MODELING DUCTILE DAMAGE OF METALLIC MATERIALS

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MODELING DUCTILE DAMAGE OF METALLIC MATERIALS

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In this dissertation, a comprehensive study of ductile damage of metallic materials is presented, covering constitutive modeling, numerical implementation and model calibration and verification.

As the first part of this dissertation, a pressure-insensitive plasticity model, expressed as a function of the second and third invariants of the stress deviator ($J_2$ and $J_3$), is presented. Depending on whether the power of the $J_3$ term is odd or even, the proposed model can capture either the tension-compression strength-differential (S-D) effect or the torsion-tension strength-differential effect of the material. The plasticity model with an odd power to the $J_3$ item has been calibrated and validated using measured experimental data of a $\beta$-treated Zircaloy-4 with a wide range of triaxiality and Lode parameter values. Results show that this model captures the strong strength-differential (S-D) effect in the material. The plasticity model with an even power to the $J_3$ item is able to capture the isotropic plastic behavior of a stainless steel Nitronic 40, under various stress states with good accuracy and computational efficiency. Next, the effect of the material’s plasticity behavior on the ductile damage process is studied by conducting a series of unit cell analyses of a void-containing representative material volume (RMV), where the plastic response of the matrix material is governed by the $J_2$-$J_3$ dependent plasticity model.

To simulate the ductile damage process in anisotropic materials, a new constitutive model, which combines the models proposed by Zhou et al. (2014) and
Stewart and Cazacu (2011), is developed and employed to study the plasticity and ductile fracture behavior of a commercially pure titanium (CP Ti). In particular, a Gurson-type porous material model is modified by coupling two damage parameters, accounting for the void damage and the shear damage respectively, into the yield function and the flow potential. The plastic anisotropy and tension-compression asymmetry exhibited by CP Ti are accounted for by a plasticity model based on the linear transformation of the stress deviator. The theoretical model is implemented in the general purpose finite element software ABAQUS via a user defined subroutine and calibrated using experimental data. Good comparisons are observed between model predictions and experimental results for a series of specimens in different orientations and experiencing a wide range of stress states. The model is shown to capture the effect of stress state and the change of fracture mechanism. The results also reveal the important effect of the plastic anisotropy and tension-compression asymmetry on the ductile damage process. Literature review indicates that this is the first time to simulate failure under shear dominated conditions in an anisotropic and tension-compression asymmetric material.
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CHAPTER I
INTRODUCTION

1.1 Motivations and Background

Fracture is one of the most important concepts in many fields of engineering and our daily life. Although not always noticeable, any material may be involved with fracture. Convincingly, Figure 1.1 shows several failure examples in the various situations. Back in 1983, the National Bureau of Standards estimated the costs for failure due to fracture alone to be $119 billion dollars per year in 1982 (Roylance, 2001). The costs in dollars are important, but the cost of many failures causing human life loss and injury is infinitely more so. Failures have occurred for many reasons, including uncertainties in the loading or environment, defects in the materials, inadequacies in design, and deficiencies in construction or maintenance. There is never overstressing the issues of how the greatest importance of the fracture failure will affect many people’s life and sometimes cause huge amount of property and even life loss, especially during disasters as shown in the following Figure 1.1: Figure 1.1a shows the USS Cole under attack by a skiff loaded with explosives; Figure 1.1b shows the plastic ductile deformation of a pipe ductile failure; Figure 1.1c shows I-35W bridge aerial view collapse conditions in Minnesota; and Figure 1.1d shows the Southwest Boeing 737 fuselage Mode - I fracture failure due to long cycles of fatigue crack propagation.
Figure 1.1 (a) The USS Cole (b) Pipe ductile failure (c) Bridge collapsed in Minnesota on the Interstate 35W (d) Crack propagation inside metal Boeing 737 fuselage

Prediction of ductile fracture of metal in engineering structure is a very important topic in the aerospace, automotive and military industries. For example, the automotive industry has put many efforts on the research of finding a material to reduce weight of the automotive and has a good material strength but at the reduced cost. One solution to this problem is to use high strength steels as the main body part materials. Because the material has a high strength, the ductility of the material tends to be low. Therefore, fracture will be a challenge when using these materials.

The main concerns of fracture are crack initiation, crack path and crack bifurcation. On a macro-scale fracture is characterized by surface discontinuities in the displacement field. The three fundamental fracture mechanisms are represented in Figure 1.2. Mode I, II and III correspond to an opening or tensile mode, Figure 1.2a, to a sliding
model, Figure 1.2b, and to a tearing mode, Figure 1.2c, respectively. Any other fracture mechanism is simply a combination of these elementary modes.

![Fracture Mechanisms](image)

Figure 1.2 Fracture mechanisms and fracture modes

On the other hand, on a micro-scale the fracture phenomenon becomes more complicated than it appears to the human eye. The best evidence is given by fractography. Fractography highlights how the inter-atomic structure as the crystalline structure and the material defects crucially affects the crack mechanism. A microscopic investigation of the fracture surfaces and their neighborhoods reveals numerous different crack states. Regardless of the fracture paths, there are essentially only four principal fracture modes: dimple rupture, decohesion rupture, cleavage, fatigue (Dudrová and Kabátová, 2008). Figure 1.3 shows typical fractographs of dimple rupture, cleavage, and decohesion rupture respectively.
Generally speaking, the intimate interrelation among the materials, the fracture modes (Figure 1.2), the load conditions and the microscopic failure mechanisms contribute to the macroscopic fracture. For metallic materials, it is reasonable to acknowledge that plasticity, damage and fracture are often closely related to each other. To highlight this point and characterize the complete ductile fracture process in metals, Milza et. al. (1997) conducted a series of tensile test for the cooper bar. Figure 1.4 shows the true stress-strain curves for notched specimen where the notch radius is 2 mm. The stress-strain responses are compared with optical micrographs in Figure 1.5, which allows to be identified by the marked points (circle, triangle, square) on the curve.
Figure 1.4 True stress-strain curve for 2mm notch round cooper bar specimen (after Milza et al., 1977), circle indicates void nucleation; triangle indicates void coalescence; square indicates crack formation.

(a) Void nucleation (circle point);

(b) Void coalescence (triangle point);

(c) Crack formation (square point)

Figure 1.5 Micrographs images (200μm) (Milza et al., 1997) corresponding to the different points on the stress strain curves of the notched round copper bar in Figure 1.4.

With advanced computer hardware and software, it is possible to model material processing, product design, product manufacturing, product performance in service, and
to analysis the failure mechanism. Moreover, modeling can be used as a research tool for a more fundamental understanding of physical phenomena that can result in the development of improved or new products. In any of these cases, a constitutive model, a mathematical description of a material behavior, is needed. The central theme of this dissertation will focus on modeling the ductile plastic deformation and damage accumulation and evolution. Obviously, it is impossible to develop constitutive models for industrial applications that can capture all the macroscopic and microscopic phenomena involved in plastic deformation and ductile fracture. Plasticity can be studied at various scales. Bottom-up approaches model the motion of the basic particles that constitute the material and the interaction among them. On the other hand, top-down approaches couple continuum mechanics descriptions to phenomenology and experimental calibration. In top-down approaches, mechanism-based concepts provide key insights for development of models and experiments are used to provide calibration of these models. For industrial applications, macroscopic models appear to be more appropriate. Because of the scale difference between the microstructure and an engineered component, the amount of microscopic material information necessary to store in a simulation would be enormous. It is not possible anyways to track all of the relevant microstructural features in detail. Therefore, it seems more appropriate to integrate them all in a few macroscopic variables. Microscopic models are more suitable as a guide for material design, as a tool for fundamental understanding of plasticity and for inferring suitable formulations at the macroscopic scale. The following sections provide briefly review metal plasticity models and ductile fracture models which provides general research background for the proposed dissertation research study.
1.2 Metal plasticity models

Two of the oldest and well accepted yield criteria for isotropic materials are due to Tresca and von Mises. Tresca proposed the first yield criteria in 1864, i.e. the maximum shear stress yield criterion. It states that yielding occurs when the maximum shear stress reaches a certain value $\tau_{\text{max}}$ (Tresca, 1864). Von Mises proposed a function for the constant distortion energy criterion in 1913 (von Mises, 1913). This has been the most widely used yield criteria and also known as the $J_2$ flow theory. In structural analysis, the classical $J_2$ flow plasticity has been overwhelmingly adopted to describe the plastic response of metallic alloys. Drucker (1949) proposed the yield criterion to describe the behavior of certain metals such as aluminum alloys for which the yield loci are located between the Tresca and Von Mises yield loci. These isotropic yield criteria mentioned above were all postulated on the basis of macroscopic experiments. Hershey (1954) and Hosford (1972) performed the polycrystalline simulations to achieve a microscopic yield criterion which is dependent on the principal value of the Cauchy stress. This yield criterion can be reduced to Tresca criterion and to von Mises criterion. In von Mises criterion, only the second invariant of the deviatoric stress tensor ($J_2$) controls yielding and plastic flow. Although the $J_2$ flow plasticity theory provides an extremely useful description of a large number of engineering materials, the key assumption made in this theory may not be valid for many other materials. Spitzig et al. (1975, 1976, 1984) conducted extensive experiments of high-strength metals under uniaxial tension and compression and found the plastic responses of these materials are pressure sensitive. Mohr-Coulomb and Drucker-Prager discussed the pressure dependent yield conditions which originally were developed for soil material but a lot of
publications show that these yield condition are capable of describing the metal plasticity (Spitzig and Richmond, 1984; Brownrigg et al. 1983). Brunig (1999) presented also an $I_1$-$J_2$ yield criterion similar to the Drucker-Prager (1952) yield condition in soil mechanics to describe the hydrostatic stress effect on the plastic response of materials. Bai and Wierzbicki (2008) postulated a general form of asymmetric metal plasticity by considering both the pressure sensitivity and the Lode dependence. The Lode angle parameter is related to the third deviatoric stress invariant ($J_3$). The proposed plasticity model is calibrated and validated on material aluminum 2024-T351. Cazacu and Barlat (2004) proposed an isotropic yield criterion to describe the asymmetry in yielding for tension and compression. Lee et al. (2008) modified the Drucker–Prager model to describe the high tension-compression asymmetry of magnesium sheet material. Gao et al. (2009, 2011) found that the plastic response of a 5083 aluminum alloy is stress state dependent and proposed an $I_1$-$J_2$-$J_3$ plasticity model that accurately predicts a wide range of experiments where specimens experience various stress states. Zhang et al. (2012) presented a series of first-order homogeneous functions of stress expressed in terms of the first invariant of the stress tensor ($I_1$) and the second and third invariants of the deviatoric stress tensor ($J_2$ and $J_3$) and demonstrated the application of these functions as a general form of the yield function for isotropic materials.

Under the thermo-mechanical processing, e.g. in rolled or extruded sheet, the convenient assumption of plastic isotropy was found to be unsuitable to a large extent. Metal sheets exhibit orthotropic symmetric propriety with the rolling direction, the transverse direction and the normal direction to the plane of the sheet. There are many anisotropic metal plasticity models which have been proposed in recent years, including
both phenomenological models at the macroscopic level and crystal plasticity models at the microstructure level. Crystal plasticity models at the microstructure level are beyond the scope of this dissertation. Barlat et al. (1991) adapted the isotropic Hershey (1954) and Hosford (1972) yield function for orthotropic materials through the linear transformation of the stress tensor. Similarly, Karafillis and Boyce (1993) obtained a more general anisotropic yield function by using the linear transformation of the stress tensor in conjunction with a linear combination of two isotropic yield functions. This criterion is a phenomenological plasticity model which has been developed for plastic anisotropy of metals after the original Hill model (1948). Cazacu and Barlat (2001) extended Drucker’s isotropic yield criterion to orthotropy. Cazacu et al. (2006) spread their isotropic yield criterion proposed in 2004 (Cazacu and Barlat, 2004) for orthotropic and asymmetry between yielding in-plane tension and compression materials.

1.3 Ductile fracture models

In the particular case of ductile materials, modeling the progressive internal material degradation and failure process has been the main focus of extensive research efforts over the past several decades. The formation of macroscopic cracks in metals is often considered as the result of the accumulation of damage within the material at the macroscopic and/or microscopic level (Lemaitre, 1985). In the ductile fracture theory, damage is generally described as a process of comprising void growth, nucleation and coalescence (McClintock, 1968; Rice and Tracey, 1969).

Continuum damage mechanics (CDM) is useful to model the degradation of a mechanical body leading up to macrocracks. CDM models have been developed as an
alternative to porous plasticity models using a rigorous thermodynamics framework (Chaboche, 1988; Dhar et al., 2000; Lemaitre, 1985; Voyiadjis and Dorgan, 2007; Wang, 1992). CDM model does not explicitly show the changes in the microstructure, while a thermodynamic dissipation potential is introduced to obtain the damage evolution law. In the CDM models, the material degeneration is described by a phenomenological damage parameter.

Gurson (1977) proposed a widely used homogenized yield criterion for void-containing materials based on the maximum plastic work principle, where the matrix material is assumed to obey the von Mises isotropic yield criterion. Tvergaard (1981, 1982) introduced two adjustment parameters into the Gurson model to account for the effect of void interaction and material strain hardening. Chu and Needleman (1980) proposed void nucleation models controlled by the local stress or plastic strain. Tvergaard and Needleman (1984) introduced a simplified method to provide for rapid deterioration of stiffness after localization has occurred in the material. The Gurson model, with the additional development by Tvergaard and Needleman, is often referred to as the GTN model by the fracture mechanics community. Gologanu et al. (1993, 1994) derived a yield function for materials containing prolate and oblate voids. This model reduces to the form of the Gurson model when the void shape remains spherical. Gao et al. (2011) postulated to extend the Gurson model to include the effects of hydrostatic stress and the third invariant of stress deviator on the matrix material. Benzerga et al., (2001, 2004); Bron and Besson, 2006; Brunet et al., 2005; Rivalin et al., 2000 extended the Gurson model to take account of the effect of anisotropy of matrix materials and the effect of non-spherical voids. Recently, Stewart and Cazacu (2011) developed a macroscopic
anisotropic yield criterion for porous materials when the matrix material is incompressible, anisotropic and displays tension-compression asymmetry. This model degenerates to the form of the Gurson model when the matrix material obeys the von Mises plasticity theory. For the Gurson-type models, the prediction of ductile fracture comes out naturally through the progressive loss of load carrying capacity at the material level. Despite the apparent success and wide popularity of the Gurson-type models in predicting ductile fracture, a major drawback is their inapplicability to model localization and fracture under low stress triaxiality, shear dominated deformations since these models do not predict void growth and damage evolution under shear loading. Under low stress triaxiality, rather than void growth, shear localization becomes the mechanism of ductile fracture (Rice, 1976; Yamamoto, 1978; Mear and Hutchinson, 1985; Barsoum and Faleskog, 2007; Mohr and Marcadet, 2015). To overcome this problem, Xue (2008) and Nahshon and Hutchinson (2008) modified the GTN model (Gurson, 1977; Tvergaard, Needleman, 1984) by treating the void volume fraction in the model as a generalized damage parameter which includes the shear damage contribution. Nielsen and Tvergaard (2009) modified the Nahshon-Hutchinson model by pre-multiplying the shear damage contribution by an ad hoc triaxiality-dependent factor to improve the model performance in the medium to high triaxiality region. Zhou et al. (2014) discussed the issues of using a single damage parameter in the GTN yield function and presented a modified model by combining the damage mechanics concept of Lemaitre (Lemaitre, 1985) with the Gurson-type porous plasticity model. Malcher et al. (2014) proposed an extended GTN model, which has two independent damage parameters: the first one is driven by the hydrostatic stress and the second other is driven by the deviatoric stress. Jiang et al. (2016) modified
the GTN model in a similar way to study the ductile fracture behavior under high, low and negative stress triaxiality loadings, in which two distinctive damage parameters, respectively, related to void growth mechanism and void shear mechanism, are introduced into the yield function as internal variables of the degradation process. In all these modified GTN models, the matrix material is always treated as isotropic.

In order to balance between the complexity of the underlying physics and the simplicity needed for industrial applications, uncoupled phenomenological models have been developed besides the previous CDM (Continuum Damage Mechanics) and Gurson type models. Any plasticity model and a separate fracture model can be used together without taking account of the effect of damage on the elasto-plastic material behavior before fracture. Assumption is often made that fracture initiates at which a weighted cumulative equivalent plastic strain reaches a critical value (e.g. Fischer et al., 1995). The weighting function usually depends on the Cauchy stress tensor or describes the effect of the stress state on fracture initiation. Bao and Wierzbicki (2004) published a comparative study on eight models with different weighting functions based on the respective work of McClintock (1968), Rice and Tracey (1969), LeRoy et al. (1981), Clift et al. (1990) and the modified Cockcroft and Latham criterion (1968) by Oh et al (1979). Most early ductile fracture models assume the stress triaxiality (ratio of hydrostatic stress to von Mises stress) is the only stress state parameter controlling the onset of fracture. More recent studies (Coppola et al., 2009; Zhang et al., 2001) suggested that the ductility of metals also depend on the third deviatoric stress invariant (Lode parameter). Kim et al. (2003, 2004), Gao et al. (2005) and Gao and Kim (2006) have demonstrated the significant effects of the Lode parameter on the ductile fracture process.
Xue (2007) addressed this issue and introduced a general ductile fracture model by taking an account of the effect of both pressure and Lode parameter into the weighting function. Gao et al. (2009; 2011) studied the effect of the hydrostatic stress and the third invariant of deviatoric stress tensor on both plasticity and ductile fracture. Bai and Wierzbicki (2010) proposed the Modified Mohr-Coulomb (MMC) model where the weighting function is dependent on stress triaxiality and Lode angle that is obtained from transforming the stress-based Mohr-Coulomb failure criterion. Other approaches to predicting ductile fracture involve the modeling of the localization of deformation using theoretical bifurcation analysis (e.g. Li and Karr, 2009), micromechanics based analysis (e.g. Sun et al., 2009).

1.4 Dissertation structure

In this dissertation, a comprehensive study of ductile damage of metallic materials is presented, covering constitutive model, numerical implementation and model calibration and verification. This dissertation consists of six chapters. Each chapter, except for Chapter 1 and Chapter 6, addresses one specific research topic.

Chapter 2 develops a pressure-insensitive, continuum plasticity model, dependent on the second and third invariants of the stress deviator ($J_2$ and $J_3$). One form of the yield function where the $J_3$ term is raised to an even power is shown to be able to describe the tension-torsion strength differential (S-D) effect of the plastic response, while the other form of the yield function where the $J_3$ term is raised to an odd power is shown to be able to describe the tension-compression strength differential effect of the plastic response. The plasticity model with an odd power of the $J_3$ calibrated and validated using measured
experimental data of a β-treated Zircaloy-4 with specimens experiencing a wide range of stress triaxiality and Lode parameter values. Results show that the proposed model captures the strong strength-differential effects in the material. The plasticity model with an even power of the $J_3$ item is calibrated and validated using measured experimental data of a stainless steel Nitronic 40, with specimens experiencing different stress-state. The calibrated model results in good agreement between numerical predictions and experimental measurements.

Chapter 3 studies the effect of the plastic behavior on the ductile damage process by conducting a series of unit cell analyses of a void-containing representative material volume, where the plastic response of the matrix material is governed by the plasticity model described in Chapter 2. It is clearly shown that the plastic flow of the matrix, described by the plasticity model in terms of the invariants of the stress deviators $J_2$ and $J_3$, has a very strong influence on all aspects of the response of the porous solids.

Chapter 4 provides a brief overview of the yield criterion developed by Cazacu et al. (2006) for anisotropic metals. The criterion includes the general aspects of a linear transformation operating on the Cauchy stress tensor. The method to calibrate the anisotropy and strength differential coefficients is developed and described in detail. This provides a model to describe the matrix plastic response of a commercially pure titanium whose plasticity and ductile damage process is investigated in Chapter 5.

Chapter 5 presents a constitutive model, which combines the models proposed by Stewart and Cazacu (2011) and Zhou et al. (2014), to describe the ductile damage process in commercially pure titanium (CP Ti) and to simulate its mechanical response. The
ductile damage model, which includes both void damage and shear damage, and the matrix plasticity model, which accounts for both plastic anisotropy and tension-compression asymmetry, are developed and described. The evolution law for void volume fraction remains the same as in the original GTN model and the shear damage evolution law proposed by Xue (2008) is adopted. The model is shown to capture the effect of stress state and the change of fracture mechanism. The results also reveal the important effect of the plastic anisotropy and tension-compression asymmetry on the ductile damage process.

Chapter 6 summarizes the main conclusions of this dissertation research and provides some pertinent recommendations for future studies.
CHAPTER II
MODELING THE EFFECT OF STRESS STATE ON PLASTICITY FOR ISOTROPIC MATERIAL

2.1 Introduction

Recently, structural numerical simulation is of great interest in industrial situations for which full scale experimental approaches are either too costly or even impracticable. For such application, it often demands constitutive models that can accurately describe a material's plasticity behavior. A complete plasticity model consists of a yield criterion, a flow rule and a hardening law. The yield criterion determines the stress-state when yielding occurs, the flow rule describes the plastic strain increment after yielding, and the hardening law characterizes the evolution of the flow stress with increased plastic deformation. The most popular continuum plasticity model is called $J_2$ flow plasticity theory (i.e. the Mises plasticity theory). For a material that obeys $J_2$ flow plasticity theory, its plasticity behavior is characterized by the (von Mises) equivalent stress-strain curve which can be obtained by conducting a uniaxial tension test, a uniaxial compression test, or a pure torsion test. The stress-strain curve obtained from either one of these three tests can be then used to predict the material’s plastic response under various states of stress. Actually, the assumption made in von Mises theory is not valid for some materials. Increasing experimental evidence shows $J_3$ item also plays a very important role in the plasticity behavior for some engineering materials. In this
chapter, two $J_2$-$J_3$ (the second and third invariant of stress deviator) dependent plasticity models are introduced. One plasticity model which has the even power of $J_3$ item is related to the stress-strain response difference between tensile and torsion strength, another plasticity model which has the odd power of $J_3$ item is to describe the yielding asymmetry in tension and compression strength. These new plasticity models are calibrated and verified for a β-treated Zircaloy-4 and a stainless steel Nitronic 40 to simulate the plasticity behaviors of specimens experiencing different stress-state tests. Test matrix includes round bar tensile specimens, notched round bar tensile specimens, compression specimens, grooved plains train specimens, modified Lindholm torsion specimens, modified Lindholm torsion-compression specimens and the flat notched tensile specimen.

2.2 Stress tensor and its invariants

Let $\sigma_{ij}$ be the stress tensor and $\sigma_1$, $\sigma_2$ and $\sigma_3$ be the principal stress values. The hydrostatic stress (or mean stress) can be expressed as

$$\sigma_h = \frac{1}{3} I_1 = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) \quad (2.1)$$

where $I_1$ represents the first invariant of the stress. For the tensile test, $I_1$ is positive value while for the compression test, $I_1$ is negative. Let $\sigma'_{ij}$ be the stress deviator tensor and $\sigma'_1$, $\sigma'_2$ and $\sigma'_3$ be principal values of the deviatoric stress tensor, i.e.

$$\sigma'_{ij} = \sigma_{ij} - \sigma_h \delta_{ij} \quad (2.2)$$

where $\delta_{ij}$ represents the Kronecker delta. The first invariant of the deviatoric stress tensor is calculated by $\sigma'_{ii}$, and the summation convention is adopted for repeated indices. It is
obvious that the first invariant of the deviatoric stress tensor is zero. The second and third invariants of the stress deviator tensor are defined as

\[ J_2 = \frac{1}{2} \sigma'_{ij} \sigma'_{ji} = - (\sigma'_1 \sigma'_2 + \sigma'_2 \sigma'_3 + \sigma'_3 \sigma'_1) \]

\[ = \frac{1}{6} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \]

(2.3)

\[ J_3 = \det(\sigma'_{ij}) = \frac{1}{3} \sigma'_{ij} \sigma'_{jk} \sigma'_{ki} = \sigma'_1 \sigma'_2 \sigma'_3 \]

The von Mises equivalent stress is related to the second invariant of stress deviator tensor as

\[ \sigma_e = \sqrt{3J_2} \]  

(2.4)

Figure 2.1  The stress state represented in (a) principal stress space (b) the \( \pi \) plane

A principal stress state \((\sigma_1, \sigma_2, \sigma_3)\), can be mathematically represented by a vector \(\overrightarrow{OP}\) in the principle stress coordinate system, with the three principal stresses as axes \(\sigma_1, \sigma_2, \sigma_3\), as shown in Figure 2.1(a). Consider a vector \(\overrightarrow{ON}\) passing through the origin and having equal angles with the coordinate axes. \(\overrightarrow{ON}\) is called the hydrostatic axis, where every point corresponds to \(\sigma_1 = \sigma_2 = \sigma_3\).
The plane passing through the origin and perpendicular to $\overrightarrow{ON}$ is called the $\pi$ plane and the hydrostatic stress is zero on this plane. Consider an arbitrary stress state at point $P$ with stress components $\sigma_1$, $\sigma_2$, and $\sigma_3$. The stress vector $\overrightarrow{OP}$ can be decomposed into two components, the component $\vec{r}$ parallel to $\overrightarrow{ON}$ and the $\vec{\rho}$ on the octahedral plane whose normal direction is along the vector $\overrightarrow{ON}$ direction, then

\[
r = \sqrt{3} \sigma_h = \frac{l_1}{\sqrt{3}}
\]  
\[
\rho = \sqrt{\frac{2}{3}} \sigma_e = \sqrt{2} j_2
\]

where $\sigma_h$ and $\sigma_e$ represent the hydrostatic stress (mean stress) and the equivalent stress, respectively. Consequently, the stress triaxiality ratio is

\[
T = \frac{\sigma_h}{\sigma_e} = \frac{\sqrt{2} r}{3 \rho}
\]  

Therefore, for a given stress triaxiality ratio $T$, there are a number of stress states, each corresponds to a point on the surface of a cone with $\overrightarrow{ON}$ as the axis, Figure 2.1(a). To distinguish these stress states having the same $T$ (triaxiality ratio) value, consider the location on the $\pi$ plane of the projection of point $P$, Figure 2.1(b). The angles between the projections of the coordinate axes $\sigma_1$, $\sigma_2$, and $\sigma_3$ on the $\pi$ plane are $\frac{2\pi}{3}$. Let $\theta$ be the angle measured from the horizontal axis, angle $\theta$ is called Lode angel, then

\[
tan \theta = \frac{2\sigma_3 - \sigma_2 - \sigma_1}{\sqrt{3}(\sigma_2 - \sigma_1)}
\]

or

\[
\cos \left(3\theta + \frac{\pi}{2}\right) = \frac{3\sqrt{3}j_3}{2j_2^{3/2}} = \xi
\]
ξ is called the Lode parameter. There is another way to define Lode parameter in literatures, Bai and Wierzbicki (2008), Barsoum and Faleskog (2007) defined Lode parameter \( \mu \) as

\[
\mu = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} \quad (2.10)
\]

This parameter can be easily related to the Lode angle \( \theta \) with

\[
tan\theta = \frac{\mu}{\sqrt{3}} \quad (2.11)
\]

Therefore, when von Mises equivalent stress \( (\sigma_e) \) is known, the stress triaxiality ratio \( (T) \) together with the Lode angle \( (\theta) \) can be used to specify the stress state.

2.3 Modeling of the plasticity response of isotropic materials

For isotropic materials, the general forms of the yield function \( (F) \) and the flow potential \( (G) \) can be expressed as functions of \( I_1, J_2 \) and \( J_3 \), where \( I_1 \) is the first invariant of the stress tensor and \( J_2 \) and \( J_3 \) are the second and third invariants of the stress deviators, respectively. The yield condition can be written as

\[
F(I_1, J_2, J_3) - \bar{\sigma} = 0 \quad (2.12)
\]

where \( \bar{\sigma} \) represents the hardening parameter. When the material deforms plastically, the inelastic part of the deformation is defined by following flow rule

\[
\dot{\varepsilon}_{ij}^p = \lambda \frac{\partial G(I_1, J_2, J_3)}{\partial \sigma_{ij}} \quad (2.13)
\]

where \( \dot{\varepsilon}_{ij}^p \) are the rates of the plastic strain components and \( \lambda \) is a positive scalar called the plastic multiplier.

If the flow potential and the yield function are identical, \( i.e., F = G \), Eq. (2.13) becomes the so-called associated flow rule. Otherwise, it is the non-associated flow rule.
Various forms of $F$ and $G$ functions result in different plasticity models. In particular, $F$ and $G$ can be taken as first order homogeneous functions of stress and an equivalent stress can be defined as $\sigma_e = F$

The hardening parameter depends on the strain history. By applying the principle of plastic work equivalence, \textit{i.e.,}

$$\bar{\sigma} \dot{\varepsilon}^p = \sigma_{ij} \dot{\varepsilon}_{ij}^p$$

(2.14)

The equivalent plastic strain increment can be defined as

$$\dot{\varepsilon}^p = \sigma_{ij} \dot{\varepsilon}_{ij}^p / \bar{\sigma}$$

(2.15)

Therefore, the hardening behavior can be described by a stress vs. plastic strain relation $\bar{\sigma}(\varepsilon^p)$, where $\varepsilon^p = \int \dot{\varepsilon}^p \, dt$.

Since the flow potential is taken to be a first order homogeneous function of stress, Euler's homogeneous function theorem results in

$$\sigma_{ij} \varepsilon_{ij}^p = \dot{\lambda} \sigma_{ij} \frac{\partial g}{\partial \sigma_{ij}} = \dot{\lambda} G$$

(2.16)

From (2.14) and (2.16), the plastic multiplier and the equivalent plastic strain rate can be related through

$$\dot{\lambda} = \dot{\varepsilon}^p \frac{\sigma}{G} = \dot{\varepsilon}^p \frac{F}{G}$$

(2.17)

If the material follows the associate flow rule ($F = G$), it is obvious that $\dot{\lambda}$ and $\dot{\varepsilon}^p$ are equal. For materials in which the non-associated flow rule applies, the equivalent plastic strain rate defined by Eq. (2.15) differs from $\dot{\lambda}$ by a factor.
2.3.1 The yield function with the $J_3$ term having an odd power

For most metals, during the plastic deformation, the volume change is trivial. So the first invariant of the stress tensor $I_1$ in the plastic flow potential can be neglected. After investigating various forms of first order homogeneous functions, the formulation of the $J_2$-$J_3$ plasticity model is presented. The influence of $J_3$ are also shown with two formulation with the $J_3$ term has the odd power and even power, the following two sets of $F$ and $G$ pressure-insensitive functions where the $J_3$ term has the odd power are studied

$$F = c_1 \left(3\sqrt{3} J_2^2 + b_1 J_3\right)^{\frac{1}{3}}$$

$$G = c_2 \left(3\sqrt{3} J_2^2 + b_2 J_3\right)^{\frac{1}{3}}$$

where $(b_1, c_1)$ and $(b_2, c_2)$ are material constants and $b_1$ and $b_2$ need to lie between $[-6.75, 6.75]$ to guarantee convexity for $F$ and $G$. With $F$ defined as above, an equivalent stress, $\sigma_e$, can be defined from Eq. (2.18) as $\sigma_e = F$. The constant $c_1$ given above is to ensure the equivalent stress defined by $\sigma_e = F$ equals to the applied stress under the uniaxial tensile condition.

The above yield function gives an asymmetric yield locus in the $\pi$-plane (see Figure 2.2). The shape of the yield surface depends on the sign of the coefficient $b_1$, and the size of the yield surface depends on the magnitude of $b_1$. When $b_1 = 0$, the yield function returns to von Mises criterion. Figure 2.2(a) shows the shape of the yield surface when $b_1$ is positive and describes the case of compressive strength greater than tensile strength. In this case as $b_1$ increases, the predicted strength difference increases. Figure 2.2(b) shows the shape of the yield surface when $b_1$ is negative and describes the case of
tensile strength greater than compressive strength. In this case as $b_1$ decreases, the predicted strength difference increases. Since the $J_3$ term in the yield function has an odd power, the yield locus exhibits a three-folded symmetry about the $\sigma_1$, $\sigma_2$ and $\sigma_3$ axes independent of the sign of coefficient $b_1$.

The difference between the tension and compression yield stress is controlled by the value of $b_1$. Figure 2.2 (a) shows the yield loci given by the model with $b_1$ taking the value of 0, 2.2 and 4.8 respectively. Figure 2.2(b) shows the yield loci given by the model with $b_1$ taking the value of 0, -2.2 and -4.8, respectively. These particular $b_1$ values presented here are to illustrate the trends of how $b_1$ values affect the yield surface and the predicted strength difference. (Figure 2.2(c))

Figure 2.2. The yield function projected to the $\pi$-plane: (a) compression strength is greater than tensile strength; (b) compression strength is less than tensile strength; (c) $b_1 = \pm 4.8$ in comparison with $J_2$ model (circle, and $b_1=0$)
2.3.2 The yield function with the $J_3$ term having an even power

The following two sets of $F$ and $G$ pressure-insensitive functions where the $J_3$ term has the even power are studied

$$F = c_1(27J_2^3 + b_1J_3^2)^{1/6} \quad c_1 = \left(\frac{4b_1}{729} + 1\right)^{-1/6}$$

$$G = c_2(27J_2^3 + b_2J_3^2)^{1/6} \quad c_2 = \left(\frac{4b_2}{729} + 1\right)^{-1/6} \quad (2.19)$$

where $(b_1, c_1)$ and $(b_2, c_2)$ are material constants and $b_1$ and $b_2$ need to lie between [-60.75, 91.125] to guarantee convexity for $F$ and $G$. With $F$ defined as above, an equivalent stress, $\sigma_e$, can be defined from Eq. (2.19) as $\sigma_e = F$. The constant $c_1$ given above is to ensure the equivalent stress defined by $\sigma_e = F$ equals to the applied stress under the uniaxial condition.

Since the $J_3$ term in the yield function has an even power, the yield locus exhibits a six-folded symmetry about the $\sigma_1$, $\sigma_2$ and $\sigma_3$ axes independent of the sign of coefficient $b_1$ in the $\pi$-plane (Figure 2.3). Similarity, the shape of the yield surface depends on the sign of the coefficient $b_1$, and the size of the yield surface depends on the magnitude of $b_1$. When $b_1 = 0$, the yield function returns to von Mises criterion ($J_2$ model). The difference between the tension and torsion yield stress is controlled by the value of $b_1$. Figure 2.3(a) shows the shape of the yield surface when $b_1$ is positive and describes the case where the true (von Mises) stress vs. plastic strain curve obtained using the torsion data is greater than the true stress-strain curve obtained using the tension data. In this case as $b_1$ increases, the predicted strength difference increases. Figure 2.3(b) shows the shape of the yield surface when $b_1$ is negative and describes the case where the true (von Mises)
stress vs. plastic strain curve obtained using the torsion is less than the true stress-strain curve obtained using the tension data. In this case as $b_1$ decreases, the predicted strength difference increases. Figure 2.3(c) illustrates the trends of how these particular $b_1$ values affect the yield surface and the predicted strength difference.

![Yield function projection](image)

Figure 2.3. The yield function projected to the $\pi$-plane: (a) $b_1 = +91$ and (b) $b_1 = -60$ in comparison with $J_2$ model (circle, and $b_1=0$); (c) $b_1 = +91$, $b_1 = -60$ in comparison with $J_2$ model (circle, and $b_1=0$).

2.4 Applications of the $J_2$-$J_3$ dependent plasticity model for isotropic material

In this section, the $J_2$-$J_3$ plasticity models are employed to study the plastic response of a stainless steel Nitronic 40 and a $\beta$-treated Zircaloy-4. These plasticity models are calibrated and validated for these two materials and good comparison between the numerical predictions and experimental data have been achieved.
2.4.1 Materials, Specimens and Experiments

The materials considered in this study are stainless steel Nitronic 40 and β-treated Zircaloy-4. A stainless steel Nitronic 40 consists of C, Cr, Mn, N, Ni, P, S, Si, and Fe; β-treated Zircaloy-4 consists of Sn, Fe, Cr, O and Zr. These chemical compositions in weight percentages of these alloys are listed in Table 2.1. The Nitronic 40 specimens were extracted in the radial direction from a forged disk. All the Zircaloy specimens were extracted from wrought material in the longitudinal direction and were beta heat-treated to produce a random texture on a macroscopic scale. Cockeram and Chan (2009) conducted extensive mechanical tests of the same material in different orientations and found the mechanical properties can be considered as isotropic. The Young’s modulus of the β-treated Zircaloy-4 material is 99.6 GPa and the Poisson’s ratio is 0.34. The Young’s modulus of the stainless steel Nitronic 40 material is 194.5 GPa and the Poisson’s ratio is 0.288.

Table 2.1 Nominal Chemical Composition of Nitronic 40 and Zircaloy-4 (wt %)

<table>
<thead>
<tr>
<th>Zircaloy-4</th>
<th>Element</th>
<th>Sn</th>
<th>Fe</th>
<th>Cr</th>
<th>O</th>
<th>Zr</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Composition</td>
<td>1.53</td>
<td>0.21</td>
<td>0.11</td>
<td>0.13</td>
<td>Balance</td>
</tr>
<tr>
<td>Nitronic 40</td>
<td>Element</td>
<td>C</td>
<td>Cr</td>
<td>Mn</td>
<td>N</td>
<td>Ni</td>
</tr>
<tr>
<td></td>
<td>Composition</td>
<td>0.03</td>
<td>20.2</td>
<td>8.8</td>
<td>0.34</td>
<td>6.8</td>
</tr>
</tbody>
</table>

All the tests reported in this section were performed at room temperature and are considered as quasi-static. The test matrix for both materials includes smooth round tensile bars, notched round tensile bars, cylindrical compression specimens, Lindholm-
type torsion specimens subjected to pure torsion and combined torsion-compression, and flat grooved plane strain tensile specimen. The flat notched tensile specimen is only for $\beta$-treated Zircaloy-4 material. Figure 2.4 shows the sketches of the specimens considered in this study. The diameter of the gage section of the smooth round tensile bar is 12.7 mm and the gage length is 50.8 mm. For the notched round bars, the diameter at the notch section is 7.62 mm, the notch radius is 2.54 mm, and the gage length is 25.4 mm. Two types of compression specimens, one with a length/diameter (L/D) ratio of 1.5 and the other with L/D = 0.75, are tested. The diameter of both compression specimens is 8.0 mm. The Lindholm-type torsion specimen is a hollow cylinder having an inner diameter of 13.1 mm and outer diameter of 25.4 mm. The gage section length and wall-thickness are 2.54 mm and 0.7366 mm respectively. One group of the torsion-compression tests was performed with a central pin to prevent inward buckling and other group of the torsion-compression tests was performed without the central pin. The dimensions for the flat notched tensile specimens are $30.5mm \times 7.1mm \times 2.3mm$. The thinnest cross section is 1.07 mm, the notch radius is 0.25mm and the gage section length of 12.7 mm. The dimensions for the flat grooved plane strain specimens are $101.6mm \times 31.8mm \times 6.1mm$. The thickness of the flat grooved plane strain specimen at the groove is 2.032 mm, the radius of the groove is 2.032 mm, and the length of gage section is 12.7 mm.
2.4.2 Finite element procedure

The finite element software ABAQUS is used to analyze all the specimens where the plasticity model is implemented via a user defined subroutine. The round tensile specimens and compression specimens are axis-symmetric and 4-node, axisymmetric, hybrid elements (CAX4H) are used in the finite element analyses. For the torsion specimen, torsion-compression specimen, flat notched tensile specimen and the flat grooved plane strain tensile specimen, the 3D, 8-node brick elements with reduced integration (C3D8R) are used. To improve model efficiency, symmetry conditions are applied whenever available.

Figure 2.5 shows typical finite element meshes of a round tensile specimen, a compression specimen with L/D = 0.75, a torsion specimen, a flat notched tensile specimen, a notched round tensile specimen, and a flat grooved plane strain specimen. For the compression tests, the compression platens are modeled as two analytically rigid surfaces and frictional surface contact models the interaction between the platen and the specimen. Friction coefficients between 0.05 and 0.2 are shown to have little effect on the
numerical predictions. Since the exact friction coefficient is unknown and difficult to obtain, a value of 0.08 is used in the analyses.

Figure 2.5 Finite element mesh of (a) a smooth round tensile specimen, (b) a compression specimen with $L/D = 0.75$, (c) a pure torsion specimen, (d) a flat notched tensile specimen, (e) a notched round tensile specimen, and (f) a flat grooved plane strain tensile specimen.
2.4.3 Modeling Plasticity Behavior of a β-Treated Zircaloy-4

Figure 2.6 shows the stress-strain curves of a β-treated Zircaloy-4 considered in this study obtained from uniaxial tension test and uniaxial compression test (the length/diameter ratio of the compression specimen is 1.5). Figure 2.6(a) shows the engineering stress-strain curve and Figure 2.6(b) shows the true stress vs. true plastic strain curve. Assuming plastic incompressibility, the true stress-strain curve is obtained from the measured engineering stress-strain curve on and before the loss of uniaxial stress/strain states due to localized deformation. After necking occurs in the tensile specimen or barreling occurs in the compression specimen, the true stress-strain relation is obtained from each test specimen by an inverse, iterative method: (1) perform numerical simulations of the uniaxial (tension or compression) test specimen using an assumed stress–strain curve; (2) compare the predicted load vs. displacement response with experimental measurements; (3) adjust the stress–strain curve based on the error shown in step (2); (4) repeat steps (1)–(3) until the error between the numerical and experimental results reaches an acceptable level.

![Figure 2.6 Stress-strain curves obtained from uniaxial tension and uniaxial compression tests: (a) the engineering stress-strain curve; (b) the true stress vs. true plastic strain curve.](image-url)
Results from above method, shown in Figure 2.6, indicate that the β-treated Zircaloy-4 exhibits different elastic-plastic response in tension and compression (e.g., the yield strength is much higher for compression than for tension). For example, the initial yield stress obtained from the compression test is 27% higher than that obtained from the tension test. The stress-strain curves shown in Figure 2.6 suggest that the material considered in this study exhibits a higher yield stress in compression than in tension. The experimental results presented in this section demonstrate that the $J_2$-$J_3$ plasticity model defined in Section 2.2.1 where the $J_3$ is odd power can describe the plastic response of the β-treated Zircaloy-4, including the observed tension compression asymmetry. Further close examination of Figure 2.6 reveals that the difference in plastic flow stress between tension and compression varies with the plastic strain. Consequently $b_1$ is treated as a function of plastic strain and indicates distortional hardening of the yield surface.

Substituting the tensile and compressive yield stresses and the corresponding $J_2$ and $J_3$ values for uniaxial tension and uniaxial compression into the equivalent stress definition respectively, we can solve for $b_1$

$$b_1 = \frac{27 \sigma_c^2 - \sigma_t^2}{2 \sigma_c^2 + \sigma_t^2} \quad (2.20)$$

where $\sigma_c$ represents the yield stress in compression and $\sigma_t$ represents the yield stress in tension. At a specified plastic strain level, a $\sigma_c$ value and a $\sigma_t$ value can be obtained from Figure 2.6 (b) and used to calculate a $b_1$ value. The variation of $b_1$ versus plastic strain is shown in Figure 2.7.
The difference between tensile and compressive yield stresses for this material is almost a constant at small plastic strain levels ($\varepsilon^p \leq 0.1$) as shown Figure 2.6(b), where the segments of the two curves are nearly parallel. The point of plastic instability (or necking) in the tensile test occurs at about a strain value of 0.1 for this material (Cockeram and Chan, 2009), which also corresponds to the uniform elongation. This difference increases gradually as the plastic strain increases from 0.1 to 0.25. As the material further plastically deforms and strain localization occurs in the tensile test, the difference between tensile and compressive yield stresses appears to saturate at another constant level. As a result, the $b_1(\varepsilon^p)$ function can be divided into three segments. Figure 2.7 suggests that $b_1$ varies linearly with $\varepsilon^p$ when $\varepsilon^p$ is between 0.1 and 0.25. Therefore, the $b_1(\varepsilon^p)$ function can be expressed as

$$
\begin{align*}
b_1 &= 2.2 & \text{for } \varepsilon^p \leq 0.1 \\
b_1 &= 17.3 \times (\varepsilon^p - 0.1) + 2.2 & \text{for } 0.1 < \varepsilon^p < 0.25 \\
b_1 &= 4.8 & \text{for } \varepsilon^p \geq 0.25
\end{align*}
$$

The plasticity model described above is implemented in ABAQUS via a user defined subroutine for explicit finite element analyses. Using this plasticity model with
the calibrated $b_1(\epsilon^p)$ function given by Eq. (2.21) and the tensile stress-strain curve shown in Figure 2.4(b), finite element analyses of all the specimens are conducted.

Figures 2.8(a), (b) and (d) compare the computed load vs. displacement curves using the calibrated $J_2$-$J_3$ plasticity model with experimental data for the round tensile specimen, the notched round tensile specimen, and the flat groove plane strain tensile specimen respectively. Figure 2.8(c) compares the torque vs. twist angle response between the numerical simulation and the experiment data for the pure torsion specimen. In Figure 2.8, the dashed/dotted lines represent the experimental data and the solid black lines represent the numerical predictions. The numerical results match the experimental records very well for all these specimens.

Figure 2.8 Comparison of the computed load vs. displacement curve using the calibrated $J_2$-$J_3$ plasticity model with experimental data for (a) the round tensile specimen; (b) the notched round tensile specimen; (c) the pure torsion specimen; (d) the flat grooved plane strain tensile specimen
Figures 2.9(a) and (b) compare the load vs. displacement response from the numerical prediction and the experimental data of the upset testing of the L/D = 1.5 compression specimen and the L/D = 0.75 compression specimen respectively. The calibrated $J_2$-$J_3$ plasticity model predicts the approximate hardening rate and initiation of buckling for both compression specimens. Tests for the L/D = 0.75 specimens ended before failure occurs, and there is one specimen, denoted as “Com2-2”, that experienced an unloading and reloading process as shown in Figure 2.9(b). Figures 2.9(c)-(f) provide comparisons of the load vs. displacement and torque vs. twist angle responses between numerical simulations and experimental data for the torsion-compression specimens without central pin and with central pin respectively. Again, very good agreement between the model predictions and the experimental data is shown in these figures.

Figure 2.10 compares the model predicted load vs. displacement curve with experimental data for the flat notched tensile specimen. Similar to other cases, the numerical prediction agrees reasonably well with experimental measurement.
Figure 2.9 Comparison of the computed load vs. displacement responses using the calibrated $J_2-J_3$ plasticity model with the experimental data for compression specimens with (a) L/D = 1.5, and (b) L/D = 0.75; the torsion-compression specimen (no pin): (c) axial force vs. axial displacement; (d) torque vs. twist angle; the torsion-compression specimen (with pin): (e) axial force vs. axial displacement; (f) torque vs. twist angle.
Figure 2.10 Comparison of the computed load vs. displacement responses using the calibrated $J_2$-$J_3$ plasticity model with experimental data for flat notched tensile specimen

In this section, a pressure insensitive, continuum plasticity model which is shown in section 2.2.1, is developed to describe the tension-compression asymmetry response of a $\beta$-treated Zircaloy-4. Since the difference between the tensile and compressive yield stresses increases with the accumulated plastic strain, the coefficient of the $J_3$ term in the yield function is treated as a function of the plastic strain, i.e., distortional hardening of the isotropic yielding surface. The resulting model is implemented into general purpose finite element software, ABAQUS, via a user defined material subroutine. To calibrate and verify the plasticity model, a series of experiments are conducted quasi-statically at room temperature, including tests of smooth round tensile bars; notched round tensile bars; cylindrical compression specimens; Lindholm-type torsion specimens subjected to pure torsion and combined torsion-compression; flat notched tensile specimen; and flat grooved plane strain tensile specimen. The calibrated model results in good agreement between numerical predictions and experimental measurements for all of these specimens, suggesting it is capable of describing the plasticity behavior of the $\beta$-treated Zircaloy-4.
2.4.4 Modeling Plasticity Behavior of a Nitronic 40

Figure 2.11 (a) shows the true stress-true plastic strain curves of Nitronic-40 obtained from the round bar uniaxial tension test. When the material experiences large plastic deformation, the true stress-strain curve should be used in the finite element analysis. Assuming plastic incompressibility, the true stress-strain curve can be obtained from the measured engineering stress-strain curve according to \( \sigma_{true} = \sigma_{eng} (1 + \varepsilon_{eng}) \) and \( \varepsilon_{true} = \ln(1 + \varepsilon_{eng}) \) before necking or barreling. After necking occurs in the tensile specimen or barreling occurs in the compression specimen, the true stress-strain relation can be obtained by using an inverse, iterative method. The method is mentioned in section 2.4.3. To this end, the stress-strain curve obtained from the uniaxial tensile test is used to check whether it can predict the responses of the compression specimen and the flat groove plane strain specimen using the classical \( J_2 \) flow plasticity theory. Considering the problematic measuring during the torsion experiment, here we use the flat grooved plane strain tensile test instead of the pure torsion test because both tests exhibit the same Lode parameter (Bai and Wierzbicki, 2008). Figure 2.11 (b), (c) and (d) compare the numerical and experimental load-displacement curves of the round bar tensile specimen, the compression specimen with \( L/D = 1.5 \), and the flat grooved plane strain tensile specimen.
Figure 2.11 (a) Stress-strain curves obtained from tension test; Comparison of the measured load vs. displacement response and the numerical prediction using $J_2$ plasticity model for (b) the round bar tensile specimen, (c) the compression specimen with $L/D = 1.5$, and (d) the flat groove plane strain tensile specimen.

In Figure 2.11 (b), the numerical result of the round bar tensile specimen shows excellent agreement with experimental data, which is no surprising because the stress-strain curve used in the finite element analysis was extracted from the measured load-displacement curve of the tensile specimen. Figure 2.11 (c) also shows a good agreement between the numerical and experimental results, indicating that there is no significantly strength difference between tension and compression for Nitronic-40. However, Figure 2.11 (d) shows that the $J_2$ plasticity theory over-predicts the load-displacement response of the plane strain specimen. Considering all of this, a yield locus need to be selected in the $\pi$-plane which matches the $J_2$ theory at the Lode angle $\xi$ equal to $\pm 1$ but lies inside the $J_2$ theory at the load angle $\xi$ equal to zero. The equation 2.19 proposed in the section
2.2.2 can satisfy all of the above requirements. The parameter values $b_1 = b_2 = -40$ are determined by comparing the model predicted load-displacement curves with experimental measurements. In these finite element simulations, the associative plastic flow rule is adopted and the tensile stress-strain curve shown in Figure 2.11 (a) is employed.

This calibrated model is then applied to predict the responses of all the specimens, including the round tensile specimen, the compression specimen with $L/D = 0.75$ and $L/D=1.5$, pure torsion specimen, the torsion-compression specimen with central pin, the torsion-compression specimen without central pin, the notched round tensile specimen and the flat groove plane strain tensile specimen. Figure 2.12 compares the computed load vs. displacement curve using the calibrated $J_2$-$J_3$ plasticity model with experimental data for the round tensile specimen, showing an almost perfect agreement. Figures 2.13 (a) and (b) show the comparison of the load vs. displacement response between the numerical prediction and the experimental data of the $L/D = 1.5$ specimen and the $L/D = 0.75$ specimen, respectively. The calibrated $J_2$-$J_3$ plasticity model predicts the load vs. displacement responses very well for both compression specimens. The tests for the $L/D = 0.75$ specimens ended before failure occurs. Failure was not observed in some of compression specimens $L/D = 1.5$ and for both of the non-failure tests were ended when the applied load reached the machine capacity which is the 324.72 kN.
Figure 2.12 Comparison of the computed load vs. displacement curve using the calibrated $J_2$-$J_3$ plasticity model with experimental data for the round tensile specimen

Figure 2.13 Comparison of the computed load vs. displacement responses using the calibrated $J_2$-$J_3$ plasticity model with the experimental data for compression specimens with (a) L/D = 1.5, and (b) L/D = 0.75

Figure 2.14. Comparison of the computed torque vs. twist angle curve using the calibrated $J_2$-$J_3$ plasticity model with experimental data for the pure torsion specimen

Figure 2.14 compares the torque vs. twist angle response between the numerical simulation and the experiment data for the pure torsion specimen. Figures 2.15 provide
the comparisons of the load vs. displacement and torque vs. twist angle responses between numerical simulations and experimental data for the torsion-compression specimen without central pin in (a) & (b) and the torsion-compression specimen with central pin in (c) & (d) respectively. Figure 2.16 compares the computed load vs. displacement using the calibrated $J_2-J_3$ plasticity model with experimental data for the notched round tensile specimen. The predicted result is excellent. Figure 2.17 compares the computed load vs. displacement using the calibrated $J_2-J_3$ plasticity model with experimental data for the flat groove plane strain tensile specimen. The predicted result is very good.

![Figure 2.15 Comparisons between the numerical predictions using the calibrated $J_2-J_3$ plasticity model and the experimental data for the torsion-compression specimen (no pin): (a) axial force vs. axial displacement; (b) torque vs. twist angle (with pin); (c) axial force vs. axial displacement; (d) torque vs. twist angle](image-url)
In this section, the experimental and numerical work presented show that the stress state has significant effect on the plasticity behaviors of Nitronic-40. To account for the stress state effect, a $J_2-J_3$ plasticity yield criteria with $J_3$ term in even power mentioned in 2.2.2 section is present and implemented into ABAQUS via a user defined subroutine. The calibrated model results in good agreement between numerical predictions and experimental measurements for all of these specimens.
CHAPTER III
THE EFFECT OF THE MATERIAL PLASTICITY BEHAVIOR ON VOID GROWTH
AND DUCTILE FRACTURE

3.1 Introduction

Ductile fracture is commonly associated with the process of void nucleation, growth and coalescence. In order to accurately predict the ductile material failure, the method to accurately describe void evolution law is of critical importance. Two types of approach have been proposed in the published literature. In the first approach, the voids are considered to be implicit. All the void nucleation, progressive growth and eventual coalescence are considered by using void containing elements based on the continuum damage models. The most popular macroscopic constitutive model in this approach is the GTN model. In the second approach, the voids are considered to be explicit, the so called unit cell model. In this approach, the shape of voids is modeled using refined finite elements. An exact implementation of void growth behavior is the distinct advantage of this approach. Recently, in most of these micromechanical studies, it was assumed that solid material in the cell model is governed by the $J_2$ plasticity (von Mises) theory. In this chapter, a micromechanical FEA (unit cell FE simulation) is studied to investigate the influence of those two proposed plasticity yield criteria in chapter II on the ductility and the void growth of porous material. The matrix materials are the pressure insensitive and
dependent on second and third invariant of the deviatoric stress tensor ($J_2$ and $J_3$), with a scalar material parameter, $b_1$ is considered in each yield criteria. One criterion which has the even power of $J_3$ item is capable of describing tensile and torsion strength-differential effects in plastic flow, another criterion which has the odd power of $J_3$ item is capable of describing tension and compression strength-differential effects in plastic flow. The associated flow rule is investigated. The porous solid is represented as a three dimensional (3-D) regular spatial array of initially spherical voids packed in a fully dense matrix. Using the FE unit-cell model, the porosity evolution and its effects on the mechanical response of the porous materials for tensile loading corresponding to stress triaxiality $T=1.2$ and Lode angle, $\theta$, is $-30^\circ$, $0^\circ$ and $30^\circ$ are examined, respectively. This stress triaxiality corresponds to tensile loading of notched round bar made of material 5083 (Zhou et al., 2012). To detect the onset of void coalescence, the procedure method outlined in W.H. Wong, T.F. Guo (2015) is used. This method is based on monitoring, examining and comparing the elastic and plastic energies of a voided cell throughout its deformation history. The distribution of the plastic zone and the triaxiality within the domain will be explored.

3.2 The voided cell model

It is assumed that the porous material contains a regular and periodic array of cube cells with an initially spherical void at each individual cube center. The inter-void spacing is considered to be the same in any direction. Each cell is a voided representative material volume (RMV). The voided RMV is a three dimensional element which is initially a cube with dimension $D_0$. It contains a spherical void of radius $r_0$ and the initial
volume fraction of the each cell is 

$$f_0 = \frac{4}{3} \pi \left( \frac{r_0}{D_0} \right)^3$$

. The material outside the layer of voided cells is modeled as the same characteristics as the matrix material in the cells. This matrix material plasticity model is isotropic as described in chapter section 2.2.1 and section 2.2.2. The origin of the coordinate system is taken at the center of the void (see Figure 3.1). The center voided RMV subjected to the macroscopic stresses $\Sigma_1$, $\Sigma_2$, and $\Sigma_3$. For given values of the stress triaxiality, $T$, and the Lode angle, $\theta$, the principal stress ratios, $\rho_1 = \frac{\Sigma_1}{\Sigma_2}$ and $\rho_2 = \frac{\Sigma_2}{\Sigma_2}$, can be determined uniquely. Therefore, in order to maintain the same values of $T$ and $\theta$ throughout the entire deformation history, boundary conditions must be prescribed such that the values of $\rho_1$ and $\rho_2$ remain constants. In the FE implementation, at the end of each time increment, the condition of constant proportionality is strictly verified. Faleskog et al. (1998) and Kim et al. (2004) provide details of how to prescribe such boundary conditions.

Figure 3.1 (a) Dimensions of the unit cell (b) macroscopic stresses acting on the unit cell referring to a Cartesian coordinate system with origin at the center of the void.

The macroscopic principal strains $E_1$, $E_2$, and $E_3$ are given by equation 3.1,
where $D_1$, $D_2$ and $D_3$ are the length of the deformed cell in x, y and z directions respectively. $D_0$ is the undeformed cell length. The macroscopic equivalent strain $E_e$ is calculated as following equation:

$$E_e = \sqrt{\frac{2}{9} ((E_1 - E_2)^2 + (E_2 - E_3)^2 + E_3^2)}$$

The work conjugate stresses to the macroscopic principal strains $E_1$, $E_2$, and $E_3$ are the macroscopic (true) principal stresses $\Sigma_1$, $\Sigma_2$, $\Sigma_3$, i.e., the average reaction forces per unit area of the deformed cell boundary. The macroscopic equivalent stress $\Sigma_e$ can be expressed by

$$\Sigma_e = \sqrt{\frac{1}{2} ((\Sigma_1 - \Sigma_2)^2 + (\Sigma_2 - \Sigma_3)^2 + (\Sigma_3 - \Sigma_1)^2)}$$

The void volume fraction, $f$, is evaluated at the end of each time increment as:

$$f = \frac{V_{total} - V_{matrix}}{V_{total}}$$

Where $V_{total} = D_0^3$ and $V_{matrix}$ is determined from the FEA simulation result, $V_{matrix} = \sum_{i=1}^{N} V_i$. $V_i$ is the FEA simulation result of volume of the element i and $N$ is the total element number in the mesh.

3.3 The matrix material model

The matrix material is assumed to act as an isotropic elastic-plastic material following the yield functions described in chapter II section 2.2.1 and section 2.2.2 and
the associate flow rule. The true stress-strain curve in uniaxial tension is assumed to follow a power-law relation

\[
\begin{align*}
\varepsilon &= \frac{\sigma}{E} \quad \sigma \leq \sigma_0 \\
\varepsilon &= \frac{\sigma_0}{E} \left(\frac{\sigma}{\sigma_0}\right)^{1/n} \quad \sigma > \sigma_0
\end{align*}
\]  

(3.5)

Here, the material parameters are taken to be \( E = 69 \) GPa, \( \sigma_0 = 207 \) MPa, \( \nu = 0.3 \), and \( n = 0.125 \). In all the analysis calculations, all the input material parameters are kept the same.

The numerical analysis has been carried out using finite element program ABAQUS. Only one eighth of the unit cell with an initial void volume fraction \( f_0 = 0.005 \) is analyzed numerically. Symmetry conditions are imposed on the planes \( X = 0 \), \( Y = 0 \), and \( Z = 0 \), respectively. To simulate the constraints of the surrounding material, the faces of the unit cell is enforced that are initially planes parallel to the coordinate planes, remain planes and are shear free. The void is also considered to be traction-free. The 3D 8-node brick elements with reduced integration (C3D8R) are used. Since reduced integration is used, the hourglass control is needed to be employed in the ABAQUS input files to increase resistance to hourglass. A mesh refinement study was carried out to ensure that the results are mesh independent.

3.4 The \( J_3 \) term having an odd power in the yield function

Specifically, two matrix materials characterized by different tension-compression asymmetry strength with the material parameter \( b_1 \) given by \( b_1 = -4.0 \), \( b_1 = +4.0 \) are studied in this section. The micromechanical FEA of the three dimensional unit cell is conducted. Simulation results are presented for two axisymmetric tensile loadings
corresponding to fixed value of the stress triaxiality $T = 1.2$ and the two possible values of the Lode angles $\theta = -30^0$ or $\theta = +30^0$.

3.4.1 Analysis of the porosity evolution and the stress-strain behavior of porous materials for tensile loading corresponding to $T=1.2$ and $\theta = -30^0$

An axisymmetric loading is considered first, where $\Sigma_2 \geq \Sigma_1 = \Sigma_3 (\theta = -30^0)$. The applied macroscopic loading following ratios between the macroscopic principal true stresses, i.e., the stress triaxiality $T$ is 1.2 and load angel $\theta$ is $-30^0$. Figure 3.2 shows a comparison between the macroscopic effective stress and macroscopic effective strain ($\Sigma_e$ vs. $E_e$) curves of the three porous materials by $b_1 = -4.0$, $b_1 = 0$, $b_1 = +4.0$; Figure 3.3 shows the evolution of the void volume fraction, $f$, as a function of the macroscopic effective strain $E_e$ for those three porous materials. Void Coalescence will occur when the macroscopic deformation of the cell shifts to a uniaxial strain mode (Koplik and Needleman, 1988). Detailed explanation of the uniaxial straining mode can be found in references Koplik and Needleman (1988), and Kim et al. (2004)). In order to determine the Macroscopic strain at void coalescence, an energy-based criterion for the onset of the void coalescences are adopted, which is proposed by W.H. Wong, T.F. Guo (2015). Wong and Guo (2015) identify the point with minimum $\frac{dW_e}{dW_p}$ as the onset of void coalescence. Here we use $E_c$ to denote the macroscopic effective strain at the onset of void coalescence. The macroscopic strain $E_c$ at void coalescence is shown in the filled point on both curves in Figure 3.2 and Figure 3.3.
When the maximum macroscopic equivalent stress is reached, and the maximum strain strongly depend on $b_1$ (see Figure 3.2). It is the fact that the porous material characterized by $b_1= +4.0$ has the highest ductility, but the stress drop is also the most rapid. This indicates that this material failure will cause sudden greater damage than in a porous von Mises material or in a porous material with matrix characterized by $b_1= - 4.0$. Void growth is fastest in the material with $b_1= - 4.0$ and slowest in the material with $b_1=4.0$. It is worth noting that for material with $b_1= +4.0$ the void growth is very slow for most of the deformation process. In contrast, for porous von Mises material by $b_1= 0$ and for porous material characterized by $b_1= - 4.0$, damage accumulation is more gradual. That is, for the porous material characterized by $b_1= +4.0$, the rate of void growth is significantly lower (see Figure 3.3) than in the other two materials. Of the three materials, in the material with $b_1= - 4.0$, void growth is the fastest and consequently the decrease in relative ligament size is most rapid. It has the smallest strain localization value ($E_c$). This explains that for this loading, this material has much more reduced ductility than the other two materials. The macroscopic strain $E_c$ at coalescence (strain localization) is more than three times higher in the material with $b_1= +4.0$ than in the material with $b_1= - 4.0$ (see Figure 3.2 and Figure 3.3).
Figure 3.2 Comparison between the macroscopic stress-strain response for porous materials with matrix characterized by different $b_1$ value.

Figure 3.3 Evolution of the void volume fraction with the macroscopic equivalent strain $E_e$, for porous materials with matrix characterized by different $b_1$ value.

To figure out the mechanism why the voids grow differently among the three porous materials, the contour plots of local equivalent plastic strain and stress triaxiality values corresponding to the same level of macroscopic strain at which $E_e$ is 0.15 are output, analyzed and discussed. Notice that this macroscopic strain level corresponds to the early stages of the deformation process where only a very slight difference between the macroscopic stress-strain responses of the three materials can be observed (see Figure 3.2). Figure 3.4 shows the contours of constant local equivalent plastic strain corresponding to a macroscopic effective strain $E_e = 0.15$. The black regions in Figure 3.4 represent the elastic zones. It is particular importance that only in the material with $b_1 = +4.0$, the entire domain (whole cell) is plastic zone. However, for the porous von
Mises material and the porous material characterized by $b_1= -4.0$, there exists a zone above the void along the vertical axis where yielding did not occur. Specifically, for the porous von Mises material ($b_1=0$), the elastic zone is adjacent to the void while for the material characterized by $b_1= -4.0$, the elastic zone is slightly shifted upwards away from the void. Moreover, the distribution of the local stress triaxiality is also different among these three matrix materials, i.e. porous materials with matrix characterized by different $b_1$ value. Figure 3.5 shows the contour plot of the local stress triaxiality value for each material. Notice that in the porous material with matrix characterized by $b_1= -0.4$, the triaxiality value is positive in the entire domain while in the material with $b_1= +4.0$ whose entire domain (whole cell) is plastic zone, there is negative stress triaxiality zone close to the void, i.e. negative (compressive) mean stress zone. As a consequence, for the latter material void growth is slowed down as compared to the porous material with matrix characterized by $b_1= -4.0$. This correlates very well with the results presented in Figure 3.3, in particular it explains the different porosity evolution between the three materials.

Figure 3.4 Contour plot of the local equivalent plastic strain value for porous materials with matrix characterized by different $b_1$ value. (a) $b_1= -4.0$; (b) $b_1=0$; (c) $b_1=+4.0$;
Figure 3.5 Contour plot of the local triaxiality value for porous materials with matrix characterized by different $b_1$ value. (a) $b_1 = -4.0$; (b) $b_1 = 0$; (c) $b_1 = +4.0$;

3.4.2 Analysis of the porosity evolution and the stress-strain behavior of porous materials for tensile loading corresponding to $T=1.2$ and $\theta = +30^0$

In this section, the macroscopic loading that is imposed corresponds to the same value of the stress triaxiality as in the previous case analyzed in Section 3.4.1, such as the stress triaxiality $T=1.2$, but the Lode angle $\theta$ is taken as $\theta = +30^0$. Typically, the applied macroscopic loading following ratios between the macroscopic principal true stresses $\frac{\Sigma_1}{\Sigma_2} = 0.3478$ and $\frac{\Sigma_3}{\Sigma_2} = 1.0$. The macroscopic equivalent stress $\Sigma_e$ vs. macroscopic equivalent strain $E_e$ curves for the three materials is shown in Figure 3.6. The evolution of the void volume fraction vs. the macroscopic effective strain $E_e$, respectively is given in Figure 3.7.
Figure 3.6 Comparison among the macroscopic stress-strain responses for porous materials with matrix characterized by different $b_1$ value.

Figure 3.7 Evolution of the void volume fraction with the macroscopic equivalent strain $E_e$, for porous materials with matrix characterized by different $b_1$ value.

Comparison with the previous analysis result in section 3.4.1, where the macroscopic loading correspond to the stress triaxiality $T=1.2$ and Lode angle $\theta= -30^0$, the fastest void growth was predicted for the porous material with matrix characterized by $b_1= -4.0$, in this section, for the applied loading which corresponds to the stress triaxiality $T=1.2$ and Lode angle $\theta=30^0$, the fastest void growth is observed in the porous material characterized by $b_1 = 4.0$ and this material shows enhanced strength which is the same as the analysis result reported in section 3.4.1. Furthermore, the slowest void growth is observed in the porous material characterized by $b_1 = -4.0$ and this material shows enhanced ductility.
To understand the difference in the porosity evolution between the three materials, the distribution of the local equivalent plastic strain (Figure 3.8) and local stress triaxiality (Figure 3.9) in each material are plotted. The contour plot correspond to the same level of macroscopic effective strain $E_e = 0.15$. Notice that for the porous material with matrix characterized by $b_1 = -4.0$, local equivalent plastic strain value has the smallest range, i.e. the difference between the maximum and minimum plastic strain value is smallest and the distribution is more uniform. For this material, the void growth is slow and the local triaxiality is also more uniform. In contrast, in the material with $b_1 = 4.0$, the local plastic strain range is much larger, the range of triaxiality is also higher than in the other materials. The void growth is fastest in this material.

Figure 3.8 Contour plot of the local equivalent plastic strain value for porous materials with matrix characterized by different $b_1$ value. (a) $b_1 = -4.0$; (b) $b_1 = 0$; (c) $b_1 = +4.0$;

Figure 3.9 Contour plot of the local triaxiality value for porous materials with matrix characterized by different $b_1$ value. (a) $b_1 = -4.0$; (b) $b_1 = 0$; (c) $b_1 = +4.0$;
In conclusion, the effects of the tension-compression asymmetry of the plastic flow of the incompressible matrix on void growth were investigated. To achieve this goal and purpose, FE unit cell calculations were conducted. The plastic flow of the incompressible matrix was considered to obey the isotropic form yield criterion which is described in section 2.2.1. This yield criterion is pressure-insensitive, it involves Cauchy stress deviator invariants $J_2$ and $J_3$, and a scalar material parameter, $b_1$. If $b_1=0$, the criterion reduces to von Mises yield criterion. If $b_1$ is different from zero, the criterion accounts for strength differential effects. The imposed axisymmetric tensile loadings were followed by a prescribed proportional loading history corresponding to a specified value of the stress triaxiality and Lode angle. It was clearly shown that the plastic flow of the matrix, described by stress deviator invariants $J_2$ and $J_3$ and the parameter $b_1$, has a very strong influence on all aspects ductile damage. Furthermore, in particular of the stress triaxiality and the Lode angle, the void evolution and ultimately the material's ductility was observed. Specifically, this is very strong relation between the sign of material parameter $b_1$, which is related to the tension compression differential strength ratio of the matrix, and the void growth rate. While the fixed stress triaxiality $T=1.2$ is considered, for the matrix material characterized by $b_1=+4.0$, the void growth rate is slower for the Lode angle $\theta=-30^0$ than the Lode angle $\theta=+30^0$. On the other hand, for the matrix material characterized by $b_1=-4.0$, the opposite holds true. Most importantly, although at the macroscopic level there is very little difference in the macroscopic stress-strain response between the porous materials while the differences in the local state fields are very marked.
3.5 The $J_3$ term having an even power in the yield function

In this section, the $J_3$ term has the even power in the yield function as it is mentioned in section 2.2.2. Two matrix materials characterized by different tension-torsion strength with the material parameter $b_1$ given by $b_1= -60, b_1= +91$ are studied. The same micromechanical FEA of the three dimensional unit cell analyzed in section 3.4 is used. Simulation results are presented for two axisymmetric tensile loadings corresponding to fixed value of the stress triaxiality $T=1.2$ and the two possible values of the Lode angles $\theta=-30^0$ or $\theta=0^0$.

3.5.1 Analysis of the porosity evolution and the stress-strain behavior of porous materials for tensile loading corresponding to $T=1.2$ and $\theta=-30^0$

An axisymmetric loading is considered in this section, where $\Sigma_2 \geq \Sigma_1 = \Sigma_3$ ($\theta= -30^0$). The applied macroscopic loading following ratios between the macroscopic principal true stresses $\frac{\Sigma_1}{\Sigma_2} = 0.3478$ and $\frac{\Sigma_2}{\Sigma_2} = 1.0$. i.e., the stress triaxiality $T$ is 1.2 and load angle $\theta$ is $-30^0$.

In order to define the critical value of effective strain ($E_c$) for the onset of void coalescence, an energy-based criterion which is proposed by W.H. Wong, T.F. Guo (2015) is adopted similar to the section 3.4.1. The corresponded macroscopic effective critical strain is $E_c$ indicated by the filled points on the macroscopic effective stress macroscopic effective strain ($\Sigma_e$ vs. $E_e$) curves and the evolution of the void volume fraction curves ($f$ vs. $E_e$). Figure 3.10 shows a comparison between the macroscopic effective stress macroscopic effective strain ($\Sigma_e$ vs. $E_e$) curves of the three porous materials; it provides an overview of the competition between matrix material strain
hardening and porosity induced softening. As loading increases, a maximum macroscopic equivalent stress is reached the corresponding ultimate points in $\Sigma_e$ vs. $E_e$ curves, and then $\Sigma_e$ decreases as strain-hardening of matrix material is not “hard” enough to compensate for the reduction in ligament area caused by void growth. As the macroscopic effective strain reaches $E_c$ (indicated by the open circle), a rapid drop in macroscopic effective stress occurs. Figure 3.11 shows the evolution of the void volume fraction, $f$, as a function of the macroscopic effective strain $E_e$. The void expands in size as the effective strain increases and the void volume fraction increases rapidly once $E_e$ reaches $E_c$. The $f$ value corresponding to $E_c$ is referred to as the critical void volume fraction (solid filled shape in Figure 3.11) for void coalescence and is denoted as $f_c$.

The porous material characterized by $b_1= +91$ has the highest ductility. Void growth is fastest in the material with $b_1= -60$ and slowest in the material with $b_1= +91$ (as shown in Figure 3.10). Of the three materials, in the material with $b_1= -60$, void growth is the fastest and consequently the decrease in relative ligament size is most rapid. It has the smallest strain localization value ($E_c$).

![Figure 3.10](image)

Figure 3.10 Comparison between the macroscopic stress-strain response for porous materials with matrix characterized by different $b_1$ value.
Figure 3.11 Evolution of the void volume fraction with the macroscopic equivalent strain \( E_e \), for porous materials with matrix characterized by different \( b_1 \) value.

To find out the reasons or mechanism that the voids grow differently among the three porous materials, the local equivalent plastic strain and triaxiality values corresponding to the same level of macroscopic strain at which \( E_e \) is 0.15 are contour plot out as shown Figure 3.12 and Figure 3.13, respectively. Notice that the macroscopic strain level represents the early stages of the deformation process where macroscopically only a little difference between the stress-strain responses of the three materials can be observed (see Figure 3.10). Figure 3.12 shows the contour plots of constant local equivalent plastic strain corresponding to a macroscopic effective strain \( E_e = 0.15 \). The black regions in this Figure 3.12 represent the elastic zones. The matrix material with \( b_1 = +91 \) has the smallest plastic zone area. All the zone areas exist above the void along the vertical axis and adjacent to the void. Figure 3.13 shows the contour plot of the local triaxiality value for each material. Note that in the porous material with all three different matrix materials, the triaxiality values are all positive in the entire domain. While in the matrix material with \( b_1 = -60 \), there is the largest which is close to the void, in the matrix material with \( b_1 = +90 \) whose triaxiality values is more uniform and the triaxiality zone has smallest value in all these three materials. As a consequence, for the matrix material represented by \( b_1 = +91 \) whose void growth is slowed down as compared to the porous
material with matrix characterized by $b_1 = -60$. This correlates very well with the results presented in Figure 3.10 and 3.11.

Figure 3.12 Contour plot of the local equivalent plastic strain value for porous materials with matrix characterized by different $b_1$ value. (a) $b_1 = -60$; (b) $b_1 = 0$; (c) $b_1 = +91$;

Figure 3.13 Contour plot of the local triaxiality value for porous materials with matrix characterized by different $b_1$ value. (a) $b_1 = -60$; (b) $b_1 = 0$; (c) $b_1 = +91$;

3.5.2 Analysis of the porosity evolution and the stress-strain behavior of porous materials for tensile loading corresponding to $T=1.2$ and $\theta = 0^0$

In this section, the macroscopic loading that is imposed corresponds to the same value of the stress triaxiality as in the previous case analyzed in Section 3.5.1, such as the stress triaxiality $T=1.2$, but the Lode angle $\theta$ is taken as $\theta=0$. Typically, the applied macroscopic loading following ratios between the macroscopic principal true stresses

$$\frac{\Sigma_1}{\Sigma_2} = 0.3503 \text{ and } \frac{\Sigma_2}{\Sigma_2} = 0.6752.$$  

The macroscopic equivalent stress $\Sigma_e$ vs. macroscopic
equivalent strain $E_e$ curves for the three materials are shown in Figure 3.14. The evolution of the void volume fraction vs. the macroscopic effective strain $E_e$, respectively is given in Figure 3.15. The filled points on the macroscopic effective stress macroscopic effective strain ($\Sigma_e$ vs. $E_e$) curves or the evolution of the void volume fraction curves ($f$ vs. $E_e$) represent the macroscopic effective critical strain $E_c$. In this loading state, the porous material characterized by $b_1 = -60$ has the largest critical strain value while under the loading state applied in section 3.5.1, the porous material characterized by $b_1 = +91$ has the largest critical strain value. The porous material characterized by $b_1 = +91$ still has the higher ductility.

To better understand the reasons that the voids growth differently among the three porous materials, the local equivalent plastic strain and triaxiality values corresponding to the same level of macroscopic strain at which $E_e$ is 0.2 are discussed. Figure 3.16 shows the contours of constant local equivalent plastic strain corresponding to a macroscopic effective strain $E_e = 0.2$. Then all of these three materials are in the plastic zone area at this macroscopic effective strain level. The porous material characterized by $b_1 = -60$ has the uniform plasticity zone in all these three materials. Figure 3.17 shows the contour plot of the local triaxiality value for each material. Note that the higher triaxiality value zone is not adjacent to void in the unit cell model for all these three materials. The porous material characterized by $b_1 = -60$ has the largest triaxiality $T$ which is closer to the void than the other two porous materials characterized by $b_1 = 0$ and $b_1 = +91$. 
Figure 3.14 Comparison between the macroscopic stress-strain response for porous materials with matrix characterized by different $b_1$ value.

Figure 3.15 Evolution of the void volume fraction with the macroscopic equivalent strain $E_e$, for porous materials with matrix characterized by different $b_1$ value.

Figure 3.16 Contour plot of the local equivalent plastic strain value for porous materials with matrix characterized by different $b_1$ value. (a) $b_1 = -60$; (b) $b_1 = 0$; (c) $b_1 = +91$;
In this chapter, void growth and its influence on ductility in specified materials whose material plasticity model is considered isotropic as described in section 2.2.1 and section 2.2.2. The three dimensional unit cell FEA simulation results are calculated under axisymmetric tensile loadings corresponding to fixed values of the stress triaxiality for the two possible values of the Lode angle. The entire results showed that relations among the specificities of the plastic flow of the matrix, the local plastic strain and the local triaxiality lead to different void growth rates which strongly affect the ductility of the porous materials. In view of industrial applications, studies devoted to the investigation of the ductile damage in such materials may further help provide additional valuable insights.
CHAPTER IV
AN ORTHOTROPIC PLASTICITY MODEL

All real materials exhibit some anisotropic behavior and when this behavior is significant enough it is not accurate to make the assumption that a material is isotropic. Material models for these materials behavior must incorporate the anisotropic nature of yielding. Some materials, such as those with a hexagonally close packed (hcp) crystal structure, exhibit strong anisotropic behavior both at the single crystal and polycrystal level. Furthermore, such materials may display a strength differential, or non-symmetry between tensile and compressive strengths. The plasticity model developed by Cazacu et al. (2006) can capture the tension/compression asymmetries and material anisotropic. In this chapter, a brief overview of the criterion is given, which includes the general aspects of a linear transformation operating on the Cauchy stress tensor. The method to obtain the input data needed for the calculation of the anisotropic yield function coefficients is discussed. The theoretical model is implemented in ABAQUS via a user defined subroutine. The parameter identification procedure based on commercially pure titanium (CP Ti) experimental results is given in chapter five. A comparison between model predictions and the experiment data is shown in Chapter five, also.
4.1 Yield criterion

The plasticity model developed by Cazacu et al. (2006) is adopted to describe the material plasticity behavior. This model is based on a linear transformation of the deviatoric part of the Cauchy stress tensor, similar to previous studies by Barlat and coworkers (Barlat et al., 1991, 1997) and Lademo et al. (1999). The yield condition of this plasticity model is expressed as

\[
\sigma_e (\Sigma_i, k) = \sigma_M
\]

\[
\sigma_e = m \sqrt{(|\Sigma_1| - k \Sigma_1)^2 + (|\Sigma_2| - k \Sigma_2)^2 + (|\Sigma_3| - k \Sigma_3)^2}
\]  

(4.1)

With

\[
m = \frac{1}{\sqrt{(|\theta_1| - k \theta_1)^2 + (|\theta_2| - k \theta_2)^2 + (|\theta_3| - k \theta_3)^2}}
\]

\[
\theta_1 = \frac{2}{3} L_{11} - \frac{1}{3} L_{12} - \frac{1}{3} L_{13}
\]

\[
\theta_2 = \frac{2}{3} L_{12} - \frac{1}{3} L_{22} - \frac{1}{3} L_{23}
\]

\[
\theta_3 = \frac{2}{3} L_{13} - \frac{1}{3} L_{23} - \frac{1}{3} L_{33}
\]

where \( \sigma_M \) is the uniaxial tensile yield strength along an axis of orthotropy, \( k \) is used to capture the material strength differential effect, \( m \) is defined so that Eq. (4.1) is identically satisfied for uniaxial tensile loading along this orthotropy axis, \( \Sigma_i \) are the principal components of the transformed stress tensor and \( \Sigma = L : K : \sigma \), \( \sigma \) is the Cauchy stress tensor, \( K \) is the 4th order deviatoric projection operator, and \( L \) is the 4th order tensor which satisfies the major and minor symmetric and the requirement of invariance with respect to the orthotropy group. This linear transformation “weights” the different components of the stress tensor of the anisotropic material in order to account for the anisotropy of the material. The transformed tensor is called the “isotropic plasticity equivalent (IPE) deviatoric stress tensor” (Karafillis and Boyce, 1993) and is used as the
argument in the yield function (4.1). To ensure that it reduces to the fourth-order identity
tensor for isotropic conditions, the following constraints are imposed:

\[
\begin{align*}
L_{11} + L_{12} + L_{13} &= 1 \\
L_{12} + L_{22} + L_{23} &= 1 \\
L_{13} + L_{23} + L_{33} &= 1
\end{align*}
\] (4.2)

The additional constraints of Eq. (4.2) also ensure that the transformed stress tensor \( \Sigma \) is
deviatoric.

When the material deforms plastically, the inelastic part of the deformation is defined by
the flow rule

\[
\dot{\varepsilon}_{ij}^p = \dot{\lambda} \frac{\partial \phi}{\partial \sigma_{ij}} = \dot{\lambda} \frac{\partial \phi}{\partial \sigma_{mn}} L_{mnkl} K_{klij}
\] (4.3)

where \( \dot{\varepsilon}_{ij}^p \) are the rates of the plastic strain components and \( \dot{\lambda} \) is a positive scalar called
the plastic multiplier.

Let \((x,y,z)\) be the reference directions associated with the orthotropy. In the case
of a sheet material, \(x, y \) and \(z\) represent the rolling direction, the long transverse direction,
and the normal direction (through thickness direction), respectively. Relative to the
orthotropy axes the transformed stress tensor \( \Sigma \) (in vector form) can be expressed in terms
of the Cauchy stress tensor according to
\[
\begin{bmatrix}
\Sigma_{xx} \\
\Sigma_{yy} \\
\Sigma_{zz} \\
\Sigma_{xy} \\
\Sigma_{yz} \\
\Sigma_{xz}
\end{bmatrix} = 
\begin{bmatrix}
L_{11} & L_{12} & L_{13} \\
L_{12} & L_{22} & L_{23} \\
L_{13} & L_{23} & L_{33}
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
\frac{2}{3} & \frac{2}{3} & \frac{2}{3} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3}
\end{bmatrix} \quad 1
\end{bmatrix}
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz}
\end{bmatrix}
\]
(4.4)

or

\[
\begin{bmatrix}
\Sigma_{xx} \\
\Sigma_{yy} \\
\Sigma_{zz} \\
\Sigma_{xy} \\
\Sigma_{yz} \\
\Sigma_{xz}
\end{bmatrix} = 
\begin{bmatrix}
\theta_1 & \phi_1 & \Omega_1 \\
\theta_2 & \phi_2 & \Omega_2 \\
\theta_3 & \phi_3 & \Omega_3
\end{bmatrix}
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz}
\end{bmatrix}
\]
(4.5)

where

\[
\begin{align*}
\theta_1 &= \frac{2}{3} L_{11} - \frac{1}{3} L_{12} - \frac{1}{3} L_{13}, \\
\phi_1 &= \frac{2}{3} L_{12} - \frac{1}{3} L_{11} - \frac{1}{3} L_{13}, \\
\Omega_1 &= \frac{2}{3} L_{13} - \frac{1}{3} L_{11} - \frac{1}{3} L_{12} \\
\theta_2 &= \frac{2}{3} L_{12} - \frac{1}{3} L_{22} - \frac{1}{3} L_{23}, \\
\phi_2 &= \frac{2}{3} L_{22} - \frac{1}{3} L_{12} - \frac{1}{3} L_{23}, \\
\Omega_2 &= \frac{2}{3} L_{23} - \frac{1}{3} L_{12} - \frac{1}{3} L_{22} \\
\theta_3 &= \frac{2}{3} L_{13} - \frac{1}{3} L_{23} - \frac{1}{3} L_{33}, \\
\phi_3 &= \frac{2}{3} L_{23} - \frac{1}{3} L_{13} - \frac{1}{3} L_{33}, \\
\Omega_3 &= \frac{2}{3} L_{33} - \frac{1}{3} L_{13} - \frac{1}{3} L_{23}
\end{align*}
\]
(4.6)

It is easy to observe that the following relationships exist

\[
\begin{align*}
\Omega_1 &= -(\theta_1 + \phi_1) \\
\Omega_2 &= -(\theta_2 + \phi_2) \\
\Omega_3 &= -(\theta_3 + \phi_3)
\end{align*}
\]
(4.7)
4.2 Identification of Material Parameters

Using the plane stress condition for a thin sheet as an example, the only non-zero stress components are the in-plane stresses ($\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{xy}$). Consider uniaxial loading in the plane (x,y) of the sheet along an arbitrary direction having an angle $\alpha$ with the rolling direction. Then, the Cauchy stress components $\sigma_{xx}$, $\sigma_{yy}$ and $\sigma_{xy}$ can be calculated and consequently $\Sigma_1$, $\Sigma_2$ and $\Sigma_3$ can be obtained as

$$
\begin{align*}
\Sigma_1 &= \frac{1}{2}(C_{xx} + C_{yy} + \sqrt{\left(C_{xx} - C_{yy}\right)^2 + 4C_{xy}^2}) \sigma_\alpha = R_1 \sigma_\alpha \\
\Sigma_2 &= \frac{1}{2}(C_{xx} + C_{yy} - \sqrt{\left(C_{xx} - C_{yy}\right)^2 + 4C_{xy}^2}) \sigma_\alpha = R_2 \sigma_\alpha \\
\Sigma_3 &= C_{zz} \sigma_\alpha = R_3 \sigma_\alpha
\end{align*}
$$

(4.8)

with

$$
\begin{align*}
C_{xx} &= \theta_1 (\cos \alpha)^2 + \phi_1 (\sin \alpha)^2 \\
C_{yy} &= \theta_2 (\cos \alpha)^2 + \phi_2 (\sin \alpha)^2 \\
C_{zz} &= \theta_3 (\cos \alpha)^2 + \phi_3 (\sin \alpha)^2 \\
C_{xy} &= L_{44} \sin \alpha \cos \alpha
\end{align*}
$$

(4.9)

where $\sigma_\alpha$ is the applied stress and $\sigma_\alpha^T$ and $\sigma_\alpha^C$ denote the tensile and compressive yield stresses in the loading direction respectively. Then, the matrix plasticity model, Eq. (4.1), leads to

$$
\begin{align*}
\sigma_{eT}^T &= \sigma_\alpha^T \sqrt{\frac{\left(|R_1|-kR_3\right)^2+\left(|R_2|-kR_3\right)^2+\left(|R_3|-kR_3\right)^2}{\left(|\theta_1|-k\theta_4\right)^2+\left(|\theta_2|-k\theta_4\right)^2+\left(|\theta_3|-k\theta_4\right)^2}} \\
\sigma_{eC}^C &= \sigma_\alpha^C \sqrt{\frac{\left(|R_1|+kR_3\right)^2+\left(|R_2|+kR_3\right)^2+\left(|R_3|+kR_3\right)^2}{\left(|\theta_1|-k\theta_4\right)^2+\left(|\theta_2|-k\theta_4\right)^2+\left(|\theta_3|-k\theta_4\right)^2}}
\end{align*}
$$

(4.10)  (4.11)

In particular, let $\sigma_{0T}^T$ and $\sigma_{0C}^C$ be the tensile and compressive yield stresses along the rolling direction ($\alpha = 0$) respectively. Then Eq. (4.10) becomes $\sigma_{e0T}^T = \sigma_0^T$ and Eq. (4.11) becomes
Similarly, if \( \sigma^T_{90} \) and \( \sigma^C_{90} \) are tensile and compressive yield stress, respectively, in the transverse direction (y-direction), then

\[
\sigma^T_{e90} = \sigma^T_{90} \sqrt{\frac{(\phi_1 - k\theta_1)^2 + (\phi_2 - k\theta_2)^2 + (\phi_3 - k\theta_3)^2}{(\phi_1 - k\theta_1)^2 + (\phi_2 - k\theta_2)^2 + (\phi_3 - k\theta_3)^2}} \tag{4.13}
\]

\[
\sigma^C_{e90} = \sigma^C_{90} \sqrt{\frac{(\phi_1 + k\theta_1)^2 + (\phi_2 + k\theta_2)^2 + (\phi_3 + k\theta_3)^2}{(\phi_1 - k\theta_1)^2 + (\phi_2 - k\theta_2)^2 + (\phi_3 - k\theta_3)^2}} \tag{4.14}
\]

Yielding under equibiaxial tension occurs when \( \sigma_{xx} \) and \( \sigma_{yy} \) are both equal to \( \sigma^T_b \). The equibiaxial tensile stress can be replaced by the uniaxial compression stress value in the normal direction \( \sigma^C_n \) because of the relation given by Eq. (4.7)

\[
\sigma^T_{eb} = \sigma^T_b \sqrt{\frac{(\Omega_1 - k\theta_1)^2 + (\Omega_2 - k\theta_2)^2 + (\Omega_3 - k\theta_3)^2}{(\theta_1 - k\theta_1)^2 + (\theta_2 - k\theta_2)^2 + (\theta_3 - k\theta_3)^2}} \tag{4.15}
\]

Yielding under equibiaxial compression occurs when \( \sigma_{xx} \) and \( \sigma_{yy} \) are both equal to \( \sigma^C_b \). The equibiaxial compressive stress can be replaced by the uniaxial tension stress value in the normal direction \( \sigma^T_n \) because of the relation given by Eq. (4.7)

\[
\sigma^C_{eb} = \sigma^C_b \sqrt{\frac{(\Omega_1 + k\Omega_1)^2 + (\Omega_2 + k\Omega_2)^2 + (\Omega_3 + k\Omega_3)^2}{(\theta_1 - k\theta_1)^2 + (\theta_2 - k\theta_2)^2 + (\theta_3 - k\theta_3)^2}} \tag{4.16}
\]

Under the pure shear condition, for example, if the torque is applied along the axial direction (z-direction), in the local cylindrical coordinate system (\( r, \beta, z \)) the only non-zero stress component is the shear stress \( \sigma_{2\beta} \). Let \( \sigma_l \) denotes the stress tensor expressed in the local coordinate system, \( \sigma_g \) denotes the stress tensor expressed in the global \( (x, y, z) \) coordinate system, and \( T \) denotes the transformation matrix between the local and global coordinates. \( \sigma_g \) and \( \sigma_l \) can then be related by
If $\beta$ is the twist angle in pure torsion experiment (i.e., one that no axial stress develops), the only non-zero stress components in the global coordinate system are $\sigma_{xz}$ and $\sigma_{yz}$

\[
\sigma_{xz} = \sigma_z \beta \sin \beta \\
\sigma_{yz} = \sigma_z \beta \cos \beta
\]

Therefore, if the axis of the torsion specimen is along the normal direction of the plate, the non-zero stress components in the global coordinate system are $\sigma_{zx} = \tau_{n\beta} \sin \beta$ and $\sigma_{zy} = \tau_{n\beta} \cos \beta$ and by substituting them into equation (4.4) and (4.1), the equivalent stress $\tau^{n\beta}_e$ can be calculated; if the axis of the torsion specimen is along the transverse direction, the non-zero stress components in the global coordinate system are $\sigma_{yx} = \tau_{90-\beta} \sin \beta$ and $\sigma_{yz} = \tau_{90-\beta} \cos \beta$ and substituting them into (4.4) and (4.1), the equivalent stress $\sigma^{90\beta}_e$ can be calculated; and if the axis of the torsion specimen is along the rolling direction, the non-zero stress components in the global coordinate system are $\sigma_{xy} = \tau_{0-\beta} \sin \beta$ and $\sigma_{xz} = \tau_{0-\beta} \cos \beta$ and substituting them into equation (4.4) and (4.1), the equivalent stress $\sigma^{0\beta}_e$ can be calculated. If the pure shear data is not available, the uniaxial tensile data along directions other than the axis directions can be used.

The Lankford coefficient $r$ is defined as the width to thickness strain ratio under uniaxial loading. In the $x$-direction the Lankford coefficient is

\[
r_{xx} = \frac{d\varepsilon_{yy}}{d\varepsilon_{zz}} = -\frac{d\varepsilon_{yy}}{d\varepsilon_{xx}+d\varepsilon_{yy}}
\]

or

\[
r^T_0 = -\frac{(1-k)^2\psi_1+(-1-k)^2(\theta_2\phi_2+\theta_3\phi_3)}{(1-k)^2\psi_1(\phi_1+\theta_1)+(-1-k)^2(\theta_2\phi_2+\theta_3\phi_3+\theta^2+\theta^2)}
\]
When yield stresses measured at different orientations with respect to the rolling direction are available, an error function can be defined as

$$\text{Error}(L, k) = \sum_i w_i \left( \frac{\sigma_{T_i}}{\sigma_0} - 1 \right)^2 + \sum_j w_j \left( \frac{\sigma_{C_j}}{\sigma_0} - 1 \right)^2 + \sum_k w_k \left( \frac{\sigma_{\alpha_k}}{\sigma_0} - 1 \right)^2 \quad (4.21)$$

where $i$ represents the number of experimental tensile yield stress, $j$ represents the number of experimental compressive yield stress, $k$ represents the number of experimental shear yield stress, $w_i$, $w_j$ and $w_k$ are the weighting factors given to the respective experimental values, $\sigma_0^T$ is the experimental tensile yield stress along the rolling direction and the theoretical values $\sigma_{T_e}$ and $\sigma_{C_e}$ are calculated according to the Eqs. (4.10) - (4.18).

If the experimental $r$-values (Lankford coefficients) are available, an additional contribution to the error function should be included

$$\text{Error}(L, k) = \sum_i w_i \left( \frac{\sigma_{T_i}}{\sigma_0} - 1 \right)^2 + \sum_j w_j \left( \frac{\sigma_{C_j}}{\sigma_0} - 1 \right)^2 + \sum_k w_k \left( \frac{\sigma_{\alpha_k}}{\sigma_0} - 1 \right)^2 + \sum_m w_m \left( r_{T,\alpha_m} - 1 \right)^2 \quad (4.22)$$

Barlat et al. (2005) suggested that larger weighting factors should be given to yield stress terms than the $r$-value terms because a difference of a few percent in the flow stress is much more significant than in the $r$-values.

The anisotropy and strength differential coefficients, $L$ and $k$, involved in the matrix plasticity model can be determined by the minimizing the error function given by Eq. (4.22). A MATLAB program is written to compute the error function and the MATLAB built-in minimization function “fminsearch” is used to determine the anisotropy coefficients.
4.3 Anisotropic Hardening

The anisotropic hardening, which is due to the evolving texture, cannot be described with the classical isotropic or kinematic hardening laws. Isotropic hardening implies a proportional expansion of the surface, without any changes in shape or position. The classical linear kinematic hardening law describes the pure translation of the initial yield surface. Recently, physically-based models of the evolution of the anisotropic work-hardening under arbitrary strain path changes that involve several tensorial hardening variables have been proposed (e.g. Teodosiu et al., 1995 and Li et al., 2003). It is to be noted that none of these models account for the evolution of texture during work-hardening. The interpolation method has to be used to account for the texture evolution. The method allows for the variation of the anisotropy coefficients with accumulated plastic deformation. For example, the linear transformation operator $L$ is not a constant. $L$ is dependent on the plastic strain, i.e. the linear transformation operator $L$ is a function of the equivalent plastic strain and it will be calculated for a discrete set of values.

The hardening behavior can be described by a stress vs. plastic strain relation. Choosing the equivalent plastic strain $\bar{\varepsilon}^P$ as the hardening parameter, the current material yield strength $\sigma_M$ and the current equivalent stress $\sigma_e$ are found by interpolation based on the current level of the equivalent plastic strain $\bar{\varepsilon}^P$. The procedure is as follows:

First, choosing a particular stress vs. plastic strain curve. Usually, the uniaxial tensile curve in the rolling direction is used as reference hardening curve. An alternative uniaxial tensile curve in transverse or normal direction could be chosen if this particular loading direction would be given in a test.
Second, for a discrete set of strain levels, the yield strength $\sigma_M^i$ associated to the corresponding equivalent plastic strain levels $\bar{\varepsilon}_i^P$ are determined. The anisotropic parameters for the yield model such as the linear transformation operator $L(\bar{\varepsilon}_i^P)$ and the material strength differential effect $k(\bar{\varepsilon}_i^P)$ are identified at the same set of discrete strain levels.

Third, during the FE simulation the current effective plastic strain level is used an interpolation factor between two bounding levels of strain from the discrete set. The interpolation factor is found as

$$\bar{\varepsilon}_i^P \leq \bar{\varepsilon}_i^P \leq \bar{\varepsilon}_{i+1}^P$$

$$\xi = \frac{\bar{\varepsilon}_{i+1}^P - \bar{\varepsilon}_i^P}{\bar{\varepsilon}_{i+1}^P - \bar{\varepsilon}_i^P}$$

Fourth, using the interpolation parameter $\xi$, the current equivalent stress $\sigma_e$ is computed as

$$\beta_e(\sigma_{ij}, \bar{\varepsilon}_i^P)_{\text{current}} = \xi \sigma_e^i + (1 - \xi)\sigma_e^{i+1}$$

Fifth, the current yield strength $\sigma_M$ is found similarly as

$$\sigma_M(\bar{\varepsilon}_i^P)_{\text{current}} = \xi \sigma_M^i + (1 - \xi)\sigma_M^{i+1}$$

Finally, the current yield criteria is taken as

$$\beta_e(\sigma_{ij}, \bar{\varepsilon}_i^P)_{\text{current}} = \sigma_M(\bar{\varepsilon}_i^P)_{\text{current}}$$

4.4 FE Implementation of the Plasticity Model

Before FEA simulation can be performed, the yield criterion of Eq.(4.1) must be implemented into commercial FE code Abaqus. In order to implement the criteria into the
FE code, a stress update algorithm must be used to correctly update the stress at each time increment based on the plastic flow in the increment. An associated plastic flow rule is assumed that the flow potential \( G \) is the same as the yield function \( F \). The hardening parameter depends on the strain history. By enforcing the equivalent of plastic work and the flow potential is first order homogeneous function of stress, the plastic multiplier and the equivalent plastic strain rate is equal to each other, \( i.e. \)

\[
\dot{\lambda} = \dot{\epsilon}^p \tag{4.28}
\]

Within the stress update subroutine, an updated trial stress is computed assuming an elastic increment, \( i.e. \) \( \dot{\sigma} = C: (\dot{\epsilon} - \dot{\epsilon}^p) \), where \( C \) is the elastic compliance tensor. \( \dot{\sigma} \) is trial stress, \( \dot{\epsilon} \) is total strain increment and \( \dot{\epsilon}^p \) is plastic strain increment. Next an. equivalent stress \( \beta_e (\sigma_{ij}, \epsilon^p) \) is computed according to the proposed yield criterion and checked against the current yield stress \( \sigma_M(\epsilon^p) \). If \( \beta_e < \sigma_M \) the current stress state is elastic, the trial stresses are set to the actual stresses. If \( \beta_e > \sigma_M \), the stress state is plastic. In order to update the stress state for this case an iterative scheme is used to return the stress state to the yield surface. The initial guess for the iteration is the updated trial stresses. The total plastic strain \( (\dot{\epsilon}^p) \) is taken as the hardening parameter, the yield criterion being

\[
\Phi = \beta_e (\sigma_{ij}, \epsilon^p) - \sigma_M(\epsilon^p) \tag{4.29}
\]

In the elasto-plastic region, the forward Euler scheme is used to computer the plastic strain increment. The aim is to find the plastic multiplier \( \lambda \). The consistency condition is applied into Eq.4.19, the following equation is obtained:

\[
\dot{\Phi} = \frac{\partial \beta_e}{\partial \sigma} \dot{\sigma} + \frac{\partial \beta_e}{\partial \epsilon^p} \dot{\epsilon}^p - \frac{\partial \sigma_M}{\partial \epsilon^p} \dot{\epsilon}^p = 0 \tag{4.30}
\]

Noticed that

\[
\dot{\sigma} = C: (\dot{\epsilon} - \dot{\epsilon}^p) = C: \left( \dot{\epsilon} - \dot{\lambda} \frac{\partial \beta_e}{\partial \sigma} \right) = C: \dot{\epsilon} - \dot{\lambda} C: \frac{\partial \beta_e}{\partial \sigma} \tag{4.31}
\]
Substitute above Eq. 4.31 into 4.30,
\[ \frac{\partial \beta}{\partial \sigma} : \mathbb{C} : \dot{\varepsilon} - \dot{\lambda} \frac{\partial \beta}{\partial \sigma} : \mathbb{C} \frac{\partial \beta}{\partial \sigma} + \frac{\partial \beta}{\partial \varepsilon^p} \dot{\varepsilon}^p - \frac{\partial \sigma_M}{\partial \varepsilon^p} \dot{\varepsilon}^p = 0 \] (4.32)

Because the flow potential is first order homogeneous function of stress, the plastic multiplier and the equivalent plastic strain rate is equal to each other. The rate form of plastic multiplier can be calculated from the above equation and the plastic multiplier \( \dot{\lambda} \) is given by Eq.4.33.

\[ \dot{\lambda} = \frac{\frac{\partial \beta}{\partial \sigma} : \mathbb{C} \dot{\varepsilon}}{\frac{\partial \beta}{\partial \sigma} : \mathbb{C} \frac{\partial \beta}{\partial \sigma} + \frac{\partial \beta}{\partial \varepsilon^p} \dot{\varepsilon}^p + \frac{\partial \sigma_M}{\partial \varepsilon^p} \dot{\varepsilon}^p} \] (4.33)

where

\[ \frac{\partial \sigma_M}{\partial \varepsilon^p} = \frac{\sigma_M(\bar{\varepsilon}^p_{i+1}) - \sigma_M(\bar{\varepsilon}^p_i)}{\bar{\varepsilon}^p_{i+1} - \bar{\varepsilon}^p_i} \] (4.34)

\[ \frac{\partial \beta}{\partial \varepsilon^p} = \frac{\sigma_{i+1}^e - \sigma_i^e}{\bar{\varepsilon}^p_{i+1} - \bar{\varepsilon}^p_i} \] (4.35)

\[ \frac{\partial \beta}{\partial \sigma} = \xi \frac{\partial \sigma_i^e}{\partial \sigma} + (1 - \xi) \frac{\partial \sigma_{i+1}^e}{\partial \sigma} \] (4.36)

Note: \( i \) and \( i+1 \) represent discrete levels of the yield criterion

The incremental form \( \Delta \lambda \) is obtained through forward-Euler integration, which assumes all derivatives are computed at the beginning of plastic loading:

\[ \Delta \lambda = \frac{\frac{\partial \beta}{\partial \sigma} : \mathbb{C} \Delta \varepsilon}{\frac{\partial \beta}{\partial \sigma} : \mathbb{C} \frac{\partial \beta}{\partial \sigma} + \frac{\partial \beta}{\partial \varepsilon^p} \frac{\partial \varepsilon^p}{\partial \sigma} + \frac{\partial \sigma_M}{\partial \varepsilon^p} \frac{\partial \varepsilon^p}{\partial \sigma}} \] (4.37)

\[ \sigma^1 = \sigma^0 - \Delta \lambda : \mathbb{C} \frac{\partial \beta}{\partial \sigma} : \sigma^0 \] (4.38)

The new stress (Eq.4.38) is evaluated using \( \Delta \lambda \) to see if it lies on the yield surface, i.e. if Eq.4.29 has converged within a specified tolerance. If not, this value is used as the
starting stress for the next iteration. When the stress has converged, the global stress
tensor is updated and returned to the main program.
CHAPTER V
THE GURSON-TYPE-DUCTILE FRACTURE MODEL

This chapter presents a constitutive model, which combines the models proposed by Stewart and Cazacu (2011) and Zhou et al. (2014), to describe the ductile damage process in commercially pure titanium (CP Ti) and to simulate its mechanical response. In particular, a Gurson-type porous material model is modified by coupling two damage parameters, accounting for the void damage and the shear damage respectively, into the yield function and the flow potential. The plastic anisotropy and tension-compression asymmetry exhibited by CP Ti are accounted for by a plasticity model based on the linear transformation of the stress deviator. The theoretical model is implemented in the general purpose finite element software ABAQUS via a user defined subroutine and calibrated using experimental data. Good comparisons are observed between model predictions and experimental results for a series of specimens in different orientations and experiencing a wide range of stress states.

5.1 The ductile damage model

Ductile fracture is usually attributed to a process of void nucleation, growth and coalescence under triaxial stress state (McClintock, 1968; Rice and Tracey, 1969; Van Stone et al., 1985; Garrison Jr. and Moody, 1987) and a process due to shear localization
when the stress triaxiality becomes low (Rice, 1976; Yamamoto, 1978; Mear and Hutchinson, 1985; Barsoum and Faleskog, 2007; Mohr and Marcadet, 2015). One of the most widely used micromechanical models for ductile fracture is due to Gurson with subsequent development by Tvergaard and Needleman (Gurson, 1977; Tvergaard and Needleman, 1984). The yield function of the GTN model takes the following form

\[
\Phi = \left( \frac{\sigma_e}{\sigma_M} \right)^2 + 2q_1f \cosh \left( \frac{q_2 \sigma_{kk}}{2 \sigma_M} \right) - 1 - (q_1f)^2 = 0, \tag{5.1}
\]

where \( f \) is the current void volume fraction, \( \sigma_e \) is the macroscopic effective stress, \( \sigma_{kk} \) is the hydrostatic stress, and \( \sigma_M \) is the current yield stress of the matrix material. The adjustment parameters \( q_1 \) and \( q_2 \) were introduced by Tvergaard (1981, 1982) to improve model predictions. The evolution of the void volume fraction is given by

\[
\dot{f} = \dot{f}_{\text{growth}} + \dot{f}_{\text{nucleation}} \tag{5.2}
\]

where \( \dot{f}_{\text{growth}} \) and \( \dot{f}_{\text{nucleation}} \) represent the growth and nucleation of the voids. Evaluation of the void growth rate is based on the bulk material incompressibility under plastic deformation

\[
\dot{f}_{\text{growth}} = (1 - f) \dot{\varepsilon}_{kk}^P \tag{5.3}
\]

where \( \dot{\varepsilon}_{kk}^P \) represents the first invariant of the plastic strain rate tensor, which defines the rate of volume change. Void nucleation can be stress or strain controlled. A commonly used strain controlled void nucleation law follows a normal distribution in a statistical way as suggested by Chu and Needleman (1980)

\[
\dot{f}_{\text{nucleation}} = A_N \varepsilon_M^P, \quad A_N = \frac{f_n}{s_n \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\varepsilon_M^P - \varepsilon_n}{s_n} \right)^2 \right] \tag{5.4}
\]
where $\varepsilon_M^p$ represents the matrix plastic strain, $S_n$ and $\varepsilon_n$ are the standard deviation and the mean value of the distribution of the plastic strain, and $f_n$ is the total void volume fraction that can be nucleated. Parameters $f_n$, $\varepsilon_n$ and $S_n$ can be treated as material constants.

The effect of rapid void coalescence after the onset of localization is taken into account by replacing $f$ in Eq. (5.1) with an effective porosity $f^*$ defined by the following bilinear function (Tvergaard and Needleman, 1984)

$$f^* = \begin{cases} 
  f & \text{for } f \leq f_c \\
  f_c + \frac{1}{q_1^* - f_c} (f - f_c) & \text{for } f_c \leq f \leq f_f
\end{cases} \quad (5.5)$$

where $f_c$ is the critical void volume fraction at which void coalescence begins and the material softening is accelerated thereafter. As $f$ reaches $f_f$, the material loses all stress carrying capacity.

In the original GTN model, the matrix material obeys the $J_2$ flow plasticity theory, where $\sigma_e$ is the von Mises equivalent stress. Stewart and Cazacu (2011) extended GTN model to account for the plastic anisotropy and tension-compression asymmetry exhibited by the matrix material. The macroscopic yield criterion of this extended model is expressed as

$$\Phi = \left(\frac{\sigma_e}{\sigma_M}\right)^2 + 2q_1 f \cosh \left(\frac{q_2}{h} \frac{\sigma_{kk}}{\sigma_M}\right) - (1 + q_1 f^2) = 0 \quad (5.6)$$

where $h$ is a material parameter depending on the anisotropy coefficients as well as the strength differential coefficient and $\sigma_e$ is defined by Eq. (3.1). Here the plasticity model developed by Cazacu et al. (2006) is adopted to describe the matrix plasticity behavior. This model is based on a linear transformation of the deviatoric part of the Cauchy stress tensor, similar to previous studies by Barlat and coworkers (Barlat et al., 1991, 1997) and
Lademo et al. (1999). The plasticity model developed by Cazacu et al. (2006) is briefly presented in chapter IV, while a detailed description can be found in Cazacu et al. (2006).

The Gurson-type porous material models regards void volume increase as the only contribution to ductile damage and disregards the distortion of voids and inter-void linking in the evolution of the material internal degradation. Consequently these models cannot predict shear induced fracture. Recently Xue (2008) and Nahshon and Hutchinson (2008) proposed similar modifications to the original GTN model to incorporate the shear induced damage. In these modifications, the void volume fraction that appears in Eq. (5.1) is replaced by a general damage parameter containing contributions of both volumetric damage and shear damage while the form of the GTN yield function is retained. Nahshon and Hutchinson (2008) claimed that \( f \) is no longer directly tied to the plastic volume change but rather should be regarded as a damage parameter, and introduced an additional term in the evolution equation of \( f \) to account for shear damage. Xue (2008) directly introduced a new damage parameter, \( D \), which contains both void damage and shear induced damage, and substituted the \( q_1f \) term in Eq. (5.1) with \( D \). Zhou et al. (2014) discussed the drawbacks of using a unified single damage parameter in the GTN yield function and suggested that two damage parameters, the volumetric damage (\( f \)) and the shear damage (\( D_s \)), should be included in the modified GTN model. This is done by combining the damage mechanics concept of Lemaitre with the GTN void growth model. When the total damage \( (q_1f + D_s) \) becomes unity, the material loses its load carrying capacity completely. It is shown that this new model not only is capable of predicting damage and fracture under low (even negative) triaxiality conditions but also suppresses
spurious damage that has been shown to develop in earlier modifications of the GTN model for moderate to high triaxiality regimes (Zhou et al., 2014).

In this work, we combine the models of Stewart and Cazacu (2011) and Zhou et al. (2014) to describe damage evolution in CP Ti. The macroscopic yield criterion is expressed as

$$\phi = \left(\frac{\sigma_e}{\sigma_{M}}\right)^2 + 2q_1 f \cosh\left(\frac{q_2 \sigma_{kk}}{\bar{h} \sigma_{M}}\right) - [1 + (q_1 f + D_s)^2 - 2D_s] = 0 \quad (5.7)$$

where $f$ represents the void volume fraction (volumetric damage), which grows due to the hydrostatic tension and the evolution equation for $f$ is the same as in the original GTN model as described in Eqs. (5.2)-(5.5); $D_s$ represents the shear damage, which accumulates under the deviatoric stress state. This modified model degenerates to the form of the Stewart and Cazacu model (2011) when shear damage does not exist. It reduces to the model of Zhou et al. (2014) when the matrix material is isotropic.

In establishing the shear damage evolution law, Xue (2008), Nahshon and Hutchinson (2008) as well as Zhou et al. (2014) all derived the evolution of shear damage under the pure shear or simple shear state and then extended it to other stress states by introducing a Lode angle dependent function. Nahshon and Hutchinson (2008) proposed a phenomenological shear damage law that assumes linear dependence on the porosity and the effective strain increment. Inspired by the solution for coalescence of holes in a shear band by McClintock et al. (1966), Xue (2008) developed his shear damage law based on the change of the void ligament of a unit cell model under simple shear deformation. Zhou et al. (2014) assumed that shear damage is not directly linked to the void volume fraction and regarded it as an accumulation of plastic deformation.
By comparing the numerical predictions with the experimental data, we find that the shear damage evolution law proposed by Xue (2008) is suitable to describe the current material. This evolution law is expressed as

$$\dot{D}_s = q_3 f_4 g(\theta_L) e_M^p \varepsilon_M^p$$  \hspace{1cm} (5.8)

with

$$g(\theta_L) = \left(1 - \frac{6|\theta_L|}{\pi}\right)$$  \hspace{1cm} (5.9)

In the above equations the Lode angle ($\theta_L$) is defined as

$$\theta_L = \tan^{-1}\left[\frac{1}{\sqrt{3}}\left[2\left(\frac{s_2 - s_3}{s_1 - s_3}\right) - 1\right]\right]$$  \hspace{1cm} (5.10)

where $s_1$, $s_2$ and $s_3$ are the principal values of the deviatoric stress tensor. The Lode angle can be related to the third invariant of the deviatoric stress tensor ($J_3$) through

$$\cos\left(3\theta_L + \frac{\pi}{2}\right) = \frac{27J_3}{2\sigma_e} = \xi$$  \hspace{1cm} (5.11)

Here shear damage is considered as a weighted integration of the equivalent plastic strain increment. According to Xue (2008), $q_4$ should be taken as 1/2 for 2D problems and 1/3 for 3D problems. The $q_3$ term is an adjustable parameter to scale the growth rate of the shear damage.

In summary, the ductile damage model is described by the following equations

$$\Phi = \left(\frac{\sigma_e}{\sigma_M}\right)^2 + 2q_1 f_s \cosh\left(\frac{q_2 \sigma_{kk}}{\sigma_M}\right) - \left[1 + (q_1 f_s + D_s)^2 - 2D_s\right] = 0$$

$$\dot{f} = \dot{f}_{\text{growth}} + \dot{f}_{\text{nucleation}} \quad \dot{f}_{\text{growth}} = (1 - f) \varepsilon_{kk}^p \quad \dot{f}_{\text{nucleation}} = A_N \varepsilon_M^p;$$

$$A_N = \frac{f_n}{s_n \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\varepsilon_M^p - \varepsilon_n}{s_n}\right)^2\right];$$
\begin{equation}
\dot{D}_s = q_3(f^*)^q g(\theta_L)\varepsilon_M^p \varepsilon_M^p ; \ g(\theta_L) = \left(1 - \frac{6|\theta_L|}{\pi}\right); \quad (5.12)
\end{equation}

\begin{equation}
D = q_1 f^* + D_s ;
\end{equation}

\begin{equation}
f^* = \begin{cases} 
  f & \text{for } D \leq q_1 f_c \\
  f_c + \frac{1}{q_1 - f_c} (f - f_c) & \text{for } q_1 f_c \leq D \leq q_1 f_f 
\end{cases}
\end{equation}

where $D$ is the total damage parameter and $\sigma_e$ is defined by Eq. (3.1). When $D$ reaches unity, the material is said to have completely failed.

In this model, two damage parameters, accounting for the void damage and the shear damage respectively, are coupled in the yield function and the flow potential. The material is assumed void free initially and nucleation of voids follows a strain-controlled criterion suggested by Chu and Needleman (1980). Since the void nucleation model suggested by Chu and Needleman takes the form of a normal distribution, strictly speaking void starts to nucleate as soon as material deforms plastically. The majority of void nucleation takes place when the matrix plastic strain reaches the range $\varepsilon_n \pm S_n$. After void nucleation, the evolution of void volume fraction is described by the void growth rate, which is evaluated based on the bulk material incompressibility under plastic deformation. Therefore the void growth rate is driven by the plastic deformation and the stress triaxiality. Because the matrix material is plastically anisotropic and asymmetric with respect to tension and compression, the void growth rate is influenced by the anisotropy coefficients as well as the strength differential coefficient. The shear damage is considered as a weighted integration of the equivalent plastic strain increment. The evolution of the shear damage is driven by the matrix plastic strain and plastic strain rate and is also a function of the void volume fraction and Lode angle.
By enforcing equality between the rates of macroscopic plastic work and the matrix plastic dissipation, the matrix yield stress $\sigma_M$, and the matrix plastic strain rate $\dot{\varepsilon}_M^p$, are coupled through

$$\left(1 - \frac{\rho}{\dot{q}_1}\right)\sigma_M \dot{\varepsilon}_M^p = \sigma_{ij} \dot{\varepsilon}_{ij}^p$$  \hspace{1cm} (5.13)

where the matrix material follows a prescribed hardening function $\sigma_M(e_M^p)$.

5.2 Material and Experiment

The material considered in this research is the commercially-pure Titanium (CP-Ti). It was purchased from TIMET in the form of a hot-rolled plate with an initial thickness of 12.7 mm. All specimens were machined from this plate. The initial anisotropic texture of the CP titanium plate material was measured using Electron Back Scattered Diffraction (EBSD). Strong crystallographic texture was observed with very little grain shape anisotropy (Figure 5.1). Optical microscopy showed that the material has an average grain size of about 38 $\mu$m in R-T plane, 32 $\mu$m in N-T plane and 32 $\mu$m in R-N plane (where R denotes the Rolling Direction, T the in-plane direction orthogonal to rolling and N the normal to the plane of rolling).
Figure 5.1 Initial texture of the CP titanium plate material: (a) R-T plane, color corresponds to crystallographic direction in the normal direction; (b) N-T plane, color corresponds to crystallographic direction in the rolling direction; (c) R-N plane, color corresponds to crystallographic direction in the transverse direction.

The test matrix includes uniaxial tensile specimens (both round and square cross-sections) and compressive specimens (round) in the rolling, transverse and normal (through thickness) directions respectively, tensile notched round bars in the rolling and transvers directions, grooved plane strain specimens in the rolling and transverse directions, and thin wall torsion specimens with the axis in rolling and transverse directions respectively and subjected to pure torsion and combined tension-torsion loading. All tests were performed at room temperature and are considered to be quasi-static. Sketches of selected specimens are shown in Figure 5.2.
Figure 5.2 Sketches of a smooth round bar, a notched round bar, a compression specimen, a torsion tension specimen, and a flat grooved plane strain specimen.

The uniaxial tensile specimens in the rolling and transverse directions were extracted at 0° and 90° respectively from the rolling direction (RD) of a plate. The length of the gauge section is 36 mm and the diameter is 8.89 mm. The specimen geometry followed the ASTM E-8 standard (subsize specimens). Due to the small thickness of the plate (12.7 mm), the uniaxial tensile specimens in normal direction are the miniature specimens extracted in the rolling direction (RD) and normal direction (ND) of the CP-Ti plate, where the square cross section of the gauge area of the bar is 1.58 mm × 1.58 mm and the gauge length is 4.80 mm. The compression specimens in the rolling, transverse and normal directions are cylinders with a diameter of 8 mm and the length/diameter (L/D) ratio of 1.27. For the notched round bars in the rolling and transverse directions, the minimum diameter at the notch section is 4.57 mm, and the notch radius for the B, D and E notched specimens is 1.52, 0.76 and 3.81 mm, respectively. The thin wall torsion specimen is a hollow cylinder with an inner diameter of 6.73 mm and outer diameter of 12.7 mm. The gauge section length and wall-thickness are 2.54 mm and 1.47 mm respectively and the rotation axis is along the rolling direction and the transverse direction. The plane strain specimen is a rectangular plate with a pair of semi-circular
notches at each side of the center line of the plate. Two different groove radii, G-groove and H-groove, and two specimen orientations, rolling and transverse directions, are considered. The minimum thickness at the groove of these flat grooved, plane strain specimens is 2.03 mm, the groove radius of the G-groove specimen is 5.08 mm and radius of the H-groove specimen is 16.26 mm, the plate thickness at the specimen shoulder is 6.1 mm,

The uniaxial tensile experiments in the rolling and transverse directions were performed on an MTS Landmark 370 tension/compression servo-hydraulic testing machine with Flextest software and controller. The capacity of the machine is 250 kN and the stroke is 176 mm. It is equipped with hydraulic grips, which were used to clamp the specimens in these experiments. During the experiments, the load and displacement of the actuator, the average strain over a 25.4 mm gage-length were recorded. The machine was used in displacement control. All uniaxial tensile experiments were performed at a cross-head displacement of 0.04 mm/s, which induced a nominal strain-rate of $10^{-3}$ /s. The compression experiments were performed on an Instron Model # 1350 tension/compression servohydraulic testing machine with DAX software and controller. The capacity of the machine is 100 kN and the stroke is 101.6 mm. The machine is fitted with a custom compression fixture with two cylindrical compression dies aligned vertically along the axis of the actuator. The specimen was placed in the testing machine in between the two compression dies and centered properly, using a custom fixture. The specimen is then secured within the grips by manually pre-loading the specimen up-to few hundreds Newton force. During the experiments, the load and displacement of the actuator was recorded. A special lubricant, Molykote, primarily composed of
Molybdenum disulfide (MoS$_2$) was used to minimize the effects of the contact friction between the die and the specimen. To make the friction minimizing procedure even effective, each test was interrupted three times (a total of four passes) to allow re-lubricating the specimen in the beginning of each pass. The experiment is run in displacement control, with an actuator displacement rate of 0.013 mm/s, which induced a nominal strain-rate of $10^{-3}$ /s. The data acquisition was set at 20 Hz for each of the test. The miniature tensile specimen experiments (uniaxial tensile test in normal direction) were performed on a Meso-Scale Tensile Testing Machine (μTS) from Psylotech with NI LabVIEW software and controller. The μTS frame is equipped with two fixed dovetail structures for sliding the removable grips. The ordinary procedure for running a tensile experiment in this setup is to first clamp the specimen inside the grips and then slide the grips over the two dovetails and attach them to the μTS frame. Considering the size of the miniature specimens and the relatively small gripping length (3 mm), it’s impractical to place the specimen between the grips inside the frame and almost impossible to prevent specimen slipping. To fix these issues, it was decided to make use of extension plates which would come out of the grips and be attached to the specimen with a dowel pin. As seen in Figure 5.3(a), a total of four rectangular steel plates (2 mm in thickness) of suitable dimensions were made and two holes were drilled on each plate in order to run the dowel pins through the grips and through the specimen. Two spacers (same thickness as that of the specimen) are placed between the extension plates in each side of the grips in order to minimize the bending effects on the pins due to the possible misalignment of the plates. A preliminary calculation was done to verify the deformations in the extension
plates at the maximum load are within the elastic range and will not tear the plates near the holes.

Figure 5.3 Experiment set-up: (a) μTS grips and specimen with extension setup (side view); (b) axial-torsion local transducer shown mounted on a specimen in the test frame

The notched round tensile specimens were tested on a 150 kN servo-hydraulic load frame at a nominal cross-head displacement rate of 0.25 mm/min. An extensometer with a gage length of 15 mm was attached such that it spanned the notch. The plane strain tensile experiments were conducted on a 100 kN servohydraulic load frame with hydraulic wedge grips at a nominal cross-head displacement rate of 0.75 mm/min, and an extensometer with a gage length of 25 mm was used to measure the specimen extension.

Pure torsion and combined torsion-tension experiments were conducted using a two-axis MTS servo-hydraulic load frame that is capable of independent and simultaneous application of tension or compression via an axial actuator and torsion via a rotational actuator. The force capacity of this machine is 222 kN and the torque capacity is 2825 kN-mm. In order to more accurately monitor and control the deformation in the specimen gage section, a local transducer that measures extension plus rotation across the gage section was used. The transducer, shown in Figure 5.3(b), uses two capacitive
displacement probes to achieve non-contacting measurement of both extension and rotation. The upper and lower parts of transducer are attached to the specimen just above and just below the notch. The axial extension is measured between the axial probe (right) and a flat surface that is normal to the specimen axis. The rotation is determined by measuring the gap between the angle probe (left) and a spiral ramp, and subtracting the axial extension from that gap. The transducer is calibrated to obtain the relationship between the rotation angle and the corrected gap. The spiral ramp was designed to provide a rise of 6.35 mm in 180-degrees, thereby providing a ramp rate of 28 deg/mm. During the test, the axial extension probe is used to control the axial actuator, while a Rotary Variable Differential Transformer (RVDT) on the rotary actuator is used to control rotation. The RVDT is used to control rotation instead of the angle probe because it provides direct control (does not require correction) and experience has shown that during plastic deformation there is not much difference between the RVDT and the local gage because the ends of the specimen have minimal twist at the torques applied. The extension and rotation rates were chosen to achieve an axial strain rate of $10^{-3}$ /s and the specific tension–torsion ratio for the test.

5.3 Modeling the ductile fracture behavior of Commercially Pure Titanium

The ductile damage model described above is implemented in ABAQUS via a user defined subroutine and is calibrated in this section for commercially pure titanium. Among the experiments conducted, the uniaxial tensile and compressive data in rolling, transverse and normal directions, the in-plane tensile data along the 45° direction, the pure torsion data with the axis of the specimen along the rolling and transverse directions are used to calibrate the matrix plasticity model, and the round tensile specimen and the
pure torsion specimen in the transverse direction are used for the ductile damage model calibration. Other specimens, including the notched round tensile specimens, the flat grooved plane strain tensile specimens and the tension-torsion specimens are used for model validation.

5.3.1 Calibration of the matrix plasticity model

The engineering stress-strain curves and true stress vs. true plastic strain curves obtained from the uniaxial tension and the uniaxial compression tests in the rolling, transverse and normal directions respectively are shown in Figures 5.4-5.6. (1) stress-strain curve which is obtained from the uniaxial tension experiment data; (2) compare the predicted load vs. displacement response with experimental measurements; (3) adjust the stress-strain curve based on the error shown in step (2); (4) repeat steps (1) – (3) until the error between the numerical and experimental results reaches an acceptable level. Because the specimen is under uniaxial loading, the finite element simulation is conducted using the $J_2$ plasticity (von Mises) model. The three-dimensional, 8-node brick elements with reduced integration (C3D8R) are used in the finite element simulations.

Figure 5.7 shows an example of how the tensile, true stress vs. true plastic strain curve in the rolling direction (RD) is obtained. Figure 5.7(a) shows three different stress-strain curves used in the finite element simulations, denoted as DATA1, DATA2 and DATA3 respectively. The portion of the three curves for the range of plastic strain less than 0.1 is obtained by converting the engineering stress and strain values into true stress and strain values. The three stress-strain curves differ after the plastic strain reaches 0.1 and beyond. Figure 5.7(b) compares the computed load-displacement curves using the three stress-strain curves with the experimental data, where the experimental data is denoted as SRB-
RD, the numerical result using stress-strain curve DATA1 is denoted as FEA1, the numerical result using stress-strain curve DATA2 is denoted as FEA2, and the numerical result using stress-strain curve DATA3 is denoted as FEA3. As shown in Figure 5.7(b), all three stress-strain curves give excellent prediction of the load-displacement response in the early stage, but stress-strain curve DATA1 leads to an under-prediction while stress-strain curve DATA3 leads to an over-prediction of the applied force after the nominal strain exceeds 0.1. The stress-strain curve DATA2 results in an excellent match with the experimental data and thus is chosen as tensile stress-strain curve of the material in the rolling direction.

The tensile, true stress-strain curves in the transverse and normal directions are obtained in the same way. Specimen barreling is not observed in the compression tests. The compressive, true stress-strain curves in the rolling, transverse and normal directions are obtained by converting the corresponding engineering stress and strain values into true stress and strain values respectively.

![Stress-strain curves in tension and compression along the in-plane rolling direction (RD): (a) the engineering stress-strain curve; (b) the true stress vs. true plastic strain curve.](image)

Figure 5.4 Stress-strain curves in tension and compression along the in-plane rolling direction (RD): (a) the engineering stress-strain curve; (b) the true stress vs. true plastic strain curve.
Figure 5.5 Stress-strain curves in tension and compression along the in-plane transverse direction (TD): (a) the engineering stress-strain curve; (b) the true stress vs. true plastic strain curve.

Figure 5.6 Stress-strain curves in tension and compression along the normal direction (ND): (a) the engineering stress-strain curve; (b) the true stress vs. true plastic strain curve.

Figure 5.7 (a) Three different stress-strain curves used in the finite element simulations; (b) Load-displacement curves of the smooth round tensile specimen in rolling direction predicted by using different stress-strain curves
Figures 5.4-5.6 confirm that the material is anisotropic and displays tension-compression asymmetry. Furthermore, the extent of plastic anisotropy and strength differential response evolves as the plastic deformation increases. In Figures 5.4-5.6, the symbols, for which the numerical values are listed in Table 5.1, indicate the data point used in the optimization process to calculate the linear transformation matrix $L$ and the strength differential coefficient $k$ which are involved in the expression of the matrix plasticity model introduced by Cazacu et al. (2006).

Table 5.1 Stress-strain data in rolling, transverse and normal directions at different plastic strain levels

<table>
<thead>
<tr>
<th>Plastic Strain</th>
<th>True stress (MPa)</th>
<th>Uniaxial Tension in Rolling Direction</th>
<th>Uniaxial Compression in Rolling Direction</th>
<th>Uniaxial Tension in Transverse Direction</th>
<th>Uniaxial Compression in Transverse Direction</th>
<th>Uniaxial Tension in Normal Direction</th>
<th>Uniaxial Compression in Normal Direction</th>
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</thead>
<tbody>
<tr>
<td>0.01</td>
<td></td>
<td>405.07</td>
<td>365.65</td>
<td>445.04</td>
<td>451.71</td>
<td>444.99</td>
<td>451.98</td>
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<tr>
<td>0.05</td>
<td></td>
<td>487.76</td>
<td>461.73</td>
<td>515.96</td>
<td>548.27</td>
<td>523.38</td>
<td>561.92</td>
</tr>
<tr>
<td>0.1</td>
<td></td>
<td>530.59</td>
<td>538.25</td>
<td>550.00</td>
<td>609.23</td>
<td>573.94</td>
<td>623.87</td>
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<tr>
<td>0.15</td>
<td></td>
<td>555.62</td>
<td>619.20</td>
<td>568.64</td>
<td>660.00</td>
<td>598.09</td>
<td>672.11</td>
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<td></td>
<td>579.71</td>
<td>704.65</td>
<td>591.62</td>
<td>709.26</td>
<td>617.11</td>
<td>717.44</td>
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<tr>
<td>0.25</td>
<td></td>
<td>603.92</td>
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<td>758.32</td>
<td>632.69</td>
<td>763.12</td>
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<td>0.3</td>
<td></td>
<td>628.28</td>
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<td>642.09</td>
<td>804.44</td>
<td>647.22</td>
<td>803.43</td>
</tr>
</tbody>
</table>

Using the tensile and compressive flow stress data in rolling, transverse and normal directions, the in-plane tensile flow stress data along the 45° direction, the pure torsion data with the axis of the specimen along the rolling and transverse directions, and the $r$-values obtained in the uniaxial tensile test along the rolling direction, the anisotropy coefficients $L$ and $k$ involved in the expression of the matrix plasticity model (Cazacu et al. (2006)) are calculated following the procedure outlined in section 4.2 and the
parameters are listed in Table 5.2. The experimental data indicate that there is distortion of the yield surface even for the simplest monotonic loading paths. As a result, the anisotropy coefficients vary with the plastic strain.

Figure 5.8 plots the predicted yield loci given by Eq. (3.1) in comparison with the experimental data at different strain levels, where the open circles represent the RD tensile flow stress data, the solid circles represent the RD compressive flow stress data, the open triangle represent the TD tensile flow stress data, the solid triangle represent the TD compressive flow stress data, the open diamonds represent the ND tensile flow stress data, and the solid diamonds represent the ND compressive flow stress data. It is shown that the criterion describes well the asymmetry and anisotropy in yielding. Furthermore, the yield loci changes shape as the plastic strain increases, from the ellipse-like shape to the triangle-like shape, reflecting the material texture evolution.

Table 5.2 Calibrated anisotropy and strength differential coefficients

<table>
<thead>
<tr>
<th>$\varepsilon^p$</th>
<th>$k$</th>
<th>$L_{11}$</th>
<th>$L_{12}$</th>
<th>$L_{13}$</th>
<th>$L_{22}$</th>
<th>$L_{23}$</th>
<th>$L_{33}$</th>
<th>$L_{44}$</th>
<th>$L_{55}$</th>
<th>$L_{66}$</th>
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<tbody>
<tr>
<td>0.01</td>
<td>-0.0167</td>
<td>0.9997</td>
<td>-0.0017</td>
<td>0.0020</td>
<td>0.0655</td>
<td>0.9362</td>
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<td>1.0525</td>
<td>0.7544</td>
<td>1.0250</td>
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<tr>
<td>0.05</td>
<td>-0.0850</td>
<td>1.0556</td>
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<td>-0.0242</td>
<td>1.0030</td>
<td>0.0284</td>
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<td>0.0224</td>
<td>0.9539</td>
<td>0.0475</td>
<td>0.9302</td>
<td>1.0362</td>
<td>0.8087</td>
<td>1.0350</td>
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<td>1.0359</td>
<td>-0.0342</td>
<td>-0.0018</td>
<td>1.0185</td>
<td>0.0157</td>
<td>0.9861</td>
<td>1.1191</td>
<td>0.9590</td>
<td>1.1919</td>
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<tr>
<td>0.2</td>
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<td>0.9515</td>
<td>0.0118</td>
<td>0.0367</td>
<td>0.9390</td>
<td>0.0492</td>
<td>0.9141</td>
<td>0.9737</td>
<td>0.8644</td>
<td>1.1677</td>
</tr>
<tr>
<td>0.25</td>
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<td>1.0152</td>
<td>-0.0155</td>
<td>0.0003</td>
<td>0.9999</td>
<td>0.0156</td>
<td>0.9841</td>
<td>1.0613</td>
<td>0.9924</td>
<td>1.5451</td>
</tr>
<tr>
<td>0.3</td>
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<td>1.0078</td>
<td>-0.0075</td>
<td>-0.0003</td>
<td>0.9933</td>
<td>0.0142</td>
<td>0.9861</td>
<td>1.0386</td>
<td>0.9983</td>
<td>1.7473</td>
</tr>
</tbody>
</table>
5.3.2 Calibration of the ductile damage model

Parameters $q_1$ and $q_2$ were introduced by Tvergaard (1981, 1982) to the original Gurson model to improve model predictions. Here the values suggested by Tvergaard (1981, 1982) are adopted, i.e., $q_1 = 1.5$, $q_2 = 1$. The ductile damage model calibration follows a two-step strategy. For specimens where the onset of fracture was dominated by the void damage mechanism, e.g., the round tensile specimens, the calibration of void related parameters are conducted. The shear damage parameters are calibrated using the test data where fracture is dominated by shear damage, e.g., the pure torsion specimen. Cockeram and Chan (2009, 2013) conducted in-situ and ex-situ experimental studies over a range of positive stress triaxialities on $\beta$-treated Zircaloy-4 and Zircaloy-2, which have similar crystal structures as CP titanium at room temperature. They observed that the
materials are initially void-free and void nucleation occurs at precipitates located at lath boundaries as well as within the lath at the intersection of slip bands. Their experimental studies indicate that the critical local strain at the initiation of void nucleation is almost constant for all the stress states studied. They also observed that void nucleation process in the tensile specimens occurs at the UTS and once the voids are nucleated, little growth is required before the voids coalesce. In the present study, the void nucleation model is motivated by the work of Cockeram and Chan (2009, 2013). In the tensile tests of the smooth round specimens it is observed that necking occurs at a global strain of about 0.1. Finite element simulations reveal that the local strain in the center element when noticeable necking takes place is around 0.22, which is taken as the mean strain for void nucleation, $\varepsilon_n$. A relatively small standard deviation of the void nucleation strain ($S_n$) is chosen to facilitate a rapid void nucleation process. Parameters $f_n$, $f_c$ and $f_f$ are calibrated using the round tensile specimen, and the detailed calibration process is shown in Figure 5.9.

Figure 5.9 (a) compares the predicted load-displacement curves using three different $f_n$-values with the experiment data. The experiment data of the smooth round tensile specimens in the transverse direction is denoted as SRB-TD. Numerical results using three different $f_n$-values are shown in Figure 5.9(a), where the curve FEA1 represents the result using a larger value of $f_n$, the curve FEA3 represents the result using a smaller value of value of $f_n$, and the curve FEA2 represents the result using an intermediate value of $f_n$. As shown in the figure, a larger $f_n$-value results in a softer material response and predicting a lower load carrying capacity. Using an $f_n$-value of
0.002, the predicted load-displacement curve best matches the experiment data and thus the calibrated $f_n$-value is taken as 0.002.

Figure 5.9 (b) compares the predicted load-displacement curves using three different $f_c$-values with the experiment data, where curves FEA1, FEA2 and FEA3 correspond to numerical predictions using a large, an intermediate and a small $f_c$-value respectively. As shown in the figure, a smaller $f_c$-value leads to an earlier fracture initiation and sudden drop of the load-displacement curve. By comparing multiple simulation results with the experiment data, the $f_c$-value is calibrated as 0.035.

Parameter $f_f$ corresponds to the value of void volume fraction when the material completely loses stress carrying capacity. It only affects the slope of the final portion of the predicted load-displacement curve, after the point of sudden load drop. The value of parameter $f_f$ is set to be 0.12.

![Figure 5.9](image.png)

Figure 5.9 Comparison of the predicted load vs. displacement curve with the experimental data for the smooth round tensile specimen in transverse direction (a) with different values of $f_n$; (b) with different values of $f_c$.

It is worth noting that during the review process of this manuscript, Revil-Baudard et al. (2016) published a paper for a high-purity titanium, which contains ex-situ and in-situ X-ray tomography measurements of damage distribution and evolution as well
as modeling using an anisotropic elastic/plastic damage model. The above calibrated $f_n$-value is roughly in agreement with the void volume fraction determined by Revil-Baudard et al. (2016).

Parameters $q_3$ and $q_4$ are related to the shear damage. Xue (2008) suggested $q_4$ should be set to $1/3$ for 3D problems. Here we calibrate $q_3$ using the pure torsion test data. Figure 5.10 compares the numerically predicted the torque vs. twist angle responses using three different $q_3$ values with the experiment data. Here the experimental data is denoted as PureTorsion-T and the numerical results using a large, an intermediate and a small $q_3$-value are denoted as FEA1, FEA2 and FEA3 respectively. As shown in Figure 5.10, the smaller value of $q_3$ delays the sudden load drop on the predicted torque vs. twist angle response. The calibrated $q_3$ value is 1.6.

![Figure 5.10 Comparison of the predicted torque vs. twist angle response using different values of $q_3$ with the experiment data for the pure torsion specimen (rotation axis is along the transverse direction)](image)

Table 5.3 lists the calibrated damage model parameters. To validate the calibrated model, the B-notched, the D-notched, E-notched round bar, the flat grooved plane strain tensile specimen and tension-torsion specimen are simulated using the calibrated model
parameters and the numerical results are compared with the experimental data in section 4. All the specimens are modeled in three dimensions because the material is anisotropic.

Table 5.3 Calibrated damage model parameters

<table>
<thead>
<tr>
<th>$f_n$</th>
<th>$f_c$</th>
<th>$q_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.002</td>
<td>0.035</td>
<td>1.6</td>
</tr>
</tbody>
</table>

5.4 Comparison between the numerical and the experimental results

This section shows the comparisons between the model predictions and the experimental data. In the numerical simulations, the tensile stress-strain curve obtained from the smooth round specimen in the rolling direction is used together with the calibrated model parameters listed in Tables 5.2 and 5.3. It is worth noting that the portion of the stress-strain curve, where the plastic strain is greater than 0.1 is obtained using an inverse, iterative method, as described in section 4.

ABAQUS/Explicit was used to analyze all the specimens, where the material model was implemented via a user defined subroutine VUMAT. A forward-Euler scheme for the stress integration proposed by Crisfield (1997) was adopted. The numerical implementation procedure is shown in a below flow chart (Figure 5.11).
Figure 5.11 The flow chart of numerical implementation procedure
5.4.1 Finite element models of specimens

In the finite element analyses, three-dimensional, 8-node brick elements with reduced integration (C3D8R) are used for all specimens, the element size is 63.5μm × 63.5μm × 63.5μm in the region around the center of the specimen where failure is expected to occur. To improve model efficiency, symmetry conditions are applied whenever available. Figure 5.12 shows typical finite element meshes of the notched round tensile specimen, the plane strain tensile specimen and the torsion specimen. Note that axisymmetry was not used in creating these models, because of the material anisotropy.

Figure 5.12 Typical finite element meshes of the notched round tensile specimen, the plane strain tensile specimen and the torsion specimen

5.4.2 Load vs. displacement and torque vs. twist angle response

Figures 5.13(a) and (b) compare the computed load vs. displacement curves of the smooth round tensile specimens in the rolling and transverse directions with the experimental data while Figure 5.13(c) compares the computed load vs. displacement
curve of the miniature tensile specimen in the normal direction with the experimental data. Figure 5.14 compares the load vs. displacement response between the numerical simulations and the experimental data for the compression specimens in the rolling, transverse and normal directions respectively. There was no failure observed in these compression specimens before the experiments were stopped. In these figures, “RD” refers to the rolling direction, “TD” refers to the transverse direction, “ND” refers to the normal direction, the thicker lines (in red) represent the simulation results and the thinner lines (in black) represent the corresponding experiment data. Figure 5.15 compares the computed torque vs. twist angle responses of the pure torsion specimens with the experimental data. Since all these specimens were used to identify the anisotropic and strength differential coefficients of the plasticity model, the predicted mechanical responses of these specimens agree very well with the experimental measurements. The model captures the plastic anisotropy and tension-compression asymmetry exhibited by CP Ti. The round tensile specimen and the pure torsion specimen are used to calibration the ductile damage parameters. As a result, the predicted onset of fracture (sudden load drop) of these specimens shows very good agreement with experimental observations.
Figure 5.13 Comparisons between the numerical predictions and the experimental data for the tension specimen: (a) smooth round bar in rolling direction; (b) smooth round bar in transverse direction; (c) tensile bar in normal direction.
Figure 5.14 Comparisons between the numerical predictions and the experimental data for the compression specimen: (a) rolling direction; (b) transverse direction; (c) normal direction.

Figure 5.15 Comparisons of the computed torque vs. twist angle response with the experimental data for the pure torsion specimen ("R" refers to the rolling direction and "T" refers to the transverse direction)

To validate the calibrated model, the notched round tensile specimens, the flat grooved plane strain tensile specimens and the tension-torsion specimens are analyzed.
Figure 5.16 Comparisons of the computed force vs. displacement responses with the experimental data for the notched round tensile specimens in rolling and transverse directions: (a) Specimen D (notch radius = 0.762mm); (b) Specimen B (notch radius = 1.524mm); (c) Specimen E (notch radius = 3.81mm)
Figure 5.16 compares the computed load vs. displacement responses with the experimental data for the notched round tensile specimens having different notch radii. The comparisons are made in both rolling and transverse directions. The load displacement discrepancy is partly due to material hardening and plastic anisotropic effects without explicitly considering the different values of stress triaxility. Generally, the model is able to accurately predict the load-displacement responses of these specimens.

Figure 5.17 compares the computed load vs. displacement responses with the experimental data for the flat grooved plane strain tensile specimens having different groove radii. The comparisons are made in both rolling and transverse directions. The load and displacement curves are higher than the experimental results especially along the rolling directions. While assigning material directions to plane strain experiment specimens, ND direction is along the thickness direction of the specimen. During the tensile loading, in the ND direction, the specimen experience compression as similar to necking effects shown in tensile round bar test specimen. According to Figure 5.8, the yield loci for plastic strain larger than 0.1 falls outside the corresponding experimental points. This is the reason that primarily leads to the FEA predictions of force and displacement curve higher than the experimental ones. In general, the model is reasonably capable of predicting the load-displacement responses of these specimens. The failure predictions of the round notched specimens show better agreement with experimental observations than the flat grooved specimens.
Figure 5.17 Comparisons of the computed force vs. displacement responses with the experimental data of plane strain tensile specimens in rolling and transverse directions: (a) Specimen G (groove radius = 5.08mm); (b) Specimen H (groove radius = 16.256mm)
Figures 5.18-5.25 provide the comparisons of the axial force vs. axial displacement and torque vs. twist angle responses between the numerical simulations and the experimental data for the tension-torsion specimens with various applied tensile displacement/twist angle ratios. These tensile displacement/twist angle (mm/radian) ratios are listed in Table 5.4. Different applied tensile displacement/twist angle ratio leads to different stress states experienced by the specimen (Graham et al., 2012). It can be seen that model predictions agree very well with experiments. Several specimens (especially those with lower applied tensile displacement/twist angle ratios) see a sharp decrease in axial force in the plastic region. This is because the experiment is conducted under displacement-control and the material exhibits the so-called Swift effect, i.e., the material elongates naturally under torsional loading. When the applied tensile displacement is less than the natural elongation of the specimen, a compressive force is actually imposed on the specimen. The model is able to predict this complex behavior.

Table 5.4 Ratio of the applied tensile displacement over the applied twist angle used in the tension-torsion test

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Rotation axis along the rolling direction</th>
<th>L9</th>
<th>L8</th>
<th>L7, L10</th>
<th>L6, L11</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Rotation axis along the transverse direction</td>
<td>T5, T11</td>
<td>T10</td>
<td>T9, T6</td>
<td>T8, T1</td>
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<tr>
<td>Tensile displacement/twist angle (mm/radian)</td>
<td>0.10668</td>
<td>0.2794</td>
<td>0.5334</td>
<td>1.1684</td>
<td></td>
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</tbody>
</table>
Figure 5.18 Comparisons between the numerical predictions and the experimental data for the tension-torsion specimen (tensile displacement/twist angle (mm/radian) = 1.1684) in rolling direction: (a) torque vs. twist angle; (b) axial force vs. axial displacement

Figure 5.19 Comparisons between the numerical predictions and the experimental data for the tension-torsion specimen (tensile displacement/twist angle (mm/radian) = 1.1684) in transverse direction: (a) torque vs. twist angle; (b) axial force vs. axial displacement
Figure 5.20 Comparisons between the numerical predictions and the experimental data for the tension-torsion specimen (tensile displacement/twist angel (mm/radian) = 0.5334) in rolling direction: (a) torque vs. twist angle; (b) axial force vs. axial displacement

Figure 5.21 Comparisons between the numerical predictions and the experimental data for the tension-torsion specimen (tensile displacement/twist angel (mm/radian) = 0.5334) in transverse direction: (a) torque vs. twist angle; (b) axial force vs. axial displacement

Figure 5.22 Comparisons between the numerical predictions and the experimental data for the tension-torsion specimen (tensile displacement/twist angel (mm/radian) = 0.2794) in rolling direction: (a) torque vs. twist angle; (b) axial force vs. axial displacement
Figure 5.23 Comparisons between the numerical predictions and the experimental data for the tension-torsion specimen (tensile displacement/twist angle (mm/radian) = 0.2794) in transverse direction: (a) torque vs. twist angle; (b) axial force vs. axial displacement

Figure 5.24 Comparisons between the numerical predictions and the experimental data for the tension-torsion specimen (tensile displacement/twist angle (mm/radian) = 0.10688) in rolling direction: (a) torque vs. twist angle; (b) axial force vs. axial displacement

Figure 5.25 Comparisons between the numerical predictions and the experimental data for the tension-torsion specimen (tensile displacement/twist angle (mm/radian) = 0.10688) in transverse direction: (a) torque vs. twist angle; (b) axial force vs. axial displacement
5.4.3 Prediction of fracture initiation and propagation

The contour plots of equivalent plastic stain, triaxiality, porosity and total damage are presented in Figure 5.26 for the smooth round tensile specimen in the transverse direction before fracture initiation. The equivalent plastic strain contour shows higher values at the center of the specimen and the maximum value for porosity is observed at the center of the specimen. The crack initiates in the center of specimen where the triaxiality is the highest.

In Figure 5.27, the contour plots of equivalent plastic stain, triaxiality, porosity and total damage are presented for the smooth round tensile specimen in the rolling direction before fracture initiation. It is shown that the region with high values of equivalent plastic strain, triaxiality, porosity and total damage shifts from the center of the specimen outwards when compared with the TD specimen (in Figure 5.26). Similar phenomenon was observed by Revil-Baudard et al. for isotropic materials exhibiting strong strength differential effect (Revil-Baudard et al. 2012) as well as a high-purity titanium material (Revil-Baudard et al. 2016).

Figure 5.28 shows the contour plots of the G-grooved plane strain specimen (groove radius is 5.08 mm) in the rolling direction. Both the stress triaxiality and the total damage have high values in a region at the specimen center, shown in Figure 5.28(a), (b). Figure 5.28(a) and Figure 5.28(b) are the center section view of the specimen. Consequently fracture initiates at the center of the specimen and propagates to the sides of the specimen, Figure 5.28(c). This figure is a front view of the specimen. Figure 5.28(d) shows the photo of a fractured specimen which verifies the simulation result shown in Figure 5.28(c).
Figure 5.26 Contour plots for the smooth round tensile specimen in the transverse direction: (a) equivalent plastic strain; (b) triaxiality; (c) porosity; (d) total damage.

Figure 5.27 Contour plots for the smooth round tensile specimen in the rolling direction: (a) equivalent plastic strain; (b) triaxiality; (c) porosity; (d) total damage.
Figure 5.28 Crack initiation and growth in the flat grooved plane strain tensile specimens (groove radius is 5.08 mm) in the rolling direction: (a) contour plot of triaxiality before fracture initiation; (b) contour plot of total damage before fracture initiation; (c) final fracture; (d) photo of a fractured specimen.

Figure 5.29 Crack initiation and growth in the pure torsion specimen (specimen axis is along the transverse direction): (a) equivalent plastic strain contour; (b) shear damage contour; (c) final fracture; (d) photo of the fractured specimen.
Figure 5.30 Crack initiation and growth in the tension-torsion specimen (specimen axis is along the transverse direction; applied tensile displacement/twist angle ratio is 0.10688 mm/radian): (a) equivalent plastic strain contour; (b) shear damage contour; (c) final fracture; (d) photo of the fractured specimen.

Figure 5.31 Crack initiation and growth in the tension-torsion specimen (specimen axis is along the transverse direction; applied tensile displacement/twist angle ratio is 1.1684 mm/radian): (a) equivalent plastic strain contour; (b) shear damage contour; (c) porosity; (d) total damage; (e) final fracture; (f) photo of the fractured specimen.
Figure 5.29 shows the fracture initiation and propagation process in the pure torsion specimen with the axis along the transverse direction. The maximum plastic strain occurs in the transition region due to strain concentration, and as a result, the shear damage is highest in the transition region. The model predicts fracture initiates in the transition region and propagates circumferentially, Figure 5.29(c), which agrees with the experimental observation, Figure 5.29(d).

Figure 5.30 shows the fracture initiation and propagation process in a tension-torsion specimen with a small applied tension-torsion ratio. Here the specimen axis is along the transverse direction and the applied tensile displacement/twist angle (mm/radian) ratio is 0.10688. Similar to the pure torsion specimen, fracture initiate at the transition region where the plastic strain is the highest and propagates circumferentially. As the applied tension-torsion ratio changes, the stress state experienced by the material changes, which affects the ductile damage evolution process. Figure 5.31 shows the fracture initiation and propagation process in a tension-torsion specimen with a larger applied tension-torsion ratio. Here the specimen axis is along the transverse direction and the applied tensile displacement/twist angle (mm/radian) ratio is 1.1684. The contour plots of equivalent plastic strain, shear damage, porosity and total damage are shown in Figures 5.31(a)-(d). In this case, fracture initiates at the outer equator in the middle of specimen gauge section and propagates circumferentially, Figure 5.31(e), which agrees with the experimental observation, Figure 5.31(f). Also a similar conical phenomenon in gage section is observed from both the simulation and experiment test result.
The change of fracture initiation location is a reflection of the damage evolution process in the specimen, which contains two contributions, void damage and shear damage. Figures 5.32-5.34 show the damage evolution in the element where fracture initiates in the pure torsion specimen, the tension-torsion specimen (applied tensile displacement/twist angle ratio = 1.1684 mm/radian), and the B-notch specimen respectively. The axes of these three specimens are in the transverse direction. In the pure torsion specimen, void nucleates at a plastic strain value around 0.22 but does not grow as plastic deformation increases. Therefore, the total damage is solely due to shear damage accumulation. In the tension-torsion specimen, both void growth and shear deformation contribute to the total damage. In the B-notch specimen, the total damage is almost entirely due to void growth. The model is able to capture the effect of stress state and the change of fracture mechanism.

Figure 5.32 Pure torsion specimen in transverse direction: (a) damage evolution in a critical element in the transition region; (b) specimen torque vs. twist angle response and damage evolution in the critical element.
Figure 5.33 Tension-torsion specimen in transverse direction (the ratio of tensile displacement vs. twist angle = 1.1684 mm/radian): (a) damage evolution in the element at the outer equator of the specimen; (b) specimen torque vs. twist angle response and damage evolution in the critical element; (c) specimen axial force vs. axial displacement response and damage evolution in the critical element.

Figure 5.34 B-notch tensile specimen in transverse direction: (a) damage evolution in the center element; (b) specimen load-displacement response and damage evolution in the critical element.
5.5 Conclusion

In this chapter, the constitutive models of Stewart and Cazacu (2011) and Zhou et al. (2014) are combined to describe the ductile damage process in a commercially pure titanium (CP Ti), where the plastic anisotropy and tension-compression asymmetry of the matrix material are addressed and two damage parameters accounting for void damage and shear damage are coupled into the yield function and flow potential. In Stewart and Cazacu (2011) the Gurson model is extended to account for the plastic anisotropy and tension-compression asymmetry of the matrix material, while in Zhou et al. (2014) shear damage is introduced but the matrix plastic anisotropy is not considered. The model presented in this paper not only accounts for matrix plastic anisotropy and tension-compression asymmetry but also considers both void volume damage and shear damage. It is able to simulate damage in pure torsion specimen or torsion-tension tests or any specimen includes shear damage. Literature review indicates that this is the first time to simulate failure under shear dominated conditions in an anisotropic and tension-compression asymmetric material.

To quantify the plastic behavior of the material, uniaxial tension and compression tests were conducted at different orientations. The material exhibits strong plastic anisotropy and tension-compression asymmetry. Furthermore, the extent of plastic anisotropy and strength differential response evolves as the plastic deformation increases. Additional tests of notched round specimens, grooved plane strain specimens, pure torsion specimens and tension-torsion specimens were conducted to study the effect of stress state on ductile damage evolution. The model is shown to capture the effect of stress state and the change of fracture mechanism. The results also reveal the important
effect of the plastic anisotropy and tension-compression asymmetry on the ductile damage process. For example, unlike what was observed for conventional isotropic materials such as steels or aluminum alloys, fracture in round tensile specimens of CP titanium in the rolling direction actually propagates from the surface to the specimen center.

The procedures for model calibration are detailed in the chapter and the model parameters are calibrated using experimental data. The calibrated model is shown to be capable of predicting the deformation behavior and damage evolution process in various specimens. The numerical results, including the predicted load-displacement or torque-twist angle curves and the fracture initiation locations, show very good agreement with the experiment data and observations.
CHAPTER VI

CONCLUSION AND FUTURE WORK

6.1 Conclusions

This dissertation intends to develop models to describe the ductile damage process in metallic materials. The conclusions from the study are summarized as below:

1. A pressure-insensitive, continuum plasticity model, dependent on the second and third invariants of the stress deviator ($J_2$ and $J_3$) is proposed. Depending on whether the power of the $J_3$ term is odd or even, the proposed model can capture either tension-compression strength-differential (S-D) effect or the torsion-tension strength differential effect of the material. Considering the tension-compression asymmetry of a $\beta$-treated Zircaloy-4 and its distortional hardening of the non-Mises yield surface, the plastic response is described by the proposed model with the $J_3$ term having an odd power in conjunction with an internal variable related to the plastic strain. The plasticity model has been calibrated and validated using measured results from an experimental test program. Results show that the model captures the complex elastic-plastic response observed in measured load-displacement and torque-rotation curves over a wide range of triaxiality and Lode parameter values. The plasticity model with an even power of the $J_3$ term is calibrated and validated using measured experimental data of a stainless steel, Nitronic
40, with specimens experiencing different stress states. The calibrated model results in good agreement between numerical predictions and experimental measurements.

2. The effect of the material plastic behavior on the ductile damage process is studied by conducting a series of unit cell analyses of a void-containing representative material volume, where the plastic response of the matrix material is governed by the plasticity model described in Chapter 2. It is clearly shown that the plastic flow of the matrix, described by the plasticity model in terms of the invariants of the stress deviators $J_2$ and $J_3$, has a very strong influence on all aspects of the dilatational response of the porous solids.

3. A constitutive model, which combines the models proposed by Stewart and Cazacu (2011) and Zhou et al. (2014), is presented to describe the ductile damage process in commercially pure titanium (CP Ti) and to simulate its mechanical response. In particular, a Gurson-type porous material model is modified by coupling two damage parameters, accounting for the void damage and the shear damage, respectively, into the yield function and the flow potential. The plastic anisotropy and tension-compression asymmetry exhibited by CP Ti are accounted for by a plasticity model based on the linear transformation of the stress deviator. The model is shown to capture the effect of stress state and the change of fracture mechanism. The results also reveal the important effect of the plastic anisotropy and tension-compression asymmetry on the ductile damage process.
6.2 Future work

Although the extended GTN model discussed in Chapter 5 is shown to capture the ductile damage process in CP Ti with great success, there are still a number of unresolved issues that need further consideration.

1. Anisotropic ductile damage model: one way to account for anisotropic ductile damage is to take into account the void shape and orientation. Gologanu et al. (1993, 1994, 1995) extended the Gurson model to account for prolate and oblate voids and this model is further extended by Benzer (2004) to account for the void orientation change. Further research is needed to extend this model to include shear damage and matrix anisotropy.

2. Continuum damage mechanics model: The continuum damage mechanics approach provides an alternative method to model ductile fracture. To account for anisotropic damage, a damage tensor, rather than the scalar damage parameter used to describe isotropic damage, needs to be developed.

3. Crystal plasticity models: Polycrystalline plasticity models can reveal that the development of the crystallographic texture is the underlying mechanism of plastic anisotropy. Polycrystalline ductile models can offer a lot of insights into shear band formation and information of void formation and growth.

4. Mesh-size effect: Mesh dependency is an important issue in damage simulation using the local approach. In this dissertation, the element size is used to serve as the length scale and is fixed in the region where failure occurs. The non-local approach and gradient theories may offer more robust solutions to this problem.
BIBLIOGRAPHY


