STOCHASTIC MATERIAL CHARACTERIZATION OF HETEROGENEOUS MEDIA
WITH RANDOMLY DISTRIBUTED MATERIAL PROPERTIES

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STOCHASTIC MATERIAL CHARACTERIZATION OF HETEROGENEOUS MEDIA
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ABSTRACT

In the field of computational mechanics, there has been a very challenging problem, which is the characterization of heterogeneous media with randomly distributed material properties. In reality, no material is homogeneous and deterministic in nature and it has been well-known that randomness in microstructures and properties of materials could significantly influence scatter of structural response at larger scales. Therefore, stochastic characterization of heterogeneous materials has increasingly received attention in various engineering and science fields. In order to deal with this challenging problem, two major challenges need to be addressed: 1) developing an efficient modeling technique to discretize the material uncertainty in the stochastic domain and 2) developing a robust and general inverse identification computational framework that can estimate parameters related to material uncertainties.

In this dissertation, two major challenges have been addressed by proposing a robust inverse analysis framework that can estimate parameters of material constitutive models based on a set of limited global boundary measurements and combining the framework with a general stochastic finite element analysis tool. Finally a new stochastic inverse analysis framework has been proposed, which has a novel capability of modeling effects of spatial variability of both linear and nonlinear material properties on macroscopic material and structural response. By inversely identifying statistical parameters (e.g. spatial mean, spatial variance, spatial correlation length, and random
variables) related to spatial randomness of material properties, it allows for generating statistically equivalent realizations of random distributions of linear and nonlinear material properties and their applications to the development of probabilistic structural models.

First, a robust inverse identification framework, called the Self-Optimizing Inverse Method (Self-OPTIM), has been developed. Unlike other signal matching approaches used in model updating, in the course of two parallel finite element simulations, Self-OPTIM automatically minimizes an implicit objective function defined as a function of internal full-field stresses and strains. The performance of Self-OPTIM has been proven by both numerical and experimental verifications. Second, in the stochastic finite element method (SFEM), the spatially varying random material properties, such as linear elastic modulus, Poisson’s ratio and initial yield strength, are discretized by the Karhunen-Loève expansion. This computational tool allows for Monte Carlo simulations considering material uncertainties and their propagations through forward simulations. By combining SFEM with Self-OPTIM, a novel algorithm named as stochastic Self-OPTIM for stochastic material characterization of heterogeneous media with randomly distributed material properties is invented. Its performance has been verified by applying to reconstruction problems and stochastic inverse identifications of statistical parameters of random fields related to material properties. To demonstrate promising applications of the stochastic Self-OPTIM, a probabilistic low-cycle fatigue life prediction model has been developed with considerations of spatial variability of Young’s modulus, Poisson ratio and the initial yield strength. For this purpose, a
probabilistic surrogate model has also been developed by using the polynomial chaos expansion.
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TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LIST OF TABLES.</td>
<td>xi</td>
</tr>
<tr>
<td></td>
<td>LIST OF FIGURES.</td>
<td>xii</td>
</tr>
<tr>
<td>I.</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1</td>
<td>Research Background</td>
<td>1</td>
</tr>
<tr>
<td>1.1.1</td>
<td>Material Uncertainty</td>
<td>1</td>
</tr>
<tr>
<td>1.1.2</td>
<td>Stochastic Finite Element Method (SFEM)</td>
<td>3</td>
</tr>
<tr>
<td>1.1.3</td>
<td>Inverse Identification Problem</td>
<td>5</td>
</tr>
<tr>
<td>1.2</td>
<td>Objectives</td>
<td>7</td>
</tr>
<tr>
<td>1.3</td>
<td>Outline of the Dissertation</td>
<td>7</td>
</tr>
<tr>
<td>II.</td>
<td>LITERATURE REVIEW</td>
<td>10</td>
</tr>
<tr>
<td>2.1</td>
<td>Inverse Identification Algorithm and Problem</td>
<td>10</td>
</tr>
<tr>
<td>2.1.1</td>
<td>Inverse Identification Framework</td>
<td>10</td>
</tr>
<tr>
<td>2.1.2</td>
<td>Optimization Algorithm</td>
<td>11</td>
</tr>
<tr>
<td>2.1.3</td>
<td>Inverse Identification Problem</td>
<td>14</td>
</tr>
<tr>
<td>2.2</td>
<td>Stochastic Finite Element Method</td>
<td>20</td>
</tr>
<tr>
<td>2.2.1</td>
<td>Uncertainty Representation Method</td>
<td>21</td>
</tr>
<tr>
<td>2.2.2</td>
<td>Stochastic Finite Element Analysis Framework</td>
<td>23</td>
</tr>
<tr>
<td>2.3</td>
<td>Stochastic Material Characterization</td>
<td>25</td>
</tr>
<tr>
<td>2.4</td>
<td>Current Work</td>
<td>27</td>
</tr>
</tbody>
</table>
III. SELF OPTIMIZATION INVERSE IDENTIFICATION METHOD .................... 30
   3.1 Self-Optimizing Inverse Method (Self-OPTIM) .................................. 30
      3.1.1 Theoretical Background .................................................. 30
      3.1.2 Algorithms for Inverse Analysis in Self-OPTIM ...................... 36
      3.1.3 Implicit Formulation of Objective Function .......................... 39
   3.2 Optimization Algorithm ........................................................ 41
   3.3 Advantages of Self-OPTIM Inverse Method ................................. 43

IV. A GENERAL PLATFORM FOR STOCHASTIC FINITE ELEMENT SIMULATION ................................................................. 45
   4.1 Spectral Decomposition of Random Field by Karhunen-Loève Expansion ................................................................. 45
      4.1.1 Random Field Discretization by Karhunen-Loève Expansion ...... 45
      4.1.2 Galerkin Finite Element Techniques for Karhunen-Loève Expansion ................................................................. 48
   4.2 General Mapping-Interpolation Method between Random Field and Finite Element Meshes .................................................. 53
      4.2.1 Separation of Random Field Mesh from Finite Element Mesh .... 54
      4.2.2 General Mapping-Interpolation Method ................................. 54
   4.3 Intrusive Formulation of Stochastic Finite Element ..................... 59
   4.4 Implementation of Stochastic Finite Elements for Monte Carlo Simulation ........................................................................... 61
      4.4.1 Monte Carlo Simulation with SFE and KL Expansion .............. 63
      4.4.2 Post-Processing for Stochastic Characterization of Material Response ........................................................................... 64
   4.5 Conclusion .................................................................................... 64

V. VERIFICATIONS OF SELF-OPTIM .................................................. 66
   5.1 Cyclic Elasto-Plastic Constitutive Model ...................................... 66
   5.2 Numerical Verification .................................................................. 69
5.2.1 Example 1: Verification with A Simple Model .........................69
5.2.2 Example 2: Verification with A Complex Model ......................79
5.3 Effects of Image Noises and Boundary Force Errors on Self-OPTIM ...87
5.4 Verification of Self-OPTIM Identification with Experiment Test Results.93
5.4.1 Material Tests and Predefined Boundary Loading Data .............94
5.4.2 Identification Results and Discussions ....................................95
5.5 Conclusion ..................................................................................99
5.6 Micromechanics and Fracture Mechanics Based Damage Model ....101
5.6.1 Microcrack Nucleation ..............................................................102
5.6.2 Microcrack Growth .................................................................103
5.7 Validation with Synthetic Data ....................................................104
5.7.1 Reference Simulation with Rate-Dependent Damage Constitutive Model .................................................................104
5.7.2 Numerical Example .................................................................108
5.7.3 Discussions .............................................................................114
5.8 Damage Parameter Identification Using Actual Experiment Data ......115
5.9 Conclusion ..................................................................................119
VI. NUMERICAL EXAMPLES OF APPLYING THE GENERAL STOCHASTIC FINITE ELEMENT SIMULATION PLATFORM FOR RELIABILITY ANALYSIS ...........................................................................121
6.1 A Square Plate with Randomly Distributed Young’s Modulus .......121
6.1.1 Results of Static Stochastic Finite Element Analysis .................124
6.1.2 Monte Carlo Simulation for Analysis of Statistical Material Response ........................................................................128
6.2 A Quarter Circular Tube with Randomly Distributed Young’s Modulus 131
6.3 A Retaining Wall with Randomly Distributed Young’s Modulus .......133
A dog-bone specimen with random distributed Young’s Modulus, Poisson’s ratio, and yield strength

Conclusion

VII. STOCHASTIC CHARACTERIZATION OF MATERIAL UNCERTAINTIES FOR PROBABILISTIC LOW-CYCLE FATIGUE MODELS

7.1 Stochastic Material Characterization

7.2 Stochastic Self-OPTIM

7.3 Reconstruction of Heterogeneous Material Properties by Stochastic Self-OPTIM

7.3.1 A square Domain with Spatially Varying Young’s Modulus

7.3.2 Reconstruction Results

7.4 Stochastic Characterization of Material Uncertainties by Stochastic Self-OPTIM

7.4.1 Construction of A Database of Random Heterogeneous Material Properties

7.4.2 Results of Stochastic Self-OPTIM

7.5 Probabilistic Surrogate Modeling by Using PCE

7.6 Effects of Variability of Material Properties on Low Cycle Fatigue Life Prediction

7.6.1 Deterministic Low-Cycle Fatigue Life Prediction Model

7.6.2 Development of A Probabilistic Low-Cycle Fatigue Life Prediction Model

7.7 Conclusion

VIII. SUMMARY AND FUTURE WORK

8.1 Summary

8.2 Future Work

REFERENCES
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>Summary of the data of the simple model</td>
<td>77</td>
</tr>
<tr>
<td>5.2</td>
<td>Summary of the results of the complex model</td>
<td>86</td>
</tr>
<tr>
<td>5.3</td>
<td>Summary of the data of the simple model</td>
<td>92</td>
</tr>
<tr>
<td>5.4</td>
<td>Identification results from Self-OPTIM</td>
<td>96</td>
</tr>
<tr>
<td>5.5</td>
<td>Self-OPTIM identification results by Simplex method ($X_T$ indicates reference parameters)</td>
<td>109</td>
</tr>
<tr>
<td>5.6</td>
<td>Identification results by using SSGA</td>
<td>112</td>
</tr>
<tr>
<td>5.7</td>
<td>Identification results at strain rate 300 (/sec)</td>
<td>116</td>
</tr>
<tr>
<td>5.8</td>
<td>Identification results at strain rate 490 (/sec)</td>
<td>118</td>
</tr>
<tr>
<td>6.1</td>
<td>The two-point correlation before and after the nonlinear transformation</td>
<td>123</td>
</tr>
<tr>
<td>6.2</td>
<td>The stochastic parameters of the random fields</td>
<td>135</td>
</tr>
<tr>
<td>7.1</td>
<td>Test matrix (BNP: Biaxial non-proportional loading path)</td>
<td>154</td>
</tr>
<tr>
<td>7.2</td>
<td>Summary of the identification results</td>
<td>154</td>
</tr>
<tr>
<td>7.3</td>
<td>Identification results of Young’s modulus</td>
<td>157</td>
</tr>
<tr>
<td>7.4</td>
<td>Identification results of Poisson’s ratio</td>
<td>158</td>
</tr>
<tr>
<td>7.5</td>
<td>Identification results of initial yield stress</td>
<td>158</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Two boundary conditions from experimental testing on a solid deformable body Subjected to body force, b and with material parameters unknown (dashed line indicates kinematically admissible virtual displacements)</td>
</tr>
<tr>
<td>3.2</td>
<td>Differences in internal virtual work functionals between traction- and displacement-driven simulated testing</td>
</tr>
<tr>
<td>3.3</td>
<td>Schematic flowchart of Self-OPTIM inverse analysis method</td>
</tr>
<tr>
<td>3.4</td>
<td>Flow chart of chaotic firefly algorithm</td>
</tr>
<tr>
<td>4.1</td>
<td>Plots of covariance functions where $x_1$=$y_1$=0.5: a) $b_{c1}=b_{c2}$=1, b) $b_{c1}=b_{c2}$=0.1, c) $b_{c}$=1, and d) $b_{c}$=0.1</td>
</tr>
<tr>
<td>4.2</td>
<td>First 50 Eigenvalues of the covariance function (Equation (4.3))</td>
</tr>
<tr>
<td>4.3</td>
<td>Eigenfunctions of covariance function (Equation (4.3)) with $N_{RF}$=9, $L_{D1}$=$L_{D2}$=1, $b_{c1}$=$b_{c2}$=1 and $\sigma$=1</td>
</tr>
<tr>
<td>4.4</td>
<td>a) numerically reproduced covariance function by using 6 by 6 square elements in RF mesh referring to $x_2$=0.5, $y_2$=0.5; b) analytical covariance function referring to $x_2$=0.5, $y_2$=0.5</td>
</tr>
<tr>
<td>4.5</td>
<td>Illustration of the transformation of coordinates within a certain RF element (Green dotted lines are FE mesh)</td>
</tr>
<tr>
<td>4.6</td>
<td>A demonstration of the coordinate transformation between two fully independent RF mesh and FE mesh on non-rectangular domain</td>
</tr>
<tr>
<td>4.7</td>
<td>Configuration of UEL SFEQ8</td>
</tr>
<tr>
<td>4.8</td>
<td>Schematic of Monte Carlo Simulation using SFE and post-processing with UEL framework interfaced with MATLAB codes (RF: Random Field; KL: Karhunen-Loève)</td>
</tr>
<tr>
<td>4.9</td>
<td>Schematics of the RF mesh and the corresponding FE mesh</td>
</tr>
</tbody>
</table>
5.1 Cyclic hardening behavior and effects of combined nonlinear hardening model parameters

5.2 (a) A simple model and reference stress-strain response and (b) a half-cycle displacement loading

5.3 Objective function values v.s. No of iterations during Self-OPTIM analysis

5.4 Parameter estimations by Self-OPTIM using synthetic test data under half-cycle displacement loading (a) $C$, Young’s modulus; (b) $\mu$, Poisson ratio; (c) $S_{y0}$, yield stress; (d) $H$, saturated yield stress by kinematic hardening; (e) $\gamma$, rate of change of yield stress by kinematic hardening

5.5 Convergence to false values of parameters related to nonlinear isotropic hardening by using half-cycle loading profile (a) $Q$, saturated yield stress by isotropic hardening and (b) $b$, rate of saturation of the yield stress by isotropic hardening

5.6 (a) 8-cycle displacement profile and (b) reference stress and strain response

5.7 Isotropic hardening parameter identification by Self-OPTIM under using 8-cycle displacement profile (a) $Q$, saturated yield stress by isotropic hardening and (b) $b$, rate of saturation of the yield stress by isotropic hardening

5.8 (a) Reconstructed strain-stress response (b) reconstructed back stress (c) yield surface plotted in different iteration step

5.9 (a) FE model for the complex case and the boundary displacement profile for reference simulation (b) 25 load steps in linear elastic region (c) 160 load steps in half cycle large strain region (d) 320 load steps in eight cycles

5.10 Parameter estimations by Self-OPTIM using synthetic test data under biaxial cyclic displacement loading (a) $C$, Young’s modulus; (b) $\mu$, Poisson ratio; (c) $S_{y0}$, yield stress; (d) $H$, saturated yield stress by kinematic hardening; (e) $\gamma$, rate of change of yield stress by kinematic hardening; (f) $Q$, saturated yield stress by isotropic hardening and (g) $b$, rate of change of saturation of $Q$

5.11 The displacement field introduced by Gaussian white noise on a nodal point

5.12 Parameter estimations by Self-OPTIM using synthetic test data with experimental noises and errors (a) $C$, Young’s modulus; (b) $\mu$, Poisson ratio; (c) $S_{y0}$, yield stress; (d) $H$, saturated yield stress by kinematic hardening; (e) $\gamma$, rate of change of yield stress by kinematic hardening; (f) $Q$, saturated yield stress by isotropic hardening and (g) $b$, rate of saturation of the yield stress by isotropic hardening

5.13 Geometry and FE mesh of the test specimen (all dimensions shown are in mm)

5.14 Mean value and standard error of the identified parameters in homogenous and heterogeneous test
5.15 Comparison of the force-displacement curves reconstructed by using the parameters prior to Self-OPTIM, after Self-OPTIM and experiment data. ...............................97

5.16 a) Reconstructed force-displacement curve of the dog-bone specimen by the parameter set identified by notched specimens; b) reconstructed force-displacement curve of the notched specimen by the parameter set identified from dog-bone specimens .......................................................................................98

5.17 Plots of normalized axial stresses and strains of a notched specimen from force-driven and displacement-driven simulations using the parameter set prior to Self-OPTIM, at the 5th optimization iteration, and after Self-OPTIM .........................................................98

5.18 Geometry of the specimen used in tensile impact tests (unite mm) .................105

5.19 (a) The ABAQUS model of the simulated specimen (b) reference stress-strain response (strain rate 10 s⁻¹) (c) reference microcrack growth (ζ) and (d) reference microcrack nucleation (N) ........................................................................................................107

5.20 Comparisons of (a) stress-strain curve, (b) mean crack radius growth and (c) nucleation between simulation using the Self-OPTIM identified DPS and reference simulation ........................................................................................................111

5.21 Comparisons of (a) stress-strain response, (b) mean crack radius growth and (c) nucleation between simulations using the upper bound DPS, lower bound DPS, identified DPS, and reference DPS. (d) degradation of the material stiffness ....114

5.22 Comparisons of the stress/strain curves of three simulations with the experiment data of strain rate 300 (/sec) ........................................................................................................117

5.23 Identified mean crack radius growth of strain rate 300 (/sec) .........................117

5.24 Comparison of the stress/strain curve of simulation result by using identified DPS₄₉₀ and the experiment data of strain rate 490 (/sec) .................................................118

6.1 225 Q8 stochastic finite element mesh overlapped with nine Q9 random field elements of a square plate .................................................................................................121

6.2 The locations of the FE nodal points used to evaluate the two-point correlation (the number is the label of the nodes) ............................................................................123

6.3 Normalized a) \(E_{N}^{36}\) vs. \(E_{N}^{93}\) and b) \(E_{L}^{36}\) vs. \(E_{L}^{93}\) (5000 samples) ..........................124

6.4 Randomly distributed elastic modulus for four different cases: a) \(L_{c1}=1\) m, \(\sigma_{L1}=1\) GPa; b) \(L_{c2}=1\) m, \(\sigma_{L2}=0.1\) GPa; c) \(L_{c3}=0.5\) m, \(\sigma_{L3}=1\) GPa; d) \(L_{c4}=0.5\) m, \(\sigma_{L4}=0.1\) GPa ........................................................................................................125

6.5 Representations of the approximated RF by using different number of KL expansion terms (M): a) \(M=5\); b) \(M=20\); c) \(M=35\); d) \(M=49\) (\(L_{c}=1, \sigma_{L}=1\) GPa)126
a) random distribution of elastic modulus; c) stress in \( X \) direction; c) stress in \( Y \) direction; d) in-plane shear stress; e) displacement in \( X \) direction; f) displacement in \( Y \) direction

Stochastic responses to material uncertainty a) displacement with \( \mu_{SR}=1.201 \text{mm}, \sigma_{SR}=0.0283 \text{mm} \); b) strain with \( \mu_{SR}=1.201 \times 10^{-3}, \sigma_{SR}=2.68 \times 10^{-5} \); and c) stress with \( \mu_{SR}=36 \text{MPa}, \sigma_{SR}=3.64 \times 10^{-3} \text{MPa} \) in \( x \) direction at Point A by using \( \sigma_L=1 \text{ GPa} \) and \( L_c=1 \text{ m} \)

Probabilistic distribution of stresses and strains at the final load step and at Point A

Probabilistic distribution of the position of Point A at the final loading step

Finite element mesh intersected with random field mesh for the quarter circular tube model

The lognormal Young’s modulus spatial distribution from one realization

Stochastic displacement response from simulation at Point A in \( X \) direction with \( \mu_{SR}=-0.0715 \text{m}, \sigma_{SR}=0.006 \text{m} \) by using \( \sigma_L=0.026 \text{ MPa} \) and \( L_c=1 \text{ m} \)

The problem definition of a retaining wall with randomly distributed Young’s Modulus

Deformation of the right edge AB predicted from Monte Carlo simulation with 2000 samples (the deformation is magnified by 6 times)

Geometry and FE model of the dog-bone specimen (unit mm)

Random field distribution of the \( E, \mu, \) and \( S_{xy0} \) obtained from one sample

Axial strain distribution with homogeneous and heterogeneous material

Stress/strain curves predicted from Monte Carlo simulation with 2000 samples

The schematic of stochastic material characterization

The mesh scheme of the square domain

Reference Young’s modulus distribution

Boundary condition and three different loading paths for the displacement-controlled test

Represented random field distributions: a) uniaxial tension on \( x \) direction; b) uniaxial tension on \( y \) direction; c) biaxial non-proportional loading
7.6 Represented random field by applying full-field displacement in a) objective function with $U_{DIC_{Ref}}$; b) displacement control data with $U_{DIC_{Ref}}$; and c) $U_{DIC_{Ref}}$ applied in both objective function and displacement control data.................... 154

7.7 Distribution of random fields of a certain reference simulation ......................... 156

7.8 Displacement amplitude ..................................................................................... 165

7.9 Cyclic effective stress vs. effective strain curve ............................................... 165

7.10 Steady-State cyclic stress and strain curve ....................................................... 166

7.11 Statistical distribution of a) $\{K'(\theta)\}_{l=1}^{2000}$, b) $\{n'(\theta)\}_{l=1}^{2000}$, and c) $\{E_e(\theta)\}_{l=1}^{2000}$ ....... 167

7.12 Statistical distribution of random variables: a) $K'(\theta_l)$, b) $n'(\theta_l)$, and c) $E_e(\theta_l)$ fitting with a polynomial chaos ................................................................. 168

7.13 Probabilistic low cycle strain-life curve .......................................................... 169
CHAPTER I
INTRODUCTION

1.1 Research Background

In reality, no material is homogeneous and deterministic in nature and it has been well-known that randomness in microstructures and properties of materials could significantly influence scatter of structural response at larger scales. Therefore, stochastic characterization of heterogeneous materials has increasingly received attentions in various engineering and science fields.

1.1.1 Material Uncertainty

All models used in simulation are just approximations to reality since two types of uncertainties exist, epistemic or system uncertainty and aleatory or statistical uncertainty, respectively. Epistemic uncertainty is due to lack of information and knowledge of physics behind the target phenomena, plus the limited accuracy of the selected mathematical model, which describes the system. It can be reduced by gathering data or by refining simulation models. Aleatory uncertainty is an inherent variation associated with the physical system or the environment. It cannot be reduced but should be handled in modeling.

The sources of aleatory uncertainty in simulating material mechanical behavior are: 1) The loads (either magnitude or location or both) are not known in a precise manner; 2) Boundary conditions have an associated uncertainty since they are usually
obtained either from experience, historical data, or from assumption; 3) All materials have inherent dispersions in material properties due to lack of complete control over the manufacturing, fabrication and processing techniques used. In this dissertation, the uncertainty quantification is carried out by focusing only on linear and non-linear material behaviors.

Traditionally, material behavior is assumed and modeled as deterministic and homogeneous in macroscale. A deterministic model is one in which every set of variable states is uniquely determined by parameters in the model and by sets of previous states of these variables. Homogeneity usually used to describe a material or system that has the same properties at every material point; in other words, the material properties do not vary with position. Based on these assumptions, most of the engineering and scientific mechanical problems are successfully solved in a perfect system by using the mean value of the material properties uniformly distributed over the medium while neglecting the uncertainties and heterogeneity. Unfortunately, no material is deterministic and homogeneous in reality.

At the micro/meso length scale where multiresolution complexity reaches its zenith, it is obvious that the uncertainty and heterogeneity of the material should not be ignored. Furthermore, as the resolution becomes finer in the medium, uncertainty always increases. Therefore, one-scale continuum mechanics no longer suffices the simulation need since advanced materials such as light weight metal alloy and ceramic matrix composites used in high temperature environment require insights into the local material behavior. In order to study the influence of the uncertainty in the global material behavior, it is necessary to deeply tunnel into the material microstructures. Recent developments in
micromachining and microscale material characterization are calling for stochastic approach based predictive models, due to the considerable influence of inherent uncertainties on material behavior. Clearly, providing for these random effects at the modeling level can introduce more robustness and realism in the simulation and design purpose. The ultimate goal of stochastic analysis is to include the uncertain material parameters of a system based on rigorous mathematical and statistical terms, and to reveal the nature and degree of influence of these parameters on the response variability as well.

1.1.2 Stochastic Finite Element Method (SFEM)

The unmodeled randomness at the microstructure level in heterogeneous media initiated the need for an approach to quantify and propagate the material uncertainty in a physical system. It is a critical path for the improvement of the prediction of the material mechanical behavior. The development of computational mechanics and extremely powerful computers make the stochastic approaches feasible in engineering applications. To this end, various numerical approaches have been developed to characterize and propagate uncertainties from data to system response through solving boundary value problems containing stochastic parameters. Conventional methods for uncertainty propagation can be broadly classified into four categories: 1) sensitivity testing, 2) analytical methods, 3) sampling based methods, and 4) computer algebra based methods. Among all the alternatives, the SFEM, which is one of the analytical methods, exhibits the appealing capacities to describe and simulate the material uncertainties in various research fields and received growing attentions in last two decades. The fundamental issue in SFEM is to discretize an infinite dimensional stochastic function
(processes/fields), which is employed to model the material property by using a suitable basis set in the space of square-integrated random variables. Karhunen-Loeve (KL) expansion can be used to deal with this issue based on prior known covariance function of the random field/process, which is represented in terms of orthogonal functions of a set of standardized random variables. These random variables completely characterize the uncertainty in the underlying system. The spectral stochastic finite element method (SSFEM) has been introduced by Ghanem and Spanos (Ghanem and Spanos, 1993) as an extension of the deterministic finite element method for the solution of boundary value problems with random material properties. The unknown nodal response is expanded in a series of random Hermite polynomials and the resulting representation is called the polynomial chaos expansion (PCE). By combining KL expansion and PCE within finite element framework, the stochastic problem is translated into a deterministic equation. This equation is used to estimate the chaos coefficients when a Galerkin projection is used to minimize the residual of the governing equation. Although this method is intrusive, the computational cost required for the solution of this system is huge. This is why the use of SSFEM has been limited in the past to the solution of uncertain systems with a small number of degrees of freedom. Alternatively, the PCE can also be used as a non-intrusive surrogate model to represent the response of the system, which is obtained from Monte-Carlo simulation (MCS). While MCS generally needs a huge amount of computational time to obtain converged results, the use of a small number of simulations will not provide accurate results in this procedure. In this dissertation, the KL expansion is used to discretize the random fields and non-intrusive PCE is used to represent the response after MCS. In this dissertation, a general purpose stochastic finite element
simulation framework is developed. This computational framework is seamlessly combined with FEM to treat any parameter as a random field/process in a certain constitutive model for both linear and non-linear problems.

1.1.3 Inverse Identification Problem

Generally, products must be designed with parameters that ensure high levels of reliability and quality. Therefore, a large number of tests have to be carried out to determine the parameters of the prototype of the product based on some special requirement. The “Build it, Test It, Fix It” approach is known to be very costly as an iterative design cycle is centered on the construction of real prototype components. Accordingly, the forward simulation serves as an important alternative to carry out the virtual experiment instead of the real test. The forward simulation model includes design parameters, which completely describe a physical system used to predict the outcome of some measurements. However, in case of optimal geometry design and constitutive law identification, using only the forward simulation in a trial and error way to find the demanded parameters in high dimensional space is still ineffective and even impossible. Therefore, the inverse identification problem, which consists of using the results of actual observations to infer the values of the parameters characterizing the system under investigations, has been increasingly receiving attention.

An inverse identification method is a general framework, which is used to convert observed measurements into information about a physical object or system that we are interested in. It exists in many research fields such as calibration of material properties, shape design or optimization, boundary valuesitial values identification, external loading identification, etc. Typically, an inverse identification framework contains three
parts: forward simulation model, objective function, and optimization algorithm. In the case of material identification, the objective function (OF) is built up in the form of the bias between the predicted quantities and the measured experimental data of these quantities. By employing the optimization algorithm, the parameters are tuned to minimize the objective function. When the OF reaches the minimum value, the identified design parameters are obtained. These parameters will let the prediction results from simulation model have minimum error when compared with the experimental data. More often, some specific constraints from the physical definition of the problem will be applied to define the acceptable space of variables for the optimization process.

It is well known that the inverse problem is always ill-posed, which makes it hard to solve. The difficulties include at least three aspects: 1) Due to the iterative computational procedures carried out in an optimization algorithm, the computational cost is enormous for the large-scale engineering system, which has a large number of degrees of freedom and parameters that need to be identified; 2) The non-uniqueness problem may exist in the identification; in another words, several local minima may exist in the OF; 3) The process of computing an inverse solution can be, and often is, extremely unstable because of large variation of parameters under small perturbation in measurement. Furthermore, an inherently more difficult family of inverse problems, such as non-linear inverse problems, has more complex physical model and parameters, which make the inverse identification a very tough problem. In the case of the optimization algorithm, it is not always evident which optimization approach is suitable for use with a particular problem. The performance of the algorithm may be highly dependent upon the choice of the formulation of the objective function. In this dissertation, a robust and
reliable inverse identification framework, Self Optimization Inverse Method (Self-OPTIM), is developed to tackle the problem of inverse identification of the parameters of non-linear constitutive law and the stochastic material properties in a heterogeneous media.

1.2 Objectives

The overall objective in this dissertation is to develop a general and robust inverse identification framework for stochastic material characterization of a heterogeneous media with random distributed material properties, which is caused by the randomness of the material microstructures. The achievement of the objective can be split into the following main objectives:

- To develop a deterministic inverse identification method to characterize the material property.
- To develop a general stochastic finite element framework to simulate the uncertainties in material properties for linear and nonlinear problems.
- To develop a stochastic inverse analysis framework that can characterize spatial variability of material properties and construct stochastic material data base. On the basis of this data base, the probabilistic surrogate model is developed for the purpose of reliability design.

1.3 Outline of the Dissertation

The dissertation is structured into the following chapters:

- Chapter 1 is the introduction. It describes the overview and background of this dissertation, and the main objectives of the research.
• Chapter 2 is the comprehensive literature review of the state-of-the-art of inverse identification algorithms, stochastic finite element modeling techniques, and stochastic material characterization.

• Chapter 3 presents a novel model-independent and general purpose inverse identification framework, Self-OPTimizing Inverse Method (Self-OPTIM).

• Chapter 4 presents the development of a general purpose stochastic finite element method (SFE) for simulating the probabilistic structural responses caused by material uncertainties in large-scale engineering problems by using Karhunen-Loeve expansion and Monte Carlo simulation.

• Chapter 5 presents a novel methodology for stochastic material characterization with consideration of spatial varying material properties. New capabilities of the proposed method, such as development of probabilistic surrogate models by polynomial chaos expansion method on a basis of the constructed stochastic material data base, will be described in more detail.

• On the basis of Chapter 3, Chapter 6 shows verifications of Self-OPTIM by using a cyclic elasto-plasticity model, a micromechanics and fracture mechanics based damage model, and experimental testing.

• Chapter 7 presents the effects of variability of nonlinear material properties on structural responses by using the SFEM developed in Chapter 4 for several numerical examples.

• Chapter 8 presents stochastic inverse material characterization. Based on the constructed stochastic material data base, the probabilistic surrogate model for
low cycle fatigue prediction is presented for the purpose of reliability based analysis.

- Chapter 9 presents the summary and future work.
2.1 Inverse Identification Algorithm and Problem

As computer and computational technologies have rapidly advanced, complex nonlinear constitutive models have become more and more feasible to use in the design of complex structures. Consequently, such complicate simulation models potentially pose difficulties on determination of their parameters from experiments. Therefore, inverse identification method has been increasingly used for the purpose of parameter estimation especially for the nonlinear problems.

2.1.1 Inverse Identification Framework

All of the existing inverse methods for identification of material properties enforce equilibrium conditions, that is, either in weak or strong form, the constitutive relationships that relate full-field displacements to the stresses, and the boundary conditions that are essential to all boundary value problems. Based on applying forward simulation or not, the inverse identification method can be split into two distinct categories of: 1) Model updating methods (Hild and Roux, 2006; Mahnken, 2000; Pagnacco et al., 2005); and 2) virtual field method (VFM) (Avril et al., 2004; Avril et al., 2008; Grediac et al., 2006; Grediac and Vautrin, 1990; Pierron et al., 2007).

The model updating methods iteratively update constitutive parameters so as to minimize an objective function that represents the error, or gap, between a measured
quantity and the same quantity computed from the forward simulation, which is carried out by using either finite element method or analytical model. The model updating approach has been applied to parameter identification for materials with linear elastic (Lecompte et al., 2007), viscoelastic (Kim and Kreider, 2006; Moreau et al., 2006), elasto-plastic (Kajberg and Lindkvist, 2004a) and viscoplastic (Kajberg and Wikman, 2007) behavior. Model updating based methods do not require full-field measurements; partial measurements can be sufficient to determine the constitutive parameters. However, it requires iterative forward simulation, which takes a great deal of computational time.

In VFM, a chosen set of kinematically admissible virtual displacement fields is assumed and substituted into the virtual work equation along with full-field displacements measured by the digital image correlation (DIC) technique. This leads to a system of linear equations that is solved for constitutive parameters (Grediac and Vautrin, 1990). Avril and Pierron compared the VFM with model updating methods based on minimization of a variety of gaps, such as displacement gap, constitutive equation gap, and equilibrium gap. They concluded that model updating on a basis of “displacement gap” minimization yields equations that are similar to those used by the VFM (Avril and Pierron, 2007). VFM has an advantage because it permits faster computation times than model updating approaches; however, it requires full-field measurements obtained by using costly equipment. In addition, measurements of full-field displacement data are subject to potential measurement errors.

2.1.2 Optimization Algorithm

Based on the iterative computational method, optimization algorithms can be roughly split into two categories, gradient based and none-gradient based, respectively.
Both of these two methods have been widely used in various branches of scientific and engineering problems.

The negative of the gradient vector denotes the direction of steepest descent from any point in n-dimensional space, the function value decrease at the fastest rate. In steepest descent method, the use of the negative of the gradient vector as a direction for minimization was first made by Cauchy. This method can be used to solve the minimization problem of a differentiable objective function (Snyman, 2005). The convergence characteristics of the steepest descent method can be improved greatly by modifying it into a conjugate gradient method since it is quadratically convergent. The Levenberg-Marquardt method attempts to take advantage of both the steepest descent and Newton methods (Madsen et al., 1999). It is a more robust method since the initial search point can be very far from the final minimum. The LM method needs a modified Hessian matrix. The Broyden–Fletcher–Goldfarb–Shanno (BFGS) method has been proven that it has good performance of solving nonlinear optimization problems even for non-smooth OF surface. The BFGS requires the inverse of the Hessian matrix. As we know, any gradient based methods need an arbitrary initial point; therefore, the performance strongly depends on the initial selection. Furthermore, in case of non-smooth OF or rapidly changed Hessian, some of the gradient based methods are technically hard to apply.

The recent advances in computing power have inspired non-gradient based method that receives its fast development since 1960s. One of them is the Simplex method (Nelder and Mead, 1965), which is used to solve the multidimensional nonlinear optimization problems. It includes two steps: (1) Establish the initial simplex. (2) Perform
the simplex transformation algorithm by using four linear algebra operations, reflection, expansion, contraction and reduction on the simplex under different conditions. Simulated annealing (SA) (Kirkpatrick et al., 1983) is a generic probabilistic metaheuristic, which is often used for the global optimization problem with discretized search space. Although the metaheuristic methods are computational expansive, they provide more robust solution for large scale engineering problems. Most metaheuristic algorithms are nature-inspired as they have been developed based on some abstraction of nature. Two major characteristics of any metaheuristic algorithms are: selection of the best solutions and randomization (Yang, 2010). The convergence is guaranteed by selecting the best one among all the solutions and the randomization avoids the solution being trapped at local minimum. The well known Classic Genetic algorithm (GA) is the mimic of the processes of biological evolution in order to solve the minimum search problems (Beasley et al., 1993a, b). Unlike standard genetic algorithms that need to evaluate the whole individuals per each generation, the steady state genetic algorithm (SSGA) (Whitley and Kauth, 1988) only needs to evaluate the whole individuals once at the beginning. Instead, SSGA selects four best individuals and two pairs are made by them for crossover. Subsequently, SSGA replaces two worst individuals with the off-spring and evaluates the new off-spring after applying mutation process to them. Therefore, SSGA can significantly reduce computational costs. Several interesting metaheuristic algorithms mimic the behavior of the animal or insects, such as particle swarm optimization (PSO) (Kennedy and Eberhart, 1995), firefly method (FA) (Yang, 2009a), ant colony method (Dorigo and Di Caro, 1999), and Honey bee method (Nakrani
and Tovey, 2004). It is worth noting that, all of these nature-inspired methods have been proven that they have very good performance in solving nonlinear optimization problems.

2.1.3 Inverse Identification Problem

As mentioned in the introduction, inverse identification problem are widely investigated in several of engineering research fields. In this dissertation, the parameter inverse identification of constitutive model is extensively reviewed.

2.1.3.1 Parameter Inverse Identification of Elasto-Plasticity Model

The simulation of the metallic material under large deformation needs accurate constitutive models to describe the material behavior. Most of the advanced elasto-plastic constitutive models contain several either physical or phenomenological parameters that should be calibrated on a basis of the observations obtained from costly and complex direct experimental test. For example, the elasto-plasticity constitutive laws studied in (Bari and Hassan, 2000) for predicting ratcheting responses under uniaxial and biaxial loading contains many parameters, which needs several independent experiments to determine. Therefore, inverse techniques and methodologies for identifying constitutive model parameters are considered as being of great importance for better calibration tool (Cooreman et al., 2007; Ghouati and Gelin, 1998; Grediac and Pierron, 2006; Pannier et al., 2006; Ponthot and Kleinermann, 2006).

Most of the existing model updating inverse parameter estimation methods applies optimization techniques in order to minimize cost functions, which express differences between FE models and experimental test data in terms of forces, strain fields, displacements and modal properties.
In (Yoshida et al., 2003), an approach is proposed to the identification of the mechanical properties of individual component layers of a bimetallic sheet. A set of material parameters in two different cyclic elasto-plasticity constitutive models, Chaboche-Rousselier model and Prager model, are identified for the two layers of the metallic sheet simultaneously by minimizing the difference between measured tensile load, moment, and strain data and the corresponding results of finite element simulation. An optimization technique based on the iterative multipoint approximation concept is used. Similar to (Yoshida et al., 2003), a hybrid-inverse method (Zhang et al., 2010) for the identification of the parameters of Ramberg-Osggod model. The tensile load and bending moment obtain from experiment and simulation of a stainless steel clad copper sheet are integrated into the objective function, which is minimized by using the specific form of the gradient-based method on a basis of quasi-Newton algorithm.

Material parameters estimation of two types of elasto-plastic models, which both are based on isotropic hardening is carried out in (Kajberg and Lindkvist, 2004b). The objective function is formulated as a least-square functional based on the difference between the pointwise experimental displacement, which is captured by using digital speckle photography (DSP) and strain field is computed on a basis of the numerical differentiation of the displacement field and FE-calculation. An optimization algorithm, the unconstrained subspace-searching simplex method (SUBPLEX), is used to minimize the objective function by adjusting the constitutive parameters.

An FEM based inverse method is developed in (Cooreman et al., 2007) to identify the elasto-plastic material parameters in an iterative way by minimizing the cost function, which expresses the discrepancy between the experimentally measured and the
numerically computed strain fields. Linear kinematic hardening combined with linear isotropic hardening law is employed to simulate the material behavior under large deformation. The gradient based iterative method is employed by using the derived sensitivity matrix. The methods for determine the sensitivity matrix are reviewed.

The elastic-plastic model is calibrated for a thin film which exhibits transversely isotropic material response (Nakamura and Gu, 2007). The inverse identification method post-processes load–displacement records simulated by power-law hardening law of depth-sensing indentations to obtain best estimates of the young’s moduli and plastic parameters on longitudinal and transverse directions. Prior to actual testing, detailed simulations were performed to verify the method’s applicability and robustness. In the inverse analysis technique, Kalman filter is utilized.

In order to solve the computing time problem in model updating approaches A hybrid identification method based on finite elements method, neural network computations and multi-objective genetic algorithm is proposed (Aguir et al., 2011) The strategy suggested is then applied to identify the Karafillis and Boyce criterion and the Voce law hardening parameters of the Stainless Steel AISI 304 using two tests (plane tensile test and bulge test with a circular die) at the same time.

In (Ponthot and Kleinermann, 2006), The parameters of a finite strain elasto-plasticity model based on maximum plastic dissipation and the multiplicative decomposition are identified by minimizing the difference between measured external force and necking radius and simulation results. A hybrid optimization method, Levenberg-Marquardt plus globally convergent modified method of moving asymptotes
plus conjugate gradient method, are developed on a bases of the comparison of different performance of using several optimization algorithms alone.

Recently, the VFM had been extended from solving identification of linear elastic problem to elasto-plasticic problem using an incremental formulation and the methodology that splits the elastic strain and the plastic strain from the measured total strain (Grediac and Pierron, 2006; Sutton et al., 2008). The most important feature of VFM is that neither finite element analysis nor analytical forward simulation is required. This fact makes the computation of the inverse identification problem relatively efficient. In (Pierron et al., 2010), VFM has been specifically developed for solving inverse problems of a combined linear kinematic/isotropic hardening law for predicting the material under cyclic loading. Different load paths (tension, compression, notched specimen) have been studied.

2.1.3.2 Parameter Inverse Identification of Damage Model

Since the damage model is derived based on fracture mechanics and microscale considerations, the calibration of damage parameters by direct observations needs expensive measuring machines and complex measurement techniques that challenge the role of numerical simulations (Lauro et al., 2001). For example, in order to accurately simulate the effective behavior of the composites, the microscopic damage mechanism of constituents which strongly influence the macroscopic constitutive behavior of the composite must be taken into account. The cost-effectiveness of virtual experimental tests by computer simulations has motivated developments and advances of numerous micromechanics-based damage constitutive models of composites to simulate the matrix cracks, fiber fractures, fiber/matrix debonding, and delamination (Barbero et al., 2005;
Chaboche et al., 1998; Chaboche and Maire, 2002; Chen et al., 2010; Gudmundson, 2000; Haj-Ali, 2009; Ju and Lee, 2001; Ju and Yanase, 2008; Kim et al., 2007; Koimtzoglou et al., 2001; Lamon, 2001; Liu et al., 2004; Liu et al., 2005; Mishnaevsky and Brondsted, 2009; Murari and Upadhyay, 2012; Okabe et al., 2010; Pyo and Lee, 2009; Skolnik et al., 2008; Subramanian et al., 1995; Tekoglu and Pardo, 2010; Tian and Fu, 2011; Wang et al., 2009; Yang et al., 2012). For that reason, damage parameter inverse estimation techniques and methodologies have received significant attentions in the field of computational damage modeling in microscale. The importance of the identification of physical damage parameters associated with advanced material constitutive and damage models is two-fold: 1) it allows us to access the current damage state of the material to determine the damage initiation; and 2) it also allows us to predict evolutionary behavior of the material damage and failure.

A damage model which is based on the description of the nucleation, growth and coalescence of the microvoids is integrated into the explicit finite element framework to predict the damage evolution which occurs under dynamic loading in the crash or stamping process (Lauro et al., 2001). The model is adapted to take the material behavior anisotropy and damage anisotropy into account. An inverse method is developed to identify the material parameters by minimizing the discrepancy between experimental and numerical measurements of the inner radius of the notch with the tensile test on a double notched specimen.

The material parameters of Rousselier model which is based on isotropic hardening as well as isotropic damage is calibrated by using an gradient based inverse identification method (Springmann and Kuna, 2003). The objective function is
formulated as the difference between the measured non homogeneous displacement on certain points and the simulated one. A so called optimizer OPTB2L, based on an iterative method with a conjugated gradient method and a BFGS method, is used.

An algorithm for parameter identification which is based on the dual boundary control method is modified to improve the stability (Xiang et al., 2002). The algorithm is subsequently extended to identify the parameters in a three-dimensional anisotropic elastic damage model by matching the simulated displacement to the observed displacement on certain observational points while tuning the partakers by using Kalman filter approach.

The identification of the material parameters of the Gurson–Tvergaard–Needleman damage law has been done in (Abendroth and Kuna, 2006). FE simulation is carried out to train a neural network until the neural network can approximate the direct boundary value problem. Then the material parameters can be found by an gradient based optimization routine to minimiz the error between an experimental load displacement curve and one predicted by the network function of a small punch test. The great advantage of this identification approach is that no more FE simulation is needed since the NN contains all the knowledge.

In (Lauwagie et al., 2002), the local damage of the beam made by brittle material is simulated by using a reduction of the bending stiffness. Selected stiffness parameters in the finite element model are adjusted in such a way that the computed modal quantities match the measured quantities. It has been found that the modal based inverse identification method is indeed a possible tool to reconstruct the damage patterns.
The problem of parameter identification of elastic-damage time-dependent model for the interlaminar surfaces of layered composites is carried out in (Coriglano and Mariani, 2001) on the basis of the responses of standard interlaminar fracture tests. This constitutive model can be used to simulate both pure-mode and mixed-mode crack propagation phenomena. The indirect identification procedure is based on the use of the extended Kalman filter and on an approximate, simplified computation of the sensitivity of the structural response to model parameters.

The parameters identification problem of the gradient-enhanced continuum damage model is examined by means of an inverse analysis (Iacono et al., 2006). It shows that the unique identification solution is obtained by involving additional averaged local experimental information such as the width of the damaged.

VFM is extended to the identification of an anisotropic damage law in (Perie et al., 2009). The reconditioned Equilibrium Gap Method is then used to retrieve a damage law that accounts for shear softening. The example of axial shear test performed on a cruciform specimen is considered. A good agreement is observed between the prescribed and identified laws for distinct parameter settings, even when significant noise is added to the displacement fields. The complete scheme is finally tested on a carbon/carbon composite.

2.2 Stochastic Finite Element Method

All materials in general (examples include porous media, biological materials, etc.), are heterogeneous and random at the microstructural length scale. They exhibit variability and uncertainties in their material properties with relation to their compositions and manufacturing processes. The random heterogeneous media (RHM) is
defined as the heterogeneous media which has the randomly distributed physical or mechanical properties. The randomness is coming from the stochasticity of the microstructures of the material at the microscopic level. The variability in their composition dramatically affects their overall properties, resulting in materials with lower weight, increased stiffness and higher strength. The random heterogeneous media (RHM) can be classified into two categories: 1) consist of more than one phases which have different physical or mechanical properties and are distributed within the volume of the material; 2) consist only one kind of material, however, the physical or mechanical properties are spatially varying.

During last two decades, considerations of random nature of physical systems are gradually and widely recognized of importance by industry and researchers. As mentioned in (Oden et al., 2003) “During the next decade, probabilistic modeling of mechanical problems will be a topic of great importance and interest”. Nowadays, the uncertainty assessment of engineering structures has increasingly become an important topic of interests in various engineering communities. Stochastic finite element method (SFEM) has been recognized as one of the primary computational techniques in probabilistic analysis of structures with uncertainties.

2.2.1 Uncertainty Representation Method

The space-varying material properties such as Young’s modulus and strength can be represented by the concept of the random field. The problem encountered is that discretizing a random field to be implemented in a finite element formulation of the physical problem. To accomplish this, one efficient way is the direct use of stochastic expansions. From the wide variety of methods developed for the simulation of Gaussian
stochastic processes and fields, two expansion algorithms are most often used in applications: KL expansion (Huang et al., 2001; Li et al., 2007; Phoon et al., 2004, 2005; Phoon et al., 2002; Stefanou and Papadrakakis, 2007a) and PCE (Debusschere et al., 2005; Eldred, 2009; Ghanem and Spanos, 1990; Hosder et al., 2006; Xiu and Karniadakis, 2003a).

In case of prior known its covariance function, the random field can be represented as a series expansion involving a complete set of deterministic functions with corresponding random coefficient. Karhunen–Loeve (KL) expansion is one of the general orthogonal series expansion which is based on the eigen-decomposition of the covariance function. In the KL expansion method, the 2D random fields with non-zero means are decomposed into a deterministic part and a stochastic part as follows

\[
\begin{align*}
\mathbf{u}(\mathbf{x}, \theta) &= \overline{\mathbf{u}}(\mathbf{x}) + \sum_{i=1}^{\infty} \sqrt{\lambda_i} \phi_i(\mathbf{x}) \xi_i(\theta) \\
\end{align*}
\]

where \(\mathbf{u}(\mathbf{x}, \theta)\) is a random field; \(\mathbf{x}\) is the position vector defined over the physical domain \(D\); \(\theta\) represents primitive randomness that belongs to the space of random events; \(\xi_i\) is the statistically uncorrelated Gaussian random variable with zero mean and unit standard deviation and \(\overline{\mathbf{u}}(\mathbf{x})\) is the mean value of \(\mathbf{u}(\mathbf{x}, \theta)\). In this study, \(\overline{\mathbf{u}}(\mathbf{x})\) is defined as a constant and does not vary over the domain \(D\). Therefore, \(\overline{\mathbf{u}}(\mathbf{x})\) is simplified as \(\overline{\mathbf{u}}\). \(\lambda_i\) and \(\phi_i(\mathbf{x})\) are the eigenvalue and the eigenfunction of the covariance function assumed. \(\lambda_i\) and \(\phi_i(\mathbf{x})\) can be obtained by solving the homogeneous Fredholm integral equation as follows

\[
\int_{\Omega_{2e}} C(\mathbf{x}_1, \mathbf{x}_2) \phi_k(\mathbf{x}_2) dA_{2e} = \lambda_k \phi_k(\mathbf{x}_1) \tag{2.2}
\]
The polynomial chaos approach has become a well-known and widely applied for uncertainty quantification method in many branches of computational science and engineering (Das et al., 2009; Ghanem and Spanos, 1991; Schoefs et al., 2011; Soize, 2010). It is a polynomial expansion of orthogonal polynomials in terms of random variables to approximate the uncertainty distribution of the output. A simple definition of the PCE for a Gaussian random response $u(\theta)$ as a convergent series is as follows

$$u(\theta) = \sum_{i=0}^{p} b_i \psi_i(\xi(\theta))$$  \hspace{1cm} (2.3)

where $b_i$ is the coefficient of the polynomial chaos, $\psi_i$ is the corresponding multidimensional orthogonal Hermite polynomials in terms of an n-dimensional vector of random variables $\xi(\theta)$, and $p$ is the order of PCE. The original method employs the Hermite polynomials as the basis in random space. Later on, the generalized PCE is proposed to various continuous and discrete distributions using orthogonal polynomials from the so called Askey-scheme (Xiu and Karniadakis, 2003b), the maximum entropy (Das et al., 2006), minimum relative entropy (Arnst et al., 2008) method of moments (Arnst and Ghanem, 2008; Soize et al., 2008), maximum likelihood (Desceliers et al., 2006; Ghanem et al., 2008; Huebner and Rozovskii, 1995), and Bayesian inference (Marzouk and Xiu, 2009; Marzouk and Najm, 2009; Marzouk et al., 2007) can be applied to identify the coefficients of functional expansions.

2.2.2 Stochastic Finite Element Analysis Framework

Stefanou provided a state-of-the-art review of past and recent developments in the SFEM area indicating future directions as well as open issues to be examined in the future (Stefanou, 2009). A vast amount of research has been conducted for tackling
different problem definitions where uncertainties have been suggested to accomplish this in various ways and with various objectives: evaluation of response variability due to uncertain geometrical parameters (Bielewicz and Gorski, 2002; Papadopoulos et al., 2009; Schenk and Schueller, 2003), uncertain linear elastic properties (Argyris et al., 2002; Chung et al., 2005; Noh, 2005, 2006; Noh and Kwak, 2006; Rahman and Chakraborty, 2007; Rahman and Xu, 2005a; Xu and Graham-Brady, 2005), uncertain elasto-plastic parameters (Anders and Hori, 1999, 2001; Sett et al., 2007), both geometrical parameters and linear elastic properties (Lagaros and Papadopoulos, 2006; Stefanou and Papadrakakis, 2004), uncertain initial damage (Arun et al., 2010; Rahman and Rao, 2001), fracture properties (Yang and Frank Xu, 2008); multiscale uncertainty propagation (Steven Greene et al., 2011; Yin et al., 2008; Yin et al., 2009); uncertainty in dynamic system (Johnson et al., 1997; Wojtkiewicz, 2010; Wojtkiewicz and Johnson).

One of the most urgent needs for applications of SFEM in engineering problems is to develop a robust and efficient user-friendly computational framework that can interact with powerful third party codes. The first attempt to integrate stochastic finite elements into general purpose FE software can be found in (Ghanem and Abras, 2003). Several representative examples of structural reliability software can be found in literatures (Chen and Soares, 2008; Der Kiureghian et al., 2006; Sachdeva et al., 2007; Schueller and Pellissetti, 2006; Schueller and Pradlwarter, 2006; Thacker et al., 2006). Noteworthy features and capability of these examples in interacting with the third party FE code have been studied in Reference (Schueller and Pellissetti, 2006). Two computer codes namely FERUM (Finite Element Reliability Using MATLAB)/OpenSees have been developed at the University of Berkeley. FERUM is a MATLAB toolbox for
structural reliability analysis and OpenSees, a widely used computational framework, contains reliability analysis tools in conjunction with static and dynamic nonlinear time history analysis. Numerical evaluation of stochastic structures under stress (NESSUS) is also a general-purpose tool for computing the probabilistic response or reliability of complex systems (Thacker et al., 2006). The NESSUS is a highly interactive software that can be used in conjunction with commercial FE analysis programs such as ABAQUS, ANSYS, NASTRAN, etc. to solve problems in stochastic mechanics (Schueller and Pellissetti, 2006). A stochastic FE library (StoFEL) was successfully coupled with ANSYS for approximating the structural response statistics and carrying out graphical post-processing (Yongke, 2003). Finite element software ATENA and the stochastic package Feasible Reliability Engineering Tool (FReET) are integrated into software system SARA to carry out nonlinear analysis of reinforced concrete structures with uncertainties and randomness of structural input parameters (Pukl et al., 2003). On the commercial company side, ANSYS Inc. has integrated probabilistic design capabilities in its recent releases, namely the ANSYS Probabilistic Design System and the ANSYS DesignExplorer. More applications of the software in various industrial example problems can be found in Reference (Reh et al., 2006).

2.3 Stochastic Material Characterization

The conventional deterministic identification methods of material properties mentioned in section 2.1.3 do not account for the associated spatial variability and uncertainties in the identified properties. The technique used to characterize spatially-stochastic macroscopic material properties of random heterogeneous material fabrics from limited-size macro-scale experimental measurement is desirable. The studies of
stochastic material characterization can be divided into two categories as 1) digital image based characterization method and 2) model updating based characterization method.

The methods belong to the first category doesn’t apply any mechanical information but use the digital image processing based stochastic optimization technique. Several recent studies can be referred to in (Bochenek and Pyrz, 2004; Eschricht et al., 2005; Jiao et al., 2007; Patelli and Schueller, 2009; Politis et al., 2008; Rozman and Utz, 2002; Sundararaghavan and Zabaras, 2005). In this method, one starts with a given, arbitrarily chosen, initial configuration of a random medium and a set of target functions. The target functions describe the desirable statistical properties of the medium of interest, which can be various correlation functions taken from experiments or theoretical considerations. The method proceeds to find a realization configuration, in which calculated correlation functions best match the target functions. There are many different types of statistical descriptors that can be chosen as target functions; the most basic one is two-point correlation function, which is obtainable from small-angle x-ray scattering (Jiao et al., 2007). Although this method can reconstruct the material microscopic structures, it can only indirectly characterize the RHM.

The methods belong to the second category directly apply the mechanical experimental test and use the simulation-experiment based method to update the parameters included in a stochastic finite element model. This approach is advantageous in the way it helps to avoid involving the microscopic complexity of the composite structure and the required instrumentations to obtain reliable data at this level (Chen et al., 2006). A dynamic stochastic composite material properties characterization method is proposed in (Mehrez et al., 2012a; Mehrez et al., 2012b). First, the deterministic inverse
analysis are incorporated to ascertain spatial nodal Young’s modulus realizations based on the experimental frequency response functions measured on a group of coupon shaped composite specimen. Each inverse analysis, associated with the ensemble of FRFs measured on one specimen, considers a spatial randomness in the Young’s modulus field as opposed to traditional identification numerical–experimental techniques where effective values have been sought. Secondly, a probabilistic approach is developed to fit the experimental database into a stochastic model of the Young’s modulus of the heterogeneous composite structure. The stochastic characterization method developed in this dissertation belongs to this category. It can directly characterize the RHM.

2.4 Current Work

A new inverse constitutive parameter identification framework, self-optimization inverse method (Self-OPTIM), that was inspired by the autoprogessive training algorithm (Ghaboussi et al., 1998) is developed. Ghaboussi and his co-workers developed an autoprogessive training algorithm (Ghaboussi et al., 1998) that enables neural network based material models to be trained through a course of two parallel nonlinear finite element analyses under predefined boundary loading conditions. The autoprogessive training algorithm is an innovative inverse analysis method that can extract in-situ local constitutive behavior from the global response. However, it is limited to the use of neural network (NN) based material constitutive models. The Self-OPTIM inversely estimates constitutive model parameters using global forces and displacements on partial boundaries and full-(or partial-) field displacement data. A novelty of the methodology is that it automatically self-estimates material parameters by updating reference local stresses and strains through two parallel nonlinear finite element
simulations nested into an optimization framework. More importantly, it must be emphasized that the proposed method is model-independent, which means that it can be naturally integrated with all other types of material constitutive models other than NN. In this dissertation, a classical plasticity model with nonlinear isotropic and kinematic hardening behavior and a micromechanics and fracture mechanics based damage model for damage tolerant composite have been chosen to demonstrate the proposed inverse analysis method.

A SFEM was presented along with implementations into a general-purpose finite element analysis program ABAQUS in order to simulate the probabilistic response of structures with material uncertainty. Discretization and quantification of random fields associated with material uncertainties are accomplished through Karhunen–Loève (KL) expansion. In particular, issues on the separation of RF mesh from FE mesh have been addressed by suggesting a general mapping-interpolation method for stochastic finite element simulations. Further applications of the presented SFE computation techniques in ABAQUS/standard element library will significantly improve the research and practices in the field of probabilistic computational mechanics. In the numerical demonstrations, Young’s modulus, Poisson’s ratio, and initial yield stress of two-dimensional structures are considered as spatially varying random fields. The probabilistic material responses from both linear elastic and elasto-plastic cases were presented to show the flexibility and fidelity of this computational framework. It is worth noting that the application of this general SFEM is able to be extended to simulate any other spatially distributed material parameters such as initial damage, the coefficient of thermal expansion, and density for heterogeneous materials.
Finally, a novel stochastic material characterization method, Stochastic Self-OPTIM, has been developed by combining Self-OPTIM with the general SFEM. This method is able to inversely identify the material property distribution and the statistical parameters (e.g. spatial mean, spatial variance, and correlation length) of the material properties of RHM. The stochastic material data base is constructed by using Stochastic Self-OPTIM based on MCS. By using this stochastic material data base, the probabilistic surrogate model for low cycle fatigue life prediction has been built up to present the application of this stochastic material characterization method.
CHAPTER III
SELF OPTIMIZATION INVERSE IDENTIFICATION METHOD

3.1 Self-Optimizing Inverse Method (Self-OPTIM)

In this dissertation, a novel inverse analysis methodology, Self-Optimizing Inverse Method (Self-OPTIM), has been presented, which inversely estimates constitutive model parameters using global forces and displacement on the same partial boundaries and full-(or partial-) field displacement data. A novelty of this methodology is that it automatically self-estimates material parameters by updating local stresses and strains through two parallel nonlinear finite element simulations. It also has to be emphasized that the proposed Self-OPTIM method is a model-independent method, which means that any advanced model can be naturally integrated with the proposed methodology.

3.1.1 Theoretical Background

Well-posed boundary value problems in solid mechanics considering small displacement and linear elastic material behavior are described as follows.

\[
\begin{align*}
\text{div}(S) + b &= 0 \quad \text{in } B \\
Sn &= \tau_b \quad \text{in } \Omega_F \\
S &= S^T \quad \text{in } \Omega_F \\
u &= u_b \quad \text{in } \Omega_U
\end{align*}
\]

(3.1)

where \( S \) is the Cauchy Stress and \( b \) is the body force in \( B \); \( n \) is the outward unit normal vector on the surface \( \Omega_F \) where the traction vector \( (\tau_b) \) is prescribed and \( u_b \) is the boundary displacement prescribed on \( \Omega_U \). The linear elastic behavior is expressed as
\[
S = \lambda tr(E)I + 2G(E)
\]
\[
E = \frac{1}{2}(\nabla u + \nabla u^T)
\]  
(3.2)

Considering the free surface as traction boundaries where the traction is zero, \( \partial B = \Omega_f \cup \Omega_u \). It is notable that \( \Omega_f \cap \Omega_u = 0 \) in well-posed theoretical boundary value problems. Thus, for analytical solutions, two boundary conditions (traction and displacement boundary conditions) are not allowed to be imposed on the same boundary. Although \( \Omega_f \cap \Omega_u = 0 \) in well-posed theoretical boundary value problems, it is possible to measure both tractions and displacements along the same boundary from laboratory testing. These experimentally obtainable conjugate boundary conditions are key inputs to the Self-OPTIM methodology as shown in Figure 3.1. It will sufficiently alleviate non-uniqueness issues often encountered in inverse problems as demonstrated in this dissertation.
Figure 3.1 Two Boundary Conditions from Experimental Testing on a Solid Deformable Body Subjected to Body Force, $b$, and with Material Parameters Unknown (Dashed line indicates kinematically admissible virtual displacements)

Theoretical fundamentals of the Self-OPTIM methodology can be articulated in terms of the principle of virtual work and the theorem of calculus of variations. First, parameters associated with material constitutive laws are assumed to be unknown. In addition, the experimentally measured tractions and displacements are considered as “reference” values containing information on true material constitutive behavior since they are from real testing. Let’s consider two loading conditions under the experimentally measured tractions and displacements, respectively as shown in Equation (3.3) and Equation (3.4). It naturally creates two parallel nonlinear finite element simulations: one is a traction-driven problem and the other is a displacement-driven problem. Subjected to the body force ($b$), two virtual work functionals under the traction and displacement boundary conditions can be written as follows, respectively.

$$G_F (\mathbf{D}_{ep}, \mathbf{u}_F, \overline{\mathbf{u}}_F) = - \int_{\Omega_F} (\mathbf{D}_{ep} \nabla \mathbf{u}_F : \nabla \overline{\mathbf{u}}_F - \mathbf{b} \cdot \overline{\mathbf{u}}_F)dV - \int_{\Omega_F} \mathbf{t}_b \cdot \overline{\mathbf{u}}_F dA$$

for $\forall \overline{\mathbf{u}}_F \in V(\mathbf{u})$ (3.3)

$$G_U (\mathbf{D}_{ep}, \mathbf{u}_U, \overline{\mathbf{u}}_U) = - \int_{\Omega_U} (\mathbf{D}_{ep} \nabla \mathbf{u}_U : \nabla \overline{\mathbf{u}}_U - \mathbf{b} \cdot \overline{\mathbf{u}}_U)dV$$

for $\forall \overline{\mathbf{u}}_U \in V(\mathbf{u})$ (3.4)
where $\overline{u}_F$ and $\overline{u}_U$ are the admissible virtual displacement fields under traction and displacement boundary conditions, respectively and $D_{ep}$ is the fourth-order inelastic material stiffness tensor. $V(u)$ is a set of kinematically admissible displacement fields. Both in Equation (3.3) and Equation (3.4), the first integrations on the right hand side indicate internal virtual work functional. The second and third integration in Equation (3.3) denotes external virtual work functional. The second integration in Equation (3.4) denotes the external virtual work functional. The virtual displacement field should satisfy homogeneous essential boundary condition as $\overline{u}_F = \overline{u}_U = 0$ on $\Omega_U$ under both loading conditions but the real displacement on $\Omega_U$ (where prescribed displacements are applied) is $u_b$, which is non-zero. The virtual displacement is not vanished on $\Omega_F$, that is, $\overline{u}_F \neq 0$ and $\overline{u}_U \neq 0$ on $\Omega_F$. From the theorem of calculus of variations, if the two virtual work functionals in Equation (3.5) and (3.6) are zero for any admissible virtual displacement, then the equilibrium condition is satisfied equivalently. However, difference of either external or internal virtual work functional between the traction-driven and displacement-driven problems would have an inherent virtual work functional error when material constitutive laws are unknown as follows

$$
\overline{W}_{I_error}(D_{ep}, u_F, u_U, \overline{u}_F, \overline{u}_U) = -\int_B D_{ep} \nabla u_F : \nabla \overline{u}_F dV + \int_B D_{ep} \nabla u_U : \nabla \overline{u}_U dV
$$

$$
\overline{W}_{E_error}(D_{ep}, u_F, u_U, \overline{u}_F, \overline{u}_U) = \int_B b \cdot \overline{u}_F dV - \int_{\Omega_V} \tau_b \cdot \overline{u}_F dA - \int_B b \cdot \overline{u}_U dV
$$

Theoretically, if the unknown material constitutive law is identical to true constitutive law, then the virtual work functional error will be negligible. We capitalize minimizing this error to determine unknown material parameters since the error is mainly caused by unknown material parameters. For this purpose, the displacement field is
discretized by the Galerkin method and finite element concept resulting in two linearized
incremental equilibrium equations as

\[
\begin{align*}
[K]_{Uep} \{\Delta U\}_U &= \{\Delta P\}_U - \int_B B^T S_U dV \quad \text{from } G_U(D_{ep}, u_U, \bar{u}_U) \\
[K]_{Fep} \{\Delta U\}_F &= \{\Delta P\}_F - \int_B B^T S_F dV \quad \text{from } G_F(D_{ep}, u_F, \bar{u}_F)
\end{align*}
\] (3.7)

where \(\{\Delta U\}_U\) and \(\{\Delta U\}_F\) are non-prescribed displacement vectors that contain \(N\)-DOF
values under two different boundary conditions; \(\{\Delta P\}\) indicates the incremental force
vector; \(B\) is the strain-displacement matrix in FE analysis and \([K]_{Uep}\) and \([K]_{Fep}\) are the
global stiffness matrices with assumed material parameters. Separate enforcement of the
displacement and traction boundary conditions on the same boundary in laboratory
testing can obviously generate two different sets of solutions \((U_U, U_F)\), stresses \((S_F, S_U)\)
and strains \((E_F, E_U)\).

A simulated testing is conducted to verify that the difference of either internal or
external virtual work functional between the traction-driven and displacement-driven
testing is mainly caused by unknown material parameters. First, a pre-determined
monotonic displacement profile is applied on a partial boundary of a finite element model
with the cyclic elasto-plastic material parameters assumed. In the first simulation,
material constitutive model parameters used are set to be reference values and reactions
and displacements along the partial boundary are saved. Second, the saved reaction and
displacement boundary values are imposed in subsequent traction-driven and
displacement-driven simulations, respectively. Then accumulated error of the internal
virtual work functional \((\bar{W}_{I\_error})\) defined in Equation (3.5) is computed both when the
material constitutive parameters are the same as the reference values and when they have
5% errors compared to the reference values. As shown in Figure 3.2, the internal virtual
work functional error is induced purely by the error in material constitutive parameters. The error percentage defined as \( | \left( \overline{W}_{I,F} - \overline{W}_{I,U} \right) | \times 100 / \overline{W}_{I,F} \) is deemed very small (less than 1%) if material constitutive parameters used in the traction- and displacement-driven simulations are assumed to be the same as the reference values, however if the constitutive model parameters have 5% error, then errors of the internal virtual work functional between traction- and displacement-driven simulations are significantly increased up to 45% at the end of load step. This implies that we can estimate material constitutive parameters by minimizing differences of stresses and strains from the two parallel simulations.
Figure 3.2 Differences in Internal Virtual Work Functionals between Traction- and Displacement-driven Simulated Testing

The proposed Self-OPTIM method capitalizes such theoretical rationale linked to the error of the virtual work functionals defined in Equation (3.5) or (3.6). In the Self-OPTIM inverse method, we assume that the constitutive tensor $D_{ep}$ is unknown in terms of model parameters. Such consideration naturally allows us to use experimentally measured displacement and traction boundary values in the Self-OPTIM analysis.

3.1.2 Algorithms for Inverse Analysis in Self-OPTIM

Unique feature of the proposed Self-OPTIM method is that they do not require stress and strain measurements from experimental testing to identify constitutive parameters. Only global forces and displacements on the same partial boundaries and full-(or partial-) field displacements are required. Although boundary displacements are related to average strain fields, the boundary displacement is insufficient to identify parameters of the constitutive law because of lack of information. Thus, full-field displacements have been significantly improving existing inverse parameter estimation.
techniques. Compared to the existing inverse methodologies in which error functions of displacement (or strain) fields are minimized by adjusting constitutive parameters, the proposed Self-OPTIM method minimizes an error function directly defined in terms of “full-field” stresses and strains (not observable displacement fields). However, the inputs to the Self-OPTIM method are global forces and displacements on the same partial boundary and full-(or partial-)field displacements. In that sense, fundamentally, our approach is different from existing finite element model updating techniques. Considering the fact that “full-field” stresses and strains in the objective function are used and they satisfy equilibrium, compatibility and boundary conditions, it is considered to be remarkable technology advancement in inverse parameter identifications. In the first step, the targeted material or structural component is tested under quasi-static cyclic loading. It could be tension or compression or bending cyclic testing. From the testing, the required data are measured. Depending on the parameters to be identified, loading paths measured from the experimental test can be subdivided into multiple sub-paths called a load step window. For example, if the first load step window is set up to linear elastic range (for example, from load step 1 to load step 20), Young’s modulus \( E \) and Poisson ratio \( \mu \) could be easily estimated.
Figure 3.3 Schematic Flowchart of Self-OPTIM Inverse Analysis Method

Figure 3.3 shows a schematic computational flowchart of the Self-OPTIM analysis method. Global force \((F_U)\) and displacement data \((U_U)\) on the boundary measured from experimental tests will be applied to the force-controlled (FEMA) and displacement-controlled (FEMB) FE simulations, respectively. Two different parallel nonlinear FE simulations, FEMA and FEMB, are performed using the parameter set which is tuned by using the optimization operator inside the optimization iteration. Two sets of stress and strain data \((S_U, E_U, S_F, E_F)\) are extracted at all the Gauss points of specified elements through post-processing. In addition, the displacement from FEMA \((U_F)\) and force from FEMB \((F_U)\) of the boundary nodal points are also extracted from the parallel nonlinear FE simulations.
3.1.3 Implicit Formulation of Objective Function

An important step of the inverse identification problem is to define the objective function that is going to evaluate the fitness of the solutions. Within each optimization process in a selected load step window, two different finite element analyses are conducted using updated material model parameters under the experimentally measured traction and displacement boundary values. From displacement-driven FE simulations, stresses ($S_U$) and strain ($E_U$) are saved at every Gauss point and load step. Correspondingly, from the traction-driven FE simulations, stresses ($S_F$) and strains ($E_F$) are saved at every Gauss point and load step. According to the theoretical rationale previously mentioned, the Self-OPTIM method uses an implicit objective function (OF) (See Equation(3.8)) defined as a function of stresses ($S_F, S_U$) and strains ($E_F, E_U$) since the stress and strain are directly related by the constitutive law within internal virtual work functionals.

Global force ($F_U$) and displacement data ($U_U$) on the boundary measured from experimental tests will be applied to the force-controlled (FEMA) and displacement-controlled (FEMB) FE simulations, respectively. Two different parallel nonlinear FE simulations, FEMA and FEMB, are performed using the parameter set which is tuned by using the optimization operator inside the optimization iteration. Two sets of stress and strain data ($S_U, E_U, S_F, E_F$) are extracted at all the Gauss points of specified elements through post-processing. In addition, the displacement from FEMA ($U_F$) and force from FEMB ($F_U$) of the boundary nodal points are also extracted from the parallel nonlinear FE simulations. Computed stress and strain fields along with the extracted boundary
displacements and forces are used to evaluate various types of implicit-explicit objective functions. For example, the objective functions can be formulated as

$$OF_{\text{implicit}} = W_1 \cdot \sum_{k=1}^{LS} \left[ 1 - \frac{\sum_{i=1}^{GP} S_{k,UF}^i}{\sum_{i=1}^{GP} S_{k,IF}^i} \right]^2 + W_2 \cdot \sum_{k=1}^{LS} \left[ 1 - \frac{\sum_{i=1}^{GP} E_{k,UF}^i}{\sum_{i=1}^{GP} E_{k,IF}^i} \right]^2 + W_3 \cdot \sum_{k=1}^{LS} \left[ 1 - \frac{\sum_{i=1}^{NP} U_{k,UF}^i}{\sum_{i=1}^{NP} U_{k,IF}^i} \right]^2 + W_4 \cdot \sum_{k=1}^{LS} \left[ 1 - \frac{\sum_{i=1}^{NP} F_{k,UF}^i}{\sum_{i=1}^{NP} F_{k,IF}^i} \right]^2$$

where $W_1$, $W_2$, $W_3$, and $W_4$ are weight factors whose values indicate the relative importance of each term relative to the other (Rao, 2009) and $||\cdot||$ signifies the Euclidean norm. $S_{k,IF}^i$, $E_{k,IF}^i$, and $S_{k,UF}^i$, $E_{k,UF}^i$ are the simulated local stresses and strains from FEMA and FEMB, respectively. $U_{k,IF}^i$ and $F_{k,IF}^i$ are the simulated displacements and forces from FEMA and FEMB, respectively. $U_{k,UF}^i$ and $F_{k,UF}^i$ are the measured displacement and force data from experiments. In addition, $LS$, $GP$, and $NP$ denote the number of time steps in current time step window, the total number of Gauss points in current finite elements, and the number of boundary nodal points, respectively. The Self-OPTIM method uses the implicit-explicit objective function that takes a least square form as a function of two sets of stresses, strains, displacements, and forces. It is notable that two parallel nonlinear finite element simulations with the updated parameter set are nested within the selected optimization algorithm and executed independently. Therefore, the Self-OPTIM analysis method can be easily implemented into various global optimization techniques that do not need to calculate sensitivity of the objective function.
Within the selected optimization algorithm, nonlinear FE simulations are repeated while specific optimization algorithms are applied. If the objective function value is reduced below the predetermined criterion or the number of iterations reaches a preset maximum value, then the iteration will stop; otherwise, the updated parameter set will be input to the next optimization iteration. Three different optimization techniques are tested for Self-OPTIM: Nelder-Mead simplex method, SSGA, and Chaotic firefly algorithm (CFA).

3.2 Optimization Algorithm

The simplex method is one of the gradient-free and computationally compact optimization techniques frequently used for multi-dimensional unconstrained nonlinear optimization problems. Simplex method does not call for any information about gradients or Hessian matrix and it can thus be suitable for problems with either non-smooth objective function or rapidly changed Hessian matrix. Multiple termination criteria can be applied to stop the optimization iteration. Although the simplex method has a very fast converging speed of finding the minimum, it appears to have limitations in escaping from the neighborhood of the local minimum. Therefore, the selection of the initial DPS is of great impotence in the simplex method.

Steady-state genetic algorithm (SSGA), randomly generated initial population at the beginning is evolved through evolutionary process such as selection, crossover and mutation like standard genetic algorithms. Initially, each individual consists of randomly generated binary codes that correspond to real parameters within preset damage parameter bounds. Preset upper bound and lower bounds of the parameters ensure that selected individual is permissible for the problem. Unlike standard genetic algorithms
that need to evaluate the whole individuals per each generation, SSGA needs to evaluate the whole individuals only once at the beginning. Instead, SSGA selects four best individuals and two pairs are made by them for crossover. Subsequently, SSGA replaces two worst individuals with the off-spring and evaluates the new off-spring after applying mutation process to them. Therefore, SSGA can significantly reduce computational costs. This iterative process continues until a maximum number of generations have been reached. Although initially randomized, SSGA evaluates only two off-spring made by the best DPSs in each generation and superior genotypes gradually become dominant as the generation proceeds. It has been proven to converge faster than traditional genetic algorithm (GA) with less computational times (Caicedo, 2010; Yun et al., 2009). SSGA is capable of searching for a global minimum throughout a high dimensional parameter space in an evolutionary way. In particular, the mutation process enables an individual to avoid being trapped in the region around the local minimum and continue to search for the global minimum.

The chaotic firefly algorithm (CFA) (Gandomi et al., 2012), which is developed based on the firefly algorithm (Yang, 2008), is used as the optimization algorithm in the stochastic Self-OPTIM. The computational procedure of CFA is shown in Figure 3.4. At first, the FA will generate a group of locations of fireflies (the initial generation of the individuals) and then the fireflies will move based on the light intensity, which is a function of the location of each firefly and their corresponding objective function values. The primary studies show that it is capable to outperform the well-known algorithms such as GA (Yang, 2009b). This method has also been verified with structural
engineering problems in (Gandomi et al., 2011a) in comparisons with several methods such as GA and Simulated annealing.

Figure 3.4 Flow chart of chaotic firefly algorithm

3.3 Advantages of Self-OPTIM Inverse Method

It is worth noting that the Self-OPTIM approach is distinguishable from existing FE model updating based techniques that use predicted global responses such as forces and displacements in the objective function. Unlike conventional FE model updating approaches, the Self-OPTIM method uses full-field stresses and strains as shown in the first and second term of Equation (3.8). The third term in the objective function accounts for the displacement difference between the reference boundary displacement and the simulated displacement from FEMB. Likewise, the force difference between the reference boundary force and the force from FEMA is added to the objective function.
is worth noting that Self-OPTIM is fully model independent. Especially, any material parameters of ABAQUS build-in model and user defined material (UMAT) can be easily calibrated in Self-OPTIM. The performance of the Self-OPTIM method will be verified numerically in the following section.
CHAPTER IV
A GENERAL PLATFORM FOR STOCHASTIC FINITE ELEMENT SIMULATION

4.1 Spectral Decomposition of Random Field by Karhunen-Loève Expansion

The material property is modeled as a random field by using Karhunen-Loève Expansion. The Galerkin-based finite element approach is applied to obtain the discretized random field.

4.1.1 Random Field Discretization by Karhunen-Loève Expansion

Uncertainties of material properties (e.g., elastic modulus, thickness, density) are often assumed to be Gaussian due to its simplicity and the lack of relevant experimental data although most material uncertainties appearing in engineering systems are non-Gaussian in nature according to Lumley (Lumley, 1984; Spanos and Zeldin, 1998; Stefanou, 2009). There are many random field discretization methods such as spatial average method (Vanmarcke, 1977), midpoint method (Deodatis and Micaletti, 2001; Der Kiureghian and Ke, 1988; Shinozuka and Dasgupta, 1986), expansion optimal linear estimation method (Li and Der Kiureghian, 1993), Karhunen-Loève (KL)-expansion (Stefanou and Papadrakakis, 2007b), and etc. A comprehensive review and comparison of these methods can be found in (Sudret and Der Kiureghian, 2000). In this dissertation, KL-expansion is employed for discretizing spatially varying random fields in the two-dimensional domain. KL expansion is able to use a very few terms to capture the characteristic of the strongly correlated random fields. In the KL expansion method,
unlike the deterministic case, the 2D random fields with non-zero means are decomposed into a deterministic part and a stochastic part as follows

$$E(x, \theta) = \bar{E}(x) + \sum_{i=1}^{\infty} \sqrt{\lambda_i} \phi_i(x) \xi_i(\theta)$$

(4.1)

Several covariance functions are available in different forms (Rahman and Xu, 2005b). Choice of covariance functions is dependent on the nature of stochasticity of the random field. The covariance function (CF) for spatially varying random fields can also be constructed from field measurement data (Vanmarcke, 1988). Two commonly used exponential forms are presented as follows

$$C(x_1; x_2) = \sigma^2 \exp\left(-\frac{|x_1 - x_2|}{b_{c1}L_{D1}} - \frac{|y_1 - y_2|}{b_{c2}L_{D2}}\right) \quad x_1, x_2 \in \Omega$$

(4.2)

$$C(x_1; x_2) = \sigma^2 \exp\left(-\frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{b_c L_D}\right) \quad x_1, x_2 \in \Omega$$

(4.3)

where $b_{c1}$ and $b_{c2}$ are the correlation length on different directions; $L_{D1}$ and $L_{D2}$ are the physical characteristic length on different directions; $\sigma$ is the standard deviation; $b_c$ and $L_D$ are the correlation length and physical length of the symmetric CF kernel. Since Equation (4.2) is a separable integral kernel, the analytical solution of the integral function can be derived in Reference (Roger G. Ghanem, 2003). However, Equation (4.3) is an inseparable kernel (Rahman and Xu, 2005b). Therefore, numerical solutions have to be provided for Equation (4.3). The covariance function surfaces are plotted in a unit square domain using different correlation lengths as shown in Figure 4.1. It can be seen in Figure 4.1 that the covariance function by Equation (4.2) is piecewise smooth whereas
the covariance function by Equation (4.3) is smooth over the entire domain except the singular peak point.

\[ \text{Equation 4.2} \]

\[ \text{Equation 4.3} \]

Figure 4.1 Plots of covariance functions where \( x_1 = y_1 = 0.5 \): a) \( b_{c1} = b_{c2} = 1 \), b) \( b_{c1} = b_{c2} = 0.1 \), c) \( b_{c} = 1 \), and d) \( b_{c} = 0.1 \).

Since the covariance function is bounded, symmetric and positive definite, it is expressed in the spectral decomposition as follows
\[
C(x_1; x_2) = \sum_{i=0}^{\infty} \sqrt{\lambda_i} \varphi_i(x_1) \varphi_i(x_2)
\]  \hspace{1cm} (4.4)

By multiplying both sides of Equation (4.4) by \( \varphi_k(x_2) \) and integrating over the domain \( D \), the homogeneous Fredholm integral equation is obtained as follows

\[
\int_{\Omega_{2e}} C(x_1; x_2) \varphi_k(x_2) dA_{2e} = \lambda_k \varphi_k(x_1)
\]  \hspace{1cm} (4.5)

where \( A_{2e} \) indicates the elemental domain \( (\Omega_{2e}) \) in terms of position vector \( x_2=(x_2, y_2) \), that is, \( dA_{2e}=dx_2dy_2 \). The orthogonality of the eigenfunctions is utilized in the above derivation. The eigenvalue \( \lambda_i \) and eigenfunction \( \varphi_i(x_2) \) are obtained from solving the Fredholm integral equation. Eigensolutions of the Fredholm integral equation can be solved either numerically or analytically for an assumed covariance function. Due to statistical independence of the uncorrelated random variables \( \{\xi_i(\theta)\} \), \( E(x, \theta) \) can readily be realized at a certain point for Monte Carlo simulations.

4.1.2 Galerkin Finite Element Techniques for Karhunen-Loève Expansion

It is worth noting that the analytical solution of Equation (4.5) does not always exist for most of covariance functions, especially in the case that the RF domain \( \Omega \) takes irregular shapes. Therefore, Galerkin-based finite element approach is more preferred than analytical solutions in solving the Fredholm integral equation. Galerkin-based finite element approach for the RF mesh allows us to seamlessly integrate material variability into the finite element formulation. RF elements will have one degree of freedom per node, which corresponds to a scalar value of the eigenfunction. For computations, finite terms in Equation (4.3) will be obtained by numerical methods; in other words, the truncated KL expansion is used in this dissertation.
\[ E(x, \theta) = \bar{E} + \sum_{i=1}^{M} \sqrt{\lambda_i} \varphi_i(x) \xi_i(\theta) \]  

(4.6)

where \( M \) denotes the number of KL expansion terms.

In Galerkin finite element approach, the eigenfunction is approximated by discretizing the problem domain. The approximation of the eigenfunction is expressed as

\[ \varphi_i(x) = \sum_{j=1}^{nnod_{RF}} L_j(x) d_{ij} = \langle L(x) \rangle \{ d \} \]  

(4.7)

where \( d_{ij} \) is the \( j \)th nodal value of the \( i \)th eigenfunction of the covariance function; \( nnod_{RF} \) is the number of nodes per RF element. Basis function \( L_j \) can be any complete set of 2D functions in the Hilbert space \( H \). In this paper, shape functions of a nine-node quadratic quadrilateral finite element (Q9) (i.e. Lagrange interpolation function) have been selected as the basis function. Galerkin finite element procedures are carried out by substituting Equation (4.7) into Equation (4.5). For each RF element, Equation (4.5) can be written in terms of a nodal vector \( \{ d \} \rangle \) and Q9 shape function \( \langle L(x) \rangle \) of the eigenfunction as follows.

\[ \int_{\Omega_e} C(x_1; x_2) \langle L(x_2) \rangle \{ J_e \} dA_e \{ d \} \rangle = \lambda_i \varphi_i(x_1) \]  

(4.8)

where \( |J_e| \) is the Jacobian that maps from parametric coordinates to physical coordinates.

For each RF element, a weighted residual of Equation (4.8) by the same shape function \( \langle L(x_1) \rangle \) can also be written as follows

\[ \left[ \int_{\Omega_e} \int_{\Omega_e} C(x_1; x_2) \langle L(x_1) \rangle^T \langle L(x_2) \rangle |J_e|^2 dA_e \right] \{ d \} \rangle = \lambda_i \int_{\Omega_e} \langle L(x_1) \rangle^T \langle L(x_1) \rangle dA_e \{ d \} \rangle \]  

(4.9)
After all RF elements are assembled, a generalized algebraic eigenvalue problem is obtained as follows

\[ \mathbf{CD} = \Lambda \mathbf{BD} \]  

(4.10)

\[ \mathbf{C} = \sum_{e=1}^{N_{ef}} \sum_{i=1}^{N_{fg}} \int_{\Omega_e} \int_{\Omega_g} \mathbf{C}(x_1; x_2) \mathbf{L}(x_1)^T \mathbf{L}(x_2) |\mathbf{J}_e|^2 \, dA_e \, dA_{te} \]  

(4.11)

\[ \Lambda_y = \delta_{yj} \lambda_j \]  

(4.12)

\[ \mathbf{B} = \sum_{e=1}^{N_{ef}} \int_{\Omega_e} \mathbf{L}(x)^T \mathbf{L}(x) |\mathbf{J}_e| \, dA_e \]  

(4.13)

where values in the \( j \)th column of \( \mathbf{D} \) matrix are \( j \)th eigenfunction values at nodal points; \( \mathbf{C}, \mathbf{D}, \Lambda, \) and \( \mathbf{B} \) are \( ndof_{RF} \) by \( ndof_{RF} \) dimensional matrices, where \( ndof_{RF} \) is the total DOFs of the RF mesh; \( \delta_{ij} \) is the Kronecker Delta. Elemental matrices \( \mathbf{C}_e \) and \( \mathbf{B}_e \) have to be computed for each RF element and assembled into the global matrices \( \mathbf{C} \) and \( \mathbf{B} \). Detail information for computing the elemental matrices can be found in (Allaix and Carbone, 2009). The property of symmetry holds for matrices \( \mathbf{B} \) and \( \mathbf{B}_e \) if the covariance function is symmetric. However, only if both of the covariance function and RF domain \( \Omega \) are symmetric, the global matrix \( \mathbf{C} \) will be symmetric. After the general eigenvalue problem is solved, eigensolutions of the Fredholm integral equation are obtained. The selected eigenvalues and eigenfunctions are depicted in Figure 4.2 and Figure 4.3.
Figure 4.2 First 50 Eigenvalues of the covariance function (Equation (4.3))

Figure 4.3 Eigenfunctions of covariance function (Equation (4.3)) with $N_{RF}=9$, $L_{D1}=L_{D2}=1$, $b_{c1}=b_{c2}=1$ and $\sigma=1$
The size of finite elements should be selected to adequately capture the essential features of the stochastic spatial variability of the material properties. The strategy used to select the number or the size of the element used in decomposition is comprehensively studied in (Allaix and Carbone, 2009). For KL expansion method, it is recommended to choose the number of elements satisfying the following condition:

\[ N_{RF} \geq \frac{L_D}{L_c} \]  

(4.14)

where \( L_D \) is the length of the physical domain \( D \) and \( L_c \) is the correlation length. This inequality is not only suitable for one-dimension domain but can also be applied to 2D domain. Following Equation (4.14), a reasonable random field mesh can be selected for the KL expansion to accurately capture the random field fluctuations. All the numerical examples presented in this paper satisfy this condition. It can be observed that if the correlation length decreases, \( N_{RF} \) has to be increased.

A unit square domain is presented in the following example to show the numerically reproduced covariance function and the decomposition error between it and the analytical covariance function. The covariance function presented in Equation (4.2) is used. The physical area \( A_D=1 \), correlation length \( L_c=b_cL_d=1 \), and standard deviation \( \sigma=1 \). 36 square elements are used in RF mesh. Numerically reproduced covariance function and analytical covariance function referring to a certain point are plotted in Figure 4.4 (a) and (b). To calculate the decomposition error, the global error estimator shown in Equation (4.15) is used. In this example, the calculated decomposition error \( \bar{\varepsilon}_d \) equals to \( 3.1934 \times 10^{-4} \).
\[\bar{E}_d = \frac{\int_D \int_D \left| C(x_1, x_2) - \sum_{i=1}^{M} \lambda_i \phi_{i}^{RF}(x_1, y_1) \phi_{i}^{RF}(x_2, y_2) \right|}{A^2 \sigma^2} \]  

(4.15)

Figure 4.4  a) numerically reproduced covariance function by using 6 by 6 square elements in RF mesh referring to \(x_2=0.5, y_2=0.5\); b) analytical covariance function referring to \(x_2=0.5, y_2=0.5\)

4.2 General Mapping-Interpolation Method between Random Field and Finite Element Meshes

Refinement of the FE mesh generally depends on geometrical features or stress states by the given loadings. However, distribution of random material properties is not necessarily identical to the FE mesh because inherent material heterogeneity is mainly related to manufacturing process or post-event internal defects. A general mapping-interpolation method is developed to completely separate the RF mesh and the FE mesh. It is beneficial that different criteria in terms of mesh density can be assigned to RF and FE meshes for reasonable accuracy.
4.2.1 Separation of Random Field Mesh from Finite Element Mesh

Two different kinds of meshes: 1) RF element mesh and 2) FE mesh will be used. Two different meshes are assumed to have identical boundary shape and size but can have different number of elements. Since the RF mesh is different from the FE mesh, a method is needed to transfer the spatial distributed material properties from RF mesh to FE mesh. From a computational point of view, the discretized value of the random field has to be finally transferred to the Gaussian points in FE mesh, because the Gaussian integration of the elemental stiffness matrix is carried out by using the material properties defined at Gaussian points. In this dissertation, a general mapping and interpolation method is presented to obtain the random field in FE mesh. The value of eigenfunctions at RF nodal points, which is obtained from KL expansion, is transferred to FE nodal points. Then, the discretized value of the random field at FE nodal points is calculated based on the eigenfunctions at FE nodal point. At last, the random field is interpolated to the location of the Gaussian points in FE mesh by using the eight-node shape function. An example that shows the mapping and interpolation error of the eigenfunctions based on a global error estimator is presented.

4.2.2 General Mapping-Interpolation Method

There have been very few studies that employ completely independent meshes for physical domain and random field domain, respectively, due to lack of numerical methods that can seamlessly interconnect RF mesh with FE mesh (Stefanou, 2009). However, in order to deal with large-scale and stochastically complex engineering problems, a general mapping-interpolation method needs to be developed. First, let’s assume that randomly distributed material properties are already determined at all the
nodal points of the RF mesh through KL expansion. Then, material properties at FE mesh nodal points are interpolated from nodal values of the RF mesh by using nine-node shape functions associated with the RF mesh. Because two meshes are independent, FE nodal points that are located inside or on the edge of each RF element must be identified. Within each RF element, RF mesh isoparametric coordinate values corresponding to the FE nodal points are calculated for subsequent interpolations. For a certain RF element, a nonlinear equation that relates coordinates \((x_{ij}, y_{ij})\) of FE nodes with coordinates \((X_{mj}, Y_{mj})\) of RF element nodes can be formulated as follows

\[
\sum_{m=1}^{9} N_m(\xi^j, \eta^j)X_m^j - x_i^j = 0
\]
\[
\sum_{m=1}^{9} N_m(\xi^j, \eta^j)Y_m^j - y_i^j = 0
\]  

(4.16)

where \(x_i^j\) and \(y_i^j\) are the physical coordinates of \(i\)th nodal point of current finite element which is located within \(j\)th RF element; \(X_m^j\) and \(Y_m^j\) is the physical coordinates of \(m\)th RF nodal point that belongs to \(j\)th RF element. \(\xi_i^j\) and \(\eta_i^j\) are the isoparametric coordinates attached to \(j\)th RF element, which correspond to \(i\)th FE nodal point. Figure 4.5 shows the coordinate transformation from physical coordinates to isoparametric coordinates within a certain RF element.
Since both of the RF and FE physical nodal coordinates are known, the system of non-linear equations can be solved for the isoparametric coordinates $\xi_{ij}$ and $\eta_{ij}$. By using the trust region method (‘fsolve’ built-in function of MATLAB), the nonlinear equation is solved as an unconstrained nonlinear optimization problem. An initial guess of every isoparametric coordinate is set as $[\xi_{ij}, \eta_{ij}]=[0.0,0.0]$. According to numerical examples presented in this dissertation, solutions are always converged within less than three optimization iterations with a terminating criterion that the objective function value is smaller than a preset tolerance ($10^{-7}$). One demonstration is shown in Figure 4.6.
Figure 4.6 A demonstration of the coordinate transformation between two fully independent RF mesh and FE mesh on non-rectangular domain

After solving all the optimization problems, $\zeta_{ij}$ and $\eta_{ij}$ are substituted to Equation (4.17) to calculate eigenfunction value at the location of $i$th FE nodal point within $j$th RF element

$$
\varphi_{ij}^{FE} = \sum_{m=1}^{9} N_m(\xi_i, \eta_i)^k \varphi_{ij}^{RF}
$$

(4.17)

where $\varphi_{m}^{jRF}$ is the value of $k$th eigenfunctions at $m$th nodal point of the $j$th RF element; $\varphi_{i}^{jFE}$ is the value of $k$th eigenfunctions value at $i$th nodal point in FE mesh that is located within $j$th RF element. The Young’s modulus distribution at FE nodal points $E^{FE}$ can be calculated based on the obtained $\varphi^{FE}$ using Equation (4.18).
\[ E^{FE}(\mathbf{x}, \theta) = \bar{E} + \sum_{i=1}^{M} \sqrt{\lambda_i} \varphi_i^{FE}(\mathbf{x}) \xi_i(\theta) \]  

(4.18)

It is worth noting that the optimization problem needs to be solved only one time as long as the RF and FE meshes do not change during simulations. The values of \( \xi_{ij} \) and \( \eta_{ij} \) will be saved and retrieved later at the time of the numerical integration of the stochastic part of elemental stiffness matrices. In the case of Monte Carlo Simulation (MCS), each realization requires reevaluation of Young’s modulus at the nodal points of FE mesh.

The proposed method mapped and interpolated the decomposed eigenfunctions obtained from KL expansion based on RF mesh to the FE mesh. And then, the Young’s modulus is generated by using the eigenvalue obtained from KL expansion and eigenfunctions in FE mesh. To access the mapping and interpolation error, an error estimator of the reproduced covariance function and the covariance function can be defined as

\[ \varepsilon_i(x_1, y_1; x_2, y_2) = \sum_{i=1}^{M} \left| \lambda_i \phi_i^{RF}(x_1, y_1) \phi_i^{RF}(x_2, y_2) - \lambda_i \phi_i^{FE}(x_1, y_1) \phi_i^{FE}(x_2, y_2) \right| \]  

(4.19)

Since the location of the nodal points in RF mesh and FE mesh are mismatch, a global error estimator has to be used as

\[ \bar{\varepsilon}_i = \sum_{i=1}^{M} \int_D \int_D \left| \lambda_i \phi_i^{RF}(x_1, y_1) \phi_i^{RF}(x_2, y_2) - \lambda_i \phi_i^{FE}(x_1, y_1) \phi_i^{FE}(x_2, y_2) \right| \frac{A_D^2 \sigma^2}{A_D^2 \sigma^2} \]  

(4.20)

To demonstrate the mapping and interpolation error evaluation, an example is presented using a RF mesh with nine elements and the FE mesh with 100 elements. It assumes that the physical area \( A_D=1 \), correlation length \( L_c=1 \), and standard deviation \( \sigma=1 \).
We got $\varepsilon_i$ equals to $3.8 \times 10^{-8}$, which means the mapping and interpolation error between the eigenfunctions on RF mesh and FE mesh is very small and negligible.

4.3 Intrusive Formulation of Stochastic Finite Element

The ABAQUS/Standard UEL subroutine named SFEQ8 (stochastic Q8 finite element) is coded in FORTRAN. As an 8-node plane stress finite element, the element totally contains eight nodes with 16 dofs (translation on 1 and 2 directions on each node). Labels of nodes, faces and integration points for output are shown in Figure 4.7.

![Figure 4.7 Configuration of UEL SFEQ8](image)

SFEQ8 is based on the isoparametric formulation. Shape functions used are same as the one for Q8 finite element (Cook et al., 2002). Elemental stiffness matrix of the stochastically grade finite element is decomposed into deterministic and stochastic parts as follows.

$$[K_e] = [K_{ed}] + [K_{es}]$$

(4.21)

The deterministic element stiffness matrix $[K_{ed}]$ can be calculated by a standard method in finite element procedures. Full integration scheme in Gaussian quadrature is used as in Equation (4.22).
\[
[K_e]_s = \int \int [B]^T[D][B]dx\,dy = \int_1^1 J_1^1 [B]^T[D][B] \det J \, d\xi d\eta
\]
\[
= \sum_{i=1}^3 \sum_{j=1}^3 [B]^T[D][B] \det J \, W_i W_j
\]  
(4.22)

where \([B]\) is the strain-displacement matrix; \(|J|\) is the determinant of Jacobian matrix; \(W\) is the weight factor of Gaussian quadrature; \(t\) is the thickness, and \([D]\) is the material constitutive matrix where the mean value of Young’s modulus is used. In the case of the plane stress condition, the stochastic elemental stiffness matrix is formulated by substituting the second term of Equation (4.6) into Young’s modulus in the material constitutive matrix.

\[
[K_e]_s = \int \int [B]^T[\hat{D}(\xi, \eta; \theta)][B]dx\,dy = \int_1^1 J_1^1 [B]^T[\hat{D}(\xi, \eta; \theta)][B] \det J \, d\xi d\eta
\]
\[
= \sum_{i=1}^3 \sum_{j=1}^3 [B]^T[\hat{D}(\xi, \eta; \theta)] [B] \det J \, W_i W_j
\]  
(4.23)

\[
[\hat{D}(\xi, \eta; \theta)] = \frac{\hat{E}_G(\xi, \eta; \theta)}{1 - \mu^2} \begin{bmatrix}
1 & \mu & 0 \\
\mu & 1 & 0 \\
0 & 0 & (1 - \mu) / 2
\end{bmatrix}
\]

\[
\hat{E}_G(\xi, \eta; \theta) = \sum_{i=1}^8 N_i(\xi, \eta)\hat{E}_{i,\text{FE}}(x, \theta)
\]  
(4.24)

where \([K_e]_s\) is the stochastic element stiffness matrix; \(x\) is the position vector of FE nodal points, and \(\mu\) is the Poisson’s ratio; \(\hat{E}_{i,\text{FE}}(x, \theta)\) is the randomly distributed elastic modulus at FE nodal points which is calculated by using the general mapping-interpolation method introduced in Section 3; \(\hat{E}_G(\xi, \eta; \theta)\) is the interpolated Young’s modulus at Gauss points. As shown in Equation (4.24), Young’s modulus values at Gauss points are interpolated from nodal Young’s modulus values using eight-node shape functions. It is
worth noting that instead of using a constant value of elastic modulus over the entire elements, every Gauss point has different elastic modulus realized based on the stochastic parameters of the random field. This numerical integration technique allows us to simulate stochastic material behavior in this element. ABAQUS assembles element stiffness matrices into a global stiffness matrix and delivers the matrix to the numerical solver with prescribed boundary conditions. The detailed implementation of SFE in the MATLAB-ABAQUS combined framework will be introduced in Section 5.

4.4 Implementation of Stochastic Finite Elements for Monte Carlo Simulation

Though expensive computationally, Monte Carlo simulation is the only currently available universal methodology for accurately solving problems in stochastic mechanics involving strong nonlinearities and large variations of non-Gaussian uncertain system parameters Reliability-based design. In this section, detailed implementations for Monte Carlo Simulation (MCS) using the SFE are presented to predict probabilistic structural response caused by material uncertainties. A flow chart showing the MCS is depicted in Figure 4.8. ABAQUS is one of the most widely used sophisticated finite element analysis tools and MATLAB is also a commonly used tool for numerical simulations. Therefore, SFE is implemented in user-defined element (UEL) in ABAQUS and a stochastic decomposition tool developed in MATLAB codes is interfaced with the UEL.
Figure 4.8 Schematic of Monte Carlo Simulation using SFE and post-processing with UEL framework interfaced with MATLAB codes (RF: Random Field; KL: Karhunen-Loève)

Finite element models are prepared by ABAQUS. Nine-node RF element mesh can be created by other preprocessor such as PATRAN. One example of RF mesh and the corresponding FE mesh is shown in Figure 4.9. Stochastic data that include the standard deviation, the correlation length and the mean value are predetermined to be associated with a continuous covariance function.
4.4.1 Monte Carlo Simulation with SFE and KL Expansion

For MCS that considers the material uncertainty, zero-mean and unit-variance independent Gaussian random variables ($\xi_i$) in Equation (4.6) need to be generated. In MCS, a set of these random variables is described as realizations, observations, or samples. Therefore, the stochastic problem is converted to a large number of deterministic problems. If computations for solving the deterministic problem are computationally costly, the MCS will be very time-consuming because the simple sample random sampling scheme needs a large number of samples to draw statistically meaningful results. In order to reduce computational times in MCS, an efficient sampling approach has to be applied. In $j$th Gaussian realization, $\{\xi_{n_\text{dof}_{ij}}\}_j$ is generated by using Latin hypercube sampling (LHS) technique that preserves marginal probability distributions for each variable simulated. This LHS method allows for a small number of
samples maintaining feasible representations of statistical characteristics of random fields (Huntington and Lyrintzis, 1998). The spatial random distribution of the elastic modulus on FE nodal points $E_{FE}(x, \theta)$ is calculated by using Equation (4.6) combining with the general mapping-interpolation method.

The user subroutine UEXTERNALDB is used to import the $E_{FE}(x, \theta)$ at the beginning of every elemental calculation carried out within ABAQUS. For each element, the code in UEL searches for $E_{FE}(x, \theta)$ at corresponding eight nodal points of SFE. After that, Equation (4.23)-(4.24) are used to compute the elemental stiffness matrix.

4.4.2 Post-Processing for Stochastic Characterization of Material Response

When an UEL subroutine is used, no information such as field output data, history output data, and model data (i.e. element connectivity and nodal coordinates) is available for visualization in ABAUQS odb files. Therefore, a Python script is used to import the model data, the field output data, and Young’s modulus into ABAQUS odb files. In this way, spatial distributions of the elastic modulus, stress/strain, displacement/force, etc. are visualized by contour plots in ABAQUS Viewer. The stochastic response of the material from MCS such as Probability Density Function (PDF), Cumulative Density Function (CDF), etc. are computed and visualized in MATLAB codes.

4.5 Conclusion

In this dissertation, a stochastic finite element (SFE) is proposed in an intrusive way that employs Karhunen-Loève (KL) expansions for spectral decomposition of random fields. To the best of authors’ knowledge, there has been no attempt to integrate the SFE into third party software such as ABAQUS/Standard. The SFE is able to account for uncertainties of mechanical properties of isotropic randomly graded materials. Two
different meshes are defined in the proposed work: 1) finite element (FE) mesh and 2) random field (RF) mesh. By a general mapping-interpolation method proposed in this paper, the RF and FE meshes are systematically mapped to each other. Monte Carlo simulation (MCS) is used to simulate the stochastic structural response under material uncertainty. Most importantly, the proposed methodology is able to easily cooperate with any working deterministic code, which is developed for handling very complex problem such as visco-elasto-plasticity. The method requires only repetitive calls to an existing deterministic solver. Furthermore, the parallel processing is suitable for the method.
CHAPTER V
VERIFICATIONS OF SELF-OPTIM

5.1 Cyclic Elasto-Plastic Constitutive Model

Cyclic plasticity models considering Bauschinger effect, cyclic hardening or softening with strain range effect, non-proportional hardening, and strain ratcheting under asymmetric stress cycling has been substantially advanced in the past several decades (Abdel-Karim, 2009; Chaboche, 1986, 1991; Chaboche et al., 1991; Chen and Jiao, 2004; Chen et al., 2005; Jiang and Kurath, 1997a; Khan and Cheng, 1996; Krishna, 2009; Mayama et al., 2004; Moosbrugger et al., 2008; Ohno and Wang, 1991; Ohno and Wang, 1993; Sai and Cailletaud, 2007; Taleb et al., 2006; Taleb and Hauet, 2009; Wolff and Taleb, 2008; Yoshida, 2000; Zhang and Jiang, 2008). Particularly, significant research and studies to better understand complex cyclic plastic behavior of various engineering materials and improve capabilities of the cyclic plasticity model have been undertaken focusing on the superposed (Bari and Hassan, 2000; Chaboche, 1989; Johansson et al., 2005; McDowell, 2000), strain ratcheting (Abdel-Karim, 2009; Chen and Kim, 2003; Johansson et al., 2005; Mayama et al., 2004; Nakane et al., 2008; Ohno and Wang, 1991; Rahman et al., 2008; Taleb et al., 2006) and the non-proportional hardening (Doring et al., 2003; Jiang and Kurath, 1997b; Krishna, 2009; Krishna et al., 2009; Moosbrugger and Mc Dowell, 1989; Taleb and Hauet, 2009; Tanaka, 1994). According to literatures (Choi et al., 2006; Jiang and Zhang, 2008; Krishna et al., 2009; Shamsaei et al., 2010a; Wang et
al., 2006), materials subjected to complex, multi-axial and non-proportional strain paths could show very different response compared to that subjected to rather simple deformational states often observed in traditional material testing.

In this dissertation, a non-linear isotropic/kinematic combined hardening law based cyclic elasto-plasticity constitutive model is applied. For the kinematic hardening behavior, the Armstrong-Frederick type model (A-F model) (Armstrong and Frederick, 1966) is used for simulating evolutions of the back stress under cyclic loadings. However, it is known that the model has limitations in reproducing ratcheting response of the material and the decomposed nonlinear kinematic hardening models can better predict stresses under cyclic loadings than the conventional A-F model does (Bari and Hassan, 2000). As previously mentioned, it is notable that such limitations of the A-F model cannot be claimed as limitations of the proposed Self-OPTIM methodology. To reproduce the Bauschinger effect, the A-F model defines evolutionary equation of the back stress as follows

\[
\dot{\alpha}_{ij} = \frac{2}{3} H n_{ij} \dot{\sigma} - \gamma \alpha_{ij} \dot{\sigma}
\]  

(5.1)

As the effective plastic strain increment (\(\dot{\sigma}\)) increases, the rate of the back stress (\(\dot{\alpha}_{ij}\)) also increases. \(n_{ij}\) is a component form of the normal direction tensor (\(n\)). Two model parameters \(H\) and \(\gamma\) determines the maximum saturated yield stress (\(H/\gamma\)) along the kinematic hardening evolution and \(\gamma\) influences the rate of saturation of the yield stress due to kinematic hardening. Considering such effects of the H and \(\gamma\) parameters on the kinematic hardening and translational evolutions of the yield surface in stress space, at least half-cycle (loading-yielding-unloading-yielding) stress-strain path with a sufficiently large strain range is required to identify the parameters.
A nonlinear isotropic hardening model suggested by Chaboche (Chaboche, 1986) is adopted to simulate expansion of the yield surface under increasing plastic deformations. The rate equation of the isotropic hardening parameter is expressed as

\[ \dot{\tau}(p) = b(Q - r) \dot{p} \]  

(5.2)

In Equation (5.2), \( r \) is the drag stress showing isotropic hardening behavior and \( Q \) determines the saturated yield stress \((Q+S_0)\) to be reachable during cyclic straining. The value of \( b \) determines the rate of the saturation of the yield stress. Equation (5.2) is expressed as \( r(p) = Q(1-e^{-bp}) \) if integrated with initial value \( r=0 \). Thus, depending on the strain range and the parameter \( b \), local stress-strain paths could be sufficient or insufficient for identifying the \( Q \) parameter. In actual material behavior, cyclic hardening is frequently observed from symmetric stress or strain controlled experiments of metallic materials. In this case, both isotropic and kinematic hardening effects are combined to generate the cyclic hardening behavior. Figure 5.1 illustrates the effect of combined nonlinear isotropic and kinematic hardening on the cyclic hardening response. Based on this observation, cyclic hardening stress-strain paths experienced by the material can provide comprehensive information to identify the hardening parameters.
As long as sufficient stress-strain data within the linear elastic range are generated, Young’s modulus and Poisson ratio could be also identified. Since error of the initial yield stress can create large errors in subsequent stress updates, the yield stress can be identified as long as the material is yielded. In the following, the proposed Self-Optimizing inverse method will be introduced with details.

5.2 Numerical Verification

Inaccurate reproductions of experimental results could be dominantly caused by inherent shortcomings of the chosen model. Thus such limitations of the chosen model should not be considered as defects of the proposed Self-OPTIM methodology. To validate the proposed Self-OPTIM analysis method, two numerical examples are selected: 1) a simple model subjected to uniaxial cyclic tension-compression loading and 2) a complex model subjected to biaxial cyclic tension-compression loading.

5.2.1 Example 1: Verification with A Simple Model

A simple two-dimensional FE model (2 m by 2 m) consists of four 8-node plane strain elements which are restrained on the translational DOFs in Y-direction on the bottom edge and the translational DOFs in X-direction on the left edge as shown in .
Synthetic test data, full-field in-plane reference displacements and global force data along the right edge boundary, are generated by a displacement-driven simulation under an half-cycle (160 steps) uniaxial displacement profile and a material parameter set $X_t=[C=210,000\text{MPa}, \mu=0.3, S_{y0}=200\text{MPa}, b=0.26, Q=2000 \text{MPa}, H=17000 \text{MPa}, \gamma=21]$ is considered as true values. The material constants are from low-carbon steel (AISI 1010) in a laminated state (Doghri, 1993). Thus the uniaxial stress component is prevailing compared to shear and transverse stresses generated under these boundary conditions. Although the linear elastic Young’s modulus can be found from conventional optimization fitting of the parameter under uniaxial loadings, it would be difficult to determine Poisson ratio without information on displacements normal to the direction of the uniaxial loadings. In this example, it is intended to demonstrate performances of the Self-OPTIM method in finding out two linear elastic and five elasto-plastic material constitutive parameters under cyclic loading conditions. Full integration strategy has been applied in which 9 Gauss points per element are considered. A strain and stress response curve on a Gauss point shown in Figure 5.2 indicates that it contains information on the linear elastic behavior, the yielding point, the hardening behavior and the kinematics of yielding surface. In order to estimate the parameters which control the plastic deformation, a cyclic testing with a large strain range (2.5% in amplitude) was required.
The Self-OPTIM methodology could successfully estimate the true constitutive parameters using the full-field reference displacement, and the partial boundary force data. The initial parameter values are set to $X_0=[C=170,000\text{MPa}, \mu=0.15, S_{y0}=150\text{MPa}, b=0.35, Q=1,900\text{MPa}, H=15,000\text{MPa}, \gamma=25]$. In Figure 5.3, the implicit OF value was observed to rapidly decrease from 147 to 0.01. Traces of the updated parameters are depicted in Figure 5.4 where the blue solid horizontal line represents true values and the
red curve represents converging parameter values. The linear elastic parameters $C$ and $\mu$, yield stress $S_y$, non-linear kinematic hardening parameter $H$ and $\gamma$ were well estimated after 200 iterations and eventually converged.

Figure 5.3 Objective Function Values v.s. No of Iterations During Self-OPTIM Analysis
Figure 5.4 Parameter Estimations by Self-OPTIM using Synthetic Test Data under Half-Cycle Displacement Loading (a) $C$, Young’s Modulus; (b) $\mu$, Poisson Ratio; (c) $S_y0$, Yield Stress; (d) $H$, Saturated Yield Stress by Kinematic Hardening; (e) $\gamma$, Rate of Change of Yield Stress by Kinematic Hardening
Figure 5.5 Convergence to False Values of Parameters Related to Nonlinear Isotropic Hardening by Using Half-Cycle Loading Profile (a) $Q$, Saturated Yield Stress by Isotropic Hardening and (b) $b$, Rate of Saturation of the Yield Stress by Isotropic Hardening

The half-cycle tension-compression displacement-driven simulation supplied sufficient information about those parameters. However the loading profile does not contain enough information required to identify the isotropic hardening parameters ($Q$ and $b$). Accordingly in Figure 5.5, even though isotropic hardening parameters are initially set close to the true value, they could not be estimated and they converged to false values in the end. Even if the implicit OF value was relatively very small, the identified parameter set was not the global minimum and the simplex was trapped in local minima. In order to identify the isotropic hardening parameters, a multi-cycle loading profile was required. By adopting the estimated five parameters as fixed values, the parameter space for the Self-OPTIM analysis reduces to a two-dimensional space, $X = [b, Q]$. Eight-cycle displacement loading profile and a matching reference response on a particular Gauss point are shown in Figure 5.6.
With an initial parameter set $X_0 = [b=0.4, Q=1500\text{MPa}]$, $b$ and $Q$ were eventually well estimated within 110 iterations as shown in Figure 5.7. Final Self-OPTIM analysis.
results are summarized in Table 5.1. All of the parameters with initially large errors were successfully estimated by the Self-OPTIM analyses and the final errors were less than 1%.

![Graphs](image-url)

(a) Figure 5.7 Isotropic Hardening Parameter Identification by Self-OPTIM under Using 8-Cycle Displacement Profile (a) $Q$, Saturated Yield Stress by Isotropic Hardening and (b) $b$, Rate of Saturation of the Yield Stress by Isotropic Hardening
Table 5.1 Summary of the Data of the Simple Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$C$ (MPa)</th>
<th>$\mu$</th>
<th>$S_{y0}$ (MPa)</th>
<th>$b$</th>
<th>$Q$ (MPa)</th>
<th>$H$ (MPa)</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Value</td>
<td>210000</td>
<td>0.3</td>
<td>200</td>
<td>0.26</td>
<td>2000</td>
<td>17000</td>
<td>21</td>
</tr>
<tr>
<td>Initial Guess</td>
<td>170000</td>
<td>0.15</td>
<td>150</td>
<td>0.5</td>
<td>1400</td>
<td>15000</td>
<td>25</td>
</tr>
<tr>
<td>Initial Error (%)</td>
<td>19</td>
<td>50</td>
<td>25</td>
<td>53.8</td>
<td>30</td>
<td>11.8</td>
<td>19</td>
</tr>
<tr>
<td>Calibrated Value</td>
<td>209303.90</td>
<td>0.3005</td>
<td>200.02</td>
<td>0.2602</td>
<td>1998.19</td>
<td>17086.98</td>
<td>21.2082</td>
</tr>
<tr>
<td>Final Error (%)</td>
<td>0.33</td>
<td>0.17</td>
<td>0.01</td>
<td>0.08</td>
<td>0.06</td>
<td>0.51</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Stress-strain response along the x-axis and principal back stress along the 1-1 direction at a Gauss point are reconstructed by forward FE simulations using the estimated parameter set $X_i$ at different iteration steps. In the early stage, significant differences between reference and reconstructed response in both loading and unloading region were observed as shown in Figure 5.8 (a) and (b). However, the response curves gradually approaches to the reference curves as iterations are further carried out and finally overlapped with the reference response. It manifests that the Self-OPTIM method is potentially capable of estimating model parameters uniquely. The yield surface is also reconstructed at different iteration steps in 1-2 principal stress space as shown in Figure 5.8 (c). Since the isotropic and non-linear kinematic parameters are not identified in the early iteration, the center and size of the yield surface are different from the reference one. Applying the final identified parameter set, it converged to the true yield surface curve.
Figure 5.8 (a) Reconstructed Strain-Stress Response (b) Reconstructed Back Stress (c) Yield Surface Plotted in Different Iteration Step

5.2.2 Example 2: Verification with A Complex Model

Further demonstration of capabilities of the Self-OPTIM is presented by using a complex model shown in Figure 5.9. Primary aim for the biaxial loading case with more complex model is to show that the Self-OPTIM method is stable with larger number of finite elements under biaxial loading conditions. It has the same geometry as the simple one but a different mesh of 32 8-node plane strain elements with a total of 288 Gauss points was used. The boundary condition also changed to a uniformly distributed biaxial displacement profile which yields obvious biaxial stress states. However, the loading is a proportional loading. A parameter set $X_t=[C=210,000\text{MPa}, \mu=0.3, S_{y0}=200\text{MPa}, b=0.26, Q=2,000\text{MPa}, H=17,000\text{MPa}, \gamma=21]$ was utilized in the reference simulation to generate synthetic reference data. Young’s modulus and Poisson’s ratio are well known for most of engineering materials. Although existing test values for the elasticity properties can be
found, they are masked in this example since the elasticity properties could be different depending on the environment and manufacturing process. In the case of the complex model, 3 steps were carried out to identify the unknown parameters: 1) Estimate $C$ and $\mu$ by using the data in linear elastic region. 2) With the estimated $C$ and $\mu$, use half-cycle tension-compression data under a large strain range to estimate the rest of the five parameters. 3) Since $b$ and $Q$ could not be estimated in Step 2, Step 3 is designed to use an eight-cycle displacement profile. The displacement profiles for these three steps are shown in Figure 5.9 (b), (c) and (d).
Figure 5.9 (a) FE Model for the Complex Case and The Boundary Displacement Profile for Reference Simulation (b) 25 Load Steps in Linear Elastic Region (c) 160 Load Steps in Half Cycle Large Strain Region (d) 320 Load Steps in Eight Cycles

Finally the traces of the converging parameters are presented in Figure 5.10. According to the results Poison’s ratio $\mu$ was shown to have a relatively large final error. It is because the vertical and horizontal displacements are controlled separately, resulting in reduction of the natural Poisson effect. Compared with the results from the simple model case, overall error level of the estimated parameters except $S_{\gamma 0}$ increased. Results from the complex model are summarized in Table 5.2.
Figure 5.10 Parameter Estimations by Self-OPTIM using Synthetic Test Data under Biaxial Cyclic Displacement Loading (a) $C$, Young’s Modulus; (b) $\mu$, Poisson Ratio; (c) $S_{y0}$, Yield Stress; (d) $H$, Saturated Yield Stress by Kinematic Hardening; (e) $\gamma$, Rate of Change of Yield Stress by Kinematic Hardening; (f) $Q$, Saturated Yield Stress by Isotropic Hardening and (g) $b$, Rate of Change of Saturation of $Q$

Table 5.2 Summary of the Results of the Complex Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$C$ (MPa)</th>
<th>$\mu$</th>
<th>$S_{y0}$ (MPa)</th>
<th>$b$</th>
<th>$Q$ (MPa)</th>
<th>$H$ (MPa)</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Value Initial Guess</td>
<td>210000</td>
<td>0.3</td>
<td>200</td>
<td>0.26</td>
<td>2000</td>
<td>17000</td>
<td>21</td>
</tr>
<tr>
<td>Initial Error (%)</td>
<td>28.6</td>
<td>33.3</td>
<td>25</td>
<td>46.2</td>
<td>15.0</td>
<td>11.8</td>
<td>19</td>
</tr>
<tr>
<td>Calibrated Value</td>
<td>201295.36</td>
<td>0.2461</td>
<td>200.0073</td>
<td>0.2596</td>
<td>2002.61</td>
<td>16987.93</td>
<td>20.2255</td>
</tr>
<tr>
<td>Final Error (%)</td>
<td>4.14</td>
<td>18</td>
<td>0.00</td>
<td>0.15</td>
<td>0.13</td>
<td>0.07</td>
<td>3.69</td>
</tr>
</tbody>
</table>

The four parameters ($Q$, $b$, $S_{y0}$ and $H$) could be estimated with high accuracy. Their errors were less than 1%. However, the errors of $C$ and $\gamma$ were 4.14% and 3.69%, respectively.
respectively. Although the parameters values in the uniaxial and biaxial cases are both the same, final estimated values were different. It is due to a net effect of the model complexity and differences of the loading condition. However, the observed results are very promising for the purpose of parameter estimations under any arbitrary loading profile as long as the reference response contains sufficient contents on the material constitutive behavior. Moreover, general theoretical rationale of the Self-OPTIM methodology concludes that it is model-independent and applicable to other material constitutive models. For example, a cyclic plasticity model by Zhang et al. (Zhang and Jiang, 2008) can be integrated with this Self-OPTIM for inversely identifying a set of material parameters in the hardening rule. The model is known to successfully capture the strain range effect and the dependence of cyclic hardening and non-proportional hardening on the loading magnitude.

5.3 Effects of Image Noises and Boundary Force Errors on Self-OPTIM

With the previous Example 1, experimental noises and errors are considered to further verify the Self-OPTIM method. The full-field displacement based on the digital image correlation is primary input information for the proposed Self-OPTIM method. Image noises due to CCD camera are usually in the form of undesirable random deviations of the brightness or color information in the color image produced by the digital optic equipments when they capture the images in harsh environments. In the case of grayscale image, the noise presents in the form of variation of the gray level which is a single integer number and carries the light intensity information of the image at each pixel. The noises will degrade the quality of the image and bring difficulties to the DIC based inverse identification algorithm. To show the noise robust feature of the proposed
Self-OPTIM method, the image is assumed to be corrupted by a Gaussian white noise in different gray levels at every pixel. From a theoretical analysis (Besnard et al., 2006; Roux and Hild, 2006), the standard deviation of the displacement field due to camera noises is expressed as

\[ \sigma_u = \frac{12\sqrt{2}\sigma_g P}{\frac{7\left(\nabla f\right)^2}{l}} ^{1/2} \]  

(5.3)

where \(\sigma_u\) is the standard deviation of the full-field displacement field introduced by the Gaussian noise, \(\sigma_g\) is the standard deviation of the gray level, \(P\) is the physical pixel size, \(\left(\nabla f\right)^2\) is a spatially averaged gradient of the gray level within the zone of interest (ZOI), and \(l\) is the smallest dimension of ZOI in pixel. From Equation (5.3) it is realized that the larger single pixel size and smaller size of ZOI will result in a larger noise. \(\left(\nabla f\right)^2\) represents the averaged level of contrast of the image. It is notable that the small contrast level leads to a high noise level of an image. In this dissertation, the physical pixel size is 0.02 in/pixel and \(\left(\nabla f\right)^2=5,340\) /pixel. The displacement field caused by the noise has zero mean and the standard deviation \(\sigma_u\) is superposed to the original displacement field. Figure 5.11 shows time history data of typical displacement errors generated with noise level of three pixels. According to technical specifications of normal camera, three pixels are typical noise level.
Figure 5.11 The Displacement Field Introduced by Gaussian White Noise on a Nodal Point

Loading errors are also considered by adding randomly distributed noises to the boundary loading data. Boundary force measurement errors are usually dependent on the ratio of measured forces to max capacity of actuators. For the present case, 5% uncertainties on the boundary forces are assumed in terms of the standard deviation. Considering the fact that measurement errors in typical load cells are within 1%, current errors are very severe. According to current studies on the noise effects, noises due to camera images are less significant than the measurement errors of restoring forces on the boundary. The identification procedures were the same as the Example 1.
Figure 5.12 Parameter Estimations by Self-OPTIM using Synthetic Test Data with Experimental Noises and Errors (a) $C$, Young’s Modulus; (b) $\mu$, Poisson Ratio; (c) $S_{y0}$, Yield Stress; (d) $H$, Saturated Yield Stress by Kinematic Hardening; (e) $\gamma$, Rate of Change of Yield Stress by Kinematic Hardening; (f) $Q$, Saturated Yield Stress by Isotropic Hardening and (g) $b$, Rate of Saturation of the Yield Stress by Isotropic Hardening

Table 5.3 Summary of the Data of the Simple Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$C$ (MPa)</th>
<th>$\mu$</th>
<th>$S_{y0}$ (MPa)</th>
<th>$b$</th>
<th>$Q$ (MPa)</th>
<th>$H$ (MPa)</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Value</td>
<td>210000</td>
<td>0.3</td>
<td>200</td>
<td>0.26</td>
<td>2000</td>
<td>17000</td>
<td>21</td>
</tr>
<tr>
<td>Initial Guess</td>
<td>180000</td>
<td>0.15</td>
<td>150</td>
<td>0.4</td>
<td>1500</td>
<td>15000</td>
<td>25</td>
</tr>
<tr>
<td>Initial Error (%)</td>
<td>14</td>
<td>50</td>
<td>25</td>
<td>53.8</td>
<td>25</td>
<td>11.8</td>
<td>19</td>
</tr>
<tr>
<td>Calibrated Value</td>
<td>213972.38</td>
<td>0.2962</td>
<td>202.385</td>
<td>0.3378</td>
<td>1569.3</td>
<td>17368.00</td>
<td>22.345</td>
</tr>
<tr>
<td>Final Error (%)</td>
<td>1.89</td>
<td>1.27</td>
<td>1.19</td>
<td>29.96</td>
<td>21.53</td>
<td>2.16</td>
<td>6.41</td>
</tr>
</tbody>
</table>

As shown in Figure 5.12, the model parameters are gradually converging to true values even under experimental noises and errors. However, compared to Example 1 without noises and errors, the identified values showed less accuracy as seen in Table 5.3.
Particularly, the nonlinear isotropic hardening parameters showed large errors due to the accumulated errors subjected to eight cycles. There is also a computational technique available in the Self-OPTIM method. This technique has not been addressed in this dissertation. In most of cases in lab or field test, engineers can predict highly stressed regions in the structures depending on the boundary conditions and geometrical shapes. In computing the implicit objective function, Self-OPTIM users can count for the finite elements only within the high stress regions. In other words, when extracting stresses and strains, users can exclude elements that are still within linear elastic range. In that way, the objective function will get better and the isotropic hardening parameters will be calibrated better.

5.4 Verification of Self-OPTIM Identification with Experiment Test Results

In this section, a cyclic plasticity model with a non-linear kinematic hardening law (Armstrong–Frederick type, as presented in (Armstrong, 1966) has been adopted. The evolution law of the kinematic hardening is shown in an incremental form as follows:

$$\Delta \alpha_{ij} = \frac{2}{3} C n_{ij} \Delta p - \gamma \alpha_{ij} \Delta p$$  \hspace{1cm} (5.4)

where $\Delta p$ is the effective plastic strain increment; $\alpha_{ij}$ is the back stress component of the nonlinear kinematic hardening law; material parameters $C$ and $\gamma$ determine the maximum saturated yield stress ($C/\gamma$) along the kinematic hardening evolution; and $\gamma$ influences the rate of saturation of the yield stress due to kinematic hardening (Yun and Shang, 2011). Also including elastic Young’s modulus ($E$) and yield stress ($\sigma_{y}$), the parameter set to be identified is selected as $x = [E, \sigma_{y}, C, \gamma]$. 

93
5.4.1 Material Tests and Predefined Boundary Loading Data

Two types of test specimens, dog-bone specimens (for homogeneous stress tests) and notched specimens (for heterogeneous stress tests), are used for the purpose of experimentally verifying the Self-OPTIM methodology. Three specimens of each type were cut from the same carbon steel plate (AISI 1095 steel) with 2 mm thickness. The geometry of the specimens and the corresponding finite element models using eight-node plane stress elements are shown in Figure 5.13. Uniaxial tension loading and unloading tests were carried out in Y direction by an Instron material test machine at a quasi-static strain rate $1.5 \times 10^{-5}$. In the homogeneous stress test, three dog-bone specimens were loaded up to a maximum displacement 4.318 mm and unloaded to 4.064 mm. In the heterogeneous stress tests, three notched specimens were loaded up to maximum displacement 2.032 mm and unloaded to 1.778 mm. The displacement and force data were recorded at a sampling frequency of 5 Hz. Data of the full loading path can be divided into three parts: linear elastic, plastic deformation, and linear unloading regions. Due to the measurement noise and errors, the experimental force-displacement data had to be curve-fitted in order to be used in the Self-OPTIM analyses. Approximately 200 load steps of data (pairs of displacement and force data) were selected for application to parallel nonlinear FE simulations. For applying the boundary force, it was assumed that the boundary force is imposed as uniformly distributed along the gripped line. A group of white-colored elements indicated in Figure 5.13 was selected as ZOI from which stresses and strains are extracted during identification processes.
Figure 5.13 Geometry and FE mesh of the test specimen (all dimensions shown are in mm)

It must be noted, however, that data in the plastic regime of the unloading course were not included in the predefined boundary loading because the test data included the geometric softening effect due to buckling under compression in addition to the effect of pure material deformation. According to our tests, inclusions of the unloading hardening curve hindered Self-OPTIM from identifying correct constitutive parameter sets. It could be judged that the identified parameter set was very far from the true parameters by reproduction of experimental force-displacement curve by nonlinear FE simulations with the identified parameters. Therefore, it is important to Self-OPTIM analyses that no geometric softening effect be included in the predefined boundary loading data (i.e. reference data).

5.4.2 Identification Results and Discussions

A total of six Self-OPTIM inverse analyses were conducted to identify the constitutive model parameters: three for dog-bone shape specimens and three for notched specimens. Good convergence was observed in all of the Self-OPTIM inverse analyses. Identification results of the unknown parameter set are summarized in Table 5.4. The
mean value and the standard error (\(=\sigma/\sqrt{n}\) where \(\sigma\) is the standard deviation and \(n\) is the number of samples) of each identified parameter are plotted in Figure 5.14.

Table 5.4 Identification results from Self-OPTIM

<table>
<thead>
<tr>
<th></th>
<th>Dog-bone Specimen</th>
<th>Notched Specimen</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Standard error</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>(E) (GPa)</td>
<td>92.582</td>
<td>95.430</td>
<td>97.450</td>
</tr>
<tr>
<td>(\sigma_y) (MPa)</td>
<td>276.604</td>
<td>295.633</td>
<td>268.254</td>
</tr>
<tr>
<td>(C) (GPa)</td>
<td>6.618</td>
<td>5.528</td>
<td>6.197</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>51.251</td>
<td>45.591</td>
<td>48.801</td>
</tr>
</tbody>
</table>

Figure 5.14 Mean value and standard error of the identified parameters in homogenous and heterogeneous test

It can be observed that the identified parameters have good agreement with each other in both specimen types. The six force-displacement curves were reconstructed by using nonlinear FE simulations in which the model parameters identified by Self-OPTIM were used. When compared with the corresponding curves from experimental tests, all of them were observed to be identical. Comparison results of the first notched specimen are shown in Figure 5.15. Therefore it was demonstrated that the Self-OPTIM is able to identify the parameter set which can represent the experimental data accurately.

To further examine the identification results, two simulations were conducted for cross comparisons in which the mean values of the parameters identified from dog-bone
shape specimens were used to predict the force-displacement response of notched specimens and vice versa. Because the specimens are made of the same material, the force-displacement curves are expected to be identical in an ideal case. However, the discrepancy between the reconstructed curves observed in Figure 5.16 (a) and Figure 5.16 (b) is attributable to the stochastic process associated with materials. In considering the stochastic process in detail, it must first be noted that the material has inherited variability; in reality, every material point has different material properties. Second, the residual stress caused during the manufacturing process could be different between the dog-bone and notched specimens; this could lead to different yield stresses and hardening behavior in the two specimen types. Finally, the modeling error caused by gripping is inevitable in each test; it is impossible to guarantee that the grip position is exactly the same as designed.

Figure 5.15 Comparison of the force-displacement curves reconstructed by using the parameters prior to Self-OPTIM, after Self-OPTIM and experiment data
Figure 5.16 a) Reconstructed force-displacement curve of the dog-bone specimen by the parameter set identified by notched specimens; b) reconstructed force-displacement curve of the notched specimen by the parameter set identified from dog-bone specimens.

Figure 5.17 Plots of normalized axial stresses and strains of a notched specimen from force-driven and displacement-driven simulations using the parameter set prior to Self-OPTIM, at the 5th optimization iteration, and after Self-OPTIM.

In order to show progress of self-calibration by the Self-OPTIM analysis,
normalized axial stresses and strains extracted from force-driven and displacement-driven simulations were plotted at different stages of Self-OPTIM analyses as shown in Figure 5.17. Stresses and strains were normalized by using the maximum values from the corresponding simulation. A notched specimen was used for this verification in the local scale because the notched specimen shows heterogeneous stress states. First, a parameter set prior to Self-OPTIM was used in the simulation. As shown in the first column of Figure 5.17, the stresses and strains resulting from FE simulations subjected to two boundary loading conditions were very different. If those points are gathered along the solid diagonal line, the constitutive parameters are considered to be identified. After five iterations in Self-OPTIM, the two sets of stresses and strains have better agreements. Finally, the two sets of stresses and strains become identical since the parameter set is well identified after the Self-OPTIM analyses.

5.5 Conclusion

The Self-OPTIM has been carried out to inversely estimate cyclic elasto-plastic constitutive model parameters in both numerical and experimental examples. The global forces and displacement on the same partial boundaries and full-field displacement data are used in numerical cases. While, only the global forces and displacement on the same partial boundaries are used in experimental cases. An implicit OF has been defined as a function of “full-field” stresses and strains from two parallel nonlinear finite element simulations.

In the numerical verification, a cyclic elasto-plasticity constitutive model with combined non-linear isotropic and kinematic hardening has been employed. The performance of the Self-OPTIM methodology has been verified in two challenging
parameter estimation problems shown both in a simple FE model consisting of 4 elements subjected to uniaxial tension and compression loading and a complex FE model consisting of 32 FEs subjected to biaxial tension and compression loading. All of the seven parameters in the material model were successfully estimated in the simple case and six out of seven parameters are well identified in the complex case except the Poisson ratio which is deemed little contribution to the global reference data. In addition, the effects of experimental noises from CCD camera and measurement errors of boundary forces are also investigated for the Self-OPTIM method.

In the experimental verification, by using only the global force and displacement boundary loadings that can readily be measured from experimental tests, parameters of the elasto-plastic model, which is on a basis of non-linear kinematic hardening law were successfully identified. Two different geometric types of specimens were used with intentions for verification of generality of the identified parameters. For statistical analysis of test results, three specimens were tested for each geometric type. Parameters identified through the Self-OPTIM were evaluated for reproducing global and local material response. The Self-OPTIM analysis provided consistent parameter sets, and it was proven that the identified parameters could predict in-situ material response of different types of materials. While Young’s modulus and yield stress were very consistent for multiple tests and different geometries of the specimens, two hardening parameters showed relatively large difference between dog-bone shape specimens and notched specimens. It is considered that this difference is a result of the stochastic process associated with materials and the insensitivity of the parameters to stresses and strains.
5.6 Micromechanics and Fracture Mechanics Based Damage Model

The evolution of microcrack is one of the main causes of degradation of effective mechanical properties in damage-tolerant brittle composites (Lawn, 1993). Principal mechanisms are the nucleation, growth, and coalescence of microcracks and these mechanisms are often sequenced into separate stages (Lee and Shin, 2003; Ortiz, 1988). The microcrack nucleation is dominant during early stages of load applications, followed by the growth of microcracks which eventually leads to coalescence of microcracks (Arndt and Nattermann, 2001; Lee and Shin, 2003; Ortiz, 1988). Several damage models have been developed to describe the microcrack evolution (Addessio and Johnson, 1990; Horii et al., 1989; Karihaloo et al., 1991; Lee and Shin, 2003; Ortiz, 1988). In particular, Lee and Shin (Lee and Shin, 2003) developed a damage constitutive model based on micromechanics and fracture mechanics to predict overall response and damage evolution in brittle composites. The damage model proposed by Lee and Shin (Lee and Shin, 2003) has a flexibility to represent a wide range of stress-strain curves with a small number of model parameters. In addition, it is capable of representing the softening effect caused by uncorrelated growth of microcracks. The micromechanics and fracture mechanics based damage model proposed by Lee and Shin (Lee and Shin, 2003) is recapitulated in this section.

For the damage model formulation, it is assumed that evolutionary damage occurs through the nucleation and growth of microcracks, where the two mechanisms occur sequentially and independently (Lee and Shin, 2003). The basic concept of the cumulative effect of microcracks can be qualitatively and quantitatively described by the
number of cracks in the unit volume $N$ and the average microcrack size $\bar{c}$ as (Lee and Shin, 2003)

$$\bar{\sigma} = C(\bar{c}, N) : \bar{\varepsilon}$$

(5.5)

where ":" signifies the tensor contraction; $C$ denotes the overall stiffness tensor; and $\bar{\sigma}$ and $\bar{\varepsilon}$ are the macroscopic stress and strain, respectively. Here, assuming proportional damage, the rate form of constitutive relation and equivalent incremental form can be expressed, respectively, as follows (Lee and Shin, 2003)

$$\dot{\sigma} = \frac{\partial C}{\partial N} \dot{N} : \dot{\varepsilon} + \frac{\partial C}{\partial \varepsilon} \dot{\bar{c}} : \dot{\varepsilon} + C : \dot{\varepsilon}$$

(5.6)

in which $\dot{N}$ and $\dot{\bar{c}}$ are the ratios of microcrack nucleation and growth, respectively.

5.6.1 Microcrack Nucleation

The process of microcrack nucleation is assumed to be stochastic and the microcracks initiate uniformly (Lee and Shin, 2003). Microcrack nucleation model is described in (Seaman et al., 1976) in which the relationship between the ratio of microcrack nucleation and the applied stress is exponential as

$$\dot{N} = \begin{cases} \dot{N}_0 \exp \left[ \frac{P_s - P_{n0}}{P_1} \right], & P_s > P_{n0} \\ 0, & P_s \leq P_{n0} \end{cases}$$

(5.7)

where $\dot{N}$ and $P_1$ denote experimentally determined material parameters; $P_{n0}$ designates the threshold stress for the nucleation of microcracks. In addition, $P_s = \sigma_{kk}/3$ is the tensile pressure in the material. The incremental form of Equation (5.7) for the backward Euler integration scheme can be expressed as

$$N_{n+1} = N_n + \Delta t_{n+1} \dot{N}_0 \exp \left[ \frac{P_s - P_{n0}}{P_1} \right]$$

(5.8)

When $n=0$, $N_0$ denotes the initial number of microcracks in unit volume.
5.6.2 Microcrack Growth

When the crack-driving force exceeds the fracture toughness of material, the balance tilts from nucleation to growth of microcracks (Krajcinovic and Vujosevic, 1998). Microcrack growth model is described in (Addessio and Johnson, 1990) where the single crack instability criteria were introduced in order to consider the energy balance surrounding the microcracks. The rate of microcrack growth is assumed to be a function of overstress, i.e., the distance in the stress space between the current stress and the damage surface as follows

\[ \dot{c} = \beta \dot{c}_{\text{max}} \tanh(S^2 - F_d) \] (5.9)

where \( \dot{c}_{\text{max}} = \sqrt{E^*/\rho^*} \) signifies the shear wave speed; \( E^* \) and \( \rho^* \) are the overall Young’s modulus and the density, respectively; \( \beta \) is the scale factor for crack speed and is assumed to be an independent constant. In addition, \( S \) denotes the deviatoric stress and \( F_d \) is the damage surface. In case of tension, the damage surface \( F_d \) based on energy balance on isolated crack can be found in (Addessio and Johnson, 1990)

\[ F_d(p,q,\bar{c}) = q^2 + \frac{45}{4(5-v)}[(2-v)p_s^2 - \frac{\pi(2-v)}{\bar{c}(1-v)}]G \] (5.10)

where \( q \) is the effective stress, \( \gamma \) is the crack surface energy of the solids. In Equation (5.10), if the stress state locates outside of the damage surface, microcracks grow. For the backward Euler method, Equation (5.9) can be expressed in an incremental form

\[ \bar{c}_{n+1} = \bar{c}_n + \beta \Delta t_{n+1} \dot{c}_{\text{max}} \tanh(S^2 - F_d) \] (5.11)

The effective moduli which is the function of the damage state and the micromechanical framework with the fracture mechanics based evolutionary damage
model was implemented into the ABAQUS user defined material (UMAT) that accounts for the two-dimensional plane-stress state (Lee and Shin, 2003). Details of step by step computational algorithms for the evolutionary damage model can be found in (Lee and Shin, 2003) and (Lee and Simunovic, 2006).

5.7 Validation with Synthetic Data

In this section, the synthetic experiment data is generated by using reference simulation. Two different optimization methods, simplex method and steady state genetic algorithm (SSGA) are used in Self-OPTIM. The preference of these two optimization methods is discussed.

5.7.1 Reference Simulation with Rate-Dependent Damage Constitutive Model

A specimen used for the impact tension test in (Xia and Wang, 1996) is as shown in Figure 5.18 and modeled in Figure 5.19(a). The model is a 3 mm by 2 mm coupon that is one quarter of a 6 mm by 4 mm rectangular plate. Totally 24 plane-stress eight-node elements (CPS8) in ABAQUS/Standard are used for the simulation. By applying the symmetry boundary conditions, the left boundary $Γ_1$ and bottom boundary $Γ_2$ are constrained in the normal direction. Boundary $Γ_3$ is under uniformly distributed loads in $X$ direction. Therefore, every nodal point in the model will have the consistent stress-strain behavior in $X$ direction due to symmetric boundary and loading conditions. The damage model involves seven model parameters to simulate the material behavior subjected to tension impact load. Here, the material properties is adopted according to (Meraghni and Benzeggagh, 1995): density $ρ=1403$ (km/m$^3$), bulk modulus $K=14.02$ (GPa), and shear modulus $μ=6.28$ (GPa). The boundary displacement and force data are generated from a displacement-controlled reference simulation yielding the reference
DPS $x_t=[\bar{c}_0=0.014\text{cm}, N_0=1\times10^5/\text{cm}^3, \gamma=10 \text{ Pa-m}, \beta=1.0\times10^{-5}, \dot{N}_0=1\times10^5/\text{s/cm}^3, P_{n_0}=5 \text{ MPa}, P_1=10 \text{ MPa}]$. The strain rate can be controlled by setting different time intervals $\Delta t$ and number of intervals. Then the strain rate is computed by the relationship $\dot{\varepsilon} = \frac{\Delta \varepsilon}{\Delta t}$.

For example, if the total strain $\varepsilon_{\text{tot}}=0.1$ is caused by $n=200$ time steps deformation with the time increment $\Delta t=0.005 \text{ sec}$, then $\dot{\varepsilon} = \frac{\Delta \varepsilon}{\Delta t} = \frac{(\varepsilon_{\text{tot}}/n)}{\Delta t} = 10 \text{ s}^{-1}$. The reference stress-strain response has a total of 131 steps prior to failure is plotted in Figure 5.19(b).

Figure 5.18 Geometry of the specimen used in tensile impact tests (unite mm)
Figure 5.19 (a) The ABAQUS model of the simulated specimen (b) reference stress-strain response (strain rate 10 s\(^{-1}\)) (c) reference microcrack growth (\(\overline{\varepsilon}\)) and (d) reference microcrack nucleation (N)

Nonlinearity in the stress-strain curve is initiated at a very early stage (\(\varepsilon<0.02\)) due to crack nucleation and subsequent crack growth as evidenced in Figure 5.19(c) and
(d). Since the damage modes are modeled in a sequential way, when microcracks begin to grow the nucleation process is terminated. With accumulations of the damage caused by crack growth and remaining damage caused by crack nucleation, the material goes through softening behavior and eventually fails. Two numerical examples of damage parameter identification are studied in the following sections by applying the same reference global boundary displacement and force data from the reference simulation presented here.

5.7.2 Numerical Example

In order to verify Self-OPTIM, numerical examples are carried out by applying the Simplex method and SSGA. The initial set up of the optimization algorithms and inverse identification results are presented.

5.7.2.1 Damage Parameter Identification by Self-OPTIM Using Simplex Method

In this section, the Nelder-Mead method (or downhill Simplex Search Method) has been adopted. Totally three termination criteria have been applied: 1) maximum iteration number that is 1000; 2) termination tolerance on the objective function value that is $10^{-8}$, and 3) termination tolerance on DPS that is $10^{-8}$. In order to evaluate performance of the simplex method in Self-OPTIM analyses, several numerical examples under different weight factors of the objective function and initial DPS have been studied using the same termination criteria.

According to the numerical studies, parameter estimations are not satisfactory without displacement and force errors in the objective function, that is, when weight factors in the objective function Equation (3.8) were selected as $W_1=W_2=1/2$ and $W_3=W_4=0$. In order to improve the performance of Self-OPTIM with the simplex method,
boundary displacement and force errors were considered by choosing weight factors \( W_1=W_2=1/8 \) and \( W_3=W_4=3/8 \). It was observed that including the displacement and force errors in the objective function significantly enhanced the identification results. Since the optimization problem presented in this dissertation is solved in a seven-dimensional parameter space, it is hard to visualize the objective function surface (OFS). After the optimization iterations, most of damage parameters could converge to reference values as summarized in Table 5.5.

Table 5.5 Self-OPTIM identification results by Simplex method (\( X_T \) indicates reference parameters)

<table>
<thead>
<tr>
<th>( X_i )</th>
<th>( X_T )</th>
<th>Initial Error (%)</th>
<th>Final Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_0 ) (cm)</td>
<td>0.0132</td>
<td>0.0140</td>
<td>7.1</td>
</tr>
<tr>
<td>( N_\theta ) (/cm3)</td>
<td>1.06x10^5</td>
<td>1.00x10^5</td>
<td>10</td>
</tr>
<tr>
<td>( \gamma ) (Pa-m)</td>
<td>9.18</td>
<td>10.00</td>
<td>10</td>
</tr>
<tr>
<td>( \beta ) (/s/cm3)</td>
<td>0.97x10^{-5}</td>
<td>1.00x10^{-5}</td>
<td>10</td>
</tr>
<tr>
<td>( N_\theta ) (/s/cm3)</td>
<td>0.91x10^5</td>
<td>1.00x10^5</td>
<td>10</td>
</tr>
<tr>
<td>( P_{m0} ) (MPa)</td>
<td>6.60</td>
<td>5.00</td>
<td>10</td>
</tr>
<tr>
<td>( P_l ) (MPa)</td>
<td>9.34</td>
<td>10.00</td>
<td>10</td>
</tr>
</tbody>
</table>

Forward simulation results are plotted in Figure 5.20 by using the identified DPS from current example and the reference DPS. The identified stress-strain response was very close to the reference curve except in the region of softening behavior.

According to this example, the optimized DPS can reconstruct the material behavior of the macro stress-strain response and microcrack growth well. However, it failed to reproduce modeling features of the microcrack nucleation because parameters associated with the microcrack nucleation has minimal influence both on the macro stress-strain response and global force-displacement response. It is also partly because of possible limitations of the selected optimization algorithm. Therefore, in the following
section, another global optimization method is employed for damage parameter identification with Self-OPTIM analyses.
Figure 5.20 Comparisons of (a) stress-strain curve, (b) mean crack radius growth and (c) nucleation between simulation using the Self-OPTIM identified DPS and reference simulation.

5.7.2.2 Damage Parameter Identification by Self-OPTIM Using Steady State Genetic Algorithm

Steady-State Genetic Algorithm is employed in this example for the purpose of improving the performance of the Self-OPTIM damage parameter identification. The preset upper and lower bounds of the damage model parameter are set to have sufficiently large parameter space that includes the true DPS. The algorithm is terminated when the 6,000th generation is reached. In each generation of the SSGA, 90% crossover and 30% mutation probabilities are applied. The identification results are summarized in Table 5.6. Table 5.6 shows that all the parameters except the nucleation rate were accurately identified. The reason is that when the damage model includes two damage modes together, the influence from micorack nucleation on the stress-strain behavior becomes
minor as demonstrated in (Lee et al., 2004). Furthermore, insensitivity of the nucleation rate is partly because the number of microcracks increases only in a small strain region (0<ε<0.014) after microcrack starts to growth and maintains a constant crack density until the moment of failure Figure 5.19(d).

Table 5.6 Identification results by using SSGA

<table>
<thead>
<tr>
<th></th>
<th>$\bar{C}_o$(cm)</th>
<th>$N_o$/cm$^3$</th>
<th>$\gamma$(Pa-m)</th>
<th>$\beta$</th>
<th>$\dot{N}_o$/(s/cm$^3$)</th>
<th>$P_{\alpha}$(MPa)</th>
<th>$P_f$(MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_U$</td>
<td>0.0180</td>
<td>1.30x10$^5$</td>
<td>12.00</td>
<td>1.40x10$^{-5}$</td>
<td>1.30x10$^5$</td>
<td>7.00</td>
<td>13.00</td>
</tr>
<tr>
<td>$X_L$</td>
<td>0.0100</td>
<td>0.60x10$^5$</td>
<td>8.00</td>
<td>0.60x10$^{-5}$</td>
<td>0.7x10$^5$</td>
<td>4.00</td>
<td>8.00</td>
</tr>
<tr>
<td>$X_I$</td>
<td>0.0143</td>
<td>1.04x10$^5$</td>
<td>9.50</td>
<td>0.97x10$^{-5}$</td>
<td>0.85x10$^5$</td>
<td>4.57</td>
<td>9.71</td>
</tr>
<tr>
<td>$X_T$</td>
<td>0.0140</td>
<td>1.00x10$^5$</td>
<td>10.00</td>
<td>1.00x10$^{-5}$</td>
<td>1.00x10$^5$</td>
<td>5.00</td>
<td>10.00</td>
</tr>
</tbody>
</table>

Initial Error

<table>
<thead>
<tr>
<th></th>
<th>(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_U$</td>
<td>28.6</td>
</tr>
<tr>
<td>$X_L$</td>
<td>-28.6</td>
</tr>
<tr>
<td>$X_I$</td>
<td>4.0</td>
</tr>
<tr>
<td>$X_T$</td>
<td>2.1</td>
</tr>
</tbody>
</table>

Final Error

<table>
<thead>
<tr>
<th></th>
<th>(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_U$</td>
<td>-40.0</td>
</tr>
<tr>
<td>$X_L$</td>
<td>-20.0</td>
</tr>
<tr>
<td>$X_I$</td>
<td>5.3</td>
</tr>
<tr>
<td>$X_T$</td>
<td>3.0</td>
</tr>
</tbody>
</table>

The identified response and the damage behavior are compared with results from forward simulations using the upper bound DPS $X_U$ and the upper bound DPS $X_L$ as shown in Figure 5.21. Obviously, the stress-strain response matches well with the reference response. Both of the initial state of the microcrack growth and damaging behavior could be identified within the selected damage constitutive modeling framework.
(a) Stress (GPa) vs. Strain

(b) Mean Crack Radius (cm) vs. Strain

- Lower bound DPS
- Upper bound DPS
- Identified DPS
- Reference DPS
Figure 5.21 Comparisons of (a) stress-strain response, (b) mean crack radius growth and (c) nucleation between simulations using the upper bound DPS, lower bound DPS, identified DPS, and reference DPS. (d) degradation of the material stiffness.

5.7.3 Discussions

The selected damage constitutive model provides two nonlinear damage modes in the micro scale, such as microcrack growth and crack nucleation. Since such micro
damaging mechanisms naturally link to the degradation of stiffness that dominates the material behavior, it is feasible to apply the macro stress-strain based Self-OPTIM method to the damage parameter identification. It is notable that successful parameter estimation by the Self-OPTIM analysis brings significant impacts to damage constitutive modeling. Identification of two physical parameters 1) the initial mean crack radius $\bar{c}_0$ and 2) the initial number of cracks in unit volume $N_0$ means that Self-OPTIM analyses can provide information on initial material states in terms of physical damage. Moreover, the Self-OPTIM approach can identify the progressive damage characteristics in micro-scale by applying the global boundary displacement and force data that can be easily acquired from laboratory tests.

5.8 Damage Parameter Identification Using Actual Experiment Data

In order to verify the proposed damage parameter identification method, experimental data from existing impact tension tests of glass fiber reinforced epoxy composites presented in (Xia and Wang, 1996) were used. Experimental test data at two different strain rates: 300 (/sec) and 490 (/sec) were selected. The specimen used in (Xia and Wang, 1996) is simulated by using the same FE model under the same boundary condition shown in Figure 5.19. Since the tested specimen has a relatively small size (6mm by 4mm), it can be assumed that the distribution of stress/strain within the tested specimen is uniform.

By a careful study of the damage model, it was found that no microcrack nucleation will happen above strain rate 300 (/sec). The density which has influence on microcrack growth (Equation (5.9)) is included as one of the parameters to be identified since it is not provided in (Xia and Wang, 1996). Therefore, five parameters are selected
as the components of the $DPS_{300} = [\rho, \overline{c}_0, N_0, \gamma, \beta]$, respectively. Boundary displacement and force data along 88 times steps are reproduced from the stress/strain curve (Xia and Wang, 1996) at the strain rate 300 (/sec) and used as the reference input data to the Self-OPTIM. SSGA was used in the Self-OPTIM analyses. A maximum iteration number (6000) was used as a termination criterion. Identification results are summarized in Table 5.7. Three direct forward simulations using the upper and lower bounds of DPS, and the identified DPS were carried out for comparisons. Stress/strain curves are plotted in Figure 5.22. Simulation results obtained by using the identified $DPS_{300}$ have a good agreement with the measured experimental data. The identified DPS is also used to reconstruct the microcrack growth in Figure 5.23.

Table 5.7 Identification results at strain rate 300 (/sec)

<table>
<thead>
<tr>
<th></th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$\overline{c}_0$ (cm)</th>
<th>$N_0$ (/cm$^3$)</th>
<th>$\gamma$ (Pa-m)</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_U$</td>
<td>2200.00</td>
<td>0.01200</td>
<td>1.70x10$^8$</td>
<td>11.00</td>
<td>8.00x10$^{-5}$</td>
</tr>
<tr>
<td>$X_L$</td>
<td>1600.00</td>
<td>0.00600</td>
<td>1.30x10$^8$</td>
<td>7.00</td>
<td>4.00x10$^{-5}$</td>
</tr>
<tr>
<td>$DPS_{300}$</td>
<td>1832.64</td>
<td>0.00984</td>
<td>1.35x10$^8$</td>
<td>7.60</td>
<td>6.79x10$^{-5}$</td>
</tr>
</tbody>
</table>
Figure 5.22 Comparisons of the stress/strain curves of three simulations with the experiment data of strain rate 300 (/sec)

Figure 5.23 Identified mean crack radius growth of strain rate 300 (/sec)
The displacement and force data were reproduced from the experimental stress and strain curve at strain rate 490 (/sec) and entered to Self-OPTIM. Since the strain rate will have no influence of the density of the material, ρ=1832.64 kg/m³ identified at strain rate 300 (/sec) is adopted in this case. Therefore, there are four parameters in the $DPS_{490} = [c_0, N_0, \gamma, \beta]$. The identification results are summarized in Table 5.8. Reconstructed stress/strain curve has a good agreement with the experiment data as shown in Figure 5.24.

Table 5.8 Identification results at strain rate 490 (/sec)

<table>
<thead>
<tr>
<th></th>
<th>$\rho$ (kg/m³)</th>
<th>$C_0$ (cm)</th>
<th>$N_0$ (/cm³)</th>
<th>$\gamma$ (Pa-m)</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_U$</td>
<td>N/A</td>
<td>0.01200</td>
<td>1.70x10⁸</td>
<td>11.00</td>
<td>8.00x10⁻⁵</td>
</tr>
<tr>
<td>$X_L$</td>
<td>N/A</td>
<td>0.00600</td>
<td>1.30x10⁸</td>
<td>7.00</td>
<td>4.00x10⁻⁵</td>
</tr>
<tr>
<td>$DPS_{490}$</td>
<td>1832.64</td>
<td>0.01101</td>
<td>1.42 x10⁸</td>
<td>7.21</td>
<td>7.30x10⁻⁵</td>
</tr>
</tbody>
</table>

Figure 5.24 Comparison of the stress/strain curve of simulation result by using identified $DPS_{490}$ and the experiment data of strain rate 490 (/sec)
5.9 Conclusion

The damage parameters which are defined in microscale of a micromechanics and fracture mechanics based damage model for brittle damage-tolerant composites are identified by using the newly formulated model-independent parameter estimation framework Self-OPTIM. The damage model includes two rate-dependent damage mechanisms, microcrack nucleation and growth, respectively. Numerical examples are studied to show the capacity of the proposed algorithm that the seven microscopic damage parameters that govern the two damage behaviors are identified simultaneously. The implicit-explicit objective function that contains two types of the weighted least squares formulations: 1) the accumulated error of the global displacements and forces between the one from experiment and the one simulated from FEMA and FEMB on the boundary nodes in every time step; 2) the accumulated error between the full-field stresses/strains which are simulated from FEMA and FEMB on every Gaussian point in every time step is applied within Self-OPTIM. Two optimization algorithms, simplex method and SSGA, are implemented into the damage identification framework. Performances of these two algorithms are evaluated in the numerical examples and their advantages and disadvantages are remarked in the discussion. Furthermore, the proposed framework is applied to identify the damage parameters of the unidirectional composites tested in high strain rate impact tension experiments in the reference by considering the microcrack growth and initial number of microcracks in the unit volume. Two sets of damage parameters are estimated by applying the reproduced experiment data that are obtained at different strain rates. Compared with experiment data, the identified $DPS_{300}$ is able to predict the material damage behavior well at strain rate 300 (\$/sec). The
microcrack growth is reproduced by using the identified damage parameters. Again, the damage identification algorithm is carried out again based on the experiment data obtained at strain rate 490 (sec). By using the identified $DPS_{490}$, the predicted stress-strain behavior has a good agreement with the experimental one.
CHAPTER VI
NUMERICAL EXAMPLES OF APPLYING THE GENERAL STOCHASTIC FINITE ELEMENT SIMULATION PLATEFORM FOR RELIABILITY ANALYSIS

6.1 A Square Plate with Randomly Distributed Young’s Modulus

A physical model of a thin square plate with one meter length shown in Figure 6.1 is selected as one of the numerical examples. Boundary edges indicated by the surfaces $\Gamma_1$ and $\Gamma_2$ are restrained in the direction normal to the edge. The edge $\Gamma_3$ is subjected to a uniformly distributed loading of magnitude 1800 kN. Random field domain $\Omega$ is discretized into nine Q9 Lagrange elements ($N_{RF}=9$), thus the RF mesh has a total of $ndof_{RF}=49$ DOFs. Physical domain $D$ has a different mesh that consists of 225 Q8 stochastic elements ($N_{FE}=225$) with a total of $ndof_{FE}=736$ DOFs.

Figure 6.1 225 Q8 stochastic finite element mesh overlapped with nine Q9 random field elements of a square plate
In order to have non-negative elastic modulus values in realizations, the lognormal distribution was assumed. Mean value ($\bar{E}_L$) and standard deviation ($\sigma_L$) of Young’s modulus that follows lognormal distribution are first assumed. Using Equation (6.1), statistical parameters in lognormal distribution are converted to those in the normal distribution. Because the KL expansion assumes that the random field follows Gaussian distribution, variance ($\sigma^2_N$) and mean values ($\bar{E}_N$) in normal distribution are used as in Equation (4.1) and Equation (4.3) for generating multiple realizations of spatially varying Young’s modulus through KL expansion. Realizations of the random field in normal distribution are converted to realizations in lognormal distribution by Equation (6.2). The transformation technique was used in Monte Carlo Simulation.

$$\sigma_N = \sqrt{\ln \left( \left( \frac{\sigma_L}{\bar{E}_L} \right)^2 + 1 \right)}; \quad \bar{E}_N = \ln \bar{E}_L - \frac{1}{2} \sigma_N^2$$

(6.1)

$$\hat{E}_L(\mathbf{x}, \theta) = \exp \left[ \bar{E}_N(\mathbf{x}, \theta) + \sum_{i=1}^{N} \sqrt{\lambda_i} \varphi_i(\mathbf{x}) \xi_i(\theta) \right]$$

(6.2)

Although such a mapping guarantees the proper probability distribution, it distorts the correlation structure. The methods for correcting this problem can be found in (Deodatis and Micaletti, 2001). As mentioned in (Phoon, 2008; Popescu, 2004), if the coefficient of the variance (CV) is small, the distortion of the correlation structure caused by the nonlinear transformation will be small. To evaluate the change of the correlation structure, an example is presented. The Equation (4.3) is used as the covariance function. $\bar{E}_L = 30$ GPa and the standard deviation $\sigma_L = 3$ GPa. Four pair of arbitrary locations has
been selected as shown in Figure 6.2. The two-point correlation of the random field is calculated before and after the exponential transformation based on the MCS with 5000 samples and shown in Table 6.1. Figure 6.3 shows the normalized Young’s modulus sampled from nodal point 36 versus the one from nodal point 93 before and after the transformation. It can be concluded that, if the CV is small, the nonlinear transformation used in Equation (6.2) will change the correlation structure very slightly. Therefore, the largest CV used is equal to 0.1 and no correction method is used in this paper.

![Diagram showing FE nodal points](image)

**Figure 6.2** The locations of the FE nodal points used to evaluate the two-point correlation (the number is the label of the nodes)

**Table 6.1** The two-point correlation before and after the nonlinear transformation

<table>
<thead>
<tr>
<th></th>
<th>(21, 338)</th>
<th>(36, 93)</th>
<th>(49, 73)</th>
<th>(31, 41)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_N$</td>
<td>0.3375</td>
<td>0.5937</td>
<td>0.8200</td>
<td>0.9094</td>
</tr>
<tr>
<td>$\rho_L$</td>
<td>0.3354</td>
<td>0.5930</td>
<td>0.8206</td>
<td>0.9092</td>
</tr>
<tr>
<td>$100\ast(\rho_L-\rho_N)/\rho_L$</td>
<td>0.6261</td>
<td>0.1180</td>
<td>0.0731</td>
<td>0.0220</td>
</tr>
</tbody>
</table>
6.1.1 Results of Static Stochastic Finite Element Analysis

In this example, materials are assumed to be high-strength concrete which has a lognormal mean value of elastic modulus $\bar{E}_L = 30$ GPa (4351.13 ksi). The Poisson’s ratio is set to be $\mu = 0.15$ as a deterministic parameter. The exponential CF in Equation (4.3) is adopted in this example. Four combinations of different correlation lengths and standard deviations of the random field were assumed: 1) $L_{c1}=1.0$ m, $\sigma_{L1}=1.0$ GPa; 2) $L_{c2}=1.0$ m, $\sigma_{L2}=0.1$ GPa; 3) $L_{c3}=0.5$ m, $\sigma_{L3}=1.0$ GPa; 4) $L_{c4}=0.5$ m, $\sigma_{L4}=0.1$ GPa.

Figure 6.4 shows realizations of spatially varying Young’s modulus for each of four different statistical distributions. One important observation in the discretization of the random field by KL expansion is the effect of correlation length ($L_c$) on the distribution. As the correlation length decreases, the level of fluctuation of spatial variation apparently increases based on comparisons between Figure 6.4 (a)-(b) and Figure 6.4 (c)-(d). Comparing Figure 6.4 (a)-(c) and Figure 6.4 (b)-(d), variance effects could also be observed. Young’s modulus ranges from 30.82 GPa to 29.39 GPa in the
case of $\sigma_L=1.0$ GPa (Figure 6.4 (a)-(c)) whereas it ranges from 30.02 GPa to 29.92 GPa in the case of $\sigma_L=0.1$ GPa (Figure 6.4 (b)-(d)) as expected.

Figure 6.4 Randomly distributed elastic modulus for four different cases: a) $L_{c1}=1$ m, $\sigma_{L1}=1$ GPa; b) $L_{c2}=1$ m, $\sigma_{L2}=0.1$ GPa; c) $L_{c3}=0.5$ m, $\sigma_{L3}=1$ GPa; d) $L_{c4}=0.5$ m, $\sigma_{L4}=0.1$ GPa

For any physical domain that is discretized into $n_{dofRF}$, representations of the spatially varying Young’s modulus by using the different number of terms in the truncated KL expansion are shown in Figure 6.5. As more terms are included in the truncated KL expansion, discretization errors in the approximate realization of random
distribution tend to decrease as usual in Galerkin-based finite element approaches. Comparing random distribution of Young’s modulus with the different number of terms, it can be observed that by using more terms in computation, the approximated random field becomes more detailed.

Figure 6.5 Representations of the approximated RF by using different number of KL expansion terms (M): a) $M=5$; b) $M=20$; c) $M=35$; d) $M=49$ ($L_c=1$, $\sigma_L=1$ GPa)

Stochastic finite element analysis results from one realization with $L_c=0.5$ m and $\sigma_L=1.0$ GPa are shown in Figure 6.6. Results of the stochastic finite element analysis were verified through equilibrium and compatibility conditions. Although the structure is
under uniformly distributed loading, it does not show constant stress states due to material uncertainties. Figure 6.6 (a)-(d) indicate such non-uniform stress states.
Monte Carlo Simulation for Analysis of Statistical Material Response

In order to demonstrate the capability of SFE for simulating the stochastic material response, MCS is performed. In this example, 1000 samples are generated by using LHS and assuming Gaussian distribution. Equation (4.3) is used as the covariance function with $\sigma_L=1$ GPa and $L_c=1$ m. The PDFs and CDFs of the material response: a) displacement ($U_x$); b) strain ($\varepsilon_{xx}$); c) stress ($\sigma_{xx}$) at Point A are plotted in Figure 6.7. $\mu_{SR}$ and $\sigma_{SR}$ in lognormal distribution are calculated from the simulated responses (SR) under the realized material uncertainty. It can be seen that the computed analytical PDFs are reasonably comparable with the simulated PDFs. The stresses and strains at Point A in $X$ direction are plotted in Figure 6.8. Due to the spatially varying material uncertainty, stresses and strains appear to be uncorrelated in Figure 6.8. However, as shown in Figure 6.9, two displacement components $U_x$ and $U_y$ are strongly correlated due to displacement compatibility conditions. The possible locations of Point A after deformation are plotted in Figure 6.9.
Figure 6.7 Stochastic responses to material uncertainty a) displacement with $\mu_{SR}=1.201\text{mm}$, $\sigma_{SR}=0.0283$ mm; b) strain with $\mu_{SR}=1.201\times10^{-3}$, $\sigma_{SR}=2.68\times10^{-5}$; and c) stress with $\mu_{SR}=36 \text{MPa}$, $\sigma_{SR}=3.64\times10^{-3}$ MPa in x direction at Point A by using $\sigma_L=1$ GPa and $L_c=1$ m
Figure 6.8 Probabilistic distribution of stresses and strains at the final load step and at Point A

Figure 6.9 Probabilistic distribution of the position of Point A at the final loading step
6.2 A Quarter Circular Tube with Randomly Distributed Young’s Modulus

In order to show the generality of the proposed SFE, a quarter-circular tube model was used in this example. Figure 6.10 shows an illustration of FE mesh intersected with RF mesh for the quarter circular tube that has inner radius of two meters and outer radius of three meters. Displacements along the edge $\Gamma_1$ and $\Gamma_2$ are restrained in their normal directions. An external concentrated load of magnitude 100 N is applied at Point B in horizontal direction. The RF domain $\Omega$ is discretized into $N_{RF}=10$ RF elements, resulting in a total of $n_{dof_{RF}}=55$ DOFs. The physical domain $D$ is meshed into $N_{FE}=210$ SFEs, resulting in a total of $n_{dof_{FE}}=705$ DOFs. The exponential CF in Equation (4.3) is used for the second-order statistical feature of the random field. Lognormal distribution described in Section 4.1 is used for generating multiple realizations of Young’s modulus distribution. Young’s modulus of the material are assumed to have $E_L = 0.26$ MPa (lognormal mean), $\sigma_L=0.026$ MPa (standard deviation), $\mu = 0.3$ (Poisson ratio), and $L_c=1$ m. Young’s modulus distribution from one realization is shown in Figure 6.11. 1000 samples are generated for MCS to obtain the stochastic material response. The stochastic displacement in X direction at Point A is plotted in Figure 6.12.
Figure 6.10 Finite element mesh intersected with random field mesh for the quarter circular tube model

Figure 6.11 The lognormal Young’s modulus spatial distribution from one realization
133

Figure 6.12 Stochastic displacement response from simulation at Point A in X direction with \( \mu_{SR} = -0.0715 \) m, \( \sigma_{SR} = 0.006 \) m by using \( \sigma_L = 0.026 \) MPa and \( L_c = 1 \) m

6.3 A Retaining Wall with Randomly Distributed Young’s Modulus

To demonstrate the potential of the proposed approach in dealing with large scale complex engineering problems, a retaining wall is used as a more realistic example which is shown in Figure 6.13. The geometry, boundary condition, RF mesh, and FE mesh are presented.

Figure 6.13 The problem definition of a retaining wall with randomly distributed Young’s Modulus
The RF domain $\Omega$ is discretized into $N_{RF}=184$ RF elements, resulting in a total of $ndof_{RF}=811$ DOFs. The physical domain D is meshed into $N_{FE}=2962$ SFEs, resulting in a total of $ndof_{FE}=9179$ DOFs. As shown in Figure 6.13, both of the RF and FE mesh use the free mesh control. Displacements along edge $\Gamma_1$ is restrained in X and Y direction. The lateral earth pressure is applied on edge $\Gamma_2$ with the magnitude of 60MPa. The same exponential CF used in section 6.2 is still applied here. Young’s modulus of the material are assumed to have $E_L=30$ GPa (lognormal mean), $\sigma_L=3$ GPa (standard deviation), $L_c=2$ m and $\mu = 0.15$ (Poisson ratio). Figure 6.14 shows the deformation of the right edge predicted form MCS with 2000 samples. The red dash line presents the deformation predicted by considering the retaining wall as homogeneous and the Young’s modulus on every material point has the same value as the mean.

![Figure 6.14 Deformation of the right edge AB predicted from Monte Carlo simulation with 2000 samples (the deformation is magnified by 6 times)](image)
6.4 A Dog-Bone Specimen with Random Distributed Young’s Modulus, Poisson’s Ratio, and Yield Strength

The elastic–plastic modeling and simulations have been studied extensively in the last century. However, the uncertainties in material properties, which probably have the largest influence on many aspects of structural and solids behavior, has received very little attention. Despite its importance, effects of uncertainties of material properties on overall response of structures and solids have rarely been studied. To demonstrate the proposed methodology is able to easily cooperate with any working deterministic code, the Associative J2 plasticity simulation model with nonlinear Isotropic/Kinematic combined hardening rules explained in section 5.1 is implemented into the SFE simulation platform to access the influence of the spatial variability of material properties on both linear and nonlinear material behaviors. In this example, the Young’s modulus, Poisson’s ratio, and initial yield stress are modeled as independent random fields, $E(x, \theta)$, $\mu(x, \theta)$, and $S_{y0}(x, \theta)$, respectively. The covariance function in Equation (4.3) is applied for all of the random fields but with different stochastic parameters (mean value, standard deviation, and correlation length), which are listed in Table 6.2. The deterministic hardening parameters are set as, $b=0$, $Q=0$, $H=17095$ MPa, and $\gamma=-216$

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Correlation length</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(x, \theta)$</td>
<td>131.128 GPa</td>
<td>2.623 GPa</td>
<td>0.05</td>
</tr>
<tr>
<td>$\mu(x, \theta)$</td>
<td>0.37</td>
<td>0.0074</td>
<td>0.05</td>
</tr>
<tr>
<td>$S_{y0}(x, \theta)$</td>
<td>870 GPa</td>
<td>43.5 GPa</td>
<td>0.05</td>
</tr>
</tbody>
</table>

The dog-bone-shaped specimen designed by ASTM E646–07 is shown in Figure 6.15. The thickness of the specimen is 4 mm. Since the material is heterogeneous, the
symmetric condition cannot be applied for the simulation. Therefore the entire region of the specimen within the grip line is modeled as shown in Figure 6.15. The RF mesh and FE mesh are identical in this example, which have totally 88 elements resulting in \( ndof_{FE} = 317 \) DOFs. The MCS is carried out with 2000 samples for displacement controlled uniaxial monotonic tension test. The maximum amplitude of displacement on the moving end is 3.75 mm. The distributions of the random fields of one sample are presented in Figure 6.16. To compare with the homogeneous material, the axial strain distributions is compared in Figure 6.17 based on both the homogeneous material, which uses the mean value of the three random fields and the heterogeneous material, which uses the distributions of the material properties shown in Figure 6.16. The strain distribution of the homogenous material is uniform within the gage length. However, in the heterogeneous material case, the strain is concentrated at the position where the yield stress is low. In this example, the overall material response is studied instead of the local responses studied in section 7.1 to 7.3. Therefore, the effective stress/strain curves of 2000 tested samples are recorded. The effective stress is calculated as \( \sigma_e = F/A \), where \( F \) is the total load and \( A \) is the initial area of the cross-section of the specimen. The effective strain is calculated as \( \varepsilon_e = (l-L)/L \), where \( L \) is the initial gage length and \( l \) is the gage length after deformation.
Figure 6.15 Geometry and FE model of the dog-bone specimen (unit mm)

Figure 6.16 Random field distribution of the $E$, $\mu$, and $S_{y0}$ obtained from one sample
Figure 6.17 Axial strain distribution with homogeneous and heterogeneous material

Based on the simulation result, which is presented in Figure 6.18, one can conclude that by considering three uncertainties sources (Young’s modulus, Poisson’s ratio, and initial yield stress) the nonlinear material behavior can be very different among all the samples. Since the standard deviation of Young’s modulus is small, the material behaviors within linear elastic region are similar. However, with a relatively large standard deviation of yield stress combined with the variation of Young’s modulus, the material behaviors in plastic deformation region are very different. The resulting differences between the responses in terms of effective stress and effective strain at the last load step are significant. Therefore, mechanical effects induced by material heterogeneity are more pronounced for phenomena governed by highly nonlinear laws.
Figure 6.18 Stress/strain curves predicted from Monte Carlo simulation with 2000 samples

6.5 Conclusion

In the numerical demonstrations, the material properties of two-dimensional structures are modeled as spatially varying random fields. Numerical examples include square plate, quarter circular tube, retaining wall, and dog-bone-shaped test specimen were presented to show the flexibility and fidelity of the SFE analysis in both linear and nonlinear problem. MCS showed that material uncertainties resulted in reasonable probabilistic distributions of the structural response. Although problems presented in this dissertation are defined in isotropic linear elasticity and elasto-plasticity, the potential application of this SFE framework is not limited to this. It is general enough to be used with any constitutive models such as damage, thermodynamics, and fatigue with uncertain material parameters. Different constitutive models will only yield different implementations in the UEL. Furthermore, it provides a promising model to be used in
stochastic inverse analysis which is capable of identifying and characterizing the uncertainty in material property on a basis of experiment test.
CHAPTER VII

STOCHASTIC CHARACTERIZATION OF MATERIAL UNCERTAINTIES FOR
PROBABILISTIC LOW-CYCLE FATIGUE MODELS

7.1 Stochastic Material Characterization

As mentioned in Section 2.2, all the material in nature is heterogeneous with randomness in material properties. In this regard, there is an inevitable need of a thorough characterization of the RHM to achieve reliable structural designs. In general, realizations could be obtained from direct measurements of the field or indirectly from measuring the response of the physical system and solutions of a set of inverse problems. For the direct measurement approach, one can refer to the work done by (Sriramula and Chryssanthopoulos, 2009). Many coupon specimens were cut out of a test panel at different locations and tested in the compression test. Afterwards, the spatial variation of the compression strength is obtained and visualized base on the test data. However, this direct measurement approach could be very time consuming. Moreover, accuracies of the realization depend on the cutting strategy and results are limited to sample-by-sample variations. The latter inverse approach, whereby direct observations of the material property are assumed to be unavailable, is pursued in this thesis. A realistic inverse material characterization should consider the variability of material properties presented at the macroscopic level.
Figure 7.1 shows the schematic of the stochastic material characterization method we proposed. The SFEM is integrated into Self-OPTIM to create a novel data-drive stochastic inverse identification algorithm, called Stochastic Self-OPTIM, which is based on limited experimental information to characterize material uncertainties. Within the stochastic finite element setting, the quantities to be identified contain the macro-scale Young’s modulus, Poisson’s ratio, strengths, etc. that are mathematically described as random fields. Owing to the generality of the proposed methodology, any other random field could be included for stochastic characterization. The test data (displacements and forces) from multiple samples have to be obtained from experiments, for example, uniaxial, biaxial testing, etc. Afterwards, the parameter set, $X=[\bar{E}, \sigma, L_c, \{\xi(\theta)\}]$, which is comprised of the mean value, the standard deviation, the correlation length, and a set of zero mean-unit variance Gaussian random variables, will be identified by carrying out stochastic Self-OPTIM for each specimen by using the corresponding test data. Afterwards, the random field can be reconstructed by using the identified parameter set.
The identified parameter set $X_i (i=1,\ldots,n)$ from multiple specimens will be averaged to determine sample means of the spatial mean, the variance, and the correlation length of the heterogeneous material properties can be reconstructed. It is worth noting that the epistemic uncertainty can be reduced from these multiple stochastic characterizations. Therefore, infinite Gaussian realizations of the random fields can be generated by using MCS with the averaged parameter set $\bar{X}$. The stochastic material data base is constructed by these Gaussian realizations of the random fields. This data base is very useful for probabilistic modeling for the purpose of reliability design such as building up a probabilistic low-cycle fatigue life prediction model which is demonstrated in this chapter. MCS of cyclic test on dog-bone specimens with the material properties sampled from the stochastic material database will be carried out to obtain the
probabilistic distribution of the strength coefficient $K'(\theta)$, strain-hardening exponent $n'(\theta)$, and effective Young’s modulus $E_e(\theta)$ which are random variables. PCE is used to build up the probabilistic surrogate model for these random variables. Based on an empirical model, a low-cycle probabilistic fatigue life prediction model will be developed.

The parameter set $X=[\bar{E}, \sigma, L_c, \{\xi_i(\theta)\}]$ will be in a very high dimensional space if a fine RF mesh is applied. For example, a RF mesh with nine nine-node elements will results in $M=49$, which means 49 random variables in $\{\xi_i(\theta)\}$ have to be identified. The dimension of the parameter space can be even higher, if the problem defines multiple random fields to be reproduced at the same time. Although the performance of CFA method is excellent in global optimization, the identification problem in such a high dimensional space could be very challenging and need more individuals and generations in order to converge to the global minimum. However, the parameter space can be reduced by truncating the KL expansion. The first $N$ terms can be selected from $\{\xi_i(\theta)\}$, and the rest of them will be neglected because the eigenvalues calculated from KL expansion of the strong correlated random field will decay very fast (as shown in Figure 4.2). It implies that the effect of the eigenvalue and the eigenfunction in the higher order terms can be neglected. It is obvious that it’s a trade off with computational efficiency and truncation error. And it is only a matter of choice that if the problem itself is simple (simple constitutive relationship, less number of elements, simple loading, only one random field, and et al) more terms can be included to increase the identification accuracy.
7.2 Stochastic Self-OPTIM

It is well known that the formulation of the objective function has significant influence on the inverse identification performance. In general, a least square form with weight factors, which is shown in Equation (3.8), has been widely used. However, for characterizing the RHM, this least square form doesn’t work well. The reason is that unlike the homogeneous material, in which the material properties are identical at every material point, the distribution of the heterogamous material properties spatially varies. As a result, the local material responses such as the stress and strain also spatially vary. In other words, the stresses and strains turn into random field quantities. The least square formulation is only able to account for the summation of differences of the randomly varying stress and strain fields between two parallel finite element simulations. It does not reflect the difference of statistical random fields in terms of statistical measures such as the level of fluctuations (i.e. correlation) and the level of amplitude (i.e. mean values). In this regard, another type of the implicit objective function has to be carefully formulated. The implicit objective function used in stochastic Self-OPTIM is proposed referring to (Gandomi et al., 2011b). The formulation is shown in Equation (7.1)

\[
OF_{\text{implicit}} = \sum_{i=1}^{LS} \left( \frac{RMSE(S_A, S_B) + MAE(S_A, S_B)}{R(S_A, S_B) + 1} + \frac{RMSE(E_A, E_B) + MAE(E_A, E_B)}{R(E_A, E_B) + 1} \right) \quad (7.1)
\]

where LS is the number of load steps; \( RMSE \), \( MAE \), and \( R \) are the root mean squared error, the mean absolute error and the correlation coefficient between the simulated field quantities such as \( S_A/S_B \) or \( E_A/E_B \), respectively; \( (E_A, S_A) \) and \( (E_B, S_B) \) are two sets of the strains and stresses at Gaussian points simulated from force-driven FEA and

\[
145
\]
displacement-driven FEA, respectively. To minimize the implicit objective function, \(RMSE\) and \(MAE\) that account for the difference between the field quantities in an average sense will be reduced. Furthermore, \(R\) that accounts for the difference of the level of fluctuation between two field quantities will be increased. In conclusion, compare to the formulation in Equation (3.8), the implicit objective function in Equation (7.1) is much more suitable evaluating Self-OPTIM based inverse analyses. In order to avoid the scale difference between the different stresses and strains, the normalization has to be carried out. There are many choices of normalizations and we select the one presented below

\[
S_{ij\_norm} = \frac{S_{ij} - S_{ij\_min}}{S_{ij\_max} - S_{ij\_min}}, \quad E_{ij\_norm} = \frac{E_{ij} - E_{ij\_min}}{E_{ij\_max} - E_{ij\_min}}
\]

(7.2)

where the \((S_{ij\_min}, S_{ij\_max})\) and \((E_{ij\_min}, E_{ij\_max})\) are defined as the maximum and minimum values of stresses and strains from the initial generation by CFA.

As explained previously, the full-field displacement enriches the information of the material that will improve the performance of the inverse identification. The Stochastic Self-OPTIM can also take this advantage. The full-field displacement (DIC displacement) obtained by DIC technique can be added into Equation (7.1)

\[
OF_{implicit} = \sum_{n=1}^{LS} \left( \frac{RMSE(S_A, S_B) + MAE(S_A, S_B)}{R(S_A, S_B) + 1} + \frac{RMSE(E_A, E_B) + MAE(E_A, E_B)}{R(E_A, E_B) + 1} \right)
\]

\[
+ \frac{RMSE(U_{DIC\_Ref}, U_{DIC\_A}) + MAE(U_{DIC\_Ref}, U_{DIC\_A})}{R(U_{DIC\_Ref}, U_{DIC\_A}) + 1}
\]

(7.3)

where \(U_{DIC\_Ref}\) is a vector that contains the full-field displacements at the FE nodal points from experimental testing or reference simulation and \(U_{DIC\_A}\) is a vector that contains the full-field displacements at the FE nodal points from the force-driven simulation. For the
displacement-driven simulation, the \(u_{\text{DIC,Ref}}\) can also be applied instead of only using the boundary displacement data.

7.3 Reconstruction of Heterogeneous Material Properties by Stochastic Self-OPTIM

Two-fold significances of the proposed stochastic Self-OPTIM methodology are deserved to be highlighted: 1) Homogeneous random field regularized by the KL expansion can efficiently be parameterized with the predetermined covariance function. Therefore, the stochastic Self-OPTIM is capable of inversely reconstruct one specific realization by identifying those stochastic parameters; 2) Currently, to the best knowledge of the author, there has been lack of efficient methodologies that can experimentally obtain the correlation length of random fields within materials. Depending on the form of covariance function, the correlation length has to be assumed for probabilistic simulations. However, the proposed stochastic Self-OPTIM can inversely identify the correlation length at the local material scale. This probabilistic characterization by the stochastic Self-OPTIM is less challenging than the inverse reconstruction problems. In this section, the reconstruction problem is demonstrated by the proposed stochastic Self-OPTIM.

7.3.1 A Square Domain with Spatially Varying Young’s Modulus

Young’s modulus is considered to be a random field of a target square domain by using the KL expansion. The geometry and FE/RF meshes of the square domain are shown in Figure 7.2. There are 9 RF elements, resulting in \(n_{\text{dofs}}=49\) and 100 FE elements are used. In this example, the reference RHM can be generated from Equation (4.18) by using the reference parameters, which is selected as \(E_{\text{Ref}}=30\) GPa, \(\sigma_{\text{Ref}}=3\) GPa, and \(L_{c\text{Ref}}=0.7\) m. \(\{\xi_i(\theta)\}\) is sampled from a Gaussian distribution with zero mean and unit variance. For Monte Carlo simulations, the multiple Gaussian realizations of the random
field are generated by randomly sampling the \( \{ \xi_i(\theta) \} \) defined in the KL expansion. However, in this inverse reconstruction problem, the Gaussian realization of the reference target is unique. Therefore, the Gaussian realization of the reference heterogeneous random field distribution can be obtained by using a certain set of determined random variables \( \{ \xi_i(\theta) \}_{\text{Ref}} \) and it is shown in Figure 7.3.

Figure 7.2 The mesh scheme of the square domain

Figure 7.3 Reference Young’s modulus distribution
7.3.2 Reconstruction Results

It is well known that the loading path, the objective function, and the specimen geometry are critical factors that can considerably influence the performance of experiment-simulation based inverse identification algorithms. The objective function formulation has already been discussed in Section 7.1. The effect of the chosen multidimensional loading path(s) on inverse identification of the heterogeneous elastic moduli estimation is examined. As shown in Figure 7.4, three different loading paths (i.e. uniaxial loadings in X and Y directions, and biaxial non-proportional loading) for displacement-controlled tests are accessed. For uniaxial tension tests in X and Y directions, the controlled displacement is equal to 0.0015m. For the non-promotional loading test, the controlled displacement is equal to 0.0015m in X direction and 0.001m in Y direction. The first three loading scenarios can be found in a test matrix shown in Table 7.1. The inputs for stochastic Self-OPTIM are only boundary displacements for the displacement-driven simulation and boundary forces for the force-driven simulation. For each case, the stochastic Self-OPTIM is carried out multiple times, and only the best result is presented. The initial population size for CFA equals to 50, and the total number of generations equals to 150. Upper and lower bounds of each parameter are set to ±30% initial error of the reference value.

Reconstructed random field distributions are shown in Figure 7.5. Although the identified stochastic parameters ($E$, $\sigma$, and $L_c$) can be directly compared with the reference values, an evaluation of the identified distribution of the heterogeneous material property is much more important. Therefore, we introduced a point-wise distribution
error estimator that is formulated to evaluate the error between the reproduced random field and the reference one

\[ \varepsilon_R = \frac{RSME(r_{\text{iden}}, r_{\text{Ref}}) + MAE(r_{\text{iden}}, r_{\text{Ref}})}{R(r_{\text{iden}}, r_{\text{Ref}})} \]  \hspace{1cm} (7.4)

where \( r_{\text{iden}} \) is a vector that contains the reproduced random field values at the FE Gauss points by using the identified parameter set and \( r_{\text{Ref}} \) is a vector that contains the reference random field values at the FE Gauss points. The identification results are presented in Table 7.2. According to the identification results, it can be concluded that the non-proportional biaxial square loading path can most benefit the stochastic Self-OPTIM. It is obvious that the non-proportional biaxial loading path provides more comprehensive information about the random material distribution.

Figure 7.4 Boundary condition and three different loading paths for the displacement-controlled test
Figure 7.5 Represented random field distributions: a) uniaxial tension on x direction; b) uniaxial tension on y direction; c) biaxial non-proportional loading

As shown in Table 7.1, three more cases (Case 4, 5 and 6), are created to investigate the effect of the full-field displacements on the stochastic Self-OPTIM performances. The reproduced random fields are presented in Figure 7.6. Totally six identification results of the parameter set and the distribution error can be compared based on Table 7.2.

It can be observed that Case 6 provides the best reproduced random field, which has the smallest distribution error comparing with the reference distribution. It uses $U_{DIC_{Ref}}$ from simulated reference testing under the biaxial loading path in the displacement-controlled simulation and also in the objective function as in Equation (7.1). The max/min values and their locations identified in Case 6 are presented in Figure 7.6 (c). It shows a very good matching with the reference one. At this point, the random distribution of the material property in the RHM could be well identified by using the global experimental measurements and the stochastic Self-OPTIM.
c)

Figure 7.6 Represented random field by applying full-field displacement in a) objective function with $U_{DIC_{-}Ref}$, b) displacement control data with $U_{DIC_{-}Ref}$, and c) $U_{DIC_{-}Ref}$ applied in both objective function and displacement control data.

Table 7.1 Test matrix (BNP: Biaxial non-proportional loading path)

<table>
<thead>
<tr>
<th>Case No.</th>
<th>$U_{DIC_{-}Ref}$ in OF</th>
<th>$U_{DIC_{-}Ref}$ in disp. control analysis</th>
<th>Loading path</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>no</td>
<td>no</td>
<td>X</td>
</tr>
<tr>
<td>2</td>
<td>no</td>
<td>no</td>
<td>Y</td>
</tr>
<tr>
<td>3</td>
<td>no</td>
<td>no</td>
<td>BNP</td>
</tr>
<tr>
<td>4</td>
<td>yes</td>
<td>no</td>
<td>BNP</td>
</tr>
<tr>
<td>5</td>
<td>no</td>
<td>yes</td>
<td>BNP</td>
</tr>
<tr>
<td>6</td>
<td>yes</td>
<td>yes</td>
<td>BNP</td>
</tr>
</tbody>
</table>

Table 7.2 Summary of the identification results

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Reference 1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (GPa)</td>
<td>30</td>
<td>32.63</td>
<td>29.28</td>
<td>31.15</td>
<td>31.37</td>
<td>30.64</td>
</tr>
<tr>
<td>$\sigma$ (GPa)</td>
<td>3</td>
<td>3.07</td>
<td>2.73</td>
<td>2.56</td>
<td>2.81</td>
<td>2.57</td>
</tr>
<tr>
<td>$L_c$</td>
<td>0.7</td>
<td>0.796</td>
<td>0.127</td>
<td>0.722</td>
<td>0.723</td>
<td>0.725</td>
</tr>
<tr>
<td>$\varepsilon_R$ / $\bar{E}_{ref}$ (%)</td>
<td>0</td>
<td>8.18</td>
<td>6.63</td>
<td>2.16</td>
<td>1.66</td>
<td>1.76</td>
</tr>
</tbody>
</table>
7.4 Stochastic Characterization of Material Uncertainties by Stochastic Self-OPTIM

To quantify the influence of the material uncertainty on nonlinear material behavior, a stochastic material database has to be built up by using the proposed stochastic Self-OPTIM.

7.4.1 Construction of A Database of Random Heterogeneous Material Properties

Compared to the deterministic material data base that has only mean values of material properties, a stochastic material database consists of multiple realizations of spatially varying material properties that are statistically equivalent to actual distributions of material properties. To construct the stochastic material database, the material property is modeled as random fields and stochastic Self-OPTIM is carried out for a finite number of specimens. For each specimen, the mean value, the standard deviation, and the correlation length of spatially random distributed material properties are inversely identified. By combining all the identification results, averages of the mean value, the standard deviation, and the correlation length of the random field can be obtained. Therefore, statistically equivalent Gaussian realizations of the random field can be generated with the average values of the identified statistical parameters and different sets of random variables defined in KL expansion. All of the Gaussian realizations of the random field build up the stochastic material database. It is obvious that the stochastic Self-OPTIM plays an important role in constructing stochastic database of the material properties. In this dissertation, instead of using test specimens, the simulated reference dog-bone-shaped specimen shown in Figure 6.15 is used to demonstrate the proposed stochastic material characterization method.
In this section, the stochastic material database contains Young’s moduli, Possion ratio, and the initial yield stress. These three material properties are modeled as uncorrelated random fields (no correlation between these random fields). The FE and RF mesh is identical to previous example as shown in Figure 6.15. There are 88 FE/RF elements in the mesh, which result in \( n_{dofs} = 317 \). The reference parameter sets of the random fields, \( X_{ref} = [\bar{E} = 131.128 \text{ GPa}, \sigma_E = 2.622 \text{ GPa}, L_{Ec} = 0.05 \text{m}, \bar{\mu} = 0.37, \sigma_\mu = 0.0074, L_{\mu c} = 0.05 \text{m}, \bar{\sigma}_{y0} = 920 \text{ MPa}, \sigma_{y0} = 46 \text{ MPa}, L_{y0c} = 0.035 \text{m}] \) assumed based on the Titanium alloy BT9 (Shamsaei et al., 2010b) are shown in Table 7.3-Table 7.5. For modeling the elasto-plastic material behavior, the constitutive model introduced in Section 5.1 is applied. The deterministic hardening parameters are set as \( b = 0, Q = 0, H = 17095 \text{ MPa}, \) and \( \gamma = -216 \). Reference distributions of the three random fields of a certain specimen are shown in Figure 7.7.

![Figure 7.7 Distribution of random fields of a certain reference simulation](image-url)
The specimens are tested under displacement-controlled uniaxial monotonic test, in which the specimens will undergo plastic deformation. The displacement of the moving end is equals to 3mm with 200 loading steps. Used as the input data of stochastic Self-OPTIM, full-field displacement $U_{\text{DIC, ref}}$ and boundary force data at 40 selected loading steps are acquired from the reference simulations. For each of the inverse identification simulations, $X_I = [\bar{E}, \sigma_E, L_{Ec}, \bar{\mu}, \sigma_\mu, L_{\mu c}, \sigma_{\gamma_\theta}, L_{\gamma c}]$ is the parameter set to be identified.

7.4.2 Results of Stochastic Self-OPTIM

CFA is used as the optimization method. The objective function shown in Equation (7.3) is applied and $U_{\text{DIC, ref}}$ is used as the displacement control data for the displacement-controlled simulation in stochastic Self-OPTIM. The upper and lower bounds of each parameter are set to ± 30% initial error of the reference value. The stochastic Self-OPTIM was carried out for each specimen multiple times to obtain a converged solution. The best identification results, the averaged value of the parameters, and the absolute identification error between the reference value and the averaged value are presented in Table 7.3 to Table 7.5.

<table>
<thead>
<tr>
<th></th>
<th>$\bar{E}$ (GPa)</th>
<th>$\sigma_E$ (GPa)</th>
<th>$L_{Ec}$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>131.128</td>
<td>2.622</td>
<td>0.05</td>
</tr>
<tr>
<td>Lower bound</td>
<td>91.790</td>
<td>1.835</td>
<td>0.035</td>
</tr>
<tr>
<td>Upper bound</td>
<td>170.466</td>
<td>3.409</td>
<td>0.065</td>
</tr>
<tr>
<td>Sample 1</td>
<td>127.5267</td>
<td>2.6316</td>
<td>0.0494</td>
</tr>
<tr>
<td>Sample 2</td>
<td>128.9631</td>
<td>2.5984</td>
<td>0.0485</td>
</tr>
<tr>
<td>Sample 3</td>
<td>130.5288</td>
<td>2.5815</td>
<td>0.0495</td>
</tr>
<tr>
<td>Sample 4</td>
<td>129.7449</td>
<td>2.6124</td>
<td>0.0491</td>
</tr>
<tr>
<td>Avg.</td>
<td>129.1909</td>
<td>2.6060</td>
<td>0.0491</td>
</tr>
<tr>
<td>Error (%)</td>
<td>1.5</td>
<td>0.6</td>
<td>1.8</td>
</tr>
</tbody>
</table>
Table 7.4 Identification results of Poisson’s ratio

<table>
<thead>
<tr>
<th></th>
<th>$\bar{\mu}$</th>
<th>$\sigma_\mu$</th>
<th>$L_{\mu c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>0.37</td>
<td>0.0074</td>
<td>0.05</td>
</tr>
<tr>
<td>Upper bound</td>
<td>0.259</td>
<td>0.0052</td>
<td>0.035</td>
</tr>
<tr>
<td>Lower bound</td>
<td>0.481</td>
<td>0.0096</td>
<td>0.065</td>
</tr>
<tr>
<td>Sample 1</td>
<td>0.379112</td>
<td>0.00767</td>
<td>0.0523</td>
</tr>
<tr>
<td>Sample 2</td>
<td>0.377109</td>
<td>0.00772</td>
<td>0.0530</td>
</tr>
<tr>
<td>Sample 3</td>
<td>0.375209</td>
<td>0.00767</td>
<td>0.0517</td>
</tr>
<tr>
<td>Sample 4</td>
<td>0.37915</td>
<td>0.00748</td>
<td>0.0510</td>
</tr>
<tr>
<td>Avg.</td>
<td>0.377645</td>
<td>0.007635</td>
<td>0.0520</td>
</tr>
<tr>
<td>Error (%)</td>
<td>2.1</td>
<td>3.2</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 7.5 Identification results of initial yield stress

<table>
<thead>
<tr>
<th>$\bar{S}_{y 0}$ (MPa)</th>
<th>$\sigma_{Sy0}$ (MPa)</th>
<th>$L_{Sy0c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>920</td>
<td>46</td>
</tr>
<tr>
<td>Upper bound</td>
<td>644</td>
<td>32.2</td>
</tr>
<tr>
<td>Lower bound</td>
<td>1196</td>
<td>59.8</td>
</tr>
<tr>
<td>Sample 1</td>
<td>989.542</td>
<td>45.618</td>
</tr>
<tr>
<td>Sample 2</td>
<td>986.983</td>
<td>45.565</td>
</tr>
<tr>
<td>Sample 3</td>
<td>959.108</td>
<td>44.777</td>
</tr>
<tr>
<td>Sample 4</td>
<td>961.528</td>
<td>45.102</td>
</tr>
<tr>
<td>Avg.</td>
<td>974.290</td>
<td>45.266</td>
</tr>
<tr>
<td>Error (%)</td>
<td>5.9</td>
<td>1.6</td>
</tr>
</tbody>
</table>

From the identification results, we can conclude that the performance of stochastic Self-OPTIM in stochastic material characterization is excellent even with considering the nonlinear material behavior. The stochastic material data base of Young’s modulus, Poisson’s ratio, and initial yield stress can be constructed by using MCS with the averaged identified parameter set. The application of this data base to developing a probabilistic low cycle fatigue life prediction model will be demonstrated. This stochastic material property based model is of great importance for reliability analyses.
7.5 Probabilistic Surrogate Modeling by Using PCE

Following the last section, the probabilistic distributions of the random variables (material response) can be obtained by using the MCS with the Gaussian realizations of the material properties sampled from the identified stochastic material data base. PCE is used in order to build up a surrogate model for representing these random variables. Let’s consider a random variable \( \{U(\theta_i)\}_{i=1}^{M} \) with \( M \) samples. A representation with PCE is written as

\[
U(\theta_i) = \sum_{i=0}^{p} k_i h_i(\xi(\theta_i))
\]  \hspace{1cm} (7.5)

where \( k_i \) is the unknown \( p+1 \) coefficients of the polynomial chaos, \( h_i \) is the Hermite polynomial of degree of \( i \), and \( \xi(\theta_i) \) is the Gaussian random variable that, will be sampled by using LHS technique. For instance, the one-dimensional third order PCE can be written as

\[
U(\theta_i) = k_0 h_0(\xi_1) + k_1 h_1(\xi_1) + k_2 h_2(\xi_1) + k_3 h_3(\xi_1);
\]

\[ h_0 = 1; \]
\[ h_1 = \xi_1; \]
\[ h_2 = \xi_1^2 - 1; \]
\[ h_3 = \xi_1^3 - 3\xi_1; \]  \hspace{1cm} (7.6)

The key procedure is to determine the coefficients \( k_i \) of the polynomial chaos expansion. In this dissertation, the selected identification method to determine the PCE coefficients is the maximum likelihood method (Desceliers et al., 2007), in which \( k_i \) are the solution of the optimization problem

\[
\mathbf{k} = \arg\min(- \log(L(\mathbf{k})))
\]  \hspace{1cm} (7.7)

where \( \mathbf{k} \) is the vector of coefficients \( k_i \) and \( L \) is the likelihood function.
\[ L(k) = \prod_{i=1}^{M} p_k(U(\theta_i); k) \]  \hspace{1cm} (7.8)

where \( M \) is the size of the sample of \( U(\theta_i) \) and \( p_k(U(\theta_i); k) \) is the probability density function of the variable \( U(\theta_i) \) depending on the set of PCE coefficients. To systematize the algorithm, it is suggested to reduce the optimization problem by stating conditions

\[
k_0 = \mu_k \\
\sum_{i=1}^{p} k_i^2 = \sigma_k^2 \tag{7.9}
\]

where \( \mu_U \) and \( \sigma_U \) are the mean value and the standard deviation of \( \{U(\theta_i)\}_{i=1}^{M} \), respectively. The first condition reduces the number of unknown coefficients from \( p+1 \) to \( p \) and the second one allows for searching other coefficients on a hypersphere with radius. By denoting the ratio \( k_i/\sigma_U \) with \( k_i^* \), the second condition in Equation (7.9) is modified to

\[
\sum_{i=1}^{p} (k_i^*)^2 = 1 \tag{7.10}
\]

The CFA is used to solve this optimization problem. To deal with the constraint conditions in Equation (7.9) and (7.10), the penalty function approach adds a penalty-like term to the cost function that consists of a penalty parameter and a measure of violation of the constraints. The measure of violation is nonzero when the constraints are violated and is zero in the region where constraints are satisfied. By doing this, the constrained optimization problem is transformed to an unconstrained optimization problem that is simple to solve. After the optimization problem is solved, the random variable \( U(\theta_i) \) can be represented by using Equation (7.5).
Effects of Variability of Material Properties on Low Cycle Fatigue Life Prediction

The purpose of this research is to develop a probabilistic low-cycle fatigue life model with systematic considerations of stochastic variability of materials properties. Although most of the designed components may appear to have nominally cyclic elastic stresses, notches or other stress concentrations present in the component may result in local cyclic plastic deformation. Under these conditions, an approach that uses the strains as the governing fatigue parameter is required. The strain-life relationship is typically represented as a curve of strain versus fatigue life and is generated by conducting strain-controlled axial fatigue tests on smooth, polished specimens of the material.

The transient cyclic response of a material describes the process of change in the resistance of a material to deformation due to cyclic loading. If a material is repeatedly cycled under fully reversed strain-controlled loading, the material may respond in one of the following ways: cyclic hardening or cyclic softening. Here, the transient cyclic hardening of the material is studied. The stress increases in each successive strain reversal increases as the number of cycle increases. In this case, the rate of change of the applied stress will gradually reduce and the stress magnitude will reach a stable level, which is called a steady-state condition and remain stable for the rest of the fatigue life until the detection of the first fatigue crack. Fatigue life can be determined by the cyclic steady-state cyclic stress-strain behavior for constant strain amplitude-controlled testing.

Deterministic Low-Cycle Fatigue Life Prediction Model

Based on (Morrow, 1965), the relation of the total strain amplitude $\varepsilon_a$ and the fatigue life in reversals to failure $2N_f$ can be expressed in the following form
where $E$ is the averaged Young’s modulus, $\sigma_{f}^\prime$ is the fatigue strength coefficient, $b$ is the fatigue strength exponent, $\varepsilon_{f}^\prime$ is the fatigue ductility coefficient, and $c$ is the fatigue ductility exponent. Among these six parameters, only four of them are independent since they satisfy the following equation

$$K' = \frac{\sigma_{f}^\prime}{(\varepsilon_{f}^\prime)^{m}}, \quad n' = \frac{b}{c}$$

(7.12)

where $K'$ is the strength coefficient and $n'$ is the strain-hardening exponent. By following ASTM E646-07, the $K'$ and $n'$ can be determined from a steady-state cyclic stress and strain curve. In this test standard, the constant strain-amplitude controlled testing is carried out. Pairs of true stresses and true plastic strains are obtained when the cyclic behavior reaches the saturation at each strain-amplitude. Afterwards, the $K'$ and $n'$ is determined from fitting these data pairs by using an empirical equation, whose mathematical representation is a power curve

$$\sigma_{t} = K'\varepsilon_{t}^{n'}$$

(7.13)

where $\sigma_{t}$ is the true stress and $\varepsilon_{t}^{p}$ is the true plastic strain. If $\sigma_{f}^\prime$ and $b$ or $\varepsilon_{f}^\prime$ and $c$ are known, the other pair of parameters can be calculated from Equation (7.12). Finally, Equation (7.11) is able to be used to predict the low-cycle fatigue life.

7.6.2 Development of A Probabilistic Low-Cycle Fatigue Life Prediction Model

In previous section, the deterministic low-cycle fatigue life prediction model is introduced. However, it is well known that the low-cycle fatigue life prediction is a very tough problem since there are many factors that have to be considered, especially the
uncertainties in the material. Uncertainties of the material properties naturally propagate to the material elasto-plastic behavior to obtain a realistic and more accurate prediction of the fatigue life. By using the stochastic material data base constructed in Section 7.3.2 and the stochastic elasto-plasticity finite element method developed in Section 6.4, we proposed a probabilistic low-cycle fatigue life prediction model, which considers the influence of the uncertainty of spatially varying material properties such as Young’s modulus, Poisson’s ratio, and yield strength to the fatigue life of metallic materials. It is notable that this probabilistic fatigue life model follows the empirical fatigue model although microscale defects in materials are main causes of scatter in the observed fatigue life of actual materials. Therefore, current approach for developing probabilistic low-cycle fatigue life model is based on the empirical fatigue life model which is widely accepted.

7.6.2.1 Probabilistic Distribution of Random Variables

At first, MCS of the strain amplitude-controlled cyclic simulations are carried out with $M$ samples. In each Gaussian realization, the three random fields, Young’s Modulus, Poission's ratio, and initial yield stress are sampled from the stochastic material data base constructed by applying stochastic Self-OPTIM. Different steady-state cyclic strain and stress curves can be obtained by fitting the true stress and plastic strain data pairs in each Gaussian realization. In this way, the uncertainties of material properties are propagated to $K'$ and $n'$; in other ward, $K'$ and $n'$ become to random variables, $\{K'(\theta_i)\}_{i=1}^{M}$ and $\{n'(\theta_i)\}_{i=1}^{M}$, respectively. Since the material properties are random and heterogeneous, the average Young’s modulus in Equation (7.11) cannot be a deterministic value. The effective Young’s modulus on the sample scale that is also a random variable $\{E_c(\theta_i)\}_{i=1}^{M}$
is obtained from the cyclic simulation by using the stress and strain data within early
loading stages. The surrogate probabilistic models of these three random variables $E_e(\theta_i)$,
$K'(\theta_i)$, and $n'(\theta_i)$ are created by using PCE. The maximum likelihood method is used to
determine the coefficients of the polynomial chaos expansion based on $\{E_e(\theta_i)\}_{i=1}^M$, 
$\{K'(\theta_i)\}_{i=1}^M$, and $\{n'(\theta_i)\}_{i=1}^M$. By assuming $\sigma_j'$ and $b$ are known and deterministic, we can get
\[
\varepsilon_j'(\theta_i) = \left( \frac{\sigma_j'}{K'(\theta_i)} \right)^{\frac{1}{\mu'(\theta_i)}} , \quad c(\theta_i) = \frac{b}{n'(\theta_i)} \tag{7.14}
\]
Therefore, the probabilistic low-cycle fatigue life prediction model is presented as
\[
\varepsilon_a = \varepsilon_a^e + \varepsilon_a^p = \frac{\sigma_j'}{E_e(\theta_i)} (2N_f)_b + \varepsilon_j'(\theta_i)(2N_f)^{c'(\theta_i)} \tag{7.15}
\]
$K'$ and $n'$ are obtained from simulation by using the elasto-plastic constitutive model with
nonlinear isotropic/kinematic hardening. The MCS are carried by using the Gaussian
realizations sampled from the stochastic material data base of Young’s modulus,
Poission’s ratio and yield strength. The displacement control data, which contain five
different strain amplitudes, are shown in
Figure 7.8. Five full reversals are contained at each of displacement amplitudes to let the
stress reach the saturation state. In the displacement-controlled simulation, the effective
stress $\sigma_c=F/A$ and effective strain $\varepsilon_c=(l-L)/L$ are calculated in the same way as introduced
in Section 6.4. Figure 7.9 shows the cyclic stress-strain curve from one sample in the
MCS.
Figure 7.8 Displacement amplitude

Figure 7.9 Cyclic effective stress vs. effective strain curve

The steady-state cyclic stress-strain curves are presented in Figure 7.10

Figure 7.10 Steady-State cyclic stress and strain curve

from the MCS with 2000 samples. By curve fitting all these steady-state cyclic stress-strain curves based on Equation (7.13), the probabilistic distribution of the random
variables, \( \{K'(\theta_i)\}_{i=1}^{2000} \), \( \{n'(\theta_i)\}_{i=1}^{2000} \), and \( \{E_e(\theta_i)\}_{i=1}^{2000} \) can be obtained as shown in Figure 7.11.

Figure 7.10 Steady-State cyclic stress and strain curve
Figure 7. 11 Statistical distribution of a) \( \{K'(\theta)\}_{1=1}^{2000} \), b) \( \{n'(\theta)\}_{1=1}^{2000} \), and c) \( \{E_c(\theta)\}_{1=1}^{2000} \).

For each distribution of the random variables, the one-dimensional and the third order PCE is carried out to obtain the corresponding probabilistic surrogate model. The identified coefficient of PCE for \( K'(\theta) \), \( n'(\theta) \), and \( E_c(\theta) \) is listed in Table 7.6.

Comparison of the PCE model and the MCS result is shown in Figure 7.12

Table 7.6 PCE coefficients solution of the optimization problem

<table>
<thead>
<tr>
<th></th>
<th>( \mu )</th>
<th>( \sigma )</th>
<th>( k_1 )</th>
<th>( k_2 )</th>
<th>( k_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K'(\theta) ) MPa</td>
<td>1272.1990</td>
<td>40.4368</td>
<td>-0.9955</td>
<td>0.0362</td>
<td>-0.0351</td>
</tr>
<tr>
<td>( n'(\theta) )</td>
<td>0.0380</td>
<td>0.0010</td>
<td>0.9927</td>
<td>-0.0972</td>
<td>0.0121</td>
</tr>
<tr>
<td>( E_c(\theta) ) GPa</td>
<td>131.8034</td>
<td>2.2518</td>
<td>0.9961</td>
<td>0.0226</td>
<td>-0.0454</td>
</tr>
</tbody>
</table>
Figure 7.12 Statistical distribution of random variables: a) $K'(\theta_l)$, b) $n'(\theta_l)$, and c) $Ee(\theta_l)$ fitting with a polynomial chaos

7.6.2.2 Probabilistic Low-Cycle Fatigue Life Prediction

Assuming $\sigma_f' = 1180$ MPa and $b = -0.025$ (Shamsaei et al., 2010b), the probabilistic low-cycle fatigue life can be calculated based on Equation (7.15) by using MCS with 2000 samples from Gaussian realizations of the PCE model of $K'(\theta_l)$, $n'(\theta_l)$, and $Ee(\theta_l)$. The probabilistic total strain-life curve is plotted in Figure 7.13. The deterministic fatigue life-strain amplitude curve is overlapped by using the deterministic parameters, $\sigma_f' = 1180$ MPa, $b = -0.025$, $\varepsilon_f = 0.278$, $c = -0.665$, and $E_e = 131.13$ GPa.
From Figure 7.13, it is observed that the fatigue life at the same strain level has a large scatter. The variation will increase when the strain amplitude becomes larger. The reason is that when the effective strain amplitude is small, most of the material points (due to heterogeneity, material points will behavior differently) are undergo only the elastic deformation, therefore, only the uncertainty of Young’s modulus majorly influence the fatigue life. Nevertheless, when the effective strain amplitude become larger, most of the material points are undergo plastic deformation, accordingly, the predicted fatigue life is influenced by both of the uncertainties of Young’s modulus and initial yield stress. The proposed probabilistic low-cycle fatigue life prediction method quantitatively propagates the uncertainties from the material properties to the probabilistic low-cycle fatigue life prediction. This research is of great importance for the reliability based design.
7.7 Conclusion

A realistic inverse material characterization that considers variability of material properties is proposed in this chapter. Based on stochastic Self-OPTIM, which integrated the SFEM into the deterministic inverse identification framework Self-OPTIM, the distribution of the heterogeneous material property can be accurately reproduced by using the identified parameter set. Furthermore, the stochastic material database is constructed on a basis of limited information from the test of multiple specimens. Instead of using real experimental test, the demonstration is successfully made by using synthetic data. A suitable objective function and appropriate application of the full-field displacement are also discussed. It shows that, the newly formulated objective function and the full-field displacement test data can improve the performance of stochastic Self-OPTIM. By using the stochastic data base, a probabilistic low-cycle fatigue life prediction model is developed based on probabilistic surrogate modeling applying PCE. The prediction result shows that by considering the uncertainties of the material properties, the predicted fatigue life has variations at the same strain amplitude. This research is very important for the reliability based design of the structure that has elasto-plastic material behavior under cyclic loadings.
CHAPTER VIII
SUMMARY AND FUTURE WORK

8.1 Summary

The research presented in this dissertation proposes numerical investigations into the stochastic nature of the material properties by implementing a general purpose stochastic finite element method into a novel inverse identification framework, Self-OPTIM. The stochastic material data base is characterized for the purpose of reliability design by using the numerical-experimental based stochastic material characterization method, Stochastic Self-OPTIM. The conclusions gained from this work are summarized below.

A comprehensive literature review of the state-of-the-art of inverse identification algorithms, stochastic finite element modeling techniques, and stochastic material characterization is carried out. It clearly identified the lack of techniques for material identification of random heterogeneous media and stochastic material characterization.

Self-Optimizing Inverse Method (Self-OPTIM), which is a model-independent inverse analysis methodology, has been developed. It automatically self-estimates the material parameters the material properties by updating local stresses and strains through two parallel nonlinear finite element simulations, which are controlled by global forces and displacement measured from experiment. No local stress and strain measurements from experimental testing are required. Full-field displacements measurement obtained
from digital image correlation technique can be integrated into Self-OPTIM to improve its performance on identification. Compared to the traditional FE model updating based techniques the proposed Self-OPTIM method minimizes an implicit objective function that contains stresses and strains fields from the two parallel FE simulations. It is worth noting that any material parameters of ABAQUS build-in model and user defined material (UMAT) can be easily calibrated in Self-OPTIM.

In order to simulate the probabilistic structural response of the material with uncertain properties, an intrusive stochastic finite element method is developed. A new stochastic finite element is implemented within a general purpose finite element analysis program ABAQUS. Karhunen–Loève expansion is used to represent the random field, which is used to model the randomly distributed material property. The issues on the separation of two different mesh, RF and FE mesh, have been addressed along with an general mapping-interpolation method. This method completely separates the RF and FE meshes. By using this method, the discretized random field, which is distributed at the nodal points in RF mesh, is seamlessly and accurately transformed to the Gaussian points in FE mesh. Therefore, the mesh scheme of these two different meshes can be very flexible. The proposed method can significantly reduce dimensionality of the stochastic domain and efficiently predict probability density functions of the structural response under material uncertainties through Monte Carlo simulations combined with the Latin hyper cube sampling technique.

By combining the Self-OPTIM and SFEM, a novel stochastic inverse material characterization method is presented in Chapter 5. The Stochastic Self-OPTIM is a realistic material inverse characterization that considers the random heterogeneous
distribution of the material properties at macroscopic level that caused by the intrinsic scatter in the microscopic structures that is present at the microscopic level. This proposed method helps to avoid involving the microscopic complexity of the material structure and the required instrumentations to obtain reliable data at microscopic level. The advances in Self-OPTIM are automatically inherited. The quantities to be identified can be the macro-scale Young’s modulus, Poisson’s ratio, strengths, and etc. which are mathematically described as random fields. Based on the limited information, which is obtained indirectly by measuring the global force and displacement response of several geometrically identical structures made of the same material, the Stochastic Self-OPTIM can not only be used to reconstruct the material property distribution of the random heterogeneous media, but also to construct the stochastic material database. The surrogate probabilistic model of the random variable obtained from MCS can be build up based on the stochastic material database and polynomial chaos expansion technique. Therefore, the reliability based design can be carried out by using MCS and LHS.

Numerical and experimental verifications of Self-OPTIM are carried out for an elasto-plastic model for cyclic material behavior and a micromechanics and fracture mechanics based damage model for damage tolerance composite material in Chapter 6. For the elasto-plastic model, the reconstructed stress/strain and force/displacement curves perfectly matching with the reference one from both reference simulation and the experimental measurement from the homogenous and non homogenous specimens. For the damage model, the microscopic damage behavior is successively captured by identifying the parameter set of the microcrack nucleation and growth laws with the global force and displacement from reference simulation. The experimental test data is
also acquired from the literatures to verify the Self-OPTIM. It can be conclude that the performance of Self-OPTIM is excellent and it can be used to identify the material parameters embedded in any constitutive model.

Numerical simulation examples are presented to demonstrate the application of the proposed SFEM. The probabilistic structural responses are obtained from a square plate and a quarter circular cube with spatial varying linear elastic material property. To demonstrate the potential of the proposed approach in dealing with large scale complex engineering problems, a retaining wall is used as a more realistic example with both of RF and FE mesh use the free mesh control containing a large number of elements. The probabilistic deformation of the front surface under a lateral earth pressure is presented. Furthermore, the elasto-plastic model is integrated into the SFEM to show the fidelity and capacity of the algorithm in nonlinear reliability analysis, which considering Young’s modulus, Poisson’s ratio, and initial yield stress as independent random fields.

The application of the proposed stochastic material characterization method, Stochastic Self-OPTIM, is demonstrated by using the synthetic data in Chapter 8. Firstly, the random distribution of the linear material property over a square RHM is accurately characterized. It is proved that the formulation of the objective function, optimized loading path, and the full-field displacement can improve the performance of Stochastic Self-OPTIM. Furthermore, based on limited experimental information, a group of parameter sets can be obtained by carrying out Stochastic Self-OPTIM on multiple specimens, whose geometry design is identical. Therefore, a stochastic material database is formed up by infinite Gaussian realizations of the random fields, which are generated from MCS with an averaged parameter set. Finally a probabilistic low-cycle fatigue life
prediction model is developed based on the deterministic one. The deterministic parameters of the model are replaced by random variables, which are represented by PCE on a basis of the probabilistic material response from MCS with sampling the material properties from the constructed stochastic material database. This probabilistic low-fatigue life prediction model can be applied in reliability based structural design.

8.2 Future Work

The realistic experimental verification will be carried out to verify the proposed stochastic material characterization method. Two groups of specimens will be tested identically. The test results from the first group of specimens will be used to construct the stochastic material database by using the proposed method. Afterward, the test results from the second group will be used to confirm the accuracy of the constructed material database.

Since material properties of composite material has large uncertainties introduced by the defects in microscopic scale and manufacturing process. The stochastic material characterization is very suitable to be applied on laminated composites. The Young’s modulus, failure strength, and initial damage can be modeled as random fields and characterized. This ongoing research will significantly influence the damage tolerance analysis, which is the key for designing the modern aircrafts that use the composites material in both major and minor load-bearing structures.

At the end, some ideas are discussed for the potential contributions that current research can do, based on the knowledge of the author. Although problems presented in this paper are defined in isotropic linear elastic and elasto-plastic, various potential applications of SFE are expected. It is able to be extended to simulate stochastic
responses under uncertainties related to damage, fracture mechanics, and thermodynamics of anisotropic materials. Furthermore, the stochastic material characterization method can be applied on damage localization and quantification since it can describe the distribution of the material property.
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