DESIGN OF ROBUST SUPERDIRECTIVE RECEIVING ANTENNA ARRAY
FOR CIRCULAR, HEXAGONAL AND ELLIPTICAL GEOMETRIES

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DESIGN OF ROBUST SUPERDIRECTIVE RECEIVING ANTENNA ARRAY
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ABSTRACT

Superdirective beamforming deals with designing an antenna array that gives high gain, narrow radiation pattern and also cancels the signal in unwanted directions. It is not practically possible to achieve the same properties as an array antenna without use of an array; to do so would require a huge antenna that is too complex and expensive to be practical.

Use of an array antenna induces a white noise gain into the output of an antenna array due to the mismatches between the antenna elements in an array. In optimum array gain method array gain optimization is considered as eigenvalue problem and antenna weights are the eigenvector of the largest eigenvalue. Optimum array gain method gives the maximum value of array gain and white noise gain which is not possible in practice, due to induced white noise gain. In this thesis, we present a design for a superdirective receiving circular antenna array. In the design process, complex (amplitude and phase) weights for each antenna element in the array are calculated in order to reduce white noise gain using a constraint for white noise gain depending on white noise gain from optimum array gain method. The output beam pattern of an array antenna is a function of the weights and antenna element positions. Beam patterns of each individual antenna element are multiplied by their
respective complex weights and then summed together to give the output radiation pattern. Antenna arrays using the weights found with this design process have lower white noise gain than antennas designed using optimum array gain method.

The circular antenna array has a drawback in that it has high side lobe levels in the radiation pattern. In order to design antennas with a more narrow radiation pattern, the design method is further extended to different antenna array geometries namely elliptical, concentric elliptical, cylindrical elliptical, hexagonal and concentric hexagonal array geometries. The array gain and white noise gain are calculated and the radiation patterns of these geometries are plotted using MATLAB. All these simulation results are compared with the optimum array gain method which does not consider white noise gain for calculation of antenna weights. Use of other geometries in place of circular array geometry has advantages like reduced side lobe level and increase of array gain considering same number of antenna elements in elliptical array compared to circular array. Then, simulation results for optimum array gain and constrained optimum array gain are generated at 60 GHz and 30 GHz frequency waves which are millimeter waves and results are compared and discussed with 30 MHz wave results. These results show that use of millimeter wave frequency reduces the size of the array antenna.
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CHAPTER I
INTRODUCTION

Use of an antenna array improves the receiving and transmitting radiation pattern by combining and processing patterns from all the antenna elements. By adjusting the phasings and distance between the antenna elements a narrow beampattern is obtained, which makes an antenna array a superdirective antenna array. Superdirective arrays are used in some applications which need high directivity and antenna elements with less dimensions. In this chapter, the history of Maxwell’s equations, motivation for the thesis, the goal and organization of the thesis are discussed.

1.1 History

In 1873, James Clerk Maxwell proposed a set of partial differential equations called Maxwell’s Equations. Antennas operate on the basic principles of Maxwell’s Equations. Electric and magnetic field equations of antennas are derived using Maxwell’s equations. There are various types of antennas: wire antennas, aperture antennas, and micro strip antennas. Electrical and mechanical constraints and operating costs should be considered while selecting the type of antenna. In this thesis, the infinitesimal straight wire (dipole) antenna is considered as an antenna element in array antenna [1].
1.2 Literature Review

In this section, a literature review which gives a clear understanding of the project by summarizing and describing the relation of the other papers in the present area of work is given.

1.2.1 Robust Superdirective Beamforming for HF Circular Receive Antenna Arrays [2]

In this paper, Qingchen Zhou, Huotao Gao, Huajun Zhang and Fan Wang, describes the design of a robust superdirective receive antenna array using circular geometry. Antenna array weights are designed using an optimum directive gain method and constrained optimum directive gain method and are compared with each other. An optimum directive gain method is solved as eigenvalue method, where as constrained optimum directive gain method develops a cost function which has a distortion less constraint and sensitivity constraint. This paper shows results which confirm that constrained optimum directive gain method is effective and convenient than the optimum directive gain method. The present thesis solves the array gain as eigenvalue problem and compares with antenna weights derived using the Hamiltonian cost function considering distortion less constraint and quadratic in equal white noise gain constraint.
1.2.2 Theoretical and Practical Solutions for High-Order Superdirectivity of Circular Sensor Arrays [3]

This paper solves array gain of an antenna array as eigenvalue problem initially. Thereafter, the antenna beampattern is decomposed into eigenbeams. The sum of these eigenbeams gives the antenna beampattern. This paper includes the construction of a 16-hydrophone circular array and water tank tests. Practical results show distortions from sixth-order eigenbeam. These distortions are due to improper positioning of hydrophones, amplitude and phase errors. These errors can be considered as white noise gain. The present thesis consists of the similar mathematical formulation for solving the array gain as an eigenvalue problem. Then array gain is optimized with white noise gain to reduce distortions in the practical environment.

1.2.3 Gain Optimization for Arbitrary Antenna Arrays [4]

This article explains the optimization of gain for arbitrary antenna arrays. Eigenvalue theorem is used in this paper for maximization or minimization of gain. Here, largest eigenvalue is considered as the maximum gain and eigenvector corresponding to large eigenvalue gives antenna weights. Optimal array gain method uses the same concept to maximize the array gain in the present thesis.

1.2.4 Spatial Correlation in Arbitrary Noise Fields With Application to Ambient Sea Noise [5]

In this paper, Henry Cox developed expression for normalized cross-spectral density between two sensors as \( \frac{\sin(kd)}{(kd)} \) and wavenumber-frequency spectrum for two-
dimensional and three-dimensional fields. "d" is the distance between two sensors and k is wavenumber. Spatial fourier transform of cross-spectral density gives a wavenumber-frequency spectrum. This expression helps to find cross spectral density in any geometry between any two sensors. Normalized cross spectral density is found using this expression in the present thesis.

1.2.5 Investigation of Hybrid Elliptical Antenna Arrays [6]

In this paper, A.A. Lotfi Neyestanak, M. Ghiamy, M. Naser-Moghaddasi and R.A. Saadeghzadeh derived the array factors for elliptical, concentric elliptical, cylindrical elliptical, and elliptical coaxial arrays. Array factor of the cylindrical elliptical array is obtained as the product of array factors of linear and elliptical arrays. Then, directivity and sidelobe levels (SLL) are compared for different geometries followed by simulations of the effects of spacing between the elements and number of elements on the directivity and SLL. In this thesis, comparison of the optimum array gain and constrained optimum array gain methods is extended to the elliptical, concentric elliptical and cylindrical elliptical geometries.

1.2.6 Comparative Study of Circular and Hexagonal Antenna Array Synthesis Using Improved Particle Swarm Optimization [7]

This paper use Improved Particle Swarm Optimization (IPSO) method in optimization procedures. IPSO is not discussed in detail in this paper, but two cost functions to reduce side lobe levels using IPSO are designed. The hexagonal array is considered as concentric circular array and array factor is derived. The purpose of this paper is
to compare circular and hexagonal array sidelobe levels (SLL). This paper shows that hexagonal array excels circular array with simulation results. In this thesis, hexagonal array is considered as a concentric elliptical array with vertices on the outer elliptical array and sensors between the vertices are on the inner ellipse and array factor is derived. Then, optimum array gain and constrained optimum array gain methods are applied to the hexagonal and concentric hexagonal antenna array.

1.3 Motivation for Thesis

An antenna is defined as a metallic device which can receive and transmit signals. Antenna arrays are used broadly in many applications such as radars, sonar, and wireless communications. For applications that need high directivity with a narrow main beam, antenna arrays are used instead of a single antenna element.

Adaptive antenna beamforming systems are used mostly to model the antenna array properties. Minimum mean square error algorithm and Least mean square algorithm are some of the adaptive antenna beamforming procedures [8, 1].

Antenna measurements are the other significant parameters to be considered for designing weights for the antenna array irrespective of the geometry. Radiation pattern shows the direction of the main beam of the antenna array. Directivity, array gain, and Q-factor are some of the antenna parameters, along with the shape of the beam pattern that depict the output antenna array receiving capacity.

In recent years, several cost functions were designed to improve the robustness of the superdirective array. A cost function designed without constraint becomes an
MVDR Beamformer [9]. Adding a constraint to the cost function design weights which make the antenna array outperform compared to the MVDR weights. A large array using the conventional beamformer weights gives the results similar to the small array using the superdirective weights [2, 10]

1.4 Goal of Thesis

Theoretical antenna measurements are not necessarily the same as practical measurements. In practice, there are important errors between array elements like the improper positioning of the antenna elements, sensor mismatches. These errors are similar to the white noise [11]. Antenna array weights are designed in order to reduce these errors.

The objective of this thesis is to design a superdirective receiving antenna array complex weights using a cost function in order to maximize the array gain. In this method, the array gain of the antenna array is maximized and white noise gain is decreased by increasing the denominator of the white noise gain more than the inverse of white noise gain from the optimum array gain method. This makes the white noise gain less than the theoretical white noise gain value calculated using the optimum array gain method. This thesis deals with circular array geometry initially. Then it is extended to the elliptical and a hexagonal array of antennas. The simulation results are then performed at millimeter frequency range and compared with low frequency simulations.
1.5 Organization of Thesis

This thesis document is organized as follows. Chapter 1 consists of the history of antenna arrays, motivation, and the goal.

Chapter 2 reviews the antenna arrays and beamforming concepts. Electromagnetic field equations of the infinitesimal dipole which is used as an antenna is derived, followed by the derivation of the array factor of the linear array. The direction of arrival algorithm used in the adaptive antenna beamforming systems is discussed in detail. Some of the weighting methods such as Minimum Mean Square Error (MMSE), Least Mean Square (LMS) algorithm, Phased tapered weights, Schelkunoff polynomial method, Dolph-chebyshev method, and Minimum Variance Distortion less Response (MVDR) beamforming for calculating the weights depending on a few requirements of the antenna array are presented.

In Chapter 3, constrained optimum array gain method is implemented on the circular array. Formulation of the circular array for the steering vector, array factor, and the propagation range difference are presented. The formulas used to calculate array gain and white noise gain are derived. An optimum array gain method derived as an eigenvalue problem and constrained optimum array gain method are discussed in detail followed by the simulations and comparison of the results.

Chapter 4 extends the constrained optimum gain method in Chapter 3 to the various array geometries like elliptical and hexagonal arrays. The advantages of these array geometries are discussed. Derivations of the array factors of these geometries
are elucidated. The simulation results of the proposed method with the respective geometric arrays are presented.

In Chapter 5, basics of millimeter waves are discussed. Then, optimum array gain method and constrained optimum array gain method are extended to millimeter wave frequency. Radiation patterns at lower frequency 30 MHz and millimeter wave frequency (60 GHZ and 30 GHz) are compared. With these comparisons, advantages and applications of millimeter waves are discussed.

Finally, the conclusion of the work done combined with the future work that can be done are explained in Chapter 6.
CHAPTER II

BACKGROUND ON ARRAY ANTENNA BEAMFORMING AND WEIGHTING METHODS

This chapter presents an introduction to the antenna arrays and beamforming methods. Several weighting methods associated with the adaptive antenna array beamforming are also discussed.

2.1 Antenna Arrays

An antenna array is a set of antennas. All antennas will be of the same type and the same size in most cases. Antenna arrays are preferable to single antennas for directive characteristics. The geometry of the array, the distance between the elements, complex weights of the antennas and relative pattern of individual antenna elements are the few controls to manage the antenna array pattern.

In phased array antennas, desired radiation pattern is produced by steering the signal in the desired direction, by changing the phase of the elements in the array.

In adaptive array antennas, the weights of the antennas in the array can be adapted to various conditions such as the direction of transmission or required power, minimum mean square error [12].
Antenna arrays consist of a number of the antenna elements arranged in respective geometry depending on the requirements. Usually, an infinitesimal dipole element is considered as the antenna element. Therefore, to derive the field intensity and the array factor of the array antennas, the field intensity of the dipole antenna is derived initially.

2.1.1 Infinitesimal Dipole

The infinitesimal dipole has a length less than the associated wavelength, i.e. \( l << \lambda \).

A time-dependent current must flow in the dipole in order to serve as an antenna. An infinitesimal dipole is placed along the z-axis and it is symmetric about the origin as shown in the figure 2.1 [1].

Differentiation of the vector potential gives the radiated fields. Electric and magnetic fields can be calculated by knowing the current in the dipole. The vector potential for electric current \( I_e \) due to a linear wire dipole is given as [1, 12]

\[
\mathbf{A}(x, y, z) = \frac{\mu}{4\pi} \int_c I_e(x', y', z') \frac{e^{-jkR}}{R} dl' \tag{2.1}
\]

\[
I_e(x', y', z') = \hat{a}_z I_0 \tag{2.2}
\]

\( R \) is the distance between the observation point and any point in the source. The infinitesimal dipole placed symmetric at the origin has coordinates \((x', y', z') = 0\). \( r \) is the distance between the origin and the observation point. Here, \( R = r \), \( k \) is wavenumber, \( \mu \) is the magnetic permeability and \( \epsilon \) is electric permittivity. Hence,
magnetic vector potential due to the infinitesimal dipole can be written as [12]

\[
\mathbf{A}(x, y, z) = \hat{a}_z \frac{\mu I_0}{4\pi r} e^{-jkr} \int_{\frac{z}{2}}^{\frac{z}{2}} dz' = \hat{a}_z \frac{\mu I_0 l}{4\pi r} e^{-jkr}
\]

Here, \( A_x = 0 \), \( A_y = 0 \), \( A_z = \frac{\mu I_0 l}{4\pi r} e^{-jkr} \) (2.4)

The translation between the cartesian and spherical coordinates is given as

\[
\begin{bmatrix}
A_r \\
A_\theta \\
A_\phi
\end{bmatrix} = 
\begin{bmatrix}
\sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\
\cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\
-\sin \phi & \cos \phi & 0
\end{bmatrix}
\begin{bmatrix}
A_x \\
A_y \\
A_z
\end{bmatrix}
\]

(2.5)
From 2.3 and 2.5 we obtain the components of vector potential in the spherical coordinate system as follows:

\[ A_r = A_z \cos \theta = \frac{\mu I_0 l}{4\pi r} e^{-jkr} \cos \theta \]  
\[ A_\theta = -A_z \sin \theta = -\frac{\mu I_0 l}{4\pi r} e^{-jkr} \sin \theta \]  
\[ A_\phi = 0 \]  

Vector potential then in the spherical coordinate system is

\[ A = r \frac{\mu I_0 l}{4\pi r} e^{-jkr} \cos \theta - \hat{\theta} \frac{\mu I_0 l}{4\pi r} e^{-jkr} \sin \theta \]  

The magnetic field intensity can be calculated from the magnetic flux \( B \) as \[13\]

\[ B = \mu H = \nabla \times A \]  

Since vector potential has only \( A_r \) and \( A_\theta \) the \( \nabla \times A \) can be written in the simplified form

\[ \mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A} = \hat{\phi} \frac{1}{\mu r} \left[ \frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right] \]  

The magnetic field intensity should be in planes perpendicular to the direction of the current,

\[ H_r = H_\theta = 0 \]  
\[ H_\phi = \frac{jkl I_0 \sin \theta}{4\pi r} [1 + \frac{1}{jkr}] e^{-jkr} \]  

Upon substitution of \( \mathbf{J} = 0 \) in the Maxwell’s equation, the electric field is expressed as \[1\]

\[ j \omega \varepsilon \mathbf{E} = \nabla \times \mathbf{H} \]
\[ E = \frac{1}{j\omega \varepsilon} \left[ \hat{r} \frac{1}{r \sin \theta} \left( \frac{\partial}{\partial \theta} (H_\varphi \sin \theta) - \frac{\partial H_\varphi}{\partial \theta} \right) - \hat{\theta} \frac{1}{r} \left( \frac{1}{\sin \theta} \frac{\partial H_\varphi}{\partial \varphi} - \frac{\partial (r H_\varphi)}{\partial r} \right) + \hat{\varphi} \frac{1}{r} \left( \frac{\partial (r H_\theta)}{\partial r} - \frac{\partial H_\varphi}{\partial \theta} \right) \right] \]  

(2.15)

Substituting 2.12 and 2.13 into 2.15 yields

\[ E = \hat{r} \eta I_0 \cos \theta \frac{1}{2\pi r^2} \left[ 1 + \frac{1}{jkr} \right] e^{-jkr} + \hat{\theta} \frac{k I_0 \sin \theta}{4\pi r} \left[ 1 + \frac{1}{jkr} - \frac{1}{(kr)^2} \right] e^{-jkr} \]  

(2.16)

where, \( \eta \) is an intrinsic impedance given as \( \eta = \sqrt{\frac{\mu}{\varepsilon}} \).

Equations 2.13 and 2.16 yield the electric and magnetic field intensity at any point in the space. The field equations depend on the distance \( r \) and \( \theta \). We can further simplify the field equations depending on the distance between origin and observation point \( r \).

2.1.1.1 Near Field Region

The near field region is a region close to the source. The field equations consist of the terms like \( \frac{1}{jkr} \), \( \frac{1}{(jkr)^2} \) and \( \frac{1}{(jkr)^3} \) in which \( k = \frac{2\pi}{\lambda} \) is constant for a given wave. In the near field for the condition \( kr << 1 \) these terms can be arranged as follows

\[ \frac{1}{jkr} << \frac{1}{(jkr)^2} << \frac{1}{(jkr)^3} \]  

(2.17)

The electric and magnetic field equations in the near field region \( kr << 1 \) are given as [1]

\[ E_r \approx \frac{\eta I_0 e^{-jkr}}{2\pi jkr^3} \cos \theta \]  

(2.18)

\[ E_\theta \approx \frac{\eta I_0 e^{-jkr}}{4\pi jkr^3} \sin \theta \]  

(2.19)

\[ H_\varphi \approx \frac{I_0 e^{-jkr}}{4\pi r^2} \sin \theta \]  

(2.20)

\[ E_\varphi = H_\varepsilon = H_\theta = 0 \]  

(2.21)
2.1.1.2 Far Field Region

The far field region is a region away from the source in which \( r \) is very large compared to \( \lambda \) i.e., \( kr >> 1 \). In other words, to be in far field the condition to be satisfied is \( r >> \lambda \) [14]. Here \( \frac{1}{(jkr)^2} \) and \( \frac{1}{(jkr)^3} \) terms are neglected The electric and magnetic field intensities can be rewritten as

\[
E_\theta \approx \frac{jk\eta I_0 e^{-jkr}}{4\pi r} \sin \theta 
\]
\[
H_{\varphi} \approx \frac{jkI_0 e^{-jkr}}{4\pi r} \sin \theta 
\]
\[
E_r \approx 0 \quad E_\varphi = H_r = H_\theta = 0 
\]

The ratio of the amplitude of the \( E_\theta \) and \( H_{\varphi} \) is an intrinsic impedance. There is another field called intermediate field (\( kr > 1 \)). This field is considered as the transition between the near field and the far field regions.

2.1.2 Radiation Pattern

The radiation pattern is a graphical representation of the electric or magnetic field strength or power density with respect to the direction for a fixed distance. The radiation pattern is determined usually in the far field region. Field and power patterns are two types of radiation patterns. The plot of the magnitude of the electric or magnetic field gives the field pattern. Power pattern is the plot of the square of the magnitude of the electric or magnetic field. The radiation patterns can be normalized with respect to the maximum value [1].

The radiation pattern of a dipole is doughnut shaped as shown in the figure 2.2, symmetric about the axis in a three-dimensional spherical coordinate system.
These three-dimensional patterns are difficult to describe. Thus, we use $\varphi = 0^\circ$ and $\theta = 90^\circ$ planes to plot the planar plots.

In the far field region, electric field is in $\theta$ direction and also depends only on $\theta$ when $r$ is constant. The magnitude of $E$ is [12]

$$| E | = \frac{j k \eta I_0 e^{-jkr}}{4\pi r} \| \sin \theta \| \quad (2.25)$$

In order to normalize the electric field intensity, the magnitude of electric field intensity is divided by the maximum value of the E-field intensity. The normalized electric field intensity is

$$| E_{normalized} | = | \sin \theta | \quad (2.26)$$
A vertical plane through dipole is an **E-plane**, which contains electric field vector. The vertical \((\varphi = 0)\) plane gives the E-plane pattern. Figure 2.3 shows the normalized electric field radiation pattern (eq. 2.26) in the E-plane considering \(\theta\) from 0 to 180°. **H-plane** contains magnetic field vector and maximum radiation direction. H-plane is 90° perpendicular to the E-plane. The H plane is a \(\theta = 90°\) plane passing through the center of a dipole. For \(\theta = 90°\) the normalized electric field is equal to one. Hence, the field radiation pattern in the horizontal plane is a unit circle. Figure 2.4 is a normalized dipole electric field radiation pattern in the H-Plane for \(\varphi = 0\) to 360°.

![Polar plot in E-Plane (phi=0)](image1)

![Polar plot in H-Plane (THETA=90)](image2)

**Figure 2.3: E-Plane**

**Figure 2.4: H-Plane**

Usually, all the radiation patterns are plotted in the horizontal plane. Antenna arrays are a group of dipole elements arranged in required order. Each dipole element will have a circular radiation pattern in the horizontal plane. All the dipole radiation patterns are multiplied by the respective complex weights calculated depending on the requirements and added together to get the radiation pattern of an antenna array.
Array antennas can be arranged in many geometries. Derivation of the array factor for a linear array is discussed for simplicity.

2.1.3 Two Element Linear Array

Two element linear array constitutes two infinitesimal dipoles placed horizontally along the z-axis as shown in figure 2.5 [1].

![Figure 2.5: Two Element Linear Array](image)

The total electric field due to two elements is given as

\[
E_t = E_1 + E_2 = \frac{j\eta k I_0}{4\pi} \left\{ \frac{e^{-jkr_1-(\beta/2)}}{r_1 \cos\theta_1} + \frac{e^{-jkr_2+(\beta/2)}}{r_2 \cos\theta_2} \right\}
\]  

(2.27)
Here, $l$ is the length of dipole element, and $\beta$ is the phase difference between two elements. In the far field observations case, $\theta_1 \approx \theta_2 \approx \theta$. $r_1$ and $r_2$ written in terms of $r$, $d$ and $\theta$ are [1]

$$r_1 \approx r - \frac{d}{2} \cos \theta$$

$$r_2 \approx r + \frac{d}{2} \cos \theta$$

Using 2.28 and 2.29 equation 2.27 can be rewritten as

$$\mathbf{E}_t = \hat{a}_\theta \frac{j\eta k I_0 e^{-jkr}}{4\pi r} \cos \theta \left\{e^{j(kd\cos \theta + \beta)/2} + e^{-j(kd\cos \theta + \beta)/2}\right\}$$

(2.30)

Upon Simplification

$$\mathbf{E}_t = \hat{a}_\theta \frac{j\eta k I_0 e^{-jkr}}{4\pi r} \cos \theta \{2\cos\left[\frac{(kd\cos \theta + \beta)}{2}\right]\}$$

(2.31)

Array factor of a two element linear antenna array is given as

$$AF = 2\cos\left[\frac{(kd\cos \theta + \beta)}{2}\right]$$

(2.32)

Array factor depends on the distance between the elements and the phase difference.

For a two-element linear array, array factor multiplied with the electric field of the dipole element at origin gives the total electric field [1].

2.1.4 N-Element Linear Array

N-Element linear array is similar to the two element array. The only difference between the two is a number of elements in an array. N-Element linear array is
an array of N-dipole elements placed along the z-axis. Here, distance and phase difference between two consecutive elements is $d$ and $\beta$ respectively [1].

$$\begin{align*}
AF &= 1 + e^{j(kd\cos\theta + \beta)} + e^{j2(kd\cos\theta + \beta)} + \ldots + e^{j(N-1)(kd\cos\theta + \beta)} \\
&= \sum_{n=1}^{N} e^{j(n-1)(kd\cos\theta + \beta)} \\
&= \sum_{n=1}^{N} e^{j(n-1)\psi} \\
\psi &= kd\cos\theta + \beta
\end{align*}$$

(2.33)

(2.34)

(2.35)

(2.36)

Multiplying both sides of equation 2.35 by $e^{j\psi}$

$$AF(e^{j\psi}) = e^{j\psi} + e^{j2\psi} + \ldots + e^{j(N-1)\psi}$$

(2.37)

$$AF = e^{j[(N-1)/2]\psi} \left[ \frac{\sin(N\psi/2)}{\sin(\psi/2)} \right]$$

(2.38)

The simplified array factor of N-element linear array is

$$AF = \left[ \frac{\sin(N\psi/2)}{\sin(\psi/2)} \right]$$

(2.39)

An array is a broadside array if the maxima of the total field pattern is directed perpendicular to the array axis i.e., $\theta = 90^0$.

In the case of N-element linear array maxima occurs when $\psi = 0$ and axis of the array is along the z-axis. To get the radiation pattern in broadside direction $\theta = 90^0$ implies $\beta = 0$ for broadside N-element linear antenna array.
An array which gives the maximum radiation along the axis of an array is an End-fire Array i.e, $\theta = 0^\circ$. When $\theta = 0^\circ$ and $\psi = 0$, the phase excitation $\beta$ is given as $\beta = -kd$.

2.2 Introduction to Beamforming

Beamforming is placing the maxima in the desired direction. The fields from different antenna elements are combined constructively in the desired direction and destructively in the remaining directions.

There are two different types of beamforming techniques

1. Switched-Beam Systems

2. Adaptive Array Systems

Switched beam systems have multiple beam patterns already defined in the system. In order to improve the gain of the system, a decision is made to choose one of the fixed patterns, depending on the requirements of the system. Switched beam systems use simple algorithms to assign a user to an exact beam pattern. However, switched beam systems show its inefficiency to cancel interferers, also main beam cannot be adjusted in desired direction because the beams are already predefined.

Adaptive array system allow the antenna to direct the beam towards the signal while simultaneously nulling interferers. Adaptive array systems increase capacity while synchronously identifying, and minimizing interferers [1, 15].
Figure 2.6: Comparison of Switched and Adaptive Array Systems

Figure 2.7 shows the block diagram of the adaptive array system. Here, the received signal is first converted to a baseband signal. Then the signal was digitized using an analog to digital converter. Direction of Arrival (DOA) Algorithm estimates the direction by calculating the phase difference between the sensors. Thereafter, adaptive array system calculates the weights depending on the requirements of the system using the cost function.

2.2.1 Direction of Arrival

Direction of arrival is determining the direction in which the plane wave reaches the antenna. There are four types of DOA estimation techniques: conventional tech-
Figure 2.7: Block Diagram of Adaptive Array System

...
Maximum likelihood techniques (ML) can perform well in low signal to noise ratio situation. ML techniques are computationally intensive, which makes them less prominent for DOA estimation.

Integrated techniques associate subspace based techniques with the property restoral method, which is an iterative least square projection based constant modulus algorithm. It is used to separate the multiple signals and estimate the spatial signature by which direction of arrival is determined [1].

2.2.2 MUltiple SIgnal Classification (MUSIC)

Conventional MUSIC algorithm was introduced by Schmidt in 1979. The MUSIC algorithm uses eigenstructure decomposition of the input covariance matrix to estimate the signal parameter. The MUSIC algorithm gives information about the number of incident signal, direction of arrival, cross correlation and noise power.

\[ x = S\alpha + n \]  \hspace{1cm} (2.40)

\( n \) is a noise vector. \( \alpha \) is a signal column vector generated by \( D \) different sources.

\[ R_{xx} = E[|x|^2] \]

\[ = E[xx^H] \]

\[ = E[S\alpha\alpha^H S^H] + E[nn^H] \]

\[ = SAS^H + \sigma^2 I \]

\[ = R_s + \sigma^2 I \]  \hspace{1cm} (2.41)
$A$ is a signal covariance matrix, and $\sigma^2$ is a noise covariance matrix given as

$$\sigma^2 = E[nn^H] \quad (2.42)$$

$I$ is an $M \times M$ identity matrix. $M$ is number of antenna elements.

$$R_s = SAS^H \quad (2.43)$$

Let $\lambda_0, \lambda_1, \ldots, \lambda_{M-1}$ be the eigenvalues of the covariance matrix $R_{xx}$

$$| R_{xx} - \lambda_i I | = 0 \quad (2.44)$$

$$| SAS^H + (\sigma^2 - \lambda_i)I | = 0 \quad (2.45)$$

$S$ is a steering matrix due to $D$ signals incident on the array. Hence, each column in $S$ is linearly independent. $A$ is a positive definite matrix. $M - D$ eigenvalues of the $SAS^H$ are zeros. All the eigenvalues are arranged in the descending order as

$$\lambda_0 \geq \lambda_1 \cdots \geq \lambda_M \quad (2.46)$$

Therefore, when the incident signals ($D$) are less than the number of array antennas ($M$), $M - D$ of the smallest eigenvalues of the $R_{xx}$ are equal to the noise covariance $\sigma^2$.

$$\lambda_{D+1} = \lambda_{D+2} = \cdots = \lambda_M = \sigma^2 \quad (2.47)$$

$K$ is the multiplicity of the smallest eigenvalue. The estimate of the incident signals is found as

$$D = M - K \quad (2.48)$$
The eigenvector associated with the particular eigenvalue, \( \lambda_i \), be \( q_i \) are

\[
(R_{xx} - \sigma^2 I)q_i = SAS^H q_i + \sigma^2 I - \sigma^2 I = 0
\]  
(2.49)

\[
SAS^H q_i = 0 \quad i = D + 1, \ldots, M
\]  
(2.50)

\[
S^H q_i = 0 \quad i = D + 1, \ldots, M
\]  
(2.51)

From the above equation, \( M - D \) eigenvectors are orthogonal to the steering vectors \( S \). \( V_n \) is a set of noise eigenvectors.

\[
V_n = [q_{D+1} \quad q_{D+2} \ldots \quad q_M]
\]  
(2.52)

The direction of arrival (DOA) of the signals can be estimated by finding the peaks of \( P_{MUSIC}(\theta) \) [17]

\[
P_{MUSIC}(\theta) = \frac{1}{a^H(\theta)V_n V_n^H a(\theta)}
\]  
(2.53)

The largest peak from the \( P_{MUSIC} \) spectrum gives the direction of arrival (DOA). Several algorithms such as Root-MUSIC algorithm, and Cyclic MUSIC were developed in order to reduce the computational complexity of MUSIC algorithm.

2.2.3 Spatial Division Multiple Access (SDMA)

SDMA is used to allocate many users, by using multiple beams. SDMA concept increases system performance, frequency reuse and capacity. As shown in figure 2.8 there are \( N \) beamformers at the base station to allocate \( N \) different users. Depending on the user requirements, each beamformer will determine the weights such that
maxima is placed in the direction of the user. In this technology, just by changing the angle of the main beam in the beampattern, we can allocate a many number of users in the same cell and to the same channel \([1, 18]\).

![Block Diagram of SDMA system](image)

**Figure 2.8: Block Diagram of SDMA system**

2.3 Weighting Methods

Array factor of an antenna array depends on the weights of the respective elements. Hence, it is very important to determine the appropriate weights for each element in the antenna array. The design of these weighting methods can be for steering
the beam in the desired direction, nulling the interfering signals, and minimizing the
Mean Squared Error (MSE).

2.3.1 Minimum Mean Square Error (MMSE) Criterion

MMSE weighting method is used to minimize the mean square error. The desired
output of an array is given as \( s_k \). The actual output of an array is given as \( y \).

\[
y = w^H x_k
\]  

(2.54)

where \( H \) is a hermitian operator. MMSE criterion uses error between the desired
output of an array and the actual output of an array, \( e_k \).

\[
e_k = s_k - w^H x_k
\]  

(2.55)

Mean square error (MSE) is given as an expectance of the square of an error.

\[
MSE = E[(s_k - w^H x_k)^2]
\]  

(2.56)

\[
= E[s_k^2 - 2s_k w^H x_k + w^H x_k x_k^H w]
\]

\[
= s_k^2 - 2w^H E[s_k x_k] + w^H E[x_k x_k^H] w
\]

\( E[\cdot] \) is the expectation operator.

\( E[s_k x_k] \) is a cross-correlation given as,

\[
r_{xs} = E[s_k x_k]
\]  

(2.57)

The covariance matrix, \( R_{xx} \), is given as

\[
R_{xx} = E[x_k x_k^H]
\]  

(2.58)
From 2.57 and 2.58, equation 2.56 can be written as

\[ MSE = s_k^2 - 2w^H r_{xs} + w^H R_{xx} w \]  

Now, we need to find weights \( w \) which gives minimum MSE.

The derivative of equation 2.59 with respect to the weights (\( w \)) is

\[ \frac{dMSE}{dw} = -2r_{xs} + 2R_{xx} w \]  

setting equation 2.60 to zero gives the weights.

\[ -2r_{xs} + 2R_{xx} w = 0 \]  

\[ w_{opt} = R_{xx}^{-1} r_{xs} \]  

Above equation 2.62 gives the optimized weights to get minimum mean square error. In order to find these optimized weights we require to find the auto correlation, \( r_{xs} \) and covariance matrix, \( R_{xx} \) [1].

2.3.2 Least Mean Square Algorithm

The introduction of the LMS Algorithm was done by Widrow and Hoff in 1959 [19]. Least Mean Square (LMS) Algorithm is another adaptive algorithm. It uses an iterative correction of weights which makes the mean square error minimum with less complexity.

In figure 2.9, \( s(k) \) is a reference signal. All the signals from the sensors are summed up after multiplying them with the respective weights of the sensors, which
Figure 2.9: LMS Adaptive Beamforming

gives the output $y(k)$. $y(k)$ is given as $w^H x_k$. Error $e$ is the difference between the $s(k)$ and $y(k)$. Weights are calculated using the LMS algorithm.

Gradient $\nabla (MSE)$ is given by equation 2.60 from minimum mean square error criterion. The update equation for weight vector used in the direction of the reduced MSE is given as

$$w_{k+1} = w_k - \lambda \nabla (MSE)$$  \hspace{1cm} (2.63)

Here, $\lambda$ is a scalar to control the step size. Substituting the gradient vector from equation 2.60 in 2.63

$$w_{k+1} = w_k + 2\lambda x_k(s_k - x_k^T w_k)$$  \hspace{1cm} (2.64)
Initially the random value $w_0$ is used in case of the LMS algorithm. After successive updates of the weight vector using LMS algorithm leads to reduced minimum mean square error.

The bound in which weight vectors converge and stay stable is [19]

$$0 < \lambda < \frac{1}{\Lambda_{\max}}$$ (2.65)

$\Lambda_{\max}$ is the largest eigenvalue of the covariance matrix $(R_{xx})$ i.e., $R_{xx} = E[x_kx_k^H]$. If $\lambda$ is small convergence becomes slow. If $\lambda$ is chosen to be large, convergence will be high but there will be less stability [8].

2.3.3 Phase Tapered Weights

In this method, the direction of the main beam is steered in the required direction by introducing phase taper to the weights. This phase taper is the delay of the signal with respect to the desired direction. If the desired direction of the beam is $\theta_d$. The weights for the linear array are [20]

$$w_n = e^{jnkd\cos\theta_d}$$ (2.66)

Array factor of linear array is given as

$$AF = \sum_{n=0}^{M} w_n e^{-jnkd\cos(\theta)}$$ (2.67)

Substituting the weights from equation 2.66 in 2.67. The array factor is

$$AF = \sum_{n=0}^{M} e^{jnkd(\cos \theta_d - \cos \theta)}$$ (2.68)
Figure 2.10 shows the example of the phase taper weight method. Here, number antenna elements chosen is \( M = 7 \) and \( \theta_d = 80^\circ \). The magnitude of the array factor given in equation 2.68 is plotted. We can see that the main lobe of the linear antenna array steers in the \( 80^\circ \) direction.

![Figure 2.10: Example of Phase Taper Array Method](image)

### 2.3.4 Schelkunoff Polynomial Method

Schelkunoff polynomial method is used to steer the nulls in the direction which is not desired.

Considering the array factor of the linear array

\[
AF = \sum_{n=0}^{N-1} w_n e^{-jkd\cos\theta}
\]  
\[ (2.69) \]

Let

\[
z = e^{-jkd\cos\theta}
\]  
\[ (2.70) \]
Using 2.70, 2.69 can be written in the polynomial as [21]

\[ AF = \sum_{n=0}^{N-1} w_n z^n \]  \hspace{1cm} (2.71)

Equation 2.71 is an \( N - 1 \) degree polynomial which can be written as

\[ AF = w_{N-1} \prod_{n=0}^{N-2} (z - z_n) \]  \hspace{1cm} (2.72)

In the schelkunoff polynomial method, \( z_n \) is the \( n \)th root of a polynomial. Firstly \( z_n \) is calculated as \( \rho_n \exp(j\phi_n) \). Here, \( \rho_n \) and \( \phi_n \) are magnitude and phase of \( n \)th root.

As a second step, \( z_n \)'s from the first step are used with equation 2.72. Finally, weights are found by equating 2.72 after substituting \( z_n \), to the equation 2.71 [22].

2.3.5 Dolph-Chebyshev Method

In 1946, Dolph proposed this method to reduce the sidelobe level. All the sidelobes should be of equal magnitude. Reduction of the sidelobe level of the highest sidelobe will increase the level of the lower sidelobes.

Array factor of an even and odd number of an antenna array is simply an addition of \( M \) and \( M + 1 \) cosine terms. Array factors of even and odd numbers of the linear array are given as [1]

\[ AF = \sum_{m=1}^{M} w_n \cos[(2n - 1)u](\text{even array}) \] \hspace{1cm} (2.73)

\[ AF = \sum_{m=1}^{M+1} w_n \cos(2(n - 1)u)(\text{odd array}) \] \hspace{1cm} (2.74)
Here,

\[ u = \frac{kd}{2} \cos \theta \]   \hspace{1cm} (2.75)

This method is designed for a broadside uniformly spaced linear arrays. Here, chebychev polynomials are used to get the pattern [23, 24].

\[ m = 0 \quad T_0(z) = \cos(0) = 1 \]   \hspace{1cm} (2.76)

\[ m = 1 \quad T_1(z) = \cos(u) = z \]   \hspace{1cm} (2.77)

\[ m = 2 \quad T_2(z) = \cos(2u) = 2z^2 - 1 \]   \hspace{1cm} (2.78)

where, \( z = \cos u \). An iterative formula for the chebychev polynomial is given as

\[ T_m(z) = 2zT_{m-1}(z) - T_{m-2}(z), m = 2, 3, \ldots \]   \hspace{1cm} (2.79)

Chebyshev polynomials can also be defined as

\[ T_m(z) = \cos[m \cos^{-1}(z)] \quad -1 \leq z \leq +1 \]

\[ T_m(z) = \cosh[m \cosh^{-1}(z)]^\dagger \quad z < -1, \: z > +1 \]   \hspace{1cm} (2.80)

Since \( | \cos(mu) | \leq 1 \), chebycheff polynomial \( | T_m(z) | \leq 1 \). Let

\[ \cos(u) = \frac{z}{z_0} \]   \hspace{1cm} (2.81)

\( S \) is sidelobe level after converting to linear units from decibels. Considering a point \( z = z_0 \) such that \( T_m(z_0) = S \). Let \( z_1 \) be a null near to +1 such that the pattern contains minor lobes from -1 to \( z_1 \) whereas, major lobe is from \( z_1 \) to \( z_0 \). \( z_0 \) can be
determined using equation 2.80 as

\[ T_m(z_0) = S = \cosh[m \cosh^{-1}(z_0)] \]
\[ z_0 = \cosh[\frac{1}{m} \cosh^{-1}(S)] \]

After calculating the \( z_0 \), equation 2.81 is replaced in array factor 2.73 or 2.74 after expanding the array factor using trigonometric identities. Chebychev polynomial \( T_m(z) \) is calculated using the iterative formula where \( m \) is one less than the total number of antenna elements. Equating the array factor to the chebychev polynomial gives the weights of the respective antenna elements.

2.3.6 MVDR Beamforming

MVDR Beamformer is an unconstrained optimization problem. Capon’s minimum variance method [25], a conventional adaptive beamforming method, developed by capon lead to the introduction of the adaptive beamforming technique using a minimum-variance distortionless response (MVDR). Here, output of the array antenna is given as

\[ y = w^H x \quad (2.82) \]

Where \( x \) is a complex steering vector and \( w \) is a complex weight vector.
Output power of an antenna is given as

\[ P_{\text{output}} = E[|y|^2] \]

\[ = E[(w^H x)(w^H x)^H] \]

\[ = E[w^H x x^H w] \]

\[ = w^H E[x x^H] w = w^H R_{xx} w \] \hspace{1cm} (2.83)

Array Gain (AG) is given as

\[ \text{AG} = \frac{w^H x_0 x_0^H w}{w^H R_{nn} w} \] \hspace{1cm} (2.84)

\( x_0 \) is the steering vector in the preset direction. \( R_{nn} \) is a noise covariance matrix.

MVDR beamformer minimizes the output power of an antenna by maintaining the gain. \( w^H x_0 = 1 \) is a distortionless constraint to maintain gain of an antenna. Hence, the cost function will be

\[ \min \{ w^H R_{xx} w \} \quad \text{subject to} \quad w^H x_0 = 1 \] \hspace{1cm} (2.85)

The weight function with the MVDR beamformer is found to be

\[ w = \frac{R_{xx}^{-1} x_0}{x_0^H R_{xx}^{-1} x_0} \] \hspace{1cm} (2.86)

MVDR beamformer has low nulling capacity, which is the main disadvantage [9, 26].
CHAPTER III

MINIMIZING THE WHITE NOISE GAIN BY MAINTAINING THE ARRAY GAIN

In this chapter, an optimization concept is used with antenna arrays to minimize the white noise gain at the same time maintaining the array gain. This method is implemented on a circular antenna array.

3.1 Steering Vector

The output of an antenna array is the sum of the steering vectors after multiplying them with the weights. Let \( s_n \) be steering vector output of an antenna element \( n \). \( w_n \) is the weight with respect to \( s_n \). The radiation pattern of the antenna array is given as

\[
Y = \sum_{n=1}^{N} w_n s_n
\]  \hspace{1cm} (3.1)

Figure 3.1 shows the schematic of the antenna array output. If there are \( N \) antenna elements in an antenna array, \( w \) is \( N \times 1 \) weight vector and \( s \) is an \( N \times M \) matrix in \( M \) different directions. Considering \( w \) as a vector and \( s \) as a matrix equation 3.1 can be written as

\[
Y = w^T s
\]  \hspace{1cm} (3.2)
Considering a plane wave arriving in the direction \((\theta, \varphi)\). The phase delays of a plane wave at each antenna element in an antenna array put together in a vector form is a steering vector which is given as

\[
s(\theta, \varphi) = [e^{jk\psi_0}, e^{jk\psi_1}, e^{jk\psi_3}, \ldots, e^{jk\psi_{N-1}}]^T
\]  

(3.3)

\(\psi_n\) is the phase difference between the \(n\)th antenna element and original point which depends on distance between antenna elements and \((\theta, \varphi)\). In this method, initially circular geometry of the antenna arrays is considered to calculate \(\psi_n\).
3.1.1 Circular Array Formulation

Circular array is considered over a linear array most of the time because circular array utilizes the total $360^\circ$ horizontal plane. Figure 3.2 shows the circular geometry of an antenna array. All the antenna elements are distributed uniformly in the circular arrangement in the XY-plane. Assuming the first element is on the X-axis, $\phi_n$ is an angle made by nth sensor at the X-axis. $\varphi_n$ is given as

$$\varphi_n = n \frac{2\pi}{N} = n\beta$$  \hspace{1cm} (3.4)

Here, $\beta = \frac{2\pi}{N}$ and $n = 0, 1, 2, \ldots, N - 1$ referring a respective sensor.

![Figure 3.2: Geometry of a Circular Array](image-url)
Here $R_n$ is the distance between the observation point and the $n$th sensor. $R_n$ is found using the cosine rule.

$$R_n = (r^2 + a^2 - 2ar \cos \Psi_n)^{\frac{1}{2}}$$ (3.5)

The radius of a circular antenna array "$a$" is less than the distance from the observation point to origin '$r$' i.e., $a << r$. Equation 3.5 can be written as

$$R_n = r^2\left(1 + \frac{a^2}{r^2} - 2\frac{a}{r} \cos \Psi_n\right)^{\frac{1}{2}} = (r - 2a \cos \Psi_n)^{\frac{1}{2}} = (r - a \cos \Psi_n)$$ (3.6)

The electric field equation of an antenna array can be written as

$$E_n(r, \theta, \varphi) = \sum_{n=1}^{N} w_n e^{-jkR_n}$$ (3.7)

$\cos \Psi_n$ is found as the dot product of the normalized position vectors of the signal direction and the $n$th sensor $\hat{a}_s$ and $\hat{a}_n$ respectively.

$$\cos \Psi_n = \hat{a}_n . \hat{a}_s$$ (3.8)

$$= (\hat{a}_x \cos \varphi_n + \hat{a}_y \sin \varphi_n). (\hat{a}_x \sin \theta \cos \varphi + \hat{a}_y \sin \theta \sin \varphi + \hat{a}_z \cos \theta)$$ (3.9)

$$= \sin \theta (\cos \varphi \cos \varphi_n + \sin \varphi \sin \varphi_n)$$ (3.10)

$$= \sin \theta \cos (\varphi - \varphi_n)$$ (3.11)

$R_n$ becomes as $R_n = r - a \sin \theta \cos (\varphi - \varphi_n)$. The electric field equation after substituting $R_n$ is [1]

$$E_n(r, \theta, \varphi) = \sum_{n=1}^{N} w_n e^{-jkr} e^{jka \sin \theta \cos (\varphi - \varphi_n)}$$ (3.12)
In the above equation 3.12, radius $a$ is fixed and $k$ is constant for the assumed plane wave. $e^{jka \sin \theta \cos(\varphi - \varphi_n)}$ changes with $\theta$, $\varphi_n$ and $\varphi$.

Hence, the array factor of a circular array can be written as

$$AF_{\text{circular}} = \text{Beampattern}(B) = \sum_{n=1}^{N} w_n e^{jka \sin \theta \cos(\varphi - \varphi_n)}$$ (3.13)

Using equation 3.13, the steering vector of circular array can be written in the form of equation 3.3, where $\psi_n$ is

$$\psi_n = a \sin \theta \cos(\varphi - \varphi_n)$$ (3.14)

Array pattern (3.13) can be rewritten using 3.2 as

$$\text{Beampattern}(B) = w_n^T s(\theta_0, \varphi_0)$$ (3.15)

### 3.2 Array Gain and White Noise Gain

Array gain is the ratio of signal to noise ratio at output to the signal to noise ratio at input [3].

$$\text{ArrayGain} = \frac{\text{SNR}_{\text{output}}}{\text{SNR}_{\text{input}}}$$ (3.16)

Let $x$ be the input to a receiver antenna array after adding noise ($n$) to the steering vector ($s$) [27, 28].

$$x = s + n$$ (3.17)

The output power at the receive antenna array is

$$\text{Power} = w^H R_s w + w^H R_n w$$ (3.18)
Signal and noise cross spectral density matrices are $R_s$ and $R_n$ respectively. Signal and noise power strength at the input are $\sigma_s^2$ and $\sigma_n^2$ respectively.

$$\text{Array Gain} = \frac{w^H R_s w}{w^H R_n w} \frac{\sigma_s^2}{\sigma_n^2} = \frac{w^H P w}{w^H \delta_{mn} w} \quad (3.19)$$

Since, the radiation pattern is plotted in a constant horizontal plane for a fixed plane wave signal arriving an antenna array in the preset direction $(\theta_0, \varphi_0)$, the cross spectral density matrix of a signal can be written as $R_s = \sigma_s^2 s^*(\theta_0, \varphi_0) s^T(\theta_0, \varphi_0)$, where, $P = s^*(\theta_0, \varphi_0) s^T(\theta_0, \varphi_0)$. Therefore, Array Gain becomes

$$\text{Array Gain} = \frac{w^H s^*(\theta_0, \varphi_0) s^T(\theta_0, \varphi_0) w}{w^H \delta_{mn} w} \quad (3.20)$$

Here, $\delta_{mn}$ is a normalized cross spectral density matrix such that all the main diagonal elements are one and the trace of a matrix is $N$. There are several cases of noise cross spectral density matrix $\delta_{mn}$. Array gain becomes a white noise gain, when the $\delta_{mn}$ matrix is an identity matrix [29, 30]

$$\text{White Noise Gain} = \frac{w^H s^*(\theta_0, \varphi_0) s^T(\theta_0, \varphi_0) w}{w^H w} \quad (3.21)$$

$\delta_{mn}$ is given as

$$\delta_{mn} = \begin{bmatrix}
\delta_{0,0} & \delta_{0,1} & \ldots & \delta_{0,N-1} \\
\delta_{1,0} & \delta_{1,1} & \ldots & \delta_{1,N-1} \\
\vdots & \vdots & \ldots & \vdots \\
\delta_{N-1,0} & \delta_{N-1,1} & \ldots & \delta_{N-1,N-1}
\end{bmatrix} \quad (3.22)$$
\( \delta_{0, N-1} \) is the noise spectral density between the antenna array element number 0 and \( N-1 \). Performance of array depends on noise cross spectral density matrix. Cross spectral density rely on directional properties of noise field and distance between elements. Considering isotropic noise field in which noise intensity arriving from all the directions is same and directional density function extends uniformly. These noise fields are composed of summation of uncorrelated plane waves from different directions. Ocean noise is represented as sum of uncorrelated plane waves from independent noise sources on the surface of ocean. The cross spectral density between the two elements, say \( m \) and \( n \) in the isotropic noise field is given as [5, 31]

\[
\delta_{mn} = \frac{\sin k \Delta d_{mn}}{k \Delta d_{mn}} \tag{3.23}
\]

Where, \( \Delta d_{mn} \) is the distance between antenna elements \( m \) and \( n \). Here, a circular antenna array is considered. In a circular array all the antenna elements are equidistant i.e., the distance of the 0th antenna element to 1st antenna element is equal to the distance between the 0th element and \( (N - 1) \)th element. Hence, \( \delta_{0,1} = \delta_{0,N-1} \) which makes the cross spectral matrix a circulant matrix. A circulant matrix is a square matrix in which \( ith \) row of the matrix is obtained by shifting the \( (i-1) \)th row cyclically to the right [32]. The matrix \( \delta_n \) can be rewritten as a circulant matrix.
\[
\delta_n = \begin{bmatrix}
\delta_0 & \delta_1 & \ldots & \delta_{N-1} \\
\delta_{N-1} & \delta_0 & \ldots & \delta_{N-2} \\
\vdots & \vdots & \ddots & \vdots \\
\delta_1 & \delta_2 & \ldots & \delta_0
\end{bmatrix}
\] (3.24)

Therefore, \( \delta_n \) can be obtained using the circulant matrix properties. The eigenvectors of a circular matrix are

\[
v_n = M^{1/2} \begin{bmatrix} 1 & e^{jn\beta} & e^{j2n\beta} & \ldots & e^{j(N-1)n\beta} \end{bmatrix}^T
\] (3.25)

Angles of distributed sensors are given as \( 0, \beta, 2\beta, \ldots, (N-1)\beta \). Here, \( \beta = \frac{2\pi}{N} \).

Eigenvalues of a circulant matrix are real values and are obtained as follows

\[
\Lambda_n = \sum_{i=0}^{N-1} \delta_n e^{jni\beta}
\] (3.26)

The matrix \( \delta_n \) is obtained from the eigenvectors \( v_n \) and the eigenvalues \( \Lambda_n \) as

\[
\delta_n = \sum_{n=0}^{N-1} \Lambda_n v_n^* v_n^T
\] (3.27)

Since \( \delta_n \) is a hermitian matrix. Inverse of \( \delta_n \) can be written as

\[
\delta_n^{-1} = \sum_{n=0}^{N-1} \frac{1}{\Lambda_n} v_n^* v_n^T
\] (3.28)

Eigenvalues \( \Lambda_n \) and eigenvectors \( v_n \) follow the following conditions \([32, 3]\)

\[
\Lambda_n = \Lambda_{N-n}
\] (3.29)

\[
v_n = \bar{v}_{N-n}
\] (3.30)
3.3 Optimum Array Gain Method

The problem of optimization of array gain (3.19) can be viewed as an eigenvalue problem, since both the matrices $P$ and $\delta_n$ are the hermitian matrices. The maximum array gain is given by the largest eigenvalue of the matrix $\delta_n$ and the eigenvector corresponding to the largest eigenvalue is the weight vector of this method. [2, 4]

Weight vector is a $N \times 1$ column vector. Equation 3.20 can be rewritten in terms of eigenvalues and eigenvectors as

$$Pw - \lambda \delta_n w = 0$$  (3.31)

Above equation 3.31 can be reorganized as below

$$(P - \lambda \delta_n)w = 0$$  (3.32)

$\lambda_1, \lambda_2, \ldots, \lambda_N$ are the $N$ real eigenvalues in the order $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$. With this relation between the eigenvalues it is clear that the value of the array gain is between $\lambda_1$ and $\lambda_N$.

$$\lambda_N \leq AG \leq \lambda_1$$  (3.33)

Since $P = s^*(\theta_0, \varphi_0)s^T(\theta_0, \varphi_0)$ and using the equation 3.32, maximum array gain can be written as

$$AG = s(\theta_0, \varphi_0)^T \delta_n^{-1} s(\theta_0, \varphi_0)^*$$  (3.34)

The eigenvector corresponding to the array gain, i.e., largest eigenvector is attained using 3.34 and 3.32 which is an $N \times 1$ weight vector.
\[ w_{opt} = \delta_n^{-1}s^*(\theta_0, \varphi_0) \]  

Substituting equation 3.28 in equation 3.35.

\[ w_{opt} = \sum_{n=0}^{N-1} \frac{1}{\lambda_n} v_m^* v_m^T s^*(\theta_0, \varphi_0) \]  

Using the above weight vector in 3.13 the beam pattern of an antenna array is obtained as [3, 33]

\[ B = w_{opt}^T s(\theta, \varphi) \]  

Polar plot of the above beam pattern is plotted using Matlab to check the direction of the radiation pattern and is shown in figure 3.6 and the array gain and white noise gain are calculated using 3.20 and 3.21 respectively.

3.4 Constrained Optimum Array Gain Method

In this method, a cost function to decrease the value of the white noise gain, while maximizing the array gain in order to improve the performance of an antenna array is designed. Array gain equation is given in equation 3.20. In order to maximize array gain, the denominator of the array gain is minimized by imposing an equality constraint \( w^H s^*(\theta_0, \varphi_0) = 1 \), which makes the numerator of the array gain equal to unit value i.e.,

\[ \min\{w^H \delta_n w\} \quad \text{w.r.to} \quad w^H s^*(\theta_0, \varphi_0) = 1 \]  

The main goal of this optimization problem is to find the weight vector which minimizes the denominator of the array gain by minimizing the white noise gain. To
minimize the white noise gain, an inequality constraint is included in the cost function. Considering white noise gain equation 3.21, numerator of this equation becomes equal to one due to equality constraint 3.38 and denominator should be made greater than some value \( G \) to make white noise gain low. \( G \) is a value greater than the inverse of white noise gain from optimal array gain method. The inequality constraint is

\[
w^H w \geq G
\] (3.39)

The weights determined using the optimization method should satisfy this equality and inequality constraints. The performance index that is to be minimized and all the constraints put together to form a hamiltonian function [34].

\[
F = w^H \delta_n w + \gamma (w^H s^*(\theta_0, \varphi_0) - 1) + \nu (G - w^H w)
\] (3.40)

\( \gamma \) and \( \nu \) are lagrange multipliers. The solution to this optimization problem is found using three conditions.

\[
\frac{dF}{dw^H} = \delta_n w + \gamma s^*(\theta_0, \varphi_0) - \nu w = 0
\] (3.41)

\[
\frac{dF}{d\gamma} = w^H s^*(\theta_0, \varphi_0) - 1 = 0
\] (3.42)

\[
\frac{dF}{d\nu} = G - w^H w = 0
\] (3.43)

From 3.41 weight vector can be found

\[
(\delta_n - \nu I)w + \gamma s^*(\theta_0, \varphi_0) = 0
\] (3.44)
The optimizing weight vector is

\[ w = -\gamma (\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0) \] (3.45)

\( \gamma \) is found by substituting equation 3.45 in 3.42

\[ (-\gamma (\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0))^H s^*(\theta_0, \varphi_0) - 1 = 0 \] (3.46)

\[ \gamma = -\frac{\frac{(\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0)}{s^T(\theta_0, \varphi_0)(\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0)} \cdot \frac{(\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0)}{s^T(\theta_0, \varphi_0)(\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0)}}{\frac{(\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0)}{s^T(\theta_0, \varphi_0)(\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0)}} \] (3.47)

\[ w = \frac{(\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0)}{s^T(\theta_0, \varphi_0)(\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0)} \] (3.48)

\[ w^H w \geq G \] (3.49)

\[ w^H w = \left[ \frac{(\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0)}{s^T(\theta_0, \varphi_0)(\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0)} \right]^H \left[ \frac{(\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0)}{s^T(\theta_0, \varphi_0)(\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0)} \right] \] (3.50)

\[ w^H w = \frac{s^T(\theta_0, \varphi_0)(\delta_n - \nu I)^{-2} s^*(\theta_0, \varphi_0)}{[s^T(\theta_0, \varphi_0)(\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0)]^2} \] (3.51)

From 3.49 and 3.51 can be written as [35]

\[ G > \frac{s^T(\theta_0, \varphi_0)(\delta_n - \nu I)^{-2} s^*(\theta_0, \varphi_0)}{[s^T(\theta_0, \varphi_0)(\delta_n - \nu I)^{-1} s^*(\theta_0, \varphi_0)]^2} \] (3.52)

\( \nu \) is selected such that the above inequality 3.52 is satisfied. If \( \nu = 0 \) the optimization method becomes the MVDR method.

3.5 Simulation Results

Table 3.1 shows the comparison between the array gain, the white noise gain and time elapsed for \( N = 12 \) and \( N = 24 \) antenna elements. The white noise gain was reduced
from 11.6737 to 6.6279, by maintaining the array gain at 8.5174 when there are 12 antenna elements in the antenna array. Also, the white noise gain reduced from 23.8677 to 13.7237 when \( N = 24 \) elements. Therefore, as the number of antenna elements increases the reduction factor of the white noise gain also increases.

Table 3.1: Comparison of the Array Gain and White Noise Gain for \( N = 12 \) and \( N = 24 \)

<table>
<thead>
<tr>
<th>No. of elements</th>
<th>Optimum Array Gain</th>
<th>Constrained Optimum Array Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Array Gain</td>
<td>White Noise Gain</td>
</tr>
<tr>
<td>N=12</td>
<td>12.4799</td>
<td>11.6737</td>
</tr>
<tr>
<td>N=24</td>
<td>30.9782</td>
<td>23.8677</td>
</tr>
</tbody>
</table>

Figure 3.3, 3.4 and 3.5 are the plots of array gain and white noise gain for different values of number of antenna elements, frequency, and the preset direction \( \theta \) respectively. Array gain increases with the increase in the number of antenna elements and the \( \theta \).

Figure 3.6 and 3.7 are the radiation patterns of an antenna array using the optimum array gain Method and the constrained optimum array gain method respectively. These radiation patterns show the direction of the beam. In order to plot this patterns, the preset direction \((\theta_0, \varphi_0) = (90^\circ, 0^\circ)\) is used. Number of antenna
elements is $N = 12$ and frequency is 30 MHz. The radiation patterns are plotted in fixed theta plane, i.e., horizontal plane $\theta_d = 90^\circ$. Since the given desired direction is
$(\theta_0, \varphi_0) = (90^0, 0^0)$ the radiation patterns of both the methods have maxima in the $0^0$ direction. The radiation pattern is more directional in the constrained optimum array gain method. Figures 3.8 and 3.9 are the radiation patterns for 24 antenna elements in the antenna array.
Figure 3.6: Radiation Pattern Optimum Array Gain Method

Figure 3.7: Radiation Pattern Constrained Optimum Array Gain Method
Figure 3.8: Optimum Array Gain Method $N = 24$

Figure 3.9: Constrained Optimum Array Gain Method $N = 24$
CHAPTER IV

EXTENDING CONSTRAINED OPTIMUM ARRAY GAIN METHOD TO THE ELLIPTICAL AND HEXAGONAL ARRAY GEOMETRIES

In this chapter, constrained array gain method, and optimum array gain method are used to find the weights for elliptical and hexagonal geometries and the radiation patterns. The linear array has high directivity and the main lobes are very narrow, but the disadvantage of the linear array is that it cannot use the total azimuthal plane. The circular array uses the total azimuthal, but circular array has problems with the side lobes. Elliptical and hexagonal arrays combine with the linear array are used to reduce the side lobe levels [6].

4.1 Elliptical Antenna Array

The geometry of an elliptical antenna array is shown in figure 4.1. All the antenna elements are arranged in an elliptical geometry in the XY-plane. The angle made by the \( n^{th} \) element with the X-axis is given as \( \varphi_n = \frac{2\pi n}{N} \). \( N \) is the number of antenna elements. The first element \( n = 0 \) is considered to be on the X-axis. \( a \) is the semi-major axis and \( b \) is the semi-minor axis of the ellipse. The eccentricity is given as

\[
e = \sqrt{1 - \frac{b^2}{a^2}} \tag{4.1}
\]
When this eccentricity becomes equal to zero, elliptical geometry becomes a circular geometry.

Figure 4.1: Geometry of Elliptical Array

Derivation of the array factor of an elliptical array is similar to circular array. The position vector of the nth element is given as \( \hat{a}_n = [a \cos \varphi_n, b \sin \varphi_n, 0] \). Position vector of the signal direction is given as \( \hat{a}_s = [\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta] \) [36].

Therefore, the propagation range difference between the fixed plane wave and the nth element is

\[
\psi_n = \hat{a}_s.\hat{a}_n = \sin \theta[a \cos \varphi \cos \varphi_n + b \sin \varphi \sin \varphi_n]
\]
The steering vector of an elliptical array is

\[ S = [e^{jk \sin \theta (a \cos \varphi \cos \varphi_0 + b \sin \varphi \sin \varphi_0)} e^{jk \sin \theta (a \cos \varphi \cos \varphi_1 + b \sin \varphi \sin \varphi_1)} \ldots e^{jk \sin \theta (a \cos \varphi \cos \varphi_n + b \sin \varphi \sin \varphi_n)}]^T \quad (4.4) \]

The array factor of the elliptical antenna array becomes

\[ AF = w^H S \quad (4.5) \]

\[ AF = \sum_{n=0}^{N} w_n^H e^{jk \sin \theta (a \cos \varphi \cos \varphi_n + b \sin \varphi \sin \varphi_n)} \quad (4.6) \]

Weights \( (w_n) \) are calculated using the methods described in section 3.3 and section 3.4. Sections 3.3 and 3.4 refers to the optimum array gain and constrained optimum array gain method respectively.

Figure 4.2 and 4.3 are the beampatterns of the optimum array gain and constrained optimum array gain method respectively. Radiation patterns are plotted in the horizontal plane. The desired direction used is \((\theta_0, \varphi_0) = (90^0, 0^0)\) at frequency 30 MHz and the semi-major axis is 8m and the semi-minor axis is 3m and the number of antenna elements is 12.

Array gain and white noise gain using the optimum array gain method is 13.5726 and 11.0580 respectively. After using the weights determined by the constrained optimum gain method, white noise gain is reduced to 6.02 by maintaining the array gain at 6.74. The radiation pattern of a circular antenna array in case of constrained optimum array gain method consists of side lobes beside main lobe.
which leads to interference. Use of elliptical geometry reduces side lobes and gives
more array gain by using the same number of antenna elements in antenna array than
circular array.

Figure 4.2: Radiation Pattern Optimum Array Gain Method

Figure 4.3: Radiation Pattern Constrained Optimum Array Gain Method
Table 4.1: Array Gain and White Noise Gain versus Ellipse Eccentricity

<table>
<thead>
<tr>
<th>Eccentricity</th>
<th>Optimum Array Gain</th>
<th>White Noise Gain</th>
<th>Constrained Optimum Array Gain</th>
<th>White Noise Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11.86</td>
<td>11.74</td>
<td>7.605</td>
<td>6.07</td>
</tr>
<tr>
<td>0.3</td>
<td>12.05</td>
<td>11.7034</td>
<td>7.62</td>
<td>6.06</td>
</tr>
<tr>
<td>0.5</td>
<td>12.118</td>
<td>11.6627</td>
<td>7.6</td>
<td>6.36</td>
</tr>
<tr>
<td>0.7</td>
<td>11.8271</td>
<td>11.8118</td>
<td>7.24</td>
<td>6.422</td>
</tr>
<tr>
<td>0.9</td>
<td>12.97</td>
<td>11.57</td>
<td>11.86</td>
<td>4.02</td>
</tr>
</tbody>
</table>

Table 4.1 shows the array gain and white noise gain for both optimum array gain and constrained optimum array gain methods for different values ellipse eccentricity. In case of an elliptical antenna array, when eccentricity \((e = 0)\), i.e., a circular array array gain is less. Maximum array gain is attained when the eccentricity is 0.9. Elliptical array has nulls in some directions beside main lobe from radiation pattern which makes it very suitable for beam steering.

### 4.2 Concentric Elliptical Array

Here, in this section derivation of the array factor of a concentric elliptical array is given. Figure 4.4 shows the geometry of a concentric elliptical array. Equation 4.6 gives the array factor of an elliptical antenna array using equation 4.3. Equation 4.3 is the propagation range difference between the element on the ellipse 1 and the
signal direction. Let $\psi_{2n}$ be the propagation range difference between the element $n$ on the ellipse no. 2 concentric with ellipse 1 and plane wave direction.

$\psi_{2n}$ is given as $\sin \theta [a_m \cos \varphi \cos \varphi_{2n} + b_m \sin \varphi \sin \varphi_{2n}]$ whereas $m$ is the number of the ellipse when there are $M$ concentric elliptical array antennas are present.

![Figure 4.4: Geometry of Concentric Elliptical Array](image)

Array factor of a concentric elliptical array is the sum of the array factors of all the $M$ ellipses and can be written as [6]

$$AF = \sum_{m=1}^{M} \sum_{n=0}^{N-1} w_{mn} e^{jk \sin \theta (a_m \cos \varphi \cos \varphi_{mn} + b_m \sin \varphi \sin \varphi_{mn})}$$

(4.7)
Here, $w_{mn}$ is the weight of the $n$th antenna element in the $m$th ellipse in the concentric elliptical antenna array. $a_m$ and $b_m$ are the semi-major and semi-minor axis of the $m$th ellipse given as

$$a_m = a + (m - 1)d$$

$$b_m = a_m\sqrt{1 - e^2}$$

Here, $e$ is the eccentricity of an ellipse given as $e = \sqrt{1 - \frac{b^2}{a^2}}$. When $e = 0$ concentric elliptical array becomes a concentric circular array.

Weights $w_{mn}$ are calculated using the two methods mentioned in Chapter 3. Figure 4.5 and 4.6 are images of the radiation patterns of a concentric elliptical array using an optimum array gain method and constrained optimum gain method. The simulations are done with the preset direction as $(\theta_0, \varphi_0) = (90^0, 0^0)$ and frequency used is 30 MHz. Initial semi-major and semi-minor axis used are 8 meters and 3 meters and the distance between the two ellipses is $d = 2$ meters and there are 12 antenna elements on each elliptical array. The radiation pattern of concentric elliptical array with constrained optimum array gain method is more directional with less side lobes compared with radiation pattern in the optimum array gain method.

Array gain and white noise gain of a concentric elliptical antenna array with optimum array gain method is 24.98 and 20.36 respectively. White noise gain decreases to 13.45 when the array gain is 11.46.
Figure 4.5: Radiation Pattern of Concentric Elliptical Array Optimum Array Gain Method

Figure 4.6: Radiation Pattern of Concentric Elliptical Array Constrained Optimum Array Gain Method
4.3 Cylindrical Antenna Array

In this section, an antenna array in which the elements of an antenna are arranged in a cylindrical geometry is discussed. In this geometry, ellipses are arranged along the z-axis different from the concentric array. All the ellipses are of equal semi-major and semi-minor axis $a$ and $b$ respectively. The distance between any two ellipses along the $z$-axis is $d$.

![Geometry of Cylindrical Array](image)

Figure 4.7: Geometry of Cylindrical Array

Cylindrical structure has ellipses along the $z$-axis. Hence, there exists a phase difference between elements along $z$-axis of a cylindrical array. These elements form a linear array. The array factor of a cylindrical array becomes [37, 38]
\[ AF = \sum_{m=1}^{M} e^{j(m-1)(kd\cos\theta + \beta)} \sum_{n=0}^{N-1} w_{mn} e^{jk\sin\theta(a \cos \varphi \cos \varphi_n + b \sin \varphi \sin \varphi_n)} \]  \hspace{1cm} (4.10)

\( \beta \) is the phase excitation which is equal to zero for broadside antenna array as discussed in chapter 2 and section 2.1.4.

Radiation patterns of a cylindrical antenna array using an optimum array gain method and constrained optimum array gain method are shown in figures 4.8 and 4.9 respectively. These radiation plots were generated at 30 MHz frequency, semi major and semi minor axis is 8m and 3m respectively, with the spacing between ellipses i.e., \( d \) is 2 meters. Radiation pattern in figure 4.9 looks more directional and has low side lobe level compared to the figure 4.8. Constrained optimum array gain method gives less white noise gain and good radiation pattern compared to an optimum array gain method for cylindrical elliptical antenna array.

By using the constrained optimum gain method, white noise gain reduced to 9.61 from 16.65 and the array gain at this point is noted as 10.9. Array gain with optimum array gain method is 25.

4.4 Hexagonal Antenna Array

Figure 4.10 shows the hexagonal geometry of an antenna array. The hexagonal geometry can be viewed as a concentric elliptical structure. All the elements at the vertices of a hexagon antenna array are considered to be on an outer elliptical antenna array and antenna elements which are considered to be between the vertices of a hexagon...
Figure 4.8: Radiation Pattern of Cylindrical Elliptical Array Optimum Array Gain Method

Figure 4.9: Radiation Pattern of Cylindrical Elliptical Array Constrained Optimum Array Gain Method
are on the inner elliptical antenna array. In contrast to hexagonal antenna array, elements on each ellipse of a concentric elliptical antenna array form a linear array along the radial direction.

Figure 4.10: Geometry of Hexagonal Antenna Array

Array factor of a hexagonal antenna array is given as [7]

\[
AF = \sum_{n=0}^{N-1} \left[ w_{1n} e^{jk \sin \theta (a_1 \cos \phi \cos \varphi_{1n} + b_1 \sin \varphi \sin \varphi_{1n})} + w_{2n} e^{jk \sin \theta (a_2 \cos \phi \cos \varphi_{2n} + b_2 \sin \varphi \sin \varphi_{2n})} \right]
\]

\[(4.11)\]

Here, \( \varphi_{1n} = \frac{2\pi(n-1)}{N} \) and \( \varphi_{2n} = \varphi_{1n} + \frac{\pi}{N} \) which are the angle between the elements of the ellipse 1 and 2 with x-axis respectively. \( \psi_{1n} \) and \( \psi_{2n} \) are range difference with ellipse 1 and 2.
$w_{1n}$ and $w_{2n}$ are the weights of an antenna elements on the vertices and the antenna elements between the vertices of a hexagon in hexagonal array geometry. Let $a_1$ and $b_1$ be the semi major and semi minor axis of the outer ellipse consisting of elements on the vertices of the hexagon and $a_2$ and $b_2$ are are semi major and semi minor axis of the inner ellipse consisting of elements on the sides of hexagon.

\[ a_2 = a_1 \cos(\pi/N) \]  \hspace{1cm} (4.12)

\[ b_2 = a_2 \sqrt{1 - e^2} \]  \hspace{1cm} (4.13)

Where, $e$ is the eccentricity of an ellipse given as $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{b_2^2}{a_2^2}}$.

Figure 4.11 and 4.12 are the images of the radiation patterns of hexagonal array using an optimum gain method and constrained optimum gain method. The simulations are done with the preset direction as $(\theta_0, \varphi_0) = (90^0, 0^0)$ and frequency used is 30 MHz. Initial semi major and semi minor axis used are 8m and 2m and the distance between the two ellipses is $d = 2m$.

Hexagonal antenna array designed is a 12 element antenna array with 6 elements on the vertices of the hexagon i.e., the outer ellipse and 6 elements are on inner ellipse. Weights calculated using an optimum array gain method gives the array gain and white noise gain as 12.39 and 11.47 respectively. Array gain and white noise gain using constrained optimum array gain method are 6.9 and 6.6 respectively. White noise gain is reduced to 6.6 and the radiation pattern becomes more directional as is seen in figure 4.12 by using constrained optimum array gain method.
Figure 4.11: Radiation Pattern of Hexagonal Array Optimum Array Gain Method

Figure 4.12: Radiation Pattern of Hexagonal Array Constrained Optimum Array Gain Method
4.5 Concentric Hexagonal Array

In this section, array factor of a concentric hexagonal geometry of antenna array which is shown in figure 4.13 is derived. This geometry can be viewed as a concentric elliptical antenna array with four elliptical antenna arrays except an angle made by the antenna elements on the sides of the hexagon at the x-axis is \( \varphi_{2n} = \varphi_{4n} = \frac{2\pi(n-1)}{N} + \frac{\pi}{N} \) and the elements on the vertices of the hexagon make an angle of \( \varphi_{1n} = \varphi_{3n} = \frac{2\pi(n-1)}{N} \)
at the x-axis. Array factor of the concentric hexagonal antenna array is given as

\[
AF(\theta, \varphi) = \sum_{m=1}^{M} \sum_{n=0}^{N-1} \left[ w_{1mn} e^{jk \sin \theta (a_{1m} \cos \varphi_{1n} \cos \varphi + b_{1m} \sin \varphi_{1n} \sin \varphi)} + w_{2mn} e^{jk \sin \theta (a_{2m} \cos \varphi_{2n} \cos \varphi + b_{2m} \sin \varphi_{2n} \sin \varphi)} \right] 
\]

(4.14)

where \( a_{1m} \) and \( b_{1m} \) are the semi-major and semi-minor axis of the first ellipse consisting of the vertices of a hexagon. \( a_{2m} \) and \( b_{2m} \) are the semi-major and semi-minor axis of the concentric ellipse consisting of antenna elements on the sides of the hexagon.

\[
a_{1m} = a + (m - 1)d 
\]

(4.15)

\[
b_{1m} = a_{1m} \sqrt{1 - e^2} 
\]

(4.16)

\[
a_{2m} = a_{1m} \cos(\pi/N) 
\]

(4.17)

\[
b_{2m} = a_{2m} \sqrt{1 - e^2} 
\]

(4.18)

\( a_{1m}, b_{1m}, a_{2m} \) and \( b_{2m} \) are calculated using 4.15, 4.16, 4.17 and 4.18 respectively. M is the number of ellipses which is 4 in case of concentric hexagon. N is the number of elements on the ellipse.
The radiation pattern of the concentric elliptical array for both the optimum array gain and constrained optimum array gain method are shown in figure 4.14 and 4.15.

Radiation pattern looks directional for the constrained array gain method compared to the optimum array gain method. Array gain and white noise gain are 17.8 and 18.95 respectively. White noise gain reduces to 8.17 using the constrained optimum gain method with the semi major and semi minor axis as 8m and 2m respectively, and spacing between two ellipses is considered to be $d = 2$ at 30 MHz.

This chapter extends the constrained optimum array gain method and op-
Figure 4.14: Radiation Pattern of Concentric Hexagonal Array Optimum Array Gain Method

timum array gain method to different geometrical arrays like elliptical, concentric elliptical, cylindrical elliptical, hexagonal and concentric hexagonal arrays. Use of circular array introduce a disadvantage of high side lobe levels beside the main lobe. Using these geometrical arrays gives less side lobe levels, nulls beside main lobe which makes them very suitable for beam steering and more array gain. All these geometrical arrays give more narrow radiation pattern with constrained optimum array gain method.
Figure 4.15: Radiation Pattern of Concentric Hexagonal Array Constrained Optimum Array Gain Method
CHAPTER V

EXTENDING OPTIMUM ARRAY GAIN AND CONstrained OPTIMUM ARRAY GAIN METHOD TO MILLIMETER WAVES

In this chapter, antenna weights used with steering vector to plot array pattern are calculated using optimum array gain and constrained optimum array gain methods, at Extremely High Frequency (EHF).

5.1 Millimeter Waves

Extremely High Frequency (EHF) occupies the electro magnetic spectrum from 30 to 300 GHz. Wavelengths calculated for waves in this frequency range are from 10 to 1 millimeter. Hence, EHF waves are also called as Millimeter waves. Applications of millimeter waves are transmission of large amount of data, radar, cellular communications, and personal area wireless networks. These waves suffer high propagation losses in the atmosphere due to oxygen, water vapour and blocked by the wall. Signal losses in oxygen makes 60 GHz frequency range more suitable for short distance wireless communications. These waves loose signal strength in rain even in short distance wireless communications. Use of 60 GHz waves gives narrow beamwidth antenna radiation patterns [39].
5.2 Optimum Array Gain and Constrained Optimum Array Gain at 60 GHz

In this section, consider a 60 GHz frequency wave. In previous chapters, optimum array gain and constrained optimum array gain methods are applied to antenna arrays considering 30 MHz frequency. These methods work well at 30 MHz. Now these methods are simulated using 60 GHz in place of 30 MHz. Optimum array gain and constrained optimum array gain methods are discussed in detail in sections 3.3 and 3.4 from chapter 3.

![Figure 5.1: Optimum Array Gain Method at 60 GHz Frequency](image)

Radiation patterns using millimeter wave frequency for optimum array gain method and constrained optimum array gain method are shown in figures 5.1 and 5.2. Simulations are performed considering a circular array with radius \( a = 0.008 \) meters, frequency \( f = 60 \text{ GHz} \), \( \theta = 90^0 \) with preset direction as \( (\theta_0, \varphi_0) = (90^0, 0^0) \),
Figure 5.2: Constrained Optimum Array Gain at 60 GHz Frequency

and number of antenna elements in circular array are 12. Radius of a circular array at frequency $f = 30$ MHz is in terms of meters and same number of antenna elements in a circular array considering $f = 60$ GHz is in terms of centimeters. This confirms that size of array antenna is reduced by using millimeter waves. From these figures, it can be observed that main beam in beam pattern is very narrow with millimeter waves compared to 30 MHz, by using the same number of antenna elements. Array gain and white noise gain of a circular array using millimeter wave are 12.1827 and 11.9595 respectively. Array gain and white noise gain using the weights derived by constrained optimum array gain method are 8.0381 and 8.45 respectively. Constrained optimum array gain method works for a millimeter frequency range also. Due to reduced array size and atmospheric absorption use of this frequency is very well suitable for indoor personal area communications.
5.3 Optimum Array Gain and Constrained Optimal Array Gain at f=30 GHZ and a=.008 meters

Figures 5.3 and 5.4 are radiation patterns of the circular array antenna using optimum array gain and constrained optimum array gain methods respectively. These two radiation patterns are exactly similar to the radiation patterns of the circular array with 30 MHz frequency. Array factor of any geometry depends on $ka$. Here, $k = 2\pi/\lambda$. Considering $a$ is the radius of circular array geometry. Use of millimeter waves increases frequency by ($\approx 10^3$). This increase in frequency is compensated by decreasing the radius in $ka$ making size of antenna array small. Radiations patterns of antenna array with 60 GHz frequency and $a = 0.004$ meters radius also produce the same results of radiation patterns, array gain, and white noise gain.

Figure 5.3: Optimum Array Gain Method at 60 GHz Frequency
Figure 5.4: Constrained Optimum Array Gain at 60 GHz Frequency
CHAPTER VI
CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

In this method, an optimal control cost function was designed to determine the weights of each antenna array element to decrease the white noise gain in order to improve the antenna array performance. Several other adaptive antenna array weighting methods such as MVDR Beamforming, Minimum mean square error criterion, LMS Algorithm, Dolph-chebyshev Method were discussed. All the weighting methods were designed depending on the requirements of an antenna array.

The weights are derived using an optimum array gain method. In this method, array gain equation is solved as eigenvalue problem without any restriction to white noise gain. The calculations using these weights are compared with the weights derived using the cost function designed for constrained optimum array gain method. It can be observed that the white noise gain reduces using the constrained optimum array gain method. The radiation pattern becomes more directional using the weights derived using constrained optimum array gain method. It was observed that the array gain and the white noise gain increase with the increase in the number of antenna elements and theta. All the simulations were done using Matlab scripting.
software. It can also be noticed that the time required for the simulation of the constrained optimum array gain method is less than the optimum array gain method. Simulation results were generated corresponding to an antenna that is operating at 30 MHz frequency. Later, simulation results were developed for antennas operating on millimeter waves. Reducing the radius of the circular antenna array geometry will have the same results generated with antenna array operating at 30 MHz frequency wave. Therefore, use of millimeter wave reduces the area of the antenna array which make them very useful for cell splitting, and indoor wireless networks.

The developed method is used with several other geometries of antenna arrays like elliptical, concentric elliptical, cylindrical elliptical, hexagonal, and concentric hexagonal antenna array geometries. Elliptical geometries are studied in detail and array factor of the hexagonal geometries is developed from the basics of the elliptical array factor. The advantage of these geometrical arrays is the reduced side lobe level. Use of same number of antenna elements in elliptical antenna array give large array gain compared to circular array. Use of constrained optimum array gain method allow the advantages like decrease in the white noise gain, and directional radiation pattern for these geometries also similar to the circular array geometry.

6.2 Future Work

There is a huge scope of research in the field of antenna array optimization. The research can be extended to reduce the side lobe level of the circular antenna array
along with minimizing the white noise gain. Another possible adaptive antenna array that can be designed is interference suppressing antenna array.

Antenna arrays designed in this thesis use an infinitesimal dipole element as an antenna element and all the elements are considered as identical. The same procedure can be done for different antenna elements. For example, an antenna element having triangular variation of length is called small dipole which can be used as the antenna elements in the place of infinitesimal dipole antenna elements in an antenna array then performance and gains of an antenna array can be observed.
BIBLIOGRAPHY


