INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

UMI
University Microfilms International
A Bell & Howell Information Company
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
313/761-4700 800/521-0600
Characterization of microstrip discontinuities by a dynamic source reversal technique using potential theory

Toncich, Stanley Slavko, Ph.D.
Case Western Reserve University, 1991

Copyright ©1991 by Toncich, Stanley Slavko. All rights reserved.
CHARACTERIZATION OF MICROSTRIP DISCONTINUITIES
BY A DYNAMIC SOURCE REVERSAL TECHNIQUE
USING POTENTIAL THEORY

by

STANLEY SLAVKO TONCICH

Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

Thesis Advisor: Prof. Robert E. Collin

Department of Electrical Engineering and Applied Physics
Case Western Reserve University
January, 1991
CASE WESTERN RESERVE UNIVERSITY

GRADUATE STUDIES

We hereby approve the thesis of

Stanley S. Tuncich

candidate for the Ph.D

degree.*

Signed: [Signature]
(Chairman)

[Signature]

[Signature]

Date Sept 18/90

*We also certify that written approval has
been obtained for any proprietary material
contained therein.
I grant to Case Western Reserve University the right to use this work, irrespective of any copyright, for the University's own purposes without cost to the University or to its students, agents and employees. I further agree that the University may reproduce and provide single copies of the work, in any format other than in or from microforms, to the public for the cost of reproduction.

[Signature]
CHARACTERIZATION OF MICROSTRIP DISCONTINUITIES
BY A DYNAMIC SOURCE REVERSAL TECHNIQUE
USING POTENTIAL THEORY

Abstract
by
STANLEY SLAVKO TONCICH

This thesis presents the dynamic source reversal technique based on potential theory, for the full wave characterization of microstrip discontinuities. The technique is used to characterize open ends and asymmetrical gaps in microstrip.

The discontinuity to be analysed is enclosed in a waveguide whose dimensions are chosen such that the guide is cut off at the propagating frequency of the microstrip. The substrate is assumed lossless and isotropic, and the microstrip has zero thickness.

A complete set of sources are included in the analysis, the longitudinal and transverse current, as well as the charge on the line. The correct edge conditions are included. From these sources the scalar and vector potentials, \( \Phi \) and \( \mathbf{A} \), respectively, are determined. The electric field in the waveguide can be
determined from the potentials, and is expressed as an integral equation. The method of moments is used to reduce the integral equation to a matrix equation which can be solved for the unknown input admittance.

In this technique, there is no need to model the source excitation, and the admittance for the discontinuity can be solved for directly in the case of an open end. For a gap, the tangent plane method is used to extract equivalent circuit values.

The computer program that was developed to implement this technique can be executed on a personal computer with run times of three minutes for an open end and 15 minutes for a tightly coupled gap. Data generated by this technique are compared to that of several other investigators. The technique provides a clear improvement in program execution time while also providing very accurate representation of the physical sources on the microstrip discontinuity.
This thesis is dedicated to my father,

Stanko Vjekoslav Toncich
ACKNOWLEDGEMENTS

The research reported in this thesis was supported by NASA Lewis Research Center under Cooperative Agreement NCC3-29.

I would like to thank my advisor, Professor Robert E. Collin, for his patience, help, and support on this thesis. Without his assistance this work would not have been possible.

I would like to acknowledge the absolute love and support I received from my wife Ankica, and my mother Sonja Toncich.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter 1</th>
<th>Literature Review</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.1) Introduction</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>(1.2) Literature Review</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>(1.3) Quasi-static analysis</td>
<td></td>
<td>9</td>
</tr>
<tr>
<td>(1.4) Full wave analysis</td>
<td></td>
<td>35</td>
</tr>
<tr>
<td>(1.5) Dynamic source reversal method using potential theory</td>
<td></td>
<td>65</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 2</th>
<th>Green's functions</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2.1) Green's functions</td>
<td></td>
<td>73</td>
</tr>
<tr>
<td>(2.2) Description of sources</td>
<td></td>
<td>88</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 3</th>
<th>Discontinuities</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3.1) Open end</td>
<td></td>
<td>107</td>
</tr>
<tr>
<td>(3.2) Asymmetric gap</td>
<td></td>
<td>127</td>
</tr>
<tr>
<td>(3.3) Inclusion of $J_x$ modes</td>
<td></td>
<td>155</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 4</th>
<th>Results and Conclusions</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4.1) Program development</td>
<td></td>
<td>172</td>
</tr>
<tr>
<td>(4.2) Open end results</td>
<td></td>
<td>174</td>
</tr>
<tr>
<td>(4.3) Gap results</td>
<td></td>
<td>184</td>
</tr>
<tr>
<td>(4.4) Conclusions</td>
<td></td>
<td>200</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS (CONT)

(4.5) Recommendations for further work 203

Appendix 1  Summing series 207

Appendix 2  Flow charts
(A2.1) Common program elements 215
(A2.2) Main program for open end 217
(A2.3) Main program for gap 218

References 219
CHAPTER 1- MICROSTRIP DISCONTINUITIES

(1.1) INTRODUCTION

In the design and development of microwave circuits and systems, it is desirable to represent the effects of discontinuities in microstrip inter-connects in terms of lumped parameter element networks. By doing so, a designer can apply well known circuit theory concepts to the equivalent circuits representing the discontinuity rather than becoming involved in the electromagnetic boundary value problem inherent with the analysis of such discontinuities. If the equivalent circuit elements of a certain discontinuity can be expressed in terms of the microstrip width to height ratio and the dielectric constant of the substrate, then a designer can work backwards, that is, choose the lumped parameter circuit values needed for a desired circuit performance, and then determine the relevant parameters required for the discontinuity.

Early techniques developed for characterizing microstrip discontinuities were based on quasi-static methods, which considered wave propagation on the line to be pure TEM, or static methods, which assume only a charge distribution on the strip. The techniques gave good results at low frequencies (up to 1-2 GHz.) for
stubs and gaps, but gave inaccurate results for inductive and/or high frequency effects.

The more sophisticated full-wave techniques that followed gave better results at higher frequencies, but are uniformly more computationally intensive. As a result of this complexity, the computer programs developed required greater computer resources, such as mainframes, along with long run times to generate results, even at one frequency. This limits the usefulness of many of these techniques in microwave circuit design.

This thesis will present a new approach to characterizing microstrip discontinuities, based on potential theory. It features an accurate representation of the sources on the microstrip line(s) being analyzed, and places emphasis on employing analytical and numerical techniques that minimize the run time needed to calculate equivalent circuit parameters. The formulation developed can be run on a personal computer, so that only minimum computer resources are required.

Chapter one will present a review of existing techniques, both quasi-static and full wave, with an accompanying discussion of the results and limitations of these methods. An introduction of a new technique
which we refer to as the dynamic source reversal method will be given as well.

This new method offers a number of advantages over other proposed techniques. In addition to the development of this new technique for solving a class of microstrip discontinuity problems, it is shown how the equivalent circuit parameters for a microstrip discontinuity can be extracted from a bilinear relationship between the tangent of the electrical length of the transmission line connecting the discontinuity and a short circuit on the output line and the input susceptance. By utilizing this relationship only a single matrix inversion is needed for a two port junction. In more conventional methods the input susceptance has to be found for three different terminations of the output line and thus requires three matrix inversions. The method developed in this thesis is thus far more efficient from a computational viewpoint.

Chapter two will describe the technique in detail and derive the pertinent equations, while chapter three will discuss how this technique is used to solve the problem of a stub (i.e., an open end), and an asymmetrical gap in microstrip. Chapter four will present the results for the above discontinuities and
will also discuss the merits of this approach compared with existing techniques. Also presented there is a discussion of possible future work, as well as other problems that could be solved by using this method.
1.2 LITERATURE REVIEW

As the complexity and operating frequency range of microwave systems increases, microstrip technology offers the designer the advantages of small size, weight reduction, ease of component interconnection, and the realization of integrated passive circuits, over other forms of waveguiding systems.

Microstrip discontinuities such as junctions, gaps, bends, and impedance steps are used to synthesize the circuit elements and matching networks required in the design of microwave systems. To facilitate the design of these circuit elements, an accurate knowledge of discontinuity equivalent circuits along with their frequency dependence is required. When the dimensions of the discontinuity are much smaller than the wavelengths being considered, the discontinuity may be represented by low frequency lumped parameter equivalent circuits. However, at the higher frequencies the equivalent circuit elements depend on frequency.

A microstrip line consists of a strip conductor over a ground plane separated by a dielectric slab which also provides structural support for the line. Since the field lines between the conductor and the ground plane are not entirely confined within the dielectric
material, a pure TEM wave cannot be supported by the line. Instead, a quasi-TEM wave mode will propagate with longitudinal field components present on the line, as a result of fringing components in $E_x$. Figure (1.1) shows the typical microstrip geometry.

A discontinuity is caused by an abrupt change in the geometry of the conductor. As a result, the electric and magnetic fields are modified in the vicinity of the discontinuity. The change in electric field distribution gives rise to an equivalent junction capacitance, while the change in magnetic field results in an equivalent inductance associated with the discontinuity. Most techniques used to analyze microstrip discontinuities fall into two general categories, (1) quasi-static analysis, or (2) full wave analysis.

Quasi-static analysis is a low frequency technique used to calculate static capacitances and inductances under the assumption of a pure TEM mode of wave propagation on the line. Equivalent circuits may then be derived from these results.

Full wave analysis is a more rigorous approach to the analysis of discontinuities than the quasi-static method since it is based on a complete solution of Maxwell's equations.
\( \varepsilon \)-effective dielectric constant of substrate
\( \mu_r \)-relative permeability of substrate
\( d \)-substrate thickness
\( w \)-strip width

Fig. (1.1). Typical microstrip geometry
Fig. (1.2). (a) Longitudinal current distribution and (b) transverse current distribution on a microstrip line.
Full wave analysis involves solving the wave equation with sources and leads to a more complete characterization of microstrip discontinuities at higher frequencies.

Several representative techniques for each approach will be briefly discussed in the next few paragraphs, followed by a summary of some of the published data pertaining to discontinuity analysis.

(1.3) QUASI-STATIC ANALYSIS

Static capacitance associated with a discontinuity may be determined by calculating the excess charge distribution near the discontinuity. Several commonly used techniques for doing this are:

(i) Numerical solution of the integral equation for the charge distribution using the method of moments in the spatial domain.

(ii) Variational methods to find the capacitance.

(iii) Solution of the integral equation in the Fourier transform domain, (with Galerkin's method).

(iv) Line sources with source reversal.

In (i), the potential at a point \( p=(x,y,z) \) due to a charge distribution \( \sigma(p_0) \) is given by the integral equation

\[
\Phi(p) = \int \sigma(p_0)G(p;p_0)dp_0 \tag{1.1}
\]

where \( G(p;p_0) \) is the Green's function. The Green's
function is the solution of Poisson's equation for a point charge source

\[ \nabla^2 G(p;p_0) = -\frac{1}{\epsilon_r \cdot \epsilon_0} \delta(p-p_0) \]  \hspace{1cm} (1.2)

where \( \delta(p-p_0) = \delta(x-x_0) \cdot \delta(y-y_0) \cdot \delta(z-z_0) \), and \( \epsilon_r \) is the relative permittivity, along with the appropriate boundary conditions for the problem being considered. If the conductor is assumed to be infinitely thin and located at a height \( h \) above the ground plane, then \( \delta(z-z_0) \) becomes \( \delta(z-h) \) and the volume integral becomes a surface integral. Once a suitable expression for the Green's function is determined, the integral equation may be solved for the unknown charge distribution \( \sigma \) by converting (1.1) to a matrix equation and inverting it numerically. This technique has been used by Farrar and Adams [1], [2] to calculate the equivalent capacitances of gaps, steps, and open ends. The matrix equation corresponding to (1.1) may be written as:

\[ [V] = [D] \cdot [\sigma] \]  \hspace{1cm} (1.3)

Since the conductors may be assumed to be at a known potential, such as 1.0 volt, Eq. (1.3) may be inverted to find the unknown \( [\sigma] \) in terms of \( [D] \). The capacitance \( C \) may be found by summing over all the \( \sigma_j \),

\[ C = \sum_{j=1}^{N} \sigma_j \sum_{i=1}^{M} \sum_{j=1}^{M} D_{ij} \]  \hspace{1cm} (1.4)
where \( N \) is the number of expansion functions used to approximate the charge distribution and \( D_{ij} \) is an element of \( [D]^{-1} \).

Since the equivalent capacitance of a discontinuity is due to excess charge in the vicinity of the discontinuity, the TEM mode capacitance must be subtracted from the computed capacitance as determined in (1.4). This results in the subtraction of two nearly equal numbers, which can lead to significant errors in determining the discontinuity capacitance.

Another limitation of this technique is the slowly convergent nature of the Green's functions, in the form used by Farrar and Adams. As a consequence, considerable computer time is required in the calculations, in order to achieve accurate results.

Also, Farrar and Adams assume that for each subsection of conductor the charge density is constant. However, as shown by Delinger [3], Silvester and Benedek [7], and recently by Kobayashi [4], the source distribution on a conductor above a ground plane is as shown in Fig. (1.2). As a result of the singular behavior at the edges, smaller and smaller subsections are required as the edges of the conductor are approached, which also increases the computation time. Figure (1.3) shows the results of Farrar and Adams'
Fig. (1.3). (a) Discontinuity capacitance for a step in width (from [1]).
Fig. (1.3). (b) Gap capacitance for a gap in microstrip
(from [1])
Fig. (1.3). (c) Shunt capacitance for a gap in microstrip (from [1]).
technique for the equivalent capacitance of steps in width and open ends as a function of width to height ratio and various values of dielectric constant.

Horton [14], [15] used a matrix representation of the integral equation to determine the equivalent capacitance of bends and steps. However, little data was presented in useful design form, also the technique relies on charge estimates and does not seem to be based on a rigorous technique.

(ii) A variational technique was used by Maeda [5], to characterize gaps and open ends. This method relies on the fact that the capacitance can be given by a variational expression which is stationary with respect to first order variations in the charge distribution on the conductor. The capacitance may be expressed as [6]

$$\frac{1}{C} = \frac{\int \int p(r)G(r;r')p(r')dv\,dv'}{[\int p(r')dv']^2} \quad (1.5)$$

where $G(r;r')$ is a three dimensional Green's function for the potential. The exact nature of the charge distribution need not be known in this technique. The capacitance can be obtained by minimizing (1.5) with a suitable choice of charge distribution as a trial function. A symmetric gap is modeled as a capacitive pi network, and the analysis is carried out by placing an electric and then a magnetic wall, representing a short
and open circuit respectively, in the plane of symmetry of the gap. As a result, (1.5) must be solved twice, once with a Green's function corresponding to the electric wall, and again for the magnetic wall. The Green's functions in these two cases are determined by solving a three dimensional Poisson's equation with appropriate boundary conditions. An open end is characterized by allowing the spacing to go to infinity. This approach has been limited to characterization of gaps and open ends. Figure (1.4) shows the results of this technique for gap and open end capacitance.

The major disadvantages of this technique is that the two Green's functions take the form of slowly convergent series, and that an expression for \( \rho(r') \), the charge density, must be suitably chosen and this choice will affect the numerical accuracy and computation time.

James and Henderson [32] used a variational technique and also included surface wave effects in determining the excess capacitance for open ends.

(iii) The integral equation relating the potential on the microstrip line to the charge distribution in the spatial domain may be converted into an algebraic expression in the spectral domain by specifying the Fourier transform of the potential, charge distribution, along with Poisson's equation, whose solution is now
Fig. (1.4). Gap and open end capacitance (from [5]).

(a) Open end capacitance as a function of \( w/h \).
Fig. (1.4) (b) Gap capacitance as a function of spacing, fixed \( h \). Where \( s \) is the gap spacing, \( C_a \) is the shunt and \( C_b \) is the gap capacitance in a pi network.
Fig (1.4) (c) Gap capacitance as a function of spacing, fixed $\varepsilon_r$. Where $s$ is the gap spacing, $C_A$ is the shunt and $C_B$ is the gap capacitance in a pi network.
carried out in the spectral domain along with boundary conditions appropriate to the problem.

At this point, Galerkin's method [8] may be applied to the problem by defining an inner product space and by expanding the unknown charge distribution into a suitable set of basis functions. Taking the inner product of the algebraic expression relating the potential to the charge distribution reduces the expression to a matrix problem which may be inverted to determine the unknown coefficients of the charge expansion. From these coefficients the capacitance can be calculated, given a specified applied potential.

It should be mentioned that Galerkin's method is a general technique that can be applied to any linear operator equation and is not limited to the spectral domain. When used in the spatial domain it is a special case of the method of moments. The spectral domain approach using Galerkin's method has been used by Itoh et. al. [9] to calculate edge capacitance, and by Rahmat-Samii et. al. [10] for open ends and gaps. The major advantage of this technique is that the Green's function has a closed form in the spectral domain, as opposed to a slowly converging series form. The major disadvantages are that the inverse Fourier transform must be evaluated numerically, the excess capacitance is
found by subtracting two nearly equal numbers, the procedure is numerically time consuming, and the accuracy is very dependent on the choice of basis functions used to expand the charge distribution. As shown in Fig. (1.2) the charge distribution on the microstrip line becomes infinite at the edges, making it difficult to expand the charges in terms of simple orthogonal functions. Very little data, useful in design, has been found using this technique.

(iv) The charge reversal technique uses a bipolar charge distribution to cancel a suitable portion of an infinite line charge distribution, so as to create an applied potential field. Near the discontinuity, excess charge is added. The amount of excess charge needed to satisfy the boundary conditions can be used to determine the equivalent circuit values. The major advantage of this technique is that it avoids the subtraction of two nearly equal numbers in determining discontinuity capacitance. The dynamic source reversal method introduced in this thesis is an extension of this method and will be discussed more completely later on.

Results have been obtained by Silvester and Benedek [11] for stubs, Benedek and Silvester [12] for gaps and steps, shown in Fig. (1.5), and by Silvester and Benedek [13] for bends, junctions, and crossings.
Fig. (1.5). (a) Open end capacitance (from [11])
Fig. (1.5) (b) Even and odd capacitance as a function of gap spacing. Where $C_{even} = 2C_{shunt}$ and $C_{odd} = 2C_{gap} + C_{shunt}$ (from [12]).
Fig. (1.5) (c) Even and odd capacitance as a function of gap spacing. Where $C_{\text{even}} = 2C_{\text{shunt}}$ and $C_{\text{odd}} = 2C_{\text{gap}} + C_{\text{shunt}}$ (from [12]).
Discontinuities such as steps in width, bends, and cross junctions have inductance associated with them, in addition to capacitance. An early attempt to characterize the inductance of finite length strips vs. frequency, using a quasi-static model with Galerkin's method was carried out by Gopinath and Silvester [16]. This technique involved calculating the inductance by finding the magnetic vector potential \( \vec{A} \) using a Green's function of the form

\[
G = \frac{1}{4\pi\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}}
\]

(1.6)

The inductance may be found from

\[
LI^2 = \int \vec{A} \cdot \vec{J} dV
\]

(1.7)

and where

\[
I^2 = [\int \vec{J} dV]^2
\]

(1.8)

The current density \( \vec{J} = J_x \vec{a}_x + J_y \vec{a}_y \) was expanded in terms of a basis function set. The singularity in the Green's function is eliminated by spatial discretization of the strip along with a suitable coordinate transformation.

This approach has the same problem as with any technique using Galerkin's method or the segmentation technique, that is, it is difficult to represent the charge or current distribution on a microstrip line by basis functions of low order. This technique only
provided values of the inductance per meter for lengths of microstrip but not for discontinuities. Gopinath and Easter [17] used an extension of the moment method, using current loops as the elements to represent the excess current near right angle bends, to calculate the inductance associated with a bend. The microstrip is divided into narrow strips through which current continuity is maintained with that of infinite unperturbed lines. Superimposed on this are small circulating current loops representing the excess currents near the junctions. This excess current is assumed to extend a distance $l_1$ and $l_2$ away from the discontinuity, as shown in Fig. (1.6). The bend inductance is then calculated as the difference in inductance arising from the excess currents and the TEM mode inductance per unit length of an unperturbed line. This subtraction of two nearly equal numbers can lead to large errors in bend inductance and is one of the major disadvantages of this technique. Accuracy is also dependent on the size of the current loops used in the expression for the excess current, with smaller loops (especially near the edge's of the line, where the tangential current is singular) giving greater accuracy at the expense of greatly increased computer time. Also, this formulation does not give any information
Fig. (1.6). Geometry of a right angle bend for inductance calculation (from [16] and [17]).
on the variation of inductance with frequency.

Gopinath et. al. [18], [19], [20] used Galerkin's method to determine the inductance of a number of microstrip discontinuities. In their approach, discontinuity inductance was determined from the subtraction of two nearly equal numbers, and their expressions for the current distributions are sensitive to choice of basis functions, which is typical when using Galerkin's method and a small number of basis functions. These two papers do, however, present a wide range of data on junctions and their associated inductances. Figure (1.7) shows the inductances associated with a step change in width from [19], as well as for a cross junction.

Anders and Arndt [21] determine the equivalent capacitance and inductance using the moment method for mitered bends and other unusual geometries. The reference planes in each configuration are chosen to include a sufficient amount of the fringing field of the discontinuity. Capacitances are found using the subsection technique described before with the assumption of constant charge over each subsection, and the matrix inversion is used to solve for the equivalent capacitance. Since the TEM mode capacitance must be subtracted out, this technique is subject to potentially
Fig. (1.7). Normalized inductance for a step in width (from [19]).
Fig. (1.7). (b) Normalized junction induction for various width/height ratios (from [19]).
large errors as mentioned earlier in this section. Inductances are calculated using the excess current loop approach, which other authors also used for analysis of bends and junctions. The results obtained are geometry dependent and must therefore be plotted over a wide range of geometries. The accuracy is dependent on the number of subsections used and is therefore inversely proportional to the computation time.

Most microstrip discontinuities are predominantly capacitive in nature over the useful operating frequency range. Inductive effects are normally high frequency corrections to capacitive effects and so they should be determined by a self-consistent theory that includes capacitive effects. The quasi-static inductance calculations described above are based on a separate inductance calculation which does not include capacitive effects. Thus it is an open question whether or not the results obtained are physically meaningful or not.

Garg and Bahl [34], derived closed form expressions for several symmetric and unsymmetric cross junctions as well as for mitered bends, which allow for the calculation of equivalent capacitances and inductances for a given equivalent circuit from information on the microstrip discontinuity geometry. These expressions are quoted as being accurate to within 5%. Since
these expressions were derived from quasi-static analysis, their validity over a wide range of frequencies is not known.

Finally, Easter [22], characterized several types of discontinuities such as corners, T-junctions, and cross-over junctions using experimental techniques. Although his data seems to agree with some of the numerical results obtained by other techniques already described above, the experimental approach is limited in its repeatability and by the experimenter's experience and laboratory technique in making measurements. Along with this, the fabrication of test circuits used for making measurements can vary in quality, resulting in non-repeatable data.

Easter et. al. [35], Douville and James [36], and Stephenson and Wolf [37], used resonant circuit techniques to determine the equivalent circuits for right angle and mitered bends. This technique requires that resonant circuit measurements be carried out where the coupling of the line is located at different points along a square resonator's perimeter so as to induce voltage minima and maxima at the corners of the resonator. From the measurements of the two frequencies at which the voltage minimum and maximum occur, the normalized susceptance for a bend may be determined. To
determine an equivalent electrical length of a corner, a third measurement must be made to determine the resonant frequency of a circular resonator, fabricated on the same substrate, and which has a mean circumference that is twice the length of one side of the square loop [36].

In a similar manner, linear resonators have been used to characterize a limited class of discontinuities such as gaps and steps in width [35], [37]. As in the case with rectangular and circular resonators, the coupling gap becomes part of the test circuit and its effects are not calibrated out. The linear resonator technique for measuring step changes in width also requires two lines to be designed, with one 1/2 wavelength longer than the other, so that a voltage maximum is obtained at the step in the first line, a corresponding voltage minimum is obtained at the step in the second resonator, since it is 1/2 wavelength longer. As with all experimental techniques, design and measurement errors may be large and the results obtained may not be easily repeatable. Since in resonator measurements such as these, two resonators are required, even small errors in tolerance and layout can introduce significant errors in the data obtained.

Douville and James [36], made measurements such as the ones just described over a frequency range of 0.2 to
3.0 Ghz and they claim that their results may be scaled to frequencies as high as 24 Ghz. The disadvantages of this approach are the same as those discussed with the experimental techniques described by Easter [22], as well as the resonator techniques.

In their approach, two resonators must be designed and built, along with a movable coupling line, to make the measurements required to determine the equivalent circuit parameters. This approach is complicated by the need for a movable coupling line, which is physically difficult to realize, as well as introducing an unavoidable air gap along with the break in the dielectric material. It is also doubtful that the results obtained in a frequency range of 0.2 to 3.0 Ghz can be accurately scaled to provide equivalent circuit parameters at 24 Ghz.

Although the quasi-static techniques which have been described so far have been used extensively to characterize microstrip discontinuities at low frequencies (typically valid for frequencies below 1.0 to 2.0 Ghz), they do not lend themselves well to the inclusion of dispersive effects or frequency dependent results, since the quasi-static approach assumes a TEM mode of wave propagation along the line. The ability to include dispersion and to obtain results which are valid
over a wide range of frequencies becomes increasingly important as the operating frequency range of microwave devices and systems increases. For high frequency analysis of microstrip discontinuities, a full wave analysis needs to be carried out. In a full wave analysis, the static fields (pure TEM mode) assumption is abandoned, instead, the wave equation, with sources, is solved, leading to a more complete characterization of discontinuities at higher frequencies.

(1.4) FULL WAVE ANALYSIS

The two main techniques used to carry out a full wave analysis of microstrip discontinuities are:

(i) Fourier Transform Domain (FTD) Analysis,

(ii) Planar waveguide method.

(i) The FTD technique has been used by Itoh [23] to calculate the capacitance of open ends and gaps by analyzing a resonator enclosed by conducting walls and a cover plate. This geometry is shown in Fig. (1.8). The fields in the resonator are assumed to be a superposition of TE and TM (with respect to z) fields and are given in terms of scalar potentials \( \Phi(x,y,z) \) and \( \phi(x,y,z) \). Rather than deriving a set of coupled homogeneous integral equations in the spatial domain, the field equations along with the wave equation are
Fig. (1.8). End and top view of the microstrip resonator model (from [23]).
transformed into the spectral domain, where the integral
equations relating the electric and magnetic fields
inside the structure to the currents on the resonator
are reduced to algebraic equations. The Green's
functions required to relate these field equations are
expressed in series form, as in the quasi-static case.
In order to find the unknown current, Galerkin's method
is applied. This is done by expanding the currents on
the resonator (assumed to have x and z components) in
terms of known basis functions. Proceeding as in the
quasi-static case the wave number \( k_0 \) is determined from
which the resonant frequency of the resonator may be
determined.

A dispersion relationship may be incorporated by
solving for the propagation constant at several
frequencies from the transformed equations. Open end
discontinuities may be characterized by determining the
effective increase in the electrical length of a
specific resonator.

For gaps, two types of resonators are used, one
with electric walls at \( z=0 \) and \( z=2l \), and another with
magnetic walls. Equivalent lengths \( \Delta l_e \) and and \( \Delta l_m \) are
then found, from which two equivalent capacitances are
determined. These capacitances are related to the
capacitance of the pi equivalent networks,
\[ C_g = 0.5(C_n - C_m) \]  
\[ C_p = C_m \]  

where \( C_p \) is the shunt, and \( C_g \) the gap capacitance. This technique has been used for gaps and open ends.

The results track well with quasi-static approaches, however the \( \Delta l \) computed by this method is smaller. Itoh attributes this to the reduction of capacitive susceptance due to the inductance associated with the current distribution near the two ends [23]. Since very few expansion functions were used it seems more likely that the discrepancy is due to insufficient spatial resolution of the current and field near the open ends. Accuracy and computational time are inversely related to the choice of basis functions and the number of terms used in the expansion. Accuracy can be increased by increasing the number of higher order modes incorporated in the field equations.

(ii) As shown by Wheeler [41], [42], and Miyoshi [43], in the planar waveguide approach the microstrip line is modeled as an equivalent dielectric filled waveguide with magnetic side walls, as shown in Fig. (1.9a), using the effective width and dielectric constant of the line as determined analytically, as in [24] or [26]. The planar waveguide model for microstrip represents an intermediate stage of complexity between
Fig. (1.9), (a) Model used for representing a microstrip line as an equivalent waveguide. The effective width and dielectric constant are determined from [24].

Fig. (1.9). (b) Typical geometry for a planar circuit.
transmission line and waveguide models. Discontinuities in microstrip are then considered to generate higher order modes in their vicinity (the fundamental TEM mode of propagation is assumed in the planar waveguide model). These discontinuities may be characterized by matching the modes, on both sides of the discontinuity, for the electric and magnetic fields present [25]. Accuracy of characterization is determined by the number of modes used, however this also increases the amount of computer time involved [27].

Sorrentino [44] presents a planar waveguide model for a general N port passive network. The junction, is enclosed by magnetic walls and perfect conductors on the cover plates. The enclosed volume is assumed to be driven by equivalent surface currents on the side walls, corresponding to the terminal planes of the N lines terminating inside the volume, this geometry is shown in Fig. (1.9b). The voltage wave equation which describes the field inside the junction can be solved using resonant mode expansions [45] or a Greens function approach [46]. The voltage and currents on the planar circuit can be related to each other by means of a generalized impedance matrix as follows

$$V_i^{(m)} = \sum_{j=1}^N Z_{ij}^{(mn)} I_j^{(n)}$$  \hspace{1cm} (1.10)
where \( n \) and \( m \) refer to the order of the mode used in the expansion, and \( j=1,N \) refers to each of the \( N \) ports as defined on the planar model. From the determined \( Z_{ij}^{mn} \) the scattering matrix for the model may be determined. The equivalent circuit for such a model can, in the case of a two port network \( (N=2) \), be represented as a series connection through a transformer of anti-resonant LC circuits. For symmetrical structures, a transformerless lattice network may be used [47].

The mode matching technique is particularly suited for the characterization of symmetric discontinuities such as steps in width [27] and \( T \)-junctions [25]. Scattering matrices may be used to describe the frequency dependent behavior of these discontinuities.

Menzel and Wolff [28] used the equivalent waveguide model for the microstrip line, along with a field matching technique proposed by Kuhn [29] to characterize a large number of non-symmetrical junctions using scattering parameters. The frequency dependent effective width of the waveguide model was taken into account in this approach by using the expression [24]

\[
W_{\text{eff}}(f) = W + \frac{W_{\text{eff}} - W}{1 + (f/f_c)^2} \tag{1.11}
\]

where \( W \) is the width of the microstrip line, \( W_{\text{eff}}(0) \) is the static value of the effective width, and
\[ f_c = \frac{Z_0}{(8\pi h)} \], where \( Z_0 \) is the impedance of the line and \( h \) is the substrate thickness, in cm.

Since at the top and bottom of the waveguide equivalent model, the tangential electric field must vanish, only TEM and \( \text{TE}_{n,0} \) modes, consisting of \( E_y \), \( E_x \), and \( H_z \) components exist. The field components are derived from a scalar potential and are assumed to consist of a number of higher order modes, which will be used to determine the scattering coefficients. For each geometry, the field components are made equal at the interface, and are scaled as necessary for different impedance levels, corresponding to different line widths. Figure (1.10) shows the data obtained by Menzel and Wolff for various geometries.

The planar waveguide model was also used by Kompa and Mehran [38], Kompa [39], and Mehran [40], to determine the scattering matrix for bends, T-junctions, and crossings. This technique can give good results for the magnitude of the scattering matrix elements. However, the planar waveguide models do not give accurate results for the phase angles of the scattering matrix elements.

Jackson and Pozar [30] used a moment method to characterize open ends and gaps. Near the actual discontinuity, piecewise sinusoidal modes were used as
Fig. (1.10). (a) Calculated scattering parameters for an unsymmetrical T-junction vs. frequency (from [28]).
Fig. (1.10). (b) Calculated scattering parameters for an unsymmetrical T-junction vs. frequency (from [28]).
Fig. (1.10) (c) Reflection and transmission coefficient for an unsymmetrical T-junction (from [28]).
expansion functions to evaluate the integral equation using matrix methods. The accuracy of this method depends on the number of piecewise modes used to represent the excess current at the discontinuity, and this number is probably frequency dependent. As more sections are used however, the complexity of the calculations increases. Also, their initial results were based on a uniform transverse current distribution, this was later modified to a distribution that goes to infinity as the transverse edge of the line is approached.

It is doubtful that they used a sufficient number of expansion functions to obtain an accurate characterization of the open end as a circuit element for design purposes. However, they were primarily interested in determining the amount of radiation from the open end, both in the form of space waves and surface wave modes.

Katehi and Alexopoulos [31] characterized open ends, gaps, and coupled resonators in terms of lumped parameter equivalent circuits, while including surface wave effects and radiation. Their technique takes into account radiation losses, as well as surface wave losses, which become more important loss mechanisms as the frequency range of operation is increased into the
millimeter wavelength region. These extra loss mechanisms are modeled as shunt conductances.

The radiated field due to the presence of the discontinuities is given by Pocklington's integral equation, which relates the radiated electric field to the current distribution on the microstrip line. This expression may be written as

\[
\vec{E}(\vec{r}) = \int \hat{G}(\vec{r}/\vec{r}') \cdot \vec{J}(\vec{r}') d\vec{s}' \tag{1.12}
\]

where the Dyadic Green's function, \( \hat{G}(\vec{r}/\vec{r}') \) is given by

\[
\hat{G}(\vec{r}/\vec{r}') = \int [k_0 \hat{I} + \nabla \nabla] \cdot (J_0(\lambda|\vec{r}-\vec{r}'|) \hat{F}(\lambda)) d\lambda \tag{1.13}
\]

where \( \hat{I} \) is the unit dyadic, \( k_0 = 2\pi/\lambda_0 \), \( \hat{F} \) is a known dyadic function, \( J_0 \) is the zero order Bessel function, and \( \hat{G} \) represents the spectral expansion of the scalar Greens function in cylindrical coordinates in the \( z=0 \) plane.

To solve for \( \vec{J} \) in (1.12) the method of moments is used. The microstrip line is divided into a number of segments and the current density is expressed as a sum of piecewise sinusoidal functions. Equation (1.13) is reduced to a matrix equation by using Galerkin's method. The unknown coefficients for the current can be solved
for by matrix inversion.

The equivalent circuits for the various discontinuities can be derived by determining the equivalent guide wavelength and the characteristic impedance of the line, along with using proper excitation for the discontinuity being considered. Figure (1.11) shows the variation of characteristic impedance and guide wavelength as a function of frequency. Figure (1.12) shows some of the results obtained by this technique for open ends and gaps.

The spectral domain approach to the solution of frequency dependent microstrip discontinuity problems, along with various numerical techniques applicable to the spatial and/or spectral domain were discussed by Jansen [33]. Of particular interest is the use of the Tangent method which was developed as an experimental method to characterize the discontinuity by an equivalent circuit containing three parameters, as shown in Fig. (1.13).

The measurements should be made at reference planes located sufficiently far away from the discontinuity so that only dominant modes are present on the line.
Fig. (1.11). Characteristic impedance and guide wavelength as a function of frequency (from [31]).

Fig. (1.12). (a) Open circuit excess length and capacitance as a function of frequency for $\varepsilon_r=9.6$ (from [31]).
Fig. (1.12) (b) Normalized gap discontinuity capacitance as a function of frequency for $\varepsilon_r=9.6$ and $w/h=1.0$.
(c) Gap discontinuity admittance matrix elements as a function of frequency for $\varepsilon_r=9.6$, $w/h=1.0$ (from [31]).
A plot of the null positions $\phi_1$ as a function of the short circuit position $\phi_2$ will yield a curve like the one shown in Fig. (1.14). An analysis of the equivalent circuit shown in Fig. (1.13) will give expressions for the equivalent circuit parameters $\theta_1$, $\theta_2$ and $n:1$ in terms of the parameters of the curve of $\phi_1$ vs. $\phi_2$ [48].

Jansen extended this approach to include $N$ port junctions in conjunction with a three dimensional spectral domain resonator analysis. In this approach, the $N$ port junction is placed inside a shielded box, with the field inside this volume constrained by short circuited stubs of length $l_i$. Associated with this is an equivalent circuit represented by scattering parameters calculated at specific reference planes $RP_i$ for $i=1$ to $N$. For a fixed frequency $\omega$, the field is determined inside the resonator for $n$ different lengths of stubs. A flow chart for this technique is shown in Fig. (1.15). The $N$ hypothetical resonator experiments determine the the resonant lengths as well as the stub current density distributions. These results can be used to determine the longitudinal current distribution on the stubs. The complex wave amplitudes on the stubs are derived for each of the $N$ experiments and these amplitudes may be used to determine the scattering.
Fig. (1.13) Waveguide discontinuity and equivalent circuit used in the Tangent method (from [48]).
Fig. (1.14) Plot of $\phi_1$ vs. $\phi_2$ as determined by the Tangent method (from [48]).
matrix of the corresponding equivalent circuit. This technique does not include radiation effects, that is, this technique can't be applied as used here for systems where the radiation loss mechanism is appreciable. This technique has been used to characterize several types of discontinuities [49], [50], [51], [52], and [56].

Chada and Gupta [53] describe a technique by which common microstrip discontinuity reactances may be compensated for by removing appropriate triangular sections from the discontinuity configuration. They employ a planar waveguide model [43] for the analysis and the results are expressed in terms of the scattering parameters of the configuration under study. A two dimensional analysis was used employing the segmentation and desegmentation method [54], [55], for the analysis of T-junctions, steps, and bends.

Figure (1.16) shows a 2:1 step in width, and the response for various types of linear tapers. By considering the response for no taper (θ=90°) it can be seen that as the frequency of operation is increased, the magnitude of $S_{11}$ begins to diverge from its low frequency value of 0.333. This results from the excitation of the next higher TE mode (TE$_{20}$ in this case) on the line, while the taper of 60° gives a near
Fig. (1.15) Flow chart for the modified Tangent method for characterization of discontinuities (from [33]).
Fig. (1.16) 2:1 change in width as a function of taper angle (from [53]).
Fig. (1.17) Variation of $\Delta l$ with frequency (from[53]).
constant response across the range of frequencies.

Normally when a microstrip discontinuity is characterized by a scattering matrix, the reference planes are chosen on the microstrip lines sufficiently far from the discontinuity so that only the dominant TEM modes are present at these points on the lines, and the effect of the discontinuity on the dominant mode is that of a disturbance in the phase angle as measured relative to a given reference plane where the particular $S_{ii}$, $i=1, N$ is being measured. In this technique, the authors calculate the change in electrical length, $\Delta l_i$, which is the distance between the step reference plane from the magnetic wall model of the line, and the effective step plane, which is where the computed $S_{ii}$ become real. Figure (1.17) shows the normalized shift in the effective step reference plane for the compensated and uncompensated line, whose magnitude response was shown in Fig. (1.16).

In each case, the relative degree of improvement using this technique can be determined by the condition that the normalized $\Delta l_i$ are small or that they remain constant over the frequency range of interest. This same technique was used for T-junctions, where a symmetric wedge was removed from the plane of symmetry, and for right angle bends. This technique relies on
there being a sufficient amount of symmetry present in
the geometry under consideration.

All of the more recent published articles dealing
with microstrip discontinuities focus on full wave
analysis techniques which are valid at higher
frequencies. This is in line with current design trends
and reflect the need for more accurate characterization
of discontinuities at these frequencies.

Rautio and Harrington [56] use the Galerkin method
to analyze microstrips of arbitrary geometry which are
enclosed in a cavity. They use "roof top" functions to
expand the unknown longitudinal current amplitudes,
which are defined as triangles with a rectangular base.
Two modes are used to expand the current in the lateral
direction, but there are no edge conditions built into
these expansions. No transverse current is assumed on
the line. The source for the cavity is modeled as a
equivalent magnetic current element. Data is presented
for only the open end and for only a single geometry.
The accuracy of their model relies on the number of sub-
sections used to expand the unknown current, that is,
the base width of the roof top function. On a personal
computer, a small circuit of 12 subsections takes
several minutes to evaluate, while a large one, up to
100 sections, takes several hours.
Uzunoglu et. al. [57] use a mode matching technique to analyze a step in width enclosed in a waveguide structure. The fields on both sides of the discontinuity are expanded in terms of the normal modes of a dielectric loaded waveguide. These modes are matched at the location of the step.

Chen and Gao [58] use the method of lines to analyze a step width. The method offers the advantage over other methods in that it doesn’t require a choice of basis functions and therefore avoids the associated convergence problems.

Yang et. al. [59] use a numerical solution of integral equations to evaluate open ends and gaps. Radiation and surface wave losses are included. They use an accurate representation for the longitudinal currents on the microstrip(s) in question, but neglect the transverse currents on the lines. Figure (1.18) shows their results for gap capacitance and conductance.

Dunleavy and Katehi [60] use the method of moments to solve the integral equation representing the electric field inside a cavity. The longitudinal current is expanded in overlapping sinusoids, with edge currents built in, from the location of the discontinuity back to the aperture in the cavity wall. The field in the aperture is represented as a magnetic current. They present
Fig. (1.18) (a) Gap conductance vs. frequency: $\varepsilon_r = 9.6$, $w/h = 1.0$ (from [60]).
Fig. (1.18) (b) Normalized gap capacitance vs. frequency, w/h=1.0 (from [60]).
this work mainly to discuss the numerical accuracy of the technique and the distribution of currents on the line, for an open end.

Zhang and Mei [61] use a time domain finite difference technique to analyze several types of discontinuities. They give S parameter data for open ends, gaps, T-junctions, and steps. However, except for the open end, only magnitude data is given.

Jackson [62] uses finite element expansion currents in a full wave analysis of stubs, bent stubs, and steps in width. While the approach is good for characterization of some otherwise complex geometries like the bent stub, the technique is limited to strip dimensions that are multiples of the element size. Figure (1.19) gives Jacksons' results for a step in width.

This completes the review of published work. The next section will present an introduction to the technique developed in this thesis.
(a) Transmission phase \((S_{21})\).

(b) Reflection phase \((S_{22})\).

Fig. (1.19) Comparison of step phase calculation (from [62]).
(1.5) DYNAMIC SOURCE REVERSAL METHOD USING

POTENTIAL THEORY

The method developed in this thesis for solving microstrip discontinuity problems is based on using a dynamic source reversal method formulated in terms of vector and scalar potentials. The essential features are described below.

The basic geometry used for all the discontinuities to be characterized is shown in Fig. (1.20). The discontinuity is placed inside a dielectric loaded waveguide whose dimensions are such that at the frequency of operation for the dominant microstrip mode, all the waveguide modes are non-propagating.

Inside the waveguide, let $G_x$, $G_z$, and $G$ be the scalar Green's functions corresponding to the components $A_x$ and $A_z$ of the vector potential, and of $\phi$, the scalar potential, respectively. Define $J_{0x}$, $J_{0z}$, and $\rho_0$ as the current and charge on the microstrip for a propagating dominant mode on the strip. The potentials, for an infinite strip are then related to the sources by

$$A_{0z} = \mu_0 \int_{-\infty}^{\infty} \int_{-w}^{w} G_z(x, z; x', z') J_{0z}(x') e^{-j\beta z'} dx' dz'$$  \hspace{1cm} (1.14)
(a) Front view.

(b) Top view.

(c) End view.

Fig. (1.2X) Shielded microstrip geometry.
$$A_{0x} = \mu_0 \int_{-\infty}^{\infty} \int_{-w}^{\infty} G_x(x,z;x',z') J_{0x}(x') e^{-j\beta z'} dx' dz' \quad (1.15)$$

$$\varepsilon_0 \Phi = \int_{-\infty}^{\infty} \int_{-w}^{\infty} G(x,z;x',z') \rho_0(x') e^{-j\beta z'} dx' dz' \quad (1.16)$$

In these expressions and in all others the primed coordinates are the source points and the unprimed coordinates are the field points.

For the purpose of this introductory description, the expressions presented here will pertain to the special case of an open end discontinuity.

The electric field \( \vec{E} \) may be expressed in terms of the potentials via the expression \( \vec{E} = -j\omega \vec{A} - \vec{\nabla} \Phi \). On the microstrip, the tangential components of the field, \( E_x \) and \( E_z \) in this case, must vanish. As a result it is required that

$$E_z = -j\omega A_z + j\beta \Phi_0 = 0 \quad (1.17)$$

$$E_x = -j\omega A_x - \frac{\partial \Phi_0}{\partial x} = 0 \quad (1.18)$$

For the special case of no transverse current, \( J_x = 0 \). For simplicity, this will be assumed in the following discussion.
Now assume that instead of an infinite line, the line is terminated at some point in a discontinuity. Since the point of termination is arbitrary, it may be taken to be at \( z = 0 \). The presence of a discontinuity will cause a reflected dominant mode charge and current to appear on the line, along with a perturbation in the charge and current localized in the vicinity of the discontinuity. The total charge and current distribution on this terminated line may be written as

\[
J_z(x', z') = J_{0z}(x') (e^{-j\beta z'} - \text{Re}e^{j\beta z'}) + J_{1z}(x', z') \quad (1.19)
\]

\[
\rho(x', z') = \rho_0(x') (e^{-j\beta z'} + \text{Re}e^{j\beta z'}) + \rho_1(x', z') \quad (1.20)
\]

for \( z' \leq 0 \), with \( R \) as an unknown reflection coefficient. The perturbation of the current and charge are represented as \( J_{1z} \) and \( \rho_1 \). Equations (1.19) and (1.20) may be written as

\[
J_z(x', z') = J_{0z}(x') (1 + R) \left( \frac{1-R}{1+R} \cos(\beta z') - j\sin(\beta z') \right)
\]

\[
+ J_{1z}(x', z')
\]

\[
= j(1 + R) J_{0z}(x') (B_{in} \cos(\beta z') - \sin(\beta z')) + J_{1z}(x', z') \quad (1.21)
\]
\[ \rho = (1+R)(B_{in}\sin(\beta z') + \cos(\beta z')) + \rho_i(x',z') \quad (1.22) \]

where \( jB_{in} = (1-R)/(1+R) \) is the normalized input susceptance for the line. Since the amplitude of the incident mode is arbitrary, \( 1+R \) may be chosen as its amplitude and set equal to unity. Since the amplitudes of \( J_{1z} \) and \( \rho_i \) are yet to be determined, they can be defined in such a way so as to absorb the \( 1+R \) factor.

For this line, \( A_z \) and \( \Phi \) may be written as

\[
A_z = \mu_0 \int_{-\infty}^{\infty} G_z J_z dx'dz' \quad \text{and} \quad \varepsilon_0 \Phi = \int_{-\infty}^{\infty} G \rho dx'dz'
\]

where \( J_z \) and \( \rho \) are zero for \( z' > 0 \).

At this point, use is made of the fact that

\[
\int_{-\infty}^{0} f(z') dz' = \int_{0}^{\infty} f(z') dz' - \int_{-\infty}^{0} f(z') dz'.
\]

Now, \( A_z \) and \( \Phi \) may be written as

\[
A_z = \mu_0 \left\{ \int_{-\infty}^{\infty} j(B_{in}\cos(\beta z') - \sin(\beta z')) J_{0z}(x') G_z dz' + \right. \\
\left. \int_{-\infty}^{0} J_{1z}(x') G_z dz' - \int_{0}^{\infty} j(B_{in}\cos(\beta z') - \sin(\beta z')) J_{0z}(x') G_z dz' \right\} dx'
\]

\[ (1.23) \]

and
\[
\phi = \frac{1}{\varepsilon_0} \int_{-\infty}^{\infty} \int_{-W}^{W} \left\{ \int_{-\infty}^{\infty} (B_{in} \sin(\beta z') + \cos(\beta z')) \rho_0(x') G dz' + \int_{-\infty}^{0} \rho_1(x') G dx' - \int_{0}^{\infty} (B_{in} \sin(\beta z') + \cos(\beta z')) \rho_0(x') G dz' \right\} dx'
\]

(1.24)

where the \( x, z \) and \( x', z' \) dependence of \( G \) and \( G_z \) is understood. In effect, we assume that the dominant mode current and charge exists over the infinite interval \(-\infty \leq z' \leq \infty \) and then subtract the effect of these sources in the interval over \( 0 < z' < \infty \) since these sources actually do not exist. When \( E_x \) and \( E_z \) are calculated from the above potentials, the contributions from the dominant mode sources on the infinite line (\(-\infty \) to \(+\infty \)) already satisfy the boundary conditions for the dominant mode standing waves on the line, and so these terms may be dropped. The terms involving integrals from \( 0 \) to \(+\infty \) may be considered "source reversed" terms, and produce the impressed field on the line in the region \( z \leq 0 \). Since these source reversed terms are given in terms of the known dominant mode amplitudes, they now make up a known forcing function. This is the basis of the technique presented in the following chapters.

The boundary conditions require that the electric
field produced by the perturbed current $J_{iz}$ and charge $\rho_1$ cancel the tangential component of the applied field on the microstrip line in the region $z \geq 0$. For simplicity in explaining the basic formulation, we have not included the transverse current in the x direction. For many microstrip discontinuities the transverse currents are very small relative to the longitudinal currents and can be neglected.

The method described above is a dynamic version of the static source reversal method introduced by Silvester and Benedek in [7], [11], [12], and [13].

The use of potential theory has the advantage that it places in evidence the important role played by the charge distribution $\rho_1$ in establishing the dominant capacitive effects associated with most microstrip discontinuities. The impressed field from the reversed line sources are confined to the near vicinity of the discontinuity. The unknown perturbed current $J_{iz}$ and charge $\rho_1$ can be determined, along with the unknown input susceptance $B_{in}$ by solving appropriate integral equations using the method of moments. It is also not necessary to specify any external source of excitation which would introduce additional perturbations in the region of the source. As a result, substantial computational savings are achieved.
In Chapter 2, the required Greens functions will be developed. In Chapter 3, the stub and gap problems will be developed, the explicit expressions for the perturbed charges and currents will be given for these two cases. Chapter 4 will discuss the results using the data generated from the implementation of this technique and will compare these results with published data.
CHAPTER 2 - GREEN'S FUNCTIONS

In this chapter the necessary Green's functions will be developed. The Green's functions will satisfy the boundary conditions for a dielectric loaded waveguide.

Also presented in this chapter will be the expressions for the sources, the charges and currents, which exist on the microstrip. The potentials are then determined by integrating the Green's functions over the sources. From the potentials, the electric fields that exist inside the enclosure may be determined.

(2.1) DERIVATION OF GREEN'S FUNCTIONS

With reference to Fig. (1.22c), the dielectric constant may be expressed as a function of \( y \) in the following manner

\[
\begin{align*}
\varepsilon(y) &= \varepsilon & 0 &\leq y \leq h \\
\varepsilon(y) &= 1 & h &< y \leq c
\end{align*}
\]  

(2.1)

The microstrip is assumed to be located at a height \( y = h^+ \), just above the slab, possessing negligible thickness. The sources on the strip are defined as
having an x dependence (transverse) and z dependence (longitudinal). There are no y directed current sources. The electromagnetic fields existing inside the waveguide must satisfy Maxwell's equations. It is desired to derive the equations for the potentials $\vec{A}$ and $\Phi$, in terms of the sources $\vec{J}$ and $\rho$ on the strip. In everything that follows, all source and field quantities are assumed to have a time dependence given by $e^{j\omega t}$.

The constitutive relations $\vec{D}=\varepsilon \vec{E}$ and $\vec{B}=\mu \vec{H}$ are assumed. Since $\varepsilon=\varepsilon_\infty$ and $x=x(y)$, therefore $\varepsilon=\varepsilon(y)$. The dielectric is assumed to be isotropic.

The magnetic flux vector $\vec{B}$ is always solenoidal and therefore it may be derived from a vector potential function $\vec{A}$ as follows,

$$\vec{B}=\nabla \times \vec{A} \quad (2.2)$$

Since $\nabla \cdot (\nabla \times \vec{A})=0$, this satisfies the requirement that $\vec{B}$ have zero divergence, i.e. $\nabla \cdot \vec{B}=0$. The electric field satisfies the equation

$$\nabla \times \vec{E}=-\frac{\partial \vec{B}}{\partial t} \quad (2.3)$$

When (2.2) is substituted into (2.3) it gives $\nabla \times (\vec{E}+j\omega \vec{A})=0$. Integrating this expression gives
$$\vec{E} = -j \omega \vec{A} - \nabla \Phi \quad \text{(2.4)}$$

where $\Phi$ is a scalar function, as yet unspecified. Beginning with the equation $\nabla \times \vec{E} = j \omega \mu_0 \epsilon_0 \chi(y) \vec{E} + \mu_0 \vec{J}$, and substituting (2.2) and (2.4) into this equation gives

$$\nabla \cdot \vec{A} - \nabla^2 \vec{A} = j \omega \mu_0 \epsilon_0 \chi(y)(-j \omega \vec{A} - \nabla \Phi) + \mu_0 \vec{J}$$

$$= \omega^2 \mu_0 \epsilon_0 \chi(y) \vec{A} - j \omega \mu_0 \epsilon_0 \chi(y) \nabla \Phi + \mu_0 \vec{J} \quad \text{(2.5)}$$

If the Lorentz condition, $\nabla \cdot \vec{A} = -j \omega \mu_0 \epsilon_0 \chi(y) \Phi$, is used to define a relationship between $\vec{A}$ and $\Phi$, (2.5) becomes

$$-j \omega \mu_0 \epsilon_0 \nabla(\chi(y) \Phi) - \nabla^2 \vec{A} = k_0^2 \chi(y) \vec{A} - j \omega \mu_0 \epsilon_0 \chi(y) \nabla \Phi + \mu_0 \vec{J}$$

or

$$\nabla^2 \vec{A} + \chi(y) k_0^2 \vec{A} = -j \omega \mu_0 \epsilon_0 \frac{\partial}{\partial y}(\chi(y) \Phi) - \mu_0 \vec{J} \quad \text{(2.6)}$$

Equation (2.6) gives one relationship needed between $\vec{A}$, $\Phi$, and $\vec{J}$. In terms of individual components, and using the fact that $\vec{J} = J_x \vec{a}_x + J_z \vec{a}_z$, this equation can be written as three scalar equations

$$\nabla^2 A_x + \chi(y) k_0^2 A_x = -\mu_0 J_x \quad \text{(2.7)}$$
\[ \nabla^2 A_z + \chi(y) k_0^2 A_z = -\mu_0 J_z \quad (2.8) \]

\[ \nabla^2 A_y + \chi(y) k_0^2 A_y = j\omega \mu_0 \varepsilon_0 (x-1) \Phi(h) \delta(y-h) \quad (2.9) \]

where \( \partial \chi(y)/\partial y = -(x-1) \delta(y-h) \) has been used.

Now, using Gauss's law, \( \nabla \cdot (\chi(y) \vec{E}) = \rho/\varepsilon_0 \) with (2.4) gives \( \vec{E} \cdot \nabla \chi(y) + \chi(y) \nabla \cdot \vec{E} = \rho/\varepsilon_0 \). Substituting for \( \vec{E} \) gives

\[ (-j\omega A_y - \nabla \Phi) \cdot \nabla \chi(y) + \chi(y) \nabla \cdot (-j\omega A_y - \nabla \Phi) = \frac{\rho}{\varepsilon_0} \]

By using \( \nabla \chi(y) = (\partial \chi(y)/\partial y) \vec{a}_y \), and the Lorentz condition, we obtain

\[ \frac{\partial \Phi}{\partial y} \frac{\partial \chi(y)}{\partial y} + \chi(y) \nabla^2 \Phi + \chi^2(y) k_0^2 \Phi = -\frac{\rho}{\varepsilon_0} - j\omega A_y (x-1) \delta(y-h) \]

or

\[ \chi \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) + \chi \left( \partial \chi(y) \frac{\partial \Phi}{\partial y} \right) + \chi^2(y) k_0^2 \Phi = -\frac{\rho}{\varepsilon_0} + j\omega (x-1) A_y (h) \delta(y-h) \quad (2.10) \]

as the last required relationship between the sources and the potentials.

A dielectric loaded waveguide can support both longitudinal section electric (LSE) and longitudinal section magnetic (LSM) modes. An LSE mode has no
component of \( \mathbf{E} \) normal to the air-dielectric interface, while an LSM mode has no \( \mathbf{H} \) component normal to this interface. As a result, \( \mathbf{H}_y \) may be expressed in terms of LSE modes, while \( \mathbf{E}_y \) requires LSM modes. Both sets of modes produce \( x \) and \( z \) components of the field.

Since

\[
E_x = -j\omega A_x - \frac{\partial \Phi}{\partial x} \quad (2.12a)
\]

\[
E_y = -j\omega A_y - \frac{\partial \Phi}{\partial y} \quad (2.12b)
\]

\[
E_z = -j\omega A_z - \frac{\partial \Phi}{\partial z} \quad (2.12c)
\]

it can be seen that \( \Phi \), which appears in all three components of \( \mathbf{E} \) must be represented by both LSE and LSM modes. The Green's functions corresponding to \( A_x \), \( A_z \), and \( \Phi \), namely, \( G_x \), \( G_z \), and \( G \), may now be determined, in such a way so as to satisfy the appropriate boundary conditions. Since \( G_x \) and \( G_z \) are similar, they will be determined first. \( G_x \) and \( G_z \) must satisfy the following

\[
(\nabla^2 + \kappa^2) G_x = -\delta(x-x') \delta(y-h) \delta(z-z') \quad (2.13a)
\]

\[
(\nabla^2 + \kappa^2) G_z = -\delta(x-x') \delta(y-h) \delta(z-z') \quad (2.13b)
\]

from which \( A_x \) and \( A_z \) may be calculated as follows.
\[ A_x(x, z) = \mu_0 \int G_x(x, z; x', z') J_x(x', z') dx' dz' \quad (2.14a) \]

\[ A_z(x, z) = \mu_0 \int G_z(x, z; x', z') J_z(x', z') dx' dz' \quad (2.14b) \]

Due to the similarity of (2.14a) and (2.14b), the expression for \( G_z \) will be determined first, then \( G_x \).

The boundary conditions that \( G_z \) must satisfy are:

\[ G_z = 0 \quad \text{at } y = 0, \; b \]
\[ G_z = 0 \quad \text{at } x = \pm a \]
\[ G_z(h^+) = G_z(h^-) \]

along with the boundary condition on \( \partial G_z / \partial y \) which will be given in equation (2.18c), further on in the derivation.

Expressions for the Green's functions will be obtained by a Fourier transform method. Assume a general solution for \( G_z \) of the form

\[ G_z = \frac{1}{2\pi} \sum_{n=1, 3, 5 \ldots} \int_{-\infty}^{\infty} \cos (u_n x) f_n(y) e^{i\omega y} d\omega \quad (2.15) \]

where \( u_n = n\pi x / (2a), \; n = 1, 3, 5 \ldots \). The \( x \) dependence for \( G_z \)
has been chosen as a Fourier series expansion in terms of \( \cos(u_n x) \), this ensures that the B.C.'s are satisfied at \( x = \pm a \). The Fourier transform of (2.13a) yields

\[
\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + (y)k_0^2 - \omega^2 \right) G_z = -\delta(x-x') \delta(y-h) e^{-j\beta z'}
\]  \hspace{1cm} (2.16)

The operator \( \frac{\partial^2}{\partial x^2} \) may be replaced with its eigenvalues, \( -u_n^2 \). If both sides of (2.16) are multiplied by \( \cos(u_n x) \) and integrated between \( \pm a \) the equation becomes

\[
a(\frac{\partial^2}{\partial y^2} + (y)k_0^2 - u_n^2 - \omega^2) f_n(y) = -\cos(u_n x') e^{-j\omega z'} \delta(y-h)
\]  \hspace{1cm} (2.17)

Assume the following solutions for \( f_n(y) \)

\[
f_n(y) = A_n \sin(p c) \sin(ly), \hspace{1cm} p^2 = k_0^2 - \omega^2 - u_n^2 \hspace{1cm} y < h \]  \hspace{1cm} (2.18a)

\[
f_n(y) = A_n \sin(l h) \sin(p(b-y)), \hspace{1cm} l^2 = k_0^2 - \omega^2 - u_n^2 \hspace{1cm} y > h \]  \hspace{1cm} (2.18b)

where the requirement that \( G_z(y) \) be continuous at the \( y = h \) interface has been enforced. If (2.17) is integrated over a vanishingly small interval in \( y \) centered at \( y = h \), the following expression results
\[
A_n = \cos(u_n x') e^{-j\omega z'} \cdot DZ_n^\ast \cdot \frac{a}{\pi} \left( \frac{\pi}{y^2 + h^2} \right) = -A_n (\sin(\pi h) \cos(p \gamma) + \sin(p \gamma) \cos(\pi h)) = -\cos(u_n x') e^{j\omega z'} (2.18c)
\]

which is the required boundary condition on \( \partial G_z / \partial y \).

Now, the coefficient \( A_n \) may be written as

\[
A_n = \cos(u_n x') e^{-j\omega z'} \cdot DZ_n \quad (2.19)
\]

where \( DZ_n = 1/a(\sin(\pi h) \cos(p \gamma) + \sin(p \gamma) \cos(\pi h)) \).

Substituting the expression for \( f_n(h) \) into (2.15) gives the desired expression for \( G_z \) which is

\[
G_z(x, z; x', z') = \frac{1}{2\pi} \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \cos(u_n x) \cos(u_n x') \cdot DZ_n \sin(p \gamma) \sin(\pi h) e^{-j\omega(z-z')} d\omega \quad (2.20)
\]

The derivation of \( G_x \) follows the same steps except for the \( x \) dependence, where \( \sin(u_n x) \) is used in place of \( \cos(u_n x) \), to satisfy the requirement that \( \partial E_x / \partial x = 0 \) at \( x = \pm a \). As a result, the Green's function \( G_x \) is

\[
G_x(x, z; x', z') = \frac{1}{2\pi} \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \sin(u_n x) \sin(u_n x') \cdot DZ_n \sin(p \gamma) \sin(\pi h) e^{-j\omega(z-z')} d\omega \quad (2.21)
\]
It still remains for \( G \), corresponding to \( \Phi \), to be determined. To do this, it is necessary to simultaneously solve for \( G \) and \( G_y \), since as (2.9) and (2.11) show, these equations are coupled. As in the case of \( G_x \) and \( G_z \), a Fourier transform solution will be used. Let

\[
A_y = \frac{1}{2\pi} \sum_{n=1,3}^{\infty} \int_{-\infty}^{\infty} \cos(u_x x) g_n(y) e^{-j\omega z} d\omega 
\]

(2.22a)

\[
\Phi = \frac{1}{2\pi} \sum_{n=1,3}^{\infty} \int_{-\infty}^{\infty} \cos(u_x x) f_n(y) e^{-j\omega z} d\omega 
\]

(2.22b)

where once again, the choice of expansion in \( x \) using \( \cos(u_x x) \) is made to satisfy the B.C.'s on \( \Phi \) and \( E_y \) at \( x = \pm a \). Substituting (2.22a) into (2.9) and (2.22b) into (2.11), and following the same procedure for the reduction of the \( x \) and \( z \) variables as done for \( G_z \) gives

\[
a(\frac{\partial^2}{\partial y^2} + \chi(y) k_0^2 - u_n^2 - \omega^2) g_n(y) = \\
\quad a' j \omega \mu \epsilon_0 \chi(1) f_n(h) \delta(y-h) e^{j\omega z'}
\]

(2.23a)

\[
a(\frac{\partial}{\partial y} \chi(y) \frac{\partial}{\partial y} + \chi^2(y) k_0^2 - \chi(u_n^2 + \omega^2)) f_n(y) = \\
- \frac{1}{\epsilon_0} \cos(u x') e^{j\omega z'} + j \omega \epsilon(1) g_n(h) \delta(y-h)
\]

(2.23b)
The $y$ dependence may be taken as

$$f_n(y) = A_n \sin(ly) \quad 0 \leq y \leq h$$

$$f_n(y) = A_n \sin(lh) \sin(p-b-y)/\sin(pc) \quad h \leq y \leq b \quad (2.24)$$

$$g_n(y) = B_n \cos(ly) \quad 0 \leq y \leq h$$

$$g_n(y) = B_n \cos(lh) \cos(p-b-y)/\cos(pc) \quad h \leq y \leq b \quad (2.25)$$

where $k^2 = x_n^2 - u_n^2 - \omega^2$, and $p^2 = k_n^2 - u_n^2 - \omega^2$. Note that the choice of $f_n(y)$ ensures that $\Phi(y) = 0$ at $y=0, b$, while that of $g_n(y)$ ensures that $\partial E_y/\partial y = 0$ at $y=0, b$, as it should, and that $\Phi(y)$ and $A(y)$ are continuous at $y=h$.

Integrating (2.23a) and (2.23b) across a vanishingly small interval in $y$ around $y=h$ gives

$$\left. \frac{\partial f_n}{\partial y} \right|_{h^+} = j\omega \mu_0 \varepsilon_0 (\varepsilon-1) f_n(h) \quad (2.26a)$$

$$a \kappa(y) \left. \frac{\partial g_n}{\partial y} \right|_{h^-} = j\omega a(\varepsilon-1) g_n(h) - \frac{1}{\varepsilon_0} \cos(u_n x') e^{j\omega z'} \quad (2.26b)$$

Substituting (2.24) and (2.25) into (2.26a) and (2.26b) and recalling that $\kappa(h^+)=1$ while $\kappa(h^-)=\varepsilon$ allows (2.26a) and (2.26b) to be solved for the unknown $B_n$ in terms of $A_n$ giving
\[
B_n = A_n \frac{j \omega \mu_0 \epsilon_0 (\pi - 1) \sin lh}{\cos lh \sin pc + l \sin lh \cos pc}
\]

Substituting \(B_n\) back into either equation in (2.26) gives the desired solution for \(A_n\)

\[
A_n = \frac{p \cos lh \sin pc + l \sin lh \cos pc}{a \epsilon_0 \cdot D1 \cdot D2} \cos (u_n x') e^{j \omega z'}
\]

(2.28)

where \(D1\) and \(D2\) in (2.28) are

\[
D1 = p \sin lh \cos pc + l \cos lh \sin pc
\]

(2.29)

\[
D2 = x p \cos lh \sin pc + l \sin lh \cos pc
\]

(2.30)

Using a partial fraction expansion, \(A_n\) can be expressed as the sum of two terms, the first with \(D1\) as its denominator, the second with \(D2\) as its denominator. Note that \(D1\)’s zeros correspond to LSE mode eigenvalues, while those of \(D2\) correspond to LSM eigenvalues.

If the numerator of (2.28) is called \(N\), then \(N/(D1 \cdot D2) = C1/D1 + C2/D2\), which can be solved for \(C1\) and \(C2\) as follows. Let \(N = C1(p \sin lh \cos pc + l \cos lh \sin pc) + C2(x p \cos lh \sin pc + l \sin lh \cos pc)\). This requires \(p = C2 \cdot l + C1 \cdot x \cdot p\), and \(l = C2 \cdot p + C1 \cdot l\). Solving for \(C1\) and \(C2\) gives
C2 = -\frac{(\gamma-1)p_1}{1^2 - \kappa p^2}  \\
C1 = -\frac{(\gamma-1)k_0^2}{1^2 - \kappa p^2}

For the LSE modes, the first root of D1 occurs for \( m=1 \), while for the LSM modes, the first root of D2 is for \( m=0 \). For the \( m=0 \) root \( I_0 \) is real and \( p \) is imaginary, so for \( m=0 \) wherever there is a \( \sin(p_0 c) \) it is replaced by \( j \sinh(p_0 c) \) and \( \cos(p_0 c) \) is replaced by \( \cosh(p_0 c) \). While not shown explicitly, it will be understood that this has been done in the material that follows. Regardless of the guide geometry or operating frequency, the \( m=0 \) LSM mode always exists.

Now, the Green’s function \( G \), evaluated at \( \gamma=h \), may be identified as

\[
G = \frac{1}{2\pi a} \sum_{n=1,3}^{\infty} \int_0^\infty \cos(u_n x) \cos(u_n x') \sinh(h) \sin(p c) e^{-j\omega(z-z')}. 
\]

\[
\left[ \frac{(\gamma-1)k_0^2}{(1^2 - \kappa p^2)D1} - \frac{(\gamma-1)p_1}{(1^2 - \kappa p^2)D2} \right] \cdot d\omega \tag{2.31}
\]

Equations (2.20), (2.21), and (2.31) are the Green’s functions, in the transform domain, for the dielectric loaded waveguide. However, the integrals remain to be evaluated. Note that the denominators of \( G_x \) and \( G_z \),
(2.21) and (2.20) respectively, are \( D_1 \).

The integrals over \( \omega \) may be evaluated using the theorem of residues, which says \( \oint f(z)dz = 2\pi j \cdot \text{residues} \). Since the integral is from \(-\infty\) to \(+\infty\), the contour may be closed at infinity in the upper half plane if \( z < z' \), or in the lower half plane if \( z > z' \). Let

\[
H_n(\omega) = \frac{(\chi-1)k_0^2}{(1^2-\lambda p^2)D_1} - \frac{(\chi-1)p_1}{(1^2-\lambda p^2)D_2}
\]

and let the residues of \( H_n(\omega) \) at \( \gamma_{nm} \) be \( R_{nm} \) and those at \( \bar{\gamma}_{nm} \) be \( \bar{R}_{nm} \), where

\[
\gamma_{nm} = (u_n^2 + l_m^2 - \lambda k_0^2) \quad \text{and} \quad \bar{\gamma}_{nm} = (u_n^2 + l_m^2 - \lambda k_0^2) \quad (2.32)
\]

are the values of \( \omega \) at which \( D_1(\omega) = 0 \) and \( D_2(\omega) = 0 \) respectively. \( R_{nn} \) and \( \bar{R}_{nn} \) are given by

\[
R_{nm} = \frac{\text{Num}(j\gamma_{nm})}{(\partial D_1/\partial \omega)|_{\omega=j\gamma_{nm}}} \quad \text{and} \quad \bar{R}_{nm} = \frac{\text{Num}(j\bar{\gamma}_{nm})}{(\partial D_2/\partial \omega)|_{\omega=j\bar{\gamma}_{nm}}}
\]

The \( \gamma_{nm} \) are the roots of \( D_1 \) in (2.29) while \( \bar{\gamma}_{nm} \) are the roots of \( D_2 \) in (2.30). Note that \( 1^2-\lambda p^2 = (\chi-1)(k_0^2 - p^2) \) in (2.31) so the residues at the poles \( p = \pm k_0; \ l = \pm \sqrt{\lambda} \cdot k_0 \) for the two terms cancel.

For \( z > z' \), \( \omega^2 = \lambda k_0^2 - u_n^2 - l^2 = k_0^2 - u_n^2 - p^2 \), \( \partial l/\partial \omega = -\omega/l \), and
\[ \frac{(\omega^2 - 1)k_0^2p_m^2 \sin(p_m c) \sin(l_m h)}{l_m^2 - \omega^2p_m^2} \gamma_{nm} \cdot \text{DR1} \]

where

\[ \text{DR1} = (\omega^2 - 1)k_0^2 \cos(l_m h) \sin(p_m c) + p_m^2 \sin(l_m h) \sin(p_m c) \cdot -p_m(c_1^2 + h^2) \cos(l_m h) \cos(p_m c) \]  

(2.33)

The second factor D2, gives the \( \tilde{R}_{nm} \) with \( \omega = j\gamma_{nm} \)

\[ \frac{\gamma_{nm}^{-3} \sin(p_m c) \sin(l_m h)}{\gamma_{nm}^{-2} - 1} \gamma_{nm} \cdot \text{DR2} \]

where

\[ \text{DR2} = (\omega^2 - 1)k_0^2 \sin(l_m h) \cos(p_m c) - p_m^2 \sin(l_m h) \cos(p_m c) \cdot + \left( p_m c + p_m^2 \right) \sin(l_m h) \sin(p_m c) \]  

(2.34)

The Green's function \( G \), evaluated at \( y = h \), may now be written as

\[ G = \sum_{n=1,3} \sum_{m=1,2} \cos(u_n x) \cos(u_n x') \cdot \frac{k_0^2}{k_0^2 - p_m^2} \frac{p_m^2 \sin(p_m c) \sin(l_m h)}{\gamma_{nm}^{-1}} e^{-\gamma_{nm} |z-z'|} - \gamma_{nm}^{-1} \frac{\gamma_{nm}^{-1} \sin(l_m h) \sin(p_m c)}{\gamma_{nm}^{-1}} e^{-\gamma_{nm} |z-z'|} \]  

(2.35)
It will be useful to define two new variables $F(m)$ and $\bar{F}(m)$ in (2.35), so that $G$ may be written as follows:

$$G = \frac{1}{a} \sum_{n} a \sum_{m} \cos(u_n x) \cos(u_n x') \cdot \frac{k_0^2 F(m)}{k_0^2 - p_m^2} e^{-\gamma_{nm}|z-z'|} - \frac{k_0^2 \bar{F}(m)}{k_0^2 - p_m^2} e^{-\bar{\gamma}_{nm}|z-z'|}$$  \hspace{1cm} (2.35)

and define $H(m)$ and $\bar{H}(m)$ as

$$H(m) = \frac{k_0^2 F(m)}{k_0^2 - p_m^2}$$ \hspace{1cm} (2.37a)

$$\bar{H}(m) = -\frac{\bar{F}(m)}{k_0^2 - p_m^2}$$ \hspace{1cm} (2.37b)

From here on the subscripts on the $\gamma$'s will be dropped in order to simplify the notation. The final desired expressions for the Green's functions are

$$G = \frac{1}{a} \sum_{n} a \sum_{m} \cos(u_n x) \cos(u_n x') \left[ \frac{H(m)}{\gamma} e^{-\gamma|z-z'|} + \frac{\bar{H}(m)}{\bar{\gamma}} e^{-\bar{\gamma}|z-z'|} \right]$$  \hspace{1cm} (2.38)
\[ G_z = \frac{1}{a} \sum_{n} \sum_{m} \cos(u_n x) \cos(u_n x') \frac{F(m)}{\gamma} e^{-\gamma |z-z'|} \tag{2.39} \]

\[ G_x = \frac{1}{a} \sum_{n} \sum_{m} \sin(u_n x) \sin(u_n x') \frac{F(m)}{\gamma} e^{-\gamma |z-z'|} \tag{2.40} \]

These are the required Green's functions for the dielectric loaded waveguide. From these, the potentials may be found using

\[ A_z = \mu_0 \int_{z'}^{z} \int G_z(x, z; x', z') J_z(x', z') dx' dz' \tag{2.41a} \]

\[ A_x = \mu_0 \int_{z'}^{z} \int G_x(x, z; x', z') J_x(x', z') dx' dz' \tag{2.41b} \]

\[ \varepsilon_0 \Phi = \int_{z'}^{z} \int G(x, z; x', z') \rho(x', z') dx' dz' \tag{2.41c} \]

(2.2) DESCRIPTION OF SOURCES

In this section, the dominant mode and perturbed sources used in the dynamic source reversal method will be presented and discussed. This will begin with a description of the dominant mode charge and current sources, followed by the perturbed sources. Reasons for the particular choices of these representations will be
given.

An accurate expression for the $x$ dependence of the dominant propagating charge and current waves on the microstrip may be given by (see (1.19), (1.20)) [63]

$$J_{0z}(x) = \left[ I_0 - I_1 T_2(x/w) + I_2 T_4(x/w) \right] \cdot \left[ 1 - (x/w)^2 \right]^{-1/2}$$  \hspace{1cm} (2.42)

$$p_0(x) = \left[ Q_0 - Q_1 T_2(x/w) + Q_2 T_4(x/w) \right] \cdot \left[ 1 - (x/w)^2 \right]^{-1/2}$$  \hspace{1cm} (2.43)

where the edge conditions are built into the expressions, and $T_n(x/w)$ is the $n$th Chebychev polynomial. In the dynamic source reversal method, the initial values for the $I_i$ and the $Q_i$ ($i=0,1,2$) are taken from [63] for a given dielectric constant and frequency. However, since the technique described in [63] does not use a top cover, and uses fixed sidewalls, a perturbation technique has been developed to determine the proper amplitudes for the $I_i$ and $Q_i$, as well as to find the effective dielectric constant for a given waveguide geometry. With this information the propagation constant $\beta$ can be determined.

There is no dominant mode transverse current $J_x$ assumed on the strip, since its effect is negligible for a wide range of useful geometries [64]. When a
component of \( J_x \) is required, such as for the case of an asymmetrical gap, it will be included as a perturbation term, localized near the discontinuity. It will be described in the appropriate sections.

The sources given in (2.42) and (2.43) may be transformed by a change of variable \( x = w \sin \Theta \), into the following expressions

\[
J_{0z}(\Theta) = (I_0 + I_1 \cos(2\Theta) + I_2 \cos(4\Theta)) / \cos \Theta \tag{2.44}
\]

\[
\rho_0(\Theta) = (Q_0 + Q_1 \cos(2\Theta) + Q_2 \cos(4\Theta)) / \cos \Theta \tag{2.45}
\]

The sources are expressed by a Fourier series expansion

\[
J_z = \sum_{n=1}^{\infty} J_{zn} \cos(u_n x) \quad J_{zn} = \frac{1}{a} \int_{-w}^{w} J_z(x) \cos(u_n x) \, dx \tag{2.46}
\]

and for the charge

\[
Q = \sum_{n=1}^{\infty} Q_n \cos(u_n x) \quad Q_n = \frac{1}{a} \int_{-w}^{w} Q(x) \cos(u_n x) \, dx \tag{2.47}
\]

When the substitution \( x = w \sin \Theta \) is used, \( J_{zn} \) and \( Q_n \) become

\[
J_{zn} = \frac{2w}{a} \sum_{n=1}^{\pi/2} I_i \int_{0}^{\cos(2i\Theta) \cos(u_n \sin \Theta) \, d\Theta} \tag{2.48}
\]
\[ Q_n = \frac{2w}{a} \sum_{i=0}^{\pi/2} \int_0^{\pi/2} \cos(2i\theta) \cos(u_n \sin \theta) d\theta \]  \hspace{1cm} (2.49)

Since the integrals are used many times, they are evaluated once and stored as

\[ P_{ni} = 2w \int_0^{\pi/2} \cos(2i\theta) \cos(u_n \sin \theta) d\theta \]  \hspace{1cm} (2.50)

This integral has a closed form solution

\[ P_{ni} = (\pi/2) J_{2i}(n\pi w/2a), \]  where \( J_{2i} \) is the Bessel function of order \( 2i \). Rather than trying to evaluate the \( P_{ni} \) by a Simpson's rule integration, it turns out to be easier to calculate them by means of a backward recursion formula for the Bessel functions. In all that follows \( n \) is taken to go from \( n=1,3,\ldots,199 \) and \( i=0,1,2 \). The recursion uses the relationship \( Z_{n-1}(x) = (2n/x)Z_n(x) - Z_{n+1}(x) \) for the Bessel functions.

The perturbation technique used here to include the effects of a top plate follows closely the technique described by Collin [64]. In this approach, the potential \( \Phi(y=h) \) is set equal to \( 1/\varepsilon_0 \) on the strip. As a result, the potential is an absolute value. From the integration of the continuity equation, \( \nabla \cdot J = -j\omega p \) across the strip, the relationship \( \beta I_{TOT} = \omega Q \) is obtained,
where $I_{TOT}$ is the total $z$ directed current on the line and $Q$ is the total charge on the line. As a result, the vector potential $A_{zr}$ is a reference value relative to the relation $\beta I_{TOT} = \omega Q$. From the boundary condition $E_z = 0$ on the strip, we can obtain the relationship $\omega A_z = \beta \Phi$.

Since an infinite line is being considered, the sources exist for $z'$ from $-\infty$ to $+\infty$.

The expressions for $J_z$ and $\rho$ are

$$J_{zr}(x', z') = \sum_{i=0}^{2} I_i \frac{\cos(2i\theta')}{\cos \theta'} e^{-i\beta z'}$$

(2.51)

$$\rho(x', z') = \sum_{i=0}^{2} Q_i \frac{\cos(2i\theta')}{\cos \theta'} e^{-i\beta z'}$$

(2.52)

where the relative current, $J_{zr}$ is related to the true current by the factor $J_z = (\beta c/k_0) J_{zr}$, where $c = 1/\sqrt{\mu_0 \varepsilon_0}$. Similarly, $A_z = (\beta c/k_0) A_{zr}$ and $A_x = (\beta c/k_0) A_{xr}$. The charge amplitudes are chosen to be absolute values since $\Phi$ is an absolute value. This allows the effective dielectric constant $\beta/k_0 = \kappa_e$ to be obtained from $Q_0/I_0$ by using the relationships $\beta \Phi = \omega A_z$ and $\beta I_{TOT} = \omega Q$. This is due to the fact that $I_{TOT} = \pi I_0$ and $Q = \pi Q_0$ [63].

For an infinite line the potentials may be written as follows
\[ A_{z} = 2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G_z(x, z; x', z') J_{z'}(x', z') \, dx' \, dz' \]  
\hspace{1cm} (2.53a)

\[ \varepsilon_0 \Phi = 2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(x, z; x', z') \rho(x' z') \, dx' \, dz' \]  
\hspace{1cm} (2.53b)

Referring to (2.38) and (2.39) for the \( z' \) dependence

\[ \int_{-\infty}^{\infty} e^{-\gamma |z-z'|} e^{-j\beta z'} \, dz' = \frac{2\gamma}{\gamma^2 + \beta^2} e^{-j\beta z} \]  
\hspace{1cm} (2.54a)

\[ \int_{-\infty}^{\infty} e^{-\overline{\gamma} |z-z'|} e^{-j\beta z'} \, dz' = \frac{2\overline{\gamma}}{\overline{\gamma}^2 + \beta^2} e^{-j\beta z} \]  
\hspace{1cm} (2.54b)

The \( e^{-j\beta z} \) dependence may be dropped since its presence does not affect the solution. Substituting (2.51) and (2.52) into (2.53a) and (2.53b) respectively, along with the \( x, x' \) dependence of \( G \) from (2.38) and \( G_z \) from (2.39), and using \( x = ws \sin \theta \) with (2.50) gives

\[ \frac{4w}{a} \sum_{n} \sum_{m} \sum_{l} I_{l} P_{n_{l}} \cos(u_{n_{l}} ws \sin \theta) \frac{F(m)}{\gamma^2 + \beta^2} = 1 \]  
\hspace{1cm} (2.55a)

\[ \frac{4w}{a} \sum_{n} \sum_{m} \sum_{l} Q_{l} P_{n_{l}} \cos(u_{n_{l}} ws \sin \theta) \left[ \frac{H(m)}{\gamma^2 + \beta^2} + \frac{\overline{H(m)}}{\overline{\gamma}^2 + \beta^2} \right] = 1 \]  
\hspace{1cm} (2.55b)

The Galerkin method is now used to solve these two
equations. Multiplying both sides of (2.55a) and (2.55b) by \cos(2j\theta) and integrating over \theta results in

\[
\frac{4w}{a} \sum I_i \sum P_{ni} \sum m \frac{F(m)}{\gamma^2 + \beta^2} P_{nj} = \delta_{0j} \frac{\pi}{2} \tag{2.56a}
\]

\[
\frac{4w}{a} \sum Q_i \sum P_{ni} \sum m \left[ \frac{H(m)}{\gamma^2 + \beta^2} \right] P_{nj} = \delta_{0j} \frac{\pi}{2} \tag{2.56b}
\]

where \(\delta_{0j} = 1\) for \(j = 0\) and 0 for \(j > 0\).

Equations (2.56a,b) are solved for the desired value of \(\beta\) for the enclosed microstrip by an iteration technique. On the first pass, the value of \(\beta\) used in the equations is the value obtained for a microstrip with no top plate from [64]. Using this value of \(\beta\), (2.56a,b) are solved for \(I_i\) and \(Q_i\), the dominant mode current and charge amplitudes for an enclosed microstrip. A new estimate of \(x_e\) is obtained from the relationship \(x_e = Q_0/I_0\), from which a new \(\beta\) may be calculated from \(\beta^2 = x_e^2 \cdot k_0^2\). If the new value of \(x_e\) differs from the old value of \(x_e\) by less than 1%, the iteration is terminated, and the most recent values of \(\beta\), \(I_i\), and \(Q_i\) are used as the propagation constant and dominant mode current and charge amplitudes respectively, for the enclosed microstrip.

If the new value of \(x_e\) differs from the old value by more than 1%, the iteration is repeated, this time
using the latest value for $\beta$ as determined from the latest value of $\gamma$. Typically, a converged value for $\gamma$ is obtained in two or three passes. This is due to the fact that the original estimate for $\beta$ from [64] is usually quite good, since the top plate location is typically 10 times the substrate height, so that the presence of a top plate has little effect on the fields associated with the microstrip under consideration.

Note that the double sums over $n$ and $m$, involving terms like $P_{ni}P_{nj}/(\gamma^2+\beta^2)$, will converge, but very slowly. The asymptotic value of $P_{ni}P_{nj}$ will decay as $1/n$ but $n$ must be quite large relative to the order of the Bessel function involved. These terms represent the dominant parts of $G$ and $G_z$ which arise when the field and source points coincide. Fortunately, these terms may be efficiently summed into closed form. The details are described in Appendix 1.

This completes the determination of the dominant mode sources and propagation constant for the enclosed microstrip. The perturbed sources are discussed next. The total charge and current distribution on the microstrip was given in (1.19) and (1.20), where $J_{1z}$ and $\rho_1$ represent the perturbation in the dominant mode current and charge due to the presence of a discontinuity. As Fig. (2.1) shows, triangle functions
(a) Current triangles $T(z')$

(b) Charge pulses $P_k(z')$

Fig. (2.1) Current and charge expansion functions. Create the required matrix elements from these elements.
were chosen for expanding the perturbed current, while unipolar pulses were chosen for the charge expansion. From the unipolar pulses, bipolar pulses are formed to represent the perturbed charge on the line. Unipolar pulses are used in the calculations for their simplicity in calculating the fields arising from the charge pulses on the line. The discontinuity is chosen to be located at $z=z'=0$.

There is one charge pulse associated with each current triangle, bipolar pulses were chosen since they satisfy the continuity equation $\nabla \cdot J=-j\omega p$. For $J_x=0$ this becomes $\partial J_z/\partial z= -j\omega p$, and the derivative of a triangle will give a bipolar pulse. The perturbed sources may be expressed as (the index $i$ goes from 0 to 2, as before)

$$J_{1zr}(\theta', z') = j(1+R) \sum_{i}^{K} C_{ki} \frac{\cos(2i\theta')}{\cos\theta} T_k(z')$$

$$\rho_1(\theta', z') = (1+R) \sum_{i}^{K} D_{ki} \frac{\cos(2i\theta')}{\cos\theta} P_k(z')$$

(2.57a) (2.57b)

where the transformation $x=wsin\theta$ has been used. $C_{ik}$ and $D_{ik}$ are unknown amplitudes, and $(1+R)$ is the same scale factor that appears in (1.21) and (1.22) for the dominant modes. The number of expansion modes used in $x'$ (or $\theta'$) is three, the same as for the dominant mode. The number of pulses used to expand in $z'$, namely $K$, is
arbitrary, although the more pulses used, the better is the approximation.

The discontinuity will introduce a $1/|z|^{1/2}$ singularity in the charge distribution $\rho_1$ near the open end. As a result of this singularity, the expansion pulses that are used to represent the charge near the open end should have very narrow pulse widths near the open end. Farther away from the open end, the effect of the discontinuity are less significant, and so wider pulses would suffice. To satisfy these contradictory requirements on pulse width, the basic pulse width was chosen to give an accurate representation of the charge near the open end, while farther down the line, these pulses are compressed, in groups of three or five, to give broader pulse widths.

As a result of computations carried out for an air dielectric open circuit line, it was found that the total number $K$ of required pulses, was at least 27, compressed down to 12, in order to obtain converged values of $B_{in}$ when the unit pulse width was taken to be .1h, where $h$ is the substrate height. A further discussion of the pulse widths and the compression technique used will be discussed in Chapter 3 and 4.

While a greater number of pulses leads to a more accurate representation of the effects of the
discontinuity on the charge near the open end, it also leads to an increased matrix size. As a result, there is a trade off as to the number of expansion pulses used and the array size and fill time. It should be noted that 27 basic pulses are used since the calculation of matrix elements is simplified if there is no change in the pulse width along the line. Compressed pulses may be created by appropriate weighted summing of rows and columns in the coefficient array.

With each expansion mode in \( x \), there is associated a number of expansion pulses used along \( z \). The number of expansion pulses in \( z \) is not the same for each mode in \( x \). For the \( i=0 \) mode in \( x \), a total of 27 pulses in \( z \) was used, while for modes 1 and 2 \((i=1,2)\) only 5 pulses were used. This acknowledges the fact that the \( i=0 \) mode is the most important in describing the dominant charge or current mode on the line. The higher order modes, \( i=1 \) or 2, are generally 10 and 100 times smaller in amplitude, respectively relative to the \( i=0 \) mode, and so they represent small corrections to it.

For the \( i=0 \) mode, the 27 pulses are compressed into 12, while the five pulses associated with modes 1 and 2 are uncompressed. This compression technique allows for the creation of wider pulses as the distance away from the junction is increased, where the effects of the
discontinuity are less pronounced.

In the absence of a $J_x$ mode, the $D_{ki}$ are related to the $C_{ki}$ by the continuity equation. Since $J_{zr}$ is a relative value, $\partial J_z/\partial z = -j\omega \rho$ may be expressed as

$$j\beta cC_{ki} P_k(z') = -j\Delta k_0^2 c D_{ki} P(z')$$

or

$$D_{ki} = -\frac{\beta}{k_0^2 \Delta} C_{ki}$$

(2.58)

(2.59)

So when there is no $J_x$ the number of unknowns is reduced by a factor of two.

With the charge and current sources so defined, and assuming that the discontinuity is located at $z=0$, the expressions for the potentials $A_z$ and $\Phi$ as given by (1.23) and (1.24) may be expressed as

$$A_z = \mu_0 \int_{-\infty}^{\infty} J_{1z} G_z dx'dz' -$$

$$-\Phi$$

$$j\mu_0 \int_{-\infty}^{\infty} (B_{in} \cos(\beta z') - \sin(\beta z')) J_{0z} G_z dx'dz'$$

(2.60a)

and
\[
\varepsilon_0 \Phi = \int_{-\infty}^{\infty} \rho_1 G dx' dz' - \int_{-\infty}^{\infty} (B_{in} \sin(\beta z') + \cos(\beta z')) \rho_0 G dx' dz' 
\]

(2.60b)

The boundary condition that \( E_z = 0 \) on the strip may now be enforced to give \(-j\omega A_z - \partial \Phi / \partial z = 0\), or

\[
-B_{in} \int_{0}^{\infty} \int_{-w}^{w} [\omega \mu_0 \cos(\beta z')] J_0 G_z - \frac{1}{\varepsilon_0} \sin(\beta z') \rho_0 \frac{\partial G}{\partial z} dx' dz' = 
\]

\[
-\int_{-\infty}^{0} \int_{-w}^{w} [j\omega \mu_0 J_{1z} G_z + \frac{1}{\varepsilon_0} \rho_0 \frac{\partial G}{\partial z}] dx' dz' = 
\]

\[
= \int_{0}^{\infty} \int_{-w}^{w} [\omega \varepsilon_0 \sin(\beta z') J_0 G_z + \frac{1}{\varepsilon_0} \cos(\beta z') \rho_0 \frac{\partial G}{\partial z}] dx' dz' 
\]

(2.61)

Note that \( B_{in} \), the normalized input susceptance which appears in (2.61), represents the factor \(-j(1-R)/(1+R)\). The above integral equation, which is actually for an open end, can be reduced to a matrix equation which can be solved for \( B_{in} \), by applying the method of moments. The integrals over the sources, \( x' \) and \( z' \), may be
carried out analytically, leaving the expression in terms of \( x \) and \( z \), the field points. Suitable testing functions in \( x \) and \( z \) will reduce (2.61) to a set of equations of the form \([A] \cdot [x] = [y]\), where \([x]\) is the unknown amplitude vector, \([y]\) is the source vector and \([A]\) is the coefficient array.

By a suitable choice of testing functions, the convergence of various sums involved in filling the coefficient array may be greatly improved. Since the expansion of the fields inside the waveguide is carried out in terms of the evanescent modes of the waveguide, most terms will have an exponential decay. Those terms arising when the source and field points coincide will have no exponential decay, but they can be efficiently summed into closed form, as will be shown.

The \( x \) dependence of these expressions is of the form \( \cos(u_n x) \), or \( \cos(u_n w \sin \theta) \). When the dominant mode charge and current amplitudes were calculated for an for an infinite line, the testing functions used were \( \cos(2j \theta)/\cos \theta \), i.e., Chebychev polynomials with the edge condition built in. Thus, the testing integrals were the \( P_{nj} \) of (2.50). Testing with the edge condition, however, places a greater emphasis on the field near the edge of the strip, and de-emphasises the rest. As a result, the edge condition was dropped, and the
\[ \cos(u_n w \sin \theta) \] are tested with just the Chebychev polynomials, as follows

\[ PP_{nj} = 2w \int_0^{\pi/2} \cos(u_n w \sin \theta) \cos(2j\theta) \cos \theta d\theta \]  \hspace{1cm} (2.62)

Figure (2.2) shows the testing pulses in \( z \). They are rectangles, offset by a \( \Delta /2 \) relative to the expansion pulses. The charge pulses are shown here as half pulses for the sake of clarity, and because the integrals involving the charge pulses are more easily carried out using half pulses. Bipolar pulses can be easily formed by adding adjacent pulses, with the appropriate sign.

The test pulses are offset since the field arising from a bipolar charge pulse is symmetric about its center, as shown in Fig. (2.3). If the testing pulses were centered as shown in Fig. (2.3c), the result would be nearly zero. But an offset pulse, like that shown in Fig. (2.3d) tests near a field maximum. This is especially important, since it is the overlapping source and test pulses that produce the largest contributions to the coefficient array.

When testing is done on a non overlapping pulse, the exact location of the test pulse is no longer
important, but to be consistant, and to insure that testing is performed over continuous sections of the line, offset pulses are used in all cases.

In the next chapter, each of the discontinuities to be characterized will be described in detail, using the material presented in this chapter as a starting point.
Fig. (2.2) Testing pulses used with the expansion pulses. Half pulses are shown for the charge pulses for the sake of clarity.
(a) Potential due to a half charge pulse, $\phi(z)$.

(b) Electric field due to a half charge pulse, $\partial \phi/\partial z$.

(d) Electric field from a bipolar charge pulse, along with a centered and offset testing pulse

Fig. (2.3) Fields from charge pulses.
CHAPTER 3- DISCONTINUITIES

In this chapter the equations for the open end and the gap will be developed in detail. Section (3.1) will describe the equations for the open end while section (3.2) will cover the gap with the assumption that $J_x$ is negligible. Section (3.3) will include the effects of $J_x$ in the characterization of the asymmetrical gap.

(3.1) OPEN END

The first discontinuity to be considered is the open end, or stub. The open end is taken to occur at $z=0$. To recap, the Green's functions involved are

$$G_z = \frac{1}{a} \sum_n \sum_m F(m) \phi_n(x,x') \frac{e^{-\gamma|z-z'|}}{\gamma_{nm}}$$

$$G = \frac{1}{a} \sum_n \sum_m \left[ \frac{H(m) e^{-\gamma|z-z'|}}{\gamma_{nm}} + \frac{\overline{H(m)} e^{-\gamma|z-z'|}}{\gamma_{nm}} \right] \phi_n(x,x')$$

where $\phi_n(x,x') = \cos(u_n x) \cos(u_n x')$, and

$$H(m) = \frac{k_0^2 F(m)}{k_0^2 - p_m^2} \quad ; \quad \overline{H(m)} = \frac{\overline{F(m)}}{k_0^2 - p_m^2}$$

$$F(m) = \frac{1}{R} \frac{p^2 \sin^2(lh) \sin^2(pc)}{Dl}$$
\[ D_1 = l^2 p ( \sin^2(p c) + \sin^2(l h)) - .5 \cdot k_0^2 \sin^2(l h) \sin^2(p c) \]

\[ \bar{F}(m) = \frac{p^2 I^2 \sin(p c) \sin(l h)}{D2} \]

\[ D2 = (I^2 c + \chi h p^2) \sin(p c) \sin(l h) - \bar{p} \cos(p c) \sin(l h) - \]
\[ - \bar{I} \sin(p c) \cos(l h) - \bar{p} l (\chi c + h) \cos(p c) \cos(l h) \]

and it is understood that the \( l, p, \bar{I}, \) and \( \bar{p} \) have the index \( m, \) and \( \gamma \) and \( \bar{\gamma} \) have the indicies \( n, m \) even though not explicitly shown.

The potentials for the open end structure are

\[ A_z = \mu_0 \int_{-\infty}^{0} \int_{-w}^{w} G_z J_z dx' dz' \quad (3.1a) \]

\[ \varepsilon_0 \Phi = \int_{-\infty}^{0} \int_{-w}^{w} G \rho dx' dz' \quad (3.1b) \]

where \( J_z \) and \( \rho \) are the total current and charge on the line, dominant mode plus perturbation terms. It has been assumed that there is no dominant mode \( J_x, \) which is a good approximation for most microstrips of practical interest.

The total current \( J_z \) may be expressed in terms of the relative current and potential by \( J_z = (\beta c / k_0) J_{zr} \)
and \( A_z = (\beta c/k_0) A_{zr} \).

The boundary condition on the strip is \( E_z = 0 = -j\omega A_z \frac{\partial \Phi}{\partial z} \). This gives \(-(j\omega\beta c/k_0) A_{zr} = \frac{\partial \Phi}{\partial z}, \) or

\[
\begin{align*}
0 &= \int_{-\infty}^{0} \int_{-W}^{W} G_z J_{zr} \, dx' \, dz' + \int_{0}^{0} \int_{-\infty}^{-W} G \rho dx' \, dz' = 0 \\
0 &= \int_{-\infty}^{0} \int_{-W}^{W} G_z J_{zr} \, dx' \, dz' + \int_{0}^{0} \int_{-\infty}^{-W} G \rho dx' \, dz' = 0
\end{align*}
\]

(3.2)

The sources for the open end are

\[
\begin{align*}
J_{zr}(x', z') &= j(1+R) J_z(x') [B_{i\infty} \cos(\beta z') - \sin(\beta z')] + \\
J_{z1}(x', z')
\end{align*}
\]

(3.3a)

\[
\rho(x', z') = (1+R) \rho(x') [B_{i\infty} \sin(\beta z') + \cos(\beta z')] + \rho_1(x', z')
\]

(3.3b)

where, as before \( J_z(x') \) and \( \rho(x') \) may be expressed as \( J_z(\theta) \) and \( \rho(\theta) \) to give

\[
\begin{align*}
J_{zr}(\theta') &= \sum_{i \neq 0} I_i \cos(2i\theta') / \cos \theta' \\
\rho(\theta') &= \sum_{i \neq 0} Q_i \cos(2i\theta') / \cos \theta'
\end{align*}
\]

(3.4a)

(3.4b)

\[
\begin{align*}
J_{z1r}(\theta', z') &= j(1+R) \sum_{i \neq 0} \sum_{k=1}^{K} C_{ki} \frac{\cos(2i\theta')}{\cos \theta'} T_k(z') \\
\rho_1(\theta', z') &= (1+R) \sum_{i \neq 0} \sum_{k=1}^{K} D_{ki} \frac{\cos(2i\theta')}{\cos \theta'} P_k(z')
\end{align*}
\]

(3.4c)

(3.4d)
where $T_k(z')$ and $P_k(z')$ are the triangle and pulse functions as previously defined.

The boundary conditions on the stub, (3.2), may now be written as

$$
0 \leq w \int_{-\infty}^{0} \int_{-w}^{0} G_z J_{1z'} dx'dz' + \frac{\partial}{\partial z} \int_{-\infty}^{0} G_0 dx'dz' \\
-0 \leq w \int_{0}^{\infty} \int_{-w}^{0} G_z (jB_{ln} \cos(\beta z') - j\sin(\beta z')) J_{1z'} dx'dz' \\
-0 \leq w \int_{0}^{\infty} \int_{0}^{-w} G(B_{ln} \sin(\beta z') + \cos(\beta z')) \rho dx'dz' = 0 \quad (3.5)
$$

The source for this equation consists of the known dominant mode terms, which are given in terms of the dominant mode amplitudes for the semi-infinite line, starting at $z'=0$, and extending to $z'=\infty$. These source terms are

$$
\text{Source} = \int_{0}^{\infty} \int_{-w}^{0} G_z \sin(\beta z') J_{1z'} dx'dz' + \int_{0}^{\infty} \int_{0}^{-w} G_0 \cos(\beta z') dx'dz' \quad (3.6)
$$
The integrals over $x'$ have already been defined in (2.50) as

$$P_{ni} = 2w \int_0^{\pi/2} \cos(u_n \sin \theta') \cos(2i\theta') d\theta'.$$

The dominant mode integrals over $z'$ in (3.5) and (3.6) may also be evaluated to give

$$\int_0^\infty e^{-\gamma |z-z'|} \cos(\beta z') dz' = \frac{Ye^{yz}}{\gamma^2 + \beta^2}$$  \hspace{1cm} (3.7a)

$$\int_0^\infty e^{-\gamma |z-z'|} \sin(\beta z') dz' = \frac{Be^{yz}}{\gamma^2 + \beta^2}$$  \hspace{1cm} (3.7b)

Integrals involving $\gamma$ give the same results except that $\gamma$ replaces $\gamma$. Note that for these integrals, the source points $z'$ are always greater than or equal to zero, while for the field points, $z \leq 0$.

Equations (3.5) and (3.6) can now be written as

$$-\beta \sum_i \sum_k C_{ki} \sum_n \sum_m \frac{F(m)}{\gamma} P_{ni} \cos(u_n x) \int_0^\infty e^{-\gamma |z-z'|} T_k(z') dz' +$$

$$\sum_i \sum_k D_{ki} \sum_n \sum_m P_{ni} \cos(u_n x).$$
\[
\frac{\partial}{\partial z} \left[ \mathcal{H}(m) \int_{-\infty}^{0} e^{-\gamma |z-z'|} \mathcal{P}_k(z') dz' + \frac{\mathcal{H}(m)}{\gamma} \int_{-\infty}^{0} e^{-\gamma |z-z'|} \mathcal{P}_k(z') dz' \right] + B_{in} \beta \left[ \sum_i \sum_n \sum_m P_n \cos(u_n x) \frac{F(m)}{\gamma^2 + \beta^2} e^{\gamma z} - \sum_i Q_i \sum_n \sum_m P_n \cos(u_n x) \left[ \frac{H(m)}{\gamma^2 + \beta^2} e^{\gamma z} + \frac{\overline{H}(m)}{\gamma^2 + \beta^2} \overline{e^{\gamma z}} \right] \right]
\]

\[
= \left[ \beta^2 \sum_i \sum_n \sum_m \frac{F(m)}{\gamma(\gamma^2 + \beta^2)} e^{\gamma z} + \sum_i Q_i \sum_n \sum_m \left[ \frac{\gamma H(m)}{\gamma^2 + \beta^2} e^{\gamma z} + \frac{\gamma \overline{H}(m)}{\gamma^2 + \beta^2} \overline{e^{\gamma z}} \right] P_n \cos(u_n x) \right]
\]

(3.8a)

From here on, the sums over \(n\) and \(m\) will be understood, and so the summation symbols will be left out.

Testing the dominant mode \(B_{in}\) and source terms may now be done quite easily. Let

\[
E_s(n,m) = E_s = \int_e^{\gamma z} dz = \frac{1}{\gamma} \left( e^{-(2s-1)\gamma \Delta/2} - e^{-(2s+1)\gamma \Delta/2} \right)
\]

\[-(2s+1)\Delta/2 \]

(3.8b)

and likewise

\[
\overline{E_s} = \frac{1}{\gamma} \left( e^{-(2s-1)\gamma \Delta/2} - e^{-(2s+1)\gamma \Delta/2} \right)
\]

(3.8c)
For simplicity, let \( DK = 1/k_0^2 \Delta \), while \( \beta \) will remain explicit in the expressions. This is the factor that relates the charge amplitudes, the \( D_{ki} \), to the current amplitudes, the \( C_{ki} \), when there is no \( J_x \) on the line.

At the open end, the higher order mode currents must cancel the dominant mode current, so as to assure that the total current at the open end is zero. This may be expressed as

\[
I_i B_{in} + C_{1i} = 0 \quad (3.9a)
\]

or

\[
C_{1i} = -B_{in} I_i \quad (3.9b)
\]

For testing in \( x \), the unweighted Chebychev polynomials are used, and the resulting integrals are given in equation (2.62) as

\[
\pi/2
P_{nj} = 2w \int_0^{\pi/2} \cos(u_n \omega \sin \theta) \cos(2j\theta) \cos \theta d\theta
\]

Offset pulses are used for testing in \( z \), and are given the label \( P_s(z) \), for \( s = 1 \) to \( K \). A sufficient number of testing pulses are used so as to generate as many linear equations as there are unknowns to be solved for. Note that \( B_{in} \), the desired normalized input susceptance appears explicitly as an unknown.
The result of testing in $z$ are double integrals of the form

$$
\int_{s_1}^{s_2} e^{-\gamma |z-z'|} T_k(z') P_s(z) dz' dz
$$

for currents: $C_{k_s} = \int_{s_1}^{s_2} e^{-\gamma |z-z'|} P_k(z') P_s(z) dz' dz$

where $s_1 = -(2s+1)\Delta/2$ and $s_2 = -(2s-1)\Delta/2$.

For current triangles, refer to Fig. (3.1). Figure (3.2) shows the $k=1$ half pulse at the open end, along with several test pulses, $s=1, 2, 3$. The electric field from a half triangle must be calculated for two cases;

**case 1**: $z < -\Delta$, the field point lies outside of the triangle, so

$$
e^{\gamma z} \int_{-\Delta}^{0} e^{-\gamma z'} T_1(z') dz' = -\frac{e^{\gamma z}}{2} \left[ \gamma \Delta + 1 - e^{\gamma \Delta} \right] \quad (3.10a)
$$

**case 2**: $-\Delta < z < 0$

$$
e^{-\gamma z} \int_{-\Delta}^{0} e^{\gamma z} T_1(z') dz' + e^{\gamma z} \int_{-\Delta}^{z} e^{-\gamma z'} T_1(z') dz' =
\frac{2T_1(z)}{\gamma} - \frac{e^{\gamma z}}{\gamma} + \frac{e^{-\gamma z - \gamma \Delta} - e^{\gamma z}}{\gamma^2 \Delta} \quad (3.10b)
$$
Fig. (3.1). Half triangle \((k=1)\) with first four test pulses \((s=1 \text{ to } 4)\).

Fig. (3.2). Full current triangle \((k=3)\) with five test pulses \((s=1 \text{ to } 5)\).

Fig. (3.3). Electric field from a current triangle.
The matrix element $CI_{11}$ may now be obtained by integrating over $z$ between $-1.5\Delta$ and $-0.5\Delta$ to give

$$CI_{11} = \frac{d}{2\gamma} + \frac{1}{\gamma^2} \left[ e^{-3\gamma d} - e^{-\gamma d} \right] + \frac{1}{2\gamma^3 d} \left[ 2 - 3e^{-\gamma d} + e^{-3\gamma d} \right]$$  \hspace{1cm} (3.11)$$

$CI_{12}$ can be integrated to give

$$CI_{12} = \frac{1}{\gamma^2} \left[ e^{-5\gamma d} - e^{-3\gamma d} \right] + \frac{1}{2\gamma^3 d} \left[ e^{-\gamma d} - 2e^{-3\gamma d} + e^{-5\gamma d} \right]$$ \hspace{1cm} (3.12)$$

In $CI_{11}$ and $CI_{12}$, $\Delta=2d$ was used. It should also be noted that there will be an extra $1/\gamma$ factor that will multiply all these terms which arises from the Green's function. Also, there are corresponding $\bar{\gamma}$ terms in all cases. All the remaining $C_{1s}$, $s > 2$, may be obtained from $CI_{12}$ by multiplying $CI_{12}$ by $e^{-2(s-2)\gamma d}$.

The electric field from a current triangle is shown in Fig. (3.3), note that it is symmetric about the apex of the triangle. As a result, testing to the left gives the same result as testing to the right for equal distances from the triangle. In Fig. (3.2) this means that $CI_{31} = CI_{35}$. For the triangle shown in Fig. (3.2), the electric field may be calculated as follows; for $z < -4\Delta$
\[ e^{\gamma z} \int_{-\gamma z}^{\gamma z} T_3(z') dz' = \frac{e^{-\gamma z}}{\gamma^2 \Delta} (e^{4\gamma \Delta} + e^{2\gamma \Delta} - 2e^{3\gamma \Delta}) \]  

(3.13a)

while for \( z > -2\Delta \) replace \( \gamma \) with \(-\gamma\) to get

\[ E(z) = \frac{e^{-\gamma z}}{\gamma^2 \Delta} (e^{-4\gamma \Delta} + e^{-2\gamma \Delta} - 2e^{-3\gamma \Delta}) \]  

(3.13b)

When \( z \) lies between \(-4\Delta\) and \(-3\Delta\)

\[ E(z) = \frac{2T(z) + \frac{1}{\gamma}}{2\gamma^2 \Delta} (e^{-\gamma z - 4\gamma \Delta} + e^{\gamma z + 2\gamma \Delta} - 2e^{\gamma z + 3\gamma \Delta}) \]  

(3.14)

To calculate a self matrix element such as \( CI_{33} \), use (3.14) to get

\[ 2 \int E(z) dz = \frac{3d}{\gamma} \frac{1}{\gamma^2 d} (2 + e^{-3\gamma d} - 3e^{-\gamma d}) \]  

(3.15)

This expression is valid for all self elements \( k = s \). The matrix element for an adjacent pulse such as \( CI_{34} \) is found by using (3.13b) and (3.14)

\[ CI_{34} = \frac{d}{2\gamma} + \frac{1}{2\gamma^3 d} (2 - 4e^{-\gamma d} + 3e^{-3\gamma d} - e^{-5\gamma d}) = CI_{32} \]  

(3.16)

The last unique term arising from testing full triangles
can be represented by CI_{35} in Fig. (3.2), which can be obtained by using (3.13b)

\[
CI_{35} = \frac{1}{\gamma^2 \Delta} (e^{-4\gamma \Delta} - 2e^{-3\gamma \Delta} + e^{-2\gamma \Delta}) \int e^{-\gamma z} dz - 5.5 \Delta
\]

\[
= \frac{1}{2\gamma d} (e^{-\gamma d} - 3e^{-3\gamma d} + 5e^{-5\gamma d} - 7e^{-7\gamma d})
\]  

(3.17)

If the second current triangle \((k=2)\) had been used instead, this would be the same result for CI_{24}. Testing with pulses further away would only multiply CI_{24} by \(e^{-2(s-2)\gamma d}\), as was the case for testing the half triangle.

Notice that in CI_{11}, CI_{kk}, and CI_{k,k+1} there are terms with no exponential decay. When the extra \(1/\gamma\) (or \(1/\gamma\)) from the Green's functions are included, along with the \(P_{ni}PP_{nj}\), these terms are of the form

\[
\sum \sum \frac{F(m)}{\gamma^2} P_{ni}PP_{nj}
\]

(3.18)

This term is analogous to (2.56a), with \(\beta^2=0\), and instead of a \(P_{nj}\) there is a \(PP_{nj}\). Equation (3.18) may be summed over \(m\) in the same fashion as was (2.56a), as shown in Appendix 1. The only difference is that \(PP_{nj}\) has an extra \(\cos \theta\) term in the integral as shown in
(2.62). This results in slightly different values for the various terms in the i's and j's, but otherwise the summation is carried out in the same way.

All the other terms that appear in the $C_{ik}$ have exponential decay and will therefore converge quickly. This is also true for the tested charge pulses, which will now be considered.

Figure (2.3b) shows the electric field due to a charge half pulse, while Fig. (2.3d) shows the field from a bipolar pulse. It can be seen that the half pulse produces an anti-symmetric field, while a bipolar pulse generates a symmetric field relative to its midpoint. To calculate the matrix elements arising from the testing of bipolar pulses, it proves to be easier to test the field produced by a unipolar pulse and then form tested, bipolar pulses by appropriately summing adjacent pairs of pulses.

Figure (3.4a) shows the first charge half pulse at the open end, while Fig. (3.4b) shows the same pulse shifted to the left by one unit. Notice that the pulse in Fig. (3.4b) tested with the second pulse is the same as the pulse in Fig. (3.4a) tested with the second testing pulse. Call these tested half pulses $C_{ik}$. Consider the field generated by the second half pulse as shown in Fig. (3.4b).
(a) Charge half pulse at open end (k=1) with first four test pulses (s=1 to 4).

(b) Second charge half pulse with first three test pulses.

(c) First bipolar charge pulse (k=2) with first three test pulses.

Fig. (3.4). Charge pulses.
for $z<-2\Delta$; \[
\frac{\partial}{\partial z} e^{\gamma z} \int e^{-\gamma z'} dz' = e^{\gamma z} (e^{2\gamma \Delta} - e^{\gamma \Delta})
-2\Delta
\]

for $z>\Delta$; \[
\frac{\partial}{\partial z} e^{-\gamma z} \int e^{\gamma z'} dz' = -e^{-\gamma z} (e^{-\gamma \Delta} - e^{-2\gamma \Delta})
-2\Delta
\]

for $-2\Delta < z < -\Delta$; \[
\frac{\partial}{\partial z} \left[ \frac{z}{-2\Delta} \int e^{\gamma z'} dz' + \frac{-\Delta}{z} \int e^{-\gamma z'} dz' \right] = e^{-\gamma z - 2\gamma \Delta} - e^{\gamma z + \gamma \Delta}
\]

$CT_{22}$ now becomes \[
CT_{22} = (e^{2\gamma \Delta} - e^{\gamma \Delta}) \int e^{\gamma z} dz + \int (e^{-\gamma z - 2\gamma \Delta} - e^{\gamma z + \gamma \Delta})
-2.5\Delta -2\Delta
\]

CT_{22} = CT_{11} = CT_{kk} = \frac{1}{\gamma} (2 - 3e^{-\gamma d} + e^{-3\gamma d}) \quad (3.19)

with a corresponding term for $\bar{\gamma}$, and an extra $1/\gamma$, $1/\bar{\gamma}$ coming from $G$. With reference to Fig. (3.4a), $CT_{12}$ becomes \[
CT_{12} = (e^{\gamma \Delta} - 1) \int e^{\gamma z} dz = \frac{1}{\gamma} (e^{-\gamma d} - 2e^{-3\gamma d} + e^{-5\gamma d}) \quad (3.20)
-2.5\Delta
\]

with a corresponding $1/\bar{\gamma}$ term. For all the remaining
pulses, the $C_{T_{1s}}$ can be obtained from the $C_{T_{12}}$ by multiplying it by $e^{-2(s-2)\gamma_d}$ for $s > 2$. Note that $C_{T_{21}} = -C_{T_{11}}$, and that if the testing of bipolar pulses is defined as $CQ_k$, then from Fig. (3.4c)

$$CQ_{21} = C_{T_{21}} - C_{T_{11}} = -2C_{T_{11}} \quad (3.21)$$

Note that the first full bipolar pulse is the $k=2$ pulse, to be consistent with the notation used with the current triangles. In a similar fashion

$$CQ_{22} = C_{T_{22}} - C_{T_{12}} = C_{T_{11}} - C_{T_{12}}$$
$$CQ_{23} = C_{T_{12}} - C_{T_{13}}$$
$$CQ_{24} = C_{T_{12}} - C_{T_{13}}$$

Therefore

$$CQ_{2s} = C_{T_{1,s-1}} - C_{T_{1,s}} \quad (3.22)$$

Since the electric field from a bipolar pulse is symmetric, testing a bipolar pulse the same distance from it on the other side will yield the same results.

There is one term in (3.19) which has no exponential decay, it is of the form
\[
\sum_{n} \sum_{m} \left[ \frac{H(m)}{\gamma^2 + \beta^2} \frac{\bar{H}(m)}{\bar{\gamma}^2 + \bar{\beta}^2} \right] p_{ni} p_{nj}
\] (3.23)

As was the case for (3.18) it can be summed over \( m \), the details of this summation are also in Appendix 1. All the other terms arising from testing the charge pulse have some exponential decay, and so they may be summed directly.

It is convenient to express the charge and current matrix elements in the way that was described in the last few pages. By doing so, certain patterns emerge regarding the individual quantities that need to be summed. This allows for an efficient structuring for the portion of the computer program that evaluates these sums.

Since the calculation of the \( CI_{k_s} \) and \( CQ_{k_s} \) depends only on the absolute distance between the testing and expansion pulses, only a limited number of terms actually need to be calculated. The half pulses at the open end give unique matrix elements for each successive test pulse, so all of these must be calculated. However, the expressions required for the testing of the first \((k=2)\) full expansion pulse will generate all of the required matrix elements for a given testing and expansion mode in \( x' \) and \( x \). For example, if the total
number of test and expansion pulses is K, then at one extreme, \( CI_{j,k} = CI_{2,k-1} \) while at the other extreme \( CI_{j1} = CI_{22} \), and similarly for the \( CQ_{ks} \) terms. This may be restated by saying that only the first half and full expansion pulses need to be tested for a given test and expansion mode. All the other matrix elements may be expressed in terms of those for the first full expansion pulse.

Associated with the \( CI_{ks} \) and \( CQ_{ks} \) are the \( P_{ni}PP_{nj} \) terms. It is now useful to define two new variables

\[
R(i,j,l,s) = \sum_n \sum_m CI_{ls}P_{ni}PP_{nj} \tag{3.24a}
\]

\[
T(i,j,l,s) = \sum_n \sum_m CQ_{ls}P_{ni}PP_{nj} \tag{3.24b}
\]

where \( l=1 \) represents the half pulse, and \( l=2 \) represents the first full pulse from the open end. Referring to (3.8a), the quantities that multiply \( B_{in} \) and the source term may be written as

\[
BI(i,j,s) = \sum_n \sum_m \frac{F(m)}{\gamma^2 + \beta^2} E_s(n,m)P_{ni}PP_{nj} \tag{3.25a}
\]

\[
BQ(i,j,s) = \sum_n \sum_m \left[ \frac{H(m)E_s(n,m)}{\gamma^2 + \beta^2} + \frac{\bar{H}(m)\bar{E}_s(n,m)}{\gamma^2 + \beta^2} \right] P_{ni}PP_{nj} \tag{3.25b}
\]
\[ SI(i,j,s) = \sum_n \sum_m \frac{F(m)}{\gamma^2 + \beta^2} E_s(n,m) P_{n i} P_{n j} \]  

\[ SQ(i,j,s) = \sum_n \sum_m \left[ \frac{\gamma H(m) E_s(n,m)}{\gamma^2 + \beta^2} + \frac{\gamma H(m) E_s(n,m)}{\gamma^2 + \beta^2} \right] P_{n i} P_{n j} \]  

Equation (104a) may now be written as

\[ \beta B_{in} \left[ \sum_{i=0}^{2} I_i BI(i,j,s) - \sum_{i=0}^{2} Q_i BQ(i,j,s) \right] = \]

\[ \beta \sum_{k=1}^{K} \sum_{i=0}^{2} \left[ C_{ki} R(i,j,1,s) - D_{ki}(i,j,1,s) \right] = \]

\[ \beta^2 \sum_{i=0}^{2} I_i SI(i,j,s) + \sum_{i=0}^{2} Q_i SQ(i,j,s) \]  

(3.27)

for \( s = 1 \) to \( K \) and \( j = 0 \) to \( 2 \). Using (3.9b) and (2.59) gives \( C_{1i} = -B_{in} I_i \) and \( D_{ki} = -\beta \cdot DK \cdot C_{ki} \) so

\[ D_{1i} = \beta \cdot B_{in} \cdot I_i \cdot DK \cdot C_{1i} \]  

(3.28)

where \( DK = 1/(2k_0^2d) \). As a result, (3.27) becomes

\[ \beta B_{in} \left[ \sum_{k=1}^{K} \sum_{i=0}^{2} I_i (R(i,j,1,s) + DK T(i,j,1,s)) + \sum_{i=0}^{2} I_i BI(i,j,s) + \sum_{i=0}^{2} Q_i BQ(i,j,s) \right] - \]

---
\[
-\sum_{k=1}^{\infty} \sum_{i=0}^{\infty} C_{k_i} (R(i,j,2,s)+DKT(i,j,2,s)) = \\
\beta^2 \sum_{i=0}^{\infty} I_I SI(i,j,s) + \sum_{i=0}^{\infty} Q_I SQ(i,j,s) \tag{3.29}
\]

for \(s=1\) to \(K\) and \(j=0\) to 2.

As was previously mentioned, there are 27 pulses used to represent the perturbed dominant mode sources for the \(i=0\) mode on the strip, and that these 27 pulses are compressed down to 12. This may be seen as follows, consider three pulses, \(k-1\), \(k\), and \(k+1\), each with unit amplitude. If pulse \(k-1\) and \(k+1\) is multiplied by \(.5\) and added to pulse \(k\), a new pulse is formed, which has unit amplitude, but a base width which is three times that of a unit pulse. This summation technique need not be done before the matrix is filled, rather it can be done by a suitable combination of rows and columns that have been filled in the systematic way described earlier using unit pulses. The procedure is the same for charge pulses.

Three modes in \(x\), with 12, 5, and 5 pulses in \(z\) are used. The boundary condition at the open end eliminates three of the unknowns, so there are 20 left, including \(B_{in}\), so a 20x20 matrix must be inverted.

The results obtained for the open end using this
technique will be presented and discussed in the next chapter, and will be compared with the results of other investigators. The next section will develop the required expressions needed to characterize an asymmetric gap.

(3.2) ASYMMETRIC GAP

Figure (3.5) shows the geometry for an asymmetric gap. The gap spacing will be called \( \delta \), line 1 is the input line and line 2 is the output line, which is terminated in a short circuit plane a distance \( l \) from the origin. It is assumed to be an ideal short so that its reflection coefficient is -1. This will produce a dominant mode standing wave on line 2. The actual location of the short is at \( z \) approaching infinity.

As was the case for the stub, it is assumed that there is no dominant mode \( J_x \) on either line. However, for a wide variety of gaps, especially those which are tightly coupled with a large step in width (\( W_2 \gg W_1 \)), a \( J_x \) mode will be needed as part of the perturbed current.

The need for a transverse current (\( J_x \)) can be seen by considering the case of tight coupling between widely dissimilar lines. In such a case, most of the field lines originating on the narrow line terminate on the
Fig. (3.5). Geometry for an asymmetric gap.
adjacent area of the wide line, creating an excess charge in that region. This excess charge is a source for current on line 2, which must ultimately diverge from this source area to assume its dominant mode characteristics further down the line. This requirement that the dominant mode z directed current diverge means that there will be a significant J \(_x\) component near the gap. This transverse current is therefore taken into account in the expressions for the perturbed currents on the two lines.

The sources on line 1 are the same as for the stub (3.3a) and (3.3b)

\[ J_{z1}(x',z') = \text{j}(1+R)J_{z1}(x')(B_{in}\cos(\beta_1z')-\sin(\beta_1z')) + I_1(x',z') \]  \hspace{1cm} (3.32a)

\[ \rho(x',z') = (1+R)\rho(x')(B_{in}\sin(\beta_1z') + \cos(\beta_1z')) + q_1(x',z') \]  \hspace{1cm} (3.32b)

\( I_1(x',z') \) and \( q_1(x',z') \) are used here to denote the perturbed current and charge distributions rather than the \( J_{z1} \) and \( \rho_1 \) used in (3.3a) and (3.3b) since in this problem the subscripts 1 and 2 are needed to distinguish between line 1 and line 2. As was the case for the stub, the factor \( (1+R) \) is absorbed into the dominant mode amplitudes.
On line 2, define a relative current \( J_{zr2} \) given by \( J_{zr2} = (\beta_2 c/k_0) \cdot J_{zr2} \) and a relative vector potential \( A_{zr2} \) by \( A_{zr2} = (\beta_2 c/k_0) \cdot A_{zr2} \). If this convention is used, then the boundary condition on line 2 is the same as for line 1, (3.31) but with \( \beta_1 \) replaced with \( \beta_2 \). The current and charge on line 2 are given by

\[
J_{z2}(x', z') = j\tau J_{z2}(x') \cos(\beta_2(z_2' - l)) + I_2(x', z') \tag{3.33a}
\]

\[
\rho_2(x', z') = j\tau Q_2(x') \sin(\beta_2(z_2' - l)) + q_2(x', z') \tag{3.33b}
\]

where the terms multiplied by \( \tau \) are the dominant mode standing waves and \( I_2 \) and \( q_2 \) are the perturbed sources on line 2. The parameter \( \tau \) is an unknown transmission coefficient.

The boundary condition on line 1 is

\[
j\beta_1 \int_{-\infty}^{0} G_{z1} dx' dz' + j\beta_2 \int_{-w_2}^{0} G_{z2} dx' dz' + \frac{\partial}{\partial z_1} \int_{-\infty}^{0} q_{z1} dx' dz' + \frac{\partial}{\partial z_2} \int_{-\infty}^{0} q_{z2} dx' dz' + j\beta_2 \int_{-w_2}^{\infty} G_{z2} dx' dz' + \frac{\partial}{\partial z_2} \int_{-w_2}^{\infty} q_{z2} dx' dz' + j\beta_1 \int_{0}^{\infty} G_{z1} dx' dz' + \frac{\partial}{\partial z_1} \int_{0}^{\infty} q_{z1} dx' dz' + \beta_1 \int_{-w_1}^{\infty} G_{z1} \left( B_{in} \cos(\beta_1 z_1') - \sin(\beta_1 z_1') \right) J_{z1} dx' dz' + \frac{\partial}{\partial z_1} \int_{-w_1}^{\infty} q_{z1} dx' dz' \tag{3.34a}
\]
\[ \int_0^\infty w_1 \int G(B_{i1}\sin(\beta_1 z_1') + \cos(\beta_1 z_1'))Q_1 dx'dz' \]
\[ + \frac{\partial}{\partial z_1} \int GZ_{I1} dx'dz' + \frac{\partial}{\partial z_1} \int GQ_1 dx'dz' \]
\[ = 0 \quad (3.34) \]

The source terms for line 1 are

\[ \text{Source 1} = \int G_{z1}'(\beta_1 z_1')J_{z1} dx'dz' \]
\[ + \frac{\partial}{\partial z_1} \int G_{z1}'(\beta_1 z_1')Q_1 dx'dz' \quad (3.35) \]

The boundary conditions on line 2 are

\[ j\beta_1 \int G_{z2} I_1 dx'dz_1' + \frac{\partial}{\partial z_1} \int G_{z1} Q_1 dx'dz_1' \]
\[ = 0 \quad (3.36) \]
\[ j\beta_2 \int G_{z2} I_2 dx'dz_2' + \frac{\partial}{\partial z_2} \int G_{z2} Q_2 dx'dz_2' \]
\[ = 0 \quad (3.37) \]
\[ + j\beta_1 \int_{-\infty}^{0} \int_{-w_1}^{w_1} G_z(jB_{1n} \cos(\beta_1 z_1') - j\sin(\beta_1 z_1')) J_{z_1} dx'dz_1' \]
\[ - j\beta_2 \int_{-\infty}^{0} \int_{-w_2}^{w_2} G_z j\tau \cos(\beta_2(z_2' - l)) J_{z_2} dx'dz_2' \]
\[ + \frac{\partial}{\partial z_1} \int_{-\infty}^{0} \int_{-w_1}^{w_1} G(z_1') \cos(\beta_1 z_1') Q_1 dx'dz_1' = 0 \tag{3.36} \]

The source for line 2 is

\[ \text{Source 2} = -\beta_1 \int_{-\infty}^{0} \int_{-w_2}^{w_2} G_z \sin(\beta_1 z_1') J_{z_1} dx'dz_1' \]
\[ - \frac{\partial}{\partial z_2} \int_{-\infty}^{0} \int_{-w_2}^{w_2} G(z_2') \cos(\beta_2 z_2') Q_1 dx'dz_2' = 0 \tag{3.37} \]

The source reversal expression \[ \int_{-\delta}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz_2' = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz_2 \]
\[ \int_{-\delta}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz_1' = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz_1 \]
was used in (3.36), while \[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz_1' = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz_1 \]
was used in (3.34). This is analogous to the technique used
in the last section. The infinite line boundary conditions were used to cancel the integrals from $-\infty$ to $+\infty$.

For line 1, the dominant mode integrals include (3.7a) and (3.7b) from the stub problem, except $\beta=\beta_1$, $z=z_1$, along with the following new terms

\[ e^{-\gamma|z_1-z'_1|\cos(\beta_2(z'_2-l))}dz'_2 = \frac{e^\gamma(z_1-\delta)}{\gamma^2+\beta^2_2}(\gamma\cos\theta-\beta_2\sin\theta) \]

(3.37a)

\[ e^{-\gamma|z_1-z'_1|\sin(\beta_2(z'_2-l))}dz'_2 = \frac{e^\gamma(z_1-\delta)}{\gamma^2+\beta^2_2}(\beta_2\cos\theta+\gamma\sin\theta) \]

(3.37b)

where $\theta=\beta_2(\delta-1)$. For line 2 the dominant mode integrals include (3.7a) and (3.7b) from the stub problem but with $\beta=\beta_2$ and $z=z_2$, along with the following new terms

\[ e^{-\gamma|z_2-z'_2|\cos(\beta_2(z'_2-l))}dz'_2 = \frac{e^{-\gamma(z_2-\delta)}}{\gamma^2+\beta^2_2}(\gamma\cos\theta+\beta_2\sin\theta) \]

(3.38a)

\[ e^{-\gamma|z_2-z'_2|\sin(\beta_2(z'_2-l))}dz'_2 = \frac{e^{-\gamma(z_2-\delta)}}{\gamma^2+\beta^2_2}(-\beta_2\cos\theta+\gamma\sin\theta) \]

(3.38b)

and appropriate $\gamma$ terms for (3.37a,b) and (3.38a,b).

Figure (3.6) shows the perturbed current and charge
(a) Half pulses at the open end of the gap.

(b) First full pulses.

Fig. (3.6). Source expansion pulses for the gap.
expansion functions for the two lines. Note that the \( D_{1i} \), the first half charge pulse on line 2, is defined as negative, so as to satisfy the continuity equation. These terms may be described as

\[
I_1(\theta', z') = \sum_{i=0}^{2} \sum_{k=1}^{K} C_{ki} T_i(z') \cos(2i\theta') / \cos \theta' \quad (3.39a)
\]

\[
q_1(\theta', z') = \sum_{i=0}^{2} \sum_{k=1}^{K} D_{ki} P_k(z') \cos(2i\theta') / \cos \theta' \quad (3.39b)
\]

\[
I_2(\theta', z') = \sum_{i=0}^{2} \sum_{k=1}^{K} C_{ki} T_i(z') \cos(2i\hat{\theta}') / \cos \theta' \quad (3.39c)
\]

\[
q_2(\theta', z') = \sum_{i=0}^{2} \sum_{k=1}^{K} D_{ki} P_k(z') \cos(2i\hat{\theta}') / \cos \theta' \quad (3.39d)
\]

where the \( \hat{\theta} \) on the \( \theta \)’s in (3.39c) and (3.39d) are used to indicate that \( x = \omega_2 \sin \theta \) was used to generate these expressions for the \( x' \) dependence on line 2. From here on, for convenience they will be dropped. It will be understood that a different transformation is used on line 2.

The boundary condition at the two open ends of the gap require that the total current vanish, therefore

\[
j B_{in} I_{ni} + j C_{1i} = 0 \quad \text{or} \quad C_{1i} = -B_{in} I_{ni} \quad (3.40a)
\]
\[ j\hat{I}_i \cos \theta + j\hat{C}_{1i} = 0 \quad \text{or} \quad \hat{C}_{1i} = -\tau \hat{I}_i \cos \theta \quad (3.40b) \]

When there is no \( J_x \) associated with the perturbed sources, the charge amplitudes may be expressed in terms of the current amplitudes by

\[ D_{ki} = -\frac{\beta_1}{2k_0} C_{ki} \quad \text{and} \quad \hat{D}_{ki} = -\frac{\beta_2}{2k_0} \hat{C}_{ki} \quad (3.41) \]

A major difference between the stub and the gap problem is that there is cross coupling in the gap problem. Sources on line 1 produce fields on line 2 and vice versa. Testing the fields produced must now include the field produced by sources on the other line as well as on the same line.

In the stub example, the integral over the sources in \( x \) was called \( P_{ni} \) while the testing integrals were called \( PP_{nj} \) where

\[
P_{ni} = 2W \int_0^{\pi/2} \cos(u_n \omega \sin \theta) \cos(2i\theta) \cos \theta \, d\theta
\]

\[
PP_{nj} = 2W \int_0^{\pi/2} \cos(u_n \omega \sin \theta) \cos(2i\theta) \cos \theta \, d\theta
\]
For the gap problem, the following definitions will be used

\[
P_{ni} = 2W_1 \int_0^{\pi/2} \cos(u_n w_i \sin \theta) \cos(2i \theta) d\theta (3.42a)
\]

\[
P_{nj} = 2W_1 \int_0^{\pi/2} \cos(u_n w_1 \sin \theta) \cos(2j \theta) \cos \theta d\theta (3.42b)
\]

where \( l=0 \) refers to line 1 while \( l=1 \) refers to line 2.

Figure (3.7) shows the fields produced by the charge pulses. Notice that the field produced by the inverted charge pulse \( \hat{D}_{1i} \) is the negative of that from \( D_{1i} \) while the field produced by the other \( \hat{D}_{ki} \), \( k>1 \), have the same sign and symmetry as the \( D_{ki} \). As a result, the self interaction matrix elements (source and field points on the same line), will produce the same matrix elements as in the stub problem. The fields produced by the current triangles are symmetric in \( z \) about the apex and so they generate the same matrix elements as well.

In the case of the symmetric gap, \( 2W_1=2W_2 \), these elements will also be equal in magnitude since \( P_{ni0} P_{nj0} = P_{ni1} P_{nj1} \) as well. Only the signs may be different between the \( B_{in} \) and \( \tau \) coefficients, since the \( \tau \) coefficients are multiplied by a \( \cos \theta = \cos \beta_2 (\delta - l) \) term, if \( \theta = \pi, 3\pi, 5\pi, \) etc.
(a) For half pulses.

(b) For full pulses.

Fig. (3.7). Fields produced by charge expansion pulses.
The cross coupling elements are new. Figure (3.8) shows sample expansion and testing pulses for this case. From symmetry considerations, it may be seen that, so far as expansion and testing in $z'$, $z$ is concerned, the cross coupling terms are the same for the half and full pulse expansions. For a symmetric gap they are in fact the same, since $P_{ni0}P_{nj0}=P_{ni1}P_{nj1}$. This results in a symmetric coefficient matrix, as well as a source vector. This feature insures that as long as the same number of unknowns are used to represent the sources on both sides of a symmetric gap, the resulting model of the gap will be symmetric as well.

Since the self coupling matrix elements are of the same form for line 1 and line 2, and therefore the same as for the stub problem already considered, they will not be repeated here. The cross coupling terms are new, and so the evaluation of the unique elements will now be presented. It should be noted, that since no test pulse overlaps a source pulse in this case, none of the terms will have a dominant part, instead all will have some exponential decay along with an extra factor of $e^{-\gamma_0}$ and $e^{-\gamma_8}$, due to the gap.

With reference to Fig. (3.8a), consider the element $CI_{11}^1=CI_{11}^2$, where the superscript 1 or 2 denotes line 1 or 2, while the subscripts represent the indices $k$ and
(a) Current triangles $k=1,2$ on line 1, and first three test pulses on line 2, $s=1,2,3$.

(b) Current triangles $k=1,2$ on line 2, and first three test pulses on line 1.

Fig. (3.8). Expansion half pulses, with first three test pulses, for the cross coupling terms.
\[ \delta_{+3d}^0 \]

\[ C_{11}^1 = \int e^{-\gamma_d} e^{-\gamma_2} \int e^{\gamma_2 T_1(z')}dz' dz \]

\[ \delta_{+d}^{-2d} \]

\[ C_{11}^1 = e^{-\gamma_2} \left[ \frac{(e^{-\gamma_d} - e^{-3\gamma_d})}{\gamma^2} - \frac{(e^{-\gamma_d} - e^{-3\gamma_d})}{2\gamma^3} \right] \]

\[ (3.43) \]

\[ C_{1s}^1 = C_{1s}^2 \text{ will be the same as } C_{11}^1 \text{ but multiplied by } e^{-2(s-1)\gamma_d}, \text{ or} \]

\[ C_{1s}^1 = e^{-2(s-1)\gamma_d} C_{11}^1 \]

\[ (3.44) \]

As in the case of the stub, the Green's functions will add an extra $1/\gamma$ or $1/\tilde{\gamma}$ to all the $C_{1s}^{ILP}$, and $CQ_{k_s}^{ILP}$ terms. Notice the superscript ILP, this will designate the line on which expansion and testing is done in the following manner

ILP=0, expand on line 1, test on line 1
ILP=1, expand on line 1, test on line 2
ILP=2, expand on line 2, test on line 1
ILP=3, expand on line 2, test on line 2

For the full pulses, consider the terms $C_{21}^1 = C_{21}^2$, 
$\delta_{3d}^{+d} \int_{-d}^{0} e^{-yz} T_2(z') dz' dz$

$CI_{21}^1 = e^{-\gamma \delta} \int_{-d}^{0} e^{-yz} T_2(z') dz' dz$

$CI_{21}^1 = \frac{e^{-\gamma \delta}}{2\gamma^3 d} (e^{-\gamma d} - e^{-3\gamma d} + 3e^{-5\gamma d} - e^{-7\gamma d})$  \hspace{1cm} (3.45)

while the rest of the $CI_{23}^1$ may be found by multiplying $CI_{21}^1$ by $e^{-2(s-1)\gamma d}$. Notice that $CI_{21}^1 = e^{-\gamma \delta} \cdot CI_{35}^1$ from (3.17) for the stub. This could be anticipated since $CI_{35}^1$ in (3.17) represents a full triangle tested by the nearest non overlapping pulse. The only difference in the case of a gap is the factor $e^{-\gamma \delta}$, if the gap spacing were reduced to zero, $CI_{21}^1 = CI_{21}^2 = CI_{35}^1$ of (3.17). Also note that terms like $CI_{35}^1$ in (3.17) are of the same form as $CI_{35}^0$ or $CI_{35}^3$, that is, the self interaction terms in the gap problem. As a result of this similarity, only one new term had to be calculated, namely, $CI_{11}^{ILP}$ for ILP=1 or 2. The rest of the expressions may be derived from the existing expressions.

Testing the charge pulses proceeds the same way. Figure (3.9) shows the half pulses at the open ends with several test pulses on the other line. Since the pulse on line 2 is inverted, the field generated by the two source pulses have the same type of z dependence as Fig.
(a) ILP=1 case, source on line 1, testing on line 2.

(b) ILP=2 case, source on line 2, testing on line 1.

Fig. (3.9). Half charge pulse with the field produced, at the open ends, cross coupling terms.
(3.6a) and Fig. (3.6b) show. As a result, $CQ_{1s}^2 = CQ_{1s}^1$. Consider the term $CQ_{11}^1$, that is, the $k=1$, $s=1$, and ILP=1 case

$$CQ_{11}^1 = \int_{\delta+1.5d}^{\delta+d} e^{-\gamma z}dz \int_{-2d}^{0} e^{\gamma z'}dz'$$

$$CQ_{11}^1 = \frac{e^{-\gamma \delta}}{\gamma} (e^{-\gamma d} - e^{-2\gamma d} + e^{-3\gamma d} + e^{-5\gamma d})$$

(3.46)

note that the $CQ_{11}^1$ has the same form, except for the $e^{-\gamma \delta}$ term, as the $CT_{12}$ term given in (3.20) for the stub. The other $CQ_{1s}^1$'s can be expressed in terms of $e^{-2(s-1)\gamma d}CQ_{11}^1$, and $CQ_{1s}^2 = CQ_{1s}^1$.

For full charge pulse expansion and testing, again define $CT_{2s}^{ILP}$, with ILP=1 or 2. The $k=2$ pulse is the first full expansion pulse, and $s$ represents the testing pulses. As in the case of the stub, $CQ_{2s}^{ILP}$ can be formed as in (3.22)

$$CQ_{2s}^{ILP} = CT_{1,s}^{ILP} - CT_{1,s+1}^{ILP}$$

(3.47)

For the cross coupling terms, only the half pulse and the first full pulse matrix element need to be
evaluated, the rest may be obtained by shifting. If \( K \) represents the last test pulse on the opposite line, the relationship between expansion pulse \( k=1 \) and \( k=2 \) may be expressed as

\[
CI_{2,k-1}^{ILP} = CI_{1,K}^{ILP} \quad \text{and} \quad CQ_{2,k-1}^{ILP} = CQ_{1,K}^{ILP}
\]

Notice that the \( k=2 \) pulse is not tested to the end of the opposite line, rather only to the \( s=k-1 \) pulse, while the \( k=3 \) pulse is tested only to the \( s=k-2 \) pulse, and so on. Finally, the \( k=K \) expansion pulse is tested only by the \( s=1 \) pulse. In this way, the contributions from the cross coupling terms are tested to the same degree of accuracy as are contributions from single line coupling elements.

As before, all the \( CQ_{ks}^{ILP} \) and \( CI_{ks}^{ILP} \) are multiplied by an extra \( 1/\gamma \) or \( 1/\bar{\gamma} \) as well as a \( F(m) \), or \( H(m) \) and \( H(m) \) from their appropriate Green's functions, and the associated \( P_{n1l} \) and \( PP_{n1l} \). As in (3.24a) and (3.24b), the matrix elements may be written as

\[
R_{ijls}^{ILP}(i,j,l,s) = \sum_{m} \sum_{s} CQ_{is}^{ILP} P_{n1l}^{PP} P_{n1l}^{PP} \quad (3.48a)
\]

\[
T_{ijls}^{ILP}(i,j,l,s) = \sum_{m} \sum_{s} CI_{is}^{ILP} P_{n1l}^{PP} P_{n1l}^{PP} \quad (3.24b)
\]

Testing the fields which arise from the dominant
mode sources proceeds the same way as for the stub, as in Eq. (3.8a) and Eq. (3.8b). Testing on line 1 gives

\[
E_s(n,m) = \int e^{\gamma d} \, dz = \frac{1}{\gamma} \left( e^{-(2s+1)\gamma d} - e^{-(2s-1)\gamma d} \right)
\]

and similarly,

\[
E_s^*(n,m) = \frac{1}{\gamma} \left( e^{-(2s-1)\gamma d} - e^{-(2s+1)\gamma d} \right)
\]

while testing on line 2 gives the same results, except that the \( P_{n_{ij}} \) terms are different if the problem is an asymmetrical gap.

The quantities that are the coefficients of the \( B \) and source terms may be written as

\[
BI(i,j,s) = \sum_n \sum_m \left[ \frac{F(m)}{\gamma^2 + \beta^2} E_s(n,m) P_{ni_{ij}} \right]_{ILP} \quad (3.49a)
\]

\[
BQ(i,j,s) = \sum_n \sum_m \left[ \frac{H(m)}{\gamma^2 + \beta^2} E_s(n,m) + \frac{H(m)}{\gamma^2 + \beta^2} E_s^*(n,m) \right] P_{ni_{ij}} \right]_{ILP} \quad (3.49b)
\]

\[
SI(i,j,s) = \sum_n \sum_m \left[ \frac{F(m)}{\gamma^2 + \beta^2} E_s(n,m) P_{ni_{ij}} \right]_{ILP} \quad (3.49c)
\]
\[ SQ(i,j,s) = \sum_n \sum_m \left[ \frac{H(m)E_s(n,m)}{\gamma^2 + \beta^2} + \frac{H(m)\beta E_s(n,m)}{\gamma^2 + \beta^2} \right] p_{ni} p_{nj} \]_ILP

(3.49d)

where the ILP subscript indicated that \( \beta^2 = \beta^2_1 \) for ILP = 0 or 1, and \( \beta^2 = \beta^2_2 \) for ILP = 2 or 3, and it also selects the proper choice of \( p_{ni} p_{nj} \).

The \( B_{ln} \) term may be written as

\[ B_{lm}^0 = \beta_1 \sum_l \left[ I_{1l} B_{l0}^0(i,j,s) - Q_{1l} B_{l0}^0(i,j,s) \right] \] (3.50a)

\[ B_{lm}^1 = -\beta_1 \sum_l \left[ I_{1l} B_{l0}^0(i,j,s) - Q_{1l} B_{l0}^0(i,j,s) \right] \] (3.50b)

The \( \tau \) terms are

\[ \tau^2 = -\sum_l \left[ \beta_2 I_{2l} (B l^2(i,j,s)\cos\theta - \beta_2 SI^2(i,j,s)\sin\theta) - Q_{2l} (\beta_2 BQ^2(i,j,s)\cos\theta + SQ^2(i,j,s)\sin\theta) \right] \] (3.51a)

\[ \tau^3 = \sum_l \left[ \beta_2 I_{2l} (B l^3(i,j,s)\cos\theta + SI^3(i,j,s)\sin\theta) + Q_{2l} (\beta_2 BQ^3(i,j,s)\cos\theta + SQ^3(i,j,s)\sin\theta) \right] \] (3.51b)

The source terms are
\[ S_0^0 = \sum_{i} \left[ \beta_1^2 I_{11}^0 SI_{01}^0(i,j,s) + Q_{11}^0 SQ_{01}^0(i,j,s) \right] \] (3.52a)

\[ S_1^0 = \sum_{i} \left[ \beta_1^2 I_{11}^1 SI_{01}^1(i,j,s) + Q_{11}^1 SQ_{01}^1(i,j,s) \right] \] (3.52b)

The final expressions relating the unknown amplitudes to the sources on the strip and satisfying the B.C.'s may be written as

\[
\beta_1^2 B_{in} \left[ B_{in}^{0,1} + \sum_{i=0}^{2} (I_{11}^0 R_{11}^{0,1}(i,j,1,s) + DK \cdot T_{11}^{0,1}(i,j,1,s)) \right]
- \sum_{i=0}^{2} \sum_{k=1}^{K} C_{ki}(R_{11}^{0,1}(i,j,2,s) + DK \cdot T_{11}^{0,1}(i,j,2,s)) + \\
+ \beta_2^2 \left[ \sum_{i=0}^{2} \sum_{k=1}^{K} C_{ki}(R_{21}^{2,3}(i,j,1,s) + DK \cdot T_{21}^{2,3}(i,j,1,s)) \right] - \\
- \sum_{i=0}^{2} \sum_{k=1}^{K} C_{ki}(R_{21}^{2,3}(i,j,2,s) + DK \cdot T_{21}^{2,3}(i,j,2,s)) = S_{01}^0
\] (3.53)

where use has been made of (3.40a) and (3.40b) to establish the boundary condition on the half pulses at the open ends, and (3.41) was used to relate the charge amplitudes to the current amplitudes in the absence of a perturbed \( J_\times \) mode(s) on the line.

As in the case of the gap, there are 20 unknowns...
involving $B_{in}$ and the $C_{ki}$ on line 1 and an extra 20 on line 2 for the $\hat{C}_{ki}$. This results in a 40 by 40 matrix which may be solved for $B_{in}$.

Since a gap is a two port network, it must be represented by a three element equivalent circuit, such as the capacitive pi network shown in Fig. (3.10). Since there are three unknowns, $C_1$, $C_2$, and $C_3$, the above set of equations (3.53) would have to be solved three times for three different positions of the short circuit plane terminating line 2. A more efficient way to extract equivalent circuit values would be to use the Tangent, or Weissfloh method [64], discussed in chapter 1. With reference to Fig. (1.13) and (1.14), a bilinear relationship between electrical null positions $\phi_1$ and $\phi_2$ may be given by

$$\tan\phi_1 = \frac{A + B\tan\phi_2}{C + D\tan\phi_2}$$  \hspace{1cm} (3.54)

where $\phi_1$ is the location of an electric field null on the input side of the discontinuity for a given short circuit position $\phi_2$ on the output side.

In the expressions (3.53) developed for the gap, it may be seen that the transmission coefficient $\tau$ is a function of $\cos\theta$ and $\sin\theta$. If a new transmission coefficient is defined as $T = \tau \cos\theta$, the $\tau$ column will
Fig. (3.10). Pi model for a gap.

Fig. (3.11). Equivalent circuit for a gap, with line 2 terminated in a short circuit.
consist of terms of the form \((T\cdot\text{constants})\) and \((T\tan\theta\cdot\text{constants})\). The terms in \(T\tan\theta\) may be brought over to the right hand side of (3.53) and considered as a second source term. The matrix equation may now be written as

\[
[A] \cdot \begin{bmatrix}
B_{in}
X_i
T
X_i
\end{bmatrix} = \begin{bmatrix} y_i \end{bmatrix} + T\cdot\tan\theta[u_i] \tag{3.55}
\]

where \([A]\) is the coefficient matrix, \([y_i]\) is the original source vector containing \(S^{0.1}\) in (3.53), and \([u_i]\) contains the terms that multiply \(T\cdot\tan\theta\). Assume that \(B_{in}\) is in row one and \(T\) is in row \(K\), and \([A]^{-1} = [C_{ij}]\), then

\[
B_{in} = C_{ij}y_i + T\cdot\tan\theta\cdot C_{ij}u_i \tag{3.56a}
\]

\[
T = C_{ki}y_i + T\cdot\tan\theta\cdot C_{ki}u_i \tag{3.56b}
\]

Equation (3.56b) gives \(T = C_{ki}/(1-\tan\theta\cdot C_{ki}u_i)\), while (3.56a) gives

\[
B_{in} = \frac{C_{ij}y_i + [(C_{ij}u_i)(C_{ki}y_i) - (C_{ij}y_i)(C_{ki}u_i)]\tan\theta}{1-(C_{ki}u_i)\tan\theta} \tag{3.57}
\]
\[ \frac{A + B \tan \theta}{C + D \tan \theta} \]  

(3.58)

Note that \( \tan \theta = \tan(\beta_2(\delta - l)) = -\tan(\beta_2 d) \). For the circuit shown in Fig. (3.11) \( B_{in} \) can be evaluated as

\[
B_{in} = \frac{(B_1 + B_2) - (B_1 \cdot B_2 + B_1 \cdot B_3 + B_2 \cdot B_3) \tan \beta_2 d / Y_c}{1 - (B_2 + B_3) \tan \beta_2 d / Y_c} \quad (3.59)
\]

Comparing (3.59) with (3.58) and using \( \tan \theta = -\tan \beta_2 d \) gives

\[
B_1 + B_2 = A, \quad B_1 \cdot B_2 + B_1 \cdot B_3 + B_2 \cdot B_3 = B, \quad B_2 + B_3 = D
\]

Solving for \( B_1, B_2, \) and \( B_3 \) gives

\[
B_1 = A \pm (-B \cdot Y_c + A \cdot D \cdot Y_c)^{-1/2} \quad (3.60a)
\]

\[
B_2 = \mp (-B \cdot Y_c + A \cdot D \cdot Y_c)^{-1/2} \quad (3.60b)
\]

\[
B_3 = Y_c \cdot D \pm (-B \cdot Y_c + A \cdot D \cdot Y_c)^{-1/2} \quad (3.60c)
\]

In (3.57), \( A = C_{i1} y_i \), \( B = (C_{i1} u_i)(C_{k1} y_i) - (C_{k1} y_i)(C_{k1} u_i) \), and \( D = -C_{k1} u_i \). Equation (3.55) may be solved for \( B_{in1} \) and \( T_1 \), due to \( y_i \) and \( B_{in2} \) and \( T_2 \) due to \( u_i \) simultaneously, requiring the matrix to be inverted only once. The solution for \( B_{in2} \) and \( T_2 \) yield \( B_{in2} = C_{i1} u_i \) and \( T_2 = C_{k1} u_i \). Note that \( B_{in1} = B_{in} |_{\theta = 0} \) and \( B_{in2} = B_{in} |_{\theta = \pi/2} \) where
$\theta=0$ represents an open circuit at the input of line 2 (that is, $z=0$), and $\theta=\pi/2$ represents a short circuit at the input of line 2.

As a result, $A=B_{in1}$, $B=B_{in2}$, and $D=-T_2$ with $C=1$. For a symmetric gap, $B_1=B_3$ since $C_1=C_3$, in this case $A=D$ or $B_{in1}=-T_2$ so

$$B_1=B_{in1} \cdot \left( -T_1 \cdot B_{in2} \cdot Y_c \right)^{-1/2} = B_3 \quad (3.61a)$$

$$B_2=\left( -T_1 \cdot B_{in2} \cdot Y_c \right)^{-1/2} \quad (3.61b)$$

$B_1$, $B_2$, and $B_3$ are normalized admittances, and since $\bar{B}=\omega C Z_0$, the capacitances may be related to $B_1$, $B_2$, and $B_3$ as follows

$$C_1=\bar{B}_1/\omega Z_0 \quad ; \quad C_2=\bar{B}_2/\omega Z_0 \quad ; \quad C_3=C_1 \quad (3.62)$$

Up to this point, no $J_x$ modes were assumed on either line. However, when an asymmetric gap is considered, it is no longer valid to neglect $J_x$ since on the wider line, the dominant mode current must spread out from a location at the open end where the coupling to the narrow line is great, to a point farther down the line, where the unperturbed dominant mode prevails. This $x$ directed current also effects the source
distribution on the narrower line so it is appropriate
to include its effects there as well. The next section
will include the effects of $J_x$ on the gap problem.
(3.3) INCLUSION OF Jx MODES

Since it is assumed that there is no dominant mode \( J_x \) on the lines, the only effect of a \( J_x \) mode(s) will appear in the expressions for the perturbed modes. All of the perturbed amplitudes must satisfy \( \nabla \cdot J = -j \omega \rho \), therefore, \( J_x \) must satisfy

\[
\frac{\partial J_x}{\partial x} + j \omega \rho \frac{\partial J_z}{\partial z} = -j \omega \rho \quad \text{or} \quad \frac{\partial J_x}{\partial x} = -(j \omega \rho + \frac{\partial J_z}{\partial z})
\]

(3.63a)

Recall that the expressions for \( I_1(x',z') \) and \( q_1(x',z') \) were given as

\[
I_1(x',z') = \sum_{i=0}^{2} \sum_{k=1}^{K} C_{ki} v_i(x') T_k(z')
\]

\[
q_1(x',z') = \sum_{i=0}^{2} \sum_{k=1}^{K} D_{ki} v_i(x') P_k(z')
\]

shown here for sources on line 1, and \( v_i(x') \) represent the Chebychev polynomial's with edge conditions, used to expand the \( x \) dependence of the sources. Similar expressions hold for sources on line 2. Now, \( \frac{\partial T_k(z')}{\partial z'} = \frac{P_k(z')}{\Delta} \) and \( J_x = (\beta c/k_0) J_xr \), \( J_z = (\beta c/k_0) J_zr \), where the \( P_k \) are bipolar pulses, so (3.63a) becomes
\[
\frac{\partial J_{xr}}{\partial x'} = -\frac{1}{\beta_1} \sum_{k=1}^{K} \left( k_0^2 D_{k1} + \frac{\beta_1}{\Delta} C_{k1} \right) v_{1}(x') P_{k}(z')
\] (3.63b)

This expression for \( \partial J_{xr}/\partial x' \) has the same \( x' \) dependence as \( \rho \) and \( J_x \), and is for line 1. For \( J_x \) on line 2, replace \( C_{k1}, D_{k1}, \) and \( \beta_1 \) with \( \hat{C}_{k1}, \hat{D}_{k1}, \) and \( \beta_2 \). Let

\[
B_{k1} = (k_0^2 D_{k1} + \beta_1 C_{k1}/\Delta) \text{ for line 1}
\] (3.63c)

\[
\hat{B}_{k1} = (k_0^2 \hat{D}_{k1} + \beta_2 \hat{C}_{k1}/\Delta) \text{ for line 2}
\] (3.63d)

Since triangles are used to expand \( J_{z1}(z') \), as in Fig.(2.1), \( J_x(z') \) must be expressed in terms of pulses.

The boundary condition that must be satisfied is

\[
E_x = -j\omega A_x \frac{\partial \Phi}{\partial x} = 0
\] (3.64)

on the strip. \( A_x \) may be found from \( G_x \) by

\[
A_x = \mu_0 \int G_x(x, z; x', z') J_x(x', z') dx' dz'
\] (3.65)

\( A_{xr} \), like \( A_{zr} \) is a relative value, where \( A_x = (\beta c/k_0) A_{xr} \).

Since \( \omega = k_0 c \), (3.64) becomes \( j\beta A_{xr} + \partial \Phi/\partial x = 0 \).

The \( x' \) dependence of \( J_x(x') \) may be expressed as
\[ J_x(x') = j(1-(x'/w)^2)^{1/2} \sum_{i=1}^{l} I_i U_{2i-1}(x'/w) \]  

(3.66)

where the \( U_{2i-1}(x'/w) \) are Chebychev polynomials of the second kind. The first two modes, \( i=1,2 \) are \( x'\sqrt{1-(x'/w)^2} \) and \( -w\sqrt{1-(x'/w)^2}(2(x'/w)^3-x'/w) \). Note that there is no \( J_x \) mode corresponding to the \( i=0 \) mode in \( J_z \) or \( \rho \), which has the form \( (1-(x'/w)^2)^{-1/2} \).

The expression for \( G_x \) is given in (2.40), and (3.65) gives \( \lambda_x \) in terms of \( J_x(x',z') \). The integration over \( x' \) in (3.65) may be converted to an integral involving \( \partial J_x/\partial x' \) explicitly, by means of an integration by parts as follows

\[
2 \int_0^w G_x(x,x') J_x(x') \, dx' = 2 \left[ J_x(x') \int_0^w G_x(x') \, dx' \right]_{x'}^{x'} - \left[ \int_0^w G_x(x') \frac{\partial J_x(x')}{\partial x'} \, dx' \right] \]

(3.67)

the first term vanishes because \( J_x(x') = 0 \) for \( x' = 0 \) and \( w \), while the second term gives

\[
\frac{1}{u_n} \int_0^w \cos(u_n x') \frac{\partial J_x}{\partial x'} \, dx' \]

(3.68)
while $\partial \phi / \partial x$ gives a $-u_n \sin(u_n x)$ for the $x$ dependence.

Define the following

$$G(z, z') = \left[ \frac{H(m)}{\gamma} e^{-\gamma |z-z'|} + \text{Re} \left( \frac{H(m)}{\gamma} e^{-\gamma |z-z'|} \right) \right] P_k(z') \quad (3.69a)$$

$$G_x(z, z') = \frac{F(m)}{\gamma} e^{-\gamma |z-z'|} P_k(z') \quad (3.69b)$$

the $P_k(z')$ are bipolar pulses used to expand $J_x$ in $z'$.  

Now the expression that satisfies $E_x = 0$ on the strip (3.64), may be written using (3.60a-c) as

$$\sum_{i=1}^{K} \sum_{k=1}^{K} \sum_{m} \frac{1}{u_n} \int_{z'x'} B_{ki} G(x, z') v_i(x') \cos(u_n x') \sin(u_n x) dx'dz'$$

$$- \sum_{i=0}^{K} \sum_{k=1}^{K} \sum_{m} \int_{z'x'} D_{ki} u_n \int_{z'x'} G(z, z') v_i(x') \cos(u_n x') \sin(u_n x) dx'dz'$$

$$= 0 \quad (3.70)$$

on line 1.  For line 2, replace the $B_{ki}$ and $D_{ki}$ by $\hat{B}_{ki}$ and $\hat{D}_{ki}$, and note that the integration over $x'$ will be over line 2.  Note that the second term in (3.70), involving the $D_{ki}$'s which arise from $\partial \phi / \partial x$, contribute to $E_x$ for all modes $(i=0,1,2)$ while the $A_x$ terms contribute only for those modes of $J_x$ on the line.

Using the transformation $x' = w \sin \theta'$ and $dx' = w \cos \theta'$ in (3.70), the integral over $x'$ may be replaced by $P_n$.  

as was done previously, since \( v_i(\theta') = \cos 2i\theta' / \cos \theta' \).

Therefore, (3.70) becomes

\[
\sum_{i=1}^{K} \sum_{k=1}^{K} \sum_{m} \frac{B_{ki}}{u_n(z')} \int G_x(z, z') P_n \sin(u_n x) dz' -
\sum_{i=0}^{K} \sum_{k=1}^{K} \sum_{m} \frac{D_{ki}}{u_n(z')} \int G(z, z') P_n \sin(u_n x) dz' = 0
\] (3.71)

Since the \( x \) dependence in (3.71) is \( \sin(u_n x) \) rather than \( \cos u_n x \) for \( E_z \), a different test function in \( x \) will be used. A good choice is \( \sin(l \pi x / w) \), since it equals 0 at \( x=0, w \). Testing (3.71) in \( x \) gives

\[
PE_n = 2 \int_{0}^{w} \sin(u_n x) \cdot \sin(\pi x / w) dx \quad \text{(3.72a)}
\]

or

\[
PE_n = -\frac{2 \pi j}{w} \frac{1}{u_n^2 \left( j \pi / w \right)^2} \cdot \sin(u_n w) \quad \text{(3.72b)}
\]

where the index \( j \) denotes the mode in \( x \), not the complex operator. If only one \( J \) mode is used in the expansion, \( I=1 \) in (3.70) and (3.71) then \( j=1 \) in (3.72b). If two modes are used, then \( j=1,2 \).

Using (3.72b), (3.71) becomes, for sources on line 1
\[
\sum_{i=1}^{l} \sum_{k=1}^{K} \sum_{n} \sum_{m} \frac{B_{ki}}{u_{n}} \int_{z} G_{x}(z, z') P_{ni} P_{nj} d z' \]

\[
- \sum_{i=0}^{l} \sum_{k=1}^{K} \sum_{n} \sum_{m} \alpha_{ki} u_{n} \int_{z} G_{x}(z, z') P_{ni} P_{nj} d z' = 0 \quad (3.73a)
\]

and for sources on line 2

\[
\sum_{i=1}^{l} \sum_{k=1}^{K} \sum_{n} \sum_{m} \hat{B}_{ki} \int_{z} G_{x}(z, z') P_{ni} P_{nj} d z' - \]

\[
- \sum_{i=0}^{l} \sum_{k=1}^{K} \sum_{n} \sum_{m} \hat{D}_{ki} u_{n} \int G(z, z') P_{ni} P_{nj} d z' = 0 \quad (3.73b)
\]

The field as a function of \( z \), \( E_{x}(z) \), that is produced by \( J_{x} \) is not the same as \( E_{z}(z) \). Figure (3.12) shows the field produced by a bipolar pulse for \( E_{x} \). For a bipolar pulse, the field is anti-symmetric about the midpoint of the pulse. As a result, the offset testing pulses that are used for \( E_{z}(z) \) can not be used here, as Fig. (3.13a) shows for the case of \( s=k \), i.e. the dominant term in the matrix element. Figure (3.31b) shows a better choice of testing pulse, since the result of such an integration in \( z \) will produce a non zero result. When test and expansion pulses do not overlap, either type of testing pulse would be acceptable.
(a) Field produced by a unipolar pulse.

(b) Field produced by a bipolar pulse, for $E_x(z)$.

Fig. (3.12). Z dependent fields produced by $J_x$. 
(a) Poor choice for testing pulse for $E_x(z)$.

(b) Good choice for testing pulse for $E_x(z)$.

Fig. (3.13). Testing pulses for fields produced by $J_x$. 
Since there is no \( J_x \) mode corresponding to the \( i=0 \) mode for \( J_z \) the relationship between \( D_{k0} \) and \( C_{k0} \) is still determined by the relationship \( D_{k0} = -(\beta/2k_0d)C_{k0} \) and similarly for \( \hat{D}_{k0} \) and \( \hat{C}_{k0} \) on line 2. If only one \( J_x \) mode is assumed to exist on the lines \( (i=1) \), then \( D_{k1} \) and \( C_{k1} \) are no longer related and must therefore be treated as independent variables. If two \( J_x \) modes are assumed to exist, then for the \( i=2 \) modes, \( D_{k2} \) and \( C_{k2} \) must also be treated as independent variables.

For one mode of \( J_x \) on the lines, the charge amplitudes \( \hat{D}_{k1} \) and \( \hat{D}_{k1} \) become independent of the \( C_{k1} \) and \( C_{k1} \) for \( k=1 \) to 5. This creates 10 new unknowns to be solved for. As a result, 10 new testing pulses must be used to generate 10 new equations relating the fields to the sources on the lines. This is done by testing (3.73b) to enforce the condition \( E_x = 0 \) on the lines.

Figure (3.14) shows the \( k=1 \) half pulses and the \( k=2 \) bipolar expansion pulses along with the test pulses used. The signs of the pulses are determined by their relationship to the current triangles. As a result, the first half pulse on line 2 is inverted. Once again, it's easier to evaluate the integrals involving bipolar pulses \( (k>1) \) by testing unipolar pulses of the type shown in Fig. (3.14a) and then form the bipolar terms by combining adjacent pulses. All of the integrals to be
(a) Half pulses, \( k=1 \) terms.

(b) First full pulses, \( k=2 \) terms.

(c) Testing pulses, \( s=1, 2, 3 \).

Fig. (3.14). First few expansion and testin pulses for \( E_x \).
evaluated are of the form

\[ C_{k}^{\text{ILP}} = \int \int e^{-\gamma|z-z'|} P_{k}(z')P_{\varepsilon}(z)dz'dz \]  \hspace{1cm} (3.74)

the \( \gamma \) terms are of the same form, and ILP=0,1,2, or 3 determines the integration interval, as before.

For ILP=0 the expansion and testing are done on line 1. When \( k=s=1 \), (3.74) becomes

\[ C_{11}^{0} = \int_{-\Delta}^{0} \left[ e^{-\gamma z} \int_{-\Delta}^{0} e^{\gamma z'}dz' + e^{\gamma z} \int_{z}^{0} e^{-\gamma z'}dz' \right]dz \]

\[ = \frac{2\Delta}{\gamma} - \frac{2}{\gamma}(1-e^{-\gamma \Delta}) = \frac{4d}{\gamma} - \frac{2}{\gamma^{3}}(1-e^{-2\gamma d}) \]  \hspace{1cm} (3.75)

This is the same result for all \( k=s \) on line 1. For ILP=3, expanding and testing on line 2, \( C_{11}^{3} = -C_{11}^{0} \), but \( C_{kk}^{3} = C_{kk}^{0} \) for \( k>1 \).

For \( s=2, k=1 \), and ILP=0, \( C_{12}^{0} \) becomes

\[ C_{12}^{0} = \int_{-2\Delta}^{-\Delta} e^{\gamma z}dz \int_{-\Delta}^{0} e^{-\gamma z'}dz' = \frac{1}{\gamma^{2}}(1-2e^{-2\gamma d}+e^{-4\gamma d}) \]  \hspace{1cm} (3.76)

Equation (3.76) holds for all \( s=k \neq 1 \), for ILP=0 or 3, since the field produced by a bipolar pulse is symmetric. For testing pulses \( s>k+1 \), the corresponding
matrix element is (3.76) times a factor of $\text{e}^{-2(s-2)\gamma d}$, or

$$CT_{1s}^0 = e^{-2(s-2)\gamma d} CT_{12}^0 \text{ for } s>2$$ \hspace{1cm} (3.77)

To evaluate the testing of bipolar pulses, consider the $k=2$ pulse, along with several testing pulses, as shown in Fig. (3.14). In this case, $CT_{21}^0 = CT_{12}^0 - CT_{11}^0$, $CT_{22}^0 = CT_{11}^0 - CT_{12}^0$, and $CT_{2s}^0 = CT_{1,s-1}^0 - CT_{1s}^0$ for $s>2$. The $CT_{2s}^0$ are the same form as the $CT_{2s}^0$ terms. Once the first bipolar pulse has been tested, the others may be obtained in the same manner as was described for $E_z$ earlier.

For the cross coupling terms, consider ILP=1 first. With reference to Fig. (3.14), $CT_{11}^1$ is the first half pulse on line 1 tested with the $s=1$ pulse on line 2. By symmetry, this should be the same as $CT_{12}^0$ with an extra factor of $\text{e}^{-\gamma \delta}$, so

$$CT_{11}^1 = \frac{e^{-\gamma \delta}}{\gamma^2} (1 - 2e^{-2\gamma d} + e^{-4\gamma d})$$

and so $CT_{1s}^1 = e^{-2(s-1)\gamma d} CT_{11}^1$ \hspace{1cm} (3.78)

For the ILP=2 case, where the source is on line 2 with testing on line 1,
\[ CT_{11}^2 = -CT_{11}^1 \]  
\[ \text{and} \]  
\[ CT_{1s}^2 = -CT_{1s}^1 \]  
\[ CT_{2s}^{1,2} = -CT_{2, s+1}^{1,2} - CT_{2, s}^{1,2} \]  

For testing the first bipolar pulse, \( k=2 \), the expressions are:

for \( s>1 \). All the \( CT_{ks}^{ILP} \) expressions are the same for the terms involving \( \bar{\gamma} \), and the Green's functions will contribute an extra \( 1/\gamma \) or \( 1/\bar{\gamma} \) term. All of the \( CT_{ks}^{ILP} \) corresponding to the vector potential \( A_x \) have associated with them a \( F(m) \), while those corresponding to \( \partial \psi / \partial x \) in (3.73b) have a \( H(m) \) associated with the \( \gamma \) terms, and a \( \bar{H}(m) \) associated with the \( \bar{\gamma} \) terms. This may be illustrated more clearly by defining:

\[ CQ_{ks}^{ILP} = \frac{H(m)}{\gamma} CT_{ks}^{ILP} + \frac{\bar{H}(m)}{\overline{\gamma}} CT_{ks}^{ILP} \]  
\[ CI_{ks}^{ILP} = \frac{F(m)}{\gamma} CT_{ks}^{ILP} \]  

where \( CT \) implies that the \( \bar{\gamma} \) terms are used. As was done for the matrix elements for \( E_z \), two arrays may be
defined as follows

\[
RX_{\text{ILP}}^{1}(i,j,l,k) = \sum_{n} \sum_{m} C_{\text{ILP}}^{1} \frac{1}{u} P_{n} PE_{nj} \quad (3.83a)
\]

\[
TX_{\text{ILP}}^{1}(i,j,l,p) = \sum_{n} \sum_{m} C_{Q_{\text{ILP}}}^{1} \frac{1}{u} P_{n} PE_{nj} \quad (3.83b)
\]

where \( l=1 \) refers to the half pulse, and \( l=2 \) to the full pulse expansion. Now (3.73b) may be written as

\[
\sum_{i=1}^{l} \sum_{k=1}^{k} B_{ki'}RX_{0,1}^{1}(i',j,l,k) - \sum_{i=0}^{k} D_{ki}TX_{0,1}^{1}(i,j,l,k) + \]

\[
\sum_{i=1}^{l} \sum_{k=1}^{k} \hat{B}_{ki'}R_{2,3}^{1}(i',j,l,k) - \sum_{i=0}^{k} \hat{D}_{ki}TX_{2,3}^{1}(i,j,l,k) = 0 \quad (3.84)
\]

Since there is no dominant mode \( J_{x} \) on either line, the corresponding source terms are zero, and the \( B_{in} \) and \( \tau \) terms are defined in (3.84). The \( B_{ki} \) and \( \hat{B}_{ki} \) were defined in (3.63c) and (3.63d), and may be expressed in terms of the charge and current amplitudes

\[
B_{ki'} = k_{0}^{2}D_{ki'} + \beta_{1}C_{ki'} / 2d \quad (3.84a)
\]

\[
\hat{B}_{ki'} = k_{0}^{2}D_{ki'} + \beta_{2}\hat{C}_{ki'} / 2d \quad (3.84b)
\]
The boundary condition imposed on $J_z$ requires that the $C_{1i}$ and $\hat{C}_{1i}$ cancel the dominant modes at the open ends. As a result, the $C_{1i}$ and $\hat{C}_{1i}$ in (3.84a) and (3.84b) are constrained in the same manner. The $D_{ki}$ and $\hat{D}_{ki}$ that multiply the $TX(i,j,1,k)$ in (3.84), for $i \neq i'$, are related to the charge amplitudes by $D_{1i} = -\beta_i B_{in} I_{1i} \cdot DK$ and $\hat{D}_{1i} = -\beta_2 \tau I_{2i} \cdot DK \cos \theta$, where $DK = 1/2k_0^2d$, while $C_{1i} = -B_{in} I_{1i}$ and $\hat{C}_{1i} = -\tau I_{2i} \cdot \cos \theta$.

As a result, the coefficients of $B_{in}$ and $\tau$ may be written as

\[
B^{0,1}_{in} = -\frac{\beta_1}{\Delta} \sum_{i=1}^{\ell} I_{1i} \cdot RX^{0,1}(i',j,1,k) + \sum_{i=0}^{2} I_{1i} \cdot TX^{0,1}(i,j,1,k)
\]

(3.85)

\[
\tau^{2,3}_x = -\frac{\beta_2}{\Delta} \sum_{i=1}^{\ell} I_{2i} \cdot RX^{2,3}(i',j,1,k) + \sum_{i=0}^{2} I_{2i} \cdot TX^{2,3}(i,j,1,k)
\]

(3.86)

so the final expression for $E_x = 0$ may be written as

\[
B^{0,1} = \sum_{k=2}^{\infty} \left[ \sum_{i=1}^{\ell} (k_0^2 D_{ki} + \frac{\beta_i}{\Delta} C_{ki} \cdot) RX^{0,1}(i',j,2,k) - \sum_{i=0}^{2} D_{ki} \cdot TX^{0,1}(i,j,2,k) \right]
\]
\[ +\tau^{2,3} + \sum_{k=2}^{\infty} \left( k \frac{D_{k}}{k} + i, j, 2, k \right) \]

\[ = 0 \quad (3.87) \]

Since there are five expansion pulses in \( z' \), \( K=5 \) in (3.87). This is true regardless if there are one or two modes in \( J_x \) (\( I=1 \) or 2) on the strips. Equation (3.87) provides the required number of equations for the number of unknowns.

If there are two modes in \( J_x \), then a new testing function is defined for \( x \), which is \( \sin(2\pi x/w) \). This insures that there will be sufficient testing functions for the given number of unknowns in (3.87).

For one \( J_x \) mode, there are 50 unknowns, while for two there are 60. The method for extracting the equivalent circuit values is the same whether or not any \( J_x \) modes are present on the strips, only the coefficient array is larger so as to accommodate the greater number of unknowns involved.

The next chapter will present results for a number of asymmetric gaps that where characterized using the technique presented here, along with results for the
open end. Chapter four will also compare these results using this technique with those obtained by other investigators.
CHAPTER 4-RESULTS AND CONCLUSIONS

(4.1) PROGRAM DEVELOPMENT

The programs that were developed to implement the techniques described in the previous chapters were done in BASIC and then compiled using a compiler that accommodates the use of an 8087 math coprocessor. This language was chosen since it is almost universally available as well as being well known. The programs were run on an Epson Equity II personal computer operating at a clock rate of 7.16 Mhz. along with an 8087 math coprocessor to speed up the calculations. This satisfied the requirement that the technique to be developed could be implemented with limited computer resources, rather than require the use of a mainframe computer.

The primary constraint that the BASIC language placed on the technique was its 64K block size. This meant that no single program could exceed 64K of memory. The 64K of memory had to store all arrays and variables besides the program lines themselves. The only way to get around this limitation is to break up large programs into smaller blocks and then chain them together during the linking stage. This approach to program development was rejected in this particular case since the greatest amount of space needed in the program was taken up in

172
the arrays that had to be defined. In chaining together several modules, certain arrays, of large overall dimension would have to be passed from one module to the next. This would require that each module set aside suitable memory space for the arrays to be passed through. As a result, the only real savings would have occurred in the reduced number of program lines that each individual module would contain.

An in-line compiler could not be used on the programs that were developed since the in-line compiler generates an object code that is significantly larger than the object code generated in the standard BASIC compiler. As a result of this, the compilation using the in-line compiler could not be completed, since its object code over ran the address boundaries that the microprocessor could handle. This is unfortunate, since an in-line code would execute much faster than the code generated by the standard compiler.

The results for the open end will now be presented, followed by those for the gap. These results will be compared with those of other investigators. Suggestions for future work will then be discussed. A flow chart for the programs will be given in an appendix.
(4.2) OPEN END RESULTS

Figure (4.1) shows the results obtained for the open end using this technique plotted against those obtained by Silvester and Benedek [11]. Silvester and Benedek's results were chosen for comparison since they seemed to be the most accurate and complete, and they appear in a form that is most useful for design purposes. Also, if the frequency dependent variables in this technique were set to zero, i.e., a purely static solution desired, the formulation presented here would collapse to that used by Silvester and Benedek, except for the exact representation of the charges on the open end. In this way, the potential theory approach may be considered as a frequency dependent version of their charge reversal method.

Since their results were obtained using static methods, the data presented here was obtained at a frequency of .5 Ghz. The program was written so as to present the results in the form of a normalized susceptance, $B_{in}$. This value had to be converted into a capacitance so that it could be plotted against their results. This was done using the relation, $C_{oc} = B_{in}/\omega Z_0$, where the characteristic impedance $Z_0$ was was taken from Collin's data [64] for infinite microstrips. While his expressions for $Z_0$ were obtained using sidewalls and no

174
Fig. (4.1). Open circuit capacitance for various width to height ratios and dielectric constants.


Dotted lines from this technique. Dimensions are h=1mm, 2A=20mm, b=11mm, d=.1mm for test and expansion pulses.
top cover, the difference in $Z_0$ for a top cover are probably quite small for a cover located of 10 units above the substrate. Typical calculated values for $Z_0$ for the shielded structure with air air dielectric were 1 to 2% smaller than for an unshielded structure.

In generating this data, a substrate thickness of $h=1\text{mm}$, top cover $b=11\text{mm}$, and sidewall spacing of $20\text{mm}$ was chosen. Several runs were made for different placements of the top cover and sidewall location, and the values given above were found to give results that minimize the effects of the enclosure dimensions, while insuring that all sums dependent on these values will converge quite rapidly. These dimensions also insure that only weak field lines will terminate on the enclosure walls, thereby giving little interaction between the enclosure and the discontinuity. The enclosure must not be too large however, so as to allow the fundamental waveguide mode to propagate. At a frequency of $0.5 \text{Ghz}$, this is not a real problem, but at higher frequencies it should be considered. The sidewall spacing chosen here is the same as that used by Collin to calculate the characteristic impedance, as a result, the addition of the top cover will result in a small reduction in impedance, but should not greatly affect the results presented here.
It should be noted that if the frequency is increased, \( B_{in} \) should vary linearly with frequency until the frequency approaches the propagating frequency of the first waveguide mode, which is the \( E_{11} \) mode. If there were no top cover, as the frequency is increased radiation losses would become noticeable. However, most microstrip designs are enclosed by some sort of shielding, often with absorbing material to reduce the coupling with the enclosure, so this technique of using an enclosure to carry out the analysis is justified.

As the cutoff frequency of the enclosure is approached and \( B_{in} \) begins to diverge from its low frequency value, the value of \( C_{oc} \) will also diverge from a constant value. To ensure that the dimensions of this enclosure is suitable, the program was executed at frequencies of 1 and 2 Ghz with no noticable deviation of \( C_{oc} \) from its constant value.

The basic pulse width used to calculate these \( C_{oc} \) values was \( 2d = .2 \text{mm} \), or \( d = .1h \). This was deemed to give the best balance between correctly representing the source distribution near the open end while still accounting for the perturbed distribution farther down the line. Table (4.1) shows the variation of \( C_{oc} \) as a function of \( d \), which is varied by 20\%, while table (4.2) gives the data at 6Ghz.
Table (4.1). $C_{oc}$ values for several different values of pulse width. Dimensions are $h=1$mm, $2A=20$, $b=11$, $f=2$ Ghz.

<table>
<thead>
<tr>
<th>$2w/h$</th>
<th>.25</th>
<th>.5</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d=.08$</td>
<td>28.13</td>
<td>22.47</td>
<td>18.99</td>
<td>16.97</td>
<td>15.84</td>
</tr>
<tr>
<td>$x=2.5$</td>
<td>27.95</td>
<td>22.35</td>
<td>18.92</td>
<td>16.95</td>
<td>15.80</td>
</tr>
<tr>
<td>$d=.10$</td>
<td>27.65</td>
<td>22.12</td>
<td>18.76</td>
<td>16.86</td>
<td>15.76</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$2w/h$</th>
<th>.25</th>
<th>.5</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d=.08$</td>
<td>39.94</td>
<td>32.10</td>
<td>27.30</td>
<td>24.48</td>
<td>22.80</td>
</tr>
<tr>
<td>$x=4.2$</td>
<td>39.41</td>
<td>31.70</td>
<td>27.00</td>
<td>24.28</td>
<td>22.68</td>
</tr>
<tr>
<td>$d=.10$</td>
<td>38.75</td>
<td>31.21</td>
<td>26.64</td>
<td>24.01</td>
<td>22.48</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$2w/h$</th>
<th>.25</th>
<th>.5</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d=.08$</td>
<td>76.86</td>
<td>62.11</td>
<td>52.99</td>
<td>47.60</td>
<td>44.15</td>
</tr>
<tr>
<td>$x=9.6$</td>
<td>75.17</td>
<td>60.83</td>
<td>52.00</td>
<td>46.77</td>
<td>43.43</td>
</tr>
<tr>
<td>$d=.10$</td>
<td>73.37</td>
<td>59.46</td>
<td>50.94</td>
<td>45.89</td>
<td>42.62</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$2w/h$</th>
<th>.25</th>
<th>.5</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d=.08$</td>
<td>120.4</td>
<td>97.53</td>
<td>83.36</td>
<td>74.89</td>
<td>69.42</td>
</tr>
<tr>
<td>$x=16$</td>
<td>117.2</td>
<td>95.10</td>
<td>81.44</td>
<td>73.26</td>
<td>67.88</td>
</tr>
<tr>
<td>$d=.10$</td>
<td>113.9</td>
<td>92.60</td>
<td>79.46</td>
<td>71.56</td>
<td>66.26</td>
</tr>
</tbody>
</table>
\[ C_{\infty} \text{ values in pF/meter, } f=6 \text{ Ghz} \]

<table>
<thead>
<tr>
<th></th>
<th>2w/h=</th>
<th>.25</th>
<th>.5</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>d=.08</td>
<td>28.12</td>
<td>22.43</td>
<td>18.93</td>
<td>16.90</td>
<td>15.82</td>
</tr>
<tr>
<td>x=2.5</td>
<td>d=.10</td>
<td>27.86</td>
<td>22.24</td>
<td>18.81</td>
<td>16.85</td>
<td>15.74</td>
</tr>
<tr>
<td></td>
<td>d=.12</td>
<td>27.46</td>
<td>21.95</td>
<td>18.60</td>
<td>16.70</td>
<td>15.66</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>2w/h=</th>
<th>.25</th>
<th>.5</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>d=.08</td>
<td>39.84</td>
<td>32.07</td>
<td>27.34</td>
<td>24.67</td>
<td>23.25</td>
</tr>
<tr>
<td></td>
<td>d=.12</td>
<td>38.30</td>
<td>30.94</td>
<td>26.52</td>
<td>24.09</td>
<td>22.88</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>2w/h=</th>
<th>.25</th>
<th>.5</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>d=.08</td>
<td>77.31</td>
<td>63.10</td>
<td>54.76</td>
<td>50.76</td>
<td>49.84</td>
</tr>
<tr>
<td>x=9.6</td>
<td>d=.10</td>
<td>75.38</td>
<td>61.74</td>
<td>53.83</td>
<td>50.18</td>
<td>49.65</td>
</tr>
<tr>
<td></td>
<td>d=.12</td>
<td>73.42</td>
<td>60.32</td>
<td>52.79</td>
<td>49.41</td>
<td>49.11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>2w/h=</th>
<th>.25</th>
<th>.5</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>d=.08</td>
<td>122.9</td>
<td>101.5</td>
<td>94.60</td>
<td>85.74</td>
<td>86.57</td>
</tr>
<tr>
<td>x=16</td>
<td>d=.10</td>
<td>119.8</td>
<td>99.42</td>
<td>93.10</td>
<td>84.81</td>
<td>86.06</td>
</tr>
<tr>
<td></td>
<td>d=.12</td>
<td>116.7</td>
<td>97.14</td>
<td>91.20</td>
<td>83.19</td>
<td>86.08</td>
</tr>
</tbody>
</table>

Table (4.2). \( C_{\infty} \) values for several different values of pulse width. Dimensions are \( h=1 \text{mm}, 2A=20, b=9, f=6 \text{ Ghz} \).
If more expansion pulses in \( z \) had been used for all of the modes in \( x \), especially for the \( i=0 \) mode, the effects of the discontinuity would have been modeled more accurately. The charge density near the open end has an edge behavior that will be proportional to \( |z|^{-1/2} \) near the open end. Therefore, a greater number of pulses would have allowed for a smaller pulse width near the discontinuity, along with a larger number of wider pulses farther away. Figure (4.2) shows the variation of \( B_{in} \) vs. the number of expansion pulses used in \( z \) for the \( i=0 \) mode, for the case of a low and high dielectric constant. This data shows that a converged value for \( B_{in} \) may be obtained with fewer pulses if the pulse width is wide. For narrower pulses (smaller \( d \)), a greater number of pulses are needed to obtain a converged \( B_{in} \) value, and that \( B_{in} \) value is larger than the one obtained with wider pulses. This data shows that narrow pulses are needed to represent the charge distribution near the open end.

One feature of the data presented here is that for narrow strip widths, the calculated data gives values of \( C_{oc} \) that are clearly lower than those of Silvester and Benedek. At the low frequency at which this data was calculated the sources on the open end are charge dominated, and since edge conditions are
Fig. (4.2a). Variation of $B_{in}$ as a function of number of expansion pulses used. Pulse width $d$ as a parameter. Low dielectric case, $\nu=4.2$, $f=2$ Ghz, $2w/h=1$, $h=1$, $b=11$, 51 pulses in $z$ compressed to 18, only $i=0$ mode in $x$. 
Fig. (4.2b). Variation of $B_{in}$ as a function of number of expansion pulses used, pulse width $d$ is a parameter. High dielectric case, $\kappa = 9.6$, $f = 2$ G Hz, $2w/h = 1$, $h = 1$, $b = 11$, 51 pulses in $z$ compressed to 18, only $i = 0$ mode in $x$. 
included in this technique, the charge distribution can be expected to be accurate. It is not entirely clear whether Silvester and Benedek's method took adequate account of the charge density singular behavior at the microstrip edges. A smaller enclosure along with suitably smaller expansion and testing pulse widths was tried, but yielded results that closely agree with those presented in Table (4.1).

One possible explanation for the difference may be the way in which Silvester and Benedek checked the convergence of their own data. They used polynomials up to degree two to expand the unknown charge across the strip (in x), and used fixed intervals in z to calculate the excess charge. When they checked to see if their results had converged sufficiently, they did not reduce the size of the interval in z to gain a better approximation of the excess charge at the open end while simultaneously adding extra intervals farther down the line. As a result, their values may not be entirely accurate.

Since the difference between their results and those based on the technique presented here are greatest for narrow strip widths, while the two sets of results get much better for wide strips, it could be that their representation of the excess charge near the open end
was poor.

While low frequency results were presented here, the user may of course run the program at any desired frequency. The pertinent parameters required to do this are available as input variables so that nothing in the geometry presented here is fixed in any way.

With the math coprocessor, an open circuited line can be characterized in just over three minutes. This includes the time required to calculate the dominant mode source amplitudes for an infinite line along with the propagation constant for a given enclosure, as well as finding the first 100 roots of the LSE and LSM modes for the dielectric loaded waveguide.

(4.3) GAP RESULTS

Figure (4.3) shows the results of this technique as compared to Silvester and Benedek's results [12] for a symmetric gap in microstrip for three different width to height ratios. For the data generated by this technique, the same geometry, substrate thickness, and pulse width was used as in the open end example.

One major difference between the open end data and the gap data, is that in the gap data one \( J_x \) mode is assumed to exist on both lines. At the frequency used to generate this data however, the effect of this \( J_x \) is
Fig. (4.3a). Symmetric gap, $2w/h=.5$, $C_{even}$, Silvester & Benedek's data, dashed lines, this technique, points.
Fig. (4.3b). Symmetric gap, 2w/h=.5, $C_{\text{odd}}$, Silvester & Benedek’s data, dashed lines, this technique, points
Fig. (4.3c) Symmetric gap, 2w/h=1.0, $C_{even}$, Silvester & Benedek's data, dashed lines, this technique, points
Fig. (4.3d). Symmetric gap, $2w/h=1.0$, $C_{odd}$, Silvester & Benedek's data, dashed lines, this technique, points.
Fig. (4.3e). Symmetric gap, 2w/h=2.0, C_{even}, Silvester & Benedek's data, dashed lines, this technique, points.
Fig. (4.3f). Symmetric gap, $2w/h=2.0$, $C_{\text{odd}}$. Silvester & Benedek's data, dashed lines, this technique, points.
negligible.

As Fig. (4.3) shows, the agreement with Silvester and Benedek's results is excellent.

Figure (4.4) shows the results compared to Jansen's [51] for \( x = 10.4 \), \( h = 0.635 \text{mm} \), and \( f = 2.54 \text{ GHz} \). Jansen's technique is probably the most complete and comprehensive full wave technique for characterizing a wide range of microstrip discontinuities. In this case the placement of the side walls was chosen to be \( 2(5 + \max(w_1, w_2)) \), since this is the way Jansen specifies the location of the side walls. The top plate was chosen to be at \( b = 11 \text{mm} \), Jansen does not state the location of his top cover.

The results of this technique compare quite well to Jansen's, especially for the series values of capacitance. There is somewhat greater discrepancy in shunt values. One important feature of Jansen's results should be noted in this regard however, and that is his results for the shunt values for the case of a symmetric gap, when \( w_1 = w_2 \). For this case, \( C_{s1} \) should equal \( C_{s2} \), but from the plot of his data, this seems not to be the case. The difference in values is greatest for small gap values, \( g/h = 0.15 \) and 0.3, while they seem to get better for more lightly coupled gaps. While this may be due to the difficulty of plotting data on a log-log
Fig. (4.4a). Gap capacitance, Jansen's data solid lines, this technique dashed lines. F=2.54 Ghz, b=11mm, 2a=2(5+max(w1,w2)), h=.635mm.
Fig. (4.4b). Shunt capacitance, line 1. Jansen's data, solid lines, this technique dashed lines. F=2.54 GHz, b=11mm, 2A=2(5+max(w1,w2)), h=.635mm.
Fig. (4.4c). Shunt capacitance, line 2. Jansen's data, solid lines, this technique dashed lines. $f=2.54$ Ghz, $b=11\text{mm}$, $2A=2(5+\max(w1,w2))$, $h=.635$mm.
graph, it may also be due to the manner in which he calculates the lumped circuit capacitance from the S parameters which is usually his way to present the data.

It may also be due to the technique used to characterize discontinuities. In Jansen's technique, the location of the discontinuity is shifted relative to the resonator walls in which it is located, which allows a new resonant frequency to be calculated for each location of the discontinuity. This shifting may have resulted in an asymmetry of the sources on the lines to occur. This would lead to different values of $C_{s_1}$ and $C_{s_2}$ to be calculated even when the gap is symmetrical.

Figure (4.5) shows the results obtained by this technique plotted against Jansen's results for a low dielectric case, $\varepsilon = 2.35$, with all other parameters unchanged. Notice that the dashed lines for certain values of $C_{s_1}$ indicate an inductive, rather than capacitive value for these cases. This occurs for tight coupling when line 2 is wider than line 1. This is due to the fact that the model used to represent the discontinuity is a capacitive pi network which represents the excess capacitance near the gap. For cases of tight coupling, some of the electric field lines from line 1 that would have terminated in the vicinity of the open end, now terminate on line 2. This
Fig. (4.5a). Gap capacitance. Jansen's data, solid lines, this technique in dashed lines. F=2.54 Ghz, b=11mm, 2A=2(5+max(w1,w2)), h=.635mm.
Fig. (4.5b). Shunt capacitance, line 1. Jansen's data, solid lines, this technique, dashed lines. $F=2.54$ Ghz, $b=11$mm, $2A=2(5+\max(w1,w2))$, $h=.635$mm.
Fig. (4.5c). Shunt capacitance, line 2. Jansen's data, solid lines, this technique, dashed lines. F=2.54 GHz, b=11mm, 2A=2(5+max(w1,w2)), h=.635mm.
has the effect of decreasing the capacitance in the vicinity of the gap below that of the capacitance per unit length that would normally be associated with a uniform microstrip line. This results in a negative value for $C_{s1}$ in these particular cases.

Regardless of the degree of asymmetry between the two lines however, when the gap spacing is increased, the capacitive effect will eventually predominate, since the behavior of a lightly coupled gap will more closely resemble two uncoupled open ends. When the gap spacing becomes greater than approximately five times the substrate thickness, the results for the gap become nearly that of two uncoupled open ends, as might be expected.

This feature allows the user of this program to calculate the open end capacitance for a stub that includes the effect of $J_x$. If a gap value of $10 \cdot h$ is input, the program will omit the calculation of all cross coupling terms and fill the array with only the self interaction terms. The result of the inversion of this matrix will be the open end capacitance of two stubs.

The effect of $J_x$ for the stub will not be significant except at high frequencies and for wide strip widths. It's importance is much greater in the
case of an asymmetrical gap. Table (4.3) shows the difference in $C_{s1}$, $C_{s2}$, and $C_g$ when there is no $J_x$ present compared to the case when it is included. As can be seen from this table, the effect of $J_x$ is greater when the difference in line widths is greater, and quite small when the line widths are comparable.

(4.4) CONCLUSIONS

The dynamic source reversal technique based on potential theory seems to be a good technique for characterizing microstrip discontinuities in terms of the actual sources involved. The Green's functions can be expressed in a simple form, the sources can be accurately described including their appropriate edge conditions, and the dominant parts of the matrix elements can be summed into closed form without resorting to sophisticated mathematical or numerical techniques to treat them. The sources are related to the actual fields inside the waveguide enclosure in a straightforward manner, and the perturbation in the sources due to the discontinuity are represented to a very accurate degree. Only a single matrix inversion is required to find all three of the equivalent network parameters.

For all this, the computer resources required to
Capacitive pi values, in pF

2wL/h=1.0, g/h=.15, x=2.35

<table>
<thead>
<tr>
<th>2wL/h</th>
<th>C_{s1}</th>
<th>C_{s2}</th>
<th>C_g</th>
<th>C_{s1}</th>
<th>C_{s2}</th>
<th>C_g</th>
</tr>
</thead>
<tbody>
<tr>
<td>.25</td>
<td>7.30</td>
<td>-2.30</td>
<td>8.58</td>
<td>6.97</td>
<td>-2.10</td>
<td>8.29</td>
</tr>
<tr>
<td>1.0</td>
<td>1.87</td>
<td>1.87</td>
<td>17.10</td>
<td>1.85</td>
<td>1.85</td>
<td>16.40</td>
</tr>
<tr>
<td>4.0</td>
<td>-3.19</td>
<td>28.27</td>
<td>25.14</td>
<td>-7.50</td>
<td>27.23</td>
<td>24.95</td>
</tr>
</tbody>
</table>

2wL/h=1.0, g/h=.5, x=2.35

<table>
<thead>
<tr>
<th>2wL/h</th>
<th>C_{s1}</th>
<th>C_{s2}</th>
<th>C_g</th>
<th>C_{s1}</th>
<th>C_{s2}</th>
<th>C_g</th>
</tr>
</thead>
<tbody>
<tr>
<td>.25</td>
<td>8.14</td>
<td>.423</td>
<td>4.37</td>
<td>7.92</td>
<td>.467</td>
<td>4.30</td>
</tr>
<tr>
<td>1.0</td>
<td>5.05</td>
<td>5.05</td>
<td>8.26</td>
<td>4.95</td>
<td>4.95</td>
<td>8.01</td>
</tr>
<tr>
<td>4.0</td>
<td>-1.06</td>
<td>28.28</td>
<td>14.41</td>
<td>-.910</td>
<td>28.25</td>
<td>14.10</td>
</tr>
</tbody>
</table>

2wL/h=1.0, g/h=1.0, x=2.35

<table>
<thead>
<tr>
<th>2wL/h</th>
<th>C_{s1}</th>
<th>C_{s2}</th>
<th>C_g</th>
<th>C_{s1}</th>
<th>C_{s2}</th>
<th>C_g</th>
</tr>
</thead>
<tbody>
<tr>
<td>.25</td>
<td>9.44</td>
<td>2.22</td>
<td>2.13</td>
<td>9.22</td>
<td>2.22</td>
<td>2.11</td>
</tr>
<tr>
<td>1.0</td>
<td>7.77</td>
<td>7.77</td>
<td>4.00</td>
<td>7.61</td>
<td>7.61</td>
<td>3.90</td>
</tr>
<tr>
<td>4.0</td>
<td>4.14</td>
<td>30.80</td>
<td>7.80</td>
<td>4.04</td>
<td>30.97</td>
<td>7.67</td>
</tr>
</tbody>
</table>

Table (4.3a). Results from this technique for capacitive pi values with no J_x vs. one J_x mode, 2wL/h=1.0 in all cases.
Capacitive pi values, in pF

\(2w1/h=1.0, \ g/h=.15, \ x=10.4\)

<table>
<thead>
<tr>
<th>(2w2/h)</th>
<th>(C_{s1})</th>
<th>(C_{s2})</th>
<th>(C_g)</th>
<th>(C_{s1})</th>
<th>(C_{s2})</th>
<th>(C_g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>22.50</td>
<td>-5.66</td>
<td>23.10</td>
<td>21.60</td>
<td>-5.11</td>
<td>22.39</td>
</tr>
<tr>
<td>1.0</td>
<td>6.88</td>
<td>6.88</td>
<td>45.84</td>
<td>6.78</td>
<td>44.56</td>
<td>6.78</td>
</tr>
<tr>
<td>4.0</td>
<td>-17.22</td>
<td>82.79</td>
<td>63.61</td>
<td>-15.70</td>
<td>81.31</td>
<td>64.06</td>
</tr>
</tbody>
</table>

\(2w1=1.0, \ g/h=.5, \ x=10.4\)

<table>
<thead>
<tr>
<th>(2w2/h)</th>
<th>(C_{s1})</th>
<th>(C_{s2})</th>
<th>(C_g)</th>
<th>(C_{s1})</th>
<th>(C_{s2})</th>
<th>(C_g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>26.32</td>
<td>3.09</td>
<td>10.62</td>
<td>25.57</td>
<td>3.20</td>
<td>10.40</td>
</tr>
<tr>
<td>1.0</td>
<td>18.20</td>
<td>18.20</td>
<td>20.33</td>
<td>17.80</td>
<td>17.80</td>
<td>19.70</td>
</tr>
<tr>
<td>4.0</td>
<td>5.27</td>
<td>88.35</td>
<td>32.68</td>
<td>5.49</td>
<td>88.75</td>
<td>31.98</td>
</tr>
</tbody>
</table>

\(2w1=1.0, \ g/h=1.0, \ x=10.4\)

<table>
<thead>
<tr>
<th>(2w2/h)</th>
<th>(C_{s1})</th>
<th>(C_{s2})</th>
<th>(C_g)</th>
<th>(C_{s1})</th>
<th>(C_{s2})</th>
<th>(C_g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>30.70</td>
<td>8.59</td>
<td>4.19</td>
<td>29.89</td>
<td>8.57</td>
<td>4.12</td>
</tr>
<tr>
<td>1.0</td>
<td>27.12</td>
<td>27.12</td>
<td>8.04</td>
<td>26.46</td>
<td>26.46</td>
<td>7.80</td>
</tr>
<tr>
<td>4.0</td>
<td>21.07</td>
<td>98.75</td>
<td>14.16</td>
<td>20.57</td>
<td>99.33</td>
<td>13.83</td>
</tr>
</tbody>
</table>

Table (4.3b). Results from this technique for capacitive pi values wit no \(J_x\) vs. one \(J_x\) mode, \(2w1/h=1.0\) for all cases.
execute the program are minimal, and execution times are quite short relative to other techniques.

In summary, this technique provides a clear improvement over existing techniques with regards to speed and accuracy in characterizing some of the commonly used microstrip discontinuities.

(4.5) RECOMMENDATIONS FOR FURTHER WORK

An immediate improvement in execution time could be obtained for this approach by running the programs on a faster personal computer. Further improvements could be obtained if the 64K block size that limits the BASIC program could be increased. If it were doubled to 128K, this would allow for array size to be increased to the point where no redundant calculations would have to be carried out, as is the case now. The BASIC language is not an optimum scientific programming language. If these programs were written in another language, the execution time could be decreased to some extent.

A greater block size would also allow for the addition of extra expansion modes representing the perturbations in the sources, leading to greater accuracy. Therefore, running this more accurate program on a faster machine would allow for a greater accuracy
at speeds comparable or faster than those discussed here.

Beyond these system level improvements, there are other things that can be done to expand the scope and utility of this approach.

For one, a greater number of commonly used discontinuities can be characterized, such as a step in width, a T-junction, and cross junctions. The increased level of complexity that these discontinuities represent, primarily with regards to the extra dominant mode and perturbed sources needed, would require a much larger computer program to implement. A more versatile approach would involve segmenting the program into smaller, self-contained blocks that could be linked together thus forming a much larger routine. As the size of the coefficient array increases, different matrix inversion techniques may be required to avoid numerical instabilities.

More complex junctions could be done as well, such as asymmetrically coupled gaps or asymmetric steps. This would require the inclusion of extra terms in the various Green's functions, since the original transverse symmetry inherent when the strips are centered inside the waveguide is no longer present. This type of effort would fully characterize a wide range of commonly used
discontinuities.

Anisotropic substrates could also be included. This would require an extra term in the Green's function associated with the potential, and would allow for the characterization of some commonly used substrate materials. Dielectric loss could also be included.

The primary factor that determines the size of the coefficient array is the number of local expansion functions used in $z$. It would be useful to replace these local expansion functions with global expansion functions, such as exponentials involving the first few evanescent waveguide propagation constants, for example, $e^{-\gamma_{nm}z}$ where $n,m=1,2,3$, or 4. These exponentials could be transformed into orthogonal functions by mean of the Gram Schmitt orthogonalization procedure. This would significantly reduce the number of unknown amplitudes while still giving an accurate representation of the perturbed sources in the vicinity of the discontinuity. This approach would have the advantage that the long exponential decay could automatically account for the perturbation at a great distance from the junction while giving a good approximation in the region near the discontinuity where the perturbed sources vary more rapidly.

This approach could be carried out for a simple
discontinuity such as the open end first, and this would allow easy comparison of the results thus obtained to those presented in this approach. Among other things, it could help in determining how many exponential terms would be needed to achieve a certain level of approximation or accuracy.

Finally, the technique presented here could be applied to other types of waveguiding systems, such as suspended substrates, slot lines, or microstrips imbedded in multiple substrates. This would extend the use of this technique to include the use of multiple layer p.c. boards.
APPENDIX 1- SUMMING SERIES

Of all the various sums that arise in the development of the equations in this technique, there are two sets of double sums that do not have any exponential decay associated with them, and so their convergence would be quite slow if they had to be summed over several hundred terms in \( n \) and \( m \). These sums are of the form

\[
\sum_{n} \sum_{m} \frac{F(m)}{\gamma^2 + \beta^2} P_{ni} P_{nj} \quad \text{and} \quad \sum_{n} \sum_{m} \left[ \frac{H(m)}{\gamma^2 + \beta^2} + \frac{\overline{H}(m)}{\gamma^2 + \beta^2} \right] P_{ni} P_{nj} \quad (A1a)
\]

which appear in the expressions for the infinite line equations, and

\[
\sum_{n} \sum_{m} \frac{F(m)}{\gamma^2 + \beta^2} P_{ni} P_{nj}^* \quad \text{and} \quad \sum_{n} \sum_{m} \left[ \frac{H(m)}{\gamma^2 + \beta^2} + \frac{\overline{H}(m)}{\gamma^2 + \beta^2} \right] P_{ni} P_{nj}^* \quad (A1b)
\]

which appear in the expressions for the discontinuities. These terms can be summed over the index \( m \) using the following expression. Consider the term containing \( F(m) \) in (A1a), and a function of spatial frequency, \( f(\omega) \)

\[
f(\omega) = \frac{\omega \sin(pc) \sin(lh)}{(\omega^2 + \beta^2) \cdot (pc \cos(pc) \sin(lh) + l \sin(pc) \cos(lh))} \quad (A2)
\]
which is even in $p$ and $l$, where $p = k_0^2 - \omega^2 - u_n^2$ and $l = k_2 - \omega^2 - u_n^2$. $f(\omega)$ has poles at $\omega = \pm j\gamma_{nm}$ which give $p = p_m$ and $l = l_m$, and poles at $\omega = \pm \beta$. Now, use can be made of \( \oint f(\omega) d\omega = 2\pi j \sum \text{Residues} \), where the contour is closed with a semicircle at infinity. The path along the $\omega$ axis is closed so as to enclose the poles at $\pm \beta$. The contour integral may be evaluated by finding the roots of the derivative of the denominator as follows, consider

\[
\frac{\partial}{\partial \omega} (pcos(pc)sin lh + lsin(pc)cos lh) =
\]

\[
= \frac{-\omega}{p} (cos(pc)sin lh - pcsin(pc)sin lh + lccos(pc)cos lh) -
\]

\[
- \frac{\omega}{l} (phcos(pc)cos lh + sin(pc)cos lh - lhsin(pc)sin lh)
\]

Evaluating this expression at $\omega = j\gamma_{nm}$ gives

\[
\frac{j\gamma_{nm}}{p_m l_m} \left[ h(p_m^2 cos(p_m)cos(l_m) - p_m l_m sin(p_m)sin(l_m)) +
\right.
\]

\[
+ c(l_m^2 cos(p_m)cos(l_m) - l_m p_m sin(p_m)sin(l_m)) +
\]

\[
+ (l_m cos(p_m)sin(l_m) + p_m sin(p_m)cos(l_m)) \right]
\]
The residue at \( \omega = j \gamma_{nm} \) is found from \( f(\omega) \bigg|_{\omega = j \gamma_{nm}} \) or

\[
\frac{p_{m}^{1/2} \sin(p_{m} c) \sin(l_{h})}{D(\beta + \gamma_{nm}^{2})} = \frac{-F(m)}{\gamma_{nm}^{2} + \beta^{2}} \quad \text{where}
\]

\[
D(\beta + \gamma_{nm}^{2}) = \left[ (\beta + \gamma_{nm}^{2}) \left( h(p_{m}^{2} \cos(p_{m} c) \cos(l_{h}) - p_{m} l_{m} \sin(p_{m} c) \sin(l_{h})) \right. \right.
\]
\[
+ c(l_{m}^{2} \cos(p_{m} c) \cos(l_{h}) - p_{m} l_{m} \sin(p_{m} c) \sin(l_{h})) +
\]
\[
+ (l_{m} \cos(p_{m} c) \sin(l_{h}) + p_{m} \sin(p_{m} c) \cos(l_{h})) \right]\]

The residue at \(-j \gamma_{nm}\) is the same. To evaluate the residues at \( \omega = \pm \beta \), define the following

\[
\gamma_{nm}^{2} = u_{n}^{2} + \beta^{2} - k_{0}^{2} \quad \text{and} \quad \gamma_{1n}^{2} = u_{n}^{2} + \beta^{2} - k_{0}^{2} \quad \text{(A3)}
\]

so, \( p = j \gamma_{n} \) and \( l = j \gamma_{1n} \) which will give the residues at \( f(\omega) \bigg|_{\omega = \beta} \) of (where the subscript \( n \) has been dropped)

\[
f(\omega) = \frac{1}{2} \frac{\beta \sin(j \gamma) \sin(j \gamma_{1})}{\beta(j \gamma \cos(j \gamma) \sin(j \gamma_{1}) + j \gamma_{1} \sin(j \gamma) \cos(j \gamma_{1}))} \quad \text{or}
\]
\( f(\omega) = \frac{1}{2} \frac{\sinh(\gamma c) \sinh(\gamma_1 h)}{\gamma \cosh(\gamma c) \sinh(\gamma_1 h) + \gamma_1 \sinh(\gamma c) \cosh(\gamma_1 h)} \) \hspace{1cm} (A4)

Therefore, the sum over m may be expressed as

\[
\sum_{m} \frac{F(m) \cdot \frac{1}{2} \sinh(\gamma c) \sinh(\gamma_1 h)}{\gamma^2 + \beta^2} \frac{\sinh(\gamma c) \sinh(\gamma_1 h)}{\gamma \cosh(\gamma c) \sinh(\gamma_1 h) + \gamma_1 \sinh(\gamma c) \cosh(\gamma_1 h)} = FZ(n)
\] \hspace{1cm} (A5)

for \( n \to \infty \) this sum approaches \( 1/4u_n = a/(2n\pi) = AF(n) \).

In a similar manner the charge terms may be summed over m. In this case, begin with

\[
f_1(\omega) = \frac{(x-1)k_0^2}{1^2 - x^2} \frac{\omega \sin(\ell h) \sin(p c)}{(\omega^2 - \beta^2)(\omega \sin(\ell h) \cos(p c) + \ell \cos(\ell h) \sin(p c))}
\] \hspace{1cm} (A6a)

\[
f_2(\omega) = \frac{(x-1)pl}{1^2 - x^2} \frac{\omega \sin(\ell h) \sin(p c)}{(\omega^2 - \beta^2)(\beta \cos(\ell h) \sin(p c) + \ell \sin(\ell h) \cos(p c))}
\] \hspace{1cm} (A6b)

Following the same procedure as for the \( F(m) \) sum, the charge terms will sum over m to give

\[
\sum_{m} \left[ H(m) + \bar{H}(m) \right] = \frac{1}{2} \frac{\sinh(\gamma_1 h) \sinh(\gamma c)}{(u_n^2 + \beta^2)} \left[ \frac{\gamma \gamma_1}{D2} + \frac{k_0^2}{D3} \right]
\] \hspace{1cm} (A7)
where
\[ D_2 = \gamma \cosh(\gamma_1 h) \sinh(\gamma c) + \gamma_1 \sinh(\gamma_1 h) \cosh(\gamma c) \]
\[ D_3 = \gamma \cosh(\gamma c) \sinh(\gamma_1 h) + \gamma_1 \sinh(\gamma c) \cosh(\gamma_1 h) \]
as \( n \to \infty \) the sum goes to \( 0.5/(u_n(x+1)) = a/(n\pi(x+1)) = AG(n) \).
Note that these sums over \( m \) are independent of the expression used for testing in \( x \), that is, whether its \( P_{nj} \) or \( PP_{nj} \).

While the sums over \( n \) could now be summed directly, a better technique is available. Note that for larger \( n \) values, \( n > 30 \) has been found adequate, the sums in (A1a) will go as \( \sum P_{ni} P_{nj}/n \). This sum may be evaluated in closed form as follows

\[ \sum_{n=1,3} \frac{P_{ni} P_{nj}}{n} = \pi/2 \]

\[ = \int_0^{\pi/2} \int_0^{\pi/2} \sum_n \frac{1}{n} \cos\left(\frac{n\pi w}{2a}\sin\theta\right) \cos\left(\frac{n\pi w}{2a}\sin\theta'\right) \cos(2i\theta) \cos(2j\theta') d\theta d\theta' \]

(A8)

where the term in the sum may be summed in closed form by noting that [64]

\[ \sum_{n=1,3} \frac{e^{jn/n}}{n} = -0.5 \ln(\tan(\pi/2)) + j\pi/4 \]

(A9)
and \[ \cos(u_n \omega \sin \theta) \cos(u_n \omega \sin \theta') = \cos(n \alpha) \cos(n \alpha') \] or
\[ 0.5 \cos(n(\alpha - \alpha')) + \cos(n(\alpha + \alpha')) \text{, so (A9) becomes} \]

\[ \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{2} (\cos(n(\alpha - \alpha')) + \cos(n(\alpha + \alpha'))) = \frac{1}{4} \ln(\tan \frac{\alpha}{2} \tan \frac{\alpha'}{2}) \]

\[ = \frac{1}{4} \ln \left( \frac{\pi w}{4a} \right)^2 \quad \frac{1}{4} \ln \left| \sin^2 \theta - \sin^2 \theta' \right| \quad \text{(A10)} \]

where the small angle approximation was used \[ \tan(\alpha/2) = \alpha/2 \text{ and } \tan(\alpha'/2) = \alpha'/2. \] The Schwinger transformation may now be used on the second term in (A9) so that

\[ -\frac{1}{4} \ln \left| \sin^2 \theta - \sin^2 \theta' \right| = \frac{1}{4} \ln(4) + \frac{1}{4} \sum_{n=1,2}^{\infty} 2 \cos(2n\theta) \cdot \cos(2n\theta') \]

so that the final sum may be expressed as

\[ \frac{1}{2} \ln \left( \frac{8a}{\pi w} \right) + \frac{1}{4} \sum_{n=1,2}^{\infty} 2 \cos(2n\theta) \cdot \cos(2n\theta') \quad \text{(A11)} \]

For \( i \neq j \) a good approximation is that \( \sum_{n=1}^{\infty} P_n^i P_n^j / n = 0 \), while for \( i = 0,1,2 \), the sums are

for \( i = j = 0 \); \[ \frac{\pi^2}{2} \ln \left( \frac{8a}{\pi w} \right) \]

for \( i = j = 1 \); \[ \frac{\pi^2}{32} \]
for \( i = j = 2; \quad \frac{\pi^2}{96} \)

The sums may now be evaluated as follows

\[
\sum_{n} (FZ(n) - AF(n)) P_{n_i} P_{n_j} \frac{a}{2\pi} \sum_{n} \frac{1}{n} P_{n_i} P_{n_j} \tag{A12a}
\]

\[
\sum_{n} (GZ(n) - AG(n)) P_{n_i} P_{n_j} \frac{a}{\pi(x+1)} \sum_{n} \frac{1}{n} P_{n_i} P_{n_j} \tag{A12b}
\]

This method applies equally well for sums involving \( P_{n_j} \) that appear in the testing of the fields due to the discontinuity. The only difference in the double integral appearing in (A8) would be the presence of an extra \( \cos\theta \) factor. The evaluation of the summation over \( n \) goes through in the same manner as described above, only the integrations over \( \theta \) are modified.

In the discontinuity problems, there are dominant terms like those in (A1b) except that \( \beta = 0 \). In this case the only change is in the \( \gamma_n \) and \( \gamma_{1n} \)'s that appear in (A3), in this case they would have \( \beta = 0 \). All of the other steps are carried out in the manner discussed above.

The ability to sum the series given in (A1a) and
(Alb) over the index $m$ and the ability to extract the asymptotic values of the resulting series in the index $n$ lead to an efficient means of summing these series into closed form. This results in a much quicker execution time with regards to the programs developed in this thesis.
APPENDIX 2- FLOW CHARTS AND PROGRAMS

This appendix gives the flow charts for the programs, as well as a listing of the programs developed in implementing this technique.

There are two main programs, called STUB and GAP. The gap program is set up to include one \( J_x \) mode on each line, the stub program assumes no \( J_x \).

The flow charts begin with that part of the two programs that are common, and then there is one chart for the stub and one for the gap which picks up where the common flow chart leaves off.

(A2.1) COMMON PROGRAM ELEMENTS

- Input parameters for enclosure and microstrip.
- Calculate the \( P_{n_i} \)'s and \( PP_{n_j} \)'s for \( n=1,3,5 \) to 199, \( i,j=0 \) to 2
- Determine the LSE and LSM eigenvalues for dielectric loaded waveguide for \( m=1 \) to 100

215
Calculate the $F(m)$, $H(m)$, and $\bar{H}(m)$
Calculate the dominant parts of the sums

Find the dominant mode sources for an infinite line
LOOP=1

Use original $x_{\text{eff}}$ as an initial estimate

Calculate new $x_{\text{effN}}$, $Q_i$ and $I_i$

$\Delta x_{\text{effN}} = x_{\text{eff}} - x_{\text{effN}} < 0.01$

NO

LOOP=LOOP+1
$x_{\text{eff}} = x_{\text{effN}}$

YES

$x_{\text{eff}} = x_{\text{effN}}$
$Q_i = Q_{i\text{LOOP}}$
$I_i = I_{i\text{LOOP}}$

Enter main program
Sums over index m for terms with exponential decay

Evaluate sums for all n, i, and j and store in appropriate arrays

Fill the R(i,j,l,s) and T(i,j,l,s) arrays; these are the actual current and charge matrix elements

Fill the coefficient array, A(20,20) and source array, Y(20) with suitably compressed pulses

Invert coefficient array find $B_{in}$

END
(A2.3) MAIN PROGRAM FOR GAP

ILP=0 for first pass

Sum over index m for terms with exponential decay

Evaluate all sums for n, i, and j and store in appropriate arrays

Fill the $R(i,j,l,s)$ and $T(i,j,l,s)$ arrays for charge and current matrix elements

Fill proper portion of coefficient array, $A(50,50)$, and source array, $Y(50)$, depending on value of ILP with compressed pulses

NO

ILP=ILP+1

YES

Invert array calculate 3 values for $B_{in}$

END
REFERENCES


11. Silvester, P. and Benedek, P., "Equivalent


42. Wheeler, H. A., "Transmission Line Properties of Parallel Wide Strips Separated by a


