USE</td>
<td>10</td>
</tr>
</tbody>
</table>

Note: Prices are in $/MWh, electricity consumption in TWh (10^6 MWh), customer investments in conservation (K) are in 10^6 $, and I values are dimensionless.

Table 8.10: Results of the LCP1, VBP, LCP2 Models for the BLP2 Case
In this example, we find that utility subsidies are recommended for sectors 2 and 4 (with higher values of distortion) under all three objectives. We find the overall results are different for the BLP2 model even though the inputs are the same as for the BLP1 example in Subsection 8.2.5. The base case results are the same but the optimal policies vary. BLP2 leads to greater utility expenditure on DSM through subsidies and the overall results are in each case better than the BLP1 formulation. The reason for this occurrence is the fact that, under the BLP2 formulation, the customer perceives the utility subsidies to be worth L times the actual DSM expenses of the utility. This, in effect, lowers the distortion in capital for the end-use sector. For example, under the LCP1 objective, utility subsidies for sector 4 halves the distortions in the capital market to 3.7 ($L_4(1-L_4)$) and induces end-users to invest more capital in conservation. In contrast, the BLP1 formulation considers utility investment as additional capital in the production of energy services and not L times that amount.

LCP1 recommends the most investment in conservation subsidies under the BLP2 formulation. The high utility subsidies to sectors 2 and 4 raises the rates for all ratepayers. This forces nonparticipants to conserve more, which in turn leads to lower revenue requirements. Thus the additional investment in conservation subsidies is more than made up for by the lowering of generation costs in nonparticipating sectors. But this causes a lowering of benefits for nonparticipating end-users and causes an
excess of capital investment in these sectors. This explains the fact that LCP1 leads to lower net benefits and higher societal costs than the other two objectives.

The VBP and LCP2 results are similar as can be observed from Table 8.10. VBP leads to more subsidies than LCP2 because of the benefits due to rebound in sectors 2 and 4. This is similar to the trend that was observed for the BLP1 model in Subsection 8.2.5. Thus, we find that the rebound benefits in participating sectors outweigh the loss in benefits due to rate changes in the nonparticipating sectors. In contrast, from the perspective of societal cost, lower utility subsidies lower the rate effect of rate changes and less inefficient use of capital in sectors 1 and 3.

8.4 Closing Remarks

This chapter presented some examples for the bilevel models presented in Chapter 6. The branch and bound algorithm provided in Section 7.3 was tested for two problems in Section 8.2. We found that about half the possible combinations of reduced problems can be eliminated using this algorithm for the example problem. We could expect a lot more fathoming for a larger sized problem. But, as observed by Edmunds and Bard (1990), there are fewer nodes fathomed for the nonlinear bilevel model as compared to the integer linear bilevel problems. The heuristic presented in Section 7.4 was also tested for these two examples and led to defensible
results. Section 8.3 presented an example for the BLP2 formulation and a sample of the type of issues that can be analyzed using this bilevel model.

The examples presented in this chapter are not meant to provide substantive conclusions. The main objective of this chapter was to illustrate the solution procedures recommended in Chapter 7 to solve BLP1 models. There is a need for more sophisticated representations of productions costs and demands before such conclusions can be made. A flavor of the type of issues that can be analyzed was presented in Subsection 8.2.4 and a more detailed analysis can be found in Nelson (1989). Further analyses are also required in order to fully explain the differences found between the results of Sections 8.2.4, 8.2.5 and 8.3.
Chapter 9

CONCLUSIONS AND DIRECTIONS FOR FURTHER RESEARCH

9.1 Introduction

This thesis considers one of the most important resources available to electric utilities called demand-side management (DSM). DSM programs are those utility measures taken to alter the load shape of customers. These include conservation, load management, load shifting, load building, and peak clipping. Increasing capital costs and environmental concerns have persuaded electric utilities to consider options such as energy conservation and renewable energy sources to meet their demand growth. Moreover, regulators, in recent years, have started to offer incentives to utility shareholders to invest in DSM programs. Thus, DSM programs figure to be an integral component of electric utility plans to meet growing demand.

The purpose of this thesis is to analyze various issues that complicate evaluations of electric utility DSM programs. A nonlinear bilevel model is formulated and solved in this thesis that considers explicitly the interactions between the electric
utility and its multiple customer classes. This model is the first nonquadratic nonlinear bilevel model solved in literature and helps address several planning issues that are not explicitly considered by present large-scale utility planning models. This explicitness permits a forum for analysis of many questions addressed earlier in this thesis in Chapter 8. The conclusions of such analyses contradict those of previous studies whose logical flaws are hidden behind qualitative arguments. These assumptions and issues complicating DSM programs need to be analyzed so that utility planners can develop a better understanding about the DSM parameters that they feed as inputs to their planning models and resource screening models.

Current screening criteria have several shortcomings and there is a need for a more comprehensive and theoretically correct criterion. Such a criterion, called the most value/general total resource cost (MV/GTRC) test, is derived in this thesis for the first time. Several versions of the MV/GTRC test are presented in this thesis to correct various factors that are ignored by other screening criteria are discussed in section 9.2.

An underlying theme of this thesis has been what is the proper objective for an investor owned utility should be? As a regulated entity, utilities are generally assumed to be maximizing societal benefits while minimizing their revenue requirements but the results of this thesis consistently show that this is not the case
when there is utility investment in DSM programs. This thesis contains several examples to illustrate the effect of each objective on the optimal selection of resources that are paid for by utility ratepayers. The public policy debate finally boils down to what the mandate of the utility regulator is. Is it the maximization of societal benefits (VBP), the minimization of utility revenue requirements (LCP1), minimization of societal costs (LCP2) or the minimization of rates? This thesis advocates the usage of the objective "maximization of value" as a criterion for selecting demand-side resources.

Section 9.2 provides a discussion of the screening tests derived in this thesis and to show the advantage of this criterion over all others. Section 9.3 summarizes the features of the bilevel model presented in this thesis and the solution technique to solve the ensuing nonconvex model. Finally, Section 9.4 presents a brief description of what remains to be done and general areas of applicability of the models formulated in this thesis.

9.2 The MV/GTRC Test

Most electric utilities in the United States adopt, and regulatory commissions recommend, a procedure called integrated resource planning (IRP) in devising an optimal resource plans. IRP can be defined as a process which considers supply-side
and demand-side alternatives simultaneously and attempts to arrive at an optimal mix of these alternatives. The initial step in this process is the screening of these resource options.

Electric utilities have had very little experience in screening DSM options as compared to evaluation of supply-side options. There are many screening tools devised for evaluating generation options over the past half century together with many capacity expansion models to evaluate the "least cost" plan of the most viable options (e.g., Bloom, 1982). A lot of information has been made available during this period on the various issues that affect supply-side planning, though environmental issues like acid rain, global warming etc. have forced utility planners to re-evaluate their screening processes.

There can be several hundred DSM options that can be considered by an electric utility and it would be impossible to evaluate all of them simultaneously in an IRP framework. So screening of DSM programs is an integral aspect on utility planning. The total resource cost (TRC) test of the California Standard Practice (California Public Utilities Commission and California Energy Commission, 1983 and 1987) is the most widely accepted criterion for evaluating DSM options. But this test, also called as the LC/TRC test in this thesis, has several shortcomings as discussed in Chapter 2 and it is possible that valuable resources are being spent on nonoptimal
resources selected under this test. Electric utilities expend two billion dollars annually on DSM resource alone and there is a need for a theoretically correct and applicable criterion to evaluate these options.

The LC/TRC test could lead to erroneous program selection under any of the following three conditions:

- DSM programs have a non-zero price elasticity of demand for energy services, i.e., they alter the amount of energy services they consume in response to changes in the effective price in providing these services.

- Program non-participants have a non-zero price elasticity of demand for energy services and the price they pay for energy does not equal the marginal cost of electricity supply.

- The program is a load building program.

In each of these cases above, DSM affects the value received by utility customers. The above conditions necessitate the derivation of a more comprehensive economic efficiency test which accounts for the benefits of the changes in amount and value changes provided by electricity in addition to the various cost categories considered by the LC/TRC test. A detailed discussion of the value changes due to DSM programs can be found in Section 2.4 of Chapter 2.

The derivation of the criterion is presented in Section 3.2 of chapter 3. The simplest form of the criterion presented in Chapter 3 is formulated as follows:
max:

Benefits of consumption of energy services by participants
+ Benefits of consumption of electricity by nonparticipants
- Costs of power generation
- DSM Program Costs
- External Environmental Costs.

subject to:

Revenue recovery constraint.

The benefits of consumption of energy services and electricity can be defined as the area under the demand curve for these quantities and is equal to what economists refer to as "consumer surplus." Taking the total derivative of the objective (defined as the value function), yields the change in value due to the implementation of the program if the program is small. A critical term in the total derivative that needs to be estimated. This factor is the derivative of the price with respect to the implementation of the program (i.e. the change in rates caused by the implementation of the program). This derivative is obtained from the total derivative of the revenue recovery constraint with respect to the implementation of the program. After considerable manipulation, we derive the MV/GTRC test in section 3.3. This criterion
is a multiple customer class, multiple period, multiyear version of the MV/GTRC test.

This derivation calculates the effects of rebound, the benefits of rate changes, and value changes that occur due to load building program, all of which are ignored by the LC/TRC tests. MOSTVALUE, a spreadsheet model developed by Nelson and Hobbs (1990a) incorporates both the MV/GTRC as well as the LC/TRC tests. This model was used by a midwestern utility to screen potential DSM programs in their integrated resource plan. Four different types of programs are evaluated in Section 3.6 to illustrate the differences between the MV/GTRC and the LC/TRC tests.

In this derivative, we use the total derivative of the value function to represent the value or net benefits due to the DSM program. A more accurate calculation of the net benefits would involve calculating the finite difference in the value function with and without the program. Costello and Galen (1984) use such an approach in their report. This leads to a complicated expression for the changes in value that is not easy to interpret in a concise form. We use the total derivative approach for the following reasons: (1) the programs screened by this model are not so large that the curvature of the value function changes between the cases with and without the program, and (2) there is a need for a criterion that is as easy to interpret as the existing while offering significant enhancements.
The ratepayer impact measure (RIM) of the California Standard Practice estimates the effect on all ratepayers due to the adoption of the DSM program. It equals the revenue gain due to the program minus the costs of the DSM program. But this test, which is presented as an equity test in the Standard Practice, also ignores the change in consumption by nonparticipants due to rate changes. Thus the general ratepayer impact measure (GRIM) is derived in Section 3.4 and is presented as a measure of equity in this thesis. GRIM equals RIM if we assume that the benefits due to rate changes are zero. This is not true if nonparticipants have a non-zero price elasticity of demand for electricity or if price does not equal the marginal cost of supply.

In the version of the MV/GTRC test presented in Chapter 3, it is assumed that the implementation of a DSM program does not affect energy consumption in any other year except the year in which the DSM program expenses occur. This is most often not true. There is a lag in demand response to a price change in any particular year. The change in consumption due to a price change in earlier years is very much dependent on the cross-price elasticity of demand for electricity. Cross price elasticity can be defined as the ratio of the change in energy consumption in any particular year to a change in price in some other year. If these values are zero, then the MV/GTRC tests with and without time lags are the same. In this thesis, we derive the MV/GTRC test for the case with time lags in section 4.2 and an example is provided.
to illustrate this phenomenon. This thesis is the first attempt at incorporating time lag
effects using a geometric lag structure in DSM screening criteria. It is important
because, in the long run, most of the change in energy use is due to changes in energy
using equipment rather than shifts in the amount of energy services consumed.
Knowing the short term and the long term price elasticity of demand, and assuming
a geometric lag structure, leads to easy determination of the effects of time lag.

Section 4.3 derived the version of the MV/GTRC test for programs which
involves fuel switching. The basic form of the MV/GTRC test presented in Chapter
3 assumed that there were no distortions in the alternate fuel market. But there are
distortions due to regulation in the natural gas market too which are ignored by the
basic form. The derivation procedure of the MV/GTRC criterion for fuel switching
programs is analogous to the derivation in Chapter 3 for the basic form of the
MV/GTRC test. This derivation, for the first time in literature, estimates the value
of rebound and rate benefits due to fuel switching. A gas chiller program is also
evaluated in section 4.3 of that chapter to illustrate the differences between the
MV/GTRC and the LC/TRC tests.

The MV/GTRC and the LC/TRC tests are economic efficiency tests and do not
consider any other attribute of electric power except economic efficiency. As
presented in Section 4.4 of Chapter 4, there are several other criteria (shareholder
benefits, environmental benefits, effects on low-income groups, etc.) that may be important in selection of a DSM option. We also present an additive utility multiattribute model developed by Nelson and Hobbs (1991) to consider the multiple attributes of electric power.

9.3 Bilevel Models

The MV/GTRC test is a static test in that it screens DSM programs given a certain scenario of prices, marginal costs, program savings, free riders, customer costs, and utility program costs. It accepts the above data assumptions as given in evaluating DSM programs. Such information is generally obtained from empirical studies. What we lack in the utility industry is the framework for interpreting the results of these empirical studies. There is a definite need to model customer-utility interactions to decipher the results of surveys and other data collection efforts. For instance, if the objective is to maximize net societal benefits, at what level of distortion in the customer's capital market should the utility invest in conservation measures? What if the objective is to minimize revenue requirements? What level of free riders or rebound can be expected at what levels of distortions? The answers to such "what if" questions can help electric utilities decide what objective is important, what parameters are essential, and what data is needed for program evaluation. Such questions are addressed by the bilevel model presented in this thesis.
The bilevel model has the electric utility at the upper level and the various customer classes at the lower level. The electric utility at the upper level seeks to optimize its objective, be it least cost planning (LCP) or value-based planning (VBP), by fixing the rates and its investment in conservation either through direct investment or through subsidies. The customer classes, on the other hand, seek to maximize their net benefits (i.e., area under the demand curve for energy services minus power bills minus their perceived cost of capital) by their consumption of energy services by deciding on the quantity of electricity to be purchased and their investment in energy efficiency measures. The utility is constrained by the requirement that they recover all their costs through electric rates. A more detailed description of the features of this model can be found in Section 5.2 of Chapter 5.

Bilevel programming is a relatively new branch of mathematical programming in which the two levels of the model are fundamentally in conflict with each other. It differs from traditional decomposition in that the latter methods essentially solve a single objective over a fixed feasible region (Bialas and Karwan, 1983). Multiobjective programming, on the other hand, finds a compromise among the various goals of planners. But this method still does not account for possible independent actions by the players or the order in which these decisions are taken. The fundamental difference between bilevel programming and multiobjective programming is that the former prohibits cooperation between players.
The bilevel model in this model is essentially a non-cooperative Stackelberg model with the utility at the upper level as the leader and the customer classes at the lower level as followers. Under this type of formulation, the leader makes his decision first and the followers react by optimizing their objectives conditioned on the leader’s decisions. There is no direct interaction between the various followers except through the leader’s decision variables.

There are several techniques to solve bilevel problems. These techniques are discussed in detail in section 5.3 of Chapter 5. The basic technique employed in this thesis is based on replacing the Kuhn-Tucker (K-T) conditions obtained by solving the lower level model as constraints in the upper level model. But the complementary slackness conditions of the K-T equations make even the simplest bilevel problem nonconvex. This is one major factor that impedes the solution of a large-scale bilevel model and motivates this first attempt in the literature to solve a bilevel model which is nonlinear and nonquadratic.

We consider two types of bilevel problems in this thesis. BLP1 is a model where the utility investment in conservation in each customer class is a variable. The second model, BLP2, defines the utility’s subsidies to consumers who invest in capital expenditures on conservation as decision variables. BLP1 is more suitable for programs such as low income weatherization in which the utility invests directly in
insulation and window treatment. On the other hand, BLP2 is suitable for programs such as air-conditioning rebates in which the utility offers rebates to customers who buy energy-efficient air-conditioners.

The bilevel models, after incorporating the Kuhn-Tucker (KT) conditions, lead to single level non-convex problem. The complementary slackness (CS) equations of the KT conditions for the consumers are the main reason for the nonconvexity of the BLP1 problem. If the CS conditions are removed, the BLP1 problem reduces to a convex problem with convex equality and inequality constraints (Section 6.4). But we cannot prove that the models are convex because of the nonlinear equality constraints. We solve the reduced problem with an augmented Lagrangian (AL) method which solves convex minimization or concave maximization unconstrained problems at each iteration (Section 6.2). Thus, when the optimal point for the problem is found for a finite penalty parameter, we can conclude that the solution is a global optimum. There is an extensive literature available to show that we obtain the global solution for every reduced problem that is solved (Powell, 1972 and Rockafellar, 1973).

A branch and bound solution approach was presented to solve the VBP and LCP2 problems of the BLP1 formulation in section 7.3. The algorithm proceeds at each node by either fixing the customer investment in conservation equal to zero or
constraining the marginal benefits of customer investment in conservation equal to the distortion factor $L$. This procedure satisfies the complementary slackness conditions for that particular class. The optimal value is updated or a node is fathomed at each stage when a feasible problem is found. If the solution value at any node is worse than the current optimal value, the node is fathomed. Thus, several nodes can be fathomed in this method.

The customer class to be selected depends on the absolute value of the complementary slackness conditions for each class. If the $L$ value is less than a prefixed value, the customer investment is set to zero. The variables associated with the constraint is set to zero if instead the opposite is true. This approach was applied to an example problem with four customer classes in section 8.2.1. Out of a possible 31 subproblems to be solved, the optimal solution for the bilevel model was determined in 9 iterations.

The LCP objective requires a different approach to be used. Since the customer investment in conservation $d_c$ does not appear in the objective, if complementary slackness are not enforced for any node, the value $d_c$ for that particular class tends to infinity. Therefore, we need to solve the problem by remaining in the region of feasibility in all cases. A heuristic, used to solve this problem, is presented in section 7.4 which negates the need to solve all $2^m$ possible
combinations of the CS conditions. This heuristic is based on two assumptions: (1) if utility investment in any sector fails to lead to a better optimum alone, it would not be justified in combination with any other program, and (2) at high level of distortions in the capital market, utility investment will not be recommended together with customer investment in conservation. Our computational experience indicates this to be the case for all scenarios with price much greater than marginal cost. An optimal solution for the four customer class problem in Section 8.2 was estimated in six steps. This approach leads to a quick solution of the problem, if the number of customer classes that need utility subsidies are much lesser than the total number of classes. But this heuristic does not guarantee global optimality, unlike the VBP and LCP2 objectives.

Examples were provided in Chapter 8 to illustrate the solution techniques for a four end-use model where it was assumed that all end-uses were in the same customer class. Two examples were analyzed for the BLP1 problem. One is for a model with price slightly greater than marginal cost and one with prices much greater than marginal cost.

The BLP2 model, presented in Section 6.5, automatically ensures that the complementary slackness conditions were satisfied. This eliminates the need for a special method to solve the model. The LCP2 and the VBP models of the BLP2
formulation are both convex problems with convex equality and inequality constraints whose global optimum can be obtained using the AL method. But the LCP1 problem is nonconvex as proved in Section 6.5. An example for the BLP2 problem is presented in Section 8.3. A brief discussion in that section also showed the effect of the objective on various factors that complicate demand-side management planning.

A brief analysis of various issues that complicate the analysis of DSM program is provided in Subsections 8.2.4-8.2.5 and 8.3 and illustrates the need for the bilevel model. No definite conclusions can be inferred from the results of the example for a general problem. The objective of the examples was to illustrate the usefulness of the algorithms of Chapter 7 in solving multiple customer class bilevel models. But even this multiple class model cannot replace traditional utility models. Data difficulties preclude the development of a version of the model to help make decisions on both the supply-side and demand-side measures. Instead the purpose is to obtain insights on those factors are considered implicit in most planning models.

9.4 Directions for Further Research

This thesis addresses several basic questions about the implementation of DSM programs in electric utility planning processes. But there are a myriad of other issues that remain to be considered. We will discuss a few of these in the discussion below.
The main criticism of the type of analysis performed in this thesis is the assumption that consumers are rational economic actors operating in a market. The biggest danger of this type of reliance on economic concepts of customer behavior is that it directs attention selectively to some important determinants of customer behavior while ignoring other behavioral factors that are also critical for policy analysis. For example, by focusing attention on prices, the elasticity concept leaves obscure the behavioral phenomena that underlie the response to price and other nonprice factors that influence energy use. In the opinion of Stern (1986), "central theoretical concepts such as elasticity, discount rate, and lag provide convenient mathematical shorthand for describing important economic processes but direct attention away from behavioral variables that offer powerful levers for public policy."

There is certainly some truth to the fact that demand models ignore or superficially consider certain other important determinants of customer behavior like information acquisition and motivation. But economic models such as the bilevel model estimated under this patently false assumption of rationality do provide intuition on and bounds for benefits of utility conservation programs (see Quigley, 1986). For example, L which is related to the implicit discount rate, could be a proxy for lack of information which is not uniform for all customer classes. Thus, segmenting of customer classes according to their end-uses and analyzing the model results for different values of L could result in intuition about where and when the
utility should invest in conservation measures. The screening tests on the other hand assume that the input parameters are fixed and account only for price effects on participant and nonparticipant demands. Thus economic analysis models such as the bilevel model and the screening tests such as the MV/GTRC test go hand in hand in predicting customer responses to utility policies.

One drawback of the bilevel models is the fact that they do not incorporate dynamics. A multiple load block version of the bilevel model was solved in Nelson (1989) but that version is only for a single year. The formulation in this thesis ignore the effects of changing marginal costs and distortions in the capital market. It also distorts the fact that price and other nonprice effects affect energy usage not just in the year they occur but also in later years. One modification that can be made to the bilevel model is to incorporate multiple year horizon and a time lag process. The MV/GTRC test was modified in this thesis to account for time lags in energy usage changes. More complicated versions than the geometric lag assumed in this thesis can be tested in the criterion. But this thesis is the first attempt at incorporating time lags in energy usage and energy using equipment changes, in screening tests, though they are well recognized in the economics literature (Nerlove, 1972). End-use models of energy use can estimate the extent and pace with which manufacturers and households improve the energy efficiency of equipment in response to changes in fuel prices (e.g., Hirst and Carney, 1978).
Another important area of research is the calibration of behavioral parameters such as production function coefficients, demand function coefficients, production exponents, and elasticities of energy services. A lot of work is being done in this field in many quarters (for example, Wilson and Gamponia, 1990, and the proceedings of the Program Evaluation Conferences of 1985, 1987, and 1989) and much remains to be done. Increasing experience with DSM pre- and post-program evaluation would lead to better empirical data on these parameters.

Finally, this thesis is the first example in literature which solved a nonlinear bilevel problem. There are many other situations in the electric utility in which bilevel programming is amenable. Morse (1986) presents an example in the form of a Stackelberg game for an excess energy marketing problem in the Pacific Northwest where the Bonneville Power Administration is the leader and the California utilities are the followers. Primarily BPA wants to maximize its revenue by selling its excess energy to California utilities which seek to minimize their operating costs by purchasing energy from BPA.

An ideal bilevel problem in the electric utility industry is the relationship between the utility and the cogenerators in its service territory. Cogeneration is the process by which both thermal energy and electrical energy can be obtained from the same source. Cogeneration is attractive for industries with large needs of thermal
energy or availability of waste fuels. The cogeneration potential in the US is estimated at about 50,000 MW in 2000 (Naill et al., 1988).

Under the Public Utilities Regulatory Policies Act (PURPA) of 1978, section 210 relates to the interaction between, on one hand qualifying facilities (QFs) (small power producers and cogenerators) and on the other hand, electric utilities. The law, in essence requires that:

- Utilities purchase all available excess electricity at a price corresponding to the utility’s avoided cost. This is usually interpreted as the marginal cost of the QF class (Ravid, 1987, and Parmesano, 1987).

- Utilities provide QFs electric power at just, reasonable, and non-discriminatory rates.

- Utilities provide data concerning present and future costs of capacity and energy on their systems.

The avoided costs are close to the long term marginal cost of power generation. The non-discriminatory rates help QFs buy power just like the rest of the customers in that class. The main inefficiency in this market is the asymmetry between the utility marginal cost (selling price) and the utility average cost which is the buying price for power. The type of questions that need to be addressed by the bilevel model are the following:

- What is the effect on the utility if the electricity is instead sold at the QF’s marginal cost of production?
0 Which is more efficient for society, buying at the utility’s marginal cost or at the QF’s marginal cost?

0 Is there economic efficiency if the price of energy equals the utility’s marginal cost of production and the energy is bought by the utility at the PURPA mandated marginal cost of power?

The bilevel PURPA model can be set up in the following manner. The electric utility is at the upper level controlling the price of power and their marginal cost of power generation. The various cogenerators at the lower levels fix their consumption of electricity (energy services) and the amount of cogenerated power they sell in such a way to maximize their net benefits (gross benefits of consumption of energy services minus their electricity bills plus their revenues from sales). The QF has an incentive to sell power if its cost of power generation does not exceed the marginal cost of power generation. Each level should satisfy their demand for electricity and the utility should recover its costs. Thus the utility and the cogenerator are fundamentally in conflict. Haurie et al. (1990) solve such a problem as an successive approximation problem rather than in a bilevel framework.
REFERENCES


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Economics, pp. 521-531, August 1981.


Hansen, P., Jaumard, B., and Savard, G., "New Branching and Bounding Rules for


