A MODEL FOR CHOOSING A FOUR-YEAR UNIVERSITY OR A TWO-YEAR COMMUNITY COLLEGE WITH THE PRESENCE OF A GOVERNMENT SUBSIDY

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A MODEL FOR CHOOSING A FOUR-YEAR UNIVERSITY OR A TWO-YEAR COMMUNITY COLLEGE WITH THE PRESENCE OF A GOVERNMENT SUBSIDY

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ABSTRACT

Students today face the decision of choosing either a four-year university or a two-year community college when pursuing higher education. This choice as well as the government’s choice of subsidizing the four-year university or the two-year community college is analyzed using game theory. The two players of the game are the individual and the government. The individual, who intends on receiving a four-year degree, can choose to either directly attend a four-year university or first attend a two-year community college before enrolling in a four-year institution. The government can choose to subsidize the four-year university and the two-year community college either at the same level or different levels. The goal of both the individual and the government is to optimize their lifetime earnings. We find that looking at the immediate future, the individual should directly attend a four-year institution regardless of ability. However, looking at the distant future, we find the choice does depend on the individual’s own merit, and based upon ability the individual should choose to attend community college first before enrolling in a university unless the subsidy for the four-year university is greater than the subsidy for the two-year community college. We also find that at retirement age, the lifetime earnings of the individual are not significantly affected by the presence or absence of a government subsidy.
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CHAPTER I

INTRODUCTION

Game theory is a method used to represent complex real-life situations in terms we can understand and to analyze the possibilities of outcomes. Game theory is often expressed in mathematical terms, but game theory is not necessarily a branch of mathematics. According to Osborne and Rubinstein [1], game theory uses mathematical expressions because of the resulting ease to define problems, be consistent, and make assumptions. Osborne and Rubinstein [1] describe four types of games, including strategic, extensive with and without perfect information, and coalitional, where a game is defined as “a description of strategic interaction that includes the constraints on the actions that the players can take and the players’ interests, but does not specify the actions that players do take.” A player of such a game is simply an individual or a group of individuals that make a decision as presented in the game.

The differences in the four types of games are in the setup of each game. A strategic game has all players making a decision simultaneously, where the decision of one does not influence the other, while the order of events is specified for an extensive game, allowing a player to consider the decision in both the beginning and end of the game. A coalitional game differs from the previous types of games in that it is viewed as “cooperative” versus “non-cooperative,” because it is looking at “joint actions of
groups of players” as opposed to those of individual players. Because game theory can be used to model real-life situations, here it will model the pursuit of higher education for the individual.

Upon graduating high school, students face the decision of whether or not to pursue higher education. Decreuse and Granier [2] say higher education typically equals a higher salary with the completion of a degree as well as providing applicable life skills for the student while also increasing his learning capacity. The most important decision for a student is likely choosing an area of study, while the next most important decision, that could actually influence the former decision, is where to go to school. Students today have the option of attending a four-year university or a two-year community college.

While students can obtain a degree at a two-year school, many go on to pursue a four-year degree and simply transfer to an applicable school. A four-year degree usually, but not always, results in a higher income for the recipient. However, the choice of whether to go straight into the four-year university or to go to community college first can affect the overall lifetime earnings of an individual. The difference in the cost of a four-year university and a two-year community college is enough to make a student consider his options and determine that community college can be a more logical choice, depending on the academic advancement of the student. With a four-year university, an unprepared student might struggle with the workload and must also deal with a higher tuition. Regardless of a students’ preparedness, a two-year community college still has lower tuition and a student still has the option of
obtaining a four-year degree. According to Dur and Glazer [3], any education would be an investment for students and they would benefit even without the completion of a degree.

Kleiman [4] says that for students today, going to college has simply become “common sense,” and it is almost an expectation rather than a privilege as it once was. Students faced with this expectation then proceed to choose their majors and schools with a specific career goal in mind. Even though many students fail out and do not make it to their sophomore year, many give college a try because of the philosophy “more ed, more bread.” Since those with a college degree tend to have higher lifetime earnings than those without, the decision takes little or no thought. Since the decision is easy, prospective students believe that college should therefore be easy, but what many students find is that they are unprepared for the new workload which results in taking remedial classes.

Since students want to avoid taking such classes, community colleges exist for those who could otherwise be denied the opportunity of higher education according to Hoachlander et al. [5]. Community colleges not only have this “open-access” policy, but typically lower tuition which attracts many students with financial difficulties. Hoachlander et al. [5] say that these advantages along with the diverse opportunities offered, such as certification, are what attract so many students to community colleges and 90% of the students attending intend to either complete the two-year degree at the current institution or a four-year degree at another institution later. However, even with such promising programs, there is still a large part of the entering student
body that does not complete even a two-year degree. These students did not receive a degree, but reported that just having attended a higher education institution for even a short period of time had a positive effect on their employment [5].

A 1995 study in Hong Kong found that the low number of college graduates is not caused by the lack of opportunities for the students, but rather the motivations of the students today. Cheng [6] says that students were not motivated to complete additional training for such high-paying careers, such as those in engineering, and instead find “jobs in the financial sector or in the marketing field.” An unfortunate reality is that since students today had so much more handed to them, they lack the work ethic to succeed on their own. The parents in Hong Kong had more interest in their children’s education than the children themselves. The children did not lack the ability to succeed, only the motivation.

Some students who lack the motivation to pursue higher education immediately after graduating high school, may very well enter the workforce and decide later to get a degree. A student in this situation of coming back to earn a degree is defined as a non-traditional student. Since they have been in the workforce for a period of time, non-traditional students are older than the majority of students at these institutions. In choosing an institution at which to earn this degree, Jepsen and Montgomery [7] say that non-traditional students tend to go to a community college because of the distance between their home and the institution. Since universities are typically in large, urban areas, it might be farther for a non-traditional student to commute, even for the benefits of a bachelor’s degree over an associate’s
degree [7]. However, even an associate’s degree can improve lifetime earnings, and non-traditional students save money for their education by attending the community college. Even though community colleges are viewed as easier course loads than universities, Munro [8] says that non-traditional students typically already have a disadvantage since there was a reason for not choosing higher education sooner such as financial difficulties, familial obligations, and prior job commitments. These non-traditional students therefore attend school part-time trying to balance their studies as well as their other commitments, and their chances of success decrease due to this lack of focus and do not depend on their ability [8].

Black and Smith [9] say that students defined as having a high ability will attend higher quality colleges, even if they transfer from a lower quality college later in their academic career with the quality of the college being measured by the resources and/or selectivity of the school. Black, Smith, and Stange [9, 10] agree that if a college has a higher quality according to these measures, the student will benefit in the quality of education and resulting income. However, Stange [10] points out that the schools with a higher quality will most likely have a higher tuition, so the students therefore pay for this higher quality. As a result, community colleges could be considered as low quality because of the low tuition, and this could deter students from enrolling.

Malchow-Moller et al. [11] say that while education is considered an important part of national wealth by many, the value of the type of education can be different. By placing values on the types of education, it can affect how professions
are viewed and influence how students choose an area to study. Alstadsaeter et al. [12] say that students choose a discipline on not only the projected income, but also the chances of getting a job in their field after graduation. Unemployment has become so common that students have to account for a long period of job searching after receiving their degree. The 2008 study said that this rampant unemployment could be alleviated by subsidizing only the disciplines that the economy is in need of to encourage students to study a specific area. However, as stated, not all types of education are equal, which leaves students confused and reluctant to make a choice. Alstadsaeter [13] says that there are three factors in making a choice including “preferences, returns, and costs.” For some students however, the first factor is the most important and can outweigh the other two. In order to pursue an education type of their choice, some were willing to give up higher wages and returns.

According to Ensor [14], the quality of a college may be affected by the disorganization of the programs to the point where even the professors are unsure of where their teachings fit into the curriculum. By restructuring the programs and giving the students more course choices, especially in electives, students can benefit more from the education offered by choosing what they want to take. Students can also benefit from their education by simply being more prepared for the workload, though this unpreparedness is also one of the biggest problems of why students do not succeed in higher education.

Kleiman [4] says that students are likely to struggle with the transition because the exit exams of high school and the entrance exams of higher education do
not correlate except in certain areas, so it is difficult for a student to prepare for what they have not yet learned. In 2002, Bailey et al. [15] theorized about how dual enrollment could help students better adjust to higher education. The proposition was to create a “K-14 system” that would help students transition from high school to higher education. This system could be argued as the basic K-12 system followed by a two-year community college, because community college is seen already as a sort of transition for those not ready for whatever reasons to attend a four-year university.

For some students, the reason they might be initially unable to attend a four-year university is that they have financial difficulties, which is why the government offers subsidies. Correa [16] says the economy requires workers with specific levels of education, and with subsidies, the government can help control those numbers by choosing either the students or the schools that receive subsidies. Since some students will require subsidies to attend higher education, the government also can control where they attend school. Lee et al. [17] found that currently there is a skewed distribution of students from higher-income families being in universities and those from lower-income families being in community colleges, regardless of the students’ ability.

According to Dur and Glazer [3], dumb rich kids and smart poor kids have the same chances of going to college. It just happens that the financial difficulties for the smart poor kids deter more potential students than the disadvantage of being dumb rich kids. To fix such difficulties, in 2010, Rey and Racionero [18] explored four different financing schemes. Rey and Racionero [18] compared “the traditional tax-subsidy,
pure loans, income-contingent loans with risk-sharing, and income-contingent loans with risk-pooling.” Each scheme can be argued as the most effective and beneficial, but Shindo [19] had the simple conclusion that the economic growth of a region will be higher if there are more subsidies in higher education for that region.

In a 2008 study by Sanusi and Oyama [20], the government unfairly distributed subsidies to private universities skewed toward the universities with both engineering and medical programs. While these programs along with other science, technology, engineering, and mathematics (STEM) programs typically show the most promise for high wages and low unemployment, the uneven distribution of subsidies will cause smaller universities without STEM programs to struggle to maintain a high quality of education with no additional subsidies.

Kirchsteiger and Sebald [21] argue that the general attitude toward higher education in a region is transmitted through generations and that parents who received a certain level of education are likely to aid their children in attaining that same level. Parents are the greatest sociological influences for their children and their attitude as well as their financial support toward the education of their children can affect the next generation’s view of higher education. Lefebvre et al. [22] researched the effect of subsidizing private high schools and how the parents’ decision to send their children there resulted in improved performance in mathematics. As seen in Bailey et al. [15] and Kirchsteiger and Sebald [21], the views of elementary and secondary education can affect the views of higher education. The benefits of higher education are clear and Malchow-Moller et al., Kirchsteiger and Sebald [11, 21] agree that the
returns of higher wages and more knowledge should be enough to motivate students to pursue higher education to at least some level.

While the goal lately for students has become to receive a college degree, this push may be the cause of unemployment and the lower high school standards. According to Banskton III [23], since so many people are now getting college degrees, there are less jobs for those qualified in the rush to match the race of education with the boom of technology, and with our focus set on higher education, high school students are more unprepared for college. Bankston calls college an industry making credentials like any other product to be sold.

With such high unemployment and unprepared students, White’s proposal [24] could prove interesting. Instead of students receiving higher education subsidies after graduating high school, White says that students should receive a grant that they can do with as they wish and is not restricted to higher education. The grant would aid students in transitioning into adulthood whether it is through higher education or as long as they remain in good standing with the law, figuring out what they want to do with their life through other outlets.

White’s proposal [24] would be beneficial to poor children especially, because as Schneider [25] says, poor parents cannot invest in the education of their children, so giving the choice to the child whether to pursue academics or simply start a life would positively affect possible future income. However, Schneider says that only education subsidies will not always help poor children become educated and that the situations need specific governmental attention and intervention to truly fix the
problem. While subsidies are overall, a good offer to many, there are times when much more is required of the government.

Our model, developed from an extension of Correa [16] by Macavei [26], incorporates a factor that will be based on merit that will determine whether a student should attend either a four-year university or a two-year community college first. Macavei determined the optimal number of years of education and the optimal amount of subsidy that would optimize the income of the two players that are the same in this game, one being the student and the other being the government.

The questions that will be addressed in this paper are: 1) To optimize the income of the student, should the student directly attend a four-year university or first attend a two-year community college before enrolling in a four-year institution, and 2) To optimize the income of the government, should the government subsidize a four-year university and a two-year community college at the same level or at different levels. To answer these questions, we modify the equations from Macavei [26] to include the projected incomes from both a four-year university and a two-year community college.

The key assumptions for our model are:

- The student graduates high school and seeks higher education,

- The student intends to go for a four-year degree,

- The student can transfer to a four-year program upon completing two years at a community college,
• The student completing two years at a community college is equal to the student completing one year at a university,

• The probability of successfully completing a year of a two-year community college, $\alpha$, remains constant for each of the two years,

• The probability of successfully completing a year of a four-year university, $\beta$, remains constant for each of the four years,

• The student does not work while attending a higher education institution,

• The discipline the student chooses to study is not included in the model,

• The government and student wish to optimize income.

Our model, like Macavei’s [26], aims to optimize the income of both the student and government with the presence of a government subsidy. However, our model focuses on the difference in the income of the student based on the type of education, whether it is a direct admission to a four-year university or to attend a two-year community college first before transferring to a four-year institution. This model does not account for chosen disciplines, but is based on the merit of the student in determining the student’s choice and will have a factor that incorporates this value.

In our game, we determine the dominant strategy for the student and the government. We therefore determine which type of higher education the student should attend, and at which levels the government should subsidize.
CHAPTER II

MODEL DEVELOPMENT

In this chapter, the model and the dominant strategies are defined. The model contains four sets of equations, which will then form the payoff matrix. The dominant strategies for the players of the game will also be determined.

2.1 The Model

Our model is setup like Correa’s [16] and Macavei’s [26], and we have two players: the individual and the government. The goal of the game for each player is to maximize his lifetime income.

The model is defined as a two-by-two game so that each player has two choices. The individual has to decide to either directly attend a four-year university or first attend a two-year community college before enrolling in a four-year institution in the pursuit of a four-year degree, and the government has to decide whether to subsidize a four-year university and a two-year community college at the same level or at different levels.

The work extends the model of Macavei [26], and uses some of the base variables from Correa’s [16] model, including the salary given to those with only a high school education, the additional income that comes from any portion of higher
education, the cost of attending any type of institution, and the amount of the cost that is subsidized by the government. We will be calculating the expected values for the income of the individual, the cost of tuition, and the subsidy provided by the government. The model does not account for a competition factor that causes an individual to lose income. The model considers lifetime earnings at the three levels: having only a high school education, having an associate’s degree, and having a bachelor’s degree or higher. The cost of a two-year community college tuition as a proportion of the cost of a four-year university tuition, and the different subsidy levels of a four-year university and a two-year community college are also considered.

The model is set up with the four sets of equations that form the game and the payoff functions that represent the two players and the two choices that each player has. With four sets of equations, there are four subgames. The first subgame is when the individual decides to directly attend a four-year university and the government decides to subsidize the four-year university and two-year community college at the same level. The second subgame is when the individual decides to first attend a community college before enrolling in a four-year institution and the government decides to subsidize the university and community college at the same level. The third subgame is when the individual decides to directly attend a university and the government decides to subsidize the university and community college at different levels, and the fourth subgame is when the individual decides to first attend a community college before enrolling in a four-year institution and the government decides to subsidize the university and community college at different levels. The
variables used in the four subgames are presented with their units and definitions in Tables 2.1-2.2.

Table 2.1: List of variables with units and definitions

<table>
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<th>Variable</th>
<th>Units</th>
<th>Definition</th>
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<td>$\alpha$</td>
<td>0 &lt; $\alpha$ &lt; 1</td>
<td>Probability of success for one year at a community college</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0 &lt; $\beta$ &lt; 1</td>
<td>Probability of success for one year at a university</td>
</tr>
<tr>
<td>$t$</td>
<td>years</td>
<td>Number of years worked</td>
</tr>
<tr>
<td>$LE_t$</td>
<td>years</td>
<td>Total number of years worked in lifetime</td>
</tr>
<tr>
<td>$LE_0$</td>
<td>dollar</td>
<td>Lifetime earnings of an individual with a high school education</td>
</tr>
<tr>
<td>$LE_2$</td>
<td>dollar</td>
<td>Lifetime earnings of an individual with an associate’s degree</td>
</tr>
<tr>
<td>$LE_4$</td>
<td>dollar</td>
<td>Lifetime earnings of an individual with a bachelor’s degree or higher</td>
</tr>
<tr>
<td>$I_2$</td>
<td>dollar</td>
<td>Calculated yearly income of a person holding a 2-year degree</td>
</tr>
<tr>
<td>$I_4$</td>
<td>dollar</td>
<td>Calculated yearly income of a person holding a 4-year degree</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0 &lt; $\tau$ &lt; 1</td>
<td>Taxation rate</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0 &lt; $\gamma$ &lt; 1</td>
<td>Base subsidy rate</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0 &lt; $\gamma_2$ &lt; 1</td>
<td>Subsidy rate for community college</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0 &lt; $\mu$ &lt; 1</td>
<td>Proportion of university tuition</td>
</tr>
</tbody>
</table>
Table 2.2: List of variables with units and definitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_2$</td>
<td>dollar</td>
<td>Cost of tuition for one year at community college</td>
</tr>
<tr>
<td>$T_4$</td>
<td>dollar</td>
<td>Cost of tuition for one year at university</td>
</tr>
</tbody>
</table>

The probabilities of success for the individual are defined as $\alpha$ and $\beta$ for a community college and a university respectively. The probabilities for each year as well as the time and incomes associated are presented below in Tables 2.3-2.4. There is a combination of $\alpha$ and $\beta$ for the individual first attending a community college and transferring to a university to account for the difference in the probabilities of success. The time associated with each year is the additional time that the individual would work at each point, because the individual who fails the first year will work longer than the individual who fails the fourth year. Since two years at a community college are set to equal one year at a university, the relation

$$\alpha^2 = \beta$$  \hspace{1cm} (2.1)

will be important when finding the dominant strategies for the individual and the government because the strategy will change going across the curve. Equation (2.1) defines the two years at a community college to be equal to the first year at a university, and means that for $\alpha^2 = \beta$, the individual has an equal likelihood at obtaining
a four-year degree regardless of directly attending a four-year university or first attending a two-year community college before transferring to a four-year institution. We hypothesize that the curve is the dividing line between these two choices for the individual. The individual will first attend a two-year community college if the point \((\alpha,\beta)\), where \(\alpha\) and \(\beta\) are calculated using the individual’s ACT score, falls below the curve. If the point \((\alpha,\beta)\) falls above the curve for an individual, then he should directly attend a four-year university.

Table 2.3: List of probabilities of success

<table>
<thead>
<tr>
<th>Year in School</th>
<th>Probability of Success</th>
<th>Time</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fail first year in 2-yr</td>
<td>((1 - \alpha))</td>
<td>(t + 4)</td>
<td>LE0</td>
</tr>
<tr>
<td>Pass first year in 2-yr</td>
<td>(\alpha(1 - \alpha))</td>
<td>(t + 3)</td>
<td>(LE0 + LE2)</td>
</tr>
<tr>
<td>Pass second year in 2-yr</td>
<td>(\alpha^2(1 - \beta))</td>
<td>(t + 2)</td>
<td>LE2</td>
</tr>
<tr>
<td>Pass third year in 2-yr to 4-yr</td>
<td>(\alpha^2\beta(1 - \beta))</td>
<td>(t + 1)</td>
<td>LE2</td>
</tr>
<tr>
<td>Pass fourth year in 2-yr to 4-yr</td>
<td>(\alpha^2\beta^2(1 - \beta))</td>
<td>(t)</td>
<td>LE2</td>
</tr>
<tr>
<td>Pass fifth year in 2-yr to 4-yr</td>
<td>(\alpha^2\beta^3)</td>
<td>(t)</td>
<td>LE4</td>
</tr>
<tr>
<td>Fail first year in 4-yr</td>
<td>((1 - \beta))</td>
<td>(t + 3)</td>
<td>LE0</td>
</tr>
<tr>
<td>Pass first year in 4-yr</td>
<td>(\beta(1 - \beta))</td>
<td>(t + 2)</td>
<td>LE2</td>
</tr>
</tbody>
</table>
Table 2.4: List of probabilities of success

<table>
<thead>
<tr>
<th>Year in School</th>
<th>Probability of Success</th>
<th>Time</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass second year in 4-yr</td>
<td>$\beta^2(1 - \beta)$</td>
<td>$t + 1$</td>
<td>$LE^2$</td>
</tr>
<tr>
<td>Pass third year in 4-yr</td>
<td>$\beta^3(1 - \beta)$</td>
<td>$t$</td>
<td>$LE^2$</td>
</tr>
<tr>
<td>Pass fourth year in 4-yr</td>
<td>$\beta^4$</td>
<td>$t$</td>
<td>$LE^4$</td>
</tr>
</tbody>
</table>

2.1.1 First Subgame

In this subgame, the individual decides to directly attend a four-year university with the government subsidizing the four-year university and the two-year community college at the same level, $\gamma = \gamma_2$.

In this case, the payoff function for the individual is

$$W_4 = I_4 t(1 - \tau) - (1 - \gamma)T_4(1 + \beta + \beta^2 + \beta^3). \quad (2.2)$$

The equation has two components. The first component is the income of the individual after taxes at any time $t$. The second component is the cost of education subtracted from the income component. The cost of education includes the presence of a government subsidy and the probabilities of success at each of the four years at a university. To form the second component, the subsidy is multiplied by the cost of tuition which is then multiplied by the sum of all the probabilities of an individual succeeding at each year at a university multiplied with the respective times and incomes associated with each probability.

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One of the components of the income of the individual is the calculated yearly income of the individual, $I_4$, which is found by first examining the relation

$$ tI_4 = \frac{LE_0}{LEt}(t+3)(1-\beta) + \frac{LE_2}{LEt}(t+2)\beta(1-\beta) + \frac{LE_2}{LEt}(t+1)\beta^2(1-\beta) $$

$$ + \frac{LE_2}{LEt}t\beta^3(1-\beta) + \frac{LE_4}{LEt}t\beta^4, $$

(2.3)

where the probabilities in Tables 2.3-2.4 are multiplied by the time and income associated with each probability. Through simplification by multiplying and combining similar terms, we obtain

$$ tI_4 = \frac{LE_0}{LEt}(t + 3 - t\beta - 3\beta) + \frac{LE_2}{LEt}(t\beta + 2\beta - \beta^2 - \beta^3 - t\beta^4) + \frac{LE_4}{LEt}t\beta^4. $$

(2.4)

Equation (2.4) was defined for $tI_4$, and so we divide by $t$ to finally obtain

$$ I_4 = \frac{LE_0}{LEt} \left( 1 + \frac{3}{t} - \beta - \frac{3\beta}{t} \right) + \frac{LE_2}{LEt} \left( \beta + \frac{2\beta}{t} - \frac{\beta^2}{t} - \frac{\beta^3}{t} - \beta^4 \right) + \frac{LE_4}{LEt}t\beta^4, $$

(2.5)

where $LE_0$, $LE_2$, and $LE_4$ are the respective lifetime earnings of an individual who graduated high school, earned an associate’s degree, and earned a bachelor’s degree or higher, and the terms of $\beta$ are the probabilities that the individual will succeed at each of the four years at a university. $LEt$ is the total number of years worked in a lifetime and divides the lifetime earnings, so that the generated yearly incomes are used in the game. The original equation was actually solved for $tI_4$, and so the additional $t$ was divided out, leaving some terms over $t$. The income equation was created so that the individual benefits from each year that he attends a higher education institution. The probabilities were multiplied by the potential income so that the individual received the relative earnings of a high school graduate until the year
that an associate’s degree would be obtained, and that the individual then received
the relative earnings of an associate program graduate until the year that a bachelor’s
degree would be obtained.

The first term of the income is the lifetime earnings of an individual who
graduated high school with the respective time and probability associated. This term
has only one time and probability because this income is representative of an individ-
ual who fails the first year of higher education, since even one year of study provides
added value for the individual. The second term of the income is the lifetime earnings
of an individual who earned an associate’s degree with multiple times and probabil-
ities because an individual who passes one year of higher education will benefit and
that benefit will increase with each year passed until the next benchmark of a bache-
lor’s degree is reached. Each respective time and income is multiplied and then added
together to create the second term with leading variable $LE_2$. The third and final
term of the income is the lifetime earnings of an individual who earned a bachelor’s
degree with the respective time and probability associated. This term has only one
time and income because once the individual reaches this point in the model, he has
completed a four-year degree and he has received the maximum income.

In this case, the payoff function for the government is

$$G_4 = I_4 t \tau - \gamma T_4 (1 + \beta + \beta^2 + \beta^3).$$

(2.6)

The equation has two components. The first component is the income of the govern-
ment from taxes on the income of the individual with taxation rate $\tau$. The second
component is the cost of the subsidy subtracted from the income component. The cost of the subsidy includes the cost of education and the probabilities of success for the individual at each of the four years at a university. The second component uses the sum of the probabilities of each year at a university to evaluate the potential total cost of the individual’s college career.

2.1.2 Second Subgame

In this subgame, the individual decides to first attend a two-year community college before enrolling in a four-year institution with the government subsidizing the four-year university and the two-year community college at the same level, $\gamma = \gamma_2$.

In this case, the payoff function for the individual is

$$W_2 = I_2(t - 1)(1 - \tau) - (1 - \gamma)[T_2(1 + \alpha) + T_4\alpha^2(1 + \beta + \beta^2)]. \quad (2.7)$$

The equation has two components. The first component is the income of the individual after taxes at any time $t$. However, it is assumed that the individual who first attends a community college will be in school one year longer and therefore work one year less than someone who first attends a university, because two years at a community college is equal to one year at a university. Therefore, for a consistent retirement age in all of the games, the term is $(t - 1)$ rather than $t$. The second component is the cost of education. Since the cost of tuition and the probabilities for a community college and a university both differ, the cost of education has two components, one for each type of institution. The first component of the cost of education includes the presence of a government subsidy and the probabilities of success at each of the two
years at a community college. The second component of the cost of education also includes the presence of a government subsidy, but also the probabilities of success at each of the three years at a university. To form the first component, the subsidy is multiplied by the cost of tuition at a community college, which is then multiplied by the sum of all the probabilities of an individual succeeding at each year at a community college. To form the second component, the subsidy is multiplied by the cost of tuition at a university, which is then multiplied by the sum of all the probabilities of an individual succeeding at each year at a university.

One of the components of the income of the individual is the calculated yearly income of the individual, $I_2$, which is found by first examining the relation

$$tI_2 = \frac{LE_0}{LE_t}(t + 4)(1 - \alpha) + \frac{LE_0 + LE_2}{2LE_t}(t + 3)\alpha(1 - \alpha) + \frac{LE_2}{LE_t}(t + 2)\alpha^2(1 - \beta) + \frac{LE_2}{LE_t}(t + 1)\alpha^2\beta(1 - \beta)$$

$$+ \frac{LE_2}{LE_t}t\alpha^2\beta^2(1 - \beta) + \frac{LE_4}{LE_t}t\alpha^2\beta^3,$$

where the probabilities in Tables 2.3-2.4 are multiplied by the time and income associated with each probability. Through simplification by multiplying and combining similar terms, we obtain

$$tI_2 = \frac{LE_0}{LE_t}(t + 4 - t\alpha - 4\alpha) + \frac{LE_0 + LE_2}{2LE_t}(t\alpha + 3\alpha - t\alpha^2 - 3\alpha^2)$$

$$+ \frac{LE_2}{LE_t}(t\alpha^2 + 2\alpha^2 - \alpha^2\beta - \alpha^2\beta^2 - t\alpha^2\beta^3) + \frac{LE_4}{LE_t}t\alpha^2\beta^3.$$

Equation (2.4) was defined for $tI_2$, and so we divide by $t$ to finally obtain

$$I_2 = \frac{LE_0}{LE_t} \left(1 + \frac{4}{t} - \alpha - \frac{4\alpha}{t}\right) + \frac{LE_0 + LE_2}{2LE_t} \left(\alpha + \frac{3\alpha}{t} - \alpha^2 - \frac{3\alpha^2}{t}\right)$$

$$+ \frac{LE_2}{LE_t} \left(\alpha^2 + \frac{2\alpha^2}{t} - \frac{\alpha^2\beta}{t} - \frac{\alpha^2\beta^2}{t} - \alpha^2\beta^3\right) + \frac{LE_4}{LE_t}\alpha^2\beta^3,$$

(2.10)
where $LE0$, $LE2$, and $LE4$ are, as before, the respective lifetime earnings of an individual who graduated high school, earned an associate’s degree, and earned a bachelor’s degree or higher, and the terms of $\alpha$ and $\beta$ are the probabilities that the individual will succeed at each of the two years of community college and each of the four years at a university, respectively. $LEt$ is the total number of years worked in a lifetime and divides the lifetime earnings, so that the generated yearly incomes are used in the game. The income equation was created so that the individual benefits from each year that he attends a higher education institution. The probabilities were multiplied by the potential income so that the individual received the relative earnings of a high school graduate until the year that an associate’s degree would be obtained, and that the individual then received the relative earnings of an associate program graduate until the year that a bachelor’s degree would be obtained. The difference is not only the added probabilities associated with community college, but since the individual is first attending a community college before a university, the individual generates more income with one year of community college than the income associated with only graduating high school. This added income is found in the average of the potential incomes of a high school graduate and of an associate’s program graduate.

To find equation (2.10), the probabilities in Tables 2.3-2.4 were multiplied by the time and incomes associated with each probability and the resulting relations were then added. The income terms were isolated to have four main variables added. The original equation was actually solved for $tI_2$, and so the additional $t$ was di-
vided out, leaving some terms over $t$. The first term of the income is the lifetime earnings of an individual who graduated high school with the respective time and probability associated. This term has only one time and probability because this income is representative of an individual who fails the first year of higher education and therefore does not gain any benefit from attending an institution. The second term of the income is the average of the lifetime earnings of a high school graduate and the lifetime earnings of someone who earned an associate’s degree. This term is for the year where there is some additional income from attending higher education, but a degree has not yet been earned and so this term also has only one time and probability associated. The third term of the income is the lifetime earnings of an individual who earned an associate’s degree with multiple times and probabilities because an individual who passes one year of higher education will benefit and that benefit will increase with each year passed until the next benchmark of a bachelor’s degree is reached. Each respective time and income is multiplied and then added together to create the second term with leading variable $LE^2$. The fourth and final term of the income is the lifetime earnings of an individual who earned a bachelor’s degree with the respective time and probability associated. This term has only one time and income because once the individual reaches this point in the model, he has completed a four-year degree and he has received the maximum income.

In this case, the payoff function for the government is

$$G_2 = I_2(t - 1)\tau - \gamma[T_2(1 + \alpha) + T_4\alpha^2(1 + \beta + \beta^2)].$$  \hspace{1cm} (2.11)
The equation has two components. The first component is the income of the government from taxes on the income of the individual with taxation rate \( \tau \) that is only gained from the \((t - 1)\) years that an individual who first attends community college will work. The second component is the cost of the subsidy that is subtracted from the income component. The first of the two components of the cost of the subsidy includes the probabilities of success for the individual at each of the two years at a community college, while the second component includes the probabilities of success for each of the three years at a university. To form the first component of the subsidy, the cost of the subsidy is multiplied by the cost of tuition at a community college, which is then multiplied by the sum of all the probabilities of an individual succeeding at each year at a community college. To form the second component of the subsidy, the cost of the subsidy is multiplied by the cost of tuition at a university, which is then multiplied by the sum of all the probabilities of an individual succeeding at each year at a university.

2.1.3 Third Subgame

In this subgame, the individual decides to directly attend a four-year university with the government subsidizing the four-year university and the two-year community college at different levels, \( \gamma \neq \gamma_2 \). When \( \gamma \neq \gamma_2 \), \( \gamma_2 \) is found by first examining the relation

\[
\gamma_2 T_2 = \gamma_2 \mu T_4, \tag{2.12}
\]
and then defining

\[ \gamma_2 \mu T_4 = \frac{\gamma T_1}{2}, \]  

(2.13)

so the subsidy of a two-year community college is half that of a four-year university due to the difference in length of study at each institution. By solving equation (2.13), \( \gamma_2 \) is defined as

\[ \gamma_2 = \frac{\gamma}{2\mu}. \]  

(2.14)

The subsidy level is the only variable that changes for the third and fourth subgames, therefore, the third subgame has identical payoff functions to the first subgame, where an individual decides to first attend a four-year university with a subsidy level \( \gamma \).

In this case, the payoff function for the individual remains

\[ W_4 = I_4 t(1 - \tau) - (1 - \gamma)T_4(1 + \beta + \beta^2 + \beta^3), \]  

(2.15)

where the yearly income term, \( I_4 \), is

\[ I_4 = \frac{LE_0}{LE_t} \left(1 + \frac{3}{t} - \beta - \frac{3\beta}{t}\right) + \frac{LE_2}{LE_t} \left(\beta + \frac{2\beta}{t} - \frac{\beta^3}{t} - \beta^4\right) + \frac{LE_4}{LE_t} \beta^4, \]  

(2.16)

and the payoff function for the government is

\[ G_4 = I_4 t \tau - \gamma T_4(1 + \beta + \beta^2 + \beta^3). \]  

(2.17)

The components of these equations are not explained here since the equations are identical to the first subgame.

2.1.4 Fourth Subgame

In this subgame, the individual decides to first attend a two-year community college before enrolling in a four-year institution with the government subsidizing the four-
year university and the two-year community college at different levels, $\gamma \neq \gamma_2$. The subsidy level again is the only variable that changes for the third and fourth sub-games, and while the two subgames that have the individual choosing to first attend a four-year university are the same, the fourth game, like the second game, has the individual first attending a community college but with different levels of subsidy for a university and a community college. This introduces a new variable for the different level subsidy, $\gamma_2$, since the levels of subsidy are no longer equal.

In this case, the payoff function for the individual becomes

$$\hat{W}_2 = I_2(t - 1)(1 - \tau) - (1 - \gamma_2)T_2(1 + \alpha) - (1 - \gamma)T_4\alpha^2(1 + \beta + \beta^2). \quad (2.18)$$

The equation has two components similar to those of the second subgame. The first component is the income of the individual after taxes at any time $t$, but since it is still assumed that the individual who first attends a community college will be in school one year longer and therefore work one year less, the term remains $(t - 1)$. The second component is the cost of education with two subsidy levels. The first component of the cost of education includes the presence of a government subsidy at level $\gamma_2$ and the probabilities of success at each of the two years at a community college, while the second component includes the presence of a subsidy at level $\gamma$, as well as the probabilities of success at each of the three years at a university. The components of the cost of education were formed using a similar method for the second subgame with the introduction of the new variable, $\gamma_2$, that is multiplied to the cost of tuition for a community college and the probabilities of succeeding in each year there, since
a community college and a university are subsidized at different levels.

The yearly income term $I_2$, however, remains the same as in the second subgame because the income is still only dependent on attending a community college first and is

$$I_2 = LE0 \left( 1 + \frac{4}{t} - \alpha - \frac{4\alpha}{t} \right) + \frac{LE0 + LE2}{2} \left( \alpha + \frac{3\alpha}{t} - \alpha^2 - \frac{3\alpha^2}{t} \right) \tag{2.19}$$

$$+ LE2 \left( \alpha^2 + \frac{2\alpha^2}{t} - \frac{\alpha^2\beta}{t} - \frac{\alpha^2\beta^2}{t} - \alpha^2\beta^3 \right) + LE4\alpha^2\beta^3,$$

The components of this expression are identical to those appearing in the second subgame.

In this case, the payoff function for the government becomes

$$\hat{G}_2 = I_2(t - 1)\tau - \gamma_2 T_2(1 + \alpha) - \gamma T_1 \alpha^2(1 + \beta + \beta^2). \tag{2.20}$$

The equation has two components. The first component is the income of the government from the taxes on the income of the individual with taxation rate $\tau$ for $(t - 1)$ years. The second component is the cost of the subsidy subtracted from the income component with two subsidy levels. The first component of the cost of the subsidy is at level $\gamma_2$ and includes the cost of education and the probabilities of success for the individual at each of the years at a community college. The second component of this cost is at level $\gamma$ and includes the cost of education and the probabilities of success for the individual at each of the years at a university. The components of the cost of the subsidies were formed using a similar method for the second subgame, with the introduction of the new variable, $\gamma_2$, so that this new subsidy level is multiplied to the cost of tuition for a community college and the probabilities of succeeding in
each year there, since a community college and a university are subsidized at different levels. The third component that includes the cost of tuition at a university and the sum of the probabilities of succeeding in each year there is still being multiplied by the original subsidy level, \( \gamma \).

2.1.5 The Payoff Matrix

The payoff matrix shows the two players and the two choices of each player and is used to show how the equations are dependent on the choices of the players. The payoff matrix used for the four subgames is presented below in Table 2.5.

Table 2.5: Payoff Matrix

<table>
<thead>
<tr>
<th>Government</th>
<th>Individual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Four-year university</td>
</tr>
<tr>
<td>Same level subsidy</td>
<td>( \gamma = \gamma_2 )</td>
</tr>
<tr>
<td>Different level subsidy</td>
<td>( \gamma \neq \gamma_2 )</td>
</tr>
</tbody>
</table>

2.2 The Dominant Strategies

The goal of the model is to determine the choice that will optimize the income for each player, so the dominant strategy for each player is found.
2.2.1 The Dominant Strategy of the Individual

The individual has the two choices of either directly attending a four-year university or first attending a two-year community college before enrolling in a four-year institution and then based on the choice of the government, there can be different levels of subsidy. The equations for first attending a four-year university and the equations for first attending a two-year community college with and without the same level of subsidy will be compared to find a dominant strategy for the individual.

To find the dominant strategy when the government is subsidizing the university and the community college at the same level, $\gamma = \gamma_2$, the difference of $W_4$ and $W_2$ is evaluated,

$$W_4 - W_2 = (1 - \tau)[t(I_4 - I_2) + I_2] - (1 - \gamma)[T_4(1 - \alpha^2 \beta) + T_2(1 + \alpha)].$$  

(2.21)

When equation (2.21) is solved, if the result is positive, then the individual should first attend a four-year university, but if the result is negative, then the individual should first attend a two-year community college.

To find the dominant strategy when the government is subsidizing the university and the community college at different levels, $\gamma \neq \gamma_2$, the difference of $W_4$ and $\tilde{W}_2$ is evaluated,

$$W_4 - \tilde{W}_2 = (1 - \tau)[t(I_4 - I_2) + I_2] - (1 - \gamma)[T_4(1 + \alpha^2 \beta) + T_2(1 + \alpha)].$$  

(2.22)

When equation (2.22) is solved, if the result is positive, then the individual should first attend a four-year university, but if the result is negative, then the individual should first attend a two-year community college.
When equation (2.22) is solved, if the result is positive, then the individual should first attend a four-year university, but if the result is negative, then the individual should first attend a two-year community college.

2.2.2 The Dominant Strategy of the Government

The government has the two choices of either subsidizing a four-year university and a two-year community college at the same level or at different levels. The equations for subsidizing a four-year university and a two-year community college at the same and different levels will be compared to find a dominant strategy for the government.

To find the dominant strategy when the community college is being subsidized at different levels, $\gamma \neq \gamma_2$, the difference of $G_2$ and $\hat{G}_2$ is evaluated,

$$G_2 - \hat{G}_2 = T_2(1 + \alpha)(\gamma_2 - \gamma). \tag{2.23}$$

This equation no longer depends on $\beta$. Equation (2.23) only depends on $\alpha$ and the two levels of subsidy. The dominant strategy for the government becomes subsidize at the same level $\gamma = \gamma_2$ if $\gamma_2 > \gamma$, and subsidize at different levels $\gamma \neq \gamma_2$ if $\gamma_2 < \gamma$.

If $\gamma_2 < \gamma$, then the government would make more money since $\hat{G}_2 > G_2$.

When comparing the difference of $G_4$ and $G_2$ or $G_4$ and $\hat{G}_2$, the government needs to determine which choice of the individual generates more income. The difference of $G_4$ and $G_2$ is evaluated,

$$G_4 - G_2 = \tau[t(I_4 - I_2) + I_2] - \gamma[T_4(1 - \alpha^2) + (1 - \alpha^2)(\beta + \beta^2) + \beta^3] - T_2(1 + \alpha)] \tag{2.24}$$
The difference of $G_4$ and $\hat{G}_2$ is evaluated,

$$G_4 - \hat{G}_2 = \tau[t(I_4 - I_2) + I_2] - \gamma[T_4(1 - \alpha^2)$$

$$+ (1 - \alpha^2)(\beta + \beta^2) + \beta^3)] + \gamma_2 T_2 (1 + \alpha).$$

(2.25)

When equations (2.24)-(2.25) are solved, the region where the results are positive will generate more income for the government than the region where the results are negative. The dominant strategy for the government would then be to meet the conditions where they would fall in the positive region.
In this chapter, the results are presented in the form of contour plots. The dominant strategies for the players of the game will be determined by analyzing the contour plots. To generate the contour plots, values had to be assigned to the variables, so data was collected that reasonably modeled real-life situations. We found that for an individual who directly attends a higher education institution after graduating high school and works until retirement age, the individual will have worked an average of forty-five years in his lifetime. The lifetime earnings of a high school graduate, an individual with an associate’s degree and an individual with a bachelor’s degree were found using census data and matched expected yearly incomes when calculated. The lifetime earnings were $0.9 million for a high school graduate, $1.1 million for an individual with an associate’s degree and $1.8 million for an individual with a bachelor’s degree. The income tax rate was between ten percent and twenty-five percent in 2012 for our potential incomes, and we chose to use the ten percent. The values for $\gamma$ and $\gamma_2$ were chosen specifically for the model. According to the Ohio Board of Regents, we found that in 2012, the average undergraduate tuition at a community college was $3,484, which we rounded to $3,000, and the average undergraduate tuition at a university was $9,608, which we rounded to $10,000. Since
\(\mu\) is the proportion of university tuition for a community college, we know that \(\mu\) is thirty percent or 0.3. The variables and the chosen values used in the contour plots are presented below with their reference in Tables 3.1-3.2.

Table 3.1: List of variables with assigned values

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<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
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<tr>
<td>(LEt)</td>
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</tr>
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<td>[27]</td>
</tr>
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<td>[27]</td>
</tr>
<tr>
<td>(\tau)</td>
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<td>[28]</td>
</tr>
<tr>
<td>(\gamma)</td>
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<tr>
<td>(\gamma_2)</td>
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<td></td>
</tr>
<tr>
<td>(\mu)</td>
<td>0.3</td>
<td>[29]</td>
</tr>
</tbody>
</table>
Table 3.2: List of variables with assigned values

<table>
<thead>
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<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
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<td>[29]</td>
</tr>
<tr>
<td>$T_4$</td>
<td>$10,000$</td>
<td>[29]</td>
</tr>
</tbody>
</table>

3.1 The Dominant Strategy of the Individual

The individual has the choice to either directly attend a four-year university or first attend a two-year community college before enrolling in a four-year institution and the government either subsidizes both schools with the same level of subsidy or with different levels of subsidy. Equations (2.21) and (2.22) are used to simulate the comparisons. While our main focus is on the lifetime earnings of an individual with our specified conditions, we also examine two other time intervals, $t = 10$ and $t = 25$, to see the changes as they progress. At $t = 10$, we want to see if the individual who made his decision trying to make the most money in the quickest way is successful and then we can see if goals should be made for the short-term or the long-term. At $t = 25$, the individual is about halfway through his career before the option to retire, and at this point, we should see a balance start to appear, as the benefits of the choice take effect. At $t = 45$, we are looking at net lifetime earnings to determine the best choice for the long-term. Another variable that is defined at multiple points is the
tuition of a community college. For the tuition of a community college we take when 
\( \mu = 0.3 \) or \( T_2 = 3,000 \), but we also explore \( \mu = 0.7 \) and 1 or \( T_2 = 7,000 \) and 10,000 
[29]. The separate line that appears on Figures 3.1-3.18 is where \( \alpha^2 = \beta \), since two 
years at a community college equal one year at a university. For an individual, \( \alpha^2 \) is 
the probability of succeeding in the second year at a community college and going in 
to year two at a university, while \( \beta \) is the probability of succeeding in the first year 
at a university and going in to year two at a university. Therefore, when \( \alpha^2 = \beta \), an 
individual has an equal probability of succeeding in one year at a community college 
or in one year at a university. When the differences in the wage equations (2.21) and 
(2.22) equal zero, we have the zero level set that appears on the figures. We anticipate 
the \( \alpha^2 = \beta \) curve and the zero level set should be similar, because intuition suggests 
that if the individual has an equal likelihood of succeeding in obtaining a four-year 
degree regardless of the starting point, then the individual should receive the same 
lifetime earnings regardless of the starting point. In the contour plots, we focus on 
those two curves to draw conclusions.

3.1.1 Using the Same Subsidy Level

For the individual, the choice between directly attending a four-year university and 
first attending a two-year community college before enrolling in a four-year institu-
tion with the same level of subsidy will be analyzed first. Figures 3.1-3.3 show the 
comparisons of the choice at \( t = 10 \), or ten years after attending school, with the cost 
of the community college at three different levels to see how the different proportions
would affect income. At ten years after school, the entire contour plot is positive, meaning that the individual should choose to first attend the four-year university regardless of the probability of success. For the individual in the short run, the dominant strategy is therefore to “go for broke” and directly attend a four-year university, since the future payoff will be greater.

Figure 3.1: Comparing $W_4$ and $W_2$ through a contour plot of the difference $W_4 - W_2$ with $t = 10$, $T_2 = 3,000$, and $\gamma = 0.3$. Ten years is insufficient time for the individual to overcome the cost of education.
Figure 3.2: Comparing $W_4$ and $W_2$ through a contour plot of the difference $W_4 - W_2$ with $t = 10$, $T_2 = 7,000$, and $\gamma = 0.3$. Ten years is insufficient time for the individual to overcome the cost of education.
Figure 3.3: Comparing $W_4$ and $W_2$ through a contour plot of the difference $W_4 - W_2$ with $t = 10$, $T_2 = 10,000$, and $\gamma = 0.3$. Ten years is insufficient time for the individual to overcome the cost of education.

Figures 3.4-3.6 show the comparisons of the choice at $t = 25$, or twenty-five years after attending school, with the cost of the community college at three different levels to see how the different proportions would affect income. After twenty-five years after school, the zero level set is starting to follow the $\alpha^2 = \beta$ curve more. This curve is important because it is the point at which the individual has equal likelihood of getting a four-year degree regardless of the starting institution. We see that the zero level set is defined on $[0,1] \times [0,1]$ in each contour, and we observe that if an individual has the probabilities of $\alpha$ and $\beta$ that fall above the zero level set, the individual should directly attend a four-year university because the individual
has a greater probability of succeeding at a four-year university, and an individual with probabilities that fall below the zero level set should first attend a two-year community college because the individual has a greater probability of succeeding at a two-year community college. We also see that after twenty-five years, there is a less noticeable change for the different levels of tuition for a community college, so the additional time washes out the cost of education. The dominant strategy for the individual would be to directly attend a four-year university if the tuition of a community college is high, because he will be in school longer if he first attends a community college.

Figure 3.4: Comparing $W_4$ and $W_2$ through a contour plot of the difference $W_4 - W_2$ with $t = 25$, $T_2 = 3,000$, and $\gamma = 0.3$. 
Figure 3.5: Comparing $W_4$ and $W_2$ through a contour plot of the difference $W_4 - W_2$
with $t = 25$, $T_2 = 7,000$, and $\gamma = 0.3$. 

Figure 3.6: Comparing $W_4$ and $W_2$ through a contour plot of the difference $W_4 - W_2$ with $t = 25$, $T_2 = 10,000$, and $\gamma = 0.3$. Greater chance of success at a university with equal tuition.

Figures 3.7-3.9 show the comparisons of the choice at $t = 45$, or forty-five years after attending school, with the cost of the community college at three different levels to see how the different proportions would affect income. After forty-five years after school, we now see the lifetime earnings of an individual and how the zero level set is starting to follow the $\alpha^2 = \beta$ curve more. The zero level set is defined on $[0,1] \times [0,1]$, and we see that if an individual has the probabilities of $\alpha$ and $\beta$ that fall above the zero level set, the individual should first attend a four-year university, and an individual with probabilities that fall below this curve should first attend a two-year community college. We also see that after forty-five years, there is only a
slight change for the different levels of tuition for a community college, meaning that once an individual reaches retirement, the cost of education has little or no effect on lifetime earnings because the cost is washed out. The dominant strategy remains to directly attend a four-year university if the tuition of a community college is high. However, if an individual has a greater chance of succeeding at a community college, meaning $\alpha > \beta$, then the individual should first attend a community college.

Figure 3.7: Comparing $W_4$ and $W_2$ through a contour plot of the difference $W_4 - W_2$ with $t = 45$, $T_2 = 3,000$, and $\gamma = 0.3$. 
Figure 3.8: Comparing $W_4$ and $W_2$ through a contour plot of the difference $W_4 - W_2$ with $t = 45$, $T_2 = 7,000$, and $\gamma = 0.3$. 
Figure 3.9: Comparing $W_4$ and $W_2$ through a contour plot of the difference $W_4 - W_2$ with $t = 45$, $T_2 = 10,000$, and $\gamma = 0.3$. Greater chance of success at a university with equal tuition.

3.1.2 Using Different Subsidy Levels

For the individual, the choice between directly attending a four-year university and first attending a two-year community college before enrolling in a four-year institution with different levels of subsidy will now be analyzed. We define the different level of subsidy for the community college as seen in equation (2.14), so that it is dependent on $\mu$, the proportion that the tuition of a community college is of the tuition of a university. Figures 3.10-3.12 show the comparisons of the choice at $t = 10$, or ten years after attending school, with the cost of the community college again at three
different levels to examine the effect on income. At ten years after school, the entire contour plot is positive, meaning that the individual should choose to first attend the four-year university regardless of the probability of success. These plots are very similar to those at the same level of subsidy, so we can conclude that ten years after attending school is not sufficient time to compare either set of data. Therefore, the dominant strategy remains to directly attend a four-year university, since the individual will have greater lifetime earnings.

Figure 3.10: Comparing $W_4$ and $\hat{W}_2$ through a contour plot of the difference $W_4 - \hat{W}_2$ with $t = 10$, $T_2 = 3,000$, $\gamma = 0.3$ and $\gamma_2 = 0.5$. Ten years is insufficient time for an individual to overcome the cost of education.
Figure 3.11: Comparing $W_4$ and $\hat{W}_2$ through a contour plot of the difference $W_4 - \hat{W}_2$ with $t = 10$, $T_2 = 7,000$, $\gamma = 0.3$ and $\gamma_2 = \frac{0.3}{1.4}$. Ten years is insufficient time for an individual to overcome the cost of education.
Figure 3.12: Comparing $W_4$ and $\hat{W}_2$ through a contour plot of the difference $W_4 - \hat{W}_2$ with $t = 10$, $T_2 = 10,000$, $\gamma = 0.3$ and $\gamma_2 = 0.15$. Ten years is insufficient time for an individual to overcome the cost of education.

Figures 3.13-3.15 show the comparisons of the choice with the different levels of subsidy at $t = 25$, or twenty-five years after school, with the cost of the community college at three different levels to see how the different proportions would affect income. We see that these contours are very similar to those from when the subsidy levels are equal, so our conclusions are the same. We can also conclude that after some time, the different levels of subsidy are irrelevant to lifetime earnings, since $\gamma_2$ is actually dependent on the changing tuition. For high tuition, the dominant strategy for an individual would be to directly attend a four-year university unless the subsidy for a community college is significantly greater than the subsidy of a university.
Figure 3.13: Comparing $W_4$ and $\hat{W}_2$ through a contour plot of the difference $W_4 - \hat{W}_2$

with $t = 25$, $T_2 = 3,000$, $\gamma = 0.3$ and $\gamma_2 = 0.5$. 
Figure 3.14: Comparing $W_4$ and $\hat{W}_2$ through a contour plot of the difference $W_4 - \hat{W}_2$

with $t = 25$, $T_2 = 7,000$, $\gamma = 0.3$ and $\gamma_2 = \frac{0.3}{1.4}$. 
Figure 3.15: Comparing $W_4$ and $\hat{W}_2$ through a contour plot of the difference $W_4 - \hat{W}_2$ with $t = 25$, $T_2 = 10,000$, $\gamma = 0.3$ and $\gamma_2 = 0.15$. Greater chance of success at a university than Figure 3.6.

Figures 3.16-3.18 show the comparisons of the choice at $t = 45$, or forty-five years after attending school, with the cost of the community college at three different levels to see how the different proportions would affect income. After forty-five years after school, we now see the lifetime earnings of both a graduate of a four-year university and a graduate of a university who first attended a two-year community college are almost equal. The zero level set is defined on $[0,1] \times [0,1]$ for each contour. The conclusions are similar to when the subsidy level is the same. We again see that after forty-five years, there is only a slight change for the different levels of tuition for a community college. The zero level set is the closest to $\alpha^2 = \beta$ of any
case when the tuition levels are equal, and further reiterates that for high tuition, the
dominant strategy of the individual would be to directly attend a four-year university,
if there is a sufficient probability of success, to have the highest net lifetime earnings.
If the individual cannot expect at least a forty percent success at a two-year school,
or $\alpha = 0.4$, then the individual should go to a two-year community college.

Figure 3.16: Comparing $W_4$ and $\hat{W}_2$ through a contour plot of the difference $W_4 - \hat{W}_2$
with $t = 45$, $T_2 = 3,000$, $\gamma = 0.3$ and $\gamma_2 = 0.5$. 

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Figure 3.17: Comparing $W_4$ and $\hat{W}_2$ through a contour plot of the difference $W_4 - \hat{W}_2$

with $t = 45$, $T_2 = 7,000$, $\gamma = 0.3$ and $\gamma_2 = \frac{0.3}{1.4}$. 
Figure 3.18: Comparing $W_4$ and $\hat{W}_2$ through a contour plot of the difference $W_4 - \hat{W}_2$ with $t = 45$, $T_2 = 10,000$, $\gamma = 0.3$ and $\gamma_2 = 0.15$. Slightly better approximation as tuition increases and is similar to results in Figure 3.9.

3.1.3 Comparing the Same Subsidy Level and Different Subsidy Levels

We isolate the zero level sets for the functions $W_4 - W_2$ and $W_4 - \hat{W}_2$ and graph on one contour to more closely compare the curves. We define the normal case as having the usual values of $\gamma$ and $\gamma_2$ as defined in Table 3.2. We examine this case at $t = 10, 25$ and $45$ as seen below in Figures 3.19-3.21 respectively. At $t = 10$, we see that the individual does not always directly attend a four-year university as we originally concluded. We see that with a sufficient subsidy, the individual should first attend a two-year community college before transferring to a four-year institution. At both $t = 25$ and $t = 45$ the two zero level sets are almost the same line. In
this case, we see that $W_4$ beats both $W_2$ and $\hat{W}_2$, and the individual would directly attend a four-year university if $\alpha$ and $\beta$ fell in the area above the zero level sets because that would imply a greater chance of success at a four-year university than at a two-year community college. The individual would more often first attend a two-year community college if $\alpha$ and $\beta$ fell beneath the zero level sets because the individual would have a greater chance of success at a two-year community college than at a four-year university.

![Graph](image)

Figure 3.19: Comparing $W_4 - W_2 = 0$ and $W_4 - \hat{W}_2 = 0$ with $t = 10, \gamma = 0.3, \gamma_2 = 0.5$
Figure 3.20: Comparing $W_4 - W_2 = 0$ and $\widehat{W}_4 - \widehat{W}_2 = 0$ with $t = 25, \gamma = 0.3, \gamma_2 = 0.5$
Figure 3.21: Comparing $W_4 - W_2 = 0$ and $W_4 - \hat{W}_2 = 0$ with $t = 45$, $\gamma = 0.3$, $\gamma_2 = 0.5$. At forty-five years, the different level subsidy only has small effect on the total wages earned.

We now look at the four extreme cases for each time $t = 25$ and $t = 45$. The extreme cases are the combinations of when $\gamma$ and $\gamma_2$ equal 1 and 0, including the cases where they are both 1 or both 0. In the first extreme case we will examine when $\gamma = 0$ and $\gamma_2 = 1$ as seen in Figures 3.22 and 3.23. There is much greater difference for $t = 25$ between the two zero curves. We see that $\hat{W}_2$ beats $W_2$ because $\hat{W}_2$ received a subsidy and $W_2$ did not. For $t = 45$, we see a general shift down and the curves are closer together. For both time intervals, $\hat{W}_2$ is dominant in the area between the two
curves and $W_4$ is dominant above the solid line, so the individual who either directly attends a four-year university or first attends a two-year community college will have a positive income. We see that with a large subsidy for the community college, the individual should attend a two-year community college.

Figure 3.22: Comparing $W_4 - W_2 = 0$ and $\hat{W}_4 - \hat{W}_2 = 0$ with $t = 25, \gamma = 0, \gamma_2 = 1$.

Two-year community college receives a subsidy and four-year university does not.
Figure 3.23: Comparing $W_4 - W_2 = 0$ and $W_4 - \hat{W}_2 = 0$ with $t = 45, \gamma = 0, \gamma_2 = 1$. At forty-five years, the subsidy level of a two-year community college has a small effect on the total wages earned.

In the second extreme case we will examine when $\gamma = 1$ and $\gamma_2 = 0$ as seen in Figures 3.24 and 3.25. We still observe a much greater difference for $t = 25$ between the two zero curves, but we see that the curves have switched dominance and that $W_4$ stills beats $W_2$ and $\hat{W}_2$, but $W_2$ now beats $\hat{W}_2$, because $W_2$ received the subsidy and $\hat{W}_2$ did not. For $t = 45$, we see a general shift up and the curves are again closer together. The individual will first attend a four-year university if, for $t = 25$
or \( t = 45, \alpha \leq \beta \) meaning that the individual has a greater chance of succeeding at the four-year university. However, the individual is more likely to attend a two-year community college if there is still a good subsidy equal to the four-year university.

![Graph comparing W_4 - W_2 and \( \hat{W}_2 \) with t = 25, \( \gamma = 1, \gamma_2 = 0 \).]

Figure 3.24: Comparing \( W_4 - W_2 = 0 \) and \( W_4 - \hat{W}_2 = 0 \) with \( t = 25, \gamma = 1, \gamma_2 = 0 \). Four-year university and two-year community college both receive a subsidy at the same level.
Figure 3.25: Comparing $W_4 - W_2 = 0$ and $W_4 - \hat{W}_2 = 0$ with $t = 45, \gamma = 1, \gamma_2 = 0$. At forty-five years, the same level subsidy only has small effect on the total wages earned.

In the third extreme case we will examine when $\gamma = \gamma_2 = 0$. In this case, no institution received a subsidy. Figures 3.26 and 3.27 show that the third extreme case is very similar to the normal case at both $t = 25$ and $t = 45$ where the lines are so close, they appear to be one continuous line. With the lines so close, the conclusion is as before and that the individual would more often first attend a two-year community college if $\alpha > \beta$.  

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Neither institution receives a subsidy. As a result, the decision of attending a four-year university versus a two-year community college is the same.

Figure 3.26: Comparing $W_4 - W_2 = 0$ and $W_4 - \hat{W}_2 = 0$ with $t = 25, \gamma = 0, \gamma_2 = 0$. 

Neither institution receives a subsidy. As a result, the decision of attending a four-year university versus a two-year community college is the same.
Figure 3.27: Comparing $W_4 - W_2 = 0$ and $\hat{W}_4 - \hat{W}_2 = 0$ with $t = 45$, $\gamma = 0$, $\gamma_2 = 0$.

The decision of attending a four-year university versus a two-year community college is the same.
In the fourth and final extreme case we will examine when $\gamma = \gamma_2 = 1$ as seen is Figures 3.28 and 3.29. In this case, both institutions received a subsidy. We see that this case is similar to the normal case and the third case, where the two zero curves are so close that it appears there is one curve. However this curve is shifted down more than the other cases, and so it is clear to say that the individual will attend a four-year university if $\alpha < \beta$, because the zero curve is below the line $\alpha = \beta$.

![Graph](image)

Figure 3.28: Comparing $W_4 - W_2 = 0$ and $\hat{W}_4 - \hat{W}_2 = 0$ with $t = 25, \gamma = 1, \gamma_2 = 1$.

Both institutions receive a subsidy at the same level.
Figure 3.29: Comparing $W_4 - W_2 = 0$ and $W_4 - \hat{W}_2 = 0$ with $t = 45, \gamma = 1, \gamma_2 = 1$. The decision of attending a four-year university versus a two-year community college is the same.

By graphing all four extreme cases at $t = 45$ in Figure 3.30, we observe that whenever $\gamma = \gamma_2$, it coincides with $W_4 - W_2 = 0$ and we can no longer see that data, because we are just showing that the government is choosing to subsidize a four-year university and a two-year community college at the same level and therefore the lines are the same. The subsidy levels used are just extreme cases where each set of numbers pointing to a line shows the value of $\gamma$ followed by the value of $\gamma_2$ to graph.
each particular line. We see that for $\gamma = 0$ and $\gamma_2 = 1$, a two-year community college is subsidized well compared to a four-year university, so it would be beneficial for the individual to attend a two-year community college, but for $\gamma = 1$ and $\gamma_2 = 0$, both a four-year university and a two-year community college are subsidized well and then it depends on the individual’s ability.

![Figure 3.30: Comparing $W_4 - W_2 = 0$ and $\hat{W}_4 - \hat{W}_2 = 0$ with $t = 45$, $(\gamma, \gamma_2) = (0, 0), (0, 1), (1, 0), (1, 1)$.](image)

As a specific example use data from the University of Akron that show the four-year, five-year, and six-year graduation rates for new first-time, full-time freshmen entering two-year and four-year programs for students with specific ACT scores ranging from 9 to 34 [30] to estimate values for $\alpha$ and $\beta$. We calculate $\alpha$ by using the
square root of the two-year success rate, and \( \beta \) by using the fourth root of the four-year success rate for the two-year and four-year data. Once \( \alpha \) and \( \beta \) are calculated, these values are plotted with the corresponding ACT score and a linear regression is then applied to the data points. The weighted linear fit of the four-year data, where higher data points were weighted more, does not have as good of fit as that of the two-year data, and the two-year data has a greater rate of change. Figures 3.31-3.32 show ACT score versus the relative probability of success. Figure 3.33 shows the two sets of data together and we see that overall, the four-year programs have a greater probability of success than the two-year programs. We also see that while both sets of data have a range of ACT scores, the two-year programs have more lower scores and the four-year programs have more higher scores.
Figure 3.31: $\alpha$ calculated by the square root of the two-year success rate. The linear regression $f_2 = 0.0334x - 0.0757$. 
Figure 3.32: $\beta$ calculated by the fourth root of the four-year success rate. The linear regression $f_4 = 0.0177x + 0.4104$. 
Figure 3.33: Comparing $\alpha$ modeled by the square root of the two-year success rate with $f_2 = 0.0334x - 0.0757$ and $\beta$ modeled by the fourth root of the four-year success rate with $f_4 = 0.0177x + 0.4104$
We also made a power law as seen in Figure 3.34. We observe that the fit for the two-year data is better than the fit for the four-year data, which has a few outlying high ACT scores.

Figure 3.34: Power Law of Graduation Rates: $\alpha$ modeled by the square root of the success rate with $f_2^2 = 0.0011x^2 - 0.0051x + 0.0057$ and $\beta$ modeled by the fourth root of the success rate with $f_4^4 = 9.815x10^{-8}x^4 - 9.103x10^{-6}x^3 + 0.0003x^2 - 0.0049x + 0.0284$

We also use the data to find an $\alpha$ and $\beta$ based on the average math ACT score for the state of Ohio [31], as an example of how accurate the equations are.
According to profile report [31], the average math ACT score for the state of Ohio in 2012 was 21.5. This score of 21.5 will be rounded off to 22, so that the success rates from [30] can be used to calculate an $\alpha$ and $\beta$. For $\alpha$, we will take the square root of the six-year graduation rate for the score of 22 in both the two-year program, 47.8%. The $\alpha$ for the two-year program is found to be 0.69. We know $\alpha$ is not required for the four-year program because the individual in the four-year program only uses $\beta$ to calculate lifetime earnings. For $\beta$, we will use the same value of the two-year program and the six-year graduation rate of 44.1% for the four-year program and take the fourth root of both to obtain a value of 0.83 for the two-year program and a value of 0.82 for the four-year program. The evaluations of the individual and government earnings functions defining the game using the calculated values of these $\alpha$ and $\beta$ are listed in Table 3.3.

Table 3.3 shows the evaluations of the six different equations we had in the game. Although the game consisted of eight equations, the two pairs for directly attending a four-year university did not change with the different levels of subsidy. For these evaluations, we set $t = 45$, $\gamma = 0.3$, $\gamma_2 = 0.5$, and $\mu = 0.3$, which means $T_4 = 10000$ and $T_2 = 3000$ and we have $\alpha = 0.69$ and $\beta = 0.83$ for the two-year program and $\beta = 0.82$ for the four-year program. We see that the individual who first attends a two-year community college before enrolling in a four-year institution makes about $150,000 less than the individual who directly attended a four-year university. We observe that the government’s wages for the four-year and two-year programs differ by roughly $15,000, which makes sense since our taxation rate, $\tau$,
was defined to be 0.1. The difference between the two-year programs with different level subsidies is minimal, although the individual gains a bit more with a different subsidy level, while the government gains a bit more with the same level subsidy. All of the figures put the break even point around the line $\alpha = \beta$, so these results are not a surprise, and we find that for $\alpha = 0.69$ and $\beta = 0.83$ or $\beta = 0.82$, we are in the positive wage zone.

Table 3.3: List of function values with defined $\alpha$ and $\beta$

<table>
<thead>
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<th>Four-year program</th>
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</thead>
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<td>-</td>
<td>$1,240,007.44$</td>
</tr>
<tr>
<td>$W_2$</td>
<td>$1,089,655.30$</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{W}_2$</td>
<td>$1,090,679.30$</td>
<td>-</td>
</tr>
<tr>
<td>$G_4$</td>
<td>-</td>
<td>$131,014.67$</td>
</tr>
<tr>
<td>$G_2$</td>
<td>$117,282.26$</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{G}_2$</td>
<td>$116,268.26$</td>
<td>-</td>
</tr>
</tbody>
</table>
3.2 The Dominant Strategy of the Government

The government has the choice to subsidize a four-year university and a two-year community college at the same level or at different levels. Equations (2.23), (2.24), and (2.25) are used to simulate the comparisons. Although our focus is on the optimal income for the government under the specified conditions, we also examine one other time interval, \( t = 25 \), as we did for the individual, to see the changes as time increases, but we do not examine \( t = 10 \), since not enough time has passed for the individual to overcome the costs of education. Another variable that is defined at multiple points is the tuition of a community college. We take the tuition of a community college to be \( \mu = 0.3, 0.7 \) and 1 or \( T_2 = 3,000, 7,000 \) and 10,000. We explore the effect the difference in the cost of education could have on income. However, we explore this effect only at \( t = 45 \), because the effect at \( t = 25 \) is minimal anyway. The separate line that appears on Figures 3.35-3.42 is where \( \alpha^2 = \beta \), as for the analysis of the individual.

For the government, the first relation to be analyzed will be equation (2.23). As we can see this equation no longer depends on \( \beta \). Equation 2.23 only depends on \( \alpha \) and the two levels of subsidy. The dominant strategy for the government becomes subsidize at the same level \( \gamma = \gamma_2 \) if \( \gamma_2 > \gamma \), and subsidize at different levels \( \gamma \neq \gamma_2 \) if \( \gamma_2 < \gamma \). If \( \gamma_2 < \gamma \), then the government would make more money since \( \hat{G}_2 > G_2 \).
3.2.1 Using the Same Subsidy Level

Figure 3.35 shows the comparisons of equation (2.24) with the same level of subsidy for a university and a community college at $t = 25$, or twenty-five years after the individual attended school. The tuition level used for the community college is where $\mu = 0.3$. We see that for twenty-five years, the area above the zero level set where the government makes money, is smaller than in the other contours, so the government is not seeing a quick return for the money they invested in subsidies.

Figure 3.35: Comparing $G_4$ and $G_2$ through a contour plot of the difference $G_4 - G_2$ with $t = 25, T_2 = 3,000$ and $\gamma = 0.3$. Smaller region in which the government makes money.

Figures 3.36-3.38 compare the choice at $t = 45$, or forty-five years after the individual attended school. Here, we present contours using all three different
levels of the cost of tuition for a community college, including $\mu = 0.3, 0.7, \text{ and } 1$. All three contours have the zero level set defined on $[0,1] \times [0,1]$ and as the cost of tuition for a community college approaches the cost of tuition for a university, we see the that the government will earn the same amount of money regardless of where the individual starts. The greater difference between the cost of tuition, the more money the government loses as their region for making money gets smaller. For the government, by subsidizing a community college and a university with near equal tuition at the same level subsidy, the optimal income would be when the individual retires after working for forty-five years.

Figure 3.36: Comparing $G_4$ and $G_2$ through a contour plot of the difference $G_4 - G_2$ with $t = 45, T_2 = 3,000$ and $\gamma = 0.3$. 

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Figure 3.37: Comparing $G_4$ and $G_2$ through a contour plot of the difference $G_4 - G_2$ with $t = 45$, $T_2 = 7,000$ and $\gamma = 0.3$. 
Figure 3.38: Comparing $G_4$ and $G_2$ through a contour plot of the difference $G_4 - G_2$ with $t = 45$, $T_2 = 10,000$ and $\gamma = 0.3$. Government makes more money as the tuition of a community college approaches that of a university.

3.2.2 Using Different Subsidy Levels

Figure 3.39 shows the comparisons of equation (2.25) with different levels of subsidy for a university and a community college at $t = 25$, or twenty-five years after the individual attended school. We take the tuition level for the community college to be $\mu = 0.3$. We see that for twenty-five years, the area above the zero level set where the government makes money, is smaller than in most of the other contours, so the government is seeing a slower return for the money they invested in subsidies. The area above the zero level set is larger than the area for comparing equation (2.24), so
the government sees more money in a period of time with different levels of subsidy than a period of time with the same level of subsidy.

Figure 3.39: Comparing $G_4$ and $\tilde{G}_2$ through a contour plot of the difference $G_4 - \tilde{G}_2$ with $t = 25$, $T_2 = 3,000$, $\gamma = 0.3$ and $\gamma_2 = 0.5$. Smaller region in which the government makes money.

Figures 3.40-3.42 show the comparisons of the equation (2.25) at $t = 45$, or forty-five years after the individual attended school. Here, we present contours using all three different levels of the cost of tuition for a community college, including $\mu = 0.3, 0.7, \text{ and } 1$. All three contours have the zero level set defined and as the cost of tuition for a community college approaches the cost of tuition for a university, we see the government will have an equal likelihood of making the same amount of money.
regardless of where the individual first attends school. This means that as the tuition
level for a community college approaches that of a university, the greater chance that
the individual will have equal likelihood of succeeding at either a four-year university
or a two-year community college. The contours for $\mu = 0.7$ and $\mu = 1$ for equations
(2.24) and (2.25) are very similar, while the contour for $\mu = 0.3$ with different levels of
subsidy, has a larger area for the government making money. The greater difference
between the cost of tuition, the more money the government loses as their region for
making money gets smaller. However, since the contour graphs are so similar, the
government would have optimal income when subsidizing a community college and a
university with near equal tuition at different levels of subsidy when the individual
retires after working for forty-five years.
Figure 3.40: Comparing $G_4$ and $\hat{G}_2$ through a contour plot of the difference $G_4 - \hat{G}_2$ with $t = 45$, $T_2 = 3,000$, $\gamma = 0.3$ and $\gamma_2 = 0.5$. 
Figure 3.41: Comparing $G_4$ and $\hat{G}_2$ through a contour plot of the difference $G_4 - \hat{G}_2$

with $t = 45$, $T_2 = 7,000$, $\gamma = 0.3$ and $\gamma_2 = \frac{0.3}{1.4}$. 
Figure 3.42: Comparing $G_4$ and $\hat{G}_2$ through a contour plot of the difference $G_4 - \hat{G}_2$ with $t = 45$, $T_2 = 10,000$, $\gamma = 0.3$ and $\gamma_2 = 0.15$. Equal probabilities of success at equal tuition. Government makes more money as the tuition of a community college approaches that of a university. No noticeable improvement from the fit with Equation (2.24).

3.2.3 Comparing the Same Subsidy Level and Different Subsidy Levels

We isolate the zero level sets for the functions $G_4 - G_2$ and $G_4 - \hat{G}_2$ and graph on one contour to more closely compare the curves. We define the normal case as having the usual values of $\gamma$ and $\gamma_2$ as defined in Table 3.2. We examine this case at both $t = 25$ and $t = 45$ as seen below in Figures 3.43 and 3.44 respectively. For these comparisons, we don’t include $t = 10$, since not enough time has passed for the profits of the individual to overcome the costs of education. For $t = 25$, the contours
have a small bend in them which they have not exhibited before, but we observe
dthat with this shape, the individual with any $\alpha$, but a high $\beta$ would first attend a
four-year university, because a high $\beta$ equals likely success at a university, and the
individual with any $\alpha$, but a low $\beta$ would first attend a community college, because
a low $\beta$ equals probable success at a community college. In this case, we see that $G_4$
beats both $G_2$ and $\hat{G}_2$ and that $G_2 > \hat{G}_2$.

Figure 3.43: Comparing $G_4 - G_2 = 0$ and $G_4 - \hat{G}_2 = 0$ with $t = 25, \gamma = 0.3, \gamma_2 = 0.5$
We now look at the two extreme cases for each time $t = 25$ and $t = 45$. The extreme cases are the combinations of when $\gamma$ and $\gamma_2$ equal 1 and 0. For the government, we will not examine when $\gamma$ and $\gamma_2$ are both 1 or both 0. In the first extreme case we examine when $\gamma = 0$ and $\gamma_2 = 1$ as seen in Figures 3.45 and 3.46. In Figure 3.45, we see the largest gap between the two relations in any of the contours. In this case, the base level subsidy, $\gamma = 0$, while the subsidy rate for the community college, $\gamma_2 = 1$, and as it is defined as an extreme case, we finally see the results of having such different values for the subsidies. Between the two curves is where the individual would go to a community college at a different level of subsidy since
$\hat{G}_2 > G_2$ because $\hat{G}_2$ received a subsidy. In the area above the $G_4 - G_2$ curve, if the individual chose to go to a four-year university, then the government would make money. For $t = 45$ in Figure 3.46, we see the curve for $G_4 - \hat{G}_2$ shifts up.

![Graph](image)

Figure 3.45: Comparing $G_4 - G_2 = 0$ and $G_4 - \hat{G}_2 = 0$ with $t = 25, \gamma = 0, \gamma_2 = 1$.

Government only subsidizes the two-year community college.
In the second extreme case we will examine when $\gamma = 1$ and $\gamma_2 = 0$ as seen is Figures 3.47 and 3.48. We still observe a much greater difference for $t = 25$ between the two zero curves, but we see that the curves have switched dominance and that $G_4$ stills beats $G_2$ and $\hat{G}_2$, but $G_2$ now beats $\hat{G}_2$. For $t = 45$, we see a general shift up and the curves are again closer together. The individual will first attend a four-year university if, for $t = 25$ or $t = 45$, his $\alpha < \beta$ meaning he is more likely to succeed in getting a four-year degree by directly attending a four-year institution. Here we start to see the positive area shrinking which means the government is making less money.
After only twenty-five years, the government is actually making very little profit, and it is only until later that the government recovers.

Figure 3.47: Comparing $G_4 - G_2 = 0$ and $G_4 - \hat{G}_2 = 0$ with $t = 25, \gamma = 1, \gamma_2 = 0$.

Government subsidizes both the four-year university and the two-year community college.
Figure 3.48: Comparing $G_4 - G_2 = 0$ and $G_4 - \hat{G}_2 = 0$ with $t = 45, \gamma = 1, \gamma_2 = 0$. The region where the government makes more money increases over time since in Figure 3.47, the government saw only a slight return on the subsidy investment.

3.3 The Dominant Strategy Cases

We will now examine the players strategies together to find where they equal. Table 3.4 shows the two cases as two rows. We know that we operate in Row 1 if $\gamma \geq \gamma_2$. Row 1 depends on $\alpha$ and $\beta$, tuition levels and time frame $t$. There is no subsidy effect in Row 1 because the subsidy level is the same for the four-year university and the two-year community college. Since this case depends on $\alpha$ and $\beta$, it also depends on the relations $\alpha = \beta$ and $\alpha^2 = \beta$. Intuition suggests that for an individual, $\alpha \geq \beta$, but
not by much. We know that we operate in Row 2 if $\gamma_2 > \gamma$. Row 2 depends on $\alpha$ and $\beta$, tuition levels, time frame $t$, as well as the subsidy effect that is encountered with the different levels of subsidy.

Table 3.4: Dominant Strategies

<table>
<thead>
<tr>
<th>Subgames</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 1</td>
<td>I $(W_4, G_4)$ II $(W_2, G_2)$</td>
</tr>
<tr>
<td>Row 2</td>
<td>III $(W_4, G_4)$ IV $(\hat{W}_2, \hat{G}_2)$</td>
</tr>
</tbody>
</table>

Figure 3.49 shows the dominant strategies for Row 1 when $\gamma \geq \gamma_2$. We see that when we are above the government curve, region I is dominant and so the government is making money when the individual goes to a four-year university. We see that when we are below the individual curve, region II is dominant and so the individual should first attend a community college before transferring to a four-year institution. Between these curves is not a mixed strategy but rather a region where the government is not at an optimal payoff level. Since the government controls the subsidy level, the government would do so to optimize its payoff and this clearly occurs when $\gamma = 0$. 

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Figure 3.49: Finding the dominant strategy in Row 1 where $\gamma \geq \gamma_2$, $\gamma = 1$ and $\gamma_2 = 0$. Dominant areas I and II surrounding zone where the government is not at an optimal payoff level.
Figure 3.50 shows where the two curves meet, which gives the government a value of $\gamma$ to set up a policy to not only further higher education for students, but also make a profit in the process. The curves are identical when $\gamma = 0.1$, or when a ten percent subsidy is given to both a four-year university and a two-year community college. In this case, everyone benefits.

![Graph](image)

Figure 3.50: Finding the value of $\gamma$ to make Equations (2.21) and (2.24) equal. $\gamma$ is found to be 0.1 for a match.

Figure 3.51 shows the dominant strategies for Row 2 when $\gamma_2 > \gamma$. We see that when we are above the individual curve, region III is dominant and so the individual should go to a four-year university. We see that when we are below the government curve, region IV is now dominant and so the individual should first
attend a community college before transferring to a four-year institution. Between these curves is not a mixed strategy but rather a region where the individual is not at an optimal payoff level. In this region, the government wants the individual to go to a four-year university so that there is a profit, but also in this region, the individual should first be going to a two-year community college based on ability. If the individual would take the risk on a four-year university, he might not succeed and he would be in a worse spot than before.

Figure 3.51: Finding the dominant strategy in Row 2 here $\gamma_2 > \gamma$, $\gamma = 0$ and $\gamma_2 = 1$. Dominant areas III and IV surrounding zone where the individual is not at an optimal payoff level.
Figure 3.52 shows the closest that the two curves can get, which gives the government a relative value of $\gamma_2$ to set up a policy to not only further higher education for students, but also make a profit in the process. The curves are very close when $\gamma_2 = 0.01$, or when a one percent subsidy is given to a two-year community college. This case does not benefit as many as before and certainly not as well.

\[
\begin{align*}
\gamma_2 & = 0.01 \\
\text{for as close together as possible.}
\end{align*}
\]

**Figure 3.52:** Finding the value of $\gamma_2$ to make Equations (2.22) and (2.25) equal. $\gamma$ is found to be 0.01 for as close together as possible.
CHAPTER IV
CONCLUSIONS

This paper used game theory to explore the differences between attending a four-year university and attending a community college. The game has two players each with two choices. The first player is the individual, who in the pursuit of a four-year degree, has to choose to either attend a four-year university or to first attend a two-year community college. The second player is the government who has to choose to subsidize a four-year university and a two-year community college at the same level or at different levels. The goal of both players is to maximize income. With two players who each had two choices, the game has four sets of equations, and therefore, four subgames.

In following the models of Correa [16] and Macavei [26], our model used many of the same variables and added new variables for lifetime earnings at the three levels of having only a high school education, having an associate’s degree, and having a bachelor’s degree or higher, the cost of a two-year community college tuition as a proportion of the cost of a four-year university tuition, and the different subsidy levels of a four-year university or a two-year community college.

The key assumptions for our model are:

- The student graduates high school and seeks higher education,
- The student intends to go for a four-year degree,

- The student can transfer to a four-year program upon completing two years at a community college,

- The student completing two years at a community college is equal to the student completing one year at a university,

- The probability of successfully completing a year of a two-year community college, \( \alpha \), remains constant for each of the two years,

- The probability of successfully completing a year of a four-year university, \( \beta \), remains constant for each of the four years,

- The student does not work while attending a higher education institution,

- The discipline the student chooses to study is not included in the model,

- The government and student wish to optimize income.

The results obtained from the model are:

- The individual should directly attend a four-year university if he is interested in maximizing income over a short time frame because he will have an extra year of income by graduating in four years instead of five unless the subsidy for a community college is sufficiently high, in which case the individual should first attend a two-year community college,

- The individual should directly attend a four-year university when the tuition of a community college is high,
• The individual should directly attend a four-year university with $\beta \geq \alpha$,

• The effect that the cost of education has on income lessens over time to the point where it has little or no effect on net lifetime earnings,

• The government should subsidize at the same level ($\gamma = \gamma_2$) if $\gamma_2 > \gamma$ and subsidize at different levels ($\gamma \neq \gamma_2$) if $\gamma_2 < \gamma$,

• The government has optimal income when subsidizing at the same level with equal tuition,

• The government should subsidize ten percent of both a four-year university and a two-year community college to optimize income when $\gamma \geq \gamma_2$,

• The government should subsidize one percent of a two-year community college to optimize income when $\gamma_2 > \gamma$.

Some future work for the model that would enhance the analysis would be differentiating between disciplines or even differentiating between STEM and non-STEM disciplines, which would have groups of disciplines rather than a myriad of individual disciplines. One modification could be to have the cost of education vary according to discipline, so that the individual would pay more for a certain discipline. Another addition that would enhance this study would be to factor in postsecondary and advanced placement courses, so that the individual could have some requirements already fulfilled before attending a higher education institution. Another possible modification to the model would be to have the probabilities of success for the student, $\alpha$ and $\beta$, vary each year.
BIBLIOGRAPHY


The following code was executed in MatLab to create the contour plots and graphs in this thesis. In this code, to increase the readability of the equations by taking into account the restrictions of the MatLab programming language, $\alpha$ is represented by $a$, and $\beta$ is represented by $b$. The lines preceded by a % are different cases that were each done on its own. This code is a compilation of all codes used to present a compact form.

```matlab
[a,b] = meshgrid(0:0.01:1,0:0.01:1);
t = 10;
% t = 25;
% t = 45;
LEt = 45;
LE0 = 900000/LEt;
LE2 = 1100000/LEt;
LE4 = 1800000/LEt;
Inc2 = LE0*(1+4/t-a-(4*a)/t) ... 
    + ((LE0+LE2)/2)*(a+(3*a)/t-a.^2-(3*a.^2)/t) ... 
    + LE2*(a.^2+(2*a.^2)/t-(a.^2.*b)/t-(a.^2.*b.^2)/t-a.^2.*b.^3) ... 
    + LE4*(a.^2.*b.^3);
Inc4 = LE0*(1+3/t-b-(3*b)/t) ... 
    + LE2*(b+(2*b)/t-b.^2/t-b.^3/t-b.^4) ... 
    + LE4*b.^4;
tau = 0.10;
gamma = 0.3;
gamma2 = 0.3/(2*mu);
mu = 0.3;
%mu = 0.7;
%mu = 1;
T4 = 10000;
T2 = mu*T4;
W4 = Inc4*t*(1-tau) - (1-gamma)*T4*(1+b+b.^2+b.^3);
```
W2 = Inc2*(t-1)*(1-tau) - (1-gamma)*(T2*(1+a) + T4*a.^2.*(1+b+b.^2));
W2hat = Inc2*(t-1)*(1-tau) - (1-gamma)*T2*(1+a) - (1-gamma)*T4*a.^2.*(1+b+b.^2);
Gov4 = Inc4*t*tau - gamma*T4*(1+b+b.^2+b.^3);
Gov2 = Inc2*(t-1)*tau - gamma*(T2*(1+a) + T4*a.^2.*(1+b+b.^2));
Gov2hat = Inc2*(t-1)*tau - gamma2*T2*(1+a) - gamma*T4*a.^2.*(1+b+b.^2);
f = W4-W2;
g = W4-W2hat;
f = Gov4-Gov2;
g = Gov4-Gov2hat;
figure;
[C,h] = contourf(a,b,f)
clabel(C,h);
h = findobj('Type','patch');
set(h,'LineWidth',2)
hold on
x = 0:.01:1;
y = x.^2;
p = plot(x,y);
set(p,'Color','red','LineWidth',2)
g = Gov4-Gov2hat;
figure;
[C,h] = contourf(a,b,f)
clabel(C,h);
h = findobj('Type','patch');
set(h,'LineWidth',2)
hold on
x = 0:.01:1;
y = x.^2;
p = plot(x,y);
set(p,'Color','red','LineWidth',2)