TRANSITION TO TURBULENT FLOW IN FINITE LENGTH CURVED PIPE

USING NEK5000

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TRANSITION TO TURBULENT FLOW IN FINITE LENGTH CURVED PIPE

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ABSTRACT

Transition to turbulent flow in curved pipe has been well studied through experiments and numerical simulations. Numerical simulations often use helical pipe geometry with infinite length such that the inlet and outlet boundary conditions can be modelled as periodic which reduces computational time. In the present study, we examined a finite length curved pipe with a Poiseuille flow imposed at the inlet and a stress-free boundary condition at the outlet. Direct numerical simulation of the Navier-Stokes equations for rigid walls and a Newtonian fluid was performed using nek5000. Straight extensions were added to the inlet and the outlet such to diminish the impact of boundary conditions on the flow field in the region with curvature. The examined model has a pipe radius of curvature that is three times that of the pipe radius. The model has over 300 million nodes and required an order of magnitude greater computational time when compared to the infinite length curved pipe. Results show that the critical Reynolds number (initiation of instabilities) is greater compared to a straight pipe and occurs near Re=5000-5200. This Re is also larger than the critical Reynolds number typically reported for an infinite length curved pipe (Re= 4200-4300). As expected, flow patterns in the finite length curved pipe were shown to be evolving through the curvature as opposed to that of an infinite
length curved pipe where it remains constant. In addition, the initial instabilities observed in the flow did not originate from a Dean flow instability, initiated through secondary flow, but rather were first observed near the outer wall.
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TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>LIST OF FIGURES</th>
<th>vii</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHAPTER</td>
<td>1</td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Literature review</td>
<td>1</td>
</tr>
<tr>
<td>II. SIMULATION OF FINITE LENGTH CURVED PIPE</td>
<td>7</td>
</tr>
<tr>
<td>2.1 Describing the problem</td>
<td>7</td>
</tr>
<tr>
<td>2.2 Computational method</td>
<td>9</td>
</tr>
<tr>
<td>2.3 Results and Discussion</td>
<td>14</td>
</tr>
<tr>
<td>2.4 Instability Analysis and Turbulent Fluctuations</td>
<td>25</td>
</tr>
<tr>
<td>2.5 Velocity distribution along the curved pipe</td>
<td>28</td>
</tr>
<tr>
<td>III. CONCLUSIONS</td>
<td>38</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>41</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>2.1</td>
<td>Toroidal coordinate system</td>
</tr>
<tr>
<td>2.2</td>
<td>Curved pipe with inlet and outlet extensions</td>
</tr>
<tr>
<td>2.3</td>
<td>Schematic of the problem set-up and Reynolds numbers examined for both increasing (forward set) and decreasing (backward set) Reynolds number during the sequence of CFD simulation restarts</td>
</tr>
<tr>
<td>2.4</td>
<td>Schematic view of chosen point in cross sectional view for pressure gradient and velocity profile</td>
</tr>
<tr>
<td>2.5</td>
<td>Impact of inlet and outlet extension length at Re=2000. Inlet and outlet are both 3 (Left) and 10 (Right) diameters</td>
</tr>
<tr>
<td>2.6</td>
<td>Hexahedral mesh in cross sectional view (Right) Mesh in mid-plane (Left)</td>
</tr>
<tr>
<td>2.7</td>
<td>Selected point locations in cross sectional view for DMD analysis</td>
</tr>
<tr>
<td>2.8</td>
<td>DMD amplitude based on the natural frequencies, Left) Re=5000 forward set, Right) Re=5200 forward set</td>
</tr>
<tr>
<td>2.9</td>
<td>DMD amplitude based on real part of eigenvalues, Left) Re=5000 forward set, Right) Re=5200 forward set</td>
</tr>
<tr>
<td>2.10</td>
<td>DMD amplitude based on the natural frequencies, Left) Re=5000 backward set, Right) Re=5000 forward set</td>
</tr>
<tr>
<td>2.11</td>
<td>DMD amplitude based on real part of eigenvalues, Left) Re=5000 backward set, Right) Re=5000 forward set</td>
</tr>
<tr>
<td>2.12</td>
<td>Number of the effective modes vs. the performance of loss Left) Re=5000 backward set, Right) Re=5200 forward set</td>
</tr>
</tbody>
</table>
2.13 Bifurcation diagram for the maximum velocity in the axial direction at a location 20 degrees from start of the curved pipe at a point near the side wall (point 7 as shown in figure 2.7) where the high fluctuation was observed (Top) Fluctuation velocity at the same point for Re=5200 (Bottom) ........................................ 22

2.14 Axial velocity bifurcation diagram at curved pipe end (180 degrees) near the inner wall (point 1 or 3 as shown in Figure 2.7) Supercritical Hopf bifurcation at two different Reynolds numbers (Re = 5000-5200 and 5800-6000) ........................................ 23

2.15 Turbulence intensity diagram ........................................ 25

2.16 Instantaneous axial velocity at Re=5200 show instabilities on outside wall prior to Deans flow instabilities ........................................ 27

2.17 Velocity fluctuations on unstable region Left Top) 20-degree, Right Top) 30-degree, Left Bottom) 40-degree, Right Bottom) 50-degree . . 27

2.18 Vector plots of velocity Left) Re=5200 Right) Re=6200 . . . . . . . . 28

2.19 Comparison of velocity distribution in cross sectional views for Re=5200 forward set ........................................ 30

2.20 Comparison of velocity distribution in cross sectional views for Re=6200 forward set ........................................ 30

2.21 Contour of instantaneous axial velocity for Re= 5000 forward set . . 32

2.22 Contour of instantaneous axial velocity for Re= 5200 forward set . . 32

2.23 Contour of instantaneous axial velocity for Re= 5600 forward set . . 33

2.24 Contour of instantaneous axial velocity for Re= 6200 forward set . . 33

2.25 Fluctuation of axial velocity at different locations (every 10 degrees starting at 40 degrees) separated by 0.4 on the y-axis for point 1 . . 35

2.26 Fluctuation of axial velocity at different locations (every 10 degrees starting at 40 degrees) separated by 0.4 on the y-axis for point 7 . . 36

2.27 RMS distribution ........................................ 37

2.28 TKE distribution ........................................ 37

viii
CHAPTER I

INTRODUCTION

1.1 Literature review

Fluid flow in a curved pipe is considerably more complex than that in a straight pipe because the effect of secondary flow exists on planes normal to the primary flow direction. In a curved pipe, flow near the mid-plane shifts towards the outer wall due to centrifugal force as flow turns through a radius of curvature. A pressure gradient from the outer to the inner surface balances this centrifugal force. The secondary flow pattern is known as Dean’s flow. The Dean number is defined as a dimensionless quantity that describes the importance of secondary flow in a toroidal pipe or an infinite curved pipe. Dean [1] mathematically developed an expression for secondary flow patterns in a toroidal pipe. In his study, he showed the secondary flow changes with the Dean number. One of the most important conclusions from his work was that pressure drop in the stream-wise direction is increased by two factors: the onset of turbulence and the intensity of secondary flow. Dean showed that the magnitude of pressure drop is greatly influenced by secondary flow in centrifugal direction. Taylor [2], White [3], and Adler [4] were the first to conduct experiments to corroborate Dean’s work. Their results showed a delay in transition to turbulent flow in the helical
pipe due to the presence of secondary flow. It was also shown that significantly larger flow rate is necessary to maintain turbulence in a curved pipe than in a straight pipe. White [3] achieved his conclusion by measuring the resistance and friction factors for both a curved and straight pipes. Taylor investigated the criteria for turbulent flow and compared them with the data obtained from White’s experiments. After it became clear that the secondary flow was a primary reason for both pressure drop and the delay in turbulence, researchers shifted to determine the transiting point between turbulent and laminar regimes.

Experiments by Ito [5] measured the friction factor $f$ for laminar and turbulent flow with respect to the Dean number and curvature ratio, $\delta$, pipe radius of curvature divided by the pipe radius. Through numerous experiments, he found an empirical formula that correlates the critical Reynolds number with friction and resistance. His results were verified by Cionilini and Satini [6] through experiments with a wider range of Reynolds numbers and additional curvature ratios. Results showed critical Reynolds number gradually gets larger as curvature is increased. Similar to Cionilini and Satini studies, many other studies used a helical pipe to model an infinite length curved or toroidal pipe. Helical pipes have an additional factor on turbulence formation due to the spacing between loops (pitch) [7, 8, 9, 10, 11]. Compared to numerical simulations, experiments have a potential altering flow effect due to the spacing between loops which is finite for a helical pipe and zero for a toroidal pipe. Humphrey et al. [12] did experiments on square cross-section curved ducts with
smooth walls to study secondary flow and compared his results with computational methods (finite difference using k-epsilon turbulence model). They performed their experiments with a 90 degree bent curved duct with 2.3 ratio which was extended long enough with straight extensions in upstream and downstream. They observed that the production of turbulent kinetic energy (TKE) is predominately near the outer walls. They have concluded that the inner wall and outer wall act as stabilizer and destabilizer of the fluid flow near walls. Wall shear stress increases as it goes along the curved duct however toward the core of the duct it diminishes. The second conclusion was that the k-epsilon turbulence model was unable to represent the negative contributions on the generation of turbulent kinetic energy and it required the accurate representation of the stress distribution in numerical modeling. Webster and Humphrey [12] did a number of experiments on helical pipes and measured turbulent instability in a curved pipe with curvature ratio 18.2 near the inner and outer walls based on Strouhal number (St). They observed periodic oscillation that for the range of Reynolds number is between 5060 and 6330 and the St = 0.25. They reported that the oscillations occur only near inner walls and the flow is steady near the outer wall for such high Reynolds number. They concluded that the turbulent instabilities originate in inner half of the cross section for low Reynolds number and extends to the mid-plane of the pipe near the inner wall for higher Reynolds numbers. They discussed the instabilities near inner walls and mid-plane which is the most unstable region.
While Humphrey et al. and Patankar et al. [13] did not closely match the experimental results, the development of numerical methods and computational power provided better tools to model turbulent flow in the years that followed. Improved solutions were presented by Webster and Humphrey [12] for turbulent flow in a curved pipe with the same curvature ratio and Re=5042 which was located in transitional regime. The object of their work was to find turbulent traveling waves and compare the numerical simulation with the results of their experimental work in [12]. The results well matched with the conclusions from the experimental model which included origination of the instabilities in the inner half of the curved pipe. They showed that the flow near outer wall remains non-fluctuated with low frequency and all traveling waves are about the inner half where the Dean vortices get strength. This study was important since the numerical simulation was reasonably matched with the experimental data. Hüttl and Friedrich [14, 15] performed the direct numerical simulation (DNS) of fully developed turbulent flow of an incompressible flow for straight, toroidal and helical pipes for Reynolds number 230 and curvature ratio of 10 in order to find effects of torsion and curvature. They showed that large curvature ($\delta >10$) has a great impact on TKE while the large torsion effect (pitch $>100$) has a weak physical effect comparatively on the turbulence. They also showed that the TKE increases for helical pipe with the same curvature ratio which implies that the critical Reynolds number in the case of helical pipe is lower than toroidal pipe. Recently, Di Piazza and Ciofalo [16] performed a DNS simulation on a toroidal pipe with quasi-periodic boundary condition. They implemented their simulations
for different Reynolds numbers corresponding to flow that is stationary to periodic, quasi-periodic and chaotic for two curvature ratios (10 and 3.33). For low curvature ($\delta = 3.33$), the critical Reynolds number for transitional turbulent occurs at 4556 to 4605 compared to 5139 to 5208 for high curvature ($\delta = 10$) where the stationary flow becomes a transitional flow. In the case of low curvature, steady flow turns to periodic flow and increasing Reynolds number led the periodic flow to become quasi periodic flow for higher Reynolds number (5042-5270). For $\delta = 10$ transition occurs between stationary to quasi-periodic flow first. Periodic flow also was observed for higher Reynolds number. They showed that the transitioning to chaotic flow occurs at high Reynolds number as 7850-8160 for $\delta = 3.33$ and 6280-8160 for $\delta = 10$. The transitional mechanism for lower curvature and abrupt breakdown of TKE occurred in the outer region of the pipe caused by centrifugal instabilities. However, for the lower curvature case, the outer regions remain stationary and transition to turbulence initially occurs in the inner region. Noorani et al. [17] also looked at the evolution of turbulent flow in toroidal pipe through DNS simulations for incompressible flow and two of curvature ratios (100 and 10) and three Reynolds numbers 5300, 6926 and 11700 which all above the transitional regime. They reported detailed characteristics of these transitional flows.

Although there are many studies as experiments and numerical simulations on curved pipe, most have focused on the infinite length curved pipe, toroidal pipe for numerical and helical pipe for experimental. We are not aware of any numerical
studies that examined the curved pipe at transition turbulence non-periodic boundary conditions. This geometry is somewhat representative of the aortic arch where transition to turbulent flow has been observed. Our group has studies transition to turbulence in biological applications for the carotid bifurcation and arteriovenous graft [18, 19, 20, 21, 22, 23, 24, 25]. Experimental and numerical comparison for transitional flow was conducted in the arteriovenous graft geometry and good agreement was obtained.

Most numerical simulations of the curved pipe employ periodic boundary conditions for the inlet using the velocity field at the outlet. The driving force for fluid motion is obtained either by adding a constant pressure or mass as a source term to the Navier-Stokes equation. This balances the friction in turbulent flow. In this paper, we aim to study differences in transitioning to turbulent flow between finite length and infinite length curved pipes. This amounts to a difference between periodic with non-periodic boundary conditions on a curved pipe. We employ DNS to simulate transitional regime with high resolution numerical schemes and mesh density. A common geometry with biological prospective is simulated for a wide range of Reynolds numbers starting from laminar flow and increasing to transitional flow. Flow parameters such as TKE, velocity fluctuation and friction factors were determined to better understand the process of the development to a transitional regime.
CHAPTER II

SIMULATION OF FINITE LENGTH CURVED PIPE

2.1 Describing the problem

The curved pipe is a 180 degree curvature with curvature ratio of three which approximately the same as a human ascending and descending aorta [26, 27]. In addition, there are many studies on infinite length curved pipe with this ratio or close to it which they provide for comparison [7, 8, 9, 10, 11]. The governing incompressible equations, continuity and Navier-Stokes equations, were solved on a Cartesian grid for fluid flow with constant properties:

\[
\nabla \cdot u = 0 \tag{2.1}
\]

\[
\frac{\partial u}{\partial t} + (u \cdot \nabla) u = -\nabla p + \frac{1}{Re_b} \nabla^2 u \tag{2.2}
\]

Where \( u \) is the velocity vector, \( p \) is the pressure and \( Re \) is the Reynolds number of the fluid that indicates scale of the main flow based on the bulk mean velocity, kinematic viscosity and pipe diameter. In secondary flow, the radial pressure gradient (created due to centrifugal forces) forces fluid flow to move in the perpendicular direction to the primary flow (non-axial) toward outer wall which induces two counter
rotating circulations on either side of the mid-plane. In Dean’s work, the assumption was that the curvature ratio is very large compared to the radius of pipe. This assumption leads to an equation to describe the secondary flow in which the scale of the secondary velocity can be estimated with the bulk mean velocity given the relation \( u_{sec} = u_b \sqrt{\delta} \). The Dean’s number is an indication of the scale of secondary velocity and given as: \( De = Re_b \sqrt{\delta} \), however, this relationship was developed for an infinitely large curved pipe and is not necessarily applicable to finite length curved pipes.

Radial coordinate system for this curved pipe geometry is shown in figure 2.1. All the calculations of the central moments were transformed from the Cartesian coordinate system to a toroidal coordinate.

![Toroidal coordinate system](image)

Figure 2.1: Toroidal coordinate system

For the boundary conditions, Poiseuille flow velocity profiles was imposed at the pipe inlet and stress free boundary conditions was set for the pipe outlet with no-slip conditions imposed at all wall boundaries. End boundaries were placed far enough from the beginning and the end of the curved part as added straight extensions
in order to reduce their effect on the fluid dynamics with the pipe curvature region. Figure 2.2 shows the curved pipe with extended parts.

![Curved pipe with inlet and outlet extensions](image)

**Figure 2.2: Curved pipe with inlet and outlet extensions**

2.2 Computational method

In this work, the spectral element method (SEM) was employed using an open-source code, *nek5000*. This Navier-Stokes equation solver is a high fidelity parallel code which breaks the computational domain to a number of elements [28, 29]. The code is based on the spectral element method, which represents velocity and pressure as Nth-order tensor-product polynomials within each of K computational mesh cells-bricks. The total number of grid-points is approximately $KN^3$. The polynomials can be differentiated to compute derived quantities and provide for accurate high-Reynolds number solutions with minimal numerical dissipation and dispersion. In turbulent flows, fluctuations are present with many different wavelengths that propagate in all directions. DNS simulation requires high resolution grid density to capture the entire range of these turbulent wavelengths. The computational mesh or node
spacing size must be less than the wavelength or the number of nodes in each direction should be more than wave numbers of the traveling waves. The advantage of the SEM is a rapid exponentially convergence in space which is exaggerated this performance with scalable multigrid solvers and power of parallel processing.

In our work, the setup for the DNS simulations was designed to go from the laminar to the transitional regime in steps (Figure 2.3). The advantage of this stepwise method is to reduce the numerical unsteadiness at each step. Convergence criteria for laminar flow was based on numerical error less than $10^{-7}$ where error is defined as the volume integral of velocity norm between sequential time steps. Numerical simulations start from low Reynolds number in laminar flow and it increases the Reynolds number in a stepwise manner that uses the results of the previous simulation as the initial condition. Convergence in transitional regime was defined as the point at which a statistically stationary solution state was detected. Statistically stationary turbulence was evaluated based on the central moments of velocity for many selected points throughout the curved pipe to be monitored subsequently. As flow gets close to transition, step size is smaller. In addition to stepwise increasing the Reynolds number, called a forward set, a second stepwise set was performed in which each simulation is initiated from the results of a simulation from a higher Reynolds number which is called a backward set. The backward set is valuable to investigate the influence of the initial conditions. Critical Reynolds number for a curved pipe with curvature ratio of three is expected to occur near 5000. So the design of the
stepwise simulation set incorporated smaller steps at the beginning of the transitional regime. Reynolds number was increased by 500 or 1000 in each step in the laminar regime and by 200 in each step from 5000 to 6000 for the forward set of simulations. Figure 2.3 shows the simulation setup.

Figure 2.3: Schematic of the problem set-up and Reynolds numbers examined for both increasing (forward set) and decreasing (backward set) Reynolds number during the sequence of CFD simulation restarts

2.2.1 Geometry and Mesh

Straight extensions were added to the curved pipe to reduce impact of end conditions. At the start of the curved pipe, the pressure gradient between the inner and outer wall causes secondary flow. This pressure gradient can also alter the flow field within the upstream inlet extension and this effect is dependent on Reynolds number. In addition, the length of outlet extension has an impact on the flow field within the
curved pipe. Thus, simulations were conducted to establish the necessary extension lengths in order to greatly diminish the influence of boundary conditions on the secondary flow inside the curved pipe. The goal was to insure no discontinuous behavior of the velocity or pressure gradient along the pipe length. Depending on the extension length, discontinuities were present near the beginning and the end of the curved pipe. The pressure gradient between inner and outer walls and the velocity in the axial and radial directions for a single point near the mid-plane were monitored at a high Reynolds number (6200) and various extension lengths (Figure 2.4).

Figure 2.4: Schematic view of chosen point in cross sectional view for pressure gradient and velocity profile

Figures 2.5 show the discontinuity of variables with various extensions for two different extension length conditions (inlet and outlet are 3 and 10 diameters). The longer extension case show the discontinuity to be absent at the same Reynolds number as the short extension case. Note that velocity profiles are the same, only pressure gradient showed a discontinuity at a layer position near 15. At higher Reynolds number (6200), this methodology determined the necessary extensions lengths to be 5 and 20 diameters for the inlet and outlet, respectively.
Hexahedral grid was generated for the 25D+3πD length curved pipe. Grid spatial resolution was determined based on the Kolmogorov dissipation scale in order to resolve all scales of turbulent fluctuations for the highest desired Reynolds number based (6200). This was based on the wall friction for a straight pipe with the calculation of the viscous characteristic lengths in radial and axial directions \((r^+, z^+)\). The requirement in cross sectional segments should have \(\Delta r^+ \leq 5\) maintaining at least four grid points near the wall with \(\Delta r^+ = 1\); the requirement in the axial direction was chosen to be as twice that of the cross sectional requirements \(\Delta z^+ \leq 10\). This requirement remains for the entire curved pipe and within the five diameters inlet and outlet extensions. However, to reduce computational resource requirements, grid size in the axial direction beyond five diameters of the outlet extension was stretched out by the factor of two.
Since nek5000 utilizes a high order polynomial, Gauss-Lobatto-Legendre (GLL), on the local elements. The grid size requirements must take into account the polynomial order. The polynomial order of seven was chosen since it was reported as an efficient order of accuracy for orthogonal polynomial basis [28]. Thus, element size requirements are seven fold larger than the grid size requirement for this polynomial order. Prenek was used to create a hexahedral mesh based on these requirements. The number of elements and number of nodes were 94752 and 335277105 for curved pipe with ratio three. Figure 2.6 shows the mesh at the mid-plane of the curved pipe (Left) and in the cross-sectional view (Right). Note that radial mesh spacing is different near the wall as indicated by line thickness with thin lines indicating where the GLL local grid approximation are located.

![Hexahedral mesh](image)

Figure 2.6: Hexahedral mesh in cross sectional view (Right) Mesh in mid-plane (Left)

### 2.3 Results and Discussion

Transition phenomena occur in the existence of turbulent instabilities and the amplitude of turbulent flow is calculated from the scale of those instabilities. These instabilities undermine the asymptotic behavior of fluid flow and react as attractors
to disturb the flow field. The complexity of turbulent flow depends on these instability attractors which can be studied with two different prospective: one is the behavior of these attractors and the second is the number of effective attractors. In order to characterize these turbulent instabilities, dynamic modes decomposition (DMD) will be used to find the number of effective attractors. In addition, bifurcation analysis will be used to address the behavior of these turbulent attractors.

2.3.1 Dynamic Modes Decomposition

In contrast to the infinite length curved pipe, fluid flow along the finite length curved pipe carries different dynamical motions in the axial direction. Understanding this transport process requires analysis of the data both in time and space coupled together. DMD is a coherent tool to calculate the dynamic modes of the fluid flow using discrete-time linear time-invariant system. The main goal of using DMD in our simulation is to compare dynamic modes between the last simulation of the laminar regime and the first of the transitional regime in order to detect the leading modes that identify the laminar and transitional features of the flow. We refer to these modes as effective turbulent modes which correspond to specific eigenvalues and natural frequencies. Identification of these effective frequencies allow for bifurcation diagram analysis.

DMD was first introduced by Schmid [30] and has obtained a great deal of attention recently. DMD constructs the matrices of the snapshot sequences $\Phi_1^N = \ldots$
\{\phi_1, \phi_2, ..., \phi_N\} using flow field data, \(\phi_i\) for the \(i\) th field. In this method, it assumes that the snapshots are equispaced in time. The dynamic time inherent is mapped constantly between the different time intervals. Consequently, a linear mapping system of any two consecutive time steps can be formulated as \(\phi_2 = A\phi_1\) where \(A\) is a linear mapping connector between two flow fields. The goal of DMD is to provide a procedure to characterize matrix \(A\). The DMD algorithm offers an optimal approximation based on proper orthogonal decomposition (POD) [31] modes of the data sequence and uses singular value decomposition (SVD) of the flow field to decompose matrix \(A\) with an optimal linear representation. Mathematical details and calculations are well explained in [32, 33, 34] and the resulting expression for an optimal representation was given as: 
\[ \bar{A} = U^* \Phi_2^N V \Sigma^{-1} \]
where \(U^*, V\) and \(\Sigma^{-1}\) are obtained from the SVD of \(\phi_1\). Now to find an optimal solution \(\bar{A}\) as low dimensional representation of the matrix \(A\), \(\bar{A}\) should be rearranged with the set of linearly independent eigenvectors and eigenvalues which take place in the linear mapping system. The modal structure can then be extracted from this derivation. Here modal structure indicates ”amplitude” of the DMD modes. For formulation details, see [30]. This amplitude is valuable as it allows comparison of the decomposed DMD modes. In the next sub-section, discussion regarding the weighting on DMD amplitude will be completed.

2.3.2 Implementation of DMD

DMD was implemented with more time snapshots at limited data locations in order to decline the computational time of DMD calculations. Nine point locations on each
cross section were selected at 19 cross sections distributed equally at axial locations along the curved pipe separated by 10 degrees (Figure 2.7).

Figure 2.7: Selected point locations in cross sectional view for DMD analysis

The DMD matrix was constructed based on the instantaneous axial velocity at different times and spatial locations to analyze the characteristic parameters. In addition, amplitudes of different DMD modes and effectiveness were computed. Amplitudes of DMD modes based on the natural frequency and the real part of the eigenvalue demonstrated in figures 2.8 and 2.9 for flow near the critical Reynolds number. One figure shows a Reynolds number in the laminar regime (5000) and the other in the transitional regime (5200). For Reynolds number 5000, all DMD modes have zero amplitude except one with a finite amplitude which is considered as the leading mode.

The difference between the amplitude for Reynolds numbers 5200 and 5000 reveals the attracting modes with small eigenvalues. For higher Reynolds numbers, other DMD modes amplitudes are elevated from zero but still with magnitude much smaller than the leading mode (<2.5 %) which indicates the statistical repetitive
Figure 2.8: DMD amplitude based on the natural frequencies, Left) Re=5000 forward set, Right) Re=5200 forward set

Figure 2.9: DMD amplitude based on real part of eigenvalues, Left) Re=5000 forward set, Right) Re=5200 forward set
distribution of the disturbing attractors. The magnified plots of the DMD amplitudes highlight these differences. Comparing the results at Reynolds number 5000 for the forward and backward sets demonstrates that the disturbing attractors are present only for the backward set which is transitional (Figures 2.10 and 2.11). Natural frequencies of the disturbing modes are close to (0,0.2), (0,-0.2) frequencies.

Sparsity-Promoting DMD (SPDMD) introduced in [35, 36] was used to select the non-zero DMD amplitudes and weight on each mode by defining a performance degradation parameter. Performance degradation approximates the percentage of lost for each mode if that mode is cut out from other DMD modes.

Figures 2.12 shows the performance loss for Reynolds numbers 5000 was non-existent which is expected since this laminar flow reached a steady state condition. However, the performance loss for the same Reynolds number for backward set illustrates ten non-zero amplitudes of which two modes had a performance loss greater than one percent of the leading mode. Thus, the transitional nature of the flow can be accessed using only two modes. The number of influential modes increases greatly which Reynolds number is increased to 5200. Many (22) distinct modes were observed with performance loss higher than one percent with the most influential mode loses above four percent.

The results of the DMD analysis demonstrates that transition was first observed at Reynolds number 5200 for the forward set of simulations. In contrast, the backward set simulations showed transition at a Reynolds number as low as 5000.
Figure 2.10: DMD amplitude based on the natural frequencies, Left) Re=5000 backward set, Right) Re=5000 forward set

Figure 2.11: DMD amplitude based on real part of eigenvalues, Left) Re=5000 backward set, Right) Re=5000 forward set
2.3.3 Bifurcation Analysis

Bifurcation analysis is used to determine the transitional behavior of a flow field from a time-independent solution, steady state, to a time dependent solution. In other words, it allows detection of the point where flow transitions from laminar to turbulent. The transition point and the behavior of this process may provide insight towards a method to control the transitional nature of the flow. The bifurcation diagram is an efficient way to understand the transition process based on the amplitude of a chosen solution parameter at a given location [37, 38]. This amplitude is then related to a characterized factor which is the Reynolds number in our case. The solution parameter examined herein is the axial instantaneous velocity.

Curved pipes are often studied using period boundary conditions. Examination of those flow fields is less complicated since the mean velocity profiles do not change in the axial direction. For our non-periodic geometry, velocity profile develops
in the axial direction and thus, bifurcation diagrams were made for many locations since the amplitude will vary along the axial direction.

Figure 2.13: Bifurcation diagram for the maximum velocity in the axial direction at a location 20 degrees from start of the curved pipe at a point near the side wall (point 7 as shown in figure 2.7) where the high fluctuation was observed (Top) Fluctuation velocity at the same point for Re=5200 (Bottom)

Roughly the first 20 degrees of the curved pipe is smooth and free of fluctuations even at high Reynolds number (<6200). After 30 degrees, the bifurcation diagram reveals that the upstream fluid flow is affected by turbulent travelling waves originating downstream from that location (Figures 2.14). This behavior remains sustainable for many points in the first quarter. In addition, the bifurcation is more
recognizable for the radial velocity amplitude since it is smaller and more sensitive than the axial velocity amplitude.

Figure 2.14: Axial velocity bifurcation diagram at curved pipe end (180 degrees) near the inner wall (point 1 or 3 as shown in Figure 2.7) Supercritical Hopf bifurcation at two different Reynolds numbers (Re = 5000-5200 and 5800-6000)

Velocity was monitored as a function of Reynolds number at a selected point in the flow field near the entrance of the pipe curvature (20 degrees from start) and located near the side wall (point 7 as shown in figure 8). Figure 2.11 (right) demonstrates the velocity changes with Reynolds number at this point. In laminar regime, velocity increases linearly up to Reynolds 4000. Increasing the Reynolds number from this point up until 5000 shows velocity to increase with Reynolds number but at a lower rate (smaller slope). A transitioning point is defined as intersection of time independent part (linear behavior) with the time dependent part (fitted as third order polynomial). The estimated transition point is approximately 5150-5170 based on
this analysis. A sudden increase in slope occurs after Reynolds number 5000 which indicates the first bifurcation. This bifurcating point is absent from the backward set of simulations since the region between Re=5000 and 5200 is where the level of fluctuations is dependent on the amplitude of an initial disturbance. In other words, fluctuation is present in the simulation of higher Reynolds number do not allow the flow field to fully laminarize at these Reynolds numbers. This behavior is similar for many points within the first half of the pipe curvature (<90 degrees). For Re >5800, there is an abrupt change in the slope of axial velocity with Reynolds number which indicates a secondary bifurcating point where fluctuations move from periodic to chaotic. Larger fluctuations are present at Re=6000 in such a way that the chaotic behavior is present at the end of the curved pipe (Figure 2.14).

2.3.4 Turbulence Intensity

Turbulence intensity is another type of analysis for the behavior of the attractors which highlights largeness of the fluctuations over the number of Reynolds numbers. Turbulence intensity is defined as the ratio of the root mean square of the velocity fluctuations to the mean free stream velocity which for internal flow (Figure 2.15).

Below Reynolds number 5000, the turbulence intensity is less than 1 percent. The discontinuity between Reynolds number 5000 and 5200 indicates that the transition process begins in this range. Above Reynolds number 5800, a slight greater slope
of the turbulence intensity is observed which may indicate the fluctuations move from periodic to chaotic.

2.4 Instability Analysis and Turbulent Fluctuations

Previous discussion was directed towards quantitative detection of transitional instabilities for various Reynolds numbers both for forward and backward set simulations. In this present section, these instabilities were examined at various cross-sections to better understand the localization of the initial fluctuations that progressed toward transition. Here we would like to take a view on instabilities pattern itself once the turbulent flow launches. Instability in the curved pipe is typically examined as an

Figure 2.15: Turbulence intensity diagram
instability arising from fluid interaction within a concave or convex surface [39, 40]. Rayleigh, Taylor, Dean and boundary-layer instability has been described previously as part of the transition process in a curved pipe. However, Saric [40] indicated that there is not a clear distinction between different types of instabilities. Most studies have examined the curved pipe study as an infinite length curved pipe in which the Dean instabilities were dominant. Since this geometry precludes a boundary layer development, any instability created with the boundary layers is not present far from the inlet for an infinite length curved pipe. In contrast, the present study has a boundary layer and the earliest instabilities within a fixed length curved pipe were shown not to originate from the secondary flow or Dean instabilities but rather from instabilities generated near the side walls which are skewed towards the outer wall. This is an important observation given the importance of flow control for transitional flows. The instantaneous axial and radial velocity at the beginning of transition for a number of cross-sections (20, 30, 40 and 50 degrees) provides visible evidence of early instabilities in the flow (figure 2.16). Near wall fluctuations between the side walls at 30 degrees is evident well before Dean vortices have formed. Radial velocity near the wall reaches over 60 percent of the bulk velocity magnitude directed radially inward which is associated with large velocity gradients that increases kinetic energy dissipation.

Time traces that reveal the velocity fluctuations at selected locations on the cross-sections described previously are shown in figures 2.17. The three near wall points are at the outer wall (1), side wall (4), and inner wall (7) at each cross-section.
At the 20-degree cross-section, point 7 has fluctuation with twice the amplitude of that at point 4 while point 1 has almost no fluctuations. However, shortly downstream from this location at 30 degrees, much smaller velocity fluctuations at points 4 and 7 are observed with fluctuations increased at point 1. This trend continues downstream at 40 and 50 degrees where point 1 has by far the largest velocity fluctuation amplitude. In addition, all fluctuations disappear at point 7.

2.5 Velocity distribution along the curved pipe

Mid-plane velocity profiles do not allow for precise understanding of the instabilities within the flow field. However, examination of the velocity magnitude and direction at the mid-plane provide insight into location where complex flow pattern arise (Figures 2.18). Vector plots of the instantaneous velocity for two Reynolds numbers of 5200 and 6200 are shown below for the transitional regime. The magnitude of the near wall radial velocity is observed to increase dramatically at a given point along
Figure 2.17: Velocity fluctuations on unstable region Left Top) 20-degree, Right Top) 30-degree, Left Bottom) 40-degree, Right Bottom) 50-degree

the inner wall and remains small along the outer wall. This effect is great at Reynolds number 6200 compared with 5200 indicating the boundary layer is unstable.

Poiseuille flow enters the straight pipe and remains nearly parabolic at the start of the pipe curvature and begins to skew towards the outer wall over the first quarter of the curved section for both Reynolds number 5200 and 6200. Low and high pressure regions form near inner and outer walls as expect for Deans flow. The small axial velocity (low kinetic energy) near the inner wall after the first quarter of the curved pipe coupled with the low pressure region provides a potentially unstable region. Velocity in this region becomes complicated and potentially indicates instability both for Re=5200 and 6200. Radial velocity is shown to change sign in a short space indicating larger velocity gradients. As the flow goes along the curved pipe, the flow near inner wall accelerates the unstable region moves towards the center such that the velocity profile appears divided. In the second half of the curved pipe, there
exists an interaction between two flows near inner and outer walls near the center. Figures 2.19 and 2.20 show the velocity vector plot in different cross sections along the curved pipe for the same Reynolds numbers. The cross sections are in the first half of the curved pipe every 10 degrees and show clearly the evolution of secondary flow.

Secondary flow begins as two large vortices on each half of the curved pipe. Further downstream, these vortices diminish in size and becomes more centric. At the zero degree cross-section, velocity vectors are uniformly skewed towards the inner wall for transitional and laminar Reynolds numbers. The centrifugal force is absent in the beginning of the curved pipe, however the secondary flow downstream creates a pressure gradient between the inner and outer walls which affects the flow upstream.
Figure 2.19: Comparison of velocity distribution in cross sectional views for Re=5200 forward set

Figure 2.20: Comparison of velocity distribution in cross sectional views for Re=6200 forward set
and creates a small radial velocity skewed towards the inner wall. This effect extends a few diameters upstream to the start of the pipe curvature with dependence the Reynolds number magnitude.

In the first quarter of the curved pipe (20 and 30 degrees), velocity near the inner wall accelerates in region where flow is unstable and appears more scattered as one move further downstream (40, 50, and 60 degrees). These scattered velocity vectors disappear as the Dean vortices become concentrated at the center and reappear near the center.

2.5.1 Instantaneous Axial Velocity Distribution:

Contour plots of the axial velocity for Reynolds numbers 5000, 5200, 5600, and 6200 describe the velocity development from laminar to transitional regime. Contour plots are given at the mid-plane with a cross sectional view at 10 degree increments. The Poiseuille velocity profile is shown to be slightly skewed towards the inner wall at the second and third cross sections (Figures 2.21-24). Figure 2.21 shows the contour plot for largest Reynolds (Re=5000) simulated that remains laminar.

At Reynolds numbers 5200 (Figure 2.22), flow clearly displays mixing which indicates transitional flow. Between Reynolds number 5200 and 5800, the flows remains symmetric about the mid-plane and the mushroom shaped Dean vortices development and reduction are observed (Figures 2.21 and 2.22). At Reynolds numbers above 5800, the structure of the flow turns chaotic (Figure 2.24). Asymmetry about the mid-plane and a complex velocity field at each cross section is present after 50-
Figure 2.21: Contour of instantaneous axial velocity for Re= 5000 forward set

Figure 2.22: Contour of instantaneous axial velocity for Re= 5200 forward set
degrees. The mushroom shaped Dean vortices are not visible and the flow appears to be twisted to the side walls.

2.5.2 Axial Velocity Fluctuation Time Trace

The change between quasi periodic velocity fluctuations to periodic and from periodic to chaotic as Reynolds number is increase is described here for four Reynolds numbers. First is Reynolds number 5000 backward set simulation and three forward set simulations (Re=5200, 5600, and 6200). In figures 2.25 and 2.26, fluctuations are shown for points 7 and 1 starting at the 30-degree cross-section until the end of the curved pipe at 10 degree increments. Point 7 is a point between inner and side wall near the initial instability/fluctuation location while point 1 is near a Dean vortex that ultimately dominates the instabilities. Note, the amplitude of fluctuations at point 7 were shown to 5 fold smaller than those at point 1 at Re=5200 (Figure 2.25). The simulations were performed for at least 200 time units for each case. Time traces show only the last cycle of flow passing through the entire curved pipe with extensions. The plot of axial velocity fluctuations was obtained by subtraction of the mean velocity from the unsteady velocity trace for the last cycle.

For the backward set simulation of Reynolds number 5000, both quasi-periodic and periodic fluctuations were seen at different points. Quasi-periodic fluctuations are extremely slow convergent signals. The required time to become a convergent solution is incredibly high. The same Reynolds number for the forward simulation is a time independent solution that converged well before 200 time units. However,
Figure 2.23: Contour of instantaneous axial velocity for Re= 5600 forward set

Figure 2.24: Contour of instantaneous axial velocity for Re= 6200 forward set
Figure 2.25: Fluctuation of axial velocity at different locations (every 10 degrees starting at 40 degrees) separated by 0.4 on the y-axis for point 1

it was seen that below Reynolds number 5000, the solution converged to time independent solution irrespective of the initial condition imposed (i.e. either forward or backward set). Flow basically starts to fluctuate in the first quarter of the curved pipe, these fluctuations decrease in the second quarter until 90 degrees. After this point, amplitude increases again moves from the inner walls towards the pipe center.

2.5.3 Root Mean Square (RMS) and Turbulent Kinetic Energy (TKE)

Root mean square (RMS) of the axial velocity fluctuations and total TKE for three Reynolds numbers (5200, 5600 and 6200) in order to describe the transition to turbulent flow. The contour plots show the cross sections of the curved pipe at 30 degree increments starting from 30 degrees until the end of the curved pipe. These
Figure 2.26: Fluctuation of axial velocity at different locations (every 10 degrees starting at 40 degrees) separated by 0.4 on the y-axis for point 7

results follow the previous observations on the instabilities. At 30-degrees, vortices are located between the outer and side walls close to the boundary layer. Even for Reynolds number as high as 6200, fluctuations elsewhere remain low. A search for the location of the highest fluctuations reveals then to be at 30-degrees begins near the wall skewed to the outer wall for all Reynolds numbers. At 60-degrees, the fluctuations move toward the inner wall due secondary flow and its strength, and thus movement, depend on the Reynolds number magnitude. For Reynolds number 6200, chaotic flow appears early in the curved pipe (60-degrees) and dissipates the TKE in such a way that undermines the symmetric flow.
Figure 2.27: RMS distribution

Figure 2.28: TKE distribution
CHAPTER III
CONCLUSIONS

In this study, the transitional process was investigated for a semi-circle curved pipe with curvature ratio of three. The results strongly support a further delay in turbulent flow for a finite length curved pipe compared with delay in an infinite length curved pipe. The critical Reynolds number varied between 5000 and 5200 based upon the initial condition. In addition, the reference location examined inside the curved pipe can also alter the the critical Reynolds number by <0.1 %. Our study reveals that the turbulent flow begins with instabilities near the side walls and presumably from boundary layer development. Previous studies on curved pipes showed the Dean instability to be identified as the initial instability. Herein, it was shown that boundary layer instabilities appear prior to Dean vortices and ultimately cause a disturbance in the secondary flow. Further downstream in the pipe curvature, Dean instabilities become the primary disturbance source that dominates all other instabilities and pushes the disturbance towards the inner wall where the Dean vortices are strong. This represents the first time simulations have demonstrated these boundary layers instability precede the Dean instabilities. Effective turbulent instability was introduced through distinction between the DMD analyses of two Reynolds number simulations close to the critical Reynolds number. Data for DMD analysis was col-
lected from time snapshots of selected data points coupled together which revealed the effective disturbing mode is as low as 2.5% of the leading mode. In addition, the amplitude of fluctuation for a point close to the region where the instability began is also as low as 2.5% of leading amplitude. The DMD analysis could important in terms of controlling the transitional flow since it indicates the amplitude of repetitive distribution. Bifurcation diagram as a product of step-wise set of simulations illustrates the pitchfork/Hopf type bifurcation at many points. Intersection between the linear time independent solution and the non-linear time dependent solution denotes the tipping between laminar and transitional flow. The range of critical Reynolds numbers was calculated to be 5150-5172 for the set of points inside curved pipe. Looking at bifurcation diagrams identifies four different flow states. The first state is for Reynolds number lower than 5000 where the solution is time independent and the instantaneous and mean velocity are the same. The second state is between 5000 and 5200 where transition to turbulent flow depends upon the initial condition; the simulation could initiate backward or forward sets and fluctuations would behave either in periodic and quasi periodic behavior, respectively. As we get closer to the tipping/transitioning point for laminar flow, the quasi-periodic fluctuations acquire low frequencies for which damping requires an extremely long period of time. The next state is where the periodic turbulent disturbance dominates the nature of the solution and it becomes time dependent which occurs between Re 5200 and 5800. As the Reynolds number increases above 5800, the non-linearity of the time dependent solution is revealed by the chaotic behavior of the velocity fluctuations. In the
fourth state, velocity fluctuations become even more unpredictable and random which implies the transitional flow is moving towards turbulence.


