STUDY OF LIQUID DROP MIGRATION ON FIBERS AND MATS DUE TO LIQUID FLOW IN A THIN SLIT GEOMETRY

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STUDY OF LIQUID DROP MIGRATION ON FIBERS AND MATS DUE TO LIQUID FLOW IN A THIN SLIT GEOMETRY

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ABSTRACT

The movement of liquid drops on fibers and mats commonly occurs in fabrics used in household and commercial applications. Removal of the liquid droplets from the gas/liquid and liquid/liquid streams will be of benefit for the health of the workers, safety of the environment, and protection of equipment. For example, the presence of water in diesel fuel may lead to some problems such as plugging of fuel injectors, reduce fuel flow rate, reduce fuel lubricity, and cause corrosion of engine parts. It is better to design a self-cleaning filter, which could be used to collect the liquid droplets and drain to a collecting device. This filter has advantages of reducing the cleaning cost and extending the filter life. The quality factor is used to evaluate the performance of the filter, which is related to the separation efficiency and pressure drop. The separation efficiency is strongly related to the velocity of drops on the fibers and mats. The drag coefficient correlation is used to estimate the average velocity of drop moving on the fiber/mat surfaces.

In this work, the drag coefficient is correlated to Reynolds and capillary numbers from experiments and models. The flow direction is parallel the fibers and mats. In the gas-liquid experiment, the experiments are conducted for different types of fibers and different liquids. In the liquid-liquid experiment, the experiments are conducted for water drop moving on different types of mat surfaces. Also, other correlations have been derived to estimate the minimum gas/liquid flow Reynolds
numbers to initiate the drop movement on fiber and mat surfaces. Finally, when the flow direction is through the filter medium (perpendicular to the fiber mat surface), another correlation is derived to estimate the minimum flow rate to move the water droplets through the pores of the mats.

These correlations are unique to determine the average velocities of the liquid drops on fiber/mat surfaces. The motion of drops on fibers and woven mats are not widely discussed in literature. Very few papers have been published on the development of such correlations to estimate the drop migration on fibers/mats which makes this work unique.

*Keywords*: filter media, coalescence, wettability, drag coefficient, drag force, drop motion
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My beloved parents
Hao Fang and Jiali Huang
And
My beloved grandparents
Ran Fang and Huijiao Wu
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CHAPTER I
INTRODUCTION

1.1 Theory of filtration

Filtration is a mechanical or physical operation which is used for separation of dispersed phase from a continuous phase by interposing a medium through which only the continuous phase can pass. The continuous phase that passes through is called the filtrate. The dispersed phase can be solid or liquid and continuous phase can either be a liquid or gas. The filtration process uses a filter medium to separate the unwanted solid particles or liquid droplets from the continuous phase. If the diameter of the unwanted solid or liquid is bigger than the pore size, the solid or liquid particles will be captured on the surface. If the diameter of the unwanted solid or liquid is smaller than the pore size, the solid or liquid particles may be captured inside of filter. The core part of filtration process is the filter medium, which is a permeable medium due to the pressure difference between inlet and outlet. The filter medium is usually evaluated by quality factor, which is related to the separating efficiency of the filtration process and the pressure drop. As a result, the selection of filter medium is very important for the filtration process.

1.2 Types of filtration

Filtration can be divided into two parts: depth filtration and surface filtration. In depth filtration, the dispersed phase particle size should be less than the pore size of
the filter media [1]. Depth filters use different single fiber capture mechanisms to capture the disperse particles, such as direct interception, diffusion, inertial impaction, gravitational deposition etc. These filters use several types of media to achieve the goal of holding disperse particles. The fluid has to take a longer path through the filter before exiting from the filter. Normally, these filters have large holding capacities and initially have a higher resistance to flow. When the dispersed phase particle size is bigger than the pore size of the filter medium, surface filters work by straining of disperse particles. Dispersed phase particles are trapped on the upstream side of the medium with the holding capacity limited by the space available for formation of a filter cake on the surface of the membrane. Depending on the applications, filtration includes following combinations of dispersed phase and continuous phase: gas-liquid, liquid-liquid, solid-liquid, and solid-gas.

1.3 Problem statement

The migrations of droplets on fibers/mats are important to a number of industrial applications. In one particular application, the movements of the drops control the performance of coalescing filters. Coalescing filters are widely used in the petrochemical industries, wastewater industries and so on. Examples include removal of dispersed water drops from diesel fuel [2, 3], and the removal of aerosol droplets from air to protect the environment and worker health [4, 5]. The convection of small droplets carried by a flowing gas and their capture due to collision with fibers are similar to those solid particles and are well understood [6-8]. The movements of the drops attached to fibers and woven mats are less understood and require study to develop theory, correlations, and data to validate models. The common underlying
need is a model to estimate the velocity of a moving drop on fibers/woven mats. The fundamental models and experiments are carried out with measurement of controlling the parameters, which are related to the motion of drops on fiber and mat, including flow rate, contact angle, surface tension, drop size, fiber size, pore size, viscosity, etc. In this work, specific models are developed and fitted to experiments that are conducted to study drop migration on fibers and mats. The models provide fundamental understanding about how the drops migration on the fibers and mats. A wide range of flow conditions, materials properties and surface geometries are considered in development of the correlations. One correlation can determine the minimum Reynolds number needed to initiate the movement of the drops on the fibers and mats. Another correlation determines the minimum liquid Reynolds number to push the drop through the mats with different pore diameters. These correlations take into account a range of flow condition, material properties, drop size, fiber size, and pore size.

1.4 Objective

The objectives of this work include:

1. Designing pressure driven experimental apparatus to study drop migration on fibers and mats.

2. Analytical models for the drop drag coefficient.
   
   a. To determine the axial drop velocity of the liquid drops on fibers with help of drag coefficient model.

   b. To determine the drop velocity of the liquid drops on mats with help of drag coefficient model.
c. Comparison of the results between pressure driven device and Couette air flow device.

3. Drop size, flow rate, fiber diameter and the surface properties influence the drop migration. Developing drag coefficient correlation taking into account all these factors with the help of dimensional analysis.
   a. Drag coefficient correlation of predict axial motion of liquid drops on fibers.
   b. Drag coefficient correlation of predict motion of liquid drops on mats.
   c. Comparison of the correlation results from dimensional analysis with experimental data.

4. Minimum Reynolds number to initiate the movement of drop on fiber/mat with the factors affecting it.
   a. A correlation to determine the minimum Reynolds number of gas to initiate the drops moving on the fibers.
   b. A correlation to determine the minimum Reynolds number of liquid to initiate the drops moving on the mats.
   c. Comparison of correlation results of the minimum Reynolds number with experimental data.

1.5 Dissertation outline

This dissertation is organized in seven chapters.

a. CHAPTER I gives an introduction to filtration theory, objectives of this work and dissertation outline.
b. CHAPTER II reviews the background information on gas-liquid, liquid-liquid filtration, the factors affect the coalescence filtration, and industrial applications of the coalescence filtration.

c. CHAPTER III discuss about the design of the pressure driven device for the gas-liquid experiment, involving the calculations for the velocity profile, the length of the laminarization mesh.

d. CHAPTER IV illustrates the drag coefficient of gas-liquid results, and it will be compared with the result from Couette device data. The drag coefficient correlation developed using dimensional analysis and the experimental data evaluated for drag coefficient under different conditions. Another correlation will be obtained related to estimate the minimum Reynolds number of gas could initiate the movement of liquid drops.

e. CHAPTER V discusses about the design of the pressure driven device for the liquid-liquid experiment, involving calculating the length of the laminarization mesh. The correlations for the drag coefficient of mat and the minimum liquid Reynolds number to initiate the movement of the drop will be derived.

f. CHAPTER VI includes the experiment setup for the liquid drop go through the mats with different pore sizes. Another correlation will be developed to estimate the minimum flow rate to push the drop go through the pore. Also, the result will be compared with the analytical model.

g. CHAPTER VII contains the overall conclusions from this research work and also discusses the future direction of the experimental and modeling work.
CHAPTER II
BACKGROUND

This chapter reviews the coalescing filter, capture mechanisms, aerosol filtration, wettability, drag theory, applications of filters and drop motion on fibers.

2.1 Coalescing filter

Coalescence is the process by which two or more droplets, bubbles or particles merge during contact to form a bigger droplet, bubble or particle. Coalescence filtration is happened when droplets carried by a flowing liquid stream are captured by the fibers in a filter medium. The dispersed liquid droplets, entering with the continuous phase, will capture by fibers due to various capture mechanisms, coalesce with other droplets, grow big and eventually drain out of the filter medium as shown in Fig 2.1. There are many industrial applications for the coalescing filters, such as removal water drop from diesel fuel, removal of liquid aerosols from metal cutting, dehumidification[2,9,10].
Figure 2.1 The coalescence mechanism in the filter is described as (a) The operation of a coalescing filter showing the difference in the inlet and outlet liquid stream due to the capture of liquid droplets inside the media. (b) The coalescing process occurs within the medium as the liquid drops combine to form larger drops that migrate and drain from the filter medium.

There are several factors that influence the coalescence of the liquid droplets captured on the fibers. The performances of filter media depend upon properties of the emulsion such as the drop sizes, the concentration, the density and the viscosity [11]. The performances of filter media also depend on the filter properties including fiber surface wettability, fiber size, fiber structure and orientation, and porosity. The flow diagram in Fig 2.2 indicates the scales at which different parameters affect the coalescing filter process. The parameters at the larger scales are dependent upon the
parameters at the lower scales. The macroscale performance of the coalescing filter is determined by the pressure drop across the media and the capture efficiency. The performance of the filter has several definitions, such as filtration index [12], figure of merit [13], or quality factor [12]. The larger $QF$ number indicates the better filter performance, and the $QF$ is defined as [6, 14]:

$$QF = -\frac{C_{OUT}}{C_{IN}} \frac{\Delta P}{\Delta P}$$  \hspace{1cm} (2.1)

Where $C_{IN}$ and $C_{OUT}$ are the inlet and outlet concentrations through the media and $\Delta P$ is the pressure drop across the media.

Figure 2.2 Different properties affect the performance of the coalescing filter

In gas-liquid/liquid-liquid filtration, the separation efficiency of a filter is usually accompanied by an increase with pressure drop [15]. The pressure drop increasing could be divided into three stages: the liquid drop trapped in the filter and the pressure drop increased quickly, the pressure drop stabilized when reaching an equilibrium between the drainage and liquid flow entering in the filter, the pressure
drop became constant when the liquid stored in the filter equals to the rate of drainage exiting filter balances the liquid droplet capture rate [16]. Shagufta and Chase [14] found the drainage channels in the filter provided the pathways for coalesced liquid to rapidly flow out of the filter without compromising the capture efficiency. This method could benefit to reduce the pressure drop and increase the filtration index and filter life.

2.2 Capture mechanisms

When the dispersed liquid droplets flow through the filter media, the liquid droplets are captured in the depth of the fiber media. There are four dominant single fiber capture mechanisms [7] used to explain the above capture phenomenon and the submicron size droplets are captured by the mechanisms as shown in Fig 2.3. The capture on a single fiber is dependent on the local flow conditions around the fiber and the filter properties are uniform so that the capture in a single fiber could be used to representation the entire filter medium. The assumption of the single fiber concept is that all fibers are identical [4]. The single fiber mechanism has been widely accepted by the filtration industry as it leads to comparable results with experiments.
a) Inertial impaction: A particle with a dense or large mass is transported in the gas stream, the particle’s inertia may be large enough that the particle’s trajectory will deviate from the air streamline as the air bends around a fiber. The distance between the particle and the fiber surface is less than the particle radius, the particle will be captured. Particles or droplet usually larger than 1µm follow this mechanism of capture.

b) Direct interception: In direct interception, a particle following the streamlines of the flow and be captured by a fiber because both the particle and fibers have finite sizes. Streamlines more than one particle radius away from a filter fiber will not contribute to the interception mechanism. Particle in the range of 0.3-1µm in diameter usually follow the air streamline. The particle diameter is proved to be an important factor in the effectiveness of interception mechanism [7, 17, 18].
c) Diffusion: A small particle moving with a fluid stream tends to depart from the streamline because the collision with the fluid is random. As a result, they travel in random motion superimposed on the streamline flow due to the collisions with air molecules. The random motion may lead the small particle captured by fibers. This mechanism is fit for the particles smaller than 0.3µm have very little mass.

d) Gravitational Deposition: At low gas velocity, the larger diameter droplets may separate from the fluid stream due to the influence of gravity before reaching the medium. When the larger droplet reaches at the fiber, it will be captured by gravitation deposition. This mechanism is suitable for the particles, which sizes are between 20 to 50µm.

The above capture mechanisms described are most effective on particles in a certain size. But the different air velocities may cause the capture efficiency to vary [7]. The detail of fibrous filtration and theory, including efficient of fibrous filters, loading characteristics of solid particles, forces controlling the filtration process are given by Brown [7].

2.3 Aerosol filtration

Aerosol filtration has been studied by a number of researchers [19-22]. Aerosol is a suspension of solid particles or liquid particles, and it can be generated in following primary ways [23-30], evaporation-condensation and mechanical atomization. A liquid could evaporate at a high temperature and re-condense around the liquid in locations of slightly lower temperature [28]. Also the heat can evaporate oil at metal working processes and the vapor oil can condense on dust particles to form small oil droplets. In the other way, the small droplets were shared by the
liquids in contact with high rotation rate equipment acquire mechanic high enough energy [23].

The typical aerosol droplets have sizes between 0.01 and 50µm and these aerosol droplets play a bad role to the human health. The health of people is in danger when they spend long time with metal working fluids. Medical evidence proved that the people exposed to the aerosol have higher possibility to get respiratory irritation, bronchitis, loss of lung function, cancer. In order to avoid the above problems, it is better to reduce worker exposure time to mist, and use air filters and mist collectors.

2.4 Wettability

Wettability refers to the interaction between fluid and solid phases. The following parts include the wetting phenomena and effect of wettability on coalescence.

2.4.1 Wetting phenomena

The wettability of a solid surface is an important physical property and depends on the chemical composition and the micro structure of the surface. The angle, at which a liquid/vapor interface meets a solid surface, is the contact angle, which is commonly used to evaluate wettability [31]. The contact angle (in Fig 2.4) is specific for any given system and is determined by the interactions across the three interfaces. The shape of the droplet is determined by the Young’s relation [32, 33]:

\[ \gamma_{SV} - \gamma_{SL} = \gamma_{LV} \cos \theta \]  

(2.2)

Where, \( \gamma_{SV} \) = the surface free energy of the solid in contact with vapor; 
\( \gamma_{SL} \) = the surface free energy of the solid covered with liquid; 
\( \gamma_{LV} \) = the surface free energy of the liquid-vapor interface;
\( \theta = \) the contact angle.

If \( \theta = 0^\circ \), the liquid completely spread on solid surface and surface is called complete wetting. If \( \theta < 90^\circ \) then solid surface is mostly hydrophilic and if \( \theta > 90^\circ \), then solid surface is hydrophobic.

2.4.2 Effect of wettability on coalescence

In fibrous filter media, when the liquid/gas stream carries the dispersed droplets to contact with a fiber surface, sometimes it attaches to the fiber surface, and sometimes not. Whether the droplet attaches to the fiber or not is determined by both drop and surface wetting properties. The Fig 2.5 explains the behavior of water droplet on fiber surfaces different wetting properties.
Figure 2.5 Behavior of water droplet on fiber surface with different wetting properties

If the fiber/mat surface is hydrophilic, then water droplets will form a thick film on the surface. If the fiber/mat is hydrophobic then water droplets do not attach to the surface and simply get carried over with flow.

Shin and Chase [34] investigated the fiber size and wettability’s effect to the performance of coalesce filter media. They found that high wettability materials for water-in-oil dispersion promote coalescence. The smaller fiber size improves the overall separation efficiency of the process and nanofibers caused a significant increase in the pressure drop through the filter.

Shin and Chase [35] tested the effect of the wettability on the coalescence mechanism. The experiment ran on the coated glass rods with three different silence coating. They found that coalescence of small drops into larger drop was observed
when the continuous phase (petroleum ether) had a higher contact angle than the dispersed phase (water). No coalescence was observed with Viscor 1487 as the continuous phase. The largest drops are produced by fibers that are highly wetted by the dispersed drops and the drop phase should preferentially wet the fiber surfaces over the continuous liquid phase.

Kocherginsky et al [36] demonstrates a hydrophilic polymer membrane for the demulsification of surfactant-stabilized water-in-oil emulsions. The good operability and high efficiency were investigated and it was found that membrane material, pore size and trans-membrane pressure has the strong effect on demulsification. Demulsification is only possible with hydrophilic membrane having pore size smaller than the emulsion droplet diameter. Also, the smaller the pore size, the better the demulsification efficiency as well as leading a high pressure drop. The membrane thickness does not play essential role and the membrane acts as a coalescer with simultaneous permeation of emulsions through porous.

Clayfield et al [37] studied the effect of wettability and surface energy to the coalescence of secondary dispersions. They found that surface treatment has a considerable effect, which means the coating could increase the separating efficiency from 50% to 90%. Coalescence performance does not always correlate with surface energy.

Bitten [38] studied the coalescence rate of water droplet on single fibers. They found the droplet growth rates follow the orders: glass, Teflon, nylon, and Dacron. Also, a considerable amount of chain formation occurred on the nylon fiber. The
maximum-size droplets that could be held by a single fiber were 400 to 500µm diameter for the glass fibers, and less than 100 micron diameter for the plastic fibers.

2.5 Flow through fibrous filters

Darcy’s law [39, 40] is a phenomenological derived constitutive equation which can be used to describe the flow of a fluid through a porous medium, and defines permeability, \( k \), Darcy’s law is

\[
k = \frac{Q \mu L}{A \Delta P}
\]  

(2.3)

Where, \( k \) = permeability of the porous media;

\( Q \) = flow rate;

\( A \) = cross-section area;

\( L \) = thickness of filter media;

\( \mu \) = air viscosity;

\( \Delta P \) = pressure drop.

Permeability is an important characteristic for a filter, and it is a measure of the rate of liquid/gas flowing through a porous media. The permeability of air flow in the fibrous filters will be affected by porosity, size of the fibers and the specific surface area.

Li et al [41] developed a new method to determine the permeability of filter cake. There are two methods could be used to calculate the permeability of filter cake now. First method is testing the liquid flowing through an already formed cake approach, and the second method is the cake filtration approach. Second method required less time and more complicated calculation compared to the first method. Li
developed a new and simplified filter cake permeability test method based on cake filtration, and the consistent results indicated the new method is reliable. The new method is based on the first method but with only one run of the filtration test.

Pich [42] summarized two types of permeability applied to the aerosol filter: channel theory and drag theory. In channel theory, permeability assumes that the porous fibrous media can be modeled as flow through an arrangement of channels and the air flow is described as the Poiseuille’s flow around an isolated circular fiber or through a capillary. This approach is based on the concept of hydraulic radius. In drag theory, permeability is related to a velocity field around the fiber as determined in a porous media and the velocity distribution is then used in calculating the drag forces. Those drag forces acting on individual fibers are further used for determining the permeability.

2.6 Drag theory

In fluid dynamics, drag forces normally acting opposite to the relative motion of any object moving with respect to a surrounding fluid. This can exist between two fluid/surfaces layers, or a solid and a fluid. Normally, there are three mainly drag forces, and they are parasitic drag, lift-induced drag, and wave drag. Unlike other resistive forces, which are independent of velocity, drag forces depend on velocity. Also, the drag force depends on the properties of the fluid and on the size, shape, and speed of the object. One way to express this is by means of the drag equation [43, 44, 45]:

\[ F_D = \frac{1}{2} \rho v^2 C_D A \]  

(2.4)
Where, \( \rho \) = density of fluid;

\( \nu \) = the speed of the object relative to the fluid;

\( C_D \) = drag coefficient;

\( A \) = cross-section area.

The drag force investigated by Muller [46] with low Reynolds number of a cylindrical fiber within a parallel array is expressed as:

\[
F_D = \frac{1}{2} C_D \mu \nu L_f \quad (2.5)
\]

where \( \nu \) is the undisturbed gas velocity and \( L_f \) is cylinder of length.

The drag force investigated by Krafcik [47] for evaluating drag force exerted on a spherical particle suspended in Stokesian flow field (\( \text{Re} \) less than 1) can be expressed as:

\[
F_D = 3\pi \mu_{\text{gas}} D_p (V_f - V_p) CV / C \quad (2.6)
\]

where the suffix \( CV \) stands for evaluation of the air flow at the particle’s center of volume. \( C \) is the slip correction factor for very small particles, which for bigger particles it approaches to 1.

The drag force investigated by Lo [48] for case of wall effect on drag force per length in molding flow using optical fiber sensing data could be expressed by:

\[
F_D = \frac{4\pi \mu_{\text{liq}} \mu}{\ln(b/a) - 0.9156 + 1.7243(a/b)^2} \quad (2.7)
\]

where \( a \) is the radius of the circular cylinder and \( b \) is the distance between the circular cylinder.
The drag force given by Kuwabara [49] for case of structure of the fibers in the media using the cell model with velocity free boundary can be expressed as:

\[ F_D = \frac{4\pi}{-0.75 + 0.5\ln \beta - 0.25\beta^2} \] (2.8)

where \( \beta \) is defined as the solidity of the fibers in the fibrous filter media.

The drag force given by Kinrsch [50] for the case on a droplet on a fiber or array of fibers in a parallel-sided channel could be calculated as:

\[ F = 6\pi b U \mu \left[ 1 - 1.004 \left( \frac{b}{l} \right) + 0.418 \left( \frac{b}{l} \right)^3 + 0.210 \left( \frac{b}{l} \right)^4 - 0.169 \left( \frac{b}{l} \right)^5 \right] \] (2.9)

where \( l=2h-a \), and \( a \) is the fiber radius. \( 2h \) is the distance between fiber axes or the distance between the fiber axis and the edge of the channel; \( 2l \) is the channel width surrounding each fiber, and \( b \) is the droplet radius.

2.7 Application of filters

Filters are widely used in the industry and people’s daily life. The following parts include the application of liquid-liquid filtration filters and gas-liquid filtration filters.

2.7.1 Application of liquid-liquid filtration filters

Liquid-liquid separation is important to many industrial applications such as, removal of water drop from diesel, removal of contaminants to get clean water, removal toxic chemical from waste water. Some of industrial applications are listed below.
Automotive industry [51]: The water is present in fuels as free water, dissolved water and emulsified water and the emulsified water is the major reason of concern because it combines with chemicals in fuels such as sulfur and chlorine, to form corrosive compounds and corrodes sensitive engine parts. Precipitation, humidity and condensation of atmospheric moisture are the normal ways the water enters to the fuels. The free water could be separated by gravity settle because of the large drop size. The emulsified water often has drop sizes less than 100µm and is commonly separated using a coalescing filter.

Aviation industry [52]: In aviation industry, safe and efficient operation of aircraft requires fuel which is clean and dry. Water is a serious hazard compared to solid contaminations. Before refueling the tank, all the contaminants should be removed from the fuel. Free water commonly could entry the tank by precipitating under the low temperatures. When an aircraft operates at different altitudes and at ground level, it will meet extreme variations in temperature. When aircraft operates at below 30F, the precipitated free water can freeze into ice.

Produced water industry [53, 54, 55]: Coalescing filters are used for water exploration applications. Instead of complicated stripping and purging procedures the use of coalescers is preferred, which are more efficient, simpler and produce clean water [56].

Wastewater treatment industry [57, 58]: Coalesce filter can be used to remove chemical and biological waste and toxic chemicals from the waste water, in order to save of polyelectrolytes and flocculants and improve wastewater treatment plant
downstream. Membrane, ultrafiltration, nano-filtration are the predominantly used technologies in wastewater treatment industries.

2.7.2 Application of gas-liquid filtration filters

The control of solid and liquid contaminants in air is a key parameter to many industrial applications such as, metal-cutting, automobile, semiconductor, food process, biotechnology process because these businesses require centralized air conditioning in production environment, clean gases.

*Food industry*: carbon dioxide is used as a key cryogenic agent in cooling, chilling and freezing applications, which is benefit for protecting the taste and texture of food products by maintain proper temperature control. Also, clean carbon dioxide could reduce the need of preservatives in packaged products, and is an essential ingredient in carbonated beverage [59].

*Welding & Metal fabrication*: Clean air is a key factor for welding and metal fabrication industries, which place a huge priority on clean air because welding smoke has been linked to a variety of illness. A well-design dust and fume collection system is normally used in industries to prevent respiratory problems and keep facilities in compliance with current air-quality requirements.

2.8 Drop motion on fibers

The drop motion on fibers is important to the filter performance. The following part will discuss the drop motion on single fiber, and drop motion on superhydrophobic surfaces.
2.8.1 Drop motion on single fiber

Few papers discuss the motion of individual drop motion on fibers. The study of drop motion on fibers is of scientific and economic interest for many applications, such as printing, coatings, drug delivery and release, and filters to remove or neutralize harmful chemicals or particulates from air streams. Gas convection induced drop motion in fibrous materials occurs in coalescing filters, clothes dryers, textile manufacturing, convection ovens, and dewatering of filter cakes. Droplet removal can significantly reduce drying costs by reducing the free moisture contained in fibrous materials prior to applying thermal drying techniques.

Mullins et al [61] incorporated a microscope study of the effect of fiber orientation on the fiber wetting process and flow of liquid droplets along filter fibers when subjected to airflow and gravity force. They found three distinct cases or shapes occur when a drop is contacted to a fiber, as shown in Fig 2.6: (a) film flow, (b) axisymmetric “barrel” shaped drops, (c) axially asymmetric “chamshell” shaped drops.

Figure 2.6 Examples of droplet attach to the fiber (a) film, (b) barrel-shape, (c) clamshell shape
Mead-Hunter et al [62] presented a theoretical model describing the force required to move a coalesced liquid droplet along filter fiber. The forces were measured using atomic force microscope. Also, the influence of droplet displacement perpendicular to the fiber on the force required to move the droplet experimentally. Finally, this work established empirical relationship can be used to predict the force, which was related to the drop volume and the liquid type.

Sahu et al [63] applied pressurized air flow to move drops on fibers and to blow drops off fibers. The displacement and velocity along the filament are measured. There are several phenomena were observed, including drop stick-slip motion, shape oscillations, shedding of a tail along the filament.

Dawar and Chase [64, 65] used a Couette flow device (in Fig 2.7) to measure transverse and axial motion of drops on fibers and detachment of drops due to the air drag force by gas flow. They correlated a drag coefficient to the the Reynolds ($Re$) and Capillary ($Ca$) numbers of a drop moving axially along a fiber. The rotating disk device used to generate the Couette flow was limited to Reynolds numbers in the range of $10^{-7} < Re < 10^{-4}$. 
2.8.2 Drop motion on superhydrophobic surfaces

Research on hydrophobic surfaces began many years ago for different applications. The motivation of using hydrophobic surfaces in industrial applications such as separation of water drops from oil, dehumidification, and rain coats [66]. For example, water drops easily roll off from hydrophobic surfaces and leave behind little to no residue [67, 68]. The motivation of this study is related to create better hydrophobic filter to separation of water drops from oil.

The presence of water in diesel fuel in the form of droplet dispersion can cause many problems if present in fuel such as plugging of fuel injectors, reducing fuel flow rate, reducing fuel lubricity and corrosion of engine parts [69]. Hence, removing water contamination from diesel is of a vital importance for the engine integrity. When a water drop size is larger than 100 microns, the water drops can be effectively
removed by gravity settling [7]. However, when the size is smaller than 100 microns, water is commonly separated using coalescing depth filters [70] or nanofiber membrane technology [71]. The separation efficiency of the filter is strongly related to the dynamic motions of drops on the filter medium.

Madani and Amirfazli [72] presented the work with shedding of liquid drops by a liquid shear flow in experimentally. The effect of surface wettability, physical properties of the oil drop, flow velocity, and drop size on single drop detachment was studied. They also did a force balance to theoretically study to better understand this phenomenon. Their results showed that at high viscosities oil drops detach more easily from Teflon than PMMA. Different regimes of drop shedding were observed: a) drop deformation in the flow direction followed by drop sliding, b) sliding along the substrate without any significant deformation, c) drop deformation and sliding followed by life-off the surface.

Gaithier et al [73] applied the tilt angle to study the drop motion on fuel cell electrode materials. The fibrous surfaces are hydrophobic, but there is a substantial threshold force necessary to initiate water drop motion. Once the water drops start to move, the adhesive force decreases and drops move with minimal friction, similar to motion on superhydrophobic surfaces. The static coefficient of friction on these textured surfaces is comparable to that for smooth Teflon. They found that the dynamic contact angle hysteresis for carbon paper is greatly reduced compared to the static contact angle hysteresis, Enhanced dynamic hydrophobicity is suggested to result from the extent to which a dynamic contact line can track topological heterogeneities of the liquid/solid interface.
CHAPTER III
EXPERIMENTAL METHOD FOR INVESTIGATING DROPLET MIGRATION ALONG SINGLE FIBERS

3.1 Laminar flow

In fluid dynamics, laminar flow occurs when a fluid flows in parallel layers, with no disruption between the layers [74]. In the laminar flow, the motion of the particles of the fluid is very orderly with all particles moving in straight lines parallel to the pipe walls. Laminar flow is a flow regime characterized by high momentum diffusion and low momentum convection.

The type of flow occurring in a fluid in channel is important in fluid dynamics problems. The dimensionless Reynolds number is an important parameter in the equations that describe whether flow conditions lead to Stokes, laminar or turbulent flow. The Reynolds number is defined as the ration of inertial forces to viscous forces and consequently quantifies the relative important of these two types of forces for given flow conditions. The Stoke flow occurs, when the Reynolds number is much less than 1. The laminar flow occurs, when the Reynolds number is between 1 to 1000. When the Reynolds number is larger than 1000, the turbulent flow can occur.

\[
Re = \frac{\rho v L}{\mu}
\]

(3.1)
Where, $\rho$ = the density of the fluid;

$v$ = the mean velocity of the object relative to the fluid;

$L$ = a characteristic linear dimension;

$\mu$ = the dynamic viscosity of the fluid.

3.2 Design of apparatus

To observe the drop motion on fibers parallel to the flow direction, the straight fibers with drops were positioned in the center of the Teflon holder. Pre-filtered house air flowed through a flow meter and into a thin-slit channel holding the fibers and drops. A microscope was used to record the process for moving drops on the fibers. In order to construct the apparatus there are several constraints to be considered for the design of an ideal laminar flow apparatus to work as desired as the flowing:

(i) The position of the fiber and drops must fall in the focal plane of the microscope.

(ii) The length of the fiber should not be too long.

(iii) The Teflon holder should allow the light of the microscope go through.

(iv) The flow pattern in the channel should be laminar that Reynolds number should be less than 1000.

3.2.1 Calculation of the length of flow straightener

The flow straightener is used to straighten the air flow in the channel and make the velocity profile flat. In our study, a glass fiber mat from Hollingworth & Vose was used as the flow straightener in order to make the flow pattern uniform in the channel of the testing holder.
The permeability of the flow straightener was measured by a Frazier Differential Pressure Air Permeability Measuring Instrument (Frazier Precision Instrument Company, Inc), shown in Fig 3.1 and 3.2. Air passes through the medium at a known pressure drop and measured flow rate. The units of the air permeability of the filter media are $m^2$. The filter medium is placed in the sample holder. The filter medium is placed in the sample holder snugly, to avoid the air flow pass from the sides of the sample. Air flow from the side of the medium will produce erroneous results because the air is not flowing through the medium itself. The flow rate is recorded when the pressure drop rises to 0.5 psig. Three sets of readings are taken for the samples. As a result, the air permeability of the flow straightener is $3 \times 10^{-9} m^2$.

Figure 3.1 Frazier differential pressure air permeability measuring instrument
By the industry experiment, the minimum length of the flow straightener should satisfy the pressure drop between the flow straightener ($\Delta P_1$ in Fig 3.3) larger than the pressure drop between the rest of the channel ($\Delta P_2$ in Fig 3.3). $\Delta P_1$ is calculated by the Darcy’s law, with known flow rate, cross section area, gas viscosity, permeability of the filter, and the length of the flow straightener. The flow rate in the channel is related to dimensions of the sample holder. After combing the above two equations, the ratio of $L_1/L_2$ is known. The total length of the channel $L_1+L_2$ is known. Two unknowns with two equations, the minimum length of $L_1$ is calculated, which is
equal to 0.75mm. All the experiments for gas-liquid here, 5mm length flow straightener is used to conduct the gas-liquid experiments.

Figure 3.3 Schematic of the channel for calculating the length of laminarization mesh

3.2.2 Channel flow Reynolds number

By reducing the thickness of the channel, the Reynolds number becomes smaller. The thickness of the channel in our testing holder is 2 mm. For a liquid density of 870 kg/m³ and viscosity of 0.0014 kg/(m*sec), the velocity range is 0-0.8 m/sec to achieve a Reynolds number the laminar range.

3.3 Experimental apparatus

The experimental apparatus is designed based on the initial calculations and the schematic of the pressure driven experimental setup as shown in Fig 3.4 and Fig 3.5. Pre-filtered house air flowed through a flow meter and into a thin-slit channel holding the test fiber and droplets. The body of the holder was machined from a slab of Teflon to form a flow channel 2 mm deep, 3 cm wide, and 6 cm long. The bottom and sides of the channel were solid Teflon. The thickness of the bottom section under
the channel was machined to about 3 mm thickness for enough strength to be rigid when exposed to the air pressure experienced in the channel (of the range of 250Pa or less) but thin enough to allow light from below to illuminate the channel.

Figure 3.4. Schematic of the pressure driven flow experimental setup. The air flow passes through a flow meter and into slit-flow channel machined in the Teflon body. Fibers with drops are positioned in the center of the channel. A fiber mat flow straightener is placed in the channel near the inlet. The channel was illuminated from below and movements of drops on the fibers are observed from above through a glass slide covering the channel.
Figure 3.5 Components of gas-liquid drop motion test (a) flow meter (b) microscope (c) Teflon holder

A 50 mm by 70 mm by 1 mm glass slide was positioned into a machined cavity on the top of the channel and clamped into place to seal the top surface of the channel. The surface of the glass slide was coated with FTS (heptadecafluoro-1, 1, 2, 2, -tetra -hydrodecyltrichlorosilance) (Sigma) to give it a low surface energy that
repelled the liquid drops. Similarly, the Teflon body of the other walls of the channel had low surface energies. The low surface energies of these surfaces reduced the possible interference of the drop performance by attraction of the drops to the surfaces of the channel.

3.4 Experimental apparatus with cylindrical single fiber

Several fiber materials were used in this work to provide different surface properties. Polypropylene and nylon fibers were supplied by Minifibers Inc. Glass fibers were obtained from Hollingsworth & Vose Company. The glass fibers used in the experiment with the diameter 12μm, polypropylene fibers have diameter varying from 16 to 60μm, and nylon fibers have diameter varying from 18-28μm. The fibers were cleaned prior to the experiment by sonication in acetone, and distilled water, and then dried in an oven to remove moisture. An SEM image is taken to confirm the cylindrical geometry of the fibers as shown in Fig 3.6.
Figure 3.6 SEM images of the polypropylene fibers predicting the geometry of the fibers. The image shows that the fibers are cylindrical in shape.

3.5 Setup the droplet on the single fiber

One difficulty in conducting the experiments is the placement of the drops onto the fibers. Common methods are to apply the drops from a syringe or use an atomizer to spray an aerosol of drops onto the fiber. The approach with the syringe is limited to large size drops that can separate from the needle and attach to the fiber. The syringe approach does not work well with very small drops because the surface forces are stronger than gravity force and drops tend to preferentially hold onto the end of the needle. An atomizer spray can produce very small drops but it is difficult to direct the drops to the desired location on the fiber. The atomizer generates many
drops that simultaneously attach on the fiber that can interact or coalesce with each other and interfere with the desired measurements.

An approach to work around the above difficulties was developed that the authors believe to be novel. A fiber of the same material as the test fiber is inserted part way into the opening of the needle of a syringe, as shown in Fig 3.7. A droplet of the desired volume of the liquid is pushed out of the needle onto the fiber extending out of the tip of the needle. The flow rate of the liquid from the needle is slow enough to not force the fiber out of the end of the needle. A second fiber of the same material is attached as a loop to the end of a copper wire. The drop attached to the fiber at the end of the needle is transferred to the fiber loop by touching the fibers together at the drop. The drop on the loop is subsequently transferred to the test fiber in the channel by touching the fiber loop and drop to the test fiber.

In this way, smaller drops are placed on the fiber in the channel. During the transfer some of the liquid remains on the fiber in the needle or on the fiber loop hence the drop volume is slightly smaller than the volume delivered by the syringe.
Figure 3.7. Diagram showing the transfer of the drop from the needle to a fiber loop and to the test fiber positioned in the channel.

Some of the properties of the liquids used for the droplets are summarized in Tab.3.1. For comparison, the properties of deionized water are included in the table, though water drops were not tested in the experiments.

Table 3.1 Physical properties of reference liquids at 20°C

<table>
<thead>
<tr>
<th>Liquid(Source)</th>
<th>Density (g/cm³)</th>
<th>Viscosity (cP)</th>
<th>Surface tension (dynes/cm)</th>
<th>Boiling point (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscor 1487 (Rock Valley Oil &amp; Chemical Company)</td>
<td>0.832</td>
<td>0.8</td>
<td>28.34</td>
<td>160</td>
</tr>
<tr>
<td>Ultra Low Sulfur Diesel(ULSD) (Purchased locally)</td>
<td>0.87</td>
<td>1.4</td>
<td>29.83</td>
<td>180-360</td>
</tr>
<tr>
<td>WD-40 (WD-40 Company)</td>
<td>0.82</td>
<td>5</td>
<td>31</td>
<td>183-187</td>
</tr>
<tr>
<td>DI water (Obtained by ion exchange,0.1 Siemens)</td>
<td>1.0</td>
<td>1.02</td>
<td>73.5</td>
<td>100</td>
</tr>
</tbody>
</table>
3.6 Summary

The continuum flow equations have been used to design a lab scale apparatus which has been used to evaluate the drag forces acting on the liquid drops captured by the fibers. Several different types of liquids captured on the fibers and their migration on different sizes and types of fibers has been studied with the help of the pressure driven flow device. Further results and discussion of the single fiber test by air flow will illustrate in Chapter IV.
CHAPTER IV

DRAG CORRELATION FOR AXIAL MOTION OF DROPS WITH
DIMENSIONAL ANALYSIS

4.1 Drag coefficient

The movements of liquid drops on fibers commonly occur in fabrics used in household and commercial applications. The fundamental mechanisms controlling the movement of drops on the fibers are basically the same as those for flat surfaces, but the differences in the geometries cause different kinematic behaviors. Drops on surfaces are widely discussed but fewer publications discuss drops on fibers. The study of drop motion is possible used for following applications, such as coating, drug delivery, and remove harmful chemical from air stream. Gas convection induced drop motion in fibrous materials occurs in coalescing filters, drying cloth. Droplet removal can significantly reduce drying costs by reducing the free moisture contained in fibrous materials prior to applying thermal drying techniques. The movements of the drops attached to the fibers are less understood and require study to develop theory, correlations, and data to validate models. In fluid dynamic, the drag coefficient is defined as a dimensionless quantity used to quantify the drag of an object in a fluid environment. The objective of this project is to calculate the drag coefficient and correlate it to the Reynolds number for axial motion of barrel drops on fibers. With the help of the drag correlation, the average velocity of the droplet on fiber could be estimated.
4.2 Force balance to calculate drag coefficient \( C_f \)

![Figure 4.1](image)

Figure 4.1. The schematic of a droplet moving on a fiber at velocity \( U \) due to the drag of the gas flowing at velocity \( V \) parallel to the fiber axis.

The Fig 4.1 shows a droplet moving on a fiber at velocity \( U \) due to the drag force of the gas flowing at velocity \( V \) parallel to the nylon fiber axis. A force balance for the droplet on the fiber includes the drag force between gas and droplet \( (F_D) \), the drag force between droplet and fiber \( (F_f) \), the contact line force \( (F_{cl}) \), and the gravitational force \( (F_{gravity}) \).

\[
\frac{dMU}{dt} = F_D - F_f - F_{cl} + F_{gravity} \tag{4.1}
\]

Assuming this droplet is on the steady state condition and neglecting the gravitational effects, the Eq (4.1) could be reduced to

\[
F_D = F_f + F_{cl} \tag{4.2}
\]

According to the defining of the drag force, the drag force acting on the droplet due to the air flow is equal to [64, 65].
\[ F_D = C_D A_{drop} \frac{1}{2} \rho_{gas} (V - U)^2 \]  
(4.3)

where \( A \) droplet is the project area of the drop in the direction of the air flow and 
\[ A = \frac{\pi}{4} d_L^2. \]

The drag coefficient for the air flow around the droplet is equal to [26],
\[ C_D = \frac{24}{\text{Re}_{gas}} \left( 1 + 0.14 \text{Re}_{gas}^{0.7} \right) \quad (1 \leq \text{Re}_{gas} \leq 1000) \]  
(4.4)

The flow is considered as the laminar flow, when \( \text{Re}_{gas} \) is between 1 to 1000. 
\( \text{Re}_{gas} \) is the Reynolds number of the gas and it is given by,
\[ \text{Re}_{gas} = \frac{\rho_{gas} d_L V}{\mu_{gas}} \]  
(4.5)

where \( \rho_{gas} \) is the density of gas, \( d_L \) is the drop diameter and \( \mu_{gas} \) is the dynamic viscosity of gas.

The drag force acting between the droplet and fiber is given as,
\[ F_f = C_f A \frac{1}{2} \rho_{liq} U^2 \]  
(4.6)

where \( A \) stands for the contact area between the droplet and fiber, and \( A = \pi d_f d_A \).

The contact line force is given as [73, 76, 77],
\[ F_{cl} = d_L \gamma_{liq} \left( \cos \theta_R - \cos \theta_A \right) \]  
(4.7)

where \( \gamma_{liq} \) is the surface tension of the liquid, \( \theta_R \) and \( \theta_A \) stands for the receding and advancing angles on the two ends of the droplet.

After combining Eqs (4.3), (4.6) and (4.7), a working equation for calculating the drag coefficient is determined as,
\[
C_D \left( \frac{\pi}{4} d_L^2 \right) \frac{1}{2} \rho_{\text{gas}} (V - U)^2 = C_f \left( \pi d_f d_A \right) \frac{1}{2} \rho_{\text{liq}} U^2 + d_L \gamma_{\text{liq}} (\cos \theta_R - \cos \theta_A)
\] (4.8)

Then, the drag coefficient of the droplet on the fiber could be solved as follow,

\[
C_f = \frac{C_D \left( \frac{\pi}{4} d_L^2 \right) \frac{1}{2} \rho_{\text{gas}} (V - U)^2 - d_L \gamma_{\text{liq}} (\cos \theta_R - \cos \theta_A)}{\left( \pi d_f d_A \right) \frac{1}{2} \rho_{\text{liq}} U^2}
\] (4.9)

4.3 Dimensional analysis

In engineering and science, dimensional analysis is the analysis of the relationship between different physical quantities by identifying their fundamental dimensions, and tracking these dimensions as calculations or comparisons are performed. It has been used in various studies for modeling a physical phenomenon [78, 79] by checking units of the equations to solving very differential equations. It can be used for modeling physical phenomena by generating a similarity form the geometry and estimating a solution.

This work takes a unique approach by defining and determining a drag coefficient correlation. The drag coefficient allows the velocity to be calculated based on the forces acting on a drop. This correlation provides a drag coefficient to estimate the average speed of drops moving on the fibers. Also, a new correlation is obtained to predict the minimum Reynolds number of the gas flow necessary to initiate the movement of the drops on the fibers. The second correlation gives an estimate of the gas flow conditions necessary to start the movement of drops on fibers.
4.3.1 Dimensional analysis for drag coefficient

To apply dimensional analysis we consider the drag force of a moving droplet on a fiber be a function of a number of variables

\[ F_f = F_f(\rho_f, \mu, U, d_f, \gamma, \theta) \]  

(4.10)

The dependence on the contact angle \( \theta \) will be introduced later in capillary number. A balance of units of mass \((m)\), length \((l)\) and time \((t)\) for Eq (4.10) is written as:

\[
\frac{ml}{l^2} = \left( \frac{m}{l^3} \right)^a \left( \frac{m}{l} \right)^b \left( \frac{l}{t} \right)^c l^d l^e \left( \frac{m}{l^2} \right)^f
\]  

(4.11)

From Eq (4.11), we should obtain three separately equation due to the balance of exponent for each type of unit:

\[ m : 1 = a + b + f \]  

(4.12)

\[ l : 1 = -3a - b + c + d + e \]  

(4.13)

\[ t : -2 = -b - c - 2f \]  

(4.14)

These equations can be used to reduce the number of unknowns. With some algebraic manipulations and rearrangement, Eq (4.11) is rewritten as:

\[
F_f = k \left( \frac{\mu_i}{\rho_i d_f U} \right)^{b_f} \left( \frac{\gamma}{\mu_i U} \right)^l \left( \frac{d_A}{d_f} \right)^d d_i^2 U^2
\]  

(4.15)

where, the ground quantities define the Reynolds \((Re_f)\) and capillary \((Ca)\) numbers

\[
Re_f = \frac{\rho_i d_f U}{\mu_i}
\]  

(4.16)

\[
Ca = \frac{\mu_i U}{\gamma(1 + \cos \theta)}
\]  

(4.17)
Eq (4.15) may be rewritten as,

\[ F_f = \left( \frac{2k}{\pi} \right) \text{Re}_f^{-b+f} \text{Ca}^{-f} \left( \frac{d_A}{d_f} \right)^{d-1} \left( \frac{1}{2} \rho(\pi d_f d_A) U \right)^2 \]  

(4.18)

The Eq (4.19) has the familiar form analogous to Eq (4.6),

\[ F_f = C_f \frac{1}{2} \rho A U^2 \]  

(4.19)

Where \( A = \pi d_f d_A \) is the area of contact between the fiber and drop.

Hence, the drag coefficient \( C_f \) has the functional form:

\[ C_f = \left( \frac{2k}{\pi} \right) \text{Re}_f^{-b+f} \text{Ca}^{-f} \left( \frac{d_A}{d_f} \right)^{d-1} \]  

(4.20)

Or, redefining the unknown parameters, we get:

\[ C_f = H \text{Re}_f^{I} \text{Ca}^{J} \left( \frac{d_A}{d_f} \right)^{K} \]  

(4.21)

The unknown parameters \( H, I, J, K \), in Eq (4.21) must be determined from the experimental data.

4.3.2 Dimensional analysis for minimum Reynolds number to initiate the drop motion

Dimensional analysis is the analysis of the relationships between different physical quantities by identifying their dimensions. In my study, dimensionless analysis is applied to estimate the lowest Reynolds number to initiate the movement of the drop. Then, the experiment data is used to fit the undetermined parameters in the correlation function.

To apply dimensional analysis, we consider the probability of droplet moving or not on fiber to be a function of a number of variables,

\[ \text{Probability} = f(\rho_{\text{gas}}, \mu_{\text{gas}}, \theta, d_L, d_f, r, V_{\text{gas}}, \rho_{\text{liq}}, \mu_{\text{liq}}) \]  

(4.22)
The dependencies on the variables are assumed to be a power law form. In the following study, we are focusing on all the moving droplets, and the probability should equal to 1.

\[1 = k \rho_{gas}^a \mu_{gas}^b \cos \theta d_L^d d_f^e \gamma^f V_{gas}^g \rho_{liq}^h \mu_{liq}^i \]  

(4.23)

After applying the Eq. (4.24) and (4.25) into the Eq. (4.23), with some algebraic manipulations and rearrangement, Eq (4.23) could be rewritten as Eq (4.26):

\[ \text{Re}_{gas} = \frac{\rho_{gas} d_L V}{\mu_{gas}} \]  

(4.24)

\[ \text{La} = \frac{\rho_{liq} d_f \gamma}{\mu_{liq}^2} \]  

(4.25)

\[ \text{Re}_{gas} = k \text{La} \cos \theta \left( \frac{d_L}{d_f} \right)^{d-g} \left( \frac{\rho_{gas}}{\rho_{liq}} \right)^{a-g} \left( \frac{\mu_{gas}}{\mu_{liq}} \right)^{b-g} d_f^{e-f+d-g} \rho_{liq}^{h-f+a-g} \mu_{liq}^{i+2f-b-g} \]  

(4.26)

Because \( \text{Re}_{gas}, \text{La}, \frac{\rho_{gas}}{\rho_{liq}}, \frac{d_L}{d_f}, \frac{\mu_{gas}}{\mu_{liq}}, \cos \theta \) and are dimensionless, so that \( d_f^{e-f+d-g} \rho_{liq}^{h-f+a-g} \mu_{liq}^{i+2f-b-g} \) should be dimensionless. A balance of units of mess \((m)\), length \((l)\), and time \((t)\) for \( d_f^{e-f+d-g} \rho_{liq}^{h-f+a-g} \mu_{liq}^{i+2f-b-g} \) term is written as:

\[ 1 = \left( \frac{m}{l} \right)^{h-f+a-g} \left( \frac{m}{l^2} \right)^{i+2f-b-g} \]  

(4.27)

Mass, length and time must be conserved; hence a balance on the exponents for each type of units gives:

\[ l: (e - f + d - g) - 3(h - f + a - g) = 0 \]  

(4.28)

\[ m: (h - f + a - g) + (i + 2f - b - g) = 0 \]  

(4.29)
The above equations can be used to reduce the number of unknowns, finally, the equation used to estimate the lowest Reynolds number should be as following,

\[ \text{Re}_{\text{gas}} = kLa^f \cos \theta^f \left( \frac{d_L}{d_f} \right)^{d-g} \left( \frac{\rho_{\text{gas}}}{\rho_{\text{liq}}} \right)^{a-g} \left( \frac{\mu_{\text{gas}}}{\mu_{\text{liq}}} \right)^{b+g} \]  

(4.31)

Or, redefining the unknown parameters, we get:

\[ \text{Re}_{\text{gas}} = ALa^g \cos \theta^g \left( \frac{d_L}{d_f} \right)^C \left( \frac{\rho_{\text{gas}}}{\rho_{\text{liq}}} \right)^D \left( \frac{\mu_{\text{gas}}}{\mu_{\text{liq}}} \right)^E \]  

(4.32)

The unknown parameters \( A, B, C, D, E \) in Eq (4.32) must be determined from the experimental data.

4.4 Wettability test results

Nylon, polypropylene, and glass fibers were tested in the experiments [79]. Tab 4.1 summarizes typical static contact angles for water and the three organic liquids measured using a drop shape analyzer (DSA20E Kruss GmbH, Germany) for drops on flat surfaces and fibers of different diameters of the same or similar materials. Given that compositions of silica glasses can vary with trace doping elements and polymer materials can have variations in molecular weights, the compositions of the materials may not be exactly the same between the flat surfaces and the fibers but are representative of the material type. The drop diameters were in the range of about 50 to 200 \( \mu \text{m} \) for the contact angle measurements of drops on the fibers. For comparison, the contact angles with water are included in Tab 4.1 even though drop movement experiments were not conducted with water for reasons described below.
The Viscor 1487 and ULSD spread on both nylon and polypropylene flat surfaces with contact angles less than 1 deg. However, drops on the fibers had significantly larger contact angles, indicating either that the material properties of the flat surfaces were not exactly the same as the material properties of the fibers, or that the fiber curvature affected the observed contact angle. In the correlations we used the static contact angles observed for drops on the fibers. Two images of drops on the fibers are shown in Fig 4.2. The organic liquids formed symmetric barrel shaped drops on the nylon and polypropylene fibers. The water drops were barrel shaped on the nylon fibers and clamshell shaped on the polypropylene fibers.

The water drops evaporated too rapidly for use in the drop motion experiments. Also, the clamshell shape of the water drops on some of the fiber materials altered the drop-fiber geometry making the correlation more difficult to evaluate, hence water was not used in the drop motion experiments.

Table 4.1 Summary of contact angles of liquids on the fibers and flat surfaces of several materials.

<table>
<thead>
<tr>
<th>FIBER MATERIAL</th>
<th>Nylon</th>
<th>Polypropylene</th>
<th>Glass</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLAT SURFACE or FIBER</td>
<td>Flat*</td>
<td>Flat*</td>
<td>Flat*</td>
</tr>
<tr>
<td>DIAMETER, μm</td>
<td>18</td>
<td>26</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>26</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Flat*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>LIQUIDS</td>
<td></td>
<td>CONTACT ANGLES, deg</td>
<td></td>
</tr>
<tr>
<td>Viscor 1487</td>
<td>&lt;1</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>29</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt;1</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>31</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt;1</td>
<td>&lt;1</td>
</tr>
<tr>
<td>ULSD</td>
<td>&lt;1</td>
<td>30</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>31</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt;1</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt;1</td>
<td>&lt;1</td>
</tr>
<tr>
<td>WD 40</td>
<td>&lt;1</td>
<td>43</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td></td>
<td>44</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt;1</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt;1</td>
<td>&lt;1</td>
</tr>
<tr>
<td>DI Water</td>
<td>63</td>
<td>66</td>
<td>107</td>
</tr>
<tr>
<td></td>
<td>69</td>
<td>103</td>
<td>106</td>
</tr>
<tr>
<td></td>
<td>96</td>
<td></td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>107</td>
<td></td>
<td>&lt;1</td>
</tr>
</tbody>
</table>
“Flat*” means the contact angle was measured on a flat planar surface of the same material or a typical representative material type.

Figure 4.2 (a) Image of a ULSD barrel shaped drop on a polypropylene fiber surface, and (b) Image of a clamshell shaped water drop on a polypropylene fiber. Because water formed clamshell shaped drops on some the fibers, water was not used in the drop motion experiments.

4.5 Experimental observation

Fig. 4.3 shows typical images from a video recording at different moments in time for a ULSD drop on a 26μm polypropylene fiber. The recording was started at t=0 s when the drop started moving. The air velocity was 3.33 m/s as determined by dividing the flow rate by the cross sectional area of the slit. The velocity of the drop was calculated to be 170μm/s by dividing the distance moved by the difference in time between the images. The static contact angles were measured while the drop was stationary and there was no gas flow. The static contact angles were used to calculate the capillary number in Eq.(4.17) to correlate the drag coefficient for the movement of the drop on the fiber.
Figure 4.3. The positions of the drops are shown at different times on a 26μm PP fiber exposed to an air velocity of 3.33 m/s. The barrel shaped drops have a 32 deg contact angle. The droplet velocity was 170μm/s.

4.6 Drag coefficient correlation

The drop movement data were applied in Eq.(4.9) to calculate the drag coefficient, $C_f$. The data were fitted to the functional form in Eq.(4.21). The experimental data reported by Dawar and Chase [64] were included in the regression fitting of the equation. Using regression analysis [78] the best fitted correlation is [79]

$$C_f = 0.27 \text{Re}^{-0.96} \text{Ca}^{-0.69}$$

(4.33)

where the regression analysis calculated the standard errors of the fitted coefficients in Eq.(4.33) to be $\ln(a) = 1.31 \pm 0.57$, $b = -0.96 \pm 0.02$, and $c = -0.69 \pm 0.04$. The P-values were 0.023, $1.3 \times 10^{-100}$, and $8.97 \times 10^{-39}$ respectively. P-values less than 0.05 are commonly interpreted to mean the parameter is statistically significant. The parameters $b$ and $c$ have very small P-values making them statistically significant. The intercept parameter, $a$, has the largest P-value and the largest 95% confidence interval ranging from 0.09 to 0.83 indicating significant uncertainty in this parameter.

In the work by Dawar and Chase [64] the regression fitted equation was
Comparison of the coefficients of the new fitted correlation in Eq.(4.33) to Eq.(4.34) shows the exponents on the $Re_f$ term are nearly identical (-0.96 vs -0.92). However the intercept coefficients are significantly different (0.27 vs $4.3 \times 10^{-4}$) and the exponents on the $Ca$ term are significantly different (-0.69 vs -1.39).

Fig 4.4 shows a plot of $C_f Ca^{0.47}$ versus $Re_f$ for the experimental data points and for the fitted curve in Eq.(4.33) for the different fiber materials and liquids. The plot also includes the experimental data reported by Dawar and Chase [64]. The solid line in Fig 6 is a plot of $C_f Ca^{0.47}$ calculated from Eq (4.33) for different Reynolds numbers. The curve from the fitted equation passes through the center of mass of all of the data points from the prior work [64] and the new data obtained here at higher $Re_f$ values, extending the range to 6 orders of magnitude from $10^{-7}$ to $10^{-1}$. With the larger set of data points and expanded $Re_f$ range the new correlation in Eq.(4.33) is expected to be more reliable.

Fig 4.5 shows a plot of the experimental values of $C_f$ plotted versus the calculated values of $C_f$ using Eq. (4.33). The plot follows the 45 deg line as expected with some scatter around the line due to experimental errors. The drag coefficient $C_f$ spans 6 orders of magnitude from $10^4$ to $10^{10}$. 

$$C_f = 4.3 \times 10^{-4} \ Re^{-0.92} \ Ca^{-1.39}$$ (4.34)
Figure 4.4 Plot of $C_f C^a 0.47$ versus the $Re_f$ of the experimental data obtained from the pressure driven slit flow experiments, indicated by fiber material and liquid in the legend, and the prior published Couette flow experiments, marked “Dawar” [64]. The legend indicates liquid and fiber material for the other data points. The solid curve is calculated by Eq.(4.33)
Figure 4.5 Plot of $C_f$ calculated from Eq.(4.33) using the experimental values of $Re_f$ and $Ca$ versus the experimental value of $C_f$

4.7 Minimum Reynolds number to initiate the drop motion

In this work the air flow rate was gradually increased to determine the lowest Reynolds number that initiated the drop motion. Within the capacity of the apparatus at the maximum flow rate there were some drops that did not move. In most cases the drops that did not move were very small drops and the Reynolds numbers were not large enough to start the movement. Larger drops were easier to move than smaller drops when exposed to the similar gas flow conditions.

Running experimental variations to fit all of the parameters in Eq.(4.32) would require a significant effort. The effort here focused primarily on varying $La$ and $\frac{d_t}{d_f}$. 
Neglecting the other terms, the fitted equation for the correlation by the 132 moving drops, has the form

\[
\text{Re}_{\text{gas min}} = 21.5L \alpha^{0.13} \left( \frac{d_f}{d_L} \right)^{0.74} 
\]

Equation (4.35)

The regression analysis calculated the standard errors of the fitted coefficients in Eq.(4.35) to be ln(\( A \)) = 3.068 ± 0.1757, \( B \) = 0.1307 ± 0.0234, and \( D \) = 0.7371 ± 0.0292. The P-values were 5.1 x10^{-36}, 1.36 x10^{-07}, and 9.18 x10^{-52} respectively, indicating all three parameters are statistically significant.

Figure 4.6 Correlation for the minimum Reynolds number of gas to initiate drop motion. The 45 deg line represents the optimum fit. The legend notation indicates the liquid, fiber material, and fiber diameter (Viscor-PP 30 means liquid Viscor 1487 and PP fibers of 30 µm diameter)
4.8 Conclusion

Experimental data for drops on fibers exposed to pressurized air flow were fitted to two correlations. The first correlation estimates the drag coefficient between a drop and a fiber as the moves along the axis of a fiber. This correlation extends the data from a prior work to a larger range in the Reynolds number covering six orders of magnitude. The second correlation provides an estimate of the minimum Reynolds number value needed to initiate drop movement on the fiber. The second correlation is a new result.
CHAPTER V
WATER DROP MOVEMENT ON WOVEN FIBER MAT SURFACES DUE TO DIESEL

5.1 Introduction

Research on hydrophobic surfaces began many years ago for different applications such as separation of water drops from oil, dehumidification, and rain coats [80]. Depending on the applications, the materials, the mat structure, the fluid flow directions, the drops may pass through a mat or move across the surface of a mat. In this work we are interested in drops that move across the mat surface. Under appropriate conditions, water drops easily roll off hydrophobic surfaces and leave behind little to no residue [81, 82]. The long-range motivation of this study is to create better hydrophobic membranes to separation of water drops from oil.

The presence of water in diesel fuels in the form of droplet dispersions can cause many problems such as plugging of fuel injectors, reduction of fuel flow rate, reduction of fuel lubricity and corrosion of engine parts [83]. Hence, removing water contamination from diesel fuel is important to the maintenance and life of diesel engines. When a water drop size is larger than 100 microns, the water drops can be effectively removed by gravity settling [7, 84]. However, dispersions of drops smaller than 100 microns are more difficult to separate and often require the use of coalescing
depth filters [85] or a membrane technology [86]. The separation efficiency of a filter is strongly related to the dynamic motions of drops on the filter medium.

Madani et al. [87] observed the motion of a drop on flat surface can be characterized as different behavioral regimes a) sliding, b) deforming c) detachment, depending on the drop size, the drop fluid properties, the continuous fluid phase properties, and the flow conditions. Daniel and Chaudhury [88] found that a liquid drop (1-2 micron L) is moving toward the more wettability part of the gradient with speeds of 1-2mm/s on gradient surface. Subramanian et al. [89] found two methods to estimate the speed of drops on surfaces with a wettability gradient. Gather et al. [90] observed water drops moving by gravity on inclined hydrophobic surfaces. Also, the motion of drop on pattern surfaces has been reported. Sheng and Zhang [91] investigated water drop motion on ratchet-like superhydrophobic surfaces. Bhushan and Jung [92] showed the static contact angles on the patterned silicon surfaces and measured adhesive forces and coefficient of friction with the help of atomic force microscope. However, to our knowledge, correlations to predict the motion of drops on woven mat due to the drag force by liquid flow have not been reported.

In this work, the movement of water drops across surfaces of woven mats of polypropylene(PP), polyamide(nylon), and polytetrafluoroethylene(PTFE) fibers are observed due to the drag flow of ultra low sulfur diesel (ULSD) fuel. We used the ULSD flow through a thin slit to initiate the movement of water drops on the mat surfaces and found that the water droplets moved when the ULSD drag force exceeded the motion resistance forces that tended to hold the drops stationary. The ULSD flow in the thin slit was gradually increased until water drop movement was observed. The minimum ULSD velocity, or minimum Reynolds number, at which the
drop began to move, was fitted to a correlation for predicting conditions to initiate drop movement. The initial drop motion depended on the surface properties of the fiber mats, such as wettability, surface roughness and heterogeneities [34]. Another correlation was fitted to relate the drag coefficient to the Reynolds and Capillary numbers of a drop moving on the mat surfaces. This work only considered clamshell shaped drops that sat on top of the surface of the mat and did not consider drops that penetrated into the fiber mat.

5.2 Force balance to calculate drag coefficient $C_{mat}$

![Figure 5.1. Schematic of a water droplet moving on a woven PP mat at a velocity $U_{water}$ due to the drag of the ULSD flowing at velocity $V_{ULSD}$ parallel to the mat](image)

![Figure 5.2. Diagram of the forces acting on the moving drop](image)
Fig 5.1 shows a sketch of a drop moving on woven mat at velocity $U$ due to the drag force of the ULSD flowing at velocity $V$ parallel to the mat. The mat structure is represented by fibers oriented in the plane of the diagram and perpendicular to the plane, as expected of a square woven mat of fibers. The mat structure has peaks and valleys over which the drop moves. However, the model correlation assumed the mat surface to be flat and the effects of the peaks and valleys are accounted for in the correlation by the values of the fitted parameters. Figure 5.2 shows the force balance on a drop as the ULSD flows left to right. A general force balance on the drop in the horizontal direction includes the drag force between ULSD and the water drop ($F_D$), the drag force between the water drop and mat ($F_{mat}$), and the contact line force ($F_{cl}$).

$$\frac{dMU}{dt} = F_D - F_{mat} - F_{cl}$$

(5.1)

The drop rapidly accelerates due to the ULSD flow, but because of the periodic structure of the fiber mat, the drop motion becomes oscillatory about a time averaged value. Since we are interested in the drop motions on a long time scale (relative to the time period of the oscillatory motion) and on over long distances (relative to the pre size of the mat) the model can be treated as a time-average pseudo steady state motion. Hence, Eq (5.1) reduced to

$$F_D = F_{mat} + F_{cl}$$

(5.2)

According to the definition of the drag force, the drag force acting on the drop due to the ULSD flow is equal to [64, 65],

$$F_D = C_D A_{drop} \frac{1}{2} \rho_{ULSD} (V-U)^2$$

(5.3)
where $A_{\text{drop}}$ is the projected area of the drop in the direction of the ULSD flow and $A_{\text{drop}} = \frac{\pi}{4} d_{\text{c}}^2$. The drag coefficient for the ULSD flowing around the droplet is equal to [64],

$$C_D = \frac{24}{Re_{\text{ULSD}}}(1 + 0.14Re_{\text{ULSD}}^{0.7}) \quad (1 \leq Re_{\text{ULSD}} \leq 1000) \quad (5.4)$$

The flow is considered as laminar flow, when $Re_{\text{ULSD}}$ is between 1 to 1000.

$Re_{\text{ULSD}}$ is the Reynolds number of the diesel and it is given by,

$$Re_{\text{ULSD}} = \frac{\rho_{\text{ULSD}} d_{\text{c}} V}{\mu_{\text{ULSD}}} \quad (5.5)$$

The drag force acting between the drop and the mat is given as,

$$F_{\text{mat}} = C_{\text{mat}} A \frac{1}{2} \rho_{\text{water}} U^2 \quad (5.6)$$

where $A$ stands for the contact area between the droplet and mat, and can be calculated by $A = \frac{1}{4} \pi d_{\text{c}}^2$. $d_{\text{c}}$ is the diameter of the contact area between the drop and the mat and can be calculated by the diameter of the drop $d_{\text{A}}$ and static contact angle $\theta_s$.

$$d_{\text{c}} = d_{\text{A}} \sin(\pi - \theta_s) \quad (5.7)$$

The contact line force is given as [76, 77],

$$F_{\text{cl}} = d_{\text{c}} \gamma_{\text{water}} \left( \cos \theta_r - \cos \theta_a \right) \quad (5.8)$$

where $\gamma_{\text{water}}$ is the surface tension of the water, $\theta_r$ and $\theta_a$ are the receding and advancing angles on the two ends of the droplet.

After combining Eqs (5.2), (5.3), (5.6), (5.8), the drag coefficient for the drag force between the drop and the mat was determined from
5.3 Dimensional analysis

This work takes a unique approach by defining and determining a drag coefficient correlation of the mat. The drag coefficient allows the velocity to be calculated based on the forces acting on a drop. This correlation provides a drag coefficient to estimate the average speed of drops moving on the mat. Also, a new correlation is obtained to predict the minimum Reynolds number of the diesel flow necessary to initiate the movement of the drops on the mats. The second correlation gives an estimate of the diesel flow conditions necessary to start the movement of drops on mats.

5.3.1 Dimensional analysis for drag coefficient $C_{mat}$

Dimensional analysis is applied to identify the independent dimensionless numbers on which the drag coefficient depends. Experiments provide data to fit the undetermined parameters in the correlation.

The drag force $F_D$ between a moving ULSD stream and a water drop is commonly related to the kinetic energy $\frac{1}{2} \rho v^2$ per unit volume through the drag coefficient $C_D$ by the relation [64]

$$F_D = \frac{1}{2} C_D A \rho v^2$$

(5.10)
For the motion of a drop on a mat surface, the velocity \( \nu \) is equal to the velocity difference between the diesel and the drop velocity:

\[
\nu = V - U
\]  

(5.11)

To apply dimensional analysis, we consider the drag force of a moving drop on a mat be a function of a number of variables

\[
F_{\text{mat}} = F_{\text{mat}}(\rho_{\text{water}}, \mu_{\text{water}}, U, d_A, d_f, d_{\text{pore}}, \gamma, \cos \alpha)
\]  

(5.12)

The dimensional analysis results in drag force of the drop moving on the woven mat to have the form analogous to Eq.(5.10) as

\[
F_{\text{mat}} = C_{\text{mat}} \frac{1}{2} \rho_{\text{water}} AU^2
\]  

(5.13)

The drag coefficient \( C_{\text{mat}} \) has the functional form:

\[
C_{\text{mat}} = A \text{Re}_{\text{mat}} B C a C \left( \frac{d_A}{d_{\text{pore}}} \right)^D \left( \frac{d_f}{d_{\text{pore}}} \right)^E (\cos \alpha)^F
\]  

(5.14)

Where parameter A, B, C, D, E, F must be determined from the experimental data.

The Reynolds and Capillary numbers are given by

\[
\text{Re}_{\text{mat}} = \frac{\rho_{\text{water}} d_{\text{pore}} U}{\mu_{\text{water}}}
\]  

(5.15)

\[
Ca = \frac{\mu_{\text{water}} U}{\gamma(1 + \cos \theta)}
\]  

(5.16)

5.3.2 Dimensional analysis for \( \text{Re}_{\text{ULSD min on mat}} \)

All of the drops moved on the mats when exposed to a high enough flow rate of ULSD. The resistance to the drop movement at lower flow rates is attributed to the
surface roughness or irregularities in the surface properties. A correlation for predicting the lowest Reynolds number to initiate the drop motion can be useful in some applications.

The probability of a drop moving or not on a fiber is considered to be a function of the relevant variables,

\[
\text{Probability} = f(\rho_{ULSD}, \mu_{ULSD}, \sin \theta, d_L, d_f, d_{pore}, \gamma, V_{ULSD}, \rho_{water}, \mu_{water}, \cos \alpha)
\] (5.17)

In this application only data from drops that moved were considered, hence the probability is unity (1). In applying dimensional analysis the dependencies on the variables are assumed to be a power law form. The probability function is assumed to take the form

\[
1 = k \rho_{ULSD}^a \mu_{ULSD}^b \sin \theta^c d_L^d d_f^e d_{pore}^f \gamma^g V_{ULSD}^h \rho_{water}^i \mu_{water}^j \cos \alpha^k
\] (5.18)

where the coefficients \(a, b, \ldots, l\) are constants. The Reynolds number of ULSD around the drop, and Laplace numbers are

\[
Re_{ULSD_{min}} = \frac{\rho_{ULSD} d_L V_{ULSD}}{\mu_{ULSD}}
\] (5.19)

\[
La = \frac{\rho_{water} d_{pore} \gamma}{\mu_{water}^2}
\] (5.20)

where \(Re_{ULSD_{min}}\) is the ratio of the ULSD inertial forces to the viscous drag forces and \(La\) is the ratio of the interfacial forces to the viscous drag force between the drop and the mat.

Substituting \(Re_{ULSD_{min}}, La\) into Eq.(5.18), rearrangement of the equation into dimensionless form, and elimination of the quantities that cannot be combined into dimensionless terms, yields the working correlation of the form
\[ \text{Re}_{ULSD \text{ min on mat}} = H L a^I (\sin \theta)^J \left( \frac{d_L}{d_{pore}} \right)^K \left( \frac{d_f}{d_{pore}} \right)^L \left( \frac{\rho_{ULSD}}{\rho_{water}} \right)^M \left( \frac{\mu_{ULSD}}{\mu_{water}} \right)^N \cos^O \]  

(5.21)

where \( H, I, \ldots, O \) are constants to be determined from experimental data.

\( \text{Re}_{ULSD \text{ min on mat}} \) is the minimum ULSD phase Reynolds number needed to initiate drop motion.

5.4 Description of experiments

The following will discuss the equipment, experiment materials, experiment setup, and experiment plan.

5.4.1 Mat, liquid materials, equipment

Woven PP mats with squared shape pore opening sizes 105, 210, 500 and 1000\( \mu \text{m} \) (referred below as samples PP105, PP210, PP500 and PP1000, respectively) were purchased from Spectrum Labs. The nylon and PTFE mats had parallelogram shaped openings. The diagonal lengths were 1080\( \mu \text{m} \) and 490\( \mu \text{m} \) for the nylon mat and 890\( \mu \text{m} \) and 520\( \mu \text{m} \) for the PTFE mat. The mats were cleaned prior to the experiment by sonication in distilled water, and dried in an oven to remove moisture. The thin glass fiber media (used as laminarization media) were supplied by Hollingsworth & Vose Company.
Table 5.1 Physical properties of reference liquids at 20°C

<table>
<thead>
<tr>
<th>Liquid(Source)</th>
<th>Density (Kg/m³)</th>
<th>Viscosity (cP)</th>
<th>Surface tension (mN/m)</th>
<th>Boiling point (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ULSD (Purchased locally)</td>
<td>870</td>
<td>1.4</td>
<td>29.83</td>
<td>180-360</td>
</tr>
<tr>
<td>DI water (Obtained by ion exchange, 0.1 Siemens)</td>
<td>1000</td>
<td>1.02</td>
<td>73.5</td>
<td>1000</td>
</tr>
</tbody>
</table>

The equipment included a custom made Plexiglas holder, a flow meter (Cole Parmer, 034-10ST), a pump (Fasco Industries, Inc), high speed camera (Mikrontron), drop shape analyzer (DSA20E Kruss GmbH, Germany), microscope (OLYMPUS, SZ-PT), and spin coater (Laurell technologies corporation, WS-400B-6NPP/LITE/AS).

5.4.2 Experimental setup

Fig 5.3 shows a diagram of the pressure-driven flow device. ULSD was pumped through a flow meter and into the thin-slit channel holding the test mat and the water droplets. The body and cover of the holder were machined from Plexiglas. The flow channel was 4 mm deep, 3.5 cm wide, and 9 cm long. The test mats were positioned on the bottom of the channel and secured in place using an acrylic tape. To distribute the flow over the cross section of the channel, a glass fiber mat used as a laminarization mesh was positioned near the inlet of the channel.

A water droplet was injected with a calibrated syringe into the holder from an injection window located on the top cover. With the help of the calibrated scale of the syringe, the drop volume and size was controlled. After injecting the water drop, the injection window was sealed by an acrylic tape. The channel was illuminated from
below, and the movements of the drops on the woven mats were recorded by using the high-speed camera from above. Simultaneously, the movements of the drops also were recorded by the Drop Shape Analyzer from the side to obtain side view images.

![Flow diagram of the experiment](image)

**Figure 5.3 Flow diagram of the experiment**

5.4.3 Experimental plan

Due to the woven structure of the woven mats, the orientation of the weave relative to the flow direction affected the drop dynamic. The mat orientation was characterized by the acute angle formed between one of the fibers in the mat structure and the ULSD flow direction as the mat was rotated as a rigid body, preserving the structural angles between crossed fibers within the mats. Six different angles (0, 24, 30, 45, 60 and 66 deg) were evaluated. Fig 5.4 shows example images taken from the high speed camera positioned above the mats and water drops of the three different mat materials and the different angles. The ULSD flow direction was horizontal from left to the right for the images in Fig 5.4.
Figure 5.4. Example images of top views of water droplets on different mat surfaces with different angles relative to the flow direction: A) 0 degree PP mat, B) 45 degree PP mat, C) 24 degree nylon mat, D) 66 degree nylon mat, E) 30 degree PTFE mat F) 60 degree PTFE mat.
5.5 Results and discussion

The following section will discuss about the wettability results, side view observation, top view observation from experiment, and drag coefficient correlation, and minimum Reynolds number of ULSD to initial the movement of the drop.

5.5.1 Wettability Results

Contact angle of water drops on woven mat submerged in ULSD are summarized in Tab 5.2. For all contact angle measurements, the drop volume was 5µL. Four images of water drops on PP/nylon/PTFE mats are shown in Fig 5.5, and all the drops represented a clamshell shape. The contact angle results are close with the different mat materials because the fibers were all hydrophobic and the ULSD fills the pores. The pore and fiber diameters were directly obtained from the optical microscope images, such as Fig 5.6.
Figure 5.5 DSA20E photographs of clamshell shaped water drops with the same volume on different mat surfaces in diesel, (A) Flat PP (B) PP210 (C) Nylon (D) PTFE. The images are taking at different magnifications and hence are not comparable in scale.

Figure 5.6. The microscope image for PP105
Table 5.2 Summary of properties of the fiber mats and the contact angles of water on the mats submerged in ULSD

<table>
<thead>
<tr>
<th>Material</th>
<th>PP Flat</th>
<th>PP105</th>
<th>PP210</th>
<th>PP500</th>
<th>PP1000</th>
<th>Nylon</th>
<th>PTFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pore size, μm</td>
<td>Flat</td>
<td>105</td>
<td>210</td>
<td>500</td>
<td>1000</td>
<td>785</td>
<td>705</td>
</tr>
<tr>
<td>Fiber size,</td>
<td>0</td>
<td>100</td>
<td>150</td>
<td>300</td>
<td>500</td>
<td>240</td>
<td>260</td>
</tr>
<tr>
<td>Water contact</td>
<td>145.7</td>
<td>148.7</td>
<td>144.5</td>
<td>143.6</td>
<td>136.6</td>
<td>139.1</td>
<td>142.1</td>
</tr>
<tr>
<td>angle, deg</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.5.2 Side view observations from experiments

Fig 5.7 shows typical side view images at different moments in time for a water drop moving on PP500 mat in ULSD. From those images, the advancing and receding angles are summarized in Tab 5.3. From Tab 5.3, the differences between the advancing and receding angles were within a few degrees; hence the contact line force is neglected. The similar phenomenon happened on nylon and PTFE mats.

Then, Eq (5.9) could be reduced to,

\[
C_{\text{mat}} = \frac{C_D \left( \frac{\pi}{4} d_L^2 \right)^2 \rho_{\text{ULSD}} (V-U)^2}{\frac{1}{4} \pi l^2 \frac{1}{2} \rho_{\text{water}} U^2} = \frac{C_D d_L^2 \rho_{\text{ULSD}} (V-U)^2}{d_c^2 \rho_{\text{water}} U^2} \tag{5.22}
\]
Figure 5.7. The position of the drops at different times on PP500 mat exposed to flowing ULSD.

Table 5.3 Summary of advancing and receding angles of water in diesel on PP mat at different times

<table>
<thead>
<tr>
<th>Time, s</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advancing, deg</td>
<td>133.8</td>
<td>132.6</td>
<td>132.9</td>
</tr>
<tr>
<td>Receding, deg</td>
<td>127.8</td>
<td>125.3</td>
<td>124.9</td>
</tr>
</tbody>
</table>

5.5.3 Top view observations from experiments

Fig 5.8 shows typical images from a video of top view recording at different moments in time for a water drop moving on PP210 mat in ULSD. The recording was started at $t=0s$ and then gradually increase the ULSD flow rate to start the movement of water drop. The drop started moving at $t=7.25s$, when the ULSD velocity was equal to $6000\mu m/s$. The ULSD velocity was determined by dividing the volumetric flow rate by the cross sectional area of the channel. The velocity of drop was calculated to be $2100\mu m/s$ by dividing the distance moved by the difference in time between the images. The static contact angles (from Tab 5.2) were used to calculate the capillary number in Eq. (5.22) to correlate the drag coefficient for the movement of the drop on the mat surfaces.
Figure 5.8 Example images the positions of a drop, at different times on PP210 mat exposed to flowing ULSD at a velocity of 6000µm/s. The drop had a clamshell shape.

The drop's velocity was 2100 µm/s

5.5.4 Drag coefficient correlation $C_{mat}$

The drop movement data were applied in Eq.(5.22) to calculate the drag coefficient, $C_{mat}$. The data were fitted to the functional form in Eq.(5.14). Using the regression analysis [78], the best fitted correlation is

$$C_{mat} = 0.000877 \ Re_{mat}^{-1.20} Ca^{-1.18} (\frac{d_A}{d_{pore}})^{-1.61} (\frac{d_f}{d_{pore}})^{-0.41} (\cos \alpha)^{0.29}$$ (5.23)

where the regression analysis calculated the standard errors of the fitted coefficients in Eq.(20) to be $\ln(A)= -7.03 \pm 2.02$, $B = -1.20 \pm 0.20$, $C = -1.18 \pm 0.20$, $D = -1.61 \pm 0.14$, $E = -0.41 \pm 0.42$, and $F = -0.29 \pm 0.30$. The P-values for each parameter were 0.0005, 3.2*10^{-9}, 1.5*10^{-8}, 7.4*10^{-7}, 0.32, and 0.34 respectively. P-values less than 0.05 are commonly interpreted to mean the parameter is statistically significant. The parameters $B, C, D, E, F$ have very small P-values making them statistically significant. The large P-value and ± error for $E$ and $F$ shows the parameter $E$ and $F$ is subject to large experimental error.
Figure 5.9 Plot of $C_{mat}$ calculated from Eq.(5.23) using the experimental values of $Re_{mat}$, $Ca$, $d_A/d_{pore}$, $dy/d_{pore}$ versus the experimental value of $C_{mat}$. The line represents the optimum fit. The legend notation indicates the angles (0, 45, 24, 66, 30, 60 deg) and the fiber materials (PP, nylon, PTFE).

Fig 5.9 shows a plot of the experimental values of $C_{mat}$ plotted versus the calculated values of $C_{mat}$ using Eq. (5.23). The plot follows the 45 deg line as expected with some scatter around the line due to experimental errors. The drag coefficient $C_{mat}$ spans 5 orders of magnitude from $10^{0}$ to $10^{5}$.

5.5.5 Minimum $Re_{ULSD}$ for drop movement

In this work, the ULSD flow rate was gradually increased to determine the lowest Reynolds number that initiated the drop motion. Within the capacity of the apparatus at the maximum flow rate, all the water drops are moving on the PP.
surfaces. Larger drops were easier to move than the smaller drops when exposed to the similar ULSD flow conditions.

The unknown parameters $H, I, J, K, L, M, N, O$ in Eq.(5.21) were determined by regression of the data from 132 moving drops,

$$
\text{Re}_{\text{ULSD min on mat}} = 0.0011 L a^{0.79} (\sin \theta)^{0.92} \left( \frac{d_L}{d_{\text{pore}}} \right)^{-0.48} \left( \frac{d_f}{d_{\text{pore}}} \right)^{-0.16} (\cos \alpha)^{0.14}
$$

(5.24)

The regression analysis calculated the standard errors of the fitted coefficients in Eq.(5.24) to be $\ln(H)=-6.78\pm1.81, I=0.79\pm0.14, J=0.92\pm0.95, K=-0.48\pm0.13, L=-0.14\pm0.18, O=0.14\pm0.13$. The $P$-values were 0.0003, $7.5\times10^{-8}$, 0.33, 0.0003, 0.38, and 0 respectively, indicating the $La, d_f/d_{\text{pore}}, \cos \alpha$ are parameters are statistically significant.

Figure 5.10 Correlation for the minimum Reynolds number of ULSD to initiate drop motion. The 45 degree line represents the optimum fit.
5.6 Conclusion

Experimental data for drops on different mats exposed to pressurized ULSD flow were fitted to two correlations. The first correlation estimates the drag coefficient between a drop and fiber mat as it moves on the woven mat surfaces. This correlation could be used to estimate the average drop velocity. The second correlation provides an estimate of the minimum Reynolds number value needed to initiate drop movement on the mat surfaces.
CHAPTER VI
IMMISCIBLE LIQUID DROP MOTION THROUGH HYDROPHOBIC FIBER MAT

6.1 Introduction

Membrane microfiltration is a type of physical filtration process where a contaminated fluid is passed through a special pore-sized membrane to separated particles from continuous phase. Compared with a gravity separator, or centrifuges filtration, membrane microfiltration is good at reducing space requirement, higher permeate quality, and lower operating costs [93]. This technology is widely used in many industry applications, such as oil/water separation [94-96], wastewater treatment [97]. Some publications discuss using models for calculating the critical pressure required to force an oil drop into a membrane pore [95], and simulate the critical pressure in terms of geometric parameters of the pore cross section with elliptical and rectangular pores [98]. However, there are no papers that discuss the experiment method to measure the minimum flow rate to make the drop flow through the pore of the membrane. To understand how the movements of the drops go through a mat, it is important to develop theories and correlations that allow exploration of the relationships existing between different parameters and forces acting on the drops.

In this work, the movements of water drops were evaluated as they through the surfaces of woven mats of PP, nylon, and PTFE mats due to the drag flow of ULSD fuel. ULSD flow was directed through a thin slit to initiate the movement of water
drops. The water drops passed through the pores when the drag force of ULSD exceeded the resistance force from the fiber mats. The ULSD flow direction was perpendicular to the fiber mat. The ULSD flow in the thin slit was gradually increased until the water drops went through the fiber mat. The minimum ULSD velocity, or minimum Reynolds number, at which the drops went through the mat, was fitted to a correlation for predicting conditions to make the drops through the pore of mat. This work only considered clamshell shaped drops that penetrated into the fiber mat.

6.2 Description of experiments

The following will discuss the equipment, experiment materials, experiment setup, and experiment plan.

6.2.1 Mat, liquid materials, equipment

Woven PP mats with squared shape pore opening sizes 105, 210, 500 and 1000μm (referred below as samples PP105, PP210, PP500 and PP1000, respectively) were purchased from Spectrum Labs. The nylon and PTFE mats had parallelogram shaped openings. The diagonal lengths were 1080μm and 490μm for nylon mat and 890μm and 520μm for the PTFE mat. The mats were cleaned prior to the experiment by sonication in distilled water, and then dried in an oven to remove moisture. The thin glass fiber media (used as laminarization media) were supplied by Hollingsworth & Vose Company.

The liquids used in this study were ULSD and DI water. Some of the properties of these two liquids are summarized in Tab 6.1.
Table 6.1 Physical properties of reference liquids at 20°C

<table>
<thead>
<tr>
<th>Liquid(Source)</th>
<th>Density (Kg/m^3)</th>
<th>Viscosity (cP)</th>
<th>Surface tension (mN/m)</th>
<th>Boiling point (°C)</th>
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<td>ULSD (Purchased locally)</td>
<td>870</td>
<td>1.4</td>
<td>29.83</td>
<td>180-360</td>
</tr>
<tr>
<td>DI water (Obtained by ion exchange, 0.1 Siemens)</td>
<td>1000</td>
<td>1.02</td>
<td>73.5</td>
<td>1000</td>
</tr>
</tbody>
</table>

The equipment included a custom made Plexiglas holder, a flow meter (Cole Parmer, 034-10ST), a pump (Fasco Industries, Inc), high speed camera (Mikrontron), drop shape analyzer (DSA20E Kruss GmbH, Germany), microscopy (OLYMPUS, SZ-PT), and spin coater (Laurell technologies corporation, WS-400B-6NPP/LITE/AS).

6.2.2 Experiment setup

Fig 6.1 shows a diagram of the pressure-driven flow device. ULSD was pumped through a flow meter and into the thin-slit channel holding the test mat and the water droplets. The body and cover of the holder were machined from Plexiglas. The flow channel was 4 mm deep, 3.5 cm wide, and 9 cm long. The test mats were positioned in the channel and secured in place with the help of four small pieces of metal. The mats covered the entire cross section of the channel. To distribute the flow over the cross section of the channel, a glass fiber mat positioned near the inlet of the channel was used as a laminarization mesh. The flow direction was perpendicular to the mat. Water droplets were injected with a calibrated syringe into the holder from an injection window located on the top cover. With the help of the calibrated scale, the drop volume and size was controlled. After injecting the water drops, the injection window was sealed by an acrylic tape. The channel was illuminated from below, and
the movements of the drops on the woven mats were recorded by using the high-speed camera from above. Fig 6.2 lists all the components in the liquid-liquid drop motion test.

Figure 6.1 Flow diagram of the experiment
Figure 6.2 Components of liquid-liquid drop motion test (a) high speed camera (b) diesel tank (c) flow meter (d) pump (e) Plexiglas holder

6.3 Dimensional analysis for $Re_{ULSD \ min \ through \ mat}$

All drops moved through when exposed to the ULSD flow. A correlation for predicting the minimum Reynolds number to initiate the drop went through the pore could be useful in some applications.

The probability of a drop moving through the fiber mat or not is considered to be a function of the relevant variables,
Probability = \( f(\rho_{\text{ULSD}}, \mu_{\text{ULSD}}, \sin \theta, d_L, d_f, d_{\text{pore}}, \gamma, V_{\text{ULSD}}, \rho_{\text{water}}, \mu_{\text{water}}, \sin \beta) \) \hspace{1cm} (6.1)

In this application only data from drops which went through were considered, hence the probability is unity (1). In applying dimensional analysis the dependencies on the variables are assumed to be a power law form. The probability function is assumed to take the form

\[ l = k \rho_{\text{ULSD}}^a \mu_{\text{ULSD}}^b \sin \theta^c d_L^d d_f^e d_{\text{pore}}^f \gamma^g V_{\text{ULSD}}^h \rho_{\text{water}}^i \mu_{\text{water}}^j \sin \beta^l \] \hspace{1cm} (6.2)

where the coefficients \( a, b, \ldots, l \) are constants. The Reynolds number of ULSD, and Laplace numbers are

\[ \text{Re}_{\text{ULSD min}} = \frac{\rho_{\text{ULSD}} d_L V_{\text{ULSD}}}{\mu_{\text{ULSD}}} \] \hspace{1cm} (6.3)

\[ \text{La} = \frac{\rho_{\text{water}} d_{\text{pore}} \gamma}{\mu_{\text{water}}^2} \] \hspace{1cm} (6.4)

where \( \text{Re}_{\text{ULSD min}} \) is the ratio of the diesel inertial forces to the viscous drag forces and \( \text{La} \) is the ratio of the interfacial forces to the viscous drag force between the drop and the mat.

Substituting \( \text{Re}_{\text{ULSD min}}, \text{La} \) into Eq.(6.2), rearrangement of the equation into dimensionless form, and elimination of the quantities that cannot be combined into dimensionless terms, yields the working correlation of the form

\[ \text{Re}_{\text{ULSD min through mat}} = P L a^Q (\sin \theta)^R \left( \frac{d_L}{d_{\text{pore}}} \right)^S \left( \frac{d_f}{d_{\text{pore}}} \right)^T \left( \frac{\rho_{\text{ULSD}}}{\rho_{\text{water}}} \right)^U \left( \frac{\mu_{\text{ULSD}}}{\mu_{\text{water}}} \right)^V (\sin \beta)^W \] \hspace{1cm} (6.5)

where \( P, Q, \ldots, W \) are constants to be determined from experimental data.

\( \text{Re}_{\text{ULSD min through mat}} \) is the minimum ULSD phase Reynolds number needed to make drop through the mat.
6.4 Results and Discussion

The following section will discuss about the wettability results, side view observation, top view observation from experiment, and drag coefficient correlation, and minimum Reynolds number of ULSD to initial the movement of the drop.

6.4.1 Wettability results

Contact angle of water drops on woven mats submerged in ULSD are summarized in Tab 6.2. For all contact angle measurements, the drop volume was 5µL. Three images of water drops on PP/nylon/PTFE mats are shown in Fig 6.3, and all the drops represented a clamshell shape. The contact angle results are close with the different mat materials because the fibers were all hydrophobic and the ULSD fills the pores. The pore and fiber diameters were directly obtained from the optical microscope images, such as Fig 6.4.

Figure 6.3 DSA20E photographs of clamshell shaped water drops with the same volume on different mat surfaces in diesel, (A) PP210 (B) Nylon (C) PTFE. The images are at different magnifications and hence are not comparable in scale.
Figure 6.4 The microscope images for (A) PP105 (B) Nylon (C) PTFE

Table 6.2 Summary of properties of the fiber mats and the contact angles of water in diesel on the mats submerged in ULSD

<table>
<thead>
<tr>
<th>Material</th>
<th>PP105</th>
<th>PP210</th>
<th>PP500</th>
<th>PP1000</th>
<th>Nylon</th>
<th>PTFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pore size, µm</td>
<td>105</td>
<td>210</td>
<td>500</td>
<td>1000</td>
<td>785</td>
<td>705</td>
</tr>
<tr>
<td>Fiber size, µm</td>
<td>100</td>
<td>150</td>
<td>300</td>
<td>500</td>
<td>240</td>
<td>260</td>
</tr>
<tr>
<td>Water contact angle, deg</td>
<td>148.7</td>
<td>144.5</td>
<td>143.6</td>
<td>136.6</td>
<td>139.1</td>
<td>142.1</td>
</tr>
</tbody>
</table>

6.4.2 Observations from experiments

Fig 6.5 shows typical images from a video of top view recording at different moments in times for a water drop through PP500 with the flowing ULSD. The recording was started at t=0s and then gradually increase the ULSD flow rate to start the movement of water drop. The water drop started through the pore of mat at t=2.9s, when the ULSD velocity equal to 0.0265m/s. The ULSD velocity was determined by dividing the volumetric flow rate by the cross sectional area of the channel. The big water drop was split to two smaller drops, and one smaller drop went through the pore of the mat. The other part still left on the surface of the mat.
Figure 6.5 Example images the position of a water drop through PP 500 exposed to flowing ULSD at a velocity of 0.0265m/s. The red circle indicates the target water drop

6.4.3 Minimum $Re_{ULSD}$ for drop through the pore

In this work, the ULSD flow rate was gradually increased to determine the minimum Reynolds number that forced the water drop through the pore. Within the capacity of the apparatus at the maximum flow rate, all the water drops were able to go through the pore of different fiber mats. Larger drops needed smaller flow rate to go through than the smaller drops when exposed to the similar ULSD flow conditions.

The unknown parameters $P$, $Q$, $R$, $S$, $T$, $U$, $V$, $W$ in Eq.(6.5) were determined by regression of the data from 100 moving drops,

$$Re_{ULSD \text{ min through mat}} = 8.86 \times 10^{-7} La^{1.59} (\sin \theta)^{-3.64} \left( \frac{d_L}{d_{\text{pore}}} \right)^{-1.32} \left( \frac{d_f}{d_{\text{pore}}} \right)^{2.33} (\cos \beta)^{-2.77}$$

(6.6)

The regression analysis calculated the standard errors of the fitted coefficients in Eq.(6) to be $\ln(P) = -13.93 \pm 0.93$, $Q = 1.59 \pm 0.09$, $R = -3.64 \pm 0.54$, $T = -1.32 \pm 0.09$,
The parameters are statistically significant.

Figure 6.6 Correlation for the minimum Reynolds number of ULSD to initiate drop through the pore. The 45 degree line represents the optimum fit.

6.5 Conclusion

This chapter focuses on developing an empirical correlation for the minimum Reynolds number of ULSD required to force the water drop through the flat mat surfaces which were perpendicular to the flow direction. This correlation could be used to determine the minimum ULSD velocity.
CHAPTER VII

CONCLUSION AND FUTURE WORK

7.1 Conclusions

The pressure drive flow experimental apparatus is designed for laminar air/liquid flow conditions. The pressure drive flow device could provide higher air/liquid flow rate compared to the Couette flow experimental apparatus. The pressure drive flow device could give uniform flow profile under laminar flow with the help of laminazation mesh. The different fiber/mat orientations to achieve axial motion and transverse motion of the liquid drops are done by constructing different Teflon and Plexiglas holders for holding the fibers and mats. The design parameters were identified and determined for the experimental apparatus as discussed in Chapter III and V, separately.

The analytical models to determining the drag coefficient of fiber for axial motion of liquid drops on fibers has been developed in Chapter IV. First the analytical model for the axial motion of liquid drops on fiber was developed to compare it with the experimental results. Then the data from Couette device was added into the model and they were match well with each other. Then the second attempt was made to develop analytical model for the liquid drops on mats in Chapter V. Because the insufficient information was available to develop the theoretical models, dimensional analysis was applied to develop the above models.
The correlations are developed for the axial drag coefficient of fiber and mat that are used to estimate the average liquid drop velocity on the fiber and mat. The motion of the drops axially on fiber and mat is used to determine the rate of separation of the liquid drops from air/liquid with the help of a filter media. The empirical correlations have been developed using dimensional analysis by curve fitting the experimental data. Those correlations consider the inertial effect of the drop and also the effect of surface properties of the fibers in contact with the liquid drops. The correlations when compared with the experimental results give a good fiber structure with its surface properties and the known physical properties of the liquid to be separated from the air/liquid we can determine the rate of removal using the drag coefficient correlations.

Two more correlations are developed that determine the minimum Reynolds number of gas/liquid to initiate the movement of the drop on the fiber/mat surfaces. In those two correlations, dimensional analysis is also applied to the setup the model. Then, the empirical correlations have been developed using dimensional analysis by curve fitting the experimental data. In these two correlations, the effects of surface properties of the fiber/mat, and liquid properties were also considered.

One more correlation is developed to determine the minimum Reynolds number of liquid can push the water drop go through the polypropylene mats with different pore size. In this correlation, the dimensional analysis was applied and experiment data was fitted in to obtain the empirical correlation. Finally, this correlation was compared with the existing theory.
The impact of the drag coefficient microscale models is determined to have an effect on the macroscopic performance of the coalescing filter media like saturation, overall drag acting on fibers/mats which determine the capture efficiency and the quality factor of the media.

7.2 Future Work recommendations

The goal of this research is to develop the first generalized theory of coalescing filters, a significant advancement beyond the 1-D model. Meanwhile, the theory will enable the design and development of the next-generation of filters with superior performance, and improve understanding of experimentally observed phenomena within coalescing media. The outcome of the proposed activity will be the first generalized theory of coalescing filters that provides prediction of filter performance (efficiency and pressure drop), taking into account microstructure, material, and operating parameters, preferably in the form

\[ E_{ss} = f(d_{fbr}, \alpha_{fbr}, \beta_{fbr}, \gamma_{fbr}, \theta_{fbr}, d_{drp}, \rho_{drp}, \mu_{drp}, \sigma_{drp}, U_{\text{flow}}) \]  
\[ \Delta P_{ss} = f(d_{fbr}, \alpha_{fbr}, \beta_{fbr}, \gamma_{fbr}, \theta_{fbr}, d_{drp}, \rho_{drp}, \mu_{drp}, \sigma_{drp}, U_{\text{flow}}) \]

where, \( E_{ss} \) and \( \Delta P_{ss} \) are the steady-state efficiency and pressure drop of the filter, respectively. The first five variables with a subscript \( fbr \) are fiber diameter, solid volume fraction in-plane fiber orientation, through-plane fiber orientation, and contact angle. The other four variables with a subscript \( drp \) are droplet diameter, density, viscosity, and surface tension. \( U_{\text{flow}} \) stands for the face velocity. The above relationships will allow the design of coalescing filters with optimum microstructures for a controlled performance.
After investigate the drop motion on single fiber with gas, the future work is to investigate the effect of fiber/mat to the drop migration with complex geometry. Also, future investigation is required to develop computational models for drag coefficient

7.2.1 Crossing fiber test

For the crossing fibers experiments in Fig 7.1, two fibers will be attached to the channel with the help of double side tape and the intersection of two fibers is at the center point in the channel. The positions of the fibers can be adjusted to create different crossing angles and different angles relative to the flow direction in the channel. In crossing fiber test, the droplets will set on the fiber by syringe method, which was explained in Chapter V. The detail of the experiment tasks is listed in Tab 7.1.

![Diagram of crossing fiber test](image.png)

Figure 7.1 Top and side views of crossing fiber test
Table 7.1. Detail of the experimental tasks for crossing fiber test

<table>
<thead>
<tr>
<th>Exp-1</th>
<th>Set</th>
<th>Variation</th>
<th>No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crossing</td>
<td>D</td>
<td>2 fiber materials, 3 fiber angles, 3 liquids, 3 fiber diameters, 2 drop diameters</td>
<td>108</td>
</tr>
<tr>
<td>Fiber</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7.2.2 Thick mat test

In thick fibrous media test, the thick mats may be formed by layering thin mats or by vacuum molding layers of fibers within from slurry. Both of above methods are able to produce thick mats which have a planar layer of fibers. The thick mats will have significant thickness in the flow direction and they will fill the thin slit channel. Compared to thin mat tests, the thick mats (Set G) will extend over the entire cross-section of the channel when the flow in perpendicular direction (Fig 7.2). Under this direction, the drop velocity will be affected by fiber medium permeability, porosity, and capture efficiency. Also, the thick mats will be positioned in the thin slit with the layers of fibers parallel to the flow direction (Set H), as indicated in Fig 7.3. Finally, experiments will be conducted with composite thick mats (Set I, J) made of glass and polypropylene fibers of different fiber diameters and pore openings. The interface between the composite layers will be oriented 45 and 90 degree relative to the gas flow in Fig 7.4. Because of the increased geometric complexity, the experiments of the thin and thick mats will only consider one liquid for the droplets and two fiber materials (glass and polypropylene). The detail of the experiment tasks is listed in Tab 7.2.
Figure 7.2. Top and side views of thick mat perpendicular to the flow (Set G, J)

Figure 7.3. Top and side views of thick mat parallel to the flow (Set H, I)
Figure 7.4. The different angles of combination of glass and polypropylene fibers

Table 7.2. Detail of the experimental tasks for thick mat test

<table>
<thead>
<tr>
<th>Exp-2</th>
<th>Set</th>
<th>Variations</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thick Mat</td>
<td>Single Material</td>
<td>G 2 fibers, 2 fiber diameters, 1 layer direction(perpendicular), 2 depth, 2 drop diameters, gravity in plane of view</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H 2 fibers, 2 fiber diameters, 1 layer direction(parallel), 2 depth, 2 drop diameters, gravity towards bottom of view</td>
<td>32</td>
</tr>
<tr>
<td>Composite Material</td>
<td>I 2 fiber diameters(pp), 1 layer directions(parallel), 2 drop diameters, 2 composites(glass/pp, pp/glass), 2 interface angles, gravity towards bottom of view</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>J 2 fiber diameters(pp), 1 layer directions(perpendicular), 2 drop diameters, 2 composites(glass/pp, pp/glass), 2 interface angles, gravity towards bottom of view</td>
<td>16</td>
</tr>
</tbody>
</table>
REFERENCES


75. R. Shankar Subramanian et al. Motion of a drop on a solid surface due to a wettability gradient, Langmuir, 21, 11844-11849, 2005.


APPENDICES
MODELLING UNSATURATED LIQUID MOVEMENT IN FIBROUS MEIDUM

The aim of this project is to model two-phase flow of gas and water through porous media and develop a correlation for predicting movement of drop through porous medium. The velocity of the drop, saturation and concentration of water in gas phase will be showed as a function of filter bed length with considering the effect of wettability.

Figure A.1 The schematic of water droplet carried by the gas went thought the porous media

A.1 Multiphase flow through porous media

Coalescence is the process by which two or more droplets, bubbles or particles merge during contact to form a bigger droplet, bubble or particle. The coalescence filtration is process in which droplets carried by a flowing gas are captured by the fibers of a filter medium. Before coalescence, the gas phase carries the water droplet into filter media, and then the droplet is captured by the single fiber with different
capture mechanisms. For this process, the multiphase flow goes through the porous media could be described by multiphase continuum theory. Volume averaging theory is used to develop multiphase continuum equations [99]. The volume averaged for any property $\phi$ for the $\alpha$ phase can be written as follows [100]:

$$\frac{\partial}{\partial t} \langle \rho_{\alpha} \phi_{\alpha} \rangle + \nabla \cdot \langle \rho_{\alpha} \phi_{\alpha} \nu_{\alpha} \rangle + \nabla \cdot \langle i_{\alpha} \rangle - \langle \rho_{\alpha} g_{\alpha} \rangle - \langle \rho_{\alpha} g_{\alpha} \rangle + \left( E_{\alpha} + I_{\alpha} + S_{\alpha} + G_{\alpha} \right) = 0 \quad (1)$$

The excess terms are constrained as

$$\sum_{\alpha} \left( E_{\alpha} + I_{\alpha} + S_{\alpha} + G_{\alpha} \right) = 0 \quad (2)$$

A.2 Important Balance

In this work, attempt is made to develop a model for predict the velocity profile through the depth of medium. The rate of droplet capture is constant varies with time. The conservation of Species Balances for gas and water species are the start-up equation to obtain rate of droplet capture on the fibers. Mass Balance, and Momentum Balance are most important balances since pressure drop and velocity profiles can be obtained using those two balances. The constitutive relations and drag force correlations are used to solve the mass and momentum balances.

A.3 General Equations

The general conservation equations for each phase are as follows,

Gas phase water species balance

$$\frac{\partial}{\partial t} \left( \epsilon^G \rho_{w}^G \right) + \nabla \cdot \left( \epsilon^G \rho_{w}^G \nu_{w}^G \right) + \nabla \cdot \left( \epsilon^G \nu_{w}^G \right) - \epsilon^G \rho_{w}^G \left( f_{\alpha}^G + \epsilon_{\alpha}^G \right) + \left( E_{w}^G + I_{w}^G + S_{w}^G + G_{w}^G \right) = 0 \quad (3)$$

Gas phase mass balance
\[
\frac{\partial}{\partial t} \left( \rho^G \rho^G \right) + \nabla \cdot \left( \rho^G \rho^G \nu^G \right) + \left( E_m^G + G_m^G \right) = 0
\]  
(4)

Water phase mass balance

\[
\frac{\partial}{\partial t} \left( \rho^W \rho^W \right) + \nabla \cdot \left( \rho^W \rho^W \nu^W \right) + \left( E_m^W + G_m^W \right) = 0
\]  
(5)

Gas phase momentum balance

\[
\frac{\partial}{\partial t} \left( \rho^G \rho^G \nu^G \right) + \nabla \cdot \left( \rho^G \rho^G \nu^G \nu^G \right) + \rho^G \nabla P + \nabla \cdot \rho^G \epsilon_{\|}^G + E_{\perp}^G - \rho^G \rho^G \| g + I_m^G + S_m^G = 0
\]  
(6)

Water phase momentum balance

\[
\frac{\partial}{\partial t} \left( \rho^W \rho^W \nu^W \right) + \nabla \cdot \left( \rho^W \rho^W \nu^W \nu^W \right) + \rho^W \nabla P + \nabla \cdot \rho^W \epsilon_{\|}^W + E_{\perp}^W - \rho^W \rho^W \| g + I_m^W + S_m^W = 0
\]  
(7)

A.4 Assumptions

- Steady state.
- Velocity is only in Z direction (no velocity in X and Y direction).
- The process is isothermal.
- The media is incompressible.
- The filter coefficient is a constant.
- No wall effect.
- No flow in the direction of gravity.
- No phase change.
- No chemical reaction.
- No property transfer due to slip.
- The permeability of media is uniform and isotropic.
- Viscous forces are neglected.
- Viscosity of air and liquid are constant.
- Density of air and liquid are constant.
A.5 Governing Equations

Based on the assumptions above, all species balance, mass balance, and momentum balance could be simplified as following.

Gas phase water species balance

$$\nabla \cdot \left( \varepsilon^G W_w \nu^G \right) + \left( I_w^G \right) = 0$$  \hspace{1cm} (8)

Gas phase mass balance

$$\nabla \cdot \left( \varepsilon^G \rho^G \nu^G \right) + \left( E_m^G \right) = 0$$  \hspace{1cm} (9)

Gas phase momentum balance

$$\varepsilon^G \nabla P + F_z^G = 0$$  \hspace{1cm} (10)

Water phase mass balance

$$\nabla \cdot \left( \varepsilon^W \rho^W \nu^W \right) + \left( E_m^W \right) = 0$$  \hspace{1cm} (11)

Water phase momentum balance

$$\varepsilon^W \nabla P + F_z^W = 0$$  \hspace{1cm} (12)

A.6 Solutions

In gas phase mass balance, $$\nabla \cdot \left( \varepsilon^G \rho^G \nu^G \right) + \left( E_m^G \right) = 0$$, the $$E_m^G$$ term could be consider as 0, so that $$\nabla \cdot \left( \varepsilon^G \rho^G \nu^G \right)$$ is equal to 0. As a result,

$$\nu^G \varepsilon^G = Q / A$$  \hspace{1cm} (13)

Then substitute the Eq(13) into $$\nabla \cdot \left( \varepsilon^G W_w \nu^G \right) + \left( I_w^G \right) = 0$$ and the

$$I_w^G = -E_m^w = \alpha w_w^G \varepsilon^G \nu^G$$ according to constitute relation from filtration theory. After
applying boundary condition \( z=0 \), \( w^G_w =0.03 \), the relationship between \( w^G_w \) and filter position can be achieved.

\[
w^G_w = e^{-\alpha - 3.5}
\]  (14)

Figure A.2  \( w^G_w \) vs filter position  \( (\alpha = \frac{4}\pi \varepsilon \mu d_f \text{ assuming } \varepsilon = 0.9 \text{ and } d_f = 10 \mu m) \)

The water droplets move through the medium on the fibers due to a drag force by bulk phase. The two-phase flow though porous media analogous to Darcy’s law for two phases can be written as follows,

Gas phase,

\[
\varepsilon^G v^G = -\frac{k^G}{\mu^G} \nabla P
\]  (15)

Water phase,

\[
\varepsilon^W v^W = -\frac{k^W}{\mu^W} \nabla P
\]  (16)

Where, \( \mu, \varepsilon, v \) and \( k \) are the viscosity, porosity, velocity and permeability of phase flow through porous media. The permeability of each phase is referred as effective
permeability of the porous medium with particular liquids. The relative permeability of porous medium with each phase is defined as,

\[ K_r^W = \frac{k^W}{k} \]  
(17)

\[ K_r^G = \frac{k^G}{k} \]  
(18)

Then the relationships for saturation and relative permeability of water and gas phase were applied,

\[ K_r^W = S^3 \]  
(19)

\[ K_r^G = (1-S)^3 \]  
(20)

After combining eq. (16), (17), (18), (19), (20), and (21),

\[ v^W = \frac{S^2}{(1-S)^2} \frac{\mu^G}{\mu^W} v^G \]  
(21)

Applying Eq (21), \( \varepsilon^W = S \varepsilon \) and \( I_w^G = -E_m^W = \alpha \varepsilon_w^G \varepsilon^G v^G \) into \( \nabla \cdot \left( \varepsilon^W \rho^W v^W \right) + \left( E_m^W \right) = 0 \), and the boundary condition is \( z=0, \ ds/dz = 0 \),

\[ \frac{-3s^2 + 3s + 1}{3(1-s)^3} = -\frac{\mu^W}{\mu^G} e^{-s} \]  
(22)
Finally, the relationship between $v^w$ and filter position could be obtained from

$$v^w = \frac{S^2}{(1-S)^3} \frac{\mu^G Q}{\mu^w A}$$

(22)

This project found that the relationships between $w^G_w$, $S$, $v^w$ and filter position.

All $w^G_w$, $S$, $v^w$ terms decrease when the filter position increases.
APPENDIX B

AIR VELOCITY PROFILE IN THE THIN SLIT BY MODELING

Figure B.1 Air velocity profile in the thin slit with laminarization mat
Figure B.2 Air velocity x-direction along the center line in the thin slit

Code for above figures:

TITLE ‘Air pattern with laminarization mesh’

SELECT

    smoothinit

    errlim=1e-3

    ngrid=30

    stages=1

    Threads =6

COORDINATES
cartesian2

VARIABLES

vx vy p

DEFINITIONS

Lx=0.055 Ly=0.015

L=Lx  R=Ly

DL=Lx/R  DR=Ly/R

visc=1.81E-5 dens=1.18 eps=0.95 kperm=3e-10

Pout=101325   {1psi}

Vf=2   {m/s, predefined inlet velocity of channel which is also the flow velocity after passing the filter }

v=vector( vx, vy) vm=magnitude(v)

unit_x=vector(1,0) unit_y=vector(0,1)

nx=normal( unit_x) ny=normal( unit_y)

div_v=dx(vx)+dy(vy)

flow1 = line_integral(vx,'cross-section1')

flow2 = line_integral(vx,'cross-section2')

flow3 = line_integral(vx,'cross-section3')
flow4 = line_integral(vx,'cross-section4')
flow5 = line_integral(vx,'cross-section5')
flow6 = line_integral(vx,'cross-section6')
flow7 = line_integral(vx,'cross-section7')

\[ Va = \frac{\text{integral}(vx)}{\text{integral}(1)} \]  \{ Average velocity \}

\[ Va_1 = \frac{\text{flow1}}{2 \cdot DR} \]  \{ Va at 1 \}

\[ Va_3 = \frac{\text{flow3}}{2 \cdot DR} \]  \{ Va at 3 \}

\[ Va_4 = \frac{\text{flow4}}{2 \cdot DR} \]  \{ Va at 4 \}

\[ Va_5 = \frac{\text{flow5}}{2 \cdot DR} \]  \{ Va at 5 \}

\[ Va_6 = \frac{\text{flow6}}{2 \cdot DR} \]  \{ Va at 6 \}

\[ Va_7 = \frac{\text{flow7}}{2 \cdot DR} \]  \{ Va at 7 \}

\[ V_d = \frac{vx}{va} \]

\[ \text{Re\_channel} = \frac{\text{dens} \cdot va \cdot R}{\text{visc}} \]

\[ \text{Re\_filter} = \frac{\text{dens} \cdot va_6 \cdot R \cdot 1e-5}{\text{visc}} \]

\[ \text{fluid\_part} = 1 \]

\[ \text{penalty} = \text{STAGED}(1e-1) \]
natp=visc*(nx*div( grad( vx)) + ny*div( grad( vy))) { Natural boundary condition for p: }

EQUATIONS

vx: fluid_part*(dx(P)-visc*div( grad( vx))) +(1-fluid_part)*(vx+kperm*dx(P)/(visc*eps))=0

vy: fluid_part*( dy(P)-visc*div( grad( vy))) +(1-fluid_part)*(vy+kperm*dy(P)/(visc*eps))=0

p: fluid_part*(div( grad( p))- penalty*div_v)+(1-fluid_part)*(div(kperm*grad(p)))=0

BOUNDARIES

region 'domain' fluid_part=1 start 'outer' (0,-DR)

value( vx)=0 value( vy)=0 natural( p)=natp line to (DL,-DR) { Slip }

natural( vy)=0 natural(vx)=0 value(p)=Pout line to (DL,DR) { Outlet }

value( vx)=0 value( vy)=0 natural( p)=natp line to (0,DR) { Slip }

natural( vy)=0 value(vx)=Vf natural(p)=0 line to close ! natural(vx)=0

REGION 1 'filter' fluid_part=0
start 'filter' (0.5*DL,DR) natural(p)= - visc*eps/kperm*( nx*vx + ny*vy) value( vy)=0
natural( vx)=0
line to (0.54*DL,DR) line to (0.54*DL,-DR)
natural(p)= - visc*eps/kperm*( nx*vx + ny*vy) natural( vx)=0 value( vy)=0
line to (0.5*DL,-DR) line to close

FEATURE 'cross-section1' Start (0.01*DL,-DR) line to (0.01*DL,DR)
FEATURE 'cross-section2' Start (0.03*DL,-0.2*DR) line to (0.03*DL,0.2*DR)
FEATURE 'cross-section3' Start (0.05*DL,-DR) line to (0.05*DL,DR)
FEATURE 'cross-section4' Start (0.1*DL,-DR) line to (0.1*DL,DR)
FEATURE 'cross-section5' Start (0.5*DL,-DR) line to (0.5*DL,DR)
FEATURE 'cross-section6' Start (0.52*DL,-DR) line to (0.52*DL,DR)
FEATURE 'cross-section7' Start (0.6*DL,-DR) line to (0.6*DL,DR)

MONITORS
grid(x,y)

PLOTS
contour( vx) contour( vy)

vector( v) norm

contour( vm) painted

vector( v) norm zoom(0,0, 0.4*DL,1.1*DR)

vector( v) norm zoom(0.4*DL,-1.2*DR, 0.35*DL,0.6*DR)

vector( v) norm zoom(0.45*DL,-1.05*DR, 0.2*DL,0.3*DR)

vector( v) norm zoom(0.4*DL,-0.8*DR, 0.35*DL,1.6*DR)

contour( p) painted report(Re_channel) report(Re_filter) report(va)

elevation( p) on 'outer' report(Va1) report(Va3) report(Va4)

contour( div_v) contour( curl( v)) painted report(Va5) report(Va6) report(va7)

elevation(vd) from (0,0) to( DL,0)

countour( vd) painted as 'v=vx/vaverage'

elevation(vx) from (0,0) to( DL,0)

elevation(vx) from (0,0) to (0,DR)

elevation( vx, vy) on 'outer'

END
APPENDIX C

AIR VELOCITY PROFILE IN THE THIN SLIT BY MODELING

Figure C.1 Diesel velocity profile in the thin slit with laminarization mat
Figure C.2 Diesel velocity x-direction along the center line in the thin slit

Code for above figures:

TITLE ‘Diesel pattern with laminarization mesh’

SELECT

   smoothinit

   errlim = 1e-3

   ngrid = 30

   stages = 1

   Threads = 6

COORDINATES
cartesian2

VARIABLES
vx vy p

DEFINITIONS
Lx=0.055Ly=0.015
L=Lx R=Ly
DL=Lx/R DR=Ly/R
visc=1.4E-3 dens=870 eps=0.95 kperm=3e-10
Pout=101325 {1psi}
Vf=0.003 {m/s, predefined inlet velocity of channel which is also the flow velocity after passing the filter }
v=vector( vx, vy) vm=magnitude(v)
unit_x=vector(1,0) unit_y=vector(0,1)
nx=normal( unit_x) ny=normal( unit_y)
div_v=dx(vx)+dy(vy)
flow1 = line_integral(vx,'cross-section1')
flow2 = line_integral(vx,'cross-section2')
flow3 = line_integral(vx,'cross-section3')
flow4 = line_integral(vx,'cross-section4')
flow5 = line_integral(vx,'cross-section5')
flow6 = line_integral(vx,'cross-section6')
flow7 = line_integral(vx,'cross-section7')

\[ V_a = \frac{\text{integral}(vx)}{\text{integral}(1)} \]  \{Average velocity\}

\[ V_{a1} = \frac{\text{flow1}}{2 \times DR} \]  \{V_a at 1\}

\[ V_{a3} = \frac{\text{flow3}}{2 \times DR} \]  \{V_a at 3\}

\[ V_{a4} = \frac{\text{flow4}}{2 \times DR} \]  \{V_a at 4\}

\[ V_{a5} = \frac{\text{flow5}}{2 \times DR} \]  \{V_a at 5\}

\[ V_{a6} = \frac{\text{flow6}}{2 \times DR} \]  \{V_a at 6\}

\[ V_{a7} = \frac{\text{flow7}}{2 \times DR} \]  \{V_a at 7\}

\[ V_d = \frac{vx}{va} \]

\[ \text{Re}_\text{channel} = \frac{\text{dens} \times va \times R}{\text{visc}} \]

\[ \text{Re}_\text{filter} = \frac{\text{dens} \times va_6 \times R \times 1e-5}{\text{visc}} \]

\[ \text{fluid part} = 1 \]

\[ \text{penalty} = \text{STAGED}(1e0) \]
natp=visc*(nx*div( grad( vx))+ ny*div( grad( vy))) \{ Natural boundary condition for p: \}

EQUATIONS

vx: fluid_part*(dx(P)-visc*div( grad( vx))) +(1-fluid_part)*(visc/eps*div( grad(vx))-dx(P)-(visc)*vx/kperm)=0

vy: fluid_part*( dy(P)-visc*div( grad( vy))) +(1-fluid_part)*(visc/eps*div( grad(vy))-dy(P)-(visc)*vy/kperm)=0

p: fluid_part*(div( grad( p))- penalty*div_v)+(1-fluid_part)*(div(kperm*grad(p))) =0

BOUNDARIES

region 'domain' fluid_part=1 start 'outer' (0,-DR)

value( vx)=0 value( vy)=0 natural( p)=natp line to (DL,-DR) \{ Slip\}

natural( vy)=0 natural(vx)=0 value(p)=Pout line to (DL,DR) \{ Outlet\}

value( vx)=0 value( vy)=0 natural( p)=natp line to (0,DR) \{ Slip\}

natural( vy)=0 value(vx)=Vf natural(p)=0 line to close \! natural(vx)=0

REGION 1 'filter' fluid_part=0
start 'filter' (0.5*DL,DR) natural(p)= - visc*eps/kperm*( nx*vx+ ny*vy) value( vy)=0
natural( vx)=0

line to (0.54*DL,DR) line to (0.54*DL,-DR)

natural(p)= - visc*eps/kperm*( nx*vx+ ny*vy) natural( vx)=0 value( vy)=0

line to (0.5*DL,-DR) line to close

FEATURE 'cross-section1' Start (0.01*DL,-DR) line to (0.01*DL,DR)

FEATURE 'cross-section2' Start (0.03*DL,-0.2*DR) line to (0.03*DL,0.2*DR)

FEATURE 'cross-section3' Start (0.05*DL,-DR) line to (0.05*DL,DR)

FEATURE 'cross-section4' Start (0.1*DL,-DR) line to (0.1*DL,DR)

FEATURE 'cross-section5' Start (0.5*DL,-DR) line to (0.5*DL,DR)

FEATURE 'cross-section6' Start (0.52*DL,-DR) line to (0.52*DL,DR)

FEATURE 'cross-section7' Start (0.6*DL,-DR) line to (0.6*DL,DR)

MONITORS

grid(x,y)

PLOTS
contour( vx) contour( vy)

vector( v) norm

contour( vm) painted

vector( v) norm zoom(0,0, 0.4*DL, 1.1*DR)

vector( v) norm zoom(0.4*DL,-1.2*DR, 0.35*DL, 0.6*DR)

vector( v) norm zoom(0.45*DL,-1.05*DR, 0.2*DL, 0.3*DR)

vector( v) norm zoom(0.4*DL,-0.8*DR, 0.35*DL, 1.6*DR)

contour( p) painted report(Re_channel) report(Re_filter) report(va)

elevation( p) on 'outer' report(Va1) report(Va3) report(Va4)

contour(div_v) contour(curl(v)) painted report(Va5) report(Va6) report(va7)

elevation(vd) from (0,0) to (DL,0)

contour(vd) painted as 'v=vx/v_average'

elevation(vx) from (0,0) to (DL,0)

elevation(vx, vy) on 'outer'

END

The above figures and codes are donated by Yalong Li.
I would like to acknowledge Yalong Li for the modeling work in liquid droplets migration on fibers and mats.
APPENDIX D

NOMENCLATURE

$A$ area of contact between the drop and the fiber ($\mu m^2$)

$A_{\text{drop}}$ projected area of the drop in the direction of air flow ($\mu m^2$)

$C_D$ drag coefficient due to the fluid drag around the drop

$C_f$ drag coefficient due to the drop on the fiber

$C_{\text{mat}}$ drag coefficient due to the drop on the mat

$Ca$ Capillary number

$d_A$ length of contact of drop along the fiber ($\mu m$)

$d_c$ diameter of contact area ($\mu m$)

$d_f$ diameter of fiber ($\mu m$)

$d_{\text{pore}}$ diameter of pore on the mat ($\mu m$)

$d_L$ diameter of drop measured perpendicular to fiber axis ($\mu m$)

$F_{cl}$ contact line force (N)

$F_D$ drag force acting on the drop due to air flow (N)

$F_f$ drag force of moving drop on fiber (N)

$F_{\text{mat}}$ drag force of moving drop on mat (N)

$F_{\text{gravity}}$ forces acting due to gravity (N)

$La$ Laplace number

$Re_f$ Reynolds number of the drop on the fiber

$Re_{\text{gas}}$ Reynolds number of the gas

$Re_{\text{mat}}$ Reynolds number of the drop on the mat
\( Re_{ULSD} \)  Reynolds number of the ULSD

\( t \)  time scale (s)

\( U \)  drop velocity (m/s)

\( v \)  velocity difference between the gas and drop velocity (m/s)

\( V \)  gas velocity (m/s)

**Greek symbols**

\( \alpha \)  angle of one fiber relative to the flow direction

\( \beta \)  angle of fibers in one mat

\( \gamma \)  surface tension of the liquid drop (N/m)

\( \mu_{water} \) viscosity of the water drop (Kg/(m*s))

\( \mu_{gas} \) viscosity of gas (Kg/(m*s))

\( \mu_{ULSD} \) viscosity of ULSD (Kg/(m*s))

\( \theta_a \)  advancing contact angle between the drop and the mat

\( \theta \)  static contact angle between the drop and the fiber/mat

\( \theta_r \)  receding contact angle between the drop and the mat

\( \rho_{ULSD} \) density of ULSD (Kg/m^3)

\( \rho_{gas} \) density of gas flow (Kg/m^3)

\( \rho_{liq} \) density of liquid (Kg/m^3)

\( \rho_{water} \) density of water (Kg/m^3)