PREDICTION OF FATIGUE CRACK NEAR-THRESHOLD CENSORED
REGRESSIONS WITH RUN-OUT DATA

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PREDICTION OF FATIGUE CRACK NEAR-THRESHOLD CENSORED
REGRESSIONS WITH RUN-OUT DATA

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ABSTRACT

The discussion is divided into two types of prediction. In the first type statistical models for fatigue life prediction for welded joints are discussed and fitted to experimental data for fillet-welded steel joints where cracks emanate from the weld toe. The models are based on an S-N approach where the number of cycles N to failure is assumed to be directly correlated to the applied nominal stress range ΔS. The models assume the existence of a fatigue limit given as a stress range below which no failure will take place. Emphasis is laid on the modeling of the fatigue life close to this limit where the service stresses for welded details often occur. Experimental data in this stress regime are sparse and do not fit the knee point of the conventional bi-linear S-N curve found in design rules and specifications. Therefore, an alternative model where both the fatigue life and the fatigue limit are simultaneously treated as random variables is investigated.

The model parameters for this random fatigue-limit model (RFLM) are determined by the maximum likelihood method, and confidence intervals are obtained by the profile likelihood method. The advantage of the model is that it takes into consideration the variation in fatigue limit found from specimen to specimen and that run-out results are easily included. The median S-N curve obtained from the model coincides with the conventional bi-linear curves in the high stress regime (stress ranges higher than 110 MPa), but predicts longer lives as the stress range decreases below 100 MPa.
The model gives a nonlinear S-N curve for a log-log scale in the fatigue-limit area; the fatigue life is gradually increasing and is approaching a horizontal line asymptotically instead of the abrupt knee point of the bilinear curve. The nonlinear curve is more in accordance with experimental data. At stress ranges below 100 MPa, the predicted fatigue lives are between 2 to 10 times longer than predictions made by the bilinear Category E curve in AISC Steel Construction Manual. The conclusion is that the rule-based S-N curves may be un-conservative in the stress regime where service stresses frequently occur. A more correct statistical model based on a random fatigue-limit model results in S-N curves that give decreased dimensions for a given fatigue design factor under constant amplitude loading.

The second type of prediction is required if inspection is to be carried out in a damage tolerance approach. In this case fatigue process in fillet welded joints is discussed and modeled. As a first approximation, a pure fracture mechanics model was employed to describe the entire fatigue process. The model is calibrated to fit the crack growth measurements obtained from extensive testing on fillet weld joints where cracks emanate from the weld toes. Emphasis is laid on the choice of growth parameters in conjunction with a fictitious initial crack size distribution in order to obtain both reliable crack growth histories and predictions of the entire fatigue life. The model has its shortcomings in describing the damage evolution at low stress ranges due to the presence of a significant crack development period in this stress regime. As an alternative to the fracture mechanics model, a two-phase model (TPM) for the fatigue process was developed and calibrated.
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CHAPTER I
INTRODUCTION

1.1 Problem Statement

1.1.1 Preliminaries

Structural Life prediction is usually based on three technologies: Fatigue initiation or early growth, Fracture Mechanics, and Non-Destructive Evaluation. Fatigue is to estimate how long it takes for a crack to develop, Fracture Mechanics is to calculate how long it takes for the crack to grow to critical size, understanding how things break can avoid having to explain why they broke later on and NDE is to determine the probability of finding a critical life limiting flaw.

1.1.2 An Overview of Fatigue and Fracture

For over a century, before much of the physics of fatigue was understood, fatigue has been described using an S-N diagram, relating demand stress or strain, $S$ and cycles-to-failure, $N$. All engineering S-N curves use a logarithmic axis for cycles as the dependent variable, cycles, is plotted on the x-axis. S-N curves are sigmoidal over the entire life range from monotonic tensile strength and stress to a fatigue runout, if such a fatigue limit exists. A linear relationship is at best appropriate in the middle life region, but this is often the
range of interest. The micromechanics of fatigue proceeds as an involved sequence of events that begins with micro slip in shear between adjacent planes of atoms in the crystal structure of a single grain, and usually oriented 45° to the normal load, and culminates with the propagation of a macro crack many times larger than the grain size, and oriented normal to the load. The behavior of the material in aggregate can be rather different from the behavior of an individual crystal. Still it is useful to describe the aggregate behavior in gross terms by relating some measure of durability like cycles to failure, with one or more of the durability controlling parameters, such as applied stress or strain range. Stress is the applied force normalized to a unit area; strain is elongation normalized to unreformed length.

The material response is a function of more than just stress or strain range. Stress ratio, \( R = \text{min stress}/\text{max stress} \), hold times at load, and temperature if isothermal or thermal cycle also influence fatigue capability. The material’s chemistry and forming history also play a role, as does the local three dimensional geometry of the fatigued part. Finally, the relative influence of all of these factors changes throughout the fatigue process.

Unfortunately, S-N models can’t deal with cracks. After a fatigue crack is formed, the S-N curve is no longer useful depends on definition of failure. This is somewhat ironic since some fraction of the life of a fatigue specimen is comprised of a propagating a macro crack, and the mechanics of crack propagation in low cycle fatigue LCF is well understood. Fracture mechanics FM considers the stress field and its synergism with a material discontinuity. For two decades fracture mechanics has been used with great success to describe the behavior of a potential crack, and thus mitigate its threat. Because of the very
large variability in time to cracking, many component lifetimes are determined using the anticipated behavior of a propagating crack which is assumed to be present from cycle one.

Some of the emphasis on FM may change with the recognition that high cycle fatigue $N > 10^7$ behavior is not well understood. The ability to describe the behavior of a crack may not be useful, if the propagation time is measured in minutes, even if the cycle count is measured in millions. Further, the variance of fatigue lifetime increases or appears to increase for longer and longer lives as a consequence of a somewhat random fatigue-limit. In any event, the precision for predicting fatigue lives under HCF is quite poor and inadequate for component design. But we still may be able to predict the probability of having the HCF excitation. Thus a potential shift in emphasis from estimating a runout stress under low cycle fatigue to understanding the conditions under which HCF loading will produce failure.

1.2 Objectives and Scope

1.2.1 Analyzing Fatigue Data with Runouts

Here is a brief description of the statistical treatment of runouts in fatigue testing. A runout is a test which is interrupted before it fails. Examples include discontinued testing after $10^7$ cycles, or specimen failure outside the gage after $N$ cycles. Runouts do not contain the same information about the placement of an S-N curve as failures do, and ignoring this fact will lead to serious errors in estimated material response. To illustrate this, consider this thought experiment: A test is performed very near the runout strength of a material and it does not fail before $10^7$ cycles. A similar test, run at 20 ksi below runout, is also stopped after $10^7$ cycles. An ordinary least squares regression of all the data, including the first
example, but not the second, would have little effect on the final position of the curve. But
including the very low point as if it were a failure will greatly lower the resulting curve.
Similar anti conservative situations can also occur, so we can't just shrug it off as being
conservative.

As already stated, the fatigue behavior of welded joints is random by nature. Very
few load-bearing details exhibit such large scatter in fatigue life as welded joints. This is
true even in controlled laboratory conditions. As a consequence, it becomes an important
issue to take scatter into consideration, both for the fatigue process and for the final life.
Furthermore, the in-service stresses may often be characterized as stochastic processes.

There has been a trend the last two decades to treat the strength problem of a structure
by applying statistics and probability calculations. As a consequence, the probability of
failure is used as a criterion, instead of the more traditional safety factors, when checking
various design criteria. The methods and tools for performing this type of analysis have
become available. The probability of no failure during a given time period is considered to
be the reliability of the structure. The methods used for determining the probability are
often called reliability methods. Furthermore, if the associated probability of failure is
weighed against the consequence of failure we arrive at the risk concept. Achieving high
reliability and low risk levels will maintain operational capability and secure life and assets.
The more sources of uncertainty there are in a structural problem, the more appropriate will
be the application of a reliability approach. For processes where damage is accumulating
with time, the probability of failure will increase with service time, depending on decisions
made for the design concept, configurations, dimensions, material properties, and in-
service inspection strategy. For fatigue of welded joints the following sources of uncertainty are dominant:

— the service stress history;
— the global and local geometry of the joint;
— the material parameters; and
— the performance of in-service inspections.

A reliability approach pinpoints the sources of uncertainty and treats them in a rational way based on probability models and statistics. An alternative is to hide the uncertainty by fixed parameters often based on worst case assumptions. To optimize structures with respect to fatigue strength one should avoid using worst case assumptions. This may result in costly over-dimensioned structures. An optimization of design and member dimensions based on reliability calculations will give lightweight structures. Last but not least, inspection planning should be developed based on risk criteria to avoid unnecessarily costly inspection during service life. This leads to the concept of risk-based inspection.

1.2.2 Ordinary Least Squares

When all fatigue specimens fail, Ordinary Least-Squares OLS is the accepted method for estimating the parameters of the S-N model. This method has been the basis of engineering data analysis for the past 200 years since Gauss popularized it.

Some mathematical models are proposed which relate stress or strain, and temperature with cycles to failure, N. The goal is to choose parameters for the model which best fit the data. According to Gauss that best means that the summed squared error of the residuals is a minimum. A residual is the difference between an observation and the model
prediction. Another way of saying the same thing is that the variance of the observations about the predicted behavior is as small as possible.

Given this criterion for goodness of fit, the OLS method first writes the equation for the sum of the squares of the differences between the observed and expected lives. This relationship is then differentiated with respect to each of the model parameters, and these derivatives set equal to zero. The simultaneous solution provides the desired least squares estimates of the parameter values. Statisticians don't talk about measuring a parameter value; they estimate it. That's because the estimate will change slightly given different or new data, something that wouldn't happen with something that could be measured without error.

1.2.3 Censored Runout data

The forgoing is a summary of current engineering practice, used with success for quite a few years, but what about a Censored or runout observation? What about a specimen which didn't fail after N cycles? How can we calculate a residual for that? The basic quick answer is you can't. That is because it could have failed at any point after it was suspended. The error residual can't be defined, thus it can't be included in the summed squared error to be minimized, and so we can't get there from here. We can pretend that it did fail, and calculate a residual, and so on, but we would be wrong for the reasons we explored with our thought experiment.

1.2.4 R. A. Fisher's Idea

This problem was solved in the comparatively recent past, and is based on a proposal developed by R.A. Fisher in the early decades of the last century, and brought into engineering practice only about 15 years ago. Fisher looked at the problem of parameter
estimation using a different criterion for goodness. Fisher theorized that the best parameter value would be the one which maximized the likelihood that the experiment would have turned out the way it actually did. Also, Fisher theorized that you could choose any parameter values you wanted, but some would be more likely to be the true values, given the experimental results.

1.2.5 Likelihood

What's likelihood? Picture the S-N data with a best fit line through it. Now imagine a normal distribution of lives scattered about the line, for a constant stress range for example. The likelihood of the line's being in the right place is the ordinate of the probability distribution which is centered at the model value. It’s the height of the probability distribution at that value, if the line is nowhere near the data, the normal distribution won't be centered appropriately, and the ordinates evaluated at the N values will be low. We want to put the curve through the data so its likelihood is maximized.

We can utilize maximum likelihood in the same manner in which we did least-squares. It begins with the likelihood equation, which is just the product of all those individual likelihoods. For practical purposes it's helpful to take the log of the likelihood equation because it turns all those products into sums of logarithms. Next, differentiate this equation with respect the model parameters. These derivatives are set equal to zero and solved simultaneously. This usually requires an iterative solution. Now, because the logarithm is a monotone function, it reaches a maximum when the variable of the logarithm reaches a maximum, so the solution to the maximum of the log of the likelihood occurs at the same parameter values as the maximum of the likelihood function itself.
Consequently, how do parameters estimated with Fisher's maximum likelihood criterion compare with those estimated using Gauss's least squares criterion? They are exactly the same. It is important to note that the errors are normally distributed, which is usually the case. So, that means if there were no runout observations, this new method produces the identical results as the method we've been using for a number of years, a comforting situation.

It could be represented by the ordinate at N cycles where it was discontinued or at the ordinate at a few cycles more, or at even more cycles after that, since it could have failed at any of those cycle counts. Since we don't know exactly where the failure would have occurred, only that it has to be after the N observed cycles, the relative likelihood of the curve being in the right place is that fraction of the area under the normal curve to the right of the suspension, since the data were right censored. This definition of likelihood also works for left censored observations and for interval censored tests. An example of interval censoring could be a test which failed over the weekend. The cycle counter was working Friday afternoon, but the specimen was found failed Monday morning, and the cycle count is in doubt. Here the likelihood would be the area under a normal curve between the last known cycle count, and the cycle count estimated by the test frequency and the duration of the interval. In fact, we must use censored regressions with runout S-N data.

Finally, the aim of this study is to understand the statistical treatments of runouts in long life fatigue testing. If a test is discontinued testing after $10^7$ cycles, or specimen failure outside the gage after N cycles how is the data treated? A runout is a test which is interrupted before it fails. From this review, a revised set of fatigue design curves are proposed that better estimates the fatigue resistance.
This dissertation will begin with a review of the basic nature of fatigue crack growth and threshold behavior. The following chapters will cover the test data and material data and finally, an analysis of these results and the major conclusions which will be made.

1.3 Preliminaries to Welded joints fatigue behavior

Welding is the most common joining method for metallic structures. Its industrial application is extremely important and many large structures designed and erected in the last decades would not have been possible without modern welding technology. Examples include steel bridges, ship structures, and large offshore structures for oil exploitation.

The strength analysis of welded structures does not deviate significantly from that for other types of structures. Various failure mechanisms have to be avoided through appropriate design, choice of material, and structural dimensions. Design criteria such as yielding, buckling, creep, corrosion, and fatigue must be carefully checked for specific loading conditions and environments. However, welded joints may be vulnerable to fatigue damage when subjected to repetitive loading. Fatigue cracks may initiate and grow in the vicinity of the welds during service life even if the dynamic stresses are modest and well below the yield limit. The problem becomes pronounced if the structure is optimized by the choice of high strength steel. The very reason for this choice is to allow for higher stresses and reduced dimensions, taking benefits of the high strength material with respect to the yield criterion. However, the fatigue strength of a welded joint is not primarily governed by the strength of the base material of the joining members; the governing Parameters are mainly the global and local geometry of the joint and applied stress range. Hence, the yield stress is increased, but the fatigue strength does not improve significantly. This makes the fatigue criterion a major issue. Fatigue strength may govern the final
dimensions of the structural members. To overlook this fact may result in fatigue failure and serious consequences.

1.4 Report Organization

A total of ten chapters are included in this dissertation. This dissertation is confined to steel structures made by fusion welding. The dissertation is divided into three parts which cover the following subjects:

I. Introduction and Literature review:
   - Historical background with basic understanding of the fatigue behavior of welded joints based on theoretical considerations and experimental data and results (Chapters 1-5),

II. Methodology:
   - the S-N approach with reference to current design rules and specification (Chapter 6),
   - the fracture mechanics approach with numerical computations (Chapter 7).

III. new prediction description of the fatigue behavior for Category E S-N curve:
   - understanding the fatigue process and estimating the fatigue life for Category E s-N curve (Chapters 8, 9).

In fact, available models of fatigue behavior may not be perfect, they are very useful tools in engineering assessment if properly understood and used. The shortcomings of the available fatigue models are less important than the problems related to the uncertainty in the parameters included in the models. Moreover, fatigue design is experimental, empirical, and theoretical and in that order. Without testing, fatigue analysis often remains an academic speculation. In addition to that, it is important to know how to deal with
uncertainty in a logical and unified manner. Fatigue life data exhibit considerable scatter even under controlled laboratory conditions and the standard deviation is equally as important as the mean value. Furthermore, typical in-service variable loading may be stochastic in nature and stress calculations may be uncertain. These considerations call for some insight into applied statistics and probability calculations. However, Chapter 8 focuses on the statistical background of the S-N curves, whereas Chapter 9 is dedicated to the fatigue process.

Finally, Chapter 10 summery and conclusion. All these chapters present methods and models that deviate from the common practice in design rules and specification. The ultimate objective is to achieve optimized structures with respect to fatigue design, dimensions and inspection efforts without compromising reliability and safety.

The remaining chapters are organized as follows: Chapter 2 presents a summary of the literature review. In the literature review a brief overview of the nature of fatigue and common means of analyzing fatigue testing data. Also focuses on the statistical analysis of fatigue experiments, with an introduction to probability and statistics and an overview of the most applicable methods used to determine fatigue strength. Chapter 3 presents basic understanding of the fatigue damage process with reference to some failure cases, and gives an overview of parameters influencing the process. Chapter 4 presents an introduction to laboratory fatigue testing. Also includes a brief overview of common statistical methods to cope with the scatter in fatigue life results. Chapter 5 presents the definition and description of the fatigue load spectrum. Accuracy in applied loading description is crucial for the credibility of fatigue life results. Chapter 6 presents an elaborate fatigue life calculation scheme based on the S-N model according to design rules and specification. The basis is
the original S-N design rules from AASHTO American Association of State Highway and Transportation Officials, DoE and British Standard, BS 54000 (nominal stress approach) and Eurocode 3 (nominal stress approach, air environment only). Chapter 7 presents an outline of applied fracture mechanics. In this chapter important question, such as how fast a crack will grow during service loading. This is a crucial question to answer at a post-fabrication stage when cracks have been detected and the alternatives are repair or no repair. Chapter 8 presents a critical reevaluation of the validity of the conventional Category E S-N curves that are used in design rules and specification. Based on experimental fatigue life data, a new stochastic model is suggested. In this model both the fatigue life and the fatigue limit are treated as simultaneous random variables. The model results in a non-linear S-N curve for a log-log scale. These types of curves are in better accord with the experimental results than the conventional S-N curves. Chapter 9 presents a two-phase model for the damage process in welded joints. The objective of this chapter is to emphasize the importance of the crack initiation phase in welded joints. It is the authors' opinion that this phase is significant for high quality welded components subjected to in-service loading. A model not accounting for this phase may lead to wrong decisions regarding both dimensions and scheduled inspections. Chapter 10 presents summary analysis and evaluation of existing fatigue test results on welded steel bridge details. Chapter 11 presents the summary of the research, the computational tools developed to facilitate the computation of the reliability analysis, the computational method for determining fatigue crack, and the associated conclusions. This chapter also provides recommendations for future research.
In this chapter, the preliminary background pertinent to the study of high cycle fatigue testing will be presented. The first part of this chapter presents a brief overview of the nature of fatigue and common means of analyzing fatigue testing data. The second part focuses on the statistical analysis of fatigue experiments, with an introduction to probability and statistics and an overview of the most applicable methods used to determine fatigue strength. The third part of this chapter summarizes previous studies of high cycle fatigue (HCF) which are relevant to this research effort.

2.1 High Cycle Fatigue and Fatigue Strength Testing

2.1.1 Preliminaries to Fatigue

It is common for components of engineering structures to be subjected to repeated loads, also called cyclic loads. These loads induce cyclic stresses around engineering details which may result in microscopic physical damage always associated with cyclic plasticity, even when the gross stresses are well below the material’s ultimate strength. This microscopic damage accumulates over time as the cyclic stressing continues until eventually a macroscopic crack or other damage becomes evident. This process of damage accumulation due to cyclic loading is known as fatigue.
Fatigue as a branch of study encompasses a range of scientific and engineering disciplines and offers a variety of phenomena to explore as basic research or for use in engineering applications. It has been a source of study for over 170 years, with an ever-growing body of knowledge which has been analyzed primarily using three major approaches [1]. The traditional approach bases the analysis on the mean stresses and associated alternating stresses in the critical region of the component under investigation and/or stress range. The effects of stress raisers such as grooves, holes, or notches are included in this analysis. This approach, known as the stress-based approach, is considered the traditional method, and was developed to near maturity by 1955. The strain-based approach, on the other hand, involves a more detailed analysis of the deformation near stress raisers. Analysis centers on the localized yielding in these regions. The last approach is the adaptation of fracture mechanics to fatigue crack growth. Typically, the stress-based approach is used in the analysis of fatigue strength.

2.1.2 Definition and Key Terms

Many practical applications involve the cycling of a component between a minimum stress level and a maximum stress level. Using the stress-based approach, this constant amplitude stressing can be described using some common definitions. The stress amplitude, $\sigma_a$, is taken as half the stress range. Mathematically, these definitions are expressed as [1]:

$$
\Delta \sigma = \sigma_{\text{max}} - \sigma_{\text{min}}, \quad \sigma_m = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2}, \quad \sigma_a = \frac{\Delta \sigma}{2}
$$

(1)

Stress range, $\Delta \sigma$, is defined as the difference between the maximum stress ($\sigma_{\text{max}}$) and minimum stress ($\sigma_{\text{min}}$). The mean stress, $\sigma_m$, is defined as the average of the maximum
and minimum stresses. The ratio of minimum stress to maximum stress is also used when describing fatigue test results. This ratio, known as the stress ratio, is by R as shown below:

\[ R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \]  

(2)

Some useful relationships based on these definitions include:

\[ \Delta \sigma = 2\sigma_a = \sigma_{\text{max}}(1 - R) \]  

(3)

\[ \sigma_m = \frac{\sigma_{\text{max}}}{2}(1 + R) \]  

(4)

Notice that when the minimum stress is the negative of the maximum stress, the mean stress is obviously zero, and the associated stress ratio \( R = -1 \). This condition is known as fully-reversed or completely-reversed loading. The definitions in equations (1-3) can also be applied to other variables of interest such as strain (\( \varepsilon \)), load (\( P \)), bending moment (\( M \)), and nominal stress (\( S \)).

2.1.3 Stress-Life (S-N) Curves

A stress-life curve relates the nominal stress level (\( S \)) to the number of cycles until a specimen fails (\( N \), or \( N_f \)) – i.e., catastrophic failure or a crack develops and reaches some critical length. The nominal stress is usually expressed in terms of stress amplitude although other fatigue parameters are used as well. The curve is usually drawn with linear \( S \) and log \( N \) scales as shown in Figure 1, although log \( S \) and log \( N \) curves are also frequently encountered. Ever since the work of Wöhler, development of these curves for various materials details and loading conditions has been the “backbone” of fatigue data generation. As Yen describes, the S-N curve can be interpreted as the progressive structural
deterioration and gradual breaking of interatomic bonds under repeated stresses and may be analyzed as a statistical process \[2\]. The same principles and mathematical approach can be applied to the development of strain-life (\(\varepsilon\)-\(N\)) curves as well.

![Figure 1 Typical S-N curve (from Dowling [1]).](image)

When \(S-N\) data are found to be approximately linear on a log-linear plot, the curve may be modeled using the following expression over the linear region \[1\]:

\[
\sigma_a = C + D \log N_f
\]  

(5)

Where \(C\) and \(D\) are fitting constants. For linear regions on a log-log plot, the following expression is often used (also called Basquin’s equation):

\[
\sigma_a = AN_f^B
\]  

(6)

Where \(A\) and \(B\) are fitting constants. This equation is more commonly expressed in the following form \[1\]:

\[
\sigma_a = \sigma_f' \left(2N_f\right)^b
\]  

(7)
In some materials, there appears to be a noticeable stress level such that fatigue failure does not occur at stresses below this level. This stress level is known as the fatigue limit. The term endurance limit as used by Nelson [3] specifies the stress level at which the fatigue life becomes a prescribed long but finite life. The term fatigue strength specifies the stress amplitude corresponding to a particular fatigue life of interest, as defined by Collins [4] and Dieter [5]. Thus, the fatigue strength at $10^9$ cycles is merely the stress level corresponding to failure at $10^9$ cycles on the $S$-$N$ diagram. For materials which do not exhibit a clear fatigue limit, fatigue strength is used to specify the stress level corresponding to a specific long life. In practice, the terms fatigue limit and fatigue strength at a very high number of cycles (such as $10^9$) are often used interchangeably. In this work, the term fatigue strength will be used when specifying the stress corresponding to a specific number of cycles. Figure 2 illustrates fatigue limit and fatigue strength on typical $S$-$N$ curves.

![Figure 2 Fatigue limit and fatigue strength on typical S-N curves](image)

The $S$-$N$ curve is often conceptually divided into two regions: the low cycle fatigue LCF and the high cycle fatigue HCF regions. LCF corresponds to the region where failures occur at relatively short lives more often described using $\varepsilon$ life, typically less than $10^4$
cycles. The HCF region corresponds to the part of the curve associated with higher fatigue lives. Note that this division between the LCF and HCF regions is somewhat arbitrary and varies by material. In the HCF region, fatigue life is dominated by the crack development phase, which transitions to crack propagation once the crack grows to some macroscopic length. LCF, on the other hand, is more dominated by the crack development phase, with a macroscopic crack, severe stress configuration, or other defect already initially present or quickly developed due to high stresses. The term very high cycle fatigue is occasionally used as well, typically referring to lives on the order of $10^6$-$10^9$ cycles, although again this term is somewhat arbitrary.

2.1.4 Fatigue Crack Growth

As a property of crystalline solids (such as aerospace metals) a result of the motion and interaction of dislocations acted upon by cyclic stresses. These dislocations represent discontinuities in the crystal lattice. Yen describes the mechanism of fatigue crack formation in a simplified three-stage manner [2].

In the first stage, the dislocations originally present in the crystal grains multiply, thus increasing the dislocation density. Fine slip bands that tend to appear initially along favorably oriented grains become more numerous as the number of stress cycles increases. Slip bands are regions of intense deformation due to the shear motion between crystal planes. Some of these slip bands remain localized, some broaden, and the very pronounced become persistent slip bands. The crystal grains begin to become distorted and strain-hardened. Then, dislocation motion in one direction may become fully reversed in synch with the stress. New dislocations and their movements are generated only in some local slip zones where microstructural features are not consistent in both directions of motion.
In the second stage, thin ribbon-like extrusions of metal are emitted from the free surface and internal fissures called intrusions develop as the persistent slip bands are matured. These fissures are the initiations of a crack, and tend to occur along the slip planes associated with maximum resolved shear.

In the final stage, the newly formed crack propagates in a zigzag manner along slip planes and cleavage planes from grain to grain, maintaining a general direction perpendicular to the maximum tensile stress. Many factors affect the rate of crack propagation at this point, but as much as 99% of the fatigue life may be spent in the development of these internal fissures and their coalescence into macroscopic cracks.

2.1.5 Mean Stress Effects

Consideration of the effect of mean stress ratio on fatigue life is a primary concern in many fatigue analyses welded assumes high mean stress. Presentation of this data is accomplished in several different manners. The most obvious manner is to develop a family of $S$-$N$ curves for tests run at various stress ratios. These curves are often drawn on the same diagram with different $R$ ratios identified for each curve, as shown in Figure 3.
Figure 3. Stress-life curves for various stress ratios (from Dowling [1]).

An alternative means of expressing data at different mean stresses is to use a constant-life diagram. In this type of diagram, data from the various S-N curves at different stress ratios is re-plotted with each combination of mean stress and stress amplitude corresponding to a specified value of the number of cycles to failure. Various curves can then be drawn for different values of N, as illustrated by Figure 4. The stress amplitude at zero mean stress (by \(\sigma_{ar}\)) is represented by the intercept (\(\sigma_m = 0\)) of the constant-life curve. A normalized constant life curve can be drawn using values of the ratio \(\sigma_{a}/\sigma_{ar}\) plotted against \(\sigma_m\) for a particular fatigue life. This plot, known as a normalized amplitude-mean diagram, is shown in Figure 5. Various relations have been developed to fit the normalized amplitude-mean data, such as the linear fit (Goodman curve), to more elaborate means such as the Gerber parabola [1].
Figure 4 Constant life diagram for 7075-T6 aluminum (from Dowling [1]).

Figure 5. Normalized amplitude-mean diagram for 7075-T6 aluminum (from Dowling [1]).

2.1.6 Scatter in Fatigue Data

In any planned fatigue experiment, there is always some amount of scatter in the data, as depicted in Figure 4. This scatter is generally the cumulative effect of a variety of
random factors. Some of the relatively controllable factors include inconsistencies in surface finish, deviations in specimen alignment, differences in applied loading conditions, and inconsistent residual stresses. These sources of scatter are generally mitigated through careful specimen preparation and handling, integration of laboratory equipment, replicable experimental procedures, and use of identical specimens made from similar material from the same supplier. But even if these steps are taken, scatter in fatigue data is still observed due to differences in the microstructure of each specimen, and differences in fabrication which produces different conditions for crack development and propagation within each specimen.

2.2 Introduction to Statistical Analysis of Fatigue Experiments

Before investigating various statistical methods used to analyze fatigue strength data, it is important to review some of the basics of probability and statistics to lay the groundwork for discussions to follow.

2.2.1 Introduction to Probability and Statistics

Although the subject of statistics is often perceived as murky and complex by engineers, the essential elements of probability and statistics relevant to fatigue testing are fairly straightforward. This objective clearly cuts to the heart of the matter at hand as the primary objective of this research is to develop a means to estimate the fatigue limit and its dispersion (the inference) based on a limited amount of fatigue tests (the sample) with reasonable confidence.

As an introduction to probability theory, consider the roll of a die, with possible outcomes of 1, 2, 3, 4, 5, or 6. These outcomes together form the sample space. Each roll of the die is a random event. The probability of any outcome assuming a fair die is
obviously 1/6. A random variable is defined as a real valued function for which the domain is a sample space [8]. In the die example, one could define a random variable Z as a function equal to 1 if the roll is odd and 0 if the roll is even. In this case, Z would have an expected value of 0.5 since half the possible outcomes would be odd and half would be even. These concepts lead to the three basic axioms of probability theory, which define a probability function as follows [9]:

I. The probability of any random event is between 0 and 1; \(0 \leq P(A) \leq 1\), where A is a random event.

II. The sum of all probabilities of random events within a sample space is 1; \(\Sigma P(A) = 1\).

III. For a countable collection of mutually disjoint random events, the probability of any of these events occurring equals the sum of the probability for each individual event’s occurrence; \(P(\cup A_j) = \Sigma P(A_j)\).

There are two basic classes of random variables: discrete and continuous. Discrete random variables take on only a finite or countable infinite number of distinct values whereas continuous random variables take on a non-countable number of values. For example, the number of cycles to failure for a given test would be a discrete random variable as it must be an integer. Conversely, the stress level at which a specified portion of specimens fails to reach a given number of cycles would be a continuous random variable as it could be any positive real number. The probabilistic behavior of a random variable is characterized by a cumulative distribution function (CDF) which is defined as

\[
F_X(x) = P(X(\gamma) \leq x)
\]  

(8)
where \( X(\gamma) \) is a random variable, \( \gamma \) is an event in the sample space, and \( x \) is a real number [9]. By this definition, the CDF is a nonnegative, non-decreasing function over the range [0, 1]. If one denotes the probability of any discrete random event as \( p_i \), then

\[
F_X(x) = \sum_{i, x_i \leq x} P_i, \tag{9}
\]

\[
\sum_i P_i = 1. \tag{10}
\]

For continuous random variables, a probability density function (pdf) is defined such that

\[
F_X(x) = \int_{-\infty}^{x} f_X(\xi) d\xi. \tag{11}
\]

Clearly, based on these definitions the following relations exist:

\[
\frac{dF_X(x)}{dx} = f_X(x),
\]

\[f_X(x) \geq 0,\]

\[
\int_a^b f_X(x) dx = F_X(b) - F_X(a), \tag{12}
\]

\[
\int_{-\infty}^{\infty} f_X(x) dx = 1
\]

Two random events are said to be independent if the occurrence of one event has no effect on the occurrence of another event. One must be careful during experimental design to ensure outcomes are independent of each other with respect to the random variables of interest. An example of dependent events in terms of fatigue testing concerns the reuse of specimens tested at other stress levels or shorter lives. Say that one is interested in determining the fatigue strength at both \( 10^7 \) and \( 10^8 \) cycles. If a series of pass/fail tests at \( 10^7 \) cycles were run and the surviving specimens were reused for the \( 10^8 \) cycle tests, a
dependence has been introduced since the outcomes of the first test series affect the second
test series. It would be impossible to have a specimen fail before $10^7$ cycles in our second
series of tests since all the specimens have already survived that duration. Note, however,
that the probability of the specimens failing before $10^7$ cycles may be so low for the second
series of tests that one may be justified in ignoring this dependence (assuming there are no
other effects due to stressing the specimens).

With this background, the expected value of a random variable is defined in the
following manner:

$$
\mu = E(X) \int_{-\infty}^{\infty} x dF_X(x) = \begin{cases}
\sum_{i} x_i P_i, & \text{discrete} \\
\int_{-\infty}^{\infty} f_X(x) dx, & \text{continuous}
\end{cases}
$$

which is simply an average value taken over all possible outcomes weighted by the
probability of each outcome [9]. Another commonly used measure of central tendency is
the median, which is just the value for which 50% of the random variable values lie above
and 50% lie below; i.e., the value of $x$ for which $F_X(x)$ equals 0.5.

The dispersion, or spread of a random variable about its mean, is generally measured
using the variance which is defined as

$$
\sigma^2_X = \begin{cases}
\sum_{i} (x_i - \mu)^2 P_i, & \text{discrete} \\
\int_{-\infty}^{\infty} (x - \mu)^2 f_X(x) dx, & \text{continuous}
\end{cases}
$$

and $\sigma_X$ (the positive square root of the variance) is the standard deviation [9]. Several
probability distributions have proven to be of significant value in both theory and
application over the years. These distributions are merely analytical expressions for $F_X(x)$.
which are useful because of their ease in deriving analytical solutions to many problems and their applicability to represent a large number of random variables with reasonable accuracy. With respect to fatigue, some of the more relevant distributions include the Poisson, Gaussian (normal), lognormal, gamma, exponential, chi-square, inverse-Gaussian, and extreme value distributions [9]. A detailed description of each of the distributions commonly used in reliability analysis is presented by Elsayed [10], with a more complete overview given by Rice [11].

The goal of a statistical analysis of a fatigue experiment is generally to estimate some material property or behavior

— fatigue limit, S-N curve, etc.
— based on a sample of test data.

Generally, the property or behavior is modeled as a random variable using an appropriate probability distribution which fits the data reasonably well. Using this distribution or parameters of this distribution, one can test most any hypothesis regarding the underlying distribution related to the property or behavior of interest. Thus, it is important to be able to determine the key parameters of the population’s underlying distribution – such as the mean and standard deviation of a fatigue limit at $10^9$ cycles. Estimators are rules that define how to estimate values of a population based on sampled data.

It is important to differentiate between the population’s underlying distribution and the sampling distribution. For example, the fatigue life at a certain stress level may be plotted using a histogram as shown in Figure 6. This plot shows that the fatigue life in
Sinclair and Dolan’s data set is distributed approximately lognormal. The sampling distribution, on the other hand, refers to the probability distribution associated with an estimator. The standard deviation of the sampling distribution for an estimator is generally known as the standard error of the estimator. If we defined a sampling statistic \( \hat{y} \) to be the mean fatigue life, then the sampling distribution would be the distribution associated with the mean fatigue life, not the distribution associated with individual fatigue life. For large samples, sampling distributions based on mean values tend to become normal according to the central limit theorem [8].

![Histogram showing lognormal fatigue life based on Sinclair and Dolan’s data (from Sobczyk and Spencer [9]).](image)

Common, unbiased estimators valid for any underlying distribution are tabulated in most statistics texts. The most important of these estimators related to fatigue research are estimates for mean and proportion. As one would expect, the point estimates for mean and proportion are calculated by simply evaluating the sample mean and sample proportion, respectively. The standard error for mean and proportion are estimated using the following equations [8]:

\[
\text{Standard Error of Mean} = \frac{s}{\sqrt{n}}
\]

\[
\text{Standard Error of Proportion} = \sqrt{\frac{p(1-p)}{n}}
\]
\[ \sigma_\theta = \begin{cases} \frac{\sigma}{\sqrt{n}}, & \text{for mean} \\ \frac{\sqrt{pq}}{n}, & \text{for proportion} \end{cases} \] (15)

where \( \sigma \) is the true population standard deviation, \( n \) is the sample size, \( p \) is the true probability of survival, and \( q \) is the true probability of failure. Since the standard error estimates are based on true population parameters which are generally unknown, some initial estimates of the true population are needed in order to estimate standard error.

If a test series can provide a reasonably accurate estimate of the fatigue limit distribution parameters associated with a given number of cycles, then a designer can easily specify the maximum loads that a component may be subjected to for an associated level of risk. For example, say a test series shows that the fatigue limit at \( 10^9 \) cycles can be estimated using a normal distribution with mean 400 MPa and standard deviation 5 MPa, then simple statistics may be used to determine the maximum allowable load for a given risk level, say 1% risk of failure before \( 10^9 \) cycles. To determine this maximum load, a standard normal probability table (such as Table 4 of Appendix 3 in Wackerly et al [8]) is used to find the Z value associated with the risk level 0.01. The Z value represents the standard normal variate (mean 0 and standard deviation 1) associated with a probability level \( \alpha \) such that the probability of any value greater than \( Z \) is equal to \( \alpha \). In this case, the Z value associated with 0.01 would be -2.3267. We can then transform this standard normal variety to a specific normal distribution using the transformation law:

\[ Z = \frac{Y - \mu}{\sigma} \] (16)

where \( Y \) is a random normal variable with mean \( \mu \) and standard deviation \( \sigma \). Thus, for this example,
\[ Z = \frac{Y - \mu}{\sigma} \Rightarrow -2.3267 = \frac{Y - 400}{5} \Rightarrow Y = 388.4 MPa. \] 

Therefore, the maximum allowable load associated with 1\% risk of failure before \(10^9\) cycles would be 388.4 MPa for this fatigue strength distribution. If the fatigue loads on this component are less than 388.4 MPa, then it would meet a design goal of \(10^9\)-cycle fatigue life with 1\% risk of failure.

2.2.2 Probabilistic Approach to Fatigue Strength

Before discussing the methods to determine the parameters of a fatigue strength distribution, it is important to provide the conceptual model and terminology common to most all statistical analyses of fatigue data. In general, there are three fundamental variables applicable to fatigue analysis [12]:

I. \(S\) – the load level, generally a stress or strain index such as stress amplitude or Smith-Watson-Topper parameter; this parameter is generally the independent variable.

II. \(N\) – the number of cycles tested, usually to failure; this parameter is generally the dependent variable.

III. \(P\) – the proportion of specimens which fail before a specified number of cycles have been completed; this parameter is sometimes used as the dependent variable.

These three parameters taken together can be used to conceptually construct a P-S-N surface [12]. The P-S-N surface is visualized by adding a third axis to the S-N curve. This third axis represents the proportion of specimens which fail given a load level and number of cycles. From this surface, one can trace an S-N curve by fixing a specific values of \(P\). Thus, one sees that any individual S-N curve has an implicit fixed value of \(P\), and a family
of S-N curves can be drawn together corresponding to different value of P as shown in Figure 7. The typical S-N curve shown without an associated probability level represents the mean (or centerline) curve, meaning that 50% of the specimens fail above this curve and 50% below it, but one could just as easily draw a curve where say 30% of the specimens fail above and 70% below so long as multiple tests are run at each stress level.

Likewise, one can trace a P-S curve from the P-S-N surface by fixing a value of N (Figure 8) or a P-N curve by fixing a value of S (Figure 9). The P-S curve plays an important role in fatigue strength testing as it relates the proportion of failed specimens to the load level for a specified number of cycles.
In the next section, a survey of current methods to statistically design and analyze experiments to determine a material’s fatigue strength will be presented. These methods provide a means to deal with the stochastic nature of fatigue data and provide an estimate for the fatigue limit and its variance at a specified number of cycles. In general, one can

Figure 8 P-S-N surface showing P-S trace (from Little and Jebe [12]).

Figure 9 P-S-N surface showing P-N trace (from Little and Jebe [12]).
sort these methods into four broad categories: (1) conventional S-N tests, (2) quantal response tests, (3) accelerated stress tests, and (4) more advanced statistical methods.

2.3 Conventional S-N Testing

The conventional S-N test approach requires a series of tests to be conducted at a range of stress levels, preferably with replication, in order to determine the stress-life curve. Using this approach, a range in fatigue life for each load level can be estimated to produce a family of S-N curves at various probability levels. Using multiple tests at the same set of load levels provides a means to describe this range in fatigue life using a probability distribution (such as the lognormal distribution used by Sinclair and Dolan). This distribution then allows an estimate for mean and standard deviation for fatigue life for any load level within the range of data. The details of how to draw the S-N curve using a mathematical model are generally straightforward. In particular, ASTM E739 describes the statistical analysis of linear (or linearized) S-N and $\varepsilon$-N fatigue data [13]. When all fatigue specimens fail, the most common method of determining the parameters of the S-N curve is to use ordinary least squares – the same method commonly used in curve fits of engineering data, since popularized by Gauss [13; 14]. This method has one serious drawback, however. Namely, runouts cannot be handled since an actual failure time is not known. Thus, a residual (i.e., the difference between the best-fit curve and the actual data point) cannot be calculated for runouts. Therefore, conventional S-N analysis using ordinary least squares (or a similar approach) must exclude runout test data. For high cycle testing there are often a large number of specimens which do not fail in the allotted test time since it is not feasible to test every specimen to failure in this long-life regime. However, if all (or the vast majority) of tests are run to failure, the conventional S-N
approach would allow a graphical representation of the fatigue strength associated with a specified number of cycles, as well as the fatigue limit, if one exists.

In addition to being unable to handle runout test data, the conventional $S$-$N$ approach has another limitation. Although it allows an estimate of the scatter in fatigue life, it is more difficult using this approach to estimate the scatter in fatigue strength. The reason for this difficulty should be rather obvious. Because the number of cycles to failure is the dependent rather than independent variable, the researcher cannot set about to collect a certain number of failed specimens at an arbitrary value of $N$ due to the stochastic nature of fatigue life. For these reasons, a conventional $S$-$N$ approach in its most simplified form becomes impractical for the determination of fatigue strength and its dispersion in the high cycle regime.

Spindel and Haibach looked at a more elaborate means of analyzing $S$-$N$ data to better represent the true shape of the $S$-$N$ curve as well as provide a better estimate of fatigue limit [15]. Their proposed approach allowed use of runouts by using maximum likelihood principles to deal with these data points in a statistically acceptable manner. Maximum likelihood estimation is a means to determine the best parameters to fit a data set using the principle that the best parameter estimates are those which would be most likely to produce the observed experimental data. Their aim was to determine the parameters of the most likely “parent” population common to a number of data sets using similar specimens along with the associated confidence intervals for these parameters. Their first task was to investigate the shape of the $S$-$N$ curve, looking at conventional linear fits (using linear $S$ and $\log N$ or both $\log S$ and $N$) which include a horizontal portion beyond the fatigue limit, as well as $S$-shaped types of curves like those suggested by Weibull and others. Based on
an analysis of available data sets, they made the following conclusions concerning basic assumptions used in traditional S-N analysis: (1) straight-line approximations gave a generally poorer fit relative to S-shaped curves for larger data sets, and (2) the assumption of a normal distribution with constant standard deviation for the logarithm of fatigue life at all stress levels is generally untrue. Their analysis recommended a computer-based approach to determination of S-N curve shape but was rather incomplete in describing a means to better estimate the fatigue limit using S-N data. However, their work is significant in that it incorporated the concept of maximum likelihood estimation as a means to handle runout data from S-N tests.

2.3.1 Quantal Response Tests

As opposed to the conventional S-N approach, quantal response tests do not consider the actual time to failure, but rather they are by their nature a “pass/fail” approach. Thus, quantal response tests are designed to handle runout test data. In a quantal response test, specimens are tested at a certain load level and they either survive the specified number of cycles or they fail. Statistical analysis of test results allows an estimate of median and standard deviation of the fatigue strength at the specified number of cycles based on the proportion of failed specimens at each load level. Although used in fatigue strength testing and other applications, quantal response methods are historically associated with biological assay. Biological assay is a set of techniques used in comparisons of alternative but similar biological stimuli – basically, the term means the measurement of the potency of any stimulus by observing the reaction that it produces in a living organism [16]. The objectives of biological assay and fatigue strength testing are quite similar. In both cases, one wishes
to determine the appropriate stimulus level for which an acceptable proportion of specimens survive.

2.3.2 Accelerated Stress Tests

Another approach to fatigue limit testing is the use of accelerated stress tests. Accelerated stress testing includes those tests in which the stress level is increased during the test of a specimen in order to ensure it fails in a reasonable number of cycles. This approach differs from the other tests discussed which utilize a constant stress level for each individual specimen. Two such accelerated stress methods are the Prot method and the step-loading method.

In the Prot method, testing begins at a stress level below the estimated fatigue limit, with the stress level increasing at a constant rate until failure occurs [17]. Each successive test is accomplished at a reduced rate of stress increase resulting in a series of stress values corresponding to each rate of stress increase. One of the concerns regarding use of such a method is the influence of coaxing on the results. Coaxing, or “under-stressing,” refers to the phenomenon of raising the fatigue strength in a material by subjecting it to lower stresses relative to the fatigue strength. There is some debate over whether this phenomenon really occurs or is just the product of statistical skewing of the population [18; 19; 20]. Regardless, the Prot method has been found by Ward et al to be valid for welded SAE 4340 steel and applicable to ferrous metals with a well-defined fatigue limit, although some ferrous metals have been shown to display coaxing behavior [21; 22]. Overall, one of the primary disadvantages of the Prot method is the need to conduct a series of tests at non-constant stresses. This requirement adds significant complexity to the overall
test approach. In general, the Prot method is not very widely used for fatigue strength testing today.

The step-loading method demonstrated by Nicholas is similar in concept to the Prot method in that the stress is not held constant for each specimen [19]. In this procedure, specimens were tested to a given number of cycles (specifically, $10^7$) and if they survived, the stress was increased by approximately 5% and the specimens were again retested up to $10^7$ cycles. This process was repeated until each specimen failed. A linear interpolation scheme was used to calculate the fatigue limit stress for each specimen.

2.4 Advanced Statistical Methods

The final groups of fatigue strength test approaches are lumped together under the heading of advanced statistical methods. Three such approaches will be discussed: (1) random fatigue limit modeling, (2) Bayesian methods, and (3) bootstrapping methods.

Random fatigue limit (RFL) modeling explicitly assumes that each specimen has its own fatigue limit based on its unique microstructure. Thus, there is an associated fatigue strength distribution at each specified number of cycles [3]. Pascual and Meeker proposed a method incorporating maximum likelihood estimation to produce probabilistic S-N curves which account for the behavior visually represented by the distributions shown in Figure 10 [23]. There are two distributions which must be modeled using the RFL model – namely, the conditional distribution in fatigue life at a specified stress level given fatigue limit (the horizontal distributions in Figure 10) and the distribution in fatigue strength at a specified fatigue life (the vertical distributions in Figure 10). Pascual and Meeker considered several combinations of analytical distributions to model these random
variables. Based on the findings of numerous researchers (such as Sinclair and Dolan [6]), the conditional distribution of fatigue life is generally assumed to have a lognormal distribution (although this assumption is not required). The distribution in fatigue strength (or, random fatigue limit) is a bit more of an unknown, but several reasonable distributions were investigated with the Weibull distribution showing significant promise [14]. The mechanics of the RFL model are rather detailed but have been simplified by who offer available software to perform the necessary calculations.

Pascual and Meeker’s work takes a bold step in better modeling observable characteristics of S-N data using a manageable mathematical formulation. This effort builds upon Nelson’s work in fitting fatigue curves with non-constant standard deviation associated with each stress level, which also utilized maximum likelihood methods to allow use of runout data [24]. Specifically, the RFL method accounts for the increase in standard deviation of fatigue life at lower stress levels as well as the curvature and flattening of the S-N curve in the HCF regime. This flattening effect is observed in the 37 probabilistic S-N curves shown in Figure 11. By treating fatigue limit as a property specific to each specimen rather than an overall material property, their approach also provides a better estimate of fatigue limit than conventional S-N analysis allows. Conventional analysis which assumes a single-valued constant fatigue limit for all specimens must by necessity result in an estimate of fatigue limit below the lowest stress tested, thus producing an unrealistically low value.
As an alternative approach, Bayesian statistical methods are an established means of estimation which utilize prior information, whether qualitative or quantitative in nature, to augment and enhance real test data to provide a better estimate of population parameters [25]. Estimation of a median fatigue strength and its associated dispersion is a means of statistical inference based on a small sample of test data in order to make conclusions based...
on the entire population of a particular material. The typical approach to statistical inference is to use hypothesis testing or confidence interval estimation using objective data in the form of independent samples from a similar population. In a Bayesian approach, additional sources of information of generally lesser quality are used to augment this objective data in order to improve the parameter estimates. For example, one may use prototype test data to augment limited test data on a full-up article in order to provide a more complete picture of real performance.

With respect to the RFL model, Johnson et al proposed incorporation of a hierarchical Bayesian approach to provide a means for parameter estimation (in lieu of Pascual and Meeker’s maximum likelihood approach) [26]. The general approach is to specify prior distributions for each of the RFL model parameters. These prior distributions are used in conjunction with the observed fatigue test data to determine posterior parameter distributions which can then be used to draw statistical inferences on fatigue limit. Johnson et al claim that the primary advantage to this approach is the ability to use prior information in the analysis beyond that data collected in the fatigue test sample. Since development of new materials is generally evolutionary, a decent picture of what some of the parameter distributions will look like already exists based on tests of similar materials or preliminary analysis of the material under investigation. In addition, some numerical advantages are realized so long as the choices of prior distribution guarantee a proper posterior distribution which can be analyzed. Besides the disadvantage of having to choose suitable prior distributions which allow a proper posterior distribution, there is the standard pitfall of Bayesian analysis to contend with: namely, how to make generally subjective prior
distribution estimates based on limited data without compromising the objectivity of the test program.

In addition to Bayesian methods, bootstrap methods may also provide some opportunities to more efficiently model fatigue strength behavior [27]. The bootstrap method is a data-based simulation which utilizes multiple random draws from real test data to make statistical inferences about the underlying population [28]. Essentially, the repeated draws from the test data pool allow estimates of parameter confidence intervals which can be quite accurate. The main drawback to bootstrapping is the reuse of existing data which may skew the results depending on the presence of outliers as well as any other misrepresentation of the sample with respect to the true population. It is also a somewhat difficult method to explain and rationalize to decision makers.

2.5 Very High Cycle Fatigue

VHCF behavior has in the last decade become a very hot topic in the fatigue community as new testing methods have allowed considerably more test data to be generated at fatigue lives that were previously impractical to test. In engineering terminology, fatigue is progressive structural damage of materials under cyclic loads. There are a few main types of fatigue. Mechanical fatigue could be described as damage induced by application of fluctuating stresses and strains. Among other types of fatigue are: creep fatigue – cyclic loads at high temperatures; thermal fatigue – cyclic changes in material’s temperature; thermo-mechanical fatigue – a combination of mechanical and thermal fatigue; corrosion fatigue – cyclic loads applied in a chemically aggressive environment; fretting fatigue – cyclic stresses together with the oscillation motion and frictional sliding between surfaces.
Fatigue life is an important characteristic of an engineering component and is measured by a number of cycles it can withstand before fatigue failure takes place. Based on the fatigue life concept the mechanical fatigue could be sub-divided into: low cycle fatigue (LCF) – up to $10^4$ cycles to failure; high cycle fatigue (HCF) – between $10^6$ and $10^8$ cycles to failure and very high cycle fatigue (VHCF) – over $10^8$ cycles to failure. The VHCF represents the main point of interest in this research.

2.6 Summary

This section gives an overview of high cycle fatigue testing. First part presents a brief overview of the nature of fatigue and common means of analyzing fatigue testing data. The second part focuses on the statistical analysis of fatigue experiments, with an introduction to probability and statistics and an overview of the most applicable methods used to determine fatigue strength. The third part of this chapter summarizes previous studies of high cycle fatigue (HCF) which are relevant to this research effort.
CHAPTER III
FATIGUE BEHAVIOR OF WELDED JOINTS

3.1 Preliminaries

This section gives an overview of fatigue behavior of welded joints. The mechanisms of the fatigue damage process are discussed and the usual method and models for fatigue life verification are outlined. The basic parameters that are important for the fatigue process in welded joints are examined. The objective of the chapter is to give basic insight into the fatigue process in welded joints, with emphasis on the aspects that makes these joints vulnerable to fatigue.

3.2 Failures of Fatigue

Fatigue is defined as damage accumulation due to oscillating stresses and strains in the material. Therefore, fatigue cracks do occur in welded details that are subjected to repetitive loading. In significant structural items, fatigue damage may lead to failures with severe consequences. The Health and Safety Executive, UK, has listed the main causes of structural damage for installation in the North Sea (1974-1992, [29]) as fatigue 25%, vessel impact 24%, dropped objects 9%, and corrosion 6%.
3.3 Metal fatigue

Fatigue is defined as a damage process in the metal due to fluctuating stresses and strains. Although the stresses and strains may be well below the static resistance level of the metal, the damage is accumulating cycle by cycle and after a certain number of load fluctuations a failure will occur. For structures in service, the damage accumulation may take several years to reach a critical level with resulting failure. The fatigue process is usually divided into three phases:

— phase 1: crack development;
— phase 2: crack growth;
— phase 3: final fracture.

The crack development usually takes place on the surface of the metal in the vicinity of a notch elevated stresses expected. The mechanism is explained by a slip band mechanism at a microscopic level driven by the maximum shear stress. When the load is imposed, some grains will be subjected to plastic deformation involving the sliding of some of the crystallographic planes. The mechanism is limited to a few grains where these crystallographic planes have an unfavorable orientation with respect to the local maximum shear stresses. When the load is reversed, the planes will not slide back to their initial position due to the cyclic stain hardening effect. Hence, in the reversed part of the load cycle, it is the neighboring planes that will suffer yielding by sliding in the opposite direction. The final result is microscopic extrusions and intrusion on the metal surface. The intrusions act as a micro-crack for further crack extension during the subsequent loading cycles. The mechanism is schematically shown in Figure 12.
After crack development has occurred within a few grains, subsequent microscopic growth will extend the crack to pass several grain boundaries. When the crack front reaches over several grains, the crack will continue to grow in a direction perpendicular to the largest tensile principal stress. This transition from microscopic to large-scale crack growth is also indicated in Figure 12. It is important to realize that whereas the initiation phase is related to the surface condition of the metal and governed by the cyclic shear stresses, the crack growth depends on the material as a bulk property and the crack is driven by the cyclic principal stresses. A more thorough presentation of the subject is found in [31].

In crack growth phase, the process is explained by a crack opening and blunting mechanism followed by a subsequent crack closing and front sharpening mechanism during each load cycle. After one complete cycle, the crack front has advanced a small increment which may be traced by microscopy on the fatigue surface. This advancement corresponding to one load cycle is the distance between two so called striations. These striations are shown for a high-strength steel in Figure 13. The advancement depends on the range of the stress intensity factor (SIF).
The final fracture in phase 3 will take place when the crack becomes so large that the remaining ligament of the cross section is too small to transfer the peak of the load cycle. In the former case, it is the net section average stresses that are the driving force for the fracture. In the latter case, it is a local failure that is driven by the maximum SIF. This factor uniquely characterizes the magnitude of the stress field at the crack front under linear elastic conditions.

3.4 Important parameters and conditions for fatigue damage process

The fatigue process is quite complex and is influenced both by the nature of the external loading, the geometry of the structural item and its material characteristics. The following conditions and parameters are important to the damage process:

— External cyclic loading:
  o loading mode with reference to the actual structural item,
  o Time history of the external forces,
— Geometry of the item:
  o Global geometry of the item,
  o Local geometry at potential crack locus,
— Material characteristics;
— Residual stresses;
— Production quality in general;
— surface finish in particular;
— Environmental condition during service.

We must bear in mind that fatigue damage starts as a local phenomenon unless significant defects present. The cracks emanate from a small detail on the item, often in the vicinity of a notch.

3.4.1 External cyclic loading

The external forces may create normal, bending or torsion effects on a structural item with associated stresses near a hot spot. These loading and response situations are often referred to as loading and stress modes. The latter concept is defined by the stress direction relative to the crack planes. The normal and bending loading mode will give rise to normal stresses that will act as the main agents for the crack development and growth. In this case, the crack planes will be moved directly apart by the normal stresses. This situation is referred to as mode I. Therefore, these two loading modes inflict the same mode and damage mechanism. If there are several stress axes involved, it is often assumed that one of them will be dominant to avoid a more complex multi-axial fatigue analysis. Shear stresses due to torsion may involve a fatigue mechanism that is different from mode I. The crack planes will be sheared to extension. However, this is a rare mode in civil structures than the stresses normal to the crack plane.
3.4.2 Geometry, stress and strain configurations

The response of structural details to external forces is usually characterized by the stresses and strains associated with the forces. The general stress response is important, and even more so is the local stress caused by geometrical discontinuities. The geometry of the item is important and is one of the issues that must be addressed at the design stage to achieve improved durability. The main issue is to reduce the stress configuration factor $K_t$ in areas susceptible to fatigue. This will limit the stress and strain response inflicted by the external loads at potential crack sites. The stress configuration is defined as a local increase of stresses due to a geometrical change or discontinuity. A more or less abrupt change in geometry is often referred to as a notch. Typical examples are threads, cope holes, and welds. The stresses at the notch may be magnified locally by a factor of from something larger than 1 as compared to the average nominal stresses. This magnification factor is defined as the stress configuration factor $K_t$. The stresses may be reduced by increasing the general dimension of the item or improving the local geometry of the notch, typically the notch radius. Whereas the first approach will increase the weight of the item, the second improvement can often be achieved without any additional weight or costs. Furthermore, increasing the dimension of the item may also have an unfavorable effect on the fatigue resistance due to the size effect. This means that the benefits achieved by reducing the stresses will partly be lost due to poorer fatigue quality related to the increased size. All these considerations favor a design where the local notch geometry is optimized.

Typical stress configurations due to notches are shown in Figure 14. At the left of Figure 14 is shown a plate with two edge notches symmetrically on each side. As can be seen, the stresses will increase at the notched section due to the reduced cross section; but
far more important is the stress rise caused by the local disturbance of the notch itself this
local effect very much depends on the notch radius $\rho$. Cracks may appear in the root of
both notches. The same phenomenon is shown for a butt joint, at the right in Figure 14. In
this case we have no reduction of the cross section, but overfill of the weld metal will act
as a stress riser. The stress will rise at the transition between the base plate and weld metal.
This area is often referred to as the weld toe. This is the potential crack locus. The local
geometry can be characterized by its flank angle $\theta$ and radius $\rho$. The stress configuration
factor is defined as:

$$K_t = \frac{\sigma_I}{\sigma_n}$$ (18)

we should be aware of the fact that the nominal stress can be defined in the section away
from the notch or the nominal stress in the notch section. This may vary in various
handbooks, Ref [4].

3.4.3 Material parameters

Common material parameters, such as tensile strength, and modulus of elasticity,
have an impact on the fatigue strength of the pure metal. Structural details are dependent
upon classification and applied stress range. However, to characterize the fatigue resistance
more explicitly, special tests with smooth specimens are often carried out to determine the fatigue life as a function of the applied stress range. In addition, a crack growth test with pre-cracked standard specimens can be carried out to reveal crack behavior. The first approach is to present the fatigue life in number of cycles as a function of the applied constant amplitude stress range as shown in Figure 15. The key parameter for fatigue life is the stress range \( S_r \), but the results will also depend on the applied mean stress level. As can be seen from Figure 15, increased mean stress will result in shorter fatigue lives for a given \( \Delta \sigma \) welded joints have high mean stress. A linear approximation of the upper part of the curve for a log-log scale gives the following equation:

\[
\log N = \log A - m \log \Delta \sigma
\]  \hspace{1cm} (19)

or:

\[
N = \frac{A}{\Delta \sigma^m}
\]  \hspace{1cm} (20)

where \( \log A \) is the curve intercept with the vertical axis, whereas -1/m is the slope of the curve. When the stress range is sufficiently low, no fatigue damage will occur. This stress range \( \Delta \sigma_D \) is referred to as the fatigue-limit under constant amplitude loading. It may be regarded as a parameter characterizing the fatigue resistance of the detail, but it will in fact depend both on the size and geometry of the specimens, as well as on the surface conditions. This is also the case for the parameters A and m.
The crack growth test is based on tests with standard cracked specimens where the crack growth rate $\frac{da}{dN}$ (m/cycle) is measured as a function of applied stress intensity factor range (SIFR) as shown in Figure 16. The specimen is a compact tension (CT) specimen with standard dimensions for a given thickness. The SIFR is a parameter that alone governs the local cyclic stress variation at the crack front:

$$\Delta K = \Delta \sigma \sqrt{\pi a F}$$  \hspace{1cm} (21)

where $\Delta \sigma_n$ is the applied nominal stress range and $a$ is the crack size. $F$ is a geometry function accounting for the crack and component geometry. There is a limit at which the crack will stop growing. This limit, $\Delta K_0$, is referred to as the fatigue crack threshold value. For $\Delta K$ values greater than $\Delta K_0$, one will observe stable crack growth is expected region II. As $\Delta K$ increases, one will enter into region III with rapid crack growth often accompanied by unstable crack growth; see Figure 16. Again, the results will depend on the mean stress level. This level can be characterized by the R ratio defined as the minimum applied force $F$ as fraction of the maximum applied force. The growth rate will increases for a given $\Delta K$ if the R ratio increases. This is in agreement with the mean stress effect that was shown in Figure 15 S-N curve based on constant amplitude fatigue tests [5].
Figure 16 for the S-N approach. In the region II, the growth results can be approximated by a straight line for a log-log scale:

$$\log \frac{da}{dN} = \log C + m \log \Delta K$$

or:

$$\frac{da}{dN} = C \Delta K^m$$

where C and m are material parameters that characterize the crack growth. If the entire fatigue life consists of crack propagation only, the exponent m coincides with the parameter m defining the S-N curve.

In addition to the S-N tests shown in Figure 16, and the crack growth test in Figure 17, it is possible to carry out tests on small specimens to study the time taken to crack development in the material. The tests are carried out on specimens that are so small that the fracture will occur after a crack some small increment of damage, say several tenths of millimeter [3]. For this type of test the strain range during a load cycle is often used as the key parameter for the time to crack development. A typical stress-strain cycle and the

---

Figure 16 Crack growth rate versus stress intensity factor range [6].
resulting strain-life curve are shown in Figure 17. For the left part of the life curve there is a significantly large cyclic plastic strain involved and the number of cycles to fracture of the specimen is relatively short, typically 1,000 cycles or less. This is referred to as low cycle fatigue. In the right part of the life curve the global stress-strain cycle is elastic and a fatigue life of 10,000 cycles or more (is not uncommon). This is referred to as high cycle fatigue. As we have shown earlier, the results depend on the mean stress. The strain-life curve to the right is given by the Manson-Coffin equation with Morrow’s mean stress correction:

$$\frac{\Delta \varepsilon_T}{2} = \frac{\left(\sigma_f' - \sigma_m\right)}{E} (2N)^b + \varepsilon_f' (2N)^c$$

Here $\Delta \varepsilon_T$ is the total strain range and $\sigma_m$ is the local mean stress at the weld toe. The parameters $b$ and $c$ are the fatigue strength and ductility exponents, and $\sigma_f'$ and $\varepsilon_f'$ are the fatigue strength and ductility coefficients respectively. The stress and strain behavior is given by the Ramberg-Osgood stabilized cyclic strain curve:

$$\Delta \varepsilon = \frac{\Delta \sigma}{E} + 2 \left(\frac{\Delta \sigma}{2K'}\right)^{1/n'}$$

where $K'$ and $n'$ are the cyclic strength coefficient and strain hardening exponent respectively. This equation governs the hysteresis curve to the left on Figure 17.
3.4.4 Definition of the Residual stresses

Residual stresses are defined as inherent stresses present in the structural item before the external forces are applied. Residual stresses are often created by the fabrication process. They are self-equilibrated; if there are zones that have tensile stresses there must be other zones that have compressive stresses. The areas subjected to tensile stresses will be more vulnerable to fatigue. The simple explanation of this is the mean stress effect, which has been discussed above. The presence of large tensile residual stresses will increase the mean stress. Even compressive stresses caused by the external forces may, when added on to pre-existing static tensile stresses, effectively act as a tensile stress cycle in the material. The influence of residual stresses is shown in Figure 18 for a partly compressive external stress cycle.

Figure 17 Local strain cycle used as key to time to crack initiation based on tests with small-scale specimens [7].
3.4.5 Material Fabrication

Fatigue predictions must not be entirely based on ideal design and dimensions taken from an engineering drawing. It is the quality of the actual as-built structural item that, in the end, governs the fatigue strength. Important topics are:

- actual dimension and alignment;
- radius of severe notches;
- surface finish; does not matter in actual civil engineering structures
- presence of sharp flaws.

Dimension control must be carried out to check that the dimensions are within the given tolerances. If misalignment occurs it may introduce secondary bending. Sharp flaws may act as starters for fatigue crack growth and in many cases the crack development phase is lost. To ensure quality regarding these matters, dimensional control and non-destructive testing should be carried out.

Figure 18 The effect of residual stresses in the stress cycles [7].
3.4.6 The Influence of the environment

Most fatigue material properties are given for normal air environment. Other types of environments may change the fatigue behavior and related parameters dramatically. One important example is the fatigue behavior of steel in seawater, where the steel is without any protection against corrosion. In this case there will be a synergistic effect between the corrosive deterioration mechanism and the fatigue damage processes. Also, the load frequency becomes important as environmental effects are time-dependent process. Therefore, fatigue laboratory tests cannot be accelerated in the same manner as for air environment. The fatigue life may decrease by a factor of 3 to 5 compared with dry air environment. If cathodic protection is provided, the fatigue life will lay between the air and free corrosion results. These matters are extremely important to ship and offshore structures.

3.5 General overview for welded joints

The general issues described above are, of course, also important for welded joints. However, a welded joint has some peculiar features that make some parameters play a much more important role than others. Let us start with an S-N curve for a welded joint and compare it with other specimens. Figure 19 shows the S-N curves based on tests with one smooth plate, one plate with a bore hole, and one plate with a fillet welded transverse attachment (gusset). This type of fillet welded joint is often referred to as non-load-carrying due to the fact that the load is carried straight through the base plate. It is also noted that the applied force is transverse to the welding direction.
In the plate, the fatigue cracks can appear anywhere in the longitudinal edges of the plate, whereas they will appear at the inner edges of the bore hole transverse to the applied loading in the specimen with the hole. The latter phenomenon is due to the stress configuration at the edge of the hole. In the fillet welded joint, the cracks will appear at the weld toe and grow through the base plate in a direction perpendicular to the applied stresses. The cracks are indicated on the specimen’s drawings in Figure 20. Various stages in the crack development for the welded detail are shown in Figure 21. The cracks may emanate from several spots along the weld seam. Small semi-elliptical-shaped cracks are formed, grow, and coalesce to become larger cracks. In the final stage, the remaining ligament of the plate section is too small to carry the peak load. As can be seen from Figure 20, the fatigue life for the fillet welded joint is significantly shorter than both the smooth plate and the plate with a bore hole. The difference between the S-N curve for the specimen with the hole and the S-N curve for the welded joint is rather surprising considering the fact that the bore hole creates a stress configuration factor close to 3 at the edge of the hole. The relatively short fatigue life of welded detail is explained, in the main, by three factors:

![Fatigue Life Curves](image)

Figure 19 Fatigue life curves for various details reproduced after [8].
- severe notch effect due to the attachment and the weld filler metal;
- presence of non-metallic intrusions or micro-flaws along the fusion line;
- presence of large tensile residual stresses.

The notch effect has already been shown by the stress configuration occurring at the weld toe for a butt joint. In the present case, the notch effect is created by a fillet weld and this effect is often more severe due to the fact that the fillet-weld has a steeper toe flank angle. The latter is the most important one and can be characterized by the local toe geometry as shown in Figure 21. A favorable geometry is characterized by a low flank angle $\theta$ and a large toe radius $\rho$. One complication when defining the notch effect is that these two parameters may vary considerably along the weld seam. It is likely that the crack spots shown in Figure 21 coincide with unfavorable local toe geometry with high notch effect.

![Figure 20 Various stages of crack growth in the fillet welded joint [8].](image)
If the stress configuration due to the notch effect is at a comparable level for the bore hole and the welded attachment as indicated on Figure 22, then the explanation for the shorter fatigue life of the welded detail must be found from the differences in surface condition (micro-flaws) and residual stresses for the two specimens. The plate with the bore hole does not have any intrusions or micro-flaws near the edge of the hole where the stress configuration is greatest unless there are machine defects. For the welded detail, these types of imperfections are created during the welding process. It is an unfortunate coincidence that the weld toe area is subjected both to the highest stress configuration and to the presence of these imperfections. As a result, the weld toe becomes the most likely crack site and the fatigue life is significantly reduced compared to a machined component. If we add the influence of the tensile residual stresses for the welded detail, the reason for the shorter fatigue life found in welded joints is explained. These stresses are introduced as a result of differential the cooling because of welding, and over the length of the weld seam they will appear as both compression and tensile stresses. Typical stress distributions are shown in Figures 22 for residual stresses that are normal to the welding direction and parallel to the welding direction respectively. For steel the maximum residual stresses in the distributions can be close to the yield stress of the material. In the tension area the

Figure 21 Notch effect at the edge of a bore hole and at the weld toe [9].
effective mean stress will increase and the associated fatigue life decrease as has been explained in the previous section. As a result, the fatigue life of welded joints has limited sensitivity to the applied mean stress caused by external loading. When these stresses are added to the residual stresses, the stress cycle will have a maximum value near the yield stress regardless of the mean stress level associated with the applied loading. This is true for joints in the as-welded condition. For such cases, a welded joint will not be so vulnerable if the stress cycles inflicted by external loading are partly compressive.

![Diagram showing tension and compression in welded joints.](image)

Figure 22 Right welding residual stresses parallel to the welding direction. Left welding residual stresses transverse to the welding direction [10].

All the issues discussed above are now generally accepted and well understood, although there is still some dispute regarding the presence of intrusions or flaws. The traditional school of thought has been to assume the presence of a crack-like defect, with a depth of several tenths of a millimeter, right from the start. Hence, the entire fatigue process in a welded joint has been regarded as large-scale crack growth. However, this belief does not fit the experimental facts. Several test series, with welded details where the crack depth is measured during the course of the test, have shown that there is an initiation phase present in the damage process. This is especially true at low stress ranges. This means that initiation may be important under typical in-service loading conditions (initiation from an
existing defect). Instead of speaking of crack-like defects, one may speak of micro-flaws or rough surface conditions at the weld toe. One may say that the welding process leaves fracture points at the weld toe that reduces the incubation time before crack growth starts. This is the rationale for choosing a two-phase model as a more correct approach to the fatigue behavior of long life welded joints. The crack development is modeled according to Figure 17 and the crack growth according to Figure 16. The obtained results should be consistent with the total fatigue life as given in Figure 18. One argument for this approach is that the quality of welded details has improved over the last several decades due to better quality assurance and control.

3.6 Plated joints type

We have shown a full penetration butt joint and a fillet-welded joint with axial loading transverse to the welding direction. Several other possible configurations and loading modes exist. However, even large complex structures may often be divided into a finite number of elementary joints.
To verify the fatigue life and control the fatigue damage process during service, it is necessary to study in detail the fatigue behavior of these joints. When characterizing a welded detail the following topics are important:

- geometry of the detail (welded plate stiffeners, attachments etc.);
- loading direction (perpendicular or parallel to the weld seam);
- joint configuration (e.g. butt joint, fillet welded joint);
- welding procedure (full penetration, flat position);
- loading mode (axial or bending);
- production quality (tolerances, surface finish);
- extent of non-destructive inspection (visual or detailed);
- post-weld treatment (stress relieving, grinding);
- potential crack locus (weld toe, weld root).

Design rules and specifications recommend categories (classes) as a result of the characterization given above. Each category is assumed to have a given fatigue quality for a given locus for potential fatigue cracking. The fatigue strength is usually given as the number of cycles to fracture as a function of the nominal stress range under constant amplitude loading. It is noticed that the steel quality does not enter our list of important topics. We have explained why: the fatigue quality is mainly governed by geometrical aspects and the local surface condition at the weld toe, together with the welding residual stresses.

Two examples of joint categories are shown in Figure 23. To the left is shown the full penetration butt joint (welded from both sides) with axial loading perpendicular to the welding direction. This is the most common high-strength joint found. To the right is
shown a plate with a gusset attached by fillet welds. The plate is subjected to bending perpendicular to the weld seam similar to the butt joint. There are several important differences between these two details. The most obvious is that the butt joint has to transfer the loading through the weld itself, whereas for the fillet weld, the load path is through the main plate. The fillet weld is regarded as non-load-carrying.

The butt joint gives fewer geometrical changes in the direction of the load path. Hence, the stress configuration factor will be smaller than for the fillet weld which gives a larger geometrical disturbance. This is the main reason for the higher fatigue quality of the butt joint. For both these joints, a potential fatigue crack will emanate from the intersection between the weld and the plate. This weld toe region is the critical point for most welded joints.

![Figure 23 Types of welded joints: left-double-sided butt weld between plates, right-fillet welded stiffener [10].](image)

The fillet welded stiffener is shown in more detail in Figure 24 for the purpose of studying the geometrical parameters. It is convenient to distinguish between global and local geometry parameters. These are defined below:

- global parameters of the detail: the plate thickness $T$ and the characteristic joint length $L$;
- local weld geometry: weld toe angle $\theta$ and weld toe radius $\rho$.
Figure 24 Definition of geometrical parameters of a fillet welded joint [11].

The geometrical parameters given above are by far the most important ones for fatigue quality because they govern the stress configuration at the weld toe. If one adds the surface condition at the weld toe and the residual stresses, the fatigue quality is almost determined. All these considerations lead to the determination of a joint category with an associated S-N curve in design rules and specifications. However, the categories are defined in less detail than we have discussed here. Local toe geometry and surface finish are not included in the definition of a category. Residual stresses are assumed. As a result, the test population from which the S-N curve is obtained may exhibit a large scatter in fatigue life. If the population (defining a category) had been defined by local toe geometry, surface finish, and residual stresses then the fatigue life would exhibit less scatter and the design curve would be easier to define. In fact, more recent rules have defined a larger number of classes due to the issues we have discussed above.

Figure 25 shows two other welded details with the weld direction transverse to the loading direction. On the left is shown a butt weld made from one side in a V-groove. The potential crack site will in this case could be the weld root due to lack of penetration. The condition may improve by a backing bar as shown, but the fatigue quality is still much poorer than for the butt weld made from both sides in an X-groove. To the right is shown
a cruciform joint with a fillet weld. The load transfer now has to go through the weld metal. As a result this detail has two potential crack sites; one at the weld toe as discussed earlier and one at the weld root. The first crack will be driven by the stress in the plate section, whereas the crack from the weld root will be driven by the stresses in the weld throat. Hence, which one of them will be critical depends on the plate thickness $T$ versus the leg length $\lambda$ of the weld. In this case the fatigue life of the joint must be checked for two possible crack locations by using two different categories and applied stresses. Again.

All the welded details shown so far will usually be less vulnerable to fatigue damage if the loading direction is changed to be parallel to the weld seam (normal to the paper plane). The reason is that the stress configurations caused by the weld toe will disappear as this notch no longer will be transverse to the direction of the stresses. For continuous longitudinal welds in a beam subjected to bending, the most likely crack site will be a ripple as shown to the left in Figure 26. It is assumed that the bending moment decreases towards the end of the beam so the terminations of the welds are in an unstressed area. For the attachments to the right in Figure 26, this is not possible and the critical spots will in this case be found in the start and the termination of the welds. This is due to the fact that the
ends of the stiffeners give a severe stress configuration. The weld on the plate edge is usually more vulnerable than the weld in the middle of the plate.

![Diagram of welded joint](image)

Figure 26 Welds longtitudinal to the direction of the applied loading [12].

We have emphasized above that fatigue behavior of a welded joint is mainly a geometrical and production-quality issue. Welded details made from high-strength steel do not behave appreciably better than details made from normal mild steel in an as-welded condition. This is one of the peculiarities of the fatigue behavior of welded joints. The other is that the applied mean stress hardly matters. At lower stress ranges, there is an initiation phase present, and the steel grade may play a role. Although this role is a minor one compared with the toe-notch effect and the surface condition at the fusion line, it should be modeled. Furthermore, the applied mean stress will obviously play a role if the welded joint is stress relieved.

3.7 Cover-Plated (database 2)

3.7.1 Investigation of High Cycle Fatigue Studies for Cover-Plated Details (database 2 and 3)

This investigation was part of PennDOT Research Project No. 72-3, a study of high cycle fatigue of welded detail. This project involved the testing of small scale cover-plated beams (W14X30). Tests have also been undertaken at U. S. Steel Corporation on similar
cover-plated beams as part of NCHRP Project 12-12 under random variable loading. Several full scale beams were also tested to determine their fatigue and fracture resistance as part of Department of Transportation Project DOT-FH-11-8271 "Determination of Tolerable Flaw Sizes in Full Size Weldments.

3.7.2 Constant-Amplitude Tests

In NCHRP Project 12-7, twelve end-welded cover-plated beam details were tested at a stress range of 8 ksi (55.2 MPa) and two details were tested at 6 ksi (41.4 MPa). The minimum stress, stress range, and cycles to failure are summarized in Table 1.

Table 1 Constant-Amplitude Fatigue (NCHRP 12-7) [15].

<table>
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<th>$S_r$</th>
<th>N</th>
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</table>

Both rolled and welded W14X30 beams were tested. The cover plates for the CR and CW series beams were 0.563 in. (14.3 mm) thick the cover plates for the CT series beams were 0.75 in. (19.1 mm) thick. The transverse weld leg size was 0.25 in. (6.4 mm) for all beams. In PennDOT Project 72-3, thirteen rolled beams were tested at stress ranges between 4 ksi (27.6 MPa) and 8 ksi (55.2 MPa). These beams were comparable to those
tested in NCHRP 12-7. The minimum stress, stress range, and cycles to failure are summarized in Table 2.

<table>
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</table>

* No visible crack

Five beams were cycled to 100,000,000 cycles without detecting fatigue cracking. The lowest stress range at which fatigue failure was observed was 4.7 ksi (32.4 MPa). The
results of all constant-cycle fatigue tests at a stress-range of 6 ksi (41.4 MPa) or less from Projects 72-3 and 12-7 are plotted in Figure 27.

![Figure 27 Small scale beam constant-amplitude](image)

The beam failures generally fall within the extension of the 95% confidence limits of the mean regression line for all end weld cover plate failures reported in NCHRP Report 102.

\[
\log N = 9.2916 - 3.0946 \log S_r
\]

\[s = 0.1006\]  

where \(S_r\) = stress range, ksi; \(N\) cycles to failure and \(s\) is standard error of estimate. The 95% confidence limits in Figure 27 are approximated as twice the standard error of estimate on each side of the mean regression line. The only fatigue tests available on full sized cover-plated beams were conducted on DOT Project DOT-FH-11-8271. Six beams were tested at a stress range of 8 ksi (55.2 MPa). The stress range and cycles to failure are listed in Table 3.
These beams were fractured at reduced temperatures prior to complete fatigue failure, but estimates indicate that the residual fatigue life at fracture was negligible. The A36 and A588 rolled beams were W36X260 and W36X230 sections respectively. The A514 beams were welded built-up shapes with a depth of 36 in. (914.4 mm) and a 1.5 in. (38.1 mm) thick flange. All cover plates were lin. (25.4 mm) thick with a weld leg size of 0.75 in. (19.1 mm).

The results of these tests on full scale beams are plotted in Figure 28 and compared with the mean and confidence limits for the smaller beams tested.

![Figure 28 Full scale beam constant-amplitude fatigue tests [15].](image.png)
It is readily apparent that these beams exhibited less fatigue strength. The lower confidence limit for beams with cover plates wider than the flange is seen to provide a lower bound estimate of the full scale beams fatigue strength. The beam nearest the lower confidence limit had a 0.5 in. (12.7 mm) deep crack when first detected.

3.7.3 Variable-Amplitude Tests

In NCHRP Project 12-12, twelve end-welded cover plates were tested at an equivalent Miner stress range less than 6.0 ksi (41.4 MPa). The variable-amplitude stress range distribution used in this study satisfied a Rayleigh probability-density curve. The beams were the same size as those tested on NCHRP 12-7. When testing was discontinued, all of the cover plate welds were cracked, but three of the cracks had not propagated completely through the flange and exhausted all fatigue life. The test results are listed in Table 4.

<table>
<thead>
<tr>
<th>Beam</th>
<th>( S_{\text{min}} ) ksi</th>
<th>( S_{\text{RMS}} ) ksi</th>
<th>( S_{\text{MINER}} ) ksi</th>
<th>( N \times 10^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>WBC 1406C</td>
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<td>2.76</td>
<td>3.0</td>
<td>103719</td>
</tr>
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</tr>
<tr>
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<td>5.51</td>
<td>6.0</td>
<td>21822*</td>
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</table>

* Cracked but no failure
** All Tests from NCHRP12-12 except RCPB 401 (Lehigh)
The results of the variable-amplitude tests which were conducted at a Miner equivalent stress range less than 6.0 ksi (41.4 MPa) are plotted in Figure 29. These failures generally fall within the extension of the 95% confidence limits determined for end-welded cover plated beams under constant-amplitude loading.

![Figure 29 Variable amplitude fatigue tests [15].](image)

Two beams failed at a Miner equivalent stress range below the constant-amplitude threshold. These are plotted in Figure 29 as the closed dots (3 ksi) considering all stress cycles in the spectrum.
Table 5  Variable-Amplitude Stress Events Greater than 4.5 ksi (31.0 MPa) [15].

<table>
<thead>
<tr>
<th>Beam</th>
<th>$S_{\text{min}}$ (ksi)</th>
<th>$S_{\text{max}}$ (ksi)</th>
<th>$S_{\text{RMS}}$ (ksi)</th>
<th>$S_{\text{MINER}}$ (ksi)</th>
<th>$N \times 10^3$</th>
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</thead>
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<td>5.08</td>
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<td></td>
<td></td>
<td>35.0</td>
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<td>7157</td>
</tr>
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<td>16 110.3</td>
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<td>16 110.3</td>
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<td>5.08</td>
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</tr>
</tbody>
</table>

* Cracked but no failure
** All Tests from NCHRP12-12 except RCPB 401 (Lehigh)

The damage caused by stresses greater than 4.5 ksi (31.0 MPa) adequately predicted the fatigue failures for all the beams except the failures at a Miner stress range of 3.0 ksi (20.7 MPa). This indicates that at details with low average stress ranges the damage caused by those stress events less than the constant-amplitude threshold can be significant. Gurney has estimated this effect by considering an increasing crack size and the decrease in number of damaging stress cycles. This will place the data points between the extremes shown and does not appear to account for all the damage at the lower stress range.
3.7.4 Crack Shape Equations

Previous studies on welded details indicate that the mean rate of crack growth is related to the stress intensity range by the following equation.

\[
\frac{da}{dN} = 2.0 \times 10^{-10} \Delta K^3
\]  

(27)

\(\Delta K\) = stress intensity range, ksi√\(\text{in.}\).

\[
\frac{da}{dN} = \text{crack growth rate, in./cycle}
\]

Equation 28 represents a lower bound estimate for the rate of crack growth.

\[
\frac{da}{dN} = 3.6 \times 10^{-10} \Delta K^3
\]  

(28)

The calculation of stress intensity, \(K\), can be formulated in the following manner as discussed by Albrecht and Yamada.

\[K = F(a)\sigma\sqrt{\pi a}\]  

(29)

Where \(F(a) = F_E F_S F_W F_G\)

- \(F_E\) = elliptical crack front correction
- \(F_S\) = free surface correction
- \(F_G\) = stress configuration correction
- \(F_W\) = finite width correction

All of these correction factors are affected by the crack shape ratio, \(a/b\). The crack shape is especially important for details with high stress configurations. Sharp discontinuities occur frequently along a fillet weld toe due to the inclusion of nonmetallic material. Inclusions are caused by slag particles which are deposited in the melted base and weld metal. These discontinuities in the high stress configuration zone at the weld toe act
as crack development sites. The size of a typical discontinuity has been estimated to be approximately 0.015 in. (0.38 mm) long and several thousandths of an inch (hundredths of a millimeter) deep with an extremely sharp root radius of 0.0001 in. (0.0025 mm) or less.

Very little crack shape data is available for end-welded cover-plated beam details, especially for full size details and at small crack sizes. Studies have been conducted to determine the crack shape ratio at the toe of other types of fillet welds. These studies describe the major semi diameter length, b, as a function of the crack depth, a. Therefore, the crack shape ratio is simply a function of crack depth. The stress intensity correction factors indicate that the crack growth shape is dependent upon other terms such as plate thickness and weld size (18). Variations in residual stress patterns, nonmetallic inclusions, and other manufacturing conditions will have an additional effect on crack shape. Therefore, fatigue crack shapes are dependent not only on the type of detail, but also on the size of the detail.

Multiple cracks usually occur along the toe of any transverse fillet weld. Typical are the transverse weld toes of cover plates and stiffeners welded to the flange. Initially these cracks, which initiate at discontinuities, are believed to grow as single cracks. These single cracks tend to grow toward a more circular shape unless they are located in close proximity to each other. As crack growth continues these single cracks begin to coalesce. The formation of multiple cracks is random and dependent upon defect size, population, and distribution in the weldment. The crack shape ratio decreases as crack coalescence occurs along the weld toe. This results in decreased fatigue life as the crack shape approaches the edge crack condition.
In Ref. 15, crack shape data was obtained for stiffener details. The smallest crack depth found was 0.009 in. (0.23 mm). The mean relationship for single cracks (tension and compression flange) at stiffener weld toes was given as follows:

\[ b = 1.088 \ a^{0.946} \]  

(30)

\( a \) = minor semidiameter, in.

\( b \) = major semidiameter, in.

This equation is plotted in Figure 30. Only 21\% of the data base came from crack size measurements on stiffeners welded to the flange. These nine data points did not provide a large enough sample to model the single crack growth of stiffeners welded to the bottom flange.

![Figure 30 crack shape equation [15].](image)

Sixteen multiple cracks at stiffeners welded to the flange were also recorded. Including both the multiple and single crack data (tension and compression flange) listed in Ref. 15 for stiffeners welded to the flange the following equation was obtained.

\[ b = 3.284 \ a^{1.241} \]  

(31)
This equation is also plotted in Figure 30. The inclusion of multiple cracks into the crack growth equation results in a trend toward more elliptical crack shapes due to coalescence.

In a study by Maddox, crack shape data was obtained for a gusset plate with a longitudinal fillet. The cracks growing from the toe of the short transverse end of the fillet were single cracks. The mean relationship for the measured crack shapes is given as follows:

\[ b = 0.1321 + 1.29a \]  \hspace{1cm} (32)

This equation is also plotted in Figure 30. These cracks are similar to the type of cracks that would be expected at the termination of the longitudinal fillet weld of an unwelded end cover plate detail.

Crack size measurements were made on W36X230 and W36x260 cover-plated beams which had been fractured in the test program reported in Ref. 7. The observed crack sizes are listed in Table 6 and the crack shape ratio (a/b) vs. crack depth (a) is plotted in Figure 31. The measured crack shapes plotted in Figure 31 show no clear correlation with any of the crack shape equations from earlier studies.
An alternate view of the problem can be obtained by estimating the equilibrium crack shape for the cover plate detail. The equilibrium shape that a single crack tends toward can

Table 6 Crack Dimensions for Full Scale Cover-Plated Beams [15].

<table>
<thead>
<tr>
<th>Beam</th>
<th>Steel</th>
<th>Size</th>
<th>a depth in.</th>
<th>2b length in.</th>
<th>a depth mm.</th>
<th>2b length mm.</th>
</tr>
</thead>
<tbody>
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<td>B5</td>
<td>A588</td>
<td>W36X230</td>
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<td>2.41</td>
<td>13</td>
<td>61.2</td>
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<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td>0.2</td>
<td>1.42</td>
<td>5.1</td>
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All cracks in tension flange except as noted.
Cover plate thickness = 1.0 in. (25.4 mm)
Weld leg size = 0.75 in. (19.1 mm)

Figure 31 full scale beam crack shape, dots represent measurements on large cover-plated beams [15].
be approximated by calculating the stress intensity range, $\Delta K$, by Eq. 6. The crack shape is in equilibrium when $\Delta K$ is constant along the crack front. An embedded elliptical crack is in equilibrium when $a/b$ equals 1. A semielliptical surface flaw will tend toward a shape other than semicircular due to the free surface correction, $F_S$, and the stress gradient correction, $F_G$.

The free surface correction factor was assumed to be defined as a semicircular surface crack in a uniform tension field and was taken as:

$$F_S = 1.211 - 0.186 \sqrt{\sin \phi}$$

$\phi \geq 10^\circ$

$\phi$ is the angle shown in Figure 32

The crack shape correction factor, $F_E'$ is given by the following equation.

$$F_E = \frac{1}{E(k)} \left[ 1 - k^2 \cos^2 \phi \right]^{1/4}$$

$E(k)$ is the complete elliptic integral of the second kind.

$$E(k) = \int_0^{\pi/2} \left[ 1 - k^2 \sin^2 \beta \right]^{1/2} d\beta$$

where $k = 1 - (a/b)^2$

a = minor semidiameter

b = major semidiameter

Zettlemoyer developed an approximate equation for the stress gradient correction factor, $F_G$, from the toe of an end-welded cover-plated beam [33].
\[ F_G = \frac{\text{SCF}}{1 + \frac{1}{0.1473 \left( \frac{a}{T_F} \right)^{0.4348}}} \]  

(36)

where SCF = stress configuration factor

\[ a = \text{crack depth} \]

\[ T_F = \text{flange thickness} \]

The stress configuration factor was calculated at the toe of the weld for the uncracked section by the following equation.

\[ \text{SCF} = -3.539 \log \left( \frac{Z}{T_F} \right) + 1.981 \log \left( \frac{T_{cp}}{T_F} \right) + 5.798 \]  

(37)

where \( z = \text{weld leg size} \)

\[ T_{cp} = \text{cover plate thickness} \]

\[ T_F = \text{flange thickness} \]

The finite width correction, \( F_W \), was assumed to equal 1.0. The crack depths where single cracks exist are generally small enough so that the back surface correction can be neglected. Also, \( F_W \) is approximately equal to 1.0 for a large range of \( \frac{a}{T_F} \) when the crack shape is near semicircular.
An interior beam (W36X230) primary cover plate end-welded detail was selected for the study. The cover plate thickness is 1.25 in. (31.8 mm), the weld leg size is 0.5 in. (12.7 mm), and the flange thickness is 1.26 in. (32.0 mm). The stress configuration for the detail when calculated by Eq. 33 is 7.22.

Near the free surface ($\phi \leq 10^\circ$) Eq. 10 is poorly defined, therefore the variation in $K$ along the crack front was observed from $\phi$ equal to $10^\circ$ to $90^\circ$. The assumed variation in $F_G$ along the crack front is great; therefore two cases were evaluated to provide estimates of an upper and lower bound equilibrium crack shape. The crack depths for which the following results were obtained ranged from 0.02 in. (0.5 mm) to 0.50 in. (12.7 mm).

In the upper bound study the $F_G$ correction factor was set equal to 1.00, $F_S$ was defined by Eq. 10, and $F_E$ was defined by Eq. 11. The equilibrium crack shape ($a/b$) for this case was found to be $0.70 \pm 0.05$ for the range of crack sizes between 0.02 (0.5 mm) and 0.50 in. (12.7 mm). This represents an upper bound for the equilibrium crack shape.

In the lower bound study the $F_G$ correction factor was defined by Eq. 13. $F_S$ and $F_E$ were the same as in the upper bound study. The equilibrium crack shape ($a/b$) for the second
case was found to be $0.40 \pm 0.05$. This represents a lower bound for the equilibrium crack shape.

It is reasonable to assume that the initial crack sizes that correspond to the minimum fatigue life for the large cover plated beams would be in the range of 0.02 in. (0.5 mm) to 0.04 in. (1.0 mm). The large cover-plated beam crack shapes plotted in Figure 31 with crack depths less than 0.04 in. (1.0 mm) is representative of a set of initial crack sizes. During the initial stage of growth as a single crack the shape will approach the equilibrium condition.

As discussed earlier coalescence of single cracks along the crack front takes place randomly. Coalescence results in a decreased crack shape ratio ($a/b$) and a subsequent decrease in fatigue life. Coalescence takes place more rapidly along a straight weld toe than along an irregular weld toe. Single cracks at an irregular toe tend to grow out-of-plane with the adjacent single cracks, therefore coalescence is delayed.

The great variability of data points in Figure 31 represents the random nature of coalescence. Equation 38 is an approximate lower bound for the crack shapes during the coalescence phase.

$$b = 5.462 \ a^{1.133}$$

Equations 27 and 38 were used to estimate fatigue life for the primary cover plate detail of an interior beam (W36X230). Equation 27 is the mean crack growth rate relationship. An initial crack size of 0.04 in. (1.0 mm) and a stress range of 8 ksi (55.2 MPa) was used. The estimated fatigue life was 1.36 million cycles. This corresponds favorably with the fatigue lives found in Ref. 7 for similar sized cover-plated beams (see
Figure 28). The predicted fatigue life falls below the lower 95% confidence limit for the smaller W14x30 end-welded cover-plated beams reported in Ref. 16.

The lower bound estimate for the crack growth rate (Eq. 5) was also used to predict the fatigue life for the above detail. At a stress range of 8 ksi (55.2 MPa) the estimated fatigue life was 0.75 million cycles. This lower bound estimate of fatigue life corresponds favorably with the early indications of cracking for the full scale beams in Ref. 7 and the lower 95% confidence limit for cover plates wider than the flange (see Figure 28).

3.7.5 Crack Growth Threshold

The cover-plated beams which were tested in developing the AASHTO fatigue specification were rolled or welded W14X30. The design stress range threshold for Category E was estimated to be 5 ksi (34.9 MPa). The constant-amplitude fatigue tests conducted at Lehigh University on W14X30 end-welded cover-plated beams indicate a stress range threshold of 4.5 ksi (31.0 MPa).

The variable-amplitude fatigue tests shown in Fig. 3 indicate that the crack growth threshold stress range (modal, RMS, and Miner) under variable loading is below the constant amplitude threshold. Also, the root-mean-square and Miner equivalent stress range from the variable cycle tests fall within the extension of the 95% confidence limits for end-welded cover plated beams under constant-amplitude loading.

In constant-amplitude tests, Paris has shown that the stress intensity threshold, $\Delta K_{Th}$, is mean stress dependent. $R$ is defined as the ratio of minimum stress to the maximum stress. As the mean stress or $R$ ratio increases the stress intensity threshold decreases. For A533B steel at $R$ ratios of 0.1 and 0.8, $\Delta K_{Th}$ equals 7.3 ksi $\sqrt{\text{in.}}$ (8.0 MPa$\sqrt{\text{m}}$) and 2.75
ksi√in. (3.0 MPa√m), respectively. At welded details the R ratio will always be large due to the tensile residual stress field. An attempt to simulate the residual tensile stresses was made in Ref. 24 for crack growth in A36 steel. This yielded a ΔK_{Th} equal to 3.0 ksi√in. (3.3 MPa√m). Hence a value of 2.75 ksi√in. (3.0 MPa√m) appears to provide a lower bound for constant cycle loading.

The stress range, Δσ_{Th} required for crack growth was calculated for the interior beam (W36X230) primary cover plate details. Crack shapes were defined by Eq. 15 and ΔK_{Th} was assumed equal to 2.75 ksi√in. (3.0 MPa√m). A plot of Δσ_{Th} vs. crack depth is shown in Figure 33.

![Figure 33 stress range threshold for full scale beams](image)

When random variable loading occurs it appears that there is a possibility that a decrease in the crack growth threshold develops. The tests on small scale beams summarized in Figure 29 suggest this possibility. Good correlation existed when all stress cycles were assumed to contribute to fatigue crack growth.

The fatigue life and stress range threshold is very sensitive to the initial crack size. As the initial crack size increases both the fatigue life and stress range threshold decrease.
The smallest typical crack development sites are caused by slag particles deposited in the melted base and weld metal. Larger crack development sites may result from undercutting and cracking due to improper welding procedures. Environmental assisted fatigue may also contribute, to the apparent reduction in fatigue strength in the factors which affect fatigue life. If environmental effects are important. The environmental fatigue crack growth rate relationship is highly dependent on frequency. As the frequency decreases the crack growth rate will increase. Environment may also be responsible for a decrease in the stress intensity range crack growth threshold.

3.8 Summary

This section gives an overview of fatigue behavior of welded joints. The mechanisms of the fatigue damage process are discussed and the usual method models for fatigue life verification are defined. The basic parameters that important for the fatigue process in welded joints are pinpointed. The main objective the chapter is to give the reader basic insight into the fatigue process in welded joints, with emphasis on the aspects that makes these joints vulnerable to fatigue. You will be acquainted with the most common methods and models describing the fatigue problem. These methods will be elaborated further in next chapters where the S-N model and the facture mechanics approach are describe, more detail.
4.1 Introduction and objectives

As pointed out before, fatigue of welded joints is a complex problem. All theories and models have to be verified and corroborated by experimental data. As for the S-N approach, this model consists of curve-fitting between applied stress range and experimental lifetime data. A fracture mechanics model has to be based on experimental data to determine appropriate values for the parameters involved. When these parameters have been determined, the model can predict crack growth and critical crack size for similar material and environments. The only theory involved is the development of the stress intensity factor concept. In fact, all models discussed are semi-empirical models. The number of cycles to crack development, crack path history, and final fatigue life must be measured for typical steel materials, joint types, and loading conditions. Model parameters must be chosen to fit the experimental facts. In this chapter a brief overview of common testing techniques is given. Finally, some often-employed statistical methods used to deal with the scatter in experimental results are also described.

The objective of this chapter is a discussion of experimental work primarily related to the S-N approach and the fracture mechanics model. Also, some discussion of
measurement techniques and how to analyze the results is provided. Hence, life and crack
growth testing will be shown with some detail. More details are found in [32].

4.2 Common types of fatigue testing

We briefly upon the three most common types of fatigue testing:

— S-N testing using smooth specimens to characterize the base material;
— S-N testing using specimens to characterize a structural detail, e.g. a welded joint;
— crack growth testing using standard pre-cracked specimens;
— Strain-life testing with small-scale smooth specimens.

4.3 Stress-life testing (S-N testing) of welded joints

4.3.1 Test specimens and test setup

S-N evaluation is defined as testing using the stress range and detail type as the main promoters controlling fatigue life. The life, given as the number of cycles to failure, is plotted as a function of the applied stress range. Details that have nearly the same geometry, welding quality, residual stresses and loading mode define one experimental population. This experimental population forms the basis for a detail-category (class) in the building codes. A straight-line relationship between log N and log ∆σ is assumed in the finite life region, and the best-fit S-N curve is found by method of least squares linear regression analysis. As the involved scatter is one of the main characteristics for lifetime data of a welded joint, this subject will be addressed in some detail at the end of this chapter.

Hydraulic digitally-controlled testing machines are often used to subject test specimens to repetitive loading. Although it is possible to simulate variable amplitude loading, most tests are carried out using constant amplitude loading. Testing is carried out under constant amplitude loading at various stress range levels. The machine is usually in
load or displacement control mode, and it is recommended that specimen stresses are monitored by strain gauges in addition to the information provided by the machine's control panel. Regarding the definition of stresses the choices include the nominal stress range and geometrical stress range. Fatigue test specimens should usually have a geometry and a loading mode that are representative for in-service condition. Smaller specimens are tested in standard servo-hydraulic testing machines, whereas full-scale testing of larger structural parts are tested in specially built frames with adaptable hydraulic cylinders.

4.3.2 Testing Preparations and measurements

Fatigue testing includes all the phases such as test planning, the preparation of specimens and the statistical analyses of the results. It is important to keep track of all the parameters that have an influence on the test results. This is necessary to define a homogenous specimen population as a basis for the obtained design curve. If a population involves test specimens with large differences in these important parameters, the compiled test results will exhibit large scatter that can be difficult to explain and cope with. The practical matter that high-quality joints will be penalized because they have been merged with joints of lower quality. For the populations defining categories in current design rules and specifications, this is often the case and improvement in this area will lead to more accurate predictions due to reduced scatter. Thus, high-quality joints will get the fatigue life predictions they deserve but likely need fabrication standards as well. This is an important topic for industries.

The following background information should be gathered:

- Before the test:
  - steel type, chemical composition, mechanical characteristics,
- welding procedure, method, electrodes, number and sequence of passes, heat input,
- manufacturing sequence,
- global specimen geometry and local weld toe geometry,
- axial or angular distortion,
- non-destructive testing (NDT),
- estimate of residual stresses in the specimens,
- rolling direction of plates with regard to applied loading,
- microscopy of the heat-affected zone (HAZ).

Most of this information is straightforward to obtain. The chemical composition and mechanical characteristic for a C-Mn steel is given in Table 7. The data are typical for medium-strength steels with nominal yield stress close to 350 MN. This is one of the most widespread steel types used for welding. The welding has been carried out by shielded metal arc welding (SMAW). Regarding the given information for mechanical parameters, hardness measurements should be carried out after the welding has been completed. This gives important information regarding the base metal, weld deposit, and the HAZ. Figure 34 and Table 8 show typical results from hardness measurements carried out on the material in Table 7 after welding had been carried out.

**Table 7 Hardness measurement [17].**

<table>
<thead>
<tr>
<th>Spot</th>
<th>HB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base 1</td>
<td>142</td>
</tr>
<tr>
<td>Base 2</td>
<td>147</td>
</tr>
<tr>
<td>Weld 1</td>
<td>175</td>
</tr>
<tr>
<td>Weld 2</td>
<td>178</td>
</tr>
<tr>
<td>Haz 1</td>
<td>213</td>
</tr>
<tr>
<td>Haz 2</td>
<td>181</td>
</tr>
<tr>
<td>Haz 3</td>
<td>173</td>
</tr>
<tr>
<td>Haz 4</td>
<td>165</td>
</tr>
</tbody>
</table>
The local toe geometry parameters are measured by applying replica material on the weld toe and inserting cuts of replica cross sections into a profile projector with a typical magnification of 10. The replica cuts must be made transverse to the weld seam direction. Table 9 shows the results from two different test series with specimens. As can be seen, the toe profile of series 1 is much more favorable than for series 2. Series 1 has low toe angle (mean value 30 degrees) and larger radius (mean value 2.7 mm). Series 2 has a mean angle of 58 degrees and a toe radius of 0.75 mm only. The fatigue testing actually showed that the mean fatigue life of series 1 was twice as long as for series 2. If local toe geometry had not been measured, this difference in fatigue life would have been difficult to understand and explain.

Table 8 Chemical composition in % and mechanical characteristics for a C-Mn steel [16].

<table>
<thead>
<tr>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cu</th>
<th>Ni</th>
<th>Cr</th>
<th>Mo</th>
<th>Nb</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>0.15</td>
<td>1.4</td>
<td>0.006</td>
<td>0.002</td>
<td>0.01</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
<td>0.008</td>
</tr>
</tbody>
</table>

Yield strength [Mpa]  Tensile strength[Mpa]  Elongation %
416  501  25

Figure 34 Results for hardness measurements for cruciform plated joint (C-Mn steel in Table 7) [17].

The local toe geometry parameters are measured by applying replica material on the weld toe and inserting cuts of replica cross sections into a profile projector with a typical magnification of 10. The replica cuts must be made transverse to the weld seam direction. Table 9 shows the results from two different test series with specimens. As can be seen, the toe profile of series 1 is much more favorable than for series 2. Series 1 has low toe angle (mean value 30 degrees) and larger radius (mean value 2.7 mm). Series 2 has a mean angle of 58 degrees and a toe radius of 0.75 mm only. The fatigue testing actually showed that the mean fatigue life of series 1 was twice as long as for series 2. If local toe geometry had not been measured, this difference in fatigue life would have been difficult to understand and explain.
When the global and local geometry and possible distortion have been measured, a finite element analysis (FEA) should be carried out to determine the stress configuration factor at the weld toe which is the potential crack locus. A result of such an analysis is shown in Figure 35. The model is using a refined mesh of the weld profile and plates. This is the weld toe area of the test specimen analyzed under plane strain conditions. As can be seen the stress configuration factor (SCF) is close to 3.0.

![Figure 35 Results from a local FEA model for a cruciform fillet-welded joint [17].](image)

Non-destructive testing should be carried out at the same level as is typical for details entering into service conditions. Specimens with flaws that would have been rejected should also be excluded from the test series. The most frequent method is magnetic particle inspection and no crack detection is the only acceptable result. Estimating the actual
residual stresses is probably the most challenging part of the preparation work. In general, small specimens will have lesser residual stresses than larger specimens. If the specimens are properly stress relieved, the residual stresses may be set to zero. If not, they can be measured by boring a hole and measuring the change in stresses in vicinity of the hole by strain gauges. This will reveal the residual stress gradient. The problem is that the residual stresses vary over the volume of the test specimen. Finally, metallographic study of the HAZ will reveal any abnormal phases or grain size. During the test the following should be measured and registered:

— loading mode, load range and nominal stress range,
— the applied R-ratio for the applied stresses,
— load frequency,
— stresses by strain gauges monitoring,
— crack growth measurements by an electrical method,
— beach marking of the fatigue surface by temporarily decreasing the stress range,
— number of cycles to final failure.

The loading mode is usually uni-axial loading or bending. One should be aware of the fact that most S-N data are compiled without taking into account the difference between these two modes; it is only the maximum stress range that is assumed to matter. Most specimens are subjected to a positive load ratio to avoid problems with buckling. Typical values of R are in the range of 0.1 to 0.3. The loading frequency is assumed to play a minor role in air environment, and most tests are accelerated so that the test program will not consume too much time. However, special care should be taken for the environmental condition. In such cases there is an interaction between the fatigue damage process related
to the oscillating stresses and the time-dependent electro-chemical reaction at the crack front. Therefore, test frequency becomes important. The load frequency should usually not exceed 1 Hz.

Detailed knowledge of the strain distributions can be obtained by the use of electrical resistance strain gauges. These gauges operate on the principle that the resistance of a wire changes when its length changes due to stretching or compressing. Hence, when bonded to the surface of the specimen, the local strain at the surface of the material is captured. Some test specimens should be equipped with strain gauges to reveal the actual strain in vicinity of the weld toe during the test. Based on these measurements, the strain configuration can be determined and also the related stress configuration. The SCF obtained from FEA can be verified by these measurements in the hot-spot region near the assumed fatigue crack location. The stress should account for the gross stress configuration due to the global geometry of the joint, but not the local effect caused by the weld profile. The gauge distance from the weld toe should be large enough to avoid the local effect of the toe itself, typically 10-15 mm. The strain measurements are normally compared to stresses obtained by FEA analysis. FEA obtained stresses were shown in Figure 35.

Local strain distribution for a tubular joint is shown in Figure 36. The distribution is obtained with a crack of uni-axial gauges with 0.3 mm length and 2 mm spacing. As can be seen, the strains measured at the chord are higher than the measurements at the branches. Furthermore, the gauges closest to the toe are affected by the weld notch as their reading deviates from a linear distribution. If a straight line is drawn through the other points, a hot-spot strain level close to $\varepsilon_{HS} = 450 \mu\varepsilon$ is found at the weld toe on the chord side. If the
strain is assumed to perpendicular to the weld seam, the strain configuration factor is defined as:

\[ S_{NCF} = \frac{\varepsilon_{HS}}{\varepsilon_N} \]  

(39)

where \( \varepsilon_N \) is the nominal strain in the branch. If this hot-spot strain coincides with the direction of the principal stress at the hot-spot, the following transformation can be used between SCF and SNCF:

\[ SCF = S_{NCF} \frac{1 + v\varepsilon_2/\varepsilon_{HS}}{1 - v^2} \]

(40)

where \( \varepsilon_2 \) is the measured strain parallel to the weld seam. Again, the SCF value based on measurements can be compared with the stress configuration found by FEA or by parametric formulae, [34]:

\[ SCF = 0.595\gamma^{0.6} \tau^{0.8}(1.6\beta^{0.25} - 0.7\beta^2)\sin^{(1.5-1.6\beta)}\theta \]

(41)

where:

\[ \beta = \frac{d}{D} \quad \gamma = \frac{D}{2T} \quad \tau = \frac{T_B}{T_C} \]

(42)

The parameter \( \theta \) is the brace inclination angle in radians, in our case \( \pi/2 \). The SCFs for both the brace and the chord side are to be multiplied by the nominal stresses in the brace. The stress configuration can be further verified by the use of acrylic model. This has led to a clearer definition of applied hot-spot stress.
4.3.3 Testing results

Traditional fatigue life testing produced one result only: the number of cycles to failure. For small test specimens the failure is defined when a final fracture separates the specimen into two parts. For larger joints the failure state is a question of definition. It can be defined as a fracture separating the joint into two pieces or by a through-thickness crack. Consider, for example, a high-strength weldable steel material for mooring chains. For these small specimens, the failure occurred when the crack was less than 1 mm. As can see, the 18 tests are carried out at three stress levels, six tests at each level. The tests were carried out in seawater without any protection against corrosion and the load frequency was 1 Hz. Some results in air are added for comparison. The mean and the design curves are drawn. When this steel is used in a larger structural item, it must be born in mind that

Figure 36 Strain measurements at brace and chord at welded intersection (strain gauges length 0.3 mm spacing 2mm) [18].

![Graph showing strain measurements](image)
the life data in Figure 37 corresponds to early cracking of the item and to not final failure. Therefore, if the curves in Figure 37 are used directly as life data for a structural item, it will be un-conservative.

![Figure 37 Results from S-N testing with small specimens of high-strength steel [20].](image)

Although the fatigue durability, given as the number of cycles to failure, is the most important result, one may say that great effort is made, but only limited information is produced. In the last decade, the use of electrical methods has made it possible to obtain more information from the tests. Crack growth measurements are carried out at different time intervals during the test to reveal the entire crack growth history, and not only the number of cycles to failure. This is particularly interesting for full-scale tests where crack growth plays a dominant role. The alternating current potential drop (ACPD) method has been used more and more frequently the recent years. A crack monitoring system based on this principle can be fully automated; an example is shown in Figure 38 for a fillet welded joint. A constant AC current field is injected into the test specimen surface at a frequency
of 6 kHz. The surface voltage distribution is measured by a crack micro-gauge that receives the signals from fixed pin probes spot welded to the test specimens. A micro-computer controls both the micro-gauge and the test machine loading unit through a multiplexer switching unit. The test data is displayed graphically on the computer and stored for further analysis. The specimen has 10 logged stations equally spaced over the weld length. A detailed description of the system is given in [35]. If the voltage measured over pins straddling the surface crack is $V_{cr}$ and a nearby reference voltage is designated $V_r$, an estimate of the crack depth reads:

$$a_1 = \frac{e}{2} \left( \frac{V_{cr}}{V_r} - 1 \right)$$  \hspace{1cm} (43)

where $e$ is the probe distance. The measurements can be calibrated against crack-front marking by blue ink penetrant injected at a chosen crack depth for each specimen. True crack depth can then be obtained from the fracture surface reading after the final failure. These readings also give valuable information about crack-shape development the crack depth versus crack length. If several true crack depths are wanted on the same specimen, ink of different colors may be applied at different times. The stress range reduction is preferably done by increasing the minimum stress.

![Diagram of ACPD measurements](image)

Figure 38 ACPD measurements [21].
Typical a-N curves obtained from ACPD measurements are shown in Figure 39 for the cruciform joint. It is obvious that these curves bring much additional information, both on number of cycles to early cracking and crack growth rates.

![Graph showing a-N curves](image)

Figure 39 Entire crack history in cruciform joints before final fracture is reached [22].

After a test has been carried out, fatigue and fracture surface microscopy should be carried out on some specimens. By applying a scanning electron microscope, it may be possible to trace initial flaws or intrusions that act as starters for crack development.

4.4 Testing to determine the parameters in the strain-life equation

We have established the equation for the strain-life, equation (24). The tests are carried out with smooth, small-scale specimens instead of full-scale welded joints. The test could, in fact, be carried out on the specimens that were used to obtain the results given in Figure 39 if the tests were carried out under strain control. These kinds of tests determine the four fatigue parameters given in equation (24). The parameters b and c are the fatigue
strength and ductility exponents, and \( \sigma_f' \) and \( \varepsilon_f' \) are the fatigue strength and ductility coefficients respectively. To determine these parameters, the total local strain range is divided into one elastic and one plastic part. It can be shown that the number of reversals to failure can be written both in terms of the elastic strain range and the plastic strain range by the equations:

\[
\frac{\Delta \varepsilon_e}{2} = \frac{\sigma_f'}{E} (2N_f)^b
\]

\[
\frac{\Delta \varepsilon_p}{2} = \varepsilon_f'(2N_f)^b
\]

We have assumed zero mean stress. These two equations correspond to the two straight lines for a log-log scale in Figure 40. The sum of them corresponds to equation (24). The mean stress correction was included. Test results can be plotted on the two formats and linear regression can be carried out to determine \( b \) and \( \sigma_f' \) from the elastic line and \( c \) and \( \varepsilon_f' \) from the plastic strain line. The parameters are indicated on Figure 40.

Figure 40 Principal sketch for determining fatigue parameters in strain-life relationship [22].
The test results can be used to describe the fatigue nucleation at the weld toe when the initiation phase becomes important. Finally, it should be mentioned that it is a common approximation to relate the parameters in equations (44) and (45) to the hardness measurements shown in Table 8.

4.5 Crack growth tests; Test setup and specimen monitoring

Crack growth testing is based on the hypothesis that it is the stress intensity factor range (SIFR, ΔK) governs the crack growth rate. The SIFR uniquely determines the local severe stress field ahead of the sharp crack front under linear elastic conditions. Crack growth tests are usually carried out on standard test specimens for which the SIF (stress intensity factor) can be determined with great accuracy. One typical test specimen is the compact tension (CT) specimen shown in Figure 41.

\[ \Delta K \]

Figure 41 Compact tension specimen for crack growth measurements [21].

The specimen is fabricated with specified dimensions (W and H) that are given once the thickness T of the specimen is chosen. As can be seen, the specimen has a sharp prefabricated notch from which the fatigue crack will grow. Definition of the loading is shown in Figure 42. The load F varies with constant amplitude between its maximum and
minimum value. The corresponding SIF can be calculated and the SIFR is defined as the difference between them. The evolution of the SIFR with time is shown in Figure 43. As can be seen, there is a slight increase in $\Delta K$ for constant $\Delta F$. This increase is due to the increase in crack length.

The crack depth is measured as a function of applied number of cycles, as shown on the left-hand side of Figure 44. The measurements can be made optically or by an ACPD technique, as we have just described, [35]. At chosen stages, the crack growth rate $\Delta a/\Delta N$ in m/cycle is calculated and plotted against the actual range of the stress intensity factor $\Delta K$. This range has to be based on the mean crack size during the observation period $\Delta N$. The procedure is shown more detailed in Figure 44 where one point in the log $a/\Delta N$-log $\Delta K$ diagram is determined to the right on the figure. By following the same procedure, at several stages along the crack path, the crack growth rate parameters can be obtained as shown in Figure 45 for a log-log scale.
As can be seen from Figure 45, the results fall into three different regions depending on the magnitude of $\Delta K$. At low $\Delta K$ there is a threshold value $\Delta K_0$ below which a crack will stop to propagate. Furthermore, there is an almost a linear relation between $\frac{da}{dN}$ and $\Delta K$ for a log-log scale in an intermediate region. It is in this region that the results obey the Paris law. The slope $m$ and parameter $C$ defined by the intercept with the $\frac{da}{dN}$ axis are obtained. Again, we see from the figure that the linear relation for a log-log scale is an approximation. Hence, a linear regression analysis has to be carried out in this region in the same way as for the S-N data. At higher values of $\Delta K$, the maximum value of $K$ is close
to the critical value of what the material at the crack front can sustain. The consequence will be unstable fracture. As a result of our hypothesis that the SIFR governs the growth rate, components with various geometries, but made of same material, may have different a-N curves, but when they are transformed into a da/dN-ΔK diagram, as in Figure 45, the curves will coincide. This is true provided the tested components have the same loading ratio R and the same similitude environment. The great benefit is then that the curve in Figure 45 can be applied to predict the crack growth rate for various types of crack in any component if we are able to calculate the SIFR for the crack and component geometry in question. Hence, the da/dN-ΔK diagram and related equations are of great importance.

Figure 46 shows the test result for crack growth tests carried out in seawater. The CT specimens with 25 mm thickness are taken from high-strength steel used in mooring chains. Steel bars are forged and flash welded to obtain chain links from this type of steel. The results to the right in the figure pertain to a test in air, but other specimens were submerged in the small seawater basin as shown to the left in the figure. The tests were carried out with an applied SIFR in the range between 10 and 30 MP $\sigma_m^{0.5}$. As can be seen, this gives growth rates from $10^{-5}$ to $10^{-4}$ mm/cycle. To reveal the threshold value, the testing has to be carried out at smaller SIFR. Typical values for $\Delta K_0$ are found between 2 and 8 MPam$^{0.5}$, dependent on the environment, [36]. The threshold value can be defined when the rate da/dN is less than $10^{-7}$ mm/cycle.
Figure 46 Test results with high-strength steel [16].

Figure 47 shows the final fracture of the CT specimen. This will occur when either the net ligament ahead of the crack front is too small (global criterion) or the crack front has too high local stresses (local criterion). In the latter case, the final fracture will be of the brittle type.

Figure 47 Final fracture of a CT specimen after crack growth [24].
4.6 Introduction to Elementary statistical methods

As linear regression is important to almost all of the types of tests that we have described, we will briefly outline the basic equations for this method. However, natural to include linear regression and clarify statistical uncertainty related to test results in the present chapter. Details for the method are found elsewhere, [37].

4.6.1 Linear regression analyses

We have shown how the number of cycles N to failure is a nearly linear function of the applied stress range $\Delta \sigma$ for a log-log scale. Likewise, the growth rate $da/dN$ is almost a linear function of $\Delta K$ for a log-log scale with averaged values of $\Delta K$ in the actual intervals. However, both relationships are approximations and there are considerable deviations from the straight line approximations. Consider the general problem by letting a variable $y$ be dependent on a variable $x$ as shown in Figure 48.

![Figure 48](image)

Figure 48 the problem of an approximately linear relationship [25].
The mean value of y is given as a function of x by the straight line, but for each data point there is a deviation $E_i$. For a given x value, the y value is in fact a random variable designated by the bold letter y. The following equations apply:

$$E(y|x) = A + Bx$$

$$y_i = A + Bx_i + E_i$$

Where the parameter A is the intercept where the regression line cuts the y-axis, whereas B is the slope of the line. The $E_i$ gives the deviation from the mean regression line for data point “i”. The parameters A and B can be estimated by the least squares method:

$$\hat{B} = \frac{s_{xy}}{s_x^2}$$

$$\hat{A} = \mu_y - \hat{B}\mu_x$$

The correlation factor between y and x is defined as:

$$\rho = \frac{s_{xy}}{s_x s_y}$$

A high factor indicates good correlation for example if $\rho = 1.0$ then all the data points will actually be found on a straight line. The following statistical parameters are needed

$$\mu_x = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\mu_y = \frac{1}{n} \sum_{i=1}^{n} y_i$$

$$s_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \mu_x)^2$$
Where \( n \) is the total number of data points. The uncertainty in the prediction can be estimated as the square sum of the residuals:

\[
Q^2 = \sum_{i=1}^{n} (E_i)^2
\]

If the \( E_i \) values are taken to be independent, then \( y \) is normally distributed for a given \( x \). As a consequence, it can be shown that \( Q^2/s^2 \) is chi-square distributed for \( n - 2 \) degrees of freedom. The variable \( s \) is defined as the standard deviation of \( y \) for a given \( x \). Hence, the point estimate for \( s \) reads:

\[
\hat{s}^2 = \frac{Q^2}{n - 2}
\]

The confidence level for \( s \) can be determined from the chi-square distribution:

\[
P \left( s^2 \leq \frac{(n - 2)\hat{s}^2}{X^2_{1-\alpha,n-2}} \right) = 1 - \alpha
\]

Where \( X^2_{1-\alpha,n-2} \) is the tabulated value of the chi-square distribution which is exceeded with a probability of \( 1 - \alpha \). The standard deviation for \( y \) is used to define percentile curves for \( y \) given \( x \). This is done by subtracting a certain number \( k \) of Standard deviations from the right-hand side of equation (46). Hence, we use the equation:

\[
y_k = A - ks + Bx
\]

There will be a known probability for obtaining a lower value than \( y_k \), as shown Table 10.
The stress range is the explaining variable, whereas the number of cycles is the result variable. The S-N presentation is made with log N at the horizontal is, but this does not change our consideration for the explaining and the result variables. The design rules usually apply S as $k = 2$, or the mean curve minus two standard deviations. Hence, from Table 10, it can be seen that this corresponds to a probability of failure of 2.3%. The standard deviation has often been based on the point estimate for $s$. However, there has been a shift in the last decade to use 1.5 standard deviations and to require a $1 - \alpha = 75\%$ confidence level. This will penalize curves obtained from few data points. This is shown in Table 11 where the 75% confidence estimates are listed as Function for the point estimate, $\hat{S} = s_{50}$ for various numbers of test specimens in the test series.

The columns to the far right hand side of Table 11 show the 1.5 standard deviations based on the 75% confidence level. These values are often used to construct design curves.
for fatigue life. As can be seen the difference between (2 s_{50}) and (1.5 s_{75}) is small when the number of specimens are small.

4.7 Summary

This chapter is a discussion of experimental work primarily related to the S-N approach and the fracture mechanics model. Also, some discussion gain some knowledge on measurement techniques and how to analyze the results is provided. Hence, life and crack growth testing will be shown with some detail.
5.1 Introduction and objectives

The determination of the loading and related stress histories that a structural detail will be subjected to during service life plays an important role in achieving a reliable fatigue life estimate. It must be borne in mind that the number of cycles to failure is a function of the stress range level raised to a power of at least 3 for welded steel joints. This means that if the stresses are underestimated by, say, 20%, the fatigue life will be close to 100% overestimated. There are three major topics regarding service life stress history:

- the loading scenarios have to be assumed at the design stage for say 20 years into the future;
- the loading is often random by nature, e.g. wave loading for structures at sea;
- the response calculation and stress analysis leading from the applied forces to the stress response in the vicinity of a weld is often based on approximations.

The objective of the present chapter is that there is an understanding of the importance of defining the fatigue stress spectrum for a given time period.
5.2 Constant amplitude loading

Very few structures are actually subjected to constant amplitude loading during service life. The importance of constant amplitude loading is related to fatigue testing in the laboratory environment. Hence, the goal that laboratory tests should emulate realistic service condition as closely as possible is often not practical. The best approach would be to subject the structural detail in question to the actual variable amplitude load history that will occur during service. The problem is that there are various possible load histories for the details found in different types of structures. It is usually too laborious and costly to simulate all load histories with variable amplitude. There are few examples of important details where a service variable load history has been standardized and actually used testing. The inner wing section of transport airplanes is such an example. In most cases the engineer is left with fatigue test results that are based on constant amplitude loading; defined in Figure 49.

![Figure 49 Definition of constant amplitude loading](image)

Figure 49 Definition of constant amplitude loading [28].
The two most important parameters are the stress range $\Delta \sigma$ and the load ratio $R$:

$$\Delta \sigma = \sigma_{max} - \sigma_{min}$$  \hfill (56) \\

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$  \hfill (57)

The choice of carrying out almost all fatigue testing at constant amplitude leaves the engineer with the problem of how the obtained fatigue resistance determined under such loading could be "translated" into the more realistic variable amplitude loading conditions. Often the Miner summation rule is employed and is based on linear interaction assumption.

5.3 Variable amplitude loading

5.3.1 Overview

A typical in-service load history may appear as is given in Figure 50.

![Figure 50 Stress time series under variable amplitude loading](image-url)

As can be seen, the time series is quite irregular and random, although there are some dominating frequencies. We have two possibilities of how to treat this stress history:
- Times series cycle counting: the time series consist of several stress ranges which can be counted and presented in groups with the same stress range. The most common counting method is the so-called rain-flow counting technique. The results can be given in a histogram or a diagram of exceedance. This gives a clear definition of the load spectrum. The corresponding fatigue damage can then be estimated by Miner's summation rule.

- The energy spectrum approach: under stationary loading conditions, the time series can be split into harmonic components. The analysis is carried out using a Fourier transformation technique. The stress history can then be characterized entirely by the parameters defining the related energy spectrum. Both approaches are equally important and we shall summarize the methods in the following sections.

5.3.2 Rain-flow cycle counting of time series

If one looks at the stress time series in Figure 50, it is obvious that defining a complete cycle and its stress range is not straightforward. One problem is that the stresses appear in half cycles between reversals. A reversal is defined as a point where the first derivative changes signal valley or a peak. As can be seen, a given half cycle may contain smaller half cycles. The counting can become rather involved and we need some guidance on how to undertake it. As a general rule, large stress cycles must not be fragmented into smaller ones as this will lead to an underestimation of the fatigue damage. Small stress cycles should be regarded as temporary interruptions of larger stress reversals. Let us take the stress history in Figure 51 as an example.
The stress cycles can be counted by the so-called rain-flow counting method. If the time axis is oriented vertically downwards, one could imagine that the stress history forms a number of "pagoda roofs" on which the rain is running over. Cycles are then defined by the manner in which the rain is allowed to drip or fall down the roofs. A number of peculiar rules are imposed in order to carry out the counting. To computerize this type of cycle counting we will define a simpler rule based on the work by Dowling, [38]. Let us analyze the two cases shown in Figure 52. The left-hand case will not define a stress cycle, but the right-hand case will. The right-hand case will define a cycle due to the fact that, after having reached valley 2, one passes the former peak 1 on the way up again. This occurs at the point 2’. It does not occur in the left-hand case.

Figure 51 Stress time series [30].
We also see from the right-hand side of the figure that the defined cycle 1-2-2' (or briefly cycle 1-2) is characterized by its:

- stress range $\Delta \sigma$;
- mean stress $\sigma_m$ (or R-value).

Our simple counting rule is now as follows:

- when the stress history passes through a valley, a cycle will be counted when the stress level passes the former peak on its way up; and vice versa,
- when the stress passes through a peak, a cycle will be counted when the stress level passes the former valley on its way down.

The argument for this rule is that external load cycles will correspond to a closed hysteresis loop for the local stress-strain found at the weld notch. More details in this field are referred to, [38, 39]. Method 3 applicable for high cycle fatigue as treated by the S-N approach in rules and specification. We obtain what is essential for life calculations:
number of cycles at given stress range levels. In Figure 54 we have applied the technique for the complete stress history in Figure 53.

![Diagram showing cycle counting](image)

Figure 53 Cycle counting for a stress time series Ref [38]

As can be seen from Figure 53[A], we start by counting cycle 2-3-2' according to the given rules. When the stress rises from point 3 towards point 4, we notice that the former peak 2 is passed. Hence, cycle 2-3-2' is defined. To make the subsequent counting easier, we remove cycle 2-3-2' before proceeding. We then count cycle 5-6-5' as shown in Figure 53[B]. Then this cycle is removed and stored before we count cycle 7-8-7' in Figure 53[C]. Finally, the large cycle 1-4-9 is counted in Figure 53[D]. The procedure gets easier if one starts and finishes with the extreme absolute values for the stresses, in our case point 1 and 9. If the time series does not start and stop with the extreme values it should be rearranged. The counted cycles are given at the bottom of Figure 53. As can be seen, the time series
contains four complete cycles. The small cycles have been identified and registered without missing the large cycle 1-4-9. When time series are long, the cycle counting must be computerized and cycles are counted each time the ranges of two following reversals satisfy the inequality, [40]:

\[ |\sigma_i - \sigma_{i-1}| \geq |\sigma_{i-1} - \sigma_{i-2}| \]  

where \( i \) is the index identifying the reversal.

The stress range and mean stress for the identified cycle are given by:

\[ \Delta \sigma_{i-1} = \sigma_{i-1} - \sigma_{i-2} \]  

\[ \sigma_{m,i-1} = \frac{\sigma_{i-1} - \sigma_{i-2}}{2} \]  

When a cycle is identified and counted it is removed from the time series and stored, as we have shown graphically in Figures 54. The counting procedure subsequently starts at point 1 and is repeated until all the cycles have been registered. We can use the algorithm given by equation (60) and (61) to count the cycles in Figure 54. Figure 55 shows the stress-strain hysteresis loops pertaining to the counted cycles.

Figure 54 Material stress-strain response at the weld for the given stress history Ref [38]
For a longer time series, the results can be presented in a histogram. It may also be convenient to fit a probability density function (frequency function) to the histogram. One frequency function that is widely used in such cases is the Weibull model:

\[
f(\Delta \sigma) = \frac{h}{q} \left(\frac{\Delta \sigma}{q}\right)^{h-1} e^{\left(\frac{\Delta \sigma}{q}\right)^h}, \Delta \sigma \geq 0
\]

\[
f(\Delta \sigma) = 0, \Delta \sigma \geq 0
\]

where \(q\) is the scale parameter, whereas \(h\) is the form parameter. In Figure 55 the frequency function to the left pertains to \(h = 2.0\), whereas the function fitted to the right pertains to \(h = 1.0\). The first type of presentation is often used for short-term fatigue loading description defined by a stress time series with some hours of duration.

The right-hand diagram in Figure 55 illustrates a long-term stress distribution. The long-term distribution must be based on statistics where all possible missions and loading conditions are included. For weather-induced loading, all seasons must include at least one year's statistics. As can be seen for this case, this function will predict many low-stress ranges close to zero that pertain to less severe conditions.
One appealing feature of the Weibull load model is that if one uses the frequency function in the Miner summation rule in conjunction with the S-N method, a closed-form solution for the damage ratio accumulation may be found. The scale parameter \( q \) can be related to the maximum stress range occurring during the time series. The maximum stress range is found by extreme value statistics for the \( n \) or \( N \) cycles in the time series. The relation is:

\[
q = \frac{\Delta \sigma_{\text{max}}}{(\ln n)^{1/h}}
\]

5.3.3 Energy spectrum approach

The energy spectrum approach is faster and easier to use than the more cumbersome, stress-cycle counting method. On the other hand, in some cases, one may lose some accuracy in the defined stress spectrum approach as compared to cycle counting results. The obtained stress cycles are not directly related to closed hysteresis loops for the stress-strain behavior in the material as was the case for the rain-flow counting technique. The spectrum approach can only be used if we have stationary loading conditions. The method is based on the assumption that any stationary time series can be split into an infinite number of harmonic components. Hence, the time series in Figure 50 can be written in the form of:

\[
\sigma(t) = \int_{0}^{\infty} \sigma_{\omega} \sin(\omega t + \varphi_{\omega}) d\omega
\]

where \( \sigma_{\omega} \) is the amplitude of the harmonic stress component that has a frequency \( \omega \). The parameter \( \varphi_{\omega} \) is a random phase distortion between the various harmonic stress components. The task is to find the amplitudes \( \sigma_{\omega} \), as a function of \( \omega \). This type of analysis can be carried out by a Fourier transformation technique, [41, and 42] for more details see
Figure 56. We have assumed a process with zero mean stress as it is only the stress amplitude or ranges that are our concern. As can be seen, the practical result is that we have left the time-axis presentation and replaced it by a frequency axis. The energy in the stress spectrum is given as a function of the frequency to the right in the figure. The energy in a narrow frequency band is proportional to $1/2 \sigma_\omega^2$ of the corresponding harmonic stress component. Hence, the energy spectrum is defined by:

$$S(\omega)d\omega = \frac{1}{2}\sigma_\omega^2 d\omega$$

(65)

A typical shape for $S(\omega)$ can be as shown in Figure 56.

![Energy spectrum for a stress time series](Ref [41])

From the obtained spectrum we can make the definition of the moment function as follows:

$$m_n = \int_0^\infty \omega^n S(\omega)d\omega$$

(66)

where $m_n$ is a general moment function of the spectrum. In particular, $m_0$ corresponds to the area under the spectrum curve and equals the total energy in the load spectrum according to equation (65). From the second-order moment the average up-crossing time period can be determined as follows:
The stress level deviation from the zero mean value at an arbitrary time can be shown to follow a normal distribution as follows:

\[
f(\sigma) = \frac{1}{2\pi s_\sigma} \exp\left(-\frac{1}{2} \left(\frac{\sigma}{s_\sigma}\right)^2\right)
\]  

(68)

The stress history is said to be a Gaussian process with a zero mean and a standard deviation designated \(s_\sigma\). This standard deviation of the process can be related to the energy of the spectrum in the following way:

\[
S_{\sigma}^2 = m_0
\]  

(69)

As we have seen before, we are not in particularly interested in the arbitrary stress level given in equation (68), but in the stress amplitudes or ranges. It can be shown that if the energy spectrum is narrow band of frequencies is dominating, the stress ranges are Rayleigh distributed:

\[
f(\Delta \sigma) = \frac{\Delta \sigma}{4m_0} \exp\left(-\frac{\Delta \sigma^2}{2\sqrt{2m_0}}\right)
\]  

(70)

The Rayleigh distribution is a special case of the Weibull distribution that we studied earlier in equation (61-62). The Weibull distribution may be written in the following form:

\[
f(\Delta \sigma) = \frac{h}{q} \left(\frac{\Delta \sigma}{q}\right)^{h-1} \exp\left(-\left(\frac{\Delta \sigma}{q}\right)^h\right)
\]  

(71)

This equation is identical to equation (70) when:

\[
h = 2 \quad \text{and} \quad q = 2\sqrt{2m_0}
\]  

(72)

Hence we can obtain the Weibull distribution for the short-term stress ranges pertaining to a stress time series either by cycle counting as was shown before. The mean

\[
T_z \approx 2\pi \sqrt{\frac{m_0}{m_2}}
\]  

(67)
value of the 1/3 largest stress cycles is as the significant stress range level and can be written as follows:

\[ \Delta \sigma_{\text{sign}} = 4\sqrt{m_0} \]  \hspace{1cm} (73)

For marine structures with wave-induced loading, a linear relation between wave height and stresses in the structure is often assumed. Hence, the definition in equation (73) will correspond to the significant wave height which, in addition to the mean up-crossing period \( T_z \). The corresponding probability of exceeding a given level of stress range is found by integration of equation (70) as follows:

\[ Q(\Delta \sigma) = \exp \left( -\frac{\Delta \sigma^2}{2}\sqrt{\frac{2}{m_0}} \right) \quad (74) \]

If we separate the one-third of the stress cycles that contains the largest ones, the mean value of this group is by setting \( Q(\Delta \sigma) = 1/n \), where \( n \) is the total imber of cycles in the series. We then obtain:

\[ \Delta \sigma_{\text{max}} = 2\sqrt{2m_0 \ln n} \quad (75) \]

5.4 Summary

In this chapter a brief overview of common testing techniques is given. Also, treats the definition and description of the fatigue load spectrum. Accuracy in applied loading description is crucial for the credibility of fatigue life results. Both the time series approach and the energy spectrum approach are presented. Some often-employed statistical methods used to deal with the scatter in experimental results are also described. The main objective of this chapter is a discussion of experimental work primarily related to the S-N approach and the fracture mechanics model. Also, some discussion gain with measurement
techniques and how to analyze the results is provided. Hence, life and crack growth testing will be shown with some detail.
CHAPTER VI
METHODOLOGY

6.1 Methodology of the RFL Model

The RFL model was developed by Pascual and Meeker in 1999 [23]. Their method was “motivated by the need to develop and present quantitative fatigue-life information used in the design of jet engines” [23]. Although Pascual and Meeker specifically noted the staircase method as an “efficient and effective way of estimating the median fatigue limit,” they noted that “it is not used to estimate the stress-life relationship” [23]. However, this section provides a brief overview of the RFL model in order to lay the groundwork for the test design analysis performed in the next section.

6.1.1 Genealogy of the RFL Model

The genealogy of the model traces its lineage through Nelson’s work in 1984 [24]. Nelson modeled the fatigue life of a nickel-based superalloy, creating probabilistic S-N curves incorporating non-constant standard deviation in fatigue life and using censored data (runouts) through a maximum likelihood approach. Standard models for fatigue curves before Nelson’s work typically assumed a lognormal distribution in fatigue life for a given stress level. The standard deviation of this lognormal fatigue life was generally assumed to be constant, as illustrated in Figure 57. Thus, the distribution in fatigue life is the same for
both low-cycle and high cycle regimes. However, fatigue-life data have shown generally more scatter in the high cycle (low stress) regime for many materials, leading to Nelson’s analysis.

Nelson developed S-N models using non-constant standard deviations as a function of the stress level. He assumed a linear form for $\mu$, the mean fatigue life, as a function of stress (represented by the 50% line in Figure 57), with a log-linear standard deviation $\sigma$ of fatigue life, as shown below where $a$ and $b$ are parameter constants used to fit the data, $LPS$ is the log pseudo-stress, and $LPS^*$ is the mean of the log pseudo-stress values [24]:

$$\sigma(S) = e^{(a + b \cdot (LPS - LPS^*))} \tag{76}$$

In addition to the linear $\mu$ model, he used a quadratic form for $\mu$, also in conjunction with the log-linear form for $\sigma$ shown above. The quadratic form was more useful for S-N data which exhibit curvature. An illustrated pictograph of the quadratic $\mu$, non-constant $\sigma$ and P-S-N model is shown in Figure 58. The fatigue model based on these assumptions provided a generally better fit for the nickel-based superalloy data, although a problem was observed in that some percentiles of the P-S-N curve may be greater (in terms of fatigue
life) at intermediate stresses than at low stresses. This situation is of course impossible as fatigue life cannot increase as stress increases, but the increasing standard deviation as stress decreases may cause the modeled curves to “bend back” too far for some data sets.

The RFL development also built upon the work of Hirose in 1993 [42], who used maximum likelihood methods to estimate the fatigue limit of polyethylene terephthalate (PET) films as well as their mean life under service stress. Hirose fitted a Weibull inverse power relationship that included a fixed fatigue life parameter (constant). In addition, Nelson’s text [3] suggested that the fatigue limit may be considered a random parameter,
such that specimens have different fatigue limits which make up a “strength distribution.” This suggestion would later generate interest in modeling using a random variable for fatigue limit.

Before developing the RFL model, Pascual and Meeker presented a model in 1997 [43] incorporating a constant fatigue limit parameter (similar to Hirose) using non-constant standard deviation of a lognormal fatigue life (similar to Nelson), applying this model to the nickel-based superalloy data of Nelson [24]. The fatigue data is described with \( x_1, x_2, \ldots, x_n \) denoting pseudo stress levels of \( n \) specimens, and \( Y_1, Y_2, \ldots, Y_n \) denoting the associated numbers of cycles tested. The fatigue limit is given as \( \gamma \). For \( x_i > \gamma \), fatigue life \( Y_i \) was modeled as log normally distributed such that \( \log(Y_i) \) was distributed normally with mean \( \mu(x_i) \) and standard deviation \( \sigma(x_i) \). The form of the mean fatigue life and the standard deviation of fatigue life are thus given as [43]:

\[
\mu(x_i) = E[\log(Y_i)] = \beta_0^{[\mu]} + \beta_1^{[\mu]} \log(x_i - \gamma), x_i > \gamma
\]

\[
\sigma(x_i) = \sqrt{\text{Var}(\log(Y_i))} = \exp[\beta_0^{[\sigma]} + \beta_1^{[\sigma]} \log(x_i)], x_i > \gamma
\]

In this fixed fatigue limit model, \( \beta_0^{[\sigma]}, \beta_1^{[\mu]}, \beta_0^{[\sigma]}, \beta_1^{[\sigma]} \), and \( \gamma \) are unknown parameters to be estimated from the fatigue test data. The \( \beta \) constants are without restrictions, however \( \beta_1^{[\sigma]} < 0 \) corresponds to a decreasing standard deviation as stress increases (generally the case with most fatigue data). The size of the fatigue limit parameter (\( \gamma \)) determines the amount of curvature in the \( S-N \) plot. The plot approaches linearity as \( \gamma \) approaches zero. Large values of \( \gamma \) indicate significant curvature. The use of this fatigue limit parameter allows a “physically appealing alternative” to the quadratic form of Nelson’s model of \( S-N \) curvature, according to Pascual and Meeker [43]. This model still may exhibit the “bend
back” problem for some data sets as seen with Nelson’s quadratic model, but Pascual and Meeker note that this effect was not observed in the range of the superalloy data, and is minimized compared to the quadratic model. The simulation work performed to evaluate this model used constant interval stress levels with one data point per stress level. The primary shortcoming of the model, as noted by Pascual and Meeker, was the assumption of a constant fatigue limit. By using a single valued parameter to describe the fatigue limit, the model requires that \( \gamma \) is less than the lowest stress tested such that \( \log(x_i - \gamma) \) is defined, regardless of whether the specimen at the lowest stress level failed or not. As Annis and Griffiths noted [44], this model thus caused the \( \gamma \) asymptote to be “so low as to produce an unrealistic material model that had to be continually revised downward to accommodate newer, low stress data.” The incorporation of a variable fatigue limit based on the concept of Nelson’s “strength distribution” led to the refinement of this model, which then became the RFL model proposed in 1999 [23].

6.1.2 Important Formulation RFL Model

The RFL model accounts for the two main trends observable in most \( S-N \) data using engineering materials; namely, the increase in fatigue life scatter as stress level is decreased, and the curvature associated with a fatigue limit. The formulation of the model is shown in Equation 79 using the notation of Annis and Griffiths from 2001 [44] rather than the original notation used by Pascual and Meeker [23], which is less conventional for fatigue analysis. The fatigue life for each specimen tested is by \( N \) and the associated stress level is by \( S \). Fatigue life for specimen \( i \) is then modeled by the following equation:

\[
\log(N_i) = \beta_0 + \beta_1 \log(S_i - \gamma_i) + \epsilon_i, S_i > \gamma_i
\]  

(79)
In this equation, $\beta_0$ and $\beta_1$ are curve coefficients, $\gamma_i$ is the fatigue limit of specimen $i$, $\epsilon_i$ is an error term associated with specimen $i$, and $\log$ denotes natural logarithm. Unlike the constant fatigue limit formulation of Equation 77, the fatigue limit used in Equation 79 is a random variable. Note that the error term $\epsilon_i$ is the random life variable associated with scatter from specimens which have the same value for fatigue limit $\gamma$.

The logarithm of the random variable for fatigue limit $\gamma$ is also a random variable, and if $V$ is defined such that $V = \log(\gamma)$, then Pascual and Meeker assume $V$ to be distributed with probability density function (pdf) given by [23]:

$$f_V(v, \mu_\gamma, \sigma_\gamma) = \frac{1}{\sigma_\gamma} \phi_V \left( \frac{v - \mu_\gamma}{\sigma_\gamma} \right) \quad (80)$$

In this equation, $\mu_\gamma$ and $\sigma_\gamma$ are location and scale parameters for the distribution of $\gamma$, respectively, and $\phi_V$ may be the standardized smallest extreme value (sev) or normal pdf. Next, they let $x = \log(S)$ and $W = \log(N)$ so that $x$ and $W$ are the logarithms of the stress and fatigue life, respectively. Then for $V < x$ (i.e., the fatigue limit is less than the stress level tested), they assume that $W$ given $V$ (as $W | V$) has a pdf of the form [23]:

$$f_{W|V}(w, \beta_0, \beta_1, \sigma, x, v) = \frac{1}{\sigma} \phi_{W|V} \left( \frac{w - [\beta_0 + \beta_1 \log(\exp(x) - \exp(v))]}{\sigma} \right) \quad (81)$$

In this equation, $\beta_0 + \beta_1 \log(\exp(x)-\exp(v))$ acts as a location parameter and $\sigma$ acts as a scale parameter. $\phi_{W|V}$ may be the standardized sev or normal pdf. The marginal pdf of $W$ is then given by [23]:

$$f_W(w; x, \theta) = \int_{-\infty}^{x} \frac{1}{\sigma \sigma_\gamma} \phi_{W|V} \left( \frac{w - \mu(w, x, \theta)}{\sigma} \right) \phi \left( \frac{v - \mu_\gamma}{\sigma_\gamma} \right) dv \quad (82)$$
where $\theta = (\beta_0, \beta_1, \mu, \gamma)$ and $\mu(w, x, \theta) = \beta_0 + \beta_1 \log(\exp(x) - \exp(v))$. Finally, the marginal cumulative distribution function (cdf) of $W$ (the logarithm of fatigue life) can be given by [23]:

$$F_W(w; x, \theta) = \int_{-\infty}^{x} \frac{1}{\sigma_w} \phi_{w|v} \left( \frac{w - \mu(w, x, \theta)}{\sigma} \right) \phi_v \left( \frac{v - \mu_v}{\sigma_v} \right) dv$$

(83)

where $\phi_{w|v}$ is the cdf of $W$ given $V$. This rather complicated formulation is the statistical representation of the RFL model. Pascual and Meeker note that there are no closed-form solutions for the density and distribution functions of the fatigue life, or specifically, $W = \log(N)$. However, numerical means can be used to evaluate these equations.

It is important to note that there are two random variables in the model described by Equations 43 through 47 which have been specified through a probability distribution. The error term $\varepsilon$ which represents the scatter in fatigue life can be adequately modeled by the lognormal distribution for many engineering materials (and thus, the logarithm of fatigue life is normal). Then, the conditional distribution for cycles to failure ($W = \log(N)$) given $\gamma$ ($V = \log(\gamma)$) will be a lognormal distribution with mean $\beta_0 + \beta_1 \log(S - \gamma)$ and standard deviation $\sigma_\varepsilon$, such that $\varepsilon$ is lognormal($0, \sigma_\varepsilon$) [44]. As for the distribution of the random variable $\gamma$, the Weibull distribution is an adequate choice for describing the skewed downward (towards lower stress levels) strength distribution of many engineering materials [44]. The Weibull distribution introduces two parameters, namely the location parameter $\eta$ and the scale parameter $\beta$, which correspond to the location and scale parameters $\mu_\gamma$ and $\sigma_\gamma$, respectively, used by Pascual and Meeker. When the RFL model incorporates these assumptions, it includes five total parameters ($\beta_0, \beta_1, \sigma_\varepsilon, \eta, \text{and } \beta$).
6.1.3 The RFL Model Parameters

Unlike conventional S-N analysis in which all specimens are tested until failure, an ordinary least-squares approach cannot be used to estimate the parameters of the probabilistic S-N model used in the RFL approach. Ordinary least-squares fitting, although popular amongst analysts since its formulation by Gauss and having a well-established place in statistical analysis of fatigue experiments, does not have the capability to account for partial information data points (runouts). In addition, an ordinary least-squares approach assumes constant variance, which of course may not be the case for S-N data [44].

The approach taken by Pascual and Meeker [23] to solve the parametric estimation problem is the use of maximum likelihood methods, as described by Nelson for data with runouts [3]. First, the likelihood function is defined for data tested at stress levels $x_i = \log(S_i)$ and cycles of $w_i = \log(N_i)$ with $n$ samples. The likelihood is given by:

$$L(\theta) = \prod_{i=1}^{n} [f_{W}(w_i; x_i, \theta)]^{\delta_i} [F_{W}(w_i; x_i, \theta)]^{1-\delta_i};$$  \hspace{1cm} (84)

where

$$\delta_i = \begin{cases} 
1 & \text{if } w_i \text{ is a failure} \\
0 & \text{if } w_i \text{ is a runout} 
\end{cases}$$

Thus, the likelihood function $L(\theta)$ can be interpreted as the probability of observing the experimental data given a set of model parameters $\theta$. The set of parameters which maximizes this value of likelihood is taken as the best-fit set of parameters, and thus a curve fit is accomplished. In practice, a log-likelihood function is generally used so that terms may be added rather than multiplied. Maximizing the log-likelihood function
produces the same set of parameters as maximizing the likelihood function, and thus either function may be used. The log-likelihood function is shown below:

\[
\mathcal{L}(\theta) = \log[L(\theta)] = \sum_{i=1}^{n} \mathcal{L}_i(\theta)
\]

(85)

The likelihood problem has already been modeled and solved by Pascual and Meeker for combinations of normal and sev distributions for the logarithm of fatigue life and the logarithm of the fatigue limit [23].

6.2 Investigation of S-N method Approach

6.2.1 Introduction and objectives

When designing a structure against the fatigue-limit state, the aim is to ensure, to an acceptable level of probability, that the integrity of the structure is satisfactory throughout its planned service life. The structure should be unlikely to fail from fatigue or to require extensive repair of damage caused by fatigue, [47]. To carry out this type of safe life analysis, we must verify the fatigue life for each critical welded detail in the structure. Fortunately, even large, complex structures may often be divided into a relatively low number of elementary joints. The fatigue strength for these elementary joints must be known from experience or laboratory testing. Based on laboratory tests, the most common type of analysis for fatigue life prediction is based on the S-N method. For each critical joint it is verified that the predicted fatigue life (PFL) is superior to the target service life (TSL) of the structure, and there should be with a required safety margin.

As outlined, the S-N curves are based on a simple relationship between applied stress range \(\Delta S\) and the number of cycles \(N\) to failure. For constant amplitude loading, the curves are assumed to be linear for a log-log scale until the curves reach a fatigue-limit. For a
stress range below this limit it is assumed that the fatigue life is in finite. Each curve is established from laboratory tests with a given type of welded detail with a given geometry, fabrication quality, environmental, and loading condition. The loading is usually uni-axial with constant amplitude. In the finite life area above the fatigue limit, the curve is determined by linear regression. When the detail is subjected to variable amplitude loading in service, some type of damage accumulation formula has to be applied.

The main results from this analysis are the design S-N curves presented in design rules and specifications. It is an advantage that who use these curves know the experimental background for the curves and how the numerical analyses are carried out to define the design S-N curve. Most design rules and specifications only specify the design curve, whereas the mean value and the standard deviation pertaining to each curve are hidden from the user. We think that this is a pity because this information would give the design engineer more insight into the fatigue problem they are dealing with. Furthermore, design rules and specifications allow for testing to establish S-N curves for special details not found in the regulations. In such cases the statistical analysis must be carried out by the engineer before obtaining the design S-N curve, [48, and 49].

6.3 Method, assumptions and important factors

6.3.1 Statistics for the S-N approach, median and percentile curves

The problem of scatter in observed fatigue life has already been treated where details on how to carry out a linear regression analysis were outlined. Based on the obtained parameters, the total fatigue life at a given stress range level is assumed to be a stochastic variable \( N_T \) which is lognormal distributed. The probability that \( N_T \) is less than a life \( N_T \) is given by:
\[ P(N_T < N_T) = \Phi \left( \frac{\log N_T - \mu_{\log N}}{s_{\log N}} \right) \]  \tag{86}

where \( \mu_{\log N} \) and \( s_{\log N} \) is the mean and standard deviation for \( \log N_T \) at the given stress range level. The standard deviation \( s_{\log N} \) is assumed to be constant when the stress range changes. The mean value \( \mu_{\log N} \) is strongly dependent on the applied stress level:

\[ \mu_{\log N} = \log A_0 - m \log \Delta \sigma \] \tag{87}

The probability of failure at different \( N_T \) can be found from tables for the standard normal distribution. The curve pertaining to a probability of failure of 0.5 (\( \Phi(0) \)) is designated the median curve and reads as follows:

\[ N_{T0} = \frac{A_0}{\Delta \sigma^m} \] \tag{88}

where \( N_{T0} \) (\( \log N_{T0} = \mu_{\log N} \)) is the median fatigue life defined by a 50% probability of survival.

This corresponds to the median S-N curve in Figure 60. The design curve is often defined by the logarithmic mean curve minus two standard deviations. According to equation (86) this gives 43(-2). This corresponds to a probability of failure of 2.3%. In building codes this design curve is normally given directly in a logarithmic format, without explicitly applying equation (86), as:

\[ \log N_{T2} = \log A_2 - m \log \Delta \sigma \] \tag{89}

When \( N_{T2} \) it is corresponds to a probability of failure of 2.3%, as discussed above. If the consequences of fatigue failure are unacceptable, this probability of failure may be reduced by subtracting more than two standard deviations for obtaining the constants in equation (89) before predicting the fatigue life. Alternatively, equation (89) can be applied without changes, but an additional safety factor is demanded on the predicted fatigue life.
$N_{T2}$ compared to TSL. This approach is the most common one in design rules and specifications. As $N_{T2}$ is widely used in fatigue design it will be designated simply as N in what follows.

6.4 S-N curves-important factors

6.4.1 The threshold phenomenon

The bilinear S-N curve has a horizontal lower-line segment corresponding to the fatigue limit under constant amplitude (CA) loading. It is assumed that if the applied constant stress range is below this limit, no damage will ever occur; the detail will be able to withstand an infinite number of loading cycles. There are very few data that support this assumption. It is possible that the long lives observed at these low stress ranges are due to a long crack development period. It may not be a threshold phenomenon. We accept the fatigue limit as prescribed in design rules and specifications. However, the assumption of
the existence of a fatigue-limit has to be modified when the detail is subjected to variable amplitude (VA) loading.

6.4.2 Summary of stress range and loading stress ratio

The S-N curves for welded joints have just one key parameter to the fatigue life: the stress range. For a machined component, the S-N curve is a function of a given stress ratio $R$ in the way that a low stress ratio is favorable with respect to fatigue life. If some of the stress variation is partly in the compressive side ($R < 0$), this will not contribute to the fatigue damage to the same extent as variation on the tensile side. It is only tensile stress variation that will open crack and contribute to damage. The reason for not taking the $R$ ratio into account for welded joints is because large tensile residual stresses are usually present in the joint when they are in as-welded (AW) condition. This is the most common case for joints in welded structures. Hence, an external load that causes partly compressive stress variation will inflict an entirely cyclic tensile stress response in such a joint when superimposed on large, static, residual tensile stresses. As can be seen from Figure 61, the maximum stresses may approach the yield stress $\sigma_y$ of the material in such cases.
6.4.3 Stress relieving

If the joints are stress relieved, it may be appropriate to take advantage of the fact that compressive stress variation is less damaging than tensile stress variation. The usual recommendation is to multiply the compressive part of the stress range by a reduction factor of \( f = 0.6 \).

6.4.4 Some relevant joint geometry parameter

There is a reduction in fatigue life for the same applied nominal stress range if the thickness of the plate is increased. This effect can be caused by:

— a notch effect an increase in the stress configuration factor at the weld toe;
— a scale effect the local stress field at the crack tip of, say, a 1-mm-deep crack will be more severe in a thick plate than in a thin plate;
— a statistical volumetric effect larger material volume will have greater probability of containing flaws;
— a metallurgical effect the steel microstructure of welded thicker plates, may be of poorer quality than that of thinner plates.
To understand the first effect one must bear in mind that fatigue starts as a local phenomenon, usually near the notch of the weld toe. The stress configuration at this notch is strongly dependent on the weld toe geometry the toe angle $\theta$ and the toe radius $\rho$. It can be shown that the SCF (stress configuration factor) is particularly sensitive to the ratio between the toe radius $\rho$ and the plate thickness $T$. When $\rho/T$ decreases, the SCF increases and, consequently, fatigue life is reduced. Both the initiation and propagation parts of the life are reduced. The problem is due to the fact that when the plate thickness is increased, the variable toe radius is not controlled. The toe radius is related to the geometry of the weld pass, close to fusion line. This radius is likely to have the same value regardless of the chosen plate thickness. Hence, the $\rho/T$ ratio will inevitably decrease when $T$ increases.

This phenomenon is also observed for machined components. When we design a shaft we must take account for this effect. If the diameter $D$ of the shaft is increased it is good fatigue design practice to increase any notch radius correspondingly to avoid a decrease in $\rho/D$ and the corresponding increase in the SCF. The strategy is not possible for welded joints unless post-weld treatment is carried out. The weld notch can then be machined so the toe radius follows the thickness increase.

The scale effect points to the fact that the stress field in the front of a crack will more severe in thicker plates than thinner plates for a given crack depth $a = 1$ mm. This is clearly shown by the fact that the stress intensity factor (SIF) for a crack is a function of $a/T$ and not the absolute value of $a$. The SIF determines the stress field at the crack front. The stress increase due to the thickness effect, often found in design rules and specifications:

$$SCF = \left( \frac{T}{t_{ref}} \right)^k$$

(90)
where $T$ is the actual thickness of the joint in question, whereas $t_{\text{ref}}$ is the reference thickness. The reference thickness is representative for the joints to which the S-N data belong. Usually the reference thickness is close to 25 mm and the parameter $k$ may vary from 0.25 to 0.33 according to various design rules and specifications.

There is also a size effect related to increased length of the weld seam. The explanation of this effect is that the probability of having an extremely unfavorable weld toe geometry increases as the weld length increases. The flank angle $\theta$ and the toe radius $\rho$ of the weld are, in fact, random variables along the weld seam and the effect of the length can be demonstrated by extreme value statistics. It is also likely that residual stresses are higher in larger joints.

6.4.5 Fabrication Imperfections Misalignment

Because of fabrication imperfections, both out-of-plane distortion and angular distortion may appear in welded structures, or may be an argument for testing full-scale where practical; see Figure 62.

![Figure 62 Out-of-plane distortion (left) and angular distortion (right)](Ref [49])

These distortions give rise to secondary stresses. If, for example, the welded detail that has an out-of-plane-distortion (see Figure 62 on the left) is subjected to axial loading, the eccentricity $e$ will cause secondary bending and a stress configuration given by the following approximation, [47, 50]:

138
\[ SCF = 1 + \frac{3e}{T} \]  

6.4.6 Welded joints Post-weld improvement techniques

Improvement techniques quality of welded joints are characterized by a favorable global geometry and gradual transition at the weld toe. Furthermore, it is essential that the potential crack loci are proven to be crack free and that they have a smooth surface after fabrication. This can only partly be controlled by the choice of the welding procedure, whereas it can be controlled by various post-weld treatment methods. Various types of improvement techniques, [51]:

- burr grinding;
- TIG dressing;
- hammer peening.

The first two methods can be classified as weld profile improvement techniques. In the first case, this is obtained mechanically; in the second case it is obtained by re-melting the weld toe area. The hammer peening technique is the building in of compressive residual stresses in the weld toe area of the joint. The local toe burr grinding is illustrated in Figure 63. The grinding is carried out in such a manner that the weld profile is improved and sharp crack-like defects are removed. The grinding should be up to 1 mm deep and the head of the grinding tool should have a diameter of at least 5 mm for a 20-mm-thick plate. If the plate thickness is increased, the tool head should be increased correspondingly for the reasons that were explained for the thickness effect. Non-destructive inspection could be carried out after grinding in cases where buried cracks have been brought to the surface. Additional grinding must then be carried out. A weld that has been ground will have less
stress configuration and a smoother surface. Initially, the grinding procedure was the only post-weld treatment accepted to in design rules and specifications to give increased life. However, in the new guidance given DNV, [52], both hammer peening and TIG dressing are assumed to increase fatigue life. Specifications on how to carry out the improvement techniques are found in [51].

6.4.7 Corrosive environment

When welded joints are subjected to repetitive loading in a corrosive environment there is a synergy effect between the mechanical-fatigue damage process and the electro-chemical corrosion process. The corrosion may result in surface pits that shorten the crack development period. Furthermore, the corrosion process aggravates the condition within a crack near the crack front and may therefore significantly speed up the growth rate. Welded structures in seawater should always have some sort of corrosion protection. This is usually provided by cathodic protection and/or protective coatings.

The influence of cathodic protection on the fatigue behavior of a welded structure depends, in a complex manner, on the interaction of mechanical, chemical, and electro-chemical parameters that affect both crack development and crack propagation. The best way to improve fatigue performance is to delay the crack development for as long as
possible. Smooth-shaped welds, post-weld improvements, and maintenance of a moderate
cathodic potential is the best way to fight the synergy effect. During crack growth, a
moderate cathodic potential is the best measure to avoid growth acceleration. A cathodic
potential between -850 and -1,050 mV with reference to a standard AgCl cell is preferable.
For high-strength steel, cathodic protection may inflict damage mechanisms such as
hydrogen embrittlement. Informative articles on these issues are found in [52, 53]. The
principal differences in fatigue resistance between the air environment and cathodic
protection (CP) and free corrosion (PC) are shown schematically in Figure 64. As can be
seen, it was previously believed that the cathodic protection gave the same fatigue life as
for a joint in dry air. However, it was assumed that the fatigue limit was lost and the curves
were drawn with the same slope all the way down towards zero stress range. More recent
research has proven that it is the other way around; at high stress levels fatigue life under
cathodic protection is close to 2.5 times shorter than in air, whereas for low stress ranges
cathodic protection is very efficient. At small stress ranges, fatigue life is very close to the
life found in dry air and the assumption of a fatigue limit for CP is acceptable. The reason
for this behavior is that the corrosion process may blunt the crack front at low stress ranges
and leave deposits in the wake of the crack front. The last effect may lead to crack closure.
In FC, the curve gives significant shorter fatigue lives at all stress levels, as can be seen in
Figure 64.
6.5 Mathematics of damage calculations

6.5.1 Linear damage accumulation

S-N curves are based on CA test data, whereas a welded detail when appearing in a structure usually will be subjected to variable amplitude loading. Based on cycle counting it is often possible to represent the stress spectrum on a histogram format, in terms of stress blocks where each block is defined by its stress range $\Delta\sigma_i$ and corresponding number of cycles $n_i$; see Figure 65. The associated histogram is shown to the left on Figure 65 with five stress blocks. The spectrum can be valid for a given period of time or be representative for the entire service life. In the first case we are talking about a short-term load spectrum, in the latter case a long-term load spectrum.

The problem then arises that we do not have the fatigue strength for the detail when subjected to the load spectrum in question; we have to use the S-N curve that is based on CA data only. In other words, the fatigue behavior under constant amplitude has to be used to evaluate behavior under variable amplitude loading to make life predictions. To cope with this problem it is usually assumed that each stress block contributes to the fatigue
damage according to its damage ratio $n_i/N_i$. The nominator $n_i$ is the number of cycles actually occurring, whereas the denominator $N_i$ is the number of cycles to failure according to the S-N curve for the actual stress range. It is further assumed that the total damage caused by all stress blocks accumulates linearly.

Figure 65 Cycle of variable amplitude loading Ref [52]

Figure 66 Load spectrum given in six stress blocks and corresponding fatigue live Ref [52]
The damage accumulation is then given by the Miner summation rule:

\[ D = \sum_{i=1}^{k} \left( \frac{N_i}{N_i} \right) \leq 1 \]  

(92)

where \( N \) is calculated from equation (89) at the actual stress range \( \Delta \sigma_i \). The failure criterion is \( D = 1.0 \). If the histogram pertains to a time span \( L \) (say one year 3ng-term load spectrum), then the predicted fatigue life (PFL) will read:

\[ L_P = \frac{PFL}{D} = \frac{L}{D} \]  

(93)

The fatigue design factor (FDF) is defined by:

\[ FDF = \frac{PFL}{TSL} \]  

(94)

This is a safety margin that comes in addition to the safety margin inherent in the design S-N curve. For a given Target Service Life TSL (say 20 years), the required FDF will be a result of the consequences of fatigue failure and assessment of whether or not the detail in question is accessible for inspection and repair. Various rules have different requirements as to this issue.

6.5.2 Linear damage accumulation

All the damage calculations presented in this chapter are based on the assumption of linear damage accumulation. The validity of this assumption has often been questioned. As more variable amplitude-sting data have become available it has been shown that the chronological order of e stress blocks is important. The standard case is that a stress block with a low stress range may be lower than the CA fatigue-limit and thus does not contribute to fatigue damage. Stress block 1 in Figure 66 is such an example. However, if this stress block appears after several of the other more severe stress blocks, these blocks may have
created a crack and the fatigue limit is no longer valid. The detail has become more vulnerable to fatigue damage and stress block 1 may now contribute to fatigue damage. As a consequence, an S-N curve with a CAFL cannot be used in the high cycle regime area for variable amplitude loading. A conservative approach is to neglect the fatigue limit altogether and draw a line from the finite-life area down towards a zero stress range without changing the inverse slope m. However, based on Haibach, [55], a curve has been drawn to give reasonable results when applying Miner's rule for common stress spectra. The slope of the curve is set to 2 m-1, where m is the slope pertaining to the fatigue data in the finite-life area. This approach has been adopted in design rules and specifications. However, it is worth mentioning that the approach is a rather crude approximation, and the slope of the lower line will in fact be dependent on the cycle-counting method and on the type of load spectrum, [56]. For various cases of VA loading, the actual damage ratio D at fracture was recorded. It was found that the failure criterion of D = 1.0 has only 10% confidence and it is over 90% probable that the specimen subjected to stress blocks of different stress range levels will fail before Miner's sum reaches 1.0. The median value was in fact close to 0.3 and the 90% confidence level was as low as 0.1. Hence, there is a factor of 10 between the damage sum pertaining to the 10% confidence level and the sum pertaining to the 90% confidence level. This factor is used as a measure for the scatter band for the failure criterion. The large value pinpoints uncertainty in Miner's criterion. The investigation involved all types of metallic materials; no results peculiar for welded joints were given. For steel material only, Miner's sum was used both as a criterion for early cracking and as a criterion for final failure. In the first case, the number of cycles to early cracking is determined by the local strain approach, whereas the failure criterion is based on
conventional S-N curves. It was found that the nominal stress approach had a median value of $D = 0.18$ with a scatter band of 9.2, whereas the local stress strain approach had a median value of $D = 0.48$ and a scatter of 12.3. Hence, the local strain approach is closer to the theoretical median value of 1.0, but has a larger scatter. The conclusion is that uncertainty in Miner’s summation is greater than previously assumed.

6.6 Definition of the equivalent stress range

The Miner’s summation rule can also be applied by using the so-called equivalent stress concept. This is a fictitious, constant stress range $\Delta \sigma_e$ that, when applied with the total number of load cycles $n$, by definition causes the same Miner’s sum as the actual stress spectrum. The following equation applies:

$$\sum_{i=1}^{i=k} \frac{n_i}{A_2 \left(\Delta \sigma_i \right)^m} = \frac{n}{A_2 \left(\Delta \sigma_e \right)^m}$$

(95)

and for a bilinear curve:

$$\Delta \sigma_e = \left[ A_2 \left( \frac{1}{A_{2u}} \sum_{i=j}^{i=j} n_i \Delta \sigma_i^{ml} + \frac{1}{A_{2u}} \sum_{i=k}^{i=k} n_i \Delta \sigma_i^{mu} \right) \right]^{1/m}$$

(96)

where $j$ is the number of stress blocks with stress ranges below the knee point of the VA S-N curve, whereas $j + 1$ to $k$ are the number of stress blocks with stress ranges above the knee point of the S-N curve. The VA S-N curve is the fully-drawn curve $D$ to the right in Figure 66. The parameters $A_2$ and $m$ are kept at $A_{2u}$ and $m_u$ if the equivalent stress range is above knee point. Otherwise, they are set equal to $A_{2l}$ and $m_l$. The fatigue life can now be verified directly by using the equivalent stress range as the key to the S-N curve.
Alternatively the equivalent stress range can be compared with the fatigue strength of the S-N curve for a given number of cycles.

6.6.1 Load spectrum on the format of a Weibull distribution

If the stress histogram with stress blocks as shown to the left in Figure 66 is fitted to a Weibull distribution, the Miner's sum will read:

\[
D = \int_{\Delta\sigma = 0}^{\infty} \frac{nf(\Delta\sigma)d\Delta\sigma}{A_2/\Delta\sigma^m} \tag{97}
\]

where \( f(\Delta\sigma) \) is the frequency function fitted to the histogram and \( n \) is the total number of applied loading cycles. The Weibull model frequency function reads:

\[
f(\Delta\sigma) = \frac{h}{q} \left( \frac{\Delta\sigma}{q} \right)^{h-1} \exp\left[-\frac{\Delta\sigma}{q}\right] \tag{98}
\]

where \( h \) is the shape parameter and \( q \) is the scale parameter in the distribution. The Weibull distribution can be obtained both by cycle counting and by the energy-spectrum approach. The integral in equation (5.12) can be solved by introducing the auxiliary variable \( t = (\Delta\sigma/q) \) The integral can then be determined by using the well-known Gamma function:

\[
D = \frac{n}{A_2} q^m \Gamma(1 + \frac{m}{h}) \tag{99}
\]

where \( \Gamma(*) \) denotes the Gamma function. This function can be found in standard tables and Excel spreadsheets. Equation (99) is valid for single slope S-N curves. In case of a bilinear S-N curve the damage ratio will read:

\[
D = \frac{nq^{m_u}}{A_{2u}} \Gamma\left(1 + \frac{m_u}{h}; \left(\frac{S_0}{q}\right)^h\right) + \frac{q^{m_l}}{A_{2l}} \gamma\left(1 + \frac{m_u}{h}; \left(\frac{S_0}{q}\right)^h\right) \tag{100}
\]

where:

\( A_{2u}, m_u = \text{fatigue parameters for the upper S-N line segment} \)
$A_21, m_1 = \text{fatigue parameters for the lower S-N line segment}$

$\Gamma (x,y) = \text{complementary Gamma function}$

$\gamma (x,y) = \text{incomplete Gamma function.}$

The scale parameter $q$ can be related to the most likely maximum stress range $\Delta \sigma_0$ occurring during a given number of cycles:

$$q = \frac{\Delta \sigma_0}{(\ln n_0)^{1/h}}$$  \hspace{1cm} (101)

The most likely maximum stress range has by definition a probability of exceedance equal to $1/n_0$. The number of cycles need not be equal to $n$ in equation (97), but the number must be large enough to characterize the loading process so that the scale parameter $q$ becomes constant. If this expression is inserted into equation (99) or (100) and $D$ is set equal to a target value, the permissible limit for the maximum stress range can be found. This leads to the simplified approach where the fatigue-limit state can be checked by an extreme load cycle provided that the shape parameter $h$ for the long-term load spectrum is known. With this simplified approach, the fatigue-limit state criterion can be checked as easily as yield $\alpha$ or buckling criteria. This is very convenient at an early design stage; limitation on admissible stress range in the extreme load cycle can be given without going into detailed fatigue analyses that includes less severe loading cycles. These conditions are in fact accounted for by the choice of the shape parameter $h$. A high value for the shape parameter $h$ (say 1.1), indicates that the extreme stress cycle is accompanied by many other severe stress cycles. A low value of $h$ (say 0.7) indicates that the extreme load cycle is accompanied by quite a number of low stress range cycles.
So far we have assumed that one single Weibull distribution is valid for a long-term load spectrum. In other cases, several distributions must be used to characterize several short-term periods during service life. In this case the total damage accumulation is found as the sum of the damage accumulated in each of the steady state short-term periods:

\[
D = \frac{n}{1} \sum_{i=1}^{N_{\text{load}}} P_i \left[ \frac{q_i^{m_i}}{A_{2u}} \Gamma(1 + \frac{m_i}{h_i}) \left( \frac{S_0}{q_i} \right)^{h_i} + \frac{q_i^{m_i}}{A_{2l}} \Gamma(1 + \frac{m_i}{h_i}) \left( \frac{S_0}{q_i} \right)^{h_i} \right] \tag{102}
\]

where:

- \( N_{\text{load}} \) = the total number of short-term load spectra considered
- \( P_i \) = fraction of design life in load short-term condition
- \( h_i \) = Weibull stress range shape parameter for load condition no \( i \)
- \( q_i \) = Weibull scale parameter for the load condition no \( i \)
- \( n \) = number of load cycles during service life.

the knowledge of the shape parameter \( h_i \) is crucial for the calculation. The short stress spectrum can also be determined the energy-spectrum approach. The scale parameter \( q \) for the Rayleigh distribution can in this case be related to the variance of the process:

\[
q = 2s\sigma^{h_i/h} \tag{103}
\]

Hence equation (99) that is valid for a one-slope S-N curve will read:

\[
D = \frac{n_0}{A_2} (2s_\sigma)^m h_i \Gamma(1 + \frac{m}{h_i}) \tag{104}
\]

Or with \( h = 2, s_\sigma^2 = m_0 \) and \( q = (2\sqrt{2m_0}) \) we get:

\[
D = \frac{n_0}{A_2} (2\sqrt{2m_0})^m \Gamma(1 + \frac{m}{2}) \tag{105}
\]

For a bi-linear S-N curve, equation (100) can be written in a similar form as equation (105).
6.7 S-N curves related to various stress definitions

Fatigue life estimates based on S-N curves were carried out using the nominal stress range as the key parameter to the fatigue life. The nominal stress was defined as the principal stress in the plate at some distance from the discontinuity of a welded attachment and the weld bead itself. The geometry of the attachment and the weld did not affect the applied nominal stress level. Furthermore, no distinction was made between membrane forces and pure plate bending. The nominal stresses for the two conditions are shown in Figure 67.

![Figure 67 Definition of nominal stresses for a welded detail under in plane tension and plate bending Ref [52]](image)

The approach may seem awkward uses simple mechanism to determine stress range, and one may argue that it is more logical to apply the stress range actually occurring at the potential crack locus in the notch area for example at the weld toe where the crack is drawn in the Figure 67. It is the magnitude of this stress range that acts as an agent for the fatigue damage process; consequently it should be used as the key to fatigue life predictions. It is a temptation to compare the situation with the man who walked under a lamppost arching for a €50 note that he had lost. He searched underneath the lamppost not because he had lost the note there, but because the light made it easier to look for. It is true that the nominal stress range can be easily determined by simple calculation for a test specimen. However, when a joint is appearing in a welded structure, stresses are usually calculated by finite
element analysis (FEA) and it may in fact be a problem to define the actual magnitude of the nominal stress for the welded joint.

Based on these considerations, it has been a trend during the last decade to use geometrical stress range $\Delta\sigma_g$, as the key parameter to fatigue life instead of the nominal stresses. This approach explicitly takes into account the stress magnification at the potential crack locus caused by the global geometry of the structural detail. The notch effect of the weld bead is not included. If one takes one step further and includes the latter effect also, one can apply the local weld notch stress range $\Delta\sigma_w$, as a key to fatigue life. As already argued, this is the vehicle that drives the damage process. Both the stress configuration effect due the global geometry of the joint and the notch effect caused by the geometry of the weld are accounted for. They are no longer inherent and hidden in the S-N curve as is the case for the nominal stress approach. Due to the current interest and need to apply the geometrical and the weld notch stress, we shall discuss and define these stresses in the following sections. We will also show how stresses can be used for fatigue life predictions.

6.7.1 Nominal stress, geometrical stress and weld notch stresses

We will consider a plate with a gusset that is welded to the plate, as shown in Figure 68. The plate is subjected to stresses in the same direction the orientation of the gusset. The plate thickness is $T$, and the length and height of the gusset are $L$ and $H$ respectively. The local geometry of the weld bead is given by the toe angle $\theta$ and the radius $\rho$. We know from experience that the potential crack locus for this detail is the weld toe at the end of the gusset. The potential crack line is dotted as A-A in Figure 68. We have chosen the pure membrane loading mode, but the considerations that follow are also true for the bending loading mode.
There are three characteristic stresses for the detail, see Figure 68:

— $\sigma_g$ - (top of section A-A) geometrical stress at the potential crack development locus;

— $\sigma_w$ - (top of section A-A) weld notch stresses at the potential crack development locus.

— $S = \sigma_u$ - (section B-B) the in main plate at some distance from the weld nominal stress;

It is only the nominal stress $S$ and the notch stress $\sigma_n$ that actually appear in the material; the geometrical stress $\sigma_g$ is only a theoretical definition. The relation between the nominal stress, and the two other stresses $S = \sigma_n$ are given by the next equations:

\[
\sigma_g = K_g S \tag{106}
\]

and:

\[
\sigma_w = K_w \sigma_g \tag{107}
\]

and, therefore:
\[ \sigma_k = K_g K_w S = K_t S \]  

(108)

\( K_g \) = the geometrical stress configuration
\( K_w \) = the notch stress configuration.

All of these equations are built on the assumption that the stresses are in the linear elastic region of the material behavior. The first stress configuration given by \( K_g \) is due to the effect of the gusset alone. The gusset is a geometrical disturbance that increases the stress from the nominal value \( S \) to the geometrical stress \( \sigma_g \) at the location of the weld toe. This magnification will also take place when the weld itself is absent. If this geometrical stress appeared in practice one could imagine that the gusset was glued to the plate. In this case one would be able to measure the geometrical stress with a strain gauge at the locus where the weld toe is to be located. The factor \( K_g \) can more easily be revealed by a FEA of the detail. This type of analysis will, however, require special qualities for the model regarding element type and mesh, as we shall discuss later. \( K_g \) will depend on the height \( H \) of the gusset, and even more so on the gusset length \( L \). Typical values of \( K_g \) will be between 1.2 and 1.5. The longer the attachments length \( L \) is, the higher the stress configuration, see Figure 68. Due to the fact that \( K_g \) is determined by the global geometry (\( T, H \) and \( L \)), it is the geometrical stress configuration factor.

If we know that the weld bead is added onto the plate, when the stress at the weld toe will further increase due to the discontinuity of the weld bead itself. The notch stress configuration factor is \( K_w \). This configuration factor will depend on the toe angle \( \theta \) and particularly the ratio \( p/T \) low values of 0 and high values of \( p/T \) are favorable. The stress can be approximately determined by measurement using small strain gauges located at the
weld toe for a fillet welded joint. Such a refined analysis is usually not included in a FEA model for a global structure.

Before finishing, let us suppose that we carry out fatigue testing with the detail shown in Figure 69. One test has given a number of cycles \(N^*\) to failure. We must also have other test results to draw an S-N curve, but let us assume for simplicity that the data point we have chosen falls on the median line for the S-N curve. The result can alternatively be given as a function of \(\Delta S\), \(\Delta\sigma_g\), or \(\Delta\sigma_w\), as we have discussed. This is illustrated in Figure 69.

![Figure 69 Alternative presentation of test results based on nominal stress geometrical stresss, and notch stress Ref [54]](image)

We have presented the result by the three different keys on the vertical stress axis, as shown. Regardless of what approach is used, it is essential that the user of the S-N curve be aware of which stress concept has been applied and that they must use the same stress concept throughout. If the same stress concept is not used, erroneous results may be the consequence. If we uses the nominal stress for a S-N curve that is based on the geometrical stress, this will give an overestimation of the fatigue life from \(N^*\) to \(N^{**}\), as is shown by the dotted line in Figure 69. This consideration aside, one could argue that it is a matter of
choice which key should be preferred as they are proportional. However, if we test similar
details with somewhat different global geometry (given by T, H and L in Figure 68), the
nominal stress concept will not reveal the associated stress variation from specimen to
specimen, whereas the geometrical stress concept will.

As a consequence, for an S-N curve based on nominal stress, all the data points will
be plotted at one given stress level, whereas for the geometrical stress approach the points
will be plotted at slightly different stress levels. This is shown in Figure 70 for five data
points numbered from 1 (shortest fatigue life) to 5 (longest fatigue life). As can be seen,
the scatter in fatigue life when using the nominal stress plot is appreciable, whereas the
scatter is reduced when using the geometrical stress. It is to be noted that the shortest
fatigue life N1 is associated with the highest geometrical stress, whereas the longest fatigue
life N5 is associated with a smaller geometrical stress, but not the smallest; see Figure 70.
The smallest geometrical stress occurs for test number 4, but this test does not result in the
longest fatigue life. The explanation for this can be related to unfavorable weld toe
geometry and/or toe surface condition for test number 4. The same reasons explain the
difference in fatigue life between test 2 and 3 which have the same geometrical stresses.
Hence, even with the geometrical stress approach, we still have some variables and
associated scatter that we do not control. These variables are mainly related to the weld toe
profile and surface conditions, if we assume that the residual stresses are relieved.
In conclusion, the nominal stress approach will give larger scatter in fatigue life at the same given nominal stress level if the global geometry is not kept constant. The geometrical stress approach will in such cases give reduced scatter because longer lives will follow from reduced geometrical stresses whereas shorter lives will be related to increased geometrical stresses. Hence, the latter approach is more precise because it captures the actual geometrical stress configuration for each tested detail. Furthermore, the method can be used with great accuracy in connection with FEA of welded structures due to the fact that these stresses obtained by the analyses can be regarded as geometrical stresses.

6.7.2 Fatigue life estimate based on the weld notch stress approach

In light of what has been outlined above one may ask why one does not relate the fatigue life directly to the notch stress at the weld toe. As we have discussed, there are several good reasons that support this approach. Apart from the rational and logical basis, the approach will force us to explicitly account for the stress configurations $K_g$ and $K_w$ in the calculations. This will make the engineer conscious of moral design detail, welding
procedures, and fabrication quality. This urges the engineer to choose favorable global
dimensions for the detail in order that $K_g$ is as small as possible. The analysis must be based
on FEA or available parametric formulae found in standard catalogues with $K_g$ for standard
details. As we shall see in the section about design rules and specifications, DNV has given
such catalogs for details often found in ship structures.

We can account for the effect of global geometry and, in addition, we may take into
account the effect of the weld notch. Joints having an abrupt toe geometry will be
penalized, whereas there will be a reward for smooth-toe transition geometry when fatigue
life estimates are carried out. This will motivate good workmanship.

A theoretical objection to the approach is that it is based on the notch stress at me
point only; it does not take the thickness of the plate into the stress gradient rough. A more
practical drawback is that the toe geometry is highly variable even within a short length of
the weld seam. Thus, the question that arises is what values $\theta$ apply for the toe angle and
radius. Furthermore, the notch stresses are not readily available from a FEA, as are the
geometrical stresses. Consequently, the notch stress approach, although appealing, is
burdened with some theoretical doubt and practical obstacles. As a result, it has gained less
popularity than the geometrical stress approach. We shall give examples on both of the
approaches in the following sections.
Figure 71 shows the joint between two large girders with a scallop hole in the web plate to avoid the transverse weld in the flange plate and the longitudinal weld in the web plate crossing. Potential crack loci, point A and B, are given, as are $K_g$ and $K$. If the two weld strings had crossed, this would have given a three-axial residual stress situation. However, the disadvantage of the scallop hole is the stress configuration that follows with it and the related reduced fatigue resistance. In heavy-duty joints this consideration usually overrides the disadvantages of weld crossing. Consequently, scallop holes are avoided. If they are present, the stress configuration has to be accounted for. If drawings with stress configurations for details as shown in Figure 71 are not available, one will have to carry out FEA in order to determine the local stress. If the model contains both the global geometry and the weld bead geometry, the stress results will reveal both $K_g$ and $K_w$. This will require a refined model with volumetric elements. For larger slender structures the model will usually be built up by thin shell elements and the welds will not be included. If the model is properly constructed the stress results will reveal $K_g$. To determine $K_w$ one can use the formulae given in Table 12.
Table 12 Notch stress concentration for butt and fillet welds, Refs [59, 60]

<table>
<thead>
<tr>
<th>Joint Type</th>
<th>$K_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fillet weld</td>
<td>$K_w = 1 + 0.51(tan\theta)^{0.47} \left(\frac{t}{\rho}\right)^{0.47}$</td>
</tr>
<tr>
<td>Butt joint</td>
<td>$K_w = 1 + 0.27(tan\theta)^{0.25} \left(\frac{t}{\rho}\right)^{0.5}$</td>
</tr>
</tbody>
</table>

The problem with using the formulae in Table 12 is that the weld toe geometry is highly variable along the weld seam. This is a serious objection to the method; the weld bead is so irregular that it escapes a specific profile characterization. Many suggestions have been made as to how to determine the toe profile, amongst them practical coin tests in combination with wire cables as shown in Figure 72.

![Bad toe profile](image1.png) ![Good toe profile](image2.png)

Figure 72 Control of the toe profile by using a coin and a piano wire Ref [57]

Due to the irregularity, the only rational solution is to treat the geometrical parameters as stochastic variables varying randomly from one type of joint to another and randomly along the weld seam for any given type of joint. For a given weld category the geometry
parameters can be given by their mean values or expected extreme values. Once the extreme values have been found we can use equation (109) and (110) to determine the weld notch factor. Let us use the formulae for a plate thickness $T = 16$ mm. A good butt joint will typically have a mean radius of $\rho = 1$ mm and a mean angle $\theta = 30$ degrees. This gives:

$$K_w = 1 + 0.27(tan\ 30)^{0.25}(16)^{0.5} = 1.92$$

(109)

For a fillet weld it is reasonable to assume the same radius, but an increased angle of 45. This gives the following figure:

$$K_w = 1 + 0.51\left(\frac{45}{180\pi}\right)^{0.25}(16)^{0.47} = 2.76$$

(110)

The angle has to be given in radians. As can be seen, a typical butt weld has a $K_w$ in the range from 1.5 to 2.0 depending on $T/\rho$ and $\theta$, whereas a fillet weld has typical values in the region 2.0-3.0. The difference is mainly due to differences in the flank angle. A smooth transition will be characterized by $\rho = 1$ mm in both cases. If inspection after fabrication has been carried out and it reveals that the weld toe geometry is much poorer than the given values, one should reduce the life predictions or improve the weld toe by grinding. Weld beads that rise steeply from the plate with a radius close to zero (say 0.1 mm) give rise to very high stresses and result in $K_w$ values between 4 and 7. However, the affected material volume will decrease, as we shall see in the next section.

6.7.3 Conclusions on the various stress approaches

As we have pointed out in the above discussion, we have three different approaches for fatigue life estimates:

Alternative A: we can use the nominal stress range as the key parameter to fatigue life for a given detail. The approach requires that we have S-N curves available for all possible detail geometries and loading directions. This is required because the stress
configuration factors associated with the various geometries do not affect the nominal stress, but are inherent in the S-N curves. Hence, we should have an S-N curve for each, single geometry. This is, in practice, impossible. The procedure is to define one test population where the joints have very similar geometries with almost equal quality from a fatigue point of view. The associated S-N curve for this population is then defined as a class or category in design rules and specifications. If the variation in geometry for details assembled in one population is too large, the drawback will be a large scatter. This will penalize the favorable geometries within the actual category. The problem with defining the nominal stress range for a detail when integrated in a complex structure has already been mentioned. Despite the drawback of the method it is still the most used method in design rules and specifications.

Alternative B: we can use the geometrical stress range as the key parameter to fatigue life for a given detail. This approach requires only one reference S-N curve, usually a butt weld between two plane plates. This joint has, by definition, $K_g = 1.0$. The curve will also be applicable for all other details when the influence of the global geometry of the actual joint is explicitly accounted for by analytical formulae or FEA. The method is appealing, but one must bear in mind the simplified assumptions on which it is based:

1) The weld bead and local toe profile for the actual welded detail are as for the butt joint used as reference.
2) Surface conditions at the weld toe are the same as for the butt joint used as reference.
3) Welding procedure and heat treatment are the same.
4) The actual FEA model is refined enough to capture the geometrical stress configuration for the detail in question.

We should carefully check the various points when using the geometrical stress approach. A fillet weld may in fact often have less favorable toe geometry than the reference butt weld due to the toe angle, as we have just shown. Furthermore, when carrying out FEA, both element type and element mesh will play an important role.

Alternative C. we can use the weld notch stress range as the key parameter for fatigue life for a given detail. We have already pointed out that this is the most logical method from a theoretical point of view. In this case, we also need only one S-N curve as a reference curve perhaps a butt joint between two plane plates where the weld bead is ground flush. This joint has, by definition, $K_g = K_w = 1.0$. Again, this curve will also be applicable for all other details when the influence of the global geometry and weld toe geometry of an actual joint is explicitly accounted for by analytical analysis using FEA.

We have emphasized the weld notch stress approach in the present chapter due to the fact that it is an excellent pedagogical presentation of the fatigue problem for welded joints. All possible stress configurations are explicitly treated. However, we must be aware of the fact that this method also is a simplification due to the fact that it assumes that it is only the SCF at the point of the weld toe that governs the fatigue life. The method disregards the stress gradient as one departs from the weld toe through plate thickness. One must be aware of the fact that it is only a small material volume at the weld toe where the maximum stress actually occurs. The stresses will decrease rapidly at a small distance from the surface as shown in Figure 73. It is a fact that a small $p/T$ ratio will increase the SCF significantly, but over a smaller material volume as compared to the volume affected by the stress
configuration for a larger p/T ratio. This is illustrated in Figure 73 with a large p/T ratio to the left and a small p/T ratio to the right. As can be seen, the SCF increases when the p/T is decreased, but at the same time the stress gradient becomes steeper. As a result, a smaller material volume will be affected by the high stresses. Consequently, the assumption that the fatigue life is influenced and scaled by the notch SCF only will be un-conservative. This has led to the concept of the fatigue notch factor instead of the stress configuration. The fatigue notch factor is smaller than the calculated K. The concept of a fatigue notch factor remains somewhat elusive.

Figure 73 Stress concentration and stress gradient for a large P/T ratio (left) and a S-N P/T ratio (right). The geometrical stresses are assumed the same for the two cases Ref [61].

Finally, let us remind ourselves that all the geometrical and notch stress approaches are suitable for welds with loading transversal to the weld direction and with cracks that emanate from the weld toe. The stress configurations we have defined refer to the principal stresses acting normal to the weld seam. If the cracks are likely to emanate from the weld root, these approaches are not applicable. In this case or any case where the weld throat shear stress range is used as the key parameter, the nominal stress approach should be
preferred. However, such cases are less frequent compared to the numerous cases of potential toe cracking.

6.8 Current design rules and specifications

6.8.1 General considerations

There currently exist a variety of design rules and specifications for fatigue life verification of welded joints. As we shall see in what follows, the nominal stress method is still the most frequently used approach in most of the design rules and specifications. Typical examples are the AASHTO, Eurocode, DoE [96, 47, and 52]. As we have discussed, the method is quite accurate if large test series are available for exactly the same detail as the one to be analyzed. If the test series are available for similar details only, the scatter in fatigue life will increase and the method becomes less precise.

The geometrical stress approach is also an approximation, as we have discussed, but has many appealing aspects when used in conjunction with FEA of large complex structures. The method has been adopted in the Health and Safety Executive (HSE), [59]. The following guidance and rules will be discussed below:

— AASHTO American Association of State Highway and Transportation Officials
— DoE and British Standard, BS 54000 (nominal stress approach);
— Eurocode 3 (nominal stress approach, air environment only);

All the different building codes have more or less the same data basis for their S-T curves and the classification of various details is more or less the same. The differences lie in the method of presentation and application. Variations are found particularly in the location of the knee point of the bilinear S-N curves, the influence of seawater and cathodic protection, and the treatment of the thickness correction. The differences between the
British and the Norwegian codes are small, and the classification of the welded details is almost the same. In Eurocode 3, there is a different format for S-N curves, as we shall see. Regardless of which code is used, we must go through the following stages for fatigue life prediction:

1) Assess the entire structure and various load conditions.

2) Identify joints and details that are vulnerable to fatigue. Understand the load transfer for these joints, and determine the service stress history that the joint will be subjected to during service life. For variable amplitude loading, the load spectrum can be presented by stress blocks, each block defined by a given stress range and a corresponding number of cycles $n_i$, equation (98).

3) Detail classification. Determine to which experimental population the joint belongs, and select the most appropriate S-N curve to determine the fatigue capacity $N_{Ti}$ for each block, equation (89).

4) Carry out a fatigue damage calculation by comparing the applied number of cycles $L_i$; at a given stress level with the experimental number of cycles to failure $N_{Ti}$, equation (92).

There is not much problem with the calculation scheme; the complications are more in foreseeing and determining a realistic service stress history. A correct classification of the detail with related fatigue strength is usually a simpler task.

We must also be aware of the consequences of a possible fatigue failure and we should assess whether or not the details under consideration are accessible for inspection and repair during service life. If they are accessible for inspection and possible to repair, a combination of safe life and damage tolerance philosophy may be used. This means that
the PDF can be reduced and scheduled inspection can be planned and carried out. Detected cracks must be repaired to prevent failure. In these cases the required FDF will be set to three for most standards, sometimes lower. If inspection and repair are impossible and the consequences of fracture are severe, a FDF of 10 is often required. These issues must be discussed from the beginning of a rule-based analysis. We shall illustrate the various design rules and specifications in a brief overview that gives some of the characteristic features of the design rules and specifications. We is referred to the original sources for details.

6.8.2 Design Rules Given by the AASHTO Specification

As in most contemporary standards, the AASHTO fatigue life rules reflect the two issues fatigue cracking induced by stress and fatigue cracking induced by displacements within the structural system. Only the first has been explained so far, and the discussion in this section will continue to be limited to load-induced fatigue cracking.

Figure 74 shows the fatigue life curves given in the AASHTO Specification. The plot shows stress range on the vertical axis and number of cycles on the horizontal axis for seven different Detail Categories. Both axes are logarithmic representations. Over some portion of the range, each Detail Category is a sloping straight line with a slope constant m equal to 3. Beyond a certain point, which depends on the Detail Category, the fatigue life line is horizontal. This feature will be discussed subsequently.

The information in Figure 74 must be used in conjunction with information like that shown in Table 1 and Figure 18, which give only a small portion of the relevant material in the AASHTO Specification.
where one of the relatively large number of typical construction details classified by the specification is shown. Application of the information is straightforward. For example, suppose a designer proposes to use a beam made by joining three plates using continuous fillet welds parallel to the direction of stress, such as was shown in Figure 75.
According to Table 13 and Figure 75, this is Detail Category B. If the number of cycles to which the beam will be subjected is, say, $2 \times 10^6$, then the permissible range of stress for this detail is 120 MPa. This number was obtained using Figure 74 to estimate the stress range corresponding to $2 \times 10^6$ cycles. Working out the equation of the line for this detail category, the "exact" value of the permissible stress range at $2 \times 10^6$ cycles is 125 MPa. As will be seen later, the AASHTO Specification provides information that allows the calculation of the permissible stress range corresponding to a given number of cycles.

The fatigue strength curves presented in the AASHTO Specification (Figure 74) are those corresponding to the mean life of a detail, usually as obtained by physical testing, shifted horizontally to the left by two standard deviations. For reasonably large numbers of test data, the corresponding confidence limit is estimated to be approximately 97.7%.
6.8.3 The original fatigue classes and S-N curves from DoE and British standard, BS 54000 (nominal stress approach).

The original DoE curves for plated joints were designated B, C, D, E, F, F2, G, and W, (DoE, [50]). The curves are valid for high-cycle fatigue, when the number of cycles to failure is greater than $10^5$ cycles. B represents the curve with the highest strength, whereas W represents the curve with the poorest. Both categories B and C are representative of non-welded material, as well as for welds loaded parallel to the welding direction. For welded joints the classification will be a result of:

— joint type;
— joint geometry;
— loading direction;
— welding procedure;
— distortion tolerances;
— post-weld treatment;
— non-destructive inspection (NDI);
— location of the potential crack.

There is a logical explanation for why each point above will have an influence on the fatigue capacity, as we have explained in the foregoing chapters. Joint type, geometry, and loading directions will obviously affect the fatigue capacity. Different welding procedures may create different weld shapes and corresponding local notch stress configuration factors. The choice of welding consumables may also affect the quality of the fusion line and initial defects. Distortion of plates introduces secondary bending stresses and decreases
the fatigue capacity. Post-weld improvement methods, such as grinding or TIG dressing, will increase the fatigue capacity. The same will be the case after NDI and any necessary repair of initial cracks. Finally, the potential crack locus is important.

We may find it surprising that the material quality does not appear on the list for the joint classification. This is because the fatigue process in welds is dominated by geometrical parameters. The material quality will not have such a strong bearing on the fatigue capacity as it has for un-welded components where the initiation phase may play an important role.

Typical figures for various classes are given in Table 14. $A_0$ and $m$ are constants for each experimental population that corresponds to a given S-N class. The inverse slope $m$ of the curves is close to 3 for most classes. Hence, the estimated fatigue life is very sensitive to the applied stress range $A_c$. Typical values of $S_{\log N}$ range from 0.18 to 0.26. This corresponds to coefficients of variation (standard deviation divided by mean value for fatigue life) in the range from 0.4 to 0.7. These coefficients pertain to linear fatigue life, not the logarithmic values. The relation is such that $s^2_{\log} = 0.188 \ln (1 + \text{CoV}^2)$. Some of the curves are drawn in Figure 61 for variable amplitude loading. The curves are valid for high-cycle fatigue and most of the tests have endured more than $10^5$ cycles. As can be seen, the inverse slope $m$ changes from 3 to 5 at $10^7$ cycles. For constant amplitude loading there will be a fatigue limit at $10^7$ cycles. There is a cutoff stress level at $10^8$ cycles. These original curves were based on test results in air, but were also used for seawater with cathodic protection before new information came to hand; see Figure 64.

Some frequently occurring classes for plated joints are shown in Table 15. The first types are continuous welds, essentially parallel to the direction of applied stresses (type 2).
These details have high fatigue strength due to the fact that the weld bead does not give rise to stresses as for perpendicular loading. Hence, the cases are classified B, C, and D, as shown. When the loading is perpendicular to the welds (type 3), the butt weld with full penetration is classified as a D class when welded from both sides and in a flat position. This is the most common case for high-strength joints. The potential crack locus is at the weld toe. If the butt joint is welded from one side only it will be classified as an F class if it is made with permanent backing strip. The potential crack locus will, in this case, be at the root; see Figure 74, to the right. The backing strip must not be tag welded, because the tag weld will give a G class. One solution often used is to bond a ceramic backing bar to the back of the joining plates. These types of one-sided joints can also be made with a TIG root pass. If it is possible to grind the backside with a tool afterwards, it is possible to obtain a C class for this one-sided joint. If the joint configuration is changed from a butt joint with full penetration to a fillet welded attachment (type 4) the classification will drop from a D to an F class. If the weld toe is close to the plate edge, the class will drop further down to a G class. The reason is that both the weld toe and the plate edge are stress risers. Hence, one should try to avoid such configurations. For these cases we also see that increased attachment length will downgrade the joint from F to F2. Table 15, below, gives an overview of certain plated welded details.

Type 2: continuous welds essentially parallel to the direction of applied stresses. Full or partial penetration butt welds or fillet welds built up by plate or section and with no attachments.
Type 3: transverse butt welds in plate (essentially perpendicular to the direction of applied stresses). Full penetration butt welds welded from both sides and join plates of equal width and thicknesses. Differences in width or thickness will be machined to a smooth transition not steeper than 1 in 4.

Table 14-a Type 3: transverse butt welds in plate Ref [62].

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Potential crack locus</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Full penetration butt welds with the weld overfill dressed flush with</td>
<td>Any defect in weld</td>
<td>B</td>
</tr>
<tr>
<td>the surface and finish-machined in direction for stresses. Weld bead</td>
<td></td>
<td></td>
</tr>
<tr>
<td>proven to be free from defects.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) butt or fillet weld made by submerged arch welding (SAW) or shielded</td>
<td>Surface ripples</td>
<td>C</td>
</tr>
<tr>
<td>metal arc welding (SMAW) with no stop-start position within the length</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) As (b), but with welds containing stop-start positions.</td>
<td>Start-stop positions</td>
<td>D</td>
</tr>
</tbody>
</table>

Type 4: welded attachments on the surface or edge of a stressed member

Table 15-b welded attachments on the surface of a stressed member Ref [62].

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Potential crack locus</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) With the weld overfill dressed flush with the surface and finish-</td>
<td>Any defect in weld</td>
<td>C</td>
</tr>
<tr>
<td>machined in direction for stresses. Weld bead proven free from defects.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) With the weld made manually or automatically (other than SAW) provided</td>
<td>Weld toe</td>
<td>D</td>
</tr>
<tr>
<td>all runs are made down-handed.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) Weld made other than as in (a) and (b).</td>
<td>Weld toe</td>
<td>E</td>
</tr>
</tbody>
</table>

Type 4: welded attachments on the surface or edge of a stressed member
For load bearing welds, the joint classification will be different whether the crack grows from the weld toe or the weld throat, as has already been pointed out. The applied stress range is also different at these two locations. The same joint may therefore be checked against two different S-N curves and with two different stress ranges. The case is shown to the left in Figure 77, where fatigue cracking may start at the weld toe or the weld root. Different stress ranges are applied for the two cases. In the first case the joint is classified as Category E, while in the latter case the joint must be classified as W-class (Category 35 Eurocode). The ratio between plate thickness and weld leg length will govern which of the failure modes is the critical one the one having the shortest predicted fatigue life.

Table 16-c Overview of some important, plated welded details Ref [62].

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Potential crack locus</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) With attachment length parallel to the applied stresses. L&lt;150mm δ&gt;10MM</td>
<td>Weld toe</td>
<td>F</td>
</tr>
<tr>
<td>b) With attachment length parallel to the applied stresses. L&lt;150mm δ&gt;10MM</td>
<td>Weld toe</td>
<td>F2</td>
</tr>
<tr>
<td>c) All types of attachments edge distance less than 10mm</td>
<td>Weld toe/edge of plate</td>
<td>G</td>
</tr>
</tbody>
</table>

Figure 77 Important cases with possible root fatigue cracks Ref [63].
Life prediction for toe cracking must use the nominal stress range in the plate and the Category E S-N curve. The Category E applies if the weld undercuts near plate edges are dressed. For root cracking, the nominal stress range in the weld throat must be used in conjunction with a W-class S-N curve. Such a case must be regarded as a special one, and cracks that emanate from the weld toe are by far the most common. Figure 77, right, shows a one-sided butt weld that is another important case with possible cracks from the weld root. We have already discussed this case.

6.8.4 S-N life predictions according to Eurocode 3-Air environment

Eurocode 3, [57], presents 14 parallel curves that all have an inverse slope of \( m = 3 \). The curves belong more or less to the same classification groups (or categories) as the original DoE, but they are designated by figures instead of letters. The characterizing figure for each class corresponds to the stress range at \( N = 2 \times 10^6 \) cycles. The S-N curves also have a slightly different appearance. As can be seen, the curves change slope at \( N = 5 \times 10^6 \) and prescribe a cutoff limit at \( N = 10^8 \). Some details from Eurocode 3 are listed in Table 15. The closest comparable classes from DoE are also given. As an example, the E class in DoE will be close to the Category E 80 in Eurocode 3.

The calculation scheme in Eurocode may also have a somewhat different format. The Miner rule is used to calculate an equivalent stress range as follows:

\[
\Delta \sigma_e^m = \frac{\sum_{i=1}^{k} n_i \Delta \sigma_i^m}{\sum_{i=1}^{k} n_i} \tag{111}
\]

The fatigue criterion can now be written in the following format:

\[
\gamma_f \Delta \sigma_e < \frac{\Delta \sigma_R}{\gamma_m} \tag{112}
\]
Where $\Delta \sigma_R$ (fatigue strength) is taken from the S-N curve at the total number of service life stress cycles $n$, the safety factors $\gamma_f$ and $\gamma_m$ account for uncertainty in fatigue loading and fatigue quality respectively. The difference from the former codes is that the equivalent applied stress is compared to the fatigue strength at given service life, instead of comparing service life with fatigue design life at a given applied stress spectrum. Eurocode 3 format offers the opportunity to explicitly take into account the load and

<table>
<thead>
<tr>
<th>Category</th>
<th>$\Delta \sigma_c$ (Mpa)</th>
<th>$\Delta \sigma_D$ (Mpa)</th>
<th>$\Delta \sigma_I$ (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N&lt;5 \times 10^6$ (m=3)</td>
<td>$N&gt;5 \times 10^6$ (m=3)</td>
<td>$N=5 \times 10^6$ (m=3)</td>
</tr>
<tr>
<td>100</td>
<td>12.301</td>
<td>16.036</td>
<td>74</td>
</tr>
<tr>
<td>90</td>
<td>12.151</td>
<td>15.786</td>
<td>66</td>
</tr>
<tr>
<td>80</td>
<td>12.001</td>
<td>15.536</td>
<td>59</td>
</tr>
<tr>
<td>71</td>
<td>11.851</td>
<td>15.286</td>
<td>52</td>
</tr>
<tr>
<td>50</td>
<td>11.401</td>
<td>14.536</td>
<td>37</td>
</tr>
<tr>
<td>45</td>
<td>11.251</td>
<td>14.286</td>
<td>33</td>
</tr>
</tbody>
</table>
material partial safety factors. These factors are in many cases set to 1.0. Exemptions are
when welds are difficult to inspect and the consequence of fracture is severe.

6.9 Summary

Chapter 6 is prepared for an elaborate fatigue life calculation scheme based on the
S-N model according to design rules and specifications. The basis is the original S-N design
rules from the current AASHTO Specifications. Chapter 6 also gives a qualitative
assessment of what is a good detail-design of welded joints and how to obtain improvements
in fatigue resistance through post-weld treatment.
CHAPTER VII
FRACTURE MECHANICS MODEL APPROACH

7.1 Introduction

Fatigue life predictions have been carried out using the S-N approach for over a century and long before the physics of the fatigue process was properly understood. As we have seen, the model is based on simplified assumptions and statistical analysis of the entire fatigue life. The method does not try to analyze the fatigue process itself. Its only goal is to estimate the time to failure at a given probability level. One may in fact use the S-N curves without being aware of the fact that it is a crack growing to a critical size that is the cause of the fatigue failure. In the fracture mechanics approach the entire crack growth history is modeled, not only from the final fatigue life, but also from a small, initial crack to the final, critical crack leading to fracture. The formulation is based on physics, not just life statistics as is the case with the S-N model. After a crack has initiated, the standard S-N curves are no longer applicable, whereas a fracture mechanics model can describe the probable crack behavior and propagation towards final failure. Because the S-N model cannot deal with the presence of cracks, the fracture mechanics model is an indispensable tool in situations when a crack is detected and sized. Furthermore, a fracture mechanics model gives valuable support when planning scheduled in-service inspection. In such cases
we will have to know the possible damage evolution in advance in order to know what crack sizes to look for and at various time stages during the service life. The principal difference between the S-N model and the fracture mechanics model is shown in Figure 79. To the left is shown the conventional S-N plot at a given stress range level; the crack depth $a$ as a function of number of cycles for one of the tests is given to the right.

![Figure 79 Right: crack depth history of one test Left S-N curves with several tests Ref [66].](image)

The basis for the fracture mechanics approach is that it considers the stress field and not just the stress configuration at the weld notch. Furthermore, it describes the synergism between the weld notch effect and the presence of a sharp fatigue crack. This synergism is reflected in the concept of the stress intensity factor (SIF). This factor determines the local stress field in front of the crack and is the key parameter along with the material characteristics for predicting the crack behavior.

The fracture mechanics approach provides insight into the interplay of parameters that primarily influence the fatigue process and final life. However, for a long time there has been debate about the usefulness of the fracture mechanics as a tool when compared to the S-N approach. Generally, the practicing designer prefers the relatively simple and
rough estimate that the S-N curves provide, whereas academics have advocated the use of fracture mechanics. The conclusion is that the S-N model and the fracture mechanics approach should be used as complementary tools when analyzing fatigue of welded structures. Fracture mechanics has obviously matured to the point where its application is both reliable and useful. As a consequence it has gained acceptance in new design rules and specifications as an alternative to the S-N approach. The British standard (design rules and specifications) is entirely devoted to the application of fracture mechanics for fusion-welded joints in order to describe the behavior of cracks. Therefore, we should certainly have a general knowledge of fracture mechanics issues.

One may argue that as long as the S-N-estimated fatigue life exceeds the target service time with a required safety margin, one does not have to worry about the fatigue process. There are some very important cases were knowledge of the fatigue crack behavior is necessary:

— where S-N curves for the detail and loading mode in question do not exist;
— where a crack has been detected after fabrication or in-service;
— where preventative scheduled inspections are to be planned.

In the first case the best strategy is to carry out life testing to establish an S-N curve. However, because this is costly and time consuming, one may try to estimate the life by trying to predict the growth of an initial crack that likely to appear in the detail to a final, critical crack. The actual geometry of the joint must be properly modeled and the material has to be characterized. A crucial issue is that one must assume the size of an initial crack which usually is small, not measurable and is random by nature. In addition to these uncertainties it is also a question of whether the fracture mechanics approach is applicable
for small cracks. The second case listed above is more straightforward. If a crack has been detected and sized, one can verify whether or not it will grow to a critical size during the target service life. If it will reach a critical crack size, repair should be carried out before the structure enters into service. If one uses current quality standards as the basis for decision-making, they are usually overly conservative with respect to allowable defects. It is for these cases that the linear elastic fracture mechanics (LEFM) was originally developed and outlined as in design rules and specifications. Figure 80 may be used as the basic geometry for our problem formulation. Before addressing this and similar problems we will briefly outline the basic concepts of LEFM.

![Fillet weld with a semi-elliptical crack at the weld toe Ref [66].](image)

7.2 Objectives of this section

We will focus on basic concepts of fracture mechanics in this section. The theoretical part is not extensive, but we will be able to calculate the Stress Intensity Factor SIF for cracks that emanate from the weld toe and use this parameter for assessing both the danger of unstable fracture and stable crack propagation. The stable crack propagation will be based on Paris's law and it will be shown how this equation can be calibrated to describe the behavior of fatigue cracks in welds. The theory is limited and confined to LEFM.
7.3 Basic concepts of linear elastic fracture mechanics

7.3.1 The local stress field ahead of the crack front

Before applying LEFM theory to details with weld notches, as shown in Figure 80, we shall outline the general theory for simple plane components containing sharp fatigue cracks. Figure 81 shows an infinite plate with a central crack subjected to in-plane loading. The crack is slitting the plate through the thickness and has a rectangular shape. The loading is termed mode 1 and is characterized by stresses acting normal to the crack plane so that the crack surfaces are moving directly apart. Other possible loading modes are shown in Figure 82 and are referred to as mode II-shear mode and mode III tearing mode (out-of-plane shear). Mode I is by far the most important for welded joints and we shall confine our analysis to this mode in what follows. Furthermore, we shall assume linear elastic behavior of the material.

The local stress field at the crack front can be found by using the Airy stress function with complex harmonic functions. According to Westergaard, [69], the solution for the case in Figure 80 reads:

\[
\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)
\]

\[
\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)
\]

\[
\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2}
\]

where \( r \) and \( \theta \) are cylinder coordinates with origin at the crack tip. \( K_I \) is the referred to as the SIF given by:

\[
K_I = \sigma_0 \sqrt{\pi a}
\]
where $\sigma_0$ is the nominal uniform stress field as it appears in the plate without the crack. Half the crack length is designated $a$ for a through-thickness crack; see Figure 80. As can be seen from equation (113), the stress situation is two-dimensional. Ahead of the crack front for $\theta = 0^\circ$ (r-axis coincides with x-axis) we will have the normal stress in x and y direction that are equal in magnitude. Furthermore, the theoretical values will approach infinity as $r$ approaches 0. If we have a plane-strain condition at the crack front we will have a third component which reads:

$$\sigma_z = \nu(\sigma_x + \sigma_y)$$  \hspace{1cm} (115)

where $\nu$ is the Poisson ratio. Plane-strain condition will appear at the crack front in the mid-area of thick plates. The associated three-axial stress condition can sustain very high stresses before yielding. Hence, we have a three-dimensional local severe stress field ahead of the crack with a singularity at the crack tip.

Figure 81 An infinite plate with a central through-thickness rectangular crack subjected to loading mode 1 Ref [67].
As can be seen from equation (113), the Stress Intensity Factor SIF alone determines the magnitude of the local stress field ahead of the crack front. The concept of the SIF leads to both a fracture criterion under an extreme load case and a crack-propagation model under repetitive loading. It is one of the controlling factors for assessing the criticality and behavior of a crack. As can be seen from equation (114), it is proportional to the applied uniform stress $\sigma_0$ and the square root of the crack size. This means that if the applied stress is doubled, the local stress field ahead of the crack front is doubled as well; likewise, if the square root of the crack size is doubled the Stress Intensity Factor SIF must not be confused with the stress configuration factor (SCF). The latter is the stress magnification at a given spot only, whereas the SIF describes the entire stress field ahead of the crack front. It is to be noted that the dimension for the Stress Intensity Factor SIF is MPa, m$^{0.5}$ in SI units. The SIF concept is of a general nature and may be extended to a variety of geometries and loading modes, including semi-elliptical surface cracks that propagate from the weld toe. For items and crack geometries other than the one shown in Figure 79 the SIF will read:

$$K_I = \sigma_0\sqrt{\pi a} F(a)$$  

(116)
where \( F(a) \) is a geometry function taking into account the geometrical deviations from the central through-thickness crack in an infinite plate. The through-thickness crack case (see Figure 81) can be treated as a reference case with \( F(a) = 1.0 \). \( F(a) \) is dependent on both the geometry of the crack and the geometry of the component, and its various deviations from the reference case can be as follows:

- finite dimensions of the plate (the material ahead of the crack front is reduced);
- surface cracks (the crack has one front in the material and a "mouth" out to the surfaces);
- elliptical cracks (curved crack fronts);
- cracks in the vicinity of a notch (the applied stress has an SCF and a gradient).

These deviations are treated by introducing correction factors that are multiplied to obtain \( F(a) \). If these correction factors are less than 1.0, it means that the local stress field at the crack front is less severe than the field occurring for the reference case. If they are larger than 1.0 the stress field is more severe. Some cases are shown in Figure 82. The plates now have finite width \( w \) and finite thickness \( T \). To the left on the figure is shown a central through-thickness crack and two symmetrical edge cracks in the plate. Both types of cracks still have straight crack fronts, as does the reference crack in Figure 79. These cases will have an increase in \( F(a) \) compared to the reference crack. For the central crack in Figure 82, this is due to the fact that the ligaments ahead of the fronts of the crack are reduced. This will increase \( F(a) \) and the following correction term for \( F(a) \) is given:

\[
f_w = \frac{1}{\sqrt{\cos \frac{\pi a}{w}}}
\]  
(117)
In the case of the symmetrical edge cracks to the left in Figure 83, the ligament on one side has disappeared compared with the reference case. This will increase the geometry function by a multiplication factor of:

\[ M_1 = 1.1215 \] (118)

The functions given in equation (117) and (118) are correction factors due to the finite width of the plate and the presence of a crack mouth at the surface respectively. If the edge crack is located with its crack mouth at a fillet as shown to the right in Figure 84, the material in the crack area will be subjected to a stress configuration. The SCF caused by the fillet will decrease as the distance from the surface increases. This notch stress field will have an impact on the stress field ahead of the crack front. For very small cracks an approximation for this effect will read:

\[ F(a) \approx M_1 SCF \] (119)

As can be seen, we have the compound effect of a free surface (M1) and the fillet notch (SCF). As we have seen for welded joints, the SCF can typically have a value between 2 and 5. Hence, this correction factor has a very strong influence on F(a) compared to the other correction factors. As we shall see below, this gradient must be taken into account and the geometry function will decrease below the SCF value with increasing crack depth. Equation (118) assume the crack depth is small as compared to the width of the plate.
Curved crack fronts are shown in Figure 84. To the left is shown an embedded elliptical crack with long axis 2c and short axis 2a. If we assume that the crack dimensions are small compared with the plate dimensions, the only correction compared to the reference case will be due to the curved crack front. The correction factor reads:

\[ f_\phi = \frac{1}{\Phi} \left[ \sin^2 \phi + \left( \frac{a}{c} \right)^2 \cos^2 \phi \right]^{1/4} \]  

(120)

Figure 83 Left: central through-thickness crack and symmetrical edge cracks with stress front. Right: symmetrical edge cracks at a fillet [69].

Figure 84 Embedded elliptical and surface semi-elliptical cracks in a plate Ref [69].
In this case, $F(a) = f_\phi$, will vary along the crack front and the angle $\phi$ gives the actual position where the geometry function is calculated (Figure 84). The end points of the short axis and the end points of the long axis are the most interesting points. The function $\Phi$ can be found by a complete elliptical integral which may be approximated by:

$$\Phi = [1 + 1.464(a/c)^{1.65}]^{1/2} \quad (121)$$

For circular cracks the expression in equation (120) reduces to:

$$f_\phi = \frac{2}{\pi} = 0.64 \quad (122)$$

In this case, the correction factor is constant along the circular crack front. As can be seen from equation (122), an embedded circular crack has a smaller geometry factor compared to a through-thickness crack with a straight front when the crack size is the same. To the right in Figure 84 is shown a surface crack with a semi-elliptical shape. $F(a)$ will increase compared to the embedded crack because the crack has a mouth at the plate surface at one side and also because the crack front is approaching the other side of the plate. Furthermore, we have to make a correction for the curved crack front. This problem was solved by Newman and Raju, [70]. The equation for uniform tensile loading reads as follows:

$$F(a/c, a/T, c/w, \phi) = [M_1 + M_2(a/T)^2 + M_3(a/T)^3 \frac{gf_\phi f_w}{\Phi}] \quad (123)$$

The sub-functions read:
\[ M_1 = 1.13 - 0.09(a/c) \]
\[ M_2 = -0.54 + 0.89/(0.2 - a/c) \]
\[ M_3 = 0.5 - 1/(0.65 + a/c) + 14(1 - a/c)^2 \]
\[ g = 1 + [0.1 + 0.35(a/T)^2](1 - \sin \varphi)^2 \]
\[ f_\varphi = [(a/c)^2 \cos^2 \varphi + \sin^2 \varphi]^{1/4} \]
\[ f_w = \left[ \sec(\pi c/w)(a/T)^{1/2} \right]^{1/2} \]

As can be seen from the formulae in equation (124), all the correction factors are dependent on the ratios \( a/c, a/T \) or \( c/w \). They are correction factors for the curved shape of the crack, the finite thickness and finite width of the plate. Hence, the function includes all corrections except the stress gradient factor. The expressions are valid for shallow flaws with \( a/c < 1.0 \).

A typical solution for various locations along the crack front is given in Figure 85.

![Figure 85](image)

Figure 85 Geometry function along the crack front for a semi-elliptical crack Ref [70].

The values at the point at the surface (\( \varphi = 0 \)) and at the deepest point (\( \varphi = \pi/2 \)) are particularly important. At the deepest point of the crack front we will have:
\[ g = 1.0 \]

\[ f_{\phi} = 1.0 \]  \hfill (125)

and at the ends of the crack at the surface:

\[ g = 1.1 + 0.35(a/T)^2 \]

\[ f_{\phi} = (a/c)^{1/2} \]  \hfill (126)

The semi-elliptical surface crack in finite plates is particularly interesting as it has the same configuration as the fatigue surface crack that was shown in Figure 80. The only correction that has to be added is the effect of the weld notch. In this case the expression in equation (123) has to be multiplied by a stress gradient factor \( M_k \). For very small cracks \( M_k \) will be close to the SCF of the weld notch as was shown for the case in Figure 83 and in equation (119). For fatigue crack growth calculations, we need to have a more refined solution. Several investigations have been conducted to determine \( M_k \). The factor can be defined as the ratio between \( F(a) \) determined with the notch, and \( F(a) \) determined without a notch for a given crack:

\[ M_k(a) = \frac{F(a)_{\text{with notch}}}{F(a)_{\text{without notch}}} \]  \hfill (127)

This ratio can be determined for a straight surface crack \((a/c = 0)\) using a two-dimensional FEM analysis or by a three-dimensional analysis where the curved crack shape is modeled together with the footprint of the welded attachment. A 2-D analysis is a good approximation, but the 3-D analysis is considered to be more accurate. The SIF may be obtained by a FEM analysis of the joint, either directly by including the crack geometry in the FEM model (direct approach) or indirectly by an analysis of the body that does not have a crack. In the direct approach, the SIF is obtained from the stress distribution in the vicinity of the crack tip or the singular displacement field at the crack tip. The indirect
method must be used in conjunction with a weight function (WFM). Due to the fact that the WFM needs only one FEM analysis for the body without cracks, it requires much less computational effort than the direct FEM analysis which must be carried out with a number of crack sizes. Although less accurate, the WFM is therefore attractive and competitive. The method is based on the fact that there exists a function \( w(x, a) \) from which the SIF can be obtained by integration [71]:

\[
K_I = \int_{0}^{a} \sigma(x)w(a,x)dx
\]  

(128)

where \( \sigma(x) \) is the stress distribution over the crack depth center line for a body without cracks. The weight function \( w(x, a) \) is a unique property for a given body geometry. Equation (128) reflects the fact that when a crack slits over a highly-stressed area (\( \sigma(x) \) is high), the material ahead of the crack front will act as an alternative stress path and it will be highly-stressed. The results from a 3-D analysis, [72, and 73] are shown in Figure 87. \( F(a) \) is shown for a plane plate and a T-butt joint. \( M_k(a) \) can be found from equation (127). The crack has an aspect ratio of \( a/c = 0.2 \) and the solution pertains to the deepest point along the crack front (\( \varphi = 90 \) degrees). The T-butt joint has a weld toe shape characterized by an angle of 45 degrees and a very sharp toe radius. As can be seen from Figure 87, the geometry function for a shallow, surface crack (\( a/T < 0.2 \)) in a plane plate is close to 1.15. This value is obtained from equation (123) and is the same as was shown in Figure 85. The geometry function for the same crack at a weld toe in a T-butt joint is as high as 3 for very small cracks (\( a/T = 0 \)) and decreases rapidly towards 1.0 as \( a/T \) approaches 0.2.

Hence, the \( M_k(a) \) function in the 1-butt joint is close to the SCF at the weld toe for very small cracks, but drops rapidly with the crack depth. For deeper cracks (\( a/T > 0.2 \)) it
can be seen that cracks in a plane plate have higher values for the geometry function than in the T-butt joint. Hence, the welded attachment has two different effects on the geometry function, depending on the crack depth. For very small cracks there is a significant increase in $F(a)$ compared to the plane-plate solution, whereas for larger cracks, the geometry function will be smaller than for plane plates. The first effect is explained by the severe stress configuration of the attachment and the weld bead, whereas the second effect is explained by the fact that the attachment gives some additional ligament (alternative load path) for larger cracks that have extended the crack out of the stress configuration area.

The stress configuration effect from the weld notch is by far the most important, whereas the additional load path effect is benign and is usually ignored. Hence, although the alternative load path effect may give an $M_k$ slightly less than 1.0, the minimum value for $M_k$ is set to 1.0.

Figure 86 Geometry function for semi-elliptical surface cracks in plates and in T-butt with $L/T=1.25$, $\theta=45$ degrees subjected to membrane loading. Ref [71].
For as-welded joints under membrane loading, [72] suggests the following equation:

\[ M_k = f_1 \left( \frac{a}{T}, \frac{a}{c} \right) + f_2 \left( \frac{a}{T}, \theta \right) + f_3 \left( \frac{a}{T}, \theta, \frac{L}{T} \right) \]  

Equation (129)

For practical solutions, the following equations are given [74]:

\[ M_k = v (a / T)^w \]  

Equation (130)

and for \( \frac{L}{T} < 2.0 \) and \( \frac{a}{T} < 0.05 (\frac{L}{T})^{0.55} \) we get the following:

\[ v = 0.51 (\frac{L}{T}) \]  

Equation (131)

\[ w = -0.31 \]

The results are obtained from 2-D analysis. Based on 3-D analysis, for \( \theta = 45 \) degrees we will get [74]:

\[ M_k = f_1 \left( \frac{a}{T}, \frac{a}{c} \right) + f_2 \left( \frac{a}{T} \right) + f_3 \left( \frac{a}{T}, \frac{L}{T} \right) \]  

Equation (132)

which is a simplification of equation (129), [72]. The equations for the bending loading mode are also found in [72].

7.3.2 The R6 criterion and critical crack size

To illustrate the R6 criterion we will take a central crack in a plate as an example; see Figure 87 and Figure 88. In Figure 87 we recognize the check against global net section yielding. In Figure 88 we recognize the check against local brittle fracture at the crack front.
The interaction between the two failure modes is accounted for in the R6 criterion. It is obvious that each of the two criteria given in Figure 87 and 88 must be fulfilled in order to avoid net section yielding and brittle fracture when these two failure modes are considered separately. If there is a danger of a mixed-mode failure, a proper limit on the sum of the two criteria must be set. The limit line of the diagram is given by the following equation:

\[
\sigma_{\text{ref}} = \sigma_0 \frac{T - W}{T(W - 2a)}
\]

\[
S_r = \frac{\sigma_{\text{ref}}}{\sigma_y} < 1.0
\]

Figure 87 Wide plate central through-thickness crack, net section yielding check Ref [71].

Figure 88 Wide plate with central through-thickness crack, brittle fracture Ref [71].
\[ K_r = \left[ \frac{8}{\pi^2 S_R^2} \ln \left( \frac{\pi}{2} S_r \right) \right]^{0.5} \]  

From this equation the critical (biggest admissible) crack size \( a_c \) can be determined. Although we have shown the method for a through-thickness central crack in a wide plate, the verification procedure remains the same for the semi-elliptical surface cracks found near the weld toes. Opposite to the influence of the initial crack depth, the fatigue life is not so sensitive to the values of \( a_c \). For medium-strength steel with yield stress up to 400 Mpa, critical crack size \( a_c \) can, for practical purposes, be set equal to the plate thickness.

![Combined failure](image)

Figure 89 Illustration of the R6 criterion Ref [72].

7.4 Fatigue threshold and fatigue crack growth

7.4.1 Crack growth models

For a welded joint made of medium strength of fatigue crack propagation from the weld toe region is a rather complex problem, involving stress gradients due to the weld toe notch effect, residual stresses, and micro-structural non-homogeneities in the heat-affected zone (HAZ). In the present chapter we shall limit the presentation to loading model I.
Hence. The fatigue propagation is often predicted using a LEFM theory. For semi-elliptical cracks at the weld toe, the deepest point at the crack front will propagate under plane strain conditions, as we have discussed. During one external load cycle the crack front will advance by a micro-mechanism involving crack bunting and re-sharpening; [66, 67]. During each load cycle, the formation of a striation may be observed on the fatigue crack surface. It is assumed that the crack growth rate at a macroscopic level is related to the stress intensity factor range (SIFR). A semi-empirical relation is given by the Paris-Erdogan law:

\[
\frac{da}{dN} = C (\Delta K)^m, \Delta K > \Delta K_0
\]  

(134)

where C and m are treated as material parameters for a given R ratio and environmental condition. \(\Delta K\) is the SIFR at the crack tip, corresponding to the applied stress nominal range \(\Delta \sigma\). The crack depth \(a\) is measured from the plate surface to the crack tip. The crack tip is defined as the deepest point along the crack front for a semi-elliptical surface crack as was shown in Figure 78. Equation (134) is coupled to similar equations in other crack front directions. However, in the present analysis the growth model will be simplified by the one-directional approach. The shape evolution of the crack given by the aspect ratio \(a/2c\) will be derived from experimental data, and introduced in the calculations as a forcing function on the aspect ratio \(a/2c\). In what follows, only the crack opening mode I is discussed for the case where the crack surfaces move directly apart. Although many problems for welded joints are of the mixed-mode type, mode 1 is considered to be the dominant mode for fatigue propagation and fracture. The SIFR can be written in the following form:
\[ \Delta K = K_{\text{max}} - K_{\text{min}} \]

\[ \Delta K = \Delta \sigma \sqrt{\pi a} F(a) \]

(135)

The validity of equation (134) is shown in region B in the diagram in Figure 90. Region B is the regime for stable crack growth and the curve corresponds to equation (134) for a log-log scale. Region A is the near-threshold regime where the crack growth rate tends to drop sharply towards very low values. It is often assumed that \( \frac{da}{dN} \) is zero if \( \Delta K \) is less than a certain "threshold" value \( \Delta K_0 \). Another way of taking account of this behavior is to introduce a modified crack law in the threshold regime (regime A in Figure 90):

\[ \frac{da}{dN} = C(\Delta K - \Delta K_0)^m \]

(136)

The threshold values for \( \Delta K_0 \) for normal welded steel qualities are normally found in the range 3-7 MPa m^{0.5} [96]. The concept of \( \Delta K_0 \) is usually based on experiments with long cracks, typically several mm in a compact tension (CT) specimen. For shallow surface cracks at weld toes, the crack growth rate may be considerably higher. These shallow cracks do not exhibit the same retardation as long cracks for low values of the SIFR. This becomes even more pronounced if they grow in a tensile residual stress field caused by the welding process. The conditions are illustrated schematically in Figure 90. It is difficult to quantify how much faster small surface cracks at the weld notch grow compared to long cracks. Because of this uncertainty it is recommended that one extrapolate the curve from region B into region A when the small values of SIFR are due to small crack depths. For the same reason we shall use equation (134) and not equation (136) for fatigue-growth calculations.
In region C of the diagram, the crack growth rates are greater than what a simple extrapolation of the curve from region B would predict. The reason for this behavior is the onset of unstable fracture due the fact that $K_{I_{\text{max}}}$ approaches the fracture toughness of the material, $K_{IC}$. In this regime the Foreman equation has been suggested, [75]:

$$ \frac{da}{dN} = \frac{C(\Delta K)^m}{(1 - R)K_C - \Delta K} $$

(137)

It should be aware of the fact that the parameters C and m are only valid for the equation to which they are fitted. Parameters for equations (134), (136) and (137) are not interchangeable. This is the most common equation for welded joints and also the one for which the most material data are available. The number of cycles to reach given crack depths is calculated by numerical integration of equation (134) as follows:

$$ N_P = \int_{a_0}^{a_C} \frac{da}{C(\Delta \sigma \sqrt{\pi a F(a)})^m} $$

(138)

where $a_0$, is the initial crack and $a_C$, is the final crack depth giving some sort of malfunction or fracture. A large number of values for the SIFR is needed during numerical integration.
of equation (138). Clearly, the computational efforts and accuracy are crucial when selecting calculation method.

7.4.2 Parameters C and m

The material characteristics of the HAZ and the base material will influence the crack growth rate through the parameters C and m in the Paris equation. Although treated as material parameters, both the environment and the applied stress ratio R may have a strong bearing on C and m. The parameters are valid for a given environment and R-value. The dependence on the R-ratio may be partly eliminated by introducing the crack-closure concept. It is assumed that during a lower part of the loading cycle the crack remains closed. This part of the cycle does not open the crack and will not contribute to the crack growth. The effect is more pronounced for lower values of the applied R-ratio. This observation leads on to consider only the effective SIF range [66]:

\[ \Delta K_{eff} = K_{max} - K_{op} = U \Delta K \]  \hspace{1cm} (139)

where \( K_{op} \) is the value at which the crack opens (or closes). In fact, the crack may close before the nominal stresses approaches zero. Under constant amplitude loading this is mainly due to a small, plastic crack tip zone developing at top of the loading cycle. As the crack grows through a succession of these zones, plastically deformed material is left within its wake. This plastic wake is constrained by the elastic surroundings during unloading and causes the crack to close while still subjected to tensile stresses. Typical values at a stress ratio \( R = 0 \) are \( K_{op} = 0.3 \ K_m; \) [68]. If \( \Delta K_{eff} \) is used in equation (134) the C and m factors will remain almost constant for different R-values; the \( \Delta K_0 \) likewise. The great advantage of the concept is that C and m values determined from one test series at specific R-factors can be used under other loading conditions, provided that the curve
fitting is based on an effective stress range concept. The problem is to determine the effective range of the SIFR from case to case. Elber [76] proposed the following formula for aluminum:

\[ U = A + BR \]  \hspace{1cm} (140)

where \( A \) and \( B \) are constants for a given material. There is at present a lack of experimental evidence for this equation for welded steel joints. Based on a literature study carried out by Verreman, [77] it may be assumed that \( A \) and \( B \) are close to 0.75 and 0.35 respectively. This means that under full alternating loading (stress ratio \( R = -1 \)) we will have \( U = 0.4 \) which corresponds to \( K_{op}/K_{max} = 0.2 \), whereas for a stress ratio \( R = 0 \) we will have \( U = 0.75 \) with \( K_{op}/K_{max} = 0.25 \). Further experiments and theoretical studies are needed to verify these values.

7.4.3 Residual stresses

The presence of residual stresses has for a long time been a source of uncertainty for fatigue crack growth in welded joints, even under laboratory conditions. During a test series, these stresses are cumbersome to measure and difficult to control. Probably the best solution is to carry out tests with stress-relieved specimens and then apply the results to as-welded conditions by explicitly taking account of the presence of residual stresses when calculating the SIF factors. If the residual stress distribution \( \sigma(x) \) along the crack line is known, the corresponding SIF can be found by the weight function method, equation (128). By the principle of superposition, the maximum and minimum SIF during the external loading cycle is found:
\[
K_{\text{max}} = K_r (\sigma_r, a) + K_{\text{max}} (\sigma_n, a), \\
K_{\text{min}} = K_r (\sigma_r, a) + K_{\text{min}} (\sigma_n, a), \\
\Delta K = K_{\text{max}} - K_{\text{min}}, \quad R_{\text{eff}} (a) = \frac{K_{\text{min}}}{K_{\text{max}}}
\] (141)

\( R_{\text{eff}} \) may be very different from the nominal applied stress ratio. There are then two alternative ways of taking account of the residual stresses in the calculations. The first is to select \( C \) and \( m \) values fitted to equation (134) based on nominal \( \Delta K \) and various \( R \) values corresponding to \( R_{\text{eff}} \). The other is to calculate \( \Delta K_{\text{eff}} \) as described earlier and provide data based on this effective SIFR.

As seen from equation (141), \( R_{\text{eff}} \) will be a function of crack depth because the self-equilibrating residual stresses may change from large tensile stresses to compression. If this is the case, then \( R_{\text{eff}} \) and \( \Delta K_{\text{eff}} \) will decrease during crack propagation. This fully explains why as-welded test specimens may give relatively higher growth rates for smaller cracks than for the larger ones, compared with results obtained from stress relieved specimens. If these results are plotted against nominal SIF, the result will be a low exponent \( m \) in Paris equation. Small surface-breaking cracks do not obey the retardation found for longer cracks at low values of SIFR. This becomes even more pronounced if the cracks are propagating in residual tensile stress field as we have shown.

A final comment should be that if \( \sigma_r (x) \) is not known, as usually is the case, constant tensile residual stresses of magnitudes near the yield stress may be assumed for a conservative crack growth estimate.
7.4.4 Size of the initial cracks

Regarding welded joints, there has been a discussion as to whether $N_p$ in equation (138) could be treated as the entire fatigue life, ignoring the number of cycles $N_I$ to crack development. The approach is usually to assume initial flaws of such small size that $N_p$ equals $N$. However, these small, initial cracks must be regarded as fictitious cracks chosen to make LEFM describe the entire fatigue process. They are not real, physical cracks (flaws, intrusions) created by the welding procedures. It is therefore difficult to relate the derived model to actual detected flaws in the weld. Furthermore, the initial cracks are often so small that LEFM is not applicable, typical values being from $a = 0.01$ to $0.05$ mm. This crack size is close to the micro-structural features of the material. Ordinary structural steel qualities often have a grains size near $0.01$ mm. Engesvik [78] concluded that it may be dubious to apply LEFM at crack depths of less than $0.1$ mm. Before this stage the crack development phenomenon is probably better modeled by the Coffin and Manson equation which is based on a local stress-strain approach.

7.5 Geometry function and growth parameters given in design rules and specifications

Reference [74] is issued as a guide to engineering critical assessments for flaws and imperfections in fusion-welded joints. Both unstable fracture and fatigue crack growth are treated for planar crack-like defects. When using equation (134) for welded plate joints, data are available for most of the parameters. For the case shown in Figure 91, fatigue crack growth is modeled by the simple version of the Paris law:
\[
\frac{da}{dN} = C(\Delta K)^m = C (\Delta S \sqrt{\pi a} F(a))^m, \Delta K > \Delta K_0
\]

\[
N = \frac{1}{C} \int_{a_0}^{a_c} \frac{da}{(\Delta S \sqrt{\pi a} F(a))^m}
\]

In simple cases where \( F(a) \) is assumed constant we have the following:

\[
N = \frac{1}{(1 - m/2)(C(\pi \Delta \sigma F))^m}[a_f^{1-m/2} - a_i^{1-m/2}]
\]  

The equation may give reasonably good estimates for the final fatigue life for welded platted joints if \( F(a) \) is set constant to a value corresponding to \( F(0.01) \).

Figure 91 Typical geometry and crack in a fillet welded joint
Ref [73].

7.5.1 The geometry function

The geometry function is determined by equations (123) and (124). The correction factor \( M_k \) due to the stress gradient is given by equation (129). Figure 75 gives the gradient correction \( M_k \) and geometry function \( F(a/T) \) for fillet welded joints for an edge crack.
The solutions for $M_k$ to the left in Figure 92 are obtained by the method that we explained when we presented equations (131) and (132). The geometry function $F\left(\frac{a}{T}\right)$ to the right is valid for a surface crack with a straight front. The solution designated design rules and specifications-3-D has been obtained simply by multiplying $M_k$ to the left by the free surface factor $M_1=1.12$. As can be seen, the function designated design rules and specifications-3-D is very close to the solution presented by Gurney, [79] for cracks with depth greater than $a/T = 0.005$ ($a = 0.1\text{mm for a 20-mm-thick plate}$). The solution given by Gurney is based on a two-dimensional FEA analysis using a weld toe angle of 45 degrees and ignoring the weld toe radius. Hence, the stress field used had to be extrapolated toward the singularity at the weld toe. The analysis was carried out for a joint without a crack and the SIF was subsequently obtained by the weight function method. The given-3-D solution is also determined for a flank angle of 45 degrees and very small values for the toe radius. Both functions are valid for straight-edge cracks. As can be seen from the figure, the Gurney solution is approximately 10% smaller than the design rules and specifications-3-D.
D solution for cracks with a/T > 0.005. If a correction is carried out for the elliptical shape of the crack with a/c close to 0.20, the given-3-D solution for a curved crack front will approach the Gurney straight crack solution.

The design rules and specifications guide also gives values for geometry functions at other points along the crack front. However, in the present analysis the growth is simplified by considering one-direction only. The shape evolution of the crack is derived from experimental data, and introduced into the calculations as a forcing function on the aspect ratio a/2c, where 2c is the crack length at the plate surface (see Figure 91):

\[ 2c = 2.92a + 3.83 \]  

(144)

This expression is based on extensive amount of crack-path data obtained by ink staining of the fatigue crack at various stages during the tests; [80]. Based on the above equation, a crack depth of 1 mm will typically have a length at the surface that is close to 6 mm.

7.5.2 Parameters C and m

In the design rules and specifications, two alternatives are suggested for the relationship between the growth rate \( da/dN \) and the SIFR for a log-log scale. Mean values and scatter are given for the parameters \( m \) and \( C \). The first alternative is based on a single linear relationship, whereas the second alternative proposes a bilinear relationship. The main difference between the two models is that the bilinear models the gradual decrease in the growth rate for low values of the SIFR before the threshold \( \Delta K_0 \) is reached. For the simple linear relationship \( m \) is set to 3.0 and only the upper bound or the \( C \) is given. In an earlier version PD6493, [81] recommended values were equal to \( 3.0 \times 10^{43} \) (units MPa, mm), whereas design rules and specifications recommends as high as \( 5.21 \times 10^{13} \) (units...
MPa, mm): see Table 16. Hence, the value is increased as much so by 80% from the first version. The mean values are not given, and we have listed the mean value found in Ref [17] in Table 16. This is done because the figure or the mean plus two standard deviations (mean + 2SD) given in Ref [17] coincides with the upper bound PD6493.

Data for the bilinear relationship are given in Table 17. The stage A/stage B transition point is 363 N/mm$^{3/2}$ for the mean curve and 315 N/mm$^{3/2}$ for the mean plus two standard deviations curve. For shallow surface cracks the threshold value $f$ SIFR is given as 63 N/mm$^{3/2}$ as a lower limit regardless of the applied stress ratio.

![Figure 93 Sketch of one-slope and bilinear relationship Ref [74].](image)

**Table 18 Growth parameters in air one-slope curve $m=3.0$ Ref [75].**

<table>
<thead>
<tr>
<th>Growth curve</th>
<th>Ref</th>
<th>m</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>17</td>
<td>3</td>
<td>$1.85\times10^{-13}$</td>
</tr>
<tr>
<td>Mean +2SD</td>
<td>9</td>
<td>3</td>
<td>$3.00\times10^{-13}$</td>
</tr>
<tr>
<td>Upper Bound</td>
<td>16</td>
<td>3</td>
<td>$5.21\times10^{-13}$</td>
</tr>
</tbody>
</table>

For $da/dN$ in mm/cycle and $\Delta K$ in N/mm$^{3/2}$
7.6 Fracture mechanics model for a fillet welded plate joint

7.6.1 Basic assumptions and criteria for the model

Based on the above discussion of applied fracture mechanics and the recommendations given in design rules and specifications PD6493, we shall demonstrate LEFM is abilities and shortcomings for fillet welded joints. The objective is to establish a model that is consistent with design rules and specifications both based on the S-N approach and applied fracture mechanics. Emphasis is placed on how to choose growth parameters in conjunction with a fictitious, initial crack size to obtain both reliable crack growth paths and predictions of the entire fatigue life. There is no doubt that the model outlined above and the parameters given by PD6493 are well suited to describing the behavior of large cracks detected and sized during inspection. It is however more uncertain as to whether the model is capable of describing the entire fatigue process from initiation to final fatigue failure for high-quality welded joints. This will obviously be an approximation and we will pinpoint the merits and shortcomings of such a model with reference to two databases presented below. Based on these consideration, a model will be established to meet the following criteria:

<table>
<thead>
<tr>
<th>Stage A</th>
<th>Mean curve</th>
<th>Mean +2SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>m</td>
<td>C</td>
</tr>
<tr>
<td>1.21x10^{-26}</td>
<td>8.16</td>
<td>4.37x10^{-26}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stage B</th>
<th>Mean curve</th>
<th>Mean +2SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>m</td>
<td>C</td>
</tr>
<tr>
<td>3.98x10^{-13}</td>
<td>2.88</td>
<td>6.77x10^{-13}</td>
</tr>
</tbody>
</table>

For da/dN in mm/cycle and ∆K in N/mm^{3/2}.

Table 19 Growth parameter in air Ref [75].
— the model should be corroborated by S-N data for the joint in question when these
data are available (database 1 and 2);
— the model should predict a crack evolution that coincides with measured crack
growth histories before failure (database 1).

With this background we will endeavor to model the fatigue process in fillet-welded
joints. Before proceeding let us have a closer look at experimental data.

7.6.2 Data for crack growth measurements (database 1)

Database 1 contains crack growth measurements made directly on fillet-welded
joints. The results have been presented in [80, 83]. 34 non-load-carrying cruciform and T-
joint test specimens were tested under constant amplitude axial loading. All the test joints
were fabricated from C-Mn steel plate with a 25-mm thickness. The nominal yield stress
was 345 MPa. The welding procedures were taken from normal offshore fabrication
practices. The joints were proven free from cracks and undercuts. The specimens were
tested under constant amplitude axial loading at $\Delta S = 150$ MPa with a loading ratio of $R=0.3$. Experimental details are to be found in [80]. Typical crack size histories as a function
of time are shown in Figure 94.

The total fatigue lives for the 34 specimens have been plotted in Figure 95. The main
characteristics of the measurements are given in Table 18. As can be seen, the time to reach
a crack depth of 0.1 mm is close to 30% of the total fatigue life, whereas the total life is
only 10% less than the prediction of the Category E S-N curve ($N = 513,000$ cycles).
Hence, the test series is of normal quality and comparable with the population pertaining
to the Category E and F-class.
Data points pertaining to the Category E S-N curve have the center of gravity at a stress range in the region of 50–80 MPa (17.4–21.7 ksi). This regime is well represented by database 1. In addition, we have in the present work assembled results at lower stress levels from fatigue test series in several large experimental investigations,[73]. The applied stress ranges are between 30 and 70 MPa (3.6–7.2 ksi) and the thicknesses of the plates ranged from 16 to 38 mm (0.63–1.5 in.). Other thicknesses are excluded to minimize the so-called thickness effect. All the selected specimens are as-welded and the loading ratio R is between 0 and 0.3. These data-points are plotted in Figure 101. As can be seen, most of the data points have substantially longer lives than predicted by the Category E curve, with only a few exceptions. Furthermore, some of the results are run outs. The scatter

| Nᵢ, defined as time to reach 0.1 mm |
|-----|-----|-----|-----|
| Nᵢ | Nᵢ | Nᵢ | Nᵢ, Category E |
| 140 | 330 | 468 | 513 |

Figure 94 Crack growth histories in database I Ref [76].
is considerably greater than for database 1, as expected. This is partly due to the fact that
database 1 contains only one homogeneity test series, but mostly due to the fact that scatter
increases at low stress levels as already discussed, scatter increases at low stress levels.

![Plot of S-N points for databases 1, 2 and 3 with median curves in current rules](image)

7.6.4 Procedure and curve fitting

The numbers of cycles to reach given crack depth are calculated by numerical
integration of equation (134) as follows:

\[
N = \frac{1}{C} \int_{a_0}^{a_c} \frac{da}{(\Delta \sigma \sqrt{\pi a F(a)})^m}
\]

(145)

It is noted that equation (145) can be reduced to the same general form as the S-N
curve given in before for constant amplitude loading. The number of cycles is inversely
proportional to the stress range raised to a power of m in both equations. The problem
arises when using equation (145) is to choose an appropriate combination of the parameters
C, m, and a₀. For the parameters C and m, suggestions based on growth tests for C-Mn
steels are found in design rules and specifications (Tables 16 and 17), whereas the depth a₀

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is difficult to measure for welds and must be taken based on experience and previous destructive testing.

In contrast to the influence of the initial crack depth, the fatigue life is not as sensitive to the values of the critical crack depth $a_c$. In the present work, $a_c$, is set to half of the plate thickness of the joints. The $F(a)$ geometry function is based on the work by Gurney [79], who derived numerical values for $F(a)$ for an edge crack with an average weld toe profile; (see Figure 92). In the present analysis we will examine both the linear and the bilinear relationships in design rules and specifications (Figure 93) with respect to database 1. We will adopt the m values and see if the corresponding C values derived from each of the growth curves in Figure 95 are compatible with the statistics given in Tables 16 and 17.

The procedure for determining the model parameters are as follows:

1) the slope parameter m of the growth rate curve is chosen in accordance with design rules and specifications (Tables 16 and 17) for the actual value of $\Delta K$;

2) the parameter C is determined so that the measured number of cycles spent between depths of 0.1 mm and the final crack depth of 12.5 mm coincides with the measured number of cycles for each joint in database 1;

3) with the C and m values obtained above for a given test, $a_o$ is determined so that the number of cycles from $a_o$ to the first measured crack depth of 0.1 mm coincides with the measured number of cycles. Hence, for each of the 34 tests, a set of the variables m, C and $a_o$ is obtained.

Figure 96 shows experimental curves taken from Figure 94 and curves obtained from fracture mechanics using the above procedure and equation (145). As can be seen from the
figures, the a-N curves simulated by the two alternative relationships are quite close to the experimental results.

The measured curves are somewhat more irregular, but they are very "Paris-like" and the fit must be regarded as excellent considering the fact that we have kept m fixed. If we had optimized the fit by letting both m and C vary, the results would even been even closer. However, the scope of the present work is to use the recommendation in design rules and specifications.

Figure 96 Measured and fitted crack growth histories Ref [76].
7.6.5 Growth parameters C and m

A histogram for the natural logarithm of C (MPa, m^{0.5}) obtained by simulation is shown in Figure 97. The values were obtained under the assumption of a single linear relationship between crack growth rate and the SIFR for a log-log scale with m = 3. The variable in C has a rather even distribution and not a normal distribution as might have been expected.

The mean value for In C is -29.4 (units MPa, mm^{0.5}) with a standard deviation of 0.20. The statistics for C are shown in Table 19.

![Figure 97 Statistics of ln C (MPa,M) obtained from curve fitting to experimental a-N curves Ref [76].](image)

<table>
<thead>
<tr>
<th>Median</th>
<th>Median +2SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.67x10^{-13}</td>
<td>2.48x10^{-13}</td>
</tr>
</tbody>
</table>

For da/dN in mm/cycle and K in N/mm^{3/2}

The mean value in Table 19 is only 10% less than the design rules and specifications mean value given in Table 16. The mean value plus two standard deviations is only 20% less than the value given in PD6493. Figure 97, to the left-hand side, shows the experimental results plotted for an arbitrary value of the SIFR together with the design
rules and specifications mean and upper bound curves. None of the results obtained were in the vicinity of the upper bound.

A similar analysis was carried out for a bilinear relationship. The obtained results do not fit the lower line as shown to the right in Figure 98. The growth rates are close to 4 times higher than the values given in design rules and specifications. If the geometry function given by design rules and specifications had been used instead of the Gurney function, the growth rates would have been reduced to twice the values given in design rules and specifications. The reason for these high growth rates is likely due to the lower part of the bilinear curve derived from tests with relatively long cracks (several millimeters) in compact tension specimens. The fatigue process in welded joints comprises crack growth of small surface-breaking elliptical cracks with depths of less than 0.1 mm. These cracks may grow considerably faster than the lower part of the bilinear curve in design rules and specifications. The results are consistent with the sketch in Figure 90. Hence, one should take care when using the bilinear relationship given in design rules and specifications to calculate the early stage fatigue-crack growth in welded joints. These curves should only be used when larger cracks are detected.
7.6.6 The initial crack depth $a_0$

Based on the curve fitting procedure above, the initial crack depth for each test was also obtained. The statistics for the derived initial crack depths are given in Table 20. The distribution of $a_0$ is shown above in Figure 99. As can be seen from Table 20, the mean value for the initial crack depth is 0.015 mm and the upper bound is close to 0.03 mm. It should be kept in mind that the model does not take into account the variability of the local toe geometry when determining the initial crack depth we have held the geometry function $F(a)$ constant at its mean value. It should also be emphasized that this initial crack depth distribution is theoretical concept.

A corresponding analysis was carried out for the bilinear relationship and gave a mean value for the initial crack depth of close to 0.06 mm.

<table>
<thead>
<tr>
<th>Mean [mm]</th>
<th>Standard Deviation [mm]</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0151</td>
<td>0.0045</td>
<td>0.3</td>
</tr>
</tbody>
</table>
The curve in Figure 99 (b) shows the part of the growth-rate curve used when calculating the growth rate from $a_c$ to $a = 0.1 \text{mm}$. As can be seen, we are very close to the threshold value of $63 \text{N/mm}^2 \cdot \text{mm}^{0.5} (2 \text{ MPa m}^{0.5})$.

![Figure 99 (a) PDF for $a_0$. (b) Part of the relationship $da/dN$ versus $K$ uses (MPa,mm) Ref [78].](image)

7.6.7 Prediction of crack growth histories and construction of S-N curves

After curve fitting was carried out for the experimental stress range of 150 MPa, the obtained mean values for $a_0$ and $C$ were substituted into equation (149) to calculate both crack evolution and the fatigue life at various constant-amplitude stress levels. Both the linear and bilinear results were used. Crack evolution at stress ranges equal to 150 MPa and 100 MPa are shown in Figure 100.
As can be seen from Figure 100, the two relationships follow slightly different paths to arrive at the same end point for AS 150 MPa. The fatigue lives predicted at ΔS = 100 MPa are, however, very different. The linear relationship gives a fatigue life close to 1.5 x 10⁶ cycles, which is again 10% less than the Category E S-N prediction, whereas the bilinear relationship gives a fatigue life close to 7 x 10⁶ cycles, long as compared to the S-N estimate. The latter scaling of fatigue life with the applied stress range will not be consistent with the slope of the S-N curve. This is because close to half the fatigue life (from the initial crack depth to a depth of 0.6 mm) scales relative to the stress level with a power of m = 8.16 (Table 17), whereas the Category E S-N curve has an inverse slope corresponding to m = 3. The predictions made by the linear relationship will correspond to the predictions made by the S-N curve Category E, whereas the predictions made by the bilinear curve do not. It is also to be noted that both the Category E curve and the fracture mechanics model (FMM) fail to predict the long, experimental lives given by the data points in database 2 at low stress levels; see Figure 95. As for the fatigue-limit ΔS₀, this is determined from the threshold value ΔK₀ by the following equation:
\[ \Delta S_0 \sqrt{\pi a_0 F(a_0)} = \Delta K_0 \quad (146) \]

If we enter a mean value of \( a_0 = 0.015 \) ram into this equation we get \( \Delta S_0 = 95 \) MPa, which is far greater as compared with the given Category E value of 56 MPa. The fatigue limit is between 40 MPa and 56 MPa as can be seen from Figure 95. Even if we choose an initial crack depth equal to the mean value plus two standard deviations (0.025 mm), we will still get an endurance limit as high as 75 MPa. The reason is the same for the discrepancy found between the experimental growth rates and the growth rates given in the lower part of the bilinear relationship in design rules and specifications. Both the prescribed crack growth rate decrease and the final stop at low SIFR are valid for larger cracks only. They are not applicable to shallow elliptical cracks at the weld toe. The investigation supports the choice of a one-slope curve for the da/dN as a function of SIFR. Furthermore, the fatigue limit in the S-N curve is not readily explained by a crack growth threshold phenomenon.

7.6.8 Inferences for fillet joints with cracks at the weld toe

The entire fatigue process in fillet-welded joints has been modeled by a pure fracture mechanics approach. The model has been calibrated to describe the entire fatigue process. The simple version of the Paris law has been adopted. The initial crack depth and growth rate parameters have been determined to fit each experimental a-N curve in database 1. The following conclusion can be drawn:

The single linear relationship between da/dN and SIFR for a log-log scale with slope \( m = 3.0 \) gives a good fit between the calculated a-N curves and the curves measured at a stress range of 150 MPa. The growth rates found have a mean value close to the mean value given in design rules and specifications. Furthermore, all the rates are well below the upper
bound given in design rules and specifications. In fact, the mean plus two standard
deviations is quite close to the upper bound given in the document PD6493. The predicted
fatigue life is very close to the predictions given by the Category E S-N curve at any stress
level above the fatigue limit.

The initial cracks are in a range between 0.005 and 0.03 mm with a mean value of
0.015 mm. The initial crack distribution applied for the model is difficult to verify by
experimental findings. Hence, the fatigue-limit stress cannot be determined from a simple
model. The bilinear relationship between da/dN and SIFR for a log-log scale also gives
good agreement with experimental a-N curves. However, the derived growth rates are
higher than the upper bound given in design rules and specifications for low values of the
SIFR. Compared with the Category E curve, the model fails to predict the slope of the
curve and overestimates significantly the fatigue life as the stresses approach service
stresses.

In conclusion, the FMM should not be used to model the entire fatigue process in
high-quality welds that have been proven to be free from detectable initial cracks. Although
the model is capable of describing crack evolution at any one given stress level, it will fail
to predict the change in slope of the S-N curve as the stress range decreases. Furthermore,
the fatigue limit will be overly optimistic. The FMM should primarily be applied in cases
where cracks are found and sized. In other cases, a crack development phase should be
modeled before the crack-propagation phase is added on.

7.7 Summary

This Chapter is an outline of applied fracture mechanics. In this part important
questions, such as how fast a crack will grow during service loading and what is the critical
crack size that leads to unstable rupture during extreme load, are treated. These are crucial questions to answer at a post-fabrication stage when cracks have been detected and the alternatives are repair or no repair. Moreover, crack growth behavior is crucial information when planning scheduled in-service inspections.
8.1 Introduction

The S-N model is based on the assumption that the number of cycles $N$ to failure is directly correlated to the applied nominal stress range $\Delta S$. All other parameters are assumed to play a secondary role. The background for these curves is a simple linear regression analysis in the finite life region. The data for the curves are obtained by accelerated constant amplitude tests at different high stress levels. Also, the model assumes the existence of a fatigue limit, given as a stress range below which no failure will take place. This fatigue limit is subsequently added as a horizontal cut-off line to the regression line. The final curve is bilinear for a log-log scale between $N$ and $\Delta S$. These curves are presented as the conventional S-N curves given in design rules and specifications. We have shown how S-N data were analyzed to establish the median and the design S-N curve for the fatigue life in the finite life region.

The procedure and calculations are quite straightforward; apart from test specimens that are stopped at a given number of cycles before fatigue fracture has occurred. The slope of the mean curve was determined along with the coefficient of variation in fatigue life. We did not go into the problem of determining the fatigue limit. In this section, emphasis
is placed on the modeling of fatigue life close to this limit. This stress regime is of major importance because many structures will be subjected to service stresses that correspond to this stress region.

Experimental data in this stress regime are sparse and do not fit the knee point of the conventional bilinear S-N curve found in design rules and specifications. Consequently, an alternative model where both the fatigue life and the fatigue limit are simultaneously treated as random variables is investigated. The model parameters for this random fatigue-limit model (RFLM) are determined by the maximum likelihood method, and confidence intervals are obtained by the profile likelihood method. The advantages of the RFLM are that it takes into consideration the variations in fatigue limits found from specimen to specimen and run-out results are easily included.

8.2 Objectives

The objective of this work was to investigate the ability of the bilinear S-N curves (AISC, Eurocode 3, BS 5400, [96–98]) to describe the fatigue life behavior in general and in particular near the fatigue-limit regime. The validity of the upper part of these curves in the finite life area is not questioned, as these curves have been corroborated by a huge amount of data. The dilemma is that the conventional S-N curves are widely used in a stress regime where the curve changes slope to a horizontal line as described above and where very few data exist to corroborate this abrupt shift in slope. Small alterations of the position of the knee point of the curve in the fatigue-limit area will have a strong bearing on fatigue life predictions and, as a consequence, fatigue design and final dimensions of welded details.
If the ability of the bilinear curve to describe fatigue life behavior is dubious, other statistical models should be applied to obtain more reliable predictions. In this work, an investigation is made using the random fatigue-limit model (RFLM) suggested by Pascual and Meeker [99]. In this model, the scatter in finite fatigue life and in the fatigue limit is integrated into one joint model and treated simultaneously. This results in a smooth nonlinear S-N curve for a log-log scale. The model has so far been applied with success to describe the fatigue life behavior of epoxy laminate panels and smooth specimens with metal base material [99, 100]. It has so far not been applied to welded joints.

To study this problem, the present study has endeavored to collect experimental data in the actual stress regime for fillet welded joints Figure 101.

![Figure 101 Plate with fillet welded attachments Ref [96-97]](image)

For this type of joint, fatigue cracks emanate from the weld toe and through the plate thickness. The purpose of the study is to compare conventional S-N curves found in design rules and specifications and with the alternative approach based on the random fatigue-limit model for given series of test data. The analysis and discussion are limited to constant amplitude behavior in a dry air environment in the high cycle fatigue regime.
8.3 General consideration for the conventional S-N approach

8.3.1 Basic assumptions

Let us start by repeating the basic assumptions for the S-N curves of a given welded detail. These assumptions are:

— There is a strong dependence between the applied stress range $\Delta S$ and fatigue life given as the number of cycles $N$ to failure;
— The standard deviation in fatigue life is not constant and increases as the stress range decreases;
— There is a fatigue limit $\Delta S_0$ and it is assumed that below this threshold stress the joints will never fail.

8.3.2 The S-N Approach Based on design rules and specifications

In design rules and specifications, the relation between number of cycles $N$ and the stress range $\Delta S$ is assumed to be linear for a log-log scale:

$$\log(N) = \log A - m\log(\Delta S) + \varepsilon \Delta S > \Delta S_0$$  \hspace{1cm} (147)

where:

$Log$ : denotes the logarithm to base 10.
$A$ and $m$ : parameters characterize the fatigue quality for the joint in question
$\varepsilon$ : is the error term due to the inherent scatter
$\Delta S_0$ : is the fatigue limit, and it is assumed that no failure occurs under this threshold value

Hence, the design ruled-based S-N curves are bilinear for a log-log scale. The upper curve will have a slope $-1/m$, whereas the lower line will be horizontal. Examples are given in...
Figure 102, which shows the Category E curve and F-class curve taken from AASHTO - Highway Safety Manual and BS 5400 respectively. These curves are discussed below.

The parameters are determined by linear regression analysis in the finite life regime. The standard deviation for the logarithm of fatigue life is assumed constant for all stress levels. As a consequence, the coefficient of variation (COV) for fatigue life will be constant, and the standard deviation will increase toward lower stress levels. The COV in fatigue life is as high as 0.5 and this makes scatter a major issue. The S-N approach is based entirely on constant amplitude (CA) experimental fatigue data. A linear regression analysis is carried out, and the mean curve and standard deviation for the fatigue life are obtained. The design curve is drawn at the median value minus two standard deviations [96], alternatively at minus 1.5 standard deviations if the standard deviation has a 75% confidence level [97]. At a given stress level, the fatigue life is assumed to obey a log normal distribution.

This implies that the resulting mean logarithmic curve corresponds to a failure probability of $p = 0.5$, i.e. the median fatigue life. In this section we work with the median S-N curve unless otherwise stated. The S-N curve taken from design rules and specifications, [96,97,98], reads

$$\begin{align*}
N &= \begin{cases} 
A\Delta S^{-m}, & \Delta S > \Delta S_0 \\
\infty, & \Delta S \leq \Delta S_0 
\end{cases} 
\end{align*}$$

(148)

The parameters for the median Category E curve are $\log A = 12.238$ and $m = 3$. The COV is 0.54. The fatigue limit is 35 MPa and the corresponding fatigue life is $10^7$ cycles. The Category E gives almost the same predictions as F-class [97, 98], except that the latter curve becomes horizontal at $5 \times 10^6$ instead of $10^7$ cycles. This difference pinpoints the uncertainty in the stress region near the knee-point of the curves and this is why we have
collected test results in this region. The curves are drawn in Figure 102 together with collected data points.

![Figure 102 Plot of Bilinear rule-based median S-N curves together with test data.](image)

As can be seen from the figure, almost all the data are to the right-hand side of the curves near the knee points. A general problem with the S-N curves is that they are based on data compiled without regard to material quality, thickness, and loading ratio. A re-analysis of the data where these aspects are taken into account when defining the test population is given in [101]. More homogeneity classes are chosen and the scatter in fatigue life decreases significantly.

8.4 S-N Curves Based on a Random Fatigue Limit Model

Due to the uncertainty and large scatter in fatigue life in the knee-point region, an S-N curve based on a random fatigue limit approach has been proposed [99, 100]. In fact, the sparse data available indicate that there is a variation in fatigue limit from specimen to specimen. Consequently, the distribution for the fatigue limit should be sought and incorporated into the statistical model for the fatigue life. It should not be treated separately
as done for the bilinear curves. The S-N curve obtained from the RFLM will not have an abrupt change from an inclined straight line to a horizontal line, but a gradual change in slope as stress ranges get very low. Our hypothesis is that this nonlinear curve for a log-log scale is more consistent with observed fatigue life data for welded details at low stress ranges. The governing equation is

\[
\ln(N) = \beta_0 - \beta_1 \ln(\Delta S - \gamma) + \varepsilon, \Delta S > \gamma
\]  
(149)

where

\[
\ln : \text{denotes the natural logarithm}
\]

\[
\gamma = \Delta S_0 : \text{is the fatigue limit.}
\]

\[
\beta_0 \text{ and } \beta_1 : \text{are fatigue curve coefficients.}
\]

As can be seen, Equation 149 is fundamentally different from Equation 147. Let \( V = \ln(\gamma) \) and assume that \( V \) has a probability density function (PDF) given by

\[
f_V(v) = \frac{1}{\sigma_V} \phi_V \left( \frac{v - \mu_V}{\sigma_V} \right)
\]  
(150)

where

\[
\mu_V : \text{location parameter}
\]

\[
\sigma_V : \text{scale parameter}
\]

\[
\phi_V(\cdot) : \text{is the normal PDF.}
\]

The log distribution was chosen because it gave the best fit to fatigue data in [99], and due to the fact that it is the usual assumption for fatigue life distribution in design rules and specifications. Let \( x = \ln(\Delta S) \) and \( W = \ln(N) \). Assuming that, conditional on a fixed value of \( V < x \), \( W|V \) has a PDF

\[
f_{W|V}(w) = \frac{1}{\sigma} \phi_{W|V} \left( \frac{w - [\beta_0 - \beta_1 \ln(\exp(x) - \exp(v))]}{\sigma} \right)
\]  
(151)
with the location parameter \( \beta_0 - \beta_1 \ln(\exp(x) - \exp(v)) \) and scale parameter \( \sigma \). The marginal PDF of \( W \) is given by

\[
 f_W(w) = \int_{-\infty}^{x} \frac{1}{\sigma \gamma} \phi_{W|V}(w - \frac{[\beta_0 - \beta_1 \ln(\exp(x) - \exp(v))]}{\sigma}) \phi_V \left( \frac{v - \mu_V}{\sigma_V} \right) dv \tag{152}
\]

The marginal cumulative distribution function (CDF) of \( W \) is given by

\[
 F(w) = \int_{-\infty}^{x} \frac{1}{\sigma \gamma} \Phi_{W|V}(w - \frac{[\beta_0 - \beta_1 \ln(\exp(x) - \exp(v))]}{\sigma}) \phi_V \left( \frac{v - \mu_V}{\sigma_V} \right) dv \tag{153}
\]

Where

\( \Phi_{W|V} \): is the CDF of \( W|V \).

\( w_i \): given sample data

\( x_i \): various test specimens \( i = 1 \),

\( n \): the model parameters can be determined by the maximum likelihood (ML) function

\[
 L(Q) = \prod_{i=1}^{n} \left[ f_W(w_i; x_i Q) \right]^{\delta_i} \left[ 1 - F_W(w_i; x_i Q) \right]^{1-\delta_i} \tag{154}
\]

Where:

\( \delta_i = 1 \) if \( w_i \) is a failure

\( \delta_i = 0 \) if \( w_i \) is a censored observation (run out).

The vector \( Q \) contains the model parameters

\[
 Q = (\beta_0, \beta_1, \sigma, \mu_V, \sigma_V) \tag{155}
\]

Once these parameters have been determined from optimization of Equation 154, the corresponding confidence intervals can be obtained by a profile likelihood method using profile ratio of the variables together with chi-square statistics. The integration of
Equations 152 and 153 and the optimization of Equation 154 must be done numerically for more detail see [99, 100]. When the parameters are determined, we can calculate the fatigue life for a chosen probability \( p \) of failure using Equation 153. Hence, the median curve and quintiles curves for design purpose are obtained.

8.5 Experimental data for model integration

8.5.1 Data for fatigue life at high stress levels (database 1)

Database 1 contains experimental fatigue lives in the finite life regime where all tests are continued until fracture occurs. The test series consist of 34 non load-carrying cruciform and \( T \) joint test specimens (Figure 101). All the test specimens were fabricated from C-Mn steel plate 25 mm (1 in.) thick. The nominal yield stress was 345 MPa (50 ksi). The welding procedures were taken from normal offshore fabrication practice. The joints were proven free from cracks and undercuts. The specimens were tested under constant amplitude axial loading at \( \Delta S = 150 \) MPa (21.7 ksi) with a loading ratio of \( R = 0.3 \). Experimental details are found in [102]. The total fatigue lives for the 34 specimens have been plotted in Figure 102. The median life of the series (\( N = 460,000 \) cycles) is only 12% less than the prediction of the Category E (\( N = 513,000 \) cycles). Hence, the test series is of normal quality and comparable with population pertaining to the Category E and F-class. However, due to the homogeneity of the test series, the COV is much smaller \( COV = 0.22 \). In addition to recording the fatigue life, crack growth measurements were made during the course of each test. These data are important information for modeling the fatigue process, but not for the present fatigue life statistical analysis.
8.5.2 Data for fatigue lives at low stress levels (database 2)

Data points pertaining to the Category E S-N curve have the center of gravity at a stress range in the region of 50–80 MPa (17.4–21.7 ksi). This regime is well represented by database 1. In addition, we have in the present work assembled results at lower stress level from fatigue life test series in several large experimental investigations carried out in [103]. The applied stress ranges are between 30 and 70 MPa (3.6–7.2 ksi) and the thicknesses of the plates range from 16 to 38 mm (0.63–1.5 in.). Other thicknesses are excluded to minimize the so-called thickness effect. All the selected specimens are as-welded and the loading ratio R is between 0 and 0.3. These data-points are also plotted in Figure 102. As can be seen, most of the data points have substantially longer lives than the predictions of the Category E curve, with only a few exceptions. Furthermore, some of the results are run outs. The scatter is considerably greater than for database 1, as expected. This is partly due to the fact that database 1 contains only one homogeneity test series, but mostly due to the fact that scatter increases at low stress levels.

8.6 Comparison between the Category E curve, the RFLM based curve and the data

By applying Equation 150 to 155 for the databases presented above, the parameters for the RFLM are determined and given in Table 23. The 90% confidence intervals are also listed. The confidence intervals are obtained from plots of the profile ratio as shown for $\mu_\gamma$ and $\sigma_\gamma$ in Figures 104.
As can be seen, the point estimate for the fatigue limit is near 36 MPa. The AISC fatigue limit of 36 MPa is well within the 90% confidence interval and is quite close to the point estimate of 36 MPa. The BS 5400 fatigue limit of 56 MPa is far outside our confidence interval. Although the present database is limited, it is a surprise that the BS5400 fatigue limit is so far outside the 90% confidence interval. The median curve (p = 0.5) for the fatigue life pertaining to the point estimates in Table 23 is drawn in Figure 105 along with the Category E curve and the data points.

As can be seen, the Category E curve, the RFLM curve, and the data points at a stress range of 150 MPa (Database 1) are in good agreement. At lower stress ranges the RFLM curve becomes nonlinear and predicts substantially longer fatigue lives than does the Category E. At a stress range of 50 MPa the RFLM predicts more than three times longer fatigue life, and at 40 MPa, the RFLM predicts close to 10 times longer fatigue life than the Category E.

It can also be seen from the figure that the RFLM is more in accordance with the data-points. The quadratic sum of the error terms ε in equation 149 will be much less than the corresponding sum pertaining to equation 147. As the stress level approaches the Category E fatigue limit of 36 MPa, the RFLM curve has becomes nearly horizontal, so

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point estimate</th>
<th>90% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>β₀</td>
<td>(18.39)</td>
<td>(18.308) (18.456)</td>
</tr>
<tr>
<td>β₁</td>
<td>2.1</td>
<td>2.2 (2.12)</td>
</tr>
<tr>
<td>σ</td>
<td>(0.44)</td>
<td>(0.39) (0.51)</td>
</tr>
<tr>
<td>μγ</td>
<td>3.52 (40.3 MPa)</td>
<td>3.4 (37 MPa) 3.6 (43.7 MPa)</td>
</tr>
<tr>
<td>σγ</td>
<td>0.2</td>
<td>0.12 (0.22)</td>
</tr>
</tbody>
</table>
that it almost coincides with the Category E horizontal line. However, it is only for fatigue lives longer than $10^9$ cycles that the two curves are for all practical purposes identical. This illustrates that care must be taken when comparing fatigue limits as we did above. The fatigue limit for the Category E will already appear at $10^7$ cycles, whereas the RFLM curve will approach the fatigue limit of 50 MPa at $5 \times 10^9$ cycles. The reason for this major difference is that the RFLM is based on a joint distribution (random fatigue life-random fatigue limit) and this will push the point estimate for fatigue limit toward a higher number of cycles.

When comparing the RFLM curve with the F-class curve, it can be seen from Figure 105 that the discrepancy is less than was found from comparison with the Category E. In fact, the curves cross each other at $5 \times 10^7$ cycles at the fatigue limit of 36 MPa given for the Category E. Above this stress range the RFLM curve will predict longer lives than the Category E, whereas at lower stress ranges the reverse will be true. The quantile curve pertaining to $p = 0.025$ is shown in Figure 106. This curve is intended for design purposes. When comparing with the data points, it can be seen that it is only the two run outs at $2 \times 10^8$ cycles that are well below the curve, whereas two failure data points at $10^6$ and $10^7$ cycles are just beneath the curve. All these 4 points pertain to database 2. The curve obtained cannot be directly compared to the Category E design curve due to the fact that our database is more limited and has less scatter in the finite life region (database 1).
Figure 103 Profile likelihood with 90% confidence interval for the median and standard deviation of the fatigue limit
To force our model to be valid for the data pertaining to the Category E in the finite life region, we may adjust the parameters given in Table 23 so that the mean life and scatter for high stresses coincide with the Category E data. The parameter $\beta_0$ is increased from
18.39 to 22.60 and $\sigma$ is increased from 0.44 to 0.50. The latter figure is a characteristic scatter for most categories of welded joints. The other parameters are kept constant as a first approximation. The obtained quantile S-N curve ($p = 0.025$) is drawn together with the Category E design curve (based on logarithmic mean minus two standard deviations). As can be seen from Figure 107, the fit between the two curves is good in the high-stress region, with the curve coinciding at stress ranges above 110 MPa. Both curves have safety margins to all the data points in test series 1, as expected. As for the median curves, the RFLM curve will predict substantially longer lives than the Category E below 100 MPa. In fact, the RFLM curve is only slightly lower in this stress region compared with the original quantile curve in Figure 106. This is due to the fact that it is the parameters that characterize the random fatigue limit that mainly govern the curve in this area. As can be seen from Figure 107, the present design curve will approach the F-class curve at high and low stress ranges. It is only near the knee point that the discrepancy in fatigue life is significant.
8.7 Conclusions

The statistical behavior of the fatigue life of fillet welded joints has been examined and modeled with reference to conventional S-N curves found in current design rules and...
specifications. An alternative statistical model based on a joint random fatigue life and a random fatigue limit has been applied. Constant amplitude fatigue life data near the “knee point” of the rule-based bilinear SN curves are assembled to study and corroborate the model. The model has been fitted to experimental fatigue lives and the obtained S-N curve is compared with the traditional bilinear S-N curves given in design rules and specifications. The rule-based S-N curves and the RFLM based curve coincide for stress ranges above 110 MPa. For stress ranges below 100 MPa, the RFLM curve will predict fatigue lives that are from 2 to 10 times longer than the predictions made by the Category E S-N curve. It appears that the nonlinear curve obtained from the RFLM has a much better ability to model fatigue life behavior in this stress region. The abrupt knee point of rule-based bilinear curves does not fit the experimental facts for the assembled data. The fatigue life behavior in this stress regime is obviously more complex than the conventional bilinear S-N curve can describe. The discrepancy between the present RLFM curve and the Category E curve is important as it occurs in a stress region where the majority of the load cycles for a welded detail in service usually occur. The rule-based S-N curves seem un-conservative in this regime and this will have a strong bearing on practical fatigue life predictions, fatigue design and final dimensions of welded details.

8.8 Summary

Statistical models for fatigue life prediction for welded joints are discussed and fitted to experimental data for fillet-welded steel joints where cracks emanate from the weld toe. The models are based on an S-N approach where the number of cycles N to failure is assumed to be directly correlated to the applied nominal stress range $\Delta S$. The models assume the existence of a fatigue limit given as a stress range below which no failure will
take place. Emphasis is laid on the modeling of the fatigue life close to this limit where the service stresses for welded details often occur. Experimental data in this stress regime are sparse and do not fit the knee point of the conventional bi-linear S-N curve found in design rules and specifications. Therefore, an alternative model where both the fatigue life and the fatigue limit are simultaneously treated as random variables is investigated.

The model parameters for this random fatigue-limit model (RFLM) are determined by the maximum likelihood method, and confidence intervals are obtained by the profile likelihood method. The advantage of the model is that it takes into consideration the variation in fatigue limit found from specimen to specimen and that run-out results are easily included. The median S-N curve obtained from the model coincides with the conventional bi-linear curves in the high stress regime (stress ranges higher than 110 MPa), but predicts longer lives as the stress range decreases below 100 MPa.

The model gives a nonlinear S-N curve for a log-log scale in the fatigue-limit area; the fatigue life is gradually increasing and is approaching a horizontal line asymptotically instead of the abrupt knee point of the bilinear curve. The nonlinear curve is more in accordance with experimental data. At stress ranges below 100 MPa, the predicted fatigue lives are between 2 to 10 times longer than predictions made by the bilinear Category E curve in AISC Steel Construction Manual. The conclusion is that the rule-based S-N curves may be un-conservative in the stress regime where service stresses frequently occur. A more correct statistical model based on a random fatigue-limit model results in S-N curves that give decreased dimensions for a given fatigue design factor under constant amplitude loading.
CHAPTER IX

MODELING THE FATIGUE PROCESS IN A WELDED JOINT

9.1 Introduction and objectives

Earlier we discussed the merits and shortcomings of a fracture mechanics mode (FMM) when it comes to modeling the entire fatigue process in a welded joint. In conclusion, the FMM should not be used to model the entire fatigue process in high-quality welds proven free from detectable initial cracks. Although the model is capable of describing the crack evolution at one given stress level, it will fail to predict the fatigue life as the stress range decreases. Furthermore, the fatigue limit will be overly optimistic. The FMM should only be applied if cracks are found and sized. In other cases, a crack development phase should be modeled before the crack propagation phase is added. This will be addressed in the present chapter.

Let us bear in mind that a statistical model based on a joint random fatigue life and a random fatigue limit (random fatigue-limit model (RFLM)) was applied to predict the fatigue life of welded joints. The S-N curve obtained from this RFLM was non-linear for a log-log scale between the stress range ΔS and number of cycles N to failure. The S-N curve gradually changes slope as the stress range decreases. A FMM cannot describe such behavior. The model fitted the assembled S-N data in the low stress region far better than
the conventional bilinear S-N curve found in design rules and specifications. The fatigue behavior is significantly more complex at low stress ranges than the conventional curves are able to describe. The RFLM-based non-linear curves give some new physical insight into the fatigue process itself. This will be addressed in the present section by a semi-empirical physical model. The model is a complementary tool to the statistical RFLM developed. It is important to control the physical parameters that have an influence on the fatigue damage process as it evolves towards final failure. This is important when the following predictions are required:

— Predictions of fatigue life under conditions, for which experimental data do not exist;

— Predictions of likely crack growth histories leading to failure.

The need for a more a physical model for carrying out the first type of predictions is obvious: fatigue life can only be predicted by a statistical model for joints pertaining to the populations and conditions for which the model was established. If basic fatigue properties such as joint geometry or loading modes are changed it is only a physical model that can predict the effect of such changes on the fatigue life. Hence, the physical model is an important tool in safe life analysis. The second type of prediction is required if inspection is to be carried out in a damage tolerance approach. In this case it is necessary to characterize the fatigue process itself, not only to determine the final fatigue life. This is essential if in-service inspections are to be planned; we must know what crack sizes to look for at different time stages before final failure. Based on these considerations, a physical model will be established to meet the following criteria:
a) the model should be corroborated by S-N data for the joint in question when these are available (databases 1 and 2);

b) the model should predict a crack evolution that coincides with measured crack growth histories before failure (database 1).

With this background we will endeavor to model the fatigue process in fillet welded joints. The total fatigue life is considered to be the sum of the cycles spent in the crack development phase and the crack propagation phase:

\[ N_T = N_i + N_p \]  \hspace{1cm} (156)

For some time there has been a debate amongst researchers and engineers as to whether or not the crack development period is important. It has been a traditional belief that fatigue crack growth often starts from surface-breaking defects in the weld toe region. The initial flaw has often been assumed to have a depth greater than 0.1 mm, sometimes 0.25 mm. In design rules and specifications an initial crack depth, even as deep as 0.5 mm, has been recommended; [105].

These flaws directly start the fatigue crack propagation and the fatigue development period can be ignored. However, with advanced fatigue testing where the entire crack depth history is monitored—not only the final fatigue life—it has become obvious that this simple approach does not fit. Several researchers have obtained experimental evidence that supports the existence of a crack development period; see [106] and [107]. The conclusion is that rather than speaking of small micro-cracks or inclusions in the vicinity of the weld toe, it is more correct to use the notion of unfavorable surface condition which gives a rather short development period under accelerated laboratory conditions.
The early damage mechanism is a combination of crack nucleation and micro-crack growth. In welded joints subjected to high stresses in accelerated laboratory conditions (typically a stress range of 120-150 Mpa) the development period, defined as the time to reach a crack depth of 0.1 mm, is typically 30% of the entire fatigue life; see [107]. This means that if the same joint is subjected to stress levels that are typical of service conditions (equivalent stress range 50-80 MPa) the crack development will totally dominate the fatigue life. Due to this fact, a FMM will not be able to meet both of the model criteria A and B, listed above.

With this background, a two-phase model (TPM) is developed and investigated. It was Lawrence et al. who first suggested the TPM, see [108] and [109]. As the method now stands, its accuracy depends greatly on the integration experiments. Our objective is to elaborate and calibrate the model to fit the various test series. The predictions made by the model will be compared with the pure fracture mechanics model and the predictions made by the RFLM. The joint used in the analysis is shown in Figure 108.

![Figure 108 Joint configuration with crack shape parameters a and c Ref [100].](image)
9.2 Modeling the fatigue crack development period

9.2.1 Basic concept and equations for the local stress-strain approach

The predictions for the number of cycles to crack development, \( N_i \), are based on the Coffin-Manson equation with Morrow's mean stress correction [108] and [109]:

\[
\frac{\Delta \varepsilon}{2} = \frac{\sigma'_f - \sigma_m}{E} (2N_i)^b + \varepsilon'_f (2N_i)^c
\]  

(157)

where, \( \Delta \varepsilon \) is the local strain range and \( \sigma_m \) is the local mean stress at the weld toe. The parameters \( b \) and \( c \) are the fatigue strength and ductility exponents, and \( \sigma'_f \) and \( \varepsilon'_f \) are the fatigue strength and ductility coefficients respectively. The local stress and strain behavior is given by the Ramberg-Osgood stabilized cyclic strain curve:

\[
\Delta \varepsilon = \frac{\Delta \sigma}{E} = 2 \left( \frac{\Delta \sigma}{2K'} \right)^{\frac{1}{n'}}
\]  

(158)

where \( K' \) and \( n' \) are the cyclic strength coefficient and strain hardening exponent respectively.

Equation 158 is combined with the Neuber rule:

\[
\Delta \varepsilon \Delta \sigma = \frac{(K_t \Delta S)^2}{E}
\]  

(159)

where \( \Delta S \) is the nominal stress range, \( E \) is the Young modulus, and \( K_t \) is the stress configuration factor at the welded toe. Equation 159 is sometimes modified by introducing the fatigue notch factor \( K_f \) instead of \( K_t \). It is argued that the fatigue notch factor better quantifies the severity of a discontinuity in the fatigue life calculation. Yung and Lawrence conclude [108] that Peterson's equation correctly interrelates the fatigue notch factor with elastic stress configuration factors for welded joints:
\[ K_f = 1 + \frac{(K_t - 1)}{1 + \left(\frac{\alpha_p}{\rho}\right)} \]  

(160)

where, \( \rho \) is the weld toe root radius and \( \alpha_p \) is Peterson's material parameter. The latter may be approximated by the expression 1.087 \( \times \) 10\(^5\) \( S_u \cdot \rho^2 \) (in mm, N/mm\(^2\)) where \( S_u \) is the tensile strength of the steel. The following expression for \( K_t \) is used [112]:

\[ K_t = 1 + \left[ 0.5121 (\theta)^{0.572} \left(\frac{T}{\rho}\right)^{0.469} \right] \]  

(161)

Where \( \theta \) is the weld toe angle (radians) and \( T \) is the plate thickness; see Figure 108.

The definition of the local stress-strain variation is illustrated in Figure 109. The nominal stress \( S \) (left) and the local stress \( \sigma \) (right) are shown for the first reversal (0-1) and the stabilized hysteresis loop (1-2-3). The local \( \Delta \sigma \) and \( \Delta \varepsilon \) and the mean stress, \( \sigma_m \), corresponding to the cyclic loading, are determined. The effect of cyclic hardening or cyclic softening is neglected.

In the case of an elastic notch root condition at the weld toe, equation (157) reduces to the Basquin equation, ignoring the second term on the right-hand side. This equation can be rearranged:

\[ N_i = \frac{1}{2} \left( \frac{K_f \Delta S}{2(\sigma_f' - \sigma_m)} \right)^{1/b} = \frac{1}{2} \left[ 2(\sigma_f' - \sigma_m) \right]^{-1/b} \left( K_f \Delta S \right)^{-1/b} \]  

(162)

From this equation may construct an S-N curve with slope \( b \).
9.2.2 Definition of the initiation phase and determination of parameters

When applying the Coffin-Manson approach to predict time to crack development at the weld toe, three questions arise. The first is what stress configuration factor $K_t$ should choose to characterize the notch effect from the highly variable local toe geometry. The next is the definition of time to crack development what crack depth is reached at the end of the phase before propagation takes over. This depth will be referred to as the transition depth. If the chosen depth is too large, the initiation phase will not obey equation (157) due to a substantial amount of crack growth involved in the process. Several definitions are possible for the transition depth and this is probably one of the reasons why so few TPMs have been applied in practice. The last question is how to determine the cyclic mechanical parameters and the parameters in the Coffin-Manson equation such that they are valid for the weld toe condition. Usually these parameters are determined from tests with small-scale smooth specimens. However, in a full-scale welds there may be a thickness-effect involved and the surface-finish will certainly have an influence on these parameters. Tests
with smooth specimens will not be representative for the rough surface conditions found in the vicinity of the weld toe. It is our hypothesis that the actual parameters should be determined directly from full-scale tests with welded joints where early cracking is measured. Database 1 gives us this possibility. Based on the discussion above, we will emphasize three topics:

- the local toe geometry and stress configuration factor;
- the choice of the transition depth;
- the determination of the parameters in Coffin-Manson law.

9.2.3 The Local toe geometry and stress configuration factor

The statistics for the local geometry for the specimens tested in series 1 (database 1) are given in Table 24. If we substitute the mean values in Table 24 into equation (161), this will give $K_t = 2.9$. The total weld leg length is assumed to be 1.8 times the plate thickness $T$. This value is corroborated by a refined finite element analysis. However, it is highly likely that the cracks initiate at a more unfavorable geometry. It is the random variation in the weld toe radius that gives large variation for $K_t$ with values ranging from as low as 2.9 up to as high as 7.0. To circumvent this problem, Lawrence et al suggested using the fatigue notch factor $K_f$ given in equation (160) instead of $K_t$. Furthermore, a worst-case notch was defined by setting the toe radius equal to the Peterson constant in equation (160). In our case, this will approximately give $\rho = a_p = 0.3$ mm, which corresponds to $K_f = 3.1$. The problem with the $K_f$ concept is that the physical interpretation is not obvious. It has been claimed [112] that the need for a definition of a fatigue notch factor is due to the fact that, in reality, the crack development life includes an appreciable amount of crack growth. If the initiation phase were limited to pure nucleation of a micro-crack, the stress
configuration factor could have been used directly in the calculations. Furthermore, the smallest radius in a joint is a random variable and its mean value will be a function of the length of the weld seam.

Let us set the toe angle constant at the mean value and try to determine the smallest toe radius most likely to be found within one test specimen. The statistics in Table 24 were derived from 300 measurements taken with 5.5 mm spacing along a weld seam of the length of 1,650 mm. It appeared that two neighbor radii with a spacing of 5.5 mm could be very different, whereas at a closer distance there is a correlation. Based on this observation, we assume that at a 5.5 mm spacing the measured radii will be independent. Hence, a weld seam with length $W$ mm contains $k = W/5.5$ totally independent radii. The extreme distribution for the smallest value will then read:

$$1 - F_{\rho_{\text{min}}} (\rho) = (1 - F_{\rho} (\rho))^k$$

(163)

where $F_{\rho} (\rho)$ is the cumulative distribution function (CDF) of the arbitrary radius as given in Table 24, whereas $F_{\rho_{\text{min}}} (\rho)$ is the CDF for the smallest value over the length $W$. The mean and peak values for this smallest radius can be obtained from the corresponding probability density function (PDF). The peak value is, by definition, the most likely value for the smallest radius within the length $W$. If we assume that $\rho$ is Weibull distribution [114] and $W$ is set to $1,650 mm$ ($k = 300$), then our approach will give a peak value for the smallest radius close to 0.1 mm. The smallest radii found in various test series are actually close to 0.1 mm; [114]. Hence, these results support the approach. If we use the same approach within the width of one test specimen (W= 60 mm, k=60/5.5 = 10) we get $p=0.42$ mm. The PDFs for an arbitrary and extreme toe radius are shown in Figure 110. As can be seen, the mean value and the peak value are not very different for the relatively
narrow symmetrical extreme value distribution. The peak value of 0.42 mm will result in a stress configuration factor of 4.5 that will be used in our calculations.

Table 24 Statistics for local weld toe geometry, Database 1

<table>
<thead>
<tr>
<th>Weld toe angle $\theta$ (Degree)</th>
<th>Weld toe radius $\rho$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>58</td>
<td>9</td>
</tr>
</tbody>
</table>

Figure 110 Definition of the toe notch geometry in one specimen by extreme value statistics

9.2.4 Transition depth

Often in earlier work the transition depth has arbitrarily been set to 0.25 mm (0.01 in). However, the time to develop such a deep crack will include a large amount of crack growth. This has in many cases resulted in models that give reasonably good predictions at short total fatigue lives (less than $10^6$ cycles), but an over-prediction of the fatigue lives for low stress levels. The reason is that the initiation part of the fatigue life at low stress levels will have a life curve with a slope close to the parameter $b \approx -1/10$; see equation
(162). This is only true for pure initiation, whereas crack growth will have a slope, according to the Paris law, \(-1/m \cong -1/3\) relative to the applied stress range. Hence, a phase that contains both nucleation and growth will have a slope between \(-1/3\) and \(-1/10\). If a slope of \(-1/10\) is assumed for such a mixed process it will significantly overestimate the fatigue life at low stresses. In more recent work, Lawrence et al. [109] suggested that the transition depth should be between 0.05 and 0.1 mm. In this work we will investigate the results obtained by setting the transition depth equal to the lower and upper bound of this given range. The following arguments support a transition depth of 0.1 mm:

- to apply the fracture mechanics model at crack depths smaller than 0.1 mm may be dubious because such small crack depths approaches the grain size (typically 0.01 mm);
- in laboratory tests it is almost impossible to measure any crack less than 0.1 mm with sufficient accuracy without using destructive methods. Hence, integration data will not be available;
- cracks with a depth of less than 0.1 mm are not of interest in in-service inspection as no common non-destructive inspections (NDI) method can detect such small cracks. Hence, for inspection planning we do not need the notion of a crack smaller than 0.1 mm.

The actual number of cycles to reach 0.1 mm is given in Table 25 for database 1. In contrast to the fictitious initial crack depths obtained from the FMM analysis, this is a measurable quantity. The fatigue initiation life is close to 30% of the entire fatigue life. The entire fatigue life is 10% shorter than the predictions from the Category E curve.
These arguments for a transition depth as low as 0.05 mm are less obvious, apart from the fact that it will exclude any crack growth. It is to be noted that even this shallow depth is well above the initial crack distribution obtained from the FMM Figure 99. The upper bound was found to be 0.03 mm for these cracks. Hence, the transition depth of 0.05 mm is about the smallest transition depth possible based on what may be interpreted as crack size of possible initial flaws. The time to arrive at 0.05 mm crack depth was not measurable for the tests carried out for database 1. The number of cycles to reach this depth can be found by back-calculation from the first measurable crack size (0.1 min) by applying the FMM. In this way the number of cycles to reach 0.05 mm was determined to 90,000 cycles 20% of total fatigue life.

<table>
<thead>
<tr>
<th>Table 25 Measured number of cycles (in1000) spent in various phases. Accelerated laboratory condition 150 MPa (database 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_i$ is defined as time to reach 0.1 mm</td>
</tr>
<tr>
<td>$N_i$</td>
</tr>
<tr>
<td>140</td>
</tr>
</tbody>
</table>

9.2.5 Cyclic mechanical properties and parameters in Coffin-Manson equation

In general parameters determined from small-scale smooth specimens are not directly applicable for weld toe conditions. Hence, we will determine these parameters directly from the time to early cracking as given Table 25. We now seek the parameters in equations (157) and (158) that correspond to the initiation time given in Table 25. The integration is carried out by assuming a dependency between the various parameters with Brinell hardness (HB) as the key parameter. The following equations are applied [115]:

249
\[ S_u = 3.45 \text{ HB MPa} \quad n' = \frac{b}{c} \]

\[ S'_y = 0.608 S_u \quad \text{MPa} \quad K' = S'_y(0.002)^{-n'} \quad \text{MPa} \]

\[ b = -0.1667 \log \left(2.1 + \frac{917}{S_u}\right) \quad \sigma'_f = 0.95 S_u + 370 \quad \text{MPa} \]

(164)

\[ c = -0.7 < c < -0.5 \quad \varepsilon'_f = \left(\frac{\sigma'_f}{K'}\right)^{1/n'} \]

where \( S_u \) and \( S_y \), are the tensile stress and yield stress respectively, and \( S_y' \) is the cyclic yield stress. The correlation between the various parameters, using the HB as a master variable, is not exact. In [116] it was found that the relationship might lead to significant overestimation of the initiation life. Furthermore, Dowling [110] suggested that the surface condition at the weld toe would primarily alter the fatigue strength exponent \( b \). However, we accept the relationships given in equation (164), but not the absolute values. An absolute value for HB is sought such that the time to reach a given transition crack depth coincides with what actually has been measured on the welded joints in database 1. The solution gives a HB close to 202 for the cycles to reach a crack depth 0.1 mm. This HB is very close to the value actually measured in the heat-affected zone (HAZ) of the weld toe. The values measured for the base metal gave an HB = 145 and values of the HAZ gave 213. Hence, our solution HB = 202 is only 5% less than the highest value measured at the potential crack locus. If we had applied HB = 213 directly, the solution would give the time to reach a crack depth of 0.3 mm. Hence, a significant amount of crack propagation would have been included in the initiation phase. When using the search scheme for determining the parameters with a transition depth of 0.05 mm, we obtained a HB of 180 still within the range measured on the specimens, but 15% less than the value at the HAZ.
The parameters corresponding to HB = 180 (a = 0.05 mm) and HB = 202 (a = 0.1 mm) were used in equations (157) and (158) to define the first part of the TPM. The propagation phase based on fracture mechanics was subsequently added to calculate the entire fatigue life. The model predicts exactly the mean value for the fatigue lives of database 1 at a stress range of 150 MPa. When the stress range was decreased from 150 MPa to below 100 MPa, the model based on a transition depth of 0.1 mm predicted somewhat longer lives than the median line obtained from the RFLM, which is representative for database 2 in this stress region.

The model based on a transition depth of 0.05 mm predicted results with somewhat shorter lives than figures obtained from the RFLM in this region. At a stress range of 80 MPa, the model with a = 0.1 mm predicts a 28% longer life than the median line of the RFLM, whereas the model with a = 0.05 mm predicts a 20% shorter life than the same median line. These results will be discussed in more detail in the next section. The results indicate that the interval for a transition depth between 0.05 mm and 0.1 mm as proposed by Lawrence et al. [109] is a reasonable choice. Furthermore, any transition depth in this narrow band will predict fatigue life well within the scatter band of database 2. Based on the arguments stated at the beginning of this section, we have selected a transition depth of 0.1 mm in what follows. The corresponding parameters in the Coffin-Manson equation are given in Table 26. As we already have shown, the solution given in Table 26 is not unique. Other solutions without total dependency between the parameters are possible. However, these solutions will not be far from the one given in Table 26. Hence, we regard the solution representative for prediction of time to reach a depth of 0.1 mm in welds made from C-Mn steel with a yield stress close to 345 MPa.
9.3 Constructing the S-N curve from the two-phase model

One of our main goals is to construct S-N curves from the TPM that are consistent with the RFLM curves obtained. As pointed out, the RFLM curve fitted the data points far better than the Category E curve at low stress ranges. Although the TPM is semi-empirical, it has a more physical-theoretical basis than the RFLM which is based on purely statistical methods. The TPM is capable of predicting the influence of, for example, local weld toe geometry, stress ratio, and stress relieving. Thus, high-quality joints will have long fatigue lives, whereas poor quality joints will be penalized by the model. The TPM model can also be applied to calculate fatigue lives at low stress levels where experimental data do not exist and where it is dubious to extrapolate the statistical RFLM.

Let us begin by demonstrating that the model can predict fatigue lives that are in good agreement with the S-N curve obtained from the RFLM under appropriate assumptions of the quality of the joint. For joints that are stress-relieved (database 1), the TPM will predict fatigue lives as given in Table 27 at various stress levels. As can be seen from the table, the time to crack development at a test stress range of 150 MPa is 30% of the entire fatigue

<table>
<thead>
<tr>
<th>Parameter, Symbol (units)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyclic yield stress, $S'_y$(Mpa)</td>
<td>424</td>
</tr>
<tr>
<td>Ultimate strength, $S_u$(Mpa)</td>
<td>697</td>
</tr>
<tr>
<td>Young modulus, E (GPa)</td>
<td>206</td>
</tr>
<tr>
<td>Fatigue strength exponent, $b$</td>
<td>-0.089</td>
</tr>
<tr>
<td>Strain hardening exponent, $n'$</td>
<td>1032</td>
</tr>
<tr>
<td>Fatigue strength coefficient, $\sigma'_f$(Mpa)</td>
<td>-0.6</td>
</tr>
<tr>
<td>Fatigue ductility coefficient, $\varepsilon'_f$</td>
<td>0.81</td>
</tr>
<tr>
<td>Cyclic strength coefficient, $K'(Mpa)$</td>
<td>1064</td>
</tr>
<tr>
<td>Fatigue ductility exponent, $c$</td>
<td>0.148</td>
</tr>
</tbody>
</table>

Table 26 Cyclic mechanical and parameters in the Coffin-Manson equation calibrated for time to reach 0.1 mm, HB=202[77]
life, whereas it is 88% of the fatigue life at a stress range of 80 Mpa. These results pin point the importance of the crack development life at low stress ranges in the region where service usually occurs. The table also lists the fatigue lives predicted by the RFLM and the Category E. At stress ranges below 100 MPa, the TPM predicts somewhat longer lives than the S-N curve based on the RFLM and significantly longer lives than the Category E S-N curve. As can be seen from Table 27, the total TPM fatigue life is close to 1.6 times longer than the life obtained from the RFLM and 5.5 times longer than the prediction made by the Category E at 80 Mpa. When comparing these figures we must bear in mind that the figures derived from the TPM correspond to the test series in database 1stress relieved (SR) and with an applied stress ratio of R = 0.3. This stress relieving has a strong bearing on the time to crack development through the Morrow mean stress effect at long lives; see equation (157). The RFLM S-N curve is dominated by database 2 in the low stress region. These tests are carried out on non-load-carrying fillet welded joints with thicknesses in the range of 16 to 38 mm. The specimens are all in as-welded (AW) condition and with positive stress ratio there may be large residual stresses present in the specimens. Furthermore, the vast majority of tests used to determine the Category E curve are in AW conditions and are often tested at a stress ratio close to R= 0.1.

Thus, our next step is to simulate these conditions for the initiation part of the TPM by setting the residual stress equal to the actual material yield stress 400 Mpa. The results are given in Table 28. As can be seen, the TPM results are now almost identical to the non-linear S-N curve obtained from the RFLM, but the model still predicts a fatigue life 2.5 times longer than the Category E at 80 Mpa. These results are illustrated in Figure 111 where the Category E and RFLM-based S-N curves are drawn together with the TPM S-N
curve and the test results. As can be seen, we have basically two types of curves. The Category E curve is bilinear, whereas the RFLM and TPM curves are both continuously changing slope. All three S-N curves coincide at high stress levels. Hardly any discrepancy in fatigue life (less than 10%) is found above a stress range level of 120 MPa. When the stresses are lowered to under 100 MPa, the RFLM curve and the TPM curve still coincide, but they predict two to nine times longer lives than the Category E curve as long as the stress range is above the Category E fatigue limit of 36 Mpa. It is our judgment that the Category E curve is too conservative in the stress region under consideration. This is due to the fact that it is a straight line and based on test results that have the center of gravity for the stress ranges between 120 MPa and 150 Mpa. Hence, the curve fails to take into account the increasing fatigue life due to the importance of an initiation phase below 100 MPa. The experimental results plotted in this stress region corroborate the predictions made by the TPM. It has been shown how the TPM is capable of correctly taking into account the effect of residual stresses and loading ratio. This is shown in more detail in Figure 111. The life curve obtained for the AW condition coincides with the median curve (i.e. the RFLM curve) for database 2.

Table 27 Results derived from the TPM at various stress ranges; SR, R=0.3

<table>
<thead>
<tr>
<th>Stress Range (Mpa)</th>
<th>$N_I$(cycles) TPM</th>
<th>$N_P$(cycles) TPM</th>
<th>$N_T$(cycles) TPM</th>
<th>$N/N_I$(cycles) TPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>$1.3 \times 10^5$</td>
<td>$3.3 \times 10^5$</td>
<td>$4.6 \times 10^5$</td>
<td>28</td>
</tr>
<tr>
<td>120</td>
<td>$4.3 \times 10^5$</td>
<td>$6.4 \times 10^5$</td>
<td>$1.1 \times 10^6$</td>
<td>39</td>
</tr>
<tr>
<td>100</td>
<td>$1.3 \times 10^5$</td>
<td>$1.1 \times 10^5$</td>
<td>$2.4 \times 10^6$</td>
<td>54</td>
</tr>
<tr>
<td>50</td>
<td>$6.6 \times 10^6$</td>
<td>$2.2 \times 10^6$</td>
<td>$8.8 \times 10^6$</td>
<td>75</td>
</tr>
<tr>
<td>40</td>
<td>$8.1 \times 10^7$</td>
<td>$5.1 \times 10^6$</td>
<td>$8.5 \times 10^7$</td>
<td>95</td>
</tr>
</tbody>
</table>
Finally it should be noted that although the statistically-based RFLM and the physically-based TPM give the same life predictions at almost any stress range level, there is one fundamental difference between them. The RFLM prescribes a fatigue limit, whereas the TPM does not. This is illustrated in Figure 11 where the focus is on the lower stress region. The slope of the S-N curve derived from the TPM will not be smaller than \( b = -1/10 \) and will never become horizontal as is the case with the RFLM. This gives a discrepancy between the curves at very long lives (longer than \( 10^8 \) cycles) such that the RFLM is more optimistic. The TPM predicts that any joint will eventually fail if the number of cycles is high enough. It is in fact possible to build a fatigue limit into the TPM by assuming that after crack development has taken place the crack may stop growing due to the fact that it has a stress intensity factor range (SIFR) below the threshold value. However, there is no data available to corroborate such behavior for shallow surface breaking cracks. The development of the model carried out thus far is also found in [117]. In what follows we will look into the consequences of the model with regard to damage accumulation and practical results.

### Table 28 Results derived from the TPM at various stress ranges; AW, R=0.1

<table>
<thead>
<tr>
<th>Stress Range (Mpa)</th>
<th>( N_i ) (cycles)</th>
<th>( N_P ) (cycles)</th>
<th>( N_i ) (cycles)</th>
<th>( N_i/N_i% ) (cycles)</th>
<th>( N_i ) RFLM</th>
<th>Category E</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>1.4 x 10^5</td>
<td>3.3 x 10^5</td>
<td>4.7 x 10^5</td>
<td>30</td>
<td>4.6 x 10^5</td>
<td>5.1 x 10^5</td>
</tr>
<tr>
<td>120</td>
<td>5.6 x 10^5</td>
<td>6.5 x 10^5</td>
<td>1.2 x 10^5</td>
<td>47</td>
<td>1.1 x 10^6</td>
<td>1.0 x 10^6</td>
</tr>
<tr>
<td>100</td>
<td>2.1 x 10^6</td>
<td>1.1 x 10^6</td>
<td>3.2 x 10^6</td>
<td>66</td>
<td>2.5 x 10^6</td>
<td>1.7 x 10^6</td>
</tr>
<tr>
<td>50</td>
<td>1.6 x 10^8</td>
<td>2.2 x 10^6</td>
<td>1.8 x 10^7</td>
<td>89</td>
<td>11.0 x 10^6</td>
<td>3.4 x 10^6</td>
</tr>
<tr>
<td>40</td>
<td>3.7 x 10^8</td>
<td>5.1 x 10^8</td>
<td>2.9 x 10^8</td>
<td>99</td>
<td>( \infty )</td>
<td>8.0 x 10^6</td>
</tr>
</tbody>
</table>
Figure 111 plot S-N curves constructed from the RFLM and TPM together with the Category-E median curve and test data.

Figure 112 plot S-N curves constructed from the RFLM and TPM in the low stress region.
9.4 Damage accumulation using the TPM

The TPM is an interesting model with respect to damage accumulation under variable amplitude (VA) loading. The model treats crack development and crack propagation separately and does not have any fatigue limit. A logical way of accumulating fatigue damage is to first use the Miner’s linear damage sum to predict when initiation has taken place, then use the summation rule for the subsequent propagation phase:

\[
D_I = \sum_{i=1}^{k} \frac{n_i}{N_{I,i}} = 1.0
\]  \hspace{1cm} (165)

\[
D_P = \sum_{i=k+1}^{\infty} \frac{n_i}{N_{P,i}} = 1.0
\]  \hspace{1cm} (166)

The summation procedure for a given load spectrum is first to carry out the summation of \(D_I\) only, until this sum equals 1.0. The subsequent summation of \(D_P\) can then start. The fracture criterion will be that both summations are equal to 1.0. Although the summation is linear for both \(D_I\) and \(D_P\), the total damage sum will be dependent on the sequence of the applied stress spectrum. The S-N curve for initiation and propagation are given in Figure 114. This is the S-N curve given in Figure 113 for the AW condition, but now split into initiation and propagation life. As can be seen, a stress block with stress range of 70 Mpa will do much less harm in the initiation phase compared to the propagation phase. The traditional Miner's linear damage summation rule based on the S-N curve for the total fatigue life is not reliable. Predicting fatigue failure under variable amplitude loading by that approach seems like an illusion. Both equation (165) and (166) must be
calibrated against VA test results. Non-linear damage accumulation should also be investigated.

![Graph](image)

**Figure 113** the TPM S-N (AW) divided into the initiation part and the propagation part

### 9.5 The practical consequences of the TPM

#### 9.5.1 General considerations

We have constructed an S-N curve that is non-linear for a log-log scale and that predicts substantially longer lives at stress ranges below 100 MPa than does the Category E linear curve. Furthermore, at these long fatigue lives (close to \(5 \times 10^6\) cycles), the initiation life is at least 70% of the entire fatigue life. We will show by a simple example what the consequences of these results are in respect to:

- predicting fatigue life for selecting dimensions for a joint;
- predicting crack growth path for inspection planning.
We will compare our results with the results obtained by the Category E curve and a pure FMM. The latter approach is traditionally used for decision-making regarding inspection planning. We shall use our FMM before, but for simplicity we decrease the growth parameter C by 10%, for example C=1.52 \times 10^{-13} \text{ (for } \frac{da}{dN} \text{ in mm/cycle and } AK \text{ in N/mm}^3/2\text{). This small adjustment will make the predictions made by Category E and by the FMM coincide.}

9.5.2 Life predictions and dimensions

Our goal is to compare results obtained by:

- Category E and the FMM (which are equal when the endurance limit is neglected);
- the present TPM assuming AW conditions, R= 0.1.

For simplicity, we assume constant amplitude loading and choose a stress range in the region considered, e.g. $\Delta S = 60$ MPa. By using equations (157) and (158) we obtain the results in Table 29.

Table 29 Median life predictions made by the TPM and Category E

<table>
<thead>
<tr>
<th>Stress range (Mpa)</th>
<th>TPM</th>
<th>E-Category</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_i$</td>
<td>$N_p$</td>
</tr>
<tr>
<td>60</td>
<td>$6.6 \times 10^6$</td>
<td>$2.2 \times 10^6$</td>
</tr>
<tr>
<td>38</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As can be seen, the TPM predicts 2.5 times longer fatigue life than the Category E, and the initiation part is close to 70% of the entire life at 60 MPa. If dimensions are chosen according to the Category E, the dimensions must be increased by 38% to give the same predicted fatigue life as the TPM $8.8 \times 10^6$ cycles. This will correspond to an allowable
stress range of 38 Mpa only. These assessments will also be valid for a design curve if the lives predicted by the TPM have the same scatter as the Category E.

9.5.3 Predicted crack evolution and inspection planning

Let us compare the two alternatives above in relation to inspection planning. The first alternative is the TPM-based design with an allowable stress of 60 MPa; the second alternative is the Category E design with allowable stress range 38 Mpa. The two cases will have the same design life. Our task is to compare the crack evolutions before and up to final fracture based on the TPM and the FMM in each case. We shall more precisely consider the effect of a scheduled inspection program for the two growth histories. The purpose of such a program is to detect cracks so that they can be repaired before reaching the final critical crack size. We use the concept of a probability of detection (POD) curve to characterize the performance of the inspection technique. The POD is a function of the joint type, environment, and crack size, and is established based on blind tests by inspectors. The POD curve for magnetic particle inspection (MPI) under poor conditions reads:

$$POD(a) = 0.9 \left[1 - e^{-(a-1)}\right] \quad a > 1 \text{ mm}$$

(167)

The curve is shown to the left in Figure 114. The obtained crack histories for the FMM and the TPM are shown to the right. As can be seen, the curve derived from the TPM has a more hidden path that makes the crack more difficult to detect at an early stage. If the strategy is to implement $k$ inspections during the planned service life, the likelihood of all of them failing can be estimated by:
\[ P_F = \prod_{i=1}^{i=k} [1 - POD(a_i)] \]  

(168)

The expression is based on the assumption that each inspection is independent. The reliability of the inspection program is \( R = 1 - P_F \).

If we are undertaking inspections at a constant time interval corresponding to \( 5 \times 10^5 \) cycles, we will, for the two curves in Figure 113, get the figures in Table 30. An effective inspection is defined when the current crack depth is larger than \( a_B \), i.e. 1 mm. According to the TPM, there will only be two effective inspections. As can be seen, there is an important difference in achieved reliability for the given inspection program for the two different crack evolutions. The FMM predicts a reliability of 0.999, whereas the TPM predicts only 0.935. When comparing the probability of failure, the difference becomes more striking, the probability of failure pertaining to the FMM is in the acceptable range, whereas the one pertaining to the TPM is not. However, if the predictions made by the TPM are accepted as true, one would obtain acceptable reliability if the inspection efforts were concentrated in the last part of the service life with a decreased inspection interval of \( 2 \times 10^5 \) cycles. For a more fully stochastic approach one can introduce the probability of a preexisting crack in the TPM. The difference in reliability will then be less, but the revealed tendency will be the same.

In conclusion, the practical consequences of the applying a TPM will be a reduction in joint dimension of 30% and a scheduled inspection program that is progressive with the time in-service.
9.6 Conclusions

Our study in this section is primarily concentrates on non-load-carrying fillet welded joints made of C-Mn steel with nominal yield stress close to 345 MPa (50 ksi). A TPM was
used to predict the fatigue life. The initiation life was modeled by Coffin-Manson equation, whereas the crack propagation was based on the simple version of the Paris law. The models were validated and calibrated with the use of large databases. The criteria for acceptance of the model were that the model should predict both damage evolution and final fatigue life at any stress level. The FMM model failed to fulfill these criteria, whereas the TPM gave an excellent fit to both measured crack growth histories and experimental fatigue lives. The following conclusions are drawn.

The fatigue behavior of fillet welded joints is far more complex at typically in-service stresses than fracture mechanics can describe. This is due to the fact that the crack development phase dominates the fatigue life at these low stresses.

A TPM is capable of modeling the damage evolution from the initial state to the final fracture provided that the model is accurately calibrated for this purpose. The notch factor at the weld toe is based on extreme value statistics for the toe geometry, and the transition crack depth between the initiation phase and the propagation phase is set to 0.1 mm. The parameters in the Coffin-Manson equation were determined directly from early cracking in full-scale welded joints. As the TPM has a semi-empirical physical basis, the determining factors, such as residual stresses, global and local joint geometry, and loading mode, are readily accounted for.

The S-N curves constructed from the model are non-linear for a log-log scale and coincide with the curves obtained from the statistical RFLM. Both models fit experimental data far better than the conventional bilinear S-N curves.
There is a fundamental difference between S-N curves obtained from the RFLM and the TPM in the way that the latter curves do not predict any fatigue limit. At stress ranges below 60 MPa, the RFLM curve will appear flat, whereas the TPM curve will continue to fall with a small slope close to the parameter b in the Coffin-Manson law. At present there is no data to corroborate either one of these curves, but they tend to have more confidence in the prediction made by the physical-based TPM than the predictions based on the statistical RFLM when extrapolated outside the range of the data.

The first practical consequence of the present TPM is that it predicts longer lives at low stress ranges than the conventional S-N curves in design rules and specifications. With the application of the TPM-constructed S-N curve in the lower stress region it is possible to reduce dimensions by 30-40% and still achieve the same fatigue life as for the Category E S-N curve.

The second practical consequence is that in-service inspection strategy may be optimized. This is due to the fact that the crack path leading to final fracture is quite different from the path calculated by a pure FMM. The TPM with its long initiation phase will give a more hidden path for the crack evolution. Hence, an inspection program with increased inspection frequency at the end of service life is proven to be favorable.

9.7 Summary

The fatigue process in fillet welded joints is discussed and modeled. As a first approximation, a pure fracture mechanics model was employed to describe the entire fatigue process. The model is calibrated to fit the crack growth measurements obtained from extensive testing on fillet weld joints where cracks emanate from the weld toes. Emphasis is laid on the choice of growth parameters in conjunction with a fictitious initial
crack size distribution in order to obtain both reliable crack growth histories and predictions of the entire fatigue life. The model has its shortcomings in describing the damage evolution at low stress ranges due to the presence of a significant crack development period in this stress regime. As an alternative to the fracture mechanics model, a two-phase model (TPM) for the fatigue process was developed and calibrated.

The number of cycles to crack development was modeled by a local strain approach using the Coffin-Manson equation, whereas the propagation phase was modeled by fracture mechanics, adopting the simple version of the Paris law. The notch effect of the weld toe was treated by extreme value statistics for the weld toe radius. To make the model fit all test data for crack development and propagation, it is crucial to select a sufficiently low transition depth between the two phases. A transition depth of 0.1 mm (0.004 in.) was selected. Furthermore, the material parameters in the Coffin-Manson equation were determined directly from the early cracking in the weld toe for full-scale fillet welds and not from tests carried out with small-scale smooth specimens.

This is essential if the model is to account for the actual surface condition at the weld toe. The S-N curves constructed by the present physical model were compared with the median nonlinear S-N curve obtained from the statistical random fatigue-limit model (RFLM) presented in Part 1 of this investigation. The curves are almost identical in the stress region where experimental data exist and both models fit the experimental data very well. However, the TPM does not predict any fatigue limit, in contrast to the RFLM. According to the TPM, the long fatigue life found at low stress ranges is a result of a dominating long development period and is not a threshold phenomenon. The two models are complementary tools. The RFLM is a pure statistical model, whereas the TPM is a semi
empirical physical model. The TPM is capable of taking into account the effect of the global geometry of the joint, the local weld toe geometry, applied stress ratio, and the residual stress condition. Both models can be used for fatigue life predictions, but only the physical TPM can be used for planning of in-service inspection strategies where damage evolution as a function of time is needed.
10.1 Introduction

The current AASHTO Specifications [1] contain provisions for the fatigue design of steel bridge details. These provisions are based on a set of fatigue resistance curves which define the strength of different classes of details. The curves were developed from an extensive research program sponsored by the National Cooperative Highway Research Program (NCHRP) under the direction of the Transportation Research Board. The statistically designed experimental program was conducted under controlled conditions so that analysis of the test data would reveal the parameters that were significant in describing fatigue behavior. The result was the quantification of the fatigue strength of welded bridge details and the development of comprehensive design and specification provisions.

Since the adoption of the AASHTO fatigue specifications in 1974, several major fatigue studies have been carried out on similar beam type specimens. Tests were conducted in Germany, Japan, Switzerland, Office of Research and Experiments of the International Union of Railways – ORE, as well as here in the United States. The additional studies evaluated the applicability of the findings of the NCHRP test program to fabrication conditions elsewhere in the world and were used to develop similar fatigue codes. The additional tests augmented the NCHRP findings and often defined the fatigue strength of
details that were not tested under the NCHRP program. For instance, much of the Japanese
data stem from research performed to develop fatigue specifications for, the design of long-
span bridges for the Honshu-Shikoku Bridge Authority. Many of the simulated details are
typical of those found in welded box members.

In addition, during the last several years, several countries have adopted their own set of comprehensive fatigue specification provisions. Though many of these specifications base the majority of their fatigue resistance provisions on the original NCHRP fatigue data, they have tended to deviate slightly from the AASHTO criteria. Both the International Organization of Standardization (ISO) and the European Convention for Constructional Steelwork (ECCS) have developed fatigue curves that differ slightly from the AASHTO requirements. Equally spaced S-N curves with constant slopes of -3.0 have been used to define the fatigue strength of details ranging from base metal to coverplated beam members. Many of these curves are about the same as the companion AASHTO curves.

10.2 Objectives and Scope

With the addition of the fatigue data and the development of fatigue codes in other countries, it is beneficial to review and re-evaluate the existing AASHTO fatigue specifications using these additional resources. The principal objective of this section is to compile and review all available fatigue data. This allows for a re-evaluation of the existing fatigue specifications so that they more accurately reflect the current state of knowledge. In addition, it provides an opportunity to provide criteria consistent with other applications in other countries. Specifically, the project was initiated with the following objectives:

To develop a database for welded steel bridge details that includes details from all available sources. This includes the original NCHRP data used to develop the current
AASHTO fatigue provisions as well as data from test programs that have been conducted since the implementation of the specifications.

To analyze the new data on an individual basis with respect to the existing AASHTO fatigue provisions to see if changes are required in the code due to the test results. In this way the adequacy of the current specifications can be reviewed for possible inconsistencies or for detail types whose fatigue resistance may be presently misrepresented. The possibility of this exists since, in the original test program, the number and type of details were limited, though the findings have been extended for almost all types of bridge details.

To unify or standardize the AASHTO fatigue design curves with the increased database so that the curves more accurately reflect the present knowledge of the fatigue behavior of welded steel bridge details.

To provide new fatigue design specifications for use in design codes that more accurately define the fatigue resistance of welded steel bridge details. The database has been limited to test data that can be used to define the fatigue resistance of welded steel details. We do not attempt to analyze the fatigue strength of other types of structural details such as riveted and bolted components. In addition, we don’t attempt to evaluate the adequacy of weld improvement techniques. While these processes tend to increase the fatigue resistance of certain details, the objective of this section is to define the lower bound fatigue resistance for as-welded details, or the minimum level of fatigue strength that would be obtained provided that standard fabrication and inspection procedures were employed.

Since the main objective of this fatigue data review is to build on the findings in NCHRP Reports 102 and 147, the database has been primarily limited to test data obtained from fatigue testing large-scale test specimens. Small-scale specimens are used in the
review where no alternative exists, though reliance on this data has been minimized. As was extensively addressed in the NCHRP reports, small-scale specimens always provide higher cycle lives than large-scale beam type specimens. This behavior can be attributed to many factors, one of which is the decreased residual stress fields in small-scale specimens. Without sufficient base metal or geometric conditions to constrain the cooling weld metal, residual tensile stresses will not develop to the magnitude found in full-scale weld details. Other factors are the distribution of defects and their frequency of occurrence, and favorable secondary stresses due to misalignment of the specimens during testing, as well as the variability of joint design. The small-scale fatigue test data does not contribute to defining the lower bound design resistance for welded steel bridge details.

10.3 Basic concept approach

A major portion of the project was the development of a computerized database containing fatigue test results of varying detail types [2]. Included in the database is data from the original NCHRP test program, subsequent NCHRP test programs, Japanese and European (ORE) test data, as well as other sources. The database was developed with the intent to make it as comprehensive as possible. Therefore, for each data set (defined as one fatigue failure) not only were the detail type, stress range, and cycles to failure provided, but also critical dimensions of the test specimens were given. This allowed for studies involving the influence of size effects in the specimens. Each detail type was given a unique code number, unrelated to the category designation it would receive under the AASHTO fatigue specifications. Therefore, the lower bound fatigue strengths of the details were not predetermined or biased by the current fatigue provisions. Several computer programs were written to utilize the test data. These programs include sorting routines, plotting functions,
and regression analysis. The development of this database allowed for the systematic analysis of the large amount of fatigue data compiled from various sources.

Each source of test data was first compared with the appropriate existing AASHTO fatigue curves. This was done to determine if the results from the tests were consistent with the findings of the original NCHRP studies or if the fatigue resistance of a particular detail had possibly been misrepresented by the current specifications. A regression analysis was performed using all the data for a given detail type to see if any significant differences arose. Once the data was properly categorized, all data, for each detail type were compared with the appropriate fatigue resistance curve.

Although use was made of statistical methods (primarily linear regression analysis), the results were often difficult to evaluate. This can be partly attributed to the fact that the relationship between the stress range and the number of cycles to failure is log-log in nature. Also, if the database is limited in number or is not distributed along the S-N curve, the results can easily become biased. Often, data were clustered over a small increment of stress range, and any number of regression lines could have described the relationship between stress range and life. Therefore, the primary method used for comparison and analysis was the direct comparison of the fatigue test data with various fatigue resistance curves. Since the comparison was being made against a predetermined set of design curves, it was only necessary to show a distribution of failure points plotting above a particular curve in order to insure adequacy.

10.4 Findings

The findings of the fatigue test data review, conducted under NCHRP Project 12-15(5), are summarized in literature review. A detailed examination of each source of data,
as well as a comparison of all existing fatigue data to existing provisions and a revised set of fatigue resistance curves is given in literature review.

10.5 Current and Proposed Fatigue Design Curves

A comparison was made between the current AASHTO fatigue design curves and those currently under consideration for adoption by the European Convention for Constructional Steelwork (ECCS) and the International Organization for Standardization (ISO). The ECCS curves represent a major effort to re-evaluate the fatigue design specifications. The analysis was based on the original NCHRP test data, limited data obtained from a specially designed test program, as well as other sources.

The ECCS fatigue curves consist of fifteen equally spaced curves on a log-log scale. The slopes of all curves are set to a constant slope of -3.0 up to 5 \cdot 10^6 cycles. Six of these curves closely resemble the current AASHTO curves.

The results of this review indicate that adjustments should be made to the current AASHTO fatigue curves. The adjustments result in proposed curves that have a slope of -3.0 and are therefore compatible with the sloping portion of several of the ECCS curves. Only seven of the ECCS curves are suggested for the new set. The test data review does not support the need for fifteen fatigue resistance curves to define the strength of welded steel details. Six of the proposed curves are similar to the original AASHTO Categories A thru E' curves in that the 2 \cdot 10^6 intercept values are the same. Their slopes have been slightly adjusted to -3.0. The constant amplitude fatigue limits have remained unchanged with the exception of Category E. High cycle fatigue test results of cover plated beams indicated 4.0 ksi provided a better estimate of the constant amplitude fatigue limit than the current 4.5 ksi value.
10.6 Inadequacies of Current Fatigue Provisions

The fatigue test data review indicated that several detail types have not been properly accounted for in the current AASHTO fatigue provisions. This conclusion results from the analysis of test data on details not previously included in the database. The decreased strength is attributed to size effects, initial flaws, and the geometry of the detail. The review also indicated that web attachments with plate thicknesses greater than 1.0 in. (25mm) resulted in a decreased fatigue resistance which corresponds to the Category E' detail.

10.7 Proposed Fatigue Design Curves

The current AASHTO fatigue design curves (except Category E') were developed from the test results and data analysis provided in NCHRP Reports 102 and 147. Although a large number of test results were generated from these programs, the number and variety of detail types tested were limited. The design curves were developed from linear regression analyses of the data using the 95 percent confidence limits defining the lower bounds of the fatigue resistance. This resulted in a set of curves which varied slightly in slope since the actual computed slopes were used. These curves have since been used to define the fatigue strength for other types of welded bridge details based on geometric similarities and test results that correspond to the originally tested detail types.

As the database for a given detail has increased it has generally been observed that the slope of the S-N curve tends to stabilize to a slope of -3.0. This can be seen from the NCHRP studies on coverplated beams summarized in ch4 before. The regression analysis of the test data provides a slope that is -3.02. This large well distributed set of data is an exception. Generally, the specific data sets that were reviewed were found to be limited in
number, or the data were not well distributed along the S-N curve. No other test program, other than the original NCHRP studies, resulted in a data set that was sufficiently distributed for a regression analysis. Often data were clustered over a small increment of stress range, and any number of regression lines could be used to describe the relationship between stress range and life. This becomes most evident for the higher strength Categories A and B. For these higher strength details the data are bounded by the constant amplitude fatigue limit and the yield stress of the steel. This often results in a relatively limited range of stress and a grouping of the data points.

Another factor that influences the regression analysis for a particular data group is the variability that exists between test programs. For a given test series, the distribution of the initial flaws, residual stress fields, weld profile, and specimen size are controlled and are therefore similar. These factors play a major role when different sets of data are grouped together. Of particular importance is the defect size. This was noted in (Ref. 15) where each series of tests yielded a slope near -3.0. When these tests were combined, major deviations existed between series and this resulted in a wide variation of the test results and also caused the slope provided by the regression analysis to change.

As reported in NCHRP Reports 188 and 267, variable loading studies have shown that the use of an exponent of -3.0 and Miner's Rule provide a reasonable estimate of variable stress cycle cumulative fatigue damage. Variable loading is what a bridge structure experiences from normal traffic, not the constant cycle loading used in most of the tests that the fatigue design curves have been developed from. The variable load test results plotted in Figs. 15 thru 18 show that a slope of -3.0 provides good agreement with the resistance curves. This is particularly true in the long life regions applicable to most bridge
structures. All tests indicate that the straight-line extension at a slope of -3.0 is an appropriate lower bound estimate.

Recognition of the relationship between crack growth and the experimental results on welded details has led to the adoption of a slope of -3.0 for other design S-N curves. This was first adopted when the NCHRP test data was used to develop the S-N relationships used in the draft Swiss Fatigue Provisions in 1974 [40]. Since that time, a slope of negative three has been adopted in the British standard [41] and more recently in the recommendations adopted by the European Convention Constructional Steelwork (ECCS) fatigue specifications [42]. These criteria are also being considered by the International Organization for Standardization (ISO).

The "European Fatigue Strength Curves" (ECCS) are to provide uniformity to the fatigue design curves. The curves are based on most of the same fatigue data considered in this section. This includes the ORE program, the initial NCHRP Reports, as well as data from other sources. The ECCS curves define a set of equidistant log-log S-N curves that can be used to classify fatigue data regardless of origin or type. The test data for a particular detail can be compared with these curves and a specific curve chosen to define its fatigue strength.

The ECCS fatigue curves are shown in Figure 115. They consist of fifteen equally spaced curves on a log-log scale. The vertical spacing corresponds to an approximate 10 percent variation in fatigue strength. The slopes of all curves are equal to -3.0 in the life range up to $5 \times 10^6$ cycles. At $5 \times 10^6$ cycles two options are provided. One option changes the slope to -5.0 until $50 \times 10^6$ cycles where a cut off is provided. The intercept at $50 \times 10^6$ cycles establishes a fatigue limit regardless of the type of loading. All cycles below this limit can
be ignored when evaluating fatigue damage. The second option is provided by the dashed lines which correspond to a straight line extension of the -3.0 slope S-N curves. The reference fatigue strength or detail category identification is the stress range value at \(2 \cdot 10^6\) in MPa.

![Fatigue Design Curves Adopted by ECCS/ISO [142]](image)

The ECCS fatigue curves in their entirety have several shortcomings. The test data review indicates that the accuracy of a set of fifteen different classes of fatigue resistance is questionable. It does not seem reasonable to define the resistance of welded steel details with the accuracy that this number implies. The adoption of a constant cycle fatigue limit at \(5 \cdot 10^6\) cycles for all details is not compatible with the actual fatigue test data, as can be seen in the test data plots. The test data indicate that the constant cycle fatigue limit occurs at increasing cycles as the severity of the detail increases. For a high fatigue strength detail (i.e. Category A) this limit is near \(2 \cdot 10^6\) cycles. For a severe fatigue strength detail this limit is reached near \(20 \cdot 10^6\) cycles. Finally, the use of a -5.0 slope for fatigue resistance below the constant cycle fatigue limit is not in agreement with the random variable fatigue results. Test results support the use of -3.0 for all stress cycles.
When the ECCS fatigue design curves are compared to the existing AASHTO curves it becomes evident that the differences are not great, as can be seen in Figure 116.

![Figure 116 Comparison of AASHTO and ECCS (ISO) Fatigue Design Curves](image)

The seven AASHTO curves (A thru E') are the heavier lines and correspond closely to six of the ECCS curves. A tabular comparison of the two sets of curves is given in Table 31. The slope of several AASHTO curves are slightly different since they were based on the results of a regression analysis, whereas the ECCS curves all have a slope of -3.0. In general, the fatigue resistance is slightly less at higher cycle life when the fatigue strength is defined by the ECCS curves. The most significant difference is the constant amplitude fatigue limits. For Categories A thru C, the ECCS limits are lower, while for Categories D to E', the AASHTO limits are lower.
Table 31 Comparison of Current AASHTO and Proposed ECCS

<table>
<thead>
<tr>
<th>Non-Redundant Load Path Structures</th>
<th>Allowable Range of Stress $F_{sr}$, ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category</td>
<td>For 100,000 Cycles</td>
</tr>
<tr>
<td>AASHTO (ECCS)</td>
<td>( )</td>
</tr>
<tr>
<td>A</td>
<td>(160)</td>
</tr>
<tr>
<td>B</td>
<td>(125)</td>
</tr>
<tr>
<td>B'</td>
<td>----</td>
</tr>
<tr>
<td>C</td>
<td>(90)</td>
</tr>
<tr>
<td>D</td>
<td>(71)</td>
</tr>
<tr>
<td>E</td>
<td>(56)</td>
</tr>
<tr>
<td>E'</td>
<td>(40)</td>
</tr>
</tbody>
</table>

The results of this review suggest that adjustments should be made to the current AASHTO fatigue design curves. The slope of these curves should be established at -3.0 and thus be compatible with the sloping portions of the corresponding ECCS/ISO curve. This adjustment is shown in Figure 117. A cross bar has been placed above the category letter in order to distinguish between the proposed and current categories and curves. The cross bar notation will be used throughout the remainder of the Report.

Figure 117 Proposed AASHTO Fatigue Design Curves
The proposed curves were developed using the stress range intercept values at 2\( \cdot 10^6 \) cycles. The constant amplitude fatigue limits for each curve, with the exception of Category\( \bar{E} \), correspond to their current values as the review of the data did not indicate a need to change these values. High cycle fatigue test results of coverplated beams indicated 4.0 ksi (27 MPa) provided a better estimate of the fatigue limit than the current 4.5 ksi (31 MPa) value. The stress range intercept values for 1\( \cdot 10^5 \), 5\( \cdot 10^5 \), and 2\( \cdot 10^6 \) cycles as well as the constant amplitude fatigue limits for each proposed curve are given in Table 32.

Table 32 Proposed Allowable Fatigue Stress Ranges
For Redundant

<table>
<thead>
<tr>
<th>Redundant Load Path Structures*</th>
<th>Allowable Range of Stress ( F_{sr} ), (psi)(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category</td>
<td>For 100,000 Cycles</td>
</tr>
<tr>
<td>( \bar{A} )</td>
<td>60,000</td>
</tr>
<tr>
<td></td>
<td>49,000(^d)</td>
</tr>
<tr>
<td>( B )</td>
<td>48,000</td>
</tr>
<tr>
<td>( \bar{B}' )</td>
<td>40,000</td>
</tr>
<tr>
<td>( \bar{C} )</td>
<td>34,000</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>( \bar{D} )</td>
<td>27,000</td>
</tr>
<tr>
<td>( \bar{E} )</td>
<td>20,000</td>
</tr>
<tr>
<td>( \bar{E}' )</td>
<td>16,000</td>
</tr>
<tr>
<td>( \bar{F} )</td>
<td>15,000</td>
</tr>
</tbody>
</table>

The major difference between the existing and proposed curves is their slope. With the exception of Category A and E, the 2\( \cdot 10^6 \) intercept values for the two sets of curves are identical. The majority of the existing curves have a slope which is slightly greater than -3.0. Because of this slope difference the proposed curves result in a slightly higher fatigue resistance in the low cycle regime. The current AASHTO design provisions [5] were based
on Miner's Rule and the assumption that the slope of the fatigue resistance curve was -3.0. Hence, the damage estimate was compatible with the damage that would result from the relationships shown in Figure 117. This same assumption was used in Ref. 12 when evaluating the growth of cracks in steel bridge structures.

10.7.1 Comparison of Test Data with Proposed Fatigue Resistance Curves

The proposed set of fatigue design curves (Figure 117) are compared with all available test data in this section. The data are reviewed in order of detail category, beginning with Category $\overline{A}$. Where the fatigue resistance is a function of the detail geometry, as is the case with web and flange attachments, the data is plotted as a group with the corresponding curves.

Initial analysis of the database for a particular detail type indicated that a rigorous regression analysis was of limited, practical use. As discussed in the previous section, many variables influence a regression analysis. This is particularly true when results from different test programs are combined and analyzed. Complete regression analyses were performed on all detail groups and the results may be found in Ref. 210.

By comparing the test data with the proposed curves their adequacy can be analyzed. The data for a particular detail type should be distributed above the lower bound provided by these fatigue resistance curves. Since these curves represent the 95% lower confidence limits, most of the test data should plot above the curve. Furthermore, test data for a particular detail should not deviate significantly from the applicable curve. Table 33 gives the number of data points plotted for each detail type in the figures referred to in comparison of all the test data with the proposed fatigue resistance curves. Since the data
from each test program have been individually compared to the current AASHTO fatigue curves and only minor changes have been made, no significant variation should occur.

Table 33 Number of Test Data Plotted in Figs. 119 thru 135

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Detail Type</th>
<th>Category</th>
<th>Number of Test Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>119</td>
<td>Plain Rolled Beams</td>
<td>A</td>
<td>49</td>
</tr>
<tr>
<td>120</td>
<td>Longitudinal Welds</td>
<td>B</td>
<td>182</td>
</tr>
<tr>
<td></td>
<td>Welded Beams</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Flat Plate Specimens</td>
<td>B̅</td>
<td>169</td>
</tr>
<tr>
<td>121</td>
<td>Flange Splices</td>
<td>B̅</td>
<td>81</td>
</tr>
<tr>
<td></td>
<td>A514/A517 Straight Transition</td>
<td>B̅′</td>
<td>16</td>
</tr>
<tr>
<td>122</td>
<td>Box Girder Longitudinal Welds</td>
<td>B̅</td>
<td>48</td>
</tr>
<tr>
<td>123</td>
<td>Transverse Stiffeners</td>
<td>Ė</td>
<td>118</td>
</tr>
<tr>
<td>124</td>
<td>Web Attachments</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Plate thickness less than 1.0 in.</td>
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<td>Plate thickness 1.0 in or greater</td>
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<td>125</td>
<td>Web Gusset Plates</td>
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<td>Rectangular Plate</td>
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<td>Transition Radius</td>
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<td>Tapered Plate End</td>
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<td>Flange Tip Attachments, Rectangular Plate</td>
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<td>127</td>
<td>Flange Tip Attachments with Transition Radius, Groove Welded</td>
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<td>Radius between 2.0 and 6.0 in.</td>
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<td>Radius less than 2.0 in.</td>
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<td>Flange Tip Attachments with Transition Radius, Fillet Welded</td>
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<td>Flange Surface Attachments</td>
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<td>Attachment length between 2.0 and 4.0 in.</td>
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<td>Attachment length greater than 4.0 in.</td>
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<td>Coverplated Beams, Wide Plate</td>
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<td>Welded end</td>
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<td>Unwelded end</td>
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<td>135</td>
<td>Attachment Specimens</td>
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10.7.2 Base Metal

For the Category $\tilde{A}$, the results were reported in NCHRP reports 102 and 147. The database is provided by plain rolled beams without any welded connections or attachments. The fatigue design condition seldom governs as the stress range is usually limited by a detail with lower fatigue strength. The test data are plotted in Figure 118 and compared with the $\tilde{A}$. All failure points plot above the resistance curve.

![Figure 118 Fatigue Resistance of Plain Rolled Beams with Proposed Category $\tilde{A}$ Curve](image)

10.7.3 Longitudinal Welds

The data from continuous longitudinal welds are given in Figure 119. These only include data from web-to-flange longitudinal fillet welds and from large flat plate specimens with single and double bevel groove welds. The review has resulted in a sizable increase in the number of test data for these types of details. The data are well distributed above the Category $\tilde{B}$ curve with few points falling below. The constant amplitude test points shown below the constant amplitude fatigue limit are failures from the original
NCHRP test program. Several flange splice detail specimens yielded fatigue crack failures outside the splice transition zone and were therefore classified as longitudinal weld detail failures. The cracks originated at poor weld repair locations and were independent of the steel yield stress. The remaining failures plotting below the constant amplitude fatigue limit are test results from variable amplitude studies.

![Figure 119 Longitudinal Weld Test Data with Proposed Category $\bar{B}$ Resistance Curve](image)

10.7.4 Transverse Flange Splices

The flange splice data are compared with both the Category $\bar{B}$ and $\bar{B}'$ curves in Figure 120. The types of details included in the data are: flange splices with both curved and tapered transitions, and flange groove welds in both box girders and welded beams. While the majority of the failure points plot above $\bar{B}$, a number of points fall below. These are primarily the straight tapered flange splices in A514/ A517 high strength steel. The current specifications require a 2.0 ft. (0.61 in) transition radius when A514/ A517 steel is spliced. An alternative would be to classify straight tapered transitions in A514/ A517 steel as Category $\bar{B}'$. The test results for flange transverse groove welds in box and plate girders...
also provide a fatigue resistance that is consistent with the $\bar{B}$ category. All constant cycle tests at stress ranges below the constant amplitude fatigue limit showed no evidence of cracking at the time the test was discontinued.

![Figure 120 Flange Splice Test Data with Proposed Category $\bar{B}$ and $\bar{B'}$ Resistance Curves](image)

10.7.5 Box Girder Longitudinal Welds

As the data review revealed, the fatigue resistance of full size partial penetration longitudinal groove welds was overestimated by the $\bar{B}$ category. The available test data are compared with the Category $\bar{B'}$ curve in Figure 121. All but one test plots above the curve. The proposed curve provides a more accurate lower bound fatigue strength for this detail. The decreased resistance is due to size effects, both in the initial flaws and in the geometry of the detail. Larger initial defects were found to develop in the large scale sections as a result of blowholes and root gap flaws. These defects appear to be larger than the discontinuities observed in fillet welds. This same condition has been observed in longitudinal groove welds with backing bars left in place. The longitudinal fillet weld data
show no reduction in fatigue strength, indicating that Category $\bar{B}$ remains adequate for this type of weld.

10.7.6 Transverse Stiffeners and Diaphragms

The transverse stiffener data are compared with the proposed Category $\bar{C}$ curve in Figure 122. No failure points fall below the lower bound limit. Included in the plot are web stiffeners with their end cut short or welded to the flange and internal diaphragms in box girder members. Each detail type resulted in comparable fatigue resistance. The two test results plotting below the constant amplitude fatigue limit are the result of variable amplitude loading. Both tests plot beyond the straight line extension of the resistance curve. The effective stress range based on Miner's Rule provides a reliable estimate of the fatigue strength under this type of loading.
As indicated in Figure 123, the plate thickness influences the fatigue resistance of the detail. Since all tested details had an attachment length greater than 4.0 in. (100 mm) or 12.0 times the thickness, the maximum fatigue strength would correspond to Category $\bar{E}$. But with a 1.0 in. (25 mm) thickness or greater the resistance of the detail is reduced to Category $\bar{E}'$. Extensive results on 1.0 in. (25 mm) thick attachments subjected to variable amplitude loading were also obtained and are plotted in the high cycle region. These also confirm the applicability of Category $\bar{E}'$ to this detail.
10.7.8 Web Gusset Plates

Although web gusset plates are a form of web attachment, they have been plotted separately in Figure 124. These tested details are usually associated with lateral bracing elements and frequently have a vertical stiffener passing through the plate. The web gusset plates that have been included in the database all have a minimum attachment length of 24 in. (600 mm), therefore the detail corresponds to Category $\bar{E}$. The plot indicates that the failure data are well distributed above the curve. No difference in fatigue resistance was indicated when tapered ends were used, this being primarily due to the long attachment length. The details with a 4 in. (100 mm) radius transition provided a fatigue resistance equal to Category $\bar{D}$, similar to other fillet welded attachments with a radius transition.
10.7.9 Rectangular Flange Tip Attachments

Figure 125 shows the fatigue data for attachment plates welded to the flange tips. The plates were all rectangular in shape, without any treatment of the end condition. In all cases the attachment length was greater than 4.0 in. (100 mm), which would classify the detail fatigue strength as Category $\bar{E}$. 

Figure 125 Flange Tip Attachment Test Data with Proposed Category $\bar{E}$ Resistance Curve
10.7.10 Flange Tip Attachments with Transition Radius

When a ground transition radius is used at the ends of an attachment plate welded to the flange tip, the fatigue strength is increased (Figs. 126 and 127). All test specimens had the longitudinal attachment length greater than 4.0 in. (100 mm), which corresponds to Category $\bar{E}$ without any end treatment. All groove welded test data (Figure 126) plotted above the Category $\bar{C}$ curve when the termination had a radius equal to or greater than 2 in. (50 mm). Many of the tests were stopped without any evidence of cracking. These test data are identified by arrows. Fillet welded transition radius details (Figure 127) do not appear to be able to provide the fatigue resistance attainable with groove welded details, just as was observed with the web attachments.

Figure 126 Flange Tip Attachments with Groove Welded Transition Radius Test Data with Proposed Category $\bar{C}, \bar{D}$ and $\bar{E}$ Resistance Curves
10.7.11 Flange Surface Attachments

When the attachment plate is welded on the flange surface, the fatigue strength is also governed by the longitudinal length. The test data plotted in Figure 128 show reasonable correlation with the current specifications. When plates are welded transverse to the flange, the attachment length is the thickness of the plate. The length in the direction of stress was less than 2.0 in. (50 mm) for all test specimens. All test data provided a fatigue resistance that equals or exceeds Category $\bar{C}$. For intermediate attachment lengths between 2.0 and 4.0 in. (50 and 100 mm) the fatigue resistance is defined by the Category $D$. The test data for 2.0 in. (50 mm) attachments plots at the Category $\bar{C}$ curve. The 4.0 in. (100 mm) attachment test specimens plot between Category $\bar{C}$ and $D$. The long attachment details, with lengths greater than 4.0 in., gave results consistent with the Category $\bar{E}$.
10.7.12 Coverplated Beams

The fatigue data for coverplated beam details are summarized in Figs. 129 to 131. For coverplated beams with a narrow plate (width of the coverplate less than the width of the flange) and with or without transverse end welds, the fatigue strength is adequately defined by the Category $E$ curve as shown in Figure 129. All fatigue test data in this plot correspond to a beam flange thickness less than 0.8 in. (20 mm). The treatment of the end weld condition had no influence on the fatigue strength. This was found true for details with or without a transverse end weld. It can be seen that several of the variable amplitude tests fell below the resistance curve when Miner's Rule was used to determine the effective stress range.
Figure 130 shows the test data for wide coverplated beams in which the coverplate overlaps the beam flange. As indicated in the plot, the end weld condition influences the fatigue life. The details with a transverse end weld gave a fatigue resistance corresponding to Category $\bar{E}$, as did beams with a narrow coverplate. When no transverse end weld was used, the strength was decreased to $\bar{E}'$. This decrease in strength results because crack growth initiates at the flange tip. This results in a more severe crack geometry and a reduction in fatigue strength.
When the beam flange thickness is increased above 0.8 in. (20 mm), the fatigue strength is further reduced to Category $E'$ as shown in Figure 131. All data are for narrow coverplate specimens. It is not presently known if thick beam flanges with wide coverplates will result in a further reduction in fatigue resistance.
10.7.13 Simulated Specimens

Figures 133 to 135 summarize test data from smaller flat plate specimens that simulated design details. While these smaller type specimens were fabricated and tested to simulate full scale details, the majority of the test data plot significantly above the lower bound fatigue resistance curve defining the strength of the large scale detail. Figure 132 gives the non-load carrying cruciform joint data. With the exception of the English data, the remaining data all plot well above the Category $\bar{C}$ curve. In Figure 133 short longitudinal attachments (plates welded on edge) plot well above the Category $\bar{D}$. Most specimens had an attachment length of 4.0 in. (100 mm), so their classification would be Category $\bar{D}$. The specimens with the attachment welded flat and the attachment length of approximately 4.0 in. (100 mm) are given in Figure 134. These test data are also scattered significantly above the Category $\bar{D}$ curve although the distribution is more consistent and the lower bound test results plot near the Category $\bar{C}$ curve. The test data indicate that small scale specimens overestimate the fatigue resistance of full scale welded details.

Figure 132 Non-Load Carrying Cruciform Joint Test Date with Proposed Category $\bar{C}$ Resistance Curve
10.8 Conclusions

The conclusions in this chapter are based on an analysis and evaluation of existing fatigue test results on welded steel bridge details. The test data were compiled from a number of independent test programs that were conducted during the last twenty years. The
conclusions are based on a review of all available test data analyzed as a whole without relying on the results from any one particular test program.

10.8.1 Test Data Acquired

1) The review of the test data that have been produced since NCHRP Reports 102 and 147 were published has significantly increased the database for welded steel details. The current AASHTO fatigue resistance curves were based on approximately 800 fatigue test failure results. The review, as outlined before, has added 1500 additional test results to the database.

2) Types of details that were not previously considered in the original provisions have been added to the database. This includes longitudinal groove welds in both flat plate specimens and in box members. Internal diaphragms for box type members were also included. Large scale coverplate and web attachment details that provided information on size effects. Also, a wider range of flange attachment details with varying geometries and weld condition results have been added to the database. A number of large simulated test specimens were examined, such as gusset attachments and nonload carrying cruciform joints.

3) The comparison of the test data with the current AASHTO fatigue provisions did not result in any major deviations between the design fatigue strength and test results. Almost all data for each detail type plotted above the appropriate curve defining its lower bound fatigue resistance.

4) The findings that were reported in the original NCHRP reports have been supported by the subsequent test programs. No indications were given in these reports that the NCHRP results were in error.
10.8.2 Inadequacies of Current Fatigue Provisions

1) Partial penetration longitudinal groove welds such as those used in box-type or built-up members were found to exhibit a fatigue strength that was overestimated by the Category B resistance curve. The original longitudinal weld detail strength was based on test results with fillet welds providing the web-to-flange connections. The test data indicate that partial penetration groove welds can result in a more severe initial defect condition, thereby decreasing their fatigue strength below Category B.

2) Web attachments with the plate thickness greater than 1.0 in. (25 mm) resulted in a fatigue strength that was less than that provided by the Category E resistance curve. It was found that Category E' gave a more reasonable lower bound estimate of the fatigue resistance of this detail.

3) Additional fatigue tests of coverplated beams in the high cycle region have indicated that the constant amplitude fatigue limit for Category E is more accurately defined by a stress range value of 4.0 ksi rather than the current value of 4.5 ksi.

10.8.3 Proposed Fatigue Design Curves

The adjustments to the AASHTO fatigue design curves presented in this section provide a better fit to the test data, and are compatible with crack propagation concepts and cumulative damage theories. The following statements summarize the basis for these changes:

1) The test data review was generally in good agreement with the current AASHTO curves. Large samples of test data for a given detail tended towards a better fit when the slope was a constant value of -3.0.
2) The proposed curves coincide with the current resistance curves at the 2\cdot10^6 intercept values. The exception is Category A, which showed a slight change. The tabularized form of the curves showed only minor deviations at all life increments.

3) The proposed curves would provide more compatibility between the AASHTO fatigue provisions and the fatigue resistances adopted or under consideration in many other parts of the world (i.e. the ECCS and ISO provisions).

4) The proposed curves are easily described in mathematical terms. They can be defined by one equation with only a varying intercept value. The use of an equation in design and damage assessment procedures should lead to more accurate estimates. The estimated life for each category is tabulated at four discrete intervals. The equations would provide a continuous relationship between stress range and cycle life and would therefore avoid inaccurate extrapolation.

5) Comparisons of the test data with the proposed curves indicated that they adequately defined the fatigue resistance of welded steel details commonly used in the design and fabrication of bridge structures.

10.9 Summary

Since the AASHTO Specification fatigue resistance provisions were developed from test data reported in NCHRP Reports 102 and 147, several major fatigue studies have been conducted. By reviewing the results of these studies on full-scale, welded steel test specimens, the original database was broadened to include a wider range of detail types and sizes. Each data group was compared to the existing AASHTO fatigue design provisions in order to determine the adequacy of the resistance curves and to check for detail types whose fatigue strengths deviate from these curves. Most of the additional data
correlate well with the original database. Since the current (1986) AASHTO fatigue design curves were based on a limited number of detail types, the expanded database allows for a more comprehensive assessment of the fatigue strength provisions.

From this review, a revised set of fatigue design curves is proposed that better estimates the fatigue resistance of welded steel bridge details. Though they are similar to the current AASHTO curves, the curves are more uniform and parallel; each curve is set at a constant slope of -3.0. The available data have been compared with the appropriate curve in order to assess the validity of the proposed fatigue design curves.

Although the database has been significantly enlarged by the inclusion of test data, several areas have been identified that require further in-depth study. This includes a more thorough examination of size effects, so that test data can be accurately correlated with field conditions. Also, additional test data is needed in the high cycle regime. This would help to better establish the constant amplitude fatigue limits and provide a better understanding of whether or not bridge structures will experience cracking at some of these details.
CHAPTER XI
SUMMARY AND CONCLUSIONS

11.1 Summary of the Research

Since the AASHTO Specification fatigue resistance provisions were developed from test data reported in NCHRP Reports 102 and 147, several major fatigue studies have been conducted. This study is divided into two types of prediction. In the first type statistical models for fatigue life prediction for welded joints are conversed and fitted to experimental data for fillet-welded steel joints where cracks emanate from the weld toe.

Crack Initiation – we began this work by looking at crack initiation for which we did a significant amount of work in the literature review. The primary aim of this was to examine the initiation of fatigue damage with the idea that some of this information might get at a damage threshold or equivalent. In the long life regime, defects are kept small and below the damage threshold, the life of the detail or component should be infinite. By keeping cracks from initiation, the fatigue life should be infinite.

Analysis of current welded steel S-N curves – We obtained some long life fatigue data for typical welded steel details which contains long – life run-out data. The primary idea was to look at the influence of the run-out points on position of the S-N curves and if we’re fortunate, the value(s) of the constant amplitude fatigue limit(s). The way we started was to put the data into a spreadsheet and run an analysis two ways; a) without the run-
outs and b) with the run-outs included in the data set. Then we draw conclusions about the influence of the run-out data on the fatigue strength. As a follow up step, the analysis will be conducted using the ML or maximum likelihood method and compare the S-N parameters to what we found in a) and b).

The models are based on an S-N approach where the number of cycles N to failure is assumed to be directly correlated to the applied nominal stress range ΔS. The models assume the existence of a fatigue limit given as a stress range below which no failure will take place. Emphasis is laid on the modeling of the fatigue life close to this limit where the service stresses for welded details often occur. Experimental data in this stress regime are sparse and do not fit the knee point of the conventional bi-linear S-N curve found in design rules and specifications. Therefore, an alternative model where both the fatigue life and the fatigue limit are simultaneously treated as random variables is investigated.

In a parallel step, we were looking at typical flaw sizes for welded steel details. This information is likely some of the most difficult to get, as we have to search papers, case studies and similar documents. However, the second type of prediction is required if inspection is to be carried out in a damage tolerance approach. In this case fatigue process in fillet welded joints is discussed and modeled. As a first approximation, a pure fracture mechanics model was employed to describe the entire fatigue process. The model is calibrated to fit the crack growth measurements obtained from extensive testing on fillet weld joints where cracks emanate from the weld toes. Emphasis is laid on the choice of growth parameters in conjunction with a fictitious initial crack size distribution in order to obtain both reliable crack growth histories and predictions of the entire fatigue life. The model has its shortcomings in describing the damage evolution at low stress ranges due to
the presence of a significant crack development period in this stress regime. As an alternative to the fracture mechanics model, a two-phase model (TPM) for the fatigue process was developed and calibrated.

Using the information on flaw size distribution we can use simple fracture mechanics models for the detail(s) we examined in the analysis of the S-N curves. With a distribution of flaw sizes and knowledge of the constant amplitude fatigue limit, it should be a matter of a few calculations to get some estimates for the fatigue damage threshold. Analysis of all this information should result in some recommendations on the long life fatigue strength of welded steel details and may change some of the current specifications.

11.2 Category E S-N Curve New Prediction

Statistical models for fatigue life prediction for welded joints are discussed and fitted to experimental data for fillet-welded steel joints where cracks emanate from the weld toe. The models are based on an S-N approach where the number of cycles N to failure is assumed to be directly correlated to the applied nominal stress range ΔS. The models assume the existence of a fatigue limit given as a stress range below which no failure will take place. Emphasis is laid on the modeling of the fatigue life close to this limit where the service stresses for welded details often occur. Experimental data in this stress regime are sparse and do not fit the knee point of the conventional bi-linear S-N curve found in design rules and specifications. Therefore, an alternative model where both the fatigue life and the fatigue limit are simultaneously treated as random variables is investigated.

The model parameters for this random fatigue-limit model (RFLM) are determined by the maximum likelihood method, and confidence intervals are obtained by the profile likelihood method. The advantage of the model is that it takes into consideration the
variation in fatigue limit found from specimen to specimen and that run-out results are easily included. The median S-N curve obtained from the model coincides with the conventional bi-linear curves in the high stress regime (stress ranges higher than 110 MPa), but predicts longer lives as the stress range decreases below 100 MPa.

The model gives a nonlinear S-N curve for a log-log scale in the fatigue-limit area; the fatigue life is gradually increasing and is approaching a horizontal line asymptotically instead of the abrupt knee point of the bilinear curve. The nonlinear curve is more in accordance with experimental data. At stress ranges below 100 MPa, the predicted fatigue lives are between 2 to 10 times longer than predictions made by the bilinear Category E curve in AISC Steel Construction Manual. The conclusion is that the rule-based S-N curves may be un-conservative in the stress regime where service stresses frequently occur. A more correct statistical model based on a random fatigue-limit model results in S-N curves that give decreased dimensions for a given fatigue design factor under constant amplitude loading.

11.3 Modeling the fatigue process in a welded joint

The fatigue process in fillet welded joints is discussed and modeled. As a first approximation, a pure fracture mechanics model was employed to describe the entire fatigue process. The model is calibrated to fit the crack growth measurements obtained from extensive testing on fillet weld joints where cracks emanate from the weld toes. Emphasis is laid on the choice of growth parameters in conjunction with a fictitious initial crack size distribution in order to obtain both reliable crack growth histories and predictions of the entire fatigue life. The model has its shortcomings in describing the damage evolution at low stress ranges due to the presence of a significant crack development period.
in this stress regime. As an alternative to the fracture mechanics model, a two-phase model (TPM) for the fatigue process was developed and calibrated.

The number of cycles to crack development was modeled by a local strain approach using the Coffin-Manson equation, whereas the propagation phase was modeled by fracture mechanics, adopting the simple version of the Paris law. The notch effect of the weld toe was treated by extreme value statistics for the weld toe radius. To make the model fit all test data for crack development and propagation, it is crucial to select a sufficiently low transition depth between the two phases. A transition depth of 0.1 mm (0.004 in.) was selected. Furthermore, the material parameters in the Coffin-Manson equation were determined directly from the early cracking in the weld toe for full-scale fillet welds and not from tests carried out with small-scale smooth specimens.

This is essential if the model is to account for the actual surface condition at the weld toe. The S-N curves constructed by the present physical model were compared with the median nonlinear S-N curve obtained from the statistical random fatigue-limit model (RFLM) presented in Part 1 of this investigation. The curves are almost identical in the stress region where experimental data exist and both models fit the experimental data very well. However, the TPM does not predict any fatigue limit, in contrast to the RFLM. According to the TPM, the long fatigue life found at low stress ranges is a result of a dominating long development period and is not a threshold phenomenon. The two models are complementary tools. The RFLM is a pure statistical model, whereas the TPM is a semi empirical physical model. The TPM is capable of taking into account the effect of the global geometry of the joint, the local weld toe geometry, applied stress ratio, and the residual stress condition. Both models can be used for fatigue life predictions, but only the
physical TPM can be used for planning of in-service inspection strategies where damage evolution as a function of time is needed.

11.4 Proposed fatigue design curves

Since the AASHTO Specification fatigue resistance provisions were developed from test data reported in NCHRP Reports 102 and 147, several major fatigue studies have been conducted. By reviewing the results of these studies on full-scale, welded steel test specimens, the original database was broadened to include a wider range of detail types and sizes. Each data group was compared to the existing AASHTO fatigue design provisions in order to determine the adequacy of the resistance curves and to check for detail types whose fatigue strengths deviate from these curves. Most of the additional data correlate well with the original database. Since the current (1986) AASHTO fatigue design curves were based on a limited number of detail types, the expanded database allows for a more comprehensive assessment of the fatigue strength provisions.

From this review, a revised set of fatigue design curves is proposed that better estimates the fatigue resistance of welded steel bridge details. Though they are similar to the current AASHTO curves, the curves are more uniform and parallel; each curve is set at a constant slope of -3.0. The available data have been compared with the appropriate curve in order to assess the validity of the proposed fatigue design curves.

Although the database has been significantly enlarged by the inclusion of test data, several areas have been identified that require further in-depth study. This includes a more thorough examination of size effects, so that test data can be accurately correlated with field conditions. Also, additional test data is needed in the high cycle regime. This would help to better establish the constant amplitude fatigue limits and provide a better
understanding of whether or not bridge structures will experience cracking at some of these details.

11.5 Conclusions

The approaches is mainly based on the S-N method, and the chapters dealing with applied fracture mechanics. The research gives the latest updates found in design rules and specification and a more thorough presentation of the fracture mechanics approach. Some computational models based on applied fracture mechanics are also included. Based on the results of parametric studies and the reliability analysis, the following conclusions are made:

- Category E S-N Curve New Prediction

1) The statistical behavior of the fatigue life of fillet welded joints has been examined and modeled with reference to conventional S-N curves found in current design rules and specifications. An alternative statistical model based on a joint random fatigue life and a random fatigue limit has been applied. Constant amplitude fatigue life data near the “knee point” of the rule-based bilinear S-N curves are assembled to study and corroborate the model. The model has been fitted to experimental fatigue lives and the obtained S-N curve is compared with the traditional bilinear S-N curves given in design rules and specifications. The rule-based S-N curves and the RFLM based curve coincide for stress ranges above 110 MPa. For stress ranges below 100 MPa, the RFLM curve will predict fatigue lives that are from 2 to 10 times longer than the predictions made by the Category E S-N curve. It appears that the nonlinear curve obtained from the RFLM has a much better ability to model fatigue life behavior in this stress region.
2) The abrupt knee point of rule-based bilinear curves does not fit the experimental facts for the assembled data. The fatigue life behavior in this stress regime is obviously more complex than the conventional bilinear S-N curve can describe. The discrepancy between the present RLFM curve and the Category E curve is important as it occurs in a stress region where the majority of the load cycles for a welded detail in service usually occur. The rule-based S-N curves seem un-conservative in this regime and this will have a strong bearing on practical fatigue life predictions, fatigue design and final dimensions of welded details.

- Modeling the fatigue process in a welded joint

1) Our study in this section is primarily concentrates on non-load-carrying fillet welded joints made of C-Mn steel with nominal yield stress close to 345 MPa (50 ksi). A TPM was used to predict the fatigue life. The initiation life was modeled by Coffin-Manson equation, whereas the crack propagation was based on the simple version of the Paris law. The models were validated and calibrated with the use of large databases. The criteria for acceptance of the model were that the model should predict both damage evolution and final fatigue life at any stress level. The FMM model failed to fulfill these criteria, whereas the TPM gave an excellent fit to both measured crack growth histories and experimental fatigue lives. The following conclusions are drawn.

2) The fatigue behavior of fillet welded joints is far more complex at typically in-service stresses than fracture mechanics can describe. This is due to the fact that the crack development phase dominates the fatigue life at these low stresses.

3) A TPM is capable of modeling the damage evolution from the initial state to the final fracture provided that the model is accurately calibrated for this purpose. The notch
factor at the weld toe is based on extreme value statistics for the toe geometry, and the transition crack depth between the initiation phase and the propagation phase is set to 0.1 mm. The parameters in the Coffin-Manson equation were determined directly from early cracking in full-scale welded joints. As the TPM has a semi-empirical physical basis, the determining factors, such as residual stresses, global and local joint geometry, and loading mode, are readily accounted for.

4) The S-N curves constructed from the model are non-linear for a log-log scale and coincide with the curves obtained from the statistical RFLM. Both models fit experimental data far better than the conventional bilinear S-N curves.

5) There is a fundamental difference between S-N curves obtained from the RFLM and the TPM in the way that the latter curves do not predict any fatigue limit. At stress ranges below 60 MPa, the RFLM curve will appear flat, whereas the TPM curve will continue to fall with a small slope close to the parameter b in the Coffin-Manson law. At present there is no data to corroborate either one of these curves, but they tend to have more confidence in the prediction made by the physical-based TPM than the predictions based on the statistical RFLM when extrapolated outside the range of the data.

6) The first practical consequence of the present TPM is that it predicts longer lives at low stress ranges than the conventional S-N curves in design rules and specifications. With the application of the TPM-constructed S-N curve in the lower stress region it is possible to reduce dimensions by 30-40% and still achieve the same fatigue life as for the Category E S-N curve.
7) The second practical consequence is that in-service inspection strategy may be optimized. This is due to the fact that the crack path leading to final fracture is quite different from the path calculated by a pure FMM. The TPM with its long initiation phase will give a more hidden path for the crack evolution. Hence, an inspection program with increased inspection frequency at the end of service life is proven to be favorable.

❖ Proposed Fatigue Design Curves

1. The conclusions in this part are based on an analysis and evaluation of existing fatigue test results on welded steel bridge details.

2. The conclusions are based on a review of all available test data analyzed as a whole without relying on the results from any one particular test program.

11.6 Recommendations for Future Research

The study been limited to fillet welded joints subjected to constant amplitude loading. Future research should carry out an investigation on other joint shapes such as butt joints and VA loading. As for a butt joint, which is the most common shape for high load transfer, we already know that the initiation phase will play an even more important role than has been shown for the fillet welded joint [123]. This is due to the significantly lower stress concentration factor at the weld toe. When it comes to variable amplitude loading, the importance and the consequences of an initiation phase are not obvious and an investigation is necessary before design conclusions. As the TPM has no fatigue limit, it will be interesting to analyze how the model responds to variable loading by using a damage accumulation law. For load spectra with the center of gravity in the low stress range area
(e.g. exponential distributed stress ranges), the effects revealed for constant amplitude
loading will probably prevail [123].

In chapter 9 the TPM has been used to construct median S-N curves only. Future
research should focus on constructing quantile curves for design purposes as well. This can
be done by a Monte Carlo simulation treating the main determining factors as random
variables. The resulting curves should be compatible with the quantile curves obtained
from the RFLM. Finally, the practical consequences of the model in terms of joint
dimensions and scheduled inspection programs should be studied in more detail. Other
types of joints, such as butt joints, are also of great interest in this regard.
REFERENCES


Rice, John A. Mathematical Statistics and Data Analysis, Duxbury Press (Belmont, CA), 1995.


Review of Repairs to Offshore Structures and Pipelines, Publication 94/102 Marine Technology Directorate, UK, 1994

A. Almar-Nmss (ed), Fatigue Handbook, Tapir, 1985


H.P. Lieurade, Methodologie d'essais de fatigue sur composants souders. CETIM, Conference proceedings, November 1992 at Senlis


A. Almar Nwss, Fatigue Handbook, Trondheim, Tapir 1985

American Society of Testing Materials: D1141 and E 647 specifications ASME

T. Lassen, J.L. Arana and L. Canada, "Crack growth in high strength steel subjected to fatigue loading in a corrosive environment", 24th International Conference of Offshore Mechanics and Artie Engineering (OM_AE Halkidiki, Greece, June 2005


J. Bannantine, J. Corner and J. Handrock, Fundamental of Metal Fatigue Analysis, College of Engineering, University of Illinois, 1987

G. Glinka, Fatigue Life Predictions of Notched Components: A Crack development, Waterloo, Canada, University of Waterloo, 1990

A. Almar Nxss, Fatigue Handbook, Trondheim, Tapir, 1985


ENV: EUROCODE 3- Steel Structure-Fatigue

A. Almar N2ess, Fatigue Handbook, Trondheim, Tapir, 1985


DNV: Fatigue Strength Analysis of Offshore Steel Structures, Recommended practice RP-C203, 2005


X. Niue and G Glinka, "The weld profile effect on the SIF in weldments" International Journal of Fracture 1987 (35), pp 3-20

F.V. Lawrence Ho, "Fatigue Test Results and Predictions for Cruciform and Lap Welds" in Theoretical and Applied Fracture Mechanics 1984 (1)
A. Stacy and J.V. Sharp, the Revised HSE Fatigue Guidance, OMAE, 1995, pp 1-16

NORSOK Design of Steel Structures, N-004, Annex C, NORSOK standard 1998


I. Lotsberg et al, "Fatigue assessments of Floating Production Vessels", Conference proceedings, BOSS'97, Delft University of Technology, July 1997


DNV, Fatigue Assessment of Ship Structure Classification Note 30.7, Det Norske Veritas 2003

BV, Fatigue Strength of Welded Ship Structures, Bureau Veritas, July 1998


N. Recho, Ruptures par fissuratio des structures, Hermes, 1995


K. Engesvik, "Analysis of the uncertainty of the fatigue capacity of welded joints", NTNU, Trondheim, UR-82-17

T.R. Gurney, "Finite element analysis of some joints with the welds transverse to the direction of stress", Welding Research International 6, 1976, pp 40-72


G. Lebas and J.C. Fauve, Collection de donnees de fatigue, Pau, Elf Aquitaine, 1988

D. Radencovic, "Calcul de duree de vie des noeuds tubulaires Ecole Polytechnique", Laboratoire de mecanique des solides, CNRS, 1982


J.C.P. Kam and W.D. Dover, "Structural integrity of welded tubular joints in random load fatigue combined with size effect", International Conference of Offshore Structures, Glasgow, September 1987


E. Niemi (Editor) and N. Recho; "Probabilistic approach to fatigue life in T-welded tubular joints Tubular Structures, Third International Symposium, Finland 1989


NORSOK, Standard N-004, Rev 2, October 2004, Chapter 10, "Re-assessment of structures"

DNV RP-C203 Fatigue design of offshore structures - recommended practice DNV 2005

APL report, "Fatigue Analysis of Inner Flowline", report no 6330-94573-DE, Advanced Production and Loading


T. Lassen, P.H. Darcis and N. Recho, "Fatigue behavior of Welded Joints Part 1 — Statistical Methods for Fatigue Life Predictions" Welding Journal 84 (12), 2005, 183-s to 187-s

T. Lassen, "The effect of the welding process on the fatigue crack growth in welded joints" Welding Journal 69 (2), 1990, 75-s to 85-s

G. Lebas and J.C. Fauve, Collection of Fatigue Data, 1988, Elf Aquitaine, Pau


T. Lassen, "The Effect of the Welding Process on the Fatigue Crack Growth in Welded Joints" Welding Journal 96 (2), 1990, pp 75s, 85s


Ph.Darcis, et al., "A fracture mechanics approach for the crack growth in welded joints with reference to 8S7910" European Conference on Fracture (ECF 15), Stockholm 11-13 August 2004


K. Engesvik and T. Lassen, "The Effect of Weld Toe Geometry on Fatigue Life" The 7th OMAE Conference, Houston, Texas, 1988, pp 441-45
