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Flow simulations in a Banbury mixer

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Case Western Reserve University, 1990
FLOW SIMULATIONS IN A BANBURY MIXER

by

JING-JY CHENG

Submitted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy

Department of Macromolecular Science
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January 1990
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GRADUATE STUDIES

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FLOW SIMULATIONS IN A BANBURY MIXER

ABSTRACT

by

JING-JY CHENG

A fluid dynamics analysis package--FIDAP using the finite element method was implemented to simulate the flow patterns in a Banbury mixer. The flow simulations were carried out initially for the intensive mixing region of a Banbury mixer and they were limited to a two-dimensional analysis. A Lagrangian point of view was adopted and 18 different geometries were selected to represent one revolution of the rotor. A power-law model with time and temperature dependent parameters was used to characterize the rheological behavior of the mixture. The results of the analysis show good agreement with experimental observations regarding the pressure distributions and temperature profiles.

The flow analysis was then extended to the entire mixing chamber with two counter-rotating rotors. This
time, an Eulerian point of view was adopted. The flow simulations were limited again to a two-dimensional analysis, under the assumptions of pseudo steady-state and isothermal conditions. Streamline contours and velocity profiles indicating the material exchange between the two chamber lobes on the bridge region, as well as the obtained dynamic pressure profiles are in agreement with reported experimental observations.

The simulation results were used to characterize the flow field in terms of a parameter quantifying the elongational flow components. The influence of design and processing variables on the flow characteristics as well as on the average shear rate were analyzed.
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Finally, I would like to attribute this work to my parents; without their inspiration and expectation, I would not be able to overcome the frustrations and depressions.
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CHAPTER 1

INTRODUCTION

Since its invention in 1916 by Fernley Banbury, the Banbury mixer has been widely used in the rubber and plastics industries and is still one of the most popular high intensity batch mixers. The Banbury mixer consists of a mixing chamber shaped like a figure eight with a spiral lobed rotor in each chamber (Figure 1). The mixture is fed to the mixing chamber through a vertical chute in which an air or a hydraulic driven ram is located. There is a small clearance between the rotors, which usually operate at different speeds, and the chamber wall. In these clearances dispersive mixing takes place. The homogenized material is discharged through a slide at the bottom. Both rotors and chamber walls are temperature controlled.

Due to the complex geometry and the highly transient character of the flow which severly limit the hydrodynamic analysis, the mechanics of flow in the Banbury mixer is still poorly understood. Simplified models of the flow in the region of the rotor tip have been presented by Bergen [1], Bolen and Colwell [2], McKelvey [3], Guber [4,5], Udaltsov [6], Stupachenko [7] and Tadmor [8].
While providing some fundamental insight into the shearing action in the region of the rotor tip, the severe simplifications of the geometry, and of the rheological properties of the fluid as well as the isothermal assumption limit the applications of these analyses. The real flow in close-clearance mixers is complex and it is not possible to obtain general analytical solutions of the problem for non-Newtonian fluids. Kaloni [9] and White and Kondo [10] derived approximate solutions of the problem and suggested the development of vortices, and this is what is observed also experimentally in some polymer melts. More recently, White and co-authors [11] used the lubrication approximation to analyze the global flow patterns in an internal mixer by solving the field equations for different mixing regions separately. Simplified boundary conditions were used for Newtonian fluids and isothermal conditions.

Freakley and Wan Idris [12] initiated flow visualization studies in an internal mixer as an alternative approach for a better understanding of the mechanics of flow in the chamber of an internal mixer. More recently, Min and co-authors [13-18] made some investigations using a laboratory scale mixer with glass front and transverse windows to observe the flow of
elastomers and thermoplastics. Freakley and Patel [19] also used the results of mixing trials on a highly instrumented BR Banbury to analyze the flow and mixing characteristics in the region of the rotor wing. A one-dimensional flow analysis for a power-law model fluid under isothermal conditions was developed.

In the present work, a fluid dynamics analysis package--FIDAP [20] using the finite element method was implemented to simulate the flow patterns in a Banbury mixer. The actual geometry of the mixer was preserved and the different flow regions were analyzed simultaneously. A number of 18 different flow geometries were selected to represent the dynamics of the rotors during one repeated mixing cycle. 2-D isothermal and non-isothermal flow simulations were carried out for a power-law model fluid. The influence of geometry and processing variables on the flow characteristics were also analyzed.
Figure 1: Schematic representation of a Banbury mixer.
CHAPTER 2

FLOW SIMULATIONS FOR HALF OF THE MIXING CHAMBER

2.1 DESCRIPTION OF METHOD

Internal mixers have two outstanding and characteristic geometrical features: a narrow gap between the rotor wings and the mixer wall and a larger space between the rotors. In this chapter, we assume that the space between the rotors is a well mixed region in which extensive mixing takes place exclusively. The flow analysis was carried out for the intensive mixing region—the narrow space between the rotor and the chamber wall.

The simulations are limited to a two-dimensional flow analysis. Material motion in a direction normal to the rotor wing was neglected. Figure 1 shows a layout of the region of flow analysis including a schematic representation of six sequential positions of the rotor blade.

One complete revolution of the rotor was represented by 18 different positions of the rotor tip (Lagrangian point of view). The geometries selected for our flow analysis are shown in Figure 2.

A general purpose computer program—FIDAP [1], based on the finite element method was used for the flow
simulations. A detailed mesh plot for one of the 18 flow geometries is represented in Figure 3. The number of elements employed for the different geometries varied between 84 and 126, with 9 nodal points in each element.

2.2 RESULTS AND DISCUSSION

2.2.1 Example

The flow analysis was carried out for a Banbury B mixer, whose geometric characteristics were kindly provided by the Farrel Corporation.

The material used in our analysis was a masterbatch compound based on SBR 1502 with 35 parts carbon black N550. The rheological properties of this compound were studied by Freakley and Patel [2]. They found that the flow behavior of the material is adequately described by the Equation (1) below:

\[ \tau = \eta_0 \ e^{-b(T-T_0)} \left| \dot{\gamma} \right|^{n+c(T-T_0)-1} \dot{\gamma} \]

In Equation (1) \( \tau \) is the shear stress tensor, \( \eta_0 \) is a reference viscosity, \( T \) is the test temperature, \( T_0 \) is an arbitrary reference temperature and \( n \) is the power-law index. The constants \( b \) and \( c \) describe the dependence of the reference viscosity and power-law index on temperature (see Figure 4).
Freakley and Patel [2] also studied the changes in the flow behavior of the compound due to mixing. They found linear relationships of both reference viscosity and power-law index with mixing time:

\[ \eta_0 = p + q \left( t - t_0 \right) \quad \text{at } t \geq 2 \text{ minutes} \quad (2) \]

\[ n = r + s \left( t - t_0 \right) \quad \text{at } t \geq 2 \text{ minutes} \quad (3) \]

The values of the different coefficients in Equations (1)-(3) for a reference temperature \( T_0 = 100 \, ^\circ\text{C} \) and a reference time \( t_0 = 2 \text{ minutes} \) are listed below:

- \( b = 1.54 \times 10^{-2} \) (K-1)
- \( c = 2.5 \times 10^{-3} \) (K-1)
- \( p = 2.675 \times 10^5 \) (Pa.s)
- \( q = -3.7084 \times 10^2 \) (Pa)
- \( r = 0.1193 \)
- \( s = 3.2 \times 10^{-4} \) (s-1)

2.2.2 Isothermal Flow Simulations

We started our hydrodynamic analysis of the Banbury mixer for isothermal, steady-state flow conditions. Such conditions can be reached in the mixer after long enough mixing times. In our flow simulations the mixing time was set at 8 minutes and the temperature in the mixer was fixed at 80\(^\circ\text{C}\). A filling factor of 1 and a rotational speed of 50 rpm were used in the analysis.

For an incompressible fluid under steady-state conditions, the equations of continuity and motion can be written:
\[
\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \quad (4)
\]

\[
\rho \left( V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} \right) = -\frac{\partial P}{\partial x} - \left( \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{yx}}{\partial y} \right) \quad (5)
\]

\[
\rho \left( V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} \right) = -\frac{\partial P}{\partial y} - \left( \frac{\partial T_{xy}}{\partial x} + \frac{\partial T_{yy}}{\partial y} \right) \quad (6)
\]

For each of the 18 different geometries representing a complete revolution of the rotor, Equations (4)-(6) were solved for a fluid described by Equation (1). The boundary conditions used in the flow simulations were that of a moving chamber wall (at a tangential velocity corresponding to 50 rpm and in opposite direction to the rotor) and a stationary rotor.

Velocity profiles at two different positions of the rotor tip are shown in Figures 5 and 6. Velocity vectors are represented by orientation and magnitude. Note the high velocity gradients in the vicinity of the rotor tip.

Flow streamlines at 4 different positions of the rotor tip are presented in Figures 7-10. Note the changing character of the flow pattern in the regions in front and behind the rotor tip.

Isobars at a position of the rotor tip \(\alpha=90\) are displayed in Figure 11. Higher values of the pressure can be observed in the region in front of the rotor tip in
accordance with experimental observations. Freakley and Wan Idris [3] employed pressure transducers mounted in the chamber wall of a Banbury BR mixer to record pressure traces at different filling factors. In Figure 12 we compare the result of our simulations (see Appendix I for the derivation) for the pressure profile during one complete revolution of the rotor (at a fixed position in the mixer) with the experimental observations made at a filling factor of 1. The agreement is good considering our lack of knowledge for the exact position of the pressure transducer. The existent differences are certainly also due to the neglect of the material motion in a direction normal to the rotor wing in our two-dimensional flow analysis.

In carrying out the simulations, we did not assign boundary conditions to the entrance and exit of the flow region. Velocity profiles for the geometry of $\alpha=70$ ($\alpha$ is the angle between the rotor tip and the horizontal axis) are shown in Figure 13. Flow streamlines for the same geometry ($\alpha=70$) are presented in Figure 14. Although, due to the changing flow geometry, streamlines are not identical to pathlines, they are still indicative of the flow patterns in the mixer. Figure 15 displays isobars obtained by connecting nodal points of the same pressure
magnitude. As mentioned earlier, we did not assign boundary conditions for the entrance and exit of the flow region, but rather have considered for the corresponding nodal points the best solution obtained in the iterative regressive calculations. We checked on the validity of our results by repeating the flow simulations for those geometries at which a logical assignment of boundary conditions seemed possible. For the geometry of \( \alpha = 70 \), we considered the boundary conditions at the entrance and exit to the flow region to be represented by:

\[
(\mathbf{v} \cdot \mathbf{n}) = v
\]  

(7)

where \( \mathbf{v} \) is the velocity vector, \( \mathbf{n} \) is a vector of unit length normal to the entrance/exit region and \( v \) is the magnitude of vector \( \mathbf{v} \). Figure 16 and 17 display the flow streamlines and isobars for the geometry of \( \alpha = 70 \) obtained from the flow simulations making allowance for Equation (7). Except for very small differences at the entrance and exit regions, Figure 16 and 17 look very much like their counterparts, Figures 14 and 15, respectively, obtained from the flow simulations without assigned boundary conditions.

Another way to analyze the flow patterns is by the use of an Eulerian point of view. Here the flow simulations were performed considering a fixed observer
sitting on the chamber wall. A rotating rotor at 50 rpm and a filling factor of 1 were the conditions for the flow simulations. Velocity profiles for the geometry of $\alpha=70$ are shown in Figure 18, flow streamlines for the same geometry in Figure 19, whereas Figure 20 displays the isobars. Figure 18, 19 and 20 can be compared with similar figures obtained in the Lagrangian analysis, namely Figures 13, 14 and 15, respectively. As expected, only the pressure profiles which are independent of the simulations frame are similar in the Lagrangian and Eulerian analysis. We also compared the shear rate distributions and found them to be similar. Table I lists the average shear rate and maximum pressure gradients obtained for two different geometries. The comparison shows excellent agreement.

2.2.3 Non-Isothermal Flow Simulations

The transient, non-isothermal flow analysis was carried out by solving the equations of continuity, motion and energy (Equation (8)-(11)) for each of the different geometries encountered during one complete revolution of the rotor.

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$  \hspace{1cm} (8)
\[ \rho \left( \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} \right) = - \frac{\partial p}{\partial x} - \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} \right) \] (9)

\[ \rho \left( \frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} \right) = - \frac{\partial p}{\partial y} - \left( \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \right) \] (10)

\[ \rho c_v \left( \frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} \right) = k \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] - \left[ \frac{\tau_{xx}}{\partial x} \frac{\partial v_x}{\partial y} + \frac{\tau_{yy}}{\partial y} \frac{\partial v_y}{\partial x} \right] - \left[ \frac{\tau_{xy}}{\partial y} \frac{\partial v_x}{\partial x} + \frac{\tau_{xy}}{\partial x} \frac{\partial v_y}{\partial y} \right] \] (11)

Flow simulations were considered for a mixing time equal or greater than 2 minutes, when the rheological behavior of the material is described by Equations (1)-(3). An initial material temperature of 80°C and a constant cooling water temperature of 40°C at the chamber wall were assumed. The initial rotor temperature was set at 55°C. The boundary conditions for the flow simulations were a moving chamber wall at a velocity corresponding to 50 rpm and heat fluxes toward the rotor and toward the cooling water at the mixer wall. The material parameters used in the calculations are given in Table II.

Flow streamlines at a given position of the rotor tip (Q=90) for different mixing times are illustrated in Figures 21-24. Note the expansion of vortices with mixing progression. This can be explained by a lower overall viscosity of the mixture, which favors vortices
development.

Freakley and Patel [2] observed experimentally a progressive reduction with mixing time in the magnitude of the registered pressure peaks. A similar trend can be depicted from the flow simulations and is illustrated in Figure 25.

Isotherms at a given position of the rotor tip (θ=90) and shown as regions of material in a certain temperature range are plotted in Figures 26-29. With the progression of mixing, material temperature becomes more homogeneous (the regions of temperatures close to an average batch value occupy more of the mixing space).

The average material temperature was calculated based on average temperature values at different positions of the rotor tip during one complete revolution weighted by the corresponding flow rates (detailed calculation procedure can be found in Appendix II). The calculated batch temperature is plotted as a solid line in Figure 30. The dashed line on the same Figure represents the batch temperature trace as measured experimentally by Freakley and Patel [2]. The qualitative agreement is excellent. The differences can be explained in terms of uncertainties in the selection of the material parameters as well as in the initial rotor temperature.
2.3 CONCLUSIONS

The flow simulations were carried out only for the intensive mixing region and they were limited to a two-dimensional analysis. A Lagrangian point of view was adopted and 18 different geometries were selected to represent one revolution of the rotor. A power-law model with time and temperature dependent parameters was used to characterize the rheological behavior of the mixture. Steady-state, isothermal flow simulations as well as a transient, non-isothermal flow analysis were performed for a Banbury B mixer. The results of the analysis show good agreement with experimental observations regarding the pressure distributions and the temperature profiles. Flow simulations were carried out for different flow geometries in both a Lagrangian and Eulerian frame. Similar results were obtained for pressure profiles and shear rate distributions since they are quantities independent of the used coordinate system. However, velocity profiles and streamline plots are totally different because of the rotation of the observer in the Lagrangian simulations.
Table I
Comparison between the Lagrangian and Eulerian analyses for the one rotor flow simulations

<table>
<thead>
<tr>
<th>Geometry</th>
<th>θ = 70</th>
<th>θ = -110</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
<td>γ** (s⁻¹)</td>
<td>ΔP* (MPa)</td>
</tr>
<tr>
<td>Lagrangian viewpoint</td>
<td>16.36</td>
<td>4.13</td>
</tr>
<tr>
<td>Eulerian viewpoint</td>
<td>16.32</td>
<td>4.14</td>
</tr>
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</table>

* ΔP is the difference between the maximum and the minimum pressure values.

** γ was obtained by weighing the shear rates of each element area.
Table II
Material parameters used in the flow simulations

<table>
<thead>
<tr>
<th></th>
<th>SBR 1502/ N 550 carbon black</th>
<th>Stainless Steel</th>
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</thead>
<tbody>
<tr>
<td>Density [4] Kg/m³</td>
<td>980</td>
<td>7800</td>
</tr>
<tr>
<td>Thermal [4] W/m.K</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>

Overall heat transfer coefficient from mixture to cooling [5,6] = 2500 W/m².K

Heat transfer coefficient from mixture to rotor [5,6] = 3500 W/m².K
Figure 1: Layout of the simulation region with a schematic representation of four sequential positions of the rotor blade.
Figure 2: The 18 different geometries considered for one complete revolution of the rotor. \( \alpha \) is the angle between the rotor tip and the horizontal axis.
Figure 3: Mesh plot for the geometry at $\alpha = 70$. 
Rheological Behavior of Mixture

Figure 4: Dependences on temperature and mixing time of the consistency and power-law index for the mixture.

Figure 5: Velocity profiles for $\alpha = 90$ from the steady state flow simulations.
Figure 6: Velocity profiles for $\alpha = -90$ from the steady state flow simulations.
Figure 7: Streamline contours for $\alpha = 10$ from the steady state flow simulations.
Figure 8: Streamline contours for $\alpha = 90$ from the steady state flow simulations.
Figure 9: Streamline contours for $\alpha = 170$ from the steady state flow simulations.
MIN = 0.000E+0  MAX = 0.448E-2
A  0.15E-3  F  0.22E-2
B  0.40E-3  G  0.26E-2
C  0.80E-3  H  0.31E-2
D  0.13E-2  I  0.35E-2
E  0.17E-2  J  0.40E-2

Figure 10: Streamline contours for $\alpha = -70$ from the steady state flow simulations.
Figure 11: Pressure contours for $\theta = 90$ from the steady state flow simulations.
Figure 12: Pressure trace at a fixed position at the chamber wall. Solid line represents the result from the steady state flow simulations. Dashed line is the experimental data by Freakley and Wan Idris [3].
Figure 13: Velocity profiles for $\alpha = 70$ from the Lagrangian simulations, without assignment of boundary conditions at entrance and exit to the flow region.
Figure 14: Streamline contours for $\alpha = 70$ from the Lagrangian simulations, without assignment of boundary conditions at entrance and exit to the flow region.

The values of streamlines: A—$0.16E-3$, B—$0.13E-3$, C—$0.10E-3$, D—$0.7E-4$, E—$0.4E-4$, F—$-0.1E-4$, G—$0.2E-4$, H—$0.6E-4$, I—$0.1E-3$, J—$0.3E-3$, K—$0.6E-3$, L—$0.8E-3$. 
Figure 15: Pressure contours for $\alpha = 70$ from the Lagrangian simulations without assignment of boundary conditions at entrance and exit to the flow region.

Max.=4.13(MPa), Min.=0.00(MPa). The values of the contours: A-0.20, B-0.50, C-0.90, D-1.21, E-1.55, F-1.90, G-2.24, H-2.58, I-2.93, J-3.27, K-3.61, L-3.96.
Figure 16: Streamline contours for $\alpha = 70$ from the Lagrangian simulations with the assignment of boundary conditions at the entrance and exit to the flow region.

The values of the streamlines: A——$-0.12E-3$, B——$-0.10E-3$, C——$-0.80E-4$, D——$-0.50E-4$, E——$-0.20E-4$, F——$-0.10E-4$, G——$-0.40E-4$, H——$-0.28E-3$, I——$-0.52E-3$, J——$-0.76E-3$. 
Figure 17: Pressure contours for $\alpha = 70$ from the Lagrangian simulations with the assignment of boundary conditions at the entrance and exit to the flow region.

Max. = 4.13(MPa), Min = 0.00(MPa). The values of the contours are the same as in Fig. 15.
Figure 18: Velocity profiles for $a = 70$ from the Eulerian simulations.
Figure 19: Streamline contours for $a = 70$ from the Eulerian simulations.

The values of the streamlines: A$=-0.44E-2$, B$=-0.40E-2$, C$=-0.36E-2$, D$=-0.32E-2$, E$=-0.29E-2$, F$=-0.25E-2$, G$=-0.21E-2$, H$=-0.17E-2$, I$=-0.13E-2$, J$=-0.95E-3$, K$=-0.57E-3$, L$=-0.19E-3$. 
Figure 20: Pressure contours for $\alpha = 70$ from the Eulerian simulations.

Max. = 4.14 (MPa), Min. = 0.00 (MPa). The values of the contours are the same as in Fig.15.
Figure 21: Streamline contours for $\alpha = 70$ from the transient flow simulations at 2 min. mixing time.
Figure 22: Streamline contours for $a = 70$ from the transient flow simulations at 2.5 min. mixing time
Figure 23: Streamline contours for $a = 70$ from the transient flow simulations at 3 min. mixing time.
Figure 24: Streamline contours for $\alpha = 70$ from the transient flow simulations at 5 min. mixing time.
Figure 25: Pressure peaks at $\alpha = 70$ from the transient flow simulations.
Average material temperature = 80.35 °C

Figure 26: Temperature map for \( \alpha = 70 \) at 2 minutes mixing time.
Average material temperature = 90.13°C

Region of $T = 61 - 68°C$
Region of $T = 68 - 75°C$

Region of $T = 75 - 82°C$
Region of $T = 82 - 89°C$

Region of $T = 89 - 96°C$
Region of $T = 96 - 103°C$

Figure 27: Temperature map for $a = 70$ at 3 minutes mixing time.
Average material temperature = 97.51 °C

Region of $T = 70 - 76.5$ °C

Region of $T = 76.5 - 83$ °C

Region of $T = 83 - 89.5$ °C

Region of $T = 89.5 - 96$ °C

Region of $T = 96 - 102.5$ °C

Region of $T = 102.5 - 109$ °C

Figure 28: Temperature map for $\alpha = 70$ at 4 minutes mixing time.
Average material temperature = 103.04 °C

Region of $T = 77 - 83 \, ^\circ C$

Region of $T = 83 - 89 \, ^\circ C$

Region of $T = 89 - 95 \, ^\circ C$

Region of $T = 95 - 101 \, ^\circ C$

Region of $T = 101 - 107 \, ^\circ C$

Region of $T = 107 - 113 \, ^\circ C$

Figure 29: Temperature map for $\alpha = 70$ at 5 minutes mixing time.
Figure 30: Batch temperature trace. Solid line represents the result from flow simulations. Dashed line is experimental data by Freakley and Patel[2].
CHAPTER 3

2-D FLOW SIMULATIONS FOR THE ENTIRE MIXING CHAMBER

3.1 DESCRIPTION OF METHOD

In our previous study (Chap. 2), the flow analysis of the Banbury mixer was carried out only for the intensive mixing region—the narrow space between the rotor and the chamber wall, in half of the mixing chamber. One complete revolution of the rotor was represented by a series of geometries illustrating differences in relative positions of the rotor tip and the chamber wall.

In this chapter a similar concept was applied and 18 different geometries were selected to represent one repeated mixing cycle with 3 revolutions of the left rotor and 2 revolutions of the right rotor (the two rotors have different speeds with a ratio of their respective rpm's of 3:2). Figure 1 shows the different geometries used in our flow analysis.

The simulations were limited to a two-dimensional flow analysis. Material motion in a direction normal to the rotor wing was neglected. The field equations for the isothermal flow of an incompressible fluid were solved for each geometry. A pseudo steady-state and no-slip boundary conditions were assumed.
The finite element method was employed in the flow simulations. Detailed mesh plots for 2 of the 18 geometries are represented in Figures 2 and 3. The number of elements used for the different geometries varied between 420 and 476 with 9 nodal points in each element.

3.2 RESULTS AND DISCUSSION

The flow analysis was performed for a Banbury B mixer, whose geometric characteristics were kindly provided by the Farrel Corporation. The rotational speed of the left rotor was set at 60 rpm and that of the right rotor at 40 rpm. A filling factor of 1 was employed.

The material used in our analysis was the same as in chapter 2, i.e. a masterbatch compound based on SBR 1502 with 35 parts carbon black N550. The rheological behavior of the compound was studied by Freakley and Patel [1], who found that a power-law model fluid adequately describes the flow of the material. The consistency index and the power-law index are both functions of temperature and mixing time. In our analysis the temperature was fixed at 80°C (isothermal flow simulations). The isothermal assumption is less dramatic after long enough mixing times. In calculating the rheological parameters, the mixing time selected was 8 minutes. With these
conditions, the constitutive equation of the material is

\[ \tau = 98735.515 |\dot{\gamma}|^{0.2205-1} \dot{\gamma} \]  \hspace{1cm} (1)

where \( \tau \) is the shear stress tensor in Pa and \( \dot{\gamma} \) is the rate of deformation tensor in s\(^{-1}\) with \( \dot{\gamma} \) its magnitude.

Velocity profiles for 4 different geometries are shown in Figures 4, 5, 6 and 7. Velocity vectors are represented by orientation and magnitude at each nodal point. When neither of the two rotor tips is in the bridge region (Figure 4), the fluid just undergoes a circular motion following the rotation of each rotor. A close look at the magnitudes of the velocity vectors in the regions other than the rotor tips indicate that the flow is a summation of pressure flow, due to the convergence of the flow path and drag flow, due to the motion of the rotors. Experimental pressure data indicate that the values are lower in the region behind the rotor tip than in front of the tip [2]. Therefore, in the tip region the pressure flow is reversed and opposes the drag flow. This is clearly indicated in Figure 8 which is a closer lookout of the tip region for the left rotor.

Figures 5, 6, 7 show velocity plots for different flow geometries with one or two rotor tips close to the bridge region. These figures clearly indicate the material exchange between the lobes of the mixing chamber.
Essentially, one rotor wing pumps material across the bridge into the path of the wing of the other rotor. Magnifications of the bridge region for Figures 5, 6, 7 are in Figures 9, 10 and 11, respectively.

Figures 12, 13, 14 and 15 present streamlines at different flow geometries during a mixing cycle. Although, due to the changing flow geometry, streamlines are not identical to pathlines, they are still indicative of the flow patterns in the internal mixer. From the streamlines in Figures 13, 14 and 15, it can be inferred that the material exchange across the bridge occurs by sweeping the fluid particles in front of one rotor tip, by the other rotor, and incorporating them in the region behind its tip. This is in agreement with experimental observations from the flow visualization studies [2].

Figures 16, 17, 18 and 19 display isobars at different flow geometries. Note consistently higher values of the pressure in the regions in front of the rotor tips as compared to the regions behind the rotor tips. However, the pressure gradients encountered across each rotor tip are strongly dependent on the relative position of the two rotors.

Dynamic pressure profiles at two fixed positions in the mixer are presented in Figures 20 and 21. In Figure
20 the dynamic pressure profile was calculated for a fixed point (A2=150) in the left chamber wall. The numbers on the curve correspond to the different geometries (see Figure 1) from which the pressure values at the fixed position were obtained. The pressure peaks marked 1, 7 and 13 occur as left rotor tip is just starting its traverse across the fixed position in the chamber wall. The pressure peak marked 4 is caused by the passage of the right rotor tip through the bridge region between the two chamber lobes. The second passage of the right rotor tip through the bridge region is almost coincident with the left rotor tip passage across the designated position in the chamber wall. This concurrence of events explains the larger pressure value for the peak marked 13.

Figure 21 displays the dynamic pressure profile at a fixed position (A1=-120) in the right chamber wall. The peaks marked 6 and 15 are caused by the imminent passage of the right rotor tip across the fixed position on the chamber wall. The higher value of the peak 15 is explained by the simultaneous passage of the left rotor tip through the bridge region. The peaks marked 3 and 9 are caused by the first two passages of the left rotor tip through the bridge region during the simulated mixing cycle (3 revolutions for the left rotor and 2 revolutions
for the right rotor).

If we would imagine pressure transducers located at the two fixed positions in the chamber wall (A2=150 and A1=-120), the curves displayed in Figures 20 and 21 would actually represent recorded pressure traces. Typical pressure traces obtained in experiments and reported in the literature (e.g. Figure 10 of ref.2 and Figure 7 of ref.1) shows excellent qualitative agreement with the curves displayed in Figures 20 and 21.

3.3 SUMMARY AND CONCLUSIONS

The flow patterns in the entire mixing chamber of a Banbury B mixer were obtained by using a fluid dynamics analysis package based on the finite element method. A number of 18 geometries were selected to represent different relative positions of the two rotors during one repeated mixing cycle with 3 revolutions for the left rotor and 2 revolutions for the right one. The flow simulations were limited to a two-dimensional analysis, assuming pseudo steady-state and isothermal conditions. The rheological behavior of the mixture was characterized by a power-law model constitutive equation. The streamline contours and velocity profiles indicate the material exchange between the two chamber lobes on the
bridge region and the development of small vortices in the tip regions, in agreement with reported experimental observations from flow visualization studies [2]. The higher pressure values in the region in front of the rotor tip, as compared to the region behind the rotor tip, as well as the dynamic pressure profiles obtained from the simulations are also in agreement with reported experimental data.
Figure 1: The 18 different geometries considered for one repeated mixing cycle. $A_1$ is the angle between the right rotor tip and the horizontal axis. $A_2$ is the angle between the left rotor tip and the horizontal axis.
MESH DESIGN FOR A1=-120 AND A2=0

Figure 2: Mesh design for the geometry at A1=-120 and A2=0. Total number of nodal points is 1811, that of mesh elements is 427.
MESH DESIGN FOR A1=40 AND A2=120

Figure 3: Mesh design for the geometry at A1=40 and A2=120. Total number of nodal points is 2027, that of mesh elements is 476.
Figure 4: Velocity profiles for A1=40 and A2=120.
Figure 5: Velocity profiles for $A_1=120$ and $A_2=0$. 
Figure 6: Velocity profiles for A1=160 and A2=-60.
Figure 7: Velocity profiles for $\alpha_1=160$ and $\alpha_2=-120$. 

Velocity plot for $\alpha_1=-160$ and $\alpha_2=-120$. 
Figure 8: Magnification of Fig. 4 around the left rotor tip.
Figure 9: Magnification of Fig. 5 around the left rotor tip.
Figure 10: Magnification of fig.6 around the right rotor tip.
Figure 11: Magnification of Fig. 7 around the right rotor tip.
Figure 12: Streamline contours for $A_1=0$ and $A_2=180$. The values of streamlines: $a$ = -0.85E-2, $b$ = -0.79E-2, $c$ = -0.74E-2, $d$ = -0.68E-2, $e$ = -0.62E-2, $f$ = -0.57E-2, $g$ = -0.51E-2, $h$ = -0.45E-2, $i$ = -0.39E-2, $j$ = -0.26E-2, $k$ = -0.22E-2, $l$ = -0.18E-2, $m$ = -0.14E-2, $n$ = -0.11E-2, $o$ = -0.085E-2, $p$ = -0.04E-2.
Figure 13: Streamline contours for $A_1=120$ and $A_2=0$. The values of streamlines: $a = -0.60E-2$, $b = -0.55E-2$, $c = -0.50E-2$, $d = -0.45E-2$, $e = -0.415E-2$, $f = -0.38E-2$, $g = -0.36E-2$, $h = -0.34E-2$, $i = -0.30E-2$, $j = -0.26E-2$, $k = -0.22E-2$, $l = -0.18E-2$, $m = -0.14E-2$, $n = -0.11E-2$, $o = -0.085E-2$, $p = -0.04E-2$. 
Figure 14: Streamline contours for A1=160 and A2=-60. The values of streamlines: a — -0.74E-2, b — -0.68E-2, c — -0.62E-2, d — -0.56E-2, e — -0.50E-2, f — -0.44E-2, g — -0.38E-2, h — -0.32E-2, i — -0.26E-2, j — -0.23E-2, k — -0.20E-2, l — -0.17E-2, m — -0.13E-2, n — -0.10E-2, o — -0.06E-2, p — -0.02E-2.
Figure 15: Streamline contours for $A_1=160$ and $A_2=120$. The values of streamlines: $a=-0.62E-2$, $b=-0.56E-2$, $c=-0.52E-2$, $d=-0.44E-2$, $e=-0.38E-2$, $f=-0.32E-2$, $g=-0.26E-2$, $h=-0.20E-2$, $i=-0.16E-2$, $j=-0.14E-2$, $k=-0.11E-2$, $l=-0.08E-2$, $m=-0.04E-2$, $n=-0.01E-2$, $o=0.02E-2$, $p=0.06E-2$. 
Figure 16: Pressure contours for A1=40 and A2=120. Max.=5.01(MPa), Min.=0.00(MPa). The values of the contours: a— 0.40, b— 0.57, c— 0.87, d— 1.27, e— 1.67, f— 2.07, g— 2.47, h— 2.87, i— 3.17, j— 3.27, k— 3.37, l— 3.47, m— 3.67, n— 3.72, o— 3.77, p— 4.07, q— 4.37, r— 4.57.
Figure 17: Pressure contours for $A_1=120$ and $A_2=0$. Max. = 4.68(MPa), Min. = 0.000(Mpa). The values of the contours: a—0.27, b—0.57, c—0.87, d—1.27, e—1.67, f—2.07, g—2.47, h—2.87, i—3.17, j—3.27, k—3.37, l—3.47, m—3.67, n—3.72, o—3.77, p—4.07, q—4.37, r—4.57.
Figure 18: Pressure contours for $A_1=160$ and $A_2=60$. Max. $=5.979$ (MPa).

The values of the contours: $a = 0.193$, $b = 0.593$, $c = 0.993$, $d = 1.393$, $e = 1.793$, $f = 2.193$, $g = 2.593$, $h = 2.993$, $i = 3.393$, $j = 3.793$, $k = 3.893$, $l = 4.193$, $m = 4.493$, $n = 4.893$, $o = 5.093$, $p = 5.393$, $q = 5.893$. 
Figure 19: Pressure contours for $A_1=-80$ and $A_2=-60$. Max. = 5.288(Mpa), Min. = 0.000(MPa). The values of the contours: a = 0.30, b = 0.70, c = 1.00, d = 1.10, e = 1.20, f = 1.30, g = 1.40, h = 1.70, i = 2.10, j = 2.40, k = 2.70, l = 3.10, m = 3.40, n = 3.80, o = 4.10, p = 4.50, q = 4.80, r = 5.20.
Figure 20: Dynamic pressure profile at a fixed position (A2=150) in the left chamber wall.
Figure 21: Dynamic pressure profile at a fixed position (A1=-120) in the right chamber wall.
CHAPTER 4
FLOW FIELD CHARACTERIZATION IN A BANBURY MIXER

4.1 SIMULATION METHOD

The finite element method (software package FIDAP) was used for the flow simulations. The mixing chamber was divided into a number of quadrilateral elements with 9 nodal points in each element. The mesh density was non-uniform over the surface, with higher values in the regions where significant changes of the flow patterns were expected (e.g. the region of the rotor tip).

A power law model was used to describe the rheological behavior of the fluid:

\[ \boldsymbol{\tau} : \dot{\gamma} = m |\dot{\gamma}|^{n-1} \dot{\gamma} \]

(1)

where \( \boldsymbol{\tau} \) is the dynamic stress tensor, \( \dot{\gamma} \) is the rate of strain tensor, \( \dot{\gamma} \) is the magnitude of the rate of strain tensor, \( m \) is the consistency index and \( n \) is the power-law index. The specific material used in this work was a masterbatch compound based on SBR 1502 with 35 parts carbon black N550. The rheological properties of this compound were studied by Freakley and Patel [1]. Considering an average temperature of 80°C and a mixing time of 8 minutes, the values of the parameters \( m \) and \( n \) are found to be \( 9.87 \times 10^4 \) N.s\(^{-0.22}\).m\(^{-2}\) and 0.22.
respectively. The density of the fluid was considered to be 980 kg/m³ [2].

The rotational speed of the left rotor was set at 60 rpm, and that of the right rotor at 40 rpm. The nodal points on the rotor surface have velocities tangential to imaginary circles, centered as the rotor and passing through the respective points, rotating in the same direction and with the same rpm as the rotor itself. The nodal points on the chamber have zero velocity. To represent one repeated mixing cycle with 3 revolutions of the left rotor and 2 revolutions of the right rotor we selected 18 different geometries for flow analysis. The number of elements for various geometries varied between 428 and 476. A filling factor of 1 for the mixing chamber was employed.

The equations of continuity and motion together with Equ.(1), expressed in rectangular coordinates were solved for all the nodal points of each geometry simultaneously. The flow simulation results in terms of velocity profiles, streamline contours, isobars plots and dynamic pressure profiles were presented in the previous chapter.

In this chapter, the characteristics of the flow field were analyzed in terms of a parameter $\lambda$ defined by

$$\lambda = \frac{\gamma}{\dot\gamma + \omega}$$

(2)
where $\dot{\gamma}$ is the magnitude of the rate of strain tensor and $\omega$ is the magnitude of the vorticity tensor. Obviously, the parameter $\lambda$ quantifies the elongational and rotational flow components and it can assume values between 0 for pure rotation and 1 for pure elongation. A value of $\lambda$ of 0.5 represents the familiar case of simple shear flow. The values of $\lambda$ for each element were evaluated from the velocity gradients at the Gaussian points. The average value of the parameter $\lambda$ for the entire geometry was calculated by weighing the corresponding parameter for each element by the surface area of the element itself.

The parameter $\lambda$ is important in assessing the efficiency of a flow field in dispersive mixing. Indeed, Elemendorp [3] reported experimental observations showing that for the dispersion of liquids with high viscosity ratio and low interfacial tension, elongational flows are more effective than shearing flows. Also Manas-Zloczower and Feke [4,5] posited that elongational flows are more efficient than simple shear for dispersing solid agglomerates into liquids. However, it has to be pointed out that the parameter $\lambda$ has to be interpreted in close correlation with the effective shear stress, in order to assess correctly the efficiency of different flow fields in mixing.
4.2 RESULTS AND DISCUSSION

Distribution plots of the parameter λ, for two different geometries are shown in Figures 1 and 2. The flow fields show more elongational flow components in the regions close to the chamber wall and in the bridge region. Table I lists average values for the parameter λ and the shear rates calculated for different geometries. While there is little variation in the average shear rate for the entire mixing chamber from one geometry to another, the parameter λ slightly increases for the geometries with one or two rotor tips in the bridge region.

We studied the influence of design and processing variables on the flow characteristics. By increasing the gap size (the distance between the rotor tip and the chamber wall) the vortices in the tip region become larger as can be seen from Figures 3-6. Figure 7 shows the streamlines for a geometry with no path contraction (the gap size is so large that the rotors take the shape of cylinders). There is no material exchange between the two chamber lobes and the fluid elements undergo a circular motion around the rotors (cylinders). Figures 8-15 show distribution plots of the parameter λ for one geometry (A1=140 and A2=-30) at different gap sizes. As the gap
size increases, more elongational flow components can be found in the regions close to the rotors than in the regions next to the chamber wall. Figure 16 shows the changes incurred in the parameter \( \lambda \) by increasing the gap size, for the whole mixing chamber and for the regions close to the rotor tips. While there is not much change of the flow behavior in the regions of the rotor tips, the elongational flow components for the overall flow show a maximum at a gap size of 1.2 cm. Figure 18 shows similar results obtained for another flow geometry. Obviously, by increasing the gap size the average shear rate decreases. This decrease is more dramatical for the regions of the rotor tips as illustrated in Figures 17 and 19 for two different geometries.

Increasing the rotational speed while maintaining the same rpm ratio for the two rotors, does not affect the characteristics of the flow field. This is illustrated in Figures 20 and 21 which display distribution plots for the parameter \( \lambda \) at two different rotational speeds. Figure 22 shows the no dependency of the parameter \( \lambda \) on the rotor velocity. By increasing the rpm's, the shear rate increases both in the regions of the rotor tip, as well as for the entire mixing chamber (Figure 23). However, the effective shear stress might not necessarily increase
proportionally, since one has to remember the adverse effect of viscous dissipation on the material viscosity.

Changing the speed ratio between the two rotors will affect the distribution plots of the parameter $\lambda$. This is illustrated for one flow geometry in Figures 24-26. Despite the changing character of the flow observed especially in the bridge region, the average value of the parameter $\lambda$ shows only little change with the rotational speed ratio of the two rotors. This is illustrated in Figures 27 and 29 for two different flow geometries. The average shear rate, for the entire mixing chamber and for the region of the right rotor tip, decreases with increasing the speed ratio (Figures 28 and 30). The speed ratio was changed by changing the right rotor velocity while keeping the left rotor at a constant rpm (60).

Mixing in a Banbury mixer is a highly non-isothermal process. Although in this study the flow simulations were limited to isothermal conditions, we wanted to check on the effect of temperature rise on the flow characteristics. We used the batch temperature profile as measured by Freakley and Patel [1] in their mixing experiments and calculated the values of the power-law fluid parameters in Equ.(1). These values are listed in Table II. Figures 31 and 32 show the distribution plots
of the parameter $\lambda$ at 2 minutes and 8 minutes of mixing time, respectively. Not too much difference can be observed between these two figures. The change in time of average values for the parameter $\lambda$, the shear rate and shear stress are illustrated in Figures 33-35. Mostly affected by the increasing temperature with the mixing time is the average shear stress in the mixer, due to the decrease in the viscosity of the fluid.

As mentioned before, in assessing the influence of design and processing parameters on mixing efficiency, one should correlate the characteristics of the flow field with the effective shear stress in the mixer. Our flow simulations were limited to a two-dimensional analysis. Therefore, the results obtained and their interpretation carry over the limitations coming from this simplification.

4.3 CONCLUSIONS

In this chapter we analyzed the results of flow simulations carried out for a Banbury mixer. The flow patterns were obtained by using a fluid dynamics analysis package based on the finite element method. A number of different flow geometries were selected to represent the dynamics of the rotors during one repeated mixing cycle.
The simulations were limited to a two-dimensional analysis under steady-state and isothermal conditions. A power-law model was used to characterize the rheological behavior of the fluid. The flow field was characterized in terms of a parameter quantifying the elongational flow components. Design variables such as the size of the gap between the rotor tips and the chamber wall have a more pronounced effect on the flow characteristics as compared to processing variables (rotational speed, rpm's ratio, temperature). The influence of the above mentioned variables on the average shear rate and shear stress in the mixer for different flow geometries was also analyzed. In spite of the limitations coming from a 2-D flow analysis, the results obtained are important for a better understanding of the influence of design and processing variables on mixing efficiency.
### Table I

Flow field characterization for the entire mixing chamber

<table>
<thead>
<tr>
<th>Geometry no.</th>
<th>$\lambda$</th>
<th>$\dot{\gamma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>3</td>
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<tr>
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<tr>
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Table II
Dynamics of the rheological behavior of the mixture (from References 9 and 11)

<table>
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<tr>
<th>Time (min.)</th>
<th>Temperature (°C)</th>
<th>$m$ (Pa.s$^n$)</th>
<th>$n$</th>
</tr>
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<tr>
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<td>148892</td>
<td>0.2050</td>
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<tr>
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<td>98483</td>
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<td>8</td>
<td>98.0</td>
<td>75536</td>
<td>0.2676</td>
</tr>
</tbody>
</table>
Figure 1: Distribution plot of the parameter $\lambda$ for $A1=160$ and $A2=60$. 
Figure 2: Distribution plot of the parameter $\lambda$ for $A_1=40$ and $A_2=60$. 

\[ \lambda = 0.00, 0.25, 0.50, 0.75, 1.00 \]
Figure 3: Streamline contours for A1=140 and A2=-30; gap size = 0.3cm; rpm(LR)=60; rpm(RR)=40. Abbreviations—LR: left rotor, RR: right rotor. The values of streamlines: A—-0.73E-2, B—-0.69E-2, C—-0.65E-2, D—-0.61E-2, E—-0.57E-2, F—-0.52E-2, G—-0.48E-2, H—-0.44E-2, I—-0.40E-2, J—-0.36E-2, K—-0.31E-2, L—-0.27E-2, M—-0.23E-2, N—-0.19E-2, O—-0.15E-2, P—-0.10E-2, Q—-0.63E-3, R—-0.21E-3.
Figure 4: Same as Fig.3, but for gap size = 0.6cm. The values of streamlines:
A— -0.59E-2, B— -0.56E-2, C— -0.52E-2,
D— -0.49E-2, E— -0.45E-2, F— -0.42E-2, G— -0.39E-2,
H— -0.35E-2, I— -0.32E-2, J— -0.28E-2, K— -0.25E-2,
L— -0.21E-2, M— -0.18E-2, N— -0.14E-2, O— -0.11E-2,
P— -0.74E-3, Q— -0.39E-3, R— -0.46E-3.
Figure 5: Same as Fig. 3, but for gap size = 1.5 cm. The values of streamlines: A — -0.26E-2, B — -0.21E-2, C — -0.18E-2, D — -0.16E-2, E — -0.14E-2, F — -0.125E-2, G — -0.117E-2, H — -0.11E-2, I — -0.103E-2, J — -0.95E-3, K — -0.85E-3, L — -0.60E-3, M — -0.35E-3, N — -0.10E-3.
Figure 6: Same as Fig. 3, but for gap size = 2.1cm. The values of streamlines: A = -0.152E-2, B = -0.123E-2, C = -0.108E-2, D = -0.90E-3, E = -0.797E-3, F = -0.76E-3, G = -0.73E-3, H = -0.715E-3, I = -0.70E-3, J = -0.685E-3, K = -0.67E-3, L = -0.64E-3, M = -0.58E-3, N = -0.51E-3, O = -0.36E-3, P = -0.10E-3.
Figure 7: Same as Fig. 3, but for gap size = 2.6 cm. The values of streamlines: A — -0.62E-3, B — -0.54E-3, C — -0.46E-3, D — -0.425E-3, E — -0.41E-3, F — -0.404E-3, G — -0.396 E-3, H — -0.89E-3, I — -0.38E-3, J — -0.36E-3, K — -0.32 E-3, L — -0.20E-3.
Figure 8: Distribution plot of the parameter $\lambda$ for conditions similar to Fig. 3. Gap size = 0.3 cm.
Figure 9: Distribution plot of the parameter $\lambda$ for conditions similar to Fig. 3. Gap size = 0.6cm.
Figure 10: Distribution plot of the parameter $\lambda$ for conditions similar to Fig. 3. Gap size = 0.9cm.
Figure 11: Distribution plot of the parameter $\lambda$ for conditions similar to Fig.3. Gap size = 1.2cm.
Figure 12: Distribution plot of the parameter $\lambda$ for conditions similar to Fig. 3. Gap size = 1.5cm.
Figure 13: Distribution plot of the parameter $\lambda$ for conditions similar to Fig. 3. Gap size = 1.8 cm.
Figure 14: Distribution plot of the parameter $\lambda$ for conditions similar to Fig. 3. Gap size = 2.1cm.
Figure 15: Distribution plot of the parameter $\lambda$ for conditions similar to Fig. 3. Gap size = 2.6cm.
CHARACTERIZATION FOR A1=140 AND A2=−30

Figure 16: Parameter $\lambda$ vs. gap size for A1=140 and A2=−30. rpm(LR) = 60 and rpm(RR) = 40.
SHEAR RATE FOR $A_1=140$ AND $A_2=-30$

Figure 17: Average shear rate vs. gap size for $A_1=140$ and $A_2=-30$, $\text{rpm}(LR)=60$ and $\text{rpm}(RR)=40$. 

- ○ AVERAGE
- □ RIGHT ROTOR TIP
- △ LEFT ROTOR TIP
Figure 18: Parameter $\lambda$ vs. gap size for $A_1=0$ and $A_2=180$. rpm(LL)=60 and rpm(RR)=40.
Figure 19: Average shear rate vs. gap size for A1=0 and A2=180. 
rpm(LR)=60 and rpm(RR)=40.
Figure 20: Distribution plot of the parameter $\lambda$ for $A_1=0$ and $A_2=180$. Gap size = 0.3cm; rpm(LR)=75; rpm(RR)=50.
Figure 21: Distribution plot of the parameter $\lambda$ for $A1=0$ and $A2=180$. Gap size = 0.3 cm; rpm(LR)=90, rpm(RR)=60.
CHARACTERIZATION FOR A1=0 AND A2=180
SPEED RATIO L:R = 1.5:1.0

Figure 22: Parameter $\lambda$ vs. the rotational speed. The speed ratio between the left rotor and the right rotor is 1.5:1. Gap size = 0.3cm.
SHEAR RATE FOR A1=0 AND A2=180
SPEED RATIO L:R = 1.5:1.0

Figure 23: Average shear rate vs. rotational speed. The speed ratio between the left rotor and the right rotor is 1.5:1. Gap size = 0.3cm.
Figure 24: Distribution plot of the parameter $\lambda$ for $A1=0$ and $A2=180$. Gap size = 0.3cm; rpm(LR)=rpm(RR)=60.
Figure 25: Distribution plot of the parameter $\lambda$ for $A_1=0$ and $A_2=180$. Gap size = 0.3 cm, rpm(LR)=60 and rpm(RR)=40.
Figure 26: Distribution plot of the parameter $\lambda$ for $A_1=0$ and $A_2=180$. Gap size = 0.3cm; rpm(LR)=60 and rpm(RR)=30.
CHARACTERIZATION FOR A1=0 AND A2=180

Figure 27: Parameter $\lambda$ vs. the rotational speed ratio for A1=0 and A2=180. Gap size = 0.3cm; rpm(LR)=60.
Figure 28: Average shear rate vs. the rotational speed ratio for \( A_1=0 \) and \( A_2=180 \). Gap size = 0.3cm; rpm(LR)=60.
Figure 29: Parameter $\lambda$ vs. the rotational speed ratio for $A_1=140$ and $A_2=-30$. Gap size = 0.3 cm; rpm(L/R) = 60.
Figure 30: Average shear rate vs. the rotational speed ratio for $A_1=140$ and $A_2=-30$. Gap size = 0.3cm; rpm(LR)=60.
Figure 31: Distribution plot of the parameter $\lambda$ at 2 min. mixing time for $A1=140$ and $A2=-30$. Gap size = 0.3cm; rpm(LR) = 60 and rpm(RR) = 40.
Figure 32: Same as Fig. 31 at 8 min. mixing time.
CHARACTERIZATION FOR A1=140 AND A2=-30

Figure 33: Parameter $\lambda$ vs. mixing time for $A1=140$ and $A2=-30$. Gap size = 0.3 cm; rpm(LR)=60 and rpm(RR)=40.
Figure 34: Average shear rate vs. mixing time for $A_1=140$ and $A_2=-30$.
Gap size = 0.3cm; rpm(LR)=60 and rpm(RR)=40.

SHEAR RATE FOR $A_1=140$ AND $A_2=-30$
Figure 35: Average shear stress vs. mixing time for A1=140 and A2= -30. Gap size = 0.3cm; rpm(LR)=60 and rpm(RR)=40.
SUMMARY

The flow patterns in a Banbury mixer were analyzed by using a fluid dynamics analysis package--FIDAP based on the finite element method. Different flow regions (in front of the rotor tip, behind the rotor tip, at the rotor tip and across the bridge) were simultaneously analyzed while preserving the real geometry of the mixing chamber. A power law model fluid with time and temperature dependent parameters was used in the flow simulations.

The problem of a changing geometry caused by the continuous rotation of the rotors was solved by using 18 different geometries to represent a repeated mixing cycle. For polymer processing operations with laminar flows of highly viscous materials, the overall effect caused by a changing geometry can be analyzed from the results obtained separately in the selected sequential geometries. The good agreement between the simulation results and the experimental data confirms the validity of this method.

The analysis of flow characteristics was carried out by using a parameter quantifying the elongational flow components. To predict the mixing efficiency, the characteristics of the flow field should be interpreted in conjunction with the shear stress distribution in the
mixing chamber. The simulation results showed that geometric variables have a much more pronounced influence on the flow characteristics than the processing variables.

This work opens the way for a more comprehensive analysis of the flow patterns in a Banbury mixer.
APPENDIX I

CALCULATION OF THE PRESSURE TRACE AT A FIXED POSITION IN THE CHAMBER WALL

When using the FIDAP package for the simulations, it is not required to assign pressure magnitude for any nodal point. During the execution of the programs, the computer will assign a value "0" to the first nodal point automatically, and the pressure values for the rest of the nodal points will be calculated by solving the appropriate equations based on the reference value at the first nodal point. Therefore, in the output file, the pressure values listed for each point are actually relative values. A few of them may go below 0 or sometimes, all of them are less than zero, depending on the flow geometry.

In carrying out the flow simulations for a Banbury mixer, we used a number of different geometries to represent the dynamics of the rotors. In this way, the sequence of the different flow geometries can be interpreted in terms of the mixing time during on complete cycle. Thus, we are able to get the dynamic pressure profile at a fixed position in the chamber wall from the readings of the sequentially simulated flow geometries.

Before we record the readings from the simulation results, the pressure values for each nodal point should
be normalized. In our work, the normalization was made by setting the minimum pressure value to 0 for each flow geometry. The normalized values can be used for comparison among different flow geometries as well as for obtaining the dynamic pressure profile at a fixed position in the chamber wall.
APPENDIX II

CALCULATION PROCEDURE TO OBTAIN THE AVERAGE TEMPERATURE

To apply the finite element method for the flow simulations of the Banbury mixer, we selected a number of different geometries to represent the dynamic rotation of the rotors during a complete mixing cycle such that the simulation results would show the characteristics of the flow field. Then in order to obtain an average quantity for the entire mixing chamber as a function of time, we need to combine the results of flow simulations for all of the geometries representing one mixing cycle.

We calculated the average temperature step by step with a fixed time interval. The fixed time interval, $\Delta t$, was chosen to be the amount of time for a complete revolution of the rotor. In our simulations, the rotational speed of the rotor was 50 rpm, therefore, the time interval was 1.2 seconds. Non-isothermal flow simulations were performed for each flow geometry. Assuming that after a time interval $n\Delta t$, the average temperature of the mixing chamber is $T_n$, those of the cooling water flowing through the chamber wall and the rotor are $T_{w}$ (constant) and $T_{r,n}$ respectively, then the procedure to go from time $t_n (=n\Delta t)$ to $t_{n+1}$


\(=(n+1)\Delta t\) is:

(i) Calculate the parameters of the power-law model \(m\) and \(n\) with time \(t_n\) and average temperature \(T_n\).

(ii) Carry out isothermal simulation for average temperature \(T_n\) with the power-law model from (i) and appropriate boundary conditions (a moving chamber wall with the same rpm as the rotor but in opposite direction, Lagrangian analysis).

(iii) Use the results obtained from (ii) as the initial velocities for each nodal point and set the initial temperature for all of the nodal points as \(T_n\). The time in the power-law model equals \(t_n\). Keep the temperatures of the cooling water and rotor constant at \(T_w\) and \(T_{r,n}\) (boundary condition) and that of the entrance edge at \(T_n\). Perform the non-isothermal flow simulations with heat fluxes to the cooling water and the rotor for a time interval \(\Delta t\).

(iv) Calculate an average temperature \(T_a\) for the flow geometry based on the temperature values of each element and the corresponding surface areas.

(v) Calculate the amount of heat flux through the interface of the rotor \(Q_H\).

(vi) Calculate the flow rate of the mixture through each radial cross section \(Q_m\) (the same for all of the
cross sections because of the mass conservation).

After executing all the steps from (i) to (vi) for all of the 18 flow geometries, we can calculate the average temperature for the entire mixing chamber, \( T_{n+1} \), and the temperature of the rotor \( T_{r,n+1} \) at time \( t_{n+1} \). Since we made the assumption that for the space between the two rotors, the material undergoes extensive mixing exclusively, the energy dissipated by the shearing action in this region was neglected as compared to the energy dissipated in the region between the rotor and the chamber wall. Only those fluid elements which flow through the intensive mixing region will exit with higher temperatures. The values of \( T_{n+1} \) and \( T_{r,n+1} \) were calculated from:

\[
T_{n+1} = \Delta t \left[ \frac{\sum Q_m}{18} \right] \cdot \frac{\sum T a Q_m}{Q_m} + \left( A - \Delta t \cdot \frac{\sum Q_m}{18} \right) \cdot T_n \tag{1}
\]

\[
T_{r,n+1} = T_{r,n} + \left( \Delta t \cdot \frac{\sum Q_H}{18} \right) / ( \bar{\rho}_r \bar{C}_p, r A_r ) \tag{2}
\]

where the summation accounts for the results from the 18 different flow geometries, \( A \) is half of the total surface area of the mixing chamber (the mixing chamber has a symmetrical geometry), \( \bar{\rho}_r \) is the material density of the rotor, \( \bar{C}_p, r \) is its heat capacity and \( A_r \) its surface
area. The term I accounts for the average flow rate of the fluid elements affected by viscous dissipation. The term II calculates the average temperature of the shearing area. Term III represents the amount of fluid whose temperature does not change during the time interval $\Delta t$. The total heat flux through the rotor interface is represented by term IV.

After the values of $T_{n+1}$ and $T_{r,n+1}$ were obtained, the procedures from (i) to (vi) were repeated for the next time interval until a steady state value for the temperature was reached.
APPENDIX III

CALCULATION OF THE AVERAGE VALUE FOR THE PARAMETER $\Lambda$

The dispersion of liquid droplets or solid agglomerates in a liquid matrix depends on many factors, among which the characteristic of the flow field is an important one. According to Elmendorp [1] and Manas-Zloczower and Feke [2,3], the elongational flows are more efficient than simple shear flows for dispersion. Figure 1 shows the comparison of the droplet dispersion in different flow fields. As can be seen from the figure, when the viscosity ratio between the two phases is greater than 1, the critical shear to breakup the droplets in an elongational flow field is much smaller than that required in a simple shear flow.

In our work, the characteristic of a flow field was analyzed in terms of a parameter defined by

$$\Lambda = \dot{\gamma} / (\dot{\gamma} + \Omega)$$  \hspace{1cm} (1)

where $\dot{\gamma}$ is the magnitude of the rate of strain tensor and $\Omega$ is the magnitude of the vorticity tensor. The magnitude of a tensor is defined by

$$A = \sqrt{\frac{1}{2} \, II_A}$$  \hspace{1cm} (2)

$$II_A = \sum_i \sum_j a_{ij}$$  \hspace{1cm} (3)

in which $A$ is the magnitude of tensor $A$, $II_A$ is the second invariant of $A$ and can be calculated from the
values of the components $a_{ij}$. For rotation, simple shear and elongational flows, the values of $\dot{\lambda}$ are 0, 0.5 and 1, respectively. The detailed calculations are shown below:

(i) Rotational flow field

\[
V_x = -ay \\
V_y = ax
\]

\[
\dot{\gamma} = \dot{\omega} + (\dot{\omega}^T) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \Rightarrow \quad \dot{\gamma} = 0
\]

\[
\omega = \dot{\omega} = \begin{bmatrix} 0 & -2a \\ 2a & 0 \end{bmatrix} \quad \Rightarrow \quad \omega = 2a
\]

\[
\lambda = \frac{\dot{\gamma}}{(\dot{\gamma} + \omega)} = 0
\]

(ii) Simple shear flow field

\[
V_x = ay \\
V_y = 0
\]

\[
\dot{\gamma} = \dot{\omega} + (\dot{\omega}^T) = \begin{bmatrix} 0 & a \\ a & 0 \end{bmatrix} \quad \Rightarrow \quad \dot{\gamma} = a
\]

\[
\omega = \dot{\omega} = \begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix} \quad \Rightarrow \quad \omega = a
\]

\[
\lambda = \frac{\dot{\gamma}}{(\dot{\gamma} + \omega)} = 0.5
\]

(iii) Elongational flow field

\[
V_x = ax \\
V_y = -ay
\]

\[
\dot{\gamma} = \dot{\omega} + (\dot{\omega}^T) = \begin{bmatrix} 2a & 0 \\ 0 & -2a \end{bmatrix} \quad \Rightarrow \quad \dot{\gamma} = 2a
\]

\[
\omega = \dot{\omega} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \Rightarrow \quad \omega = 0
\]

\[
\lambda = \frac{\dot{\gamma}}{(\dot{\gamma} + \omega)} = 1
\]
The average value of the parameter $\lambda$ in our simulations was calculated by the corresponding value of each element and a weighting factor, its surface area. The value of parameter $\lambda$ for each element was the average from the 16 Gaussian points within the element (we used Quadrilateral rectangular elements with 9 nodal points in the simulations). The information of the velocity gradients for each Gaussian point was obtained by adding an output command in the subroutine "USRVSC", a subroutine used for the calculation of the viscosity. The average value of $\lambda$ together with the average shear rate (calculated by the same procedures as $\lambda$) can then be used together to interpret the mixing efficiency in the mixing chamber.
Figure 1: Comparison of dispersion in different flow fields. Solid line stands for simple shear flow, dashed line stands for elongational flow.

NOTATIONS

A_1 : angle between the right rotor tip and the horizontal axis in the two-rotor flow geometry

A_2 : angle between the left rotor tip and the horizontal axis in the two-rotor flow geometry

A_r : surface area of the rotor

a_{ij} : components of a tensor A

b, c : constants describing the dependence of \( \theta \) and \( n \) on the test temperature in the power-law model

C_{p,r} : heat capacity of the rotor

C_V : heat capacity of the mixture

k : thermal conductivity of the mixture

m : consistency index in the power-law model

n : power-law index in the power-law model

n : normal direction of the entrance or exit boundary in the one-rotor flow geometry

\( \Delta P \) : difference between the maximum and the minimum pressure values

Q_H : heat flux from the mixing chamber to the rotor

Q_m : flow rate of the mixture through each radial cross section in the one-rotor flow geometry

T : temperature in the mixing chamber

T_0 : arbitrary reference temperature used in the power-law model

T_n : average temperature of the entire mixing chamber after \( n \) time intervals

T_{r,n} : average temperature of the rotor after \( n \) time intervals
T_{n+1} : average temperature of the entire mixing chamber after n+1 time intervals

T_{r,n+1} : average temperature of the rotor after n+1 time intervals

T_w : average temperature of the cooling water flowing through the mixing chamber wall

T_a : average temperature of the entire mixing chamber for the simulated flow geometry

Δt : fixed time interval used in the non-isothermal simulations

v : magnitude of vector \( \vec{v} \)

\( \vec{v} \) : velocity vector

V_x : velocity component in the x-direction

V_y : velocity component in the y-direction

\( \alpha \) : angle between the rotor tip and the horizontal axis in the one-rotor flow geometry

\( \dot{\gamma} \) : shear deformation tensor

\( \dot{\gamma} \) : magnitude of \( \dot{\gamma} \), defined by eqs.(2) and (3) in Appendix III.

\( \rho \) : density of the mixture

\( \tau \) : shear stress tensor

\( \tau_{xx}, \tau_{xy} \) : components in the shear stress tensor

\( \tau_{yx}, \tau_{yy} \) : components in the shear stress tensor

0 : reference viscosity calculated at a reference temperature \( T_0 \) and a reference mixing time \( t_0 \)

\( \psi \) : streamline function defined by \( V_x = -\left( \frac{\partial \psi}{\partial y} \right) \) and \( V_y = \left( \frac{\partial \psi}{\partial x} \right) \)

\( \lambda \) : parameter quantifying the elongational component in the flow field
\( \omega \): magnitude of the vorticity tensor

\( \rho_r \): density of the rotor

\( II_A \): the second invariant of a tensor \( A \), defined by eq. (3) in Appendix III
REFERENCES

CHAPTER 1


CHAPTER 2


CHAPTER 3


CHAPTER 4


APPENDIX III

