NUMERICAL SIMULATIONS OF MAGNETOHYDRODYNAMIC FLOW AND HEAT TRANSFER

A Thesis

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Magnetohydrodynamic (MHD) natural convection in a porous medium with low-magnetic Reynolds number ($R_{em}$) is investigated in a rectangular cavity with isothermal walls on the left and right and adiabatic walls on the top and bottom. The validity of Darcy’s law is addressed for high-Rayleigh number ($Ra$) flows with high permeability, where the velocity-pressure gradient relationship transitions from linear (i.e. the Darcy law) to nonlinear, due to the fact that the form drag due to solid obstacles is now comparable with the surface drag due to friction, which in turn results in the Darcy-Forchheimer law. In addition, the effect of different magnetic field strengths in terms of Hartmann numbers ($Ha$) is also investigated for cavities for varying aspect ratios to analyze how the flow and thermal characteristics in a porous medium are influenced by the applied magnetic field. Here, the interaction between the fluid velocity and the electromagnetic forces gives rise to different flow scenarios. In particular, the influence of magnetic field under the varying conditions of convective currents (through $Ra$) and length scales (through aspect ratios) on quantities such as stream function, temperature and Nusselt number, $Nu$ is studied. Assessment of three regularization-based models and two eddy-viscosity-based subgrid-scale (SGS) turbulence models for large eddy simulations (LES) are carried out for MHD
decaying homogeneous turbulence (DHT) and MHD transition to turbulence for the Taylor-Green vortex (TGV) through comparisons to direct numerical simulations (DNS). Simulations are conducted using the low-magnetic Reynolds number approximation ($Re_m << 1$) and the initially-isotropic turbulence problem has a Taylor scale Reynolds number ($Re_\lambda$) of 120. LES predictions using the Leray-\(\alpha\), LANS-\(\alpha\), and Clark-\(\alpha\) regularization-based SGS models are compared to the classic non-dynamic Smagorinsky and the dynamic Smagorinsky models. Regarding the regularization models, this work represents their first application to MHD decaying turbulence or transition-to-turbulence problems. Analyses of turbulent kinetic energy decay rates, energy spectra, and vorticity fields are made between the varying magnetic field cases. Overall, the regularization models did poorly compared to the eddy-viscosity models for all MHD cases, but the comparisons improved as the magnetic field increase in magnitude.
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td>viii</td>
</tr>
<tr>
<td><strong>CHAPTER</strong></td>
<td></td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. LITERATURE REVIEW</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Numerical simulations of non-Darcy magnetohydrodynamic natural convection in a porous rectangular cavity</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Assessment of five subgrid-scale (SGS) models for low-$Re_m$ magnetohydrodynamic (MHD) turbulence</td>
<td>8</td>
</tr>
<tr>
<td>III. SCOPE AND STRUCTURE</td>
<td>13</td>
</tr>
<tr>
<td>3.1 Scope of work</td>
<td>13</td>
</tr>
<tr>
<td>3.2 Thesis structure</td>
<td>14</td>
</tr>
<tr>
<td>IV. FORMULATION</td>
<td>16</td>
</tr>
<tr>
<td>4.1 Numerical simulations of non-Darcy magnetohydrodynamic natural convection in a porous rectangular cavity</td>
<td>16</td>
</tr>
<tr>
<td>4.2 Assessment of five subgrid-scale (SGS) models for low-$Re_m$ magnetohydrodynamic (MHD) turbulence</td>
<td>24</td>
</tr>
<tr>
<td>V. PROBLEM DESCRIPTION</td>
<td>30</td>
</tr>
<tr>
<td>5.1 Numerical simulations of non-Darcy magnetohydrodynamic natural convection in a porous rectangular cavity</td>
<td>30</td>
</tr>
<tr>
<td>5.2 Assessment of five subgrid-scale (SGS) models for low-$Re_m$ magnetohydrodynamic (MHD) turbulence</td>
<td>31</td>
</tr>
</tbody>
</table>
VI. COMPUTATIONAL DETAILS ........................................ 33

6.1 Numerical simulations of non-Darcy magnetohydrodynamic natural convection in a porous rectangular cavity .................. 33

6.2 Assessment of five subgrid-scale (SGS) models for low-$Re_m$ magnetohydrodynamic (MHD) turbulence ......................... 34

VII. RESULTS AND DISCUSSION .................................. 36

7.1 Numerical simulations of non-Darcy magnetohydrodynamic natural convection in a porous rectangular cavity .................. 36

7.2 Assessment of five subgrid-scale (SGS) models for low-$Re_m$ magnetohydrodynamic (MHD) turbulence ......................... 57

VIII. CONCLUSIONS .............................................. 80

8.1 Numerical simulations of non-Darcy magnetohydrodynamic natural convection in a porous rectangular cavity .................. 80

8.2 Assessment of five subgrid-scale (SGS) models for low-$Re_m$ magnetohydrodynamic (MHD) turbulence ......................... 81

BIBLIOGRAPHY .................................................. 84
LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>Schematic of the rectangular enclosure.</td>
<td>31</td>
</tr>
<tr>
<td>7.1</td>
<td>Comparison of the average $Nu$ calculated from the current study with that from the correlation [1]. The subscripts c and n are for correlation and numerical values respectively.</td>
<td>41</td>
</tr>
<tr>
<td>7.2</td>
<td>Numerical results for stream function (left) and temperature (right) for aspect ratio 1:1 and $Ra_D = 10000$ in the absence of magnetic field ($Ha_e = 0$).</td>
<td>42</td>
</tr>
<tr>
<td>7.3</td>
<td>Numerical results for stream function (top) and temperature (bottom) of table 7.1 for aspect ratio AR 4:1 and $Ra_D = 500$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature.</td>
<td>46</td>
</tr>
<tr>
<td>7.4</td>
<td>Numerical results for stream function (top) and temperature (bottom) of table 7.1 for aspect ratio AR 4:1 and $Ra_D = 5000$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature.</td>
<td>47</td>
</tr>
<tr>
<td>7.5</td>
<td>Numerical results for stream function (top) and temperature (bottom) of table 7.1 for aspect ratio AR 4:1 and $Ra_D = 10000$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature.</td>
<td>48</td>
</tr>
<tr>
<td>7.6</td>
<td>Numerical results for stream function (top) and temperature (bottom) of table 7.2 for aspect ratio AR 6:1 and $Ra_D = 500$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature.</td>
<td>49</td>
</tr>
<tr>
<td>7.7</td>
<td>Numerical results for stream function (top) and temperature (bottom) of table 7.2 for aspect ratio AR 6:1 and $Ra_D = 5000$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature.</td>
<td>50</td>
</tr>
</tbody>
</table>
7.8  Numerical results for stream function (top) and temperature (bottom) of table 7.2 for aspect ratio AR 6:1 and $Ra_D = 10000$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature. ......................................................... 51

7.9  Numerical results for stream function (top) and temperature (bottom) of table 7.3 for aspect ratio AR 8:1 and $Ra_D = 500$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature. ......................................................... 52

7.10 Numerical results for stream function (top) and temperature (bottom) of table 7.3 for aspect ratio AR 8:1 and $Ra_D = 5000$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature. ......................................................... 53

7.11 Numerical results for stream function (top) and temperature (bottom) of table 7.3 for aspect ratio AR 8:1 and $Ra_D = 10000$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature. ......................................................... 54

7.12 Variation of the average Nusselt number with effective Hartmann number for different values of the Rayleigh Darcy number a) AR 4:1 b) AR 6:1 c) AR 8:1. ................................................................. 55

7.13 Variation of the average Nusselt number with Rayleigh Darcy number for different values of the effective Hartmann number a) AR 4:1 b) AR 6:1 c) AR 8:1. ................................................................. 56

7.14 Evolution of the turbulent kinetic energy (tke) with time, for five different SGS models compared with DNS, with different values of $N$. N0, N0.1, N0.5, N1 represent $N = 0, 0.1, 0.5$ and 1 respectively. ................................................................. 64

7.15 Evolution of rate of decay of turbulent kinetic energy, dK/dt, with time, for five different SGS models compared with DNS; with different values of $N$. N0, N0.1, N0.5, N1 represent $N = 0, 0.1, 0.5$ and 1 respectively. ................................................................. 65

7.16 Spectra, E(k), at four different times; N0 represents magnetic interaction parameter, $N = 0$. ................................................................. 66

7.17 Spectra, E(k), at four different times; N0.1 represents magnetic interaction parameter, $N = 0.1$. ................................................................. 67

7.18 Spectra, E(k), at four different times; N0.5 represents magnetic interaction parameter, $N = 0.5$. ................................................................. 68
7.19 Spectra, E(k), at four different times; N1 represents magnetic interaction parameter, $N = 1$.

7.20 Grey scale contours of vorticity magnitude on a $\pi^3$ domain at time, $t = 6$ s, for DNS and different SGS models without magnetic field i.e. magnetic interaction parameter $(N) = 0$. (a) DNS (b) NDSMAG (c) DSMAG (d) LERAY (e) LANS (f) CLARK. The legends and the axes for the contours are same.

7.21 Grey scale contours of vorticity magnitude on a $\pi^3$ domain at time, $t = 6$ s, for DNS and different SGS models with magnetic field i.e. magnetic interaction parameter $(N) = 0.5$. (a) DNS (b) NDSMAG (c) DSMAG (d) LERAY (e) LANS (f) CLARK. The legends and the axes for the contours are same.

7.22 Evolution of the turbulent kinetic energy (tke) of decaying Taylor - Green Vortex (TGV) with time, for five different SGS models, with different values of $N$. N0 and N0.05 represent $N = 0$ and 0.05, respectively.

7.23 Decay rate of turbulent kinetic energy (tke) of decaying Taylor - Green Vortex (TGV) with time, for five different SGS models, with different values of $N$. N0 and N0.05 represent $N = 0$ and 0.05, respectively.

7.24 Spectra, E(k), of decaying Taylor - Green Vortex (TGV) at two different times; N0 and N0.05 represent magnetic interaction parameter, $N = 0$ and $N = 0.05$ respectively.

7.25 Grey scale vortical structures in a $\pi^3$ domain at time, $t = 10$ s, in terms of iso-surface at value $C = 12$ for the different SGS models without magnetic field i.e. magnetic interaction parameter, $N = 0$ (a) LERAY (b) LANS (c) CLARK (d) NDSMAG (e) DSMAG.

7.26 Grey scale vortical structures in a $\pi^3$ domain at time, $t = 10$ s, in terms of iso-surface at value $C = 12$ for the different SGS models with magnetic field i.e. magnetic interaction parameter, $N = 0.05$ (a) LERAY (b) LANS (c) CLARK (d) NDSMAG (e) DSMAG.
Magnetohydrodynamic (MHD) is an extremely extensive (covering or affecting a large area) word which ranges from phenomena realizable in the laboratory with liquid metals or ionized gases, to the behavior of the mass of conducting matter which make up planets or stars. MHD concerns those phenomena, where, in an electrically conducting fluid, the velocity field \( \mathbf{u} \) and the magnetic field \( \mathbf{B} \) are coupled. The coupling between fields \( \mathbf{u} \) and \( \mathbf{B} \) can be more or less strong. The movement of an electrically conducting fluid in a magnetic field generates an electromotive force, and causes an electric current density \( \mathbf{J} \) to flow; this induces its own magnetic field. In our study we have neglected the effect of this induced magnetic field, which is negligible in comparison with the applied magnetic field in some cases such as technological and industrial applications like steel production and processing, solidification and casting, and the design of heat transfer devices for future nuclear fusion reactors. When we consider a magnetohydrodynamic (MHD) turbulence of liquid metals, there is always an assumption of a low magnetic Reynolds number \( Re_m \) quasi-static approximation which is an assumption of negligible induced magnetic field in comparison with the imposed one. In these cases, \( Re_m \) is assumed to be small \( (Re_m << 1) \), rather than in astro and geophysical applications, which has high value of \( Re_m \) \( (Re_m >> 1) \). Here,
\[ Re_m \] is given by,

\[ Re_m = \frac{U_0 L_0}{\eta} \]

where \( U_0 \) and \( L_0 \) are the typical velocity and length scales respectively, \( \eta \) is the magnetic diffusivity given by \( \eta = (\sigma \mu_0)^{-1} \), \( \sigma \) and \( \mu_0 \) being electric conductivity of the fluid and magnetic permeability of the vacuum, respectively. In majority of processes \( Re_m \ll 1 \), the results are valid for technological and laboratory flow than that for astrophysics and geophysics. In our case, the electrically conducting fluid is assumed to be incompressible and the physical properties like magnetic permeability \( (\mu) \), the electrical conductivity \( (\sigma) \), and the kinematic viscosity \( (\nu) \) are assumed to be constant.

Two different problems are chosen here: 2D porous cavity and 3D homogeneous turbulence. A porous medium is defined as a fixed solid matrix with connected void spaces through which an electrically conducted fluid can flow. Here, the porosity or void fraction is a measure of void spaces in a material, and is a fraction of the volume of the voids over the total volume. The value for the porosity lies in between 0-1 (0.8 in our case). In a natural porous medium the distribution of pores with respect to shape and size is irregular. Examples of natural porous media can be taken as beach sand, sandstone, limestone, wood and the human lungs. Different ceramics, composite materials and high porosity metallic foams include man made porous media. Natural convection defined as the convection motion driven by buoyancy forces. When an electrically conducting fluid is introduced through porous media, it has
large number of technical applications like in geophysics, casting and solidification, crystal growth control, etc.

In this thesis, the effects of a magnetic field on the natural convection in a 2-D cavity filled with porous media using non-Darcy or Brinkman-Forchheimer extended Darcy law model including convective inertial term for different aspect ratios have been numerically investigated. The magnetic field is applied in the perpendicular direction to the isothermal walls and a grid size of $64 \times 64$ is employed for an effective Hartmann number ranging from 0 to 10.

When we consider turbulence, flow is unsteady, irregular, random and chaotic where we can observe the motion of many scales. Homogeneous turbulence is the simplest class of flow to study turbulence where pseudo-spectral methods are preferred numerical approach due their superior accuracy. Equations are solved for a time dependent velocity field, $U(x, t)$, in a turbulent flow simulation which varies significantly and irregularly both with position and time. So for homogeneous turbulence the mean velocity gradient $\partial \langle U_i \rangle / \partial x_j$ is non-zero, but uniform. The turbulent flow is said to be isotropic if rotation and buoyancy are of no importance. In a RANS turbulence model, like $k-\varepsilon$ equations are solved for mean quantities like $\langle U \rangle$, $\langle u_i u_j \rangle$. In Direct Numerical Simulation (DNS) all the length scales and time scales have to be resolved and Navier Stokes equation is solved to determine $U(x, t)$ for one realization of the flow. In DNS we can determine all the information of the flow but this approach is restricted to flow with low-to-moderate $Re$ since the computational cost increases as the cube of the $Re$ i.e. $Re^3$. Due to this high computational ex-
pense for high-$Re$ flows, large eddy simulation (LES) is used. In LES the large-scale motions are resolved/simulated whereas the effects of the smaller-scale motions are modeled by using models like subgrid- scale models (SGS). Different SGS models like Smagorinsky (dynamic and non-dynamic) and regularization (Leray-$\alpha$, Lans-$\alpha$ and Clark-$\alpha$) models are used in our case. The fluctuations in turbulence become anisotropic at low $Re_m$ in the presence of a sufficiently strong magnetic field. When the fluctuations are anisotropic then there is an elongation of flow structures along the lines of the magnetic field.

Here, the performance of the five different SGS models is presented for LES of MHD turbulence and comparisons are made to in-house DNS results at varying magnetic field strengths (different values of magnetic interaction parameter, $N$). Two different cases are considered here: First is a homogeneous turbulence case, and the second the Taylor-Green vortex case, which is perhaps the simplest system to study the generation of small scales and the resulting turbulence [2]. Energy spectra, vortex line stretching including the vortical structures are analyzed with varying values of $N$. In next section description of the computational methods, SGS models and key results are presented and discussed.
CHAPTER II

LITERATURE REVIEW

2.1 Numerical simulations of non-Darcy magnetohydrodynamic natural convection in a porous rectangular cavity

Darcy’s law [3] is an empirical law which is valid only for low-speed flows, specifically for Reynolds number \((Re) < 1\). It consists of a linear relationship between the velocity and the pressure gradient in the direction of flow. So the law begins to fail when inertial effects dominate, which leads to more non-linearity. Hence this law is not valid for many practical applications, where boundary and inertial effects are important. In addition, the Darcy law cannot account for the no-slip boundary condition at the interface of a porous medium and solid boundary, since the Darcy model is the simplest and is limited to either low porosity or low-speed flows. So its main limitation is that the no-slip boundary condition cannot be applied. Even though the use of Brinkman-Darcy model enables the use of no-slip boundary condition, this is not suitable for high porosity or high velocity media as presented previously [4]. So a more general model, Brinkman-Forchheimer extended Darcy model has to be considered.
The Brinkman extension of the Darcy relationship gets around this obstacle by adding a viscous-like term in the governing equations. Proposed by Brinkman \[5\], this term is proportional to the Laplacian of the fluid velocity in the purely Darcy-based governing equations. There has been some uncertainty in the validity of the Brinkman’s equation for large-porosity cases \[1\]. Durlofsky and Brady \[6\] concluded that the Brinkman equation was valid only for porosity \(\phi > 0.95\). Rubinsten \[7\] introduced a porous medium having a very large number of scales, and concluded that the Brinkman equation could be valid for \(\phi\) as small as 0.8. In our present study, we have used porosity of 0.8. Even though one could make the argument that the applications of such a high value of porosity is limited, since most of the naturally occurring phenomena has a porosity less than 0.8 \[8\], practically, there are a lot of high-porosity media that have increasingly become of interest \[9, 10, 11\]. In recent years, high porosity metal foams have gained attention for meeting the high thermal dissipation demands in electronic industry. Forchheimer \[12\] investigated fluid though the porous media at high velocity and observed that as the flow velocity increases, the inertial effects start dominating the flow. He suggested to include an inertial term in order to account for these inertial effects. In fact, to account for the porous inertia effect on the pressure drop, Muskat \[13\] added this term a velocity square term. After the inclusion of Forchheimers extension the inertial effects have also been studied by Poulakakos \[14\], and Georgiadis and Catoon \[15\], and both those studies found that these effects were important where the high- Darcy number cases. Other studies involving natural convection in a rectangular porous cavity include those by Nield and
Bejan [16], where the Forchheimer-extended Darcy equation was used to investigate the effect of the inertia parameter on the fluid flow and heat transfer. It was found that both the average Nusselt number and the strength of fluid circulation decreased when the inertial effects parameter was increased. Saeid and Pop [17] numerically investigated Non-Darcy (Darcy-Forchheimer) natural convection in a square cavity filled with a porous media and found that increasing the inertial effects parameter slowed down the process of natural convection in the cavity and for the constant value of Rayleigh number, the average Nusselt number was again reduced.

The studies of Raptis et al. [18, 19] were some of the first who considered the interaction of an external magnetic field with the convection currents in a porous media. Raptis [18] studied how the constant horizontal magnetic field influenced the free convection flow thorough the porous media in a semi infinite region bounded by two vertically-infinite surfaces. Later on they [19] extended their investigation bounded by two horizontal plates. Singh and Dikshit [20] investigated the effect of the natural convection of the Couette motion in a porous media by an electrically conducting fluid. Takhar and Ram [21] theoretically analyzed the effects of the hall current on hydrodynamic free-convection in a porous medium bounded by vertical plate. The effect of different inclination of magnetic field on hydro-magnetic natural convection through a porous media was investigated [22]. Nithiarasu et al. [23] analyzed the effects of heat transfer coefficient on the cold wall of the cavity and heat transfer in the porous medium. The study investigated the difference in Darcy and non-Darcy flow regime for different values of Darcy, Rayleigh and Biot number
at different aspect ratios. Grosan et al. [24] discussed how natural convection in a rectangular cavity filled with a porous medium is effected by magnetic field and internal heat generation.

2.2 Assessment of five subgrid-scale (SGS) models for low-$Re_m$ magnetohydrodynamic (MHD) turbulence

The flow of an electrically conducting fluid under the influence of a magnetic field is relevant to many technological and industrial applications, including steel production and processing [25], and the design of heat transfer devices for future nuclear fusion reactors. In these cases, the magnetic Reynolds number, $Re_m$, is usually very small, which allows considering the electromagnetic effect as an additional body force in the momentum equations of the flow. $Re_m$ is defined as,

$$Re_m = U_o L_o / \eta_m$$  \hspace{1cm} (2.1)  

where $\eta_m$ is the magnetic diffusivity, and $U_o$ and $L_o$ are the typical velocity and length scales, respectively. If $Re_m \geq 1$, there is a two-way coupling between fluctuations of the magnetic field and velocity. This happens for astrophysical problems and in geophysics where $Re_m \gg 1$ [26, 27]. The other extreme case of $Re_m \ll 1$ occurs in the majority of technological processes as explained above, where a strong steady magnetic field is imposed on an electrically conducting fluid. See some of the classic papers for experimental [28], theoretical [29] and numerical [30] studies related to low-$Re_m$ MHD. One of the main features of magnetohydrodynamic (MHD) turbulence at
low-$Re_m$ is that the turbulent fluctuations become anisotropic in the presence of a sufficiently strong magnetic field, which has important consequences for the properties of turbulence. The principal manifestation of anisotropy is mostly an elongation of the flow structures along the lines of the magnetic field [29, 31].

Direct numerical simulation (DNS) involves a full numerical solution of the Navier-Stokes (NS) equations, resolving all the scales of the motion. The approach of DNS was infeasible before 1970s until computers of sufficient power was unavailable [32]. DNS is computationally expensive and because the computational cost increases rapidly as the cube of the Reynolds number ($Re$) [32], this approach is inapplicable to high-$Re$ flows. Homogeneous MHD turbulence illustrates how an applied magnetic field increases the decay rate and introduces some anisotropy. For example in some of the MHD studies [33, 34, 35, 36], homogeneous turbulence calculations were performed to investigate the development of flow anisotropy. While a majority of studies [33, 35, 36] have concentrated on forced turbulence simulations, there have been some decaying turbulence investigations as well [34, 37, 38, 39, 40]. Out of the later set, studies by Agullo et al. [37], Knaepen and Moin [34], and Burattini et al. [40] focused mainly on LES and subgrid scale (SGS) models, while Kassinos et al. [39] investigated passive scalar mixing in MHD turbulence. In the study by Knaepen et al. [38], DNS was conducted for homogeneous MHD turbulence at moderate $Re_m$, thus solving the magnetic induction equation in addition to the incompressible NS.

Large eddy simulation (LES) attempts to overcome this limitation by resolving the transport equations for all scales of motion larger than the grid size $\Delta$.
(cutoff filter width), while the effects of the subgrid (smaller than $\Delta$) on the resolved field are parameterized using subgrid-scale (SGS) models. In case of LES applied to MHD turbulence, earlier work has shown the need to incorporate MHD effects on them. Shimomura [41] theoretically calculated the Reynolds stress of weakly conducting turbulent shear flow and found that the effect of magnetic field should be incorporated into the SGS model in the case of LES. Improvement of the dynamic Smagorinsky model over classical non-dynamic Smagorinsky model was shown by Knaepen and Moin [34]. More recently a detailed numerical investigation of MHD turbulence at magnetic interaction parameter, $N$ ranging from 0 to 50 was conducted by [40] and found that the flow evolution was rapidly dominated by nonlinearity and hence was unable to be described by any type of linear theory.

In the last twenty years, various of SGS models have been developed [42]. Firstly, if implicit LES [43] is considered, no SGS model is implemented and the numerical effects of the discretization are assumed to mimic the physics of unresolved turbulent motion and molecular diffusion effects. The second kind is the most widely used eddy viscosity-type model (e.g. the Smagorinsky model) [32, 44, 45], which is needed for the anisotropic residual stress tensor to close the equations for the filtered velocity. The third approach, which is rather novel in modeling turbulent flow, employs regularization modeling as a SGS model [46, 47, 48]. These models are surrogates to the Navier-Stokes equation and have better properties from a pure mathematical point of view. Through a specific proposal for direct alteration of the convective terms (nonlinear), a regularization model expresses the smoothing of
the dynamics of NS equations [47]. This model differs from traditional, less direct approaches like viscosity models [44] and gives rise to a basic mixed formulation involving both the filtered and unfiltered solution [47].

In this study we have considered three regularization models like Leray-\(\alpha\), LANS-\(\alpha\) and Clark-\(\alpha\), where the filtered convective terms in the NS equations are directly modified so we can retain much of the mathematical properties of the filtered equations [49]. In the Leray-\(\alpha\) model, the advective operator of NS equation \((u.\nabla)u\) is replaced by \((v.\nabla)u\), where filtered velocity \((v) = H^{-1}(u)\) and \(H\) is the Helmholtz filter. It has been found that the Leray model is closest to the filtered DNS results in general, while the modified Leray model is found least accurate [49]. Reeuwijk et al. [50] compared the Leray-\(\alpha\) simulations from well resolved and coarse DNS and found that the Leray-\(\alpha\) model was not convincingly superior to coarse DNS for the flow types under consideration. Along with this, additional (dissipative) modeling seemed unavoidable to capture accurately all effects from the turbulence.

An alternative systematic method for modeling the mean circulatory effects of small scale turbulence under consideration is provided by Lagrangian averaged Navier-Stokes-\(\alpha\) (LANS-\(\alpha\)) equations (also referred to as the Casmassa-Holm equation) [51, 52, 53]. Different comparison for LANS-\(\alpha\) have been made to DNS of the Navier-Stokes equations at modest Taylor Reynolds number for decaying \((Re_\lambda = 130\ [53], Re_\lambda = 220\ [54], Re_\lambda = 300\ [46])\) forced turbulence \((Re_\lambda = 80\ \text{and} 115, [54])\). LANS-\(\alpha\) demonstrated a correct alignment between the vorticity vector, eigenvectors of the subgrid stress tensor and resolved stress tensor [55]. Comparison between Leray
and LANS-\(\alpha\) was made by [48] and found that LANS-\(\alpha\) model provides solution with more realistic variability that corresponds better to the filtered DNS results. It has also been shown that energy accumulates in the sub-filter scales for high-resolution simulation of LANS-\(\alpha\), and as a result giving only a modest computational gain at very high Reynolds number [56].

Finally, a nonlinear LES model of turbulent flows with explicit filtering as filtered Clark (Clark-\(\alpha\)) model [57] is considered. The model consists of “tensor-diffusivity model” of Leonard [58], filtered by inversion of the Helmholtz operator with width \(\alpha\). Vreman et al. [59, 60] used this explicitly filtered tensor diffusivity model combination with a dynamic Smagorinsky term as a nonlinear mixed model. The existence and uniqueness of the Clark-\(\alpha\) model solution were well demonstrated in [61]. Recently, Graham et al. [55] compared the three regularization models (Leray-\(\alpha\), LANS-\(\alpha\) and Clark-\(\alpha\)) and DNS in the context of a high Re (\(Re_\lambda = 790\)) Taylor-Green-type forced isotropic turbulence. They found that at scales larger \(\alpha\), Clark-\(\alpha\) was the best approximation regarding reproducing the total dissipation rate and the energy spectrum. LANS-\(\alpha\), which doesn’t vary much with \(\alpha\) may be consider a superior model in regard to intermittency but for Clark-\(\alpha\) intermittency may be function of filter width \(\alpha\) [55].
CHAPTER III

SCOPE AND STRUCTURE

3.1 Scope of work

Numerical investigation of the effects of a horizontal magnetic field on the natural convection in a 2-D cavity filled with porous media using non-Darcy or Brinkman-Forchheimer extended Darcy law model including convective inertial terms for different aspects ratio was conducted. An implicit Euler method is used to solve the pseudo-time derivatives in the governing equations where as all spatial derivatives are discretized using 2nd-order central difference method. In the numerical method employed here, the stream-function, vorticity and energy equations are solved separately by using in-house Fortran code. All the three equations are advanced into their next level by solving three tridiagonal systems. In case of homogeneous turbulence and Taylor-Green vortex, pseudo-spectral methods are used because of their superior accuracy. The computer codes written in Fortran 90 employed the P3DFFT which can be freely available from the San Diego Supercomputing center. The third-order Runge-Kutta time-stepping algorithm was used for temporal integration of the governing equations in spectral space. The performance of different subgrid-scale (SGS) models for LES of Magnetohydrodynamic (MHD) turbulence was presented
and compared at varying magnetic field with in-house DNS results [62] for homogeneous MHD turbulence and for the Taylor-Green Vortex (TGV) case [2, 63]. Vortex line stretching, including the vorticity structures were presented with different values of magnetic interaction parameters ($N$).

3.2 Thesis structure

The thesis structure consists of two topics for different chapters. In chapter 1, basic introduction of the topic is explained. Chapter 2, introduced the literature part of the thesis and its scope. Different terms like buoyancy, the Ohm’s law, the law of conservation of electrical charge, the Gauss’s law for magnetism and the Lorentz force are defined in chapter 3. This chapter includes the Darcy law, the Brinkman and the Forchheimer terms and governing equations like continuity, momentum and energy equation for the topic numerical simulations of non-Darcy magnetohydrodynamic natural convection in a porous rectangular cavity. The next section of the chapter includes the Fourier pseudo-spectral methods for Navier-Stokes equation and different subgrid-scale models used for five subgrid-scale (SGS) models for low-$Re_m$ magnetohydrodynamic (MHD) turbulence. The descriptions of the problem that are carried out in the thesis are discussed in chapter 4. The numerical methods and the grid size used for the simulation of the magnetohydrodynamic flow are presented in chapter 5. All the discussion about the results related to numerical simulations of non-Darcy magnetohydrodynamic natural convection in a porous rectangular cavity and assessment of five subgrid-scale (SGS) models for low $Re_m$ magnetohydrody-
namic (MHD) turbulence are described in chapter 6 followed by concluding remarks in chapter 6. Bibliography includes the details of the citation presented in the thesis.
CHAPTER IV
FORMULATION

In this study the Lorentz force and the buoyancy forces are the only external forces considered. Different laws and their extension for the porous media are presented here.

4.1 Numerical simulations of non-Darcy magnetohydrodynamic natural convection in a porous rectangular cavity

When the speed of the fluid is much below the speed of sound, the fluid can be treated as incompressible. Here in this study, it is assumed that the fluid is incompressible and the flow of fluid is governed by the N-S equations. Let \( x \) and \( y \) be denoted as horizontal and vertical spacial directions, and \( u \) and \( v \) are velocities of fluid in \( x \) and \( y \) directions. If \( t \) represents the time, \( p \) the pressure and \( \rho \), the density of fluid, the 2D N-S equations are given by,

4.1.1 Buoyancy

Buoyancy force is generated in fluid when part of the fluid is heated or cooled by a surface. This causes motion in the fluid as the warm fluid rises and the cool fluid is
then moved to the heated surface. The Buoyancy force is mentioned as,

\[ \mathbf{F_B} = g \beta (\theta - \theta_0) \]  \quad (4.1)

where, \( g \) is the acceleration due to gravity, \( \theta \) is the reference temperature, \( \theta_0 \) is the wall temperature and \( \beta \) is the coefficient of thermal expansion given by,

\[ \beta = \frac{-1}{\rho} \frac{dp}{d\theta} \Delta \theta \]  \quad (4.2)

where, \( \rho \) is the density, \( \frac{dp}{d\theta} \) is the change in pressure (\( p \)) with respect to temperature (\( \theta \)).

### 4.1.2 Electromagnetic field

The governing equations for the electromagnetic fields can be written as,

the Ohms’s law,

\[ \mathbf{J} = \sigma (-\nabla \varphi + \mathbf{u} \times \mathbf{B}) \]

the law of conservation of electric charge,

\[ \nabla \cdot \mathbf{J} = 0 \]

the Gauss’s law for magnetism,

\[ \nabla \cdot \mathbf{B} = 0 \]

and the Lorentz force

\[ \mathbf{F} = \frac{1}{\rho} \mathbf{J} \times \mathbf{B} \]

where \( \mathbf{J} \) is the electric current density, \( \sigma \) is the electric conductivity, \( \varphi \) is the electrostatic potential, \( \rho \) is the density of the fluid, and \( \mathbf{B} \) is the total (applied + induced)
magnetic field. In the flow where the $Re_m$ is small, the flow is affected by the magnetic field, but the effect of the flow on the magnetic field is negligible. As the $Re_m$ is less than unity, the induced magnetic field can be neglected which is due to induced electric current. The induced electric current interacts with the applied magnetic field and produces a force called a Lorentz force, $(\mathbf{J} \times \mathbf{B})/\rho$. This Lorentz force which is in opposite direction of flow is perpendicular to the electric and magnetic field. The magnetic field is applied in the perpendicular direction to the isothermal walls, since a horizontal magnetic field is more effective compared to a vertical magnetic field in suppressing the fluid [64]. The Lorentz force retards the motion of the flow which is a function of $\mathbf{B}$ and $\mathbf{u}$.

4.1.3 Darcy’s law and its extension

Darcy’s law states that the flow rate (velocity) is proportional to the pressure gradient. Darcy’s law is an empirical law that is known to hold only at low velocity (for the Reynolds number less than one). As the Reynolds number is as dimensional measure of the relative strength of inertial to the viscous forces. Darcy law starts to fail when inertial effects are more important, even though the flow is laminar. It is not turbulence, but inertia and that leads to non linearity. Many authors on convection in porous media used an extension of the Darcy equation, $\nabla P = -\frac{\mu}{K} \mathbf{v}$ in the form,

$$
\rho \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla P - \frac{\mu}{K} \mathbf{v} \quad (4.3)
$$

When, the Dupuit-Forchheimer relationship is used, the above equation becomes

$$
\rho \left[ \phi^{-1} \frac{\partial \mathbf{v}}{\partial t} + \phi^{-2} (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla P - \frac{\mu}{K} \mathbf{v} \quad (4.4)
$$
The Dupuit-Forchheimer relationship relates the above term by,

\[ \mathbf{v} = \phi \mathbf{V}, \]

where \( \phi \) is the porosity, \( \rho \) is the density of a fluid, \( \mathbf{v} \) is the fluid velocity vector (seepage velocity or filtration velocity) over \( V_m \) and \( \mathbf{V} \) is the fluid velocity vector (intrinsic average velocity) over a volume \( V_f \). \( V_m \) is the volume of the medium (incorporating both solid and fluid) and \( V_f \) is the volume of fluid only.

4.1.4 Brinkman

An alternative way to present Darcy’s equation, commonly known as Brinkman’s equation, with inertial terms omitted is given by,

\[ \nabla P = -\frac{\mu}{K} \mathbf{v} + \mu_{eff} \nabla^2 \mathbf{v} \]

Here, we have two viscous terms. The first is the Darcy term and the second is analogous to the Laplacian terms that appears in the Navier-Stokes equation. The coefficient \( \mu_{eff} \) is an effective viscosity. In equation (4.5) \( P \) is the intrinsic fluid pressure, so that each term in the equation represents a force per unit volume. For an isotropic porous medium,

\[ \frac{\mu_{eff}}{\mu} = \frac{1}{\phi} T^* \]  

(4.6)

where, \( T^* \) is a quantity called the tortuosity of the medium [65]. The concept of the tortuosity is used to characterized the structure of porous media. In our study we have neglected the effect of tortuosity. Several recent authors have added a Laplacian term to the Forchheimer equation to form a ”Forchheimer-Brinkman” equation. The validity is not completely clear. In order for Brinkman’s equation to be valid
the porosity must be large, and there is some uncertainty about the validity of the Forchheimer law at such large porosity.

4.1.5 Forchheimer

The Darcy’s equation is linear in the seepage velocity \( v \). As Darcy’s law hold only at low velocity means that Reynolds numbers based on a typical pore diameter is of order unity or smaller. As \( v \) increases the transition to nonlinear drag is quite smooth. This transition is not one from laminar to to turbulent flow since at such comparatively small Reynolds numbers the flow in the pores is still laminar. So, the appropriate modification to Darcy’s equation is to replace by,

\[
\nabla P = -\frac{\mu}{K} v - C\rho |v| v
\]

(4.7)

The form coefficient \( C \) can be estimated by using the model of Ward [66], \( C = \frac{0.55}{\sqrt{K}} \). Although the Ward model is not universal, they are the simplest and, therefore the most general models for estimating form coefficient \( C \) of the porous medium.

4.1.6 Brinkman-Forchheimer extended Darcy

The equation corresponding to porous medium is modeled according to the Brinkman-Forchheimer extended Darcy model including the convective inertial term. The equations governing the conservation of mass, momentum and energy are described in different subsections presented below.
4.1.6.1 Continuity

\[ \nabla \cdot \mathbf{v} = 0 \quad (4.8) \]
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4.9) \]

The vorticity defined as the curl of the velocity field \( (\omega = \nabla \times \mathbf{v}) \) and the stream function is given by,
\[ \omega = \omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad \text{and} \quad u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \]

The variables that are used to non-dimensionalize the continuity equation are,
\[ X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \Psi = \frac{\psi}{\alpha}, \quad \Omega = \frac{\omega L^2}{\alpha} \quad (4.10) \]

where, \( L \) is the length scale, \( \Psi \) and \( \Omega \) are dimensionless stream function and vorticity respectively. Therefore the non dimensional form of the continuity equation is given by,
\[ \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -\Omega \quad (4.11) \]

4.1.6.2 Momentum Equation

The momentum equation for the Brinkman-Forchheimer extended Darcy equation is written as,
\[ (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nabla p \phi^2}{\rho} + \nu \phi \nabla^2 \mathbf{v} - \frac{\nu \phi^2}{K} \mathbf{v} - C \phi^2 |\mathbf{v}| \mathbf{v} + \mathbf{F_B} + \mathbf{F} \quad (4.12) \]

\(|\mathbf{v}|\) is an absolute velocity of the velocity vector \( \mathbf{v} \), given by \( \sqrt{u^2 + v^2} \). \( \mathbf{F_B} \) and \( \mathbf{F} \) are buoyancy force and the Lorentz force respectively. The variables that we used to
non-dimensionalize the momentum equation are,

\[ X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \Psi = \frac{\psi}{\alpha}, \quad \Omega = \frac{\omega L^2}{\alpha}, \quad Pr = \frac{\nu}{\alpha}, \quad Da = \frac{K}{L^2}, \quad T = \frac{(\theta - \theta_c)}{(\theta_h - \theta_c)} \]  

(4.13)

where, \( Pr \) is the Prandtl number defined as the ratio of kinematic viscosity \( \nu \) to thermal diffusivity \( \alpha \). \( Da \) is the Darcy number, where \( K \) is the permeability of the medium. Here, \( T \) is the dimensionless temperature, \( \theta \) is the dimensional temperature, \( \theta_h \) and \( \theta_c \) are the temperatures of hot wall and cold wall respectively. The Rayleigh number \( (Ra) \) is the product of Grashoff number \( (Gr) \) and \( Pr \). The dimensionless form of the momentum equation in terms of stream function and vorticity as mentioned in continuity equation is,

\[ \frac{\partial \Psi}{\partial Y} \frac{\partial \Omega}{\partial X} - \frac{\partial \Psi}{\partial X} \frac{\partial \Omega}{\partial Y} = Pr \phi \left( \frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} \right) + \frac{Pr \phi^2}{Da} \left( \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} \right) - \Omega \frac{0.55 \phi^2}{\sqrt{Da}} \sqrt{\Psi_Y^2 + \Psi_X^2} \]

\[ + Ra Pr \phi^2 \frac{\partial T}{\partial X} + Ha^2 Pr \phi^2 \left( B^2 \frac{\partial^2 \Psi}{\partial X^2} \right) \]  

(4.14)

where, \( Ha \) is the Hartmann number, a dimensionless parameter, defines as the ratio of electromagnetic force to the viscous force is given by,

\[ Ha = B L \sqrt{\frac{\sigma}{\mu}} \]  

(4.15)

Here, \( B \) is the magnetic field, \( \sigma \) is the electrical conductivity and \( \mu \) is the dynamic viscosity. The parameters like Rayleigh-Darcy number \( (Ra_D = Ra \ Da) \), and effective Hartmann number \( (Ha_e = Ha^2 \ Da) \) are studied to explore the effects of the magnetic field in natural convection filled with a porous media. The temperature effect on various properties like \( K \), \( \sigma \) and \( \mu \) is not considered in our study.
4.1.6.3 Energy Equation

The steady state 2D energy equation is,

\[(\mathbf{v} \cdot \nabla T) = K_{eff} \nabla^2 T + \frac{J^2}{\sigma}\]  \hspace{1cm} (4.16)

It is assumed here that the thermal conductivity of the solid and liquid are equal, which leads effective conductivity ratio \(K_{eff}\) to be unity and is defined as, \(K_{eff} = \frac{k_s^{1 - \phi} k_f^\phi}{k_f}\), where subscripts \(s\) and \(f\) are used for solid and fluid respectively. The variables that are used to non-dimensionalize the equations are,

\[X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \Psi = \frac{\psi}{\alpha}\]  \hspace{1cm} (4.17)

The dimensionless form of the energy equation is,

\[\frac{\partial\Psi}{\partial Y} \frac{\partial T}{\partial X} - \frac{\partial\Psi}{\partial X} \frac{\partial T}{\partial Y} = K_{eff} \left( \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right) - J_u \left( \frac{\partial \psi}{\partial y} \right)^2\]  \hspace{1cm} (4.18)

where, \(J_u\) is the Joule heating parameter, which is the thermal energy that raises due to the reduction of kinetic energy by the Lorentz force and is given by,

\[J_u = (\gamma - 1) N M_a^2\]  \hspace{1cm} (4.19)

The heat specific ratio \((\gamma)\) is the ratio of the heat capacity at constant pressure \((C_p)\) to heat capacity at constant volume \((C_v)\), suffix \(p\) and \(v\) refer to constant pressure and constant volume conditions respectively. Mach number \((M_a)\) represents the ratio of speed of an object moving through a fluid to the speed of sound.
4.2 Assessment of five subgrid-scale (SGS) models for low-$Re_m$ magnetohydrodynamic (MHD) turbulence

The Fourier pseudo-spectral method is employed to integrate the incompressible NS equations in a cubic box of side $L$ for periodic boundary conditions:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

$$\nabla \cdot \mathbf{u} = 0$$

(4.20)

where $\mathbf{u}$ is the velocity field, $p$ is pressure, $t$ is the time, and $\nu$ is the kinematic viscosity. The Lorentz force, $\mathbf{f}$ is governed by the following equations:

$$\mathbf{f} = \frac{1}{\rho} \mathbf{J} \times \mathbf{B}$$

(4.21)

$$\mathbf{J} = \sigma (-\nabla \phi + \mathbf{u} \times \mathbf{B})$$

(4.22)

$$\nabla \cdot \mathbf{J} = 0$$

(4.23)

$$\nabla \cdot \mathbf{B} = 0$$

(4.24)

where $\mathbf{B}$ is the total (applied + induced) magnetic field, $\mathbf{J}$ is the electric current density, $\sigma$ is the electrical conductivity, and $\phi$ is the electrostatic potential. Equation 4.22 is the Ohm’s law. At low-$Re_m$, the flow is affected by the magnetic field, but the effect of the flow on the magnetic field is negligible. As a consequence, the Lorentz force term, $\mathbf{J} \times \mathbf{B}/\rho$ in the above equation is a function of only $\mathbf{B}$ and $\mathbf{u}$, where $\rho$ is the density of the fluid. We also assume that the magnetic field is directed along the $z$ direction, $\mathbf{B} = B_0 \hat{k}$. Therefore, the Lorentz force term reduces to a function, that
is linearly dependent on velocity \([29, 30, 35]\) and is given by

\[
f = -\frac{\sigma B_o^2}{\rho} \Delta^{-1} \frac{\partial^2 u}{\partial z^2}
\]  

(4.25)

For homogeneous turbulent flows, pseudo-spectral methods (pioneered by \([67, 68]\)) are the preferred numerical approach, because of their superior accuracy and the computational efficiency associated with the use of the Fast Fourier Transform (FFT). Hence the velocity field \(u(x, t)\) is approximated as a finite Fourier series

\[
u(x, t) = \sum_k e^{ik \cdot x} \hat{u}(k, t),
\]

(4.26)

where \((\cdot)\) represents a variable in spectral or FFT space, and \(k\) is the wavenumber vector. The incompressible NS equations in wavenumber space under the quasistatic approximation of the MHD equations can then be derived to be

\[
\left( \frac{d}{dt} + \nu k^2 + \frac{\sigma B_o^2}{\rho} \left( \frac{k_z}{k} \right)^2 \right) \hat{u}_j(k, t) = -ik_l \left( \delta_{jk} - \frac{k_jk_k}{k^2} \right) \sum_{k'} \hat{u}_k(k', t) \hat{u}_l(k - k', t)
\]

(4.27)

where \(\sigma\) is the resistivity, \(k\) is the wavenumber vector, \(i\) is the imaginary unit, and \(k \equiv |k|\). The nonlinear terms on the right side of the equation are solved using pseudo-spectral methods to avoid the large costs associated with solving triad interactions in spectral space.

See [69] for details of the derivation related to the quasistatic derivation. The validity of this approximation relies on the fact that the magnetic field induced by the fluid motion is negligible compared to that applied. From the above equation, it can be seen that under the quasistatic approximation, the electromagnetic force
represents an anisotropic term in the equation of motion. The non-dimensionalization of the above equation results in a non-dimensional parameter, namely the interaction parameter, \( N \), given by

\[
N = \frac{\sigma B_o^2 L_o}{\rho U_o}
\] (4.28)

On dimensional grounds, \( N \) measures the relative strengths of the magnetic damping term and the nonlinear term in Eqn. 4.27. \( N \) can also be considered as a measure of the ability of an imposed magnetic field to force the turbulence to a two-dimensional state. So the energy becomes increasingly concentrated in modes independent of the coordinate direction aligned with \( B \).

4.2.1 SGS models

The SGS models used are the non-dynamic Smagorinsky, dynamic Smagorinsky and regularization models. LES calculation using the various SGS models are compared to DNS.

4.2.1.1 Non-dynamic Smagorinsky model

Classical Smagorinsky model [44] assumes quasi-equilibrium between large and small scales. The different SGS models are related to the specific filter of width \( \Delta \). The filtered Navier-Stokes equation yields,

\[
\partial_t \bar{u}_i = 0 \quad \text{(4.29)}
\]

\[
\partial_t \bar{u}_i + \bar{u}_j \partial_j \bar{u}_i = -\partial_i \bar{p} + \nu \partial_{jj} \bar{u}_i - \nabla \cdot \tau_{ij,R} + f_i \quad \text{(4.30)}
\]
where $\langle \cdot \rangle$ is the LES-filtered quantity and the residual stress is

$$\tau_{ij,R} = -2\nu_T \tilde{S}_{ij} = -2(C_s \nabla)^2 |\tilde{S}| \tilde{S}_{ij}$$  \hspace{1cm} (4.31)$$

$C_s$ is the Smagorinsky model coefficient, which is determined independent of the specific filter. For isotropic turbulence in the inertial range, $C_s$ is fixed at its classical value, $C_s = 0.16$ [32]. $\tilde{S}_{ij} = \frac{1}{2}(\partial \tilde{u}_i / \partial x_j + \partial \tilde{u}_j / \partial x_i)$ is the resolved strain-rate tensor and $\tilde{S} = (2\tilde{S}_{mn}\tilde{S}_{mn})^{1/2}$ its modulus. This model allows only forward energy transfer from the resolved scales to the subgrid scales.

### 4.2.1.2 Dynamic Smagorinsky

In the dynamic Smagorinsky, the filtered Navier-Stokes are written in the same manner as presented above for the non-dynamic Smagorinsky, except now the model coefficient, $C_s$, for calculating the residual stress, is calculated as,

$$C_s^2 = \frac{\langle L_{ij} M_{ij} \rangle}{\langle M_{ij} M_{ij} \rangle}$$  \hspace{1cm} (4.32)$$

From the Germano identity [70]

$$L_{ij} \equiv T_{ij} - \tau_{ij} = \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j$$  \hspace{1cm} (4.33)$$

where, $\langle \cdot \rangle$ is a test-filtered quantity filtered using a filter width of $2\Delta$, $T_{ij} = \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j$ is the SGS tensor [71].

$$M_{ij} \equiv -2\Delta^2 \left[ 4 |\tilde{S}| \tilde{S}_{ij} - |\tilde{S}| \tilde{S}_{ij} \right]$$  \hspace{1cm} (4.34)$$
4.2.1.3 Regularization models

The different regularization models considered here are, Leray-\(\alpha\), LANS-\(\alpha\) and Clark-\(\alpha\). The governing equations in the Leray formulation can be written as,

\[
\partial_t u_i = 0, \quad \partial_t v_i = 0 \tag{4.35}
\]

\[
\partial_t u_i + u_j \partial_j, v_i = -\partial_i p + \nu \partial_{jj} u_i + f_i \tag{4.36}
\]

where the flow is advected by a smoothed (filtered) velocity \(v\).

\[
v = H^{-1} u = (1 - \alpha^2 \nabla)^{-1} u. \tag{4.37}
\]

\(H\) is the Helmholtz filter and \(\alpha\) is the Helmholtz length, which defines the effective width of the filter. \(H\) is the standard filter used for all the regularization models considered here. Similarly, for the LANS-\(\alpha\) model and the Clark-\(\alpha\) models, Equation 4.35 stays the same, but Equation 4.36 is replaced by,
\[ \partial_t u_i + u_j \partial_j v_i + v_j \partial_j u_i = -\partial_i p + \nu \partial_{jj} u_i + f_i \]  

(4.38)

and

\[ \partial_t u_i + \partial_j (v_j u_i + u_j v_i - v_j v_i - \alpha^2 \partial_l u_i \partial_l u_j) = -\partial_i p + \nu \partial_{jj} u_i + f_i, \]  

(4.39)

respectively.
5.1 Numerical simulations of non-Darcy magnetohydrodynamic natural convection in a porous rectangular cavity

A rectangular cavity filled with an electrically conducting fluid in a porous medium is considered here and a schematic is shown in Figure 5.1. The flow is assumed to be steady, laminar, incompressible and Newtonian. The top and bottom walls of length \( L \) are insulated, whereas left and right walls of height \( H \) are maintained at constant and uniform temperatures \( \theta_h \) and \( \theta_c \), respectively with no slip boundary conditions. A uniform and constant magnetic field is applied in the horizontal direction. The porous medium is assumed to be isotropic and homogeneous. Both viscous dissipation and Joule dissipation are neglected due to the incompressibility assumption. It is also assumed that the effective viscosity is equal to the fluid viscosity divided by porosity \([1]\). The vorticity and stream function boundary conditions are as follows:

\[
X = 0, \Psi_0 = 0, \Omega_0 = \frac{-2\Psi_1}{\Delta h^2} \quad (5.1)
\]

\[
X = 1, \Psi_0 = 0, \Omega_0 = \frac{-2\Psi_1}{\Delta h^2} \quad (5.2)
\]

\[
Y = 0, \Psi_0 = 0, \Omega_0 = \frac{-2\Psi_1}{\Delta h^2} \quad (5.3)
\]

\[
Y = 1, \Psi_0 = 0, \Omega_0 = \frac{-2\Psi_1}{\Delta h^2} \quad (5.4)
\]
Figure 5.1: Schematic of the rectangular enclosure.

The average Nusselt number ($Nu$) across the enclosure is obtained by numerical integration from,

$$Nu = \frac{\bar{h} L}{K} = \frac{1}{AR} \int_0^{AR} \frac{\partial T}{\partial X} \bigg|_{X=0} dY \quad (5.5)$$

where, $AR$ is the aspect ratio of the domain given by, $AR = H/L$, $\bar{h}$ is the convective heat transfer coefficient of the fluid and $K$ is the thermal conductivity of the fluid.

5.2 Assessment of five subgrid-scale (SGS) models for low-$Re_m$ magnetohydrodynamic (MHD) turbulence

The computational domain is a cube of length $L$. The numerical grid size is given by $h = L/N_f$, where, $L = 2\pi$ (m) and $N_f$ is the number of the Fourier modes. The medium is assumed to be homogeneous and isotropic. The boundary condition is
triple periodic and the Fourier pseudo-spectral method is employed to integrate the incompressible Navier-Stokes equations.
CHAPTER VI
COMPUTATIONAL DETAILS

6.1 Numerical simulations of non-Darcy magnetohydrodynamic natural convection in a porous rectangular cavity

A finite difference method used to solve the dimensionless steady state governing equations with boundary conditions as mentioned above. An approximate solution of the equations is obtained at finite number of grid points having \( x = i\Delta x, \ y = j\Delta y, \) where \( i \) and \( j \) are integers. All spatial derivatives are discretized using a 2\textsuperscript{nd}-order central difference method and an implicit Euler method is used to solve the pseudo-time derivatives in the governing equations. The stream function, vorticity, and the energy equation are solved separately by the tridiagonal-matrix algorithm in a line-by-line fashion by using an in-house FORTRAN code. We can see the details about the method in the paper by [72]. The results presented here are for a uniform mesh spacing of grids 64 \( \times \) 64 in cavity for aspect ratios of 4:1, 6:1 and 8:1. For the iterative convergence, the criteria used were,

\[
\max | f_{i,j}^{new} - f_{i,j}^{old} | < r_f,
\] (6.1)
where, \( f_{i,j} \) is for \( \Psi, \Omega \) and \( T \). The value of \( r_f \) is chosen as \( 10^{-6} \). The final mathematical formulation is represented as,

\[
(1 - \Delta t \frac{\partial^2}{\partial X^2}) (1 - \Delta t \frac{\partial^2}{\partial Y^2}) \Psi^{n+1} = \Psi^n + \Delta t \Omega^n + \left( \Delta t \frac{\partial^2}{\partial X^2} \right) \left( \Delta t \frac{\partial^2}{\partial Y^2} \right) \Psi^n \tag{6.2}
\]

\[
\Omega^{n+1} \left( 1 - \Delta t \frac{\partial^2}{\partial X^2} + \Delta t D a \frac{\partial^2}{\partial Y^2} \right) \left( 1 - \Delta t \frac{\partial^2}{\partial X^2} - \Delta t D a \frac{\partial^2}{\partial Y^2} \right) \left( 1 - \Delta t \frac{\partial^2}{\partial X^2} + \Delta t D a \frac{\partial^2}{\partial Y^2} \right) \left( 1 - \Delta t \frac{\partial^2}{\partial X^2} - \Delta t D a \frac{\partial^2}{\partial Y^2} \right) \Omega^n
\]

\[
= \Omega^n + \left( \Delta t \frac{\partial^2}{\partial X^2} + \Delta t D a \frac{\partial^2}{\partial Y^2} \right) \left( \Delta t \frac{\partial^2}{\partial X^2} + \Delta t D a \frac{\partial^2}{\partial Y^2} \right) \left( \Delta t \frac{\partial^2}{\partial X^2} + \Delta t D a \frac{\partial^2}{\partial Y^2} \right) \left( \Delta t \frac{\partial^2}{\partial X^2} + \Delta t D a \frac{\partial^2}{\partial Y^2} \right) \Omega^n
\]

\[
- \Delta t \Omega^2 \left( Pr + 0.55 \sqrt{Da} \left( \frac{\partial \Psi^2}{\partial Y} + \frac{\partial \Psi^2}{\partial X} \right) + \Delta t Ra Pr Da \phi^2 \frac{\partial T^n}{\partial X} 
\]

\[
+ \Delta t H a^2 Da \phi^2 Pr B_X^2 \frac{\partial^2 \Psi^n}{\partial X^2} \right) \tag{6.3}
\]

\[
\left( 1 - \Delta t K_{eff} \frac{\partial^2}{\partial X^2} + \Delta t \frac{\partial \Psi^n}{\partial Y} \frac{\partial}{\partial X} \right) \left( 1 - \Delta t K_{eff} \frac{\partial^2}{\partial Y^2} - \Delta t \frac{\partial \Psi^n}{\partial Y} \frac{\partial}{\partial X} \right) T^{n+1}
\]

\[
= T^n + \left( \Delta t K_{eff} \frac{\partial^2}{\partial X^2} + \Delta t \frac{\partial \Psi^n}{\partial Y} \frac{\partial}{\partial X} \right) \left( \Delta t K_{eff} \frac{\partial^2}{\partial Y^2} + \Delta t \frac{\partial \Psi^n}{\partial Y} \frac{\partial}{\partial X} \right) T^n
\]

\[
- J_u \left( \frac{\partial \Psi^n}{\partial Y} \right)^2 \tag{6.4}
\]

The flow is simulated by solving the continuity, momentum and energy equation as represented by equations 6.2, 6.3 and 6.4 respectively.

6.2 Assessment of five subgrid-scale (SGS) models for low-\( Re_m \) magnetohydrodynamic (MHD) turbulence

A computer code has been written to implement the Fourier pseudo-spectral method described above. The code is written in Fortran90 and employs the P3DFFT parallel three-dimensional fast Fourier transform library which makes use of 2D, or pencil, decomposition to overcome an important scalability limitations associated with other
3D parallel FFT libraries using 1D or slab decomposition. See the authors’ recent publications [73, 74] for more details. Computing time was provided by the Purdue University Supercomputing.

Dealiasing is achieved using the 3/2-rule [68]. So the grid resolutions mentioned in this paper are all values after dealiasing. A third-order Runge-Kutta time stepping algorithm [75] is used for temporal integration of the governing equations in spectral space. The numerical grid size is $h = L/N_f$ where $N_f$ is the number of Fourier modes. $L$, the side of the cubic computational domain is $2\pi \text{ m}$. The time step is calculated as $\Delta t = c.f.l. h/\sqrt{k_0}$, where $k_0$ is the initial turbulent kinetic energy and $c.f.l.$ is chosen to be 0.03 [32] in all the cases presented and the time step size is set to be a constant throughout the simulation. For all the cases presented here, the units of wavenumber and time are $\text{m}^{-1}$ and $\text{s}$, respectively and total simulation time was 10.0 s.
CHAPTER VII

RESULTS AND DISCUSSION

7.1 Numerical simulations of non-Darcy magnetohydrodynamic natural convection in a porous rectangular cavity

A numerical study of a freely convecting electrically conducting fluid in cavity filled with porous media in the presence of magnetic field is conducted here. The non-dimensional parameters that are of consideration are the Rayleigh number \( Ra \), the Darcy number \( Da \), the effective Hartmann number \( Ha_e \), and the Rayleigh-Darcy number \( Ra_D \). In the present study, computations are performed with a porosity of 0.8 and for the following parameter ranges: \( 5 \times 10^4 \leq Ra \leq 10^6 \), \( Da = 0.01 \), \( 0 \leq Ha_e \leq 10 \) with aspect ratios of 4:1, 6:1 and 8:1. The influence of the magnetic field, buoyancy and aspect ratio on the fluid flow and heat transfer characteristics is investigated here. Effects of magnetic field on the average Nusselt number are also discussed. In Table 7.1, the influence of magnetic field with varying \( Ra_D \) for aspect ratio of 4:1 can be seen. The flow field comprises of higher velocity in the absence of a magnetic field in comparison with the applied magnetic field. The maximum intensity of stream function is \( \psi_{max} = 13.575 \) for \( Ha_e = 0 \), and it decreases with increasing \( Ha_e \). For \( Ha_e = 10 \), maximum intensity of circulation is \( \psi_{max} = 0.598 \).
So, we can see that magnetic field reduces the velocities as its value is increased due to the magnetic damping created by the Lorentz force. As presented in Table 7.1, 7.2 and 7.3, stronger magnetic fields are required to cause the same kind of effects those observed in low $Ra_D$ case, because higher $Ra_D$ increases circulation due to the increased buoyancy forces and thereby increases velocities. In addition, with increase in aspect ratio the highest velocities increase for a fixed $Ra_D$ and $Ha_e$ because of a compression in circulation area and the effect increases with higher $Ra_D$ and lower $Ha_e$. For instance, from Tables 7.1, 7.2 and 7.3, for $Ha_e = 10$, $AR = 4 : 1$ and $Ra_D = 500$, $\psi_{max} = 0.598$, and $\psi_{max} = 0.596$ for aspect ratio of 6:1 and 8:1. For higher values of $Ra_D = 10000$, and for the same given condition, $\psi_{max}$ values are 10.328, 11.161 and 11.527 for aspect ratio of 4:1, 6:1 and 8:1 respectively.

Figures 7.3-7.11 show the stream function and temperature contours for the various $Ra_D$, $Ha_e$ and aspect ratios. Here, the only driving mechanism of the flow is due to the buoyancy effects. The fluid flow is mostly characterized by a primary recirculating eddy of the size of the cavity generated by the temperature gradient between the side walls. For a fixed $Ra_D$ and aspect ratio, the application of magnetic field slows down the movement of the fluid in the cavity and causes the progressive inhibition of stream function as the $Ha_e$ changes from 0 to 10, thereby causing the flow to be restricted mostly close to the wall. The same kind of stream function behavior is observed for all the different $Ra_D$ and aspect ratios. For a fixed $Ha_e$ and aspect ratio, and with increase in $Ra_D$, the center cell is strengthened due to increased buoyancy forces as expected. For a fixed $Ra_D$ and $Ha_e$, with increase in aspect ratio,
there is no significant difference between the stream function contours. With regard to
temperature contours in Figures 7.3-7.11. The legend for temperature contour lines
represents values in 18 equal intervals between 0 and 1. In general, it can be observed
that an increase in magnetic field makes the temperature distribution more linear
from the hot to cold walls. For the lower $Ra_D$ i.e., $Ra_D = 500$, and beyond $Ha_e = 6$,
the temperature distribution does not change with a further increase in magnetic field.
These are due to the much higher thermal diffusion rates with increased magnetic
field strengths, resulting in the fluid heat transfer characteristics being more similar to
that of a solid. For the $Ra_D = 5000$ and $Ra_D = 10000$ cases though, where buoyancy
forces are significantly more than the $Ra_D = 500$ case, the thermal boundary layer
extends over the entire domain for low $Ha_e$. The usual convective distortion of the
isothermal lines are formed with two thermal spots, one at the bottom of hot wall and
another at the top of cold wall. These spots have a higher magnitude of heat flux as
they have higher temperature gradient. The isothermal lines are distorted due to the
convection inside the cavity. Increase in $Ha$ eventually leads to a linear temperature
variation between the hot and cold regions, almost like heat conduction in a solid.
Again with a variation in aspect ratio, there is not much change in temperature
distributions.

Figure 7.12 shows the variation of average Nusselt number, $Nu$, with $Ha_e$
for different aspect ratios and $Ra_D$. Physically, the $Nu$ represents the ratio of the
convective and the conductive heat transfer coefficients. The $Nu$ shown here is cal-
culated as the gradient of temperature ($\partial T/\partial x$) along the hot and cold walls, and
the average value is calculated over the entire wall. With an increase in $Ha_e$, the $Nu$ decreases with increasing magnetic field for every $Ra_D$, because, convective heat transfer effects decrease with the increased magnetic field magnitudes owing to the Lorentz force-related Joule damping. Boundary layer regime is followed by asymptotic and conduction regime at progressively higher $Ha_e$ as $Nu$ approaches to unity. Furthermore, it is interesting to note that while stream function and temperature contours did not show any significant qualitative difference with different aspect ratios, the average $Nu$ does show some minor differences.

Specifically, there is a decrease in average $Nu$ values with increase in aspect ratio for a fixed $Ra_D$ and $Ha_e$, because of the reduction in effective length scale, $L$. Figure 7.13 shows the variation of average $Nu$ with $Ra_D$ for different aspect ratios and $Ha_e$. With an decrease in $Ha_e$, the increase in $Nu$ is evident due to the rise in convective effects and effects of aspect ratios are also similar to those observed in Figure 7.12. The values of the $Nu$ calculated here are compared with the correlation presented by Manole and Lage [1]. The correlation for the average $Nu$ in the absence of magnetic field ($Ha_e = 0$) for a non-Darcy flow presented in [1] is,

$$Nu \sim \frac{1}{2} \left[ -\frac{Pr\phi^2}{Da} + \left\{ \frac{Pr^2\phi^4}{Da^2} + 2\phi^2RaPr(1 + \phi A(Pr)Pr + \frac{0.143\phi^{0.5}}{Da^2}) \right\}^{\frac{3}{2}} \right]^{\frac{1}{2}}$$ (7.1)

The comparison of $Nu$ from the current study with that from the above correlation is presented in Figure 7.1. It can be seen that there are some differences between the two sets of values, which can be attributed due to the differences in non-Darcy and Darcy models applied in the two studies. The comparison goes better for the
increased value of $Ra$. The variation is about 25% for $Ra = 100,000$ which decreases to about 2.27% for $Ra = 1,000,000$. As the average $Nu$ is taken in left wall of the domain, the validation would be better if the non-uniform grid size having more number of grids in left wall is used, also to save the computational cost.

Figure 7.2 shows typical contour maps of stream function and temperature obtained numerically for $Ra = 10^6$, $Da = 10^{-2}$ and $AR = 1 : 1$ in the absence of magnetic field. These results are used to compare with the results presented by Hadim [76]. It is seen that the maximum stream function ($\Psi_{\text{max}}$) of 15.9 is obtained here, whereas $\Psi_{\text{max}} = 17.54$ in case of Hadim [76].
Figure 7.1: Comparison of the average $Nu$ calculated from the current study with that from the correlation [1]. The subscripts $c$ and $n$ are for correlation and numerical values respectively.
Figure 7.2: Numerical results for stream function (left) and temperature (right) for aspect ratio 1:1 and $Ra_D = 10000$ in the absence of magnetic field ($Ha_e = 0$).
Table 7.1: Stream function for aspect ratio 4:1 with different $Ha_e$.

<table>
<thead>
<tr>
<th>$\Psi_{\text{max}}$</th>
<th>$Ra$</th>
<th>$Ra_D$</th>
<th>$Ha_e=0$</th>
<th>$Ha_e=1$</th>
<th>$Ha_e=2$</th>
<th>$Ha_e=6$</th>
<th>$Ha_e=8$</th>
<th>$Ha_e=10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times 10^4$</td>
<td>500</td>
<td>13.575</td>
<td>11.603</td>
<td>7.939</td>
<td>1.568</td>
<td>0.914</td>
<td>0.598</td>
<td></td>
</tr>
<tr>
<td>$1 \times 10^5$</td>
<td>1000</td>
<td>18.117</td>
<td>16.286</td>
<td>12.439</td>
<td>3.107</td>
<td>1.823</td>
<td>1.191</td>
<td></td>
</tr>
<tr>
<td>$5 \times 10^5$</td>
<td>5000</td>
<td>29.500</td>
<td>28.379</td>
<td>25.612</td>
<td>12.304</td>
<td>8.285</td>
<td>5.735</td>
<td></td>
</tr>
<tr>
<td>$8 \times 10^5$</td>
<td>8000</td>
<td>33.644</td>
<td>32.572</td>
<td>29.965</td>
<td>16.659</td>
<td>11.920</td>
<td>8.639</td>
<td></td>
</tr>
<tr>
<td>$1 \times 10^6$</td>
<td>10000</td>
<td>35.788</td>
<td>34.750</td>
<td>32.176</td>
<td>18.957</td>
<td>13.935</td>
<td>10.328</td>
<td></td>
</tr>
</tbody>
</table>
Table 7.2: Stream function for aspect ratio 6:1 with different $Ha_e$.

<table>
<thead>
<tr>
<th>$\Psi_{\text{max}}$</th>
<th>$Ra$</th>
<th>$Ra_D$</th>
<th>$Ha_e$=0</th>
<th>$Ha_e$=1</th>
<th>$Ha_e$=2</th>
<th>$Ha_e$=6</th>
<th>$Ha_e$=8</th>
<th>$Ha_e$=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times 10^4$</td>
<td>500</td>
<td>15.750</td>
<td>13.218</td>
<td>8.592</td>
<td>1.570</td>
<td>0.914</td>
<td>0.596</td>
<td></td>
</tr>
<tr>
<td>$1 \times 10^5$</td>
<td>1000</td>
<td>21.866</td>
<td>19.361</td>
<td>14.221</td>
<td>3.128</td>
<td>1.826</td>
<td>1.192</td>
<td></td>
</tr>
<tr>
<td>$3 \times 10^5$</td>
<td>3000</td>
<td>32.639</td>
<td>30.685</td>
<td>25.929</td>
<td>8.950</td>
<td>5.423</td>
<td>3.564</td>
<td></td>
</tr>
<tr>
<td>$5 \times 10^5$</td>
<td>5000</td>
<td>37.864</td>
<td>36.245</td>
<td>32.103</td>
<td>13.704</td>
<td>8.779</td>
<td>5.891</td>
<td></td>
</tr>
<tr>
<td>$8 \times 10^5$</td>
<td>8000</td>
<td>42.940</td>
<td>41.592</td>
<td>38.026</td>
<td>19.250</td>
<td>13.143</td>
<td>9.164</td>
<td></td>
</tr>
<tr>
<td>$1 \times 10^6$</td>
<td>10000</td>
<td>45.617</td>
<td>44.297</td>
<td>40.940</td>
<td>22.249</td>
<td>15.649</td>
<td>11.161</td>
<td></td>
</tr>
</tbody>
</table>
Table 7.3: Stream function for aspect ratio 8:1 with different $Ha_e$.

<table>
<thead>
<tr>
<th>$\Psi_{\text{max}}$</th>
<th>$Ra$</th>
<th>$Ra_D$</th>
<th>$Ha_e=0$</th>
<th>$Ha_e=1$</th>
<th>$Ha_e=2$</th>
<th>$Ha_e=6$</th>
<th>$Ha_e=8$</th>
<th>$Ha_e=10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times 10^4$</td>
<td>500</td>
<td>500</td>
<td>17.016</td>
<td>14.098</td>
<td>8.853</td>
<td>1.570</td>
<td>0.914</td>
<td>0.596</td>
</tr>
<tr>
<td>$1 \times 10^5$</td>
<td>1000</td>
<td>1000</td>
<td>24.281</td>
<td>21.311</td>
<td>15.227</td>
<td>3.131</td>
<td>1.826</td>
<td>1.192</td>
</tr>
<tr>
<td>$3 \times 10^5$</td>
<td>3000</td>
<td>3000</td>
<td>37.870</td>
<td>35.323</td>
<td>29.282</td>
<td>9.159</td>
<td>5.45</td>
<td>3.568</td>
</tr>
<tr>
<td>$5 \times 10^5$</td>
<td>5000</td>
<td>5000</td>
<td>44.604</td>
<td>42.455</td>
<td>36.995</td>
<td>14.438</td>
<td>8.958</td>
<td>5.927</td>
</tr>
<tr>
<td>$8 \times 10^5$</td>
<td>8000</td>
<td>8000</td>
<td>50.961</td>
<td>49.193</td>
<td>44.466</td>
<td>20.874</td>
<td>13.757</td>
<td>9.359</td>
</tr>
<tr>
<td>$1 \times 10^6$</td>
<td>10000</td>
<td>10000</td>
<td>54.102</td>
<td>52.488</td>
<td>48.120</td>
<td>24.429</td>
<td>16.608</td>
<td>11.527</td>
</tr>
</tbody>
</table>
Figure 7.3: Numerical results for stream function (top) and temperature (bottom) of table 7.1 for aspect ratio AR 4:1 and $Ra_D = 500$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature.
Figure 7.4: Numerical results for stream function (top) and temperature (bottom) of table 7.1 for aspect ratio AR 4:1 and $Ra_D = 5000$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature.
Figure 7.5: Numerical results for stream function (top) and temperature (bottom) of table 7.1 for aspect ratio AR 4:1 and $Ra_D = 10000$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature.
Figure 7.6: Numerical results for stream function (top) and temperature (bottom) of table 7.2 for aspect ratio AR 6:1 and $Ra_D = 500$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature.
Figure 7.7: Numerical results for stream function (top) and temperature (bottom) of table 7.2 for aspect ratio AR 6:1 and $Ra_D = 5000$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature.
Figure 7.8: Numerical results for stream function (top) and temperature (bottom) of table 7.2 for aspect ratio AR 6:1 and $Ra_D = 10000$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature.
Figure 7.9: Numerical results for stream function (top) and temperature (bottom) of table 7.3 for aspect ratio AR 8:1 and $Ra_D = 500$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature.
Figure 7.10: Numerical results for stream function (top) and temperature (bottom) of table 7.3 for aspect ratio AR 8:1 and $Ra_D = 5000$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature.
Figure 7.11: Numerical results for stream function (top) and temperature (bottom) of table 7.3 for aspect ratio AR 8:1 and $Ra_D = 10000$. $Ha_e$ increases as 0, 1, 2, 6, 8 and 10 (left to right) for stream function and temperature.
Figure 7.12: Variation of the average Nusselt number with effective Hartmann number for different values of the Rayleigh Darcy number a) AR 4:1 b) AR 6:1 c) AR 8:1.
Figure 7.13: Variation of the average Nusselt number with Rayleigh Darcy number for different values of the effective Hartmann number a) AR 4:1 b) AR 6:1 c) AR 8:1.
7.2 Assessment of five subgrid-scale (SGS) models for low-$Re_m$ magnetohydrodynamic (MHD) turbulence

In this section the results from five different SGS models such as the non-dynamic Smagorinsky (NDSMAG), Dynamic Smagorinsky (DSMAG), Leray-$\alpha$ (LERAY), LANS-$\alpha$ (LANS) and Clark-$\alpha$ (CLARK) are compared with in-house DNS results [62] for homogenous MHD turbulence and previous simulations for the Taylor-Green Vortex (TGV) case [2, 63]. The value of $\alpha$ is taken as $1/42$, which gives the best agreement and there is no significant difference between the results by decreasing the $\alpha$ below $1/42$ [62]. These models presented here are compared with DNS results without a magnetic field, i.e. $N = 0$ and with a magnetic field using values of $N = 0.1, 0.5$ and $1$. For all the simulations presented here, the magnetic filed is applied only in the $z$-direction.

In the case of non-dynamic Smagorinsky model, the constant $C_s$ is fixed at 0.16, which is the convention for isotropic turbulence in the inertial range [77]. Energy spectra, evolution of turbulent kinetic energy (tke) and decay rate ($-dK/dt$) of tke of different above mentioned models are plotted and compared with DNS using a spectral cutoff filter with the same filter width of LES. Simulation parameters for both the homogenous MHD turbulence and MHD Taylor-Green vortex are presented in table 7.4.
Table 7.4: Simulation parameters for DNS and SGS models.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Grid</th>
<th>(N)</th>
<th>Filter width, (\Delta) (m)</th>
<th>Constants</th>
<th>Time step (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNS-MHD_N</td>
<td>256(^3)</td>
<td>0, 0.1, 0.5, 1</td>
<td>-</td>
<td>-</td>
<td>(6 \times 10^{-4})</td>
</tr>
<tr>
<td>LES: NDSMAG_N</td>
<td>96(^3)</td>
<td>0, 0.1, 0.5, 1</td>
<td>(\pi/48)</td>
<td>(\alpha = 1/42,) (C_s = 0.16)</td>
<td>(5.3 \times 10^{-3})</td>
</tr>
<tr>
<td>LES: DSMAG_N</td>
<td>96(^3)</td>
<td>0, 0.1, 0.5, 1</td>
<td>(\pi/48)</td>
<td>(\alpha = 1/42)</td>
<td>(5.3 \times 10^{-3})</td>
</tr>
<tr>
<td>LES: LERAY_N</td>
<td>96(^3)</td>
<td>0, 0.1, 0.5, 1</td>
<td>(\pi/48)</td>
<td>(\alpha = 1/42)</td>
<td>(5.3 \times 10^{-3})</td>
</tr>
<tr>
<td>LES: LANS_N</td>
<td>96(^3)</td>
<td>0, 0.1, 0.5, 1</td>
<td>(\pi/48)</td>
<td>(\alpha = 1/42)</td>
<td>(5.3 \times 10^{-3})</td>
</tr>
<tr>
<td>LES: CLARK_N</td>
<td>96(^3)</td>
<td>0, 0.1, 0.5, 1</td>
<td>(\pi/48)</td>
<td>(\alpha = 1/42)</td>
<td>(5.3 \times 10^{-3})</td>
</tr>
</tbody>
</table>
7.2.1 Homogenous Turbulence

The problem is that of an initially-isotropic decaying turbulence case of $Re_{\lambda} = 120$. The Taylor scale Reynolds number, $Re_{\lambda}$, is based on the Taylor length scale and the r.m.s. velocity in the longitudinal direction, i.e. $Re_{\lambda} = u_{1r.m.s.} \nu / \lambda$, where $\nu$ is the viscosity. The initialization for both DNS and LES starts with a divergence free velocity field and a specified energy spectrum according to [32] given by

$$E(k) = C_k k^{2/3} k^{-5/3} \left( \frac{kL_u}{(kL_u)^2 + c_L} \right)^{5/3 + p_0} \exp \left( -\beta \left\{ (k\eta)^4 + c_\eta^4 \right\}^{1/4} - c_\eta \right)$$

(7.2)

where $k$ is the wavenumber, $L_u$ is the integral length scale chosen to be 0.6 m, and $\eta$, the Kolmogorov length scale can be calculated as

$$\eta = Re_L^{-3/4} L_u = 1.8937 \text{ mm}$$

(7.3)

where $Re_L$ is the integral scale Reynolds number given by

$$Re_L = 3 \frac{Re_{\lambda}^2}{20} = 2160$$

(7.4)

In the energy spectrum function above, $p_0$, $c_L$, and $c_\eta$ are constants chosen to be 2.0, 3.75, and 0.4, respectively [32, 78]. The constants $C_k$ and $\beta$ are chosen such that the total turbulent kinetic energy of the initial velocity field is 0.55 m$^2$/s$^2$, and their values are 1.5 and 5.2, respectively. Using random phase Fourier modes, the velocity field is scaled in Fourier space so that the energy spectrum corresponds to the initial energy spectrum. However, in order to have as realistic initial conditions as possible, an initial simulation starting from random phase Fourier modes is run for a time of $t = 2$ s, a time chosen based on the idea that the velocity derivative...
skewness remains steady. The velocity field is then rescaled in spectral space so that the energy spectrum corresponds again to the initial energy distribution described by the spectrum at $t = 0$.

Figure 7.14 shows the comparison of turbulent kinetic energy (tke) evolution for the five different SGS models with DNS results. The tke of filtered turbulence (filtered using spectral cutoff filter) is obtained by integrating the 3D energy spectrum according to,

$$tke = \int_0^{k_{\text{max}}} E(k) dk$$

where $k_{\text{max}}$ is the largest wavenumber represented by, $k_{\text{max}} = \pi N/L = \pi/\Delta x$. $L$ is the side of the domain, $\Delta x$ is the grid spacing and $N$ is the grid size. Note that, in order to compare to the filtered tke obtained from LES, the DNS data is truncated using a spectral cutoff filter with the same filter width as the LES. For example, the spectral cutoff filter can be considered as the implicit numerical grid filter of filter size, $\Delta$, when using a pseudo-spectral code, in which case, $\Delta = h$. So for a LES grid resolution of $96^3$ after dealiasing, like the one employed in this paper, the filter size, $\Delta$, is $\pi/48$.

The LES calculations for the different models are compared with in-house DNS results for four different values of $N$, $N = 0, 0.1, 0.5$ and 1 in Figure 7.14. Obviously the tke decreases with time because of decaying turbulence, but it decreases faster with increasing magnetic field due to the Joule dissipation effects resulting from the magnetic Lorentz force. With regard to the performance of the SGS models, in the absence of magnetic field, the Smagorinsky models, in particular the dynamic
Smagorinsky seems to predict the tke evolution the best. All the regularization models overpredict the tke at all times, with the LANS-α being the worst. It is interesting to note that with an increase in magnetic field the agreement between the regularization models and the DNS results keeps improving until at $N = 1$, all the SGS models do a pretty good job in predicting the evolution of tke.

Comparisons for the evolution of decay rate of tke are presented in Figure 7.15. In order to emphasize the behavior at early times, results are presented for a time of $t < 4$ s. The tke decay rate increases early ($t < 0.5$) and subsequently decrease continuously. For the no-magnetic field case, the Smagorinsky models are better than the regularization models, with the dynamic Smagorinsky doing a slightly better job. All the regularization models show a delay in the peak and also severely underpredict the peak of the tke decay rate. With an increase in magnetic field, the comparisons are better with DNS results, with the SGS models all on top of DNS for the $N = 1$ case. The kind of behavior observed in the plots of tke and tke decay rate shows again, how the regularization models are unable to correct predict the energy levels at high-$Re$. But when the magnetic field is added, the effective $Re$ is reduced [31] resulting in better predictions compared to DNS. This was also seen in some of the earlier studies [62, 74].

Energy spectra displays information about the energy distribution across the entire range of the scales, so it gives a better and more specific understanding of the ability of the SGS models to correctly predict the energy with regard to the different length scales. Figures 7.16 - 7.19 shows the 3D energy spectra for the various magnetic
field cases at four different times, $t = 2 \text{ s}$, $t = 4 \text{ s}$, $t = 6 \text{ s}$ and $t = 8 \text{ s}$. Consider the $N = 0$ case, where the magnetic field is absent. Here, all the SGS models compare well with DNS for the low-wavenumbers or large length scales at both the times. But early in the simulation ($t = 2 \text{ s}$), at the high-wavenumbers ($k > 20$) or small length scales, there is an insufficient dissipation of energy resulting in a build-up at these wavenumbers. This phenomena was exhibited only to a slight extent by the dynamic Smagorinsky model, but for the regularization models, discrepancy was much worse. With regard to the non dynamic Smagorinsky model, no build-up of energy can be seen. In fact it underpredicts the energy at the small scales, but such a behavior is a function of the Smagorinsky constant, $C_s$, whose value was chosen as 0.16 in these simulations. Later at $t = 8$ the comparisons are better with DNS, with only slight build-up observed at the high wavenumbers. But that is because at this stage of the simulation, the small-scales have dissipated significantly and hence the SGS models, especially the regularization models are able to predict the energy levels better at this stage.

With the addition of magnetic field (See Figures 7.17 - 7.19), the Lorentz force comes into play, and the Joule dissipation from the magnetic field acts directly on all modes including the most energetic large-scale/low-wavenumber ones, hence causing a reduction in the amount of energy at these scales. Also, with the increase in the strength of the magnetic field, the energy cascade from large scales to small scales reduces, thus decreasing the energy content in the smaller scales as well [74]. Energy spectra for the magnetic field cases show a similar behavior as what was seen
for $N = 0$, where the regularization models exhibit a larger build-up of energy at the small scales compared to the eddy-viscosity models. But the results do show an overall improvement with increase in magnetic field. That is because the magnetic field tends to suppress the turbulent fluctuations, reduce the turbulent kinetic energy at all the scales and thereby reduce the level of turbulence itself (i.e. an overall reduction in effective $Re$).

To visualize the effect of anisotropy on the flow, contours of vorticity magnitude are presented as gray scale contours for two magnetic interaction parameters, $N = 0$ and $N = 0.5$, in Figures 7.20 and 7.21, respectively, at time, $t = 6$ s, for the six (One DNS + five LES) different cases in a $\pi^3$ domain. It can be seen that the flow becomes nearly two-dimensional in the $x - y$ plane with an increase in the magnetic field due to the damping of fluid motions in the direction of the magnetic field, i.e. the $z$-direction, thus causing a flow stretching in the same direction. While all the SGS model results qualitatively are similar to DNS, the dynamic Smagorinsky shows the better comparison quantitatively as indicated by the levels of gray scale contours and all the regularization models do a much poorer job in predicting the vorticity contours correctly.
Figure 7.14: Evolution of the turbulent kinetic energy (tke) with time, for five different SGS models compared with DNS, with different values of $N$. N0, N0.1, N0.5, N1 represent $N = 0, 0.1, 0.5$ and 1 respectively.
Figure 7.15: Evolution of rate of decay of turbulent kinetic energy, $\frac{dK}{dt}$, with time, for five different SGS models compared with DNS; with different values of $N$. $N0$, $N0.1$, $N0.5$, $N1$ represent $N = 0$, 0.1, 0.5 and 1 respectively.
Figure 7.16: Spectra, $E(k)$, at four different times; $N_0$ represents magnetic interaction parameter, $N = 0$. 
Figure 7.17: Spectra, $E(k)$, at four different times; N0.1 represents magnetic interaction parameter, $N = 0.1$. 
Figure 7.18: Spectra, $E(k)$, at four different times; $N0.5$ represents magnetic interaction parameter, $N = 0.5$. 
Figure 7.19: Spectra, $E(k)$, at four different times; N1 represents magnetic interaction parameter, $N = 1$. 
Figure 7.20: Grey scale contours of vorticity magnitude on a $\pi^3$ domain at time, $t = 6$ s, for DNS and different SGS models without magnetic field i.e. magnetic interaction parameter ($N$) = 0. (a) DNS (b) NDSMAG (c) DSMAG (d) LERAY (e) LANS (f) CLARK. The legends and the axes for the contours are same.
Figure 7.21: Grey scale contours of vorticity magnitude on a $\pi^3$ domain at time, $t = 6$ s, for DNS and different SGS models with magnetic field i.e. magnetic interaction parameter $(N) = 0.5$. (a) DNS (b) NDSMAG (c) DSMAG (d) LERAY (e) LANS (f) CLARK. The legends and the axes for the contours are same.
7.2.2 Taylor Green Vortex

The next case presented here is the classic Taylor-Green vortex problem [79], which is a 3D transition to turbulence problem. The velocity field is initialized in physical space according to

\[ u_{1,0} = \sin(x) \cos(y) \cos(z) \]
\[ u_{2,0} = -\cos(x) \sin(y) \cos(z) \]
\[ u_{3,0} = 0 \]  

(7.5)

A case of \( Re = 3000 \) is simulated here, where the \( Re \) is defined here as the inverse of the viscosity. Ever since one of the first numerical simulations of the Taylor-Green vortex by Steven Orszag in 1973 [79], several studies have focused on this problem. Studies of Brachet [2, 63] conducted DNS of Taylor-Green vortex to study small scale turbulent structures at \( Re \) of up to 3000. Bensow et al. [80] conducted LES using a more novel variational multiscale method [81] at \( Re = 1600 \) and Drikakis et al. [82] used implicit LES to compare their Taylor-Green results to Brachet’s DNS [63] at a range of \( Re \) from 400 to 5000.

LES of the TGV is presented here for two different magnetic interaction parameters, \( N = 0 \) and \( N = 0.05 \), and for the five different SGS models: non-dynamic Smagorinsky (NDSMAG), dynamic Smagorinsky (DSMAG), Leray-\( \alpha \) (LERAY), LANS-\( \alpha \) (LANS) and Clark-\( \alpha \) (CLARK). Figure 7.22 shows the evolution of the turbulent kinetic energy and Figure 7.23 shows the evolution of the rate of energy decay. From the decay rate plots at \( N = 0 \), the results are qualitatively similar to previous results including DNS [63, 62], where the evolution follows two phases. In the first phase
(t < 9), the flow essentially remains laminar and transitional, the flow structure would be highly organized and the dissipation increases from zero to a maximum value, as the flow turns turbulent. In the second phase (t > 9), starting with the maximum dissipation (or -dK/dt), the flow is of fully-developed turbulent nature and the dissipation keeps decreasing. From Figure 7.22, it is clear that in general, with an increase in magnetic field, the energy decays faster due to the added Joule dissipation from the Lorentz force.

In the no-magnetic field case, all the regularization models overpredict the tke compared to the eddy-viscosity models mostly at all times, but in the presence of a magnetic field even as small as N = 0.05, the predictions are closer to each other. This is because the magnetic field has caused the flow to be less turbulent and the regularization models perform better in these conditions. With regard to the decay rate (in Figure 7.23), the magnetic field has caused a delay in the transition to turbulence as indicated by all the SGS model results. And as expected the regularization models for reasons outlined previously, are closer to the Smagorinsky models.

Figure 7.24 shows the energy spectra for the two magnetic field cases (N = 0 and N = 0.05) at two different times, t = 8 and t = 16. All the SGS models predict the large scales (k < 20) pretty well at all times and for both magnetic fields. With regard to the small scales (k > 20), compared to the dynamic Smagorinsky model, the non-dynamic Smagorinsky overpredicts the dissipation at all times and all magnetic field levels. On the other hand, the regularization models, especially the Leray-α and LANS-α tend to underpredict the dissipation, thereby displaying
a build-up of energy at these large scales. The Clark-\(\alpha\) model though is very close to the dynamic Smagorinsky for all the cases at all times. The predictions of the regularization models get better with time for a single magnetic field case and also with an increase in magnetic field because of the increased dissipation of the small scales at those conditions.

Figures 7.25 and 7.26 present the vortical structures in the form of iso-surfaces of vorticity magnitude at \(C = 12\) for the different SGS models and the two magnetic field cases at \(t=10\) s. This time was chosen in order to indicate full-developed turbulence. It can be seen that there are radical differences in the vortical structures between the regularization-type models and the eddy-viscosity models. The obvious difference is the amount of structures visible at this particular value. For instance, the non-dynamic Smagorinsky seems to show the least amount of structures, which is directly related to the overprediction of small-scale dissipation by this model. On the other end of the spectrum is the Leray model where it can be seen that the structures are thicker and more in the number, indicating the underprediction of dissipation. The LANS and Clark models also show much more structures compared to the Smagorinsky models, but smaller compared to the Leray model, indicating a level of predicted dissipation between Leray and Smagorinsky models.
Figure 7.22: Evolution of the turbulent kinetic energy (tke) of decaying Taylor-Green Vortex (TGV) with time, for five different SGS models, with different values of $N$. N0 and N0.05 represent $N = 0$ and 0.05, respectively.
Figure 7.23: Decay rate of turbulent kinetic energy (tke) of decaying Taylor-Green Vortex (TGV) with time, for five different SGS models, with different values of $N$. $N_0$ and $N_0.05$ represent $N = 0$ and 0.05, respectively.
Figure 7.24: Spectra, E(k), of decaying Taylor - Green Vortex (TGV) at two different times; N0 and N0.05 represent magnetic interaction parameter, N = 0 and N = 0.05 respectively.
Figure 7.25: Grey scale vortical structures in a $\pi^3$ domain at time, $t = 10$ s, in terms of iso-surface at value $C = 12$ for the different SGS models without magnetic field i.e. magnetic interaction parameter, $N = 0$ (a) LERAY (b) LANS (c) CLARK (d) NDSMAG (e) DSMAG.
Figure 7.26: Grey scale vortical structures in a $\pi^3$ domain at time, $t = 10$ s, in terms of iso-surface at value $C = 12$ for the different SGS models with magnetic field i.e. magnetic interaction parameter, $N = 0.05$ (a) LERAY (b) LANS (c) CLARK (d) NDSMAG (e) DSMAG.
8.1 Numerical simulations of non-Darcy magnetohydrodynamic natural convection in a porous rectangular cavity

A numerical study of the flow field and heat transfer characteristics in non-Darcy MHD natural convection in a cavity filled a porous medium has been performed. The non-dimensional forms of the continuity, momentum and energy equation are solved numerically and the effects of $Ra_D$, $Ha_e$ and aspect ratio on flow dynamics are investigated. To obtain the realistic and accurate results at large Darcy number ($Da > 10^{-2}$), both the Brinkman and Forchheimer extensions were used simultaneously [83]. The numerical investigation presented here shows that the fluid flow velocities decreases with increasing value of the $Ha_e$, since the convection is almost suppressed and the isotherms are parallel to the vertical isothermal walls, indicating that the conduction regime was reached. Velocities also increased with increase in $Ra_D$ due to increased buoyancy forces and with increase in aspect ratio due to reduced flow areas. The average $Nu$ across the enclosure was also calculated and results showed that it reduced with increasing $Ha_e$ and aspect ratio again due to reduced convection effects.
8.2 Assessment of five subgrid-scale (SGS) models for low-\(Re_m\) magnetohydrodynamic (MHD) turbulence

A series of LES of low-\(Re_m\) MHD turbulence was examined in the context of a decaying homogeneous turbulence (DHT) case and the classic transition to turbulence case, the Taylor-Green vortex (TGV). Five different SGS models such as the non-dynamic and dynamic Smagorinsky, which constitute eddy-viscosity-type models, and the Leray-\(\alpha\), LANS-\(\alpha\) and Clark-\(\alpha\) models which constitute the regularization-type models, were assessed for varying interaction parameters. The magnetic field was imposed only in the \(z\)-direction in a periodic cubic domain of side \(2\pi\) m. Calculations at various levels of magnetic field strengths in terms of the interaction parameter, \(N = 0, 0.05, 0.1, 0.5,\) and \(1\) were carried out and comparisons were made to in-house DNS in the case of DHT and previous DNS in the case of TGV.

For the decaying MHD turbulence calculations, comparisons DNS for all the regularization models were worse than the Smagorinsky models, with the Leray-\(\alpha\) performing the worst and the dynamic Smagorinsky the best. From the energy spectra it was clear that the regularization models underpredicted the dissipation at small scales indicating a build-up of energy due to insufficient resolution. On the other hand, the non-dynamic Smagorinsky did overpredict the dissipation, which could be related to the Smagorinsky constant chosen here in these studies (which is 0.16). The predictions of the regularization-based SGS models improved with increase in magnetic field though, due to a reduction in the effective \(Re\) as a result.
of the additional Lorentz force. The vorticity contours in the box demonstrated how all the SGS models were able to qualitatively predict the stretching in the $z$-direction (due to the Lorentz force resulting from a $z$-direction magnetic field) reasonably well. However with regard to actual vorticity values, the regularization model results did not match very well with DNS, while the eddy-viscosity-based models fared better.

For the TGV case, only two magnetic field cases were simulated, $N = 0$ and $N = 0.05$ for a $Re = 3000$. Qualitative comparisons were made with previous DNS data from Brachet [63]. Again on assessing the five different SGS models, dynamic Smagorinsky seemed to perform the best and Leray the worst. All the SGS models predicted a further delay in transition to turbulence compared to the dynamic Smagorinsky as indicated by the decay rate evolution. The LANS and Clark models though did a better job in predicting the magnitudes of these decay rates while Leray and non-dynamic Smagorinsky highly underpredicted these values. The comparisons were closer to each other with an increase in magnetic field. The spectra demonstrated the same story as what was observed in homogeneous turbulence, where non-dynamic Smagorinsky showed too much dissipation at the small scales while the regularization models showed a build-up of energy at the small scales or the large wavenumbers, and again comparisons were better with an increase magnetic field. It was also interesting to note that the vortical structures characterized by iso-surfaces of vorticity magnitude also showed significant differences between the SGS models, especially with regard to the number of structures displayed at a particular value. For instance the non-dynamic Smagonrinsky showed the least amount of structures
indicating the maximum dissipation, while regularization models showed the largest amount of structures indicating the least dissipation.
BIBLIOGRAPHY


89


