NUMERICAL ANALYSIS OF DROPLET AND FILAMENT DEFORMATION FOR PRINTING PROCESS

A Thesis
Presented to
The Graduate Faculty of The University of Akron

In Partial Fulfillment
of the Requirements for the Degree

Master of Science

Muhammad Noman Hasan
August, 2014
NUMERICAL ANALYSIS OF DROPLET AND FILAMENT DEFORMATION FOR PRINTING PROCESS

Muhammad Noman Hasan

Thesis

Approved: 

Advisor 
Dr. Jae–Won Choi

Accepted: 

Department Chair 
Dr. Sergio Felicelli

Committee Member 
Dr. Abhilash J. Chandy

Dean of the College 
Dr. George K. Haritos

Committee Member 
Dr. Chang Ye

Dean of the Graduate School 
Dr. George R. Newkome

Date
ABSTRACT

Numerical analysis for two-dimensional case of two-phase fluid flow has been carried out to investigate the impact, deformation of (i) droplets and (ii) filament for printing processes. The objective of this research is to study the phenomenon of liquid droplet and filament impact on a rigid substrate, during various manufacturing processes such as jetting technology, inkjet printing and direct-printing. This study focuses on the analysis of interface capturing and the change of shape for droplets (jetting technology) and filaments (direct-printing) being dispensed during the processes. For the investigation, computational models have been developed for (i) droplet and (ii) filament deformation which implements quadtree spatial discretization based adaptive mesh refinement with geometrical Volume–Of–Fluid (VOF) for the representation of the interface, continuum–surface–force (CSF) model for surface tension formulation, and height-function (HF) curvature estimation for interface capturing during the impact and deformation of droplets and filaments. An open source finite volume code, Gerris Flow Solver, has been used for developing the computational models. The computational model has been validated with (i) literature for the droplet deformation, and (ii) with experiment for the filament deformation. This study focuses on the effects of variation of various dimensionless governing and process parameters on the deformation of (i) droplet and (ii) filament during printing process.
ACKNOWLEDGEMENTS

I would like to thank my advisor Dr. Jae-Won Choi for his constant guidance and supervision. I would also like to thank my committee members, Dr. Abhilash J. Chandy and Dr. Chang Ye for their guidance. I wish to express my gratitude to Mr. Morteza Vatani and Mr. Yanfeng Lu for their constant support for any experimental issues I have encountered through the course of this study. I also wish to express my gratitude to Cooperative R&D Program funded by Korea Small and Medium Business Association for supporting this work.
TABLE OF CONTENTS

LIST OF TABLES ............................................................................................................ vii
LIST OF FIGURES ......................................................................................................... viii
NOMENCLATURE ......................................................................................................... x

CHAPTER

I. INTRODUCTION ........................................................................................................... 1
  1.1. Overview .................................................................................................................. 1
  1.2. Thesis Objectives ..................................................................................................... 2
  1.3. Organization of Thesis ............................................................................................. 3

II. LITERATURE REVIEW ............................................................................................... 4
  2.1. General ..................................................................................................................... 4
  2.2. Droplet Deformation ............................................................................................... 4
  2.3. Filament Dispensing ............................................................................................... 9
  2.4. Problem Formulation ............................................................................................. 12

III. MATHEMATICAl MODELING ............................................................................... 17
  3.1. Governing Equation ............................................................................................... 17
  3.2. Normalization .......................................................................................................... 19
  3.3. Initial Condition ...................................................................................................... 21
  3.4. Boundary Condition .............................................................................................. 22

IV. NUMERICAL MODELING ...................................................................................... 24
  4.1. General ................................................................................................................... 24
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>13</td>
</tr>
<tr>
<td>2-2</td>
<td>15</td>
</tr>
<tr>
<td>3-1</td>
<td>22</td>
</tr>
<tr>
<td>3-2</td>
<td>23</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>26</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
</tr>
<tr>
<td>7</td>
<td>31</td>
</tr>
<tr>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>9</td>
<td>33</td>
</tr>
<tr>
<td>10</td>
<td>33</td>
</tr>
<tr>
<td>11</td>
<td>34</td>
</tr>
<tr>
<td>12</td>
<td>35</td>
</tr>
<tr>
<td>13</td>
<td>36</td>
</tr>
<tr>
<td>14</td>
<td>37</td>
</tr>
<tr>
<td>15</td>
<td>39</td>
</tr>
<tr>
<td>16</td>
<td>42</td>
</tr>
</tbody>
</table>
17 Evolution of droplet deformation for various $Fr$ at $Re = 1000$, $We = 20.0$ (a) spreading factor, $\xi$ (b) deformation ratio, $R_\delta$ ................................................................. 43

18 Droplet deformation for various Reynolds Number, $Re$ at $Fr = 10.0$, $We = 20.0$ .... 44

19 Evolution of droplet deformation for various $Re$ at $Fe = 10.0$, $We = 20.0$ (a) spreading factor, $\xi$ (b) deformation ratio, $R_\delta$. ................................................................. 45

20 Droplet deformation for various Weber number, $We$ at $Fr = 10.0$, $Re = 1000$....... 47

21 Evolution of droplet deformation for various $We$ at $Fr = 10.0$, $Re = 1000$ (a) spreading factor, $\xi$ (b) deformation ratio, $R_\delta$. ................................................................. 48

22 Filament evolution for $Fr = 1.5$, $Re = 0.01$, $We = 0.25$, $GR = 0.75$ and $VR = 2$ ........ 50

23 Vorticity field and velocity vector plot - dispensing velocity of 4.4 mm/s, substrate velocity 5 mm/s, 0.63 mm gap between nozzle and substrate .............................................. 52

24 Velocity field and velocity vector plot - dispensing velocity of 4.4 mm/s, substrate velocity 5 mm/s, 0.63 mm gap between nozzle and substrate .............................................. 53

25 Vorticity field with velocity vector for (a) leading and (b) lagging filament and velocity field with velocity vector (c) leading and (d) lagging filament ................. 54

26 Effect of Froude number on filament deformation at different combinations........... 56

27 Effect of Reynolds number variation on filament deformation at different combinations................................................................................................................. 58

28 Effect of Weber number on filament deformation at different combinations. ....... 60

29 Effect of Gap Ratio on filament deformation at different combinations............... 62

30 Effect of Velocity Ratio on filament deformation at different combinations......... 64
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Cross-sectional area</td>
</tr>
<tr>
<td>c</td>
<td>volume fraction field</td>
</tr>
<tr>
<td>c̃</td>
<td>filtered volume fraction field</td>
</tr>
<tr>
<td>D</td>
<td>diameter of droplet and filament</td>
</tr>
<tr>
<td>F</td>
<td>force</td>
</tr>
<tr>
<td>Fr</td>
<td>Froude number</td>
</tr>
<tr>
<td>g</td>
<td>gravitational acceleration</td>
</tr>
<tr>
<td>h</td>
<td>droplet height</td>
</tr>
<tr>
<td>HF</td>
<td>height function</td>
</tr>
<tr>
<td>L</td>
<td>length</td>
</tr>
<tr>
<td>n</td>
<td>normal vector</td>
</tr>
<tr>
<td>P</td>
<td>pressure</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>R₅</td>
<td>deformation ratio</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>U</td>
<td>velocity vector</td>
</tr>
<tr>
<td>u, v</td>
<td>velocity in x and y directions</td>
</tr>
<tr>
<td>x, y</td>
<td>Cartesian coordinates</td>
</tr>
<tr>
<td>VOF</td>
<td>volume of fluid</td>
</tr>
<tr>
<td>W</td>
<td>width</td>
</tr>
<tr>
<td>We</td>
<td>Weber number</td>
</tr>
</tbody>
</table>

x
**Greek symbols**

- $\delta$ deformation
- $\Delta$ difference
- $\nabla$ differential operator
- $\nabla^2$ Laplacian
- $\kappa$ surface curvature
- $\lambda$ Navier slip coefficient
- $\mu$ viscosity
- $\zeta$ spreading factor
- $\rho$ density
- $\sigma$ surface tension
- $\times$ vector product

**Superscript**

- $^*$ dimensionless parameter
- $^x$ level of refinement

**Subscripts**

- $1$ fluid one
- $2$ fluid two
- $b$ body

xi
$f$  final
$Fr$  Correspond to Froude number
$g$  gravitational
$G$  gaseous
$L$  liquid

$n, n + 1, n + \frac{1}{2}$  time steps

$o$  initial
$r$  ratio
$rd$  reduced
$R$  reference
$s$  interface
$st$  surface tension
$t$  instantaneous

**  projected
CHAPTER I

INTRODUCTION

1.1. Overview

Droplets are small body of liquid defined or bounded by a free surface. They may be bounded partially by a free surface such as pendant drop or sessile drop. However, falling droplets are entirely bounded by a free surface. On the other hand, filaments are continuous body of liquid that is always bounded partially by a free surface. Filaments are more commonly known as “jet” in fluid mechanics. These filaments are bounded by free surface as of following three ways. The filament can (i) partially bounded by free surface, and one end is attached to a solid surface such as, the edge of a dispenser, (ii) partially bounded by free surface and both ends are attached to solid surfaces such as, filament attached between the dispenser and the substrate being dispensed, (iii) partially bounded by free surface and attached by a side longitudinally with the surface being dispensed on. “Free surface” is defined as a surface of a fluid that is under constant normal stress but no shear stress in other words it is the boundary between two homogenous fluids. It exists wherever there are two immiscible fluids co–existing, and the phenomena of flow of two immiscible fluids are omnipresent in nature such as, rain, sea water, waterfall and more. They are also readily common in lots of industrial processes. It is of a great importance to understand the interaction between a body either
fully (droplet) or partially (droplet and filament) bounded by free surface and impinging surface i.e., droplet and filament impact, deformation, and coalescence (merging of droplets for a droplet train) during and after being dispensed on a solid surface.

1.2. Thesis Objectives

The objective of this research is to analyze numerically and observe the deformation of bodies with free surface i.e. droplet and filament after being dispensed, during their impact on a rigid surface and consequently after impact. Literature shows numerous works on droplet impact and spreading for various applications such as spray cooling, atomization and more. These works include both experimental and numerical studies. According to the best knowledge of the author, study of droplet deformation and spreading for curing based print system has not received much attention. For filament deformation, the availability of literature is even less. For filament based printing system, the deformation of filament during and after the impact plays an important role while making cross lines. The after impact deformation determines the dimensional accuracy of the printed lines. In this study, a numerical investigation has been carried out for the accurate capturing of the droplet and filament interface to investigate the impact and the consequent deformation. The developed numerical model validates results from the literature (for droplet deformation) and experiment (for filament deformation). These models have been implemented to conduct a parametric case study to analyze the deformation of (i) droplet and (ii) filament. The results from this parametric study will
provide insight for the optimization of manufacturing processes incorporating droplet and filament deformation.

1.3. Organization of Thesis

Chapter II provides a comprehensive review of the literature. Chapter III describes the mathematical modelling implemented for the computational work. Chapter IV presents a comprehensive discussion of the numerical modeling. Chapter V analyzes the computational results, and the conclusion is outlined in Chapter VI.
CHAPTER II

LITERATURE REVIEW

2.1. General

One of the most fundamental fields of fluid mechanics is the free surface flow. The beginning of the study of free surface flow dates long ago, and it is still an active area of research due to its fundamental nature and its countless applications in fundamental and applied study of physics as well as its presence in uncountable number of industrial processes. Processes, regardless of simple or complex, those deals with multiple liquid phases are subjected to class. From bubbly flow, open channel flow, partially filled closed channel flows to spray cooling, atomization, metal casting, vapor deposition whether simple or complex process, study of free surface flows is essential for the fundamental understanding of the process or the optimization of it. Droplet and filaments (or sometimes referred as jet, usually when the fluid is at high velocity) are two of the basic forms of free surface that exists in different industrial and manufacturing processes.

2.2. Droplet Deformation

Droplet impact and deformation is one of the fundamental studies in various fields such as printing, manufacturing, cooling, biotechnology and more. Although this fundamental
phenomenon has been studied for a long time, due to countless applications and added novelty of its various applications, the study of droplet impact and deformation is still an active field of research. State of the art processes; such as, three–dimensional cell patterning for tissue engineering [1, 2], hybrid biological–electronic–device construction, and biochip–array fabrication [3], as well as traditional processes, they all need the study of droplet impact and deformation. The purpose is to achieve better accuracy and to attain process optimization. Jetting technology is one of the fields where the extensive study of droplet formation, dispensing, impact, and deformation is essential as the whole essence of the process is to produce continuous or drop–on–demand droplet at very high frequency. Inkjet is one of the mostly used jetting technologies in the industry to produce paper print to fabrication of micro–lens [4]. For polymer deposition processes such as the manufacturing of multicolor polymer light-emitting diode (PLED) displays and other polymer electronics, inkjet printing is considered to be one of the key technologies [5]. Direct-print which is a widely used process of creating conductive patterns, complex structure, and scaffold for tissue culture uses jetting technology including inkjet printing and continuous dispensing of filament as well. Processes such as, laser-guided direct writing, highly appreciate direct-print for applications in biotechnology [6] and complex functional materials [7].

In last few decades, inkjet technology has established itself as an essential class of technologies for many industries such as three–dimensional shaping [8], flat panel display [9], printed circuit boards (PCBs) [10], semiconductor packaging, and DNA chip and biosensors [11]. Direct-print, being existent for decades, has been emerging as an advanced form of manufacturing of sensors, antenna, flexible circuits and complex 3D
structures [12–18]. Direct-print is either continuous or drop–on–demand based, hence, droplet study for impact and deformation plays a significant role for its development as this technology is being used for manufacturing micro-scale and nano-scale high precision devices.

Worthington [19] reported one of the earliest studies of liquid drop impact on a solid surface about a century ago. It is considered one of the pioneering works due to its relevance to many natural phenomena as well as industrial applications. This work made a significant progress in the understanding of the droplet impact process, and it has become a fundamental basis for impact study from the theoretical, computational, and experimental aspects. Another more recent work of Rioboo et al. [20] made a significant contribution to the understanding of the droplet deformation process. Their panoptic experimentation revealed the mechanism for the formation of crown during impact of a single droplet on the wetted surface. Yarin [21] reviewed the dynamics of drop impact comprehensively and delineated many interesting phenomena such as splashing, spreading, receding, bouncing, crown formation. A significant amount of research has been carried out to investigate the deformation process of water droplet, due to its intensive application in industry specially, in manufacturing and spray cooling processes. Wachters and Westerling [22] conducted an extensive experimental study on water droplet deformation. They investigated the consequent volume decrease of water droplet after impact and the bottom radius of the deforming droplets. Chandra and Avedisian [23] conducted an experimental investigation on the deformation of n–heptane over a hot surface at different temperatures. A computational analysis of water droplet deformation has been performed by Songoro et al. [24]. Hatta et al. [25] performed an experimental
and computational study of deformation of water and n–heptane droplet. They concluded that their zero surface tension model was in accordance with experimental results reported in Chandra and Avedisian [19]. Rioboo et al. [26] experimentally investigated the time evolution of deformation of droplet, due to its impact on a dry surface. They conducted their study for different Reynolds number, \( \text{Re} = \frac{\rho u^* L_R}{\mu^*} \), Weber number, \( \text{We} = \frac{\rho u^* t^* L_R}{\sigma^*} \), for glycerin, silicone oil and water. The performed the experiment on different solid surfaces and those, are glass, coated glass and polymer. Their work on droplet deformation has been benchmarked as one of the most extensive work for deformation of droplet. They divided the spreading factor plot into four distinctive phases: kinetic, spreading, relaxation and wetting phases. At the first phase of impact, the liquid is compressed for a sudden pressure increase and a reduction of volume takes place [22]. Formation of a shock wave occurs in case of high impact velocities. At the end of the first phase, the spreading phase begins to be apparent, which is characterized by the formation of a radially expanding film. This is literally the spreading phase as the area of a droplet increases at this stage. The following phase, the relaxation phase, is characterized by either recoiling or stabilization of the radially expanding film of liquid. At the last stage, further wetting or equilibrium state is established.

For direct-print, the dispensed droplet needs to be cured by light or heat source to create a pre-designed pathway or pattern to achieve a 3D structure. For this reason, maintaining the desired dimensional accuracy and desired shape is very crucial. Most of the experimental and computational work has been carried out for either fundamental study of this natural phenomenon or a specific industrial application. To the authors’ best knowledge very few experimental works have been done. For droplet deformation and
Figure 1: Schematic of the (a) physical domain for droplet dispensing

(b) droplet deformation: definition of the instantaneous width
analysis for direct-print application, and there is a lack of computational work for the investigation of the droplet deformation in terms of identifying the initial steady width. This initial steady width is a key factor of effective curing based direct-print technology. Because, curing of the dispensed droplet needs to be initiated and completed within this time. So the initiation and the intensity of curing depend on the small window of time which is this initial steady period. In this present work, the droplet deformation has been numerically investigated up to the spreading stage [26] for different governing parameters. Chapter VI elaborates these results of this study.

2.3. Filament Dispensing

Dispensing of filaments is the majority of the application for direct-print technology. The application of filament dispensing ranges from printing of conductive material to fabrication of 3D scaffolds, 3D standing structures and 3D printing. For direct-print, one of the pioneering works has been reported by Lewis and Gratson [27]. They discussed the capabilities of ink based writing of various methods which include Robotic Deposition, Three-dimensional printing, Ink-jet printing, Fused deposition Modeling, Micro-pen writing, Dip-pen nanolithography, Scanning probe contact printing. They also discussed the suitable materials for each of the direct print technologies and the possible minimum feature size attainable. Their work is one of the most decorative literatures for direct-print that includes filament based, contact based and droplet based printing. The majority of the applications of these direct-print techniques is for patterned structures and scaffolds which have various applications such as tissue engineering, microfluidics, and more.
Geng et al. [28] also reported a robotic system for printing of a chitosan scaffold. The dispensing system reported here is a filament based system. They reported a thorough description of the materials and fabrication process for their dual dispensing processes. They reported three primary process control parameters those are responsible for effective printing. They are dispensing pressure, dispensing speed and height, increment of each layer. The pressure dictated the dispensing or extrusion velocity of the chitosan material. The proper process control for the dispensing pressure and dispensing speed needs to be ensured for the spreading of the chitosan so that after the sodium hydroxide can stabilize the dimension of the dispensed chitosan. Khalil et al. [29] reported biopolymer deposition for freeform fabrication of three-dimensional tissue scaffolds for multi–nozzle deposition system. Their works include both the drop–on–demand deposition and the continuous deposition (or the filament deposition) of biopolymers. Their results showed the relations of process parameters on deposition flow rate and scaffold structural formability. Ahn et al. [30] showed the application of filament based direct-print for printing of origami structures with folding procedure. They presented the folding method not only for simpler geometrical objects but also for complex geometrical objects such as rolled high aspect ratio cylindrical and spiral coils as well as origami objects such as origami birds. Similar application of filament dispensing for has also been reported in [31]. Application of direct-print for hydrogel based scaffolds is reported in [32]. The guided cell growth implements this hydrogel based scaffold. There are numerous other applications of direct print technology such as printed electronics, hybrid manufacturing and more. Ahn et al. [33] reported the fabrication of transparent conductive grids with filament based direct print system. Palmer et al. [34] presented the
fabrication of high-density circuitry using direct-print technology. Hybrid manufacturing such as integration of direct-print and ultrasonic consolidation for rapid prototyped parts has been reported by Robinson et al. [35]. Similar application for printed electronics and hybrid manufacturing of filament based direct printing has been reported in [36–43]. The application of such hybrid manufacturing system includes design and fabrication of heat sink antenna, conformal electronic packaging, printing of micro-battery, other microelectronics and micro-scale manufacturing. Lopez et al. [44] reported a combination of stereolithography and direct printing technologies for 3D structural electronics fabrication. Coming back to the bio application of filament based direct printing, Chang et al. [45] reported direct print based bio-printing for the purpose of future regenerative therapies. They found that the dispensing nozzle size, material flow rate (i.e., the dispensing velocity) and linear write speed (i.e., the nozzle travel speed) are very crucial for fabrication of the designed patterns. They reported that depending on the combination of these three parameters, there may be leading, balanced and lagging dispensing of material. Leading happens when the dispensed material stays in front of the dispensing nozzle during dispensing. For the case of filament lagging, the material is falling behind that is there is no material just under the tip of the dispensing nozzle. For the balanced deposition, the dispensed material is neither leading nor lagging, rather they just move along the dispensing nozzle. This is the most optimum condition of material dispensing. The leading causes the accumulation of material while printing cross lines (see, Figure 2). The lagging is comparatively better than leading as it does not create any accumulation of materials during the printing of cross lines. However, it can cause necking of the filament or reduction of the line width. For this reason, the lagging and leading during the
dispensing should be avoided or minimized. The most desirable condition is to obtain an optimum or ideal filament position during the dispensing. The motivation behind the work of filament deformation is to conduct a case study to investigate and determine the optimum parameters for obtaining this condition for filament. For the filament dispensing, the lagging and leading of the dispensed filament have been investigated for different processes and governing parameters. The numerical analysis reveals the effect of these governing and process parameters on the formation of leading, balanced and lagging front of the filament during the filament based direct printing technology.

Figure 2: Schematic of the problem description for filament dispensing

2.4. Problem Formulation

In the present study, (i) two–phase flow has been solved to investigate the droplet impact and deformation over a solid surface (Figure 1) and (ii) two–phase flow has been solved to investigate the deformation of the filament front during material dispensing (Figure 2).
The deformation of a droplet has been studied in terms of interface contour; spreading factor, $\xi$; and deformation ratio, $R_\delta$. The spreading factor is the ratio of instantaneous width to the initial diameter of the droplet. The deformation ratio is the ratio of instantaneous diameter to the instantaneous height of the deformed droplet. These parameters have been studied for different values of Froude number, $Fr$; Reynolds number, $Re$ and Weber number, $We$. The density ratio, $\rho_r$ is the ratio of liquid phase density, $\rho_L$ to gaseous phase density, $\rho_G$ while the viscosity ratio, $\mu_r$ is the ratio of liquid phase viscosity, $\mu_L$ to gaseous phase viscosity, $\mu_G$. For all the cases, the density ratio, $\rho_r$ and viscosity ratio, $\mu_r$ of liquid to gaseous fluid were kept fixed at 500 and 25, respectively, except the cases of varying Reynolds number. For the case of varying $Re$, $\mu_r$ has been varied for $10 \leq Re \leq 2 \times 10^3$, respectively. As the change of $Fr$ indicates the change of droplet velocity. For varying $Re$ cases the $Fr$ has been kept constant, $\mu_r$ has been varied to implement the variation of Reynolds number. Figure 1(a) illustrates the

<table>
<thead>
<tr>
<th>Case</th>
<th>$Fr$</th>
<th>$Re$</th>
<th>$We$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0, 2.5, 5.0, 7.5, 10, 15, 20</td>
<td>$Re = 1000$</td>
<td>$We = 20$</td>
</tr>
<tr>
<td>2</td>
<td>10, 25, 50, 75, 100, 250, 500, 750, 1000, 1250, 1500, 1750, 2000</td>
<td>$Fr = 10.0$</td>
<td>$We = 20.0$</td>
</tr>
<tr>
<td>3</td>
<td>0.10, 0.25, 0.50, 0.75, 1.0, 2.5, 5.0, 7.5, 10.0, 15.0, 20.0, 25.0, 30.0</td>
<td>$Fr = 10.0$</td>
<td>$Re = 1000$</td>
</tr>
</tbody>
</table>
A schematic of the problem described above and the definition instantaneous width ($D_t$) and height ($h_t$) are shown in Figure 1 (b). Table 2-1 gives the details of the dimensionless parameters considered for different cases in this investigation.

![Schematic of the physical domain for filament dispensing](image)

**Figure 3:** Schematic of the physical domain for filament dispensing

The investigation of filament deformation has been studied in terms of the interface contour. Figure 3 illustrates the schematic of the physical domain. The problem schematic shows, the developed model considers the problem as the dispenser being fixed and the substrate being in motion. The model considers that the gaseous medium is also moving with the same velocity of the moving substrate. For direct-print dispensing systems, the gap between the dispenser outlet and the substrate is usually less than a millimeter. It is less than the thickness of boundary layer if the flow laminar. This is the reason behind the assumption for considering the gaseous medium to be in the same velocity of the moving substrate. The dispenser diameter and the dispensing gap both are less than a millimeter. So, the flow is in micro-scale and at this scale the gravitational
force becomes less dominant. For this reason, a reduced gravitational effect approach has been considered for the dispensing material. However for the gaseous medium, these assumptions are not applicable. Therefore, the developed model does not consider the reduced gravitational effect for gaseous medium. The values of Froude number, \( Fr \); Reynolds number, \( Re \), Weber number, \( We \), Gap ratio, \( GR \) and velocity ratio, \( VR \) are varied for this case study of filament deformation. The gap ratio is the ratio of the height of the filament to the filament diameter. The velocity ratio is the ratio of the nozzle travel velocity to the dispensing (or feed) velocity. For all the cases, the density ratio, \( \rho_r \) were kept at \( \approx 1000 \). For the case of varying \( Re \), \( \mu_r \) has been varied for \( 0.01 \leq Re \leq 0.1 \). So the maximum value of Reynolds number is considered is 0.1. Table 2-2 provides the different cases for the filament front deformation.

<table>
<thead>
<tr>
<th>Fr</th>
<th>1.0, 1.5, 2.0</th>
<th>Re</th>
<th>0.01, 0.05, 0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>We</td>
<td>0.25, 1.0, 2.5</td>
<td>GR</td>
<td>0.5, 0.75, 1.0</td>
</tr>
<tr>
<td>VR</td>
<td>2,3,4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The computational analysis considers all the possible combinations of parameters. The Investigation looked into the effects of variation of a single parameter with the variation of each of the other parameter. For instance, this study investigated the effects of variation of Reynolds number for the variation of gap ratio while, the Froude number, the Weber number, and the velocity was fixed. Similarly, this study also investigated the
effects of variation of Reynolds number with the variation of Froude number, Weber number and velocity ratio, following the same manner.
CHAPTER III

MATHEMATICAL MODELING

This section presents the equations of conservation of mass and momentum, interface capturing methodology, and appropriate boundary conditions. Presented non-dimensionalization of the governing equations reveals the parameters those affect the considered problem for this study. A brief description of the numerical code Gerris Flow solver is provided, including some detail of the adaptive discretization.

The developed model considers the following assumptions: (i) the fluids are incompressible, immiscible, and Newtonian, (ii) fluid density and viscosity may vary from phase to phase, but are assumed constant in a particular phase, (iii) surface tension is constant, and (iv) the flow is laminar.

3.1. Governing Equation

A two–dimensional, two–phase flow case has been solved to investigate the deformation of impacting droplets. Figure 1 and 3 shows the details of the geometries for (i) droplet and (ii) filament deformation. The differential equations those govern the two–phase flow are, (i) the incompressible, variable-density, Navier–Stokes equations with surface tension and (ii) an advection equation for volume fraction (Interface capturing). The
advection equation needs a scalar function which is denoted by “$\bar{c}$”, below. The value of “$\bar{c}$” is one for one fluid and zero for the other one. The governing equations [46] are –

$$\nabla \cdot \vec{U} = 0$$  \hspace{1cm} (1)

$$\rho \frac{\partial \vec{U}}{\partial t} + \rho \vec{U} \cdot \nabla \vec{U} = -\nabla P + \mu \nabla^2 \vec{U} + \vec{F}_b$$  \hspace{1cm} (2)

$$\frac{\partial \bar{c}}{\partial t} + (\vec{U} \cdot \nabla) \bar{c} = 0$$  \hspace{1cm} (3)

In equation (2), $\vec{F}_b$ is the body force. The body force includes the gravity force, $\vec{F}_g$; and the surface tension force, $\vec{F}_{st}$. Therefore, $\vec{F}_b = \vec{F}_g + \vec{F}_{st}$. Where,

$$\vec{F}_g = \rho \vec{g} \text{ and } \vec{F}_{st} = \sigma \kappa \delta_s \vec{n}$$  \hspace{1cm} (4)

The Dirac distribution function $\delta_s$ signifies that the surface tension term is concentrated on the interface; $\sigma$ is the surface tension coefficient, $\kappa$ and $\vec{n}$ are the curvature and normal to the interface, respectively. Therefore, the equation (2) becomes –

$$\rho \frac{\partial \vec{U}}{\partial t} + \rho \vec{U} \cdot \nabla \vec{U} = -\nabla P + \mu \nabla^2 \vec{U} + \rho \vec{g} + \sigma \kappa \delta_s \vec{n}$$  \hspace{1cm} (5)

For two-phase flows, volume fraction, $c(x, t)$ of the fluid is used to define and separate the properties of two different fluids. The volume weighted formulae [46] written below evaluates the density and the viscosity for the two-phase problem, as –

$$\rho(c) \equiv c\rho_1 + (1 - c)\rho_2$$  \hspace{1cm} (6)

$$\mu(c) \equiv c\mu_1 + (1 - c)\mu_2$$  \hspace{1cm} (7)
where, \( \rho_1, \rho_2 \) and \( \mu_1, \mu_2 \) are the densities and viscosities of the first and second fluids, respectively. Here, the field “c” is either identical to “\( \tilde{c} \)” or is constructed by applying a smoothing spatial filter to “\( \tilde{c} \)”. The advection equation for the density can then be replaced with an equivalent advection equation for the volume fraction

\[
\frac{\partial c}{\partial t} + (\bar{U} \cdot \nabla)c = 0
\]  (8)

In scalar form,

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  (9)

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho g_x + \sigma \kappa \delta_x n_x
\]  (10)

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial P}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g_y + \sigma \kappa \delta_x n_y
\]  (11)

\[
\frac{\partial c}{\partial t} + u \left( \frac{\partial c}{\partial x} + \frac{\partial c}{\partial y} \right) = 0
\]  (12)

\[
\frac{\partial c}{\partial t} + v \left( \frac{\partial c}{\partial x} + \frac{\partial c}{\partial y} \right) = 0
\]  (13)

3.2. Normalization

To normalize the equations (9), (10), (11), (12) and (13) into non-dimensional form, the following parameters are defined
\[ x^* = \frac{x}{L_R} \quad y^* = \frac{y}{L_R} \quad u^* = \frac{u}{U_R} \quad v^* = \frac{v}{U_R} \quad g^*_x = \frac{g}{g_R} \]

\[ x = L_R x^* \quad y = L_R y^* \quad u = U_R u^* \quad v = U_R v^* \quad g = g_R g^*_x \]

\[ g^*_y = \frac{g}{g_R} \quad \rho^* = \frac{\rho}{\rho_R} \quad p^* = \frac{p}{p_R} \quad \mu^* = \frac{\mu}{\mu_R} \quad \sigma^* = \frac{\sigma}{\sigma_R} \]

\[ g = g_R g^*_y \quad \rho = \rho_R \rho^* \quad p = p_R p^* \quad \mu = \mu_R \mu^* \quad \sigma = \sigma_R \sigma^* \]

\[ t^* = \frac{t}{t_R} \quad t_R = \frac{L_R}{U_R} \quad \kappa^* = L_R \kappa \quad n_n^* = L_R n_n \]

\[ t = t_R t^* = \frac{t^* L_R}{U_R} \quad \kappa = \frac{\kappa}{L_R} \quad n_n = \frac{n_n^*}{L_R} \]

where, \( L_R, U_R \) and \( t_R \) are the reference length, velocities and time, respectively. \( L_R \) corresponds to the diameter of the droplet and filament for respective cases. \( u^* \) and \( v^* \) are the dimensionless velocities in \( x^* \) and \( y^* \) directions. Similarly, \( g^*_x \) and \( g^*_y \) are the components of gravitational acceleration in \( x^* \) and \( y^* \) directions and \( \rho^*, \mu^*, \sigma^*, \kappa^*, n^* \) and \( P^* \) are the dimensionless density, viscosity, surface tension, surface curvature, normal vector and pressure, respectively. The \( t^* \) is the dimensionless time. The following dimensionless parameters have been defined using the above normalizing parameters –

\[ Fr = \frac{u^*}{\sqrt{g^* L_R}} \quad Re = \frac{\rho^* u^* L_R}{\mu^*} \quad \text{and} \quad We = \frac{\rho^* u^{*2} L_R}{\sigma^*} \]

Normalizing the dimensional equations, the following non-dimensional governing equations can be found –
\[
\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0
\]

(13)

\[
\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial P^*}{\partial x^*} + \left(\frac{1}{Re}\right) \left(\frac{\partial^2 u^*}{\partial x^*^2} + \frac{\partial^2 u^*}{\partial y^*^2}\right) + \left(\frac{1}{Fr}\right) g^*_x + \left(\frac{1}{We}\right) \kappa^* \delta^*_n n^*_n
\]

(14)

\[
\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{\partial P^*}{\partial y^*} + \left(\frac{1}{Re}\right) \left(\frac{\partial^2 v^*}{\partial x^*^2} + \frac{\partial^2 v^*}{\partial y^*^2}\right) + \left(\frac{1}{Fr}\right) g^*_y + \left(\frac{1}{We}\right) \kappa^* \delta^*_n n^*_n
\]

(15)

\[
\frac{\partial c}{\partial t^*} + u^* \left(\frac{\partial c}{\partial x^*} + \frac{\partial c}{\partial y^*}\right) = 0
\]

(16)

\[
\frac{\partial c}{\partial t^*} + v^* \left(\frac{\partial c}{\partial x^*} + \frac{\partial c}{\partial y^*}\right) = 0
\]

(17)

3.3. Initial Condition

The developed model initialized the computational domain for both (i) the droplet and (ii) the filament deformation with zero velocities for the gaseous medium, and velocity corresponding to the Froude number for the dispensing fluid volume. For solving this problem, the model considers a fixed density ratio of (i) 500 for droplet deformation, and (ii) 1000 for filament deformation. The model initializes the dispensing fluid’s viscosity with the corresponding viscosity of the Reynolds number. For each case of droplet and filament deformation, the model initializes the dispensing fluid’s velocity and viscosity is from the corresponding Froude number and Reynolds number, respectively. The model initializes the surface tension coefficient associated Weber number of the case.
3.4. Boundary Condition

For the deformation of droplet, the computational domain considered for the problem is a square domain. The top and side walls have been considered as no-slip boundary, and the bottom wall has been considered to have a Navier–slip condition, i.e., the computational domain has been considered as an impermeable box with a slip condition at the bottom. The Navier-slip condition has been used to model the moving contact as moving contact

Table 3-1: Boundary condition for the case of droplet deformation

<table>
<thead>
<tr>
<th>Top wall</th>
<th>$u^* = 0$</th>
<th>$v^* = 0$</th>
<th>$P^* = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left wall</td>
<td>$u^* = 0$</td>
<td>$v^* = 0$</td>
<td>$P^* = 0$</td>
</tr>
<tr>
<td>Right wall</td>
<td>$u^* = 0$</td>
<td>$v^* = 0$</td>
<td>$P^* = 0$</td>
</tr>
<tr>
<td>Bottom wall</td>
<td>$u^* = u_0^* + \lambda \frac{\partial u^<em>}{\partial y^</em>}$</td>
<td>$v^* = 0$</td>
<td>$P^* = 0$</td>
</tr>
</tbody>
</table>

lines can be more accurately approximated using this. The values for coefficients at the Navier–slip condition, $u_0$ and $\lambda$ are 0.0 and 0.05, respectively. Table 3-1 shows the boundary condition for the droplet deformation case.

For the study of the deformation of the filament front, the computational domain has been considered as rectangular domain with a width that is ten times of its height. The model implements no-slip condition at the bottom wall, unlike the case of droplet deformation. A part of the top wall is inlet and the rest of are open boundary. The model implements
open boundary for the side walls. The detail of the boundary condition for filament front
deformation is given in Table 3-2.

Table 3-2: Boundary condition for the case of filament dispensing

<table>
<thead>
<tr>
<th>Wall Type</th>
<th>Condition 1</th>
<th>Condition 2</th>
<th>Condition 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right wall (i)</td>
<td>( u^* = VR \cdot u_{Fr} )</td>
<td>( v^* = 0 )</td>
<td>( P^* = 0 )</td>
</tr>
<tr>
<td>Top wall (ii), (vi)</td>
<td>( u^* = 0 )</td>
<td>( \frac{\partial v^*}{\partial n} = 0 )</td>
<td>( P^* = 0 )</td>
</tr>
<tr>
<td>Top wall (iii), (v)</td>
<td>( u^* = 0 )</td>
<td>( v^* = 0 )</td>
<td>( P^* = 0 )</td>
</tr>
<tr>
<td>Top wall (iv)</td>
<td>( u^* = 0 )</td>
<td>( v^* = u_{Fr} )</td>
<td>( P^* = 0 )</td>
</tr>
<tr>
<td>Left wall (vii)</td>
<td>( u^* = VR \cdot u_{Fr} )</td>
<td>( v^* = 0 )</td>
<td>( P^* = 0 )</td>
</tr>
<tr>
<td>Bottom wall (viii)</td>
<td>( u^* = VR \cdot u_{Fr} )</td>
<td>( v^* = 0 )</td>
<td>( P^* = 0 )</td>
</tr>
</tbody>
</table>
CHAPTER IV

NUMERICAL MODELING

4.1. General

In this chapter, a brief description of the numerical scheme that Gerris Flow Solver uses is described. Gerris implements an intermediate face-based (MAC) projection so that the advection velocity used in evaluating the convective terms satisfies the divergence constraint. This chapter provides the computational procedure, adaptive mesh refinement, grid sensitivity and validation of the developed model.

4.2. Computational Procedure

An open source finite volume code Gerris Flow Solver [47] has been used to model and solve this two–phase fluid flow problem. The computational domain is composed of square boxes. The domain is spatially discretized with square finite volumes or cells organized hierarchically as a quadtree [48]. Finkel and Bentley [49] first introduced the quadtree. It is a tree data structure that is used to partition a 2D space, dividing in half recursively, in each direction. Thus, a parent cell is being subdivided into four child cells. Each division is labeled as a level of refinement. For the undivided domain, the level of refinement is set as zero. With the increase of each refine level, each cell undergoes a division in half for each direction. For level of development of “x”, the number of cells in
each direction per unit length is $2^x$ and the grid spacing $\Delta x$ is $1/2^x$. Figure 4 illustrates the structure mentioned above.

![Quadtree spatial discretization and tree structure](image.png)

**Figure 4: Quadtree spatial discretization and tree structure [47]**

It shows with increase in each level of refinement; each parent is subdivided into four children cells. This type of organizational structure has originally been used for computer graphics and image processing, especially to find a particular pixel in a 2D image [50]. Later, it has been introduced for solving Euler equations for flows [51, 52]. Figure 5 shows the grid distribution for the computational domain. The initial discretization is $2^6$ and it goes up to $2^8$ near the interface with adaptive refinement. For the solution of the advection equation for volume fraction, a piecewise–linear geometrical Volume–Of–Fluid (VOF) scheme, generalized for the quadtree spatial discretization, has been used.
Scardovelli and Zaleski [53] and Popinet [47] describe the details of the numerical scheme. The numerical formulations are based on finite volumes, organized hierarchically as a quadtree, have been described in Popinet [46]. Gerris Flow Solver uses dynamic adaptive mesh refinement (ARM) with the quadtree (2D) and octree (3D) discretization. This adaptive refinement facilitates the solver adaptively to follow the small structures and features of the flow. The implementation of the adaptive refinement leads to an increased concentration of the computational effort on the area where it is most needed. This dynamic adaptive mesh refinement makes the solver more efficient and less costly from the perspective of computational resource. Since, Gerris Flow Solver
has implemented a tree-based discretization for space; it is somewhat simpler, in a relative sense, to incorporate a fully flexible adaptive refinement strategy. Equation (17) describes an example of adaptive mesh refinement criteria based on the vorticity of the flow –

\[
\frac{\Delta x \| \nabla \times \vec{u} \|}{\max \| \vec{u} \|} > \Gamma
\]  

(17)

here, \( \max \| \vec{u} \| \) is evaluated over the entire domain. The \( \Gamma \) is the threshold for the refinement criteria and it triggers on and off the refinement process when the criteria are fulfilled. For this particular example, it is interpreted as the maximum acceptable angular deviation. The computational cost associated with the implementation of this algorithm is relatively small compared to the cost for the solution of the Poisson solver. This dynamic adaptive mesh refinement algorithm required less than 5% of the total cost [47]. This low cost of the computational resource makes it affordable to be applied at every time step as the increase in the overall computational cost is negligible.

For the refinement at the first stage, those cells that satisfy the refinement criterion undergo the refinement process. The neighboring cells are also refined if the refinement is needed there for computation, to satisfy imposed constrains due to applying the quadtree structure [47, 49]. With the assumption of the flow being evolved slowly, the refinement is executed once per time step. At the second stage, the adaptive refinement method coarsens the cells which do not satisfy the refinement criterion. Figure 5 helps to have better understanding of this refinement and coarsening. The scheme that Gerris Flow Solver uses follows a second order accurate temporal discretization [46] –
\[
\rho_{n+1/2} \left[ \frac{\bar{u}_{n+1} - \bar{u}_n}{\Delta t} + \bar{U}_{n+1/2} \cdot \bar{U}_{n+1/2} \right] = -\nabla P_{n+1/2} + \mu_{n+1/2} \nabla^2 \left[ \bar{u}_n + \bar{U}_{n+1} \right] + (\sigma \kappa \delta_{x} \bar{n})_{n+1/2}
\]

(18)

\[
\left[ \frac{c_{n+1/2} - c_{n-1/2}}{\Delta t} + \nabla \cdot (c_n \bar{u}_n) \right] = 0
\]

(19)

\[
\nabla \cdot \bar{U}_n = 0
\]

(20)

This system is further simplified using a classical time-splitting projection method [35]

\[
\rho_{n+1/2} \left[ \frac{\bar{u}_* - \bar{u}_n}{\Delta t} + \bar{U}_{n+1/2} \cdot \bar{U}_{n+1/2} \right] = -\nabla P_{n+1/2} + \mu_{n+1/2} \nabla^2 \left[ \bar{u}_n + \bar{U}_* \right] + (\sigma \kappa \delta_{x} \bar{n})_{n+1/2}
\]

(21)

\[
\left[ \frac{c_{n+1/2} - c_{n-1/2}}{\Delta t} + \nabla \cdot (c_n \bar{u}_n) \right] = 0
\]

(22)

\[
\bar{U}_{n+1/2} = \bar{U}_* - \frac{\Delta t}{\rho_{n+1/2}} \nabla P_{n+1/2}
\]

(23)

\[
\nabla \cdot \bar{U}_{n+1} = 0
\]

(24)

For solving of the velocity field, a solution of Poisson equation is also needs. That is –

\[
\nabla \left[ \frac{\Delta t}{\rho_{n+1/2}} \nabla P_{n+1/2} \right] \bar{U} = \nabla \cdot \bar{U}_*
\]

(25)

For the improvement of the convergence time, the momentum equation is rearranged as

\[
\frac{\rho_{n+1/2}}{\Delta t} \bar{U}_* - \nabla^2 \left[ \mu_{n+1/2} \bar{U}_* \right] = \nabla^2 \left[ \mu_{n+1/2} \bar{u}_n \right] + (\sigma \kappa \delta_{x} \bar{n})_{n+1/2} + \rho_{n+1/2} \left[ \frac{\bar{U}_n}{\Delta t} - \bar{U}_{n+1/2} \cdot \nabla \bar{U}_{n+1/2} \right]
\]

(26)
In applying the VOF methods for surface-tension-driven flows, one of the most difficult tasks is the accurate estimation of the surface tension term \((\sigma \kappa \delta_s \vec{n})_{n+1/2}\). The original Continuum–Surface–Force (CSF) approach of Brackbill et al. [55] proposes the following approximations which have been considered in the scheme for calculating the surface tension term in the equation (21)

\[
\sigma \kappa \delta_s \vec{n} \approx \sigma \kappa \nabla \vec{c} \quad \quad \kappa \approx \nabla \cdot \vec{n} \quad \text{with} \quad \vec{n} \equiv \nabla c / |\nabla c|
\]

Here, \(\vec{c}\) either identical to \(c\) or spatially filtered value of \(c\). The estimation of the curvature of the interface is another difficult task while using VOF schemes. This inspires the use of alternatives such as front-tracking schemes [56, 57]; level set method [58]; and coupled VOF & level set method [59, 60]. Here, for the curvature estimation, the height function (HF) curvature calculation has been utilized for the computational scheme. Torrey et al. [61] first proposed this. The estimated curvature from the HF curvature calculation resembles more practicality. The accuracy of the estimated curvature is comparable to results reported by Cummins et al. [62].

4.3. Grid Independence

To determine the optimum grid resolution, this investigation performed a number of runs of a test case to test the grid independency of the given model. Figure 6 shows the representation of the droplet interface at each level of refinement from 4 to 9. It shows that a level refinement of 7, i.e., a mesh size \(2^7\), at each direction, or higher is good enough for an acceptable level of accuracy in case of interface representation. Figure 7 illustrates the grid independence test result for the case of a single droplet impacting on a
solid surface. The evolution of width ($D_t$) to height ($h_t$) ratio, i.e., deformation ratio, $R_\delta$ with time has been plotted for different level of refinement. For all the cases considered for Figure 7, the initial refine level was 4 ($2^4$ meshes per unit length). The figure shows an initial refine level 4 and an adaptive level of refinement of 7 or higher provides a very good result in terms grid independency. For the presented case, the initial and maximum adaptive refine levels are 6 and 8, respectively. Thus, the selection of level of refinement ascertains the acceptability of the chosen grid level.

![Diagram of droplet interface at different level of refinement](image)

**Figure 6:** Representation of a droplet interface at different level of refinement
Figure 7: Grid sensitivity result in term of deformation ratio, $R_{\delta}$

4.4. Experimental Setup

The validation experiments were performed with a computer controlled direct write system developed by Vatani et al. [16]. The system uses a screw-driven micro-dispensing head (PCD3, GPD Global, Grand Junction, CO). This micro-dispensing head is installed on a high precision xyz translation stage (Aerotech, Pittsburg, PA). The resolution of the translation stages is 500 nanometers. Figure 8 presents a schematic and a closure snapshot of the developed DW system, consisting of the xyz stage, micro-dispensing device, and imaging system. A consistent deposition of material is obtained by controlling the speed of the xy stage, the gap distance. The input voltage to the system controls the dispensing velocity.
4.5. Validation

In this section, the validation of the developed model has been provided. For the deformation of droplet, the developed model for has been validated against the experimental results reported at [25]. The computed evolution of droplet height and width after impacting a solid surface has been compared with the mentioned reference. Figure 9(a) shows that the computed results are in an acceptable agreement with the experimental results.

For the filament deformation, an assumption of reduced gravity has been implemented. Considering the height of dispensing, which is less than one millimeter and the dimension of the filament is also at that order. The considered problem is a micro-scale fluid flow problem. It is a common practice for micro-scale flows to ignore the effect of gravity. For the model developed, the validation shows that the use of much-reduced amount of
gravity for the dispensing medium gives a better match with the experiments. The model assumes a magnitude of 50 mm/s$^2$ as gravitational acceleration constant for the

Figure 9: Validation of the droplet deformation

![Validation of the droplet deformation](image)

Figure 10: Mass flow rate versus the input voltage for nozzle orifice of 0.564 mm

![Mass flow rate vs input voltage](image)

Mass Flow Rate = (3.1513 x Voltage) + 0.0232

$33$
dispensing fluid and 9.8 m/s² for gaseous medium. For the validation, this investigation matched the amount of filament lagging and / or leading of the computational results with experimental measurements. For all the experimental runs, the dispensing tip orifice was at 0.400 mm high form the substrate and the inner diameter of the used nozzle tip was 0.564 mm, and the outer diameter was 0.635 mm. This study validated the model with experiments performed with a commercially available Newtonian monomer (CN309, Sartomer LLC, Exton, PA) at room temperature (25 °C). The substrate for dispensing was a VWR micro slide.

![Graph showing velocity versus input voltage](image)

**Figure 11:** Velocity versus the input voltage for nozzle orifice of 0.564 mm

The GPD Global PCD Control unit, which controls the dispensing of the material, drives the screw inside the micro-dispenser. The flow rate of the dispensing head is proportional to the voltage input to the system. The flow rate of the dispenser varies with the input voltage and the orifice size of the dispenser nozzle tip. If the nozzle orifice size is fixed,
the flow rate of the system is linearly proportional to the input voltage. Figure 10 shows a mass flow rate plot against input voltage for a nozzle orifice size of 0.564 mm, for a material with a density of 920 Kg/m$^3$ (a commercial monomer, CN309 from Sartomer Americas). Figure 11 plots the relation between input voltage and velocity at the nozzle tip.

![Images of nozzle tips with dispensed material](a) (b) (c)

Figure 12: Dispensing of CN309 – dispensing height 0.4 mm, nozzle diameter 0.564 mm, velocity at the nozzle orifice 5.6 mm/s and substrate travel speed

(a) 5 mm/s, (b) 10 mm/s, (c) 15 mm/s

Figure 12 shows the pictures from the experiments of dispensing of CN309 with a nozzle of 0.564 mm orifice size. The Material properties of CN309 are, density 920 Kg/m$^3$, viscosity 0.15 Kg/m-s or 150 centipoise and surface tension 0.0263 N/m. Due to the surface tension, the dispensed material climbs up along the nozzle tip after being dispensed. This ascension of material causes an increment of the initial diameter of the dispensed filament. The initial diameter of the filament is the same as the orifice diameter or the inner diameter of the nozzle tip. Due to the ascending of the filament along the nozzle tip, the final diameter of the filament becomes slightly larger than the outer
diameter of the dispensing nozzle tip. An increase of the diameter causes an increase of the cross-sectional area of the filament. Due to the continuity law of fluid, the velocity of the dispensed filament decreases with the increase of the cross-sectional area. Therefore, the initial filament velocity and height change after the initiation of the dispensing.

Figure 13: Schematic of the surface tension effect on the development of filament

During this period, the filament climbs up the nozzle tip due to the capillary effect. So due to the climbing, the filament height also gets increased. The filament stops further climbing after reaching a certain height. This is because, the gravitational effect starts to overcome the capillary effect at that point. At that stage, the final development of the filament occurs as it starts to descend. While descending, the diameter and the velocity of the dispensing have changed. Due to the increase in cross-sectional area, the filament velocity decreases. Figure 13 depicts the schematic of this phenomenon.

Figure 14 presents the comparison of the experimental and the computational result. For the computation, the model considers outer diameter of the dispensing nozzle tip as the
(a) Dispensing speed 5.6 mm/s, substrate travel speed 5 mm/s

(b) Dispensing speed 5.6 mm/s, substrate travel speed 10 mm/s

(c) Dispensing speed 5.6 mm/s, substrate travel speed 15 mm/s

Figure 14: Comparison of experimental (left) and computational result (right) for filament dispensing
increased diameter of the filament. The velocity of the filament at its developed stage has been calculated using the continuity equation –

\[ A_o v_o = A_f v_f \]  

(27)

Here, “A” is the cross-sectional area and “v” is the velocity. The subscripts “o” and “f” indicated initial and final, respectively. The initial velocity is the velocity calculated at the orifice of the dispensing nozzle tip (Figure 11). For the experiments, the input voltage was set to 0.5 volt. The corresponding initial velocity is 5.6 mm/s, approximately. The inner and outer diameters of the nozzle tip are 0.564 mm and 0.635 mm. Therefore, from equation (27) –

\[ \pi \left( \frac{0.564}{2} \right)^2 \times 5.6 = \pi \left( \frac{0.635}{2} \right)^2 v_f \]

\[ \Rightarrow v_f = \left( \frac{0.564}{0.635} \right)^2 \times 5.6 \]

\[ \Rightarrow v_f \approx 4.4 \]

Therefore, the final filament velocity is 4.4 mm/s, approximately. The approximate increased filament heights for the cases showed in Figure 14 (a), (b) and (c) are 0.630 mm, 0.593 mm and 0.573 mm, respectively. The substrate travel speed was set to 5.0 mm/s, 10 mm/s and 15 mm/s for Figure 14 (a), (b) and (c), respectively. This trend of increased height shows that with the increase in the velocity ratio of travelling to dispensing, the height ascended by the filament decreases. Figure 15 illustrates a comparison of the experimental and computational results for the filament deformation for the cases mentioned above. It shows that the experimental and the computational results are in a very good agreement. Figure 15 shows that for lower velocity i.e.
dispensing velocity of 4.4 mm/s and travelling velocity of 5.6 mm/s, the error is higher than higher velocity ratio. For higher velocity ratios, the results show good agreement with the experimental data. The velocity ratios considered for the presented study are 2, 3 and 4. The results presented in Figure 15 assure that the presented results of this study have an excellent reliability.
CHAPTER V
RESULTS AND DISCUSSION

5.1. General

In this study, two–dimensional, two–phase flow has been solved to investigate the (i) droplet impact and deformation over a solid surface and (ii) the deformation of dispensed filament and the development of the filament front is investigated to optimize the direct printing process. This investigation studies the deformation of droplets in terms of interface contour, spreading factor, $\zeta$; and deformation ratio, $R_d$. For varying Froude number case, the values of $Fr$ have been taken as 1.0, 2.5, 5.0, 7.5, 10.0, 15.0, and 20.0. $Re$ and $We$ were kept at a constant value of 1000 and 20.0, respectively. For varying Reynolds number cases, the values for Reynolds number, $Re$ has been taken as 10, 25, 50, 75, 100, 250, 500, 750, 1000, 1250, 1500, 1750 and 2000, keeping $Fr$ and $We$ at a constant value of 10.0 and 20.0, respectively. For the varying Weber number case, the values for $We$ has been taken as 0.1, 0.25, 0.50, 0.75, 1.0, 2.5, 5.0, 7.5, 10.0, 15.0, 20.0, 25.0 and 30.0, keeping $Fr$ and $Re$ at a constant value of 10.0 and 1000, respectively.

This investigation studied the deformation of the filament in terms of the interface contour. Figure 3 provides the schematic of the problem. The study has been performed for different values of Froude number, $Fr$; Reynolds number, $Re$, Weber number, $We$, Gap ratio, $GR$ and velocity ratio, $VR$. The gap ratio is the ratio of the height of the
dispensing to the filament diameter, and the velocity ratio is the ratio of the nozzle travel velocity to the dispensing (or feed) velocity. For all the cases, the density ratio, $\rho_r$ were kept fixed at 1000. For the case of varying $Re$, $\mu_r$ has been varied for $0.01 \leq Re \leq 0.1$. The values of $Fr$ for the filament deformation investigation have been taken as 1.0, 1.5, and 2.0. The Weber number has been taken as 0.25, 1.0, and 2.5. The gap ratio of the problem has been considered as 0.5, 0.75, and 1.0. The velocity ratio has been taken as 2, 3, and 4.

5.2. Droplet Deformation

In this section, the deformation of droplet is discussed for the parameters considered. The effects of the variation of these individual parameter are discussed in individual subsections –

5.2.1. Effect of Froude number

Figure 16 and Figure 17 presents the effect of Froude number for droplet deformation. Figure 16 shows the interface contour for the droplet deformation for $Fr$ values of 1.0, 5.0, 10.0 and 20.0. The interface contour for different $Fr$ has been presented at $t^* = 0.0443, 0.1772, 0.2544$ and 0.3909. Figure 17 (a) is the plot for spreading factor, $\xi$ against dimensionless time for various Froude number. For this case of varying Froude number, the density ratio, $\rho_r$ and the viscosity ratio, $\mu_r$ has been kept at 500 and 25, respectively. From Figure 16, for lower values of Froude number, such as $Fr = 1.0$;
after the impact, there is an initial compression phase and then the droplet starts to spread at the spreading phase. But, for this range of $Fr$, there is no thin spreading film is
observed which is usually visible at the spreading phase. The circular droplet tends to take a hemispherical shape at $t^* = 0.1772$. With further deformation, the drop transforms into a dome-like shape at around $t^* = 0.3909$. For $Fr \geq 5.0$, the presence of a thin spreading film is observed during the spreading phase. Within this range of Froude number, $5.0 \geq Fr \geq 20.0$, the spreading film is thicker for lower value of $Fr$. With increasing value of $Fr$, the spreading film gets thinner and wide spread for the same dimensionless time, $t^*$. This trend is obvious from Figure 17(a). During the spreading phase at a relatively higher value of $Fr$, the instantaneous height, $h_t$ seems to have the magnitude of same order, though the instantaneous width, $D_t$ has greater magnitude at any dimensionless time, $t^*$.

![Graph](a) ![Graph](b)

**Figure 17:** Evolution of droplet deformation for various $Fr$ at $Re = 1000$, $We = 20.0$

(a) spreading factor, $\xi$ (b) deformation ratio, $R_\delta$

Figure 17 (b) illustrates the deformation ratio against the dimensionless time for various Froude number. It shows the ratio of $D_t$ to $h_t$ is higher for higher values of $Fr$ at any
Figure 18: Droplet deformation for various Reynolds Number, $Re$ at $Fr = 10.0$, $We = 20.0$
dimensionless time. With the increase of $Fr$, $Dt$ is higher for any certain dimensionless time, and the $h_t$ is at a similar order. This is why $R_\delta$ increases as the magnitude of $Fr$ increases.

5.2.2. Effect of Reynolds number

Figure 18 and Figure 19 presents the effect of Reynolds number for droplet deformation. Figure 18 shows the interface contour for the droplet deformation for $Re$ values of 10, 100, 1000 and 2000. The interface contour for different $Re$ has been presented at $t^* = 0.0044, 0.1772, 0.3101$ and 0.4429. For this case of varying Reynolds number, the $Fr$ and the $We$ have been kept 10.0 and 20.0, respectively, while keeping $\rho_r$ at 500. However, $\mu_r$ has been varied with the values of $Re$. As the Froude number, $Fr = \frac{u^*}{\sqrt{gL_R}}$ has
been kept fixed for this case and varied with varying velocity, $u^*$. For having varying $Re$ the viscosity ratio has been varied. In other words, the viscosity of the liquid phase has been varied with fixed velocity, $u^*$ for having varying $Re$. For lower values of Reynolds number ($Re = 10$), the deformation trend is similar to that of lower $Fr$ values. For $10^2 \leq Re \leq 2 \times 10^3$, there is a visible spreading film in Figure 18. For $Re \leq 10^2$ at the spreading phase, there is a formation of rim at $t^*$ being as low as 0.1772. Figure 19(a) shows the steady phase for the range of $Re$ considered, is within $0.05 \leq t^* \leq 0.1$ for an decreasing trend of $Re$. With increasing value of $Re$, the steady phase diminishes more rapidly. Figure 19(b) shows the deformation ratio for the range of $Re$ considered. Figure 19(a) and 19(b) shows that both the spreading factor, $\zeta$ and the deformation ratio, $R_\delta$ increases more rapidly for $10 \leq Re \leq 5 \times 10^2$ compared to $10^3 \leq Re \leq 2 \times 10^3$.

5.2.3. Effect of Weber number

Figure 20 and Figure 21 presents the effect of Weber number for droplet deformation. Figure 20 presents the interface contour of the droplet deformation for $We$ values of 0.10, 5.0, 15.0 and 30.0. The interface contour for different $We$ has been presented at $t^* = 0.0044, 0.3101, 0.6201$ and 0.8859. For this case of varying Weber number, the $Fr$ and the $Re$ have been kept at 10.0 and 1000, respectively. The density ratio and viscosity ratio have been kept at 500 and 25, respectively. For this case of varying Weber number, for all values of $We$ there is a characteristic spreading film can be seen at the spreading phase. For lower value of Weber number ($We = 0.10$), the spreading film observed is relatively thicker, and deformation rate is lower. As $We$ increases, at higher
Figure 20: Droplet deformation for various Weber number, $We$ at $Fr = 10.0$, $Re = 1000$

dimensionless time, the spreading profile of surface contour seems to change from more spherical to more flat shape at any $t^*$. This is obvious as increasing $We$ corresponds to a
5.3. Filament Deformation

This section discusses the development of the filament and its deformation and the effects of the aforementioned dimensionless governing and process parameters on this.
The Froude number for the filament deformation analysis has been calculated considering the reduced value of gravitational acceleration that has been used for the validation of the model. So, the Froude number for the filament deformation analysis is a modified Froude number and different from the Froude number that has been used for the droplet deformation analysis. The results for all the combinations of the governing and process parameters have been organized to present the best combinations to get a better insight of their effects on filament development and its deformation. Figure 2 shows, after the filament impacts on the surface, the filament will deform, and the filament front will either deform into a lagging or leading shape. During some direct printing processes, such as, instantaneous curing direct print to create 3D structures; sometimes there are multiple layers of printing is necessary, and the lines are to drown in cross directions. In such cases, from practical experiences, it was observed that, a minimum amount of filament lagging is desired. The reason behind this is, if the filament forms a leading front; it causes some accumulation of dispensed material at the junction of the cross lines. These accumulated are then cured instantaneously. The cured accumulation of material then causes a fabrication fault. Hence, the formation of leading filament front is undesired for the printing process. The parametric study of the considered parameters was chosen to determine the region of governing and process parameters that will help to achieve a lagging filament front during dispensing process.

Figure 22 shows a case for the investigation of the filament deformation. In this figure, the development of filament has been presented. It shows how a filament is deformed before, during and after the impact. After the impact, the filament initially continues to
Figure 22: Filament evolution for $Fr = 1.5$, $Re = 0.01$, $We = 0.25$, $GR = 0.75$ and $VR = 2$
deform. this stage determines whether there will be the formation of lagging or leading filament front. At the end of this stage, the filament reaches its final developed form and continues to retain this form as the dispensing continues. The filament deformation in this section has been presented in terms of this developed filament front. Figure 22, shows the evolution of the filament front during, and after the impact with dimensionless time. From Figure 22, it can be clearly seen, after $t^* = 1.17$, there is no observable deviation of the filament front.

Figure 23 and Figure 24 presents the mechanism behind achieving the final form of the filament front. It shows the simulation of the case presented in Figure 14 (a), in the validation section. Figure 23 shows the change of the vorticity field and the velocity vector with time and Figure 24 shows the velocity field and the velocity vector with time. Figure 23 shows; there are two vortices at two sides of the filament that determine the change of the filament shape and the final form of the filament during the dispensing. Figure 23 (c) shows the vortices become stronger when the filament reaches close to the substrate. Figure 24 (c) also shows that there is a significant increase of velocity and the gaseous medium between the filament and substrate rushed outward. Figure 23 (c) shows the sudden rush that causes an increase in the vorticity field. These two vortices and the different of their strength determine the change of the filament shape during the impact. After the filament impinges on the substrate, the vortex at the upstream side (right side of the filament) affects the shape of the filament front. The interaction between the velocity fields of the dispensed phase and the gaseous phase also has a role in the evolution of and the final form of the filament. At the interface, the velocity fields of these two phases interact and the filament interface aligns itself to the resultant of these two velocity field.
Figure 23: Vorticity field and velocity vector plot - dispensing velocity of 4.4 mm/s, substrate velocity 5 mm/s, 0.63 mm gap between nozzle and substrate
Figure 24: Velocity field and velocity vector plot - dispensing velocity of 4.4 mm/s, substrate velocity 5 mm/s, 0.63 mm gap between nozzle and substrate
Figure 23 (b) and (c) shows how the velocity field of the dispensed phase near the interface is being affected by the velocity field of the gaseous phase. The change in the velocity field near the interface affects the velocity field of the center region of the filament phase. Figure 24 (d) - (j) shows that after the filament impinge on the substrate, a region at the lower front of the filament is developed. In this region, the velocity of the fluid particle is very small. Once this region is fully developed, the leading of the filament front stops and the filament attains its final form. Figure 25 (d) shows that for a lagging filament front, there is such region near the filament front. Figure 25 illustrates

![Leading Filament](image-a)

![Lagging Filament](image-b)

(a) $t = 0.340$ s  
(b) $t = 0.200$ s

(c) $t = 0.340$ s  
(d) $t = 0.200$ s

Figure 25: Vorticity field with velocity vector for (a) leading and (b) lagging filament and velocity field with velocity vector (c) leading and (d) lagging filament.
the vorticity and the velocity field with a velocity vector for the cases described in Figure 14 (a) and (b). The leading filament results are the simulation of the case described in Figure 14 (a) and lagging filament results are the simulation of the case described in Figure 14 (b). Figure 25 (d) shows that at the developed stage of filament for the case of lagging, there is a big region of minimum velocity at the upper left section of the filament. At that region, the velocity is less than the dispensing velocity. The dispensed material moves faster at the region near the interface of the filament front. This causes a thinner layer thickness in case of lagging filament front and also stabilizes a consistent filament front when the substrate velocity is higher than the dispensing velocity. Figure 25 (a) and (b) also shows that the thickness of the dispensed filament layer, for the case of filament lagging, is lesser than the case of filament leading.

5.3.1. Effect of Froude number

In this section, the effect of Froude number is presented. Figure 26, depicts the effects of Froude number on the developed filament front shape. In this figure, the developed front of dispensed filaments for different Froude number has been presented. For this case, the velocity ratio, VR for all the cases is 2. The effects of variation of Froude number at different Reynolds number has been presented while keeping other parameters constant at $We = 0.25$, $GR = 0.75$. The results show that with increasing Froude number, the filament front tends to have more lagging effect. This lagging effect is more apparent for lower Reynolds number than higher Reynolds number. Figure 26 shows that the change in
Figure 26: Effect of Froude number variation on filament deformation at different combinations.

Filament front for variation of Froude number is less apparent for higher Reynolds number. For high Froude number, if the Reynolds number is low (for, Re = 0.01) then there is more apparent lagging. When Reynolds number is high (For, Re = 0.1) and other conditions are same, there is less apparent lagging of the filament front. This is due to a decrease in viscosity for higher Reynolds number. The higher viscosity lets the filament spread less for the same process parameters. This is why, low Froude number and low
Reynolds number favors the lagging or optimum filament front. The effect of Froude number variation on filament at different gap ratio has been presented while keeping other parameters constant at $Re = 0.01$, $We = 0.25$. The results show that a decrease in the gap ratio causes a filament to show more leading tendency. As for higher gap ratio (for, $GR = 1.0$), the leading effect is less apparent. The change in Weber number does not have any visible effect on the filament front shape. Figure 26 shows that for high Weber number (for, $We = 2.5$) and for low Weber number (for, $We = 0.25$), the filament front shape change remains visibly unaltered for corresponding Weber number. That means the trend of filament front shape with various Froude number at high Weber number is visibly same for low Weber number. This lack of visible change indicates that even though surface tension is dominant at this scale of the flow, it is less determinant for the developed filament front shape. From the experimental investigation at the validation section, it was found that the domination of the surface tension at this scale of the flow causes the filament to climb along the dispenser nozzle tip diameter. However, it has no visible effects on the final form of the developed filament front.

5.3.2. Effect of Reynolds number

The section discusses the effect of Reynolds number variation on the filament development. Figure 27 illustrates the effects of Reynolds number on the developed filament front shape. The results show that the development of the filament front is quite sensitive to the change of Reynolds number. The effect of Reynolds number variation with the change of Froude number has been presented for $We = 0.25$, $GR = 0.75$, $VR = 2$. Figure 27 shows for a fixed Froude number, there is a more lagging tendency of the
filament for low Reynolds number \((Re = 0.01)\) compared to a high Reynolds number \((Re = 0.1)\). With higher Froude number, the dispensing speed gets increases. The travel speed

\[
\begin{array}{ccc}
Re = 0.01 & Re = 0.05 & Re = 0.1 \\
Fr = 1.0 & Fr = 2.0 \\
Gr = 0.5 & Gr = 1.0 \\
We = 0.25, GR = 0.75, VR = 2 \\
Fr = 1.0, We = 0.25, VR = 2 \\
Fr = 1.0, GR = 0.75, VR = 2 \\
Fr = 1.0, GR = 0.75, We = 0.25 \\
Fr = 1.0, GR = 0.75, We = 0.25
\end{array}
\]

Figure 27: Effect of Reynolds number variation on filament deformation at different combinations.
also increases with the increase of Froude number, as the velocity ratio is fixed. It results in slightly more lagging filament front for high Froude number. The effect of Reynolds number variation with the change of gap ratio has been presented for $Fr = 1.0$, $We = 0.25$, $VR = 2$. The results presented in Figure 27 shows, for low gap ratio ($GR = 0.5$) there is no lagging or optimum filament shape of the filament for $0.01 \leq Re \leq 0.1$. However, as the gap ratio increases (such as, $GR = 1.0$), for low Reynolds number ($Re = 0.01$), slightly more lagging trend is apparent. The effect of Reynolds number variation with the change of Weber number has been presented for $Fr = 1.0$, $GR = 0.75$, $VR = 2$. Similar to the discussion provided in the previous section, there is no visible effect of Weber number on the development of the filament front. The effect of Reynolds number variation with the change of velocity ratio has been presented for $Fr = 1.0$, $GR = 0.75$, $We = 0.25$. The results show that the velocity ratio is a very significant parameter for the development of the filament. Velocity ratio and Reynolds number combination is one of the most dominant deciding factor for the filament front condition during the dispensing, apart from the gap ratio. Figure 27 indicates, for low Reynolds number ($Re = 0.01$), the optimum filament front can be seen for low velocity ratio ($VR = 2$). As the velocity ratio increases (such as, $VR = 4$), an optimum filament or slightly lagging filament front seems to be achievable for higher Reynolds number too.

5.3.3. Effect of Weber number

This section illustrates the effect of the variation of Weber number for the filament front development. Figure 28, describes the effects of Weber number on the developed
filament front shape. The effect of Weber number variation with the change of Froude number has been presented for \( Re = 0.01, \ GR = 0.75, \ VR = 2 \). The effect of Weber number on filament deformation at different combinations.

- **Fr = 1.0**
  - \( Re = 0.01, \ GR = 0.75, \ VR = 2 \)
  - \( Fr = 1.0, \ GR = 0.75, \ VR = 2 \)
  - \( Fr = 1.0, \ Re = 0.01, \ VR = 2 \)

- **Gr = 0.5**
  - \( Fr = 1.0, \ Re = 0.01, \ VR = 2 \)

- **VR = 2**
  - \( Fr = 1.0, \ Re = 0.01, \ GR = 0.75 \)

**Figure 28**: Effect of Weber number on filament deformation at different combinations.
number variation with the change of Reynolds number has been presented for $Fr = 1.0$, $GR = 0.75$, $VR = 2$. The effect of Weber number variation with the change of gap ratio has been presented for $Fr = 1.0$, $Re = 0.01$, $VR = 2$. And, the effect of Weber number variation with the change of gap ratio has been presented for $Fr = 1.0$, $Re = 0.01$, $GR = 0.75$. Figure 28 presents the results which clearly show that the variation of Weber number merely dictates the filament front condition during the dispensing.

5.3.4. Effect of Gap Ratio

This section elaborates the effect of the variation of gap ratio variation on the filament development and its deformation. Figure 29 is a summary of the investigation of filament development for the variation of the gap ratio. The presented results show; the gap ratio is a crucial factor that has a significant effect on the development of filament. The developed filament, whether it will have a lagging or leading front, significantly depends on the gap ratio. The effect of the variation of the gap ratio with the change of Froude number has been presented for $Re = 0.01$, $We = 0.25$, $VR = 2$. Figure 29 shows, for low Froude number (for, $Fr = 1.0$), there is more leading tendency than for high Froude number (for, $Fr = 2.0$) with the variation of gap ratio. In other words, for the same gap ratio, an increase in Froude number yields less tendency of a leading filament front. For the same Froude number, increase in gap ratio results in more lagging tendency of the filament front. The effect of the variation of the gap ratio with the change of Reynolds number has been presented for $Fr = 1.0$, $We = 0.25$, $VR = 2$. Results indicate that increase in Reynolds number yields more tendency of leading. For high Reynolds number ($Re =$
0.1) and high gap ratio ($GR = 1.0$), an optimum filament front is found in Figure 29. For low Reynolds number ($Re = 0.01$), there are optimum filament front found for $GR = 0.75$

<table>
<thead>
<tr>
<th>GR = 0.5</th>
<th>GR = 0.75</th>
<th>GR = 1.0</th>
<th>GR = 0.5</th>
<th>GR = 0.75</th>
<th>GR = 1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fr = 1.0</td>
<td>Fr = 2.0</td>
<td>Fr = 1.0</td>
<td>Fr = 2.0</td>
<td>Fr = 1.0</td>
<td>Fr = 2.0</td>
</tr>
<tr>
<td>Re = 0.01, We = 0.25, VR = 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fr = 1.0, We = 0.25, VR = 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>We = 0.25</td>
<td>We = 2.5</td>
<td>Fr = 1.0, Re = 0.01, VR = 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VR = 2</td>
<td>VR = 4</td>
<td>Fr = 1.0, Re = 0.01, We = 0.25</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 29: Effect of Gap Ratio on filament deformation at different combinations
and $GR = 1.0$. The effect of the variation of the gap ratio with the change of Weber number has been presented for $Fr = 1.0$, $Re = 0.01$, $VR = 2$. As established in the previous sections, the Weber number variation does not have any visible effect on the development of the filament. The effect of the variation of the gap ratio with the change of velocity ratio has been presented for $Fr = 1.0$, $Re = 0.01$, $We = 0.25$. The velocity ratio plays a significant role for the developed filament front condition. The results presented in Figure 29 indicate; both the velocity ratio and the gap ratio favor the lagging condition when they are increasing. For a fixed velocity ratio, an increase in gap ratio results in a more lagging tendency during the development of the filament. Similarly, for a fixed gap ratio, an increase in velocity ratio results in a more lagging tendency during the development of the filament. In other words, Figure 29 shows; an increasing gap ratio at low velocity ratio (for, $VR = 2$) tends to form a lagging filament front. For high velocity ratio (for, $VR = 4$), this tendency is even more amplified.

5.3.5. Effect of Velocity Ratio

This section illustrates the effect of the variation of velocity ratio variation on the filament development. Figure 30 presents the effect of velocity ratio for the filament front during dispensing. The results in Figure 30 show that, velocity ratio is one dominant factor for deciding the filament front condition. The effect of variation of velocity ratio has been presented for various combinations of Reynolds number, Weber number and gap ratio. The effect of the variation of the velocity ratio with the change of Reynolds number has been presented for $Fr = 1.0$, $We = 0.25$, $GR = 0.75$. It is obvious that higher travelling speed will cause the filament to have more lagging trend. However, Figure 30
shows that for high Reynolds number ($Re = 0.1$) the impact of increasing velocity ratio is less than low Reynolds number ($Re = 0.01$). The effect of the variation of the velocity ratio with the change of Weber number has been presented for $Fr = 1.0$, $We = 0.25$, $GR = 0.75$. As found from the discussion in the previous sections, the Weber number does not have any visible impact on the filament front formation. The effect of the variation of the velocity ratio with the change of gap ratio has been presented for $Fr = 1.0$, $Re = 0.01$, $We = 0.25$.

Figure 30: Effect of Velocity Ratio on filament deformation at different combinations
1.0, $Re = 0.01$ and $We = 0.25$. For low gap ratio ($GR = 0.5$), the lower velocity ratio ($VR = 2$) results a filament with leading front. The high velocity ratio ($VR = 4$) for low gap ratio ($GR = 0.5$) results a more optimum filament front. For high gap ratio ($GR = 1.0$), even for low velocity ratio ($VR = 2$) there is an optimum filament front profile is visible. As the velocity ratio increases up to $VR = 4$, a large measure of filament lagging is found. From the point of view of direct-print optimization, such great extent of filament lagging is as much undesirable as the leading filament. This amount of excessive filament lagging causes very narrow printed lines.
CHAPTER VI
CONCLUSION

In this Study, two–dimensional, two–phase Newtonian fluid flow problem of (i) droplet and (ii) filament impact and deformation has been studied for with a parametric range for governing and process parameters. The results obtained from the computation is summarized below –

For the study of droplet deformation, an increase of Froude number results in a reduction of the initial steady period. The duration of the initial steady spreading factor period ranges between dimensionless time $0.05 \leq t^* \leq 0.08$ for a decreasing trend of Froude number. An increase of Reynolds number also results in a reduction of the initial steady period. For the case of varying Reynolds number, duration of the initial steady spreading factor period ranges between dimensionless time $0.05 \leq t^* \leq 0.1$. For varying Reynolds number case, at higher values of Reynolds number, such as $Re \leq 10^2$, rim formation occurs at the ends of spreading film. For varying Weber number, the initial steady spreading factor period does not vary with the variation of the considered range of $0.1 \leq We \leq 30$. For all values of Weber number, i.e., $0.1 \leq We \leq 30$, the initial steady period ranges between dimensionless time $0.05 \leq t^* \leq 0.06$. With constant Froude number, Reynolds number, density ratio and viscosity ratio; increase in Weber number yield in very small degree of variation in spreading factor for any dimensionless time.
From the study of filament development and deformation, the most dominant factors for determining the filament front are (i) material viscosity, (ii) gap ratio i.e. the ratio of filament height to filament diameter (iii) velocity ratio i.e. the ratio of substrate travel speed to dispensing speed. The dispensing speed which has been signified by Froude number is less dominant factor that those three most important parameters. The surface tension which has been signified by Weber number does not have any visible impact on the filament front deformation. However, the experiments run for the validation of the developed computational model showed that surface tension causes the filament to climb the dispensing nozzle tip. This causes an increase in both filament diameter and filament height. A thus in experiment, the surface tension affects one of the most dominant parameters which is the gap ratio. However, for the computational study, this capillary action has not been modeled. Rather, the gap ratio considered for the computational study is the final gap ratio of the experimental case. This is why; the computational results do not show any visible effect of the variation of Weber number in filament development and filament front deformation.

For the study of filament front deformation, for all parameter being constant, the tendency of filament front lagging increases for increasing Froude number i.e. the dispensing speed. For lower Froude number such as, \( Fr = 1.0 \), higher Reynolds number such as, \( Re \approx 0.1 \) and lower gap ratio such as, \( GR = 0.5 \), to get a desired filament front, the velocity ratio has to be higher than 3. For higher Froude number such as, \( Fr = 2.0 \), lower Reynolds number such as, \( Re \approx 0.01 \) and higher gap ratio such as, \( GR = 1.0 \), to get a desired filament front, the velocity ratio has to be around 2. To have an optimum or slightly lagging filament front, for all other parameter being constant, the velocity ratio
needs to be chosen based on the Reynolds number and gap ratio. In case of gap ratio, the results show that the optimized gap ratio, for the considered ranges of parameters is in between $0.75 \sim 1.0$. Too high velocity ratio will cause excessive lagging of the filament. This excessive filament lagging will create a considerable amount of necking that results in dimensional inaccuracy. For the ranges of Weber number considered for the study, showed no visible effect on the lagging and leading. The velocity ratio is more sensitive to low Reynolds number i.e. high material viscosity than high Reynolds number i.e. low material viscosity. For $0.01 \leq Re \leq 0.1$, at $Re \approx 0.1$, increasing velocity ratio has relatively small degree of change in the filament development and filament front shape. Compared to that at $Re \approx 0.01$, for the same changes in velocity ratio has relatively drastic effect on the development of filament and filament front deformation.
BIBLIOGRAPHY


Write and Stereolithography”, Proceedings of International Solid Freeform Fabrication Symposium.


