DETECTION OF MATERIAL PROPERTIES USING LASER OPTICS

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DETECTION OF MATERIAL PROPERTIES USING LASER OPTICS

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Thesis

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This thesis presents a method for detection of material properties such as refractive
index, permittivity and conductivity of a remote object by using laser beams. The
method is demonstrated for planar dielectric media in which properties of reflected
and refracted wave information is known via Snell’s law. The reflection coefficient of
a laser beam on an object is shown to depend on the angle of incidence, the beam
frequency and material constants such as electrical permittivity and conductivity. By
measuring the intensities of the incident and the reflected waves, reflection coefficients
are determined. The reflection coefficient is then used to calculate the refractive index
of the material at an accurate level. By varying the parameters such as the angle
of incidence and the beam frequency, non-linear complex polynomial equations are
obtained which are solved to estimate the unknown conductivity and permittivity of
the medium.

The results are generalized and applied to rough surfaces which are referred
to as diffuse media. In this case, only partial information of the reflected field can be
obtained. This partial information yields a scaling factor ‘s’, which we have termed
as the diffuse parameter. Finally, by constructing appropriate numbers of nonlinear
equations, the permittivity, the conductivity and the diffuse parameter are determined
for a diffuse media.

The estimation of the material properties, made from the proposed method, are verified through experimental results. To achieve this, a series of experiments are carried out in which the angle of incidence is changed with fixed frequency of the incident wave and vice versa. Low power laser sources with wavelengths typically between 400 and 900 nm are used for these applications. Refractive index, electrical permittivity and conductivity of an aluminium sheet are measured through the experiment. The experimental results and the theoretical results are observed to vary by less than 10%. This work realized successfully that optical and electrical properties of materials can be calculated directly through laser applications.
I would like to express my sincere gratitude to my advisor, Dr. S. I. Hariharan, whose encouragement, guidance and advice brought me successfully through this thesis. I would also like to express my gratitude to the rest of my committee members, Dr. George Giakos, who provided extensive help with the lab equipments and setup and Dr. Hamid Bahrami, who generously gave the gift of his time and support when needed. I would also like to thank department chair Dr. Alex De Abreu Garcia for his valuable suggestions and tremendous support during my stay in this department.

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# TABLE OF CONTENTS

| LIST OF TABLES | viii |
| LIST OF FIGURES | ix |

## CHAPTER

I. INTRODUCTION 1

1.1 Motivation and Goal of the Thesis 1

1.2 Related Work in Laser Optics 2

1.3 Summary of Contribution 3

1.4 Organization of the Thesis 4

II. MATERIAL PROPERTIES DETECTION FOR SMOOTH SURFACES 6

2.1 Overview 6

2.2 Specular Reflection 6

2.3 Determination of Fresnel Reflection Coefficients 13

2.4 Determination of Complex Refractive Index by Varying Angles of Incidence 16

2.5 Determination of Electrical Permittivity and Conductivity by Varying Frequency 21

III. MATERIAL PROPERTIES DETECTION FOR ROUGH SURFACES 26
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Refractive Indices of Different Metals at 830 nm</td>
<td>17</td>
</tr>
<tr>
<td>4.1</td>
<td>Experimental Results of Reflected Wave Intensities at $0^\circ$ and $5^\circ$</td>
<td>32</td>
</tr>
<tr>
<td>4.2</td>
<td>Results Summary for Smooth Surfaces</td>
<td>37</td>
</tr>
<tr>
<td>4.3</td>
<td>Results Summary for Rough Surfaces</td>
<td>39</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 A Laser Induced Breakdown Spectroscopy Process [1].</td>
<td>3</td>
</tr>
<tr>
<td>1.2 Material Property Detection.</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Oblique Incidence of Parallel and Perpendicular Polarized Waves.</td>
<td>8</td>
</tr>
<tr>
<td>2.2 Rotation with Respect to the Original Axis.</td>
<td>10</td>
</tr>
<tr>
<td>2.3 TM and TE Reflection Coefficients for Air-Silver Interface.</td>
<td>15</td>
</tr>
<tr>
<td>2.4 Reflection Coefficients for Air-Water Interface at Frequency of 1 GHz.</td>
<td>16</td>
</tr>
<tr>
<td>2.5 Reflection Coefficients for Air-Water Interface at Frequency of 100 MHz.</td>
<td>18</td>
</tr>
<tr>
<td>2.6 Reflection Coefficients for Lossy Medium A Interface.</td>
<td>19</td>
</tr>
<tr>
<td>2.7 Reflection Coefficients for Lossy Medium B Interface.</td>
<td>20</td>
</tr>
<tr>
<td>2.8 Design Flow for the Material Properties Calculation.</td>
<td>25</td>
</tr>
<tr>
<td>3.1 Scattering in Rough Surface.</td>
<td>28</td>
</tr>
<tr>
<td>4.1 Apparatus Setup for the Measurement of the Reflection Coefficient of a Smooth Aluminum Slab.</td>
<td>31</td>
</tr>
</tbody>
</table>
CHAPTER I
INTRODUCTION

1.1 Motivation and Goal of the Thesis

Laser Optics is an extensively explored field whose applications range from target detection [2], [3], to medical applications [4], to spectroscopy and ocean depth measurement [5]. In recent times, lasers have been widely used in intelligent vehicle systems [6] as well as miscellaneous applications such as laser radars and CD players [7]. By applying a laser beam, an object may be identified through its refractive index, electrical permittivity and conductivity [8]. In an application involving remote sensing, where the sensing instrumentation is not in contact with the object being observed, laser application presents a feasible and efficient method. Using lasers in remote sensing allows the observer to avoid hazardous or difficult to reach regions such as ocean depths, mountain tops and large scale geographic features.

The goal of this thesis is to introduce a method which can be used to identify material properties of a remote object by calculating its properties such as refractive index, electrical permittivity and conductivity. The algorithm will be experimentally verified in order to explore if one can use lasers to detect material properties.
1.2 Related Work in Laser Optics

Lasers have been widely used in a multitude of applications ranging from intelligent transportation systems and biomedical optics to spectroscopy. One of the key applications of a laser is in intelligent transportation systems where it is used to detect a moving or a stationary object/target. Such an application is called Detection and Tracking of Moving Objects [9]. In this application, a system is able to classify several kinds of objects and can be easily expanded to detect new ones. The application is extensively used to design and integrate a moving vehicle to avoid collision, where it computes the time-to-collision for each moving obstacle and validates the results. Other applications such as traffic-lane detection [10], [11] and long range obstacle detection [6] are performed using a laser scanner. On board laser-scanners are able to observe the vehicle’s environment in order to detect, track and classify the surrounding objects and thus provide data for active safety systems [12].

In recent years, considerable work has been done towards the development and testing of continuous emissions monitors and discrete particle detection [13]. Laser-Induced Breakdown Spectroscopy (LIBS) [1] is extensively used for such applications. LIBS is an atomic emission spectroscopy that utilizes a high-power pulsed laser beam as the excitation source. The resulting optical breakdown, also referred to as a laser-induced plasma or laser spark, excites the target, which then enables determination of elemental composition. Figure 1.1 shows a schematic flow of a LIBS process. This technique is widely used as a metal emissions monitoring technology.
In other laser applications, Grant et al [14] used pulsed laser in particle detection and sizing and successfully detected 5-m latex spheres. Bauer et al [15] used mid-infrared lasers for the detection of explosive contaminated surfaces. Lasers are also widely used in biomedical optics applications. Moritz et al [16] used laser spectroscopy to detect cancerous cells, while Song et al [4] have used it to detect small structural changes in bones.

1.3 Summary of Contribution

A laser based method for determining refractive index, electrical permittivity and conductivity of a remote object is introduced. The reflection coefficient of a laser beam on an object is shown to depend on the angle of incidence, beam frequency and
material constants such as electrical permittivity and conductivity of the material-air interface. An example of such an application is shown in Figure 1.2. An unknown remote object A, whose properties need to be identified is targeted by a laser beam. By measuring the intensity of the incident and the reflected waves, the material properties are calculated with high accuracy. By varying parameters such as the angle of incidence and the beam frequency, non-linear complex polynomial equations are obtained which are solved to estimate the unknown conductivity and permittivity of the medium.

The results are generalized and applied to rough surfaces which are referred to as diffuse media. In this case, only partial information of the reflected field can be obtained. This partial information yields a scaling factor ‘s’, which we call “diffuse parameter”. Finally, by constructing an appropriate number of nonlinear equations, the permittivity, the conductivity and the diffuse parameter are determined for a diffuse media.

1.4 Organization of the Thesis

The rest of this thesis is divided into four chapters. In Chapter II, the background for material property detection using the reflection coefficient is discussed. Various methods for detection of material and optical properties of metallic and non-metallic surfaces are presented. In Chapter III, a method is introduced which is used to calculate the refractive index, permittivity and conductivity for smooth surfaces. The method’s application is extended for property calculation for rough surfaces. In ad-
dition, a term “diffuse parameter” ‘s’ is introduced and calculated for rough surfaces. In Chapter IV, the experimental results and calculations for a thick aluminium slab are shown. In Chapter V, the results are summarized and possible future work is discussed.
CHAPTER II
MATERIAL PROPERTIES DETECTION FOR SMOOTH SURFACES

2.1 Overview

When a ray of light is incident on an interface separating two different media, part of it gets reflected back to the original medium, and part of it gets absorbed by the medium and/or travels through the medium. We can study the properties of the scattered or the reflected light to find the distance and other physical and chemical information of the remote target. Our focus here is to find physical properties of the target.

Light reflecting from a perfectly smooth surface follows the law of reflection i.e. angle of reflection is the same as angle of incidence. This phenomena is called Specular Reflection. If the remitted energy is exactly the same in each direction, then the surface is ideally matte and is described as Lambertian [17].

2.2 Specular Reflection

Consider a linearly polarized plane wave that is propagating at angle $\theta$ in the $z$-direction. The case is made simpler by taking the incident plane as the $x-z$ plane, where the plane $z = 0$ separates the two media. In this thesis, two cases of parallel
and perpendicular polarization are considered. In a parallel polarization, the electric fields lie on the plane of incidence (at angle $\theta$ with the $x$-axis) and the magnetic fields are perpendicular to that plane. This is also known as Transverse Magnetic (TM) or $p$-polarization. In a perpendicular polarization, the magnetic fields lie on the plane of incidence (at angle $\theta$ with the $x$-axis) and the electric fields are perpendicular to the plane. This is also known as Transverse Electric (TE) or $s$-polarization.

The cases of TM and TE polarization of a laser ray incident obliquely on a smooth interface separating two media are shown in the Figure 2.1. The figure shows the incidence and the reflection plane coinciding with the $x - z$ plane where both the incident and the reflected angles are equal. These are consequences of the boundary conditions. In the figure, $\mathbf{k}$ is the wave vector in the incident plane ($x - z$ plane) with $\mathbf{k}_\pm = \hat{x}k_{x\pm} + \hat{z}k_{z\pm}$, $\mathbf{k}_+$ being the wave vector in the incident direction and $\mathbf{k}_-$ being the wave vector in the reflected direction. Likewise, $\mathbf{k}'$ is the wave vector in the refracted plane and has similar form to $\mathbf{k}$.

The total incident electric field in the $\mathbf{k}_+$ direction is $E_+e^{-j\mathbf{k}_+\cdot\mathbf{r}}$, the total reflected electric field in the $\mathbf{k}_-$ direction is $E_-e^{-j\mathbf{k}_-\cdot\mathbf{r}}$ and the total transmitted electric field in the $\mathbf{k}'_+$ direction is $E_+e^{-j\mathbf{k}'_+\cdot\mathbf{r}}$, where $E_+$ is the amplitude of the electric field in the $\mathbf{k}_+$ direction and $E_-$ is the amplitude of the electric field in the $\mathbf{k}_-$ direction. Since the wave is incident at an angle $\theta$ to the normal, it is separated into transverse (tangential or $x$-component) and longitudinal components with respect to the direction in which the dielectric is placed (the $z$-axis). The net transverse component of the electric field (denoted by the subscript $T$) must be continuous
Figure 2.1: Oblique Incidence of Parallel and Perpendicular Polarized Waves.
across the interface, which is the boundary condition and is true for both polarizations [18–20] i.e.

\[
E_{T,+}e^{-jk_{x}z} + E_{T,-}e^{-jk_{x}z} = E'_{T,+}e^{-jk'_{z}z}, \text{ at } z = 0
\]

\[
E_{T,+}e^{-jk_{x}z} + E_{T,-}e^{-jk_{x}z} = E'_{T,+}e^{-jk'_{z}z}
\]  \hspace{1cm} (2.1)

The net transverse field anywhere on either medium is given by:

\[
E_{T}(x,z) = E_{T,+}e^{-j(k_{x}x+k_{z}z)} + E_{T,-}e^{-j(k_{x}x+k_{z}z)}
\]

which is the sum of the incident and the reflected field in the transverse direction.

2.2.1 Oblique Incidence Field Calculations

In order to calculate the oblique angle of incidence, we consider an oblique incidence field which can be represented by a rotated coordinate system \((x', y', z')\) with rotation angle \(\theta\) with respect to the original \(x - z\) axis and the propagation direction being the new \(z'\)-axis (i.e \(k = z'\)). Fields \(E, H\) and \(K\) form a right-handed vector system, \(K\) being the wave vector (propagation direction) in the incident plane \((x - z\) plane). The \(y\)-axis remains stationary as shown in Figure 2.2.

Representing the fields \(E\) and \(H\) in amplitude and phase as:

\[
E(z', t) = E_{0}e^{-j\omega t - jk_{z}'z'}
\]

\[
H(z', t) = H_{0}e^{-j\omega t - jk_{z}'z'}
\]  \hspace{1cm} (2.2)

where amplitudes \(E_{0}\) and \(H_{0}\) are constant vectors transverse to the direction of propagation \(z'\) such that:

\[
H_{0} = \frac{1}{\eta} \hat{z}' \times E_{0}
\]
Figure 2.2: Rotation with Respect to the Original Axis.
where $\eta$ is the characteristic impedance of the medium in which the wave is propagating. The new coordinates are written in terms of the old coordinates as:

\[
\begin{align*}
    z' &= z \cos \theta + x \sin \theta \\
    x' &= x \cos \theta - z \sin \theta \\
    y' &= y
\end{align*}
\]

Also, $k_x = k \sin \theta$ and $k_z = k \cos \theta$. Now, we can write (2.1) as:

\[
\mathbf{E}_T e^{-jx k \sin \theta} + \mathbf{E}_{T-} e^{-jx k \sin \theta} = \mathbf{E}'_T e^{-jx' k' \sin \theta'}
\]  \hspace{1cm} (2.3)

For both sides of the interface to match at all points, the phase factors of the above equation must be equal

\[
e^{-jx k \sin \theta} = e^{-jx' k' \sin \theta'}
\]

This is in accordance with Snell’s law of reflection ($k \sin \theta = k \sin \theta$) and Snell’s law of refraction ($k \sin \theta = k' \sin \theta'$) in a plane ($x - z$ plane).

As seen from Figure 2.2a and 2.2b, we have transverse components (with respect to the new $z'$-axis) of the electric field along $x'$ (for TM) and along $y$ (for TE). Hence, $\mathbf{E}_0$ and corresponding $\mathbf{H}_0$ may be written as:

\[
\begin{align*}
    \mathbf{E}_0 &= \hat{x'} A + \hat{y'} B = (\hat{x} \cos \theta - \hat{z} \sin \theta) A + \hat{y'} B \\
    \mathbf{H}_0 &= \frac{1}{\eta} (\hat{y'} A - \hat{x'} B) = \frac{1}{\eta} [\hat{y'} A - (\hat{x} \cos \theta - \hat{z} \sin \theta) B]
\end{align*}
\]  \hspace{1cm} (2.4)

where $\eta$ is the characteristic impedance of the medium in which the wave is propagating. The components $A$ and $B$ are complex constants and are referred to as
transverse magnetic and transverse electric components of the electric field, respectively. The polarization property of the wave depends on its amplitude and relative phase. Combining Equations (2.2) and (2.4), we get the desired oblique fields.

\[ E(z', t) = [(\hat{x}\cos\theta - \hat{z}\sin\theta)A + \hat{y}B]e^{-j\omega t - jkz} \]  
\[ H(z', t) = \frac{1}{\eta}[\hat{y}A - (\hat{x}\cos\theta - \hat{z}\sin\theta)B]e^{-j\omega t - jkz} \] (2.5)

The incident fields may be written as:

\[ E_+(x, z) = [(\hat{x}\cos\theta - \hat{z}\sin\theta)A_+ + \hat{y}B_+]e^{-j(k_x x + k_z z)} \]  
\[ H_+(x, z) = \frac{1}{\eta}[\hat{y}A_+ - (\hat{x}\cos\theta - \hat{z}\sin\theta)B_+]e^{-j(k_x x + k_z z)} \] (2.6)

whose transverse fields are:

\[ E_{T+}(x, z) = (\hat{x}A_+\cos\theta + \hat{y}B_+)e^{-j(k_x x + k_z z)} \]  
\[ H_{T+}(x, z) = \frac{1}{\eta}(\hat{y}A_+ - \hat{x}B_+\cos\theta)e^{-j(k_x x + k_z z)} \] (2.7)

Similarly, the reflected field may be written as:

\[ E_-(x, z) = [(\hat{x}\cos\theta + \hat{z}\sin\theta)A_- + \hat{y}B_-]e^{-j(k_x x - k_z z)} \]  
\[ H_-(x, z) = \frac{1}{\eta}[-\hat{y}A_- - (\hat{x}\cos\theta - \hat{z}\sin\theta)B_-]e^{-j(k_x x - k_z z)} \] (2.8)

whose transverse fields are:

\[ E_{T-}(x, z) = (\hat{x}A_-\cos\theta + \hat{y}B_-)e^{-j(k_x x - k_z z)} \]  
\[ H_{T-}(x, z) = \frac{1}{\eta}(-\hat{y}A_- + \hat{x}B_-\cos\theta)e^{-j(k_x x - k_z z)} \] (2.9)

The oblique incidence fields are used to obtain the Fresnel Reflection Coefficients which are discussed in Section 2.3.
2.3 Determination of Fresnel Reflection Coefficients

Transverse Impedance is defined as the ratio of the transverse components of the electric and magnetic fields [8].

\[ \eta_{TM} = \frac{E_x}{H_y} = \frac{A \cos \theta}{\frac{1}{\eta} A} = \eta \cos \theta \]  
(2.10)

\[ \eta_{TE} = -\frac{E_y}{H_x} = \frac{B}{\frac{1}{\eta} B \cos \theta} = \frac{\eta}{\cos \theta} \]  
(2.11)

The transverse refractive index is given by \( n_T = \frac{n}{\eta_T} \). Hence, we can write:

\[ n_{TM} = \frac{n}{\cos \theta} \]  
(2.12)

\[ n_{TE} = n \cos \theta \]  
(2.13)

Fresnel Reflection Coefficients for both sides of interface, as defined by (5.3.5) of [8] are reproduced below:

\[ \rho = \frac{\eta' - \eta}{\eta' + \eta} \]  
(2.14)

\[ \rho' = \frac{\eta - \eta'}{\eta + \eta'} \]  
(2.15)

In terms of the transverse refractive index, the transverse reflection coefficients may be written as:

\[ \rho_T = \frac{n_T - n_T'}{n_T + n_T'} \]  
(2.16)

\[ \rho_T' = \frac{n_T' - n_T}{n_T' + n_T} \]  
(2.17)

where T represents either the TM or TE wave. Now, from Equations (2.12) and (2.13), we can write Equation (2.16) as follows:

\[ \rho_{TM} = \frac{n}{\cos \theta} - \frac{n'}{\cos \theta'} \]  
(2.18)
\[ \rho_{TE} = \frac{n \cos \theta - n' \cos \theta'}{n \cos \theta + n' \cos \theta'} \]  

Here, \( n \) and \( n' \) are the refractive indices of the medium on the left and the right side of the interface and \( \theta \) and \( \theta' \) are the incident and the transmitted angles, respectively. Equations (2.18) and (2.19) can be written in a number of equivalent ways with some modifications using trigonometric identities. One of them, with only the angle of incidence, derived using Snell’s law (\( n \sin \theta = n' \sin \theta' \)) is shown below:

\[ \rho_{TM} = \frac{\sqrt{(\frac{n'}{n})^2 - \sin^2 \theta - (\frac{n'}{n})^2 \cos \theta}}{\sqrt{(\frac{n'}{n})^2 - \sin^2 \theta + (\frac{n'}{n})^2 \cos \theta}} \]  

(2.20)

\[ \rho_{TE} = \frac{\cos \theta - \sqrt{(\frac{n'}{n})^2 - \sin^2 \theta}}{\cos \theta + \sqrt{(\frac{n'}{n})^2 - \sin^2 \theta}} \]  

(2.21)

If the medium on the left side of the interface is assumed to be air, we can approximate \( n = 1 \) and \( n' = n_d \) where \( n_d \) can be the complex-valued refractive index of the medium on the right of the interface. With this condition, rewriting Equations (2.20) and (2.21), we obtain:

\[ \rho_{TM} = \frac{\sqrt{n_d^2 - \sin^2 \theta - n_d^2 \cos \theta}}{\sqrt{n_d^2 - \sin^2 \theta + n_d^2 \cos \theta}} \]  

(2.22)

\[ \rho_{TE} = \frac{\cos \theta - \sqrt{n_d^2 - \sin^2 \theta}}{\cos \theta + \sqrt{n_d^2 - \sin^2 \theta}} \]  

(2.23)

Using this expressions for reflection coefficients for both the TE and TM waves, refractive indices for several smooth surface interfaces can be obtained.

Studies were done to explore the variation in reflection coefficients with change in the angle of incidence. Figure 2.3 shows the reflection coefficients for both the TE and TM waves for air-silver interface at the frequency of 1GHz. Figures
2.4 and 2.5 show the reflection coefficients for air-water interface for two different frequencies of 1GHz and 100 MHz, respectively. The figures show that the reflection coefficient for a TM wave is less linear with change in the angle of incidence. Figures 2.6 and 2.7 show TE and TM reflection coefficients versus the angle of incidence for the two cases $\eta_{d1} = 1.5 - j0.15$ and $\eta_{d2} = 1.5 - j0.30$ and compares them with the lossless case of $\eta_{d1} = 1.5$. (The values for $\eta_{d}$ were chosen only for plotting purposes and have no physical significance.) The comparisons show that regardless of the refractive index and the frequency of operation, the non-linear relationship in the TM case is more prominent than in the TE case. Hence, most of our calculations are shown for the TM case, for smooth and rough surfaces reflection in the next sections, though the method is equally valid for the TE case.
2.4 Determination of Complex Refractive Index by Varying Angles of Incidence

Real materials have Complex Refractive Indices, the real part of which indicate the phase speed while the imaginary part is the extinction coefficient which indicates the amount of absorption loss when the electromagnetic wave propagates through the material. The Refractive Index is unique to a material and depends upon the frequency of the electromagnetic wave as well as temperature of operation. If we are able to know the refractive index of any object, we can identify the material. Table 2.1 shows the complex refractive indices of a few metals at 830 nm [21,22].

We begin our work of calculating refractive index ($\eta_d$) by making use of Equation (2.23). To simplify the case, we have not considered the temperature dependence of the refractive index. As we know that $\eta_d$ depends on the angle of incidence ($\theta$), first we use this dependence to calculate the complex refractive index. We then vary frequencies of the incident wave and again calculate the complex refractive index of
<table>
<thead>
<tr>
<th>metal</th>
<th>wavelength (nm)</th>
<th>Refractive Index (Re)</th>
<th>Refractive Index (Im)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>830</td>
<td>2.6712</td>
<td>8.093</td>
</tr>
<tr>
<td>Beryllium</td>
<td>830</td>
<td>3.3901</td>
<td>3.4384</td>
</tr>
<tr>
<td>Chromium</td>
<td>830</td>
<td>4.2207</td>
<td>4.2868</td>
</tr>
<tr>
<td>Copper</td>
<td>830</td>
<td>0.2389</td>
<td>5.1805</td>
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<tr>
<td>Gold</td>
<td>830</td>
<td>0.2283</td>
<td>4.7373</td>
</tr>
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<td>830</td>
<td>2.5742</td>
<td>3.549</td>
</tr>
<tr>
<td>Tungsten</td>
<td>830</td>
<td>3.5103</td>
<td>2.8031</td>
</tr>
</tbody>
</table>

Table 2.1: Refractive Indices of Different Metals at 830 nm.
The refractive index is a complex quantity (having both real and imaginary parts), but only the real reflection coefficient (amplitude) can be measured through experiments [23]. Hence, we need two equations to solve the two unknowns of the refractive index. In this section, we proceed to calculate the refractive index of a material making the frequency of operation fixed and changing the angle of incidence.

2.4.1 Case I: Perpendicular Polarized Wave

The relationship of the reflection coefficient of a Transverse Electric Wave ($\rho_{TE}$) with the angle of incidence ($\theta$) and refractive index of the material ($\eta_d$) is given by

$$\left| \rho_{TE} \right| = \left| \frac{\cos \theta - \sqrt{\eta_d^2 - \sin^2 \theta}}{\cos \theta + \sqrt{\eta_d^2 - \sin^2 \theta}} \right|$$

(2.24)
Here, we have considered the magnitude of the reflection coefficient as it is the only parameter that can be directly measured through experiments. Let

$$\sqrt{\eta_d^2 - \sin^2 \theta} = P + jQ$$

With $\eta_d = x + jy$:

$$P^2 - Q^2 = x^2 - y^2 - \sin^2 \theta$$

$$2PQ = 2xy$$

Solving for $P$ with $Q = \frac{xy}{P}$, we get:

$$P = \sqrt{\frac{(x^2 - y^2 - \sin^2 \theta) + \sqrt{(x^2 - y^2 - \sin^2 \theta)^2 + 4x^2y^2}}{2}}$$

which only depends on the angle of incidence $\theta$. The equation for $\rho_{TE}$ in terms of $P$ and $Q$ is given by:

$$|\rho_{TE}| = \left| \frac{\cos \theta - (P + jQ)}{\cos \theta + (P + jQ)} \right|$$

$$\rho_{TE}^2 = \frac{(\cos \theta - P)^2 + Q^2}{(\cos \theta + P)^2 + Q^2}$$  \hspace{1cm} (2.25)
Given the $\rho_{TE}$ for any two incident angles $\theta$, Equation (2.25) can be solved for the refractive index, $\eta_d (x + jy)$, of the material by substituting $P$ and $Q$.

2.4.2 Case II: Parallel Polarized Wave

The relationship of the reflection coefficient of a Transverse Magnetic Wave $\rho_{TM}$ with the angle of incidence $\theta$ and the refractive index of the material $\eta_d$ is given by:

$$| \rho_{TM} | = \left| \frac{\sqrt{\eta_d^2 - \sin^2 \theta - \eta_d^2 \cos \theta}}{\sqrt{\eta_d^2 - \sin^2 \theta + \eta_d^2 \cos \theta}} \right| \quad (2.26)$$

Let $\eta = \eta_d^2 = x + jy$. Hence, we get:

$$| \rho_{TM} | = \left| \frac{\sqrt{\eta - \sin^2 \theta - \eta \cos \theta}}{\sqrt{\eta - \sin^2 \theta + \eta \cos \theta}} \right|$$

Again, let:

$$\sqrt{\eta - \sin^2 \theta} = P + jQ$$

$$P^2 - Q^2 = x - \sin^2 \theta$$
Solving for $P$ with $Q = \frac{y}{2P}$, we get:

$$P = \sqrt{\frac{(x - \sin^2 \theta) + \sqrt{(x - \sin^2 \theta)^2 + y^2}}{2}}$$ \hspace{1cm} (2.27)

Which only depends on the angle of incidence $\theta$. The equation for $\rho_{TM}$ in terms of $P$ and $Q$ is given by

$$| \rho_{TM} | = \left| \frac{(P - x \cos \theta) + j(Q - y \cos \theta)}{(P + x \cos \theta) + j(Q + y \cos \theta)} \right|$$

$$\rho_{TM}^2 = \frac{(P - x \cos \theta)^2 + (Q - y \cos \theta)^2}{(P + x \cos \theta)^2 + (Q + y \cos \theta)^2}$$ \hspace{1cm} (2.28)

Given the $\rho_{TM}$ for any two incident angles $\theta$, Equation (2.28) can be solved for $\eta (x + jy)$ by substituting $P$ and $Q$, which then give the refractive index, $\eta_d$, of the material.

2.5 Determination of Electrical Permittivity and Conductivity by Varying Frequency

This method is based on varying the frequency of operation with a constant angle of Incidence. Analysis and calculations are shown only for the parallel polarization (TM) case. Similar calculations can be performed for the perpendicular polarization (TE) case to get the desired results.

2.5.1 Case I: Normal Angle of Incidence

We can find the permittivity $\epsilon'$ of a conducting material with conductivity $\sigma_m$ given the reflection coefficient $\rho_{TM}$ at two operating frequencies $\omega_1$ and $\omega_2$. Using Equation
(2.26) at normal angle of incidence,

$$| \rho_{TM} | = \left| \frac{1 - \eta_d}{1 + \eta_d} \right|$$

With $$\epsilon' = (\epsilon_m \epsilon_0 - j \frac{\sigma_m}{\omega})$$ and $$\eta_d = \sqrt{\frac{\epsilon'}{\epsilon_0}}$$, we get:

$$\eta_d = \sqrt{\frac{\epsilon_m - j \frac{\sigma_m}{\omega \epsilon_0}}{\epsilon_0}}$$  \hspace{1cm} (2.29)

Hence, we can write,

$$| \rho_{TM} | = \left| \frac{1 - \sqrt{\epsilon_m - j \frac{\sigma_m}{\omega \epsilon_0}}}{1 + \sqrt{\epsilon_m - j \frac{\sigma_m}{\omega \epsilon_0}}} \right|$$  \hspace{1cm} (2.30)

Let $$P + jQ = \sqrt{\epsilon_m - j \frac{\sigma_m}{\omega \epsilon_0}}$$:

$$P^2 - Q^2 = \epsilon_m = x (let)$$

$$2PQ = \frac{-\sigma_m}{\omega \epsilon_0}$$

Solving the above two equations for $$P$$, with $$Q = \frac{-\sigma_m}{2 \omega \epsilon_0} \frac{1}{P} = \frac{-y}{P}$$, we get

$$P = \sqrt{\frac{x + \sqrt{x^2 + \frac{4y^2}{\omega^2}}}{2}}$$

which only depends on the frequency of operation. The equation for $$\rho_{TM}$$ in terms of $$P$$ and $$Q$$ is given by:

$$\rho_{TM}^2 = \frac{(1 - P)^2 + Q^2}{(1 + P)^2 + Q^2}$$  \hspace{1cm} (2.31)

which can be obtained from Equation (2.30) by taking the absolute value of the expression. Given the $$\rho_{TM}$$ for any two values of operating frequencies, Equation (2.31) can be solved to find $$x$$ and $$y$$, which will then give the $$\epsilon_m$$ and $$\sigma_m$$ of the material.
2.5.2 Case II: Angle of Incidence other than Normal

The permittivity, $\epsilon'$, of a conducting material with conductivity, $\sigma_m$, can be found given the reflection coefficient, $\rho_{TM}$, at two operating frequencies, $\omega_1$ and $\omega_2$, and any angle of incidence, $\theta$. Rewriting Equation (2.26),

$$|\rho_{TM}| = \left| \frac{\sqrt{\eta_d^2 - \sin^2 \theta} - \eta_d^2 \cos \theta}{\sqrt{\eta_d^2 - \sin^2 \theta} + \eta_d^2 \cos \theta} \right|$$  \hspace{1cm} (2.32)

Substituting $\eta_d$ from Equation (2.29), Equation (2.32) can be rewritten as,

$$|\rho_{TM}| = \left| \frac{\sqrt{\epsilon_m - j \frac{\sigma_m}{\omega \epsilon_0} - \sin^2 \theta - (\epsilon_m - j \frac{\sigma_m}{\omega \epsilon_0}) \cos \theta}}{\sqrt{\epsilon_m - j \frac{\sigma_m}{\omega \epsilon_0} - \sin^2 \theta + (\epsilon_m - j \frac{\sigma_m}{\omega \epsilon_0}) \cos \theta}} \right|$$  \hspace{1cm} (2.33)

Now, let:

$$\sqrt{(\epsilon_m - \sin^2 \theta) - j \frac{\sigma_m}{\omega \epsilon_0}} = P + jQ$$

$$P^2 - Q^2 = \epsilon_m - \sin^2 \theta$$

$$2PQ = -\frac{\sigma_m}{\omega \epsilon_0}$$

Solving with $Q = -\frac{\sigma_m}{2P \epsilon_0 \omega}$ we get,

$$P = \sqrt{\frac{(\epsilon_m - \sin^2 \theta) + \sqrt{(\epsilon_m - \sin^2 \theta)^2 + \frac{y^2}{\omega^2}}}{2}}$$  \hspace{1cm} (2.34)

where $y = \frac{\sigma_m}{\epsilon_0}$. Equation (2.32) in terms of $P$ and $Q$ is written as,

$$\rho_{TM}^2 = \frac{(P - \epsilon_m \cos \theta)^2 + (Q + \frac{y \cos \theta}{\omega})^2}{(P + \epsilon_m \cos \theta)^2 + (Q - \frac{y \cos \theta}{\omega})^2}$$  \hspace{1cm} (2.35)

which can be obtained from Equation (2.33) by taking the absolute value of the expression. Given, the $\rho_{TM}$ for any two values of operating frequencies and particular angle of incidence, $\theta$, Equation (2.35) can be solved to find $\epsilon_m$ and $y$ (which will then
give the $\sigma_m$ of the material by substituting $P$ and $Q$ from the above equations. The calculated values can then be compared with the ideal values of the complex refractive indices [21, 22]. Figure 2.6 shows the summary of the logic used in our calculation process.
Figure 2.8: Design Flow for the Material Properties Calculation.

1. Start
2. Calculate $E$ and $H$ fields from given angle of Incidence
3. Calculate reflection coefficients from (2.23) and (2.24) for given $\theta$
4. Calculate $\varepsilon$ and $\sigma$
   - No: From (2.28) Vary angle $\theta$ to get two equations for two unknowns ($x$ and $y$) of refractive index
   - Yes: With constant $\theta$, vary the frequency in (2.30) to get two equations for two unknowns $\varepsilon$ and $\sigma$ of the material
5. Solve non linear equations to get refractive index.
6. Stop
7. Solve non linear equations to get $\varepsilon$ and $\sigma$.
8. Stop
CHAPTER III
MATERIAL PROPERTIES DETECTION FOR ROUGH SURFACES

3.1 Overview

In the analysis made so far, materials with plane smooth surfaces are considered. These surfaces have a negligible variation in surface roughness with respect to the angle of incidence and the frequency of the laser beam. However, for a rough surface, these variations are no longer negligible [24]. The analysis and the calculations of the surface parameters are shown in the following sections.

3.2 Determination of Scattering Coefficient

Scattering from a rough surface depends upon the relative size of the reflecting surface particle and the wavelength $\lambda$ of the incident radiation [8, 24]. If the particle size is much larger than the light’s wavelength, the reflection can be considered specular whereas the reflection from particles much smaller than the wavelength of the light follow Rayleigh-like scattering (increasing with decrease in wavelength) [23]. If the particle size is of the same order as $\lambda$, the scattering is described by Mie’s Theory, where particles are distant enough from each other so as not to interact [17]. The intermediate cases are more complex and cannot be treated by these theories.
The surface roughness and the angle of incidence of light have a higher effect on the directional distribution and polarization of the reflected light [24]. In this thesis, a simpler approach to calculate the roughness of the reflecting surface is used where it is assumed that the surface has a uniform roughness so that the scattering coefficient can be taken as a real constant.

Recalling the expression for the reflection coefficient for the transverse magnetic case,

\[ \rho_{TM} = \frac{E_r^x}{E_i^x} \]  

where \( E_r^x \) is the reflected electric field and \( E_i^x \) is the incident electric field in the \( x \)-direction, with the same angle of incidence and reflection. Equation (3.1) can be viewed as:

\[ \rho_{TM} = s \frac{E_r^x}{E_i^x} \]  

where \( s = 1 \). This represents a perfectly smooth reflecting surface. When the surface is not smooth, and has certain uniform roughness \( (s \neq 1, 0 < s < 1) \), the reflected energy, and hence, the reflection coefficient, at an angle equal to the angle of incidence, will be much less than that of the case of the smooth surface. We have used this idea to calculate the parameter ‘\( s \)’. From Equation (3.2) we can write Equations (2.22) and (2.23) as follows:

\[ \rho_{TM} = s \sqrt{\frac{n_d^2 - \sin^2 \theta - n_d^2 \cos \theta}{n_d^2 - \sin^2 \theta + n_d^2 \cos \theta}} \]  

\[ \rho_{TE} = s \frac{\cos \theta - \sqrt{n_d^2 - \sin^2 \theta}}{\cos \theta + \sqrt{n_d^2 - \sin^2 \theta}} \]  

Figure 3.1 represents reflection in a uniform rough surface.
We shall now introduce the Scattering Coefficient \( \sigma \), an unknown quantity, which is assumed to be a constant at one angle of incidence irrespective of frequencies, in expressions of the reflection coefficient and all derived expressions. This will aid us in obtaining the refractive index, \( \eta_d \), from the absolute value of the \( \rho_{TM} \) scattered at an angle equal to the angle of incidence. \([24]\)

3.3 Determination of Material Properties for a Rough Surface

The analysis for rough surfaces is carried out for only the parallel polarized waves. However, this is a generalized method and a similar approach and calculations can be made for the perpendicular polarized wave to get the desired results.

With the presence of the parameter \( \sigma \), three non-linear equations are required in order to solve three unknown quantities. This is accomplished by consider-
ing the case of parallel polarization. Now that we have three unknowns to solve, we need three equations, which can be written with three different frequencies. We can then find ‘s’ by solving these equations.

Rewriting Equation (2.33) by introducing the scattering coefficient, we get:

\[
| \rho_{TM} | = s \left| \frac{\sqrt{\epsilon_m - j \frac{\sigma_m}{\omega \epsilon_0} - \sin^2 \theta - (\epsilon_m - j \frac{\sigma_m}{\omega \epsilon_0}) \cos \theta}}{\sqrt{\epsilon_m - j \frac{\sigma_m}{\omega \epsilon_0} - \sin^2 \theta + (\epsilon_m - j \frac{\sigma_m}{\omega \epsilon_0}) \cos \theta}} \right| \tag{3.5}
\]

It is observed that at an angle of incidence (\(\theta\)) the constant \(s\) and \(\rho_{TM}\) depend only on the frequency of the incident light. Since we have three unknowns to solve (\(s\), \(\epsilon_m\) and \(\sigma_m\)), we need to have three equations, which can be obtained from Equation (3.5) by varying the frequencies,

\[
\rho_{TM}^2 = s^2 \frac{(P - \epsilon_m \cos \theta)^2 + (Q + \frac{\omega \epsilon_0}{\omega \epsilon_0})^2}{(P + \epsilon_m \cos \theta)^2 + (Q - \frac{\omega \epsilon_0}{\omega \epsilon_0})^2} \tag{3.6}
\]

where \(P\) and \(Q\) are the same as in Equation (2.34). Solving these three non-linear equations yields the three unknowns; that are, \((s, \epsilon_m\) and \(\sigma_m\)).
CHAPTER IV

EXPERIMENT SETUP AND RESULTS

4.1 Setup

A series of experiments were carried out to measure the reflection coefficient of a laser beam target at a 0.5" thick aluminium slab. The wavelength of the laser beam was 830 nm. The selection of 0.5" thickness was made to ensure the condition $t >> \lambda$, where $t$ is the thickness of the slab and $\lambda$ is the wavelength of the laser beam. Figure 4.1 shows the experiment setup for the measurement of the reflection coefficient. The setup consists of a laser source, an optical chopper, a polarizer, a half-wave retarder, a beam splitter, the target aluminium slab and a detector.

In the experiment, the laser beam was passed through an optical chopper. The primary function of the optical chopper is to modulate the beam in order to reject background noise. The modulation was carried out at 968 Hz. The modulated light was passed through a polarizer which was oriented to get the linearly polarized TE beam. The beam was then passed through a beam splitter to hit the target. The reflected beam was received through the beam splitter and finally received at the detector. The detected signal intensities were observed on an oscilloscope.

The properties of the aluminium slab were determined from the experiment
setup described above. The results obtained from the experiments are compared with the properties calculated from the equations derived in Chapter II for a smooth aluminum interface.

4.2 Properties Calculation for Air-Aluminum Interface

For an air-aluminum interface, $|\rho_{TE}|$ at any angle for a given $\eta_d(at 650 nm) = 1.4204 + j7.4673$ is found by using Equation (2.24). For two angles of incidence, $\theta_1 = 5^0$ and $\theta_2 = 45^0$, the reflection coefficients are $\rho_{TE}^2(5^0) = 90.81 \times 10^{-2}$ and $\rho_{TE}^2(45^0) = 93.41 \times 10^{-2}$. From Equation (2.25),

$$90.81 \times 10^{-2} = \frac{(\cos 5^0 - P1)^2 + Q1^2}{(\cos 5^0 + P1)^2 + Q1^2} \quad (4.1)$$

$$93.41 \times 10^{-2} = \frac{(\cos 45^0 - P2)^2 + Q2^2}{(\cos 45^0 + P2)^2 + Q2^2} \quad (4.2)$$
<table>
<thead>
<tr>
<th>Angle(^{(0)})</th>
<th>-0.02(^{0})</th>
<th>-0.01(^{0})</th>
<th>0(^{0})</th>
<th>0.01(^{0})</th>
<th>0.02(^{0})</th>
</tr>
</thead>
<tbody>
<tr>
<td>ReflectedIntensity((V))</td>
<td>6.116</td>
<td>6.088</td>
<td>6.218</td>
<td>6.142</td>
<td>6.010</td>
</tr>
<tr>
<td>Angle(^{(0)})</td>
<td>4.98(^{0})</td>
<td>4.99(^{0})</td>
<td>5(^{0})</td>
<td>5.01(^{0})</td>
<td>5.02(^{0})</td>
</tr>
<tr>
<td>ReflectedIntensity((V))</td>
<td>5.748</td>
<td>5.821</td>
<td>5.762</td>
<td>5.791</td>
<td>5.829</td>
</tr>
</tbody>
</table>

Table 4.1: Experimental Results of Reflected Wave Intensities at 0\(^{0}\) and 5\(^{0}\).

In Equations (4.1) (4.2), a Matlab program was executed to solve the two equations and the complex refractive index was obtained as \(\eta_d = 1.4204 + j7.4673\).

4.3 Experiment Results for Air-Aluminum Interface

The experiment was conducted for two angles of incidence 0\(^{0}\) and 5\(^{0}\). Using the setup described above, the incident wave of 8\(V\) was target at the object. The angle of incidence was varied \(\pm 0.02^{0}\) from the base angle of incidence 0\(^{0}\) and 5\(^{0}\).

Table 4.1 shows the measured intensity of the reflected wave at two different angles of incidence. The average values of the reflected wave intensities were 6.092 \(V\) for 0\(^{0}\) and 5.791 \(V\) for 5\(^{0}\). The reflection coefficients were calculated as \(\rho_{TE}^2 = 0.7616\) at 0\(^{0}\) and \(\rho_{TE}^2 = 0.7616\) for 5\(^{0}\). Using the reflection coefficients, the refractive index of the aluminum slab was calculated for both angles.

\[
\eta_d = 2.35 + j7.27.
\]
Comparing this value with the refractive index of a smooth pure aluminum, $\eta_d = 2.67 + j8.09$ from Table 2.1, the percentage error is 11%. The impurities in the aluminum slab can be attributed to the reasons behind the higher percentage error. The commercially available aluminum contains about 90% of pure aluminum. The impurities in the slab can cause the actual results to deviate from the ideal values. The other reason for the error can be the imperfection of the laser beam. The laser used had a beam pattern that was not perfectly Gaussian. Hence, the spread of the incident intensity of the laser beam might also result the deviation from the ideal values.

4.4 Properties Calculation for Smooth Surface Interfaces

The properties of several smooth surface interfaces such as air-water and air-aluminum are carried out for both the TE and TM waves. The calculation results based on the proposed method are shown in the following subsections.

4.4.1 Air-Aluminum Interface-TM Wave

Considering the air-aluminum interface, $|\rho_{TM}|$ at any angle for a given $\eta_d(al) = 1.4204 + j7.4673$ is found by using Equation (2.26). Let us take the two angles of incidence to be $\theta_1 = 20^0$ and $\theta_2 = 64^0$ for which the reflection coefficients are $\rho_{TM}^2(20^0) = 90.21 \times 10^{-2}$ and $\rho_{TM}^2(64^0) = 81.02 \times 10^{-2}$. Now, we obtain two equations (from Equation (2.28)) and we have two unknowns to solve for the refractive index $\eta_d$. 
\[ 90.21 \times 10^{-2} = \frac{(P_1 - x \cos 20^0)^2 + (Q_1 - y \cos 20^0)^2}{(P_1 + x \cos 20^0)^2 + (Q_1 + y \cos 20^0)^2} \]  \hspace{1cm} (4.3) \\
\[ 81.02 \times 10^{-2} = \frac{(P_2 - x \cos 64^0)^2 + (Q_2 - y \cos 64^0)^2}{(P_2 + x \cos 64^0)^2 + (Q_2 + y \cos 64^0)^2} \]  \hspace{1cm} (4.4) \\

For Equations (4.7) (4.8), a matlab program was executed to solve for \( \eta_d = x + jy \).

The refractive index was found to be \( \eta_d = 1.42 + j7.46 \).

### 4.4.2 Air-Water Interface- TE wave

The reflection coefficient, \( |\rho_{TE}| \), at any angle for a given \( \eta_d(1GHz) = 9.73 - j3.69 \) is found by using Equation (2.24). Taking \( \theta_1 = 5^0 \) and \( \theta_2 = 45^0 \), the reflection coefficients are \( \rho_{TE}^2(5^0) = 69.86 \times 10^{-2} \) and \( \rho_{TE}^2(45^0) = 77.52 \times 10^{-2} \).

\[ 69.86 \times 10^{-2} = \frac{(\cos 5^0 - P_1)^2 + Q_1^2}{(\cos 5^0 + P_1)^2 + Q_1^2} \]  \hspace{1cm} (4.5) \\
\[ 77.52 \times 10^{-2} = \frac{(\cos 45^0 - P_2)^2 + Q_2^2}{(\cos 45^0 + P_2)^2 + Q_2^2} \]  \hspace{1cm} (4.6) \\

Solving (4.5) (4.6), \( \eta_d \) was found to be 9.73-j3.69.

### 4.4.3 Air-Water Interface-TM wave

Considering the Air-Water interface with a TM wave, \( |\rho_{TM}| \) at any angle for a given \( \eta_d(water) = 9.73 - j3.69 \) is found by using Equation (2.26). For \( \theta_1 = 20^0 \) and \( \theta_2 = 64^0 \), the reflection coefficients are \( \rho_{TM}^2(20^0) = 68.17 \times 10^{-2} \) and \( \rho_{TM}^2(64^0) = 43.83 \times 10^{-2} \).

Now, we obtain two equations (from Equation (2.28)) and two unknowns to solve for the refractive index \( \eta_d \)

\[ 68.17 \times 10^{-2} = \frac{(P_1 - x \cos 20^0)^2 + (Q_1 - y \cos 20^0)^2}{(P_1 + x \cos 20^0)^2 + (Q_1 + y \cos 20^0)^2} \]  \hspace{1cm} (4.7)
\[43.83 \times 10^{-2} = \frac{(P2 - x \cos 64^0)^2 + (Q2 - y \cos 64^0)^2}{(P2 + x \cos 64^0)^2 + (Q2 + y \cos 64^0)^2}\]  \hspace{1cm} (4.8)

Solving Equations (4.7) (4.8) using Matlab, the refractive index \(\eta_d = x + jy\) was found to be \(\eta_d = 9.73 - j3.69\).

In addition, \(\epsilon_m\) and \(\sigma_m\) can also be calculated for water, \(\epsilon_m = 81\) and \(\sigma_m = 4\). Taking \(\omega_1 = \frac{2*\pi}{\lambda_1}\) and \(\omega_2 = \frac{2*\pi}{\lambda_2}\), where \(\lambda_1 = 400\,\text{nm}\) and \(\lambda_2 = 750\,\text{nm}\) and using these values in Equation (2.30), we get \(|\rho_{TM}|\) at two given frequencies,

\[\rho_{TM1}^2 = 98.34 \times 10^{-2}\]

\[\rho_{TM2}^2 = 98.78 \times 10^{-2}\]

Now, we get two equations (from Equation(2.31)) and we have two unknowns to solve for the permittivity \(\epsilon_m\) and the conductivity \(\sigma_m\).

\[98.34 \times 10^{-2} = \frac{(1 - P1)^2 + Q1^2}{(1 + P1)^2 + Q1^2}\]  \hspace{1cm} (4.9)

\[98.78 \times 10^{-2} = \frac{(1 - P2)^2 + Q2^2}{(1 + P2)^2 + Q2^2}\]  \hspace{1cm} (4.10)

Substituting \(Q1 = \frac{-y}{P1\omega_1}\), \(Q2 = \frac{-y}{P2\omega_2}\),

\[P1 = \sqrt{\frac{x + \sqrt{x^2 + \frac{4y^2}{\omega_1^2}}}{2}}\]

and,

\[P2 = \sqrt{\frac{x + \sqrt{x^2 + \frac{4y^2}{\omega_2^2}}}{2}}\]
4.4.4 Air-water interface for Angle of Incidence Other than Normal

Let us take the example for the air-water interface. For water, \( \epsilon_m = 81 \) and \( \sigma_m = 4 \).

Taking \( \omega_1 = \frac{2 \times \pi}{\lambda_1} \) and \( \omega_2 = \frac{2 \times \pi}{\lambda_2} \), where \( \lambda_1 = 400 \text{nm} \) and \( \lambda_2 = 750 \text{nm} \), the angle of incidence \( \theta = 5^0 \) and using these values in Equation (2.33), we get \( \rho_{TM}^2 \),

\[
\rho_{TM\omega_1}^2 = 98.337 \times 10^{-2}
\]

\[
\rho_{TM\omega_2}^2 = 98.78 \times 10^{-2}
\]

Now, we get two equations (from Equation(2.35)) and we have two unknowns to solve for the permittivity \( \epsilon_m \) and the conductivity \( \sigma_m \).

\[
98.33 \times 10^{-2} = \frac{(P_1 - \epsilon_m \cos 5^0)^2 + (Q_1 + \frac{y_2 \cos 5^0}{\omega_1})^2}{(P_1 + \epsilon_m \cos 5^0)^2 + (Q_1 - \frac{y_2 \cos 5^0}{\omega_1})^2} \tag{4.11}
\]

\[
98.78 \times 10^{-2} = \frac{(P_2 - \epsilon_m \cos 5^0)^2 + (Q_2 + \frac{y_2 \cos 5^0}{\omega_2})^2}{(P_2 + \epsilon_m \cos 5^0)^2 + (Q_2 - \frac{y_2 \cos 5^0}{\omega_2})^2} \tag{4.12}
\]

Substituting \( Q_1 = \frac{-y_2}{2P_1 \omega_1} \), \( Q_2 = \frac{-y_2}{2P_2 \omega_2} \),

\[
P_1 = \sqrt{\frac{(\epsilon_m - \sin^2 5^0) + \sqrt{(\epsilon_m - \sin^2 5^0)^2 + \frac{y_2^2}{\omega_1^2}}}{2}}
\]

and,

\[
P_2 = \sqrt{\frac{(\epsilon_m - \sin^2 5^0) + \sqrt{(\epsilon_m - \sin^2 5^0)^2 + \frac{y_2^2}{\omega_2^2}}}{2}}
\]

in Equations (4.11) (4.12), a Matlab program was executed to solve the two equations for \( \epsilon_m \) and \( y \) (which then give \( \sigma_m \)) of the material. Table 4.2 shows the calculated results for the smooth surface interfaces for different conditions.
<table>
<thead>
<tr>
<th>Smooth Surface</th>
<th>λ (nm)</th>
<th>Interface</th>
<th>∠ (°)</th>
<th>ρ²</th>
<th>η₁d</th>
</tr>
</thead>
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<tr>
<td>Angle Variation</td>
<td>(ρₑₑ)</td>
<td>0.3m air-water</td>
<td>5/45</td>
<td>0.69/0.77</td>
<td>9.73-j3.69</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>650 air-aluminum</td>
<td>5/45</td>
<td>0.90/0.93</td>
<td>1.42+j7.46</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(ρ₉₉)</td>
<td>0.3m air-water</td>
<td>20/64</td>
<td>0.68/0.43</td>
<td>9.73-j3.69</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>650 air-aluminum</td>
<td>20/64</td>
<td>0.90/0.81</td>
<td>1.42+j7.46</td>
<td></td>
</tr>
<tr>
<td>Frequency Variation</td>
<td>(ρ₉₉)</td>
<td>0° air-water</td>
<td>400/750</td>
<td>0.98/0.97</td>
<td>81/4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5° air-water</td>
<td>400/750</td>
<td>0.98/0.97</td>
<td>81/4</td>
</tr>
</tbody>
</table>

Table 4.2: Results Summary for Smooth Surfaces.
4.5 Properties Calculation for a Rough Surface-Air Interface

Let us take the example of any arbitrary interface for which, $\epsilon_m = 81$ and $\sigma_m = 400$.

Taking $\omega_1 = \frac{2 \pi}{\lambda_1}$, $\omega_2 = \frac{2 \pi}{\lambda_2}$ and $\omega_3 = \frac{2 \pi}{\lambda_3}$ where $\lambda_1 = 400\text{nm}$, $\lambda_2 = 750\text{nm}$ and $\lambda_3 = 550\text{nm}$, the angle of incidence $\theta = 5^0$ and the scattering coefficient $s = 0.8$.

Using these values in Equation (3.5), we get $\rho_{TM}^2$,

$$\rho_{TM\omega_1}^2 = 63.89 \times 10^{-2}$$

$$\rho_{TM\omega_2}^2 = 63.92 \times 10^{-2}$$

$$\rho_{TM\omega_3}^2 = 63.90 \times 10^{-2}$$

Now, we get three equations (from Equation(3.6)) and we have three unknowns to solve for the diffuse parameter $s$, the permittivity $\epsilon_m$ and the conductivity $\sigma_m$.

$$63.89 \times 10^{-2} = s^2 \left( P1 - \epsilon_m \cos 5^0 \right)^2 + \left( Q1 + \frac{y \cos 5^0}{\omega_1} \right)^2$$

$$63.92 \times 10^{-2} = s^2 \left( P2 - \epsilon_m \cos 5^0 \right)^2 + \left( Q2 + \frac{y \cos 5^0}{\omega_2} \right)^2$$

$$63.90 \times 10^{-2} = s^2 \left( P3 - \epsilon_m \cos 5^0 \right)^2 + \left( Q3 + \frac{y \cos 5^0}{\omega_3} \right)^2$$

Substituting $Q1 = \frac{-y}{2P1\omega_1}$, $Q2 = \frac{-y}{2P2\omega_2}$, $Q3 = \frac{-y}{2P3\omega_3}$,

$$P1 = \sqrt{\frac{\epsilon_m - \sin^2 5^0 + \sqrt{(\epsilon_m - \sin^2 5^0)^2 + \frac{y^2}{\omega_1}}}{2}}$$

$$P2 = \sqrt{\frac{\epsilon_m - \sin^2 5^0 + \sqrt{(\epsilon_m - \sin^2 5^0)^2 + \frac{y^2}{\omega_2}}}{2}}$$

$$P3 = \sqrt{\frac{\epsilon_m - \sin^2 5^0 + \sqrt{(\epsilon_m - \sin^2 5^0)^2 + \frac{y^2}{\omega_3}}}{2}}$$
Table 4.3: Results Summary for Rough Surfaces.

<table>
<thead>
<tr>
<th>Frequency Variation ((\rho_{TM}))</th>
<th>(\angle(0)) Interface</th>
<th>(\lambda) (nm)</th>
<th>(\rho^2)</th>
<th>(\epsilon/\sigma/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>5(^0) arbitrary</td>
<td>750 0.639</td>
<td>81/400/0.8</td>
<td></td>
</tr>
<tr>
<td>550</td>
<td>0.639</td>
<td>750</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

and,

\[
P_3 = \sqrt{\frac{(\epsilon_m - \sin^2 5^0) + \sqrt{(\epsilon_m - \sin^2 5^0)^2 + \frac{y^2}{\omega^2}}}{2}}
\]

in Equations (4.13), (4.14) and (4.15), and dividing Equations (4.13) and (4.14) by (4.15), we can eliminate ‘s’ from these equations. Hence, we will finally have two equations and two unknowns. This can be solved in Matlab to get \(\epsilon_m\) and \(\sigma_m\) of the material. Now, knowing the refractive index as well as the reflection coefficient, Equation (3.5) can be solved to get the scattering coefficient. A Matlab program that clarifies the example, is presented in the Appendix. Table 4.3 shows the calculated results for a rough surface.
5.1 Summary

The proposed process for the calculation of the refractive index, the permittivity and the conductivity of the desired target based on the properties of reflected laser beam from a surface is shown through calculations and demonstrated through experiments. Starting from the relationship between the reflection coefficient and the refractive index, non-linear equations were formed which were then solved using Matlab. The execution of the program in Matlab was fast and stable and yielded consistent results.

Two types of polarization cases were mainly studied: parallel polarization and perpendicular polarization. The results were consistent for both polarizations. The method was further extended to a generalized rough surface which has diffuse reflections. A new parameter, ‘s’, was introduced, the latter is a scaling factor referred to as diffusion parameter, for reflection from a non-smooth surfaces. Finally, partial reflection in the incident direction was used to measure the scaling factor in that direction. The process provides a new mathematical way for calculating the properties of the target material and gives exact results using only the magnitude of the reflected wave, which is easily obtainable through experiments.
5.2 Future Work

The present work can be extended in multi-dimensional and multi-directional cases. In the current work, calculations of the incident fields are made in a single dimension to obtain the scaling factor in that direction. This can be extended further by studying the two dimensional case and the scaling factor can be calculated in all other directions as well. In addition, further research can be done to implement this technique in real-time applications such as moving vehicles.
BIBLIOGRAPHY


42


APPENDIX

MATLAB CODES

A.1 Air-Water Interface

% Calculation of Refractive Index for given rte at any given % angles for Air–Water Interface

clear all;
clc;
clear, syms x y;
theta1=20*pi/180;
theta2=60*pi/180;
nd=9.73-1i*3.69;
rtel=abs((cos(theta1)-sqrt(nd^2-sin(theta1)^2))/(cos(theta1)+
          sqrt(nd^2-sin(theta1)^2)));
rtel_sq=rtel^2
rtel2=abs((cos(theta2)-sqrt(nd^2-sin(theta2)^2))/(cos(theta2)+
          sqrt(nd^2-sin(theta2)^2)));
rtel2_sq=rtel2^2

% At 5 and 45 degree %

45
eq1 = '(((cos(5*pi/180) - (sqrt(((x^2-y^2-(sin(5*pi/180))^2) + 4*x^2*y^2))/2))^2 + 
(x^2*y^2)/(((x^2-y^2-(sin(5*pi/180))^2) + 4*x^2*y^2))/2)))^2 + 
((x^2*y^2)/(((x^2-y^2-(sin(5*pi/180))^2) + 4*x^2*y^2))/2)^2 + 
((x^2*y^2)/(((x^2-y^2-(sin(5*pi/180))^2) + 4*x^2*y^2))/2))) = 0.6986' ;

eq2 = '(((cos(45*pi/180) - (sqrt(((x^2-y^2-(sin(45*pi/180))^2) + 4*x^2*y^2))/2))^2 + 
(x^2*y^2)/(((x^2-y^2-(sin(45*pi/180))^2) + 4*x^2*y^2))/2)))^2 + 
((x^2*y^2)/(((x^2-y^2-(sin(45*pi/180))^2) + 4*x^2*y^2))/2)^2 + 
((x^2*y^2)/(((x^2-y^2-(sin(45*pi/180))^2) + 4*x^2*y^2))/2))) = 0.7752' ;

% result = maple('fsolve', [eq1, eq2])
% At 20 and 60 degree %

eq1 = '(((cos(20*pi/180) - (sqrt(((x^2-y^2-(sin(20*pi/180))^2) + 4*x^2*y^2))/2))^2 + 
(x^2*y^2)/(((x^2-y^2-(sin(20*pi/180))^2) + 4*x^2*y^2))/2)))^2 + 
((x^2*y^2)/(((x^2-y^2-(sin(20*pi/180))^2) + 4*x^2*y^2))/2)^2 + 
((x^2*y^2)/(((x^2-y^2-(sin(20*pi/180))^2) + 4*x^2*y^2))/2))) = 0.6986' ;

eq2 = '(((cos(45*pi/180) - (sqrt(((x^2-y^2-(sin(45*pi/180))^2) + 4*x^2*y^2))/2))^2 + 
(x^2*y^2)/(((x^2-y^2-(sin(45*pi/180))^2) + 4*x^2*y^2))/2)))^2 + 
((x^2*y^2)/(((x^2-y^2-(sin(45*pi/180))^2) + 4*x^2*y^2))/2)^2 + 
((x^2*y^2)/(((x^2-y^2-(sin(45*pi/180))^2) + 4*x^2*y^2))/2))) = 0.7752' ;

% result = maple('fsolve', [eq1, eq2])
% At 20 and 60 degree %
\[
\frac{(x^2 + y^2)}{(\sqrt{(x^2 - y^2 - \sin(20\pi/180)^2) + 4x^2y^2})} + \frac{(\cos(20\pi/180) + \sqrt{(x^2 - y^2 - \sin(20\pi/180)^2) + 4x^2y^2})^2}{2} = 0.7129;
\]

\[
eq 2 = \frac{(\cos(60\pi/180) - \sin(60\pi/180))}{\sqrt{(x^2 - y^2 - \sin(60\pi/180)^2) + 4x^2y^2})} + \frac{(\cos(60\pi/180) + \sqrt{(x^2 - y^2 - \sin(60\pi/180)^2) + 4x^2y^2})^2}{2} = 0.8352;
\]

result = maple('fsolve', ['eq1', 'eq2'])

A.2 Air-Aluminum Interface

% calculation of refractive index for given rtm at given angles for air-aluminum interface

clc;

clear all;

clear, syms x y;
theta1 = 20*pi/180;
theta2 = 64*pi/180;
nd = 1.4204 + j*7.4673;
rtm1 = abs((sqrt(nd^2 - sin(theta1)^2) - nd^2*cos(theta1)) / (sqrt(nd^2 - sin(theta1)^2) + nd^2*cos(theta1)));
rtm1_sq = rtm1^2
rtm2 = abs((sqrt(nd^2 - sin(theta2)^2) - nd^2*cos(theta2)) / (sqrt(nd^2 - sin(theta2)^2) + nd^2*cos(theta2)));
rtm2_sq = rtm2^2

% For 20 degree and 64 degree %
eq_1 = '((((sqrt(((x-(sin(20*pi/180))^2)+sqrt(((x-(sin(20*pi/180))^2+y^2))/2)))^2) - (x*cos(20*pi/180))^2+((y/(2*(sqrt(((x-(sin(20*pi/180))^2)+sqrt(((x-(sin(20*pi/180))^2+y^2))/2)))) - (y*cos(20*pi/180))^2)/((sqrt(((x-(sin(20*pi/180))^2)+sqrt(((x-(sin(20*pi/180))^2+y^2))/2))) + (x*cos(20*pi/180))^2+((y/(2*(sqrt(((x-(sin(20*pi/180))^2)+sqrt(((x-(sin(20*pi/180))^2+y^2))/2)))) + (y*cos(20*pi/180))^2))/2)))) = 0.9021';
eq_2 = '((((sqrt(((x-(sin(64*pi/180))^2)+sqrt(((x-(sin(64*pi/180))^2+y^2))/2)))^2) - (x*cos(64*pi/180))^2+((y/(2*(sqrt(((x-(sin(64*pi/180))^2)+sqrt(((x-(sin(64*pi/180))^2+y^2))/2)))) = 0.9021';
\[
\left(\sin\left(\frac{64\pi}{180}\right)\right)^2 + \sqrt{\left(\left(\frac{x - \sin\left(\frac{64\pi}{180}\right)}{2}\right)^2 + y^2\right)/2}\right) - \left(\frac{y\cos\left(\frac{64\pi}{180}\right)}{2}\right)^2 / \left(\frac{\left(\frac{x - \sin\left(\frac{64\pi}{180}\right)}{2}\right)^2 + y^2\right)/2 + \left(\frac{y}{2\left(\sqrt{\left(\left(\frac{x - \sin\left(\frac{64\pi}{180}\right)}{2}\right)^2 + y^2\right)/2}\right)+\left(\frac{y\cos\left(\frac{64\pi}{180}\right)}{2}\right)^2}{2}\right)\right) = 0.8102;
\]

\[
\text{res = maple('fsolve', [ 'eq1', 'eq2'])}
\]

\[
eq 1 = '(x1^2 - y1^2) = -53.743';
\]

\[
eq 2 = '2*x1*y1 = 21.2131';
\]

\[
\text{final_result = maple('fsolve', [ 'eq11', 'eq22'])}
\]

A.3 Rough Surface

% calculation of s, em and sigma for given rtm at any angle
% of incidence
clc;
clear all;
theta = 5*pi/180;
ep0 = 8.854*1e-12;
w1 = (2*pi)/(400*1e-9);%(1.570e+7)
w2 = (2*pi)/(750*1e-9);%(8.377e+6)
w3 = (2*pi)/(550*1e-9);%(1.142e+7)
sigma = 400;
em=81;

s =0.8;

ep1=(em*ep0)-1i*(sigma/w1);

nd1=sqrt(ep1,ep0);

rtm1=s*abs((sqrt(nd1^2-sin(theta)^2)-nd1^2*cos(theta))/(sqrt(nd1^2-sin(theta)^2)+nd1^2*cos(theta)));

rtm1_sq=rtm1^2

ep2=(em*ep0)-1i*(sigma/w2);

nd2=sqrt(ep2,ep0);

rtm2=s*abs((sqrt(nd2^2-sin(theta)^2)-nd2^2*cos(theta))/(sqrt(nd2^2-sin(theta)^2)+nd2^2*cos(theta)));

rtm2_sq=rtm2^2

ep3=(em*ep0)-1i*(sigma/w3);

nd3=sqrt(ep3,ep0);

rtm3=s*abs((sqrt(nd3^2-sin(theta)^2)-nd3^2*cos(theta))/(sqrt(nd3^2-sin(theta)^2)+nd3^2*cos(theta)));

rtm3_sq=rtm3^2

% y=sigma/ep0;

% Dividing 1 by 2 and 2 by 3

rtmnew1=rtm1^2/rtm3^2

rtmnew2=rtm2^2/rtm3^2
%% Set the convergence tolerances

```matlab
option = optimset(’tolfun’,1e-12,’tolX’,1e-12);
```

% Select a grid of initial conditions

```matlab
[inix, iniy] = meshgrid([0 40 80 120],[10^10 10^11 10^12 10^13]);
N = numel(inix);
```

```matlab
xhat = zeros(N,2);
```

```matlab
fhat = zeros(N,1);
```

```matlab
for i = 1:N
    [xhat(i,:), fhat(i)] = fminsearch(@scalc_fun, [inix(i) iniy(i)], option);
end
```

%%% Reject solutions if terminal function value is above a
% threshold

```matlab
xhat = xhat(fhat<1e-8,:);
```

```matlab
em=xhat(:,1)
sigma=xhat(:,2)*8.854*1e-12
```

%%% Calculating s from the original equation

```matlab
sigma_calc=400;
```

```matlab
em_calc=81;
```

```matlab
ep_calc=(em_calc*ep0)-1i*(sigma_calc/w1);
```

```matlab
nd_calc=sqrt(ep_calc/ep0);
```

51
\[
s_{\text{calc}} = \frac{\sqrt{\text{nd}_{\text{calc}}^2 - \sin^2(\theta)} - \text{nd}_{\text{calc}}^2 \cos(\theta)}{\sqrt{\text{nd}_{\text{calc}}^2 - \sin^2(\theta)} + \text{nd}_{\text{calc}}^2 \cos(\theta)}
\]

\%

% Function Definition file %

function eq = scalc_fun(x)

% from rtm_em_sigma_s.m (without approximation)

eq(1) = (((((sqrt(((x(1)-(\sin(5*\pi/180))^2)+sqrt(((x(1)-(\sin(5*\pi/180))^2)\times2+8.3776e+6^2))/2)))-(\cos(5*\pi/180))\times(x(1))^2+((-x(2)/2*(sqrt(((x(1)-(\sin(5*\pi/180))^2)+sqrt(((x(1)-(\sin(5*\pi/180))^2)\times2+8.3776e+6^2))/2))\times8.3776e+6))+(\cos(5*\pi/180))\times(x(2)/1.1424e+7^2))/2))\times1.1424e+7))-(\cos(5*\pi/180))\times(x(2)/1.1424e+7^2))-0.999753587939460;

\]

\[
eq(2) = (((\sqrt(((x(1)-(\sin(5*\pi/180))^2)+\sqrt(((x(1)-(\sin(5*\pi/180))^2)\times2+8.3776e+6^2))/2)))-(\cos(5*\pi/180))\times(x(1))^2+((-x(2)/2*(\sqrt(((x(1)-(\sin(5*\pi/180))^2)+\sqrt(((x(1)-(\sin(5*\pi/180))^2)\times2+8.3776e+6^2))/2))\times8.3776e+6))+(\cos(5*\pi/180))\times(x(2)/1.1424e+7^2))/2))\times1.1424e+7))-(\cos(5*\pi/180))\times(x(2)/1.1424e+7^2))-
\]
\((\cos(5\pi/180))x(2)/8.3776e+6)^2)/((\sqrt{((x(1)-(\sin(5\pi/180))^2+x(2)^2/8.3776e+6)^2))})-(\sin(5\pi/180))^2)^2+x(2)^2/1.1424e+7)^2))+(\cos(5\pi/180))x(1)^2+((-x(2))/(2*\sqrt{((x(1)-(\sin(5\pi/180))^2)+\sqrt{((x(1)-(\sin(5\pi/180))^2)^2+x(2)^2/1.1424e+7)^2))})\cdot1.1424e+7))-(\cos(5\pi/180))x(2)/1.1424e+7)^2))=1.00;

\text{eq}=\text{norm(eq)};

% 

\text{Results:}

\text{em\_calc =}

\text{Columns 1 through 11}

\begin{array}{cccccc}
81.0000 & -2.6738 & -0.2614 & -0.0207 & 81.0000 & 81.0000 \\
81.0000 & 81.0000 & 81.0000 & 81.0000 & 81.0000 & 81.0000 \\
\end{array}

\text{Columns 12 through 16}

\begin{array}{cccccc}
81.0000 & 81.0000 & 81.0000 & 81.0000 & 81.0000 & 81.0000 \\
\end{array}

\text{sigma\_calc =}

\text{Columns 1 through 11}

\begin{array}{cccccc}
400.0009 & 399.9742 & 399.9750 & 399.9751 & 400.0009 & 400.0009 \\
400.0009 & 400.0009 & 400.0009 & 400.0009 & 400.0009 & 400.0009 \\
\end{array}

53
Columns 12 through 16

400.0009  400.0009  400.0009  400.0009  400.0009

s_calc = 0.8000