A GAME-THEORETIC ANALYSIS OF HOME COURT ADVANTAGE AND
OPTIMAL OFFENSIVE STRATEGY IN BASKETBALL

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A GAME-THEORETIC ANALYSIS OF HOME COURT ADVANTAGE AND OPTIMAL OFFENSIVE STRATEGY IN BASKETBALL

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ABSTRACT

This paper intends to construct a game-theoretic model with which to analyze the impact of home court advantage on a visiting team’s offensive strategy, specifically the balance between two and three-point shots attempted. We develop the *Home Court Advantage Model* to assist in our analysis. The model determines the optimal offensive strategy when one takes into account parameters that reflect the changes in offensive output by a basketball team when playing on the road. In this case this is manifested in lower shooting percentages, particularly on two-point shot attempts. The model also accounts for statistical data suggesting that three-point shot percentages are not affected as dramatically by home court advantage as are two-point shot percentages. We draw several conclusions, including that as a team’s two-point shot percentage decreases, the team should actually shoot more two-point shots to optimize their overall offensive efficiency. We conclude our model with a study of the home court advantage of the 2008-2009 Cleveland Cavaliers, applying the model specifically to post-season play.
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CHAPTER I
INTRODUCTION

There has been a recent increase in interest in statistics that measure overall efficiency in basketball. Statistics that have been traditionally used, such as points per game or assists per game, are being ignored in favor of new statistical measures that calculate how efficiently a team is operating. The application of game theory to basketball is a relatively new endeavor with very few published works in this area. However, it seems that the basketball organization which applies game theoretic principles to their in-game strategy may expect increased efficiency for their team on both the offensive and defensive ends of the floor. Here, we are concerned with determining the optimal offensive balance between two-point shots and three-point shots for a given team.

In basketball, the concept of “home court advantage” is as visible as in any other sport and manifests itself in the form of, among other factors, lowered shooting percentages by the team which is playing on the road. Therefore, it is beneficial for a team to employ a game-theoretic mindset when developing their offensive strategy for a given game. In this paper we examine the relationship between offensive strategies for a basketball team when playing both on the road and at home and seek to provide a game-theoretic model for determining optimal offensive strategy based on these
factors. Specifically we seek to apply our findings to the offensive strategy of a team participating in a best-of-seven playoff scenario, where the venue changes often and any increase in offensive efficiency could provide the margin between winning the series and elimination.

There have been a few relevant contributions that applied game-theoretic principles to basketball situations. Annis [1] examined offensive strategy in a specific scenario at the end of a basketball game, studying the implications for a defensive team of electing to force the offensive team to take two free throws rather than shoot a three point shot in an end-of-game scenario when the defensive team is leading by three points. Skinner [2] studied Nash Equilibria amongst the offensive “paths” to examine the optimum efficiency of an offensive team. Jones [3] examined how home court advantage is manifested over an entire game. Entine and Small [4] provided a statistical model to examine the effects of differing amounts of rest on a team playing on the road. These contributions each provide a framework for a game-theoretical analysis to be applied. In this case, we maximize the offensive efficiency of a team under the assumption that their opposition is also playing optimally.

We proceed by constructing the model in its most basic form in Chapter 2. In Chapter 3, we adapt the model to account for home court advantage and then in Chapter 4 we solve the model and discuss the theoretical implications of the results. In Chapter 5 we apply our model to analyze the impact of home court advantage in both regular and post-season play. Afterward, we summarize and discuss possibilities for extension of this research.
CHAPTER II

SETTING UP THE MODEL

We proceed by constructing the model as a two-by-two zero sum game in which the offense has a choice of either a two-point or a three-point shot and the defense has a choice to focus their defense on two-point shots or three-point shots. The payoffs in the model represent the expected point value of a shot taken under the given scenario for the offensive team. In Table 2.1, the offensive strategy sets are \( \{2, 3\} \)

Table 2.1: Initial Payoff Matrix

<table>
<thead>
<tr>
<th></th>
<th>Defense</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Defend 2</td>
</tr>
<tr>
<td>Offense</td>
<td></td>
</tr>
<tr>
<td>Shoot 2</td>
<td>( S_{2,2} )</td>
</tr>
<tr>
<td>Shoot 3</td>
<td>( S_{3,2} )</td>
</tr>
</tbody>
</table>

and the defensive strategy sets are \( \{D_2, D_3\} \). The payoff \( S_{2,2} \) represents the expected point value of a two-point shot taken against a defense defending a two-point shot, or \( (2, D_2) \). \( S_{2,3} \) represents the expected point value for \( (2, D_3) \) and the other payoffs hold for their corresponding strategic scenarios in Table 2.1.
Throughout our analysis we make the following assumptions:

\[ S_{2,2} < S_{2,3}, \quad S_{3,2} > S_{3,3}, \quad S_{2,3} > S_{3,3}, \quad S_{2,2} < S_{3,2}. \]  \hspace{1cm} (2.1)

There are numerous possibilities for the payoffs to be used in this model. Oliver developed a formula for a team’s “Offensive Rating” in [5] that measures the overall offensive efficiency of a basketball team. The payoffs that we use are not as detailed as a model using Offensive Rating because we lack the scenario-specific statistics necessary. However, it is reasonable to assume that any professional or serious amateur basketball team would have access to the necessary data.

We calculate the payoffs as an expected value of a shot taken under a given strategic scenario. This allows us to measure offensive efficiency in a simpler manner, accounting for actual shots taken and not taking into consideration free throws, turnovers, offensive rebounds, and other factors that impact a team’s Offensive Rating. For instance, to calculate \( S_{3,3} \) suppose there were \( FGA_{3,3} \) occasions in which the offense elected to shoot a three-point shot and the defense chose to defend the three-point line, and \( FGM_{3,3} \) of those shots were successful. The payoff would then be calculated as

\[ S_{3,3} = 3 \left( \frac{FGM_{3,3}}{FGA_{3,3}} \right). \]

The remainder of the payoffs in table 2.1 are calculated similarly.
CHAPTER III
THE HOME COURT ADVANTAGE MODEL

With the initial payoffs calculated for our original model we now proceed to model the implications of playing on the road instead of playing at home. We construct the new model in a way that the payoffs constructed in our original model represent the expected point value of a shot taken in a home game. We assume that the team is now playing on the road, which will cause a decrease in shooting percentages and therefore a change in the expected payoff of taking a given shot. We will reflect this in the model with a decrease in all payoffs to account for decreased shooting percentages. To determine the precedent for this we collected shooting percentage data in Table 3.1 from the past 11 NBA seasons obtained from http://www.espn.com [6].

Interestingly, the data in Table 3.1 reflects a smaller decrease in three point shooting percentages. This suggests that while teams do shoot more poorly when playing on the road, three point shooting tends to fluctuate less. This is somewhat counterintuitive because three-point shots are taken from a greater distance than two-point shots, and it seems that under the pressure of playing on the road a shot taken from a greater distance would be more difficult to convert. Another notable fact from the data in Table 3.1 is that, while teams shot more poorly in the post-season than in the regular season overall, three-point shooting was actually affected less by playing
Table 3.1: NBA Shooting Percentage Statistics, 1999-2000 to 2009-2010

<table>
<thead>
<tr>
<th></th>
<th>Home</th>
<th>Away</th>
<th>Differential</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average Overall Shooting Percentage</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular Season</td>
<td>45.67%</td>
<td>44.39%</td>
<td>+1.28%</td>
</tr>
<tr>
<td>Post Season</td>
<td>44.44%</td>
<td>42.78%</td>
<td>+1.66%</td>
</tr>
<tr>
<td><strong>Average Three Point Shooting Percentage</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular Season</td>
<td>35.74%</td>
<td>35.01%</td>
<td>+.73%</td>
</tr>
<tr>
<td>Post Season</td>
<td>34.36%</td>
<td>33.76%</td>
<td>+.60%</td>
</tr>
</tbody>
</table>

on the road during the post-season than during the regular season. This implies that two-point shots become significantly more difficult to convert in post-season play.

Additionally, we will construct this model under the assumption that success in shooting two-point shots and success in shooting three-point shots are intertwined with one another. In essence, if a team is having great success shooting three-point shots that will cause the defense to shift their focus to defending the three-point line, which will in turn open up more opportunities to score two-point baskets. Taking these facts into consideration, we construct the model in such a way that two-point shot percentages are impacted more significantly by playing on the road than three-point shot percentages.

We adopt a model similar to that of McGough’s *Improvement in Passing Model* [7] under similar assumptions. Namely, we assume that the payoff matrix
given in Table 2.1 has no dominant rows or columns, and therefore the solutions
to our games will only have mixed strategy Nash equilibria [8]. This assumption is
reasonable because it is not likely that any professional or serious amateur basketball
team would ever employ a pure shooting strategy.

To construct the new model adjusting for home court advantage, we introduce
two parameters. We let $x$ represent the decrease in overall shooting percentages for
the visiting team where $x \in [-1, 0)$. The parameter $x$ is specific to a given basketball
venue and it represents the “home court advantage” of playing in that venue. From
this we see that venues with $x$ values that are larger in magnitude are places where
the home court advantage has a greater negative impact on the shooting percentages
of the visiting team. This can be attributed to a number of factors such as the
quality of the team that plays its home games and the noise level generated by the
spectators inside the venue. Additionally, the $x$ value for a given venue changes from
season to season because many of the contributing factors such as team success and
player personnel are subject to change between seasons. For example, the $x$ value for
playing at Madison Square Garden in the 2009-2010 season is a different $x$ value than
that of the same venue in the 1999-2000 season. It is important to note that the $x$
value for a given venue and season is fixed.

We limit $x$ to the interval $[-1, 0)$, because choosing a value for $x$ less than
$-1$ would result in a negative value for the payoffs in Table 3.2. The $x$ value can be
interpreted as how much a visiting team’s shooting percentage drops when playing at
a given basketball venue. If $x \to 0$ the visiting team’s shooting percentage shows no
change, and if $x = -1$ the home court advantage is so drastic that the visiting team’s shooting percentage is 0%. A positive value for $x$ would represent a case where a given basketball venue actually has a positive impact on the shooting percentages of opposing teams. While this is theoretically possible, for the purposes of this game we limit $x$ to negative values only, and we assume at all times that playing on the road negatively impacts one’s shooting percentages based on the data given in Table 3.1.

To calculate the $x$ value, we use statistical values from the regular season. If $H_{NBA}$ represents the average home shooting percentage in the NBA and $A^*$ represents the average road shooting percentage at a given basketball venue, then $x^*$, the home court advantage factor for this venue is

$$x^* = \frac{A^* - H_{NBA}}{H_{NBA}}. \quad (3.1)$$

We also note that according to the data in Table 3.1, the $x$ values in post-season play increase slightly. Hence, when using this model to examine post-season games, we multiply the $x$ value for a given venue and season by a factor of $1.30 \approx \frac{1.66}{1.28}$. The calculation of this multiplication factor follows intuitively from the data given in table 3.1.

We also introduce the parameter $\xi$. This parameter is specific to a given team which is playing on the road, and adjusts the $x$ value to account for smaller decreases in three-point shooting percentage. In our model we take $\xi \in [0, 1]$ such that if $\xi = 0$, then the impact of the home court advantage on a specific visiting team’s three-point shooting is completely negated and if $\xi = 1$ the team’s three-point
shooting is affected equally by playing on the road in comparison to their overall shooting. This demonstrates that the $\xi$ value for a given team reflects how effectively the team shoots from the three-point line on the road compared to how the team shoots when playing at home. A higher $\xi$ value represents a greater discrepancy between three-point shooting percentages at home and on the road.

While we acknowledge that it is possible for a team to shoot three-point shots more effectively on the road than at home, for the sake of simplicity in this model we bound $\xi$ below by 0. In the event that a team’s calculated $\xi$ value is outside of the given interval, for the purposes of this model we will select $\xi$ to be the appropriate upper or lower bound. Once again we use statistical data from the regular season to calculate $\xi$. However, because it is team-specific, it is calculated in a slightly different manner than $x$.

To calculate $\xi$ for a given team $t$ we use the following parameters: $H_{3,t}$, the team’s home three-point shooting percentage; $A_{3,t}$, the team’s away three-point shooting percentage; $H_{ovr,t}$, the team’s overall home shooting percentage; and $A_{ovr,t}$, the team’s overall road shooting percentage. Team $t$’s specific value, $\xi_t$, is then calculated as

$$
\xi_t = \begin{cases} 
0, & \frac{H_{3,t} - A_{3,t}}{H_{ovr,t} - A_{ovr,t}} < 0 \\
\frac{H_{3,t} - A_{3,t}}{H_{ovr,t} - A_{ovr,t}}, & 0 \leq \frac{H_{3,t} - A_{3,t}}{H_{ovr,t} - A_{ovr,t}} \leq 1 \\
1, & 1 < \frac{H_{3,t} - A_{3,t}}{H_{ovr,t} - A_{ovr,t}}.
\end{cases}
$$

We also offer a post-season correction for the value of $\xi$ by multiplying it by a factor of $0.82 \approx \frac{60}{73}$. Just as above, the calculation of this multiplication factor
follows from Table 3.1. We now introduce these parameters into our initial model
given in Table 2.1, resulting in the two-by-two zero sum game in Table 3.2 which we
will refer to as the *Home Court Advantage Model*.

Table 3.2: The Home Court Advantage Model

<table>
<thead>
<tr>
<th>Offense</th>
<th>Defense</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Defend 2</td>
</tr>
<tr>
<td>Shoot 2</td>
<td>$S_{2,2}(1 + x)$</td>
</tr>
<tr>
<td>Shoot 3</td>
<td>$S_{3,2}(1 + \xi x)$</td>
</tr>
</tbody>
</table>
CHAPTER IV
SOLVING THE MODEL

We now solve the *Home Court Advantage Model* and determine the Nash Equilibria for the optimal proportion of two-point and three-point shots which are dependent on $x$, $\xi$, and the payoffs given in Table 2.1. To find the Nash Equilibria, we will assume that the offensive team and the defensive team are both playing at their optimum strategy. The Nash Equilibrium offensive strategy will occur when the defensive team is strategy-indifferent, and vice versa. We define $\phi$ to be the probability that the offensive team elects to shoot a two-point shot. Clearly, the probability that the offensive team elects to shoot a three-point shot is $1 - \phi$. We will seek the optimum value for $\phi$ when the expected value for points allowed is equal for both the “Defend 2” and “Defend 3” strategies in Table 3.2. Thus, $P_2(\xi, x)$, the Nash Equilibrium percentage of two point shots taken, is the value of $\phi$ which solves

$$S_{2,2}(1 + x)\phi + S_{3,2}(1 + \xi x)(1 - \phi) = S_{2,3}(1 + x)\phi + S_{3,3}(1 + \xi x)(1 - \phi). \tag{4.1}$$

In a similar fashion, we define $\sigma$ to be the probability that the defensive team elects to focus their defense on two-point shots rather than three-point shots, and by similar logic we determine that the Nash Equilibrium value for $\sigma$ solves

$$S_{2,2}(1 + x)\sigma + S_{2,3}(1 + x)(1 - \sigma) = S_{3,2}(1 + \xi x)\sigma + S_{3,3}(1 + \xi x)(1 - \sigma). \tag{4.2}$$
After performing some basic algebra to solve for $\phi$ in Equation (4.1), we determine

$$P_2(\xi, x) = \frac{(S_{3,2} - S_{3,3})(1 + \xi x)}{(S_{3,2} - S_{3,3})(1 + \xi x) + (S_{2,3} - S_{2,2})(1 + x)}, \quad (4.3)$$

and thus

$$P_3(\xi, x) = 1 - P_2(\xi, x), \quad (4.4)$$

where $P_3(\xi, x)$ is the Nash Equilibrium proportion of three-point shots.

It is notable that the Nash Equilibrium offensive strategy given by Equations (4.3) and (4.4) is dependent on the quantities $(S_{3,2} - S_{3,3})$ and $(S_{2,3} - S_{2,2})$. These quantities represent the added value of taking a shot against the opposite defensive strategy. We can see in Equation (4.3) that as the added benefit of shooting a two-point shot against a defense focusing on three-point shots increases, the Nash Equilibrium proportion of two-point shots taken actually decreases. We also note a similar property applies to the Nash Equilibrium proportion of three-point shots.

We perform a similar technique on Equation (4.2) which results in the Nash Equilibrium functions for the defensive team given in Equations (4.5) and (4.6),

$$P_{2\text{def}}(\xi, x) = \frac{S_{2,3}(1 + x) - S_{3,3}(1 + \xi x)}{(S_{2,3} - S_{2,2})(1 + x) + (S_{3,2} - S_{3,3})(1 + \xi x)}, \quad (4.5)$$

$$P_{3\text{def}}(\xi, x) = 1 - \frac{S_{2,3}(1 + x) - S_{3,3}(1 + \xi x)}{(S_{2,3} - S_{2,2})(1 + x) + (S_{3,2} - S_{3,3})(1 + \xi x)}. \quad (4.6)$$

$P_{2\text{def}}$ represents the Nash Equilibrium proportion of plays in which the defense focuses on two-point shots, and $P_{3\text{def}}$ represents the corresponding Nash Equilibrium value for defending three-point shots. Equations (4.3), (4.4), (4.5), and (4.6) provide the optimal mixed-strategy solutions to the game given in Table 3.2.
We now proceed to calculate what McGough referred to as the *breaking point* of the model [7]. Note that by our definitions of $x$ and $\xi$, it holds that $(1+\xi x) > (1+x)$. This fact implies that it is possible for there to exist some $\tilde{x}$ such that for all $x \leq \tilde{x}$, Row 2 in Table 3.2 is a dominant row. This is the point beyond which the offensive team would employ a strategy consisting entirely of three-point shots. There are some cases in which there will not exist a breaking point in $[-1, 0)$, and in this case a mixed strategy will always be implemented by the offensive team. This breaking point can be interpreted as the point at which the home court advantage causes such a drastic reduction in overall shooting percentages that it is logical to employ an offensive strategy consisting entirely of three-point shots. We also note that this breaking point provides a lower bound on the value of $x$ because it is unrealistic to expect that there exists a home court advantage so dominant in modern professional basketball.

We recall that from our original assumptions regarding the payoff values in Table 2.1, it is assumed that $S_{3,2} > S_{2,2}$ and $S_{2,3} > S_{3,3}$. From these assumptions we find that the breaking point will be the value of $x$ which causes the three-point shooting row of the payoff matrix to be a dominant row, or the value of $x$ which solves the equation $S_{2,3}(1+x) = S_{3,3}(1+\xi x)$. After some elementary algebra we obtain the breaking point to be

$$\tilde{x} = \frac{(S_{2,3} - S_{3,3})}{(\xi S_{3,3} - S_{2,3})}. \quad (4.7)$$

From this we see that the breaking point is the point at which both the offensive and defensive teams implement a pure strategy demonstrated by the fact
that \( P_{2\text{def}}(\xi, \bar{x}) = P_2(\xi, \bar{x}) = 0 \), implying that \( P_{3\text{def}}(\xi, \bar{x}) = P_3(\xi, \bar{x}) = 1 \). This motivates us to create a new Nash Equilibrium function accounting for the fact that there is a mixed strategy for all \( x \) values greater than the breaking point and a pure strategy for all \( x \) values less than the breaking point. We call this new function \( P_2^*(\xi, x) \), and it is defined by

\[
P^*_2(\xi, x) = \begin{cases} 
0, & -1 \leq x < \frac{(S_{3,3} - S_{3,2})}{(\xi S_{3,3} - S_{2,3})} \\
(S_{3,2} - S_{1,3})(1+\xi x) & \frac{(S_{3,3} - S_{3,2})}{(\xi S_{3,3} - S_{2,3})} \leq x < 0.
\end{cases}
\] (4.8)

We also denote the new Nash Equilibrium function for the percentage of three point shots as \( P_3^*(\xi, x) = 1 - P_2^*(\xi, x) \). We now seek to obtain an expression for the expected point value of the game given in Table 3.2. To do so, we substitute our original Nash Equilibrium function in Equation (4.3) into either side of Equation (4.1). After simplification, we see that the expected value function \( E(\xi, x) \) is given by

\[
E(\xi, x) = \frac{(1 + x)(1 + \xi x)(S_{3,2}S_{2,3} - S_{3,3}S_{2,2})}{(S_{3,2} - S_{3,3})(1 + \xi x) + (S_{2,3} - S_{2,2})(1 + x)}.
\] (4.9)

It is important to note that if the value of \( x \) is below the breaking point, then the expected value of this game is simply \( S_{3,3}(1 + \xi x) \) because both the offensive and defensive teams would be using a pure strategy. This leads us to the conclusion that the expected value of the game in Table 3.2 is actually a discontinuous piecewise function that we call \( E^*(\xi, x) \) and is given by

\[
E^*(\xi, x) = \begin{cases} 
S_{3,3}(1 + \xi x), & -1 \leq x < \frac{(S_{3,3} - S_{3,2})}{(\xi S_{3,3} - S_{2,3})} \\
\frac{(1+x)(1+\xi x)(S_{3,2}S_{2,3} - S_{3,3}S_{2,2})}{(S_{3,2} - S_{3,3})(1+\xi x) + (S_{2,3} - S_{2,2})(1+x)}, & \frac{(S_{3,3} - S_{3,2})}{(\xi S_{3,3} - S_{2,3})} \leq x < 0.
\end{cases}
\] (4.10)
Finally, we examine an interesting property of $P^*_2(\xi, x)$. If the value of $x$ is above the breaking point, we calculate

$$\frac{\partial P^*_2}{\partial x} = \frac{(S_{2,3} - S_{2,2})(S_{3,2} - S_{3,3})(\xi - 1)}{[(S_{3,2} - S_{3,3})(1 + \xi x) + (S_{2,3} - S_{2,2})(1 + x)]^2}.$$  \hspace{1cm} (4.11)

Our assumptions about the payoff values from Table 2.1 along with the fact that $0 \leq \xi < 1$ imply that $\frac{\partial P^*_2}{\partial x} < 0$. This suggests that $P^*_2(\xi, x)$ is decreasing and implies that as the $x$ value gets smaller, or the effect of the home court advantage increases, the team should actually shoot more two-point shots. This is somewhat surprising because the home court advantage has a greater effect on two-point shots than three-point shots. We now proceed to apply our model in Chapter 5.
CHAPTER V

ANALYZING THE EFFECT OF THE HOME COURT ADVANTAGE OF THE
2008-2009 CLEVELAND CAVALIERS

In 2008-2009, the Cleveland Cavaliers were one of the most dominant home teams in
the recent history of the NBA. While playing at home they outscored opponents by
an average margin of 10.9 points per game in finishing with an NBA-best home record
of 39 wins and only 2 losses. Their superb home play catapulted them to the top
playoff seed overall and the opportunity to have home court advantage throughout
the entire 2009 playoff season.

In this section we will utilize our model to analyze how the home court
advantage of the 2008-2009 Cleveland Cavaliers affected two of their post-season
opponents, the Atlanta Hawks and the Orlando Magic. We first discuss a limitation
of our calculations. In order to obtain the payoffs for the game depicted in Table 2.1,
it is necessary to have accurately counted the number of times the defensive team
implemented each strategy. In our calculations below we attempt to approximate
these values although it is reasonable to expect that a professional basketball team
would have access to this type of data.

We will proceed by first constructing the initial payoff matrices for both
the Hawks and the Magic using statistical data from the 2008-2009 NBA regular
season. We apply our *Home Court Advantage Model*, followed by calculation of the appropriate $\xi$ values for the Hawks and Magic and then calculation of the 2008-2009 $x$ value for Quicken Loans Arena, the home venue of the Cleveland Cavaliers. We will use our model to determine the optimal offensive strategy for both the Hawks and the Magic when playing road playoff games against the Cleveland Cavaliers. We then compare our results to the actual results of both series and discuss the implications.

To construct the initial payoff matrices for both teams we consider the data given in Table 5.1, obtained from http://www.espn.com [6].

Table 5.1: The 2008-2009 Magic/Hawks Offensive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Orlando Magic</th>
<th>Atlanta Hawks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2-Point Shots</td>
<td>3-Point Shots</td>
</tr>
<tr>
<td>Made</td>
<td>2112</td>
<td>817</td>
</tr>
<tr>
<td>Attempted</td>
<td>4269</td>
<td>2147</td>
</tr>
<tr>
<td>Shooting Percentage</td>
<td>49.47</td>
<td>38.05</td>
</tr>
<tr>
<td>Points Per Shot</td>
<td>.99</td>
<td>1.14</td>
</tr>
<tr>
<td>Percentage of Shots</td>
<td>66</td>
<td>34</td>
</tr>
</tbody>
</table>

We now construct the initial payoff matrix for the Atlanta Hawks. We calculate the payoffs under the assumption that NBA teams know how often the Hawks shoot two-point shots and how often they shoot three-point shots, and that they utilize each defensive strategy according to these percentages. Under this assumption,
the following equations hold:

\[ \begin{align*}
.75S_{2,2} + .25S_{2,3} &= .98, \\
.75S_{3,2} + .25S_{3,3} &= 1.10. 
\end{align*} \] (5.1)

Since we desire the optimal offensive strategy for the Hawks, we will assume further that the defensive team is playing optimally. This means that the expected value of defending against two-point shots is equal to the expected value of defending against three-point shots. This leads us to the following equation,

\[ .75(S_{2,2} - S_{2,3}) + .25(S_{3,2} - S_{3,3}) = 0. \] (5.2)

Equations 5.1 and 5.2 provide us with a dependent system of equations with three equations and four unknowns. We select \( S_{3,3} \) as the free variable and solve which leads us to find \( A_{ATL} \), the initial payoff matrix for the Atlanta Hawks, given as follows

\[ A_{ATL} = \begin{pmatrix}
.11 + .86S_{3,3} & 1.35 - .33S_{3,3} \\
1.47 - .33S_{3,3} & S_{3,3}
\end{pmatrix}. \] (5.3)

For our analysis we select \( S_{3,3} \) to be .90, which corresponds to the offensive team shooting 30% from behind the three-point line. This leads to the following matrix

\[ A_{ATL} = \begin{pmatrix}
.88 & 1.05 \\
1.17 & .90
\end{pmatrix}. \] (5.4)

This matrix leads to Table 5.2, the home court advantage model for the 2008-2009 Atlanta Hawks.
Table 5.2: The 2008-2009 Atlanta Hawks Home Court Advantage Model

<table>
<thead>
<tr>
<th>Offense</th>
<th>Defense</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Defend 2</td>
</tr>
<tr>
<td>Shoot 2</td>
<td>.88(1 + x)</td>
</tr>
<tr>
<td>Shoot 3</td>
<td>1.17(1 + (\xi x))</td>
</tr>
</tbody>
</table>

A similar calculation performed using the values in Table 5.1 gives

\[
A_{ORL} = \begin{pmatrix}
.89 & 1.12 \\
1.26 & .90
\end{pmatrix},
\]

which leads us to Table 5.3, the home court advantage model for the 2008-2009 Orlando Magic.

Table 5.3: The 2008-2009 Orlando Magic Home Court Advantage Model

<table>
<thead>
<tr>
<th>Offense</th>
<th>Defense</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Defend 2</td>
</tr>
<tr>
<td>Shoot 2</td>
<td>.89(1 + x)</td>
</tr>
<tr>
<td>Shoot 3</td>
<td>1.26(1 + (\xi x))</td>
</tr>
</tbody>
</table>

We now calculate the values of \(\xi\) for both the Orlando Magic and the Atlanta Hawks. In 2008-2009 the Orlando Magic shot 37.10\% from behind the three point line at home and 38.90\% on the road. The Hawks shot 36.10\% at home and 37.00\% on the road. Since both teams shot three point shots more effectively on the road than
at home, by the $\xi$ calculation outlined in Chapter 3 we choose the $\xi$ value for both teams to be 0, essentially nullifying the effect of home court advantage on three-point shots. Since both of these teams had $\xi$ values of 0 we also provide as an example the calculation of the $\xi$ value for the 2008-2009 Oklahoma City Thunder. In 2008-2009 the Thunder shot 35.1% from the three-point line at home and 34.0% on the road. Their overall home shooting percentage was 45.3% and their overall road shooting percentage was 44.1%. Using the calculation given in Chapter 3 we calculate the $\xi$ value for the 2008-2009 Oklahoma City Thunder,

$$
\xi_{OKC} = \frac{35.1 - 34.0}{45.3 - 44.1} = .917.
$$  \hfill (5.6)

We also note that if we were considering a post-season scenario, we would multiply $\xi_{OKC}$ given in Equation (5.6) by a factor of 0.82 to obtain the appropriate value.

The next step in our analysis is to determine the $x$ value for a post-season game at Quicken Loans Arena. In the 2008-2009 NBA regular season the average home shooting percentage was 46.53%. The average road shooting percentage at Quicken Loans Arena was 41.91%. To account for the games being played in a post-season scenario, we multiply our $x$ value by a factor of $1.30 \approx \frac{1.66}{1.28}$ as outlined in Chapter 3. This leads to

$$
x = \frac{41.91 - 46.52}{46.52} \times \frac{1.66}{1.28} \approx -.1285,
$$  \hfill (5.7)

and gives us the necessary parameters for our model.

We now solve to find the Nash Equilibrium proportions of two-point and three-point shots for both the Hawks and the Magic as outlined in Equations (4.3)
and (4.4). We list these proportions in Table 5.4. We note that based on the home court advantage of the 2008-2009 Cleveland Cavaliers at Quicken Loans Arena, both the Hawks and Magic should implement roughly a 65:35 ratio of two-point shots to three-point shots. For the Hawks, this represents shooting 10% more three-point shots than in the regular season, while for the Magic, it means shooting 1% more two-point shots than in the regular season based on the data in Table 5.1. Interestingly, the offensive strategy implemented by the Magic during the regular season is extremely close to their optimal offensive strategy when playing a road playoff game at Quicken Loans Arena. We calculate the following values for $E^*(\xi, x)$, the expected point value of a field goal attempt in this scenario:

$$E^*_{ATL} = .9097,$$  \hspace{1cm} (5.8)  

$$E^*_{ORL} = .9503.$$  \hspace{1cm} (5.9)  

We interpret $E^*(\xi, x)$ to be the expected value of a field goal attempt not accounting for fouls, turnovers, or any other factors. This means that if the 2008-
2009 Hawks are playing at their optimal offensive strategy and take 100 shots in a post-season road game against the 2008-2009 Cleveland Cavaliers they can expect to score approximately 91 points from those field goal attempts. Likewise, in the same scenario the 2008-2009 Magic can expect to score approximately 95 points from 100 field goal attempts.

To gain a better understanding of the behavior of $E^*$ over its domain we examine Figure 5.1, which plots the value of $E_{ORL}^*$ for six different $\xi$ values and $x \in [-.6, 0]$. We note that over this domain, $E^*$ is a piecewise-defined function as

![Figure 5.1: $E_{ORL}^*$ for different values of $\xi$ between 0 and 1.](image-url)
outlined in Equation (4.10). The "breaks" in each of the plots in Figure 5.1 can be interpreted as the location of the breaking point for the game at that particular \( \xi \) value. We note that as \( \xi \) increases \( E^* \) begins to change more drastically over its domain. This makes sense, because \( \xi \) essentially represents the percentage of the \( x \) value for a given basketball venue that is translated to a particular visiting team’s three-point shooting percentage. This causes a more drastic reduction in point production from three-point shooting which is manifested in \( E^* \), a representation of overall expected scoring in terms of the expected value of a shot taken. Essentially a team with a smaller \( \xi \) value will have greater success scoring points on the road because their three-point shooting ability is not affected as drastically as their overall shooting percentage.

Figure 5.1 also demonstrates that as \( x \) decreases, a visiting team can expect to score less points regardless of how well they shoot three point shots. We note that in Figure 5.1, the realistic operating range for most NBA teams is in the upper right-hand corner where \( x \) is greater than \(-.2\), giving expected values roughly between 80 and 100 points per 100 possessions. It is also important to note once again that \( E^* \) represents an expected value for points per possession drawn solely from field goal attempts and forsaking fouls, offensive rebounds, and other factors that could feasibly contribute to a team’s points per possession.

The Atlanta Hawks 2009 Eastern Conference Semifinal series against the Cleveland Cavaliers was one to forget. The Hawks lost the first two games at Quicken Loans Arena in an embarrassing fashion, with the average margin of defeat in the
games at 23.5 points. The Hawks struggled mightily to score points against the defense of the Cavaliers averaging 78.5 points in the two games, almost 20 points below their regular season scoring average. The Cavaliers’ momentum carried them to two more wins in Atlanta and a 4-0 sweep of the best-of-seven series, ending the Hawks’ 2008-2009 campaign.

During the games played at Quicken Loans Arena, the Hawks shot two-point shots on 75.5% of their field goal attempts and three-point shots on only 24.5%. We calculate their expected point value for a field goal attempt during these two games as

\[
E_{fga} = \frac{2(FGM_2) + 3(FGM_3)}{FGA}, \tag{5.10}
\]

where \( FGM_2 \) is their total amount of successful two point field goals, \( FGM_3 \) is their total amount of successful three point field goals, and \( FGA \) is their total field goal attempts. For these two games Atlanta’s expected value of a field goal attempt was approximately .8776 points. While this strategy mirrors their regular season offensive strategy our model suggests they should have attempted about 10% more three-point shots, which indicates that the Hawks were playing at less than their optimal level. Although these results are not a sufficient explanation for the outcome of the series, they do offer some insight into why the Hawks were unable to stay within 20 points of the Cavaliers in the two games played at Quicken Loans Arena.

The Cavaliers’ next playoff opponent in the 2009 playoffs following their victory over the Atlanta Hawks was the Orlando Magic in the NBA Eastern Conference Finals. The Magic played very well in the first two games at Quicken Loans Arena
stunning the Cavaliers by winning the first game by one point and coming within a
slim margin of winning the second, losing by one point on a miraculous three point
shot by Cavaliers’ forward LeBron James as time expired. The Magic also played
the fifth game of the series at Quicken Loans Arena losing to the Cavaliers by ten
points in a game that was close throughout. The Magic used an effective offense cen-
tered around All-NBA First Team Center Dwight Howard, surrounded by a myriad
of excellent three point shooters, with a total of 7 players on their roster who shot
over 35% from behind the three-point line while playing more than 1000 minutes in
2008-2009. The Magic’s success at Quicken Loans Arena carried over to the games
played in Orlando resulting in a 4-2 series victory and a trip to the 2009 NBA Finals.

During games played at Quicken Loans Arena in Cleveland the Magic at-
ttempted three-point shots on approximately 31% of their field goal attempts and
two-point shots on approximately 69%. We calculate their expected value for a field
goal attempt in the same manner as we did for the Hawks in Equation (5.10), re-
sulting in an expected value of 1.11 points per field goal attempt. The strategy
implemented by the Magic during this series varies by about 4% from the optimal
strategy suggested by our model. We also note that the Magic’s value for expected
points per field goal attempt was actually higher than the expected value given by
our model. This can be attributed to relatively high shooting percentages compared
to other opponents playing road games at Quicken Loans Arena. The reasons for this
are a matter of conjecture, but it seems that one of the following was true: the home
court advantage at Quicken Loans Arena had little to no effect on the Orlando Magic
or the Magic offense was very well conceived and well executed throughout the series especially during the road games. We are inclined to the latter, and although our model cannot explain the entirety of why Orlando played so well during these games, the Magic’s proximity to their optimal offensive strategy does offer some explanation as to why they had so much success when playing road playoff games against the 2008-2009 Cleveland Cavaliers.
CHAPTER VI

SUMMARY

This paper discusses the effects of home court advantage on a visiting team’s offensive strategy, specifically the balance between two-point and three-point shot attempts. Statistical data reflects that the home court advantage has a greater impact on two-point shooting percentages than three-point shooting percentages. Our results are unexpected in their suggestion that as the home court advantage grows, the visiting team should actually shoot more two-point shots. This analysis describes how a team should adapt their offensive strategy when playing on the road against a team with a strong home court advantage.

We note that the model has limitations. The model is only specific to an individual season and we acknowledge that many teams have ‘core’ groups of players that have been together for multiple seasons, thus allowing the observer to use statistical data covering more than one season. Also, while there are many occasions when a coach calls a play to lead to a specific shot we acknowledge that the shot taken by an offensive basketball team depends heavily on the opportunities available. Likewise, in many occurrences the defense “reacts” to what the opposing team is doing offensively rather than specifically predetermining what area of the floor they will defend. However, the results obtained from this analysis would be quite useful for coaches when
creating a gameplan in preparation for an opponent. The limitations on the impact of home court advantage on three-point shooting are restrictive, and could be improved by using data from more than one season. We also note that the model could become more accurate by developing a more sophisticated relationship between the parameters representing the effects of the home-court advantage on visiting teams.

The opportunities for extension of this analysis are numerous including determining strategy against a zone defense vs. man-to-man, determining strategy when playing on a neutral court, or even determining how the acquisition of new players impacts a team’s offensive strategy. It seems this type of analysis would also be valuable when applied to basketball played at the NCAA Division I level, where home court advantages are more pronounced and observable.

Finally, we envision future iterations of this model with a more sophisticated payoff matrix, using payoffs that account for factors such as fouls, free throws, and turnovers in an effort to develop a more complete result that encapsulates a team’s overall offensive efficiency rather than an expected point value per field goal attempt. We also envision the construction of a payoff matrix with a larger number of possible offensive and defensive strategies accounting for different shooting areas on the basketball court. This type of analysis, while quite complicated, would be very beneficial to a basketball organization seeking to maximize their offensive and defensive productivity.


