OCKEGHEM’S *FUGA TRIUM VOCUM*: A CASE STUDY OF MODAL ORIENTATION IN CANONS AND *CATHOLICA*

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ABSTRACT

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Johannes Ockeghem’s (c. 1410-1497) canonic chanson, known as Prenez sur moi vostre exemple amoureux [Take from me your amorous example] or Fuga trium vocum in Epidiatessaron, post perfectum tempus [Canon for three voices in the fourth above after perfect time], has received considerable attention in music scholarship from the sixteenth century to the present day. In general, those analyses focus on understanding Renaissance theorist Heinrich Glarean’s (1488-1563) designation of the canon as a catholicon—a modal piece able to be transposed to different modes—by interpreting the unique arrangement of hexachordal symbols replacing a clef on its notation. The few analyses that mention modal orientation do so by assigning one of Glarean’s twelve modes to each canonic voice without discussion. In this thesis, I investigate both the notation and the musical structure of Fuga trium vocum to determine the elements that allow the piece its catholicon status.

Seeking alternatives to traditional modal theory and contrapuntal studies, several scholars have developed ways of analyzing structure in Medieval and Renaissance music. One of these theories, Cristle Collins Judd’s theory of modal types and Ut, Re, Mi tonalities, serves as the basis for my analysis. After transcribing Fuga trium vocum from the print in Glarean’s treatise, I apply Judd’s theory, which asserts that modal pieces can represent modal types—paradigms of structural superius-tenor relationships based on characteristic melodic intervals—within hexachordal tonalities built on ut, re, and mi, to
diatonic transpositions of the piece. Then, I identify structural similarities among the individual voices and the canon transpositions.

My analysis using Judd’s theory reveals that the fifth-based construction of *Fuga trium vocum*’s guide melody persists in the realized canon and creates connections among its transpositions. This emphasis on the fifth—a defining interval of several modes—and modal ambiguity allow *Fuga trium vocum* to be transposed to different starting pitches with favorable results. A brief analysis of a comparable, but non-*catholicon*, piece confirms and refines these conclusions.
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CHAPTER I. INTRODUCTION

Johannes Ockeghem’s (c. 1410–1497) strictly imitative chanson, known as Prenez sur moi vostre exemple amoureux [Take from me your amorous example] or Fuga trium vocum in Epidiatessaron, post perfectum tempus [Canon for three voices in the fourth above after perfect time], frequently appears in music scholarship from the sixteenth century through the present day. Along with its intriguing arrangement of hexachordal symbols in place of a conventional clef, Fuga trium vocum remains a popular research subject today because of Renaissance theorist Heinrich Glarean’s (1488–1563) designation of it as a catholicon—a modal piece able to be transposed to different modes. In this thesis, I aim to supplement the current understanding of catholica by transcribing and analyzing Ockeghem’s Fuga trium vocum. Unlike other studies, however, I use systematic transposition and a modern theory, Cristle Collins Judd’s theory of modal types and Ut, Re, Mi tonalities, to determine the musical elements that allow Fuga trium vocum its ability to be modally transposed. Ultimately, my analysis using Judd’s theory reveals that Fuga trium vocum’s emphasis on the fifth—a defining interval of several modes—and modal ambiguity enables the piece its catholicon status. In chapter 3, I compare these results with a similar modal piece that does not have catholicon status.

Although documents as early as ninth-century Musica Enchiriadis include instructions to perform certain melodies in different modes, Medieval and Renaissance treatises rarely discuss the concept or mention specific catholica. Its scarcity seems logical: since each mode has a

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unique scale and characteristic interval, composing pieces that can successfully represent
different modes by simply changing their finals proves challenging.\textsuperscript{5} As Glarean mentions in his
treatise, which contains the most thorough explanation of \textit{catholica}, Ockeghem enjoyed this
challenge; besides \textit{Fuga trium vocum}, he wrote another popular \textit{catholicon}, \textit{Missa Cuiusvis toni}
[Mass in any ecclesiastical mode you like].\textsuperscript{6} Peter Urquhart argues that Ockeghem invented the
\textit{catholicon} canon with \textit{Fuga trium vocum} and inspired a trend of similar \textit{catholicon} canons by
Mouton, Verdelot, and Févin.\textsuperscript{7} I chose to analyze \textit{Fuga trium vocum} over other \textit{catholica}
because its lack of explicit pitch identities and a tenor voice adds interest to the tasks of
transcribing the piece and determining the modal orientation of its transpositions; in addition,
Glarean’s definition of \textit{catholica} includes a description of the piece.

As one of Ockeghem’s most published works, \textit{Fuga trium vocum} appears in many
chansonniers and treatises throughout the sixteenth century.\textsuperscript{8} In his thorough review of treatises
containing \textit{Fuga trium vocum}, Leeman Perkins asserts that, due to its use of six hexachordal
symbols instead of a clef, the piece “was given [after introducing solmization] as a practical
exercise that was presumably intended to help the student musician to grasp and internalize the
principles that had just been exposed.”\textsuperscript{9} Of all the chansonniers containing the canon, David

\textsuperscript{5} This understanding of mode as informing composition represents only one of several views, including mode as a
classification system, a pedagogical mnemonic device, or a prototype of the major-minor tonal system. Without
contemporaneous musicians to consult, scholars are unsure whether mode was a precompositional or
postcompositional concern. While the transposition ability of \textit{catholica} could be coincidental and unplanned, I
believe that the relative rarity of \textit{catholica} indicates that composers must have considered aspects of mode in some
way while writing them.

Compositions,” 454.

\textsuperscript{7} Peter Urquhart, “Calculated to Please the Ear: Ockeghem’s Canon Legacy,” \textit{Tijdschrift van de Koninklijke

\textsuperscript{8} For a complete list of \textit{Fuga trium vocum} publications, see Wexler, \textit{Motets and Chansons}, 87-8.

Disciplina} 44 (1990): 133.
Fallows states that most treatises reference its *Canti C* print (1504).\(^{10}\) Although the Copenhagen Chansonnier (c. 1470), which holds the first published manuscript of the piece, generally serves as the main source for contemporary analyses of *Fuga trium vocum*,\(^{11}\) I chose to transcribe the *Canti C* print because Glarean provides a reproduction of it in his discussion of *catholica*. In chapter 2, I discuss discrepancies between the Copenhagen manuscript and the *Canti C* print of *Fuga trium vocum*, including differences in the locations of several notational symbols and their effect on the realization of the canon.

Other analyses of *Fuga trium vocum* explore its *catholicon* status by focusing on notational and contextual aspects of the piece. Scholars interpret the opening hexachordal symbols in different ways according to various historical documents: Joseph Levitan, citing the table of clefs in Laurentius Ribovius’s *Enchiridion Musicum* (1638) and the *musica ficta* transposing gamut diagram in Ramis’s *Musica Practica* (1492), believes that the symbols represent accidentals for specific pitches, while Carl Dahlhaus, Richard Wexler, Leeman Perkins, Peter Urquhart, and Beth Anne Lee-De Amici believe that the symbols refer to *fa* and *mi* syllables, following descriptions of the piece by Sebald Heyden in *De Arte Canendi* (1540) and Gregorius Faber in *Musices Practicae Erotematum* (1553) that state that it can be performed using both hard and soft hexachords.\(^{12}\) Perkins and Lee-De Amici also argue that the text and decorative initial on the Copenhagen manuscript offer clues to solving the puzzle presented by

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\(^{11}\) Wexler, *Motets and Chansons*, 87.

the opening hexachordal symbols. After lengthy explorations of *Fuga trium vocum*’s notation and historical context, Dahlhaus, Urquhart, and Wexler claim that the canon can only begin on the pitches A-D-G; Levitan presents solutions on C-F-B♭ and A-D-G; Lee-De Amici acknowledges A-D-G and D-G-C as solutions; and Perkins derives solutions on A-D-G, D-G-C, and E-A-D. Despite differing conclusions, each analysis determines that Glarean’s *catholicon* designation proves true; scholars who claim that the piece has only one solution discover that each voice represents a different mode. When mentioning modal orientation, scholars merely state rather than discuss which of Glarean’s twelve modes they believe their solutions represent.

My analysis explores the *catholicon* status of *Fuga trium vocum* by looking beyond cursory modal classification to the melodic and harmonic structure of diatonic transpositions of the piece, including each voice individually, the realized canons, and how the transpositions relate to one another.

Seeking ways to understand structure and coherence in modal pieces, several early music scholars have investigated alternatives to modal classification and contrapuntal studies. While different from the ones described in treatises, these new analytical methods do not completely abandon the criteria of traditional modal categorization: they still build their theories around the final, range, hexachord system (hard, soft, or natural), and characteristic melodic intervals of a piece. Three of these contemporary theories illuminated aspects of my analysis. Jennifer Bain’s hierarchical theory focuses on the relationship between opening and ending sonorities to determine the structural pitches, or tonal centers, of a piece. Bain argues that the relative importance of opening sonorities can change as a piece unfolds in time, stating that “the initial

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and cadential sonorities of the first phrase together may indeed assert or point towards a tonal
center, but the *large-scale* implications are not certain at that point in the song."^{15} Although
developed for Machaut’s music, Bain’s idea of multiple tonal centers directly applies to the
modal ambiguity present in *Fuga trium vocum*. Also, Bain’s theory emphasizes the importance
of considering pieces in groups as well as individually, which aids my analysis of the
relationship among canon transpositions.

Like Bain’s theory, Sarah Fuller’s process-based hierarchical theory, which considers
characteristic melodic intervals and counterpoint, reveals the possibility of multiple tonal
orientations; also, I adopt her position on the use and effect of *musica ficta*. Fuller claims that in
some pieces, *ficta* choices will not necessarily alter the establishment of tonal orientations, but
can emphasize changes between them.^{16} This view applies to the medial cadence of *Fuga trium
cocum*, where performers can elect to raise the pentultimate pitch or preserve the strictly canonic
nature of the piece without altering its pitch orientation.

Finally, Cristle Collins Judd’s hierarchical theory asserts that modal pieces can represent
modal types—paradigms of structural superius-tenor relationships based on characteristic
melodic intervals—within hexachordal tonalities built on *ut*, *re*, and *mi*. Judd’s three tonalities,
which follow Glarean’s notion that *ut*, *re*, and *mi* are the only necessary and unique hexachordal
positions, reflect the different melodic possibilities of each position:

*Ut*: the full hexachord is available; it contains the characteristic species of fifth and has
implications for the upper species of fourth. *Re*: the final and upper boundary of the
hexachord emphasize the species of fifth, *re-la*, and leads to frequent but not obligatory
use of *fa super la*. *Mi*: the final and boundary of the hexachord emphasize the *mi-la*

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^{15} Jennifer Bain, “‘Messy Structure?’ Multiple Tonal Centers in the Music of Machaut,” *Music Theory Spectrum* 30,

^{16} Sarah Fuller, “Exploring Tonal Structure in French Polyphonic Song of the Fourteenth Century,” in *Tonal
Judd’s modal types and hexachoral tonalities appear on small- and large-scale analytical levels. I discuss Judd’s theory further in chapter 2, where it serves as the basis for my analysis.

In the main analysis of this study, I analyze the modal orientation of diatonic transpositions of *Fuga trium vocum* with Cristle Collins Judd’s theory of tonal coherence. I model my transcriptions after previous *Fuga trium vocum* analyses, specifically their use of Renaissance dissonance treatment with modern notation and the equal temperament tuning system. Pitch names correspond with the Guidonian gamut, which identifies pitches by their positions in surrounding hexachords (fig. 1.1). For example, “D sol re” refers to the middle line bass clef D. Although developed recently, Judd’s theory incorporates aspects of traditional modal theory to determine her modal types and tonalities, including the final and characteristic melodic intervals of a piece. According to Glarean, teachers of traditional eight-mode theory explain that pieces end on an *ut*, *re*, or *mi* position and emphasize specific interval spans; these elements help musicians define the mode of a piece (table 1.1). For example, a piece of the first mode, Dorian, ends on a *re* position and often highlights the fifth-span *re-la*. Translating this information into Judd’s theory suggests a *re-la* modal type within her *Re* tonality. As *Fuga trium vocum’s rondeau* text does not directly relate to modal orientation, I exclude it from my transcriptions and analytical discussion.

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I discuss the transcription, transposition, and modal orientation of Ockeghem’s *Fuga trium vocum* further in the next chapter, which serves as the main analysis of this study. During that chapter, I use Glarean’s definition of *catholica* to guide an analysis of *Fuga trium vocum* that addresses the notational challenges of transcribing it, the problematic dissonances certain transpositions present, and the ambiguous modal orientation Judd’s theory reveals in each transposition. In the last chapter, I summarize my findings, place them into a larger context with a similar analysis of a comparable, but non-*catholicon*, modal piece, and offer suggestions for further research of mode in canons and *catholica*. 

Table 1.1. Criteria for modal classification according to Glarean.

<table>
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<th>Final Hexachordal Position and System</th>
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<tr>
<td>1. Dorian</td>
<td>re, B-fa</td>
<td>re-la</td>
</tr>
<tr>
<td>2. Hypodorian</td>
<td>re, B-fa</td>
<td>re-fa</td>
</tr>
<tr>
<td>3. Phrygian</td>
<td>mi, B-mi</td>
<td>mi-fa</td>
</tr>
<tr>
<td>4. Hypophrygian</td>
<td>mi, B-mi</td>
<td>mi-la</td>
</tr>
<tr>
<td>5. Lydian</td>
<td>ut, B-fa</td>
<td>mi-sol</td>
</tr>
<tr>
<td>6. Hypolydian</td>
<td>ut, B-fa</td>
<td>fa-la</td>
</tr>
<tr>
<td>7. Mixolydian</td>
<td>ut, B-mi</td>
<td>ut-sol</td>
</tr>
<tr>
<td>8. Hypomixolydian</td>
<td>ut, B-mi</td>
<td>ut-fa</td>
</tr>
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Figure 1.1. The Guidonian gamut.
CHAPTER II. MODAL ORIENTATION IN *FUGA TRIUM VOCUM*

In the final chapter of *Dodecachordon*, a lengthy treatise presenting his theory of twelve modes, Heinrich Glarean mentions six composers who he thinks especially excel at polyphonic composition and describes several pieces that he believes best demonstrate their skills. Glarean’s discussion of the life and music of Johannes Ockeghem mentions *catholica*—a concept unique to the modal system—and cites a representative piece:

[Ockeghem] was indeed admirable in invention and keenness of skill. But he loved the *catholica* in song, that is, songs composed so that they would be sung in many ways, almost according to the will of the singers, yet so that the relationship of the harmony and the consonances would be observed no less; the following song of his is considered among the foremost of such a kind, in which song it is necessary to have good ears. It is the *Fuga trium vocum in Epidiatessaron* (for so they speak now), *post perfectum tempus*.1

Essentially, Glarean’s commentary states that *catholica* are pieces that can transpose. Such a distinction seems strange today, since, in the tonal system, every piece can potentially transpose; the reference to *catholica*, then, poses the question of why only certain modal pieces possess this attribute.

To explore the concept of *catholica*, I transcribe, transpose, and analyze Ockeghem’s *Fuga trium vocum* to determine the musical elements that grant its modal transposition ability. My transcriptions into modern notation used for analysis reflect the printed notation provided by Glarean in his discussion of *Fuga trium vocum* as opposed to earlier manuscripts of the piece. Transpositions comprise versions with the diatonic starting pitches D, E, F, G, A, B, and C.2 Cristle Collins Judd’s theory of modal types and *Ut, Re, Mi* tonalities, chosen for its balance of

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2 Although Glarean generally does not consider B an acceptable starting pitch or final, he admits that pieces can begin on it successfully.
Renaissance and current music theories about mode in polyphony, reveals the modal orientation of each transposition and forms a basis for comparison. Overall, I demonstrate that the *catholicon* quality of Ockeghem’s *Fuga trium vocum* comes from its structural fifth-spans and modal ambiguity since those elements lead to a large number of successful canon transpositions and the sharing of modal types among those transpositions.

In this chapter, historical and analytical investigations of *Fuga trium vocum* clarify Glarean’s description of the piece and definition of *catholica* to lead to a better understanding of the concept. Phrases from Glarean’s description guide the chapter as section titles. “For so they speak now,” alludes to manuscript changes that permit *Fuga trium vocum*’s *catholicon* status. “Almost according to the will of the singers,” which indicates that not every starting pitch produces desirable results, addresses the absence of versions beginning on *mi* and *fa* hexachordal positions. “In which song it is necessary to have good ears,” warns against the difficulty of anticipating simultaneously attacked dissonances. “So that the relationship of the harmony and the consonances would be observed no less,” refers to the structural similarities among transpositions, which Judd’s theory reveals result from *Fuga trium vocum*’s fifth-based construction and modal ambiguity. “Songs composed so that they would be sung in many ways,” states that although the transpositions preserve structural aspects, they remain distinct from one another. Finally, “among the foremost of such a kind,” inspires reflection upon the canon’s *catholicon* aspects, both basic and exceptional, to summarize and conclude the chapter.

“For so they speak now”: Notation Changes Allow *Catholicon* Status

This comment of Glarean’s suggests the presence of differences between the canon notation he presents in his treatise and earlier versions of the piece. To identify those differences
and their implications for performance and analysis, I compare and contrast the printed notation provided by Glarean with a facsimile of the manuscript notation included in the Copenhagen Chansonnier, which holds the first published notation of the piece. Upon investigation, seemingly small differences in the title of the canon and the arrangement of its realization instruction symbols have important implications for performance and analysis: these incongruities change the piece from a puzzle canon with two re-position solutions to a catholicon.

An earlier notated representation of the piece Glarean refers to as *Fuga trium vocum in Epidiatessaron, post perfectum tempus* [Canon for three voices at a perfect fourth above after a perfect breve]\(^3\) appears in the Copenhagen Chansonnier with the title *Prenez sur moi vostre exemple amoureux* [Take from me your amorous example].\(^4\) Without realization instructions explicitly stated in the title or a conventional C-, F-, or G-clef for reference, performing from this manuscript involves decoding the instructions given as hexachordal shorthand symbols and *signa congruentiae*. Although more restrictive than Glarean’s declaration of transpositional freedom, this puzzle canon notation offers pitch performance options; once decoded, the resulting instructions indicate two possible pitches at which to perform the canon.

In the Copenhagen notation of *Prenez sur moi*, the hexachordal shorthand symbols for *fa* and *mi* appear at the beginning of every staff instead of, rather than in conjunction with, a conventional clef.\(^5\) Specifically, three *fa* symbols followed by three *mi* symbols occupy the second and fourth lines at the beginning of each staff. Just as clefs show the location of a specific pitch on a particular staff line to orient the noteheads in musical space, these symbols indicate


\(^{5}\) Appendix A contains a reproduction of the Copenhagen manuscript facsimile referenced in this study.
where the hexachordal syllables fa and mi fall on the staff to assist performers in identifying the notated pitches. In his pedagogical treatise, Renaissance theorist Sebald Heyden explains that “[i]f b fa or # mi is written at the beginning on a pitch other than its usual pitch, then the normal sequence of the entire system is disregarded, and the gamut of all other syllables and tones is applied according to the new pitch of B fa or B mi.”

Supplying the remaining hexachord syllables around the fa and mi locations signified by the opening symbols in Prenez sur moi provides three identities for the first pitch, which appears on the third staff line: la re, re sol, and ut sol (fig. 2.1). Each syllable pair, separated by a perfect fourth, corresponds to two specific pitches—D and A for la re, G and D for re sol, and C and G for ut sol—which indicates that the puzzle canon has two different pitch solutions: D-G-C and A-D-G. Judging by the number and placement of the hexachordal shorthand symbols, informed performers will deduce that the canon is for three voices a perfect fourth apart starting on either D sol re or A la mi re.

To ascertain the distance of imitation from the Copenhagen notation, performers must locate two signa congruentiae near the end of the guide melody and determine the length between the signs. As Zarlino explains in a section on fugues and consequences, typically “[t]he guide is written out at length, and then at the point where the consequent is to enter there is placed the sign [signum congruentiae] . . . At the end, on the note upon which the consequent part should cease, the same sign . . . is placed.” The Copenhagen notation of Prenez sur moi, though, includes only the signs that indicate where the consequent voices end (see ex. 2.1). This lack of opening cue adds an extra challenge to determining the distance of imitation and, as music historian Willi Apel asserts, reflects common practice for mensuration canons:

The signum congruentiae: . . . serves to indicate points of coincidence in the various

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parts. . . . The sign is regularly used in canonic pieces to indicate either the fugal entrance of the imitating part or, in mensuration canons, the places where the various singers have to stop.\(^8\)

Although not marking a mensuration canon in this case, the signs still seem to signify where the consequent parts stop. With reference to the given mensuration marking of *tempus perfectum*, *prolatio minor*, the *signa congruentiae* indicate the distance of a perfect breve between voices; also, the signs show that the third voice has a different final from the other two voices. Additionally, the presence of two signs confirms that three voices, the guide plus two consequent parts, participate in the canon.

Figure 2.1. Solution to puzzle canon hexachordal symbols on Copenhagen manuscript. *Above*, hexachordal identities of opening pitches. *Below*, specific identities of opening pitches.

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Figure 2.1, continued.

The poetic title and arrangement of realization instruction symbols on the Copenhagen notation undoubtedly present the piece as a puzzle canon. Although the pitch puzzle aspect of this notation offers more than one solution, both versions—D-G-C and A-D-G—involve two re hexachordal positions followed by an ut position; therefore, they produce similar results in that both versions have the same layout of whole-steps and half-steps. The creation of both re-based and ut-based voices from the same guide melody, though, hints at Glarean’s cathomicon designation of the canon. The alternative title and arrangement of symbols on the notation provided by Glarean remove the pitch restrictions, effectively changing the piece from a puzzle canon with two similar solutions to a cathomicon that allows the piece to begin on pitches other than D or A and permits different combinations of hexachordal starting positions.

To perform the printed version in Glarean’s treatise, solving the pitch-restrictive canon realization instructions puzzle is neither necessary nor possible. A title change from the first line of the French rondeau text, Prenez sur moi vostre exemple amoureux, to the realization instructions in Latin, Fuga trium vocum in Epidiatessaron, post perfectum tempus, renders solving the puzzle no longer necessary for performance. According to Zarlino, composers typically provided realization instructions in the titles of canons, so performers likely would not attempt to derive meaning from the hexachordal symbols. Additionally, musicians who try to

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solve the puzzle will not succeed because the \textit{mi} symbols have migrated to the spaces above their original lines.\textsuperscript{10} This slightly altered placement indicates \textit{la re}, unchanged from the original notation in that it involves the unaffected \textit{fa} symbols only, \textit{re fa}, nonexistent on the gamut, and \textit{fa} alone, inadequately specific, for the opening pitches (fig. 2.2). Although the placement of two additional \textit{signa congruentiae} to show where the consequent parts enter simplifies determining the distance of imitation for the first consequent voice, the slightly displaced second sign creates undesirable harmonic results without referring to the title for clarification: dissonances occur between the voices at least once per measure (ex. 2.1). Overall, the canon’s different title and changes to the signs and symbols ensure the suppression of its original pitch restrictions and allow the freedom of performance cited by Glarean.

Figure 2.2. Solution to displaced hexachordal symbols on \textit{Dodecachordon} print. \textit{Above}, hexachordal identities of opening pitches. \textit{Below}, specific identities of opening pitches.

\begin{align*}
\textit{ut re mi fa sol la} & \quad \textit{ut re mi fa sol la} \\
\textit{ut re mi fa sol la} & \quad \textit{ut re mi fa sol la} \\
\textit{ut re mi fa sol la} & \quad \textit{ut re mi fa sol la} \\
\textit{ut re mi fa sol la} & \quad \textit{ut re mi fa sol la}
\end{align*}

\textsuperscript{10} Appendix A contains a reproduction of the \textit{Dodecachordon} notation facsimile referenced in this study.
Example 2.1. Results of *signum congruentiae* locations according to *Dodecachordon* print, mm. 1-9. Note: intervals appearing between the lower and middle staves refer to the intervals between the upper and lower voices.

By stating “for so they speak now,” Glarean acknowledges the subtle, but significant, differences between the *catholicon* notation he provides and earlier puzzle versions of the canon. His use of the Latin generic title and slightly displaced realization instruction symbols permits performers to utilize starting pitches beyond D sol re or A la mi re, which expands the sonic possibilities of the piece by offering additional combinations of opening hexachordal positions. This particular comment informs readers that Glarean’s version of Ockeghem’s canon exemplifies a broader definition of *catholicon* than the previous puzzle canon notation suggested.

“Almost according to the will of the singers”: Elimination of Hexachordal Position Possibilities and Judd’s *Mi* Tonality

Although the *Fuga trium vocum* manuscript Glarean presents in his treatise encourages
more combinations of opening hexachordal positions than earlier manuscripts allowed, this particular statement suggests that not every starting position produces successful results. While Glarean could refer to versions with starting pitches that lead to finals not included in his twelve modes, further examination of the guide melody transpositions reveals that the two versions with mi-position opening pitches—E and B—prove problematic as well. The guide melody’s fifth-centric construction, compatible with the ut and re transpositions, causes retroactive tritone outlines and mi-mi fifth spans in the mi-position versions. Applying fixed musica ficta—analagous to key signatures in modern transcriptions—to the mi-position versions simplifies avoiding their retroactive tritone outlines and changes their fifth-span hexachordal identities from unrecognized mi-mi to the characteristic melodic interval re-la. This solution affects the concept of catholica in that it effectively eliminates mi as a possible starting position and the presence of Judd’s Mi tonality in this piece.

While performers must aim to avoid tritones in each transposition, the mi guide melody transpositions provide an additional challenge: retroactive tritone outlines. As correcting B-F tritones with Bb does not make modal sense in the E and B transpositions, F# must correct those tritones instead. This solution poses no problems when F follows B in that performers, aware of potential tritone outlines involving the B, can simply change the F to F# when it arrives. The opening phrase of the second half of the B transposition illustrates this situation: the B appears one measure before the F, giving performers advance notice to avoid the tritone outline with F# (ex. 2.2). When B follows F, however, performers must be aware of the upcoming B in time to use F# to avoid creating a tritone outline retroactively with the previous F. A basic example of this scenario occurs near the end of the E transposition when the melody quickly descends by

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11 Appendix B contains transcriptions of the Fuga trium vocum guide melody transpositions.
12 For an explanation of Judd’s theory, see p. 26.
thirds from F to B (ex. 2.3). Without knowledge of the B ahead, performers will not change the F to F#. Unable to use B♭ and already past the F, the uninformed performer creates a retroactive tritone outline error at the B. The second phrase of the B transposition offers an extreme example of these retroactive tritone outlines in that a chain of two occurs (ex. 2.4). Approaching this example backwards in musical time most clearly demonstrates its complexity. First, the B in m. 7 calls for F♯ in m. 6. Then, continuing backwards in m. 6, the upper-octave F♯ forces lower-octave F♯ to avoid a dissonant leap between F♯ and F. Finally, the F♯ in m. 6 necessitates C♯ in m. 5. Altogether, performers must be aware of the B in m. 7 and its three consequences before the end of m. 4 to know to use C♯ instead of C at the beginning of m. 5. Applying fixed *musica ficta* to these two transpositions—F♯ in E and F♯ and C♯ in B—alleviates their retroactive tritone problems without causing any new errors, which greatly simplifies performing these transpositions.

Example 2.2. The pitch F follows the pitch B; B re guide melody transposition, mm. 19-21.

Example 2.3. The pitch B follows the pitch F; E re guide melody transposition, mm. 28-30.

Example 2.4. Chain of retroactive tritones; B re guide melody transposition, mm. 4-7.
The fixed ficta also rectifies another problem with the mi-position guide melodies by changing their mi-mi fifth-spans to re-la. Without fixed ficta altering the location of half-steps, mi-mi fifth-spans between E and B and B and F♯ appear frequently in the two mi transpositions. No descriptions of characteristic melodic intervals, intervals emphasized by metric placement, register, repetition, and phrase prominence, include mi-mi fifth-spans. Glarean, keeping with the established practice of his time, cites the third-span mi-fa, fourth-span mi-la, and sixth-span mi-fa as the only acceptable mi-based characteristic melodic intervals.¹³ Further, he repeatedly states his preference for the fifth-spans ut-sol and re-la over mi-mi, which he believes confuses and offends the ears by comparison.¹⁴ As characteristic melodic intervals define traditional modes and assist with the identification of Judd’s tonalities, the emphasis on mi-mi in the E and B transpositions proves problematic. By shifting the location of mi-fa half-steps ahead one pitch from E-F to F♯-G and B-C to C♯-D, the fixed ficta changes mi-mi spans into re-la and the overall hexachordal identities of the E and B transpositions from mi to re (fig. 2.3). The most straightforward illustration of this phenomenon occurs in the opening phrase of both transpositions (ex. 2.5). In both cases, the application of fixed musica ficta changes the layout of hexachordal syllables from mi-fa-sol-la-mi to re-mi-fa-sol-la. This change occurs in every remaining instance of mi-mi, therefore eradicating the problem interval from the Fuga trium vocum guide melodies by transforming it into the more desirable re-la.

Figure 2.3. Shift of E and B mi-mi fifth-spans to re-la after application of fixed musica ficta.

¹⁴ Ibid., 129-32.
Figure 2.3, continued.

Example 2.5. Hexachordal syllables before and after application of fixed *musica ficta* to the E and B guide melodies, mm. 1-4.

An alternate interpretation of Glarean’s statement infers that not every starting pitch produces unique results. In fact, while including guide melody transpositions beyond those with accepted finals seems to expand the aural possibilities of this piece and definition of *catholicon* at first, these additional transpositions do not broaden the concept in that they are intervallically identical to versions with standard finals. For example, the guide melody transpositions probably
not considered by Glarean due to their unacknowledged finals, but included in this study for equal-tempered performers—C ut, E♭ fa, F fa, and B♭ fa—behave similarly to the E and B transpositions in that they require fixed musica ficta to avoid retroactive tritone outlines and unrecognized fa-fa fifth-spans. These transpositions, however, ultimately represent the ut hexachordal position and ut-sol fifth-span. After the application of corrective ficta, the eleven different guide melody starting pitches and hexachordal positions used in the canons considered in this study—D re with B-fa, D re with B-mi, E♭ ut, E re, F ut, G re, G ut, A re, B♭ ut, B re, and C ut—represent only three unique successions of melodic intervals. After avoiding tritone errors with fixed musica ficta wherever necessary, the ut transpositions become intevallically identical to one another while the re transpositions split into two groups: essentially, those using the hexachordal layout re-mi-fa-sol-la-mi and those using the layout re-mi-fa-sol-la-fa (fig. 2.4). Overall, the fifth-centric construction of the guide melody ensures that any starting pitch or hexachordal position a performer chooses will eventually conform to one of three unique successions of melodic intervals.15

Figure 2.4. Hexachordal layout of re transpositions.

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15 Sol-based versions with sol-sol fifth-spans become ut in that sol-la-mi-fa-sol matches ut-re-mi-fa-sol; La-based versions with la-la fifth-spans become re in that la-mi-fa-sol-la matches re-mi-fa-sol-la.
When combined according to the canon realization instructions, the three guide melody types create only five distinct canons from the diatonic starting pitches D through C (fig. 2.5). As shown in fig. 2.5, the D-G-C canon forms two different versions: one using B-fa and the other using B-mi in the D and G lines. Heyden clearly has this particular transposition in mind when he presents *Fuga trium vocum* as an “example of *cantus firmus* using soft B or hard B.”\(^{16}\) The A-D-G canon does not possess this quality in that the use of B-fa in the D and G lines often forces B-fa in the A line, which leads to problematic mi-mi fifth-spans. For this reason, only one A-D-G canon appears. Additionally, avoiding dissonances in the G-C-F canon requires the use of B-fa on over half of the B pitches in the G line, so only one version of this canon appears. The remaining canon transpositions also have only one version, ranging from all one *re* type to all *ut*.

Altogether, the small number of unique guide melodies and the parameters of the canon realization instructions allow only five distinct canon versions from any selected variety of starting pitches.

Figure 2.5. Combination of the three unique guide melody transpositions into five unique canons.

<table>
<thead>
<tr>
<th>Guide Melody Combination</th>
<th>Canon Names</th>
</tr>
</thead>
<tbody>
<tr>
<td>111</td>
<td>B-E-A</td>
</tr>
<tr>
<td>112</td>
<td>E-A-D</td>
</tr>
<tr>
<td>123</td>
<td>D-G-C (B-fa), A-D-G</td>
</tr>
<tr>
<td>233</td>
<td>D-G-C (B-mi), G-C-F</td>
</tr>
<tr>
<td>333</td>
<td>F-B♭-E♭, C-F-B♭</td>
</tr>
</tbody>
</table>

As part of his general definition of *catholica*, Glarean’s conditional statement, “almost at the will of the singers,” informs readers that no *catholicon* supports total freedom of performance options. In the case of *Fuga trium vocum*, its fifth-centric guide melody construction favors *ut*.

\(^{16}\) Heyden, *De Arte Canendi*, trans. Miller, 52.
and re positions while problematizing mi and fa positions. Retroactive tritone outlines and unrecognized fifth-spans in the mi and fa transpositions call for the use of fixed musica ficta to simplify their performance and transform their fifth-spans into characteristic melodic intervals. These changes ultimately lead to a limited number of unique possibilities despite the chosen starting pitch by eliminating two hexachordal positions and one-third of Judd’s tonalities.

“In which song it is necessary to have good ears”: Simultaneously Attacked Dissonances

This remark, one of Glarean’s two that describe Fuga trium vocum specifically, indicates that the piece presents a high level of difficulty to performers. As the guide melody itself does not provide a challenge after the application of any necessary musica ficta, this difficulty must appear upon combining the guide melodies in canon. Examining the intervallic content of each canon transposition built on the diatonic pitches D through C reveals that simultaneously attacked vertical dissonances often arise between the voices. As a result, this piece requires good ears in that avoiding these dissonances requires performers to not only quickly react to the other parts, but also to anticipate them.

Dissonance treatment in the transpositions presented and analyzed below follows conventions outlined by Renaissance theorists. Essentially, the canons avoid adjacent melodic and simultaneously attacked augmented and diminished intervals. Zarlino defends and praises certain tritones not simultaneously attacked, though, and describes their desirability and frequent use:

At times musicians write a tritone between two parts so that it falls on the second part of a syncopated semibreve [quarter note] written in the bass . . . The notes of the tritone, although heard, are not actually sounded [attacked] simultaneously, and since the parts are so coordinated as to not depart from the rules [the tritone resolves to a third], the resulting effect is good. These parts are pleasing to the ear, because the small bit of dissonance heard in the tritone [augmented fourth] or semidiapente [diminished fifth]
passes quickly and adds a sweetness to the following consonance it would not possess alone. For opposites are most recognizable when compared directly. Modern composers often write such passages.\(^{17}\)

Indeed, syncopated tritones fitting Zarlino’s description appear many times in the *Fuga trium vocum* transpositions. An illustrative example from the E-A-D canon shows this type of tritone between the lower and middle parts (ex. 2.6). On the third beat of m. 6, a tritone forms when the middle voice places an F against the second half of a B quarter note in the lower voice. The lower voice then moves to an A on the second half of the middle voice’s F, forming the consonant third that excuses the tritone. All other syncopated and simultaneously attacked dissonances, however, require corrective *ficta* in accordance with Renaissance treatises.

Example 2.6. An occurrence of Zarlino’s syncopated tritones; E-A-D canon transposition, m. 6.

Attentive performers, familiar with the contour and rhythm of the other parts due to the canonic nature of the piece, can anticipate simultaneously attacked dissonances in time to avoid them. Two representative examples come from the B-\(mi\) D-G-C canon. One example shows two simultaneously attacked B-F tritones between the lower and middle parts, respectively: one on the second half of the second beat of m. 24 and the other on the second half of the first beat of m. 25 (ex. 2.7). Lower-part performers, acquainted with the middle part having just completed it a

fourth lower, can anticipate the Fs in the middle part and alter their descending passages to use B-ba instead of tritone-inducing B-mi. A similar, but more complicated example from m. 18 of the B-mi D-G-C canon requires lower-part performers to keep their past two measures in mind to avoid creating a diminished octave with the upper part Bb (ex. 2.8). By recalling their ascending perfect fourth leap two measures prior, lower-part performers can anticipate the upper-part’s use of B-ba to avoid a melodic tritone leap on the downbeat of m. 18 and respond appropriately with B-ba instead of B-mi. As these dissonance situations appear in every transposition, successful performance of this piece requires astute musicians.

Example 2.7. Tritone anticipation required in lower voice; D-G-C with B-mi canon transposition, mm. 23-25.

Example 2.8. Extended tritone anticipation required in lower voice; D-G-C with B-mi canon, mm. 15-18.

With the description, “in which song it is necessary to have good ears,” Glarean warns
readers that *Fuga trium vocum* challenges performers. Undoubtedly, Glarean refers to the
frequent appearance of simultaneously attacked dissonances that impel performers to quickly
react to and anticipate the other parts. Although the canonic nature of the piece provides
performers with the information needed to avoid these dissonances, it also requires performers to
recall their previous measures while listening to the other parts and synthesizing both sets of
information. The presence of only five unique canon transpositions, however, slightly simplifies
this task in that experienced performers will recognize patterns between intervalically identical
versions. Overall, Glarean advises that performing *Fuga trium vocum* as a *catholicon* is difficult,
but not impossible.

“So that the relationship of the harmony and the consonances would be observed no less”:  
Shared Modal Types and Modal Focus Areas

As part of Glarean’s general definition of *catholica*, this statement suggests that *catholica*
transpositions share structural elements. To determine the specific connections among *Fuga
trium vocum* transpositions that allow its *catholicon* status, this section analyzes and compares
their modal orientations with Cristle Collins Judd’s theory of modal types and *Ut, Re, Mi*
tonalities. After an explanation of Judd’s theory and brief description of the guide melody, I
explore the modal orientation of the single melody line beginning on *D re* with *B-mi*. Then, I
analyze the modal orientation of a canon containing that melody line, *A-D-G*. Finally, I consider
the remaining canon transpositions and conclude that modal ambiguity in the guide melody,
when combined twice with itself according to the canon realization instructions, persists in the
canon transpositions and creates connections among them; by representing two modal types
each, canon versions share at least one modal type with other canon transpositions. This

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18 This particular melody, chosen arbitrarily, holds no special significance over any other version.
connection, along with the guide melody’s fifth-based construction, decreases the harmonic differences between canon transpositions and contributes to the *catholicum* status of *Fuga trium vocum*.

According to Judd’s theory of tonal coherence, polyphonic pieces from about 1500 represent both modal types and *Ut, Re, or Mi* tonalities rather than traditional modes. Judd asserts that “[a] tonality and its modal type(s) as manifested in an individual work are identified by two markers: the tonal / hexachordal function of the final and its location, e.g. *Re*(D); and a characteristic melodic shape indicated by solmization syllables, e.g. *re-la*.” Modal types, which appear on levels of simple counterpoint, sectional elaboration, and *pars* or higher, represent models of common melodic-contrapuntal patterns and define the tonalities. While Judd’s *Ut, Re,* and *Mi* tonalities commonly emphasize *ut*-sol, *re*-la, and *mi*-la, respectively, other corresponding characteristic melodic intervals and final pitches can define her hexachordal tonalities; for example, any final pitch on a *re* position and either *re*-fa or *re*-la can define her *Re* tonality. The reductive graphs associated with her theory use hierarchical, but not Schenkerian, notation; stemmed pitches indicate common melodic-contrapuntal patterns, unstemmed pitches are unique to the piece, and slurs show connections between pitches, most often between the outer pitches of modal type characteristic melodic intervals. To discourage parallels with Schenkerian analysis and prolongation, Judd explains that “the bassus shown in these paradigms is a complement of a structural superius-tenor relationship, the result of the contrapuntal and registral conventions of the vocal ensemble, and not a structural determinant: it is generated as part of the counterpoint rather than a harmonic force that directs and shapes that counterpoint.”

Despite its lack of a traditional superius-altus-tenor-bassus texture, *Fuga trium vocum*’s canonic voices still form

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20 Ibid., 439.
structural voice-leading frameworks and fit paradigms of modal types, which the reductive graphs in this study reflect. Although formed with reference to Josquin’s sacred motets, Judd’s theory of modal types and Ut, Re, Mi tonalities should accurately describe modal orientation and tonal coherence in Ockeghem’s *Fuga trium vocum* in that it is a polyphonic work from about 1500, the composers worked in the same geographic area, and Judd’s illustrative examples often include imitative textures.

The guide melody, thirty-five measures long, consists of melodic figures that highlight the fifth-span characteristic melodic intervals ut-sol and re-la, depending on the hexachordal position of the first pitch, and divides into two sections of approximately equal length. This fifth-based construction favors Judd’s Ut and Re tonalities in that ut-sol and re-la often define them, respectively; the mi-based characteristic melodic intervals, indicative of Judd’s Mi tonality, never appear significantly. Further, the consistent *musica ficta* required to avoid retroactive tritone outlines in versions beginning on mi or fa hexachordal positions change unrecognized fifth-spans into characteristic melodic intervals ut-sol or re-la as discussed earlier in this chapter.

Similarities between opening and closing melodic figures determine the location of the division into two sections (ex. 2.9), which a quasi-cadential moment solidifies.²¹ Both opening figures, mm. 1-4 and 19-21, highlight the same ascending fourth followed by the same ascending fifth. The closing figures, mm. 17-18 and 33-35, resemble one another in that they also highlight the same intervals: the first figure ascends a fourth from A to D, then descends a fifth from D to G and emphasizes the C-G fourth during its descent; the second figure recalls that sequence in

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²¹ In the realized canon, performers can strengthen the medial cadence with *musica ficta* on the second beat of the middle voice in m. 18. Regarding this use of *musica ficta*, I adopt Sarah Fuller’s claim that this type of performance choice will not alter the pitch orientation of the passage, which comes from the realized notation itself, but can influence the perception of changes in pitch orientation. I forgo the *musica ficta* in my transcriptions as it has little effect on the overall modal orientation and to preserve the canonic nature of the piece. Sarah Fuller, “Exploring Tonal Structure in French Polyphonic Music of the Fourteenth Century,” in *Tonal Structures in Early Music*, ed. Cristle Collins Judd (New York: Garland Publishing, 2000), 76.
reverse by highlighting a D-A fourth during a descent from D to G, followed by an ascending fourth from G to C. The fifth-based construction, absence of one-third of Judd’s tonalities, and two-part form influence modal orientation in the guide melodies and ultimately contribute to the *catholicon* quality of this piece when combined into canons.

Analysis of the D re with B-mi guide melody reveals the presence of two strong modal areas. Although intended for use with polyphonic works, aspects of Judd’s theory, such as characteristic melodic shapes in the superius and the hexachordal function and location of the final, can suggest modal types and tonalities for monophonic lines. As theoretical treatises provide numerous guidelines for modal classification of monophonic pieces, the following guide melody analysis uses only an analytical level analogous to Judd’s simple counterpoint, which she states most closely corresponds with traditional modality. This investigation of the monophonic guide melody also corresponds with Jennifer Bain’s process-based approach to analysis in that I determine the relative importance of pitches and intervals as they unfold in time. Two kinds of simple counterpoint analysis presented by Judd, note-to-note and slight elaboration, consistently point to the ambiguity of modal orientation in the monophonic guide melody.

Example 2.9. Similarities between the opening and closing figures of each section, D re with B-mi guide melody.

*Above*, opening figures, mm. 1-4 and 19-21. *Below*, closing figures, mm. 16-18 and 33-35.

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22 Judd, “Modal Types,” 441.
24 Judd’s third type of simple counterpoint analysis, the formal unit, does not apply to *Fuga trium vocum* due to its lack of *exordium* and any other clearly defined formal units.
On the most detailed level of the D re with B-mi guide melody analysis, similar to Judd’s note-to-note simple counterpoint, surface details draw attention to the pitches A, D, and G, suggesting both Ut(G): ut-sol and Re(D): re-la modal types. First, the opening phrase rises from D sol re to a decorated G sol re ut, initially highlighting the ut-sol interval of Ut(G), but then the phrase reaches A la mi re and falls back down to D sol re to emphasize the re-la fifth-span and suggest Re(D): re-la (fig. 2.6). Later, a middle phrase suggests both Re(D): re-la and Ut(G): ut-sol by focusing attention on A re, with relatively long note values, placement of rests, and the characteristic melodic interval re-la, and G ut, by utilizing its entire hexachord, its place as the lower boundary of the melody, and its prominent role in the quasi-cadence discussed below (fig. 2.7). Finally, the ending phrase shows a shift from Re(D): re-la to Ut(G): ut-sol as a descent from A la mi re to an emphasized D sol re continues down to a relatively long A re, but ultimately reaches G ut (fig. 2.8). Although the phrase begins with two fifth-spans in support of Re(D): re-la, the presence of ut-sol, the full G ut hexachord, and G ut as the final effectively challenge the dominance of Re(D): re-la evidence.

Figure 2.6. Simple counterpoint note-to-note reduction of D re with B-mi opening phrase, mm. 1-4. Suggests Re(D): re-la.

Figure 2.7. Simple counterpoint note-to-note reduction of D re with B-mi middle phrase, mm. 15-18. Suggests both Re(D): re-la and Ut(G): ut-sol.
Figure 2.8. Simple counterpoint note-to-note reduction of D re with B-mi closing phrase, mm. 31-35. Suggests shift from Re(D): re-la and Ut(G): ut-sol.

Slight elaboration of modal types on a level analogous to simple counterpoint occurs throughout the guide melody as the pitch D sol re and near-octave register transfers from G re to A la mi re often initiate shifts between monophonic versions of the modal types Re(D): re-la and Ut(G): ut-sol. As these modal types alternate, re of the Re(D): re-la modal type becomes sol of the Ut(G): ut-sol modal type; then ut of the Ut(G): ut-sol modal type leaps to la of the Re(D): re-la modal type (fig. 2.9). These shifts between modal types and tonalities suggest Judd’s alternation of modal types elaboration. Although areas of Re(D): re-la focus open both sections and generally last longer, the modal orientation remains ambiguous in that areas of Ut(G): ut-sol focus close both sections as the finals (fig. 2.10). The modal and tonal ambiguity in the D re with B-mi guide melody, defined by the location of and alternation between ut-sol and re-la fifth-spans, contributes to similar ambiguity in the realized canons containing that line.

Figure 2.9. Simple counterpoint slight elaboration reduction of D re with B-mi melody, mm. 6-14. Suggests shifts between Re(D): re-la and Ut(G): ut-sol.
Figure 2.10. Simple counterpoint slight elaboration reduction of D re with B-mi guide melody. Suggests ambiguity between Re(D): re-la and Ut(G): ut-sol.
Above, the A section, mm. 1-18. Below, the B section, mm. 19-35.

As transpositions of the same notated melody, the remaining monophonic guide melody analyses unsurprisingly behave similarly to the D re with B-mi version discussed above in that the fifth-spans ut-sol and re-la define local areas of tonal focus in analogous places in each version. Following the grouping of exact transpositions mentioned earlier in this chapter (see fig. 2.5), the modal types and tonalities represented by each guide melody version fall into three categories: D re with B-mi and G re exhibit both Re and Ut tonalities; the remaining re transpositions exhibit two different modal types within the Re tonality; and the ut guide melodies exhibit two different modal types within the Ut tonality (table 2.1). The guide melodies that represent the same tonality and characteristic melodic interval, but different final pitches, such as Ut(G): ut-sol and Ut(C): ut-sol, illustrate that Judd’s theory allows different final pitches to define her hexachordal tonalities as distinct modal types along with different corresponding characteristic melodic intervals. The presence of two modal types, alternation between them, and their layout creates modal ambiguity in the individual lines that continues in the realized canons and creates connections between them.
Table 2.1. Modal types represented by monophonic guide melody transpositions.

<table>
<thead>
<tr>
<th>Starting Pitch</th>
<th>Modal Type 1</th>
<th>Modal Type 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>D re (#)</td>
<td>Re(D): re-la</td>
<td>Ut(G): ut-sol</td>
</tr>
<tr>
<td>D re (b)</td>
<td>Re(D): re-la</td>
<td>Re(G): re-la</td>
</tr>
<tr>
<td>E re</td>
<td>Re(E): re-la</td>
<td>Re(A): re-la</td>
</tr>
<tr>
<td>E♭ ut</td>
<td>Ut(E♭): ut-sol</td>
<td>Ut(A♭): ut-sol</td>
</tr>
<tr>
<td>F ut</td>
<td>Ut(F): ut-sol</td>
<td>Ut(B♭): ut-sol</td>
</tr>
<tr>
<td>G ut</td>
<td>Ut(G): ut-sol</td>
<td>Ut(C): ut-sol</td>
</tr>
<tr>
<td>G ♭ re</td>
<td>Re(G): re-la</td>
<td>Ut(C): ut-sol</td>
</tr>
<tr>
<td>A re</td>
<td>Re(A): re-la</td>
<td>Re(D): re-la</td>
</tr>
<tr>
<td>B ♭ re</td>
<td>Re(B): re-la</td>
<td>Re(E): re-la</td>
</tr>
<tr>
<td>B♭ ut</td>
<td>Ut(B♭): ut-sol</td>
<td>Ut(E♭): ut-sol</td>
</tr>
<tr>
<td>C ut</td>
<td>Ut(C): ut-sol</td>
<td>Ut(F): ut-sol</td>
</tr>
</tbody>
</table>

When placed in a canonic context, individual melody lines experience two changes that affect the analysis of modal types and tonalities. First, as indicated by two signa congruentiae on the original notation facsimiles, the middle and upper voices end in different places on the guide melody. While ending one measure earlier does not change the pitch of the middle voice final, ending two measures earlier changes the pitch of the upper voice final to the one on which the melody begins. By altering the most important cadential pitch, this change nearly eliminates the presence of the second modal area and further strengthens the opening modal area in the upper voice. Second, octave leaps in the melody cause frequent voice crossings (ex. 2.10). Although seemingly inconsequential at first, considering their acceptance in Renaissance music, these voice crossings drastically alter the shape of the voice parts at times, which differentiate the canon transpositions from the guide melodies they contain. Rather than representing the collective modal orientations of each of their guide melodies, the canon transpositions favor only two modal orientations as a result of altered voice shapes.

25 Appendix B contains transcriptions of the Fuga trium vocum canon transpositions.

The interval and distance of imitation, modally-ambiguous guide melody construction, and changes to the melody in its canonic context contribute to canons that represent two modal areas. Placing the two-mode melodies at an ascending fourth apart causes them to share two modal areas; for example, the canon with D re with B-mi as the middle voice, A-D-G, represents Re(D): re-la and Ut(G): ut-sol in that two voices represent both of those modal types (table 2.2). As the following analysis of that particular canon reveals, these shared modal types and tonalities create
modal and tonal ambiguity on Judd’s simple counterpoint and sectional elaboration analytical levels while a slightly stronger focus on Ut(G): ut-sol in the second half of the canon leads to an overall working out of Ut(G): ut-sol on the highest analytical level.

Table 2.2. Modal types represented by A-D-G canon transposition voices.

<table>
<thead>
<tr>
<th>Starting Pitch</th>
<th>Modal Type 1</th>
<th>Modal Type 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A re</td>
<td>Re(A): re-la</td>
<td>Re(D): re-la</td>
</tr>
<tr>
<td>D re</td>
<td>Re(D): re-la</td>
<td>Ut(G): ut-sol</td>
</tr>
<tr>
<td>G ut</td>
<td>Ut(G): ut-sol</td>
<td>---</td>
</tr>
</tbody>
</table>

Note-against-note analysis of the A-D-G canon on the simple counterpoint level shows ambiguity between Ut(G): ut-sol and Re(D): re-la modal types. A process-based approach, similar to those developed by Fuller and Bain, reveals the modal ambiguity present on this analytical level. The opening phrase attempts to establish Ut(G): ut-sol with three statements of the characteristic melodic interval in the upper voice, but does not provide consistently strong vertical support for it (fig. 2.11). After the initial D sol re, the pitches E and A appear underneath the ut-sol fifth-spans rather than the expected D and G. Later, the middle phrase ends with a quasi-cadence involving Re(D): re-la in both the middle and lower voices after asserting Ut(G): ut-sol with two strongly supported fifth-spans (fig. 2.12). Although the canon ends representing Ut(G): ut-sol with a strong cadence, a descent from A la mi re to D sol re that recalls Re(D): re-la initially supports the final ut-sol fifth-span (fig. 2.13). On this analytical level, note-to-note connections show the presence of two modal types; throughout the canon, the re-la characteristic melodic interval of Re(D): re-la challenges the strong presence of Ut(G): ut-sol.
The A section of *Fuga trium vocum* exemplifies the alternation of modal type sectional elaboration. Although sectional elaboration typically refers to alternation of modal types within the same tonality, this canon shows an extreme example in that it alternates modal types of two different tonalities. Areas of *Ut(G): ut-sol* and *Re(D): re-la* modal types alternate until the quasicadence involving the *Re(D): re-la* modal type concludes the section (fig. 2.14). The pitch D facilitates shifts between the *Ut(G): ut-sol* and *Re(D): re-la* modal types by acting as a pivot from A-D to D-G. In her article, Judd mentions that specific, significant pitches can perform this
role.\textsuperscript{26} Although modally ambiguous and a solid example of modal type alternation, the A section slightly favors Re(D): re-la over Ut(G): ut-sol in that weak harmonic support destabilizes the first two ut-sol spans, the pivot function of the pitch D elevates its importance, and Re(D): re-la participates in the quasi-cadence.

Figure 2.14. Sectional elaboration alternation of modal types reduction of A-D-G canon, A section. Alternation of Re(D): re-la and Ut(G): ut-sol.

Figure 2.15. Focus on Ut(G): ut-sol, B section.

The second section of \textit{Fuga trium vocum} fits neither the alternation of modal type nor the repeated patterning sectional elaboration. While the A section features two modal types, the B section focuses solely on Ut(G): ut-sol (fig. 2.15). This section supplies the strong harmonic support for Ut(G): ut-sol lacking in the A section and never significantly returns to Re(D): re-la. Centered around Ut(G): ut-sol, the B section does not alternate that modal type with a different one of comparable prominence or exhibit a repeating pattern of modal types that corresponds with its text form. The strong position held by Ut(G): ut-sol in this section elevates its importance slightly over Re(D): re-la in this piece, which the highest analytical level confirms.

\textsuperscript{26} Ibid., 450.
Analysis of the A-D-G canon as a whole reveals a working out of the \textit{Ut}(G): \textit{ut-sol} modal type (fig. 2.16). The \textit{Re}(D): \textit{re-la} focus in the A section becomes the \textit{sol} of an \textit{ut-sol} descent that occurs during the B section, which emphasizes the remaining pitches in order until ending strongly on G \textit{sol re ut}. Although the piece represents \textit{Ut}(G): \textit{ut-sol} on this analytical level, the relatively strong presence of \textit{Re}(D): \textit{re-la} in the A section and the appearance of D as the middle voice final challenge its prominence. Overall, the A-D-G version of \textit{Fuga trium vocum} asserts the \textit{Ut}(G): \textit{ut-sol} modal type while acknowledging the importance of \textit{Re}(D): \textit{re-la} on more detailed levels of analysis.

Figure 2.16. Working out of \textit{Ut}(G): \textit{ut-sol} in the A-D-G canon transposition.

As expected, the remaining canon transpositions behave similarly to the A-D-G version discussed above. Each canon transposition represents the two modal types shared by its modally-ambiguous melody lines and slightly emphasizes the second modal type over the first (table 2.3). Based on the combination of three unique melody lines into five unique canons, the transpositions represent three pairings of modal types and tonalities: two modal types within the \textit{Re} tonality, one modal type from both \textit{Re} and \textit{Ut}, or two modal types within the \textit{Ut} tonality. Logically, the two canons with all \textit{re}-position lines represent only the \textit{Re} tonality, the two canons with both \textit{ut}- and \textit{re}-position lines represent those two tonalities, and the four canons with all or mostly \textit{ut}-position lines represent only the \textit{Ut} tonality. Whether within the same or different tonalities, each canon version shows ambiguity between two different modal types on all three of
Judd’s analytical levels.

Table 2.3. Modal types represented by the canon transpositions.

<table>
<thead>
<tr>
<th>Starting Pitches</th>
<th>Modal Type 1</th>
<th>Modal Type 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-G-C (B-mi)</td>
<td>Ut(G): ut-sol</td>
<td>Ut(C): ut-sol</td>
</tr>
<tr>
<td>D-G-C (B-fa)</td>
<td>Re(G): re-la</td>
<td>Ut(C): ut-sol</td>
</tr>
<tr>
<td>E-A-D</td>
<td>Re(A): re-la</td>
<td>Re(D): re-la</td>
</tr>
<tr>
<td>F-B♭-E♭</td>
<td>Ut(B♭): ut-sol</td>
<td>Ut(E♭): ut-sol</td>
</tr>
<tr>
<td>G-C-F</td>
<td>Ut(C): ut-sol</td>
<td>Ut(F): ut-sol</td>
</tr>
<tr>
<td>A-D-G</td>
<td>Re(D): re-la</td>
<td>Ut(G): ut-sol</td>
</tr>
<tr>
<td>B-E-A</td>
<td>Re(E): re-la</td>
<td>Re(A): re-la</td>
</tr>
<tr>
<td>C-F-B♭</td>
<td>Ut(F): ut-sol</td>
<td>Ut(B♭): ut-sol</td>
</tr>
</tbody>
</table>

Table 2.4. Modal types shared among canon transpositions.

<table>
<thead>
<tr>
<th></th>
<th>D-G-C (B-mi)</th>
<th>D-G-C (B-fa)</th>
<th>E-A-D</th>
<th>F-B♭-E♭</th>
<th>G-C-F</th>
<th>A-D-G</th>
<th>B-E-A</th>
<th>C-F-B♭</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re(D): re-la</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Re(E): re-la</td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Re(G): re-la</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Re(A): re-la</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ut(E♭): ut-sol</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Ut(F): ut-sol</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Ut(G): ut-sol</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ut(B♭): ut-sol</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Ut(C): ut-sol</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>

Like the modal types shared between individual lines when combined in canon, all canon transpositions have at least one modal type in common with another transposition (table 2.4). In this way, the transpositions of Fuga trium vocum preserve those specific harmonic relationships. More broadly, the melodic-contrapuntal framework of each modal type that appears in the transpositions features a meaningful fifth-span, either ut-sol or re-la, so the transpositions share that general connection as well. The presence of Glarean’s description, “so that the relationship of the harmony and the consonances would be observed no less,” signals that these shared elements contribute to Fuga trium vocum’s catholicon status.
“Songs composed so that they would be sung in many ways”: Different Arrangements of Modal Types and Tonalities

This description asserts that even though catholicon transpositions share significant elements, they also differ from one another. Although seemingly contradictory, these aspects of catholica are not mutually exclusive: while preserving structural fifth-span and modal type connections, the Fuga trium vocum transpositions comprise several intervallically unique versions. Beyond these note-to-note differences, the modal ambiguity inherent in Fuga trium vocum provides an opportunity for transpositions to differ on a broader level. Judd’s theory identifies these discrepancies as three different arrangements of her Ut and Re tonalities.

As discussed above, the two hexachordal starting position options for the guide melody transpositions, ut and re, provide three unique successions of melodic intervals that, when combined in canon according to the realization instructions, produce five distinct canon versions. Overall, these transpositions represent three different arrangements of Judd’s Ut and Re tonalities: two modal types within the Ut tonality, two modal types within the Re tonality, and one modal type from the Re tonality followed by one modal type from the Ut tonality (table 2.5). As expected, the three arrangements of tonalities in the guide melody transpositions correspond with the three unique successions of melodic intervals. Following the arrangements of tonalities in the guide melody transpositions, the two canons with all Re-tonality melodies represent the Re tonality twice, the two canons with both Ut- and Re-tonality melodies represent those two tonalities, and the four canons with all or mostly Ut-tonality melodies represent the Ut tonality twice. These results agree with other Fuga trium vocum analyses that the canon has three unique versions.

Essentially, highlighting the fifth-span both unifies and distinguishes the Fuga trium vocum transpositions. As the set of characteristic melodic intervals contains two different fifth-
spans, transpositions have the same generic structural interval while maintaining varying successions of melodic and harmonic intervals. In *Fuga trium vocum*, the transpositional connections and distinctions resulting from its modal ambiguity prove extraordinary; its *catholicon* quality simply comes from its fifth-based construction.

Table 2.5. Modal types and tonalities represented by guide melody and canon transpositions.

<table>
<thead>
<tr>
<th></th>
<th>Re: re-la, Re: re-la</th>
<th>Re: re-la, Ut: ut-sol</th>
<th>Ut: ut-sol, Ut: ut-sol</th>
</tr>
</thead>
<tbody>
<tr>
<td>D re (B-mi)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D re (B-fa)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E re</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E ♯ ut</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F ut</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G ut</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G re</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A re</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B re</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B ♯ ut</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C ut</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D-G-C (B-mi)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D-G-C (B-fa)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E-A-D</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-B ♯-E ♯</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G-C-F</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A-D-G</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B-E-A</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C-F-B ♯</td>
<td>x</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

“Among the foremost of such a kind”: Conclusion

Glarean’s definition of *catholica* presents several facets of the rare attribute. Specifically, he states that *catholica* transpositions remain diverse while preserving basic harmonic relationships and that they may be performed successfully from many, but not all, hexachordal starting positions. Glarean chooses to illustrate the concept with Ockeghem’s *Fuga trium vocum*, which he commends as “among the foremost of such a kind.” Glarean, *Dodecachordon*, vol. 2, bk. 3, trans. Miller, 277.
fulfills and exceeds Glarean’s *catholicon* criteria by simply highlighting the fifth as a structural interval. Providing the option of two distinct fifth-span characteristic melodic intervals, *ut-sol* and *re-la*, allows transpositions to utilize different successions of melodic intervals while maintaining focus on a common generic interval. The modal ambiguity in each transposition further strengthens both their connections and distinctions by offering the opportunity for shared modal types and different combinations of tonalities. *Fuga trium vocum*’s fifth-based construction also permits the use of any *ut*- and *re*-position starting pitches while rendering *mi*- and *fa*-position starting pitches impossible without fixed *musica ficta*. As *sol* and *la* starting positions prove identical to *ut* and *re*, respectively, focusing on fifth-spans allows *Fuga trium vocum* the maximum number of starting pitches. Beyond meeting and surpassing Glarean’s general definition of *catholica*, *Fuga trium vocum*’s performance challenges and history as a puzzle canon elevate its prestige. Ultimately, by presenting *Fuga trium vocum* as an example of *catholica*, Glarean achieves his goal of illustrating both the concept and Ockeghem’s skill in composition.
CHAPTER III. CONCLUSION

The main analysis, discussed in chapter 2, achieves the purpose of this study—to determine the musical elements that allow modal transposition in Ockeghem’s *Fuga trium vocum*—with Glarean’s discussion of the piece and *catholica* as a guide. Using Cristle Collins Judd’s reductive theory of modal types and *Ut, Re, Mi* tonalities, the previous chapter reveals that *Fuga trium vocum*’s emphasis on fifth-spans allows the piece to be transposed to different hexachordal positions. In this chapter, I summarize my *Fuga trium vocum* analysis, place it in a larger context with a brief analysis of a similar, but non-*catholicyon*, piece, and offer suggestions for further research of mode in canons and *catholica*.

Although originally presented as a puzzle canon with two similar solutions, Ockeghem’s *Fuga trium vocum* inherently supports modal transposition with its fifth-based construction. Highlighting the fifth-span allows any *ut*- or *re*-position starting pitches in that the set of characteristic melodic intervals contains both *ut*-sol and *re*-la; also, a simple application of fixed *musica ficta* changes unusable *mi*- and *fa*-position starting pitches to successful *re* and *ut*, respectively. With the aid of reductive graphs, Judd’s theory of modal types and *Ut, Re, Mi* tonalities reveals that the *Fuga trium vocum* transpositions preserve *ut*-sol and *re*-la structural fifth-spans on several analytical levels and represent two modal types each. This modal ambiguity adds an additional point of similarity, shared modal types, and difference, varying arrangements of tonalities, among the transpositions. Ultimately, the results of analyzing *Fuga trium vocum* with Judd’s theory suggest that emphasizing characteristic melodic intervals with more than one pair of hexachordal identities facilitates modal transposition ability.

Glarean’s comment “so that the relationships of the harmony and consonances are observed
no less” implies that modal transpositions typically do not preserve those relationships. To explore this implication and place my *Fuga trium vocum* analysis in a larger context, I briefly transcribe, transpose, and analyze a similar, but non-*catholicon*, piece, *Bel acueil le sergent d’amours* by Busnoys, with Judd’s theory. This analysis not only demonstrates the results of non-*catholicon* modal transposition, but also refines the identification of *Fuga trium vocum*’s *catholicon* element. After a brief description of transcribing and transposing *Bel acueil*, I analyze *re*, *ut*, and *mi* transpositions of the piece, beginning with the original notated version of D *re* with B-*fa*, and compare the results with those of *Fuga trium vocum* in chapter 2. This analysis reveals that *Bel acueil*’s emphasis on two characteristic melodic intervals, the fifth-span *re-la* and the third-span *re-fa*, prevents transposition by only allowing starting pitches in *re* hexachordal positions. As *Bel acueil* does not transpose even though the set of characteristic melodic intervals contains more than one third-span and fifth-span, the results of this analysis also suggest that *Fuga trium vocum*’s emphasis on only one characteristic melodic interval contributes significantly to its *catholicon* designation.

To facilitate comparison between *Bel acueil* and *Fuga trium vocum*, my transcription and transpositions of *Bel acueil* follow its manuscript notation in the Dijon Chansonnier, which originally included both pieces. The conventional clefs, accidentals, and mensuration markings of *Bel acueil*’s Dijon manuscript provide straightforward transcription into modern notation. The manuscript indicates the use of B-*fa* and an opening pitch of D *sol re* for each of *Bel acueil*’s three voices, which appear in folio layout. Its imitation at the unison results in a close texture and relatively small overall range of an octave and a fifth. A strong medial cadence divides *Bel acueil*’s twenty-four measures into two sections equal in length. Although melodically

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1 Dijon no. 18, Bibliotheque Municipale, MS 517, f. 22v-23.
2 Appendix B contains transcriptions of the *Bel acueil* transpositions referenced in this study.
3 Appendix A contains a reproduction of the Dijon manuscript facsimile referenced in this study.
contrasting, both sections open with the interval re-la and cadence with re-fa in the highest voice. Transposing the notated version to the remaining diatonic pitches and retaining its use of B-fa produces additional versions beginning on Emi, F ut, G re, A mi, B♭ ut, and C ut. As Bel acueil’s imitation at the unison omits the need to investigate each voice individually, I analyze each transposition solely in its complete polyphonic rendering.

The notated version of Bel acueil, D re with B-fa, strongly represents Judd’s Re(D): re-fa. Although re-la does not earn full modal type status in that it never participates in cadences, the fifth-span characteristic melodic interval also asserts a strong presence as it opens both sections and often reappears in all three voices. Judd’s note-against-note analytical level demonstrates the prominence of re-la in the opening phrase (fig. 3.1). Aided by imitation at the unison, the initial re-la ascent remains in focus until preparation for the A section re-fa cadence. Judd’s sectional level shows the opening emphasis on re-la followed by the establishment of Re(D): re-fa in both sections (fig. 3.2). As with the B section of Fuga trium vocum, these sections do not utilize either sectional elaboration device; rather, they represent one modal type throughout. Overall, Judd’s highest analytical level shows a working out of Re(D): re-fa (fig. 3.3). The graph reflects the opening re-la and slight emphasis on D la sol re in the A section before the structural re-fa descent in the B section.

Figure 3.1. Simple counterpoint note-against-note reduction of D re transposition, mm. 1-4. Prominence of re-la.
Figure 3.2. Section level reduction of D re transposition. Opening re-la spans and establishment of Re(D): re-fa.

Figure 3.3. Working out of Re(D): re-fa in D re transposition.

The other re-position version, G re, transposes successfully as it also emphasizes the characteristic melodic intervals re-la and re-fa (fig. 3.4). Although consistently representing Re(G): re-fa, the same modal type as D re with B-fa but reflecting its final of G, the G re version is not identical to the original in that they use different hexachords (fig. 3.5). The hexachord of the original version reflects fa super la—re-mi-fa-sol-la-fa—while the G re version uses the hexachord re-mi-fa-sol-la-mi. As this slight difference in half-step location does not produce any dissonance problems, Bel acueil successfully transposes to the similar, but distinct, G re starting pitch version.

Figure 3.4. Reductive graphs of G re transposition.  
Above, prominence of re-la in simple counterpoint note-against-note reduction, mm. 1-4. Middle, opening re-la spans and establishment of Re(G): re-fa in section level reduction. Below, working out of Re(G): re-fa.
Figure 3.4, continued.

Figure 3.5. Hexachords used in re transpositions.  
Above, D re hexachord. Below, G re hexachord.

The ut transpositions—F, B♭, and C—prove problematic in that they each pose several retroactive dissonance chains and do not cadence on characteristic melodic intervals. The F ut version shows a representative example of retroactive dissonance chains (ex. 3.1). First, the B♭ in the upper voice on the third beat of m. 16 requires an E♭ in the lower voice on the same beat. Then, moving backwards in musical time, that E♭ requires an A♭ on the second beat of the lower voice to avoid a tritone leap. Eight additional dissonances occur retroactively, affecting each voice until finally ending in m. 14 with a D♭ in the middle voice. While these problematic dissonance chains do not necessarily render the ut transpositions unsuccessful, their cadences on the third-span ut-mi does. As the C ut version illustrates, these transpositions begin with the
characteristic melodic interval ut-sol, but cadence on unrecognized ut-mi (fig. 3.6). For this reason, the ut transpositions do not represent any of Judd’s modal types.

Example 3.1. Retroactive tritone chain in F ut transposition, mm. 14-16.

Figure 3.6. Section level reduction of C ut transposition. Opening ut-sol span and attempted cadence on ut-mi.

Transposing Bel acueil to the mi-position pitches—E and A—also proves unsuccessful due to their dissonance problems and emphasis on non-characteristic melodic intervals. In addition to continual retroactive dissonance chains similar to those in the ut transpositions, the mi versions lack traditional cadential formulas. Instead of the conventional cadential pattern in the original version, which consists of a major sixth moving to a perfect octave by a descending major second and an ascending minor second created by musica ficta, the mi transpositions show a major sixth moving to a perfect octave by a descending minor second and an ascending major second (ex. 3.2). Attempting to correct the erroneous cadences with musica ficta either creates tritones or changes the hexachordal identity of the transposition. Unable to conform to cadential standards, the mi transpositions prove unsuccessful. Additionally, while the cadential re-fa
translates to the characteristic melodic interval *mi-sol* in these transpositions, the opening *re-la* fifth-span becomes the undesirable *mi-fa* in the E version and *mi-mi* in the A version (fig. 3.7). Although the cadences in the *mi* transpositions melodically suggest Judd’s *Mi: mi-sol*, the strong presence of unrecognized fifth-spans supports neither this nor any of Judd’s modal types.

**Example 3.2. Cadential patterns, mm. 23-24.**

![Figure 3.7. Opening fifth-span hexachordal syllables.](image)

Ultimately, *Bel acueil* only successfully transposes from its notated version of D *re* with B-*fa* to other *re*-position pitches. Starting *Bel acueil* on an *ut* or *mi* position results in retroactive dissonance chains and weak structural intervals. Additionally, the *mi* transpositions produce
unconventional cadences. Although re-fa and re-la transpose to ut- and mi-based characteristic melodic intervals individually, highlighting both intervals in one piece prevents transposition to other hexachordal positions in that one interval out of the pair always fails to transpose to a characteristic melodic interval. These results suggest refining the identification of Fuga trium vocum’s catholicon aspect from its emphasis on a characteristic melodic interval to its emphasis on only one characteristic melodic interval as both Fuga trium vocum and Bel acueil highlight transposing intervals, but only Fuga trium vocum transposes successfully.

Additional investigation of the ways in which guide melody construction and realization instructions either challenge or assist the establishment of modal orientation will expand the understanding of mode in canons. Analyzing other strongly imitative pieces with varying analytical approaches will determine which intervals and distances of imitation affect modal orientation. In Fuga trium vocum, its fifth-based guide melody construction and ascending fourth interval of imitation contribute to modal ambiguity while Bel acueil’s emphasis on two characteristic melodic intervals and imitation at the unison consistently provide support for only one modal orientation. Also, analyzing additional imitative pieces will demonstrate whether the strength of imitation affects modal orientation.

An exploration of whether emphasizing any intervals beyond the fifth enables modal transposition will contribute to the study of mode in catholica. Fuga trium vocum shows that focusing on one characteristic melodic interval that has several hexachordal identities allows modal transposition; therefore, as the set of characteristic melodic intervals contains several third-spans and fourth-spans, these intervals potentially allow modal transposition when emphasized solely in a piece. Further, Bel acueil’s failure to successfully transpose due to its focus on two intervals suggests an exploration of whether any combination of intervals allows
modal transposition. Finally, the understanding of mode in transposing pieces will benefit from further analyses of *catholica*. Discovering the musical elements that facilitate and hinder modal transposition, such as *Fuga trium vocum*’s emphasis on one characteristic melodic interval and *Bel acueil*’s emphasis on two characteristic melodic intervals, will help illuminate the ways in which similar pieces establish modal orientation.
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APPENDIX A. FACSIMILES

Ockeghem, *Prenez sur moi vostre exemple amoureux*

Source: Copenhagen no. 33, The Royal Library, MS Thott 291 8°, f. 39v.
Ockeghem, *Fuga trium vocum in Epidiatessaron, post perfectum tempus*

Busnoys, Bel acueil le sergent d’amours

Source: Dijon no. 18, Bibliotheque Municipale, MS 517, f. 22v-23.
Ockeghem, *Fuga trium vocum in Epidiatessaron, post perfectum tempus*, guide melody

APPENDIX B. TRANSCRIPTIONS

Ockeghem, *Fuga trium vocum in Epidiatessaron, post perfectum tempus*, canon

Glarean, *Dodecachordon* Vol. II Book 3 p. 277
Fuga trium vocum in Epidiatessaron,
post perfectum tempus

Ockeghem

Glarean,
Dodecachordon
Vol. II Book 3
p. 277
Fuga trium vocum in Epidiatessaron,
post perfectum tempus
Ockeghem
Glarean,
Dodecachordon
Vol. II Book 3
p. 277
Fuga trium vocum in Epidiatessaron,
post perfectum tempus

Ockeghem

Glarean,
Dodecachordon

Vol. II Book 3

p. 277
Fuga trium vocum in Epidiatessaron, post perfectum tempus

Ockeghem

Glarean, Dodecachordon Vol. II Book 3 p. 277
Fuga trium vocum in Epidiattessaron, post perfectum tempus
Ockeghem
Glarean, Dodecachordon Vol. II Book 3 p. 277
Busnoys, *Bel acueil le sargent d’amours*
Transcriptions from source: Dijon no. 18, Bibliothèque Municipale, MS 517, f. 22v-23.