RELIABILITY-BASED DESIGN OPTIMIZATION OF CORROSION MANAGEMENT STRATEGIES FOR REINFORCED CONCRETE STRUCTURES

A Dissertation

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ABSTRACT

Chloride induced corrosion is known as the dominant cause of premature damage in reinforced concrete (RC) bridges in the United States. However, the current corrosion management strategies do not suggest a suitable procedure for performance evaluation and optimum design/repair of RC bridges in corrosive environments. Corrosion affects the integrity of the RC structure by deteriorating the material properties and the bond at the steel-concrete interface. In this study, first, a simple probabilistic model of bond strength considering corrosion effect is developed using multivariable regression technique based on a comprehensive database collected from the literature. The predictions are found to be accurate and unbiased when compared with the experimental results. The proposed bond model is employed in the nonlinear finite element models of intact and corroded RC beams to investigate the importance of steel-concrete bond modeling on evaluating flexural behavior of the beams. Then, the minimum required development length for a given corrosion level is calculated and its sufficiency is investigated through a numerical analysis.

In the next step, an analytical procedure is proposed for predicting the nonlinear flexural behavior of intact and corroded RC beams with or without lap splices using the developed bond strength. The proposed analytical procedure can facilitate the performance evaluation and reliability assessment of the existing intact/corroded RC structures. The accuracy of the proposed procedure is verified through several experimental and numerical case studies. Furthermore, the proposed procedure is applied to predict the flexural behavior of intact and corroded T-beams of an RC bridge and the results are verified though the finite element analyses.

Next, a module based on a reliability-based multi-objective design optimization (RB-MODO) technique using a non-dominated sorting genetic algorithm II (NSGA-II) is developed for the optimum design of RC bridge beams considering corrosion effects. The procedure simultaneously maximizes the reliability of the structure and minimizes the material costs, given a design service life. Note that the analytical procedure developed in the previous section can be incorporated into the RB-MODO technique for optimum design of the structures based on both ultimate and serviceability performances. As an illustration, the developed RB-MODO technique is used for optimum flexural design of an interior T-beam of an illustrated RC bridge with and without considering corrosion effects subjected to various design constraints and service lives. Three types of materials are used in the design process: normal strength concrete with black steel rebars (NSC-BS), normal strength concrete with epoxy coated rebars (NSC-EC), and high performance concrete with black steel rebars (HPC-BS). Then, the optimum design strategy is selected among the considered materials based on the Pareto front results obtained from the proposed RB-MODO procedure.
Lastly, a reliability-based life-cycle-cost-analysis (LCCA) approach is developed to compare the long-term cost effectiveness of using six commonly-used groups of materials in design and repair of RC structures, including: NSC-BS, NSC-EC, NSC-SS (NSC with stainless steel (SS) rebars), HPC-SF-BS (high performance concrete (HPC) containing Silica Fume (SF) with BS rebars), HPC-SL-BS (HPC containing Slag (SL) with BS rebars), and HPC-FA-BS (HPC containing Fly Ash (FA) with BS rebars). A reliability-based design optimization (RBDO) technique is used for optimum initial design of the structure for each group of materials through minimizing the initial costs, given a target ultimate reliability index. Then, reliability analysis is conducted to evaluate the time-dependent serviceability and ultimate performances of the structure, and to predict the time-to repair and the number of repair operations. Lastly, LCCA is conducted to select the optimum corrosion management strategy in terms of selecting an optimum material for design and repair of the RC structures in corrosive areas.
DEDICATION

To my family
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CHAPTER I

INTRODUCTION

1.1 Background and Research Significance

Chloride induced corrosion is known as the dominant cause of premature damage in reinforced concrete (RC) structures, particularly in the highway bridges exposed to deicing salts. According to the Federal Highway Administration (FHWA) report (2006), 13.7% of the total rural and urban bridges in the US are structurally deficient mainly due to corrosion, which imposes an annual cost of more than $85 billion to the nation. In another study by National Association of Corrosion Engineers (NACE) (Koch et al. 2002), the annual direct cost of corrosion (e.g., repair and maintenance of RC bridge decks, maintenance painting of steel bridges) is estimated as $8.3 billion in the US with indirect costs (e.g., traffic delays, lost productivity) of as high as 10 times that of direct costs, while 25–30% of these costs could be saved if optimum corrosion management practices were used (Koch et al. 2002). To develop optimum corrosion management strategies, information on how corrosion affects the structural performance becomes essential (Bertolini et al. 2013).

1.1.1 Structural Performance Considering Corrosion

In RC structures, corrosion initiates at the interface of rebar and concrete. Once corrosion initiates, the diameter of the rebar reduces and there is a volumetric expansion around the rebar due to the corrosion products. The magnitude of this expansion depends on the type of corrosion product that can be up to six times the original volume of the rebar (Liu & Weyers 1998). These expansive corrosion products create tensile stresses in the surrounding concrete, leading to cracking and spalling of concrete cover, and reduction of the steel–concrete bond (Zhang et al. 2012). In addition, it has been recognized that corrosion can also affects the mechanical properties of the rebars through decreasing the yield strength and ultimate elongation (ductility) of the steel rebar (Cairns et al. 2005).

With such effects, corrosion could change the stiffness, structural load carrying capacity (Al-Sulaimani et al. 1990; Almusallam et al. 1996; Castel et al. 2000; Mangat & Elgarf 1999; Sajedi et al. 2011), and even
change the ductile failure mode of the structure that the design intends to achieve to the brittle failure that can increase the risk of catastrophic failure without any warning (Almusallam et al. 1996). Note that this particular failure mode is more likely to take place in structural members without sufficient development length in rebars. Meanwhile, the minimum development length strongly depends on the steel–concrete bond behavior; thus the deterioration in the bond behavior caused by corrosion could make the development length insufficient, which can cause brittle bond failure. One example of such failure is the collapse of Ynysy-Gwas Bridge in the UK as a result of corrosion in the prestressing tendons (Woodward & Williams 1988). Therefore, modeling of corrosion effect on both the mechanical properties and the bond behavior at the steel-concrete interface is necessary in the performance evaluation of the corroded RC structures.

The performance of the RC structures itself is typically evaluated through predicting its nonlinear load-deflection behavior under transverse loading. In particular, prediction of deflection is crucial in evaluating the serviceability, deformability, and drift capacity of the structure. Serviceability deals with the performance of the structure under the service loads, which is measured in terms of various factors, such as deflections, cracks, and vibrations of structures. Serviceability is a serious concern, particularly in the recent structures with slender RC members, which are built using high-strength concrete and steel (McCormac & Brown 2015). Deformability describes the structure’s ability to absorb energy through inelastic deformation under loading, and it is usually checked by the displacement ductility index, which is defined as the ratio of maximum displacement over first-yield displacement (Azizinamini et al. 1999). Drift capacity of a structural system is also important to structural stability, damage to nonstructural components, and architectural integrity. Drift control is a critical requirement for RC structures that are located in seismic or high wind areas as well. Therefore, accurately predicting the nonlinear load-deflection behavior of RC members considering the bond stress-slip behavior between rebar and concrete is essential in performance evaluation and therefore corrosion management of the RC structures.

1.1.2. Corrosion Management Strategies

Various strategies have been proposed in the literature for corrosion management of RC bridges located in corrosive environments. These strategies include durability design of the RC structures (Gjørv 2014), inspection (e.g., visual, nondestructive testing), protection (such as coating, cathodic protection,
chloride extraction), repair (such as patching, partial replacement), rehabilitation (such as strengthening with fiber-reinforced polymers, jacketing), and replacement of the corroded RC bridge (Christodoulu et al. 2009; Huang et al. 2015; Sohangpurwala 2011; Whitmore and Ball 2004). Among these strategies, decision on selecting an optimal corrosion management strategy is usually made based on the deteriorated structural performance and the associated lifecycle costs. The deteriorated structural performance itself depends on many parameters such as properties of concrete, geometrical characteristics, environmental conditions, etc. In practice, considerable amount of uncertainties is associated with each of these parameters, which can lead to a large uncertainty in the performance evaluation of the structure. To consider these uncertainties, time-dependent reliability-based framework needs to be utilized. In the literature, most of the reliability-based research works have focused on prediction of the time-to-repair of the structure and evaluating the time-dependent reliability of the corroded structure (e.g., (Eamon et al. 2012; Grace et al. 2012; Kim et al. 2013; Lounis & Daigle 2008; Stewart et al. 2004). Very few studies can be found in reliability-based optimum design of new RC structures considering corrosion effect.

For the existing structures, patch-repair is typically used for corrosion related repair. The patch-repair approach involves removing the chloride contaminated concrete beyond the reinforcing steel, cleaning up the corroded rebars, replacing the rebars if they are significantly corroded, and applying patch-repair. This technique is straightforward to apply; however, since patch-repaired RC structures will continue to need periodic repair operations during their service lives; this technique can be considered as a short-term cost-effective approach. In other words, depending on the number of repair operations, this technique can be costly over the long term. The number of repair operations itself depends on the type of the materials used for construction and repair of the structure. For new constructions, normal strength concrete (NSC) material with conventional black steel (BS) rebars are typically used due to their low initial costs. In recent constructions, however, some designers may prefer using alternative materials such as high performance concrete (HPC), epoxy coated (EC) rebars, or stainless steel (SS) rebars. Note that although the initial costs of these materials are relatively high, they can significantly reduce the number of repair operations required for the structure, which may reduce the long-term life-cycle costs of the structure. Therefore, comparing the reliability-based cost-effectiveness of using different materials in construction and repair of the RC structures are crucial.
In brief, comprehensive researches need to be conducted in performance evaluation and development of optimum corrosion management strategies for RC bridges, including: (1) the corrosion effects on the mechanical properties and bond between rebar and concrete needs to be modelled; (2) an analytical model should be developed for predicting the nonlinear load-deflection behavior of the corroded RC bridge beams under transverse loading; (3) optimum procedure considering both safety of the designed structure and the associated life-cycle costs should be developed for durability design of new RC structures in corrosive areas; and (4) the cost-effectiveness of different corrosion management strategies in terms of using different materials with different rebars in construction and repair of the corroded structure should be compared and the optimum strategy should be suggested.

1.2. Objectives and Structure of the Dissertation

The goal of this dissertation is to develop optimum corrosion management strategies for the RC structures using a reliability-based life-cycle-cost-analysis (LCCA) approach. Such approach can be used as a suitable tool for cost-effective design and/or repair of RC bridges located in corrosive areas. This dissertation is prepared in six chapters. In Chapter I, the introduction and research significance are elaborated. In Chapter II, a probabilistic model for prediction of bond strength between intact/corroded rebar and concrete is developed based on a comprehensive database collected from the literature. The proposed model can be used in evaluating the performance of the intact, corroded, and repaired RC structures, and calculating the minimum development length required for the RC structures in corrosive environments. In Chapter III, an analytical procedure for prediction of nonlinear load-deflection behavior of the RC beams with or without lap splices located in corrosive regions is proposed considering the bond stress-slip behavior. In particular, the probabilistic bond strength model developed in Chapter II is incorporated to the procedure. Since the available techniques in literature are either complicated (such as finite element modeling) or do not consider the effects of important parameters in the analysis (such as the effect on bond stress-slip behavior or corrosion effects), the analytical procedure developed in Chapter III can facilitate the performance evaluation and risk assessment of the RC structures affected by corrosion. In Chapter IV, a reliability-based multi-objective design optimization technique is developed for design of cost-effective and durable reinforced concrete structures considering corrosion effects. This technique adopts NSGA-II algorithm for simultaneously maximizing the reliability of the structure and minimizing
the material costs, given a design service life. Furthermore, it can adopt the analytical procedure developed in Chapter III for taking into account the corrosion effects on design of the RC structures considering both ultimate and serviceability performances. In Chapter V, a reliability-based life-cycle-cost-analysis (LCCA) module is developed to compare the cost-effectiveness of using six groups of materials including NSC-BS, NSC-EC, NSC-SS, HPC-SF-BS, HPC-SL-BS, and HPC-FA-BS in construction and repair of reinforced concrete structures. Based on the results in this chapter, the optimum corrosion management strategy in terms of selecting the optimum material for an illustrated T-beam of an RC bridge is proposed. Lastly, Chapter VI represents the summary and conclusions of this dissertation.

From this dissertation, one journal paper (Sajedi and Huang 2015a) is published in Engineering Structures (Elsevier) based on the materials in Chapter II, one journal paper (Sajedi and Huang 2016) is published in Journal of Bridge Engineering (ASCE) based on the materials in Chapter III, one journal paper (Sajedi et al. 2016) is published in ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, based on the materials in Chapter IV, four conference papers are published in ASCE Structures Congress 2015 (Sajedi and Huang 2015b), NACE Corrosion Risk Management Conference 2016 (Sajedi et al. 2016, Miran et al. 2016), and NACE Corrosion Conference 2017 (Sajedi et al. 2017) based on the materials in Chapter V, and one journal manuscript is written based on the materials in Chapter V. Two other journal papers are published by the author during his Ph.D., one (Kiani et al. 2017) in Construction and Building Materials (Elsevier) and another one (Gandomi et al. 2016) in Automation in Construction (Elsevier) about predicting the time-dependent deformations of concrete, which are related to the durability design of reinforced concrete structures.
2.1. Introduction

Chloride induced corrosion is one of the prevalent causes of deterioration in reinforced concrete (RC) structures particularly in marine environments. According to the FHWA report, 13.7% of the total rural and urban bridges in United States are structurally deficient mainly due to corrosion (FHWA 2006). It is also estimated by NACE (2002) that the annual direct cost of corrosion (e.g., repair and maintenance of RC bridge decks, maintenance painting of steel bridges) is $8.3 billion in the US. Furthermore, indirect costs, such as traffic delays and lost productivity, are estimated to be as high as 10 times that of direct costs, while 25 to 30% of these costs could be saved if optimum corrosion management practices were used (NACE 2002). To develop cost-effective corrosion management strategies, information on how corrosion affects the structural integrity and service life becomes essential (Bertolini et al. 2004).

In RC structures, corrosion initiates at the interface of rebar and concrete. Once corrosion initiates, the diameter of the rebar reduces and there is a volumetric expansion around the rebar due to the corrosion products. The magnitude of this expansion depends on the type of corrosion product that can be up to six times the original volume of the rebar (Liu and Weyers 1998). These expansive corrosion products create tensile stresses in the surrounding concrete, leading to cracking and spalling of concrete cover, and reduction of the steel-concrete bond (Zhang et al. 2012). It has also been recognized that corrosion affects the mechanical properties of the rebars such as decreasing the yield strength, ultimate strength, and ultimate elongation of steel (Cairns et al. 2005).

With such corrosion effects on the material properties and the bond behavior between concrete and rebar, corrosion could change the stiffness, structural load carrying capacity (Castel et al. 2000, Al-Sulaimani et al. 1990, Cabrera & Ghoddoussi 1992, Almusallam et al. 1996, Mangat & Elgarf 1999, Maaddawy et al. 2005a) and even change the ductile failure mode that the design intends to achieve, to the brittle failure that usually increase the risk of catastrophic failure without warning Almusallam et al.
(1996). Note that this particular failure mode is more likely to take place in structural members without sufficient development length in rebars. Meanwhile, the minimum development length strongly depends on the steel-concrete bond behavior; thus the deterioration in the bond behavior caused by corrosion could make the development length insufficient, which can cause brittle bond failure. Therefore, to evaluate structural performance of corroded structures, the modeling of corrosion effect on both the material properties and the bond behavior between concrete and rebar is crucial.

Considerable research on modeling corrosion effects has been focused on the changes of rebar diameter and yielding strength, and ultimate strain (ductility) of rebar. Zhang et al. (1995) developed a linear relationship between the corrosion level and the ratio of yield strength of corroded rebar to the nominal yield strength of intact rebar. Cairns et al. (2005) proposed linear equations to estimate the yield strength, ultimate strength, and ultimate strain of corroded rebars for different exposure conditions. Yang and Zhu (2012) used the nonlinear function to consider the effect of corrosion on yield strength in finite element model (FEM) analysis. Castel et al. (2000) evaluated the effect of steel cross-section and bond strength reduction on mechanical behavior of corroded rebars. They found that the ductility (ultimate strain) of rebars decreases exponentially with steel cross section reduction and stabilizes quickly around 25% of its initial ductility. To consider the corrosion effect on the diameter reduction of rebar, it is usually assumed that corrosion forms around the rebar uniformly and the ratio of corroded rebar cross sectional area to the intact rebar cross sectional area is proportional to the corrosion level (Yang & Zhu 2012).

Few studies have studied the deterioration effect on the bond behavior in RC structures (Huang et al. 2014), while some studies have modeled the corrosion effect on bond strength. Those studies can be categorized into two groups: one uses theoretical procedures and the other one uses regression analysis on the experimental data. The theoretical approaches are usually complicated and computationally intensive. For example, the corrosion effect modeling developed by Choi and Lee (2002) involves confining pressure around the rebar due to corrosion; however, to calculate this confining pressure, a finite element analysis is needed.

Compared with theoretical approaches, regression models, on the other hand, are straightforward and easily to be used by engineers. Some regression models developed previously are based on very limited experimental data (e.g. Lee et al. 2002, Stanish et al. 1999); thus, they cannot be directly applied
to other structure members. Many researchers predict corroded bond strength by multiplying an empirical reduction factor by the bond strength of intact rebar, and the reduction factor is usually evaluated based on a regression analysis using the experimental results of corroded specimens (Rodriguez et al. 1994, Chung et al. 2004, Bhargava et al. 2007). The main shortcoming of such models is that the intact bond strength needs to be estimated first, where the model error in estimating the intact bond strength should also be considered.

In this chapter, a probabilistic model is developed to predict bond strength as a function of corrosion level using nonlinear regression analysis. Instead of modeling the reduction factor, this model predicts intact and corroded bond strength directly. A comprehensive database is collected from literature and is used for the model development so that the proposed model is valid for a wide range of structural properties. In the proposed model, the predictors selected from literature consist of parameters that influence bond strength (such as concrete compressive strength, stirrups, development length, etc.). Then the model selection procedure is applied to select the predictors that statistically contribute to the model prediction. The proposed probabilistic model considers statistical uncertainties and model error. Then, we adopt the proposed bond strength model to a FE beam model to investigate the importance of steel-concrete bond behavior on evaluating flexural behavior of corroded beams. Furthermore, to ensure ductile failure, we suggest minimum development length for intact and corroded rebars. Lastly, the calculated minimum development length is compared with the one based on ACI 318-11 design code (2011) through studying flexural behavior of intact and corroded FE beams.

2.2. Probabilistic Bond Strength Model

2.2.1. Bond Behavior

To describe bond between rebar and concrete, the relationship between bond stress and slip at the steel-concrete interface is usually modeled. One of the most widely accepted models is the MC90 bond-slip model suggested by CEB-FIP (1990) and it is defined by the following equation:

$$\tau = \begin{cases} 
\tau_{\text{max}} \left( \frac{s}{s_1} \right)^\alpha & \text{for } 0 \leq s \leq s_1 \\
\tau_{\text{max}} & \text{for } s_1 \leq s \leq s_2 \\
\tau_{\text{max}} - (\tau_{\text{max}} - \tau_f) \left( \frac{s-s_2}{s_3-s_2} \right) & \text{for } s_2 \leq s \leq s_3 \\
\tau_f & \text{for } s_3 \leq s 
\end{cases}$$

(2.1)
where $\tau$ is the bond stress, $\tau_{\text{max}}$ is the maximum bond stress, $s$ is the relative slip between rebar and concrete, $\tau_f$ is the frictional bond stress, and model parameters ($s_1$, $s_2$, $s_3$, $\alpha$) are defined based on the confinement of concrete with good or other bond conditions (CEB-FIP 1990). This model is to predict the bond stress as a function of relative displacement under monotonic loading. However, this relationship cannot be applied to describe bond behavior deteriorated by corrosion. In this study, we focus on the corrosion effect on $\tau_{\text{max}}$ by developing a probabilistic bond strength model using experimental data.

The experimental bond strength data reported in the literature is usually calculated based on the applied force on the rebar divided by the nominal area around the rebar, given an embedment length. Therefore, this bond strength is different from $\tau_{\text{max}}$ in Eq. (2.1), and refers to the average bond strength ($\tau_{\text{avg}}$) by assuming the uniform stress distribution along the rebar. To relate $\tau_{\text{avg}}$ to $\tau_{\text{max}}$, the distribution of the bond stress needs to be estimated. It has been agreed upon that the distribution of bond stress can be considered as uniform only for the cases with short embedment lengths; while in cases with long embedment lengths, the distribution is nonlinear and rather complex (Mains 1951, Perry & Thompson 1966, Somayaji & Shah 1981, Jiang et al. 1984, Kankam 1997). Despite the complexity, some researchers have proposed some simple functions to describe the distribution of bond stress along rebar. For example, Jiang et al. (1984) suggested using a parabolic function for bond stress distribution based on their experimental results which also agrees with the experimental results obtained from Perry and Thompson (1996) (as shown in Figure 2.1). Using a parabolic function, the following relationship can be obtained between $\tau_{\text{max}}$ and $\tau_{\text{avg}}$:

$$\tau_{\text{max}} = 1.5\tau_{\text{avg}}$$

(2.2)

Figure 2.1. Bond stress distribution along the rebar for eccentric pull-out tests under various loading levels (Perry and Thompson 1966)
2.2.2. Formulation of Average Bond Strength

As the experimental bond strength refers to average bond strength, we develop a probabilistic model to predict $\tau_{\text{avg}}$, and then $\tau_{\text{max}}$ can be estimated using Eq. (2.2). The formulation of the probabilistic average bond strength model uses a multivariable regression model as follows:

$$y(x, \Theta) = \sum_{i=0}^{p} \theta_i h_i(\tilde{\theta}, x) + \sigma \varepsilon$$  \hfill (2.3)

where $y(x, \Theta)$ is the response (in this study, it is the predicted average bond strength ($\tau_{\text{avg}}$) or a suitable transformation of it); $h_i(\tilde{\theta}, x)$ is the $i^{th}$ explanatory functions; $x$ is the vector of the independent selected variables; $\tilde{\theta}$ is a vector of unknown parameters to construct the explanatory functions; $\theta_i$ is unknown model coefficient; $\Theta = (\theta, \tilde{\theta}, \sigma)$ is a vector of unknown model parameters and $\theta = (\theta_0, \theta_1, \ldots, \theta_p)$; $p$ is the number of explanatory functions; $\sigma \varepsilon$ is the model error; $\sigma$ is the standard deviation and is assumed to be constant and independent of $x$ (homoscedasticity assumption); and $\varepsilon$ is the standard normal random variable (normality assumption). A variance stabilizing transformation of the response quantities of interest can be used to satisfy both assumptions (Box & Cox 1964) and diagnostic plots can be used to check the suitability of the transformation (Sheather 2008).

In this study, we let $y = \ln(\tau_{\text{avg}} / \sqrt{f'_c})$ where $f'_c$ (MPa) is the concrete compressive strength at 28 days.

To construct the explanatory functions, we select the ones that have been used in the literature and have been shown good correlation with the bond strength. Additionally, in order to ensure the practical use of the proposed model, we avoid selecting the explanatory functions involving the variables that cannot be estimated practically (e.g., confined pressure around rebar, the volume of the rust). Note that since the potential explanatory functions selected here are the ones that have been shown good correlation with the bond strength, the multivariate regression formulation (shown in Eq. (2.3)) is adopted in order to directly use those functions as predictors. Moreover, a multivariate regression model can provide the confidence interval of the prediction that reflects the accuracy of the estimation, and the statistics of the model parameters can provide insights on how each variable in the model influences the prediction (Hunag et al. 2011).
First, we let $h_0 = 1$ to capture the constant bias of the model. Then $h_1$ and $h_2$ are constructed based on the predictors used in Wang (2009) for bond strength prediction considering splitting mode of failure and they are shown as follows:

$$h_1 = \frac{C_c \mu + R_r}{d_{b0} 1 - \mu R_r} \gamma \tag{2.4}$$

$$h_2 = \frac{b_e \mu + R_r}{d_{b0} 1 - \mu R_r} \gamma \tag{2.5}$$

where $C_c$ is the concrete clear cover, $d_{b0}$ is the diameter of intact rebar, $\mu$ is the friction coefficient, $R_r$ is the relative rib area of intact rebar, $b_e$ is the effective beam width from center to center of the reinforcing bar spacing or from the edge of the beam to the center of the reinforcing bar spacing, whichever is smaller, and should be between $3C_c$ and $9C_c$. $\gamma \leq 1$ is the reduction factor for considering the effect of the development/splice length of rebar, and can be calculated using $\gamma = (8 \cdot d_{b0} / l_d)^{0.5}$ where $l_d$ is the development length of the rebar.

To capture the effect of the development length of rebar, we let $h_3 = d_{b0} / l_d$ suggested by Orangun et al. (1977). As the stirrups provide confinement to the concrete around rebar, the effect of stirrups should be considered. Following Kemp and Wilhem (1979), we let $h_4 = (1 / \sqrt{f_{y_s}}) (A_t / f_{y_s}) / (s \cdot d_{b0})$ in which $A_t$ is the cross sectional area of stirrup, $f_{y_s}$ is the yield strength of stirrup, and $s$ is the spacing between stirrups.

To capture the deterioration on the bond due to corrosion, the mechanism of corrosion process and its effect on the bond strength should be recognized. Based on some experiments (Al-Sulaimani et al. 1990, Almusallam et al. 1996), $\tau_{\text{avg}}$ increases at initial corrosion levels before cracking of concrete cover and it has been referred as promoting the internal confining pressure and increasing the roughness of reinforcing rebar. Some test results show that the increase in bond strength has been observed for the corrosion levels of 0-4% (Almusallam et al. 1996), while other test results show that this increase is only found for the corrosion levels less than 1% (Al-Sulaimani et al. 1990). Some experimental findings, however, do not confirm this phenomenon (Stanish et al. 1999, Rodriguez et al. 1994). Additionally, high scatter in bond strength test results of corroded rebars have been observed in the literature. In summary, the inconsistent/inconclusive observations for the initial stage of corrosion reported in the literature can be either due to the variations in the
material and geometrical properties of the experiment specimens, and/or the measurement errors. As the corrosion level increases, volumetric expansion of corrosion products lead to the cracking of the concrete cover, the reduction of the confining pressure around the rebar, and thus a rapid degradation of the bond strength between rebar and concrete. As the corrosion level continues increasing, the corrosion products fill the cracks and increase the confining pressure around the rebar, leading to a less degradation rate in bond strength (Hanjari et al. 2011). Therefore, to capture the corrosion effect on bond, the increase in \( \tau_{\text{avg}} \) at initial corrosion stage is ignored, and an exponential function is adopted following the suggestions by other researchers (Lee et al. 2002, Chung et al. 2004, Bhargava et al. 2007). This exponential function is used as a reduction factor and is multiplied to \( h_1, h_2, \) and \( h_3 \). In other words, we have \( h_5 = \exp(\theta_1 Q) \cdot h_1, h_6 = \exp(\theta_2 Q) \cdot h_2, \) and \( h_7 = \exp(\theta_3 Q) \cdot h_3, \) where \( Q \) is the corrosion level (%) based on the rebar mass loss. This reduction factor has not been applied to \( h_4 \), since the contribution of the stirrups on the bond strength is independent of the corrosion level on the longitudinal rebar (Rodriguez et al. 1994).

2.2.3. Experimental Data and Model Selection

The experimental data used for regression model are selected from the literature, and they are obtained from a total of 240 specimens. Specifically, 71% of these specimens are intact and 29% of them are corroded (with corrosion levels less than 20%); while 75% of the total specimens are without stirrups and 25% are confined with stirrups. During the selection, if the reported coefficient of variation of the bond strength for a specimen is larger than 20% (which indicates the test results are not reliable), we disregard that data as well. Table 2.1 shows the properties of the selected specimens.

<table>
<thead>
<tr>
<th>Reference</th>
<th>No. of spec.</th>
<th>( C_c ) (mm)</th>
<th>( b_e ) (mm)</th>
<th>( d_{50} ) (mm)</th>
<th>( R_r )</th>
<th>( f'c ) (Mpa)</th>
<th>( l_d ) (mm)</th>
<th>( Q ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darwin &amp; Graham (1993)</td>
<td>110</td>
<td>49.2−82.6</td>
<td>228.6−247.7</td>
<td>25.4</td>
<td>0.05−0.2</td>
<td>31.2−41.3</td>
<td>215.9, 304.4</td>
<td>0</td>
</tr>
<tr>
<td>Bilal (1995)</td>
<td>56</td>
<td>25.4</td>
<td>228.6</td>
<td>20.6</td>
<td>0.08−0.2</td>
<td>22.4, 43.1</td>
<td>254</td>
<td>0</td>
</tr>
<tr>
<td>Alsulaimani et al. (1990)</td>
<td>22</td>
<td>29</td>
<td>150</td>
<td>12</td>
<td>0.1&quot;</td>
<td>40</td>
<td>144, 300</td>
<td>0−4.5</td>
</tr>
<tr>
<td>Cabrera &amp; Ghoddoussi (1992)</td>
<td>11</td>
<td>25</td>
<td>75</td>
<td>12</td>
<td>0.1&quot;</td>
<td>47.9−64</td>
<td>190</td>
<td>0−7.8</td>
</tr>
<tr>
<td>Rodriguez et al. (1994)</td>
<td>12</td>
<td>15−40</td>
<td>150</td>
<td>16</td>
<td>0.12</td>
<td>53.6−86.2</td>
<td>130, 208</td>
<td>0−14.2</td>
</tr>
<tr>
<td>Almusallam et al. (1996)</td>
<td>14</td>
<td>63.5</td>
<td>190.5</td>
<td>12</td>
<td>0.1&quot;</td>
<td>30</td>
<td>102</td>
<td>0−15.65</td>
</tr>
<tr>
<td>Stanish et al. (1999)</td>
<td>8</td>
<td>20</td>
<td>112.5</td>
<td>11.3</td>
<td>0.1&quot;</td>
<td>36.4, 42.6</td>
<td>250</td>
<td>0−14.4</td>
</tr>
<tr>
<td>Mangat &amp; Elgarf (1999)</td>
<td>7</td>
<td>19</td>
<td>57</td>
<td>10</td>
<td>0.1&quot;</td>
<td>45</td>
<td>100</td>
<td>0−5</td>
</tr>
</tbody>
</table>

* the value is assumed as it is not reported in the corresponding reference
Ideally, the experimental data for each test should include the information on \( y \) (in this study, normalized average bond strength) and \( x \) (e.g., bar diameter, relative rib area) in the regression model (as shown in Eq. (2.3)) so that the model parameters \( \Theta \) can be assessed. As there are a large number of tests conducted on intact specimens, we select the ones that contain the complete information about \( y \) and \( x \). For some of corroded specimens, there are missing values for relative rib area \((R_r)\). We adopt the value of 0.1 (Wang 2004). Note that the friction coefficient between deformed rebar and concrete, \( \mu \), is not reported for the selected test results, and we set it to be 0.45 for all the specimens as suggested by Choi and Lee (2002). It is recognized that the corrosion has effects on \( x \) (such as \( d_{b0}, R_r \), and \( \mu \)); however, to maintain the simplicity of the formulation of the probabilistic model, we lump the effect of corrosion on \( x \) to the reduction factors in the explanatory functions \( h_5-h_7 \).

As described earlier, all the selected specimens can be categorized as specimens with stirrups and ones without stirrups, or can be categorized as specimens with corrosion or without corrosion. To capture the influence the presence of stirrups and corrosion, additional to the potential explanatory functions constructed early, \( h_0-h_7 \), two dummy variables are considered in the regression model. These two dummy variables are:

\[
d = \begin{cases} 
0 & \text{for specimens w/o stirrups} \\
1 & \text{for specimens w/ stirrups}
\end{cases} \quad (2.6)
\]

\[
dQ = \begin{cases} 
0 & \text{for intact specimens} \\
1 & \text{for corroded specimens}
\end{cases} \quad (2.7)
\]

To consider these two dummy variables in the regression model, the interaction terms of the dummy variables and \( h_0-h_7 \) are added as additional explanatory functions as \( h_8-h_{16} \). With all of the potential explanatory functions \((h_0-h_{16})\) (i.e., the model is called full model), a model selection process can be conducted to choose the ones that contribute statistically significantly to the model prediction. However, before the model selection, the existence of multicollinearity in the full model needs to be checked. Multicollinearity occurs when the correlations among the predictors in the multivariable regression analysis are strong. The presence of multicollinearity can lead to inaccurate estimation of model parameters; thus, it
needs to be addressed by removing the “redundant” predictors from the model before the model selection process can be conducted.

In this study, variance inflation factor (VIF) is used to detect the existence of multicollinearity and it measures how much the variance of the estimate of the model parameter is inflated by the multicollinearity. The VIF for the explanatory function \( h_i \) can be defined as:

\[
VIF_i = \frac{1}{1 - R_i^2}
\]

where \( R_i^2 \) is the coefficient of determination of \( h_i \) on the other explanatory functions. A VIF value over 5 indicates a multicollinearity problem (Sheather 2008). In other words, the explanatory functions with high VIF values have high correlation with other explanatory functions. Therefore, those explanatory functions are considered to be redundant and are removed from the full model. After removing the redundant explanatory functions, a model selection procedure is conducted on the full model.

Model selection deals with the trade-off between the goodness of fit and the complexity of the model. In this study, the all possible subsets model selection technique is applied. As mentioned, the model size for the full model is 8; thus, the possible model sizes for reduced models can vary from 1 to 7. For each model size, all possible combinations of explanatory functions are found first. Then we use Akaike’s information criterion (AIC) (Akaike 1974) to find the best model for each model size. Then among the list of the best models, the final model is selected based on the compromise between the prediction accuracy measured by standard deviation of the residuals and the model complexity measured by the number of explanatory functions (i.e. using more than a specific number of the explanatory functions has negligible effect on the prediction accuracy). Note that, when any of \( h_5-h_7 \) is in the reduced model, non-linear regression analysis is needed to evaluate the unknown parameters. As a result of the model selection, the final formulation of the proposed probabilistic model is obtained as follows:

\[
\ln \left( \frac{r_{\text{avg}}}{\sqrt{f_c}} \right) = \theta_0 + \theta_1 \cdot \exp(\theta_2 \cdot \mu + \theta_3 \cdot \exp(\theta_4 \cdot Q)) + \frac{\theta_5}{\sqrt{f_c}} + \frac{\theta_6}{\sqrt{f_c}} \cdot \frac{\theta_7}{\sqrt{f_c}} + \theta_8 \cdot \frac{1}{\sqrt{f_c}} + \sigma_e
\]  

(2.9)
Table 2.2 shows the statistics of the model parameters used in Eq. (2.9) based on SI units. It should be noted that the normality of the error distribution assumption in the model is verified through the normal Q-Q plot and the homoscedasticity of the error assumption is checked through the residuals and the roots of the standardized residuals plots versus the fitted values.

Table 2.2. Statistics of model parameters in Eq. (2.9)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>$\theta_0$</th>
<th>$\theta_1$</th>
<th>$\bar{\theta}$</th>
<th>$\theta_2$</th>
<th>$\bar{\theta}_2$</th>
<th>$\theta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>-0.897</td>
<td>0.046</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>0.476</td>
<td>0.032</td>
<td>0.03</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{\theta}_1$</td>
<td>-0.077</td>
<td>0.046</td>
<td>0.01</td>
<td>0.22</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>0.119</td>
<td>0.009</td>
<td>0.80</td>
<td>0.52</td>
<td>0.13</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{\theta}_2$</td>
<td>-0.148</td>
<td>0.148</td>
<td>0.21</td>
<td>0.00</td>
<td>0.88</td>
<td>0.15</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>0.024</td>
<td>0.370</td>
<td>0.50</td>
<td>0.08</td>
<td>0.20</td>
<td>0.36</td>
<td>0.12</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.169</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 2.2 shows the prediction of $\ln(\tau_{AVG} / \sqrt{f_c})$ obtained from our proposed probabilistic model vs. the experimental data. For the ideal prediction, all the data points will fall on the 45-degree solid line (i.e. equality line) as shown in the solid line in Figure 2.2. The dashed lines show the prediction intervals (upper- and lower-bounds), which are ± 1 standard deviation of the model error of the proposed probabilistic model additional to the mean prediction. Overall, the data points in Figure 2.2 are evenly scattered around the solid line with most of the points inside of the dashed lines, indicating the unbiased prediction.

![Figure 2.2. Prediction of proposed probabilistic model $\ln(\tau_{AVG} / \sqrt{f_c})$, vs. experimental data](image-url)
Furthermore, Figure 2.3 show the comparison of the normalized average bond strength ($\tau_{avg}$) values are predicted based on the proposed probabilistic model and the ones obtained from literature (al-sulaimani et al. 1990, Cabrera & Ghoddoussi 1992) at various corrosion levels. In Figure 2.3, the mean prediction (solid lines) is obtained by using the proposed model shown in Eq. (2.9) without considering the model error, while the prediction bounds (dashed lines) are obtained through Eq. (2.9) using the model error. It can be seen that nearly all experimental data lie within the prediction bounds, indicating that the proposed probabilistic model can predict the bond strength of corroded specimens with sufficient accuracy.

To investigate the sensitivity of the input parameters to the bond strength prediction, a parametric study is performed here. The input parameters are $Q$, $C_c/d_{b0}$, $R_r$, $d_{b0}/l_d$, $b_e/d_{b0}$, $f'_c$, and $(A_{tr}f_{yt})/(s\cdot d_{po})$ where $C_c/d_{b0}$ and $b_e/d_{b0}$ are the normalized concrete cover and effective beam width with respect to intact rebar diameter, respectively, and $(A_{tr}f_{yt})/(s\cdot d_{po})$ is the transverse reinforcement ratio. The parametric analysis is conducted by varying one of the input parameters and using mean values (that can be obtained from Table 2.1) of the other input parameters. The results of this sensitivity study are shown in Figure 2.4, where the $x$-axis is the normalized parameter by subtracting and dividing the mean value. As expected, $\hat{\tau}_{avg}$ increases with increasing $C_c/d_{b0}$, $R_r$, $d_{b0}/l_d$, $b_e/d_{b0}$, $f'_c$, and $(A_{tr}f_{yt})/(s\cdot d_{po})$, and decreases with increasing $Q$, which are consistent with the findings in the literature (Chinn et al. 1955, Rodriguez et al. 1994, Orangun et al. 1977, Kemp & Wilhelm 1979, Cairns & Jones 1995, Canbay & Frosch 2005) [22, 36, 37, and 43-45]. Additionally, it can found that the predicted $\hat{\tau}_{avg}$ is highly sensitive to the values of $f'_c$ and $Q$, and less sensitive to the values of $(A_{tr}f_{yt})/(s\cdot d_{po})$ and $R_r$. 


Figure 2.3. Comparison of the normalized average bond strength ($\tau_{avg}/\sqrt{f_c}$) values predicted based on the proposed probabilistic model and the ones obtained from literature.
2.3. Numerical Study

As described earlier, corrosion can degrade the cross sectional area and yield strength of reinforcing bar, and deteriorate the bond at the steel-concrete interface. As the deterioration on the bond behavior directly affect the minimum required development length, in this section, we use numerical study to investigate the effect of bond strength deteriorated by corrosion on the flexural performance of RC beams. More specifically, a FE beam model is constructed incorporating our developed bond strength model.

First, the modeling details in ANSYS is described, and the numerical models are verified with the experimental results. Then, the effect of steel-concrete bond on the flexural behavior of corroded RC beams is investigated. Next, to assure the ductile behavior in the corroded beams, a calculation on estimating the minimum development length of rebars is described. Lastly, the calculated minimum development length is compared with the one based on the ACI 318-11 design code (2011) using FEMs.

2.3.1. Finite Element Modeling

The commercial nonlinear finite element program, ANSYS v. 14.0 (2011), is employed to simulate the 3-D modeling of intact and corroded RC beams. An eight-node 3-D brick element, SOLID65, with the capabilities of cracking and crushing is used to simulate the concrete element. The compressive uniaxial stress-strain relationship for concrete is defined by multilinear isotropic option (MISO). MISO uses the...
Von-Mises failure criterion along with the Willam and Warnke model (Willam & Warnke 1975) to define the failure of concrete. The multilinear stress-strain relationship proposed by MacGregor (1992) is used to estimate the uniaxial concrete compressive stress-strain curve as follows:

\[
\sigma = \frac{E_c \varepsilon}{1 + (\varepsilon/\varepsilon_0)^2}
\]

(2.10)

where \(\sigma\) is stress at a given strain \(\varepsilon\), \(E_c\) is the elastic modulus of concrete and can be calculated by \(E_c = 4750\sqrt{f'_c}\) (MPa) (ACI 318-11 2011), and \(\varepsilon_0\) is strain at ultimate compressive strength \(f'_c\) and can be calculated using \(\varepsilon_0 = 2 f'_c/E_c\). In the stress-strain curve, the initial part at the stress ranging from 0 to 0.3\(f'_c\) is assumed to be linear. The Poisson ratio of concrete is assumed to be 0.2.

Other parameters needed for defining concrete materials in ANSYS are described following. The modulus of rupture of concrete \((f_r)\) is calculated using the equation proposed by Carrasquillo et al. (1981) and it is \(f_r = 0.56\sqrt{f'_c}\) (MPa). The crack interface shear coefficients for open cracks \((\beta_o)\) and closed cracks \((\beta_c)\) are set to 0.3 and 0.9, respectively. The uniaxial crushing capability of the concrete element is disabled to avoid the convergence problems (2001). Moreover, the tensile stress relaxation after the cracking of concrete is activated to avoid the sudden drop of tensile stress to ensure a converged solution.

The longitudinal rebars and stirrups are modeled using Link8 3-D spar element with the capability of plastic deformation under tension and compression. The tensile and compressive stress–strain relationship for reinforcing bars is modeled using a bilinear function, where the initial modulus, \(E_s\), is assumed as \(2\times10^5\) MPa and the plastic modulus is calculated using \(E_{sp} = 0.01E_s\) (Et-Tawil et al. 2001). The effects of corrosion on cross sectional area and yield strength of reinforcing rebars are taken into account as follows (Yang and Zhu 2012):

\[
A_b = (1 - 0.01Q)A_{b0}
\]

(2.11)

\[
f_y = \frac{1 - 0.01077Q}{1 - 0.01Q}f_{yo}
\]

(2.12)

where \(A_b\) is the cross sectional area of rebar, \(A_{b0}\) is the cross sectional area of intact rebar, \(f_y\) is the yield strength of rebar, and \(f_{yo}\) is the yield strength of intact rebar. To take into account the effect of corrosion on
ductility of reinforcing rebar, the following equation is used to calculate the ultimate strain of rebar, $\varepsilon_{su}$, based on the experimental results reported by Castel et al. (2000a, b) and formulated by Zhu and François (2013):

$$\varepsilon_{su} = \exp(-0.1Q)\varepsilon_{\varepsilon0}$$  \hspace{1cm} (2.13)

where $\varepsilon_{\varepsilon0}$ is the ultimate strain of intact rebar. Note that as reported by Castel et al. (2000a, b), $\exp(-0.1Q)$ should not be less than 0.25. For compressive and transverse rebars, perfect bonding between steel and concrete is assumed. For the tensile rebars, the bond-slip behavior between rebar and concrete is simulated using nonlinear spring elements (combin39) in three directions (two transverse directions and one longitudinal direction). It is assumed that the rebar can only slip along its longitudinal direction, thus, high stiffness values are used for the spring elements in the two transverse directions. The spring in the longitudinal direction is characterized by nonlinear stress-slip curve according to Eq. (2.1). Figure 2.5 shows the stress-slip curve.

![Figure 2.5. Bond stress-slip curve for unconfined concretes based on CEB-FIP (1990)](image)

It should be emphasized again that $\tau_{\text{max}}$ in Eq. (2.1) is calculated using Eqs. (2.2) and (2.9). The values of $s_1$, $s_2$, $s_3$, and $\alpha$ for unconfined concrete with good bond conditions are used, representing the splitting bond failure of concrete. Note that the bond force is needed for defining the spring element and it can be calculated by multiplying the bond stress to the tributary area (that is the multiplication of circumference of the rebar and the distance between two adjacent spring elements).

2.3.2. Model Verification

To verify the FE modeling described above, one intact RC beam and four corroded RC beams tested by Maaddawy et al. (2005a) are compared with the corresponding FEMs constructed in ANSYS. All five
beams were simply supported and had the same geometry configuration (as shown in Figure 2.6) using the same materials. The average 28 day compressive strength of concrete was 40 MPa. The yield and ultimate strengths of tensile rebars were 450 MPa and 585 MPa, respectively, while the yield and ultimate strengths of compressive rebars and stirrups were 340 MPa and 500 MPa, respectively. The four corroded beams were exposed to the accelerated corrosion test in the middle 1400 mm (salted zone) of the tensile rebars (as shown in Figure 2.6) for 50, 110, 210, and 310 days, respectively. As results, the percentage mass losses, $Q$, due to corrosion were reported as 8.9%, 14.2%, 22.2%, and 31.6% for these four corroded beams, respectively. Then, four point loading tests were conducted on these five beams and the force-displacement curves were obtained to compare the flexural behavior of the beams. The loading points are shown in Figure 2.6.

![Figure 2.6. Dimensions and reinforcement layout of test specimens (Maaddawy et al. 2005a)](image)

Corresponding to the five beams tested by Maaddawy et al. (2005a) described above, five FEMs are constructed in ANSYS. In order to take the advantages of geometrical and loading symmetry and to reduce the computational time, only the quarter part of each beam is modeled. Since the maximum size of the coarse aggregate used in the concrete mixture is 13 mm, the minimum mesh size for the FE models is selected as 20 mm, larger than the maximum size of the coarse aggregate (Baz’ant ZP & Oh 1983).

The nonlinear static analysis is performed based on the Newton-Raphson method with the displacement control strategy to get the converged solution. The analysis stops when the compressive strain at concrete reaches the crushing strain of 0.003 (based on the failure criteria in ACI 318-11 2011) or de-bonding occurs at the steel-concrete interface (i.e. when the bond stress reaches the mean value of $τ_{\text{max}}$), or strain at tensile rebars reaches its ultimate strain based on Eq. (2.13). To avoid the stress concentration and convergence problems, the
steel plate at the support is modeled using an eight-node 3-D element, SOLID45. The linear isotropic behavior is assumed for the element with the elastic modulus and Poisson’s ratio of $2 \times 10^5$ MPa and 0.3, respectively.

Figure 2.7 shows the comparison of load-deflection curves at the mid-span between the experimental results (Maaddawy et al. 2005a) of the five beams described earlier and the FE predictions. Overall, there is an agreement between the experimental results and the numerical predictions in terms of stiffness, yielding points, and load-carrying capacities. It should be noted that for the numerical beams with $Q = 0\%$ and $8.9\%$, they both fail due to the crushing of concrete at the compressive zone (i.e., the concrete strain reaches the crushing strain, 0.003). Compared with the experimental results, the failure points of these two beams occur earlier. This could be because that the experimental beams have an ultimate concrete compressive strain larger than 0.003 assumed in the FE modeling. For the numerical beams with $Q = 14.2\%$, $22.2\%$, and $31.6\%$, the beams failed due to the failure of tensile rebars (i.e. the tensile strain of rebars has reached the ultimate strain).

![Figure 2.7. Comparison between the experimental results and the numerical results obtained from ANSYS](image)

2.3.3. Numerical Case Study I

To study the effect of bond strength deteriorated by corrosion on the flexural behavior of RC beams, six FEMs are constructed and analyzed under four-point loading using modeling. All six beams are simply supported with a cross section of 152 x 203 mm as shown in Figure 2.8. In these beams, the development length
of tensile bars is from the loading point to the support and it is $l_d$. The distance between two point loads ($l$) is also assumed to be $l_d$. Both the diameter of tensile rebars and concrete cover thickness (from the surface of transverse rebars) are assumed as 20 mm to assure the splitting failure of concrete. Two 8 mm diameter rebars are used as compressive rebars. The shear reinforcement is assumed as 8 mm diameter round bar spaced at 80 mm on center all the way through the span. The mechanical properties of intact rebars and concrete are assumed similar to the materials used by Maaddawy et al. (2005a).

![Figure 2.8. Configurations and reinforcement arrangement of FE beams](image)

Note that both the development length provided in the beam and the bond strength could influence on the flexural behavior of the beams. Therefore, two different development lengths are used to study the effect of bond strength deteriorated by corrosion on the structural behavior. Here, we have divided the beams into two groups (three for each group): Group A and Group B. Group A has $l_d = 640$ mm, while Group B has $l_d = 780$ mm. In each group, one beam is intact (they are A0 for Group A and B0 for Group B) and the other two are uniformly corroded all through the tensile rebars with a corrosion level of $Q = 15\%$. No corrosion is assumed for compressive rebars and stirrups.

To model the corroded beams, ideally three aspects should be simulated: corrosion effect on the cross sectional area of the rebar, yield strength, and the bond behavior. To study the last aspect, for the two corroded beams in each group, one beam accounts for the first two aspects only (they are A15N for Group A and B15N for Group B), while the other one accounts for all three aspects (they are A15 for Group A and B15 for Group B). Note that beams A15N and B15N use intact bond strength. Table 2.3 summarizes the quantities used in the modeling of the studied FE beams.
Table 2.3. The quantities used in FEMs of numerical case study I

<table>
<thead>
<tr>
<th>Group</th>
<th>Beam code</th>
<th>( l_d ) (mm)</th>
<th>( f_y ) (MPa)</th>
<th>( A_b ) (mm(^2))</th>
<th>( \tau_{\text{avg}} ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A0</td>
<td>640</td>
<td>450.0</td>
<td>314</td>
<td>3.91</td>
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<td>A15</td>
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<td>A15N</td>
<td>640</td>
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<td>267</td>
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<tr>
<td>B</td>
<td>B0</td>
<td>780</td>
<td>450.0</td>
<td>314</td>
<td>3.79</td>
</tr>
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<td></td>
<td>B15</td>
<td>780</td>
<td>443.9</td>
<td>267</td>
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<tr>
<td></td>
<td>B15N</td>
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<td>443.9</td>
<td>267</td>
<td>3.79</td>
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</tbody>
</table>

2.3.4. Numerical Case Study I: Results and Discussions

Figures 2.9 and 2.10 show the load-deflection curves at the mid-span for Group A beams and Group B beams, respectively. As seen from Figure 2.9, the two corroded beams A15N and A15 show the reduction in load-carrying capacity in comparison with the intact beam A0. This indicates that the corrosion effects on the cross-sectional area and yielding strength of the rebar change the load-carrying capacity of the beam, as the two corroded beams considered these two corrosion effects. Comparing the load-deflection curves of A15 and A15N, it can be seen that the corrosion effect on bond strength can further reduce the stiffness and ultimate strength of the beam.

![Figure 2.9. The effect of bond-slip behavior on the flexural behavior of Group A beams](image)
Furthermore, failure modes are different for these three beams. Beam A0 shows the ductile behavior and fails due to crushing of concrete at the ultimate compressive strain of 0.003 after rebar yielding (this is ductile failure). The beam A15N also shows ductile behavior and fails due to crushing of concrete at the ultimate compressive strain of 0.003 (this is also ductile failure). Beam A15 fails due to de-bonding of rebar before yielding (this is brittle bond failure). This shows that the corrosion effect on the bond strength could change the failure modes of the beam. Thus, accurately accounting for the corroded bond strength is critical for evaluating structural performance.

Figure 2.10 shows the load-deflection curves for beams in Group B. All three beams show ductile behavior and failed due to crushing of concrete. As seen from Figure 2.10, the flexural behavior of corroded beam B15 is similar to corroded beam B15N, and they have less stiffness and load-carrying capacity compared with the intact beam B0. This indicates that the modeling of corrosion effect on bond strength does not affect the flexural behavior of corroded beams in Group B. The reductions in stiffness and loading-carrying capacity are due to the corrosion effect on the cross sectional area and yielding strength of rebars alone. While the study in Group A (shown in Figure 2.9) concludes that modeling of the corroded bond strength is critical, the study in Group B (shown in Figure 2.10) concludes otherwise. This shows that the effect of modeling corroded bond strength on the flexural behavior of corroded RC beams depends on the development length provided in the structure. In Group A, the provided development length is sufficient for
the intact condition, but becomes insufficient for the corroded condition; therefore, modeling of the bond strength becomes important. At the same time, the development length for Group B is sufficient for both intact and corroded conditions, thus the modeling of the bond strength can be ignored. In other words, it is important to know if the provided development length is sufficient or not under a corroded condition. Note that the ultimate strain of corroded rebars does not govern the failure point in the FE beams with flexural failure mode (A15N, B15, and B15N). The next section will study how to determine the minimum development length considering a specific corrosion level.

2.3.5. Minimum Development Length Calculation

To calculate the minimum development length, it is usually assumed that the bond stress is uniformly distributed over the length. The minimum development length can be calculated by letting the bond force (that equals to \( \tau_{\text{avg}} \pi d_b l_d \)) developed over the perimeter of the embedded rebar equal to the tensile force (that equals to \( f_y A_b \)) in the rebar. Thus, the minimum development length can be calculated as:

\[
l_{d,\text{min}} = \frac{d_b f_y}{4 \tau_{\text{avg}}} \tag{2.14}
\]

For intact beam, in Eq. (2.14), \( d_b = d_{b0}, f_y = f_{y0} \), and \( \tau_{\text{avg}} \) can be calculated using Eq. (2.9) with \( Q = 0 \). To calculate the minimum development length for corroded rebar, the corrosion effects on the cross sectional area and yielding strength of rebar and bond strength should be considered. Given a corrosion level, \( Q \), the \( d_b \) can be estimated using Eq. (2.11); \( f_y \) can be calculated using Eq. (2.12); and \( \tau_{\text{avg}} \) can be evaluated using Eq. (2.9). Note that \( \tau_{\text{avg}} \) is estimated by Eq. (2.9) and it is also a function of \( l_d \) itself. To calculate \( l_d \) using Eq. (2.14), a trial and error analysis is needed.

2.3.6. Numerical Case Study II

As shown previously, using Eqs. (2.9), (2.11), (2.12), and (2.14), \( l_{d,\text{min}} \) for a given corrosion level can be estimated. Particularly, in this study, the mean prediction (\( \hat{\tau}_{\text{avg}} \)) based on Eq. (2.9) is used in Eq. (2.14) for calculating \( l_{d,\text{min}} \). In this section, numerical FE beams will be used to exam the sufficiency of the development length for various corrosion levels. Additionally, for a comparison purpose, the minimum development length suggested by ACI 318-11 design code (2011) for intact rebars is also applied to the intact and corroded FE
beams. It should be noted that in calculation of development length based on the ACI 318-11(2011), the recommendation proposed by ACI committee 408 (2003) is applied.

The FEMs in this case study use the same material properties and geometry configurations (as shown in Figure 2.8) as the ones used in the numerical case study I. The FE beams are divided into two groups: Group AC and Group PP, corresponding to the FEMs with \( l_d = l_{d,\text{min}} \) calculated based on ACI 318-11 design code (2011) and the proposed procedure using Eqs. (2.9), (2.11), (2.12), and (2.14), respectively. Then for each group, FEMs are constructed for intact and three levels of corrosion: 5%, 10%, and 15%. It is assumed that the beams are uniformly corroded all through the tensile rebars and no corrosion occurs on the compressive rebars and stirrups. The main differences between the ACI model and the proposed procedure in calculation of \( l_{d,\text{min}} \) are: the proposed procedure take into account the corrosion effect on \( d_b, f_y \), and \( \tau_{\text{avg}} \) through Eqs. (2.9), (2.11), and (2.12), and the effects of relative rib area of rebars \( (R_r) \) and the beam width \( (b_e) \) on \( \tau_{\text{avg}} \) through Eq. (2.9), while the ACI model does not consider those effects. As shown in Figure 2.4, by using the proposed procedure, the required \( l_{d,\text{min}} \) for an RC beam increases with the increase of \( Q \) or the decrease of \( R_r \) or \( b_e \), which are consistent with the findings in the literature (Chinn et al. 1955, Al-Sulaimani et al. 1990, Darwin et al. 1996). Accordingly, for the FE beams in Group AC, \( l_d \) is the same for the intact and corroded beams, as only one value of \( l_{d,\text{min}} \) is suggested by ACI 318-11 design code (2011); while for the FE beams in Group PP, \( l_d \) varies with the corrosion level.

For a given structure, the actual value of \( \tau_{\text{avg}} \) can be estimated using Eq. (2.9) and it has 68% probability lying in the prediction interval \( [\hat{\tau}_{\text{avg}} \pm \sigma] \). Therefore, for each corrosion level, we construct three FEMs with three different \( \tau_{\text{avg}} \) values, corresponding to \( \hat{\tau}_{\text{avg}} - \sigma, \hat{\tau}_{\text{avg}}, \) and \( \hat{\tau}_{\text{avg}} + \sigma \). Thus, total 12 FEMs are constructed for each group (2 groups). Table 2.4 shows the quantities used for the constructed 24 FE models. As shown in Table 4, for the intact RC beams \( (Q = 0\%) \) in this study, Group PP suggests about 10% longer development length than Group AC. For corroded beams, Group PP suggests 24%, 31%, and 32% (corresponding to \( Q = 5\%, 10\%, \) and \( 15\% \) respectively) longer development length than Group AC. Note that in Group PP, the development lengths are similar for corrosion levels of 10%, and 15%. This is because of the change in \( \tau_{\text{avg}} \) becomes smaller when \( Q \) increases, which is captured by the exponential function used in Eq. (2.9).
Table 2.4. The quantities used in FEMs of numerical case study II

<table>
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<tr>
<th>Group</th>
<th>Beam code</th>
<th>Q (%)</th>
<th>A_b (mm²)</th>
<th>f_y (MPa)</th>
<th>l_d (mm)</th>
<th>τ_avg (MPa)</th>
<th>ε_t/ε_y</th>
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*AC = ACI, PP = proposed procedure, 0-15: corrosion level percentage, L = lower-bound, M = mean value, U = upper-bound.

2.3.7. Numerical Case Study II: Results and Discussions

Figures 2.11–2.16 show the load-deflection curves for the 24 constructed FE beams. ACI 318-11 (2011) defines a “tension-controlled” member as one with a net strain in the extreme tension steel (ε_t) greater than or equal to 0.005 and a “compression-controlled” member as one with ε_t ≤ 0.002. The beams with 0.002 ≤ ε_t ≤ 0.005 are defined as members in “transition zone”. Based on those definitions, if ε_t/ε_y < 1 (where ε_y = yielding strain of the tensile rebar), the beam will suddenly fail by the de-bonding of rebar before the rebar yielding (brittle bond failure). If ε_t/ε_y = 1, the yielding and de-bonding of rebar occur simultaneously and the provided l_d is the minimum development length required for the yielding of rebar but cannot provide any ductility (brittle failure). If ε_t/ε_y ≥ 2.3 where ε_t ≥ 0.005, the beam is tension-controlled and l_d is sufficient to provide the adequate ductility in the beam (ductile failure). Lastly, if 1 < ε_t/ε_y < 2.3, the provided l_d is sufficient for the yielding of rebar, but cannot provide the sufficient ductility in the beam (failure in transition zone). Therefore,
$\varepsilon_t/\varepsilon_y$ at the failure point of each beam is used to evaluate the sufficiency of the provided development length and the values of $\varepsilon_t/\varepsilon_y$ are listed in Table 2.4.

Figures 2.11–2.13 show the load-deflection curves and $\varepsilon_t/\varepsilon_y$ ratios for the FE beams in Group AC. Based on those figures, nearly all intact and corroded beams show the brittle bond failure ($\varepsilon_t/\varepsilon_y < 1$). This indicates that for the investigated FE beams, when the average bond strength in the FEM is laid in the prediction interval of $[\hat{\tau}_{avg} \pm \sigma]$, ACI 318-11 (2011) underestimates the $l_{d,\text{m}}$ required for the intact and corroded rebars. For the beams in Group PP shown in Figures 2.14–2.16, when $\tau_{avg}$ in the FEM is laid in the prediction interval of $[\hat{\tau}_{avg} - \sigma, \hat{\tau}_{avg}]$, all intact and corroded beams show the brittle bond failure ($\varepsilon_t/\varepsilon_y < 1$), while for $\tau_{avg}$ laid in the prediction interval of $(\hat{\tau}_{avg}, \hat{\tau}_{avg} + \sigma]$, all beams show the ductile behavior ($\varepsilon_t/\varepsilon_y \geq 2.3$).

![Figure 2.11](image1.png)

Figure 2.11. Load-deflection curves for beams of Group AC with $\tau_{avg} = \hat{\tau}_{avg} - \sigma$

![Figure 2.12](image2.png)

Figure 2.12. Load-deflection curves for beams of Group AC with $\tau_{avg} = \hat{\tau}_{avg}$
Figure 2.13. Load-deflection curves for beams of Group AC with $\tau_{avg} = \bar{\tau}_{avg} + \sigma$

Figure 2.14. Load-deflection curves for beams of Group PP with $\tau_{avg} = \bar{\tau}_{avg} - \sigma$

Figure 2.15. Load-deflection curves for beams of Group PP with $\tau_{avg} = \bar{\tau}_{avg}$
Figure 2.16. Load-deflection curves for beams of Group PP with $\tau_{avg} = \hat{\tau}_{avg} + \sigma$

Figure 2.17. Load-deflection curves for beams of Group PP with $\tau_{avg} = \hat{\tau}_{avg} - \sigma$ considering $\phi$

Figure 2.18. Load-deflection curves for beams of Group PP with $\tau_{avg} = \hat{\tau}_{avg}$ considering $\phi$
The results described above indicate that the proposed procedure can provide sufficient development length if the actual value of $\tau_{\text{avg}}$ is laid in the prediction interval of $(\hat{\tau}_{\text{avg}}, \hat{\tau}_{\text{avg}} + \sigma]$, while the proposed procedure underestimates the required $l_{d,\text{min}}$ if the actual value of $\tau_{\text{avg}}$ is laid in the prediction interval $[\hat{\tau}_{\text{avg}} - \sigma, \hat{\tau}_{\text{avg}}]$. In order to ensure the yielding of rebar ($\varepsilon_t/\varepsilon_y \geq 1$), we propose to apply a multiplication safety factor, $\phi$, to Eq. (2.13). After applying $\phi = 1.15$ to $l_{d,\text{min}}$, the stress developed in the tensile rebar can reach yielding for intact and corroded FEMs with three different $\tau_{\text{avg}}$ values corresponding to $\hat{\tau}_{\text{avg}} - \sigma$, $\hat{\tau}_{\text{avg}}$, and $\hat{\tau}_{\text{avg}} + \sigma$ as shown in Figures 2.17–2.19. Note that for the FE beams with flexural failure mode investigated in this section, the ultimate strain of corroded rebars does not govern the failure point. Furthermore, the accuracy of Eq. (2.2) is checked through extracting the bond stress distribution along the development length in all the FE beams of this study with brittle bond failure mode. It is confirmed that $\tau_{\text{max}} = 1.5\tau_{\text{avg}}$ is a good assumption.
CHAPTER III

LOAD-DEFLECTION BEHAVIOR PREDICTION OF INTACT AND CORRODED RC BRIDGE BEAMS WITH OR WITHOUT LAP SPLICES CONSIDERING BOND STRESS-SLIP EFFECT

3.1. Introduction

In the design and performance evaluation of RC structures, predicting the deflection of the structure under transverse loading is crucial for evaluating the serviceability, deformability, and drift capacity of the RC structure. Serviceability deals with the performance of the structure under the service loads, which is measured in terms of various factors such as deflections, cracks, vibrations of structures, etc. In particular, using high strength concrete and steels in design of RC structures in the last few decades has permitted the use of slender RC members that has become a serious serviceability concern (McCormac & Brown 2015). Deformability describes the structure’s ability to absorb energy through inelastic deformation under loading and it is usually checked by the displacement ductility index, which is defined as the ratio of maximum displacement over first yield displacement (Azizinamini et al. 1999). Drift capacity of a structural system is important to structural stability, damage to non-structural components and architectural integrity. Drift control is the critical requirement for RC structures that are located in seismic or high wind areas. Since the deflection plays an important role in the structural design and performance evaluation of RC structures, it is essential to accurately predict the nonlinear load-deflection behavior of RC members.

There are many parameters that affect the load-deflection behavior of an RC bridge under transverse loading (e.g., loading type, member dimensions, tension stiffening, and the bond stress-slip behavior at the steel-concrete interface) (Nilson et al. 2010, Yang et al. 2013). In recent years, considerable research has been focused on the deflection prediction of RC beams and/or decks under flexural loading by developing simplified, semi-empirical models through modifying the stiffness of a fully cracked section under service loads. Such models use the effective moment of inertia of a cracked section to take into account the tension-stiffening effect. The model developed by Branson (1963) is perhaps the most widely used model and has been adopted in different design codes such as AASHTO LRFD (2012), ACI 318-11 (2011), CSA-A23.3-04 (2004), and AS 3600-94 (1994). The procedure is simple, but it can only predict the deflection
corresponding to the service loads and cannot be used to predict the entire nonlinear load-deflection behavior of RC beams. Also, the predictions are sensitive to the type of loading, the boundary conditions, and the range of reinforcing ratio (Branson 1963, Bischoff 2007). Furthermore, it does not consider the effect of the bond stress-slip behavior at the steel-concrete interface.

Considering the effect of bond stress-slip behavior can be critical, particularly when evaluating the performance of RC bridge elements such as beams and columns in terms of brittle bond failure mode. Brittle failure occurs when such structural elements either has insufficient provided development or splice length of rebar or has sufficient provided development or splice length but with insufficient confinement (Azizinamini et al. 1999, Rakhshanimehr et al. 2014, Esfahani & Kianoush 2005). Furthermore, deteriorations such as corrosion can degrade the bond at the steel-concrete interface and also the mechanical properties of rebar, and make the development or splice length insufficient, leading to performance reduction and brittle failure (Almusallam et al. 1996, Rodriguez et al. 1996, Castel et al. 2000, Du et al. 2007). One example of such failure is the collapse of Ynysy-Gwas Bridge in UK as a result of the corrosion in the prestressing tendons (Woodward & Williams 1988). Furthermore, even when RC structures fail in a ductile mode, the bond stress-slip behavior can also affect the stiffness and deflection of a RC structure (Priestley et al. 1992, Priestley & Seible 1995, Priestley et al. 1996). Therefore, the effect of bond stress-slip behavior should be considered in predicting the flexural behavior of RC bridge members.

To consider the effect of bond stress-slip behavior between rebar and concrete on the flexural behavior of RC beams, numerical studies have been developed recently. Those studies can be grouped into two categories: nonlinear finite element (FE) analyses and analytical procedures. The FE approach usually adopts nonlinear spring elements to model the bond stress-slip behavior between rebar and concrete (e.g., Huang et al. 2014), and it also has been used to predict the flexural behavior of corroded RC beams (e.g., Kallias & Rafiq 2010). While this approach can incorporate any form of bond stress-slip behavior, FE modeling can be complicated, and the analysis is computationally expensive. Furthermore, it is not practical for the design engineers who are unable to use or do not access to commercial FE softwares.

The analytical approach, on the other hand, is computationally efficient and can be made easily available to be used by engineers. The computational efficiency can facilitate the probabilistic performance evaluation for RC structures using reliability analysis, where the deterioration in bond should be considered. Many
analytical procedures developed recently are for predicting the load-carrying capacity of the corroded RC beams (e.g., Han et al. 2014, Bhargava et al. 2007), however, most of them do not consider the slippage between rebar and concrete. Recently, Maaddawy et al. (2005b) developed an analytical procedure to predict the flexural behavior of intact and corroded RC beams based on the elongation of steel rebar between flexural cracks. However, a linear relationship between the bond stress and slippage is assumed in their procedure, which may not be the true representation in reality. Furthermore, their model is only applicable for the RC beams without lap splices and is only verified for the RC beams with sufficient development length (ductile flexural failure), where the bond stress-slip behavior effect on the flexural behavior may not be crucial.

In this study, a new analytical procedure is proposed to predict the nonlinear load-deflection behavior of RC beams based on the elongation of rebar elements between flexural cracks considering the bond stress-slip behavior at the steel-concrete interface. The three key features of the proposed analytical procedure are: (1) it uses new approach to calculate the elongation of rebar elements based on the relative longitudinal displacement of the nodes (2) it can adopt any bond stress-slip relationship; and (3) it can be applied to structural elements with lap splices. When it is applied to corroded RC structures, the procedure can consider the effects of corrosion on diameter, yield strength, and ultimate strain of rebar, and bond stress-slip behavior at the steel-concrete interface. In this study, firstly, the formulation of the proposed analytical procedure to predict the flexural behavior of RC beams with and without lap splices is described. Next, the proposed procedure is verified by predicting the load-deflection behaviors of several experimental intact and corroded RC beams found in the literature. Then, the accuracy of the procedure is further evaluated through the several numerical intact and corroded RC beams with and without lap splices modelled in ANSYS. For both experimental and numerical case studies without lap splices, the predictions of the proposed procedure are also compared with the ones obtained from the analytical procedure following Maaddawy et al. (2005b). Note that the procedure in Maaddawy et al. (2005b) is one of the few analytical procedures in the literature that incorporates the bond stress-slip relationship. Lastly, the application of the proposed procedure is illustrated in predicting the flexural behavior of intact and corroded T-beams of an RC bridge and the results are compared with FE models.

3.2. Analytical procedure

3.2.1. RC beams without Lap Splices
The proposed procedure is developed based on the calculation of steel elongation between flexural cracks under loading. The beam under loading can be considered to have a series of cracked elements, and it is assumed that the length of each cracked element is equal to the stabilized mean crack spacing, \( s_m \), with a single crack at its middle point, as schematically shown in Figure 3.1. CEB-FIP MC90 (1993) recommends calculating \( s_m \) as follows:

\[
s_m = \frac{d_b}{3.6 \rho_{eff}}
\]  

(3.1)

where \( d_b \) is the diameter of tensile rebar and \( \rho_{eff} \) is the effective reinforcement ratio (= \( n_b A_b / A_{ceff} \) in which \( n_b \) is the number of longitudinal rebars, \( A_b \) is the cross sectional area of each rebar, and \( A_{ceff} \) is the area of effective embedment zone of the concrete and can be calculated as \( 2.5b(h - d) \leq (h - c)/3 \) for a rectangular beam and \( b, h, d, \) and \( c \) refer to the width, height, effective depth, and neural axis of the beam section, respectively).

Assuming the neutral axis depth being constant for each cracked element, the mid-span deflection of an RC beam, \( \Delta \), can be predicted as follows (Maaddawy et al. 2005b):

\[
\Delta = \sum_{i=1}^{n} \frac{e_i}{d - c_i} x_i
\] 

(3.2)

where \( n \) is the number of cracked elements that for a symmetric beam, \( n \) can be calculated for the half span as \( n = l/(2s_m) \) in which \( l \) is the span length, \( e_i \) is the tensile steel elongation within Element \( i \), \( d \) is the depth of tensile steel reinforcement measured from the top face of the beam, \( c_i \) is the depth of the neutral axis at the middle of Element \( i \), and \( x_i \) (\( 0 \leq x_i \leq l/2 \)) is the distance between the center of the beam support and the middle of the Element \( i \), as schematically shown in Figure 3.1. To calculate \( c_i \), the compatibility and equilibrium equations in a beam section should be simultaneously solved (Maaddawy et al. 2005b).

![Figure 3.1. Cracked beam under four-point flexural loading (data from Maaddawy et al. 2005b)](image-url)
In this study, a new approach is used to calculate the elongation of steel rebar within Element $i$, $e_i$, as follows:

$$e_i = d_i - d_{i+1}$$  \hspace{1cm} (3.3)

where $d_i$ and $d_{i+1}$ are the longitudinal displacements of the left end points of Elements $i$ and $i+1$ with respect to the mid-span of the beam, respectively, as schematically shown in Figure 3.2. These longitudinal displacements are caused by the tensile strain in the rebar and the slippage between rebar and concrete, and they increase when the transverse loading increases.

![Figure 3.2. Schematic view of rebar elongation within Element $i$](image)

The value of $d_i$ is the summation of the displacement due to tensile strain and the slippage. For a cracked element shown in Figure 3.3, $d_i$ may be calculated as follows:

$$d_i = \sum_{j=1}^{i=n} \left( s_m e_{s,ij} - \frac{\hat{\tau}_j s_m^2}{E_s d_b} + 2\hat{s}_j \right)$$  \hspace{1cm} (3.4)

where $e_{s,ij}$ is the steel tensile strain at the middle of Element $j$ which can be calculated by the simultaneous solving of equilibrium and compatibility requirements (Maaddawy et al. 2005a), $\hat{\tau}_j$ is the average bond stress between rebar and concrete within Element $j$, $E_s$ is the elastic modulus of steel rebar, and $\hat{s}_j$ is the average slippage of rebar within half of Element $j$ as shown in Figure 3.3.

![Figure 3.3. Schematic view of cracked Element $j$](image)
Note that before flexural cracking (i.e., when tensile stress in concrete is less than the concrete modulus of rupture), due to the perfect bonding between rebar and concrete, no slippage occurs between them (i.e. $\hat{\tau}_j = \hat{s}_j = 0$). After each flexural cracking, the longitudinal displacement of the elements can be calculated considering $\hat{\tau}_j$ in Eq. (3.4). Note that beyond the development length, $\hat{\tau}_j \approx 0$ can be applied. For example, for a beam under four-point loading, $\hat{\tau}_j \approx 0$ in the constant moment region that is beyond the development length (as shown in Figure 3.1). Along the development length, $\hat{\tau}_j$ can be calculated as follows:

$$\hat{\tau}_j = \frac{f_{sj} d_b}{4 s_m \left( \frac{2j-1}{2} \right)}$$  \hspace{1cm} (3.5)

where $f_{sj}$ is the tensile stress of rebar at the middle of element $j$ and can be calculated as follows:

$$f_{sj} = \begin{cases} \varepsilon_{sj} E_s & f_{sj} < f_y \\ f_y + E_p (\varepsilon_{sj} - \varepsilon_y) & f_{sj} \geq f_y \end{cases}$$  \hspace{1cm} (3.6)

where $E_p$ is the plastic modulus of rebar (that can be assumed as $\zeta E_s$ in which $\zeta$ is a constant and depends on the type of the steel), and $f_y$ is the yield strength of rebar. To predict $\hat{s}_j$ in Eq. (3.4), a bond stress-slip relationship is needed. Model Code 2010 (MC10) (fib 2012) suggests the following equation for bond stress-slip behavior between intact rebar and concrete considering splitting bond failure:

$$s = s_{u,\text{split}} \left( \frac{\tau}{\tau_{u,\text{split}}} \right)^{\alpha}$$  \hspace{1cm} (3.7)

where $\tau_{u,\text{split}}$ is the splitting bond strength between rebar and concrete, $s_{u,\text{split}}$ is the slippage at the free end of tensile rebar which can be calculated according to the reference (fib 2012), and $\alpha = 0.4$. Note that it assumed that $\alpha$ remains constant regardless of corrosion. Following previous chapter, we can assume $\tau = \beta \hat{\tau}$, where $\hat{\tau}$ is the average bond stress along the development length and $\beta$ is a constant coefficient that depends on various factors such as embedment length of rebar; similarly, $\tau_{u,\text{split}} = \beta \hat{\tau}_{u,\text{split}}$, in which $\hat{\tau}_{u,\text{split}}$ is the average bond strength between rebar and concrete. Also, it is assumed that the total slippage along the development length is $s$, thus based on Eq. (3.7):
By predicting a distribution of $\hat{\tau}_j$ along the development length through Eq. (3.5), the distribution of $\hat{s}_j$ can be predicted through Eq. (3.8). To calculate $\hat{\tau}_{u,split}$, the prediction model proposed in Chapter II can be used by replacing $\tau_{avg}$ with $\hat{\tau}_{u,split}$ in Eq. (2.9).

To predict the flexural behavior of corroded RC beams, the effects of corrosion on the mean crack spacing, degradation of bond stress-slip between rebar and concrete, and the geometrical (cross-sectional area) and mechanical properties (yielding strength and ultimate strain) of the reinforcing bar should be considered. The effect of corrosion on cross-sectional area (Yang & Zhu 2012), yielding strength (Yang & Zhu 2012), and ultimate strain of rebar (Dang & François 2014) can be considered using Eqs. (2.11)–(2.13).

Note that the corrosion influence on the bond-stress relationship is considered on the average bond strength, $\hat{\tau}_{u,split}$, through Eq. (2.9), the average bond stress, $\hat{\tau}_j$, in Eq. (3.5) through the calculation of $d_b$ based on Eq. (2.11), and the slippage, $s_j$, in Eq. (3.8) through the calculation of $s_{1,split}$ (fib 2012), and $\hat{\tau}_j$ and $\hat{\tau}_{u,split}$ for a corroded rebar based on Eqs. (3.6) and (3.2), respectively. Furthermore, the effect of corrosion on the mean crack spacing, $s_m$, is considered in Eq. (3.1) through the calculation of $d_b$, $\rho_{eff}$ and $\hat{\tau}_{u,split}$ for a corroded rebar.

The load-deflection behavior of intact and corroded RC beams can now be predicted through the calculation of a deflection based on Eq. (3.2) for a given load. For different loading pattern, the distribution of $f_{sj}$ along the rebar will be different and can be calculated (Maaddawy et al. 2005b) together with Eq. (3.6). Note that in predicting the load-deflection behavior, the assumed initial load should be smaller than the cracking load, and it has to be gradually increased with the appropriate load step. The analysis should be stopped when it meets any one of these three criteria: (1) $\hat{\tau}_j$ reaches $\hat{\tau}_{u,split}$, (2) the maximum concrete compressive strain, $\varepsilon_{ci}$, reaches the concrete crushing strain, $\varepsilon_{cu}$ ($\varepsilon_{cu} = 0.0038$ is suggested by Park & Paulay 1975), or (3) $\varepsilon_{si}$ reaches $\varepsilon_{su}$.

### 3.2.2. RC beams with Lap Splices

To evaluate the bond strength behavior of lap splices, four-point testing is usually used by researchers. In such tests, the lap spliced rebars are located in the constant moment region (as shown in Figure 3.4).
To apply the proposed analytical procedure in the previous section to the lap spliced beams, the lap spliced rebar can be considered to be continuous with an equivalent cross-sectional area of $A_{be}$ in the lap spliced region. In the equivalent system with continuous rebars, the developed tensile force should be the same as the average tensile force in the lap splice length of the original system. It can be assumed that the distribution of $f_s$ along the spliced rebar is linear, and it starts from zero at the free end of the spliced rebar and reaches to the maximum value of $f_s$ just at the end of the splice length. To calculate $A_{be}$, first the distribution of steel stress, $f_s$, along the provided lap splice length, $l_s$, should be determined, which depends on the minimum spliced length, $l_{s,min}$, required for developing the full tensile strength (or yielding strength, $f_y$) of the rebar. And $l_{s,min}$ can be calculated by letting the bond force developed over the perimeter of the spliced rebar (that is $\tau_{u,split} \pi d_b l_s$) equals to the tensile force in the rebar (that is $f_y \pi d_b^2 / 4$) as follows:

$$l_{s,min} = \frac{d_b f_y}{4 \tau_{u,split}}$$  \hfill (3.9)

To calculate $\tau_{u,split}$ in lap spliced rebars, Eq. (2.9) can be used by replacing $l_d$ with $l_s$. Thus, there are three cases (that is, $l_s < l_{s,min}$, $l_s = l_{s,min}$, and $l_s > l_{s,min}$) to be considered as following. Note that only when $l_s \geq l_{s,min}$, $f_s$ can reach $f_y$.

1. When $l_s < l_{s,min}$, the distribution of $f_s$ is shown in Figure 3.5(a), where the steel stress is zero at the free end of the lap spliced rebar and it increases linearly to reach $f_s (= l_s/l_{s,min} f_y)$ at the end of the splice length. In the splice region, by summing the two triangular steel stress distributions along $l_s$, the average steel stress is a uniform steel stress distribution of $f_s$. In other words, to get the same tensile force developed in spliced rebar (that is $A_b f_y$) in the original system, the equivalent cross-sectional area of the continuous rebar should be $A_{be} = A_b$ in the equivalent system, as shown in Figure 3.5(a).
(2) When \( l_s = l_{s,\text{min}} \), the steel stress at rebar can reach the maximum value of \( f_s = f_y \) just at the end of the splice length, as shown in Figure 3.5(b). Therefore, the average steel stress along the splice length is a uniform distribution of \( f_s = f_y \) by summing two triangular steel stress distributions along the two spliced rebars. Therefore, \( A_{be} = A_b \) for this case.

(3) When \( l_s > l_{s,\text{min}} \), the rebar can reach the maximum value of \( f_s = f_y \) at \( l_{s,\text{min}} \), as shown in Figure 3.5(c). Since \( l_s > l_{s,\text{min}} \), based on the geometric calculations on the lap spliced rebar shown in Figure 3.5(c), the average steel stress along the splice length region in this case is \( f_y \cdot (2l_s - l_{s,\text{min}}) / l_s \). Therefore, to develop the same tensile force in the continuous rebar of the equivalent system as the one developed in the spliced length of the original system, \( A_{be} = A_b \cdot (2l_s - l_{s,\text{min}}) / l_s \) is used.

It is worth mentioning that in the lap spliced RC beams with sufficient development length, the slippage between rebar and concrete mostly occurs in the lap spliced region. Therefore, one can assume no slippage between rebar and concrete outside of the splice length region. In the original system, the slippage is at its maximum at the free end of the lap spliced rebar and zero at the end of the splice length. Therefore, in the equivalent system, the slippage distribution is uniform along the splice region. With this distribution of the slippage and considering continuous rebars with rebar area of \( A_{be} \) in the splice region, \( \hat{\tau}_j \) in Eq. (3.5) can be calculated as follows:

\[
\hat{\tau}_j = \frac{f_y d_{be}}{4 s_i l_i}
\]

(3.10)

where \( d_{be} \) is the diameter of equivalent rebar. Now the analytical procedure described in previous section can be applied to lap spliced RC beams considering the corrosion effect.
3.3. Verification of the Proposed Analytical Procedure

3.3.1. Verification through the Experimental Tests

In this section, the accuracy of the proposed procedure in predicting the flexural behavior of intact and corroded RC beams is evaluated through the experimental results found in the literature. Three experiments conducted by three different research groups are adopted here: 1) the tests conducted by Maaddawy et al. (2005a) on the flexural behavior of the intact and corroded RC beams (without lap splices), 2) the tests...
conducted by Azad et al. (2007) on the flexural behavior of the intact and corroded RC beams (without lap splices), and 3) the tests conducted by Abdel-Kareem et al. (2013) on the flexural behavior of the intact lap spliced RC beams. Note that no experimental results are found for corroded RC beams with lap splices.

3.3.1.1. The Experimental Program Conducted by Maaddawy et al. (2005a)

Maaddawy et al. (2005a) tested one intact and four corroded RC beams to evaluate the effect of different corrosion levels on the flexural behavior of RC beams without lap splices. The geometry and the reinforcement details of the beams are already illustrated in Figure 2.6. Concrete with the 28 day compressive strength of 40 MPa was used in all beams. The intact tensile rebars had yielding and ultimate strengths of 450 MPa and 585 MPa, respectively, and the compressive and shear rebars had yielding and ultimate strengths of 340 MPa and 500 MPa, respectively. The middle 1400 mm (salted zone) of the tensile rebars of the four corroded beams had corrosion levels of 8.9% (Beam CN-50), 14.2% (Beam CN-110), 22.2% (Beam CN-210), and 31.6% (Beam CN-310), respectively, through an accelerated corrosion process. Finally, force-displacement curves were obtained for all the beams under monotonic four-point loading.

Figure 3.6 shows the comparison of load-deflection curves at the mid-span between the experimental results and the numerical predictions based on the proposed procedure. With the sufficient development length provided, all beams failed in ductile mode. As shown in Figure 3.6, overall there is good agreement between the experimental results and the numerical predictions in terms of failure mode, yielding force ($P_y$), yielding displacement ($\Delta_y$), load-carrying capacity ($P_u$), and the ultimate deflection ($\Delta_u$).

![Figure 3.6. Comparison between experimental results of RC beams adapted from Maaddawy et al. (2005a) and numerical predictions based on the proposed procedure](image-url)
Figure 3.7 shows the ratio of the predictions to the test results in terms of $P_y$, $\Delta_y$, $P_u$, and $\Delta_u$, where the hat symbol indicates the quantities are predicted using the analytical procedures. Note that the ratios close to one (shown as in red dashed lines in Figure 3.7) represent the more accurate predictions by the analytical procedures. In Figure 3.7, the triangles are based on the proposed procedure, and the circles are based on the procedure developed by Maaddawy et al. (2005b). It can be seen that overall the triangles are closer to the red dashed line than the circles, indicating that the proposed procedure in this study gives more accurate predictions than the Maaddawy et al. procedure, particularly for the deflections at yielding and ultimate points. The reason of these differences is two folds that is the two limitations in the Maaddawy et al. procedure: (1) Their procedure does not fully capture the effect of bond stress-slip behavior on the load-deflection response that is critical, particularly for non-ductile beams with a development length less than or close to the $l_{d,\text{min}}$ required; and (2) the effect of corrosion on the yielding and ultimate strengths of rebar is not considered in their procedure that is important in evaluating the ductility of the RC beams affected by corrosion.

![Graphs showing the ratio of predictions to test results](image)

Figure 3.7. The ratio of the numerical predictions to the test results (triangles: based on the proposed analytical procedure; circles: based on the procedure developed by Maaddawy et al. (2005b))

3.3.1.2. The Experimental Program Conducted by Azad et al. (2007)
Azad et al. (2007) performed experimental tests to evaluate the flexural behavior of a group of intact and corroded RC beams without lap splices. Out of all the test specimens, Azad et al. (2007) only reported the load-deflection behavior of seven beams. Here, the test results of the one intact and the five corroded RC beams with corrosion levels less than 30% are used. Figure 3.8 shows the geometry and reinforcement details of these six beams.

![Figure 3.8. Dimensions and reinforcement layout of test specimens conducted by Azad et al. (2007)](image)

An accelerated corrosion test using different corrosion current densities was conducted on the whole length of tensile rebars and part of stirrups in the corroded RC beams for a period of 4 to 8 days. As a result, the tensile rebars of the five corroded beams were corroded to average corrosion levels of 5.4% (Beam BT1-2-4), 14.2% (Beam BT1-3-4), 15.2% (Beam BT1-2-6), 21.4% (Beam BT1-3-6), and 21.5% (Beam BT1-2-8), respectively. The corrosion levels of stirrups are assumed as the corrosion levels of tensile rebars, since the corrosion levels of stirrups were not reported for these five beams. The 28 day average compressive strengths of concrete, $f_{c}$, for these five corroded beams were 38.91, 36.89, 45.77, 46.45, and 33.4 MPa, respectively, while for the intact beam (Beam BT1-C), $f_{c}$ was 45.8 MPa. The yield strength, ultimate strength, and elastic modulus of 10 mm tensile rebars were 520 MPa and 551 MPa, and $1.53 \times 10^5$ MPa, respectively (Azher 2005). As the yield and ultimate strengths of shear and compressive rebars were not reported for the tested beams, the mechanical properties of shear and compressive reinforcements are assumed to be the same as the tensile rebars. As shown in Figure 3.8, the tensile rebars were well anchored in the beams and the flexural-shear failure mode was reported for all beams.

To avoid propagating measurement errors in the parameters needed in Eq. (2.9), the bond strength that is needed for the proposed procedure in this study and the Maaddawy et al. procedure (2005b) is calculated using $\hat{\tau}_{u,split} = f_{s,max} \cdot d/b/(4l_d)$, where $f_{s,max}$ refers to the maximum tensile stress along the rebar, and it can be
determined by the measured moment capacities, $M_{\text{max}}$, of the tested beams. Assuming the distribution of concrete compressive stress in the compressive zone of the section is roughly linear, $f_{s,\text{max}}$ can be calculated as (Nilson et al. 2010):

$$f_{s,\text{max}} = \frac{M_{\text{max}}}{A_b \left( 1 - \frac{\sqrt{2n_r \rho + (n_r \rho)^2}}{3} \right) d}$$  \hspace{1cm} (3.11)

where $\rho$ is the longitudinal bar ratio and can be calculated by $\rho = A_b / (b \cdot d)$, and $n_r$ is the modular ratio of steel to concrete and can be calculated by $E_s / E_c$.

Figure 3.9 shows the experimental and predicted load-deflection behaviors of the six RC beams. It can be seen based on the experimental results that corrosion reduces the stiffness and load-carrying capacity of the beams, and increases the deflection. Overall, the proposed procedure can predict the load-deflection behavior of all beams with sufficient accuracy. Some of the discrepancies between the experimental findings and the numerical results can be due to the uncertainties in the geometrical and mechanical properties of the test specimens and/or the measurement errors. It can be seen from the results that there is a considerable difference between the load-deflection behaviors of the test results with the ones predicted by Maaddawy et al.’s procedure in terms of stiffness, yielding and ultimate points. This considerable difference is due to the two limitations of the Maaddawy et al. (2005b) procedure described earlier.

![Figure 3.9. Comparison between the experimental results (solid black lines) of the beams tested by Azad et al. (2007) and the numerical predictions based on the analytical procedures](image-url)
3.3.1.3. The Experimental Program Conducted by Abdel-Kareem et al. (2013)

Abdel-Kareem et al. (2013) tested seventeen intact high-strength RC beams with lap splices under four-point bending. Figure 3.10 shows the reinforcement details and the configuration of the test specimens. Among the seventeen beams, five of them had special forms of transverse confinement in the spliced region; thus, these five beams are excluded. Table 3.1 shows the details of the twelve beams that are evaluated here. In the first three beams, no stirrups were provided in the lap spliced region. For all beams, the yielding strength of tensile rebars and stirrups were 440 MPa and 280 MPa, respectively. In the analytical procedure, the yield strength of compressive rebars is assumed as 280 MPa, similar to the yield strength of shear rebars. The plastic modulus of steel rebars are assumed as $E_{sp} = 0.03E_s$.

![Figure 3.10. Dimensions and reinforcement layout of test specimens conducted by Abdel-Kareem et al. (2013)](image)

<table>
<thead>
<tr>
<th>Beam specimen</th>
<th>$f'_{c}$ (MPa)</th>
<th>Splice length (mm)</th>
<th>Stirrups spacing (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>60.7</td>
<td>420</td>
<td>–</td>
</tr>
<tr>
<td>B2</td>
<td>81.2</td>
<td>420</td>
<td>–</td>
</tr>
<tr>
<td>B3</td>
<td>103.2</td>
<td>420</td>
<td>–</td>
</tr>
<tr>
<td>B4</td>
<td>58.7</td>
<td>420</td>
<td>150</td>
</tr>
<tr>
<td>B5</td>
<td>80.7</td>
<td>420</td>
<td>150</td>
</tr>
<tr>
<td>B6</td>
<td>102.5</td>
<td>420</td>
<td>150</td>
</tr>
<tr>
<td>B7</td>
<td>104.3</td>
<td>320</td>
<td>150</td>
</tr>
<tr>
<td>B8</td>
<td>99.6</td>
<td>320</td>
<td>125</td>
</tr>
<tr>
<td>B9</td>
<td>105.7</td>
<td>320</td>
<td>100</td>
</tr>
<tr>
<td>B10</td>
<td>108.4</td>
<td>210</td>
<td>150</td>
</tr>
<tr>
<td>B11</td>
<td>101.6</td>
<td>210</td>
<td>125</td>
</tr>
<tr>
<td>B12</td>
<td>104.3</td>
<td>210</td>
<td>100</td>
</tr>
</tbody>
</table>

Since the steel stresses at the failure point of all beams were calculated based on the maximum experimental moments and were reported in the study by the authors, $\hat{\tau}_{u,split}$ for each beam is calculated based on Eq. (3.11). Note that sufficient development lengths were provided in the beams. Thus, for different lap splice lengths, two different failure modes were observed: the splitting bond failure at the splice length region and the ductile...
flexural failure. The comparisons between the load-deflection predictions through the proposed procedure and the experimental results are shown in Figure 3.11. Overall, it is evident that the proposed procedure can accurately predict the flexural behavior of these lap spliced RC beams.

![Comparison of numerical predictions and experimental results](image)

Figure 3.11. Comparison of the experimental results (black solid lines) of lap spliced RC beams (Abdel-Kareem et al. 2013) with the numerical predictions (grey dashed lines) using the proposed procedure

3.3.2. Verification using FE Models

As mentioned earlier, the FE analysis has been widely used for flexural behavior prediction of intact and corroded RC beams considering the bond stress-slip behavior at the steel-concrete interface. Thus, in this section, the proposed analytical procedure is verified with the results obtained from the FE analysis from smeared cracking approach. There are total four cases used for this verification, including intact beams with or
without lap splices and corroded beams with or without lap splices. Several intact and corroded FE beam models, with and without lap splices, are designed and modeled using ANSYS. Then, the load-deflection results of the FE beams are compared with the results predicted from the proposed procedure. Again, to further evaluate the prediction accuracy of the proposed procedure, the prediction obtained from the analytical model proposed by Maaddawy et al. (2005b) are also compared with the load-deflection behaviors of the FE cases studies without lap splices.

3.3.2.1. Finite Element Modeling in ANSYS

ANSYS v. 14.0 (2011), a 3D FE program, is employed here to simulate the numerical beams. The details of FE modeling is already discussed in section 2.3.1. To evaluate the accuracy of the proposed procedure in predicting the flexural behavior of RC beams, numerical simply supported beams with and without lap splices are constructed in ANSYS and analyzed under four-point loading. The configuration and reinforcement arrangement of the beams with and without lap splices are shown in Figures 3.12(a) and 3.12(b), respectively, where \( l_d \) is the development length. For the beams with lap splices shown in Figure 3.12(b), sufficient development lengths are assumed to avoid debonding of rebars in the shear region. To calculate the minimum required development length, \( l_{d, \text{min}} \), for intact or corroded RC beams, Eq. (2.14) can be used. The compressive strength of concrete is assumed as 40 MPa, and the yielding and ultimate strength of all intact rebars are assumed as 340 MPa and 500 MPa, respectively.

A total of eight beams are designed: two beams (Group 1) without splice region and with a tensile rebar diameter of 25 mm are designed to have brittle failure modes, two beams (Group 2) without splice region and with a tensile rebar diameter of 20 mm are designed to have ductile failure modes, two beams (Group 3) with splice region and with a tensile rebar diameter of 25 mm are designed to have brittle failure modes, and two beams (Group 4) with splice region and with a tensile rebar diameter of 12 mm are designed to have ductile failure modes. In each group, one beam is intact and the other is uniformly corroded all the way through the tensile rebars, with a corrosion level of \( Q = 15\% \). No corrosion is assumed for compressive rebars and stirrups.
Figure 3.12. Details of numerical RC beams: (a) without splice region; (b) with splice region

Table 3.2 summarizes the quantities used in the studied FE beams. The nonlinear static analysis is conducted on the beams based on the Newton-Raphson method with the displacement control strategy. Using the same criteria adopted in the proposed procedure, the analysis stops when the concrete reaches the crushing strain of 0.0038 (Park & Paulay 1975), when de-bonding occurs at the steel-concrete interface (i.e., the bond stress reaches $\tau_{\text{max}}$), or when the steel strain reaches the ultimate steel strain.

Table 3.2. The quantities used in the numerical RC beams

<table>
<thead>
<tr>
<th>Type of beam</th>
<th>Group</th>
<th>Beam code</th>
<th>$l_d$ (mm)</th>
<th>$l_s$ (mm)</th>
<th>$Q$ (%)</th>
<th>$A_b$ (mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without splice region</td>
<td>1</td>
<td>B1$_0$</td>
<td>560</td>
<td></td>
<td>0</td>
<td>491</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1$_{corr}$</td>
<td></td>
<td></td>
<td>15</td>
<td>417</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>B2$_0$</td>
<td>560</td>
<td></td>
<td>0</td>
<td>314</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B2$_{corr}$</td>
<td></td>
<td></td>
<td>15</td>
<td>267</td>
</tr>
<tr>
<td>With splice region</td>
<td>3</td>
<td>B3$_0$</td>
<td>1000</td>
<td>400</td>
<td>0</td>
<td>491</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B3$_{corr}$</td>
<td></td>
<td></td>
<td>15</td>
<td>417</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>B4$_0$</td>
<td>1000</td>
<td>400</td>
<td>0</td>
<td>113</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B4$_{corr}$</td>
<td></td>
<td></td>
<td>15</td>
<td>96</td>
</tr>
</tbody>
</table>

3.3.2.2. Results and Discussions

Figure 3.13 shows the comparison of load-deflection curves of the four beams under four-point loading obtained from the FE analysis (shown in black curves) and the proposed procedure in this study (shown in
grey curves). As can be seen from the curves, the beams in Groups 1 and 3 have brittle bond failure, while the beams in Groups 2 and 4 have ductile failure with bond failure for corroded Beams $B_{2\text{corr}}$ and $B_{4\text{corr}}$, and concrete crushing in the compressive zone for intact beams $B_{2\text{0}}$ and $B_{4\text{0}}$. Note that for the beams that failed due to crushing of concrete, the FE analysis is unable to capture the behavior of the RC beams after peak load that is when $\varepsilon_c = 0.0038$, since the crushing capability of concrete is disabled to avoid the convergency problems in the analysis, as mentioned earlier. On the other hand, for the beams with bond failure (e.g., Beams $B_{1\text{0}}$ and $B_{3\text{corr}}$), the FE results indicate a strength drop after a peak load that is when the bond stress exceeds the bond strength. After few steps, the analysis stops due to the convergency problems. If the bond stress-slip behavior is not considered in the analytical procedure or in the FE modeling, the bond failure will not be able to be captured. Overall, the black and grey curves agree with each other very well, indicating that the proposed procedure has the same accuracy as ANSYS in predicting the flexural behavior of RC beams with and without lap splices in terms of failure mode, $P_{y\text{0}}$, $\Delta_{y\text{0}}$, $P_{u\text{0}}$, and $\Delta_{u\text{0}}$.

The predictions obtained from the procedure proposed by Maaddawy et al. (2005b) are shown in Figure 3.13(b) for Groups 1 and 2 beams (the beams without lap splices). Note that the values of bond strength, and yielding and ultimate strength of rebars used in Maaddawy et al.’s procedure are the same values used in ANSYS. Based on Figure 3.13(b), while the prediction obtained from Maaddawy et al.’s procedure agrees with the behavior of $B_{2\text{0}}$, the difference between the results predicted by Maaddawy et al.’s procedure and the results predicted by ANSYS is considerable in terms of stiffness and failure point of Beams $B_{1\text{0}}$ and $B_{1\text{corr}}$ (with brittle bond failure mode), and stiffness and yielding point of $B_{2\text{corr}}$ (with ductile flexural failure mode), which is due to the two limitations of the Maaddawy et al.’s procedure described in earlier.

![Figure 3.13. Comparison of the load-deflection curves obtained from ANSYS and the ones predicted by the analytical procedure proposed in this study](image-url)
To further evaluate the behavior of the corroded lap-spliced RC beams, the strain distributions along the lap-spliced rebar for Beams B3₀ and B3_corr are illustrated in Figure 3.14 under the same load of $P = 62.4$ kN for comparison purposes. This load corresponds to the load-carrying capacity of Beam B3_corr. As expected, in both cases, the steel strain is zero at the free end of each rebar and increases almost linearly to reach its maximum value at the end of the splice length (at $l_s = 400$ mm). Also based on the results, the increase of the strain values in the corroded beam is greater than the one in the intact beam, indicating that the corrosion can significantly increase the stress intensity along the lap spliced region mainly due to the reduction of the rebar cross section in this region.

![Figure 3.14. Distribution of rebar strain along the lap-spliced region for Beams B3₀ and B3_corr](image)

3.4. Application of the Proposed Analytical Procedure to RC Bridge Beams

In this section, the proposed procedure is applied to predict the structural performance of intact and corroded T-beam sections of an RC bridge. Figure 3.15 shows the configuration of the studied RC bridge (left) and the details of interior T-beams (right), adapted from Taly (2014). The beam was designed based on AASHTO LRFD design specifications (AASHTO 2012). The span of the beams is $l = 15.25$ m. The design used concrete with compressive strength of $f'_c = 28$ MPa and Grade 60 reinforcing rebars with yield strength of $f_y = 414$ MPa. The bridge was designed to carry two traffic lanes with traffic barriers on both sides with weighing of 7.37 kN/m in each side. The dead load due to the future wearing surface was assumed as 1.2 kN/m².
In this study, the FE models of the T-beams are constructed in ANSYS. A total of three beams are analyzed: one intact beam and two corroded beams with corrosion levels of $Q = 7.5\%$ and $15\%$, considering the uniform corrosion all the way through the tensile rebars. To take advantage of the symmetry of the beam, only one quarter of the T-beam is modelled with appropriate boundary conditions, which is setting the displacements at the plane of symmetry to be zero perpendicular to the plane. In addition, only half of the cross sectional area of the longitudinal rebars is modeled at the center of the beam due to the symmetry. Figure 3.16 shows the FE model of the quarter of the T-beam.

Figure 3.15. Configuration and details of the RC bridge and the designed interior T-beam based on AASHTO LRFD specifications (data is adapted from Taly 2014)

Figure 3.16. ANSYS FE model of the quarter of the RC bridge T-beam
Figure 3.17 shows the comparison of the load-deflection curves of the T-beams under the four-point loading obtained from the FE analysis (shown in black curves) and the proposed procedure in this study (shown in grey curves). For the investigated beams, the intact beam and the corroded beam with $Q = 7.5\%$ failed due to the crushing of concrete in the compressive zone, while the corroded beam with $Q = 15\%$, failed due to the failure of tensile rebars due to corrosion. The comparisons show that the proposed procedure can accurately predict the flexural behavior of the studied RC bridge beams in terms of failure mode, $P_y$, $\Delta_y$, $P_u$, and $\Delta_u$. Note that the proposed procedure in this study may also be applicable for predicting the load-deflection behavior of the concrete structural elements under axial loading (such as prestressed bridge girders and bridge columns carrying axial forces). To capture the effect due to the axial loading, one needs to modify the equilibrium equations (Maaddawy et al. 2005b) to include the axial compression force. If tendons are used, then also the bond stress-slip behavior for tendons should be used.

![Figure 3.17](image_url)

**Figure 3.17.** Comparison between the load-deflection behaviors obtained from ANSYS and the numerical predictions based on the proposed procedure for the intact and corroded bridge beams.
4.1. Introduction

Deterioration of reinforced concrete (RC) structures due to chloride induced corrosion is one of the main concerns for the states and the local transportation agencies in the United States. Corrosion can significantly impair the serviceability and ultimate performance of RC structures by cracking and spalling of concrete cover (Champiri et al. 2012, Champiri et al. 2016, Khanzadeh-Moradllo et al. 2015, Moradllo et al. 2016), reducing the diameter, yielding, and ultimate strengths of the reinforcing rebars, and degrading the bond at the steel-concrete interface. Such corrosion effects can significantly reduce the stiffness and load-carrying capacity of the structure, and even change the failure mode of the structure from a ductile behavior to brittle failure without any warning. This issue is more critical for the highway bridges located in the cold regions, where the annual usage of salts and deicing chemicals in those regions is considerable in winter. It is estimated that approximately 15% of RC bridges in the United States are structurally deficient due to corrosion (Koch et al. 2002). According to the Federal Highway Administration (FHWA) report, the annual cost of corrosion of RC bridges in United States is more than $85 billion (FHWA 2006). Thus, to increase the public safety and to reduce the corrosion related costs, development of optimum corrosion management strategies is crucial.

Various strategies have been proposed in the literature for corrosion management of RC bridges located in corrosive environments. These strategies include durability design of RC structures (e.g., Gjørv 2014), inspection (e.g., visual, non-destructive testing), protection (such as coating, cathodic protection, chloride extraction, etc.), repair (such as patching, partial replacement, etc.), rehabilitation (such as strengthening with fiber reinforced polymers (FRPs), jacketing, etc.), or replacement of the corroded RC bridge (Whitmore & Ball 2004, Christodoulou et al. 2009, Sohanghpurwala 2011, Huang et al. 2015). Decision on selecting an optimal corrosion management strategy is usually made based on the deteriorated structural performance and the associated life-cycle-costs. The deteriorated structural performance itself depends on many parameters such as properties of concrete, pre-corrosion cracking due to shrinkage (Kiani et al. 2012, Rahmani et al.
geometrical characteristics, environmental conditions, the model used for simulating the cracking of concrete (Cusatis et al. 2014, Rezakhani & Cusatis 2016), etc. In practice, considerable amount of uncertainties are associated with each of these parameters, which can lead to a large uncertainty in the performance evaluation of the structure. In order to consider these uncertainties, time-dependent reliability-based framework should be utilized. In the literature, most of the research works have focused on development of reliability-based modules for prediction of the time-to-repair of the corroded structure, evaluating the time-dependent reliability of the corroded structure, and selection of the optimal repair/maintenance strategies for the corroded infrastructures (e.g., Stewart et al. 2004, Lounis & Daigle 2008, Eamon et al. 2012, Grace et al. 2012, Kim & Frangopol 2012, Kim et al. 2013, Miran et al. 2016). Very few studies can be found in the durability design of RC structures considering uncertainties.

One of the well-known strategies in the corrosion management of RC structures is the durability design of cost-effective RC structures, such that the structure does not require any repairs in its design service life. This approach can further be optimized by minimizing the expected total costs subjected to various probabilistic and/or deterministic constraints through solving a single-objective optimization problem (e.g., Frangopol et al. 1997). In order to solve a single-objective optimization problem, a target reliability index for all limit states should be typically defined. However, reliability of a structure significantly depends on the considered random variables and the models adopted in the analysis. This is why different target reliabilities have been used for the RC structure design in various studies and design codes (e.g. Gulvanessian et al. 2012). Therefore, it is better to provide decision makers all the corresponding costs for a range of reliability index, such that they can observe the changes in reliability index by increasing/reducing a certain amount of cost.

In general, the expected total cost of the structure includes the initial construction cost, maintenance cost, and the life-time structural failure cost (Dyanati et al. 2015, Dyanati et al. 2016). Therefore, the reliability index and the expected total cost are not necessarily conflicting. In the durability design, however, one can assume that the maintenance costs are the same for all the design options, thus there is no need to include the maintenance cost in the optimal durability design process. According to Stewart and Val (Stewart & Val 2003), the failure cost can be assumed to be 10 times the construction cost; while Kumar et al. (Kumar et al. 2009), assumed the failure cost as 2 times the construction cost when the bridge is not critical for the transportation network. Since the minimum reliability index considered in the design
is usually pretty high that corresponds to a rather low probability failure (e.g., 3.5 of reliability index corresponds to probability of failure of $2.3 \times 10^{-4}$), the expected cost due to failure (that equals to probability of failure times the failure cost) can be ignored compared to construction cost. Therefore, one can only consider the initial construction costs in the optimal design. As such, the reliability of the designed structure and the costs become two conflicting objectives, since increasing reliability requires increasing the initial construction costs. Thus, adopting an appropriate multi-objective optimization (simultaneously maximizing the reliability of the structure and minimizing the associated initial costs) technique to find a set of optimized reliability-cost solutions through is rational and informational. Note that if the failure cost needs to be included, one could follow the studies by Gomes et al. (Gomes et al. 2013), and Gomes and Beck (Gomes and Beck 2014a, Gomes and Beck 2014b) to calculate the expected cost due to failure considering the life-time reliability of the structure.

Various techniques have been proposed in literature for handling multi-objective optimization problems. Among the current techniques, evolutionary computation (EC) has received most attention in the past two decades addressing civil engineering problems (e.g. Kiani et al. 2016, Sajjadi et al. 2016), particularly solving the multi-objective problems (Kim & Frangopol 2012, Kim et al. 2013, Gandomi et al. 2016). Multi-objective EC (MOEC) techniques are stochastic optimization techniques that can perform a global and probabilistic search for obtaining a set of multiple non-dominated optimal solutions (Pareto front) in a single run of algorithm. MOEC techniques do not require the user to prioritize or weigh the objectives and are diverse enough to represent the entire spread of the Pareto optimal set, particularly for complex multi-objective problems (Marler & Arora 2004). One of the most frequently used MOEC techniques in engineering problems is the non-dominated sorted genetic algorithm-II (NSGA-II) (Deb et al. 2002). As highlighted by several researchers, this technique can simply converge near the true Pareto-optimal set, and has a superiority over other multi-objective optimization techniques such as Pareto-archived evolution strategy (PAES) (Knowles & Corne 1999) and strength Pareto evolutionary algorithm (SPEA) (Zitzler et al. 2001).

In this section, an RB-MODO based procedure is proposed through combining the NSGA-II with the reliability-based analysis technique for optimum RC bridge design considering corrosion. The proposed procedure can optimize the design variables for a given service life by simultaneously maximizing the reliability of the structure and minimizing the associated life-cycle costs. In particular, the approximate first
order reliability method (FORM) is used in this study to estimate the time-dependent structural reliability. The prevailing uncertainties considered in the reliability analyses are the uncertainties in geometrical and mechanical properties, environmental conditions, corrosion parameters, the dead and live loads, and model errors. The effects of corrosion are captured on the deteriorations in the diameter and yielding strength of tensile rebars. As an illustration, the developed RB-MODO procedure is used to design a T-beam of an RC bridge with and without considering corrosion effect for various service lives. Two conflicting objectives are considered for the design optimization: the flexural reliability of the structure and the construction material costs. For each considered service life, three T-beams are designed using three groups of materials: 1) normal strength concrete with black steel rebars (NSC-BS), 2) normal strength concrete with epoxy coated rebars (NSC-EC), and 3) high performance concrete with black steel rebars (HPC-BS). Lastly, the optimum design strategy is selected among the considered materials based on the obtained reliability-cost Pareto front results through the proposed RB-MODO procedure.

4.2. Effects of Chloride Induced Corrosion on RC Bridge Structures

The effects of corrosion considered in this study are the time-dependent loss of the diameter and yield strength of the tensile rebars. Note that it is assumed that the rebars are properly developed in the beams, so the degradation of bond at the steel-concrete interface due to corrosion is negligible. Given the corrosion level at year $t$, $Q(t)$, the time-dependent diameter, $d_b(t)$, and yield strength of tensile rebars, $f_y(t)$, subjected to uniform corrosion can be calculated as follows (Du et al. 2005):

$$d_b(t) = \sqrt{1 - Q(t)} d_{b0}$$
$$f_y(t) = \left[1 - 0.005Q(t)\right] f_{y0}$$

(4.1)

(4.2)

where $Q(t)$ is in percent and can be calculated using the following equation (Du et al. 2005):

$$Q(t) = \begin{cases} 0 & t < t_{in} \\ 4.6 \frac{i_{corr}(t)}{d_{b0}} (t - t_{in}) & t \geq t_{in} \end{cases}$$

(4.3)

where $(t - t_{in})$ is the time elapsed since the corrosion initiation, $t_{in}$ (years) and $i_{corr}(t)$ is the corrosion current density at year $t$ ($\mu$A/cm$^2$). The corrosion rate of reinforcement in water soluble chlorides, $i_{corr}(t)$, can be predicted as follows (Liu 1996):
\[ i_{\text{corr}}(t) = 0.926 \exp\{8.37 + 0.618 \ln (1.686 C_{\text{water}}) - \frac{3034}{T} - 1.05 \cdot 10^{-4} R_c + 2.32(t - t_0)^{-0.215} + \sigma_{\text{corr}} \} \] (4.4)

where \( C_{\text{water}} \) is the water soluble chloride content at the steel surface (kg/m\(^3\)), \( T \) is the actual absolute temperature in concrete (K), \( \sigma_{\text{corr}} \) is the model error, and \( R_c \) is the ohmic resistance of concrete (Ohms) that can be calculated as:

\[ R_c = \exp\left[8.03 - 0.549 \ln\left(1 + 1.686 C_{\text{acid}}\right) + \sigma_{\text{Rc}} \right] \] (4.5)

where \( \sigma_{\text{Rc}} \) is the model error and \( C_{\text{acid}} \) is the acid soluble chloride content at the steel surface (kg/m\(^3\)) that can be calculated as: \( C_{\text{acid}} = 0.932 C_{\text{water}} - 0.272 + \sigma_{\text{acid}} \), in which \( \sigma_{\text{acid}} \) is the model error and \( C_{\text{water}} \) is the water soluble chloride content at the steel surface that can be calculated based on the Crank’s solution of the Fick’s second law of diffusion, as follows (Crank 1975):

\[ C_{\text{water}} = C_l \left\{1 - \text{erf} \left( \frac{C_b}{2 \sqrt{D_c t}} \right) \right\} \] (4.6)

where \( C_l \) is the chloride content at the surface of concrete (kg/m\(^3\)), \( \text{erf}() \) is the Gauss error function, \( C_b \) is the concrete cover (mm) and \( D_c \) is the chloride diffusion coefficient (mm\(^2\)/year), which depends on different parameters such as concrete mix design. To calculate \( D_c \) considering the mix design, the following equation developed by Boulfiza et al. (2003) for normal strength concrete (NSC) and high performance concrete (HPC) can be used (Mangat & Molloy, 1994):

\[
\begin{align*}
\log(D_c) &= -3.9(w/c)^2 + 7.2(w/c) - 14 & \text{(NSC)} \\
\log(D_c) &= -3.0(w/c)^2 + 5.4(w/c) - 13.7 & \text{(HPC with silica fume or slag)}
\end{align*}
\] (4.7)

where \( D_c \) is in m\(^2\)/s and \( w/c \) is the water-to-cement ratio that can be calculated as \( w/c = 27/(f'_{c} + 13.5) \), in which \( f'_{c} \) is the 28-day concrete compressive strength (MPa) (Vu & Stewart 2000). One may also use the recent techniques/formulations for predicting the mechanical properties of concrete based on the mix design (e.g., Khademi et al. 2015, Khademi & Jamal 2016, Khademi & Behfarinia 2016). Note that the initiation of corrosion in the reinforcing rebar takes place, only when the chloride content at the surface of the reinforcement, \( C_{\text{water}} \), reaches chloride threshold value, \( C_{\text{th}} \). Therefore, by setting \( C_{\text{water}} \) to \( C_{\text{th}} \) in Eq. (4.6) and solving for \( t \), the corrosion initiation time, \( t_{\text{in}} \) (year), in Eqs. (4.3) and (4.4) can be deduced as follows:
\[ t_{in} = \frac{C_{th}^2}{4D_c \left[ \text{erf}^{-1} \left( 1 - \frac{C_{th}}{C_{th}} \right) \right]^2} \] (4.8)

4.3. Reliability-Based Multi-Objective Design Optimization (RB-MODO)

A general form of an RB-MODO problem may be expressed as follows:

\[
\begin{align*}
\text{minimize} & \quad f(x, y) \\
\text{subject to:} & \quad \beta_i(x, y, t) - \beta_{th} \geq 0 \\
& \quad D_j(\hat{x}, \hat{y}) - D_{min,j} \geq 0 \\
& \quad \text{ranges of design parameters: } y_k^l \leq y_k \leq y_k^u
\end{align*}
\] (4.9)

where \( f(x, y) \) is the objective function that should be minimized, \( x \) refers to the input variables (e.g., chloride threshold value, geometry, material properties, model error, etc.), \( y \) refers to the design variables, \( \beta_i(x, y, t) \) is the \( i^{th} \) probabilistic performance of the structure at time \( t \), \( \beta_{th} \) is the \( i^{th} \) threshold performance value that can be determined by the design code (for example, American Association of State Highway and Transportation Officials (AASHTO) proposes the target reliability index of 3.5 for ultimate limit states) or decision maker, \( D_j(\hat{x}, \hat{y}, t) \) is the deterministic performance of the structure at time \( t \), \( \hat{x} \) and \( \hat{y} \) are the mean values of \( x \) and \( y \), \( D_{min,j} \) is the \( j^{th} \) deterministic performance limit, and \( y_k \) is the \( k^{th} \) design variable that is limited by its lower- and upper- bounds of \( y_k^l \) and \( y_k^u \), respectively. Note that a multi-objective problem shown in Eq. (4.9) has a set of non-dominated optimal solutions (Pareto front) that all are equally good and valid, if no additional preferences are considered. Therefore, solving a multi-objective problem is not straightforward and usually suffers from the non-convexity issues. In this case, multi-objective evolutionary computation (MOEC) techniques are suitable techniques to perform a probabilistic search in finding near-global solutions (Cheng & Yan 2009, Kang et al. 2009).

Various MOEC techniques have been developed in the literature for handling multi-objective optimization problems. As mentioned earlier, one of the most successful MOEC techniques that have been widely used in engineering problems is the non-dominated sorting algorithm-II (NSGA-II) technique (Deb et al. 2002). This technique uses elitism and a phenotype crowd comparison operator that keeps diversity without specifying any additional parameters (Deb & Sundar 2006). Shi and Reitz (2010) assessed the performance of different schemes for multi-objective evolutionary algorithms and concluded that NSGA-II
is very powerful in solving multi-objective optimization problems, since it performs both elite-preserving strategy and an explicit diversity-preserving mechanism. In this technique, the old and new population is sorted and partitioned into fronts, where the first front consists of a set of Pareto optimal solutions that are not dominated by any other solutions. The solutions that comprise the second front are not dominated by any other solutions, except the ones in the first front and so on. Then, a new operator called crowding factor is used in this algorithm to increase the diversity of the population by giving less priority to the solutions that are crowded together during the ranking process. Crowding factor can increase the ability of NSGA-II to explore other possible solutions within the search space. Once the crowding factor is calculated for all the solutions, in the final step, the solutions are ranked according to their positions in the fronts (i.e., those on first front are ranked above those on the second front and so on, and for the solutions that are on the same front, the crowding factor is used for ranking). This algorithm is adopted in this study to find the Pareto front for the reliability-based multi-objective problem expressed in Eq. (4.9).

As mentioned earlier, since the expected cost of failure is not considered in the design optimization process, in this study, the instantaneous probability of the failure and the instantaneous target reliability index are used for evaluating the performance of the structure, as has been used by other researchers to predict the time-dependent reliability of the structures affected by corrosion (e.g., El Hassan et al. 2010, Bigaud & Ali 2014). To evaluate the time-dependent performance of the structure, the instantaneous probability of the structure failure at time $t$, $P_{f_i}(t)$, can be expressed as follows:

$$P_{f_i}(t) = P \left( g_i \left[ C_i(x,t), D_i(x,t) \right] \leq 0 \right) = \int_{g_i(x,t)\leq 0} f_x(x) dx$$

(4.10)

where subscript $i$ denotes the failure mode in the structure, $g_i(\cdot)$ refers to the $i^{th}$ limit state function, $C_i$ and $D_i$ represent the capacity and demand for the $i^{th}$ limit state function, respectively and can be time-dependent, and $f_x(x)$ is a joint probability density function of the input variables $x$. The limit state function for the $i^{th}$ failure mode, $g_i(x,t)$, is defined as follows in this study:

$$g_i(x,t) = C_i(x,t) - D_i(x,t)$$

(4.11)

Meanwhile, the instantaneous reliability of the structure can be evaluated through a quantity called reliability index. The instantaneous reliability index for the $i^{th}$ limit state function at time $t$, $\beta_i(t)$, is determined based on the instantaneous probability of failure, $P_{f_i}(t)$, through the following equation:
\[ \beta_i(t) = \Phi^{-1} \left[ 1 - P_{f,i}(t) \right] \]  

(4.12)

where \( \Phi^{-1}(\cdot) \) is the inverse of the cumulative distribution function of the standard normal variable. The first-order reliability method (FORM) is used in this study to calculate \( P_{f,i}(t) \) and \( \beta_i(t) \).

4.4. Illustration

As an illustration, the proposed RB-MODO technique is applied to design an interior T-beam of an RC bridge. The configuration of the RC bridge and the details of the interior T-beam (encircled in red dashed line) are already shown in Figure 3.15, which is previously designed based on the AASHTO Load and Resistance Factor Design (LRFD) specifications by Taly (2014) without consideration of the effect of corrosion in the design process, as discussed earlier in section 3.5. In this session, this interior T-beam, shown in Figure 3.15(b), is re-designed based on the proposed RB-MODO technique. The variables that are re-designed here are the thickness of the slab, \( t_f \), the width of the web, \( b_w \), the height of the beam, \( h \), the cross-sectional area of longitudinal tensile reinforcements, \( A_s \), and the spacing between stirrups (\( s \)). Since using only one spacing between stirrups along the beam span is not economical, we divided the half-span of the beam into three intervals (each interval has a length of \( l/6 \) with stirrups spacing of \( s_1, s_2, \) and \( s_3 \)) and checked the shear capacity in each interval based on AASHTO. Also, note that based on the AASHTO (Art. 4.6.2.6), for \( S/l \leq 0.32 \), the effective width of the beam flange, \( b_{\text{eff}} \), is \( b_{\text{eff}} = S \), which is 230cm in this case study. Thus, \( b_{\text{eff}} \) is already known and will not be designed here. As mentioned earlier, the beams are designed using three groups of materials: NSC-BS, NSC-EC, and HPC-BS. In particular, NSC has \( f_{c}' = 30 \text{MPa} \), HPC has \( f_{c}' = 70 \text{MPa} \), and the yield strength of longitudinal tensile rebars is assumed as 414MPa (Grade 60). In addition, only flexural failure mode is considered as a reliability objective for the optimal design. The other failure modes such as serviceability failure in terms of cracking of concrete cover due to expansion of corrosion products and the shear failure are considered as constraints in this study. The ultimate limit state function in Eq. (4.11) for flexural failure is thus considered as follows:

\[ g_1(x,t) = M_f(t) - (M_D + M_L) \]  

(4.13)

where \( M_f(t) \) is the time-dependent flexural moment capacity of the T-beam at time \( t \), and \( M_D \) and \( M_L \) (demands) are the flexural moments due to the dead and live loads, respectively, which are assumed to be time-independent random variables. Note that the beams are designed with and without considering corrosion.
effect. For the beams without considering corrosion effect \( t = 0 \), and for the beams considering corrosion effect, the uniform corrosion is assumed all the way through the longitudinal tensile rebars for different service lives of \( t = 50, 75, \) and \( 100 \) years. The flexural moment capacity, \( M_f(t) \), can be calculated as follows:

\[
M_f(t) = \begin{cases} 
A_f(t) f_y(t) \left( d - \frac{a}{2} \right) & a \leq t_f \\
\left[ A_f(t) - A_{sf} \right] f_y(t) \left( d - \frac{a}{2} \right) + A_{sf} f_y(t) \left( d - 0.5t_f \right) & a > t_f
\end{cases}
\] (4.14)

where \( a \) is the equivalent compressive stress block that is \( a = A_s(t) f_y(t)/(0.85 f'c b_{eff}) \) when \( a \leq t_f \) and \( a = [A_s(t) - A_{sf}] f_y(t)/(0.85 f'c b_w) \) when \( a > t_f \), in which \( A_{sf} \) is the equivalent flange reinforcement that can be calculated as \( A_{sf} = 0.85 f'c (b - b_w) t_f / f_y(t) \), \( A_s(t) \) is the cross-sectional area of longitudinal tensile reinforcement at time \( t \) and can be calculated as \( n_b \pi d_b^2 / 4 \), in which \( n_b \) is the number of tensile rebars, and \( d \) is the effective depth of the beam section.

The flexural moment due to dead load in Eq. (4.13) can be calculated as: \( M_D = (w_{DC1} + w_{DC2} + w_{Dw}) l^2 / 8 \), in which \( w_{DC1} \) is the dead load due to the self-weight of the T-beam and depends on the geometry of the beam that will be designed based on the RB-MODO procedure, \( w_{DC2} \) is the dead loads due to the traffic barriers (7.37kN/m each), and \( w_{Dw} = 2.30 \times 1.2 \) kN/m\(^2\) is the future wearing surface. The flexural moment due to live load considering the dynamic load allowance factor of 0.33 can be calculated as: \( M_L = g_m (1.33 M_{\text{truck}} + M_{\text{lane}}) \), in which \( g_m \) is the live load distribution factor that depends on the stiffness of the T-beam and can be calculated based on AASHTO, \( M_{\text{truck}} = 80l - 380 + 162/l = 852 \) kN.m is the flexural moment due to HL-93 design truck proposed by AASHTO for the bridges with \( l > 12.27 \) m, and \( M_{\text{lane}} = 4.67(l/2 - 0.71)(l/2 + 0.71) = 269 \) kN.m is the moment due to the lane load calculated based on the influence line diagram for maximum moment. As can be seen, \( M_D \) and \( M_L \) are time-independent and are assumed as random variables in the reliability analysis.

The serviceability limit state function in Eq. (4.11) for cracking failure that is considered as a probabilistic constraint in Eq. (4.9) is predicted as follows:

\[
g_2(x,t) = w_{cr,\text{limit}} - w_{cr}(t) \] (4.15)

where \( w_{cr,\text{limit}} \) is a serviceability capacity or the crack width limit that is assumed as a constant random variable, and \( w_{cr}(t) \) is a serviceability demand that is the concrete crack width due to the expansion of corrosion products at time \( t \), and is calculated as follows (Thoft-Christensen 2001):

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where $a_d$ is the density ratio of the corrosion rust product to the reinforcing steel that can be assumed as 2 (Molina et al. 1993) and $t_{cr}$ is the crack initiation time in concrete due to the expansion of corrosion products that is calculated based on the model proposed by Chernin and Val (2011).

Table 4.1 shows the list of the deterministic and random variables used in this study. As can be seen in this table, $b_{eff}$, $d_{bb}$, $A_{sb}$, $s_1$, $s_2$, and $s_3$ are assumed as deterministic variables. Lower- and upper-bound values are assumed for all design variables, where their mean values will be found through the optimum design of the structure using the proposed RB-MODO approach. Note that the lower-bound for concrete cover, $C_b$, and the flange thickness, $t_f$, are assumed based on the practical applications. Although designing of T-beams for deflection is optional in AASHTO LRFD specifications, the lower-bound of the beam height, $h$, is assumed as $h = 0.07l$ as suggested by AASHTO (Art. 2.5.2.6.3) to satisfy the optional criteria for span-to-depth ratio. The standard deviation (SD) of 1.0cm is assumed for all geometrical random variables as shown in Table 4.1.

In Table 4.1, the standard deviations of $f_{y0}$ and $f'_{c}$ are calculated based on the 10% and 18% coefficient of variation (COV), respectively (Val et al. 2000). For environmental conditions, it is assumed that the bridge is located at a cold state (such as State of Ohio), where salt is used for de-icing on bridges during the winter season. In particular, the average annual temperature of 286K with standard deviation of 8K are used based on the average annual temperature reported for the State of Ohio, and the salt usage in the state is more than 5 tons/lane-mile/year. Therefore, according to the studies by Weyers et al. (1993), a range of $5.9 \text{kg/m}^3 \leq C_s \leq 8.9 \text{kg/m}^3$ for the state of Ohio is used for chloride surface content. Also, they propose a mean value of $1 \text{ kg/m}^3$ for chloride threshold value of black steel (BS) rebars, $C_{th,BS}$. For epoxy coated (EC) rebars, O’Reilly et al. (2011) reported a chloride threshold value of $C_{th,EC} = 4.32 \text{kg/m}^3$, when $C_{th,BS} = 0.94 \text{kg/m}^3$. To be consistent with their results, a chloride threshold value of $C_{th,EC} = 4.6C_{th,BS} = 4.6 \text{kg/m}^3$ is used here. The COV of the chloride threshold value for both BS and EC rebars is assumed as 0.19 as recommended by Bastidas-Arteaga et al. (2009). For the dead and live loads, the distribution and COV proposed by Val et al. (2000) are used. Lastly, the mean value and SD of crack width limit, $w_{cr,lim}$, are
assumed as 1.0mm and 0.1mm, respectively, which corresponds to the severe cracking (or spalling) in the concrete cover due to expansion of corrosion products.

The RB-MODO approach presented in the previous section is adopted here to find the optimum design variables for both intact and corroded RC T-beams. The population size of 150 with 500 generations is used in NSGA-II. Note that to facilitate the RB-MODO process, the bi-objective optimization is used with three deterministic constraints as follows:

\[
\text{minimize } [CM] \& \begin{bmatrix} \beta_1(t) \end{bmatrix}
\]

\begin{align*}
\text{probabilistic constraint: } & \beta_1(t) \geq \beta_{T2} \\
\text{subject to: } & \begin{cases}
(1) \frac{0.42 - \frac{c}{d}}{d} \geq 0 \\
(II) \min(M_c, 1.33M_a) - M_{cr} \geq 0 \\
(III) (\Delta F)_{II} - \gamma(\Delta f) \geq 0 \\
(IV) 4.83\gamma_e + 2d_e - s \geq 0 \\
(V) \Phi_t[V_e + V_F(t)] - V_a \geq 0
\end{cases}
\end{align*}

The ranges of design parameters: shown in Table 1

---

### Table 4.1. The deterministic and random variables used in the RB-MODO procedure

<table>
<thead>
<tr>
<th>Type</th>
<th>Random variable</th>
<th>Mean</th>
<th>SD</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometrical</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C_b$ (cm)</td>
<td>5.1</td>
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<td>Normal</td>
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<td></td>
<td>$t_f$ (cm)</td>
<td>20</td>
<td>1.0</td>
<td>Normal</td>
</tr>
<tr>
<td></td>
<td>$b_w$ (cm)</td>
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<td>1.0</td>
<td>Normal</td>
</tr>
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<td></td>
<td>$h$ (cm)</td>
<td>107</td>
<td>1.0</td>
<td>Normal</td>
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<tr>
<td></td>
<td>$s_1$</td>
<td>10</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$s_2$</td>
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<td>-</td>
</tr>
<tr>
<td></td>
<td>$s_3$</td>
<td>10</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$A_{60}$ (cm$^2$)</td>
<td>64</td>
<td>194</td>
<td>Normal</td>
</tr>
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<td></td>
<td>$b_{eff}$ (cm)</td>
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<td>-</td>
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<td></td>
<td>$d_{60}$ (cm)</td>
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<td>-</td>
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<td></td>
<td>$f_y$ (MPa)</td>
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<td>41.4</td>
<td>Normal</td>
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<tr>
<td></td>
<td>$f_y^{NSC}$ (MPa)</td>
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<td>5.4</td>
<td>Normal</td>
</tr>
<tr>
<td></td>
<td>$f_y^{HPC}$ (MPa)</td>
<td>70</td>
<td>12.6</td>
<td>Normal</td>
</tr>
<tr>
<td>Environmental</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$T$ (K)</td>
<td>286</td>
<td>8</td>
<td>Normal</td>
</tr>
<tr>
<td>Corrosion</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C_s$ (kg/m$^3$)</td>
<td>7.4</td>
<td>1.5</td>
<td>Normal</td>
</tr>
<tr>
<td></td>
<td>$C_{th,BS}$ (kg/m$^3$)</td>
<td>1</td>
<td>0.19</td>
<td>Uniform</td>
</tr>
<tr>
<td></td>
<td>$C_{th,EC}$ (kg/m$^3$)</td>
<td>4.6</td>
<td>0.87</td>
<td>Uniform</td>
</tr>
<tr>
<td>Model error</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_{corr}$</td>
<td>0</td>
<td>0.33</td>
<td>Normal</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{Re}$</td>
<td>0</td>
<td>0.12</td>
<td>Normal</td>
</tr>
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<td></td>
<td>$\sigma_{acid}$</td>
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<td>0.12</td>
<td>Normal</td>
</tr>
<tr>
<td>Load</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>dead load (N)</td>
<td>$D$</td>
<td>0.1D</td>
<td>Normal</td>
</tr>
<tr>
<td></td>
<td>live load (N)</td>
<td>$L$</td>
<td>0.408$L$</td>
<td>Normal</td>
</tr>
<tr>
<td>Crack width</td>
<td>$w_{cr, limit}$ (mm)</td>
<td>1.0</td>
<td>0.1</td>
<td>Normal</td>
</tr>
</tbody>
</table>
where $CM$ is the cost of materials and should be minimized, $\beta_1(t)$ is the reliability index at time $t$ in terms of flexural limit state function according to Eq. (4.13) and should be maximized (or $-\beta_1(t)$ should be minimized), $\beta_2(t)$ is the reliability index at time $t$ in terms of cracking limit state function based on Eq. (4.15), $\beta_{T2}$ is the target reliability index for cracking and it is assumed as $\beta_{T2} = 0$ (Stewart et al. 2004), which is corresponding to the failure probability of 50%, deterministic constraint (I) is the constraint applied for design of under-reinforced concrete beams based on AASHTO, $c$ is depth of compression stress zone, deterministic constraint (II) is the constraint applied for controlling the minimum reinforcement ratio of longitudinal rebars based on AASHTO, $M_{cr}$, $M_{fr}$, and $M_u$ are the cracking moment, factored flexural moment resistance, and the factored applied flexural moment, respectively, that can be calculated based on AASHTO, deterministic constraint (III) is the constraint applied for controlling the fatigue limit state, $\gamma$ is the load factor specified in AASHTO (Table 3.4.1.1), $\Delta f$ is the force effect specified in Art. 3.6.4.1, $(\Delta F)_{th}$ is the constant-amplitude fatigue threshold, deterministic constraint (IV) is the constraint applied for the crack controlling under service loads based on AASHTO, $s$ is the spacing of the rebars in the layer closest to the tension face (mm), $\gamma_e$ is the exposure factor, $d_c$ is thickness of concrete cover measured from the extreme tension fiber to center of the flexural reinforcement closest thereto (mm), $f_{ys}$ is the tensile stress in steel reinforcement at the service limit state (MPa), and $\beta_k = 1+d_c/[0.7(h-d_c)]$, and deterministic constraint (V) is the constraint applied for avoiding shear failure in the structure considering corrosion effect, in which in which $\Omega_1 = 0.75$ is the resistance factor, $V_c = 0.083\times2\times f_{cy}^{0.5}b_wd$ (MPa) is the shear strength of concrete, $V_d(t)$ is the shear strength of stirrups at time $t$, and $V_u$ is the factored shear load due to dead and live loads according AASHTO. Note that no limitations are specified in AASHTO for control of maximum rebar ratio and deflection of the T-beams. The shear strength of stirrups at time $t$, $V_d(t)$, can be calculated as follows:

$$V_d(t) = \frac{A_v(t)f_{ys}(t)d}{s_i} \quad (4.18)$$

where $A_v(t)$ is the cross-sectional area of double-legged stirrups at time $t$ considering corrosion effect, $f_{ys}(t)$ is the yield strength of stirrup at time $t$ considering corrosion effect that is assumed same as $f_y(t)$ in this study, and $s_i$ ($i = 1, 2, 3$) is the spacing between stirrups at $i^{th}$ interval.

The cost of the materials, $CM$, in Eq. (17) can be calculated as:

$$CM = \alpha \left(0.75V_{al}\cdot \rho_s \cdot CS\right) + \left(V_{al}\cdot \rho_s \cdot CS\right) + \left(V_{cl}\cdot CC\right) \quad (4.19)$$
where \(0.75 \cdot Vol_s \cdot \rho_s \cdot CS\) is the total cost of longitudinal steel rebars assuming that half of the total amount of bending steel is placed only within the area of maximum moment with the other half spanning the full length of the beam (Frangopol et al. 1997), \(Vol_s\) is the total volume of reinforcing rebars in one T-beam \((m^3)\), \(\rho_s\) is the steel density \((kg/m^3)\), \(CS\) is the cost of the steel rebars which is assumed as $1.91/kg for BS rebars and $2.13/kg for EC rebars (O’Reilly et al. 2011), \(Vol_{st} \cdot \rho_s \cdot CS\) is the total cost of stirrups, \(Vol_{st}\) is the total volume of stirrups along the span of the T-beam, \(Vol_c \cdot CC\) is the total cost of concrete, \(Vol_c\) is the volume of the one designed T-beam \((m^3)\), \(CC\) is the cost of the concrete which is assumed as $736/m^3 for NSC (O’Reilly et al. 2011), and 25% more in average (= $920/m^3) for HPC (Daigle & Lounis 2006), and \(\alpha\) is the modification factor to take into account the effect of steel or concrete type on the development length of the rebar. For NSC-BS, NSC-EC and HPC-BS, the values of \(\alpha\) are assumed as 1, 1.5 (according to AASHTO) and \((f'_{c,NSC}/f'_{c,HPC})^{0.5}\), respectively. Note that \(Vol_{st}\) is calculated based on the arrangement of stirrups along the beam span. For each interval corresponding to stirrup spacing of \(s_1, s_2,\) or \(s_3\), no stirrup is provided along the interval if \(Vu \leq 0.45V_c\), and thus it is assumed that \(Vol_{st} \cdot \rho_s \cdot CS = 0\) for that interval.

4.5. Results and Discussion

As mentioned earlier, the interior T-beam shown in Figure 3.15, which was previously designed based on AASHTO in the reference (Taly 2014), is re-designed in this section using the proposed RB-MODO procedure. Note that, AASHTO requires a target reliability index of \(\beta_1 \geq 3.5\) for RC bridges designed based on the flexural failure mode. For a designed beam, the minimum required stirrups were provided for shear design according to AASHTO design code. Based on the considered dimensions and the reinforcement layout of the designed section shown in Figure 3.15, the material cost of T-beam considering the cost of stirrups is approximately \(CM = $12300\) using Eq. (4.19). Furthermore, for this beam, the reliability index of the beam in terms of flexural failure mode without considering corrosion \((t = 0)\) and using the random variables shown in Table 4.1 is predicted as \(\beta_1 = 3.6\), which satisfies the AASHTO minimum requirement for the ultimate limit state.

Figure 4.1 shows the Pareto front plots (optimum results) of \(\beta_1 \cdot CM\) obtained from the proposed RB-MODO procedure after re-designing the T-beam. As can be seen in this figure, the results are presented in three plots based on the three groups of materials used in the design: NSC-BS, NSC-EC, and HPC-BS.
Each plot includes the design without considering corrosion effect \((t = 0)\) and with considering corrosion effect for different design service lives \((t = 50, 75, \text{ and } 100 \text{ years})\). Based on the results, for the all designed beams without considering corrosion effect \((t = 0)\), the initial cost of materials, \(CM\), increases with increasing the reliability index, \(\beta_i\). If the effect of corrosion is disregarded \((t = 0)\), compared with the previous T-beam shown in Figure 3.15 (which was designed based on AASHTO), the material cost of T-beam designed based on the proposed RB-MODO procedure using NSC-BS materials corresponding to \(\beta_T = 3.6\) is found to be about \(CM = $10750\) (later shown in Table 4.2), which is 12% less than the costs based on the original design, indicating the capability of the proposed RB-MODO procedure in the optimum design of the structure. Furthermore, with the \(\beta_i-CM\) curves obtained from the proposed procedure, a decision maker has a choice to select another solution considering budget constraints. For example, as shown in Figure 4.1(a), by spending an initial construction cost of \(CM = $12300\) (similar to the cost of T-beam designed based on AASHTO), the reliability of the system can be increased to \(\beta_i = 5.8\).

For the beams located in the corroded areas, Figure 4.1(a) shows the Pareto front plots of \(\beta_i-CM\) for T-beams designed based on the proposed RB-MODO procedure using NSC-BS with \(f'_c = 30\text{MPa}\). As shown in Figure 4.1(a), for \(\beta_i \geq 3.5\) required by AASHTO, the initial CM needs for the optimum design of the beam significantly increases when the expected design service life increases. In particular, when the structure is designed based on the service life of \(t \geq 50\) years, no optimum design could be found for initial cost of less than \(CM = $25000\), which means that the cost-effective design of the structure considering corrosion effect for the service life of more than 50 years is not possible, when NSC-BS materials are used in the design. Also, comparing the \(\beta_i-CM\) results of \(t = 0\) and \(t = 50\) years shows that the design service life is critical for determining the \(\beta-CM\) results. The main reason for that is because using NSC-BS leads to considerable corrosion in the structure and thus, it significantly reduces the safety of the structure after 50 years. In addition, when considering NSC-BS material as shown in Figure 4.1(a), it is not economical for a decision maker to increase the reliability of the structure (for example from \(\beta_i = 3.5\) to \(\beta_i = 4.0\)), since the initial construction costs will be significantly increased.
Figure 4.1. Pareto front plots of $\beta$-$CM$ after designing T-beam based on the RB-MODO procedure using (a) NSC-BS with $f'_{c} = 30$MPa, (b) NSC-EC with $f'_{c} = 30$MPa, and (c) HPC-BS with $f'_{c} = 70$MPa

When NSC-EC with $f'_{c} = 30$MPa is used in the design, the $\beta_{1}$-$CM$ plots (shown in Figure 4.1b) indicate that the results are less sensitive to the design service life and $CM$ for all the design service life considered smoothly increase with $\beta_{1}$. The main reason for such results is because the epoxy coating around the rebar can significantly increase the chloride threshold value (assumed as 4.6 times the chloride threshold value for BS rebars in this study), which delays the corrosion initiation and propagation processes in the structure. Similar results were found in Figure 4.1(c) when using HPC-BS with $f'_{c} = 70$MPa, since HPC materials have denser matrix and low chloride permeability compared to NSC, which effectively delay the corrosion initiation time in the structure.

Furthermore, Table 4.2 shows the summary of $CM$ required for the optimum design of the T-beam considering different procedures (i.e., AASHTO or RB-MODO) and service lives. Again no optimum design find for the beam using NSC-BS materials considering the service life of $t = 100$ years, thus it is not reported
in the table. Based on the results, if the effect of corrosion is not considered in the design process \((t = 0)\), CMS required for optimum design of T-beam using NSC-EC and HPC-BS are respectively 12% and 15% more than the CM required for optimum design of T-beam using NSC-BS. If the corrosion effect is considered in the design process, for design service life of \(t = 100\) years, the initial CM for optimum design using NSC-EC is approximately 14% more than the CM required for optimum design using HPC-BS. Therefore, HPC-BS is recommended to use in design of the structure. Additionally, considering \(t = 100\) years as a design service life, the optimum mean values of the design variables at different reliability indices obtained from the RB-MODO procedure for HPC-BS is shown in Figure 4.2. From the plots, one can choose the optimum design values for the investigated T-beam based on a target reliability index. Based on the results, the proposed RB-MODO procedure in this study can be used as a suitable tool for optimum design of safe and cost-effective RC structures considering corrosion effect.

Table 4.2. The initial material cost (CM) required for optimum design of T-beam with (for service life of \(t = 100\)) and without (\(t = 0\)) considering corrosion

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Type of material</th>
<th>(t) (years)</th>
<th>(\beta)</th>
<th>CM</th>
</tr>
</thead>
<tbody>
<tr>
<td>AASHTO</td>
<td>NSC-BS</td>
<td>0</td>
<td>3.6</td>
<td>$12300</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3.6</td>
<td>$10750</td>
</tr>
<tr>
<td>RB-MODO</td>
<td>NSC-BS</td>
<td>0</td>
<td>3.6</td>
<td>$12000</td>
</tr>
<tr>
<td></td>
<td>NSC-EC</td>
<td>100</td>
<td>4.7</td>
<td>$15200</td>
</tr>
<tr>
<td></td>
<td>HPC-BS</td>
<td>100</td>
<td>4.7</td>
<td>$13400</td>
</tr>
</tbody>
</table>

![Graph of design variables vs. reliability index](image)

Figure 4.2. The optimum mean value of the design variables obtained from the RB-MODO procedure considering a design service life of \(t = 100\) years using HPC-BS materials
CHAPTER V

RELIABILITY-BASED LIFE-CYCLE-COST COMPARISON OF DIFFERENT CORROSION MANAGEMENT STRATEGIES

5.1. Introduction

Most of the RC bridges in the United States are built of BS rebars, mainly due to their low initial construction costs. However, such bridges can be severely affected by corrosion, particularly in the cold regions where significant amount of deicing salt is used in winter. The most dominant form of corrosion-induced damage in RC bridges is the cracking and spalling of concrete cover due to the volumetric expansion of corrosion products around the corroded rebar. It is reported that almost 10 percent of the bridges in United States were structurally deficient in 2016 (ASCE 2017), mainly due to corrosion of reinforcing steel (Virmani 2002). To ensure the bridge functionality and safety, corroded RC bridges need regular monitoring, maintenance, repair, and rehabilitation, resulting in overwhelming corrosion associated costs. According to the Federal Highway Administration (FHWA), the annual direct cost of corrosion of RC bridges in United States was more than $8.3 billion in 2002. This report also highlights the tremendous indirect costs to the users due to traffic delays and lost productivity, which can be as high as 10 times that of direct corrosion costs (Virmani 2002). Therefore, to increase the public safety and to minimize the corrosion related costs, development of optimum corrosion management strategies is crucial.

Corroded RC bridges are typically being repaired by patching/overlaying approach, which involves removing the chloride contaminated concrete beyond the reinforcing steel, cleaning up the corroded rebars, replacing the rebars if they are significantly corroded, and applying patch-repair/overlay. This approach is straightforward to use, however since RC structures will continue to need periodic repair operations during their service lives; depending on the number of repair operations, this strategy can be costly over the long-run. The number of repair operations itself depends on various parameters such as the properties of repair materials, concrete cover, chloride concentration, etc. In practice, considerable amount of uncertainties are associated with each of these parameters, which can significantly affect the efficiency of this approach. Note that the improper
selection of repair materials and/or techniques may even accelerate the rate and area of the deterioration, and thus impose more repair costs (Ghasemzadeh et al. 2013). One alternative to mitigate corrosion in new constructions could be using corrosion resistant rebars. Although this strategy can significantly increase the serviceability of the RC structure in corrosive areas, the initial cost of these materials are quite high, which makes the decision making process on selection of the optimum strategy more complicated.

Current studies in the literature on LCC comparison of different corrosion management strategies can be categorized into two groups: one uses deterministic-based techniques and the other one uses reliability-based approaches. Most of the deterministic-based LCC studies have focused on evaluating the cost efficiency of using different materials in design or repair of the RC bridge decks. For example, Daigle and Lounis (2006) compared the long-term cost efficiency of HPCs containing different pozzolans with NSC for using in RC bridge decks. They concluded that using HPC is more economical in terms of agency costs, user costs, and environmental impacts. O’Reilly et al. (2011) compared the cost-effectiveness of using uncoated BS rebars, EC rebars, and SS rebars in concrete bridge decks and concluded that using of EC rebars is the most cost-effective strategy for a service life of 75 years with LCC of almost half of the BS reinforced deck. The LCC of SS reinforced deck was also estimated to be about 70% of the LCC of BS reinforced deck. The main drawback of these studies is that they do not consider the uncertainties in the decision making process, and mostly rely on local experience and expertise, which can lead to inaccurate, non-optimal, and costly solutions (Sohanghpurwala 2006).

Reliability-based studies, on the other hand, have mostly focused on evaluating the time-to-repair of the corroded structure, time-dependent performance of the corroded and/or repaired structure, and LCCA of the repaired structure using an specific repair material (e.g., Stewart et al. 2004). Very few studies can be found in the reliability-based LCC comparison of using different materials in design and repair of RC structures considering both ultimate and serviceability performances. Thus, in this chapter, we compare the time-dependent cost-effectiveness of using six groups of materials in design and repair of an illustrated interior T-beam of an RC bridge, including: NSC with BS rebars (NSC-BS), NSC with EC rebars (NSC-EC), NSC with SS rebars (NSC-SS), HPC containing SF pozzolan with BS rebars (HPC-SF-BS), HPC containing SL pozzolan with BS rebars (HPC-SL-BS), and HPC containing FA pozzolan with BS rebars (HPC-FA-BS). An RBDO technique is developed for optimizing the initial design of all bridge beams through minimizing the initial
material costs, given a target reliability index. Note that all beams are designed based on the same flexural strength subjected to the constraints provided by AASHTO LRFD design specifications (2012), without considering the corrosion effects in the design process (i.e., regular design method is adopted). While corrosion resistant materials have higher initial costs, the corrosion in BS rebars imposes periodic repair/overlay associated costs, which may increase the expenses in a long-run. To compare the long-term cost-effectiveness of using these six groups of materials and select the optimum corrosion management strategy, a time-dependent reliability-based LCC module is developed. Both serviceability and ultimate limit states are considered in the time-dependent reliability analyses. Patch-repair/overlay is applied when the reliability index becomes lower than given thresholds for each performance. It is assumed that the bridges are repaired using the materials with the mix designs similar to the parent concrete. Lastly, the time-dependent LCCs of all T-beams are compared and the optimum corrosion strategy is selected.

5.2. Corrosion Modeling

As described earlier in Section 4.2, the effects of corrosion are modelled in two phases: the corrosion initiation phase and the corrosion propagation phase. All equations used in Section 4.2 are also used here to predict the corrosion effects. The only difference is considering the effect of the type of pozzolan on the corrosion, which instead of using Eq. (4.7) to predict the time-dependent chloride diffusion coefficient, \( D_c \) (mm\(^2\)/year), the following equation is used to take into account the effect of the type and amount of pozzolans (Mangat & Molloy 1994):

\[
D_c = D_{\text{ref}} \left( \frac{t_{\text{ref}}}{t} \right)^m
\]  

(5.1)

where \( D_{\text{ref}} \) (mm\(^2\)/year) is the diffusion coefficient at reference time \( t_{\text{ref}} \) (which is typically assumed as \( t_{\text{ref}} = 28 \) days), and can be calculated as \( D_{\text{ref}} = 10^{12.06 + 2.4w/cm} \), in which \( w/cm \) is the water to cementitious ratio (EHLEN 2012) and as mentioned earlier in Section 4.2, it can be calculated as \( w/c = 27/(f'_c + 13.5) \), and \( m \) (\( \leq 0.6 \)) is the diffusion decay index that depends on the concrete mixture proportions. For NSC with ordinary Portland cement (OPC), \( m \) can be assumed as 0.2 (Ehlen 2012). For HPC with SL and/or FA, \( m = 0.2 + 0.4 \times (%FA/50+ %SL/70) \). This relationship is valid up to replacement levels of 50% FA or 70% SL. For HPC with SF replacement level of less than 15%, \( D_{c,\text{SF}} = D_{c,\text{OPC}} \exp[-0.165(%\text{SF})] \), where \( D_{c,\text{SF}} \) and \( D_{c,\text{OPC}} \) are the diffusion
coefficients of SF and OPC, respectively. It is assumed that $D_c$ doesn’t change after 25 years due to the completion of the hydration process (Ehlen 2012).

It is worth mentioning that some effects such as shrinkage (Kiani et al. 2012, Rahmani et al. 2012) or service dead and live loads (Stewart & Rosowsky 1998) may induce initial micro-cracks in the concrete before the initiation of corrosion. Recent studies, however, have shown that these surface cracks due to reinforcement stresses at shrinkage or service load levels have negligible effects on the corrosion rate (Beeby 1978, 1982, Crank 1975, Cusson & Qian 2009; Francois & Arliguie 1999). Thus, this effect has been disregarded in this study. In the presence of wide cracks, however, one may assume the equation proposed by Boufiza et al. (2003) for prediction of apparent chloride diffusion that takes into account the effects of crack width to crack spacing ratio and the diffusion coefficient inside the cracks.

5.3. Reliability-Based Design Optimization (RBDO) Procedure

As mentioned earlier, different materials impose different initial and long-term costs to the owners. Thus, it is complicated to make a decision on selection of optimum materials for construction and repair of the structures in corrosive regions. For the materials used in this study, the initial cost of HPC is quite high compared to NSC. Also, SS rebar is much more expensive than EC and BS rebars. On the other hand, however, HPC materials and SS rebars are more durable in corrosive regions and thus, they impose less repair and protection costs in the long-run. To conduct a reasonable LCCA, the first step is to minimize the initial design costs of the beams using each group of materials, as much as possible. Certainly, there exists infinite number of design solutions with different initial costs for each case. Therefore, a reliability-based design optimization (RBDO) technique should be utilized to optimize the design process through minimizing the initial construction cost for each group of materials by taking into account the prevailing uncertainties. The RBDO procedure in this case can be expressed as a single objective optimization problem (which is minimizing the initial cost of materials) subjected to different probabilistic and deterministic constraints, shown as following:

$$\text{minimize } \text{cost}(\mathbf{\hat{y}})$$

subject to:

- probabilistic constraint: $\beta_j(\mathbf{x}, \mathbf{y}) - \beta_{Ti} \geq 0$
- deterministic constraint: $D_j(\mathbf{\hat{x}}, \mathbf{\hat{y}}) - D_{min,j} \geq 0$
- ranges of design parameters: $y^l_k \leq y_k \leq y^u_k$

$$\text{(5.2)}$$
where \( y \) refer to design parameters, \( x \) refer to the other input variables (e.g., chloride threshold value, geometry, material properties, model error, etc.), \( \beta_i(x,y) \) is the \( i \)th probabilistic performance (in terms of reliability index) of the structure, \( \beta_{T_i} \) is the \( i \)th threshold reliability index that is defined by the design code (for example, as mentioned earlier, AASHTO LRFD design specifications (Lrfd 2012) propose the target reliability index of 3.5 for ultimate limit states) or a decision maker, \( D_j(\hat{x},\hat{y}) \) is the deterministic performance of the structure provided by the design code and also considering the practical design, \( \hat{x} \) and \( \hat{y} \) are the mean values of \( x \) and \( y \), \( D_{min,j} \) is the \( j \)th deterministic performance limit, and \( y_k \) is the \( k \)th design variable that is limited by its lower- and upper- bounds of \( y_k^l \) and \( y_k^u \), respectively.

In particular, Genetic Algorithm (GA) technique is used in this study to solve Eq. (10). GA, developed by Holland (Holland 1992), is an adaptive heuristic search algorithm inspired by an evolutionary process of natural selection and genetics, which uses a global and probabilistic search for finding a near-global solution. The algorithm starts with an initial set of random solutions called population and randomly selects individuals as parents from this population. Then, it uses these parents to create new off-springs for the next generation. These off-springs replace the existing individuals in the population and this process continues until the population evolves toward an optimal solution (Holland 1992).

As described in Section 4.3, here again the reliability index, \( \beta_i(t) \), is used based on Eq. (4.12) for predicting the time-dependent probabilistic performance of the structure. The first order reliability method (FORM) algorithm is used to calculate \( P_f(t) \) in Eq. (4.10) and then the corresponding \( \beta_i(t) \) is calculated based on Eq. (4.12) and used in Eq. (5.2) in the RBDO procedure. Note that the reliability of intact structure is used in Eq. (5.2), \( \beta(x,y) \), which is calculated by setting \( t = 0 \) in Eqs. (4.10) – (4.12).

5.4. Life-Cycle-Cost-Analysis (LCCA) for Selection of Optimum Corrosion Management Strategy

After optimum design of the RC structures using different groups of materials based on the illustrated RBDO procedure, the time-dependent serviceability and ultimate performances of the structures are predicted based on Eq. (4.12) considering the corrosion effects through Eqs. (4.1) – (4.2) and Eq. (4.16). Then, the repair is performed when the reliability index falls below a pre-defined threshold value for each performance. Lastly, LCCA is carried out to compare the long-term cost-effectiveness of using different materials in design and repair of the RC bridges. LCCA is typically calculated based on the present value of cost (PVC) and it
includes the costs of initial construction, routine inspections, non-destructive evaluation, protection, repair, overlaying, and replacement based on the following formulation:

\[
PVC = C_{initial} + \sum_{i=1}^{T} \frac{C_i}{(1+r)^i}
\]

(5.3)

where \( C_{initial} \) is the initial cost of construction which depends on the geometrical properties and the type of materials used in the design process, \( C_i \) is the \( i^{th} \) expenditure at time \( t \) (years) that includes the costs of inspection, evaluation, protection, repair, overlaying, and replacement required for the RC structure during its service life time, \( T \) (years), and \( r \) is the discount rate.

5.5. Illustration

5.5.1. Optimum Design of the RC Bridges using the Proposed RBDO Procedure

As an illustration, the proposed LCCA procedure is used in this section for selection of the optimum material(s) for design and repair of an interior T-beam of a RC bridge, previously illustrated in Section 3.5. This T-beam is re-designed in this study based on the RBDO procedure described in Eq. (5.2) using six different groups of materials, including three NSCs with three different rebars (they are NSC-BS, NSC-EC, and NSC-SS) and three HPCs with BS rebars with three different pozzolans (they are HPC-SF-BS that contains 10%SF as a replacement of cement, HPC-SL-BS that contains 40%SL as a replacement of cement, and HPC-FA-BS that contains 20%FA as a replacement of cement). Note that the amounts of pozzolans in the HPCs are assumed based on typical amounts used in the practical projects. The variables to be designed are the concrete clear cover, \( C_c \), the thickness of the slab, \( t_f \), the width of the web, \( b_w \), the height of the beam, \( h \), the cross-sectional area of longitudinal tensile steel reinforcement, \( A_{st} \), and the spacing between stirrups (\( s \)). Again, since using only one spacing between stirrups along the beam span is not economical, half-span of the beam is divided into three intervals (each interval has a length of \( l/6 \) with stirrups spacing of \( s_1, s_2, \) and \( s_3 \)) and the shear capacity in each interval is checked (Frangopol et al. 1997). Note that based on the AASHTO Article 4.6.2.6, for \( S/l \leq 0.32 \), the effective width of the beam flange, \( b_{eff} \), is \( b_{eff} = S = 230 \)cm, as shown in Figure 3.15(b). Thus, \( b_{eff} \) is already known and it is not assumed as a design variable.

The list of variables used in the design process are already shown in Table 4.1 and described in Section 4.4. The new variables that are used in this section include yield strength of longitudinal and transverse SS
rebars, $f_{y,SS}$, which is assumed as 621Mpa and compressive strength of HPC, which is assumed as $f'_{c, HPC} = 70$Mpa for all HPC mixtures despite the type and amount of the pozzolans used in the concrete mix design. The standard deviation (SD) and distribution for all variables are already described in Section 4.4.

In the RBDO procedure, the flexural failure mode is used as a probabilistic constraint for the optimal design of the beams. Note that the beams are designed without considering corrosion effect (i.e., $t = 0$). Therefore, the ultimate limit state function in Eq. (4.11) for flexural failure mode can be expressed as:

$$g_1(x) = M_f - (M_D + M_L)$$

(5.4)

where $M_f$ is the flexural moment capacity of the intact T-beam that is calculated based on AASHTO and by setting $t = 0$ in Eq. (4.14), $M_D$ is the flexural moment (demand) due to dead load (D), and $M_L$ is the flexural moment (demands) due to live load (L), which have been already described in Section 4.4. Thus, for this illustration, the RBDO procedure can be expressed as:

$$\begin{align*}
\text{minimize} & \quad C_{\text{initial}} \\
\text{probabilistic constraint:} & \quad \beta_1 \geq \beta_{T1} \\
\text{RBDO} & \quad \text{subjected to:} \\
\text{deterministic constraint:} & \quad \begin{align*}
(I) & \quad 0.42 \frac{C}{d} \geq 0 \\
(II) & \quad \min(M_f, 1.33M_u) - M_{cr} \geq 0 \\
(III) & \quad (\Delta F)_{TH} - \gamma(\Delta f) \geq 0 \\
(IV) & \quad 4.83 \frac{\gamma_f}{\beta_s f_{ss}} + 2d_c - s \geq 0 \\
(V) & \quad \Phi_f [V_c + V_s] - V_u \geq 0
\end{align*}
\end{align*}$$

(5.5)

where $\beta_1$ is the flexural reliability index based on the ultimate limit state function of $g_1$ defined in Eq. (5.4), $\beta_{T1}$ is the target reliability index for ultimate limit state function and is recommended as $\beta_{T1} = 3.5$ by AASHTO LRFD design code (2012). Other constraints and their parameters have been discussed in Section 4.4. The initial construction cost, $C_{\text{initial}}$, in Eq. (5.5), can be calculated as follows, which is similar to what we used in Eq. (4.19):

$$C_{\text{initial}} = \alpha \left( 0.75V_s \cdot \rho_s \cdot C_s \right) + \left( V_{st} \cdot \rho_{st} \cdot C_s \right) + \left( V_c \cdot C_c \right)$$

(5.6)

where $\alpha$ is the modification factor to take into account the effect of the type of steel or concrete on the development length of the rebar and according to AASHTO LRFD design specifications (2012), its values for NSC-BS, NSC-EC, NSC-SS, and HPC-BS are assumed as 1, 1.5, $f_{yd}f_{ySS} = 0.67$, and $(f'_{c,NSC}/f'_{c,HPC})^{0.5} =$
0.63, respectively, $0.75 \cdot V_s \cdot \rho_s \cdot C_s$ is the total cost of longitudinal steel rebars (again, assuming that half of the total amount of the bending rebar is placed only within the area of maximum moment (that is a half span) with the other half of the bending rebar spanning the full length of the beam (Frangopol et al. 1997)), in which $V_s$ is the total volume of tensile reinforcing rebars in the T-beam ($m^3$), $\rho_s$ is the density of the rebar that is assumed as 7850kg/m$^3$ for all steel rebars, $C_s$ is the cost of rebar considering the installment costs (that is assumed as $1.92/kg, $2.14/kg, and $6.33/kg for BS rebars, EC rebars, and SS rebars, respectively (O’Reilly et al. 2011)), $V_{st} \cdot \rho_{st} \cdot C_s$ is the total cost of stirrups, in which $V_{st}$ is the total volume of stirrups along the span, $V_c \cdot C_C$ is the total cost of concrete, in which $V_c$ is the volume of the one interior T-beam ($m^3$) to be designed by the RBDO procedure, and $C_C$ is the cost of the concrete that is assumed as $736/m^3$ for NSC (O’Reilly et al. 2011). For HPC materials, different factors should be considered in estimating the cost, such as the type of pozzolan, the location of the construction site, the storage space, etc. (Bouzoubaa et al. 2004). The cost of HPC containing either SL (HPC-SL) or FA (HPC-FA) pozzolans are quite similar, while the cost of HPC containing 10%SF (HPC-SF) may be assumed as 5% more than the cost of HPC-SL or HPC-FA materials (Farshadfar 2017). Also, in average, it can be assumed that the cost of HPC is 25% higher than that of NSC (Daigle and Lounis 2006). Thus, in this study, the average costs of both HPC-SL and HPC-FA are assumed as $920/m^3$, and the average cost of HPC-SF is assumed as $966/m^3$. Given that, only five T-beams out of six T-beams will be designed based on RBDO procedure, since all variables and the cost of materials for both HPC-SL-BS and HPC-FA-BS beams are same.

5.5.2. Input Data for Reliability Analyses and LCCA

After optimum design of the T-beams, in this section, the time-dependent structural performance of the beams considering the effects of corrosion is evaluated. Two limit state functions are considered for the reliability analyses: one is ultimate limit state in terms of flexural strength of the T-beam, $g_1(x, t)$, which in calculated based on Eq. (4.13), and the other one is serviceability limit state in terms of crack width, $g_2(x, t)$, which is calculated based on Eq. (4.16). Again, it is assumed that all tensile rebars are subjected to uniform corrosion all the way through the rebar. A service life of 75 years is considered in the reliability analyses as proposed by AASHTO (2012). It is expected that the reliability index of $\beta(t)$, which is used in this study to measure the time-dependent performance of the structure, decreases with time due to corrosion. The repair
is conducted on the structure when $\beta$ reaches a threshold value of $\beta_T$ for each performance. To ensure the compatibility between the repair material and substrate, it is assumed that each bridge section will be repaired using the repair material similar to the substrate concrete in the bridge (Ghasemzadeh et al. 2013).

Table 5.1 shows the list of the random variables used in the time-dependent reliability analysis. Note that SD for geometrical characteristics, mechanical properties, and the dead and live loads are already defined in Table 4.1. Again, it is assumed that the bridge is located at a northeast state (such as Ohio). Thus, the average annual temperature of 286K with standard deviation of 8K is used here based on the temperature data reported for the State of Ohio. Also, as mentioned earlier, for the State of Ohio, Weyers et al. (1993) recommended the chloride surface content of $5.9\text{kg/m}^3 \leq C_s \leq 8.9\text{kg/m}^3$ and chloride threshold value of $C_{th,BS} = 1\text{kg/m}^3$ for BS rebar. The values of $C_s$ and $C_{th,BS}$ for other states can also be found in the report (Weyers et al. 1993). For EC and SS rebar, based on the study by O’Reilly et al. (2011), one may assume $C_{th,EC} = 4.6C_{th,BS} = 4.6\text{kg/m}^3$ and $C_{th,SS} = 16.7C_{th,BS} = 16.7\text{kg/m}^3$. The SD for the chloride threshold values is calculated based on the coefficient of variation of 0.19 recommended by Bastidas-Arteaga et al. (2009). Model errors can also be found in the reference (Liu 1996).

<table>
<thead>
<tr>
<th>Type</th>
<th>Random variable</th>
<th>Mean</th>
<th>SD</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environmental</td>
<td>$T$ (K)</td>
<td>286</td>
<td>8</td>
<td>Normal</td>
</tr>
<tr>
<td>Corrosion parameters</td>
<td>$C_t$ (kg/m$^3$)</td>
<td>7.4</td>
<td>1.5</td>
<td>Normal</td>
</tr>
<tr>
<td></td>
<td>$C_{th,BS}$ (kg/m$^3$)</td>
<td>1.0</td>
<td>0.19</td>
<td>Uniform</td>
</tr>
<tr>
<td></td>
<td>$C_{th,EC}$ (kg/m$^3$)</td>
<td>4.6</td>
<td>0.87</td>
<td>Uniform</td>
</tr>
<tr>
<td></td>
<td>$C_{th,SS}$ (kg/m$^3$)</td>
<td>16.7</td>
<td>3.17</td>
<td>Uniform</td>
</tr>
<tr>
<td>Model error</td>
<td>$\sigma_{corr}$</td>
<td>0.33</td>
<td></td>
<td>Normal</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{Rs}$</td>
<td>0.12</td>
<td></td>
<td>Normal</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{acid}$</td>
<td>0.12</td>
<td></td>
<td>Normal</td>
</tr>
<tr>
<td>Crack width limit</td>
<td>$w_{cr,limit}$ (mm)</td>
<td>0.5</td>
<td>0.1</td>
<td>Normal</td>
</tr>
</tbody>
</table>

Eq. (5.3) is used for conducting LCCA and calculating PVC in the designed T-beam sections. In this equation, $C_{initial}$ is calculated based on Eq. (5.6), and $C_t$ is calculated based on the cost of routine inspections, non-destructive evaluation, protection, repair, and overlaying. No replacement of the whole structure will be considered in this study. It is assumed that the routine inspections cost is $2/\text{m}^3$ and they are scheduled at 2-year intervals for all T-beams (Cusson et al. 2010). The cost of non-destructive evaluation and protection activities is assumed as $40/\text{m}^3$ with a starting year of 5 for both NSC and HPC bridge beams, and scheduled
time interval of 5 years and 10 years for NSC and HPC bridge beams, respectively (Cusson et al. 2010). Note that the number of non-destructive evaluation and protection activities are assumed less for HPC beams compared to NSC beams, since HPC materials have denser matrix and lower permeability compared to NSCs, thus they can significantly delay the corrosion initiation and corrosion propagation phases (Ghasemzadeh et al. 2013), and perform better in terms of shrinkage cracking and freeze-thaw damage (Cusson et al. 2010). For NSC-SS beam section, no non-destructive evaluation and protection activities is assumed, since SS rebars can provide a more than 100-years maintenance-free service life (Kahl 2012). The cost of patch repair and overlaying are assumed as $200/m² and $600/m², respectively, same for both NSC and HPC materials (Daigle & Lounis 2006). Lastly, a discount rate of \( r = 2\% \) is used in calculating PVC.

5.6. Results and Discussion

5.6.1. Optimal T-beams Designed Based on the RBDO Procedure

As mentioned earlier, the interior T-beam shown in Figure 3.15 had been already designed by Taly (2014) based on the LRFD design specifications (2012). The values reported in the reference for \( C_b, t_f, b_w, h, \) and \( A_{st} \) are 5cm, 20cm, 55cm, 110cm, and 101cm², respectively. For shear stirrups, different configurations of spacing have been considered by the author, which can be found in the reference (Taly 2014). Using the random variables shown in Table 4.1 in the reliability analysis, the reliability index in terms of flexural performance for the example design by Taly (2014) will be \( \beta_1 = 3.6 \), same as the one we found through the RBDO procedure, which satisfies the minimum target reliability index of \( \beta_T = 3.5 \) provided by AASHTO (2012). Furthermore, the initial cost of materials for this design will be approximately \( C_{initial} = $12300 \) based on Eq. (5.6) and assuming the same cost of materials considered for RBDO procedure in this study.

Table 5.2 shows the optimum design parameters obtained from the RBDO procedure for the designed T-beams. As mentioned earlier, since all design variables and cost of materials are same for HPC-SL-BS and HPC-FA-BS, the T-beams designed based on these two materials have the same design parameters and the reliability index. As can be seen in Table 5.2, the RBDO procedure used in this study has designed the all beams based on \( \beta_1 \geq 3.5 \) with minimum possible initial costs. As a comparison, the initial optimal design cost of one interior NSC-BS beam is about $10700, while this value is $12300 for the same beam designed
by Taly (2014) without using any optimization techniques, which shows that the proposed RBDO procedure can save about 15% in the initial costs of materials. Note that although based on the RBDO results, it may seem that the procedure adopt only the lower-bound values for geometrical properties in optimum design of the beams, it is certainly not always the case and depending on the type of the bridge, the magnitude of applied loads, and the values used as lower-bounds, the design procedure and the optimum geometrical values can change from one case to another, as found in Section 4.5.

The results in Table 5.2 further show that for the T-beams designed based on NSC materials, using either EC or SS rebars in the design will approximately increase the initial cost of materials by 10% compared to using BS rebars. It is interesting to note that it was expected that the cost of SS rebars become significantly higher compared to BS or EC rebars. But, since these rebars have high yield strengths ($f_{y, SS} = 621$ MPa) compared to BS or EC rebars ($f_{y, BS} = 414$ MPa) and also require shorter development length, using the proposed RBDO procedure in this study, it is possible to find an optimum solution with acceptable initial cost for these types of materials. For the T-beams designed using HPC materials, the initial cost of materials increased by 18% when SF pozzolan is used in the mixture, and increased by 14% when either SL or FA pozzolans are used in the mixture. Also, the optimum design results show that HPC-SF-BS materials have higher initial costs compared to NSC-EC and NSC-SS materials. Note that this conclusion is valid, only if we assume that the HPC materials have an average initial cost of 25% more than that of NSC materials. Therefore, the readers are encouraged to input their specific data in the RBDO design procedure.

Overall, when corrosion effects are not considered in the design, as expected, NSC-BS are the cheapest materials for construction of the illustrated RC bridge, followed by NSC-EC/NSC-SS, HPC-SBS/HPC-FA-BS, and HPC-SF-BS materials.

| Table 5.2. The mean value of optimal design variables obtained from the RBDO procedure |
|-----------------------------------------------|-------------------|---------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Materials          | $C_{l}$ (cm) | $t_{l}$ (cm) | $b_{w}$ (cm) | $h$ (cm) | $A_{0l}$ (cm$^2$) | $s_{1}$ (cm) | $s_{2}$ (cm) | $s_{3}$ (cm) | $\beta_{l}$ ($t=0$) | $C_{initial}$ ($\$) |
| NSC-BS              | 5            | 20           | 35           | 110      | 82               | 15           | 15           | 25           | 3.6               | 10700              |
| NSC-EC              | 5            | 20           | 35           | 110      | 85               | 15           | 20           | 25           | 3.6               | 11800              |
| NSC-SS              | 5            | 20           | 35           | 110      | 60               | 25           | 27.5         | 27.5         | 3.7               | 11800              |
| HPC-SF-BS           | 5            | 20           | 35           | 110      | 89               | 17.5         | 17.5         | 17.5         | 3.7               | 12700              |
| HPC-SL-BS           | 5            | 20           | 35           | 110      | 91               | 17.5         | 20           | 25           | 3.6               | 12200              |
| HPC-FA-BS           | 5            | 20           | 35           | 110      | 91               | 17.5         | 20           | 25           | 3.6               | 12200              |
5.6.2. Time-Dependent Reliability Analysis of Designed T-beams considering Corrosion Effects

Before corrosion initiates ($t = 0$), the flexural reliability index, $\beta_1$, for the designed T-beams would equal the values shown in Table 5.2, which shows a reliable design for all beams. For design of RC structures based on the serviceability performance, FIB-MC10 design code (2012) recommends the minimum target reliability index of $\beta_{T2} = 1.5$. Using the random variables in Table 4.1 and assuming $t = 0$, the serviceability reliability index (crack width), $\beta_2$, based on the limit state function shown in Eq. (4.16) will be $\beta_2 = 5.0$ for all beams. Therefore, all designed beams are also reliable in terms of the serviceability performance at $t = 0$.

When corrosion initiates, the reliability of the beams may decrease with time for both ultimate and serviceability limit states due to corrosion effects. Figure 5.1 shows the time-dependent reliability indexes in terms of flexural and cracking failure modes for all designed six beams. From the results, both flexural and cracking performances for NSC-BS beam significantly decrease with time. This is due to the high permeability of NSC material and low chloride threshold level of BS rebars, which can accelerate the corrosion process. In addition, it can be seen that the decrease in the flexural reliability index is slow over time (roughly linear), while the serviceability reliability performance decreases dramatically, particularly for the first five years.

In this study, the target reliability indexes of $\beta_{T1} = 3.0$ (corresponding to failure probability of 0.1%) and $\beta_{T2} = 1.28$ (corresponding to failure probability of 10%) are defined as repair criteria for flexural and serviceability performances of the beams, respectively. These threshold reliability indexes are shown in Figure 5.1 (as solid red line for flexural performance, $\beta_{T1} = 3.0$, and dashed red line for cracking performances, $\beta_{T2} = 1.28$). With the pre-defined values of $\beta_{T1}$ and $\beta_{T2}$, the first time-to-repair of the NSC-BS beam is 5 years considering serviceability performance only and 41 years considering the ultimate performance only, as shown in Figure 5.1. Therefore, serviceability performance governs the first time-to-repair of the NSC-BS beam.

When NSC-EC materials are used in the beam, the epoxy coating increases the resistance of rebar against corrosion through isolating the steel from chloride irons. Therefore, the high chloride threshold value in EC rebars significantly delays the corrosion initiation time and its propagation. Accordingly, the first time-to-repair of the structure increases considerably. As can be seen in Figure 5.1, the time-to-repair
of NSC-EC beam is 16 years and 52 years for the serviceability and flexural performances, respectively, which indicates a significant improvement (11 years delay) in both performances compared to NSC-BS beam. Note that, in this figure the results show a sudden drop of serviceability performance of the beam after 16 years. This is because within the first 15 years, the coefficient of diffusion calculated based on Eq. (5.1) is negligible (even lower than the coefficient of diffusion of NSC beam in the second year) due to the high chloride threshold value of EC rebars. Therefore, within the first 15 years, negligible changes can be observed in the serviceability and flexural performances of the NSC-EC beam. After 15 years, however, the serviceability performance drops suddenly due to the significant increase in the amount of chloride content at the surface of rebar and reaching its value to the chloride threshold value (note that similar behavior also observed for the NSC beam, where its serviceability performance decreased suddenly in the second year). For NSC-EC beam, the serviceability performance governs the first time-to-repair of the structure as well.

When SS rebars is used, as expected, no reduction in ultimate or serviceability performances of the beam can be seen during the service life of 75 years. The reason for that is because in SS rebars, a very thin stable film on the surface of steel forms after reaction with chloride ions, which protects it from additional corrosion. Therefore, SS rebars are highly corrosion resistant materials and have very high levels of chloride threshold values compared to uncoated BS rebars, as can be found in Table 5.1. Therefore, based on the reliability analysis results shown in Figure 5.1, SS beam doesn’t need any repair operations in a service life of 75 years. This result is in consistent with a maintenance-free assumption made for this beam in this study and has been confirmed by other researches (e.g., Kahl 2012).

For HPC beams, since the water-to-cementitious ratio (w/cm) of HPCs are low, these materials have higher concrete compressive strength and lower permeability. Accordingly, the chloride diffusion process in HPCs is much slower than NSCs. Therefore, as expected, it has resulted in a significant postponement of the first time-to-repair of the structure in terms of serviceability performance from 5 years in NSC-BS beam to 60 years, 43 years, and 31 years in HPC-SF-BS, HPC-SL-BS, and HPC-FA-BS beams, respectively. Furthermore, it can be observed that no repair is required to be applied to the HPC beams in order to restore the flexural performance of the beams during the service life of 75 years. Therefore, similar to NSC-BS and NSC-EC beams, the serviceability performance governs the first time-to-repair of all HPC
beams as well. Again, the sudden drop in the serviceability performance of the beam is related to the rate of chloride diffusion change calculated based on Eq. (5.1).

Figure 5.1. Time-dependent reliability of the designed T-beams affected by corrosion in terms of flexural and cracking performances for a service life of 75 years
Furthermore, it is obvious from the results that the type of pozzolan used in the mix design of HPC is important in the long-term performance of the structure. In particular, using 10%SF in the mix design is very effective in decreasing the permeability and improving the durability of the concrete. The reason for that is because SF has ultrafine particles and hence, it has a very large specific surface area. Also, SF has a very high content of amorphous silicon dioxide. Therefore, it is a highly reactive pozzolanic material that can improve the microstructure of concrete by densifying its matrix, refining the pore structure of the cement, refining the paste-aggregate interfacial zone, and reaction with free-lime (calcium hydroxide) after hydration of cement (Ramezanianpour et al. 2015). The rate of hydration in FA is slower than SL and both are slower than SF. Therefore, they are less effective than SF in corrosive areas. For FA, it takes up to several months to fully develop the beneficial effects of hydration process such as high resistivity (Polder & Peelen 2002; Polder et al. 2012). Note that these effects have been captured in coefficient “m” of Eq. (5.1) in calculating of chloride diffusion of concrete containing different pozzolans, \( D_c \), and it verifies the reliability analysis results.

5.6.3. Time-Dependent LCCA and Selection of the Optimum Corrosion Management Strategy

After predicting the time-dependent performance and first time-to-repair of the T-beams, the number of repair operations and thus, the time-dependent PVC of the beams can be calculated based on Eq. (5.3). Note that for the repair of T-beams based on the serviceability performance, it is found from the reliability analysis that when the serviceability reliability index reaches its threshold value (i.e., \( \beta_2(t) = \beta_{T2} = 1.28 \)), the corrosion level in the rebar is not significant (less than 2%). Therefore, to re-store the serviceability performance of the structure, only changing the contaminated concrete beyond the longitudinal rebars will be sufficient and there is no need to replace the rebars or add any new rebars in the tensile zone of the beam.

On the other hand, when the ultimate reliability index reaches its threshold value (i.e., \( \beta_1(t) = \beta_{T1} = 3.0 \)), based on the reliability analysis, the corrosion level in the rebar is about 10%. In order to restore the flexural performance of the corroded structure, the corroded rebars need to be cleaned up and new rebars should be added to the tensile zone. Therefore, two repair approaches are considered in this study. The first approach refers to applying the repair after removing the damaged concrete around the corroded rebar and cleaning the surface of the corroded rebars, which will be applied when \( \beta_2(t) = \beta_{T2} = 1.28 \). The second approach refers to applying the repair after removing the contaminated concrete, cleaning the corroded
rebars, and adding new rebars, which will be applied when $\beta_1(t) = \beta_{T1} = 3.0$. In this study, we assume that the first approach refers to “patch-repair” process with a cost of $200/m^2$ and the second approach refers to “overlaying process” with a cost of $600/m^2$, as were defined earlier in the study. Also, we further assume that after applying four patch-repairs in the tensile zone of the beam based on $\beta_2(t) = \beta_{T2} = 1.28$ criterion, for the fifth repair, we apply overlay to the tensile zone of the beam (instead of another patch-repair), since the 50% of the tensile zone is already affected by corrosion and it implies a significant corroded area in the bridge (Cusson et al. 2010).

Considering these repair/overlaying criteria, the time-dependent flexural and cracking performances of the designed T-beams after applying patch-repair(s) and/or overlay(s) are shown in Figure 5.2. From this figure, as discussed earlier, no repair operations are assumed for the NSC-SS beam. For the other beams, 15 patch-repairs and 3 overlays for NSC-BS beam, 4 patch-repairs and one overlay for NSC-EC beam, only one patch-repair for either HPC-SF-BS or HPC-SL-BS beam, and only two patch-repairs for HPC-FA-BS beam are required during a service life of 75 years. An interesting result is observed for NSC-EC beam, in which the ultimate performance governs the fourth time-to-repair of the structure and as can be found later in the results (refer to Figure 5.3), it significantly increases the maintenance costs for this beam. Therefore, although in most cases the serviceability performance governs the time-to-repair of the structure, this result shows that both serviceability and ultimate performances should to be considered in time-dependent performance evaluation of the structure before and after applying repair.

It is worth mentioning that for the patch-repair operation, since the probability of failure is 10% (equivalent to $\beta = 1.28$), we may further assume that this probability of failure is equivalent to the cracking and spalling (and thus, patch-repair) in the 10% of the concrete surface in the tensile zone (Cusson et al. 2010). Therefore, in calculating the cost of the patch-repair based on serviceability performance, we assume that in each repair operation, we will apply the patch-repair on 10% of the bottom area of the T-beam. For overlaying, however, we assume that the entire area in the tensile zone will be overlaid. Considering these assumptions and based on the time and type of the repair operations shown in Figure 5.2, and assuming a discount rate of $r = 2\%$, we can calculate the time-dependent PVC of the beams based on Eq. (5.3). Figure 5.3 shows the time-dependent PVC results for the six beams investigated in this study considering a design service life of 75 years. As can be seen in this figure, although NSC-BS beam has
lower initial cost compared to other beams, due to the large number of periodic repair operations required for this beam, its maintenance costs increases dramatically over time.

Figure 5.2. Time-dependent reliability of the designed T-beams affected by corrosion in terms of flexural and cracking performances after applying patch-repair(s) and overlay(s) for a service life of 75 years.
The results of figure 5.3 further show that after 20 years, all other five groups of materials considered in this study for design of the illustrated T-beam (i.e., NSC-EC, NSC-SS, HPC-SF-BS, HPC-SL-BS, and HPC-FA-BS) can pay-off their higher initial costs, therefore they all can be used as cost-effective alternatives to NSC-BS materials. As discussed before, since NSC-SS beam does not need any repair operations within 75 years, the change in time-dependent PVC of NSC-SS beam is significantly slow over time (note that for this beam the routine inspections scheduled at every two year is considered in the cost analysis, therefore its PVC is not constant over time). For other materials, except NSC-EC, the rate of PVC increases smoothly and linearly over time. For NSC-EC materials, however, since the beam requires the overlaying operation in year 51, a sudden raise in PVC can be observed at that year.

![Figure 5.3. Time-dependent LCC of designed T-beams for a service life of 75 years](image)

The initial cost (i.e., $t = 0$) and the long-term PVC of all six beams ($t = 75$ years) are summarized in Table 5.3. Based on the results, the long-term maintenance costs of NSC-BS beam is about 64% of its initial cost, while this value is less than 2% for NSC-SS beam. If only long-term costs are considered, NSC-SS materials show lowest cost for design of the illustrated T-beam (32% less than NSC-BS materials), followed by NSC-HPC-SL-BS/HPC-FA-BS, HPC-SF-BS, and NSC-EC. Also, as can be seen in the results, both initial and long-term costs of HPC-SL-BS and HPC-FA-BS are very close (the values are rounded). Thus, the owner may decide to choose either 40%SL or 20%FA in the mix design of HPC, depending on the local prices and accessibility to the supplementary materials.
Overall, based on the results of Table 4, NSC-SS materials can be selected as the optimum materials for design of an interior T-beam of the illustrated RC bridge in Figure 3.15, since they have reasonably both low initial and long-term costs. Authors, however, would like to emphasize again that the main purpose of this study is to propose a reliability-based LCCA approach for selection of the optimum corrosion management strategy of using different materials in design and repair of the RC structures. The results here are just examples of how to use the proposed procedure. The readers are highly encouraged to incorporate their specific data and assumptions into the procedure and select the optimum strategy based on their own results.

Table 5.3. Initial costs and long-term PVC of the studied T-beams considering a service life of 75 years

<table>
<thead>
<tr>
<th>Cost</th>
<th>NSC-BS</th>
<th>NSC-EC</th>
<th>NSC-SS</th>
<th>HPC-SF-BS</th>
<th>HPC-SL-BS</th>
<th>HPC-FA-BS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost (C_{\text{initial}})</td>
<td>$10700</td>
<td>$11800</td>
<td>$11800</td>
<td>$12700</td>
<td>$12200</td>
<td>$12200</td>
</tr>
<tr>
<td>Long-term costs</td>
<td>$6900</td>
<td>$3100</td>
<td>$200</td>
<td>$1000</td>
<td>$1100</td>
<td>$1100</td>
</tr>
<tr>
<td>PVC in 75 years</td>
<td>$17600</td>
<td>$14900</td>
<td>$12000</td>
<td>$13700</td>
<td>$13300</td>
<td>$13300</td>
</tr>
</tbody>
</table>
CHAPTER VI

SUMMARY AND CONCLUSIONS

To reduce the corrosion related costs and increase the public safety, optimum corrosion management strategies considering the structural performance should be developed. In this study, first, a probabilistic model is developed to predict intact and corroded average bond strength ($\tau_{avg}$) as a function of corrosion level using nonlinear regression analysis based on a comprehensive database collected from the literature. The predictors consist of parameters selected from the literature as well and has been shown to influence bond strength. A model selection procedure is applied to select the predictors that are needed for an accurate, unbiased, and practical model. The proposed probabilistic model considers statistical uncertainties and model error. To evaluate the importance of bond modeling in the flexural behavior of intact and corroded RC beams, the proposed bond strength prediction is employed to the FE models with various development lengths. To assure the ductile behavior in the intact and corroded RC beams, the proposed bond strength prediction is then used to calculate the minimum development length ($l_{d,min}$) of tensile rebars for various corrosion levels. Then, $l_{d,min}$ calculated based on the proposed bond strength model is compared with the one based on the ACI 318-11 (2011) through the nonlinear FE analysis of intact and corroded RC beams.

Then, using the proposed probabilistic bond strength model, an analytical procedure is proposed to predict the nonlinear load-deflection behavior of RC beams under monotonic loading. This procedure can be sued for facilitating the reliability analysis in the RC bridges, since available techniques are either complicated to use (such as finite element analysis) or do not consider the effects of important parameters in the analysis such the bond stress-slip behavior at the steel-concrete interface. The proposed procedure uses the elongation of tensile rebar between flexural cracks based on the steel strain and the rebar slippage. The advantage of this proposed procedures are:

1) It considers any given bond stress-slip relationship.

2) It is capable of predicting the flexural behavior of intact and corroded RC beams with or without lap splices
3) It gives more accurate results for the RC beams with bond failure mode.

4) It can be applied to the beams under any loading pattern.

In the next step, a reliability-based multi-objective design optimization (RB-MODO) procedure is developed for optimum design of RC bridges affected by corrosion. The procedure adopts non-dominated sorting algorithm-II (NSGA-II) in combination with first-order reliability method (FORM) to simultaneously maximize the reliability of the structure and minimize the associated costs. As an illustration, the proposed RB-MODO procedure is used for optimum design of T-beams from an RC bridge through maximizing the flexural reliability of the structure and minimizing the material costs subjected to various probabilistic and deterministic random variables. The structure is designed with and without considering corrosion effect considering different design service lives. The three key features of this study are:

1) It considers both serviceability and ultimate limit states in reliability-based design of the RC bridges affected by corrosion.

2) A decision on the optimum design can be made based on the set of reliability-cost solutions through utilizing a multi-objective optimization technique.

3) It evaluates the cost-effectiveness of different materials in design of RC bridges affected by corrosion given a service life.

Lastly, a reliability-based life-cycle-cost-analysis (LCCA) module is proposed to compare the time-dependent cost effectiveness of six commonly-used groups of materials in design and repair of the RC bridges in corrosive areas, including normal strength concrete (NSC) with traditional black steel (BS) rebars (NSC-BS), NSC with epoxy coated (EC) rebars (NSC-EC), NSC with stainless steel (SS) rebars (NSC-SS), high performance concrete (HPC) containing Silica Fume (SF) with BS rebars (HPC-SF-BS), HPC containing Slag (SL) with BS rebars (HPC-SL-BS), and HPC containing Fly Ash (FA) with BS rebars (HPC-FA-BS). As an illustration, these materials are used for design of an illustrated interior T-beam of an RC bridge. A reliability-based design optimization (RBDO) technique is developed for optimum initial design of the T-beams through minimizing the initial construction cost taking into account the probabilistic and deterministic constraints. Then, the time-dependent reliability analysis in conducted to predict the time-dependent performance of the design beams using two limit states: ultimate limit state in terms of flexural resistance and serviceability limit state in terms of cracking in the concrete cover. Based on the time-dependent
reliability analysis results, the type and number of repair operations are determined for each beam. Finally, the time-dependent present value of cost (PVC) for each beam is calculated and the optimum corrosion management strategy for the illustrated T-beam is proposed.

- The probabilistic bond strength model proposed in this study has a simple formulation and considers the contribution of important parameters in the prediction (e.g. corrosion level, stirrup effect).

- The comparison between the predicted average bond strength values and the experimental results collected from the literature showed that the proposed bond strength model has sufficiently accurate and unbiased prediction.

- The importance of bond strength deterioration modeling on the flexural behavior of RC structures depends on the provided development length ($l_d$) in the beam. If $l_d$ is insufficient, an accurate modeling of the bond behavior is critical whereas it can significantly affect the stiffness, load-carrying capacity, and even failure mode of the structure. Otherwise, the effects of corrosion on cross sectional area and yielding strength of the rebars dominate the evaluation of the flexural performance.

- It is shown that the FE models with $l_{d,\text{min}}$ based on the proposed probabilistic model for $\tau_{\text{avg}}$ have the ductile behavior when the actual $\tau_{\text{avg}}$ lies in the upper-bound of the prediction.

- On the other hand, the FE models with $l_{d,\text{min}}$ based on ACI 318-11 code provisions (2011) are failed by de-bonding of rebar before yielding (brittle bond failure), indicating that the ACI 318-11 (2011) underestimates the minimum $l_d$ required for intact and corroded RC beams.

- To ensure the yielding of rebar in the intact and corroded FE beams, a multiplication safety factor, $\phi$, is suggested to apply to $l_{d,\text{min}}$ based on the proposed probabilistic model for $\tau_{\text{avg}}$, and it is found that $\phi = 1.15$ for the investigated FE models in this study.

- The proposed analytical procedure for predicting the nonlinear load-deflection behavior of the RC beams is computationally efficient and considers the contribution of important parameters in the flexural behavior prediction of RC beams (e.g., bond stress-slip behavior, corrosion deterioration).
Compared with the analytical procedure proposed by Maaddawy et al. (2005b) for the beams without lap splices, the proposed procedure in this study can provide better predictions in terms of yielding and ultimate points, particularly for the beams with brittle bond failure mode.

The proposed procedure can be successfully applied to evaluate the nonlinear structural behavior of intact and corroded RC bridge beams under transverse loading.

The initial material costs needed for optimum design of the structure considering corrosion effect increase with increasing the target reliability index and/or the design service life.

If the corrosion effect is ignored, the initial cost needed for optimum design of T-beams using NSC-EC and HPC-BS are respectively 12% and 15% more than the initial costs using NSC-BS.

When NSC-BS is used in the structure design considering corrosion effect, the cost-reliability Pareto fronts show significant sensitivity to the design service life. Therefore, this group of materials is not recommended to use in the bridges located in corrosive areas.

When either NSC-EC or HPC-BS is used in the structure design considering corrosion effect, the cost-reliability Pareto fronts are not sensitive to the design service life that shows the capability of these materials in delaying the corrosion initiation and propagation processes.

Considering the corrosion effect, for a service life of 100 years, the initial cost needed for optimum design of T-beams using NSC-EC is approximately 14% more than the $CM$ required for optimum design using HPC-BS. Therefore, HPC-BS materials are recommended for the construction of the illustrated RC bridge.

If the corrosion effects are not considered in the design of the structure, for the materials used in this study, NSC-BS materials have lowest initial costs followed by NSC-EC/NSC-SS, HPC-SBS/HPC-FA-BS, and HPC-SF-BS materials for optimum design of the illustrated T-beam.

Although serviceability performance typically governs the time-to-repair of the structure, ultimate performance can be critical, particularly in the long-run. Thus, both serviceability and ultimate performances should be considered in predicting the time-to-repair of the RC structures.

For NSC-BS beam, the serviceability and ultimate performances decrease over time due to the corrosion effects. In particular, serviceability performance decreases significantly and thus governs the first time-to-repair of the structure.
Compared with NSC-BS materials, using NSC-EC can postpone the time-to-repair of the structure in terms of both serviceability and flexural performances for 11 years.

When NSC-SS materials are used in the design, no reduction was observed in time-dependent serviceability or flexural performance of the structure. Thus, the structure will not need any maintenance during its service life.

When considering the ultimate limit state only, no repair operations are needed for HPC materials, as their lower permeability and denser matrix can significantly delay the corrosion initiation and propagation phases.

When considering both serviceability and flexural performances, the number of patch-repair operations required in a service life of 75 years for NSC-BS, NSC-EC, HPC-SF-BS, HPC-SL-BS, and HPC-FA-BS beams are 15, 4, 1, 1, and 2, respectively.

The number of overlays required in a service life of 75 years for NSC-BS and NSC-EC beams are 3 and 1, respectively. No overlays are required for other beams.

Using 10%SF pozzolan as a replacement of cement in the mix design of HPC is more effective in delaying the corrosion process than using 40%SL or 20%FA pozzolans.

Based on the PVC results, although the initial cost of NSC-BS beam is low, its long-term costs is very high due to the large number of repair operations required for this beam. In this study, the maintenance costs of NSC-BS beam is estimated to be 64% of its initial design, for a period of 75 years.

Using HPC material is effective than using EC rebars in design of the RC structures in corrosive areas.

Based on the results, among the six investigated groups of materials considered in this study, NSC-SS are the optimum ones due to their fairly low both initial and long-term costs.

The proposed reliability-based LCCA module can be used as a suitable tool for design of durable and cost-effective RC structures. It is, however, recommended that the readers incorporate their own data when using this procedure in calculating PVC.
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