ANALYTICALLY AND NUMERICALLY MODELING RESERVOIR-EXTENDED POROUS SLIDER AND JOURNAL BEARINGS INCORPORATING CAVITATION EFFECTS

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Joshua David Johnston

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ANALYTICALLY AND NUMERICALLY MODELING RESERVOIR-EXTENDED POROUS SLIDER AND JOURNAL BEARINGS INCORPORATING CAVITATION EFFECTS

Joshua David Johnston

Dissertation

Approved:                   Accepted:

Advisor
Dr. Gerald Young

Co-Advisor
Dr. Minel Braun

Committee Member
Dr. Kevin Kreider

Committee Member
Dr. Joseph Wilder

Committee Member
Dr. Alex Povitsky

Committee Member
Dr. Scott Sawyer

Committee Member
Dr. S.I. Hariharan

Advisor
Department Chair

Dr. Gerald Young
Dr. Timothy Norfolk

Co-Advisor
Dean of the College

Dr. Minel Braun
Dr. George Haritos

Committee Member
Dean of the Graduate School

Dr. Kevin Kreider
Dr. George Newkome

Committee Member
Date

Dr. Joseph Wilder

Committee Member
Dr. Alex Povitsky
The technology of porous bearings is well-known in industry. In classical cases, the porous medium acts as an external reservoir making their use ideal for applications where an external lubricant supply is undesirable or impractical or when fluid has to be delivered on a continual basis. The work considered here looks to extend the benefits of typical porous bearings to allow for the bearing to be sealed, containing, from the onset of operation, all necessary lubricant.

The goal of this work is to demonstrate a bearing that circulates the fluid between a fluid film and an eccentric reservoir, using a porous medium as an intermediary; a system that is capable of supporting a realistic load, while simultaneously pumping the fluid back and forth between the lubricating region and the reservoir.

The method used to investigate such a bearing is a mixture of analytical and numerical techniques. For the analysis, a non-dimensionalization scheme is used to analyze both the momentum and thermal governing equations at their differing orders of magnitude. Upon doing so, the governing momentum equations are reduced considerably which allows for a straight-forward numerical solution procedure. The governing thermal equations are solved using an asymptotic expansion approach, keeping the first and second order terms and equations. This is done so to more
accurately model the effects the circulating fluid has on the thermal performance of the bearing.

The phenomenon of cavitation is also discussed, utilizing a method that integrates cavitation into the governing equations and numerical solution procedure. Unlike other cavitation models that decouple cavitation from the governing momentum equations, this model accounts for mass flow continuity which leads to more realistic results.

Practical design considerations, including how to determine the effective permeability and the effective heat transfer coefficient at the exterior wall of the bearing, are discussed. These parameters, used extensively in the analytical and numerical modeling of the bearing, are essentially functions of other physical parameters. Once these relationships are established, their values can be utilized by someone looking to design a bearing considered in this work with a set of performance criteria in mind.

The combination of analytical work and numerical computations produces a comprehensive look at this new type of bearing. A long slider bearing and both long and short journal bearings are discussed in a parametric fashion, whereby the effects of varying the operational and geometric parameters are investigated by examining the accompanying pressure and temperature fields. The model presented here demonstrates the feasibility of a bearing that is capable of supporting a load while eliminating the need for an external lubricant supply and the necessary infrastructure that is required to actively feed a bearing with lubricant. It is shown that the temperatures stay within operating limits utilizing a realistic heat transfer coefficient and a
realistic thermal conductivity for the lubricant while generating pressures inside the film that can support a load and simultaneously pump fluid between the film and reservoir regions.
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13.1 Exploded View of Proposed Bearing Geometry with External Pin Fin Array

13.2 Pin Fin Array Example Geometry
1.1 Introduction

The use of journal bearings is widespread, and the scope of their applications include automotive technology, lubrication of jet engines, and many other applications where rotating machinery is used. A standard journal bearing typically makes use of a concentric metal cylinder about a rotating shaft with a lubricant feed, typically located at the location of maximum film clearance. The converging-diverging nature of the geometry of the journal bearing provides pressures that support the rotating shaft. In most cases, a pressure gradient is created along the axial direction which forces some of the lubricant out of the bearing in the axial direction. This fluid, heated by the frictional forces created by the rotating shaft and the viscosity of the lubricant, is then pumped into an external heat exchanger, whereby the fluid is cooled using either forced or natural convection. The fluid is then fed back into the journal bearing, and the cycle repeats.

A question to be asked is, 'Can this cycle be made more efficient?' For cases where the load to be supported is great, such as in an airplane jet engine, typically the lubricating fluid is quite viscous. This fact makes the fluid more difficult to pump
than a less viscous fluid, and thus, requiring more energy to pump the fluid into the bearing. Perhaps, the question just posed can then be stated as, 'Can the pump be removed from the infrastructure needed for a journal bearing to operate?' If no pump were present to pump the fluid through the heat exchanger, the energy required to move the fluid through the radiator would have to come from the bearing itself, as the pressure in the axial directions would have to be high enough to pump the fluid through the radiator and back into the bearing housing. This pressure would come at the expense of pressure in the circumferential direction and would decrease the load-carrying capacity of the bearing, which is not an ideal solution.

However, what if the question to be asked was, 'Can the pump and heat exchanger be removed from the infrastructure needed for a journal bearing to operate?' Obviously, if this question were to be answered in the affirmative, then the total amount of lubricant needed for operation would have to be present inside the bearing from the onset of operation and the bearing would have to be sealed at the axial ends. Using the jet engine example, a bearing such as the one just described would be a revolution for jet engine design, as the weight and power savings from the elimination of a powerful pump and massive heat exchanger, along with the oil lines needed to run the lubricant to and from the bearing housing would be tremendous.

If such a scenario were to exist with a typical journal bearing, clearly they would be in operation already. It stands to reason that sealed journal bearings are not capable of providing the load-carrying capacity while removing the heat created by the frictional forces inside the bearing. A possible modification to journal bearings
would be to use a liquid metal as the lubricant instead of standard hydrocarbon-based lubricants, as this would allow for the bearing to operate at much higher temperatures than oils. As a byproduct, the journal bearing may be operated without requiring an external heat exchanger.

In the 1960’s, liquid sodium was investigated for its potential use as a lubricant, as its melting point is around 371 K, and its boiling point is 1156 K, which allows for an extended thermal range of operation. The melting point of sodium is high enough that the bearing would have to be in constant operation, or at least constantly heated, as to avoid the cooling and solidification of the liquid metal. However, the downsides to using liquid sodium as a lubricant do not stop there. Sodium is highly reactive to both water and air, being potentially explosive if exposed to either. As such, liquid sodium must be kept in an inert environment, and handled extremely carefully. Weighing the advantages and disadvantages of liquid sodium, industry has chosen not to use it as a lubricant, although it is used inside some nuclear reactors as a heat transfer fluid.

The benefits of using a liquid metal inside a journal bearing are clear, though the best candidate should feature a low melting point, preferably around room temperature, a high boiling point, and be non-reactive to water or air. Liquid gallium boasts these features, with a melting point of 303 K (just above room temperature), a boiling point of 2477 K, it is considered non-toxic, and it does not react violently with air or water. However, gallium can weaken steel and other metal alloys, so for a journal bearing to use gallium as its lubricant, the choice of bearing material
properties would have to be made accordingly.

The work to be considered here describes a modification to a porous journal bearing that will feature self-circulation and will not require any external lubricant sources or the required infrastructure needed for pumping the fluid and removing the heat created by the operation of the bearing. The remainder of this chapter will discuss established theories and published works that will serve as the foundation to the work described in this paper.

1.2 Theories of Porous Medium-Fluid Interface Models and Literature Review

The study of flows through a porous medium begins with the work done by Darcy, who was the first to relate a pressure gradient with the local flow velocity relative to the porous medium [1]. He introduced the equation,

$$\nabla p = -\frac{\mu}{K} (\vec{V} - V_{porousmedium})$$ (1.1)

which is now known as Darcy’s Law. In the above equation, \(p\) represents pressure, \(\mu\) represents the viscosity of the fluid, \(K\) represents the permeability tensor, \(\vec{V}\) represents the velocity vector of the fluid, and \(V_{porousmedium}\) represents the velocity vector of the porous medium.

A shortcoming of Darcy’s law is that it cannot account for viscous shear effects at the boundary of the porous medium. Beavers and Joseph [2] attempted to correct this shortcoming by introducing a slip-flow boundary condition at the interface between the porous medium and the fluid. Through this boundary condition,
they were able to produce results that were in reasonable agreement with experimental results, however, it required an empirical constant to ensure satisfactory results. Thus, for every possible configuration of porous medium geometry and lubricating fluid, a new slip-flow coefficient would have to be produced via experimental methods.

Two analytical corrections exist for Darcy’s Law. The first was developed by Brinkman in 1947 [3]. What is now known as the Brinkman equation, or Brinkman-Extended Darcy’s Law, this equation accounts for viscous shear effects at the interface between the porous medium and the lubricating fluid as well as the viscous damping effects from Darcy’s Law:

\[
\nabla p = \mu_{\text{eff}} \nabla^2 v - \mu \frac{1}{K} (V - V_{\text{porousmedium}}), \tag{1.2}
\]

where \( v \) is the fluid velocity vector inside the porous medium. Though this accounts for viscous shear better than the empirical method of Beavers and Joseph, it still requires one to calculate the effective viscosity of the fluid, \( \mu_{\text{eff}} \), which is a function of the viscosity of the fluid and the geometry of the porous material. As of yet, there is no analytical method for determining \( \mu_{\text{eff}} \), which makes it an empirical constant, not unlike the slip-flow coefficient of Beavers and Joseph.

A second correction for Darcy’s Law is used for high speed flows through the porous medium and gives the Brinkman-Forchheimer-extended Darcy’s Law:

\[
\nabla p = \mu_{\text{eff}} \nabla^2 v - \mu \frac{1}{K} (V - V_{\text{porousmedium}}) - \frac{F \rho}{\sqrt{K}} \nabla V, \tag{1.3}
\]
where $\rho$ is the density of the fluid. Again, like Brinkman’s extension, Equation (1.3) requires an empirical constant, $F$, which depends on the properties of the fluid and the structure of the porous material.

It should be evident that one may recover Darcy’s Law from the Brinkman-Forchheimer-extended Darcy’s Law under certain circumstances, one being low velocity flows through a thick porous medium, though such a reduction is not limited to this one case. Order of magnitude analyses can be used to determine if, and how, Equation (1.3) can be reduced.

1.3 Theories of Porous Bearings and Literature Review

The use of porous journal bearings has been widespread in industry for many years. The advantages of these bearings over their standard journal bearing counterparts are numerous. Once impregnated with lubricant, the need for an exterior lubricant supply is greatly reduced or eliminated. As such, porous journal bearings are often used in applications where the bearing is inaccessible to external lubricant supplies. Another advantage of porous journal bearings is that they can be used in applications where a potential lubricant leak cannot be tolerated, such as in the food industry. Also, porous journal bearings are often cheaper than standard journal bearings.

In 1957, Morgan and Cameron were the first to analytically discuss the hydrodynamic theory of porous journal bearings [4]. Their work offered a solution to a short porous journal bearing. Several years later, Rouleau improved upon the work by Morgan and Cameron, and he also investigated a narrow press-fitted porous journal
bearing [5], [6]. In 1971 and 1972, Murti constructed analytical solutions for short, long, and finite-length porous bearings [7], [8], [9]. Also, in 1972, Cusano presented an analytical discussion for porous journal bearings [10]. In 1995, Tichy [11] discussed porous media and thin film lubrication and showed that the effect of the thickness of the porous material is to increase the load-carrying capacity while reducing the friction coefficient. He also showed that increasing the porosity parameter tends to reduce the magnitude of said increase, as slip flow can then occur in the porous layer. All of the aforementioned papers utilized Darcy’s law to govern the fluid flow inside the porous medium.

Starting in the early 1990’s, however, a paradigm shift began to occur, as most papers dealing with porous bearings began to focus on using the Brinkman-Extended Darcy’s Law to describe the fluid flow inside the porous medium and to capture the slip-flow boundary layer at the porous medium-fluid interface. Though Neale and Nader showed that Darcy’s law is valid everywhere inside a porous medium except at the boundary layer at the porous medium-fluid interface [12], the community began to adopt the Brinkman-Extended Darcy’s Law en masse, as the benefit of recovering information inside that boundary layer seemed worth the effort. In 1992, Lin and Hwang used the Brinkman-Extended Darcy’s Law to show that its use showed an increase in the pressure values and load-carrying capacity of a short journal bearing, as well as a decrease in the friction coefficient and attitude angle, when compared to using Darcy’s Law [13]. Later, Lin expanded his work to include the use of a flexible porous medium for a long porous journal bearing, again demonstrating the
benefit that the use of the Brinkman-Extended Darcy’s Law has on modeling bearing performance. In 1996, Hwang, Lin, and Yang applied their work to porous slider bearings and showed that, by using the Brinkman-Extended Darcy’s Law, the viscous shear effects increase the load-carrying capacity and decrease the friction coefficient [14]. Also, they showed that both increasing the porous medium thickness and the inclination of the slider led to greater effect of viscous shear on the bearing.

Also, in the mid-1990’s, a complementary school of thought began, concerning the boundary condition at the porous medium-fluid interface. Ochoa-Tapia and Whitaker were the first to postulate a stress jump boundary condition at the porous medium-fluid interface along with an accompanying stress jump parameter [15], [16]. Though their work was quite rigorous in its derivation, the stress jump parameter, varying from -1 to 1.47, is another empirical constant, dependent on the properties of the porous material and the lubricating fluid [16]. Li used this idea developed by Ochoa-Tapia and Whitaker to derive a modified Reynolds equation for flows between two porous layers in a wedge configuration [17]. His work aimed to correct the shortcomings of simply using Darcy’s Law or using the Brinkman-Extended Darcy’s Law with shear stress continuity at the porous medium-fluid interface, though his work showed how sensitive his results are to the value of the empirical stress jump parameter.
1.4 Theories of Cavitation and Literature Review

For moderately loaded bearings, a condition called cavitation can occur. Cavitation is the phenomenon where a fluid, when subjected to a sufficiently low pressure, is caused to rupture, forming a cavity. For a long journal bearing, typically when the length of the bearing is more than twice the diameter of the rotating journal, the solution of the Reynolds equation yields a pressure curve that is basically sinusoidal, where $p = 0$ at $\theta = 0, \pi$, and $2\pi$, the pressure is positive for $0 < \theta < \pi$ and the pressure is negative for $\pi < \theta < 2\pi$. This is known as the full-Sommerfeld solution.

For this solution to be physically accurate, operation of such a bearing would have to occur in an environment with a sufficiently high ambient pressure so as not to yield negative pressures.

The half-Sommerfeld solution, also known as the Gumbel condition, avoids this pitfall by neglecting the predicted negative pressures from the full-Sommerfeld solution. Though this yields reasonably accurate results for predicting the load-carrying capacity of the bearing, it violates the principle of continuity of flow at $\theta = \pi$. To rectify this shortcoming, Swift and Stieber independently showed that, at the cavitation angle (the angle at which cavitation begins), $\frac{\partial p}{\partial \theta} = 0$. This essentially smooths out the solution near the cavitation angle. Known as the Swift-Stieber condition, this method is generally accepted by engineers, as it yields more accurate results than the half-Sommerfeld solution. Though film rupture is more appropriately treated using the Swift-Stieber condition, reformation of the film is essentially neglected.
Jakobsson and Floberg [18] and Olsson [19] derived a set of boundary conditions which properly account for mass conservation in the cavitation region. Known as JFO theory, like the Swift-Stieber condition, this method assumed a constant pressure inside the cavitated region, and it also offered a set of boundary conditions that not only accurately predict both the cavitation angle and reformation angle (the angle at which the film reforms) but also properly account for mass conservation inside the cavitation region.

Elrod and Adams [20] introduced a computational scheme that incorporated JFO theory in a very simple manner. Their algorithm encompassed the ideas of JFO theory but it automatically predicted the region of cavitation. Later, Elrod [21] modified the finite difference portion of his algorithm and obtained much better results. This procedure, now known as the Elrod cavitation algorithm, is based on a control volume formulation and uses a switch function to eliminate the pressure term in the cavitated region. Elrod developed his algorithm using trial and error and offered little in the way of developmental details. Vijayaraghavan and Keith set out to expand on Elrod’s work and offered a similar algorithm that contained substantial developmental details [22]. Their work developed a universal cavitation equation for the fluid film for both long slider and journal bearing configurations.

1.5 Theories of Heat Transfer Inside a Porous Medium and Literature Review

The modeling of heat transfer inside a porous medium does not have nearly the amount of controversy as modeling the momentum transfer inside a porous medium.
The generally accepted method is a two-equation model called the Local Non-Thermal Equilibrium (LTNE) model,

\[ \phi(\rho c)_f \frac{\partial T_f}{\partial t} + (\rho c)_f \mathbf{v} \cdot \nabla T_f = \phi \nabla \cdot (\nabla k_f T_f) + H (T_s - T_f) + Q \]  
(1.4)

\[ (1 - \phi)(\rho c)_s \frac{\partial T_s}{\partial t} = (1 - \phi) \nabla \cdot (\nabla k_s T_s) + H (T_f - T_s) + Q, \]  
(1.5)

where \( \phi \) is the porosity of the porous medium, \((\rho c)_f\) is product of the density and specific heat inside the fluid, \((\rho c)_s\) is product of the density and specific heat inside the solid, \(k_f\) is the thermal conductivity inside the fluid, \(k_s\) is the thermal conductivity inside the solid, \(H\) is the inter-phase heat transfer coefficient, and \(Q\) is the generated heat term. This model accounts for the working fluid and the material of the porous medium having different thermal conductivities [23], [24], [25], [26].

It should be clear that for the limiting case of \(k_f = k_s\), the two equation model reduces to a one equation model, known as Local Thermal Equilibrium (LTE):

\[ \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \left( \frac{k}{\rho c} \right) \nabla^2 T, \]  
(1.6)

where \((\rho c)\) is product of the density and specific heat of the fluid and \(k\) is the thermal conductivity of the fluid. This assumption defines an equation that is similar to the advection-diffusion equation for energy transport in a homogeneous fluid.

One must note here that the energy equation and momentum equations used to describe fluid flow may be decoupled if one assumes that the affect of temperature on fluid viscosity and fluid density is insignificant. However, if the viscosity and/or the density are functions of the temperature of the fluid, the momentum and energy equations must be solved in a coupled manner, and the temperature dependence of
the fluid viscosity and/or density must be known before conducting the numerical simulation.

These equations are generally accepted, however there is room for debate concerning the boundary conditions to be used at the porous medium-fluid interfaces. Alazmi and Vafai [26] conducted an analysis of heat transfer between a porous medium and a fluid layer using three different combinations of boundary conditions shown in Table 1.1. In the table, \( k_{\text{eff}} = \phi k_f + (1 - \phi) k_s \), \( \phi \) is the porosity of the porous medium, \( k_f \) is the thermal conductivity of the fluid, \( k_s \) is the thermal conductivity of the material that composes the porous medium, \( T_+ \) corresponds to the temperature above the fluid-porous medium interface, \( T_- \) corresponds to the temperature below the fluid-porous medium interface. Their work concluded that for most practical applications, there are negligible differences between the different models shown above [26].

<table>
<thead>
<tr>
<th>Model</th>
<th>Temperature</th>
<th>Temperature Gradient</th>
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<tbody>
<tr>
<td>I</td>
<td>( T_+ = T_- )</td>
<td>( k_{\text{eff}} \frac{\partial T_-}{\partial y} = k_f \frac{\partial T_+}{\partial y} )</td>
</tr>
<tr>
<td>II</td>
<td>( T_+ = T_- )</td>
<td>( (\phi + k_f) \frac{\partial T_-}{\partial y} = k_{\text{eff}} \frac{\partial T_+}{\partial y} )</td>
</tr>
<tr>
<td>III</td>
<td>( \frac{\partial T}{\partial y}\big</td>
<td><em>+ = \frac{\alpha r}{\lambda} (T</em>+ - T_-) )</td>
</tr>
</tbody>
</table>

Table 1.1: Thermal Interfacial Boundary Conditions
CHAPTER II

SCOPE OF WORK

2.1 Overall Concept of Proposed Work

The goal of this work is to establish the viability of a sealed, self-acting, self-circulating journal bearing that can operate with no need for an external lubricant source. The geometry of the bearing proposed here can be seen in Figure 2.1. Notice that this bearing essentially takes a journal bearing and inserts a concentric porous medium between the rotor and bearing wall, producing three regions: a fluid reservoir, a porous medium, and the hydrodynamic film that supports the load placed on the rotor.

The build-up of pressure in the film near the minimum clearance between the rotor and porous medium wall (called the converging region) will create a positive pressure difference between the film and reservoir regions, and some fluid will be pushed through the porous medium into the reservoir. Then, in the region where the clearance between the rotor and porous wall increases (called the diverging region), a negative pressure difference between the film and reservoir regions will exist, drawing the fluid from the reservoir, through the porous medium, and back into the film. Thus, as a product of the geometry described here and the rotating action of the
journal, the fluid acts to support the load of the rotor while being circumferentially circulated inside the bearing.

Figure 2.2 demonstrates the high-level concept for an unwrapped journal bearing. The pressure difference between the two fluid regions circulates the flow, transferring energy created in the converging region into the reservoir where the heat will be removed using conduction into the environment and forced convection, while simultaneously maintaining high pressures inside the film region to support the load.

By using an external housing that features an array of pin fins, the bearing can act as its own heatsink. Thus, the design of the bearing acts as its own lubricant pump and heat exchanger - two pieces of equipment that are normally used in operation of a journal bearing. In a practical application, this would lead to substantial weight and energy savings.
This work takes the governing equations that would describe the momentum and heat transfer and simplifies them using a non-dimensional analysis to determine the terms that most contribute overall to the system. Though most modern computational fluid dynamics (CFD) packages can model the geometry described in this paper, they solve the governing equations using every term, not taking advantage of the disproportionality of scales that can be used. Using an asymptotic expansion approach to eliminate terms that contribute very little to the solution of the governing equations, the constructed system of equations can be numerically solved in far less time than that of traditional CFD software packages. Also, using this approach, the properties of the porous medium used can be finely tuned, instead of using an isotropic porous medium that CFD packages use.
In this work, two cases are considered: an infinitely long slider bearing and a journal bearing, for which numerical simulations will be ran for the long and short journal cases, which are differentiated by the ratios of their axial lengths to their diameters. The slider bearing discussed introduces the reader to the methods that will be used for the journal bearing and acts as a proof of concept for the operation of the journal bearing. Boundary conditions for both the momentum and thermal problems will be similar, but not identical, to those used in the corresponding long journal problem. The methods for modeling the journal bearing will then be discussed, and results will be presented for both long and short journal cases.

2.2 Analytically Modeling the Momentum Equations

Inside the fluid regions (the fluid film and reservoir), the initial governing momentum equations will be the steady-state Navier-Stokes equations. A non-dimensional scheme will be used to determine what terms will contribute to the leading order governing equations. The technique will be used for both the long slider case and for the journal cases. Darcy’s Law will be used as the initial governing equation for fluid flow inside the porous medium for the long slider momentum problem, and the Brinkman-Extended Darcy’s Law will serve as the initial governing equation for fluid flow inside the porous medium for the long and short journal momentum problems. A non-dimensional scheme will be used to determine the terms that will contribute to the leading order governing momentum equations. When matched with the boundary conditions from the porous medium, coupled with assumptions made concerning
the geometry scales of the bearing and the properties of the porous medium, a set of three coupled governing momentum equations will be constructed - one for each region - to be solved numerically.

2.3 Analytically Modeling Cavitation

After the three coupled governing momentum equations are constructed, the cavitation scheme described by Vijayaraghavan and Keith [22] will be adapted to include modeling potential cavitation inside all three regions. This is done by rewriting the governing momentum equations in terms of a new, non-dimensional density variable, $\theta$ and a switch function, $g$, to allow for an accurate cavitation model. This will produce a new coupled set of three partial differential equations that will need to be solved numerically.

2.4 Analytically Modeling the Thermal Equations

The standard energy equation will be used to describe the temperatures inside both fluid regions and a local thermal equilibrium energy equation will be used to describe the temperatures inside the porous medium. These equations will be non-dimensionalized, using the same scales used for the non-dimensionalization of the Navier-Stokes and Brinkman-Extended Darcy equations. Upon doing so, both the leading order and first correction equations will be solved, using the solution from the momentum equations as inputs to the system.
2.5 Numerical Techniques for Solving the Momentum and Thermal Equations

The coupled set of three governing momentum partial differential equations are discretized, utilizing methods similar to those used by Vijayaraghavan and Keith [22]. The final result is a system that is solved via the use of a tridiagonal solver in a block SOR iterative scheme. This approach yields values for the non-dimensional density variable, $\theta$, which is then used to calculate the pressures inside each region of the bearing. The velocities in each region are then computed using the calculated pressure values. Upon doing so, the pressure and velocity values are used as inputs to the discretized thermal equations. The methods used to discretize the thermal governing partial differential equations are discussed and the final matrix equations are generated and solved using a pentadiagonal solver.

2.6 Practical Design Considerations

The permeability used in this work, $k$, is essentially treated as a flow conductivity term, though it is a function of the geometry of the porous medium being used. As such, the relationship between the permeability, $k$, the porosity, $\phi$, and the geometry of the porous medium is established to provide a working numerical range for the permeability, $k$. Also, to discuss the feasibility of such a bearing, a realistic heat transfer coefficient at the reservoir wall must be used. A comparison between a standard journal bearing and a journal bearing whose external housing is fitted with
a pin fin array, to make the bearing housing act as a heatsink, will be drawn. The
design criteria needed to insure reliable operation of such a bearing will be discussed.

2.7 Numerical Results

Results will be displayed in a parametric fashion. The effects of varying each parameter on the pressure and temperature fields will be shown and discussed. The goal of presenting the data in this fashion is to clearly demonstrate the effect each parameter has on the resulting performance of the bearing, which will allow for one to design a bearing with certain performance characteristics in mind. Results will be shown for the long slider as well as for the short and long journal cases.

2.8 Contributions

In all, this work makes the following contributions to academia:

• We demonstrate the theoretical feasibility of the proposed bearing concept through analytical modeling and numerical simulation.

• By providing reasonable physical limits, simplified governing equations are constructed that allow for fast numerical solutions.

• We integrate a cavitation algorithm that conserves mass flow throughout the entire bearing domain.

• We provide design criteria for a designer to construct a bearing that would yield the desired performance criteria.
• We design a bearing that can represent a revolution in bearing design, yielding engine designs that are lighter and more energy-efficient than those that use standard journal bearings for lubrication.
CHAPTER III
LONG SLIDER MOMENTUM EQUATIONS

We begin our study with a look at a slider bearing that features a porous insert between the solid slider and the solid wall (Figure 3.1). This yields a bearing with three regions - a fluid reservoir region, a porous medium region, and a hydrodynamic film region. We consider the bearing to be infinitely long in the axial direction, leaving us with a two-dimensional problem, and we call the resulting configuration a long slider bearing.

We essentially are using this bearing configuration as a stepping stone to the final goal of this work, which is to accurately model a three-dimensional journal bearing with an eccentric porous insert. The point of using this simplified geometry is to understand the analytical and numerical techniques necessary to model this type of bearing without being overwhelmed by the more complex case the journal bearing represents. By starting small and modeling the long slider, we get a feel for how to non-dimensionalize, simplify, and numerically model the governing momentum and thermal equations - methods that can be extended to the journal bearing case.
3.1 Geometry

The geometry analyzed here is that of a slider bearing with a convergent-divergent film shape as shown in Figure 3.1, where the pertinent physical and operational parameters are described in Tables 3.1 and 3.2. If one examines this figure and wraps it back around onto itself it basically recreates the geometry of the journal bearing/reservoir assembly described in Figure 2.1. Thus, it follows that we are imposing periodic boundary conditions in the $x$-direction so as to mimic an unwrapped journal bearing.

![Figure 3.1: Long Slider Geometry with the Convergent-Divergent Film Profile, Porous Insert and Reservoir](image)

For this geometry slider bearing, in the converging region the pressure is higher in the film than in the reservoir and so the fluid is pumped into the reservoir. In the diverging region, the pressure is higher in the reservoir than in the film and so the fluid is pumped back into the film. This will allow for a self-lubricating slider bearing operating with no external fluid pump, as all the fluid that is needed will be
contained inside the bearing and the reservoir. In the case of a journal bearing, the fluid is entrained by the rotation of the journal and has a total velocity $U =$constant. However due to the journal curvature the horizontal and vertical components of $U$ change around the circumference. In the case presented here the fluid is entrained by the motion of the porous medium and the external wall and their velocity, $U$, contains no vertical component. Thus, one must be careful in extrapolating directly the results from this study to a journal bearing. This study is a proof of concept for a porous journal bearing with an external reservoir, seeking to show that the varying shape of the fluid film interacting with the moving porous medium/external reservoir will allow for a pumping action, in and out of the active hydrodynamic film space.

3.2 Development of the Model

A further inspection of Figure 3.1 shows that three contiguous regions were defined. Region I, defines the reservoir space while Regions II and III represent the porous medium and the active hydrodynamic film, respectively. The general mass flow and momentum governing equations represented by Equations (3.1), (3.2), and (3.3) introduced in this section will be customized for each one of these regions and the physical and mathematical connectivity relating these regions to each other is established. If one expresses the steady-state Navier-Stokes equations in two dimensions
where the body forces are neglected, one can write:

\[
\begin{align*}
    u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
    u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right).
\end{align*}
\]

(3.1) (3.2)

The corresponding continuity equation in steady state form can be written as

\[
\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0.
\]

(3.3)

In order to implement a comprehensive parametric study, Equations (3.1) and (3.2) are non-dimensionalized by adopting the following relationship between the dimensional and non-dimensional variables:

\[
\begin{align*}
    u &= UU^* \\
    v &= \left( \frac{\alpha}{L_x} \right) UV^* \\
    x &= L_x X^* \\
    y &= \alpha Y^* \\
    p &= \left( \frac{L_x \mu U}{\alpha^2} \right) P^*.
\end{align*}
\]

(3.4) (3.5) (3.6) (3.7) (3.8)

Then Equations (3.1) and (3.2) can be written in a dimensionless form as follows:

\[
\begin{align*}
\left( \frac{\alpha}{L_x} \right)^2 \left( \frac{\rho L U}{\mu} \right) \left( U^* \frac{\partial U^*}{\partial X^*} + V^* \frac{\partial U^*}{\partial Y^*} \right) &= -\frac{\partial P^*}{\partial X^*} + \left( \frac{\alpha}{L_x} \right)^2 \frac{\partial^2 U^*}{\partial X^{*2}} + \frac{\partial^2 U^*}{\partial Y^{*2}} \\
\left( \frac{\alpha}{L_x} \right)^4 \left( \frac{\rho L U}{\mu} \right) \left( U^* \frac{\partial V^*}{\partial X^*} + V^* \frac{\partial V^*}{\partial Y^*} \right) &= -\frac{\partial P^*}{\partial Y^*} + \left( \frac{\alpha}{L_x} \right)^4 \frac{\partial^2 V^*}{\partial X^{*2}} + \left( \frac{\alpha}{L_x} \right)^2 \frac{\partial^2 V^*}{\partial Y^{*2}}.
\end{align*}
\]

(3.9) (3.10)

Note that \( \frac{\alpha}{L_x} \) is a geometric aspect ratio similarity parameter where \( \alpha \ll L_x \), while \( \frac{\rho L U}{\mu} \) is the Reynolds number and thus a dynamic similarity parameter. If one defines
\[ \varepsilon_x = \frac{\alpha}{L_x} \] as a small parameter \((\ll 1)\) and the Reynolds number as \(Re_x = \frac{\rho L_x U}{\mu}\), one can rewrite Equations (3.11) and (3.12) as

\[ \varepsilon_x^2 (Re_x) \left( U^* \frac{\partial U^*}{\partial X^*} + V^* \frac{\partial U^*}{\partial Y^*} \right) = - \frac{\partial P^*}{\partial X^*} + \varepsilon_x^2 \frac{\partial^2 U^*}{\partial X^* \partial Y^*} + \frac{\partial^2 U^*}{\partial Y^*} \] (3.11)

\[ \varepsilon_x^4 (Re_x) \left( U^* \frac{\partial V^*}{\partial X^*} + V^* \frac{\partial V^*}{\partial Y^*} \right) = - \frac{\partial P^*}{\partial Y^*} + \varepsilon_x^4 \frac{\partial^2 V^*}{\partial X^* \partial Y^*} + \varepsilon_x^2 \frac{\partial^2 V^*}{\partial Y^*} \] (3.12)

Provided that the \(Re_x\) is not of Order \(\left( \frac{1}{\varepsilon_x^2} \right)\), that is \(Re_x \ll \frac{1}{\varepsilon_x^2}\), we may neglect the inertia terms, and by ignoring the non-dimensional groups that are on the order of \(\varepsilon_x^2\) and \(\varepsilon_x^4\), the resulting governing momentum equations leave the pressure forces and the viscous forces to balance each other. Thus, the leading order problem can be written as

\[ \frac{\partial P^*}{\partial X^*} = \frac{\partial^2 U^*}{\partial Y^*} \] (3.13)

\[ \frac{\partial P^*}{\partial Y^*} = 0, \] (3.14)

for both Region I and Region III. Note that under the above assumption of \(Re_x \ll \frac{1}{\varepsilon_x^2}\), this approach is possible even though the active hydrodynamic film thickness may be one order of magnitude smaller than that of the reservoir depth (for example 25 \(\mu m\) (0.001 in) versus 250 \(\mu m\) (0.01in)). Once simplified, Equations (3.13) and (3.14) can be re-dimensionalized to obtain

\[ \frac{\partial p}{\partial x} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} \] (3.15)

\[ \frac{\partial p}{\partial y} = 0. \] (3.16)
However, for the analysis performed here, Equation (3.16) will not factor into constructing the governing Reynolds equations. Instead, it lets us know that, for this first order analysis, the pressure does not vary across the thickness of the fluid regions.

For evaluation of the pressure drop in the porous medium, Darcy’s Law is employed:

\[
\frac{\partial \tilde{p}}{\partial x} = -\frac{\mu}{k_x} (\tilde{u} - U) \tag{3.17}
\]
\[
\frac{\partial \tilde{p}}{\partial y} = -\frac{\mu}{k_y} \tilde{v}. \tag{3.18}
\]

Besides the standard sources for the emergence of a pressure gradient (momentum change and friction), this equation provides an additional source of pressure drop due to the resistance introduced by the porous medium to the flow. The terms of Equations (3.17) and (3.18) are kept in their totality since they represent the Darcy pressure drops, essential to modeling flow in the porous medium; they can vary by several orders of magnitude depending on the geometry and properties of the porous medium.

Considering the assumptions and development presented above the governing equations in each one of the three regions can be defined as follows:

Region I (Reservoir):

\[
\frac{\partial p_I}{\partial x} = \mu \frac{\partial^2 u_I}{\partial y^2} \tag{3.19}
\]
\[
\frac{\partial p_I}{\partial y} = \mu \frac{\partial u_I}{\partial x} \tag{3.20}
\]
Region II (Porous Medium):

\[
\frac{\partial \tilde{p}}{\partial x} = -\frac{\mu}{k_x} (\tilde{u} - U) \tag{3.21}
\]

\[
\frac{\partial \tilde{p}}{\partial y} = -\frac{\mu}{k_y} \tilde{v} \tag{3.22}
\]

Region III (Hydrodynamic Fluid Film):

\[
\frac{\partial p_{III}}{\partial x} = \mu \frac{\partial^2 u_{III}}{\partial y^2} \tag{3.23}
\]

The governing equations in Regions I and III are subject to the following boundary conditions:

Region I:

\[
u_1(x, y = 0) = 0 \tag{3.25}
\]

\[
u_1(x, y = \alpha) = U \tag{3.27}
\]

\[
v_1(x, y = 0) = 0 \tag{3.28}
\]

The hydrodynamic fluid film height function, \(h(x)\), defined as the clearance between the moving porous medium and the stationary slider wall, is given by

\[
h(x) = \left(\frac{h_{\text{min}} + h_{\text{max}}}{2}\right) \left[1 + \left(\frac{h_{\text{max}} - h_{\text{min}}}{h_{\text{min}} + h_{\text{max}}}\right) \cos \left(\frac{2\pi x}{L}\right)\right]. \tag{3.25}
\]
Region III:

\[ u_{III}(x, y = \beta) = U \]  
\[ u_{III}(x, y = \beta + h(x)) = 0 \]  
\[ v_{III}(x, y = \beta + h(x)) = 0. \]

Pressure is continuous at the reservoir-porous medium interface, between Regions I and II, as well as at the porous medium-hydrodynamic film interface, between Regions II and III; thus one can write

\[ \rho_I(x, y = \alpha) = \tilde{\rho}(x, y = \alpha) \]  
\[ \tilde{\rho}(x, y = \beta) = \rho_{III}(x, y = \beta). \]

The mass flow at the fluid-porous medium interfaces has to be conserved; so for an incompressible fluid one must write that

\[ \rho_I v_I(x, y = \alpha) = \tilde{\rho} \tilde{v}(x, y = \alpha) \]  
\[ \tilde{\rho} \tilde{v}(x, y = \beta) = \rho_{III} v_{III}(x, y = \beta). \]

Equations (3.21) and (3.22) can be solved algebraically for the \( x \)- and \( y \)-direction fluid velocity components in the porous medium, \( \tilde{u} \) and \( \tilde{v} \), to get

\[ \tilde{u} = U - \frac{k_x}{\mu} \frac{\partial \tilde{p}}{\partial x} \]  
\[ \tilde{v} = -\frac{k_y}{\mu} \frac{\partial \tilde{p}}{\partial y}. \]

Integrating Equation (3.19) twice with respect to \( y \) and applying boundary conditions Equations (3.26) and (3.27), one obtains the velocity for the \( x \)-direction in the
reservoir as

\[ u_I = \left[ \frac{y(y - \alpha)}{2\mu} \right] \frac{\partial p_I}{\partial x} + U - \frac{k_x}{\mu} \frac{\partial \bar{p}}{\partial x} \bigg|_{y=\alpha}. \]  

(3.38)

Similarly, integrating Equation (3.23) with respect to \( y \) and applying boundary conditions, Equations (3.29) and (3.30), one finds the \( x \)-direction velocity in the hydrodynamic film as

\[ u_{III} = \left\{ \frac{[y - (\beta + h)](y - \beta)}{2\mu} \right\} \frac{\partial p_{III}}{\partial x} - \left[ \frac{y - (\beta + h)}{h} \right] \left( U - \frac{k_x}{\mu} \frac{\partial \bar{p}}{\partial x} \bigg|_{y=\beta} \right). \]  

(3.39)

In order to construct the Reynolds equations for the (Region I + Region III) system, mass continuity equations have to be satisfied in every region. This work considers using the compressible form of the continuity equations to allow for a realistic cavitation model formulation. Though the governing equations described above are steady state, the continuity equations are cast in their transient form solely for the purpose of implementing a time dependent numerical integration technique which will be discussed later.

The continuity equation to be solved in Region I is

\[ \frac{\partial \rho_I}{\partial t} + \frac{\partial (\rho_I u_I)}{\partial x} + \frac{\partial (\rho_I v_I)}{\partial y} = 0. \]  

(3.40)

Integrating Equation (3.40) across the reservoir, while applying boundary condition Equation (3.28) and the mass continuity condition at the interface, Equation (3.34) one obtains

\[ \frac{\partial}{\partial t} (\rho_I \alpha) + \frac{\partial}{\partial x} \left[ (\rho_I U \alpha) - \left( \frac{\alpha^3}{12\mu} \right) \left( \rho_I \frac{\partial \rho_I}{\partial x} \right) - k_x \left( \frac{\alpha}{2\mu} \right) \rho_I \frac{\partial \bar{p}}{\partial x} \bigg|_{y=\alpha} \right] = - \left( \rho \bar{v} \right) \bigg|_{y=\alpha}. \]  

(3.41)
The continuity equation to be solved in Region III is

\[
\frac{\partial \rho_{III}}{\partial t} + \frac{\partial (\rho_{III} u_{III})}{\partial x} + \frac{\partial (\rho_{III} v_{III})}{\partial y} = 0. \tag{3.42}
\]

Integrating Equation (3.42) across the reservoir, while applying boundary condition Equation (3.31) and the mass continuity condition at the interface, Equation (3.35), one obtains

\[
\frac{\partial}{\partial t} (\rho_{III} h) + \frac{\partial}{\partial x} \left\{ \left[ (\rho_{III}) \left( U - \frac{k_x}{\mu} \frac{\partial \bar{p}}{\partial x} \bigg|_{y=\beta} \right) \right] h \right\} - \left( \frac{\rho_{III} h^3}{12\mu} \right) \frac{\partial p_{III}}{\partial x} = (\bar{\rho} \bar{v}) \bigg|_{y=\beta}. \tag{3.43}
\]

### 3.3 Assumptions and Simplifications

The porous material model considered for this paper represents essentially a structure of capillaries parallel to each other, oriented in the \( y \)-direction and connecting the hydrodynamic film active space to the reservoir space. As a result, we are considering a heterogeneous porous material with permeability \( k_x = 0 \); then Equations (3.41) and (3.43) can be simplified to:

\[
\frac{\partial}{\partial t} (\rho_I \alpha) + \frac{\partial}{\partial x} \left[ (\rho_I U \alpha) - \left( \frac{\alpha^3}{12\mu} \right) \left( \rho_I \frac{\partial p_I}{\partial x} \right) \right] = - (\bar{\rho} \bar{v}) \bigg|_{y=\alpha} \tag{3.44}
\]

\[
\frac{\partial}{\partial t} (\rho_{III} h) + \frac{\partial}{\partial x} \left[ \left( \rho_{III} U h \right) \right] - \left( \frac{\rho_{III} h^3}{12\mu} \right) \frac{\partial p_{III}}{\partial x} = (\bar{\rho} \bar{v}) \bigg|_{y=\beta}. \tag{3.45}
\]

These two equations, together with the continuity equation applied to the inside of the porous medium,

\[
\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial (\bar{\rho} \bar{u})}{\partial x} + \frac{\partial (\bar{\rho} \bar{v})}{\partial y} = 0. \tag{3.46}
\]
yield the coupled set of differential equations that govern not only the pressure profiles but also the totality of fluid flow inside the three regions of the bearing.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Baseline value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>Bulk modulus of the fluid</td>
<td>$6.9 \times 10^7$ Pa</td>
</tr>
<tr>
<td>$h_{\text{max}}$</td>
<td>maximum film clearance</td>
<td>$1.52 \times 10^{-4}$ m</td>
</tr>
<tr>
<td>$h_{\text{min}}$</td>
<td>minimum film clearance</td>
<td>$2.54 \times 10^{-5}$ m</td>
</tr>
<tr>
<td>$k_x$</td>
<td>permeability of the porous medium in $x$-direction</td>
<td>$0$ m$^2$</td>
</tr>
<tr>
<td>$k_y$</td>
<td>permeability of the porous medium in $y$-direction</td>
<td>$1 \times 10^{-12}$ m$^2$</td>
</tr>
<tr>
<td>$L_x$</td>
<td>length in the $x$-direction</td>
<td>$7.62 \times 10^{-2}$ m</td>
</tr>
<tr>
<td>$p_{\text{I}}$</td>
<td>pressure in the reservoir</td>
<td></td>
</tr>
<tr>
<td>$\tilde{p}$</td>
<td>pressure in the porous medium</td>
<td></td>
</tr>
<tr>
<td>$p_{\text{III}}$</td>
<td>pressure in the film</td>
<td></td>
</tr>
<tr>
<td>$P^*$</td>
<td>non-dimensional $p$</td>
<td></td>
</tr>
<tr>
<td>$u_{\text{I}}$</td>
<td>$x$-component of velocity in the reservoir</td>
<td></td>
</tr>
<tr>
<td>$\tilde{u}$</td>
<td>$x$-component of velocity in the porous medium</td>
<td></td>
</tr>
<tr>
<td>$u_{\text{III}}$</td>
<td>$x$-component of velocity in the film</td>
<td></td>
</tr>
<tr>
<td>$U$</td>
<td>velocity of reservoir wall and porous medium in $x$-direction</td>
<td>$4.57$ m/s</td>
</tr>
<tr>
<td>$U^*$</td>
<td>non-dimensional $u$</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: Slider Bearing Nomenclature and Values - Part 1
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Baseline value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_I )</td>
<td>y-component of velocity in the reservoir</td>
<td></td>
</tr>
<tr>
<td>( \tilde{v} )</td>
<td>y-component of velocity in the porous medium</td>
<td></td>
</tr>
<tr>
<td>( v_{III} )</td>
<td>y-component of velocity in the film</td>
<td></td>
</tr>
<tr>
<td>( V^* )</td>
<td>non-dimensional ( v )</td>
<td></td>
</tr>
<tr>
<td>( x, y )</td>
<td>rectangular coordinates</td>
<td></td>
</tr>
<tr>
<td>( X^* )</td>
<td>non-dimensional ( x )</td>
<td></td>
</tr>
<tr>
<td>( Y^* )</td>
<td>non-dimensional ( y )</td>
<td></td>
</tr>
<tr>
<td>( \alpha )</td>
<td>height of bottom of the porous medium</td>
<td>( 1.27 \times 10^{-4} ) m</td>
</tr>
<tr>
<td>( \beta - \alpha )</td>
<td>thickness of the porous medium</td>
<td>( 2.54 \times 10^{-3} ) m</td>
</tr>
<tr>
<td>( \rho_c )</td>
<td>fluid density at cavitation pressure</td>
<td></td>
</tr>
<tr>
<td>( \rho_I )</td>
<td>fluid density in the reservoir</td>
<td></td>
</tr>
<tr>
<td>( \tilde{\rho} )</td>
<td>fluid density in the porous medium</td>
<td></td>
</tr>
<tr>
<td>( \rho_{III} )</td>
<td>fluid density in the film</td>
<td></td>
</tr>
<tr>
<td>( \mu )</td>
<td>kinematic fluid viscosity</td>
<td>( 3.9 \times 10^{-2} ) kg/m-s</td>
</tr>
</tbody>
</table>

Table 3.2: Slider Bearing Nomenclature and Values - Part 2
The cavitation model used in this work is similar to the work done by Elrod [21], which was further developed by Vijayaraghavan and Keith [22]. The basis for the work developed here is that of Vijayaraghavan and Keith. The goal of their work was to construct the governing equations and computational algorithm in such a manner that the cavitated region was automatically calculated and the accompanying finite difference numerical scheme reflected the physics of cavitation. The work described in this chapter mimics, and extends, the analytical work done in [22], whereas the numerical portion for solving the governing equations generated in this chapter will be described in Chapter 5.

4.1 Governing Equations

Recall that the three equations to be solved, using a cavitation model, are

\[
\frac{\partial}{\partial t} (\rho_1 \alpha) + \frac{\partial}{\partial x} \left[ (\rho_1 U \alpha) - \left( \frac{\alpha^3}{12 \mu} \right) \left( \rho_1 \frac{\partial p_1}{\partial x} \right) \right] = - \left( \hat{\rho} \hat{v} \right) \bigg|_{y=\alpha} \tag{4.1}
\]

\[
\frac{\partial \hat{\rho}}{\partial t} + \frac{\partial (\hat{\rho} \hat{u})}{\partial x} + \frac{\partial (\hat{\rho} \hat{v})}{\partial y} = 0 \tag{4.2}
\]

\[
\frac{\partial}{\partial t} (\rho_{III} h) + \frac{\partial}{\partial x} \left[ \left( \frac{\rho_{III} U h}{2} \right) - \left( \frac{\rho_{III} h^3}{12 \mu} \right) \frac{\partial p_{III}}{\partial x} \right] = \left( \hat{\rho} \hat{v} \right) \bigg|_{y=\beta}. \tag{4.3}
\]
Ultimately, Equations (4.1) - (4.3) will be solved in a numerical fashion utilizing a method similar to that used by Vijaraghavan and Keith. To proceed further using the concept and methodology presented in [22] one must start by recasting Equations (4.1) - (4.3) in terms of a nondimensional density variable

\[ \theta = \frac{\rho}{\rho_c} \] (4.4)

where \( \rho_c \) is the density of the lubricating fluid at cavitation pressure. In a full film, this nondimensional density takes on values greater than or equal to one, due to the fluid being compressed. In the cavitated region however, \( \theta \) takes on values less than one and represents the fractional film content of the liquid, with \( (1 - \theta) \) representing the gas content (or void fraction). The density of the fluid is related to the fluid pressure through the bulk modulus, \( B \) as:

\[ B = \rho \frac{\partial p}{\partial \rho}. \] (4.5)

Inside the cavitated regions, the pressure is essentially constant and equal to the cavitation pressure, a fact that is illustrated by the widely used Swift-Stieber condition, \( \frac{\partial p}{\partial x} = 0 \). As such, Vijaraghavan and Keith employ a switch function, \( g \), whereby

\[ gB = \rho \frac{\partial p}{\partial \rho} = \theta \frac{\partial p}{\partial \theta} \] (4.6)

where

\[ g = \begin{cases} 1 & \text{in the full film region} \\ 0 & \text{in the cavitated region} \end{cases} \]

Please note that pressures vary in the \( x \)-direction (lumped in \( y \)), and velocities vary in the \( x \)- and \( y \)-directions in the film (Region III) and the reservoir.
(Region I). The isotropic porous medium (Region II) allows a velocity gradient in the \( y \)-direction only \((k_x = 0)\). The \( \theta \) variable does not represent either pressures or velocities, and is designed to unify all domains, while varying in the \( x \)- and \( y \)-directions inside the porous medium and naturally remaining \( y \)-independent in the other two regions.

Using Equations (4.4) - (4.6) one can rewrite the governing Equations (4.1) - (4.3) in terms of \( g \), \( B \), and \( \theta \) as follows:

\[
\frac{\partial}{\partial t} \left( \rho_c \alpha \theta_I \right) + \frac{\partial}{\partial x} \left[ \left( \rho_c U \alpha \theta_I \right) - \left( \frac{\rho_c \alpha^3 B}{12 \mu} \left( \frac{\partial \theta_I}{\partial x} \right) \right) \right] = \left( \frac{\rho_c B k_y}{\mu} \right) \left( \frac{\partial \theta}{\partial y} \right) \bigg|_{y=\alpha} \tag{4.7}
\]

\[
\frac{\partial}{\partial t} \left( \rho_c \tilde{\theta} \right) + \frac{\partial}{\partial x} \left( \rho_c U \tilde{\theta} \right) + \frac{\partial}{\partial y} \left[ \left( - \frac{\rho_c B k_y}{\mu} \right) \left( \frac{\partial \tilde{\theta}}{\partial y} \right) \right] = 0 \tag{4.8}
\]

\[
\frac{\partial}{\partial t} \left( \rho_c h \theta_{III} \right) + \frac{\partial}{\partial x} \left[ \left( \frac{\rho_c h^2 B}{2} \theta_{III} \right) - \left( - \frac{\rho_c B k_y}{\mu} \right) \left( \frac{\partial \theta_{III}}{\partial x} \right) \right] = \bigg|_{y=\beta} \tag{4.9}
\]

Using again a technique proposed by Vijayaraghavan and Keith [22] one can rewrite the term \( g \frac{\partial \theta}{\partial x} \) as:

\[
g \frac{\partial \theta}{\partial x} = g \frac{\partial (\theta - 1)}{\partial x} = \frac{\partial}{\partial x} [g(\theta - 1)] - (\theta - 1) \frac{\partial g}{\partial x} \tag{4.10}
\]

Since \( g \) is a switch function that takes either the values of 0 or 1, \( \frac{\partial g}{\partial x} = 0 \) except when \( \theta = 1 \). However, when \( \theta = 1 \), \((\theta - 1) \frac{\partial g}{\partial x} = 0 \), so one can state that \( g \frac{\partial \theta}{\partial x} = \frac{\partial}{\partial x} [g(\theta - 1)] \).

The same reasoning can be used to say that \( g \frac{\partial \theta}{\partial y} = \frac{\partial}{\partial y} [g(\theta - 1)] \). As such, \( g \) does not have to be discretized separately, and a central difference numerical scheme can be
implemented on the groups \( \frac{\partial}{\partial x} [g(\theta - 1)] \) and \( \frac{\partial}{\partial y} [g(\theta - 1)] \). Thus, the coupled set of equations, Equations (4.7) - (4.9), can be rewritten to be solved numerically for \( \theta \) as:

\[
\frac{\partial}{\partial t} \left( \rho c \alpha \theta_I \right) + \frac{\partial}{\partial x} \left\{ \left( \rho c U \alpha \theta_I \right) - \left( \frac{\rho c \alpha^3 B}{12\mu} \right) \frac{\partial}{\partial x} [g_I(\theta_I - 1)] \right\} =
\]

\[
= \left( \frac{\rho c Bk_y}{\mu} \right) \frac{\partial \tilde{g}}{\partial y} \bigg|_{y=\alpha} \tag{4.11}
\]

\[
\frac{\partial}{\partial t} \left( \rho c \tilde{\theta} \right) + \frac{\partial}{\partial x} \left( \rho c U \tilde{\theta} \right) + \frac{\partial}{\partial y} \left( \left( -\frac{\rho c Bk_y}{\mu} \right) \frac{\partial \tilde{g}(\tilde{\theta} - 1)}{\partial y} \right) = 0 \tag{4.12}
\]

\[
\frac{\partial}{\partial t} \left( \rho c h \theta_{III} \right) + \frac{\partial}{\partial x} \left\{ \left( \frac{\rho c h^3 B}{12\mu} \right) \frac{\partial \tilde{g}}{\partial y} \right\} =
\]

\[
= - \left( \frac{\rho c Bk_y}{\mu} \right) \frac{\partial \tilde{g}}{\partial y} \bigg|_{y=\beta} \tag{4.13}
\]

The method for generating the numerical procedure used to solve Equations (4.11) - (4.12) will be described in Chapter 5.
CHAPTER V

NUMERICAL TECHNIQUES AND RESULTS - LONG SLIDER BEARING -
MOMENTUM EQUATIONS

5.1 Numerically Implementing Cavitation

This section of this chapter discusses how the work done by Vijayaraghavan and Keith [22] can be extended to the geometry germane to Chapter 3. The main differences between the work done in [22] and in this paper are coupling the three regions together to form a governing system of equations and also combining the three variables, \( \dot{\theta}_I, \tilde{\theta}, \) and \( \theta_{III} \), into a single variable \( \theta \).

Earlier, we used a method similar to that of Vijayaraghavan and Keith to transform Equations (3.44) - (3.46) into Equations (4.11) - (4.13),

\[
\frac{\partial}{\partial t} (\rho \alpha \theta_I) + \frac{\partial}{\partial x} \left\{ (\rho U \alpha \theta_I) - \left( \frac{\rho \alpha^3 B}{12 \mu} \right) \frac{\partial}{\partial x} [g_I(\theta_I - 1)] \right\} = \\
= \left( \frac{\rho_c B k_y}{\mu} \right) \left( \frac{\partial \tilde{\theta}}{\partial y} \right) \bigg|_{y=\alpha} 
\tag{5.1}
\]

\[
\frac{\partial}{\partial t} (\rho h \theta_{III}) + \frac{\partial}{\partial x} \left\{ \left( \rho h \frac{\theta_{III}}{2} \right) - \left( \frac{\rho h^3 B}{12 \mu} \right) \frac{\partial}{\partial x} [g_{III}(\theta_{III} - 1)] \right\} = \\
= - \left( \frac{\rho_c B k_y}{\mu} \right) \left( \frac{\partial \tilde{\theta}}{\partial y} \right) \bigg|_{y=\beta} 
\tag{5.2}
\]

\[
\frac{\partial}{\partial t} (\rho \tilde{\theta}) + \frac{\partial}{\partial x} (\rho U \tilde{\theta}) + \frac{\partial}{\partial y} \left\{ \left( - \frac{\rho_c B k_y}{\mu} \right) \frac{\partial \tilde{g}(\tilde{\theta} - 1)}{\partial y} \right\} = 0. 
\tag{5.3}
\]
that will be solved numerically using an algorithm developed by Vijayaraghavan and Keith, which will be described in detail here.

Before we begin, we would be remiss not to discuss the differences between elliptic partial differential equations and hyperbolic partial differential equations and how they must be handled when attempting to solve them using a finite difference scheme. Elliptic partial differential equations require that the dependent variable depends on all neighboring variables. Thus, a central difference finite difference scheme is allowed to solve an elliptic partial differential equation. However, hyperbolic partial differential equations do not allow for downstream information to affect values upstream, and thus a one-sided (upwind) finite difference scheme should be used.

If we examine the steady state versions of Equations (5.1) and (5.2), we note that when \( g = 1 \), which corresponds to being located inside the full film, the partial differential equations are elliptic. Thus, the numerical scheme to solve them should use central differencing. However, when \( g = 0 \), which corresponds to being located inside the cavitated region, the partial differential equations are hyperbolic. As a result, an upwinding finite difference scheme should be used to solve them numerically. The beauty of the scheme described here, is that we will use the switch function, \( g \), to construct a finite difference scheme that automatically switches the numerical scheme from central differencing to upwind differencing, depending on whether the grid point is in the cavitated region or full film region.

We begin by classifying the terms on the left hand sides of Equations (5.1) - (5.3) as Shear Flow terms or Pressure-Induced Flow terms. The main difference
between these two kinds of terms is that the shear flow terms do not feature the switch function, \( g \), whereas the pressure-induced flow terms do. The work described below will show how the switch function, \( g \), is introduced into the shear flow terms, and why it is necessary to do so.

5.1.1 Shear Flow Terms

The method described here will follow that developed by Vijayaraghavan and Keith. We begin by writing the shear flow term from Equation (5.2), \( \frac{\partial}{\partial x} \left( \frac{U \rho_c h \theta_{III}}{2} \right) \) in a simpler manner,

\[
\frac{\partial}{\partial x} \left( \frac{U \rho_c h \theta_{III}}{2} \right) = \frac{\partial E}{\partial x}. \tag{5.4}
\]

Note that the shear flow terms, \( \frac{\partial}{\partial x} (\rho_c U \alpha \theta_I) \) and \( \frac{\partial}{\partial x} (\rho_c U \tilde{\theta}) \), from Equations (5.1) and (5.3) will be discretized in the exact same manner, but the details are left out for the sake of brevity.

Since we want to use a central difference scheme when \( g = 1 \) and an upwind difference scheme when \( g = 0 \), the switch function, \( g \), should somehow be introduced into the discretization of the shear flow term, \( \frac{\partial E}{\partial x} \). We define an artificial viscosity switch function, \( \mu_a \), so that

\[
\mu_a = \begin{cases} 
0 & \text{, in the full film region} \\
1 & \text{, in the cavitated region.}
\end{cases}
\]

We then rewrite Equation (5.4) in the following way, using the artificial viscosity switch function,

\[
\frac{\partial}{\partial x} \left( \frac{U \rho_c h \theta_{III}}{2} \right) = \frac{\partial E}{\partial x} = \frac{\partial}{\partial x} \left( E - \mu_a \frac{\partial E}{\partial x} \frac{\Delta x}{2} \right). \tag{5.5}
\]
The central differencing scheme used is

\[
\left( \frac{\partial Q}{\partial x} \right)_i = \frac{Q_{i+\frac{1}{2}} - Q_{i-\frac{1}{2}}}{\Delta x},
\]

where \( Q \) is any quantity to be centrally differenced. Thus, we centrally difference Equation (5.5) to get

\[
\frac{\partial}{\partial x} \left( \frac{U \rho_c}{2} h \theta_{III} \right)_i = \frac{E_{i+\frac{1}{2}} - E_{i-\frac{1}{2}}}{\Delta x} - \frac{1}{2} \left[ \left( \mu_a \frac{\partial E}{\partial x} \right)_{i+\frac{1}{2}} - \left( \mu_a \frac{\partial E}{\partial x} \right)_{i-\frac{1}{2}} \right]
\]  

(5.6)

where

\[
Q_{i\pm\frac{1}{2}} = \frac{Q_{i\pm1} + Q_i}{2}.
\]

Thus, we may now write

\[
\frac{\partial}{\partial x} \left( \frac{U \rho_c}{2} h \theta_{III} \right)_i = aE_{i-1} + bE_i + cE_{i+1} \frac{2\Delta x}{2\Delta x}
\]  

(5.7)

where

\[
a = -\left[ 1 + \left( \mu_a \right)_{i-\frac{1}{2}} \right]
\]

(5.8)

\[
b = \left( \mu_a \right)_{i-\frac{1}{2}} + \left( \mu_a \right)_{i+\frac{1}{2}}
\]

(5.9)

\[
c = 1 - \left( \mu_a \right)_{i+\frac{1}{2}}.
\]

(5.10)

Remembering that

\[
g = \begin{cases} 
1 & \text{, in the full film region} \\
0 & \text{, in the cavitated region}
\end{cases}
\]

it should be clear that the relationship between the switch function \( g \) and the artificial viscosity switch function \( \mu_a \) is

\[
g = 1 - \mu_a.
\]

(5.11)
Thus, we may rewrite the coefficients from Equations (5.8) - (5.10) as

\[
a = - \left[ \frac{(g_{i-1} + g_i)}{2} - 2 \right] \tag{5.12}
\]

\[
b = 2 - \frac{(g_{i-1} + 2g_i + g_{i+1})}{2} \tag{5.13}
\]

\[
c = \frac{(g_i + g_{i+1})}{2}. \tag{5.14}
\]

Remember that the goal of creating such a finite difference scheme was to satisfy the finite differencing requirements for elliptic and hyperbolic partial differential equations.

We wanted a central difference scheme to be used for the elliptic partial differential equation (i.e., when \( g = 1 \)) and an upwind finite difference scheme to be used for the hyperbolic partial differential equation (i.e., when \( g = 0 \)). To verify our scheme, we let \( g = 1 \) which yields \( a = -1, b = 0, \) and \( c = 1, \) which produces

\[
\frac{\partial}{\partial x} \left( \frac{U \rho c}{2} h\theta_{III} \right)_i = \frac{E_{i+1} - E_{i-1}}{2\Delta x} = \left( \rho c U \right) \left[ \frac{(h\theta_{III})_{i+1} - (h\theta_{III})_{i-1}}{2\Delta x} \right], \tag{5.15}
\]

the desired central difference discretization for the shear flow term. We now let \( g = 0 \) which yields \( a = -2, b = 2, \) and \( c = 0, \) which produces

\[
\frac{\partial}{\partial x} \left( \frac{U \rho c}{2} h\theta_{III} \right)_i = \frac{E_i - E_{i-1}}{\Delta x} = \left( \rho c U \right) \left[ \frac{(h\theta_{III})_i - (h\theta_{III})_{i-1}}{\Delta x} \right], \tag{5.16}
\]

the desired upwind finite difference discretization for the shear flow term. Thus, the use of the switch function, \( g, \) has allowed us to construct a discretization that accurately reflects the requirements for correctly finite differencing elliptic and hyperbolic partial differential equations.
5.1.2 Pressure-Induced Flow Terms

We write the pressure-induced flow term from Equation (5.2) in terms of the variable, \( \theta \),

\[
\frac{\partial}{\partial x} \left( \frac{h^3}{12 \mu_B g} \frac{\partial \theta_{III}}{\partial x} \right)
\]  

(5.17)

and we note that this term exists in the full film region \((\theta \geq 1)\) and vanishes in the cavitated region \((\theta < 1)\). Because the switch function \( g \) removes the term completely inside the cavitated region, there is no need to introduce the artificial viscosity switch function, \( \mu_a \) for this discretization. Inside the full film, when \( g = 1 \), the governing equation is elliptic, which means that it will be subject to a central difference scheme.

A clever technique developed by Vijayaraghavan and Keith involves centrally differencing Equation (5.17) without explicitly differencing the derivative of \( g \). This is helpful because \( g \) is a switch function with an undefined derivative at the boundary between the full film and cavitated regions. Remember, from Chapter 4 that they were able to write \( g \frac{\partial \theta}{\partial x} \) as

\[
g \frac{\partial \theta}{\partial x} = g \frac{\partial(\theta - 1)}{\partial x} = \frac{\partial}{\partial x} [g(\theta - 1)] - (\theta - 1) \frac{\partial g}{\partial x}
\]  

(5.18)

but the last term on the right hand side vanishes, leaving us with the expression

\[
g \frac{\partial \theta}{\partial x} = \frac{\partial}{\partial x} [g(\theta - 1)].
\]  

(5.19)
Combining Equation (5.19) with the central difference scheme, Equation (5.6), we have

\[
\frac{\partial}{\partial x} \left[ \left( \frac{\rho cB}{12\mu} \right) h^3 \left( g_{III} \frac{\partial \theta_{III}}{\partial x} \right) \right]_i = \left( \frac{\rho cB}{12\mu \Delta x^2} \right) \left\{ \left( h_{i-\frac{1}{2}} \right)^3 [g_{III,i-1} (\theta_{III,i-1} - 1)] - \left[ \left( h_{i-\frac{1}{2}} \right)^3 + \left( h_{i+\frac{1}{2}} \right)^3 \right] [g_{III,i} (\theta_{III,i} - 1)] + \left( h_{i+\frac{1}{2}} \right)^3 [g_{III,i+1} (\theta_{III,i+1} - 1)] \right\}. \tag{5.20}
\]

5.1.3 Forcing Terms

Where this work differs from that of Vijayaraghavan and Keith is the influence of the porous material on the governing equations and the coupling together of the three regions. Note that Equations (5.1) and (5.2) have forcing terms on their right hand sides. These terms represent the flow passing from the fluid regions into the porous medium and vice versa, a phenomenon not present in the work done by Vijayaraghavan and Keith. To handle these terms, we use a three point forward difference at \( y = \alpha \) and a three point backward difference at \( y = \beta \) for differencing the term \( \frac{\partial}{\partial y} \left[ \tilde{g} (\tilde{\theta} - 1) \right] \). Before we do so, let us describe the computational grid to be used for this numerical simulation. First, we define \( i \) to be the index for the \( x \)-direction and \( j \) to be the index for the \( y \)-direction (see Figure 5.1). We index the \( x \)-direction length of the porous medium geometry, \( L_x \), from 1 to \( I \), where \( i = 1 \) denotes \( x = 0 \) and \( i = I \) denotes \( x = L_x \). Similarly, we index the \( y \)-direction thickness of the porous medium geometry, \( \beta - \alpha \), from 1 to \( J \), where \( j = 1 \) denotes \( y = \alpha \) and \( j = J \) denotes
\( y = \beta \). As a result, we may now discretize the forcing terms in the following manner

\[
\frac{\partial}{\partial y} \left[ \tilde{g} \left( \tilde{\vartheta} - 1 \right) \right] \bigg|_{y=\alpha} = \left( \frac{1}{2 \Delta y} \right) \left\{ -3 \left[ \tilde{g}_{j=1} \left( \tilde{\vartheta}_{j=1} - 1 \right) \right] + \\
+ 4 \left[ \tilde{g}_{j=2} \left( \tilde{\vartheta}_{j=2} - 1 \right) \right] - \\
- \left[ \tilde{g}_{j=3} \left( \tilde{\vartheta}_{j=3} - 1 \right) \right] \right\} \tag{5.21}
\]

\[
\frac{\partial}{\partial y} \left[ \tilde{g} \left( \tilde{\vartheta} - 1 \right) \right] \bigg|_{y=\beta} = \left( \frac{1}{2 \Delta y} \right) \left\{ 3 \left[ \tilde{g}_{j=J} \left( \tilde{\vartheta}_{j=J} - 1 \right) \right] - \\
- 4 \left[ \tilde{g}_{j=J-1} \left( \tilde{\vartheta}_{j=J-1} - 1 \right) \right] + \\
+ \left[ \tilde{g}_{j=J-2} \left( \tilde{\vartheta}_{j=J-2} - 1 \right) \right] \right\}. \tag{5.22}
\]

5.1.4 Transient Terms

A forward difference in time is used to handle the transient terms, i.e.

\[
\frac{\partial Q}{\partial t} = \frac{Q^{N+1} - Q^N}{\Delta t}, \tag{5.23}
\]

where \( N + 1 \) is the index that indicates the current time step and \( N \) is the index that indicates the previous time step.

5.1.5 Final Governing Momentum Equations

Now that every term is finite differenced, we can construct the governing equations to be solved numerically. Before we do so, we remember that the pressures do not vary in the \( y \)-direction inside the active film space and fluid reservoir regions. As a result, the pressures only need to be calculated inside the porous medium and at the boundaries of the porous medium, \( y = \alpha \) and \( y = \beta \). Thus, we may write that \( \theta_I = \theta_{j=1} \), \( \theta_{III} = \theta_{j=J} \) and we may simply replace \( \tilde{\vartheta} \) with \( \vartheta \). We may do the same to
replace \( g_I, \tilde{g}, \) and \( g_{III} \) with the single function \( g \). This yields a system where we only have to solve for one variable, \( \theta \) in one computational domain as opposed to solving for three variables in three computational domains. Using this approach, once \( \theta \) is computed inside the porous medium and at \( y = \alpha \) and \( y = \beta \), the pressures are known everywhere inside the bearing.

The resulting computational mesh can be seen in Figure 5.1, where the \( x \)-index, \( i \), varies from \( i = 1 \) (\( x = 0 \)) to \( i = I \) (\( x = L_x \)). Note, however, that computations are not performed at \( i = I(x = L_x) \), since the periodic boundary conditions in the \( x \)-direction make such calculations redundant.

![Figure 5.1](image_url)

Figure 5.1: Unique Computational Domain, Long Slider Momentum Equations, for the Three Regions: the Active Hydrodynamic Film (Region III), the Porous Medium (Region II) and the Reservoir (Region I).

If we move the \( y \)-terms to the right hand sides of the equations and consider those terms to be at the \( N \) time level, keeping the \( x \)-terms on the left hand side of
the equation and considering them to be at the $N + 1$ time level, we can construct a tridiagonal system that can be solved using a fast tridiagonal solver, such as the Thomas algorithm.

Using the aforementioned approaches, we may now write the governing equation for the variable $\theta$ at $y = \alpha$ ($j = 1$),

$$a_i \theta_{i-1,1}^{N+1} + b_i \theta_{i,1}^{N+1} + c_i \theta_{i+1,1}^{N+1} = d_i,$$  \hspace{1cm} (5.24)

where

$$a_i = - \left( \frac{\alpha^2 B \Delta t}{12 \mu \Delta x^2} \right) g_{i-1,1}$$

$$b_i = \left[ 1 + 2 \left( \frac{\alpha^2 B \Delta t}{12 \mu \Delta x^2} \right) g_{i,1} \right]$$

$$c_i = - \left( \frac{\alpha^2 B \Delta t}{12 \mu \Delta x^2} \right) g_{i+1,1}$$

$$d_i = \theta_{i,1}^{N+1} +$$

$$+ \left( \frac{B k_y \Delta t}{2 \mu \alpha y} \right) \left\{ -3 \left[ g_{i,1} \left( \theta_{i,1}^{N-1} \right) \right] + 4 \left[ g_{i,2} \left( \theta_{i,2}^{N-1} \right) \right] - \left[ g_{i,3} \left( \theta_{i,3}^{N-1} \right) \right] \right\} -$$

$$- \left( \frac{\alpha^2 B \Delta t}{12 \mu \Delta x^2} \right) \left( g_{i-1,1} - 2 g_{i,1} + g_{i+1,1} \right).$$

The governing equation for the variable $\theta$ for $\alpha < y < \beta$ ($2 \leq j \leq J - 1$) is

$$\theta_{i,j}^{N+1} = \theta_{i,j}^{N+1} +$$

$$+ \left( \frac{B k_y \Delta t}{\mu \Delta y^2} \right) \left\{ g_{i,j-1} \left( \theta_{i,j-1}^{N-1} \right) - 2 \left[ g_{i,j} \left( \theta_{i,j}^{N-1} \right) \right] -$$

$$- \left[ g_{i,j+1} \left( \theta_{i,j+1}^{N-1} \right) \right] \right\}. \hspace{1cm} (5.25)$$

Finally, the governing equation for the variable $\theta$ at $y = \beta$ ($j = J$) is

$$a_i \theta_{i-1,J}^{N+1} + b_i \theta_{i,J}^{N+1} + c_i \theta_{i+1,J}^{N+1} = d_i,$$  \hspace{1cm} (5.26)
where

\[ a_i = \left( \frac{U \Delta t}{4 \Delta x} \right) \left[ \left( \frac{g_{i-1,J} + g_{i,J}}{2} \right) - 2 \right] (h_{i-1}) - \left( \frac{B \Delta t}{12 \mu \Delta x^2} \right) \left( h_{i-\frac{1}{2}} \right)^3 g_{i-1,J} \]

\[ b_i = h_i + \left( \frac{U \Delta t}{4 \Delta x} \right) \left[ 2 - \left( \frac{g_{i-1,J} + 2g_{i,J} + g_{i+1,J}}{2} \right) \right] (h_i) + \]

\[ + \left( \frac{B \Delta t}{12 \mu \Delta x^2} \right) \left[ \left( h_{i-\frac{1}{2}} \right)^3 + \left( h_{i+\frac{1}{2}} \right)^3 \right] g_{i,J} \]

\[ c_i = \left( \frac{U \Delta t}{4 \Delta x} \right) \left[ \left( \frac{g_{i,J} + g_{i+1,J}}{2} \right) \right] (h_{i+1}) - \left( \frac{B \Delta t}{12 \mu \Delta x^2} \right) \left( h_{i+\frac{1}{2}} \right)^3 g_{i+1,J} \]

\[ d_i = h_i \theta_{i,J}^N - \]

\[ - \left( \frac{B \Delta t}{12 \mu \Delta x^2} \right) \left\{ \left( h_{i-\frac{1}{2}} \right)^3 g_{i-1,J} - \left[ \left( h_{i-\frac{1}{2}} \right)^3 + \left( h_{i+\frac{1}{2}} \right)^3 \right] g_{i,J} + \left( h_{i+\frac{1}{2}} \right)^3 g_{i+1,J} \right\} + \]

\[ + \left( \frac{B k_y \Delta t}{2 \mu \alpha y} \right) \{-3 \left[ g_{i,J} (\theta_{i,J}^N - 1) \right] + 4 \left[ g_{i,J-1} (\theta_{i,J-1}^N - 1) \right] - \left[ g_{i,J-2} (\theta_{i,J-2}^N - 1) \right] \}. \]

5.1.6 Solution Procedure

In matrix form, Equations (5.24) - (5.26) do not represent a perfectly tridiagonal matrix. Because we impose periodic boundary conditions in the \( x \)-direction, the \( i = 1 (x = 0) \) equations for the reservoir and fluid film regions are, respectively,

\[ b_1 \theta_{1,1}^{N+1} + c_1 \theta_{2,1}^{N+1} + a_1 \theta_{I-1,1}^{N+1} = d_1 \quad (5.27) \]

\[ b_1 \theta_{1,J}^{N+1} + c_1 \theta_{2,J}^{N+1} + a_1 \theta_{I-1,J}^{N+1} = d_1, \quad (5.28) \]

using each equation’s respective values for \( a_1, b_1, c_1, \) and \( d_1 \). Similarly, the \( i = I - 1(x = L_x - \Delta x) \) equations for the reservoir and fluid film regions are, respectively,

\[ c_{I-1} \theta_{1,1}^{N+1} + a_{I-1} \theta_{I-2,1}^{N+1} + b_{I-1} \theta_{I-1,1}^{N+1} = d_{I-1} \quad (5.29) \]

\[ c_{I-1} \theta_{1,J}^{N+1} + a_{I-1} \theta_{I-2,J}^{N+1} + b_{I-1} \theta_{I-1,J}^{N+1} = d_{I-1}, \quad (5.30) \]

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again, using each equation’s respective values for \( a_{I-1}, b_{I-1}, c_{I-1}, \) and \( d_{I-1} \). Note we do not calculate at the \( i = I(x = L_x) \) gridpoint since it is the same gridpoint as \( i = 1(x = 0) \). Instead of using a Gauss-Jordan procedure, which would be quite slow, the two off-diagonal terms (\( a_1 \) in the \( i = 1 \) equation and \( c_{I-1} \) in the \( i = I - 1 \) equation) were time lagged, meaning that we made the following replacements:

\[
\begin{align*}
    a_1 \theta_{I-1,1}^{N+1} &= a_1 \theta_{I-1,1}^N \\
    a_1 \theta_{I-1,J}^{N+1} &= a_1 \theta_{I-1,J}^N \\
    c_{I-1} \theta_{1,1}^{N+1} &= c_{I-1} \theta_{1,1}^N \\
    c_{I-1} \theta_{1,J}^{N+1} &= c_{I-1} \theta_{1,J}^N.
\end{align*}
\]

(5.31)

(5.32)

(5.33)

(5.34)

As a result, these quantities represent known values at the \( N + 1 \) time step and can be placed on the right hand sides of Equations (5.24) and (5.26). By doing this, the resulting matrix structure is tridiagonal and the Thomas algorithm can be applied.

The geometry was initialized to be at full film everywhere inside the bearing, where the value for \( \theta \) was such that it corresponded to the atmospheric pressure. This yields values for \( \theta^N \) and \( g \). Values for \( \theta^{N+1} \) were calculated using the values for \( \theta^N, g, \) and the physical quantities defined by the simulation (i.e. \( B, h_{\text{min}}, h_{\text{max}}, \alpha, \beta, \mu, U, \) and \( L_x \)). Upon doing so, the values for \( g \) were updated. The values for \( \theta^{N+1} \) were then set to be \( \theta^N \) and the procedure was iterated in time until a steady-state solution was achieved. The criterion used to determine if the solution had reached steady-state was that the sum of the differences in \( \frac{\partial \theta}{\partial t} \) between successive time steps for all values of \( i \) and \( j \) was no more than \( 10^{-4} \). The numerical code used to solve the matrix
equation is included in the Appendix.

5.2 Numerical Stability and Grid Convergence Test

Generally, for the discretization of time-dependent partial differential equations, a stability analysis should be performed in order to determine the ratio between $\Delta t$, $\Delta x$, and $\Delta y$, the differentials in time, the $x$-direction, and the $y$-direction, respectively.

For example, the one dimensional heat equation,

$$\frac{\partial u}{\partial t} = A \frac{\partial^2 u}{\partial x^2}$$

mandates that, for an explicit scheme, using a von Neumann analysis, we must have $\frac{A \Delta t}{\Delta x^2} \leq \frac{1}{2}$ for the scheme to be stable. Similarly, for the two-dimensional heat equation,

$$\frac{\partial u}{\partial t} = A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial y^2},$$

(5.36)

to be stable, it must be that $A \frac{\Delta t}{\Delta x^2} + B \frac{\Delta t}{\Delta y^2} \leq \frac{1}{2}$. For an implicit scheme, however, both equations, Equations 5.35 and 5.36, have no upper limit on the value of $\Delta t$ because of stability. However, a practical limit on the value of $\Delta t$ does exist due to truncation error. [27]

For systems of equations, the same stability analysis may be performed, though it is more complex. Consider the matrix equation

$$\frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = 0,$$

(5.37)

where $\mathbf{E}$ and $\mathbf{F}$ are vectors and $\mathbf{F} = \mathbf{F}(\mathbf{E})$. This system can be rewritten as

$$\frac{\partial \mathbf{E}}{\partial t} + [A] \frac{\partial \mathbf{E}}{\partial x} = 0,$$

(5.38)
where $[A]$ is the Jacobian matrix $\left[\frac{\partial F}{\partial E}\right]$. A von Neumann stability analysis then says that the stability criterion is that

$$\left|\lambda_{\text{max}} \frac{\Delta t}{\Delta x}\right| \leq 1 \quad (5.39)$$

where $\lambda_{\text{max}}$ is the largest eigenvalue of the matrix, $[A]$. [27]

For the work considered here, however, such an analysis is not straightforward. Due to the nature of the way the governing equations were constructed, where the system can transition from hyperbolic to elliptic, a von Neumann stability analysis is rather complex. For this reason, the method employed to choose the value of $\Delta t$ was largely trial and error. It was determined that the maximum value for $\Delta t$ depended on the value of the parameters being varied (i.e. $k_y, U, \alpha, (\beta - \alpha)$). The generic form for the stability criteria used for each simulation was

$$\Delta t = c(\Delta y)^2 \quad (5.40)$$

where $c$ was a factor that was varied. Depending on the parameter values chosen for each simulation, this value ranged from 0.1 to 200. It was observed that the maximum value for $c$ that yielded numerical stability decreased with increasing values of $\alpha$, decreased with increasing values of $k_y$, decreased with increasing values of $(\beta - \alpha)$, and decreased with increasing values of $U$, though with less sensitivity than the other three parameters. It was observed that too large of a value for $c$ yielded solutions where the errors increased without bound. At the same time, picking too small of a value for $c$ increased computational time as it took longer for the solution to reach steady state. Thus, the value chosen for $c$, for each simulation, was a value that
was just below the critical value that made the error increase without bound. This ensured a stable solution with the least amount of computational time necessary.

To insure that the obtained numerical solution was independent of the size of the grid used to discretize the geometry, a grid convergence test was conducted. For this test, the numerical code was ran using different values of \( I \), the number of grid points in the \( x \)-direction for the porous medium. For this test, four different values of \( I \) were used: 21, 41, 61, and 81. The benchmark case was ran for each value of \( I \) and the results can be seen in Figure 5.2.

![Figure 5.2: Pressure profiles in the hydrodynamic film for different values of \( I \) - the number of grid points in the \( x \)-direction for the porous medium. \( \alpha^* = 1.27 \times 10^{-4} \) m \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m\(^2\). All other pertinent values are given in Tables 3.1 and 3.2.](image)

Though the smoothness of the curves vary from case to case, we still recover the same profiles for the pressures, the same location for the peak pressure, and the same
location for the onset of cavitation. We can then conclude that the numerical solution obtained is indeed independent of the number of grid points used in the discretization.

5.3 Results and Discussion

The Darcy Model and some underlying simplifications and assumptions were used for the development of a model relating the flow of the fluid inside the bearing, the porous material and the reservoir, respectively. Through a non-dimensional analysis, $O(1)$ governing equations were developed for the three regions, and we also stated that, for a capillary-type porous medium proposed here, $k_x = 0$ inside the porous medium. These assumptions, together with the integration of a mass flow-conserving cavitation scheme, yielded a coupled system of partial differential equations, Equations (5.1) - (5.3), which were solved using a computer algorithm similar to that used by Vijayaraghavan and Keith [22]. The cavitation reference pressure was set at 0 Pa. Periodic boundary conditions were applied at the endpoints in the $x$-direction. These boundary conditions are similar to those imposed in the circumferential direction of a joint journal bearing, whose geometry is unwrapped for a solution in Cartesian geometry. Application of these boundary conditions to the slider together with the use of the convergent-divergent film, Figures 3.1 and 5.1, allows a reasonable extrapolation of the results obtained herein to an actual journal bearing. After each iteration the values for $\theta$ are updated while $g$ is set anew depending on the new value of $\theta$. This process is repeated until the solution for $\theta$ converges. Only then were the pressures for each region calculated. Because this method properly accounts for mass
continuity in both the full film and cavitated regions and automatically predicts the
film rupture and reformation points, the shortcomings of earlier approaches (Gum-
bel, Swift-Stieber, etc.) are overcome, yielding a cavitation model that is much more
physically accurate.

To allow for a discussion of this type of self-lubricated, self-circulation bear-
ing, it is not sufficient only to examine the pressures inside the active film space. The
active space is indeed the determining factor for the load-carrying capacity, but the
pressure differential that develops between the hydrodynamic film and the reservoir is
crucial for the self-circulation of the lubricating fluid between the regions, therefore,
for the functionality of the concept proposed herein. As a result, the pressure profiles
in both fluid regions need to be examined.

Tables 3.1 and 3.2 specify the baseline parameters used in this numerical
simulation. For algorithm validation purposes, the geometric parameters (i.e. length,
maximum and minimum film clearances, width), the properties of the lubricating
fluid (bulk modulus, $B$, and viscosity, $\mu$), and the slider velocity, $U$ were chosen to
parallel closely those of the bearing used by Vijayaraghavan and Keith [22], Table
3.1, Table 3.2]. Note that for Figures 5.3 - 5.10, the baseline values found in Tables
3.1 and 3.2 are denoted by an (*) in the legend of the graph.

5.3.1 Effects on Pressure and Fluid Circulation of Parameters $\alpha$, $k_y$, $(\beta - \alpha)$ and $U$
Effect of Reservoir Depth ($\alpha$)

Figures 5.3 and 5.4 illustrate the effect the increasing reservoir depth has on the pressure inside the lubricating film and reservoir itself, respectively. All other geometrical parameters and fluid properties are kept constant.

![Graph showing the effect of reservoir depth on pressure profiles in the active hydrodynamic film.](image)

Figure 5.3: Effect of the reservoir depth, $\alpha$, on the pressure profiles in the active hydrodynamic film. The (*$\alpha$) represents the baseline reservoir depth value. $(\beta-\alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 3.1 and 3.2.

Figure 5.3 presents the variation of pressure in the active hydrodynamic film. One can see that the shallower the reservoir becomes, the higher are the pressures induced inside the film. As the reservoir becomes deeper, the pressures inside the film reduce, eventually, to an asymptotic profile, which does not change much no matter how much deeper the reservoir becomes. This is most apparent if one looks at Figure 5.3 in conjunction with Figure 5.4. Thus in the reservoir, Figure 5.4, when it is
shallower, one can see quite a difference between its pressures when $\alpha = 6.35 \times 10^{-5}$ m and $\alpha = 1.27 \times 10^{-4}$ m. This in turn induces visible changes in the corresponding pressure curves in the hydrodynamic film, Figure 5.3. However, as the reservoir becomes deeper ($\alpha = 2.54 \times 10^{-4}$ m and $\alpha = 5.08 \times 10^{-4}$ m), its pressure profiles completely flatten, Figure 5.4, while the pressures profiles in the active space reach a limiting value, essentially falling on top of each other for the latter mentioned reservoir depths. This latter case produces a bearing geometry with the maximum observed pressure differential between the film and reservoir regions, corresponding to the highest rate of circulation, at the expense of the minimum load carrying capability. However, for reservoir depths roughly equal to the film clearance ($\alpha = 6.35 \times 10^{-5}$ m and $\alpha = 1.27 \times 10^{-4}$ m), highest pressures are created inside the film; the high
pressure is also generated inside the reservoir, causing a minimal pressure differential between the two regions. Thus, high load carrying capability is created at the expense of a good circulation between the two regions, and an optimization of the geometry appears necessary.

**Effect of Porous Medium Permeability ($k_y$)**

Figures 5.5 and 5.6 show the effect that the permeability, $k_y$, has on the pressures inside the film and reservoir, respectively. All other geometric parameters, fluid properties and velocity of the slider are kept constant.

![Figure 5.5: Effect of the $y$–direction permeability, $k_y$, on the pressure profiles in the active hydrodynamic film. The (*$k_y$) represents the baseline permeability value. ($\beta - \alpha$)$^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m. All other pertinent values are given in Tables 3.1 and 3.2.](image)

As a parallel to thermal conductivity, one may consider permeability as a sort of fluid flow conductivity inside the porous medium. A high permeability leads to an increase
Figure 5.6: Effect of the $y$-direction permeability, $k_y$, on the pressure profiles in the reservoir. The ($*k_y$) represents the baseline reservoir permeability value. $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m. All other pertinent values are given in Tables 3.1 and 3.2.

of fluid flow through the porous medium with decreased pressure potential, while a low permeability leads to reduced fluid flow accompanied by high pressure potentials necessary to drive that flow. Thus the lowest permeability of $k_y = 1 \times 10^{-13}$ m$^2$ generates the highest pressure in the active space, Figure 5.5, associated by almost a flat pressure profile in the reservoir since a large pressure drop is necessary to circulate even a reduced amount of flow through the porous medium and into the reservoir. Increasing the permeability gradually from $10^{-13}$ to $10^{-10}$ m$^2$ yields a lower pressure in the active film space, Figure 5.5, since there is less mass flow to enter the pressure-building convergent zone. This loss of mass in the active pressure-building direction can be compared to a journal bearing where the length of the bearing becomes shorter and shorter, forcing more mass out in the axial direction at the
expense of the circumferential pressure build-up. With the increase in permeability and associated with the pressure decrease in the active space there is an increase in the pressure in the reservoir. These pressures, Figure 5.6, start to follow the pattern of the pressure in the active space, though at different magnitudes. There are indeed asymptotic limits for the permeability, as too low of a permeability will result in no fluid flow through the porous bearing. This results in the highest possible pressure inside the film at the expense of no circulation inside the bearing, equivalent to the geometry of the classical slider bearing (overall solid boundaries). Alternatively, too high of a permeability creates a situation where the porous medium becomes practically transparent to the system, resulting in a practical film clearance that adds the depth of the reservoir to the clearance in the active space. Such a clearance is much too large to be capable of generating self-acting pressures.

**Effect of the Thickness of the Porous Medium, \((\beta - \alpha)\)**

Figures 5.7 and 5.8 present the effect the increase in the depth of the porous medium has upon the pressures in the films active space and reservoir, respectively.

An interesting phenomenon is displayed in these figures. In Figure 5.7, one can see that the pressures inside the film do not vary directly to the change in depth of the porous medium. It should be clear that as \(\beta - \alpha\) approaches 0, (in our simulation from 20.3 mm (thick) to 0.635 mm (thin), one recovers a bearing system where the active film depth is increased by the depth of the reservoir (imagine completely removing...
Figure 5.7: Effect of the porous medium thickness, $\beta - \alpha$, on the pressure profiles in the active hydrodynamic film. The (*$\beta - \alpha$) represents the baseline porous medium thickness. $k_y^* = 1 \times 10^{-12}$ m$^2$, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m. All other pertinent values are given in Tables 3.1 and 3.2.

the porous medium from Figure 3.1). Such a system would have too thick of a film clearance, and too low of an eccentricity to induce any pressures needed for operation. Alternatively, increasing the depth of the porous medium beyond a critical value also leads to decreasing pressure values inside the film. Thus, one may reason that for the bearing configuration, as described in Figure 3.1, as $\beta - \alpha$ increases from 0, the effective film clearance decreases to a critical value and then increases beyond such a value, yielding pressures that increase with increasing porous medium depth and then begin to decrease with further increases in the porous medium depth. Thus, again, an optimization process would have to be followed to combine the best possible dimensions for the porous medium, active film and the reservoir.
Figure 5.8: Effect of the porous medium thickness, $\beta - \alpha$, on the pressure profiles in the reservoir. The ($*\beta - \alpha$) represents the baseline porous medium thickness. $k^*_y = 1 \times 10^{-12}$ m$^2$, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m. All other pertinent values are given in Tables 3.1 and 3.2.

**Linear Velocity Effects, ($U$)**

Figures 5.9 and 5.10 present the effect of increasing the linear velocity $U$ of the reservoir wall and porous medium on both the pressures in the active film and the reservoir.

As expected, increasing the velocity increases the pressure inside the active film and is done so in approximately a linear fashion. The convergent zone, Figure 5.9, shows steadily increasing magnitudes of the pressure curves with velocity. The cavitation model indicates at the same time that as the pressures increase in the convergent zone the start of the cavitation zone shifts in the direction of the divergent zone. The pressures in the divergent zone bottom out at the cavitation pressure level prescribed as the calculations were initiated (0 Pa). The pressures in the reservoir follow in
Figure 5.9: Effect of the linear velocity, $U$, on the pressure profiles in the active hydrodynamic film. The ($*U$) represents the baseline slider linear velocity value. $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m. All other pertinent values are given in Tables 3.1 and 3.2.

magnitude and profile the trend of the pressures in the hydrodynamic film. The permeability $k_y$ provides a constant resistance to flow, so that the raise in pressures in the active space leads to almost a proportional corresponding pressure raise in the reservoir. The overall phenomenon of pressure build up in the active film, pressure drop in the porous medium in conjunction with the feedback from the reservoir is clearly non-linear so the pressures in the reservoir do not follow a linearly increasing path.

5.3.2 Comparative Parametric Results

For emphasizing the feasibility of the active film-reservoir circulation concept, Figures 5.11 and 5.12 for reservoir depth, Figures 5.13 and 5.14 for permeability and
Figure 5.10: Effect of the linear velocity, \( U \), on the pressure profiles in the reservoir. The (\( *U \)) represents the baseline slider linear velocity value. \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( k_y^* = 1 \times 10^{-12} \text{ m}^2 \), \( \alpha^* = 1.27 \times 10^{-4} \text{ m} \). All other pertinent values are given in Tables 3.1 and 3.2.

Figures 5.15 and 5.16 for the thickness of the porous medium present, superimposed, the pressure curves for the active film and the reservoir. Figures 5.11 and 5.12 present, for the effect of reservoir depth on fluid circulation for two extreme cases: the shallow reservoir \( (\alpha = 0.0635 \text{ mm}) \), Figure 5.11 and the deep reservoir \( (\alpha = 0.508 \text{ mm}) \), Figure 5.12, respectively.

For the shallow reservoir there is the pressure differential that pumps the fluid from the hydrodynamic film into the reservoir and a reverse pressure differential that pumps the fluid from the reservoir into the divergent zone of the active film. Figure 5.12 features a reservoir that is eight times deeper than that of the case of Figure 5.11. Note that even though the maximum pressure inside the active film is lower by about 10 percent when compared with Figure 5.11, there is more of a pressure
Figure 5.11: Superimposed pressures of the hydrodynamic film and the reservoir when the reservoir depth is shallowest, $\alpha = 0.0635$ mm. Fluid is pumped from the film to the reservoir where the pressure in the film is higher than the pressure in the reservoir and vice versa. All other pertinent values are given in Tables 3.1 and 3.2.

difference and a better differential average spread in Figure 5.12. One also has to remark that the pressure maxima in the film and reservoir are not in sync with each other; this lag is caused by the interference of the porous medium. This situation induces a higher rate of fluid circulation between the film and the reservoir at the cost of a somewhat lower load carrying capacity. As such, one has to be aware that there is a trade-off between the load-carrying capacity and fluid circulation capability.

Figures 5.13 and 5.14 show the pressure differential variation between the active film space and the reservoir as a result of varying the permeability of the porous medium.

Though Figure 5.13 shows for the lowest porosity limiting case of $k_y = 1 \times 10^{-13}$ m$^2$ both higher pressure values and a larger pressure differential between the two re-
Figure 5.12: Superimposed pressures of the hydrodynamic film and the reservoir when the reservoir depth is deepest, $\alpha = 0.508 \text{ mm}$. Fluid is pumped from the film to the reservoir where the pressure in the film is higher than the pressure in the reservoir and vice versa. All other pertinent values are given in Tables 3.1 and 3.2.

Regions, one has to remember from Equation (3.22) that the velocity in the $y$-direction through the porous medium is directly proportional to the permeability of the porous medium. The higher average pressure differential stands to engender an active flow circulation between the two regions of the bearing, but this situation is associated also with a much higher resistance to flow from the porous medium. For the higher permeability case seen in Figure 5.14, where $k_y = 1 \times 10^{-11} \text{ m}^2$, the fluid sees little resistance for passage through the porous medium, however there is also very little pressure difference between the two regions (see Figures 5.5 and 5.6). The low pressure differential necessary for the fluid flow from one region to the other (a positive development) is accompanied by the low pressures developing in the active film space, hence a low load carrying capacity. Again, we see that trade-offs occur in this bearing.
Figure 5.13: Superimposed pressures of the hydrodynamic film and the reservoir for a smaller permeability, $k_y = 1 \times 10^{-13}$ m$^2$. Fluid is pumped from the film to the reservoir where the pressure in the film is higher than the pressure in the reservoir and vice versa. All other pertinent values are given in Tables 3.1 and 3.2.

geometry and further studies are necessary to determine the optimal configuration for continuous operation of the bearing.

Figures 5.15 and 5.16 show the pressure differences between the film and the reservoir as a result of varying the thickness of the porous medium.

In Figure 5.15 the thickness of the porous medium is 5.08 mm. This translates into a higher resistance to flow, higher pressures in the active space and higher pressure differentials than those of Figure 5.16, where the thickness of the porous medium is just 0.635 mm. However, the total resistance to flow through the porous medium is directly proportional to the thickness of the porous medium and, as such, the configuration shown in Figure 5.16 may feature a higher rate of circulation than that of Figure 5.15. It is evident that a balancing act exists for a bearing that has
Figure 5.14: Superimposed pressures of the hydrodynamic film and the reservoir for a larger permeability, $k_y = 1 \times 10^{-11}$ m$^2$. Fluid is pumped from the film to the reservoir where the pressure in the film is higher than the pressure in the reservoir and vice versa. All other pertinent values are given in Tables 3.1 and 3.2.

been described in this paper, where high film pressures (and higher load-carrying capacities) may come at the expense of low rates of fluid circulation and vice versa.

The ultimate goal of the design described in this work is to create an optimum balance of load carrying capacity and fluid circulation to allow for a sealed bearing, utilizing the convection of the fluid to aid in overall heat transfer from the bearing to the ambient. Were no circulation to exist, the external reservoir would merely act as an insulator, and the bearing would offer no improvement over current designs. The bearing described here should trade some load carrying capability for circulation between the film and reservoir.

Figures 5.17 and 5.18 show in a carpet plot graph the pressure profiles, $p(x, y)$, inside the porous medium for the two porosity limiting cases discussed above, $k_y =$
Figure 5.15: Superimposed pressures of the hydrodynamic film and the reservoir for a thicker porous medium, $\beta - \alpha = 5.08$ mm. Fluid is pumped from the film to the reservoir where the pressure in the film is higher than the pressure in the reservoir and vice versa. All other pertinent values are given in Tables 3.1 and 3.2.

$1 \times 10^{-13}$ m$^2$ and $k_y = 1 \times 10^{-10}$ m$^2$, respectively.

It can be seen that for the low permeability case (Figure 5.17), the pressure drop across the thickness of the porous medium is steep and non-linear, as predicated in the discussion of Figures 5.5, 5.6, 5.13 and 5.14, whereas for the high permeability case (Figure 5.18), the pressure is essentially constant across the thickness of the porous medium.

Figures 5.19 and 5.20 show in a carpet plot graph the pressure profiles, $p(x, y)$, inside the porous medium for two extreme limiting cases of the porous medium studied in this paper: $\beta - \alpha = 3.17 \times 10^{-4}$ m, (thin), and $\beta - \alpha = 2.03 \times 10^{-2}$ m, (thick), respectively.
Figure 5.16: Superimposed pressures of the hydrodynamic film and the reservoir for a thinner porous medium, $\beta - \alpha = 0.635$ mm. Fluid is pumped from the film to the reservoir where the pressure in the film is higher than the pressure in the reservoir and vice versa. All other pertinent values are given in Tables 3.1 and 3.2.

The pressure trends exhibit a behavior similar to that witnessed in Figures 5.17 and 5.18. For a thicker porous medium, Figure 5.20, we observe a noticeable, nonlinear pressure drop in the $y$-direction across the porous medium, whereas Figure 5.19 shows that the pressure does not vary much in the $y$-direction for a thinner porous medium.

One of the main goals of designing the bearing with the geometry described in Figure 3.1 is to have a bearing that features self-circulation, whereby the fluid is pumped from the hydrodynamic film region, into the reservoir region, and then drawn back into the hydrodynamic film. To see how the fluid is moving, we employ the use of three-dimensional graphs (two dimensions in space, the third dimension shows the velocity value to be investigated). For completeness, we show how the velocity moves in the $x$-direction inside the bearing system (Figure 5.21), and in
Figure 5.17: Two-dimensional carpet plot of the pressure profile, $p(x, y)$, inside the porous medium for the lowest permeability limiting case, $k_y = 1 \times 10^{-13}$ m$^2$. All other pertinent values are given in Tables 3.1 and 3.2.

particular the reservoir region (Figure 5.22) and hydrodynamic film region (Figure 5.23). Since our model prescribes that $\tilde{u} = U$, such a plot is unnecessary to describe $\tilde{u}$ inside the porous medium region. Notice that the fluid slows down and speeds up in the $x$-direction inside the reservoir (Figure 5.22). Also, we observe that the velocity changes in the $x$-direction inside the hydrodynamic film region (Figure 5.23) around the point where the full film is reconstituted (where the region changes from being in a cavitated state to a full film). We also notice how the velocity changes in the $y$-direction inside the film region to satisfy the no-slip boundary condition at $y = \beta + h(x)$.

Notice that for Figures 5.21 - 5.27, we have elected only to show the middle 80% of the graph in the $x$-direction. The reason for doing so is that the numerical algorithm used by Vijayaraghavan and Keith [22], and extended in this work, produces
Figure 5.18: Two-dimensional carpet plot of the pressure profile, \( p(x, y) \), inside the porous medium for the highest permeability limiting case, \( k_y = 1 \times 10^{-10} \text{ m}^2 \). All other pertinent values are given in Tables 3.1 and 3.2.

reasonably smooth pressure curves at the onset of cavitation, however it does not generate smooth pressure curves where the film transitions from a cavitated region to a full film region. This fact, coupled with the expressions for \( u_{III} \) and \( v_{III} \) from Chapter 3, leads to a numerical oscillation about this point. However, it has been extensively tested, though not shown in this work, that this oscillation does not filter through the rest of the solution, nor does it influence the temperatures (in Chapter 7) calculated away from this point. We then consider the middle 80\% of the \( x \)-direction domain as the domain of interest, neglecting the outer 10\% at the edges that may feature a numerical anomaly that is not indicative of the actual physics of the operation of the bearing.
Figure 5.19: Two-dimensional carpet plot of the pressure profile, $p(x, y)$, inside the porous medium for the thinnest porous medium limiting case, $\beta - \alpha = 0.317$ mm. All other pertinent values are given in Tables 3.1 and 3.2.

Though, for completeness, we showed how the $x$-direction velocity, $u$, changes inside the bearing, the important aspect of the self-circulation feature of the bearing involves the $y$-direction velocity, $v$. In Figure 5.24, we see how the velocity varies inside the entire bearing system, whereas Figures 5.25 - 5.27 feature more detail, by showing how $v$ varies in each region.

In Figure 5.24, we can see what appears to be a discontinuity in $v$ at the interface $y = \beta$. However, the velocities, $\tilde{v}$ and $v_{III}$ are indeed continuous at this interface. The jump that is seen is a result of the modeling choices we used, whereby the velocities are forced to be continuous, but the derivatives are not.

Preliminary analyses that were done for this work, though not shown in this paper, considered a model without cavitation effects. As such, the mathematical model chosen allowed for negative pressures to exist in the bearing. Results from this
work did not show any kind of velocity derivative discontinuity at the porous medium-fluid interfaces. Thus, it can be said that this velocity derivative discontinuity is a byproduct of using the analytical and numerical model used to describe cavitation in this bearing. We may therefore assume that a boundary layer exists at this point, and the current methods used here do not encapture it. Future work to be considered may include capturing this boundary layer, via an altered governing model, an altered cavitation model, an altered numerical model, or any combination thereof.

The $y$-direction velocity inside the reservoir, $v_I$, shows how the fluid enters the reservoir from the porous medium (Figure 5.25). It can be seen that, inside the converging region, for the most part, $v_I$ is negative, meaning that the fluid is being drawn into the reservoir. Likewise, inside the diverging region, for the most part, $v_I$ is positive, meaning that the fluid is being drawn out of the reservoir and into
Figure 5.21: Two-dimensional carpet plot of the $x$-direction velocity, $u$, inside the bearing system. $k_y^* = 1 \times 10^{-12}$ m$^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 3.1 and 3.2.

the porous medium. Also, it can be seen that the no-slip boundary condition at the boundary, $y = 0$, is satisfied.

The $y$-direction velocity inside the porous medium, $\tilde{v}$, shows how the fluid interacts in the region between the hydrodynamic film and the reservoir (Figure 5.26). Again we note that the fluid moves in the negative $y$-direction in the converging region of the bearing, and the fluid moves in the positive $y$-direction in the diverging region of the bearing. We also see how the porous medium acts to resist the flow by noting that the magnitude of $\tilde{v}$ is reduced, in the converging region, when moving from $y = \beta$ (hydrodynamic film boundary) to $y = \alpha$ (reservoir boundary). Future work may involve designing the porous medium to have a non-constant permeability in order to direct the flow in a yet, undetermined, optimal manner.
Figure 5.22: Two-dimensional carpet plot of the $x$-direction velocity, $u_I$, inside the reservoir region. $k_y^* = 1 \times 10^{-12}$ m$^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 3.1 and 3.2.

Figure 5.27 shows how the $y$-direction velocity, $v_{III}$, changes inside the hydrodynamic film region. Here, we can see how the $y$-direction derivative of $v_{III}$ does not match up with the $y$-direction derivative of $\tilde{v}$ at the porous medium-film interface, $y = \beta$, inside the cavitated region. That aside, we see that $v_{III}$ varies with both $x$ and $y$, takes on negative values inside the converging region and positive values inside the diverging region, and satisfies the no-penetration boundary condition at $y = \beta + h(x)$.

A velocity flow field, whereby the velocity vectors indicate the direction and magnitude of the flow inside the bearing, is an excellent way to visualize how the fluid flows. For the geometry of the work considered in this paper, the disparity of scales makes generating a flow field difficult, as the velocity of the fluid in the $x$-direction is much larger in magnitude than that of the $y$-direction. However, as
Figure 5.23: Two-dimensional carpet plot of the x-direction velocity, $u_{III}$, inside the hydrodynamic film region. $k_y^* = 1 \times 10^{-12} \text{ m}^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $\alpha^* = 1.27 \times 10^{-4} \text{ m}$, $U^* = 4.57 \text{ m/s}$. All other pertinent values are given in Tables 3.1 and 3.2.

mentioned earlier, preliminary work for this project involved not using a cavitation model but still demonstrated pressure differentials between the active film space and reservoir regions. We remember that the pressure differences between the two regions are the mechanism for actively circulating the fluid. By using an exaggerated bearing geometry which featured a steep film profile and thin porous medium, a clear velocity flow field was generated and can be seen in Figure 5.28.

The circulating fluid effect is prominently displayed, whereby the positive pressure difference in the converging region pushes the fluid into the reservoir, and the negative pressure difference in the diverging region draws the fluid back into the active film space. Though the numerical values obtained from the preliminary work differ from that of the work in this paper, we can still conclude that the geometry offered in
Figure 5.24: Two-dimensional carpet plot of the y-direction velocity, $v$, inside the bearing system. $k_y^* = 1 \times 10^{-12}$ m$^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 3.1 and 3.2.

This paper does indeed actively circulate fluid between the hydrodynamic film and reservoir regions, while simultaneously providing high pressures that correlate to a substantial load carrying capacity.
Figure 5.25: Two-dimensional carpet plot of the $y$-direction velocity, $v_I$, inside the reservoir region. $k_y^* = 1 \times 10^{-12}$ m$^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 3.1 and 3.2.

Figure 5.26: Two-dimensional carpet plot of the $y$-direction velocity, $\tilde{v}$, inside the porous medium region. $k_y^* = 1 \times 10^{-12}$ m$^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 3.1 and 3.2.
Figure 5.27: Two-dimensional carpet plot of the $y$-direction velocity, $v_{III}$, inside the hydrodynamic film region. $k_y^* = 1 \times 10^{-12} \text{ m}^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $\alpha^* = 1.27 \times 10^{-4} \text{ m}$, $U^* = 4.57 \text{ m/s}$. All other pertinent values are given in Tables 3.1 and 3.2.

Figure 5.28: Velocity Flow Field From Preliminary Research
CHAPTER VI

LONG SLIDER BEARING THERMAL EQUATIONS

The governing equation used to describe the thermal characteristics of the bearing is the steady-state local thermal equilibrium equation, valid for every region inside the bearing geometry. We may justify this choice by stating that the porous medium to be used features the characteristic that its thermal conductivity is roughly that of the lubricating fluid. Though this may not always be the case, the methods used here may be easily extrapolated to that of the local thermal non-equilibrium model.

6.1 Governing Equations

The governing thermal equation, taken from Lu [28], may be written as

\[ \rho c_v \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) + p \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 
\]

\[ = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \left( \frac{2\mu}{3} \right) \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 + 
\]

\[ + 2\mu \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right] + 
\]

\[ + \left( \frac{\mu}{2} \right) \left[ \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right]. \tag{6.1} \]

In order to implement a comprehensive parametric study, Equation (6.1) is non-dimensionalized by adopting the following relationship between the dimensional and
non-dimensional variables:

\[ u = U U^* \quad (6.2) \]
\[ v = \left( \frac{\alpha}{U^*} \right) U V^* \quad (6.3) \]
\[ x = L_x X^* \quad (6.4) \]
\[ y = \alpha Y^* \quad (6.5) \]
\[ p = \left( \frac{L \mu U}{\alpha^2} \right) P^* \quad (6.6) \]
\[ T = T T^*, \quad (6.7) \]

where \( T \) is a chosen reference temperature.

Then, Equation (6.1) may be rewritten as

\[
\left( \frac{\rho c_v U \alpha^2}{k L_x} \right) \left( U^* \frac{\partial T^*}{\partial X^*} + V^* \frac{\partial T^*}{\partial Y^*} \right) + \left( \frac{\mu U^2}{k T} \right) P^* \left( \frac{\partial U^*}{\partial X^*} + \frac{\partial V^*}{\partial Y^*} \right) = \\
= \left( \frac{\alpha}{L_x} \right)^2 \frac{\partial^2 T^*}{\partial X^2} + \frac{\partial^2 T^*}{\partial Y^2} - \\
- \left( \frac{\alpha}{L_x} \right)^2 \left( \frac{2 \mu U^2}{3 k T} \right) \left( \frac{\partial U^*}{\partial X^*} + \frac{\partial V^*}{\partial Y^*} \right)^2 + \\
+ \left( \frac{\alpha}{L_x} \right)^2 \left( \frac{2 \mu U^2}{k T} \right) \left[ \left( \frac{\partial U^*}{\partial X^*} \right)^2 + \left( \frac{\partial V^*}{\partial Y^*} \right)^2 \right] + \\
+ \left( \frac{\mu U^2}{2 k T} \right) \left[ \left( \frac{\partial U^*}{\partial Y^*} \right)^2 + 2 \left( \frac{\alpha}{L_x} \right)^2 \frac{\partial U^*}{\partial Y^*} \frac{\partial V^*}{\partial X^*} + \left( \frac{\alpha}{L_x} \right)^4 \frac{\partial V^*}{\partial X^*} \right]. \quad (6.8)
\]
Again, letting $\varepsilon_x = \frac{\alpha}{L_x}$, we can rewrite Equation (6.8) as

$$
\varepsilon_x \left( \rho c_v U \alpha \right) \left( U^* \frac{\partial T^*}{\partial x^*} + V^* \frac{\partial T^*}{\partial y^*} \right) + \left( \frac{\mu U^2}{kT} \right) \frac{\partial^2 \left( \frac{\partial U^*}{\partial x^*} \right) \frac{\partial V^*}{\partial y^*}}{\partial y^2} + \frac{\partial^2 \left( \frac{\partial U^*}{\partial x^*} \right) \frac{\partial V^*}{\partial y^*}}{\partial x^2} - \varepsilon_x \left( \frac{2 \mu U^2}{3} \right) \left( \frac{\partial U^*}{\partial x^*} + \frac{\partial V^*}{\partial y^*} \right)^2 + \varepsilon_x \left( \frac{2 \mu U^2}{kT} \right) \left[ \left( \frac{\partial U^*}{\partial x^*} \right)^2 + \left( \frac{\partial V^*}{\partial y^*} \right)^2 \right] + \left( \frac{\mu U^2}{2kT} \right) \left[ \left( \frac{\partial U^*}{\partial y^*} \right)^2 + 2 \varepsilon_x^2 \frac{\partial U^*}{\partial y^*} \frac{\partial V^*}{\partial x^*} + \varepsilon_x^4 \left( \frac{\partial V^*}{\partial x^*} \right)^2 \right].
$$

By expanding $T^*$ in an asymptotic manner,

$$
T^* = T^*_0 + \varepsilon T^*_1 + O(\varepsilon_x^2)
$$

we may write the $O(1)$ and $O(\varepsilon_x)$ problems:

**Non-Dimensional Governing Equation at $O(1)$:**

$$
\frac{\partial^2 T^*_0}{\partial y^2} + \left( \frac{\mu U^2}{2kT} \right) \left[ \left( \frac{\partial U^*_0}{\partial y^*} \right)^2 \right] = \left( \frac{\mu U^2}{kT} \right) \frac{\partial^2 \left( \frac{\partial U^*_0}{\partial x^*} \right) \frac{\partial V^*_0}{\partial y^*}}{\partial y^2} + \frac{\partial^2 \left( \frac{\partial U^*_0}{\partial x^*} \right) \frac{\partial V^*_0}{\partial y^*}}{\partial x^2}.
$$

**Non-Dimensional Governing Equation at $O(\varepsilon_x)$:**

$$
\frac{\partial^2 T^*_1}{\partial y^2} = \left( \frac{\rho c_v U \alpha}{k} \right) \left( U^* \frac{\partial T^*_0}{\partial x^*} + V^* \frac{\partial T^*_0}{\partial y^*} \right).
$$

Note that the $O(1)$ equation contains conduction terms and the $O(\varepsilon_x)$ equation contains convection terms. These terms dominate the governing equation, Equation (6.9), so we only consider their contributions.
Now that the dominating terms have been determined, we may rewrite Equations (6.11) and (6.12) in their dimensional forms:

\[
\frac{\partial^2 T_0}{\partial y^2} = \frac{1}{k} \left\{ p \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - \left( \frac{\mu}{2} \right) \left[ \left( \frac{\partial u}{\partial y} \right)^2 \right] \right\} \tag{6.13}
\]

\[
\frac{\partial^2 T_1}{\partial y^2} = \left( \frac{\rho c_v}{k} \right) \left( \frac{L_x}{\alpha} \right) \left( u \frac{\partial T_0}{\partial x} + v \frac{\partial T_0}{\partial y} \right). \tag{6.14}
\]

6.2 Boundary Conditions

Akin to the work that was done in the momentum problems, we define \( T_I \) to be the temperature inside Region I (Reservoir), \( \tilde{T} \) to be the temperature inside Region II (Porous Medium), and \( T_{III} \) to be the temperature inside Region III (Hydrodynamic Film). The governing equations, Equation (6.13) and (6.14) are subject to the following boundary conditions

@ \( y = 0 \),

\[
\frac{\partial T_I}{\partial y} = \left( \frac{h_I}{k_f} \right) (T_I - T_a), \tag{6.15}
\]

where \( h_I \) is the heat transfer coefficient at the reservoir wall, \( k_f \) is the thermal conductivity of the lubricating fluid, and \( T_a \) is the ambient temperature external to the reservoir wall.

@ \( y = \alpha \),

\[
T_I = \tilde{T}, \tag{6.16}
\]

\[
k_I \frac{\partial T_I}{\partial y} = \tilde{k} \frac{\partial \tilde{T}}{\partial y}. \tag{6.17}
\]
where $\tilde{k}$ is the effective thermal conductivity of the fluid inside the porous medium.

@ $y = \beta$,

$$\tilde{T} = T_{lIII}, \quad (6.18)$$

$$\tilde{k} \frac{\partial \tilde{T}}{\partial y} = k_{III} \frac{\partial T_{lIII}}{\partial y}. \quad (6.19)$$

To describe the boundary condition at $y = \beta + h(x)$, we begin by considering the energy equation inside the solid slider. Since there is no pressure or fluid velocity inside the material, the governing energy equation inside the solid slider reduces to

$$\frac{\partial^2 T_j}{\partial y^2} = 0, \quad (6.20)$$

where $T_j$ is the temperature of the solid slider.

If one were to consider the corresponding cylindrical journal bearing, where the journal has radius, $r$, a reasonable condition to impose is that $\frac{\partial T_j}{\partial r} = 0$ at $r = 0$. Thus, in an unwrapped geometry, the related boundary condition would be $\frac{\partial T_j}{\partial y} = 0$ at $y = \beta + h(x) + r$.

Now, for the slider bearing case, which we are discussing here, instead of considering the above boundary condition at $y = \beta + h(x) + r$, we simply consider the same boundary condition at the height, $y = y_c$, with the subscript $c$ denoting 'center.' Then, we have the governing equation for the solid slider, Equation (6.20), combined with the boundary condition at $y = y_c$,

$$\frac{\partial T_j}{\partial y} = 0, \quad (6.21)$$
to get that \( T_j \) is a constant. Then, by imposing a boundary condition similar to that of Equation (6.17),

\[
k_j \frac{\partial T_j}{\partial y} \bigg|_{y=\beta+h(x)} = k_f \frac{\partial T_{III}}{\partial y} \bigg|_{y=\beta+h(x)},
\]

where \( k_j \) is the thermal conductivity of the solid slider, it follows that our final boundary condition is that, at \( y = \beta + h(x) \),

\[
\frac{\partial T_{III}}{\partial y} = 0.
\]

This is the most conservative case, and it essentially states that the slider is insulated. Define \( H(x) = \alpha h(x) \) as a non-dimensional hydrodynamic film height. Then, non-dimensionalizing boundary conditions Equations (6.15) - (6.23) yields

@ \( Y = 0 \)

\[
\frac{\partial T^*_{I}}{\partial Y^*} = \left( \frac{\alpha h_I}{k_f} \right) \left( T^*_{I} - \frac{T_a}{T} \right),
\]

@ \( Y = 1 \)

\[
T^*_{I} = \widetilde{T}^*,
\]

\[
k_f \frac{\partial T^*_{I}}{\partial Y^*} = \kappa \frac{\partial \widetilde{T}^*}{\partial Y^*},
\]

@ \( Y = \frac{\beta}{\alpha} \)

\[
\widetilde{T}^* = T^*_{III},
\]

\[
\kappa \frac{\partial \widetilde{T}^*}{\partial Y^*} = k_f \frac{\partial T^*_{III}}{\partial Y^*},
\]

@ \( Y = \frac{\beta}{\alpha} + H(x) \)

\[
\frac{\partial T^*_{III}}{\partial Y^*} = 0.
\]
We may now write the non-dimensional $O(1)$ and $O(\epsilon_x)$ boundary conditions:

@ $Y = 0$

\[ \frac{\partial T^*_{I0}}{\partial Y^*} = \left( \frac{\alpha h_I}{k_f} \right) \left( T^*_{I0} - \frac{T_a}{T} \right), \]  
(6.30)

\[ \frac{\partial T^*_{I1}}{\partial Y^*} = \left( \frac{\alpha h_I}{k_f} \right) T^*_{I1}, \]  
(6.31)

@ $Y = 1$

\[ T^*_{I0} = \tilde{T}_1^*, \]  
(6.32)

\[ k_f \frac{\partial T^*_{I0}}{\partial Y^*} = \tilde{k} \frac{\partial \tilde{T}_0^*}{\partial Y^*}, \]  
(6.33)

\[ T^*_{I1} = \tilde{T}_1^*, \]  
(6.34)

\[ k_f \frac{\partial T^*_{I1}}{\partial Y^*} = \tilde{k} \frac{\partial \tilde{T}_1^*}{\partial Y^*}, \]  
(6.35)

@ $Y = \frac{\beta}{\alpha}$

\[ \tilde{T}_0^* = T^*_{III0}, \]  
(6.36)

\[ \tilde{k} \frac{\partial \tilde{T}_0^*}{\partial Y^*} = k_f \frac{\partial T^*_{III0}}{\partial Y^*}, \]  
(6.37)

\[ \tilde{T}_1^* = T^*_{III1}, \]  
(6.38)

\[ \tilde{k} \frac{\partial \tilde{T}_1^*}{\partial Y^*} = k_f \frac{\partial T^*_{III1}}{\partial Y^*}, \]  
(6.39)

@ $Y = \frac{\beta}{\alpha} + H(x)$

\[ \frac{\partial T^*_{III0}}{\partial Y^*} = 0, \]  
(6.40)

\[ \frac{\partial T^*_{III1}}{\partial Y^*} = 0. \]  
(6.41)

Now, rewriting boundary conditions Equations (6.30) - (6.41) in dimensional form, we have our final boundary conditions.
\@ y = 0

\frac{\partial T_{I0}}{\partial y} = \left( \frac{h_I}{k_f} \right) \left( T_{I0} - \frac{T_a}{T} \right), \quad (6.42)

\frac{\partial T_{I1}}{\partial y} = \left( \frac{h_I}{k_f} \right) T_{I1}, \quad (6.43)

\@ y = \alpha

T_{I0} = \tilde{T}_0, \quad (6.44)

k_f \frac{\partial T_{I0}}{\partial y} = \tilde{k} \frac{\partial \tilde{T}_0}{\partial y}, \quad (6.45)

T_{I1} = \tilde{T}_1, \quad (6.46)

k_f \frac{\partial T_{I1}}{\partial y} = \tilde{k} \frac{\partial \tilde{T}_1}{\partial y}, \quad (6.47)

\@ y = \beta

\tilde{T}_0 = T_{III0}, \quad (6.48)

\tilde{k} \frac{\partial \tilde{T}_0}{\partial y} = k_f \frac{\partial T_{III0}}{\partial y}, \quad (6.49)

\tilde{T}_1 = T_{III1}, \quad (6.50)

\tilde{k} \frac{\partial \tilde{T}_1}{\partial y} = k_f \frac{\partial T_{III1}}{\partial y}, \quad (6.51)

\@ y = \beta + h(x)

\frac{\partial T_{III0}}{\partial y} = 0, \quad (6.52)

\frac{\partial T_{III1}}{\partial y} = 0. \quad (6.53)

Thus, we now write our \( O(1) \) and \( O(\epsilon_x) \) governing equations, in dimensional form, together with their boundary conditions, also in dimensional form:
Region I: (Reservoir)

\[
\frac{\partial^2 T_{I0}}{\partial y^2} = \frac{1}{k_f} \left[ p_I \left( \frac{\partial u_I}{\partial x} + \frac{\partial v_I}{\partial y} \right) - \left( \frac{\mu}{2} \right) \left( \frac{\partial u_I}{\partial y} \right)^2 \right],
\tag{6.54}
\]

Region II: (Porous Medium)

\[
\frac{\partial^2 \tilde{T}_0}{\partial y^2} = \frac{1}{\tilde{k}} \left[ \tilde{p} \left( \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} \right) \right],
\tag{6.55}
\]

Region III: (Hydrodynamic Fluid Film)

\[
\frac{\partial^2 T_{III0}}{\partial y^2} = \frac{1}{k_f} \left[ p_{III} \left( \frac{\partial u_{III}}{\partial x} + \frac{\partial v_{III}}{\partial y} \right) - \left( \frac{\mu}{2} \right) \left( \frac{\partial u_{III}}{\partial y} \right)^2 \right],
\tag{6.56}
\]

subject to the boundary conditions,

\[
\frac{\partial T_{I0}}{\partial y} \bigg|_{y=0} = \left( \frac{h_I}{k_f} \right) \left( T_{I0} \bigg|_{y=0} - \frac{T_a}{T} \right),
\tag{6.57}
\]

\[
T_{I0} \bigg|_{y=\alpha} = \tilde{T}_0 \bigg|_{y=\alpha},
\tag{6.58}
\]

\[
k_f \frac{\partial T_{I0}}{\partial y} \bigg|_{y=\alpha} = \tilde{k} \frac{\partial \tilde{T}_0}{\partial y} \bigg|_{y=\alpha},
\tag{6.59}
\]

\[
\tilde{T}_0 \bigg|_{y=\beta} = T_{III0} \bigg|_{y=\beta},
\tag{6.60}
\]

\[
k_f \frac{\partial \tilde{T}_0}{\partial y} \bigg|_{y=\beta} = \tilde{k} \frac{\partial T_{III0}}{\partial y} \bigg|_{y=\beta},
\tag{6.61}
\]

\[
\frac{\partial T_{III0}}{\partial y} \bigg|_{y=\beta+h(x)} = 0.
\tag{6.62}
\]

\[O(1)\]

Region I: (Reservoir)

\[
\frac{\partial^2 T_{I1}}{\partial y^2} = \left( \frac{\rho_{ICe}}{k_f} \right) \left( \frac{L_x}{\alpha} \right) \left( u_I \frac{\partial T_{I0}}{\partial x} + v_I \frac{\partial T_{I0}}{\partial y} \right),
\tag{6.63}
\]
Region II: (Porous Medium)

\[
\frac{\partial^2 \tilde{T}_1}{\partial y^2} = \left( \frac{\bar{\rho}c_v}{k} \right) \left( \frac{L_x}{\alpha} \right) \left( \tilde{u} \frac{\partial \tilde{T}_0}{\partial x} + \tilde{v} \frac{\partial \tilde{T}_0}{\partial y} \right),
\]

(6.64)

Region III: (Hydrodynamic Fluid Film)

\[
\frac{\partial^2 T_{III}}{\partial y^2} = \left( \frac{\rho_{III}c_{\alpha}}{k_f} \right) \left( \frac{L_x}{\alpha} \right) \left( u_{III} \frac{\partial T_{III0}}{\partial x} + v_{III} \frac{\partial T_{III0}}{\partial y} \right),
\]

(6.65)

subject to the boundary conditions

\[
\frac{\partial T_{II}}{\partial y} \bigg|_{y=0} = 0,
\]

(6.66)

\[
T_{II} \bigg|_{y=\alpha} = \tilde{T}_1 \bigg|_{y=\alpha},
\]

(6.67)

\[
k_f \frac{\partial T_{II}}{\partial y} \bigg|_{y=\alpha} = \tilde{k} \frac{\partial \tilde{T}_1}{\partial y} \bigg|_{y=\alpha},
\]

(6.68)

\[
\tilde{T}_1 \bigg|_{y=\beta} = T_{III} \bigg|_{y=\beta},
\]

(6.69)

\[
\tilde{k} \frac{\partial \tilde{T}_1}{\partial y} \bigg|_{y=\beta} = k_f \frac{\partial T_{III}}{\partial y} \bigg|_{y=\beta},
\]

(6.70)

\[
\frac{\partial T_{III}}{\partial y} \bigg|_{y=\beta+h(x)} = 0.
\]

(6.71)

The above governing equations and boundary conditions, Equations (6.54) - (6.71), will be solved numerically in Chapter 7.
CHAPTER VII

NUMERICAL TECHNIQUES AND RESULTS - LONG SLIDER BEARING - THERMAL EQUATIONS

7.1 Numerical Implementation for Thermal Equations

To orient the reader on the computational domains used for the thermal calculations, we define the boundary, \( y = 0 \) to correspond to the index \( j = 1 \), the boundary, \( y = \alpha \) to correspond to the index \( j = j_\alpha \), the boundary \( y = \beta \) to correspond to the index \( j = j_\beta \), and the boundary \( y = \beta + h(x) \) to correspond to the index \( j = J \), not to be confused with the computational domain index value, \( J \), from the computational mesh used in Chapter 5 for calculating the pressures inside the bearing. Note that for the momentum equations, the pressures only had to be calculated inside the porous medium and at the porous medium boundaries, \( y = \alpha \) and \( y = \beta \), whereas for the thermal equations, the temperatures have to be calculated at every point inside the bearing.

Though the analytical formulation of the governing thermal equations defined three separate temperature variables, \( T_I, \tilde{T}, \) and \( T_{III} \), for the numerical procedure defined in this section, we define a single temperature variable \( T \) whose indices in the \( y \)-direction vary from \( j = 1(y = 0) \) to \( j = J(y = \beta + h(x)) \). Thus, we now define \( T_0 \)
to be the temperature variable for the $O(1)$ problem and $T_1$ to be the temperature variable for the $O(\epsilon_x)$ problem. Also, since the only derivatives for $T$ to be discussed in this work are the $y$-direction derivatives, we define $T_{0,j}$ to be the discretized value of $T_0$ at the $y$-gridpoint $j$, not the discretized value of $T$ at the $x$-gridpoint $0$ and the $y$-gridpoint $j$. For the sake of keeping the discretizations clear, the $x(i)$ indices are not explicitly written, but they are implied.

Also, each region is defined to possess its own grid spacing in the $y$-direction, yielding a $\Delta y$ in each region. These are defined as $\Delta y_{res}$, $\Delta y_{por}$, and $\Delta y_{film}$ representing the grid spacing in the reservoir, porous medium, and film regions, respectively. Also note that the first two values for $\Delta y$ do not depend on the position of the node in the $x$-direction, but the value for $\Delta y$ inside the fluid film region does depend on position of the node in the $x$-direction, due to the shape of the height profile, $h(x)$.

Second order finite difference schemes were used to discretize every derivative term in the governing equations, Equations (6.54) - (6.56) and Equations (6.63) -
\[
\frac{\partial^2 T_{\text{I}0}}{\partial y^2} = \frac{1}{k_f} \left[ p_1 \left( \frac{\partial u_I}{\partial x} + \frac{\partial v_I}{\partial y} \right) - \left( \frac{\mu}{2} \right) \left( \frac{\partial u_I}{\partial y} \right)^2 \right] \quad (7.1)
\]

\[
\frac{\partial^2 \tilde{T}_0}{\partial y^2} = \frac{1}{k} \left[ \tilde{p} \left( \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} \right) - \left( \frac{\mu}{2} \right) \left( \frac{\partial \tilde{u}}{\partial y} \right)^2 \right] \quad (7.2)
\]

\[
\frac{\partial^2 T_{\text{III}0}}{\partial y^2} = \frac{1}{k_f} \left[ p_{\text{III}} \left( \frac{\partial u_{\text{III}}}{\partial x} + \frac{\partial v_{\text{III}}}{\partial y} \right) - \left( \frac{\mu}{2} \right) \left( \frac{\partial u_{\text{III}}}{\partial y} \right)^2 \right] \quad (7.3)
\]

\[
\frac{\partial^2 T_{\text{I}1}}{\partial y^2} = \left( \frac{\rho_I c_v}{k_f} \right) \left( \frac{L_x}{\alpha} \right) \left( u_I \frac{\partial T_{\text{I}0}}{\partial x} + v_I \frac{\partial T_{\text{I}0}}{\partial y} \right) \quad (7.4)
\]

\[
\frac{\partial^2 \tilde{T}_1}{\partial y^2} = \left( \frac{\rho c_v}{k} \right) \left( \frac{L_x}{\alpha} \right) \left( \tilde{u} \frac{\partial \tilde{T}_0}{\partial x} + \tilde{v} \frac{\partial \tilde{T}_0}{\partial y} \right) \quad (7.5)
\]

\[
\frac{\partial^2 T_{\text{III}1}}{\partial y^2} = \left( \frac{\rho_{\text{III}} c_v}{k_f} \right) \left( \frac{L_x}{\alpha} \right) \left( u_{\text{III}} \frac{\partial T_{\text{III}0}}{\partial x} + v_{\text{III}} \frac{\partial T_{\text{III}0}}{\partial y} \right) \quad (7.6)
\]
and the boundary conditions, Equations (6.57) - (6.62) and Equations (6.66) - (6.71)

\[
\frac{\partial T_{10}}{\partial y} \bigg|_{y=0} = \left( \frac{h_I}{k_f} \right) \left( T_{10} \bigg|_{y=0} - \frac{T_a}{T} \right) \tag{7.7}
\]

\[
T_{10} \bigg|_{y=\alpha} = \tilde{T}_0 \bigg|_{y=\alpha} \tag{7.8}
\]

\[
k_f \frac{\partial T_{10}}{\partial y} \bigg|_{y=\alpha} = \tilde{k} \frac{\partial \tilde{T}_0}{\partial y} \bigg|_{y=\alpha} \tag{7.9}
\]

\[
\tilde{T}_0 \bigg|_{y=\beta} = T_{III0} \bigg|_{y=\beta} \tag{7.10}
\]

\[
\tilde{k} \frac{\partial \tilde{T}_0}{\partial y} \bigg|_{y=\beta} = k_f \frac{\partial T_{III0}}{\partial y} \bigg|_{y=\beta} \tag{7.11}
\]

\[
\frac{\partial T_{III0}}{\partial y} \bigg|_{y=\beta+h(x)} = 0 \tag{7.12}
\]

\[
\frac{\partial T_{11}}{\partial y} \bigg|_{y=0} = 0 \tag{7.13}
\]

\[
T_{11} \bigg|_{y=\alpha} = \tilde{T}_1 \bigg|_{y=\alpha} \tag{7.14}
\]

\[
k_f \frac{\partial T_{11}}{\partial y} \bigg|_{y=\alpha} = \tilde{k} \frac{\partial \tilde{T}_1}{\partial y} \bigg|_{y=\alpha} \tag{7.15}
\]

\[
\tilde{T}_1 \bigg|_{y=\beta} = T_{III1} \bigg|_{y=\beta} \tag{7.16}
\]

\[
\tilde{k} \frac{\partial \tilde{T}_1}{\partial y} \bigg|_{y=\beta} = k_f \frac{\partial T_{III1}}{\partial y} \bigg|_{y=\beta} \tag{7.17}
\]

\[
\frac{\partial T_{III1}}{\partial y} \bigg|_{y=\beta+h(x)} = 0. \tag{7.18}
\]

At the boundaries in the y-direction, i.e., \( y = 0, y = \alpha, y = \beta, \) and \( y = \beta + h(x), \) three point forward and backward differences were used to discretize y-direction derivatives, with the orientation depending on the boundary in question. For example, for the boundary condition Equation (7.7), a three point forward difference was used

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for the term $\frac{\partial T_{I0}}{\partial y}$ at $y = 0$, yielding

$$\left. \frac{\partial T_{I0}}{\partial y} \right|_{y=0} = \frac{-3T_{0,1} + 4T_{0,2} - T_{0,3}}{2\Delta y_{res}}.$$  \hspace{1cm} (7.19)

Combined with setting $T_{I0}\big|_{y=0} = T_{0,1}$ and rearranging terms, boundary condition Equation (7.7) becomes

$$\left[ -3 - \left( \frac{h_I}{k_f} \right) (2\Delta y_{res}) \right] T_{0,1} + 4T_{0,2} - T_{0,3} = - \left( \frac{h_I}{k_f} \right) (2\Delta y_{res}) T_a.$$  \hspace{1cm} (7.20)

Since the governing equations, Equations (7.2) - (7.6), possess second derivatives in $y$ for the temperature variables, and these equations are valid on the interiors of the regions in question, we may use a second order center difference discretization scheme to produce

$$\frac{\partial^2 T_{I0}}{\partial y^2} = \frac{T_{0,j-1} - 2T_{0,j} + T_{0,j+1}}{(\Delta y_{res})^2}.$$  \hspace{1cm} (7.21)

$$\frac{\partial^2 T_0}{\partial y^2} = \frac{T_{0,j-1} - 2T_{0,j} + T_{0,j+1}}{(\Delta y_{por})^2}.$$  \hspace{1cm} (7.22)

$$\frac{\partial^2 T_{III0}}{\partial y^2} = \frac{T_{0,j-1} - 2T_{0,j} + T_{0,j+1}}{(\Delta y_{film})^2},$$  \hspace{1cm} (7.23)

where Equation (7.21) is valid for $2 \leq j \leq j_{\alpha - 1}$, Equation (7.22) is valid for $j_{\alpha + 1} \leq j \leq j_{\beta - 1}$, and Equation (7.23) is valid for $j_{\beta + 1} \leq j \leq j_{\beta - 1}$.

Also, we use two point centered differences on all velocity gradient terms, i.e.,

$$\frac{\partial u_I}{\partial x} = \frac{u_{I,i+1} - u_{I,i-1}}{2\Delta x}$$

$$\frac{\partial v_{III}}{\partial y} = \frac{v_{III,j+1} - u_{I,j-1}}{2\Delta y_{film}},$$
etc. We discretize Equation (7.9) using a three point backward difference for the term \( \frac{\partial T_0}{\partial y} \big|_{y=\alpha} \) and a three point forward difference for the term \( \frac{\partial T_0}{\partial y} \big|_{y=\alpha} \) to yield

\[
\begin{align*}
\frac{\partial T_0}{\partial y} \big|_{y=\alpha} &= \frac{3T_{0,j\alpha} - 4T_{0,j\alpha-1} + T_{0,j\alpha-2}}{2\Delta y_{\text{res}}} \\
\frac{\partial \tilde{T}_0}{\partial y} \big|_{y=\alpha} &= -\frac{3T_{0,j\alpha} + 4T_{0,j\alpha+1} - T_{0,j\alpha+2}}{2\Delta y_{\text{por}}}.
\end{align*}
\]

(7.24)

(7.25)

This lets us write Equation (7.9) as

\[
\begin{align*}
-T_{0,j\alpha-2} + 4T_{0,j\alpha-1} - \left[ 3 + 3 \left( \frac{\tilde{k}}{k_f} \right) \left( \frac{\Delta y_{\text{res}}}{\Delta y_{\text{por}}} \right) \right] T_{0,j\alpha} + \\
+ 4 \left( \frac{\tilde{k}}{k_f} \right) \left( \frac{\Delta y_{\text{res}}}{\Delta y_{\text{por}}} \right) T_{0,j\alpha+1} - \left( \frac{\tilde{k}}{k_f} \right) \left( \frac{\Delta y_{\text{res}}}{\Delta y_{\text{por}}} \right) T_{0,j\alpha+2} = 0.
\end{align*}
\]

(7.26)

Similarly, we discretize Equation (7.11) using a three point forward difference for the term \( \frac{\partial T_{III}}{\partial y} \big|_{y=\beta} \) and a three point backward difference for the term \( \frac{\partial \tilde{T}_0}{\partial y} \big|_{y=\beta} \) to yield

\[
\begin{align*}
\frac{\partial T_{III}}{\partial y} \big|_{y=\beta} &= \frac{3T_{0,j\beta} - 4T_{0,j\beta-1} + T_{0,j\beta-2}}{2\Delta y_{\text{por}}} \\
\frac{\partial \tilde{T}_0}{\partial y} \big|_{y=\beta} &= -\frac{3T_{0,j\beta} + 4T_{0,j\beta+1} - T_{0,j\beta+2}}{2\Delta y_{\text{film}}},
\end{align*}
\]

(7.27)

(7.28)

This lets us write Equation (7.11) as

\[
\begin{align*}
&- \left( \frac{\tilde{k}}{k_f} \right) \left( \frac{\Delta y_{\text{film}}}{\Delta y_{\text{por}}} \right) T_{0,j\beta-2} + 4 \left( \frac{\tilde{k}}{k_f} \right) \left( \frac{\Delta y_{\text{film}}}{\Delta y_{\text{por}}} \right) T_{0,j\beta-1} - \\
&\left[ 3 + 3 \left( \frac{\tilde{k}}{k_f} \right) \left( \frac{\Delta y_{\text{film}}}{\Delta y_{\text{por}}} \right) \right] T_{0,j\beta} + 4T_{0,j\beta+1} - T_{0,j\beta+2} = 0.
\end{align*}
\]

(7.29)

Finally we discretize the boundary condition Equation (7.12) using a three point backward difference for the term \( \frac{\partial T_{III}}{\partial y} \big|_{y=\beta+h(x)} \) to yield

\[
\begin{align*}
\frac{\partial T_{III}}{\partial y} \big|_{y=\beta+h(x)} &= \frac{3T_{0,J} - 4T_{0,J-1} + T_{0,J-2}}{2\Delta y_{\text{film}}},
\end{align*}
\]

(7.30)
Thus, we may rewrite Equation (7.12) as

\[-T_{0,j} + 4T_{0,j-1} - 3T_{0,j-2} = 0. \]  

(7.31)

Note that the same techniques apply for the $O(\epsilon_x)$ problem, so they will not be discussed. Derivation of the $O(\epsilon_x)$ discretized equations is left to the reader.

For every value of the index, $i$, this technique yields two matrix equation systems of equations, one matrix system of equations for the $O(1)$ problem and the other for the $O(\epsilon_x)$ problem. The first system is of the form $AT_0 = b$ where $A$ is a pentadiagonal matrix (due to the matching of derivative values at the fluid-porous medium interfaces) of coefficients that is dependent on the value of the index, $i$, $T_0$ is the $O(1)$ temperature vector in the $y$-direction, and $b$ is a vector of known coefficients from the solution of the momentum problem from Chapter 3 that depend on the values of all of the indices, $i$, and $j$.. The second system is of the form $AT_1 = b$ where $A$ is a pentadiagonal matrix of coefficients that is dependent on the value of the index, $i$, $T_1$ is the $O(\epsilon_x)$ temperature vector in the $y$-direction, and $b$ is a vector of known coefficients from the solution of the momentum problem from Chapter 3 and the calculated values of $T_0$. Note that the values that comprise the vector $b$ depend on the values of all of the indices, $i$, and $j$.

The matrix systems of equations generated using the above techniques can be directly solved using a pentadiagonal solver or using direct matrix inversion. Unlike the momentum solution which required an iterative approach, the right hand sides of the matrix equations at each order use all known values, and so the values for $T_0$
and $T_1$ are found explicitly. The code for solving for the temperatures can be found in the Appendix.

7.2 Results and Discussion

As alluded to in Chapter 5, the domain of interest is the middle 80% of the bearing length in the $x$-direction, together with the full range of values in the $y$-direction. We do this to present the smooth temperature values that are void of any numerical oscillation, which typically only spans three to four grid points in the $x$-direction. This oscillation comes from the fact that the numerical algorithm developed by Vijayaraghavan and Keith and extended in this work, does not feature a smooth pressure curve at the location where the hydrodynamic film region transitions from a cavitated region to a full film region.

The goal of presenting the data in the manner shown is two-fold. First, we elect to present the data in a parametric manner similar to how we presented the pressure curves. This lets the reader know exactly how each parameter affects the temperature field inside the bearing. Secondly, the temperatures are presented by showing the leading order solution, $T_0$, the first order correction, $\epsilon_x T_1$, and the combination, $T = T_0 + \epsilon_x T_1$. By using this approach, we see the separate temperature contributions from conduction and convection, and we see the total temperature field generated.
Before showing the data, a comment must be made on a limitation of using this approach. Using the straight-forward asymptotic expansion approach featured here, we have made an underlying assumption that $\epsilon x T_1 \ll T_0$. Thus, we are assuming at the onset that conduction dominates convection in this model. There are however circumstances that violate that assumption, including, but not limited to, high velocity flow and a small heat transfer coefficient, $h_I$. Though the limitation from high velocity flow is not discussed here (the velocities being used do not necessarily violate the assumption in question), using a small ($< 200 \frac{W}{m^2K}$) effective heat transfer coefficient (for explanation of what we call the 'effective heat transfer coefficient,' see Chapter 13) can lead to physically unrealistic temperatures (i.e. temperatures well below the ambient temperature). To model cases like these, either an alternate asymptotic expansion method or full numerical simulation must be employed.

First, we examine the leading order temperature, $T_0$, the first correction temperature, $\epsilon x T_1$, and the total temperature, $T$ for the baseline case (see Tables 3.1 and 3.2 for relevant values) at different effective heat transfer coefficients, $h_I$. By doing so, we can establish the interplay between the roles of the conduction and convection terms from the governing equation.

In Figure 7.1, we see that the leading order temperature, $T_0$, at the boundary, $y = \beta + h(x)$ is roughly 60$K$ higher than the ambient temperature, $T_a = 1000K$, and that the temperature is highest around the minimum film clearance and lowest around the maximum film clearance, both of which are expected results. Figure 7.2 shows
the influence of the first order correction temperature, $\epsilon_x T_1$. Here, we see that the convection terms lead to a maximum temperature decrease of about 13 K where the fluid is drawn into the reservoir and a maximum temperature increase of about 8 K where the fluid is drawn back into the film. Thus, the fluid circulation has a positive impact on the thermal performance of the bearing by drawing the fluid into the reservoir and cooling it via the effective heat transfer coefficient of the reservoir wall (see Chapter 13). Figure 7.3 then shows the total temperature inside the bearing, adding the contributions from the conduction terms and the convective terms.

Figures 7.4 - 7.14 show similar results, but we can see that as the effective heat transfer coefficient increases to a near infinite practical limit ($10000 \frac{W}{m^2K}$), conduction becomes more and more dominant, and the convective temperature corrections de-
Figure 7.2: Two-dimensional carpet plot of the first order correction temperature, $\epsilon_k T_1$, inside the bearing system for effective heat transfer coefficient, $h_f = 250 \frac{W}{m^2 K}$. $k_y^* = 1 \times 10^{-12} \text{ m}^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $\alpha^* = 1.27 \times 10^{-4} \text{ m}$, $U^* = 4.57 \text{ m/s}$. All other pertinent values are given in Tables 3.1 and 3.2.

crease substantially in magnitude, though not in shape. We also see that the overall temperature fields tend to the ambient temperature as the effective heat transfer coefficient increases. If we reverse the trend, and consider starting with an infinite heat transfer coefficient and work toward an effective heat transfer coefficient of $0 \frac{W}{m^2 K}$, we can see that the effects from the convective terms would increase substantially. This is one of the limits described above for this particular approach for modeling the temperatures inside the bearing. Thus, for the remainder of this work, we consider our effective heat transfer coefficient to be $1000 \frac{W}{m^2 K}$, a value that is physically realistic for the geometry we are considering (see Chapter 13).
Figure 7.3: Two-dimensional carpet plot of the total temperature, $T$, inside the bearing system for effective heat transfer coefficient, $h_I = 250 \frac{W}{m^2 K}$. $k_y^\ast = 1 \times 10^{-12}$ m$^2$, $(\beta - \alpha)^\ast = 2.54 \times 10^{-3}$ m, $\alpha^\ast = 1.27 \times 10^{-4}$ m, $U^\ast = 4.57$ m/s. All other pertinent values are given in Tables 3.1 and 3.2.

With our effective heat transfer coefficient established, $h_I = 1000 \frac{W}{m^2 K}$, we perform a parametric analysis to observe the overall temperatures inside the bearing as each parameter is varied.

Figure 7.16 shows how the temperature varies with reservoir depth, $\alpha$, at the boundary, $y = 0$. We see a few interesting phenomena occurring as $\alpha$ increases. First, we observe that the maximum temperature decreases at first, when increasing from $\alpha = 6.35 \times 10^{-5}$ m to $\alpha = 1.27 \times 10^{-4}$ m, and then increases with increasing $\alpha$. This initial decrease suggests that for even thinner values of $\alpha$, the maximum temperature would rise. At the same time, the general trend is that the minimum temperature decreases with increasing $\alpha$, denoting that the convection terms become more dominant with increasing $\alpha$. One may expect, however, that this trend will only
Figure 7.4: Two-dimensional carpet plot of the leading order temperature, $T_0$, inside the bearing system for effective heat transfer coefficient, $h_I = 500 \frac{W}{m^2K}$. $h_y^* = 1 \times 10^{-12}$ m$^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 3.1 and 3.2.

continue until a certain limiting value for $\alpha$ has been achieved whereby the reservoir becomes essentially infinitely deep and the effect of convection on the temperature will no longer be seen. We do not immediately discredit the fact that the temperature can be below the ambient temperature at $y = 0$ for a ‘deep’ reservoir, as such a case may reflect lower pressures generated in the hydrodynamic film region. This may yield a system where there is not a lot of heat generated inside the film coupled with a deep reservoir that acts as an insulator from the ambient. Such a system may produce temperatures lower than the ambient. However, were the temperature to be substantially lower than the ambient temperature, we may consider invalidating those results and turning to an alternative approach to solve for the temperature.
Figure 7.5: Two-dimensional carpet plot of the first order correction temperature, $\epsilon_x T_1$, inside the bearing system for effective heat transfer coefficient, $h_t = 500 \frac{W}{m^2K}$. $k_y^* = 1 \times 10^{-12} \text{ m}^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $\alpha^* = 1.27 \times 10^{-4} \text{ m}$, $U^* = 4.57 \text{ m/s}$. All other pertinent values are given in Tables 3.1 and 3.2.

Figure 7.17 shows how the temperature varies with reservoir depth, $\alpha$, at the boundary, $y = \beta + h(x)$. This case shows the same behavior as that of Figure 7.16 with slightly higher temperatures.

Figures 7.18 and 7.19 show how the total temperature, $T$, varies with the value of the permeability of the porous medium, $k_y$. Here, we observe a phenomenon similar to that demonstrated from Figures 7.16 and 7.17 - there is an optimal value that minimizes the maximum temperatures inside the bearing. Both figures, Figures 7.18 and 7.19, show that the temperature is high for permeability value $k_y = 1 \times 10^{-10} \text{ m}^2$, then decreases up until permeability $k_y = 1 \times 10^{-12} \text{ m}^2$, and then increases for permeability value $k_y = 1 \times 10^{-13} \text{ m}^2$. From Figures 5.5, 5.6, 5.13 and 5.14,
Figure 7.6: Two-dimensional carpet plot of the total temperature, $T$, inside the bearing system for effective heat transfer coefficient, $h_I = 500 \frac{W}{m^2K}$. $k_y^* = 1 \times 10^{-12} \text{m}^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{m}$, $\alpha^* = 1.27 \times 10^{-4} \text{m}$, $U^* = 4.57 \text{m/s}$. All other pertinent values are given in Tables 3.1 and 3.2.

we know that as the permeability of the porous medium decreases, the maximum pressure inside the film increases and the maximum pressure difference between the film and reservoir regions also increases. The higher pressures inside the film lead to higher heat generation inside the hydrodynamic film, but the larger pressure difference between the two regions leads to a higher rate of cooling of the fluid. However, for the small limiting permeability, $k_y = 1 \times 10^{-13} \text{m}^2$, the rate of cooling diminishes, and the temperatures increase. From this experiment, we conclude that there is indeed an optimal value for permeability as it pertains to the maximum temperatures generated inside the bearing, as too low or too high of a permeability leads to higher temperatures.
Figure 7.7: Two-dimensional carpet plot of the leading order temperature, $T_0$, inside the bearing system for effective heat transfer coefficient, $h_f = 750 \frac{W}{m^2K}$. $k_y^* = 1 \times 10^{-12}$ m$^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 3.1 and 3.2.

Figures 7.20 and 7.21 again make the case for optimal parameter values for the lowest peak temperature inside the bearing. We remember from Chapter 5, Figures 5.7 and 5.8 showed that too thick of a porous medium and too small of a porous medium leads to lower pressures than an ‘optimal’ value somewhere between the two. This ‘optimal’ porous medium depth value for creating the largest maximum pressure value inside the hydrodynamic film was somewhere around the value given for the benchmark case (see Tables 3.1 and 3.2). It was reasoned that too thin of a porous medium yields a system that essentially sees the entire depth of the bearing (all three regions) as the film depth, and thus low pressures were generated. Similarly, too thick of a porous medium yields a system that sees the film and a portion of the porous medium as the effective film depth, again, lowering pressures. However, a
value in the middle essentially yielded the thinnest effective film depth, and thus the highest pressures inside the film. Here, we see a similar effect. Too thin of a porous medium, though with corresponding low pressures and heat built in the film, yields a low convective cooling rate, as not enough pressure is built up to push the fluid through the porous medium at a high enough rate to cool the fluid. Similarly, a very thick porous medium offers a lot of flow resistance inside the porous medium, limiting the rate of cooling of the fluid. Thus, an optimal value between the two limiting cases must exist, and we see that in Figures 7.20 and 7.21, where a value around that of the benchmark case yields the lowest peak temperature inside the bearing.

Figures 7.22 and 7.23 show how varying the linear velocity, $U$, affects the
temperature profiles at the boundaries, \( y = 0 \) and \( y = \beta + h(x) \), respectively. It should be clear that, keeping every other notable parameter constant, increasing \( U \) should lead to higher maximum temperature values inside the bearing, and this is shown. Though a larger value of \( U \) will certainly increase the rate of convection inside the bearing, the additional heat generated by a larger value of \( U \) cannot be resolved fast enough by convection, which leads to higher temperatures. We see that for the two slower cases, a minimal amount of heat builds up inside the bearing (\(<10K\)). In these cases, the convection and conduction are essentially removing all of the heat generated by the operation of the bearing. Increasing \( U \) to \( U = 4.57 \) m/s leads to an increase of roughly 30\( K \) above ambient, with the effect of convection being more prominent (notice the dip in temperature from the leftmost endpoint of the domain,
Figure 7.10: Two-dimensional carpet plot of the leading order temperature, $T_0$, inside the bearing system for effective heat transfer coefficient, $h_f = 1000 \frac{W}{m^2K}$, $k_y^* = 1 \times 10^{-12} m^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} m$, $\alpha^* = 1.27 \times 10^{-4} m$, $U^* = 4.57 m/s$. All other pertinent values are given in Tables 3.1 and 3.2.

which corresponds to the maximum film clearance, to $x = 0.025 m$, which corresponds to the point where the fluid is exiting the film into the reservoir at its highest rate.

Then, increasing $U$ to $U = 9.14 m/s$, we see a substantial amount of heat generated (at least compared to the previous three cases). Though convection is surely at work here (again, see $x = 0.025m$), the heat generated by friction inside the film (130$K$ above ambient) far dominates the effects of convection. Please note that slightly more than 10% of the domain was removed from the right hand side of Figure 7.23, which makes the maximum temperature appear more skewed to the right of center than it actually is. This was done to eliminate the numerical anomaly around the point where the film transitions from a cavitated region to a full film region. Again, note that this oscillation did not filter through the results as the method used to calculate
the temperatures is not an iterative process.

We have shown that the geometry of the bearing lends itself to optimum parameter values that lead to the lowest maximum temperatures inside the bearing. For the baseline geometry and operational configuration described above, assuming conduction dominates convection, given an ambient temperature, $T_a$, and given an effective heat transfer coefficient, $h_I$, we have demonstrated how each parameter effects the thermal profile of the bearing. Such a system lends itself to optimum parameter values that lead to the lowest maximum temperatures inside the bearing. However, for different working fluids, different maximum and minimum film clearances, and different porous media, these optimum parameter values may change, and an anal-

Figure 7.11: Two-dimensional carpet plot of the first order correction temperature, $\epsilon_x T_1$, inside the bearing system for effective heat transfer coefficient, $h_I = 1000 \frac{W}{m^2K}$. $k_y = 1 \times 10^{-12} \text{ m}^2$, $(\beta - \alpha) = 2.54 \times 10^{-3} \text{ m}$, $\alpha^* = 1.27 \times 10^{-4} \text{ m}$, $U^* = 4.57 \text{ m/s}$. All other pertinent values are given in Tables 3.1 and 3.2.
Figure 7.12: Two-dimensional carpet plot of the total temperature, $T$, inside the bearing system for effective heat transfer coefficient, $h_I = 1000 \frac{W}{m^2 K}$, $k_y^* = 1 \times 10^{-12}$ m$^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 3.1 and 3.2.

Analysis for relating the optimum geometry values together is something that could be done in the future.

Also, we note that the results shown represent a qualitative look at a bearing geometry described in this work, and the focus should be on those results obtained, not the actual numerical values generated. What has been demonstrated here is that a straightforward asymptotic expansion method may be used to break the problem down into essentially a conduction problem and a convection problem, and the results can then be looked at separately to get an idea of how the motion of the fluid transfers the heat between the reservoir and hydrodynamic film regions and how each parameter affects the thermal profile of the bearing. For higher rates of convection and for lower effective heat transfer coefficients, the model described above may break
Figure 7.13: Two-dimensional carpet plot of the leading order temperature, $T_0$, inside the bearing system for effective heat transfer coefficient, $h_I = 10000 \frac{W}{m^2 K}$, $k_y^* = 1 \times 10^{-12} \text{ m}^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $\alpha^* = 1.27 \times 10^{-4} \text{ m}$, $U^* = 4.57 \text{ m/s}$. All other pertinent values are given in Tables 3.1 and 3.2.

down and give physically unrealistic solutions. For those cases, a different asymptotic expansion approach may be used or a full numerical simulation may be used to more accurately describe the temperature field inside the bearing. A numerical option to be considered for future work is where only the equation

$$\rho c_v \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) + p \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \left( \frac{\mu}{2} \right) \frac{\partial u}{\partial y}$$

is used to describe the temperatures inside each region. The above equation is essentially the byproduct of an order of magnitude analysis, eliminating all terms that would be considered of $O(\epsilon_x^2)$ or higher. Such an equation could be solved numerically using a block SOR iterative scheme. The obvious downside to using this method as opposed to the one described above that was used for this work is that it takes
Figure 7.14: Two-dimensional carpet plot of the first order correction temperature, $\epsilon x T_1$, inside the bearing system for effective heat transfer coefficient, $h_f = 10000 \, \text{W} / \text{m}^2 \text{K}$. $k^* = 1 \times 10^{-12} \, \text{m}^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \, \text{m}$, $\alpha^* = 1.27 \times 10^{-4} \, \text{m}$, $U^* = 4.57 \, \text{m/s}$. All other pertinent values are given in Tables 3.1 and 3.2.

much more computational time. The method described in this work does not need to be solved iteratively, whereas the work proposed above does. Such a numerical simulation would remove the restrictions placed by the straightforward asymptotic method. However, the ability to see the separate contributions from conduction and convection would be eliminated.
Figure 7.15: Two-dimensional carpet plot of the total temperature, \( T \), inside the bearing system for effective heat transfer coefficient, \( h_I = 1000 \text{ W/m}^2\text{K} \), \( k^*_y = 1 \times 10^{-12} \text{ m}^2 \), \((\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m} \), \( \alpha^* = 1.27 \times 10^{-4} \text{ m} \), \( U^* = 4.57 \text{ m/s} \). All other pertinent values are given in Tables 3.1 and 3.2.

Figure 7.16: Total temperature, \( T \), at the boundary, \( y = 0 \), for varying reservoir depths, \( \alpha \). \( k^*_y = 1 \times 10^{-12} \text{ m}^2 \), \((\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m} \), \( U^* = 4.57 \text{ m/s} \), \( h_I = 1000 \text{ W/m}^2\text{K} \), \( T_a = 1000\text{K} \). All other pertinent values are given in Tables 3.1 and 3.2.
Figure 7.17: Total temperature, $T$, at the boundary, $y = \beta + h(x)$, for varying reservoir depths, $\alpha$. $k_y^* = 1 \times 10^{-12} \text{ m}^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $U^* = 4.57 \text{ m/s}$, $h_I = 1000 \frac{W}{m^2K}$, $T_a = 1000K$. All other pertinent values are given in Tables 3.1 and 3.2.

Figure 7.18: Total temperature, $T$, at the boundary, $y = 0$, for varying permeability values, $k_y$. $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $\alpha^* = 1.27 \times 10^{-4} \text{ m}$, $U^* = 4.57 \text{ m/s}$, $h_I = 1000 \frac{W}{m^2K}$, $T_a = 1000K$. All other pertinent values are given in Tables 3.1 and 3.2.
Figure 7.19: Total temperature, $T$, at the boundary, $y = \beta + h(x)$, for varying permeability values, $k_y$. $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $h_I = 1000 \frac{W}{m^2 K}$, $T_a = 1000K$. All other pertinent values are given in Tables 3.1 and 3.2.

Figure 7.20: Total temperature, $T$, at the boundary, $y = 0$, for varying porous medium depths, $\beta - \alpha$. $k_y^* = 1 \times 10^{-12}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $h_I = 1000 \frac{W}{m^2 K}$, $T_a = 1000K$. All other pertinent values are given in Tables 3.1 and 3.2.
Temperature at $y = \beta + h(x)$ (K)

$\beta - \alpha = 6.35 \times 10^{-4}$ m

$\beta - \alpha = 1.27 \times 10^{-3}$ m

$\beta - \alpha = 2.54 \times 10^{-3}$ m

$\beta - \alpha = 5.08 \times 10^{-3}$ m

Figure 7.21: Total temperature, $T$, at the boundary, $y = \beta + h(x)$, for varying porous medium depths, $\beta - \alpha$. $k_y^* = 1 \times 10^{-12}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $h_I = 1000 \frac{W}{m^2K}$, $T_a = 1000K$. All other pertinent values are given in Tables 3.1 and 3.2.

Temperature at $y = 0$ (K)

$U = 1.14$ m/s

$U = 2.28$ m/s

$*U = 4.57$ m/s

$U = 9.14$ m/s

Figure 7.22: Total temperature, $T$, at the boundary, $y = 0$, for varying linear velocity values, $U$. $k_y^* = 1 \times 10^{-12}$ m$^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $h_I = 1000 \frac{W}{m^2K}$, $T_a = 1000K$. All other pertinent values are given in Tables 3.1 and 3.2.
Figure 7.23: Total temperature, $T$, at the boundary, $y = \beta + h(x)$, for varying linear velocity values, $U$. $k_y^* = 1 \times 10^{-12}$ m$^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $h_I = 1000 \frac{W}{m^2K}$, $T_a = 1000K$. All other pertinent values are given in Tables 3.1 and 3.2.
8.1 Geometry

The geometry considered here is the full three-dimensional model that takes into account a finite axial length.

![Diagram of a porous journal bearing with external reservoir](image)

**Figure 8.1: Porous Journal Bearing With External Reservoir**

If we then consider the thickness of the porous medium and the depth of the external reservoir to be much smaller than the radius of the bearing, we can unwrap the bearing and analyze it in a Cartesian system of coordinates. The result is shown in Figure 8.2.
The film, the porous medium, and the doughnut-shaped reservoir have been unwrapped. As a result, we will use a set of Cartesian-based simplified Navier-Stokes equations for the flow characterizing the film and the reservoir. The basic dimensions and variables characterizing the unwrapped geometry are given in Tables 8.1 and 8.2. We assume that the fluid is Newtonian, and the flow inside all three regions is laminar.
8.2 Development of the Model

To describe the flow inside the film and the reservoir, we start with the steady-state Navier-Stokes equations in three dimensions, neglecting body forces:

\[
\begin{align*}
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (8.1) \\
\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (8.2) \\
\frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\mu}{\rho} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (8.3)
\end{align*}
\]

In order to implement a comprehensive parametric study, Eqs. (8.1), (8.2), and (8.3) are non-dimensionalized by adopting the following relationship between the dimensional and non-dimensional variables:

\[
\begin{align*}
u &= UU^* \quad (8.4) \\
v &= \left( \frac{\alpha}{L_x} \right) UV^* \quad (8.5) \\
w &= \left( \frac{L_z}{L_x} \right) UW^* \quad (8.6) \\
x &= L_x X^* \quad (8.7) \\
y &= \alpha Y^* \quad (8.8) \\
z &= L_z Z^* \quad (8.9) \\
p &= \left( \frac{L_x \mu U}{\alpha^2} \right) P^*. \quad (8.10)
\end{align*}
\]
Then Equations (8.1) and (8.2) can be written in a dimensionless form as follows:

\[
\left( \frac{\alpha}{L_x} \right)^2 \left( \frac{\rho L_x U}{\mu} \right) \left( U^* \frac{\partial U^*}{\partial X^*} + V^* \frac{\partial U^*}{\partial Y^*} + W^* \frac{\partial U^*}{\partial Z^*} \right) = -\frac{\partial P^*}{\partial X^*} + \left( \frac{\alpha}{L_x} \right)^2 \frac{\partial^2 U^*}{\partial X^{*2}} + \frac{\partial^2 U^*}{\partial Y^{*2}} + \left( \frac{\alpha}{L_x} \right)^2 \left( \frac{L_x}{L_z} \right)^2 \frac{\partial^2 U^*}{\partial Z^{*2}}
\]

(8.11)

\[
\left( \frac{\alpha}{L_x} \right)^4 \left( \frac{\rho L_x U}{\mu} \right) \left( U^* \frac{\partial V^*}{\partial X^*} + V^* \frac{\partial V^*}{\partial Y^*} + W^* \frac{\partial V^*}{\partial Z^*} \right) = -\frac{\partial P^*}{\partial Y^*} + \left( \frac{\alpha}{L_x} \right)^4 \frac{\partial^2 V^*}{\partial X^{*2}} + \left( \frac{\alpha}{L_x} \right)^2 \frac{\partial^2 V^*}{\partial Y^{*2}} + \left( \frac{\alpha}{L_x} \right)^2 \left( \frac{\alpha}{L_z} \right)^2 \frac{\partial^2 V^*}{\partial Z^{*2}}
\]

(8.12)

\[
\left( \frac{\alpha}{L_x} \right)^2 \left( \frac{\rho L_x U}{\mu} \right) \left( U^* \frac{\partial W^*}{\partial X^*} + V^* \frac{\partial W^*}{\partial Y^*} + W^* \frac{\partial W^*}{\partial Z^*} \right) = -\left( \frac{L_x}{L_z} \right)^2 \frac{\partial P^*}{\partial Z^*} + \left( \frac{\alpha}{L_x} \right)^2 \frac{\partial^2 W^*}{\partial X^{*2}} + \frac{\partial^2 W^*}{\partial Y^{*2}} + \left( \frac{\alpha}{L_x} \right)^2 \left( \frac{L_x}{L_z} \right)^2 \frac{\partial^2 W^*}{\partial Z^{*2}}.
\]

(8.13)

Note that \( \frac{\alpha}{L_x} \) is a geometric aspect ratio similarity parameter where \( \alpha \ll L_x \), while \( \frac{\rho L_x U}{\mu} \) is the Reynolds number and thus a dynamic similarity parameter. If one then defines \( \epsilon_x = \frac{\alpha}{L_x} \) as a small parameter (\( \ll 1 \)) and the Reynolds number as \( Re_x = \frac{\rho L_x U}{\mu} \),
with \(Re_x = O(1)\), one can rewrite Eqs. (8.11), (8.12), and (8.13) as

\[
\varepsilon_x^2 \left( \frac{\rho L_x U}{\mu} \right) \left( U^* \frac{\partial U^*}{\partial X^*} + V^* \frac{\partial U^*}{\partial Y^*} + W^* \frac{\partial U^*}{\partial Z^*} \right) = -\frac{\partial P^*}{\partial X^*} + \\
+ \varepsilon_x^2 \frac{\partial^2 U^*}{\partial X^*^2} + \frac{\partial^2 U^*}{\partial Y^*^2} + \\
+ \varepsilon_x^2 \left( \frac{L_x}{L_z} \right)^2 \frac{\partial^2 U^*}{\partial Z^*^2} \quad (8.14)
\]

\[
\varepsilon_x^4 \left( \frac{\rho L_x U}{\mu} \right) \left( U^* \frac{\partial V^*}{\partial X^*} + V^* \frac{\partial V^*}{\partial Y^*} + W^* \frac{\partial V^*}{\partial Z^*} \right) = -\frac{\partial P^*}{\partial Y^*} + \\
+ \varepsilon_x^4 \frac{\partial^2 V^*}{\partial X^*^2} + \\
+ \varepsilon_x^2 \frac{\partial^2 V^*}{\partial Y^*^2} + \\
+ \varepsilon_x^2 \left( \frac{\alpha}{L_z} \right)^2 \frac{\partial^2 V^*}{\partial Z^*^2} \quad (8.15)
\]

\[
\varepsilon_x^2 \left( \frac{\rho L_x U}{\mu} \right) \left( U^* \frac{\partial W^*}{\partial X^*} + V^* \frac{\partial W^*}{\partial Y^*} + W^* \frac{\partial W^*}{\partial Z^*} \right) = -\left( \frac{L_x}{L_z} \right)^2 \frac{\partial P^*}{\partial Z^*} + \\
+ \varepsilon_x^2 \frac{\partial^2 W^*}{\partial X^*^2} + \frac{\partial^2 W^*}{\partial Y^*^2} + \\
+ \varepsilon_x^2 \left( \frac{L_x}{L_z} \right)^2 \frac{\partial^2 W^*}{\partial Z^*^2}. \quad (8.16)
\]

Also, as has been discussed earlier, we are using the Brinkman-Extended Darcy Model to describe the velocity components of the fluid within the porous medium.

\[
\frac{\partial \tilde{p}}{\partial x} = \tilde{\mu} \left( \frac{\partial^2 \tilde{u}}{\partial x^2} + \frac{\partial^2 \tilde{u}}{\partial y^2} + \frac{\partial^2 \tilde{u}}{\partial z^2} \right) - \frac{\mu}{k_x} \tilde{u} \quad (8.17)
\]

\[
\frac{\partial \tilde{p}}{\partial y} = \tilde{\mu} \left( \frac{\partial^2 \tilde{v}}{\partial x^2} + \frac{\partial^2 \tilde{v}}{\partial y^2} + \frac{\partial^2 \tilde{v}}{\partial z^2} \right) - \frac{\mu}{k_y} \tilde{v} \quad (8.18)
\]

\[
\frac{\partial \tilde{p}}{\partial z} = \tilde{\mu} \left( \frac{\partial^2 \tilde{w}}{\partial x^2} + \frac{\partial^2 \tilde{w}}{\partial y^2} + \frac{\partial^2 \tilde{w}}{\partial z^2} \right) - \frac{\mu}{k_z} \tilde{w} \quad (8.19)
\]
Nondimensionalizing Equations (8.17) - (8.19), we have

\[
\frac{\partial \tilde{P}^*}{\partial X^*} = \left(\frac{\tilde{\mu}}{\mu}\right) \left[\varepsilon_x^2 \frac{\partial^2 \tilde{U}^*}{\partial X^2} + \frac{\partial^2 \tilde{U}^*}{\partial Y^2} + \varepsilon_x^2 \left(\frac{L_x}{L_z}\right)^2 \frac{\partial^2 \tilde{U}^*}{\partial Z^2}\right] - \left(\frac{\alpha^2}{k_x}\right) \tilde{U}^* \tag{8.20}
\]

\[
\frac{\partial \tilde{P}^*}{\partial Y^*} = \varepsilon_x^2 \left(\frac{\tilde{\mu}}{\mu}\right) \left[\varepsilon_x^2 \frac{\partial^2 \tilde{V}^*}{\partial X^2} + \frac{\partial^2 \tilde{V}^*}{\partial Y^2} + \varepsilon_x^2 \left(\frac{L_x}{L_z}\right)^2 \frac{\partial^2 \tilde{V}^*}{\partial Z^2}\right] - \varepsilon_x^2 \left(\frac{\alpha^2}{k_y}\right) \tilde{V}^* \tag{8.21}
\]

\[
\frac{\partial \tilde{P}^*}{\partial Z^*} = \left(\frac{L_z}{L_x}\right)^2 \left(\frac{\tilde{\mu}}{\mu}\right) \left[\varepsilon_x^2 \frac{\partial^2 \tilde{W}^*}{\partial X^2} + \frac{\partial^2 \tilde{W}^*}{\partial Y^2} + \varepsilon_x^2 \left(\frac{L_x}{L_z}\right)^2 \frac{\partial^2 \tilde{W}^*}{\partial Z^2}\right] - \left(\frac{L_z}{L_x}\right)^2 \left(\frac{\alpha^2}{k_z}\right) \tilde{W}^*. \tag{8.22}
\]

Provided that \(\frac{\tilde{\mu}}{\mu} = O(1)\) and \(L_x\) and \(L_z\) do not differ by several orders of magnitude so as to negate the \(\varepsilon_x^2\), we may write the \(O(1)\) versions of Equations (8.20) - (8.22):

\[
\frac{\partial \tilde{P}^*}{\partial X^*} = \left(\frac{\tilde{\mu}}{\mu}\right) \frac{\partial^2 \tilde{U}^*}{\partial Y^2} - \left(\frac{\alpha^2}{k_x}\right) \tilde{U}^* \tag{8.23}
\]

\[
\frac{\partial \tilde{P}^*}{\partial Y^*} = - \left(\frac{\alpha}{L_x}\right)^2 \left(\frac{\alpha^2}{k_y}\right) \tilde{V}^* \tag{8.24}
\]

\[
\frac{\partial \tilde{P}^*}{\partial Z^*} = \left(\frac{L_z}{L_x}\right)^2 \left(\frac{\tilde{\mu}}{\mu}\right) \frac{\partial^2 \tilde{W}^*}{\partial Y^2} - \left(\frac{L_z}{L_x}\right)^2 \left(\frac{\alpha^2}{k_z}\right) \tilde{W}^*. \tag{8.25}
\]

Because the size of \(\left(\frac{\alpha^2}{k_y}\right)\) is unknown and can vary by several orders of magnitude, we retain the term \(-\left(\frac{\alpha}{L_x}\right)^2 \left(\frac{\alpha^2}{k_y}\right) \tilde{V}^*\) on the right hand side of Equation (8.24). We then re-dimensionalize Equations (8.23) - (8.25) to yield the governing \(O(1)\) equations inside the porous medium:

\[
\frac{\partial \tilde{p}}{\partial x} = \tilde{\mu} \frac{\partial^2 \tilde{u}}{\partial y^2} - \frac{\mu}{k_x} \tilde{u} \tag{8.26}
\]

\[
\frac{\partial \tilde{p}}{\partial y} = -\frac{\mu}{k_y} \tilde{v} \tag{8.27}
\]

\[
\frac{\partial \tilde{p}}{\partial z} = \tilde{\mu} \frac{\partial^2 \tilde{w}}{\partial y^2} - \frac{\mu}{k_z} \tilde{w}. \tag{8.28}
\]
Thus, the $O(1)$ governing equations we are considering are

Region I (Reservoir):

\[
\frac{\partial p_I}{\partial x} = \mu \frac{\partial^2 u_I}{\partial y^2} \quad (8.29)
\]

\[
\frac{\partial p_I}{\partial z} = \mu \frac{\partial^2 w_I}{\partial y^2} \quad (8.30)
\]

Region II (Porous Medium):

\[
\frac{\partial \tilde{p}}{\partial x} = \tilde{\mu} \frac{\partial^2 \tilde{u}}{\partial y^2} - \frac{\mu}{k_x} \tilde{u} \quad (8.31)
\]

\[
\frac{\partial \tilde{p}}{\partial y} = -\frac{\mu}{k_y} \tilde{v} \quad (8.32)
\]

\[
\frac{\partial \tilde{p}}{\partial z} = \tilde{\mu} \frac{\partial^2 \tilde{w}}{\partial y^2} - \frac{\mu}{k_z} \tilde{w} \quad (8.33)
\]

Region III (Hydrodynamic Film):

\[
\frac{\partial p_{III}}{\partial x} = \mu \frac{\partial^2 u_{III}}{\partial y^2} \quad (8.34)
\]

\[
\frac{\partial p_{III}}{\partial z} = \mu \frac{\partial^2 w_{III}}{\partial y^2}, \quad (8.35)
\]

where $\tilde{u}$, $\tilde{v}$ and $\tilde{w}$ represent the $x$-component, the $y$-component, and the $z$-component of the velocity of the fluid inside the porous medium, respectively.

Similar to the analysis done for the slider bearing, we note that, in the fluid regions (I, III), the pressure does not vary across the thickness of the fluid regions for this first order analysis. However this fact does not factor into the construction of the governing Reynolds equation; it just serves as a reminder that the pressure calculated at the top of the porous medium is valid for the entire lubricating film region and the pressure calculated at the bottom of the porous medium is valid for the entire fluid reservoir region.
In order to construct the Reynolds equations for the (Region I + Region III) system, mass continuity equations have to be satisfied in every region. This work considers using the compressible form of the continuity equations to allow for a realistic cavitation model formulation. Though the governing equations described above are steady state, the continuity equations are cast in their transient form solely for the purpose of implementing a time dependent numerical integration technique which will be discussed later in Chapter 10. Below are the transient form of the continuity equations:

\[
\frac{\partial \rho_I}{\partial t} + \frac{\partial (\rho_I u_I)}{\partial x} + \frac{\partial (\rho_I v_I)}{\partial y} + \frac{\partial (\rho_I w_I)}{\partial z} = 0 \quad \text{(8.36)}
\]

\[
\frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial (\tilde{\rho} u)}{\partial x} + \frac{\partial (\tilde{\rho} v)}{\partial y} + \frac{\partial (\tilde{\rho} w)}{\partial z} = 0 \quad \text{(8.37)}
\]

\[
\frac{\partial \rho_{III}}{\partial t} + \frac{\partial (\rho_{III} u_{III})}{\partial x} + \frac{\partial (\rho_{III} v_{III})}{\partial y} + \frac{\partial (\rho_{III} w_{III})}{\partial z} = 0. \quad \text{(8.38)}
\]

The hydrodynamic fluid film height function, \( h(x) \), defined as the clearance between the stationary porous medium and the rotating journal wall, is given by

\[
h(x) = C \left[ 1 + \epsilon \cos \left( \frac{2\pi x}{L_x} \right) \right], \quad \text{(8.39)}
\]

where the average film clearance, \( C \), is given by the expression

\[
C = \left( \frac{h_{\text{min}} + h_{\text{max}}}{2} \right) \quad \text{(8.40)}
\]

and the eccentricity, \( \epsilon \), may be expressed as

\[
\epsilon = \left( \frac{h_{\text{max}} - h_{\text{min}}}{h_{\text{min}} + h_{\text{max}}} \right). \quad \text{(8.41)}
\]

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Please note that this value of $\epsilon$, which is a measure of how far the centers of the journal and bearing housing differ, is not related to the value, $\epsilon_x$, that defined the geometric aspect ratio similarity parameter $\alpha_L/\epsilon_x$.

Considering boundary conditions, we note that there is no slip at the stationary reservoir wall interface. Also, observe that by non-dimensionalizing the hydrodynamic fluid film height function as $h = \alpha H$, and relating $U$ and $V$ by the expression $V = U \frac{dh}{dx}$, we have that $V = \epsilon_x U \frac{dH}{dx}$. Since this portion of the work only considers the $O(1)$ governing equations, it follows that we may just consider the $y$-direction of the fluid velocity at the journal wall to be 0. This then forces the $x$-direction velocity of the journal wall to equal the rotational velocity of the journal, $U$. Also, for an axially-aligned bearing, which we are studying here, $W$, the the $z$-direction velocity of the journal wall equals 0. Thus, the boundary conditions for the reservoir and the film are

$$u_I(x, y = 0, z) = 0$$

(8.42)

$$v_I(x, y = 0, z) = 0$$

(8.43)

$$w_I(x, y = 0, z) = 0$$

(8.44)

$$u_{III}(x, y = \beta + h(x), z) = U$$

(8.45)

$$v_{III}(x, y = \beta + h(x), z) = 0$$

(8.46)

$$w_{III}(x, y = \beta + h(x), z) = 0.$$  

(8.47)

Pressure is continuous at the reservoir-porous medium interface, between regions I and II, as well as at the porous medium-film interface, between regions II and III, so
that

\[ p_I(x, y = \alpha, z) = \tilde{p}(x, y = \alpha, z) \]  
(8.48)

\[ \tilde{p}(x, y = \beta, z) = p_{III}(x, y = \beta, z). \]  
(8.49)

The viscous shear is continuous at the reservoir-porous medium interface, between regions I and II, as well as at the porous medium-film interface, between regions II and III, so we have

\[ \mu \frac{\partial u}{\partial y} \bigg|_{y=\alpha} = \tilde{\mu} \frac{\partial \tilde{u}}{\partial y} \bigg|_{y=\alpha} \]  
(8.50)

\[ \tilde{\mu} \frac{\partial \tilde{u}}{\partial y} \bigg|_{y=\beta} = \mu \frac{\partial u_{III}}{\partial y} \bigg|_{y=\beta} \]  
(8.51)

\[ \mu \frac{\partial w}{\partial y} \bigg|_{y=\alpha} = \tilde{\mu} \frac{\partial \tilde{w}}{\partial y} \bigg|_{y=\alpha} \]  
(8.52)

\[ \tilde{\mu} \frac{\partial \tilde{w}}{\partial y} \bigg|_{y=\beta} = \mu \frac{\partial w_{III}}{\partial y} \bigg|_{y=\beta}. \]  
(8.53)

Finally, the velocities and densities are continuous at the interfaces,

\[ u_I(x, y = \alpha, z) = \tilde{u}(x, y = \alpha, z) \]  
(8.54)

\[ v_I(x, y = \alpha, z) = \tilde{v}(x, y = \alpha, z) \]  
(8.55)

\[ w_I(x, y = \alpha, z) = \tilde{w}(x, y = \alpha, z) \]  
(8.56)

\[ \tilde{u}(x, y = \beta, z) = u_{III}(x, y = \beta, z) \]  
(8.57)

\[ \tilde{v}(x, y = \beta, z) = v_{III}(x, y = \beta, z) \]  
(8.58)

\[ \tilde{w}(x, y = \beta, z) = w_{III}(x, y = \beta, z). \]  
(8.59)

\[ \rho_I(x, y = \alpha, z) = \tilde{\rho}(x, y = \alpha, z) \]  
(8.60)

\[ \tilde{\rho}(x, y = \beta, z) = \rho_{III}(x, y = \beta, z). \]  
(8.61)
Define $\gamma_x = \frac{\mu}{\mu k_x}$ and $\gamma_z = \frac{\mu}{\mu k_z}$. Then, we can solve for the $x$-direction and $z$-direction velocities inside the porous medium from Equations (8.31) and (8.33)

\[
\tilde{u} = A_1(x, z) e^{\sqrt{\gamma_x} y} + A_2(x, z) e^{-\sqrt{\gamma_x} y} - \frac{k_x}{\mu} \frac{\partial \tilde{p}}{\partial x} \quad (8.62)
\]

\[
\tilde{w} = A_3(x, z) e^{\sqrt{\gamma_z} y} + A_4(x, z) e^{-\sqrt{\gamma_z} y} - \frac{k_z}{\mu} \frac{\partial \tilde{p}}{\partial z} \quad (8.63)
\]

Observe that $A_1(x, z)$, $A_2(x, z)$, $A_3(x, z)$, and $A_4(x, z)$ are functions in $x$ and $z$ but for the remainder of this chapter they will just be referred to as $A_1$, $A_2$, $A_3$, and $A_4$.

Also, observe that if $A_1 = A_2 = A_3 = A_4 = 0$, we recover the Darcy model governing the flow inside the porous medium.

Integrating Equation (8.29) with respect to $y$ and applying boundary conditions (8.42) and (8.54), one finds

\[
u_I = \left[ \frac{y(y - \alpha)}{2\mu} \right] \frac{\partial p_I}{\partial x} + \left( \frac{y}{\alpha} \right) \left( A_1 e^{\sqrt{\gamma_x} \alpha} + A_2 e^{-\sqrt{\gamma_x} \alpha} - \frac{k_x}{\mu} \frac{\partial \tilde{p}}{\partial x} \bigg|_{y=\alpha} \right). \tag{8.64}
\]

Similarly, integrating Equation (8.30) with respect to $y$ and applying boundary conditions (8.44) and (8.56), one finds

\[
\nu_I = \left[ \frac{y(y - \alpha)}{2\mu} \right] \frac{\partial p_I}{\partial z} + \left( \frac{y}{\alpha} \right) \left( A_3 e^{\sqrt{\gamma_z} \alpha} + A_4 e^{-\sqrt{\gamma_z} \alpha} - \frac{k_z}{\mu} \frac{\partial \tilde{p}}{\partial z} \bigg|_{y=\alpha} \right). \tag{8.65}
\]

Integrating Equation (8.34) with respect to $y$ and applying boundary conditions (8.45) and (8.57), one finds

\[
u_{III} = \left\{ \frac{y - (\beta + h)}{2\mu} \right\} \frac{\partial p_{III}}{\partial x} - \left[ \frac{y - (\beta + h)}{h} \right] \left( A_1 e^{\sqrt{\gamma_x} \beta} + A_2 e^{-\sqrt{\gamma_x} \beta} - \frac{k_x}{\mu} \frac{\partial \tilde{p}}{\partial x} \bigg|_{y=\beta} \right) + \left( \frac{y - \beta}{h} \right) U. \tag{8.66}
\]
Similarly, integrating Equation (8.35) with respect to $y$ and applying boundary conditions (8.47) and (8.59), one finds

$$w_{III} = \left\{ \frac{[y - (\beta + h)](y - \beta)}{2\mu} \right\} \frac{\partial p_{III}}{\partial z} - \left[ \frac{y - (\beta + h)}{h} \right] \left( A_3 e^{\sqrt{x}y\beta} + A_4 e^{-\sqrt{x}y\beta} - \frac{k_2 \partial \tilde{p}}{\mu \partial z} \right) \bigg|_{y=\beta}. \quad (8.67)$$

We now use Equations (8.50) - (8.53) to solve for $A_1$, $A_2$, $A_3$, and $A_4$. This yields a system of four equations and four unknowns, which is solved algebraically to get

$$A_1 = \frac{CH - BI}{AH - BG} \quad (8.68)$$

$$A_2 = \frac{AI - CG}{AH - BG} \quad (8.69)$$

$$A_3 = \frac{FK - EL}{DK - EJ} \quad (8.70)$$

$$A_4 = \frac{DL - FJ}{DK - EJ} \quad (8.71)$$
where A - L are defined below:

\[
\begin{align*}
A &= 2e^{\sqrt{\gamma x} \alpha} (\mu - \tilde{\mu} \alpha \sqrt{\gamma x}) \\
B &= 2e^{-\sqrt{\gamma x} \alpha} (\mu + \tilde{\mu} \alpha \sqrt{\gamma x}) \\
C &= 2k_x \frac{\partial \tilde{p}}{\partial x} - \alpha^2 \frac{\partial p_I}{\partial x} \\
D &= 2e^{\sqrt{\gamma z} \alpha} (\mu - \tilde{\mu} \alpha \sqrt{\gamma z}) \\
E &= 2e^{-\sqrt{\gamma z} \alpha} (\mu + \tilde{\mu} \alpha \sqrt{\gamma z}) \\
F &= 2k_z \frac{\partial \tilde{p}}{\partial z} - \alpha^2 \frac{\partial p_I}{\partial z} \\
G &= 2e^{\sqrt{\gamma x} \beta} (\mu + \tilde{\mu} h \sqrt{\gamma x}) \\
H &= 2e^{-\sqrt{\gamma x} \beta} (\mu \tilde{\mu} h \sqrt{\gamma x}) \\
I &= 2k_x \frac{\partial \tilde{p}}{\partial x} + 2\mu U - h^2 \frac{\partial p_{III}}{\partial x} \\
J &= 2e^{\sqrt{\gamma z} \beta} (\mu + \tilde{\mu} h \sqrt{\gamma z}) \\
K &= 2e^{-\sqrt{\gamma z} \beta} (\mu - \tilde{\mu} h \sqrt{\gamma z}) \\
L &= 2k_z \frac{\partial \tilde{p}}{\partial z} - h^2 \frac{\partial p_{III}}{\partial z} .
\end{align*}
\]

8.3 Assumptions and Simplifications

As it stands, the complexity of \( A_1, A_2, A_3, \) and \( A_4 \) make the task of formulating a neat governing system rather difficult. However, we can take advantage of the differing orders of magnitude inside each of the terms to simplify them. First, we notice that
since $\beta - \alpha > 0$, $\alpha - \beta < 0$, $\sqrt{k_x} \ll 1$, and $\sqrt{k_z} \ll 1$, we may assume that

$$e^{\sqrt{k_z}(\beta - \alpha)} \gg 1 \gg e^{\sqrt{k_z}(\alpha - \beta)} \gg e^{\sqrt{k_z}[2(\alpha - \beta)]}$$

$$e^{\sqrt{k_z}(\beta - \alpha)} \gg 1 \gg e^{\sqrt{k_z}(\alpha - \beta)} \gg e^{\sqrt{k_z}[2(\alpha - \beta)]}.$$

This allows us to simplify $A1$, $A2$, $A3$, and $A4$ as

$$A1 = \left[ \frac{(2k_x \frac{\partial \tilde{p}}{\partial x} + 2\mu U - h^2 \frac{\partial \mu}{\partial x})}{2(\mu + \tilde{\mu} h \sqrt{\gamma_x})} \right] e^{\sqrt{k_z}\beta} - \left[ \frac{(2k_x \frac{\partial \tilde{p}}{\partial x} - \alpha^2 \frac{\partial \mu}{\partial x})}{2(\mu + \tilde{\mu} h \sqrt{\gamma_x})} \right] (\mu - \tilde{\mu} h \sqrt{\gamma_x}) e^{\sqrt{k_z}(\alpha - 2\beta)}$$

$$A2 = \left[ \frac{(2k_x \frac{\partial \tilde{p}}{\partial x} - \alpha^2 \frac{\partial \mu}{\partial x})}{2(\mu + \tilde{\mu} h \sqrt{\gamma_x})} \right] e^{\sqrt{k_z}\alpha} - \left[ \frac{(2k_x \frac{\partial \tilde{p}}{\partial x} + 2\mu U - h^2 \frac{\partial \mu}{\partial x})}{2(\mu + \tilde{\mu} h \sqrt{\gamma_x})} \right] (\mu - \tilde{\mu} h \sqrt{\gamma_x}) e^{\sqrt{k_z}(2\alpha - \beta)}$$

$$A3 = \left[ \frac{(2k_x \frac{\partial \tilde{p}}{\partial x} - \alpha^2 \frac{\partial \mu}{\partial x})}{2(\mu + \tilde{\mu} h \sqrt{\gamma_x})} \right] e^{-\sqrt{k_z}\beta} - \left[ \frac{(2k_x \frac{\partial \tilde{p}}{\partial x} - \alpha^2 \frac{\partial \mu}{\partial x})}{2(\mu + \tilde{\mu} h \sqrt{\gamma_x})} \right] (\mu - \tilde{\mu} h \sqrt{\gamma_x}) e^{-\sqrt{k_z}(\alpha - 2\beta)}$$

$$A4 = \left[ \frac{(2k_x \frac{\partial \tilde{p}}{\partial x} - \alpha^2 \frac{\partial \mu}{\partial x})}{2(\mu + \tilde{\mu} h \sqrt{\gamma_x})} \right] e^{\sqrt{k_z}\alpha} - \left[ \frac{(2k_x \frac{\partial \tilde{p}}{\partial x} - \alpha^2 \frac{\partial \mu}{\partial x})}{2(\mu + \tilde{\mu} h \sqrt{\gamma_x})} \right] (\mu - \tilde{\mu} h \sqrt{\gamma_x}) e^{\sqrt{k_z}(2\alpha - \beta)}.$$

Substituting these values for $A1$, $A2$, $A3$, and $A4$ into Equations (8.62) and (8.63), we have

$$\tilde{u} = c_1 e^{\sqrt{k_z}(y - \beta)} + c_2 e^{\sqrt{k_z}(y + \alpha - 2\beta)} + c_3 e^{\sqrt{k_z}(\alpha - y)} + c_4 e^{\sqrt{k_z}(-y + 2\alpha - \beta)} \quad (8.72)$$

$$\tilde{w} = d_1 e^{\sqrt{k_z}(y - \beta)} + d_2 e^{\sqrt{k_z}(y + \alpha - 2\beta)} + d_3 e^{\sqrt{k_z}(\alpha - y)} + d_4 e^{\sqrt{k_z}(-y + 2\alpha - \beta)}, \quad (8.73)$$

where $c_1, c_2, c_3, c_4, d_1, d_2, d_3$, and $d_4$ are constants that are inconsequential for this part of the discussion.

Note that if $y = \alpha$, we would have

$$\tilde{u} = c_1 e^{\sqrt{k_z}(\alpha - \beta)} + c_2 e^{\sqrt{k_z}[2(\alpha - \beta)]} + c_3 e^{\sqrt{k_z}(0)} + c_4 e^{\sqrt{k_z}(\alpha - \beta)} \quad (8.74)$$

$$\tilde{w} = d_1 e^{\sqrt{k_z}(\alpha - \beta)} + d_2 e^{\sqrt{k_z}[2(\alpha - \beta)]} + d_3 e^{\sqrt{k_z}(0)} + d_4 e^{\sqrt{k_z}(\alpha - \beta)}, \quad (8.75)$$
and if \( y = \beta \), we would have

\[
\tilde{u} = c_1 e^{\sqrt{\tau_x} (0)} + c_2 e^{\sqrt{\tau_x} (\alpha - \beta)} + c_3 e^{\sqrt{\tau_x} (\alpha - \beta)} + c_4 e^{\sqrt{\tau_x} [2(\alpha - \beta)]} \quad (8.76)
\]

\[
\tilde{w} = d_1 e^{\sqrt{\tau_z} (0)} + d_2 e^{\sqrt{\tau_z} (\alpha - \beta)} + d_3 e^{\sqrt{\tau_z} (\alpha - \beta)} + d_4 e^{\sqrt{\tau_z} [2(\alpha - \beta)]}. \quad (8.77)
\]

Observe that in Equations (8.74) and (8.75), the terms that feature

\[
e^{\sqrt{\gamma_x} x (\alpha - \beta)}, \quad e^{\sqrt{\gamma_x} x [2(\alpha - \beta)]}, \quad e^{\sqrt{\gamma_x} x (\alpha - \beta)}, \quad e^{\sqrt{\gamma_x} x [2(\alpha - \beta)]}, \quad e^{\sqrt{\gamma_z} y (\alpha - \beta)}, \quad e^{\sqrt{\gamma_z} y (\alpha - \beta)}, \quad e^{\sqrt{\gamma_z} z (\alpha - \beta)}, \quad e^{\sqrt{\gamma_z} z [2(\alpha - \beta)]},
\]

and

\[
e^{\sqrt{\gamma_z} z (\alpha - \beta)}, \quad e^{\sqrt{\gamma_z} z [2(\alpha - \beta)]}
\]

can be neglected, whereas in Equations (8.76) and (8.77) the terms that feature

\[
e^{\sqrt{\gamma_x} x (\alpha - \beta)}, \quad e^{\sqrt{\gamma_x} x (\alpha - \beta)}, \quad e^{\sqrt{\gamma_x} x [2(\alpha - \beta)]}, \quad e^{\sqrt{\gamma_x} x (\alpha - \beta)}, \quad e^{\sqrt{\gamma_x} x [2(\alpha - \beta)]}, \quad e^{\sqrt{\gamma_z} y (\alpha - \beta)}, \quad e^{\sqrt{\gamma_z} y (\alpha - \beta)}, \quad e^{\sqrt{\gamma_z} z (\alpha - \beta)}, \quad e^{\sqrt{\gamma_z} z [2(\alpha - \beta)]},
\]

and

\[
e^{\sqrt{\gamma_z} z (\alpha - \beta)}, \quad e^{\sqrt{\gamma_z} z [2(\alpha - \beta)]}
\]

can also be neglected. Since the only terms that ever contribute inside the domain, \( \alpha \leq y \leq \beta \), are the first and third terms in Equations (8.72) and (8.73), we may then write that

\[
\tilde{u} = \left[ \frac{2k_x \frac{\partial \tilde{p}}{\partial x} + 2\mu U - h^2 \frac{\partial p_{III}}{\partial x}}{2 (\mu + \tilde{\mu} \sqrt{\gamma_x})} \right] e^{\sqrt{\tau_x} (y - \beta)} + \left[ \frac{2k_x \frac{\partial \tilde{p}}{\partial x} - \alpha^2 \frac{\partial p_{I}}{\partial x}}{2 (\mu + \tilde{\mu} \alpha \sqrt{\gamma_x})} \right] e^{\sqrt{\tau_x} (\alpha - y)} - \frac{k_x}{\mu} \frac{\partial \tilde{p}}{\partial x} \quad (8.78)
\]

\[
\tilde{w} = \left[ \frac{2k_z \frac{\partial \tilde{p}}{\partial z} - h^2 \frac{\partial p_{III}}{\partial z}}{2 (\mu + \tilde{\mu} h \sqrt{\gamma_z})} \right] e^{\sqrt{\tau_z} (y - \beta)} + \left[ \frac{2k_z \frac{\partial \tilde{p}}{\partial z} - \alpha^2 \frac{\partial p_{I}}{\partial z}}{2 (\mu + \tilde{\mu} \alpha \sqrt{\gamma_z})} \right] e^{\sqrt{\tau_z} (\alpha - y)} - \frac{k_z}{\mu} \frac{\partial \tilde{p}}{\partial z}. \quad (8.79)
\]
We take a similar approach to simplify $u_I$, $u_I$, $u_{III}$, and $w_{III}$:

$u_I = \left[ \frac{y(y - \alpha)}{2 \mu} \right] \frac{\partial p_I}{\partial x} + \left( \frac{y}{\alpha} \right) \left\{ \frac{2k_x \frac{\partial \tilde{p}}{\partial x} - \alpha^2 \frac{\partial p_I}{\partial x}}{2 \left( \mu + \tilde{\mu} \alpha \sqrt{\gamma_x} \right)} \right\} - \left. \frac{k_x \frac{\partial \tilde{p}}{\partial x}}{\mu} \right|_{y=\alpha}$ \hspace{1cm} (8.80)

$w_I = \left[ \frac{y(y - \alpha)}{2 \mu} \right] \frac{\partial p_I}{\partial z} + \left( \frac{y}{\alpha} \right) \left\{ \frac{2k_z \frac{\partial \tilde{p}}{\partial z} - \alpha^2 \frac{\partial p_I}{\partial z}}{2 \left( \mu + \tilde{\mu} \alpha \sqrt{\gamma_z} \right)} \right\} - \left. \frac{k_z \frac{\partial \tilde{p}}{\partial z}}{\mu} \right|_{y=\alpha}$ \hspace{1cm} (8.81)

$u_{III} = \left\{ \frac{(y - \beta) [y - (\beta + h)]}{2 \mu} \right\} \frac{\partial p_{III}}{\partial x} - \left. \left[ \frac{y - (\beta + h)}{h} \right] \left\{ \frac{2k_x \frac{\partial \tilde{p}}{\partial x} - h^2 \frac{\partial p_{III}}{\partial x}}{\frac{2k_x \frac{\partial \tilde{p}}{\partial x} - h^2 \frac{\partial p_{III}}{\partial x}}{2 \left( \mu + \tilde{\mu} h \sqrt{\gamma_x} \right)} \right\} - \left. \frac{k_x \frac{\partial \tilde{p}}{\partial x}}{\mu} \right|_{y=\beta} \right\} + \left( \frac{y - \beta}{h} \right) U \hspace{1cm} (8.82)$

$w_{III} = \left\{ \frac{(y - \beta) [y - (\beta + h)]}{2 \mu} \right\} \frac{\partial p_{III}}{\partial z} - \left. \left[ \frac{y - (\beta + h)}{h} \right] \left\{ \frac{2k_z \frac{\partial \tilde{p}}{\partial z} - h^2 \frac{\partial p_{III}}{\partial z}}{\frac{2k_z \frac{\partial \tilde{p}}{\partial z} - h^2 \frac{\partial p_{III}}{\partial z}}{2 \left( \mu + \tilde{\mu} h \sqrt{\gamma_z} \right)} \right\} - \left. \frac{k_z \frac{\partial \tilde{p}}{\partial z}}{\mu} \right|_{y=\beta} \right\} \hspace{1cm} (8.83)$

Remembering that $\gamma_x = \frac{\mu}{k_x}$, it follows that

$$\tilde{\mu} \sqrt{\gamma_x} = \sqrt{\frac{\mu \tilde{\mu}}{k_x}}$$

$$\tilde{\mu} \sqrt{\gamma_z} = \sqrt{\frac{\mu \tilde{\mu}}{k_z}}$$
so that we may rewrite Equations (8.78) - (8.83) as

\begin{align*}
    u_I &= \left[ \frac{y(y - \alpha)}{2\mu} \right] \frac{\partial p_I}{\partial x} + \left( \frac{y}{\alpha} \right) \left\{ \frac{2k_x \sqrt{k_x} \frac{\partial \phi}{\partial x} - \alpha^2 \sqrt{k_x} \frac{\partial p_I}{\partial x}}{2 \left( \mu \sqrt{k_x + \alpha \mu I} \right)} \right\} - k_x \frac{\partial \bar{p}}{\mu \partial x} \bigg|_{y = \alpha} \quad (8.84) \\
    w_I &= \left[ \frac{y(y - \alpha)}{2\mu} \right] \frac{\partial p_I}{\partial z} + \left( \frac{y}{\alpha} \right) \left\{ \frac{2k_z \sqrt{k_z} \frac{\partial \phi}{\partial z} - \alpha^2 \sqrt{k_z} \frac{\partial p_I}{\partial z}}{2 \left( \mu \sqrt{k_z + \alpha \mu I} \right)} \right\} - k_z \frac{\partial \bar{p}}{\mu \partial z} \bigg|_{y = \alpha} \quad (8.85) \\
    \tilde{u} &= \left[ \frac{2k_x \sqrt{k_x} \frac{\partial \phi}{\partial x} + 2\mu U \sqrt{k_x} - h^2 \frac{\partial p_{III}}{\partial x}}{2 \left( \mu \sqrt{k_x + \alpha \mu I} \right)} \right] e^{\sqrt{\pi} \tau (y - \beta)} + \\
    &\quad + \left[ \frac{2k_x \sqrt{k_x} \frac{\partial \phi}{\partial z} - \alpha^2 \sqrt{k_x} \frac{\partial p_I}{\partial z}}{2 \left( \mu \sqrt{k_x + \alpha \mu I} \right)} \right] e^{\sqrt{\pi} \tau (\alpha - y)} - k_x \frac{\partial \bar{p}}{\mu \partial x} \quad (8.86) \\
    \tilde{w} &= \left[ \frac{2k_z \sqrt{k_z} \frac{\partial \phi}{\partial z} - \alpha^2 \sqrt{k_z} \frac{\partial p_I}{\partial z}}{2 \left( \mu \sqrt{k_z + \alpha \mu I} \right)} \right] e^{\sqrt{\pi} \tau (y - \beta)} + \\
    &\quad + \left[ \frac{2k_z \sqrt{k_z} \frac{\partial \phi}{\partial x} - h^2 \sqrt{k_z} \frac{\partial p_{III}}{\partial x}}{2 \left( \mu \sqrt{k_z + \alpha \mu I} \right)} \right] e^{\sqrt{\pi} \tau (\alpha - y)} - k_z \frac{\partial \bar{p}}{\mu \partial z} \quad (8.87) \\
    u_{III} &= \left\{ \frac{(y - \beta) \left[ y - (\beta + h) \right]}{2\mu} \right\} \frac{\partial p_{III}}{\partial x} - \\
    &\quad - \left[ \frac{y - (\beta + h)}{h} \right] \left\{ \frac{2k_x \sqrt{k_x} \frac{\partial \phi}{\partial x} + 2\mu U \sqrt{k_x} - h^2 \sqrt{k_x} \frac{\partial p_{III}}{\partial x}}{2 \left( \mu \sqrt{k_x + \alpha \mu I} \right)} \right\} - k_x \frac{\partial \bar{p}}{\mu \partial x} \bigg|_{y = \beta} + \\
    &\quad + \left( \frac{y - \beta}{h} \right) U \quad (8.88) \\
    w_{III} &= \left\{ \frac{(y - \beta) \left[ y - (\beta + h) \right]}{2\mu} \right\} \frac{\partial p_{III}}{\partial z} - \\
    &\quad - \left[ \frac{y - (\beta + h)}{h} \right] \left\{ \frac{2k_z \sqrt{k_z} \frac{\partial \phi}{\partial z} - h^2 \sqrt{k_z} \frac{\partial p_{III}}{\partial z}}{2 \left( \mu \sqrt{k_z + \alpha \mu I} \right)} \right\} - k_z \frac{\partial \bar{p}}{\mu \partial z} \bigg|_{y = \beta} \quad . \quad (8.89)
\end{align*}
The methods used below to formulate the Reynolds equations are valid for the above equations. However, at this point, we consider the case where \( k_x = k_z = 0 \), which is indicative of a porous medium that features capillaries oriented in the radial direction (solely in the \( y \)-direction for the geometry described in Figure 8.2). This simplifies Equations (8.84) - (8.89) to the following form:

\[
\begin{align*}
    u_I &= \left[ \frac{y(y - \alpha)}{2\mu} \right] \frac{\partial p_I}{\partial x} \quad (8.90) \\
    w_I &= \left[ \frac{y(y - \alpha)}{2\mu} \right] \frac{\partial p_I}{\partial z} \quad (8.91) \\
    \tilde{u} &= 0 \quad (8.92) \\
    \tilde{w} &= 0 \quad (8.93) \\
    u_{III} &= \left\{ \frac{(y - \beta)[y - (\beta + h)]}{2\mu} \right\} \frac{\partial p_{III}}{\partial x} + \left( \frac{y - \beta}{h} \right) U \quad (8.94) \\
    w_{III} &= \left\{ \frac{(y - \beta)[y - (\beta + h)]}{2\mu} \right\} \frac{\partial p_{III}}{\partial z} \quad (8.95)
\end{align*}
\]

Note that this is the same formulation as if we had used Darcy’s law for the governing equations inside the porous medium. Further research can keep the terms from Equations (8.84) - (8.89) and use an adaptive grid inside the porous medium (i.e., a finer mesh at the boundary of the porous medium transitioning to a coarser mesh for the interior of the porous medium) to recover the contribution from the Brinkman terms.

Let us make an important distinction between Darcy’s Law and Brinkman-Extended Darcy’s Law. Darcy’s Law features first order derivatives whereas Brinkman’s version features second order derivatives. However, the Navier-Stokes equations also
feature second order derivatives. Because of this fact, we cannot smoothly match the fluid flow profiles inside the fluid region with those inside the porous medium region using Darcy’s Law. Brinkman-Extended Darcy’s Law overcomes this shortcoming, however, it would require a fine computational mesh around the interface.

Conceptually, we see the difference between the two laws in Figure (8.3).

![Figure 8.3: Distinction between Darcy’s Law and Brinkman-Extended Darcy’s Law](image)

Here we see how the Brinkman extensions smooth out the fluid velocity profile at the fluid-porous medium interface. For the remainder of this work, however, the simplifications described lead to a first order approximation of the fluid velocity profile inside the fluid regions as well as the porous medium, with the subsequent use of Darcy’s Law a byproduct of these simplifications. However, the analytical work described above provides the groundwork to describe this bearing geometry using the Brinkman extensions.
We still must find the $y$-component of the velocities inside the three regions. To do this, we observe that we may algebraically find $\tilde{v}$ from Equation (8.32), yielding
\[
\tilde{v} = -\frac{k_y \partial p}{\mu \partial y}. \tag{8.96}
\]

By integrating the steady-state version of the continuity conditions, Equations (8.36) and (8.38) and applying boundary conditions Equations (8.43) and (8.46), we may find $v_I$ and $v_{III}$. Upon doing so, we find that
\[
v_I = \left(\frac{3\alpha y^2 - 2y^3}{12\mu}\right) \left[\frac{\partial}{\partial x}\left(\rho_I \frac{\partial p_I}{\partial x}\right) + \frac{\partial}{\partial z}\left(\rho_I \frac{\partial p_I}{\partial z}\right)\right] \tag{8.97}
\]
\[
v_{III} = \frac{\partial}{\partial x} \left[\left\{\frac{(-2y + 2\beta - h)(y - (\beta + h))^2}{12\mu}\right\} \left(\rho_{III} \frac{\partial p_{III}}{\partial x}\right)\right] - \frac{\partial}{\partial x} \left[\left\{\frac{(y - (\beta + h))(y - \beta + h)}{2h}\right\} \left(\rho_{III} U\right)\right] + \frac{\partial}{\partial z} \left[\left\{\frac{(-2y + 2\beta - h)(y - (\beta + h))^2}{12\mu}\right\} \left(\rho_{III} \frac{\partial p_{III}}{\partial z}\right)\right]. \tag{8.98}
\]

Then, using the transient version of the mass continuity conditions, Equations (8.36) and (8.38) and applying boundary conditions Equations (8.55), (8.58), (8.60), and (8.61), we construct the Reynolds equations for Regions I and III, respectively:
\[
\frac{\partial (\rho_I \alpha)}{\partial t} - \left(\frac{\alpha^3}{12\mu}\right) \left[\frac{\partial}{\partial x}\left(\rho_I \frac{\partial p_I}{\partial x}\right) + \frac{\partial}{\partial z}\left(\rho_I \frac{\partial p_I}{\partial z}\right)\right] = \frac{\tilde{\rho} k_y \partial p}{\mu \partial y}_{y=\alpha}. \tag{8.99}
\]
\[
\frac{\partial (\rho_{III} h)}{\partial t} + \frac{\partial}{\partial x} \left[\left(\frac{\rho_{III} U h}{2}\right) - \left(\frac{\rho_{III} h^3}{12\mu}\right) \frac{\partial p_{III}}{\partial x}\right] - \frac{\partial}{\partial z} \left[\left(\frac{\rho_{III} h^3}{12\mu}\right) \frac{\partial p_{III}}{\partial z}\right] = - \frac{\tilde{\rho} k_y \partial p}{\mu \partial y}_{y=\beta}. \tag{8.100}
\]

Equations (8.99) and (8.100), coupled with the transient version of the continuity equation, Equation (8.37),
\[
\frac{\partial \tilde{\rho}}{\partial t} = \frac{\partial}{\partial y} \left[\left(\frac{\tilde{\rho} k_y}{\mu}\right) \frac{\partial \tilde{p}}{\partial y}\right], \tag{8.101}
\]

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yield the three coupled differential equations that are to be solved numerically that will yield the pressure and velocity fields inside the bearing.

Here, we make a distinction between the governing equations for the journal bearing and the slider bearing. Although both equations describe the momentum transfer between the two fluid regions with a porous medium acting as an intermediary, the slider bearing governing equations feature the shear flow term inside the fluid reservoir governing equation, whereas the journal bearing governing equations feature the shear flow term inside the fluid film governing equation. This is due to the fact that, for the slider bearing, the porous medium and the reservoir wall are moving with constant linear velocity, $U$, whereas for the journal bearing, the journal wall is moving with constant linear velocity, $U$.

Also, we are modeling journal bearings with finite axial ($z$-direction) lengths as opposed to the slider bearing, which featured an infinite “axial” length (e.g. no $z$-dependence). The axial dependence is modeled for the journal bearings, creating a three-dimensional model ($x$, $y$, and $z$) inside the porous medium, whereas the slider bearing modeled only two dimensions inside the porous medium ($x$ and $y$).

That said, the governing equations are quite similar and the numerical techniques developed for modeling the slider bearing translate to modeling the journal bearing.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Baseline value</th>
</tr>
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<tr>
<td>$B$</td>
<td>Bulk modulus of the fluid</td>
<td>$6.9 \times 10^7$ Pa</td>
</tr>
<tr>
<td>$h_{max}$</td>
<td>maximum film clearance</td>
<td></td>
</tr>
<tr>
<td>$h_{min}$</td>
<td>minimum film clearance</td>
<td></td>
</tr>
<tr>
<td>$k_x$</td>
<td>permeability of the porous medium in $x$-direction</td>
<td>$0$ m$^2$</td>
</tr>
<tr>
<td>$k_y$</td>
<td>permeability of the porous medium in $y$-direction</td>
<td>$1 \times 10^{-12}$ m$^2$</td>
</tr>
<tr>
<td>$k_z$</td>
<td>permeability of the porous medium in $z$-direction</td>
<td>$0$ m$^2$</td>
</tr>
<tr>
<td>$L_x$</td>
<td>length in the $x$-direction</td>
<td></td>
</tr>
<tr>
<td>$L_z$</td>
<td>length in the $z$-direction</td>
<td></td>
</tr>
<tr>
<td>$p_I$</td>
<td>pressure in the reservoir</td>
<td></td>
</tr>
<tr>
<td>$\tilde{p}$</td>
<td>pressure in the porous medium</td>
<td></td>
</tr>
<tr>
<td>$p_{III}$</td>
<td>pressure in the film</td>
<td></td>
</tr>
<tr>
<td>$P^*$</td>
<td>non-dimensional $p$</td>
<td></td>
</tr>
<tr>
<td>$u_I$</td>
<td>$x$-component of velocity in the reservoir</td>
<td></td>
</tr>
<tr>
<td>$\tilde{u}$</td>
<td>$x$-component of velocity in the porous medium</td>
<td></td>
</tr>
<tr>
<td>$u_{III}$</td>
<td>$x$-component of velocity in the film</td>
<td></td>
</tr>
<tr>
<td>$U$</td>
<td>velocity of reservoir wall and porous medium in $x$-direction</td>
<td>$4.57$ m/s</td>
</tr>
<tr>
<td>$U^*$</td>
<td>non-dimensional $u$</td>
<td></td>
</tr>
</tbody>
</table>

Table 8.1: Journal Bearing Nomenclature and Values - Part 1
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Baseline value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_I$</td>
<td>$y$-component of velocity in the reservoir</td>
<td></td>
</tr>
<tr>
<td>$\tilde{v}$</td>
<td>$y$-component of velocity in the porous medium</td>
<td></td>
</tr>
<tr>
<td>$v_{III}$</td>
<td>$y$-component of velocity in the film</td>
<td></td>
</tr>
<tr>
<td>$V^*$</td>
<td>non-dimensional $v$</td>
<td></td>
</tr>
<tr>
<td>$w_I$</td>
<td>$z$-component of velocity in the reservoir</td>
<td></td>
</tr>
<tr>
<td>$\tilde{w}$</td>
<td>$z$-component of velocity in the porous medium</td>
<td></td>
</tr>
<tr>
<td>$w_{III}$</td>
<td>$z$-component of velocity in the film</td>
<td></td>
</tr>
<tr>
<td>$W^*$</td>
<td>non-dimensional $w$</td>
<td></td>
</tr>
<tr>
<td>$x, y, z$</td>
<td>rectangular coordinates</td>
<td></td>
</tr>
<tr>
<td>$X^*$</td>
<td>non-dimensional $x$</td>
<td></td>
</tr>
<tr>
<td>$Y^*$</td>
<td>non-dimensional $y$</td>
<td></td>
</tr>
<tr>
<td>$Z^*$</td>
<td>non-dimensional $z$</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>height of bottom of the porous medium</td>
<td>$1.27 \times 10^{-4}$ m</td>
</tr>
<tr>
<td>$\beta$</td>
<td>height of top of the porous medium</td>
<td>$2.67 \times 10^{-3}$ m</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>fluid density at cavitation pressure</td>
<td></td>
</tr>
<tr>
<td>$\rho_I$</td>
<td>fluid density in the reservoir</td>
<td></td>
</tr>
<tr>
<td>$\tilde{\rho}$</td>
<td>fluid density in the porous medium</td>
<td></td>
</tr>
<tr>
<td>$\rho_{III}$</td>
<td>fluid density in the film</td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>kinematic fluid viscosity</td>
<td>$3.9 \times 10^{-2}$ kg/m·s</td>
</tr>
</tbody>
</table>

Table 8.2: Journal Bearing Nomenclature and Values - Part 2
The method used in this work is similar to the work done by Elrod [21], which was further developed by Vijayaraghavan and Keith. The basis for the work developed here is that of Vijayaraghavan and Keith [22]. The goal of their work was to construct the governing equations and computational algorithm in such a manner that the cavitated region was automatically calculated and the accompanying finite difference numerical scheme reflected the physics of cavitation. The work described in this chapter mimicks, and extends, the analytical work done in Chapter 4, whereas the numerical portion for solving the governing equations generated in this chapter will be described in Chapter 10.

9.1 Governing Equations

We use the same technique as described above to develop the cavitation equations for the journal bearing configuration. Recall that the three equations to be solved,
using a cavitation model, are

\[
\frac{\partial}{\partial t} (\rho I \theta) - \left( \frac{\alpha^3}{12\mu} \right) \left[ \frac{\partial}{\partial x} \left( \rho I \frac{\partial p_I}{\partial x} \right) + \frac{\partial}{\partial z} \left( \rho I \frac{\partial p_I}{\partial z} \right) \right] = \frac{\tilde{p}_y}{\mu} \frac{\partial \tilde{p}}{\partial y} |_{y=\alpha} \quad (9.1)
\]

\[
\frac{\partial \tilde{\rho}}{\partial t} = \frac{\partial}{\partial y} \left( \frac{\tilde{p}_y}{\mu} \frac{\partial \tilde{p}}{\partial y} \right) \quad (9.2)
\]

\[
\frac{\partial (\rho_{III} h)}{\partial t} + \frac{\partial}{\partial x} \left[ \left( \frac{\rho_{III} U h}{2} \right) - \left( \frac{\rho_{III} h^3}{12\mu} \right) \frac{\partial p_{III}}{\partial x} \right] - \frac{\partial}{\partial z} \left[ \left( \frac{\rho_{III} h^3}{12\mu} \right) \frac{\partial p_{III}}{\partial z} \right] = -\frac{\tilde{p}_y}{\mu} \frac{\partial \tilde{p}}{\partial y} |_{y=\beta} \quad (9.3)
\]

Using the techniques described above, we rewrite Equations (9.1) - (9.3) as

\[
\frac{\partial (\rho_c \alpha \theta_I)}{\partial t} - \left( \frac{\alpha^3 \rho_c}{12\mu} \right) \left\{ \frac{\partial^2}{\partial x^2} [g_I (\theta_I - 1)] + \frac{\partial^2}{\partial z^2} [g_I (\theta_I - 1)] \right\} = \frac{\rho_c B k_y}{\mu} \left\{ \frac{\partial}{\partial y} \left[ \tilde{g} (\tilde{\theta} - 1) \right] \right\} \bigg|_{y=\alpha} \quad (9.4)
\]

\[
\frac{\partial \rho \tilde{\theta}}{\partial t} = \frac{\rho_c B k_y}{\mu} \frac{\partial}{\partial y} \left[ \tilde{g} (\tilde{\theta} - 1) \frac{\partial \tilde{p}}{\partial y} \right] \quad (9.5)
\]

\[
\frac{\partial (\rho_{III} h)}{\partial t} + \frac{\partial}{\partial x} \left[ \left( \frac{\rho_{III} U h}{2} \right) - \left( \frac{\rho_{III} h^3}{12\mu} \right) \frac{\partial p_{III}}{\partial x} \right] - \frac{\partial}{\partial z} \left[ \left( \frac{\rho_{III} h^3}{12\mu} \right) \frac{\partial p_{III}}{\partial z} \right] = \frac{\rho_c B k_y}{\mu} \left\{ \frac{\partial}{\partial y} \left[ \tilde{g} (\tilde{\theta} - 1) \right] \right\} \bigg|_{y=\beta} \quad (9.6)
\]

The method for generating the numerical procedure used to solve Equations (9.4) - (9.6) will be described in Chapter 10.
CHAPTER X
NUMERICAL TECHNIQUES AND RESULTS - JOURNAL BEARING -
MOMENTUM EQUATIONS

10.1 Numerically Implementing Cavitation

We also used the Vijayaraghavan and Keith method to transform Equations (8.99) - (8.101) into Equations (9.4) - (9.6),

\[
\frac{\partial (\rho c \alpha \theta)}{\partial t} - \left( \frac{\alpha^3 \rho c}{12 \mu} \right) \left\{ \frac{\partial^2}{\partial x^2} [gI (\theta_I - 1)] + \frac{\partial^2}{\partial z^2} [gI (\theta_I - 1)] \right\} = \\
\frac{\rho c B k_y}{\mu} \left\{ \frac{\partial}{\partial y} \left[ \tilde{g} (\tilde{\theta} - 1) \right] \right\} \bigg|_{y=\alpha}
\]

\[
\frac{\partial \rho c \tilde{\theta}}{\partial t} = \frac{\rho c B k_y}{\mu} \frac{\partial^2}{\partial y^2} \left[ \tilde{g} (\tilde{\theta} - 1) \frac{\partial \tilde{p}}{\partial y} \right]
\]

\[
\frac{\partial (\rho_{III} h)}{\partial t} + \frac{\partial}{\partial x} \left[ \left( \frac{\rho_{III} U h}{2} \right) - \left( \frac{\rho_{III} h^3}{12 \mu} \right) \frac{\partial p_{III}}{\partial x} \right] - \\
- \frac{\partial}{\partial z} \left[ \left( \frac{\rho_{III} h^3}{12 \mu} \right) \frac{\partial p_{III}}{\partial z} \right] = -\frac{\rho k_y}{\mu} \frac{\partial \tilde{p}}{\partial y} \bigg|_{y=\beta}.
\]

that will be solved numerically using the algorithm described above for the long slider bearing configuration.

For the sake of brevity, we note that the method described here certainly applies to the \( z \)-direction terms, but our focus, for now, will be on the \( x \)-direction terms.

We begin by classifying the terms on the left hand sides of Equations (10.1) - (10.3) as Shear Flow terms or Pressure-Induced Flow terms.
10.1.1 Shear Flow Terms

Using the method described above for the slider bearing configuration, we have that

\[
\frac{\partial}{\partial x} \left( \frac{U \rho_c h \theta}{2} \right)_{i} = a E_{i-1} + b E_{i} + c E_{i+1} \frac{2 \Delta x}{2 \Delta x} \tag{10.4}
\]

where

\[
a = - \left[ \frac{(g_{i-1} + g_{i})}{2} - 2 \right] \tag{10.5}
\]

\[
b = 2 - \frac{(g_{i-1} + 2g_{i} + g_{i+1})}{2} \tag{10.6}
\]

\[
c = \frac{(g_{i} + g_{i+1})}{2}. \tag{10.7}
\]

and \(E_{i} = \left( \frac{U \rho_c h \theta}{2} \right)_{i}\).

10.1.2 Pressure-Induced Flow Terms

Similarly, we write the pressure-induced flow term in terms of the variable, \(\theta\),

\[
\frac{\partial}{\partial x} \left( \frac{h^3}{12 \mu} B g \frac{\partial \theta}{\partial x} \right) \tag{10.8}
\]

and we remember that this term exists in the full film region (\(\theta \geq 1\)) and vanishes in the cavitated region (\(\theta < 1\)). Utilizing the central difference scheme developed for the pressure-induced flow terms for the slider bearing, we can rewrite Equation (10.8) as

\[
\frac{\partial}{\partial x} \left[ \left( \frac{\rho_c B}{12 \mu} \right) h^3 \left( g_{III} \frac{\partial \theta_{III}}{\partial x} \right) \right]_{i} = \left( \frac{\rho_c B}{12 \mu \Delta x^2} \right) \left\{ \left( h_{i-\frac{1}{2}} \right)^3 \left[ g_{III,i-1} (\theta_{III,i-1} - 1) \right] - \left[ \left( h_{i-\frac{1}{2}} \right)^3 + \left( h_{i+\frac{1}{2}} \right)^3 \right] \left[ g_{III,i} (\theta_{III,i} - 1) \right] + \left( h_{i+\frac{1}{2}} \right)^3 \left[ g_{III,i+1} (\theta_{III,i+1} - 1) \right] \right\}. \tag{10.9}
\]
10.1.3 Forcing Terms

The forcing terms from Equations (10.1) and (10.3) can be discretized in the same manner as those from the corresponding slider equations to yield

\[
\frac{\partial}{\partial y} \left[ \tilde{g} \left( \tilde{\theta} - 1 \right) \right] \bigg|_{y=\alpha} = -3 \frac{\tilde{g}_{j=1} \left( \tilde{\theta}_{j=1} - 1 \right)}{2\Delta y} + 4 \frac{\tilde{g}_{j=2} \left( \tilde{\theta}_{j=2} - 1 \right)}{2\Delta y} - \frac{\tilde{g}_{j=3} \left( \tilde{\theta}_{j=3} - 1 \right)}{2\Delta y} (10.10)
\]

\[
\frac{\partial}{\partial y} \left[ \tilde{g} \left( \tilde{\theta} - 1 \right) \right] \bigg|_{y=\beta} = 3 \frac{\tilde{g}_{j=J} \left( \tilde{\theta}_{j=J} - 1 \right)}{2\Delta y} - 4 \frac{\tilde{g}_{j=J-1} \left( \tilde{\theta}_{j=J-1} - 1 \right)}{2\Delta y} + \frac{\tilde{g}_{j=J-2} \left( \tilde{\theta}_{j=J-2} - 1 \right)}{2\Delta y} (10.11)
\]

10.1.4 Transient Terms

Just like before, a forward difference in time is used to handle the transient terms, i.e.

\[
\frac{\partial Q}{\partial t} = \frac{Q^{N+1} - Q^N}{\Delta t}. (10.12)
\]

where \( N + 1 \) is the index that indicates the current time step and \( N \) is the index that indicates the previous time step.

10.1.5 Final Governing Equations

Now that every term is finite differenced, we can construct the governing equations to be solved numerically. As in the slider bearing example, we construct the single
variables $\theta$ and $g$ instead of using $\theta_1, \tilde{\theta}, \theta_{III}, g_I, \tilde{g}$, and $g_{III}$, and we use the same computational grid, however we extend it in the $z$-direction, using $k$ to index variables in the $z$-direction.

If we move the $y$- and $z$-terms to the right hand sides of the equations and consider them at the $N$ time level, keeping the $x$-terms on the left hand side of the equation and considering them at the $N + 1$ time level, we can construct a tridiagonal system that can be solved using a fast tridiagonal solver, such as the Thomas algorithm.

Using the aforementioned approaches, we may now write the governing equation for the variable $\theta$ at $y = \alpha$ ($j = 1$),

$$a_i \theta_{i-1,1,k}^{N+1} + b_i \theta_{i,1,k}^{N+1} + c_i \theta_{i+1,1,k}^{N+1} = d_i,$$

(10.13)

where

$$a_i = -\left(\frac{\alpha^2 B \Delta t}{12 \mu \Delta x^2}\right) g_{i-1,1,k}$$

$$b_i = \left[1 + 2 \left(\frac{\alpha^2 B \Delta t}{12 \mu \Delta x^2}\right) g_{i,1,k}\right]$$

$$c_i = -\left(\frac{\alpha^2 B \Delta t}{12 \mu \Delta x^2}\right) g_{i+1,1,k}$$

$$d_i = \theta_{i,1,k}^N +$$

$$+ \left(\frac{B k_y \Delta t}{2 \mu \alpha y}\right) \left\{-3 \left[g_{i,1,k} \left(\theta_{i,1,k}^N - 1\right)\right] + 4 \left[g_{i,2,k} \left(\theta_{i,2,k}^N - 1\right)\right] - \left[g_{i,3,k} \left(\theta_{i,3,k}^N - 1\right)\right]\right\} +$$

$$+ \left(\frac{\alpha^2 B \Delta t}{12 \mu \Delta z^2}\right) \left\{\left[g_{i,1,k-1} \left(\theta_{i,1,k-1}^N - 1\right)\right] - 2 \left[g_{i,1,k} \left(\theta_{i,1,k}^N - 1\right)\right] - \left[g_{i,1,k+1} \left(\theta_{i,1,k+1}^N - 1\right)\right]\right\} -$$

$$- \left(\frac{\alpha^2 B \Delta t}{12 \mu \Delta x^2}\right) \left(g_{i-1,1,k} - 2g_{i,1,k} + g_{i+1,1,k}\right).$$
The governing equation for the variable $\theta$ for $\alpha < y < \beta$ ($2 \leq j \leq J - 1$) is

$$
\theta_{i,j,k}^{N+1} = \theta_{i,j,k}^N + \left( \frac{B k_y \Delta t}{\mu \Delta y^2} \right) \left\{ [g_{i,j-1,k} (\theta_{i,j-1,k}^N - 1)] - 2 [g_{i,j,k} (\theta_{i,j,k}^N - 1)] - [g_{i,j+1,k} (\theta_{i,j+1,k}^N - 1)] \right\}.
$$

Finally, the governing equation for the variable $\theta$ at $y = \beta$ ($j = J$) is

$$
a_i \theta_{i-1,J,k}^{N+1} + b_i \theta_{i,J,k}^{N+1} + c_i \theta_{i+1,J,k}^{N+1} = d_i,
$$

where

$$
\begin{align*}
a_i &= \left( \frac{U \Delta t}{4 \Delta x} \right) \left[ \left( \frac{g_{i-1,J,k} + g_{i,J,k}}{2} \right) - 2 \right] (h_{i-1}) - \left( \frac{B \Delta t}{12 \mu \Delta x^2} \right) \left( h_{i-\frac{1}{2}} \right)^3 g_{i-1,j,k} \\
b_i &= h_i + \left( \frac{U \Delta t}{4 \Delta x} \right) \left[ 2 - \left( \frac{g_{i-1,J,k} + 2 g_{i,J,k} + g_{i+1,J,k}}{2} \right) \right] (h_i) + \\
&\quad + \left( \frac{B \Delta t}{12 \mu \Delta x^2} \right) \left[ \left( h_{i-\frac{1}{2}} \right)^3 + \left( h_{i+\frac{1}{2}} \right)^3 \right] g_{i,j,k} \\
c_i &= \left( \frac{U \Delta t}{4 \Delta x} \right) \left[ \left( \frac{g_{i,J,k} + g_{i+1,J,k}}{2} \right) \right] (h_{i+1}) - \left( \frac{B \Delta t}{12 \mu \Delta x^2} \right) \left( h_{i+\frac{1}{2}} \right)^3 g_{i+1,j,k} \\
d_i &= h_i \theta_{i,1,k}^N - \\
&\quad - \left( \frac{B \Delta t}{12 \mu \Delta x^2} \right) \left\{ \left( h_{i-\frac{1}{2}} \right)^3 g_{i-1,j,k} - \left[ \left( h_{i-\frac{1}{2}} \right)^3 + \left( h_{i+\frac{1}{2}} \right)^3 \right] g_{i,j,k} + \left( h_{i+\frac{1}{2}} \right)^3 g_{i+1,j,k} \right\} + \\
&\quad + \left( \frac{B h_i^2 \Delta t}{12 \mu \Delta z^2} \right) \left\{ [g_{i,J,k-1} (\theta_{i,J,k-1}^N - 1)] - 2 [g_{i,J,k} (\theta_{i,J,k}^N - 1)] - [g_{i,J,k+1} (\theta_{i,J,k+1}^N - 1)] \right\} + \\
&\quad + \left( \frac{B k_y \Delta t}{2 \mu \alpha y} \right) \left\{ -3 [g_{i,J,k} (\theta_{i,J,k}^N - 1)] + 4 [g_{i,J-1,k} (\theta_{i,J-1,k}^N - 1)] - [g_{i,J-2,k} (\theta_{i,J-2,k}^N - 1)] \right\}.
\end{align*}
$$

10.1.6 Solution Procedure

In matrix form, Equations (10.13) - (10.15) do not represent a perfectly tridiagonal matrix. Because we impose periodic boundary conditions in the $x$-direction, the $i = 1$
equations for the reservoir and fluid film regions are, respectively,

\[ b_1 \theta_{1,1,k}^{N+1} + c_1 \theta_{2,1,k}^{N+1} + a_1 \theta_{I-1,1,k}^{N+1} = d_1 \]  

(10.16)

\[ b_1 \theta_{1,J,k}^{N+1} + c_1 \theta_{2,J,k}^{N+1} + a_1 \theta_{I-1,J,k}^{N+1} = d_1, \]  

(10.17)

using each equation’s respective values for \(a_1, b_1, c_1, \) and \(d_1\). Similarly, the \(i = I-1\) equations for the reservoir and fluid film regions are, respectively,

\[ c_{I-1} \theta_{1,1,k}^{N+1} + a_{I-1} \theta_{I-2,1,k}^{N+1} + b_{I-1} \theta_{I-1,1,k}^{N+1} = d_{I-1} \]  

(10.18)

\[ c_{I-1} \theta_{1,J,k}^{N+1} + a_{I-1} \theta_{I-2,J,k}^{N+1} + b_{I-1} \theta_{I-1,J,k}^{N+1} = d_{I-1}, \]  

(10.19)

again, using each equation’s respective values for \(a_{I-1}, b_{I-1}, c_{I-1}, \) and \(d_{I-1}\). Instead of using a Gauss-Jordan procedure, which would be quite slow, the two off-diagonal terms (\(a_1\) in the \(i = 1\) equation and \(c_{I-1}\) in the \(i = I-1\) equation) were time lagged,

meaning that we made the following replacements:

\[ a_1 \theta_{I-1,1,k}^{N+1} = a_1 \theta_{I-1,1,k}^{N} \]  

(10.20)

\[ a_1 \theta_{I-1,J,k}^{N+1} = a_1 \theta_{I-1,J,k}^{N} \]  

(10.21)

\[ c_{I-1} \theta_{1,1,k}^{N+1} = c_{I-1} \theta_{1,1,k}^{N} \]  

(10.22)

\[ c_{I-1} \theta_{1,J,k}^{N+1} = c_{I-1} \theta_{1,J,k}^{N} \]  

(10.23)

As a result, these quantities represent known values at the \(N+1\) time step and can be placed on the right hand sides of Equations (10.13) and (10.15). By doing this, the resulting matrix structure is tridiagonal and the Thomas algorithm can be applied.

The geometry was initialized to be at full film everywhere inside the bearing, where
the value for $\theta$ was such that it corresponded to the atmospheric pressure. This yields values for $\theta^N$ and $g$. Values for $\theta^{N+1}$ were calculated using the values for $\theta^N$, $g$, and the physical quantities defined by the simulation (i.e. $B$, $h_{\text{min}}$, $h_{\text{max}}$, $\alpha$, $\beta$, $\mu$, $U$, $L_x$, and $L_z$). Upon doing so, the values for $g$ were updated. The values for $\theta^{N+1}$ were then set to be $\theta^N$ and the procedure was iterated in time until a steady-state solution was achieved. The criterion used to determine if the solution had reached steady-state was that the sum of the relative differences for $\theta$ between successive iterations for all values of $i$, $j$, and $k$ was no more than $10^{-5}$. It was found that values less than $10^{-5}$ indicated that solution had not yet converged, yet anything over $10^{-5}$ provided no new information.

10.2 Results and Discussion

The Darcy Model and some underlying simplifications and assumptions were used for the development of a model relating the flow of the fluid inside the bearing, the porous material and the reservoir, respectively. Through a non-dimensional analysis, $O(1)$ governing equations were developed for the three regions, and we also stated that, for a capillary-type porous medium proposed here, $k_x = 0$ and $k_z = 0$ inside the porous medium. These assumptions, together with the integration of a mass flow-conserving cavitation scheme, yielded a coupled system of partial differential equations, Equations (10.1) - (10.3), which were solved using a computer algorithm similar to that used by Vijayaraghavan and Keith [22]. The cavitation reference pressure was set at 0 Pa. Periodic boundary conditions were applied at the endpoints
in the $x$-direction and atmospheric pressure boundary conditions were applied at the axial ends ($z = 0, L_z$). After each iteration the values for $\theta$ are updated while $g$ is set anew depending on the new value of $\theta$. This process is repeated until the solution for $\theta$ converges. Only then were the pressures for each region calculated. Because this method properly accounts for mass continuity in both the full film and cavitated regions and automatically predicts the film rupture and reformation points, the shortcomings of earlier approaches (Gumbel, Swift-Stieber, etc.) are overcome, yielding a cavitation model that is much more physically accurate.

To allow for a discussion of this type of self-lubricated, self-circulation bearing, it is not sufficient only to examine the pressures inside the active film space. The active space is indeed the determining factor for the load-carrying capacity, but the pressure differential that develops between the hydrodynamic film and the reservoir is crucial for the self-circulation of the lubricating fluid between the regions, therefore, for the functionality of the concept proposed herein. As a result, the pressure profiles in both fluid regions need to be examined.

Tables 8.1 and 8.2 specify the baseline parameters used in this numerical simulation. For algorithm validation purposes, the geometric parameters (i.e. length, maximum and minimum film clearances, width), the properties of the lubricating fluid (bulk modulus, $B$, and viscosity, $\mu$), and the slider velocity, $U$ were chosen to parallel closely those of the bearing used by Vijayaraghavan and Keith [22], Table 8.1, Table 8.2.

The results discussion here will be a little different than that of the slider
bearing. As we considered the slider bearing to be infinitely long in the axial direction, we presented the pressures as a function of $x$ at $y = \alpha$ and $y = \beta$ for our parametric study. However, since we are investigating a three-dimensional journal bearing, a similar study would be confusing at best, considering all of the different two-dimensional plots that would be needed for the reader to understand the resulting pressure field. Thus, every pressure figure here will be three-dimensional, with two of the axes representing cartesian axes and the third axis representing the pressure inside the bearing.

10.2.1 Long Journal Bearing - Pressure Profiles

We begin by looking at the long journal bearing, defined by $L_z = 5L_x$. For the most part, this geometry is characterized by very little pressure variation in the axial ($z$) direction for the interior 75 percent, and either a positive or negative pressure gradient near the axial ends, depending on the configuration of the numerical simulation being ran.

**Effect of Reservoir Depth ($\alpha$)**

We begin by looking at how varying the depth of the fluid reservoir changes the values and shapes of the pressure profiles inside the porous medium. Figure (10.1) shows the pressure profile for $\alpha = 6.35 \times 10^{-5}$ m at $z = L_z/2$. The $x$-axis represents the circumferential direction and the $y$-axis direction represents the thickness of the
porous medium. Figures (10.2) - (10.3) show the pressure profiles for $\alpha = 6.35 \times 10^{-5}$ m at $y = \beta$ and $y = \alpha$, respectively.
Figure 10.1: Pressure profile in the porous medium for \( z = L_z/2 \) for \( \alpha = 6.35 \times 10^{-5} \) m, \((\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m². All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.2: Pressure profile in the porous medium at \( y = \beta \) for \( \alpha = 6.35 \times 10^{-5} \) m, \((\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m². All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.3: Pressure profile in the porous medium at $y = \alpha$ for $\alpha = 6.35 \times 10^{-5}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.4) shows the pressure profile for $\alpha = 1.27 \times 10^{-4}$ m at $z = L_z/2$.

Figures (10.5) - (10.6) show the pressure profiles for $\alpha = 1.27 \times 10^{-4}$ m at $y = \beta$ and $y = \alpha$, respectively.
Figure 10.4: Pressure profile in the porous medium for $z = L_z/2$ for $\alpha = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.5: Pressure profile in the porous medium at $y = \beta$ for $\alpha = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.6: Pressure profile in the porous medium at $y = \alpha$ for $\alpha = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.7) shows the pressure profile for $\alpha = 2.54 \times 10^{-4}$ m at $z = L_z/2$.

Figures (10.8) - (10.9) show the pressure profiles for $\alpha = 2.54 \times 10^{-4}$ m at $y = \beta$ and $y = \alpha$, respectively.
Figure 10.7: Pressure profile in the porous medium for \( z = \frac{L_z}{2} \) for \( \alpha = 2.54 \times 10^{-4} \) m, \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m². All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.8: Pressure profile in the porous medium at \( y = \beta \) for \( \alpha = 2.54 \times 10^{-4} \) m, \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m². All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.9: Pressure profile in the porous medium at $y = \alpha$ for $\alpha = 2.54 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.10) shows the pressure profile for $\alpha = 5.08 \times 10^{-4} \text{ m at } z = L_z/2$. Figures (10.11) - (10.12) show the pressure profiles for $\alpha = 5.08 \times 10^{-4} \text{ m at } y = \beta$ and $y = \alpha$, respectively.

We note that we see a similar behavior to that of the slider bearing: increasing the reservoir depth, $\alpha$, leads to a decreasing, though with a limiting value, peak pressure at the active film-porous medium interface. For a very shallow reservoir, $\alpha = 6.35 \times 10^{-5} \text{ m}, we have the largest pressures inside the film and reservoir, with similar pressure profiles. For a deep reservoir, $\alpha = 5.08 \times 10^{-4} \text{ m}, there is essentially no pressure gradient in the reservoir, and the pressures inside the film space are much lower than the corresponding pressures in the case of the shallow reservoir.

Examining the figures where the $y$-value is fixed and we see the pressure curve as a function of $x$ and $z$, we see phenomena not seen in the slider case. Note, that for the shallow reservoir case, $\alpha = 6.35 \times 10^{-5}, the pressure does not increase near $z = 0$ and $z = L_z, Figures (10.2), however, for the other cases, the pressure increases then decreases near the axial ends ($z = 0$ and $z = L_z, Figures (10.5) - (10.11), indicating that a circulation zone exists near the axial ends.

**Effect of Porous Medium Permeability ($k_y$)**

We now look at how changing the permeability of the porous medium changes the values and shapes of the pressure profiles inside the porous medium. Figure
(10.13) shows the pressure profile for $k_y = 1.0 \times 10^{-10}$ m$^2$ at $z = L_z/2$. Figures (10.14) - (10.15) show the pressure profiles for $k_y = 1.0 \times 10^{-10}$ m$^2$ at $y = \beta$ and $y = \alpha$, respectively.
Figure 10.10: Pressure profile in the porous medium for \( z = \frac{L_z}{2} \) for \( \alpha = 5.08 \times 10^{-4} \) m, \((\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( U^* = 4.57 \text{ m/s}, \ k_y^* = 1 \times 10^{-12} \text{ m}^2 \). All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.11: Pressure profile in the porous medium at \( y = \beta \) for \( \alpha = 5.08 \times 10^{-4} \) m, \((\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( U^* = 4.57 \text{ m/s}, \ k_y^* = 1 \times 10^{-12} \text{ m}^2 \). All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.12: Pressure profile in the porous medium at $y = \alpha$ for $\alpha = 5.08 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.13: Pressure profile in the porous medium for $z = L_z/2$ for $k_y = 1 \times 10^{-10}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.14: Pressure profile in the porous medium at \( y = \beta \) for \( k_y = 1 \times 10^{-10} \text{ m}^2 \), \( \alpha^* = 1.27 \times 10^{-4} \text{ m}, (\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}, U^* = 4.57 \text{ m/s} \). All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.15: Pressure profile in the porous medium at \( y = \alpha \) for \( k_y = 1 \times 10^{-10} \text{ m}^2 \), \( \alpha^* = 1.27 \times 10^{-4} \text{ m}, (\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}, U^* = 4.57 \text{ m/s} \). All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.16) shows the pressure profile for $k_y = 1.0 \times 10^{-11}$ m$^2$ at $z = L_z/2$.

Figures (10.17) - (10.18) show the pressure profiles for $k_y = 1.0 \times 10^{-11}$ m$^2$ at $y = \beta$ and $y = \alpha$, respectively.

Figure 10.16: Pressure profile in the porous medium for $z = L_z/2$ for $k_y = 1 \times 10^{-11}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.17: Pressure profile in the porous medium at $y = \beta$ for $k_y = 1 \times 10^{-11}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.18: Pressure profile in the porous medium at $y = \alpha$ for $k_y = 1 \times 10^{-11}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.19) shows the pressure profile for $k_y = 1.0 \times 10^{-12} \text{ m}^2$ at $z = L_z/2$.

Figures (10.20) - (10.21) show the pressure profiles for $k_y = 1.0 \times 10^{-12} \text{ m}^2$ at $y = \beta$ and $y = \alpha$, respectively.

![Figure 10.19: Pressure profile in the porous medium for $z = L_z/2$ for $k_y = 1 \times 10^{-12}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.](image-url)
Figure 10.20: Pressure profile in the porous medium at $y = \beta$ for $k_y = 1 \times 10^{-12}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.21: Pressure profile in the porous medium at $y = \alpha$ for $k_y = 1 \times 10^{-12}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.22) shows the pressure profile for \( k_y = 1.0 \times 10^{-13} \text{ m}^2 \) at \( z = \frac{L_z}{2} \).

Figures (10.23) - (10.24) show the pressure profiles for \( k_y = 1.0 \times 10^{-13} \text{ m}^2 \) at \( y = \beta \) and \( y = \alpha \), respectively.

Figure 10.22: Pressure profile in the porous medium for \( z = \frac{L_z}{2} \) for \( k_y = 1 \times 10^{-13} \text{ m}^2 \), \( \alpha^* = 1.27 \times 10^{-4} \text{ m} \), \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m} \), \( U^* = 4.57 \text{ m/s} \). All other pertinent values are given in Tables 8.1 and 8.2.

Again, we see similar behavior to that of the slider bearing. For the high permeability case, \( k_y = 1.0 \times 10^{-10} \text{ m}^2 \), there is very little variance in pressure between the reservoir and fluid film regions. However, decreasing the permeability leads to higher pressures inside the lubricating film region and lower pressures inside the fluid reservoir. Also, as the permeability decreases, the location of peak pressure moves more toward \( x = \frac{L_x}{2} \), and the size of the cavitated region increases.

Examining the figures where the \( y \)-value is fixed and we see the pressure curve as a function of \( x \) and \( z \), we see phenomena not seen in the slider case. Note,
that for the high permeability cases, $k_y = 1.0 \times 10^{-10}$ m$^2$ and $k_y = 1.0 \times 10^{-11}$ m$^2$, the pressure does not increase near $z = 0$ and $z = L_z$, Figures (10.14) and (10.17), however, for the low permeability cases, the pressure increases then decreases near $z = 0$ and $z = L_z$, Figures (10.20) and (10.23), indicating that a circulation zone exists near the axial ends.

**Effect of the Thickness of the Porous Medium, $(\beta - \alpha)$**

Our focus now turns to what role the effect of the thickness of the porous medium has on the values and shapes of the pressure profiles inside the porous medium. Figure (10.25) shows the pressure profile for $\beta - \alpha = 6.35 \times 10^{-4}$ m at $z = L_z/2$. The $x$-axis represents the circumferential direction and the $y$-axis direc-
Figure 10.24: Pressure profile in the porous medium at \( y = \alpha \) for \( k_y = 1 \times 10^{-13} \text{ m}^2 \), \( \alpha^* = 1.27 \times 10^{-4} \text{ m} \), \( \beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m} \), \( U^* = 4.57 \text{ m/s} \). All other pertinent values are given in Tables 8.1 and 8.2.

The thickness represents the thickness of the porous medium. Figures (10.26) - (10.27) show the pressure profiles for \( \beta - \alpha = 6.35 \times 10^{-4} \text{ m} \) at \( y = \beta \) and \( y = \alpha \), respectively.
Figure 10.25: Pressure profile in the porous medium for \( z = \frac{L_z}{2} \) for \( (\beta - \alpha) = 6.35 \times 10^{-4} \) m, \( \alpha^* = 1.27 \times 10^{-4} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m². All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.26: Pressure profile in the porous medium at \( y = \beta \) for \( (\beta - \alpha) = 6.35 \times 10^{-4} \) m, \( \alpha^* = 1.27 \times 10^{-4} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m². All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.27: Pressure profile in the porous medium at \( y = \alpha \) for \((\beta - \alpha) = 6.35 \times 10^{-4}\) m, \( \alpha^* = 1.27 \times 10^{-4}\) m, \( U^* = 4.57\) m/s, \( k_y^* = 1 \times 10^{-12}\) m\(^2\). All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.28) shows the pressure profile for $\beta - \alpha = 1.27 \times 10^{-3}$ m at $z = L_z/2$. The $x$-axis represents the circumferential direction and the $y$-axis direction represents the thickness of the porous medium. Figures (10.29) - (10.30) show the pressure profiles for $\beta - \alpha = 1.27 \times 10^{-3}$ m at $y = \beta$ and $y = \alpha$, respectively.

![Figure 10.28: Pressure profile in the porous medium for $z = L_z/2$ for $(\beta - \alpha) = 1.27 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.](image)
Figure 10.29: Pressure profile in the porous medium at \( y = \beta \) for \((\beta - \alpha) = 1.27 \times 10^{-3}\) m, \(\alpha^* = 1.27 \times 10^{-4}\) m, \(U^* = 4.57\) m/s, \(k_y^* = 1 \times 10^{-12}\) m\(^2\). All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.30: Pressure profile in the porous medium at \( y = \alpha \) for \((\beta - \alpha) = 1.27 \times 10^{-3}\) m, \(\alpha^* = 1.27 \times 10^{-4}\) m, \(U^* = 4.57\) m/s, \(k_y^* = 1 \times 10^{-12}\) m\(^2\). All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.31) shows the pressure profile for \( \beta - \alpha = 2.54 \times 10^{-3} \text{ m} \) at \( z = L_z/2 \). The \( x \)-axis represents the circumferential direction and the \( y \)-axis direction represents the thickness of the porous medium. Figures (10.32) - (10.33) show the pressure profiles for \( \beta - \alpha = 2.54 \times 10^{-3} \text{ m} \) at \( y = \beta \) and \( y = \alpha \), respectively.

Figure 10.31: Pressure profile in the porous medium for \( z = L_z/2 \) for \( (\beta - \alpha) = 2.54 \times 10^{-3} \text{ m} \), \( \alpha^* = 1.27 \times 10^{-4} \text{ m} \), \( U^* = 4.57 \text{ m/s} \), \( k_y^* = 1 \times 10^{-12} \text{ m}^2 \). All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.32: Pressure profile in the porous medium at $y = \beta$ for $(\beta - \alpha) = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.33: Pressure profile in the porous medium at $y = \alpha$ for $(\beta - \alpha) = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.34) shows the pressure profile for $\beta - \alpha = 5.08 \times 10^{-3}$ m at $z = L_z/2$. The $x$-axis represents the circumferential direction and the $y$-axis direction represents the thickness of the porous medium. Figures (10.35) - (10.36) show the pressure profiles for $\beta - \alpha = 5.08 \times 10^{-3}$ m at $y = \beta$ and $y = \alpha$, respectively.

Figure 10.34: Pressure profile in the porous medium for $z = L_z/2$ for $(\beta - \alpha) = 5.08 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.35: Pressure profile in the porous medium at $y = \beta$ for $(\beta - \alpha) = 5.08 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.36: Pressure profile in the porous medium at $y = \alpha$ for $(\beta - \alpha) = 5.08 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.37) shows the pressure profile for $\beta - \alpha = 1.02 \times 10^{-2}$ m at $z = L_z/2$. The $x$-axis represents the circumferential direction and the $y$-axis direction represents the thickness of the porous medium. Figures (10.38) - (10.39) show the pressure profiles for $\beta - \alpha = 1.02 \times 10^{-2}$ m at $y = \beta$ and $y = \alpha$, respectively.

Figure 10.37: Pressure profile in the porous medium for $z = L_z/2$ for $(\beta - \alpha) = 1.02 \times 10^{-2}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.38: Pressure profile in the porous medium at \( y = \beta \) for \((\beta - \alpha) = 1.02 \times 10^{-2}\) m, \(\alpha^* = 1.27 \times 10^{-4}\) m, \(U^* = 4.57\) m/s, \(k_y^* = 1 \times 10^{-12}\) m². All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.39: Pressure profile in the porous medium at \( y = \alpha \) for \((\beta - \alpha) = 1.02 \times 10^{-2}\) m, \(\alpha^* = 1.27 \times 10^{-4}\) m, \(U^* = 4.57\) m/s, \(k_y^* = 1 \times 10^{-12}\) m². All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.40) shows the pressure profile for $\beta - \alpha = 2.03 \times 10^{-2}$ m at $z = L_z/2$. The $x$-axis represents the circumferential direction and the $y$-axis direction represents the thickness of the porous medium. Figures (10.41) - (10.42) show the pressure profiles for $\beta - \alpha = 2.03 \times 10^{-2}$ m at $y = \beta$ and $y = \alpha$, respectively.

Figure 10.40: Pressure profile in the porous medium for $z = L_z/2$ for $(\beta - \alpha) = 2.03 \times 10^{-2}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Here, we see that increasing the depth of the porous medium leads to an increase in peak pressure inside the lubricating film region, though Figures (10.37) and (10.40) show that this increase has its limit, as $\beta - \alpha = 1.02 \times 10^{-2}$ m and $\beta - \alpha = 2.03 \times 10^{-2}$ m yield very similar pressure curves. We note something peculiar for these simulations. Remembering the results for similar analyses for the slider bearing, an increase in the thickness of the porous medium led to an increase in the pressures, as seen here. However, there became a point after which increasing
the thickness of the porous medium had a negative impact on the pressures inside the porous medium. Here, we do not see such behavior, although we do see that the increase in pressures slows down for the thickest two cases. It may be that the introduction of the finite axial length means that we still have not hit the limit whereby increasing the porous medium thickness leads to a decrease in the peak pressures found in the active film space.

Examining the figures where the $y$-value is fixed and we see the pressure curve as a function of $x$ and $z$, we see phenomena not seen in the slider case. Note, that for the 'thin' porous medium cases, $\beta - \alpha = 6.35 \times 10^{-4}$ m and $\beta - \alpha = 1.27 \times 10^{-3}$ m, the pressure does not increase near $z = 0$ and $z = L_z$, Figures (10.26) and (10.29), however, for the 'thick' porous medium cases, the pressure increases then decreases near $z = 0$ and $z = L_z$, Figures (10.32) - (10.41), indicating that a circulation zone

Figure 10.41: Pressure profile in the porous medium at $y = \beta$ for $(\beta - \alpha) = 2.03 \times 10^{-2}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
exists near the axial ends.

**Journal Velocity Effects, \( U \)**

Finally, we examine our last parameter, the journal velocity, and how it affects the shape and values of the pressure profiles inside the porous medium. Figure (10.43) shows the pressure profile for \( U = 1.14 \text{ m/s} \) at \( z = L_z/2 \). Figures (10.44) - (10.45) show the pressure profiles for \( U = 1.14 \text{ m/s} \) at \( y = \beta \) and \( y = \alpha \), respectively.
Figure 10.43: Pressure profile in the porous medium for \( z = L_z/2 \) for \( U = 1.14 \) m/s, \( \alpha^* = 1.27 \times 10^{-4} \) m, \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( k_y^* = 1 \times 10^{-12} \) m². All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.44: Pressure profile in the porous medium at \( y = \beta \) for \( U = 1.14 \) m/s, \( \alpha^* = 1.27 \times 10^{-4} \) m, \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( k_y^* = 1 \times 10^{-12} \) m². All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.45: Pressure profile in the porous medium at $y = \alpha$ for $U = 1.14$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.46) shows the pressure profile for $U = 2.28 \text{ m/s}$ at $z = L_z/2$.

Figures (10.47) - (10.48) show the pressure profiles for $U = 2.28 \text{ m/s}$ at $y = \beta$ and $y = \alpha$, respectively.
Figure 10.46: Pressure profile in the porous medium for \( z = L_z/2 \) for \( U = 2.28 \text{ m/s} \), \( \alpha^* = 1.27 \times 10^{-4} \text{ m}, (\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}, k_y^* = 1 \times 10^{-12} \text{ m}^2 \). All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.47: Pressure profile in the porous medium at \( y = \beta \) for \( U = 2.28 \text{ m/s} \), \( \alpha^* = 1.27 \times 10^{-4} \text{ m}, (\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}, k_y^* = 1 \times 10^{-12} \text{ m}^2 \). All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.48: Pressure profile in the porous medium at \( y = \alpha \) for \( U = 2.28 \) m/s, \( \alpha^* = 1.27 \times 10^{-4} \) m, \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( k_y^* = 1 \times 10^{-12} \) m\(^2\). All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.49) shows the pressure profile for \( U = 4.57 \text{ m/s} \) at \( z = L_z/2 \).

Figures (10.50) - (10.51) show the pressure profiles for \( U = 4.57 \text{ m/s} \) at \( y = \beta \) and \( y = \alpha \), respectively.
Figure 10.49: Pressure profile in the porous medium for $z = L_z/2$ for $U = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.50: Pressure profile in the porous medium at $y = \beta$ for $U = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.51: Pressure profile in the porous medium at $y = \alpha$ for $U = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.52) shows the pressure profile for $U = 9.14$ m/s at $z = L_z/2$.

Figures (10.53) - (10.54) show the pressure profiles for $U = 9.14$ m/s at $y = \beta$ and $y = \alpha$, respectively.

The behavior exhibited here is very straightforward. As the velocity of the journal is increased, the pressures inside the film and reservoir also increase. We also see, with increasing velocity, the size of the cavitated region increases.

Examining the figures where the $y$-value is fixed and we see the pressure curve as a function of $x$ and $z$, we see phenomena not seen in the slider case. Note, that for the slower journal velocity cases, $U = 1.14$ m/s and $U = 2.28$ m/s, the pressure does not increase near $z = 0$ and $z = L_z$, Figures (10.44) and (10.47), however, for the faster journal cases, $U = 4.57$ m/s and $U = 9.14$ m/s, the pressure increases and decreases near $z = 0$ and $z = L_z$, Figures (10.50) and (10.53), indicating that a circulation zone exists near the axial ends.

10.2.2 Short Journal Bearing - Pressure Profiles

We now focus on the short journal bearing, defined by $L_z = 0.5 \times L_x$. For the most part, this geometry is characterized by parabolic pressure profiles in the axial direction and sinusoidal-like pressure profiles in the circumferential direction.

The pressure profiles here are not as interesting as those of the long journal, as everything behaves as expected. However, the figures are included for the sake of completeness.
Effect of Reservoir Depth ($\alpha$)

Again, we begin by examining the effects the reservoir depth has on the pressure profiles found inside the porous medium. Figure (10.55) shows the pressure profile for $\alpha = 6.35 \times 10^{-5}$ m at $z = L_z/2$. The $x$-axis represents the circumferential direction and the $y$-axis direction represents the thickness of the porous medium. Figures (10.56) - (10.57) show the pressure profiles for $\alpha = 6.35 \times 10^{-5}$ m at $y = \beta$ and $y = \alpha$, respectively.
Figure 10.52: Pressure profile in the porous medium for $z = L_z/2$ for $U = 9.14$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.53: Pressure profile in the porous medium at $y = \beta$ for $U = 9.14$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.54: Pressure profile in the porous medium at \( y = \alpha \) for \( U = 9.14 \) m/s, \( \alpha^* = 1.27 \times 10^{-4} \) m, \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( k_y^* = 1 \times 10^{-12} \) m\(^2\). All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.55: Pressure profile in the porous medium for \( z = L_z/2 \) for \( \alpha = 6.35 \times 10^{-5} \) m, \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m\(^2\). All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.56: Pressure profile in the porous medium at $y = \beta$ for $\alpha = 6.35 \times 10^{-5}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.57: Pressure profile in the porous medium at $y = \alpha$ for $\alpha = 6.35 \times 10^{-5}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.58) shows the pressure profile for $\alpha = 1.27 \times 10^{-4} \text{ m}$ at $z = L_z/2$.

Figures (10.59) - (10.60) show the pressure profiles for $\alpha = 1.27 \times 10^{-4} \text{ m}$ at $y = \beta$ and $y = \alpha$, respectively.
Figure 10.58: Pressure profile in the porous medium for \( z = L_z/2 \) for \( \alpha = 1.27 \times 10^{-4} \) m, \((\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m². All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.59: Pressure profile in the porous medium at \( y = \beta \) for \( \alpha = 1.27 \times 10^{-4} \) m, \((\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m². All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.60: Pressure profile in the porous medium at $y = \alpha$ for $\alpha = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.61) shows the pressure profile for $\alpha = 2.54 \times 10^{-4}$ m at $z = L_z/2$.

Figures (10.62) - (10.63) show the pressure profiles for $\alpha = 2.54 \times 10^{-4}$ m at $y = \beta$ and $y = \alpha$, respectively.
Figure 10.61: Pressure profile in the porous medium for \( z = L_z/2 \) for \( \alpha = 2.54 \times 10^{-4} \) m, \((\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m\(^2\). All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.62: Pressure profile in the porous medium at \( y = \beta \) for \( \alpha = 2.54 \times 10^{-4} \) m, \((\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m\(^2\). All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.63: Pressure profile in the porous medium at $y = \alpha$ for $\alpha = 2.54 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.64) shows the pressure profile for $\alpha = 5.08 \times 10^{-4}$ m at $z = L_z/2$. Figures (10.65) - (10.66) show the pressure profiles for $\alpha = 5.08 \times 10^{-4}$ m at $y = \beta$ and $y = \alpha$, respectively.

We note that we see a similar behavior to that of the slider bearing: increasing the reservoir depth, $\alpha$, leads to a decreasing, though with a limiting value, pressure inside the porous medium. For a very shallow reservoir, $\alpha = 6.35 \times 10^{-5}$ m, we have the largest pressure inside the film and reservoir, with similar pressure profiles. For a deep reservoir, $\alpha = 5.08 \times 10^{-4}$ m, there is essentially no pressure gradient in the reservoir, and the pressures inside the active film space are much lower than those found in the case of the shallow reservoir.

Examining the figures where the $y$-value is fixed and we see the pressure curve as a function of $x$ and $z$, we see that the pressure varies parabolically in the axial direction, as expected. Also, we note that, unlike the long journal cases, there is no increase in pressure at the axial ends for the thicker reservoir cases.

**Effect of Porous Medium Permeability ($k_y$)**

We turn our attention now to the permeability of the porous medium and its role in the shape and values of the pressure profiles found inside the porous medium. Figure (10.67) shows the pressure profile for $k_y = 1.0 \times 10^{-10}$ m$^2$ at $z = L_z/2$. Figures (10.68) - (10.69) show the pressure profiles for $k_y = 1.0 \times 10^{-10}$ m$^2$ at $y = \beta$ and $y = \alpha$, respectively.
Figure 10.64: Pressure profile in the porous medium for $z = L_z/2$ for $\alpha = 5.08 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.65: Pressure profile in the porous medium at $y = \beta$ for $\alpha = 5.08 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.66: Pressure profile in the porous medium at $y = \alpha$ for $\alpha = 5.08 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.67: Pressure profile in the porous medium for $z = L_z/2$ for $k_y = 1 \times 10^{-10}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.68: Pressure profile in the porous medium at $y = \beta$ for $k_y = 1 \times 10^{-10}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.69: Pressure profile in the porous medium at $y = \alpha$ for $k_y = 1 \times 10^{-10}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.70) shows the pressure profile for $k_y = 1.0 \times 10^{-11} \text{ m}^2$ at $z = L_z/2$.

Figures (10.71) - (10.72) show the pressure profiles for $k_y = 1.0 \times 10^{-11} \text{ m}^2$ at $y = \beta$ and $y = \alpha$, respectively.

Figure 10.70: Pressure profile in the porous medium for $z = L_z/2$ for $k_y = 1 \times 10^{-11} \text{ m}^2$, $\alpha^* = 1.27 \times 10^{-4} \text{ m}$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $U^* = 4.57 \text{ m/s}$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.71: Pressure profile in the porous medium at $y = \beta$ for $k_y = 1 \times 10^{-11}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.72: Pressure profile in the porous medium at $y = \alpha$ for $k_y = 1 \times 10^{-11}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.73) shows the pressure profile for $k_y = 1.0 \times 10^{-12} \text{m}^2$ at $z = L_z/2$.

Figures (10.74) - (10.75) show the pressure profiles for $k_y = 1.0 \times 10^{-12} \text{m}^2$ at $y = \beta$ and $y = \alpha$, respectively.

Figure 10.73: Pressure profile in the porous medium for $z = L_z/2$ for $k_y = 1 \times 10^{-12} \text{m}^2$, $\alpha^* = 1.27 \times 10^{-4} \text{m}$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{m}$, $U^* = 4.57 \text{m/s}$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.74: Pressure profile in the porous medium at $y = \beta$ for $k_y = 1 \times 10^{-12}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.75: Pressure profile in the porous medium at $y = \alpha$ for $k_y = 1 \times 10^{-12}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.76) shows the pressure profile for \( k_y = 1.0 \times 10^{-13} \) m\(^2\) at \( z = L_z/2 \).

Figures (10.77) - (10.78) show the pressure profiles for \( k_y = 1.0 \times 10^{-13} \) m\(^2\) at \( y = \beta \) and \( y = \alpha \), respectively.

Figure 10.76: Pressure profile in the porous medium for \( z = L_z/2 \) for \( k_y = 1.0 \times 10^{-13} \) m\(^2\), \( \alpha^* = 1.27 \times 10^{-4} \) m, \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( U^* = 4.57 \) m/s. All other pertinent values are given in Tables 8.1 and 8.2.

Again, we see similar behavior to that of the slider and long journal bearings. For the high permeability case, \( k_y = 1.0 \times 10^{-10} \) m\(^2\), there is very little variance in pressure between the reservoir and fluid film regions at the same circumferential position. However, decreasing the permeability leads to higher pressures inside the lubricating film region and lower pressures inside the fluid reservoir. Also, as the permeability decreases, the location of peak pressure moves more toward \( x = L_x/2 \), and the size of the cavitated region increases.
Figure 10.77: Pressure profile in the porous medium at $y = \beta$ for $k_y = 1 \times 10^{-13}$ m$^2$, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.

Examining the figures where the $y$-value is fixed and we see the pressure curve as a function of $x$ and $z$, we see that the pressure varies parabolically in the axial direction, as expected.

**Effect of the Thickness of the Porous Medium, $(\beta - \alpha)$**

Next, we examine how the thickness of the porous medium affects the shape and values of the pressure profiles found inside the porous medium. Figure (10.79) shows the pressure profile for $\beta - \alpha = 6.35 \times 10^{-4}$ m at $z = L_z/2$. The $x$-axis represents the circumferential direction and the $y$-axis direction represents the thickness of the porous medium. Figures (10.80) - (10.81) show the pressure profiles for $\beta - \alpha = 6.35 \times 10^{-4}$ m at $y = \beta$ and $y = \alpha$, respectively.
Figure 10.78: Pressure profile in the porous medium at \( y = \alpha \) for \( k_y = 1 \times 10^{-13} \) m\(^2\), \( \alpha^* = 1.27 \times 10^{-4} \) m, \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( U^* = 4.57 \) m/s. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.79: Pressure profile in the porous medium for \( z = L_z/2 \) for \( (\beta - \alpha) = 6.35 \times 10^{-4} \) m, \( \alpha^* = 1.27 \times 10^{-4} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m\(^2\). All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.80: Pressure profile in the porous medium at $y = \beta$ for $(\beta - \alpha) = 6.35 \times 10^{-4}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.81: Pressure profile in the porous medium at $y = \alpha$ for $(\beta - \alpha) = 6.35 \times 10^{-4}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.82) shows the pressure profile for $\beta - \alpha = 1.27 \times 10^{-3}$ m at $z = L_z/2$. The $x$-axis represents the circumferential direction and the $y$-axis direction represents the thickness of the porous medium. Figures (10.83) - (10.84) show the pressure profiles for $\beta - \alpha = 1.27 \times 10^{-3}$ m at $y = \beta$ and $y = \alpha$, respectively.

Figure 10.82: Pressure profile in the porous medium for $z = L_z/2$ for $(\beta - \alpha) = 1.27 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.83: Pressure profile in the porous medium at \( y = \beta \) for \((\beta - \alpha) = 1.27 \times 10^{-3}\) m, \(\alpha^* = 1.27 \times 10^{-4}\) m, \(U^* = 4.57\) m/s, \(k^*_y = 1 \times 10^{-12}\) m\(^2\). All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.84: Pressure profile in the porous medium at \( y = \alpha \) for \((\beta - \alpha) = 1.27 \times 10^{-3}\) m, \(\alpha^* = 1.27 \times 10^{-4}\) m, \(U^* = 4.57\) m/s, \(k^*_y = 1 \times 10^{-12}\) m\(^2\). All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.85) shows the pressure profile for $\beta - \alpha = 2.54 \times 10^{-3}$ m at $z = L_z/2$. The $x$-axis represents the circumferential direction and the $y$-axis direction represents the thickness of the porous medium. Figures (10.86) - (10.87) show the pressure profiles for $\beta - \alpha = 2.54 \times 10^{-3}$ m at $y = \beta$ and $y = \alpha$, respectively.

![Pressure profile in the porous medium](image)

Figure 10.85: Pressure profile in the porous medium for $z = L_z/2$ for $(\beta - \alpha) = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.86: Pressure profile in the porous medium at \( y = \beta \) for \((\beta - \alpha) = 2.54 \times 10^{-3}\) m, \(\alpha^* = 1.27 \times 10^{-4}\) m, \(U^* = 4.57\) m/s, \(k^*_y = 1 \times 10^{-12}\) m². All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.87: Pressure profile in the porous medium at \( y = \alpha \) for \((\beta - \alpha) = 2.54 \times 10^{-3}\) m, \(\alpha^* = 1.27 \times 10^{-4}\) m, \(U^* = 4.57\) m/s, \(k^*_y = 1 \times 10^{-12}\) m². All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.88) shows the pressure profile for $\beta - \alpha = 5.08 \times 10^{-3}$ m at $z = L_z/2$. The $x$-axis represents the circumferential direction and the $y$-axis direction represents the thickness of the porous medium. Figures (10.89) - (10.90) show the pressure profiles for $\beta - \alpha = 5.08 \times 10^{-3}$ m at $y = \beta$ and $y = \alpha$, respectively.

Figure 10.88: Pressure profile in the porous medium for $z = L_z/2$ for $(\beta - \alpha) = 5.08 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.89: Pressure profile in the porous medium at $y = \beta$ for $(\beta - \alpha) = 5.08 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k^*_y = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.90: Pressure profile in the porous medium at $y = \alpha$ for $(\beta - \alpha) = 5.08 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k^*_y = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.91) shows the pressure profile for $\beta - \alpha = 1.02 \times 10^{-2}$ m at $z = L_z/2$. The $x$-axis represents the circumferential direction and the $y$-axis direction represents the thickness of the porous medium. Figures (10.92) - (10.93) show the pressure profiles for $\beta - \alpha = 1.02 \times 10^{-2}$ m at $y = \beta$ and $y = \alpha$, respectively.

Figure 10.91: Pressure profile in the porous medium for $z = L_z/2$ for $(\beta - \alpha) = 1.02 \times 10^{-2}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.92: Pressure profile in the porous medium at \( y = \beta \) for \((\beta - \alpha) = 1.02 \times 10^{-2} \) m, \( \alpha^* = 1.27 \times 10^{-4} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m². All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.93: Pressure profile in the porous medium at \( y = \alpha \) for \((\beta - \alpha) = 1.02 \times 10^{-2} \) m, \( \alpha^* = 1.27 \times 10^{-4} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m². All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.94) shows the pressure profile for $\beta - \alpha = 2.03 \times 10^{-2}$ m at $z = L_z/2$. The $x$-axis represents the circumferential direction and the $y$-axis direction represents the thickness of the porous medium. Figures (10.95) - (10.96) show the pressure profiles for $\beta - \alpha = 2.03 \times 10^{-2}$ m at $y = \beta$ and $y = \alpha$, respectively.

![Pressure profile in the porous medium for $z = L_z/2$ for $(\beta - \alpha) = 2.03 \times 10^{-2}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.](image)

Figure 10.94: Pressure profile in the porous medium for $z = L_z/2$ for $(\beta - \alpha) = 2.03 \times 10^{-2}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Here, we see that increasing the depth of the porous medium leads to an increase in peak pressure inside the lubricating film region, although, as in the slider case, the peak pressure for the thickest porous medium is lower than that of the second thickest porous medium, indicating that after a certain point, the thicker the porous medium, the lower the pressure will be. We contrast this with the long journal case where we did not see such behavior.
Figure 10.95: Pressure profile in the porous medium at \( y = \beta \) for \((\beta - \alpha) = 2.03 \times 10^{-2} \) m, \( \alpha^* = 1.27 \times 10^{-4} \) m, \( U^* = 4.57 \) m/s, \( k_y^* = 1 \times 10^{-12} \) m². All other pertinent values are given in Tables 8.1 and 8.2.

Examining the figures where the \( y \)-value is fixed and we see the pressure curve as a function of \( x \) and \( z \), we see that the pressure varies parabolically in the axial direction, as expected.

**Journal Velocity Effects, \((U)\)**

Finally, we wrap up our parametric analysis of the pressure profiles for the short journal by examining the effects the journal velocity has on the shape and values of the pressure profiles inside the porous medium. Figure (10.97) shows the pressure profile for \( U = 1.14 \) m/s at \( z = L_z/2 \). Figures (10.98) - (10.99) show the pressure profiles for \( U = 1.14 \) m/s at \( y = \beta \) and \( y = \alpha \), respectively.
Figure 10.96: Pressure profile in the porous medium at $y = \alpha$ for $(\beta - \alpha) = 2.03 \times 10^{-2}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $k_y^* = 1 \times 10^{-12}$ m². All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.97: Pressure profile in the porous medium for $z = L_z/2$ for $U = 1.14$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m². All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.98: Pressure profile in the porous medium at $y = \beta$ for $U = 1.14$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.99: Pressure profile in the porous medium at $y = \alpha$ for $U = 1.14$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.100) shows the pressure profile for $U = 2.28$ m/s at $z = L_z/2$.

Figures (10.101) - (10.102) show the pressure profiles for $U = 2.28$ m/s at $y = \beta$ and $y = \alpha$, respectively.
Figure 10.100: Pressure profile in the porous medium for \( z = \frac{L_z}{2} \) for \( U = 2.28 \text{ m/s} \), \( \alpha^* = 1.27 \times 10^{-4} \text{ m} \), \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m} \), \( k_y^* = 1 \times 10^{-12} \text{ m}^2 \). All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.101: Pressure profile in the porous medium at \( y = \beta \) for \( U = 2.28 \text{ m/s} \), \( \alpha^* = 1.27 \times 10^{-4} \text{ m} \), \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m} \), \( k_y^* = 1 \times 10^{-12} \text{ m}^2 \). All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.102: Pressure profile in the porous medium at $y = \alpha$ for $U = 2.28 \, \text{m/s}$, $\alpha^* = 1.27 \times 10^{-4} \, \text{m}$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \, \text{m}$, $k^*_y = 1 \times 10^{-12} \, \text{m}^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.103) shows the pressure profile for $U = 4.57$ m/s at $z = L_z/2$.

Figures (10.104) - (10.105) show the pressure profiles for $U = 4.57$ m/s at $y = \beta$ and $y = \alpha$, respectively.
Figure 10.103: Pressure profile in the porous medium for $z = L_z/2$ for $U = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.104: Pressure profile in the porous medium at $y = \beta$ for $U = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.105: Pressure profile in the porous medium at $y = \alpha$ for $U = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure (10.106) shows the pressure profile for $U = 9.14$ m/s at $z = L_z/2$.

Figures (10.107) - (10.108) show the pressure profiles for $U = 9.14$ m/s at $y = \beta$ and $y = \alpha$, respectively.

The behavior exhibited here is very straightforward. As the velocity of the journal is increased, the pressures inside the film and reservoir also increase. We also see, with increasing velocity, the size of the cavitated region increases and the location of peak pressure shifts toward $x = L_x/2$.

Examining the figures where the $y$-value is fixed and we see the pressure curve as a function of $x$ and $z$, we see that the pressure varies parabolically in the axial direction, as expected.

10.2.3 Contour Plots of Cavitation

Plotting contour maps is a useful way to envision the areas of cavitation inside the bearing. Here, we examine the similarities and differences between the long and short journal bearings and their cavitated regions. To orient the reader, we are looking at the unwrapped journal configuration, at $y = \beta$, corresponding to the top of the porous medium. The $x$-axis represents the circumferential direction ($x$-direction) and the $y$-axis represents the axial direction ($z$-direction). One thing we note is that, in every case, the contour maps are symmetric about the axial centerline.
Contour Plots of Cavitation for the Parameter $\alpha$, the Depth of the Reservoir

We begin by displaying and commenting on the contour plots of cavitation for the reservoir depth, $\alpha$.

Figures 10.109 and 10.110 show the cavitated areas for the short and long journal bearings, respectively, inside the fluid film region for $\alpha = 6.35 \times 10^{-5}$ m. We note that the cavitated region in the long journal bearing is wider in the axial ($z$-) direction than that of the short journal bearing, however, the locations of cavitation and reformation of the film are essentially identical in the circumferential ($x$-) direction.
Figure 10.106: Pressure profile in the porous medium for $z = L_z/2$ for $U = 9.14$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.107: Pressure profile in the porous medium at $y = \beta$ for $U = 9.14$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 10.108: Pressure profile in the porous medium at \( y = \alpha \) for \( U = 9.14 \) m/s, \( \alpha^* = 1.27 \times 10^{-4} \) m, \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( k_y^* = 1 \times 10^{-12} \) m\(^2\). All other pertinent values are given in Tables 8.1 and 8.2.

Figure 10.109: Contour Plot of Cavitation in the Short Journal Bearing at \( y = \beta \) for \( \alpha = 6.35 \times 10^{-5} \) m, \( U^* = 4.57 \) m/s, \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( k_y^* = 1 \times 10^{-12} \) m\(^2\). All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.56.
Figure 10.110: Contour Plot of Cavitation in the Long Journal Bearing at $y = \beta$ for $\alpha = 6.35 \times 10^{-5}$ m, $U^* = 4.57$ m/s, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.2.

Figure 10.111: Contour Plot of Cavitation in the Short Journal Bearing at $y = \beta$ for $\alpha = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.59.
Figure 10.112: Contour Plot of Cavitation in the Long Journal Bearing at $y = \beta$ for $\alpha = 1.27 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.5.
Figures 10.111 and 10.112 show the cavitated areas for the short and long journal bearings, respectively, inside the fluid film region for $\alpha = 1.27 \times 10^{-4}$ m. This comparison is notably different from the previous one, in that we see a stark difference in the shapes of the cavitated regions, whereby the short journal case features a pointed reformation contour and the long journal case features a flat reformation contour. We also note that the sizes of the cavitated regions are much different, with the cavitated region in the long journal being larger than that of the short journal.

Figures 10.113 and 10.114 show the cavitated areas for the short and long journal bearings, respectively, inside the fluid film region for $\alpha = 2.54 \times 10^{-4}$ m. We note that the comparison for these figures is essentially identical to that of Figures 10.111 and 10.112.
Figure 10.113: Contour Plot of Cavitation in the Short Journal Bearing at $y = \beta$ for $\alpha = 2.54 \times 10^{-4} \text{ m}$, $U^* = 4.57 \text{ m/s}$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $k_y^* = 1 \times 10^{-12} \text{ m}^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.62.

Figure 10.114: Contour Plot of Cavitation in the Long Journal Bearing at $y = \beta$ for $\alpha = 2.54 \times 10^{-4} \text{ m}$, $U^* = 4.57 \text{ m/s}$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $k_y^* = 1 \times 10^{-12} \text{ m}^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.8.
Figure 10.115: Contour Plot of Cavitation in the Short Journal Bearing at $y = \beta$ for $\alpha = 5.08 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.65.

Figure 10.116: Contour Plot of Cavitation in the Long Journal Bearing at $y = \beta$ for $\alpha = 5.08 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.11.
Figures 10.115 and 10.116 show the cavitated areas for the short and long journal bearings, respectively, inside the fluid film region for $\alpha = 5.08 \times 10^{-4}$ m. We note that the comparison for these figures is essentially identical to that of Figures 10.111 and 10.112. This reinforces the notion of a limiting value for the depth of the reservoir, whereby, beyond a certain value, extending the reservoir further leads to no discernable differences between the pressure and cavitation plots.

Contour Plots of Cavitation for the Parameter $\beta - \alpha$, the Thickness of the Porous Medium

Next, we display and comment on the contour plots of cavitation for the porous medium thickness, $\beta - \alpha$.

Figures 10.117 and 10.118 show the cavitated areas for the short and long journal bearings, respectively, inside the fluid film region for $\beta - \alpha = 6.35 \times 10^{-4}$ m. We note that the cavitated region in the long journal bearing is wider in the axial ($z$-) direction than that of the short journal bearing, however, the locations of cavitation and reformation of the film are essentially identical in the circumferential ($x$-) direction.

Figures 10.119 and 10.120 show the cavitated areas for the short and long journal bearings, respectively, inside the fluid film region for $\beta - \alpha = 1.27 \times 10^{-3}$ m. Note that the comparison for this set of contours is essentially identical to that of Figures 10.117 and 10.118.
Figure 10.117: Contour Plot of Cavitation in the Short Journal Bearing at $y = \beta$ for $\beta - \alpha = 6.35 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.80.

Figure 10.118: Contour Plot of Cavitation in the Long Journal Bearing at $y = \beta$ for $\beta - \alpha = 6.35 \times 10^{-4}$ m, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.26.
Figure 10.119: Contour Plot of Cavitation in the Short Journal Bearing at $y = \beta$ for $\beta - \alpha = 1.27 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.83.

Figure 10.120: Contour Plot of Cavitation in the Long Journal Bearing at $y = \beta$ for $\beta - \alpha = 1.27 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.29.
For the sake of brevity, we skip ahead to the case where the thickness of the porous medium is $\beta - \alpha = 1.02 \times 10^{-2}$ m. We do this because the comparisons are so similar that we add very little to the discussion by showing every case.

Figures 10.121 and 10.122 show the cavitated areas for the short and long journal bearings, respectively, inside the fluid film region for $\beta - \alpha = 1.02 \times 10^{-2}$ m.

Finally, we show the comparison of the contour plots for $\beta - \alpha = 2.03 \times 10^{-2}$ m, Figures 10.123 and 10.124, the thickest porous medium tested.
Figure 10.121: Contour Plot of Cavitation in the Short Journal Bearing at $y = \beta$ for $\beta - \alpha = 1.02 \times 10^{-2}$ m, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.92.

Figure 10.122: Contour Plot of Cavitation in the Long Journal Bearing at $y = \beta$ for $\beta - \alpha = 1.02 \times 10^{-2}$ m, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.38.
Figure 10.123: Contour Plot of Cavitation in the Short Journal Bearing at $y = \beta$ for $\beta - \alpha = 2.03 \times 10^{-2}$ m, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.95.

Figure 10.124: Contour Plot of Cavitation in the Long Journal Bearing at $y = \beta$ for $\beta - \alpha = 2.03 \times 10^{-2}$ m, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.41.
This case is interesting in that the cavitated region for the short bearing is much smaller than that of the long bearing. The cavitated region for the short bearing for the thickest porous medium is quite small and is contained completely inside the diverging region of the bearing (Figure 10.123), whereas the cavitated region for the long bearing for the thickest porous medium consumes a majority of the area of interest, inside both the converging and diverging regions (Figure 10.124).

We observe the interesting trend that as we increase the porous medium thickness, the size of the cavitated region for the short bearing decreases and gravitates toward the center of the diverging region, while the size of the cavitated region for the long bearing increases and consumes more and more of the area of interest.

Contour Plots of Cavitation for the Parameter $k_y$, the Permeability of the Porous Medium in the $y$-direction

Next, we turn our attention to the $y$-direction permeability, $k_y$, and how varying its value changes the cavitation contour plot at $y = \beta$.

Figures 10.125 and 10.126 show the cavitated areas for the short and long journal bearings, respectively, inside the fluid film region for $k_y = 1 \times 10^{-10}$ m$^2$. Note here how much smaller the cavitated regions are here compared to previous comparisons. Again, we note that the cavitated region for the short journal bearing is considerably smaller than that of the long journal bearing, representing a common theme in this analysis.
Figure 10.125: Contour Plot of Cavitation in the Short Journal Bearing at $y = \beta$ for $k_y = 1 \times 10^{-10}$ m$^2$, $\beta - \alpha^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.68.

Figure 10.126: Contour Plot of Cavitation in the Long Journal Bearing at $y = \beta$ for $k_y = 1 \times 10^{-10}$ m$^2$, $\beta - \alpha^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.14.
Figure 10.127: Contour Plot of Cavitation in the Short Journal Bearing at $y = \beta$ for $k_y = 1 \times 10^{-11} \text{ m}^2$, $\beta - \alpha^* = 2.54 \times 10^{-3} \text{ m}$, $U^* = 4.57 \text{ m/s}$, $\alpha^* = 1.27 \times 10^{-4} \text{ m}$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.71.

Figure 10.128: Contour Plot of Cavitation in the Long Journal Bearing at $y = \beta$ for $k_y = 1 \times 10^{-11} \text{ m}^2$, $\beta - \alpha^* = 2.54 \times 10^{-3} \text{ m}$, $U^* = 4.57 \text{ m/s}$, $\alpha^* = 1.27 \times 10^{-4} \text{ m}$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.17.
Figures 10.127 and 10.128 show the cavitated areas for the short and long journal bearings, respectively, inside the fluid film region for \( k_y = 1 \times 10^{-11} \text{ m}^2 \). Here we see that decreasing the permeability increases the cavitated regions for both the short and long journal bearings. We also observe that the cavitated region for the short journal bearing is still contained inside the diverging region of the bearing, whereas, for the long journal bearing, the cavitated region consumes part of both the converging and diverging regions.

Finally, we skip to the lowest permeability value considered in this work, \( k_y = 1 \times 10^{-13} \text{ m}^2 \).

Figures 10.129 and 10.130 show the cavitated areas for the short and long journal bearings, respectively, inside the fluid film region for \( k_y = 1 \times 10^{-13} \text{ m}^2 \), representing the lowest permeability case for this study. Here, we see that decreasing the permeability to such a low level dramatically increases the size of the cavitated region inside the long journal bearing to a level where the cavitated region consumes a vast majority of the area of interest. Also, for the short journal bearing case, Figure 10.129, we see that the cavitation region has grown considerably and has extended into the converging region of the bearing geometry. It is interesting to note, however, that the contour is not completely flat at the film reformation for the long journal bearing, indicating a slight axial (\( z \)-) dependence on the cavitation contour. In earlier analyses, including the cavitation contour plot for \( k_y = 1.0 \times 10^{-11} \text{ m}^2 \) for the long
journal bearing, Figure 10.128, the cavitation contour at the film reformation was essentially constant for all z-values.

Contour Plots of Cavitation for the Parameter $U$, the Linear Speed of the Journal

We conclude our analysis of the cavitation contour plots by examining what effect the linear speed of the journal, $U$, has on the cavitated regions. Results from above indicate that as the speed of the journal increases, the cavitation region also increases in size. We now investigate and quantify this claim.

Figures 10.131 and 10.132 show the cavitated areas for the short and long journal bearings, respectively, inside the fluid film region for $U = 1.14$ m/s. As we alluded to, earlier, the cavitation regions for the slowest speed case are quite small, whereby the cavitation regions in both the short and long journal bearings are completely contained inside the diverging region.
Figure 10.129: Contour Plot of Cavitation in the Short Journal Bearing at $y = \beta$ for $k_y = 1 \times 10^{-13}$ m$^2$, $\beta - \alpha^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.77.

Figure 10.130: Contour Plot of Cavitation in the Long Journal Bearing at $y = \beta$ for $k_y = 1 \times 10^{-13}$ m$^2$, $\beta - \alpha^* = 2.54 \times 10^{-3}$ m, $U^* = 4.57$ m/s, $\alpha^* = 1.27 \times 10^{-4}$ m. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.23.
Figure 10.131: Contour Plot of Cavitation in the Short Journal Bearing at $y = \beta$ for $U = 1.14$ m/s, $\beta - \alpha^* = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.98.

Figure 10.132: Contour Plot of Cavitation in the Long Journal Bearing at $y = \beta$ for $U = 1.14$ m/s, $\beta - \alpha^* = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.44.
Figure 10.133: Contour Plot of Cavitation in the Short Journal Bearing at $y = \beta$ for $U = 2.28$ m/s, $\beta - \alpha^* = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.101.

Figure 10.134: Contour Plot of Cavitation in the Long Journal Bearing at $y = \beta$ for $U = 2.28$ m/s, $\beta - \alpha^* = 2.54 \times 10^{-3}$ m, $\alpha^* = 1.27 \times 10^{-4}$ m, $k_y^* = 1 \times 10^{-12}$ m$^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.47.
Figures 10.133 and 10.134 show the cavitated areas for the short and long journal bearings, respectively, inside the fluid film region for $U = 2.28$ m/s. As expected the cavitation regions grow with the increase in $U$, with the cavitation region for the long journal bearing entering the converging region of the bearing geometry. Note that the cavitation region for the short journal bearing is still contained inside the diverging region.

We skip to the highest velocity considered in this study, $U = 9.14$ m/s, to confirm our earlier conjecture that the size of the cavitated region increases in size as the speed of the journal increases.

Figures 10.135 and 10.136 show the cavitated areas for the short and long journal bearings, respectively, inside the fluid film region for $U = 9.14$ m/s. As expected, the cavitated regions are substantially larger in both the short and long bearing geometries. The cavitated region in the short journal bearing consumes more than half of the area of interest, and the cavitated region in the long journal bearing consumes an even larger percentage of the area. We also note a common theme that the contour of the cavitated region, at the film relocation, features an axial dependence in the case of the short journal bearing (Figure 10.135), whereas the contour of the cavitated region, at the film relocation, is essentially flat in the axial direction in the case of the long journal bearing (Figure 10.136).

Examining the contour plots of the cavitated regions gives one insight to how the size and shape of the cavitated region changes with each parameter. Though
we had a feel for this dependence by examining the pressure plots in Figures 10.1 -
10.108, the inability to interactively rotate the graphs in this work meant that certain
regions were hidden from view, obscuring the locations of where the film cavitated
and reformed. This 'top-down' view enables one to see the exact region of cavitation
and adds insight to the similarities and differences between the short and long journal
bearings.

10.2.4 Long Journal Bearing - Velocity Profiles

We now turn our attention to examining the velocity profiles inside the long journal
bearing. As one examines the pressure profiles seen for the long journal case, one
can begin to envision how the fluid should flow inside the bearing. As a reminder
of the concept of this bearing geometry, we see in Figure (10.137) that the pressure
differences between the two fluid regions circulates the flow inside the bearing.

For the sake of brevity, only several simulations will be featured here that demonstrate
the fluid velocity profiles, but they will serve as representative choices that cover the
spectrum of behaviors seen in this analysis.

Velocity Profile - Benchmark Case

We start with examining the velocity fields for the benchmark case. First we look
at \( u \), the \( x \)-direction velocity, for the entire bearing geometry at \( z = L_z/2 \), Figure
(10.138). We note several key features from this figure. First, that although the
reservoir wall and porous medium wall are stationary, there is still a pressure gradi-
ent inside the reservoir wall, with both negative and positive \( x \)-direction velocities,
indicating a flow reversal, in the $x$-direction, inside the reservoir. Secondly, the veloc-ity inside the film increases in a linear manner toward the journal wall, however, there is a slight hiccup in the curve at the location where the film recombines from its cavitated state.

Next, we look at $w$, the $z$-direction velocity, for the entire bearing geometry at several grid points along the axial direction (approximately $z = L_z/4$, $z = L_z/3$, $z = 2L_z/3$, and $z = 3L_z/4$, respectively, Figures (10.139) - (10.142).

Note that Figures (10.139) and (10.142) are mirror images of each other, as well as are Figures (10.140) and (10.141), which is to be expected, since the pairs of images are equidistant from $z = L_z/2$. We also note that for the interior figures, Figures (10.140) and (10.141), the magnitudes of the velocities are much lower than those of the exterior figures, Figures (10.139) and (10.142), which is also to be expected. Finally, we note that Figures (10.139) and (10.140) indicate that, since the fluid velocity changes from positive to negative in the $z$-direction, and vice versa for Figures (10.141) and (10.142), there is circulation in the $z$-direction as well. This is a byproduct of the no slip, no penetration boundary conditions imposed at the axial ends of the bearing geometry.

Finally, we examine the $y$-direction velocity, $v$, inside the bearing geometry for $z = L_z/2$, Figure (10.143).
Numerical oscillations aside, a byproduct of the sharp derivatives from the cavitation model used, we can see that the $y$-direction velocity changes from positive to negative and back to positive, along the $x$-axis, indicating that the fluid is indeed being transferred between the active film region and the fluid reservoir. Coupled with the results of Figure (10.138), which indicate a reversal of flow in the reservoir and a slowing of flow before the minimum clearance, we can piece together the resulting fluid flow story: as the journal rotates, at maximum clearance, the fluid flow in the reservoir and porous medium is stagnant and is moving solely in the $x$-direction inside the film. As the film recombines and begins to build pressure before the minimum clearance, the fluid is drawn from the film into the porous medium and then into the reservoir, indicated by the increase in velocity in the reservoir at the same location as the decrease in velocity in the film. Then, after the minimum clearance, the fluid is drawn back into the film, indicated by the positive velocity in the $y$-direction inside the film. As the geometry of this bearing is periodic, this action repeats periodically, with the rotation of the journal acting to pump the fluid between the two regions, with the porous medium acting as a gateway.

Another interesting comparison is the velocities for differing permeabilities. Here, we examine the $y$-direction velocities for $k_y = 1.0 \times 10^{-11}$ m$^2$, Figure (10.144), and $k_y = 1.0 \times 10^{-13}$ m$^2$, Figure (10.145). This is an interesting comparison, since, though the latter case yields higher pressures and pressure differences, the corresponding low permeability of the porous medium counteracts the pressure differences between the fluid regions in determining the rate of fluid circulation determined by

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Darcy’s law.

Here we see the interesting interplay between pressures and permeabilities, and the tradeoff between the higher load-carrying capacity/lower fluid circulation of the low permeability bearing, Figure (10.145), versus the lower load-carrying capacity/higher fluid circulation of the high permeability bearing, Figure (10.144). An engineer will have to keep these kinds of nuances in mind when designing a bearing of this configuration.

10.2.5 Short Journal Bearing - Velocity Profiles

For the short journal bearing, we only feature the benchmark case to highlight the key difference between the long and short journal bearing cases. We begin by examining the $x$-direction velocity, $u$, Figure (10.146).

Note the distinction between this short journal case and the long journal case, Figure (10.138). The pressure gradient inside the reservoir is much more pronounced here, as we see a markedly higher flow rate inside the reservoir.

Now let us examine the differences in the $z$-direction velocities, Figures (10.147) - (10.150), as we assume these to be much higher than in the long journal bearing case. The velocities are taken at the exact same sample points as those for Figures (10.139) - (10.142).
Indeed, the z-direction velocities are much higher for the short journal cases than in the long journal cases, indicating a much stronger axial flow.

We now turn our attention to the y-direction velocity profile to see if there is a distinct difference between the short and long journal bearings for the benchmark case.

It turns out that there is very little to distinguish the y-direction velocity profiles for the long and short journal bearing geometries. Thus, the main distinction between the two is that the short journal bearing features a parabolic (in the axial direction) pressure profile whereas the pressure profile for the long journal bearing is mostly constant, axially. The difference is that the long journal bearing supports more of a load whereas the short journal bearing supports higher flow rates between the three regions. This is a common theme in this work: load-carrying capacity vs. fluid flow circulation. Again, a designer must be cognizant of this fact when designing a bearing of this geometry.
Figure 10.135: Contour Plot of Cavitation in the Short Journal Bearing at $y = \beta$ for $U = 9.14 \text{ m/s}$, $\beta - \alpha^* = 2.54 \times 10^{-3} \text{ m}$, $\alpha^* = 1.27 \times 10^{-4} \text{ m}$, $k'_y = 1 \times 10^{-12} \text{ m}^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.107.

Figure 10.136: Contour Plot of Cavitation in the Long Journal Bearing at $y = \beta$ for $U = 9.14 \text{ m/s}$, $\beta - \alpha^* = 2.54 \times 10^{-3} \text{ m}$, $\alpha^* = 1.27 \times 10^{-4} \text{ m}$, $k'_y = 1 \times 10^{-12} \text{ m}^2$. All other pertinent values are given in Tables 8.1 and 8.2. This figure corresponds to Figure 10.53.
Figure 10.137: The pressure differences between the two fluid regions circulates the flow throughout the bearing.

Figure 10.138: x-direction velocity profile for the long bearing geometry at $z = L_z/2$, utilizing benchmark values. All relevant physical and geometrical values are given in Tables 8.1 and 8.2.
Figure 10.139: $z$-direction velocity profile for the long bearing geometry at $z \approx L_z/4$, utilizing benchmark values. All relevant physical and geometrical values are given in Tables 8.1 and 8.2.

Figure 10.140: $z$-direction velocity profile for the long bearing geometry at $z \approx L_z/3$, utilizing benchmark values. All relevant physical and geometrical values are given in Tables 8.1 and 8.2.
Figure 10.141: $z$-direction velocity profile for the long bearing geometry at $z \approx 2L_z/3$, utilizing benchmark values. All relevant physical and geometrical values are given in Tables 8.1 and 8.2.

Figure 10.142: $z$-direction velocity profile for the long bearing geometry at $z \approx 3L_z/4$, utilizing benchmark values. All relevant physical and geometrical values are given in Tables 8.1 and 8.2.
Figure 10.143: $y$-direction velocity profile for the long bearing geometry at $z = L_z/2$, utilizing benchmark values. All relevant physical and geometrical values are given in Tables 8.1 and 8.2.

Figure 10.144: $y$-direction velocity profile for the long bearing geometry at $z = L_z/2$, $k_y = 1.0 \times 10^{-11}$ m$^2$, utilizing all other benchmark values. All relevant physical and geometrical values are given in Tables 8.1 and 8.2.
Figure 10.145: \( y \)-direction velocity profile for the long bearing geometry at \( z = L_z/2 \), \( k_y = 1.0 \times 10^{-13} \text{ m}^2 \), utilizing all other benchmark values. All relevant physical and geometrical values are given in Tables 8.1 and 8.2.

Figure 10.146: \( x \)-direction velocity profile for the short bearing geometry at \( z = L_z/2 \), utilizing benchmark values. All relevant physical and geometrical values are given in Tables 8.1 and 8.2.
Figure 10.147: z-direction velocity profile for the short bearing geometry at $z \approx L_z/4$, utilizing benchmark values. All relevant physical and geometrical values are given in Tables 8.1 and 8.2.

Figure 10.148: z-direction velocity profile for the short bearing geometry at $z \approx L_z/3$, utilizing benchmark values. All relevant physical and geometrical values are given in Tables 8.1 and 8.2.
Figure 10.149: z-direction velocity profile for the short bearing geometry at \( z \approx 2L_z/3 \), utilizing benchmark values. All relevant physical and geometrical values are given in Tables 8.1 and 8.2.

Figure 10.150: z-direction velocity profile for the short bearing geometry at \( z \approx 3L_z/4 \), utilizing benchmark values. All relevant physical and geometrical values are given in Tables 8.1 and 8.2.
Figure 10.151: $y$-direction velocity profile for the short bearing geometry at $z = L_z/2$, utilizing benchmark values. All relevant physical and geometrical values are given in Tables 8.1 and 8.2.
The governing equation used to describe the thermal characteristics of the bearing is the steady-state local thermal equilibrium equation, valid for every region inside the bearing geometry. We may justify this choice by stating that the porous medium to be used features the characteristic that its thermal conductivity is roughly that of the lubricating fluid. Though this may not always be the case, the methods used here may be easily extrapolated to that of the local thermal non-equilibrium model.

11.1 Governing Equations

The governing thermal equation, taken from Lu [28], may be written as

$$\rho c_v \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) + \rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) =$$

$$= k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) - \left( \frac{2\mu}{3} \right) \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)^2 +$$

$$+ 2\mu \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] +$$

$$+ \left( \frac{\mu}{2} \right) \left[ \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 \right].$$

(11.1)

In order to implement a comprehensive parametric study, Equation (11.1) is non-dimensionalized by adopting the following relationship between the dimensional and
non-dimensional variables:

\[ u = UU^* \]  \hspace{1cm} (11.2)

\[ v = \left( \frac{\alpha}{L_x} \right) UV^* \]  \hspace{1cm} (11.3)

\[ w = \left( \frac{L_z}{L_x} \right) UW^* \]  \hspace{1cm} (11.4)

\[ x = L_x X^* \]  \hspace{1cm} (11.5)

\[ y = \alpha Y^* \]  \hspace{1cm} (11.6)

\[ z = L_z Z^* \]  \hspace{1cm} (11.7)

\[ p = \left( \frac{L_x \mu U}{\alpha^2} \right) P^* \]  \hspace{1cm} (11.8)

\[ T = \overline{T}T^*. \]  \hspace{1cm} (11.9)

Then, Equation (11.1) may be rewritten as

\[
\left( \frac{\rho_c U \alpha^2}{k L_x} \right) \left( U^* \frac{\partial T^*}{\partial X^*} + V^* \frac{\partial T^*}{\partial Y^*} + W^* \frac{\partial T^*}{\partial Z^*} \right) + \left( \frac{\mu U^2}{k T} \right) P^* \left( \frac{\partial U^*}{\partial X^*} + \frac{\partial V^*}{\partial Y^*} + \frac{\partial W^*}{\partial Z^*} \right) = \\
= \left( \frac{\alpha}{L_x} \right)^2 \frac{\partial^2 T^*}{\partial X^2} + \left( \frac{\alpha}{L_x} \right)^2 \frac{\partial^2 T^*}{\partial Y^2} + \left( \frac{\alpha}{L_x} \right) \left( \frac{L_x}{L_z} \right)^2 \frac{\partial^2 T^*}{\partial Z^2} - \\
- \left( \frac{\alpha}{L_x} \right)^2 \left( \frac{2 \mu U^2}{3} \right) \left( \frac{\partial U^*}{\partial X^*} + \frac{\partial V^*}{\partial Y^*} + \frac{\partial W^*}{\partial Z^*} \right)^2 + \\
+ \left( \frac{\alpha}{L_x} \right)^2 \left( \frac{2 \mu U^2}{k T} \right) \left[ \left( \frac{\partial U^*}{\partial X^*} \right)^2 + \left( \frac{\partial V^*}{\partial Y^*} \right)^2 + \left( \frac{\partial W^*}{\partial Z^*} \right)^2 \right] + \\
+ \left( \frac{\mu U^2}{2k T} \right) \left[ \left( \frac{\partial U^*}{\partial Y^*} \right)^2 + 2 \left( \frac{\alpha}{L_x} \right)^2 \frac{\partial U^*}{\partial Y^*} \frac{\partial V^*}{\partial X^*} + \left( \frac{\alpha}{L_x} \right)^4 \left( \frac{\partial V^*}{\partial X^*} \right)^2 \right] + \\
+ \left( \frac{\alpha}{L_x} \right) \left( \frac{L_x}{L_z} \right)^2 \left( \frac{\partial U^*}{\partial Z^*} \right)^2 + 2 \left( \frac{\alpha}{L_x} \right)^2 \frac{\partial U^*}{\partial Z^*} \frac{\partial W^*}{\partial X^*} + \\
+ \left( \frac{\alpha}{L_x} \right)^2 \left( \frac{L_x}{L_z} \right)^2 \left( \frac{\partial W^*}{\partial X^*} \right)^2 + \left( \frac{\alpha}{L_x} \right)^2 \left( \frac{\alpha}{L_z} \right)^2 \left( \frac{\partial V^*}{\partial Z^*} \right)^2 + \\
+ 2 \left( \frac{\alpha}{L_x} \right)^2 \frac{\partial V^*}{\partial Z^*} \frac{\partial W^*}{\partial Y^*} + \left( \frac{L_z}{L_x} \right)^2 \left( \frac{\partial W^*}{\partial Y^*} \right)^2. \hspace{1cm} (11.10)
\]
Again, letting $\epsilon_x = \frac{\alpha}{L_x}$, we can rewrite Equation (11.10) as

$$
\epsilon_x \left( \frac{\rho c_\nu U \alpha}{k} \right) \left( U^* \frac{\partial T^*}{\partial X^*} + V^* \frac{\partial T^*}{\partial Y^*} + W^* \frac{\partial T^*}{\partial Z^*} \right) + \left( \frac{\mu U^2}{kT} \right) P^* \left( \frac{\partial U^*}{\partial X^*} + \frac{\partial V^*}{\partial Y^*} + \frac{\partial W^*}{\partial Z^*} \right) =
$$

$$
= \epsilon_x^2 \frac{\partial^2 T}{\partial X^2} + \epsilon_x^2 \frac{\partial^2 T}{\partial Y^2} + \epsilon_x^2 \left( \frac{L_x}{L_z} \right)^2 \frac{\partial^2 T}{\partial Z^2} -
$$

$$
- \epsilon_x^2 \left( \frac{2 \mu U^2}{3} \right) \left( \frac{\partial U^*}{\partial X^*} + \frac{\partial V^*}{\partial Y^*} + \frac{\partial W^*}{\partial Z^*} \right)^2 +
$$

$$
+ \epsilon_x^2 \left( \frac{2 \mu U^2}{kT} \right) \left( \frac{\partial U^*}{\partial X^*} + \frac{\partial V^*}{\partial Y^*} + \frac{\partial W^*}{\partial Z^*} \right)^2 +
$$

$$
+ \left( \frac{\mu U^2}{2kT} \right) \left[ \left( \frac{\partial U^*}{\partial Y^*} \right)^2 + 2 \epsilon_x^2 \frac{\partial U^*}{\partial Y^*} \frac{\partial V^*}{\partial X^*} + \epsilon_x^2 \left( \frac{\partial V^*}{\partial X^*} \right)^2 +
$$

$$
+ \epsilon_x^2 \left( \frac{L_x}{L_z} \right)^2 \left( \frac{\partial W^*}{\partial X^*} \right)^2 + \epsilon_x^2 \left( \frac{\alpha}{L_z} \right)^2 \left( \frac{\partial V^*}{\partial Z^*} \right)^2 +
$$

$$
+ 2 \epsilon_x^2 \frac{\partial V^*}{\partial Z^*} \frac{\partial W^*}{\partial Y^*} + \left( \frac{L_z}{L_x} \right)^2 \left( \frac{\partial W^*}{\partial Y^*} \right)^2 \right].
$$

Provided that $L_x$ and $L_z$ do not differ by several orders of magnitude so as to negate

the $\epsilon_x^2$ terms, and expanding $T^*$ in an asymptotic manner,

$$
T^* = T_0^* + \epsilon T_1^* + O(\epsilon_x^2)
$$

we may write the $O(1)$ and $O(\epsilon_x)$ problems:

Non-Dimensional Governing Equation at $O(1)$:

$$
\frac{\partial^2 T_0^*}{\partial Y^2} \left[ \left( \frac{\partial U^*}{\partial Y^*} \right)^2 + \left( \frac{L_z}{L_x} \right)^2 \left( \frac{\partial W^*}{\partial Y^*} \right)^2 \right] = \left( \frac{\mu U^2}{kT} \right) P^* \left( \frac{\partial U^*}{\partial X^*} + \frac{\partial V^*}{\partial Y^*} + \frac{\partial W^*}{\partial Z^*} \right)
$$

(11.13)

Non-Dimensional Governing Equation at $O(\epsilon_x)$:

$$
\frac{\partial^2 T_1^*}{\partial Y^2} = \left( \frac{\rho c_\nu U \alpha}{k} \right) \left( U^* \frac{\partial T_0^*}{\partial X^*} + V^* \frac{\partial T_0^*}{\partial Y^*} + W^* \frac{\partial T_0^*}{\partial Z^*} \right)
$$

(11.14)
Note that the \( O(1) \) equation contains conduction terms and the \( O(\varepsilon_x) \) equation contains convection terms. These terms dominate the governing equation, Equation (11.11), so we only consider their contributions.

Now that the dominating terms have been determined, we may rewrite Equations (11.13) and (11.14) in their dimensional forms:

\[
\frac{\partial^2 T_0}{\partial y^2} = \frac{1}{k} \left\{ p \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) - \left( \frac{\mu}{2} \right) \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right] \right\}
\]

(11.15)

\[
\frac{\partial^2 T_1}{\partial y^2} = \left( \frac{\rho c_v}{k} \right) \left( \frac{L_x}{\alpha} \right) \left( u \frac{\partial T_0}{\partial x} + v \frac{\partial T_0}{\partial y} + w \frac{\partial T_0}{\partial z} \right).
\]

(11.16)

11.2 Boundary Conditions

Akin to the work that was done in the momentum problems, we define \( T_I \) to be the temperature inside Region I (Reservoir), \( \bar{T} \) to be the temperature inside Region II (Porous Medium), and \( T_{III} \) to be the temperature inside Region III (Hydrodynamic Film). The governing equations, Equation (11.15) and (11.16) are subject to the following boundary conditions

@ \( y = 0 \),

\[
\frac{\partial T_I}{\partial y} = \left( \frac{h_I}{k_f} \right) (T_I - T_a)
\]

(11.17)

where \( h_I \) is the heat transfer coefficient at the reservoir wall, \( k_f \) is the thermal conductivity of the lubricating fluid, and \( T_a \) is the ambient temperature external to the reservoir wall.
\[ y = \alpha, \]

\[ T_I = \tilde{T} \quad \text{(11.18)} \]

\[ k_I \frac{\partial T_I}{\partial y} = \tilde{k} \frac{\partial \tilde{T}}{\partial y} \quad \text{(11.19)} \]

where \( \tilde{k} \) is the effective thermal conductivity of the fluid inside the porous medium.

\[ y = \beta, \]

\[ \tilde{T} = T_{III} \quad \text{(11.20)} \]

\[ \tilde{k} \frac{\partial \tilde{T}}{\partial y} = k_{III} \frac{\partial T_{III}}{\partial y} \quad \text{(11.21)} \]

To describe the boundary condition at \( y = \beta + h(x) \), we begin by considering the energy equation inside the solid journal, similar to the method used to describe the slider bearing case. Since there is no pressure or fluid velocity inside the material, the governing thermal equation inside the solid journal reduces to

\[ \frac{\partial^2 T_j}{\partial y^2} = 0, \quad \text{(11.22)} \]

where \( T_j \) is the temperature field of the solid journal.

If one were to consider the cylindrical journal bearing, where the journal has radius, \( r \), a reasonable condition to impose is that \( \frac{\partial T_j}{\partial r} = 0 \) at \( r = 0 \). Thus, in an unwrapped geometry, the related boundary condition would be \( \frac{\partial T_j}{\partial y} = 0 \) at \( y = \beta + h(x) + r \).

Now, for the unwrapped journal bearing case, which we are discussing here, instead of considering the above boundary condition at \( y = \beta + h(x) + r \), we simply consider the same boundary condition at the height, \( y = y_c \), with the subscript \( c \) denoting ‘center.’

Then, we have the governing thermal equation for the solid journal, Equation (11.22),
combined with the boundary condition at $y = y_c$,

$$\frac{\partial T_j}{\partial y} = 0, \quad (11.23)$$

to get that $T_j$ is a constant. Then, by imposing a boundary condition similar to that of Equation (11.19),

$$k_j \left. \frac{\partial T_j}{\partial y} \right|_{y = \beta + h(x)} = k_f \left. \frac{\partial T_{III}}{\partial y} \right|_{y = \beta + h(x)}, \quad (11.24)$$

where $k_j$ is the thermal conductivity of the solid journal, it follows that our final boundary condition is that, at $y = \beta + h(x)$,

$$\frac{\partial T_{III}}{\partial y} = 0. \quad (11.25)$$

Define $H(x) = \alpha h(x)$ as a non-dimensional hydrodynamic film height. Then, non-dimensionalizing boundary conditions Equations (11.17) - (11.25) yields

@ $Y = 0$

$$\frac{\partial T_I^*}{\partial Y^*} = \left( \frac{\alpha h_I}{k_f} \right) \left( T_I^* - \frac{T_a}{T} \right), \quad (11.26)$$

@ $Y = 1$

$$T_I^* = \bar{T}^* \quad (11.27)$$

$$k_f \frac{\partial T_I^*}{\partial Y^*} = \tilde{k} \frac{\partial \bar{T}^*}{\partial Y^*} \quad (11.28)$$

@ $Y = \frac{\beta}{\alpha}$

$$\bar{T}^* = T_{III}^* \quad (11.29)$$

$$\tilde{k} \frac{\partial \bar{T}^*}{\partial Y^*} = k_f \frac{\partial T_{III}^*}{\partial Y^*} \quad (11.30)$$
\[ Y = \frac{\beta}{\alpha} + H(x) \]

\[ \frac{\partial T^*_{III}}{\partial Y^*} = 0. \quad (11.31) \]

We may now write the non-dimensional \( O(1) \) and \( O(\epsilon_x) \) boundary conditions:

@ \( Y = 0 \)

\[ \frac{\partial T^*_0}{\partial Y^*} = \left( \frac{\alpha h_I}{k_f} \right) \left( T^*_0 - \frac{T_a}{T} \right) \quad (11.32) \]

\[ \frac{\partial T^*_{I1}}{\partial Y^*} = \left( \frac{\alpha h_I}{k_f} \right) T^*_{I1} \quad (11.33) \]

@ \( Y = 1 \)

\[ T^*_0 = \tilde{T}^*_1 \quad (11.34) \]

\[ k_f \frac{\partial T^*_0}{\partial Y^*} = \tilde{k} \frac{\partial \tilde{T}^*_0}{\partial Y^*} \quad (11.35) \]

\[ T^*_1 = \tilde{T}^*_1 \quad (11.36) \]

\[ k_f \frac{\partial T^*_1}{\partial Y^*} = \tilde{k} \frac{\partial \tilde{T}^*_1}{\partial Y^*} \quad (11.37) \]

@ \( Y = \frac{\beta}{\alpha} \)

\[ \tilde{T}^*_0 = T^*_{III0} \quad (11.38) \]

\[ \tilde{k} \frac{\partial \tilde{T}^*_0}{\partial Y^*} = k_f \frac{\partial T^*_{III0}}{\partial Y^*} \quad (11.39) \]

\[ \tilde{T}^*_1 = T^*_{III1} \quad (11.40) \]

\[ \tilde{k} \frac{\partial \tilde{T}^*_1}{\partial Y^*} = k_f \frac{\partial T^*_{III1}}{\partial Y^*} \quad (11.41) \]

@ \( Y = \frac{\beta}{\alpha} + H(x) \)

\[ \frac{\partial T^*_{III0}}{\partial Y^*} = 0 \quad (11.42) \]

\[ \frac{\partial T^*_{III1}}{\partial Y^*} = 0. \quad (11.43) \]
Now, rewriting boundary conditions Equations (11.32) - (11.43) in dimensional form, we have our final boundary conditions

@ \( y = 0 \)

\[
\frac{\partial T_{I0}}{\partial y} = \left( \frac{h_I}{k_f} \right) (T_{I0} - T_a)
\]
(11.44)

\[
\frac{\partial T_{I1}}{\partial y} = \left( \frac{h_I}{k_f} \right) T_{I1},
\]
(11.45)

@ \( y = \alpha \)

\[
T_{I0} = \tilde{T}_0
\]
(11.46)

\[
k_f \frac{\partial T_{I0}}{\partial y} = \tilde{k} \frac{\partial \tilde{T}_0}{\partial y}
\]
(11.47)

\[
T_{I1} = \tilde{T}_1
\]
(11.48)

\[
k_f \frac{\partial T_{I1}}{\partial y} = \tilde{k} \frac{\partial \tilde{T}_1}{\partial y}
\]
(11.49)

@ \( y = \beta \)

\[
\tilde{T}_0 = T_{III0}
\]
(11.50)

\[
\tilde{k} \frac{\partial \tilde{T}_0}{\partial y} = k_f \frac{\partial T_{III0}}{\partial y}
\]
(11.51)

\[
\tilde{T}_1 = T_{III1}
\]
(11.52)

\[
\tilde{k} \frac{\partial \tilde{T}_1}{\partial y} = k_f \frac{\partial T_{III1}}{\partial y}
\]
(11.53)

@ \( y = \beta + h(x) \)

\[
\frac{\partial T_{III0}}{\partial y} = 0
\]
(11.54)

\[
\frac{\partial T_{III1}}{\partial y} = 0.
\]
(11.55)
Thus, we now write our $O(1)$ and $O(\varepsilon_x)$ governing equations, in dimensional form, together with their boundary conditions, also in dimensional form:

$O(1)$

Region I: (Reservoir)

$$\frac{\partial^2 T_{I0}}{\partial y^2} = \frac{1}{k_f} \left\{ p_I \left( \frac{\partial u_I}{\partial x} + \frac{\partial v_I}{\partial y} + \frac{\partial w_I}{\partial z} \right) - \left( \frac{\mu}{2} \right) \left[ \left( \frac{\partial u_I}{\partial y} \right)^2 + \left( \frac{\partial w_I}{\partial y} \right)^2 \right] \right\}$$

(11.56)

Region II: (Porous Medium)

$$\frac{\partial^2 \tilde{T}_0}{\partial y^2} = \frac{1}{k} \tilde{p} \left( \frac{\partial \tilde{v}}{\partial y} \right)$$

(11.57)

Region III: (Hydrodynamic Fluid Film)

$$\frac{\partial^2 T_{III0}}{\partial y^2} = \frac{1}{k_f} \left\{ p_{III} \left( \frac{\partial u_{III}}{\partial x} + \frac{\partial v_{III}}{\partial y} + \frac{\partial w_{III}}{\partial z} \right) - \left( \frac{\mu}{2} \right) \left[ \left( \frac{\partial u_{III}}{\partial y} \right)^2 + \left( \frac{\partial w_{III}}{\partial y} \right)^2 \right] \right\}$$

(11.58)

subject to the boundary conditions,

$$\frac{\partial T_{I0}}{\partial y} \bigg|_{y=0} = \left( \frac{h_I}{k_f} \right) \left( T_{I0} \bigg|_{y=0} - T_a \right)$$

(11.59)

$$T_{I0} \bigg|_{y=\alpha} = \tilde{T}_0 \bigg|_{y=\alpha}$$

(11.60)

$$k_f \frac{\partial T_{I0}}{\partial y} \bigg|_{y=\alpha} = \tilde{k} \frac{\partial \tilde{T}_0}{\partial y} \bigg|_{y=\alpha}$$

(11.61)

$$\tilde{T}_0 \bigg|_{y=\beta} = T_{III0} \bigg|_{y=\beta}$$

(11.62)

$$\tilde{k} \frac{\partial \tilde{T}_0}{\partial y} \bigg|_{y=\beta} = k_f \frac{\partial T_{III0}}{\partial y} \bigg|_{y=\beta}$$

(11.63)

$$\frac{\partial T_{III0}}{\partial y} \bigg|_{y=\beta + h(x)} = 0$$

(11.64)
\[ O(\varepsilon_x) \]

Region I: (Reservoir)

\[
\frac{\partial^2 T_{I1}}{\partial y^2} = \left( \frac{\rho_I c_v}{k_f} \right) \left( \frac{L_x}{\alpha} \right) \left( u_I \frac{\partial T_{I0}}{\partial x} + v_I \frac{\partial T_{I0}}{\partial y} + w_I \frac{\partial T_{I0}}{\partial z} \right) \tag{11.65}
\]

Region II: (Porous Medium)

\[
\frac{\partial^2 \tilde{T}_1}{\partial y^2} = \left( \frac{\tilde{\rho} c_v}{k} \right) \left( \frac{L_x}{\alpha} \right) \left( \tilde{v} \frac{\partial \tilde{T}_0}{\partial y} \right) \tag{11.66}
\]

Region III: (Hydrodynamic Fluid Film)

\[
\frac{\partial^2 T_{III1}}{\partial y^2} = \left( \frac{\rho_{III} c_v}{k_f} \right) \left( \frac{L_x}{\alpha} \right) \left( u_{III} \frac{\partial T_{III0}}{\partial x} + v_{III} \frac{\partial T_{III0}}{\partial y} + w_{III} \frac{\partial T_{III0}}{\partial z} \right) \tag{11.67}
\]

subject to the boundary conditions

\[
\left. \frac{\partial T_{I1}}{\partial y} \right|_{y=0} = \left( \frac{h_I}{k_f} \right) T_{I1} \tag{11.68}
\]

\[
T_{I1} \bigg|_{y=\alpha} = \tilde{T}_1 \bigg|_{y=\alpha} \tag{11.69}
\]

\[
k_f \frac{\partial T_{I1}}{\partial y} \bigg|_{y=\alpha} = \tilde{k} \frac{\partial \tilde{T}_1}{\partial y} \bigg|_{y=\alpha} \tag{11.70}
\]

\[
\tilde{T}_1 \bigg|_{y=\beta} = T_{III1} \bigg|_{y=\beta} \tag{11.71}
\]

\[
\tilde{k} \frac{\partial \tilde{T}_1}{\partial y} \bigg|_{y=\beta} = k_f \frac{\partial T_{III1}}{\partial y} \bigg|_{y=\beta} \tag{11.72}
\]

\[
\left. \frac{\partial T_{III1}}{\partial y} \right|_{y=\beta + h(x)} = 0. \tag{11.73}
\]

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CHAPTER XII

NUMERICAL TECHNIQUES AND RESULTS - JOURNAL BEARING -

THERMAL EQUATIONS

12.1 Numerical Implementation for Thermal Equations

To orient the reader on the computational domains used for the thermal calculations, we define the boundary, \( y = 0 \) to correspond to the index \( j = 1 \), the boundary, \( y = \alpha \) to correspond to the index \( j = j_\alpha \), the boundary \( y = \beta \) to correspond to the index \( j = j_\beta \), and the boundary \( y = \beta + h(x) \) to correspond to the index \( j = J \), not to be confused with the computational domain index, \( J \) from the momentum equations. Note that for the momentum equations, the pressures only had to be calculated inside the porous medium and at the porous medium boundaries, \( y = \alpha \) and \( y = \beta \), whereas for the thermal equations, the temperatures have to be calculated at every point inside the bearing.

Though the analytical formulation of the governing thermal equations defined three separate temperature variables, \( T_I, \tilde{T}, \text{ and } T_{III} \), for the numerical procedure defined in this section, we define a single temperature variable \( T \) whose indices in the \( y \)-direction vary from \( j = 1(y = 0) \) to \( j = J(y = \beta + h(x)) \). Thus, we now define \( T_0 \) to be the temperature variable for the \( O(1) \) problem and \( T_1 \) to be the temperature
variable for the $O(\epsilon_x)$ problem. Also, since the only derivatives for $T$ to be discussed in this work are the $y$-direction derivatives, we define $T_{0,j}$ to be the discretized value of $T_0$ at the $y$-gridpoint $j$, not the discretized value of $T$ at the $x$-gridpoint 0 and the $y$-gridpoint $j$. For the sake of keeping the discretizations clear, the $x(i)$ and $z(k)$ indices are not explicitly written, but they are implied.

Also, each region (I, II, and III) is defined to possess its own grid spacing in the $y$-direction, yielding a $\Delta y$ in each region. These are defined as $\Delta y_{res}$, $\Delta y_{por}$, and $\Delta y_{film}$, representing the grid spacing in the reservoir, porous medium, and film regions, respectively. Also note that the first two values for $\Delta y$ do not depend on the position of the node in the $x$-direction, but the value for $\Delta y$ inside the fluid film region does depend on position of the node in the $x$-direction, due to the shape of the height profile, $h(x)$.

Second order finite difference schemes were used to discretize every derivative
term in the governing equations,

\[
\frac{\partial^2 T_{I0}}{\partial y^2} = \frac{1}{k_f} \left\{ p_I \left( \frac{\partial u_I}{\partial x} + \frac{\partial v_I}{\partial y} + \frac{\partial w_I}{\partial z} \right) - \left( \frac{\mu}{2} \right) \left[ \left( \frac{\partial u_I}{\partial y} \right)^2 + \left( \frac{\partial w_I}{\partial y} \right)^2 \right] \right\} \quad (12.1)
\]

\[
\frac{\partial^2 \tilde{T}_0}{\partial y^2} = \frac{1}{\tilde{k}} \left[ \tilde{\rho} \left( \frac{\partial \tilde{v}}{\partial y} \right) \right] \quad (12.2)
\]

\[
\frac{\partial^2 T_{III0}}{\partial y^2} = \frac{1}{k_f} \left\{ p_{III} \left( \frac{\partial u_{III}}{\partial x} + \frac{\partial v_{III}}{\partial y} + \frac{\partial w_{III}}{\partial z} \right) - \left( \frac{\mu}{2} \right) \left[ \left( \frac{\partial u_{III}}{\partial y} \right)^2 + \left( \frac{\partial w_{III}}{\partial y} \right)^2 \right] \right\} \quad (12.3)
\]

\[
\frac{\partial^2 T_{I1}}{\partial y^2} = \left( \frac{\rho_{Ic_v}}{k_f} \right) \left( \frac{L_x}{\alpha} \right) \left( u_{I1} \frac{\partial T_{I0}}{\partial x} + v_{I1} \frac{\partial T_{I0}}{\partial y} + w_{I1} \frac{\partial T_{I0}}{\partial z} \right) \quad (12.4)
\]

\[
\frac{\partial^2 \tilde{T}_1}{\partial y^2} = \left( \frac{\tilde{\rho}_c v}{\tilde{k}} \right) \left( \frac{L_x}{\tilde{\alpha}} \right) \left( \tilde{v} \frac{\partial \tilde{T}_0}{\partial y} \right) \quad (12.5)
\]

\[
\frac{\partial^2 T_{III1}}{\partial y^2} = \left( \frac{\rho_{IIIc_v}}{k_f} \right) \left( \frac{L_x}{\alpha} \right) \left( u_{III} \frac{\partial T_{III0}}{\partial x} + v_{III} \frac{\partial T_{III0}}{\partial y} + w_{III} \frac{\partial T_{III0}}{\partial z} \right) \quad (12.6)
\]
and the boundary conditions,

\[
\frac{\partial T_{I0}}{\partial y} \bigg|_{y=0} = \left( \frac{h_I}{k_f} \right) \left( T_{I0} \bigg|_{y=0} - T_a \right) \tag{12.7}
\]

\[
T_{I0} \bigg|_{y=0} = \tilde{T}_0 \bigg|_{y=0} \tag{12.8}
\]

\[
k_f \frac{\partial T_{I0}}{\partial y} \bigg|_{y=0} = \tilde{k} \frac{\partial \tilde{T}_0}{\partial y} \bigg|_{y=0} \tag{12.9}
\]

\[
\tilde{T}_0 \bigg|_{y=\beta} = T_{III0} \bigg|_{y=\beta} \tag{12.10}
\]

\[
\tilde{k} \frac{\partial \tilde{T}_0}{\partial y} \bigg|_{y=\beta} = k_f \frac{\partial T_{III0}}{\partial y} \bigg|_{y=\beta} \tag{12.11}
\]

\[
\frac{\partial T_{III0}}{\partial y} \bigg|_{y=\beta+h(x)} = 0 \tag{12.12}
\]

\[
\frac{\partial T_{II1}}{\partial y} \bigg|_{y=0} = \left( \frac{h_I}{k_f} \right) T_{II1} \tag{12.13}
\]

\[
T_{II1} \bigg|_{y=0} = \tilde{T}_1 \bigg|_{y=\alpha} \tag{12.14}
\]

\[
k_f \frac{\partial T_{II1}}{\partial y} \bigg|_{y=\alpha} = \tilde{k} \frac{\partial \tilde{T}_1}{\partial y} \bigg|_{y=\alpha} \tag{12.15}
\]

\[
\tilde{T}_1 \bigg|_{y=\beta} = T_{III1} \bigg|_{y=\beta} \tag{12.16}
\]

\[
\tilde{k} \frac{\partial \tilde{T}_1}{\partial y} \bigg|_{y=\beta} = k_f \frac{\partial T_{III1}}{\partial y} \bigg|_{y=\beta} \tag{12.17}
\]

\[
\frac{\partial T_{III1}}{\partial y} \bigg|_{y=\beta+h(x)} = 0. \tag{12.18}
\]

Note that the left hand sides of the above equations, Equations (12.1) - (12.18), are identical to the governing equations and boundary conditions from the long slider bearing problem, so the discretization techniques developed above apply here and do not need further discussion. The only differences lie in how the z-direction velocities, \(w_I\) and \(w_{III}\) are discretized at the axial boundaries (\(z = 0, z = L_z\)).
As before, we use two point centered differences on all velocity gradient terms, i.e.,

\[ \frac{\partial u}{\partial x} = \frac{u_{I,i+1} - u_{I,i-1}}{2\Delta x} \]
\[ \frac{\partial v}{\partial y} = \frac{v_{III,j+1} - u_{I,j-1}}{2\Delta y_{film}} \],

e etc., except for \( \frac{\partial w}{\partial z} \) at the axial boundaries \( (z = 0, z = L_z) \), where a three point forward/backward difference scheme is employed, using \( w_I = w_{III} = 0 \) at the axial ends \( (k = 0, k = K) \). This yields

\[ \left. \frac{\partial T_{0}}{\partial y} \right|_{y=\alpha} = \frac{3T_{0,\alpha} - 4T_{0,\alpha+1} + T_{0,\alpha-2}}{2\Delta y_{res}} \quad (12.19) \]
\[ \left. \frac{\partial \tilde{T}_0}{\partial y} \right|_{y=\alpha} = \frac{-3T_{0,\alpha} + 4T_{0,\alpha+1} - T_{0,\alpha+2}}{2\Delta y_{por}}. \quad (12.20) \]

We discretize Equation (12.9) using a three point backward difference for the term \( \left. \frac{\partial T_{0}}{\partial y} \right|_{y=\alpha} \) and a three point forward difference for the term \( \left. \frac{\partial \tilde{T}_0}{\partial y} \right|_{y=\alpha} \) to yield

\[ -T_{0,\alpha-2} + 4T_{0,\alpha-1} - \left[ 3 + 3 \left( \frac{\Delta y_{res}}{\Delta y_{por}} \right) \right] T_{0,\alpha} + \]
\[ + 4 \left( \frac{\Delta y_{res}}{\Delta y_{por}} \right) T_{0,\alpha+1} - \left( \frac{\Delta y_{res}}{\Delta y_{por}} \right) T_{0,\alpha+2} = 0. \quad (12.21) \]
Similarly, we discretize Equation (12.11) using a three point forward difference for the term \( \frac{\partial T_0}{\partial y} \bigg|_{y=\beta} \) and a three point backward difference for the term \( \frac{\partial T_0}{\partial y} \bigg|_{y=\beta} \) to yield

\[
\frac{\partial \tilde{T}_0}{\partial y} \bigg|_{y=\beta} = \frac{3T_{0,j+1} - 4T_{0,j} + T_{0,j-2}}{2\Delta y_{por}}
\]  

(12.22)

\[
\frac{\partial T_{II0}}{\partial y} \bigg|_{y=\beta} = \frac{-3T_{0,j+1} + 4T_{0,j} - T_{0,j+2}}{2\Delta y_{film}}.
\]  

(12.23)

This lets us write Equation (12.11) as

\[- \left( \frac{\tilde{k}}{k_f} \right) \left( \frac{\Delta y_{film}}{\Delta y_{por}} \right) T_{0,j+2} + 4 \left( \frac{\tilde{k}}{k_f} \right) \left( \frac{\Delta y_{film}}{\Delta y_{por}} \right) T_{0,j-1} -
\]

\[- \left[ 3 + 3 \left( \frac{\tilde{k}}{k_f} \right) \left( \frac{\Delta y_{film}}{\Delta y_{por}} \right) \right] T_{0,j} + 4T_{0,j+1} - T_{0,j+2} = 0.
\]  

(12.24)

Finally we discretize the boundary condition Equation (12.12) using a three point backward difference for the term \( \frac{\partial T_{II0}}{\partial y} \bigg|_{y=\beta+h(x)} \) to yield

\[
\frac{\partial T_{II0}}{\partial y} \bigg|_{y=\beta+h(x)} = \frac{3T_{0,J} - 4T_{0,J-1} + T_{0,J-2}}{2\Delta y_{film}}.
\]  

(12.25)

Thus, we may rewrite Equation (12.12) as

\[-T_{0,J} + 4T_{0,J-1} - \left[ 3 + \left( \frac{h_{II1}}{k_f} \right) \right] (2\Delta y_{film}) T_{0,J-2} = 0.
\]  

(12.26)

Note that the same techniques apply for the \( O(\epsilon_x) \) problem, so they will not be discussed. Derivation of the \( O(\epsilon_x) \) discretized equations is left to the reader.

For every \( i \) and \( k \), this technique yields two matrix equation systems of equations, one matrix system of equations for the \( O(1) \) problem and the other for the \( O(\epsilon_x) \) problem. The first system is of the form \( AT_0 = b \) where \( A \) is a pentadiagonal matrix of coefficients that is dependent on the value of the index, \( i \), \( T_0 \) is the \( O(1) \) temperature
vector in the $y$-direction, and $b$ is a vector of known coefficients from the solution of the momentum problem that depend on the values of all of the indices, $i, j,$ and $k$. The second system is of the form $AT_1 = b$ where $A$ is a pentadiagonal matrix of coefficients that is dependent on the value of the index, $i$, $T_1$ is the $O(\epsilon_x)$ temperature vector in the $y$-direction, and $b$ is a vector of known coefficients from the solution of the momentum problem and the calculated values of $T_0$. Note that the values that comprise the vector $b$ depend on the values of all of the indices, $i, j,$ and $k$.

The matrix systems of equations generated using the above techniques can be directly solved using a pentadiagonal solver. Unlike the momentum solution which required an iterative approach, the right hand sides of the matrix equations at each order use all known values, and so the values for $T_0$ and $T_1$ are found explicitly. The code for solving for the temperatures can be found in the appendix.

12.2 Results and Discussion

As the methods have already been discussed, and there is very little to distinguish the characteristics of the temperature profiles seen for the journal cases from the slider cases, we will only present several select cases for both the long and short journal bearings. Please note that all figures in this Chapter are along the axial centerline, as this is the location where there should be the highest temperatures and highest amount of fluid circulation between the film and reservoir regions, leading to the most interesting conduction and convection studies.
12.2.1 Benchmark Thermal Profiles - Long and Short Journal Bearings

First, we look at the thermal profiles for the benchmark case of the long journal bearing, utilizing a heat transfer coefficient of \( h_I = 250 \frac{W}{m^2K} \), Figures (12.1) - (12.3).

![Figure 12.1: Two-dimensional carpet plot of the leading order temperature, \( T_0 \), inside the long journal bearing system for effective heat transfer coefficient, \( h_I = 250 \frac{W}{m^2K} \). \( k_y = 1 \times 10^{-12} \) m², \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \) m, \( \alpha^* = 1.27 \times 10^{-4} \) m, \( U^* = 4.57 \) m/s. All other pertinent values are given in Tables 8.1 and 8.2.]

As was true for the slider, we see that the dominant force in the bearing is conduction, with the first order correction highlighting how the convection transfers the heat from the film into the reservoir. Next we look at the short journal bearing to see what, if any, differences exist between the short and journal bearing cases.
We see a slight difference in the leading order thermal profile, although the peak temperature values and locations remain the same for both cases. We also see a slight difference in that the short journal bearing appears to have more positive cooling effects. This can be seen in that the temperature increase inside the film, for the short journal, is less than 40 \( K \) with a corresponding temperature decrease in the reservoir of almost 60 \( K \). Contrast that with the long journal where both the peak temperature increase and decrease in the film and reservoir, respectively, is around 50 \( K \), Figures (12.5) and (12.2). As was stated in the previous chapter, this difference is due to the fact that the short journal bearing features a slightly more dominant flow circulation pattern than the long journal bearing (at the expense of overall load-carrying capacity). Overall though, there is not much differentiating the thermal
Figure 12.3: Two-dimensional carpet plot of the total temperature, $T$, inside the long journal bearing system for effective heat transfer coefficient, $h_I = 250 \text{ W/m}^2\text{K}$, $k_y^* = 1 \times 10^{-12} \text{ m}^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $\alpha^* = 1.27 \times 10^{-4} \text{ m}$, $U^* = 4.57 \text{ m/s}$. All other pertinent values are given in Tables 8.1 and 8.2.

profiles of the long and short journal bearings for this benchmark case.

12.2.2 Select Thermal Profile Studies - Long and Short Journal Bearings

As seen above, not much differentiates the long and short journal bearings from a thermal profile perspective. Thus, what remains to be shown is a few comparative studies that highlight what effects geometry changes have on the thermal profiles of the bearings.

First, let us examine the role of the reservoir depth on the thermal profile of the short journal bearing for an effective heat transfer coefficient of $h_I = 250 \text{ W/m}^2\text{K}$. Though not shown here, the leading order temperature profiles for all four reservoir depths that have been studied throughout this work, are nearly indistinguishable.
Where the differences lie are in the first correction temperatures, $\epsilon_x T_1$.

Here we see an interesting trend, whereby the convection effects increase with reservoir thickness, Figures (12.7) - (12.10), due to the additional fluid volume present inside the thicker reservoirs. We now examine the same phenomenon for the long journal bearing.

Both journal bearing configurations demonstrate the same convective heat transfer behavior.
Figure 12.5: Two-dimensional carpet plot of the first order correction temperature, $\epsilon_x T_1$, inside the short journal bearing system for effective heat transfer coefficient, $h_I = 250 \text{ W/m}^2\text{K}$. $k_y^* = 1 \times 10^{-12} \text{ m}^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $\alpha^* = 1.27 \times 10^{-4} \text{ m}$, $U^* = 4.57 \text{ m/s}$. All other pertinent values are given in Tables 8.1 and 8.2.

Now, we turn our attention to the effects of varying the depth of the porous medium on the thermal profile inside the journal bearing configurations. Though not shown, the convection effects are very similar among all six porous medium depths, with a peak temperature varying by no more than 20 K from the thinnest to the thickest porous medium for effective heat transfer coefficient $h_I = 250 \text{ W/m}^2\text{K}$. However, there is a noticeable difference between the conductive heat transfer effects for the thinnest and thickest porous media, Figures (12.15) and (12.16).

Here we see, that for the short journal bearing configuration, the thicker porous medium acts to insulate the reservoir from the film, as expected, leading to higher temperatures inside the film. Note that the thin porous medium configuration
allows for the heat to transfer from the journal wall to the reservoir wall with ease. The same effects can be seen for the long journal case, though those figures are not shown since they are very nearly identical to the short journal cases.

Though the thermal simulations were ran for every configuration seen throughout this paper, with varying effective heat transfer coefficients, $h_I$, not much more can be said. In every other case, the short and journal bearings acted in a nearly indistinguishable manner, and increasing the heat transfer coefficients adds nothing more to the analysis than was already described in Chapter 7.

That said, we can make some final remarks concerning the thermal profiles of the journal bearing configurations. First, what has been described here merely outlines the methods for calculating the temperatures inside the bearing configura-
Figure 12.7: Two-dimensional carpet plot of the first order correction temperature, $\epsilon_x T_1$, inside the short journal bearing system for effective heat transfer coefficient, $h_I = 250 \frac{W}{m^2 K}$. $k_y^* = 1 \times 10^{-12}$ m$^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3}$ m, $\alpha = 6.35 \times 10^{-5}$ m, $U^* = 4.57$ m/s. All other pertinent values are given in Tables 8.1 and 8.2.

tions discussed in this paper, under the limitations imposed herein (decoupled thermal/pressure solutions, neglecting higher order terms in the energy equation, a local thermal equilibrium equation approach for describing the heat transfer inside the porous medium, etc.). Extensions can be made to this method that will allow for more subtle nuances to be seen and helping to distinguish cases from each other where this present work does not. Secondly, we see that the long and short journal bearings behave very similarly to each other at their axial centerlines. What distinguishes the short and long journal bearings are the fluid flow characteristics and their pressure profiles away from their axial centerlines, since the long bearing maintains its pressure profile away from its centerline whereas the short bearing does not.
Figure 12.8: Two-dimensional carpet plot of the first order correction temperature, $\varepsilon_x T_1$, inside the short journal bearing system for effective heat transfer coefficient, $h_l = 250 \frac{W}{m^2K}$. $k_y^* = 1 \times 10^{-12} \text{ m}^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $\alpha = 1.27 \times 10^{-4} \text{ m}$, $U^* = 4.57 \text{ m/s}$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 12.9: Two-dimensional carpet plot of the first order correction temperature, $\varepsilon_x T_1$, inside the short journal bearing system for effective heat transfer coefficient, $h_l = 250 \frac{W}{m^2K}$. $k_y^* = 1 \times 10^{-12} \text{ m}^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $\alpha = 2.54 \times 10^{-4} \text{ m}$, $U^* = 4.57 \text{ m/s}$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 12.10: Two-dimensional carpet plot of the first order correction temperature, $\varepsilon_x T_1$, inside the short journal bearing system for effective heat transfer coefficient, $h_l = 250 \frac{W}{m^2K}$. $k_y^* = 1 \times 10^{-12} \text{ m}^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $\alpha = 5.08 \times 10^{-4} \text{ m}$, $U^* = 4.57 \text{ m/s}$. All other pertinent values are given in Tables 8.1 and 8.2.

Figure 12.11: Two-dimensional carpet plot of the first order correction temperature, $\varepsilon_x T_1$, inside the long journal bearing system for effective heat transfer coefficient, $h_l = 250 \frac{W}{m^2K}$. $k_y^* = 1 \times 10^{-12} \text{ m}^2$, $(\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m}$, $\alpha = 6.35 \times 10^{-5} \text{ m}$, $U^* = 4.57 \text{ m/s}$. All other pertinent values are given in Tables 8.1 and 8.2.
Figure 12.12: Two-dimensional carpet plot of the first order correction temperature, \( \varepsilon_x T_1 \), inside the long journal bearing system for effective heat transfer coefficient, \( h_l = 250 \frac{W}{m\cdot K} \). \( k_y^* = 1 \times 10^{-12} \text{ m}^2 \), \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m} \), \( \alpha = 1.27 \times 10^{-4} \text{ m} \), \( U^* = 4.57 \text{ m/s} \). All other pertinent values are given in Tables 8.1 and 8.2.

Figure 12.13: Two-dimensional carpet plot of the first order correction temperature, \( \varepsilon_x T_1 \), inside the long journal bearing system for effective heat transfer coefficient, \( h_l = 250 \frac{W}{m\cdot K} \). \( k_y^* = 1 \times 10^{-12} \text{ m}^2 \), \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \text{ m} \), \( \alpha = 2.54 \times 10^{-4} \text{ m} \), \( U^* = 4.57 \text{ m/s} \). All other pertinent values are given in Tables 8.1 and 8.2.
Figure 12.14: Two-dimensional carpet plot of the first order correction temperature, \( \epsilon_x T_1 \), inside the long journal bearing system for effective heat transfer coefficient, \( h_I = 250 \frac{W}{m^2 K} \). \( k_y^* = 1 \times 10^{-12} \, m^2 \), \( (\beta - \alpha)^* = 2.54 \times 10^{-3} \, m \), \( \alpha = 5.08 \times 10^{-4} \, m \), \( U^* = 4.57 \, m/s \). All other pertinent values are given in Tables 8.1 and 8.2.

Figure 12.15: Two-dimensional carpet plot of the leading order temperature, \( T_0 \), inside the short journal bearing system for effective heat transfer coefficient, \( h_I = 250 \frac{W}{m^2 K} \). \( k_y^* = 1 \times 10^{-12} \, m^2 \), \( (\beta - \alpha) = 1.27 \times 10^{-4} \, m \), \( \alpha^* = 1.27 \times 10^{-4} \, m \), \( U^* = 4.57 \, m/s \). All other pertinent values are given in Tables 8.1 and 8.2.
Figure 12.16: Two-dimensional carpet plot of the leading order temperature, $T_0$, inside the short journal bearing system for effective heat transfer coefficient, $h_L = 250 \text{W/m}^2\text{K}$. $k_y^* = 1 \times 10^{-12} \text{ m}^2$, $(\beta - \alpha) = 2.03 \times 10^{-2} \text{ m}$, $\alpha^* = 1.27 \times 10^{-4} \text{ m}$, $U^* = 4.57 \text{ m/s}$. All other pertinent values are given in Tables 8.1 and 8.2.
CHAPTER XIII

PRACTICAL DESIGN CONSIDERATIONS

If one were to design such a bearing as described in this work, one would need to specify the exact parameters of the bearing in order to achieve the desired performance. The parametric study here has given a relationship between quantifiable parameters and the performance of the bearing. Though the physical parameters of the bearing, $h_{\text{min}}, h_{\text{max}}, \alpha, \beta, L_x, L_z$ and the operational parameters of the bearing, $U, \mu, B, k_f$ can be specified to construct and operate the bearing, there are still two parameters that actively define the bearing that warrant further discussion. The parameters in question are the effective permeability of the porous material, $k_y$, and the effective heat transfer coefficient for the bearing housing, $h_I$. In this chapter, we discuss how these values are determined utilizing established expressions of known quantities that can be readily evaluated.

13.1 Effective Permeability

The permeability parameter, $k_y$ has underlying relationships that we have not discussed. We have treated $k_y$ as an all-encompassing flow conductivity value, whereas it is actually a function of both the geometry and the porosity of the porous material. Thus, in order to properly design a bearing to meet desired specifications, one must
know the geometry of the porous material, the porosity of the porous material, \( \phi \), and the relationship between those two values and the effective permeability, \( k_y \).

For example, Ergun proposed that

\[
k_y = \frac{d^2 \phi^3}{150 (1 - \phi)^2}
\]  

(13.1)

defines the relationship between the permeability and the porosity for a packed bed porous material with spheres of diameter \( d \) [29].

The equation

\[
k_y = \frac{\phi d^2}{32}
\]  

(13.2)

defines the relationship between the permeability and the porosity for a capillary-type porous material with average pore diameter, \( d \) [29]. Thus, if one chooses an effective permeability, \( k_y \), for this type of bearing, using the appropriate relationship between the permeability, the porosity, and the geometry of the porous material being used, one can define the exact parameters necessary for constructing the porous sleeve to be used in the bearing.

13.1.1 Example - Porous Material Using Capillary-Type Porous Material

For the work considered in this paper, we utilized a capillary-type porous material, where the capillaries extended in the radial direction (vertically, once the geometry was unwrapped). As such, the relationship between the effective permeability, \( k_y \), the porosity, \( \phi \), and the capillary (pore) diameter, \( d \), should be given by Equation (13.2). Consider the benchmark permeability, \( k_y = 1 \times 10^{-12} \text{ m}^2 \), in a capillary configuration,
with porosity, $\phi = 0.2$. Using Equation (13.2), this yields a pore diameter of $1.265 \times 10^{-5}$ m. Changing the permeability to $k_y = 1 \times 10^{-10}$ m$^2$ yields a pore diameter of $1.265 \times 10^{-4}$ m. Utilizing modern equipment, both of these cases represent physically realistic pore diameters.

13.2 External Pin Fin Array for Determining Effective Heat Transfer Coefficient at the Reservoir Wall

In order to discuss the viability of a design discussed in this work, one must resolve the heat generated due to the sealed bearing design. Frictional forces generated by the rotating shaft and the viscous lubricating fluid lead to a build-up of heat, that for a normal journal bearing is mitigated by an external heat exchanger. The bearing discussed here, since it is completely sealed, must somehow act as its own heat exchanger. For that purpose, one proposed design is for the external housing of the bearing (the reservoir wall) to contain a pin fin array (see Figure (13.1)), in effect turning the bearing housing into a heatsink to be cooled either by forced or natural convection, depending on the application and thermal requirements. The benefits of doing so should be obvious qualitatively but must be quantified in order to determine the necessary design parameters for successful operation of the bearing. Such design parameters include the length, $L_{\text{fin}}$, diameter, $D_{\text{fin}}$, and number of fins, $N$, to be used, as well as the necessary heat transfer coefficient, $h$, to remove the heat generated by the operation of the bearing.
13.2.1 Example - Standard Journal Bearing

To begin discussion of the removal of heat of our proposed bearing, let us first examine a case of a standard journal bearing with no external pin fin array. Note, that for the remainder of this chapter, the equations and relations shown were taken from Incropera and DeWitt [30]. For this example, consider the geometric and operational parameters in Table 13.1.

The equation for determining \( q_{wo} \), the rate of heat transfer by convection from the bearing surface, is given by

\[
q_{wo} = h (2\pi r L_z) (T_b - T_a)
\]

(13.3)

and for this configuration, \( q_{wo} = 405 \) W, which is the heat transfer rate at which this journal bearing operates.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>heat transfer coefficient at the bearing wall</td>
<td>$50 \frac{W}{m^2K}$</td>
</tr>
<tr>
<td>$L_z$</td>
<td>length of the bearing in the axial direction</td>
<td>$5.08 \times 10^{-2} \text{ m}$</td>
</tr>
<tr>
<td>$q_{wo}$</td>
<td>rate of heat transfer by convection from the bearing surface</td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>radius of the bearing</td>
<td>$2.54 \times 10^{-2} \text{ m}$</td>
</tr>
<tr>
<td>$T_a$</td>
<td>ambient temperature</td>
<td>$500 \text{ K}$</td>
</tr>
<tr>
<td>$T_b$</td>
<td>temperature at the bearing wall</td>
<td>$1500 \text{ K}$</td>
</tr>
</tbody>
</table>

Table 13.1: Journal Example - Nomenclature and Values

13.2.2 Example - Journal Bearing With External Array of Pin Fins

We now take the same geometry described above but we add an external pin fin array to transform the bearing housing into a heatsink. Let us consider five equally spaced pin fins in the axial direction, where the space between the fins is equal to the diameter of each pin (see Figure (13.2)).

This yields our fin diameter value $D_{fin} = 4.62 \times 10^{-3} \text{ m}$. The circumference of the bearing is $0.1596 \text{ m}$, so if we use the same spacing in the circumferential direction as the axial direction for the pin fins, this yields approximately 17 fins in the circumferential direction, for a total of $N = 85$ pin fins. Also, we consider a fin length of $L_{fin} = 2.54 \times 10^{-2} \text{ m}$. Note that this geometry is perfectly reasonable but it does not necessarily represent an optimized configuration. Such design criteria may be
The equation for determining $q_t$, the rate of heat transfer by convection from the fins and prime surface, is given by

$$q_t = hA_t \left[ 1 - \left( \frac{NA_f}{A_t} \right)(1 - \eta_f) \right] (T_b - T_a). \quad (13.4)$$

The surface area of the prime surface (the exposed region of the bearing housing), $A_b$ is

$$A_b = 2\pi r L_z - N \left( \frac{\pi}{4} \right) D_{fin}^2 = 3.634 \times 10^{-3} \text{m}^2. \quad (13.5)$$

The surface area of each pin fin is,

$$A_f = \pi D_{fin} L_c = \pi D_{fin} \left( L_{fin} + \frac{D_{fin}}{4} \right) = 3.853 \times 10^{-4} \text{m}^2. \quad (13.6)$$

Thus, the total surface area of the prime surface plus the surface area of the pin fins, $A_t$ is

$$A_t = A_b + NA_f = 3.634 \times 10^{-3} \text{m}^2 + 85(3.853 \times 10^{-4} \text{m}^2) = 3.638 \times 10^{-2} \text{m}^2. \quad (13.7)$$
We can compare this surface area with that of the standard journal bearing of the previous example,

\[ A = 2\pi r L_z = 8.107 \times 10^{-3} \text{m}^2, \quad (13.8) \]

so the addition of these fins has yielded a surface area that is over four times greater than a standard journal bearing. Obviously, this number can change by using thinner pin fins in greater quantity, but the benefits of using the pin fins should already be apparent.

To calculate the efficiency of a pin fin, we use the expression

\[ \eta_f = \frac{\tanh (mL_c)}{(mL_c)}, \quad (13.9) \]

where \( m \) is given by

\[ m = \sqrt{\frac{h P}{k A_c}}. \quad (13.10) \]

\( P \) is the perimeter of a pin fin and \( A_c \) is the cross-sectional area of a pin fin, denoted by the relationship \( A_c = \left( \frac{\pi}{4} \right) D_{\text{fin}}^2 \). We may simplify \( m \) to get

\[ m = \sqrt{\frac{4h}{kD_{\text{fin}}}} = 20.81 \text{m}^{-1}. \quad (13.11) \]

Using this value for \( m \) and the value of \( L_c = L_{\text{fin}} + \frac{D_{\text{fin}}}{4} = 0.2655 \text{m} \), we obtain the efficiency of a single fin to be

\[ \eta_f = \frac{\tanh (mL_c)}{(mL_c)} = \frac{\tanh(0.5526)}{0.5526} = 0.9093. \quad (13.12) \]

We may now calculate the value for \( q_t \) using the expression above to find that, using this array of pin fins, \( q_t = 1670 \text{W} \). Note that this is over four times the convection
heat transfer rate of the standard journal bearing.

Table 13.3 shows how density of the pin fin array can affect the overall convection heat transfer rate. For every case, the spacing between adjacent pin fins was always equal to the diameter of each pin fin. Thus, by reducing the diameter of each pin fin, more fins are placed on the bearing housing in both the axial and circumferential directions. Let $N_{\text{circ}}$ represent the number of pin fins in the circumferential direction, $N_{\text{axial}}$ represent the number of pin fins in the axial direction, so that $N = N_{\text{circ}} \times N_{\text{axial}}$ is the overall number of pin fins in the array.

Though the relationship between the number of fins in the axial direction, $N_{\text{axial}}$, and the convection heat transfer rate, $q_t$, is not quite linear, one can see a proportional relationship between the two quantities. This relationship assumes that $h$ is not affected by the reduction of the distance between each pin fin, which is reasonable as long as the thermal boundary layers of the pin fins do not interact with each other.

Though the proof of such an assumption to validate the results shown in Table 13.3 is beyond the scope of this work, the conclusion that a higher number of thinner pin fins contributes to a higher convection heat transfer rate is still valid.

Upon determining the convection heat transfer rate, $q_t$, this value can be substituted for $q_{\text{wo}}$ in Equation (13.3). Upon doing so, one can then solve for $h$ in Equation (13.3), yielding a new effective heat transfer coefficient. This is the value that will be used for $h_I$, the heat transfer coefficient at the reservoir wall, in the thermal calculations described in Chapters 7 and 12. We observe that using a pin fin array, the effective heat transfer coefficient, $h_I$, can be over one order of magnitude
larger than the heat transfer coefficient, $h$, from the standard journal bearing example.

Thus, if a given physical value of $h$ is on the order of $50 - 100 \frac{W}{m^2K}$, the corresponding effective heat transfer coefficient at the reservoir wall, $h_I$, can be on the order of $500 - 1000 \frac{W}{m^2K}$ using the pin fin array discussed here. Notice that this is the order of values used for $h_I$ in Chapter 7 and 12.

Designers of bearings discussed in this work now have an all-encompassing set of design criteria that can be used to create a bearing to meet their specific performance targets. Upon choosing the working fluid ($\mu, B, k_f$), physical geometry of the bearing ($h_{min}, h_{max}, L_x, L_z, \alpha, \beta$) and operational parameter $U$, the only parameters left for determining bearing performance were the effective permeability, $k_y$, and the effective heat transfer coefficient at the reservoir wall, $h_I$. These quantities must be expressed in terms of physical parameters that can be controlled through construction and through operation, i.e., the effective permeability, $k_y$ is determined by the porosity, $\phi$, the pore diameter, $d$, and the geometry of the porous material whereas the effective convective heat transfer coefficient, $h_I$ is controlled by a multitude of operational and geometric parameters (speed and temperature of the air blowing over the fins, size, shape, and number of pin fins, etc.). However, once these terms are all known, every constant used in the analytical and numerical modeling of this bearing can be determined, yielding a complete set of parameters that can be adjusted to yield the desired bearing performance.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_b$</td>
<td>surface area of prime surface</td>
<td></td>
</tr>
<tr>
<td>$A_t$</td>
<td>total surface area of prime surface plus pin fins</td>
<td></td>
</tr>
<tr>
<td>$A_{fin}$</td>
<td>surface area of one pin fin</td>
<td></td>
</tr>
<tr>
<td>$D_{fin}$</td>
<td>pin fin diameter</td>
<td>$4.62 \times 10^{-3}$ m</td>
</tr>
<tr>
<td>$h$</td>
<td>heat transfer coefficient at the bearing wall</td>
<td>$50 \frac{W}{m^2K}$</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity of the pin fin</td>
<td>$100 \frac{W}{mK}$</td>
</tr>
<tr>
<td>$L_c$</td>
<td>corrected fin length</td>
<td></td>
</tr>
<tr>
<td>$L_{fin}$</td>
<td>pin fin diameter</td>
<td>$2.54 \times 10^{-2}$ m</td>
</tr>
<tr>
<td>$L_z$</td>
<td>length of the bearing in the axial direction</td>
<td>$5.08 \times 10^{-2}$ m</td>
</tr>
<tr>
<td>$N$</td>
<td>total number of pin fins</td>
<td></td>
</tr>
<tr>
<td>$q_t$</td>
<td></td>
<td>c</td>
</tr>
<tr>
<td>$q_{wo}$</td>
<td>rate of heat transfer by convection from the bearing surface</td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>radius of the bearing</td>
<td>$2.54 \times 10^{-2}$ m</td>
</tr>
<tr>
<td>$T_a$</td>
<td>ambient temperature</td>
<td>$500$ K</td>
</tr>
<tr>
<td>$T_b$</td>
<td>temperature at the bearing wall</td>
<td>$1500$ K</td>
</tr>
<tr>
<td>$\eta_f$</td>
<td>efficiency of one pin fin</td>
<td></td>
</tr>
</tbody>
</table>

Table 13.2: Journal With Pin Fins Example - Nomenclature and Values
Table 13.3: Comparison of Pin Fin Density vs. $q_t$

<table>
<thead>
<tr>
<th>$N_{circ}$</th>
<th>$N_{axial}$</th>
<th>$D_{fin}$</th>
<th>$q_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>5</td>
<td>0.004618 m</td>
<td>1670 W</td>
</tr>
<tr>
<td>33</td>
<td>10</td>
<td>0.002419 m</td>
<td>2930 W</td>
</tr>
<tr>
<td>64</td>
<td>20</td>
<td>0.001239 m</td>
<td>4970 W</td>
</tr>
</tbody>
</table>
14.1 Contributions

Accomplished here are analytical and numerical descriptions of a new kind of bearing, where the bearing acts as its own pump and heat exchanger. The novel concept of two fluid regions separated by a porous medium allows for providing a load-carrying capacity while simultaneously circulating the fluid throughout the bearing geometry. This unique combination allows for a designer to eliminate an external lubricant pump, an external heat exchanger, and the hardware necessary to circulate the fluid between the bearing and heat exchanger. In addition to the unique geometry of the bearing, the analytical model used to describe the pressure field inside the bearing is novel. We were able to adapt the work done by Keith and Vijayaraghavan to couple a mass-conserving cavitation model with the unique fluid-porous medium-fluid geometry described in this work, which provided the ability to show the fluid circulating between the fluid reservoir and the active film region. The computational solutions produced allowed us to perform a parametric analysis, which showed how the bearing performed as each parameter was changed. This analysis would give guidance to a designer on how best to design a bearing of this kind of geometry to
yield the desired performance. We also showed that this type of bearing is capable of removing a substantial amount of heat, confirming that this type of bearing is feasible. Finally, we note that if the journal bearing described here were sealed at the axial ends, the need to have its lubricant replaced would be greatly reduced or eliminated, providing an additional cost saving.

14.2 Assumptions and Simplifications Made for the Analytical Work

The analytical work done here simplified the Navier-Stokes equations and the energy equations, used to describe the pressure, velocity, and temperature fields, for both the slider and journal bearing configurations. Non-dimensional analyses were performed using an asymptotic expansion approach, which allowed for leading order governing equations and boundary conditions to be constructed. Upon constructing the governing Reynolds equations to describe the fluid flow inside the bearing, the simplification that the permeability in the $x$-direction (for the slider geometry) and the permeabilities in the $x$- and $z$-directions (for the short and long journal geometries) were zero, indicative of vertical holes drilled in the porous medium for the slider bearing and holes drilled radially in the porous medium for the journal bearings. This simplification reduced the Brinkman-Extended Darcy model to the Darcy model for the journal bearing configuration. However, we note that the methods developed for modeling the journal bearings allowed for the Brinkman-Extended Darcy model to be used using non-zero permeabilities in the $x$- and $z$-directions.
The disparities in scale that allowed for the asymptotic expansion simplification, along with negating the $x$- and $z$-direction permeabilities of the porous medium, are the major simplifications used in this work. We also assumed that the fluid flow was laminar in all three regions and that the pressure did not vary in the $y$-direction in the fluid reservoir and fluid film regions for both the slider and journal configurations. This was a result of the asymptotic expansion analysis that showed that, at leading order, the pressure was constant across the thickness of the two fluid regions.

14.3 Conclusions to the Analytical Work

The analytical work done here accomplished the following:

- Performed asymptotic expansion analyses that produced leading order momentum governing equations and the leading order and first-order correction energy governing equations;

- Constructed Reynolds equations that were then modified to incorporate a mass-conserving cavitation algorithm;

- Produced how to compute the effective heat transfer for a journal bearing housing with an external pin-fin array.

14.4 Conclusions to the Numerical Work

The numerical work done here accomplished the following:
• Produced numerical techniques that solved the Reynolds equations described in this work;

• Modified the numerical techniques of Keith and Vijayaraghavan to numerically model cavitation for a fluid-porous medium-fluid geometry;

• Produced numerical techniques that solved the conduction and convection energy equations described in this work.

14.5 Summary of Effects of Each Parameter on Bearing Performance

One of the contributions of this paper is the parametric analysis done here, allowing for a designer essentially to “dial in” the values of the geometric and operating parameters that yield the desired performance characteristics of a bearing of this type. Below, we summarize our findings of the parametric analyses performed in this work.

14.5.1 Effects of Varying the Parameter, $\alpha$, Fluid Reservoir Thickness

• As $\alpha$ increased, we saw a decrease in peak pressure accompanied by a smaller cavitated region. Note that the peak pressure had a lower bound as the depth of the reservoir decreased. The reservoir essentially appears to be infinitely deep after a certain point.

• As $\alpha$ increased, we saw a decreased rate of circulation of the fluid due to the lower pressure difference between the two fluid regions.

• As $\alpha$ increased we saw a decrease in peak temperature.
14.5.2 Effects of Varying the Parameter, $\beta - \alpha$, Porous Medium Thickness

- As $\beta - \alpha$ increased, we saw an increase in peak pressure accompanied by a larger cavitated region.

- As $\beta - \alpha$ increased, we saw an increased rate of circulation of the fluid due to the higher pressure difference between the two fluid regions.

- As $\beta - \alpha$ increased, we saw an increase in the peak temperature as well as an increase in convective heat transfer between the two fluid regions.

14.5.3 Effects of Varying the Parameter, $k_y$, Porous Medium Permeability ($y$-direction)

- As $k_y$ increased, we saw a decrease in peak pressure accompanied by a smaller cavitated region.

- As $k_y$ increased from $1.0 \times 10^{-13}$ m$^2$ to $1.0 \times 10^{-10}$ m$^2$, we saw an increased rate of circulation. Though the pressure difference between the two regions was larger for the lowest permeability, it was not enough to offset the additional flow resistance of the porous medium.

- As $k_y$ increased, we saw a decrease in peak temperature.

14.5.4 Effects of Varying the Parameter, $U$, Linear Velocity

- As $U$ increases, we saw an increase in peak pressure accompanied by a larger cavitated region.

- As $U$ increased, we saw an increased rate of circulation of the fluid between the two fluid regions.
• As $U$ increased, we saw an increase in the peak temperature as well as an increase in convective heat transfer between the two fluid regions.

14.6 Future Work

There are several projects that can be extended from this work. Several assumptions and simplifications were made to generate what can be considered a first order model of the bearing geometries considered here. Extensions to this work include, but are not limited to;

• Including the higher order terms in the momentum equations and thermal equations

• Keeping the Brinkman extensions in the governing equations for the porous medium and using an adaptive grid to recover the smooth velocity curve at the fluid-porous medium interface

• Improving on the cavitation model to generate smooth pressure curves where the fluid film recombines (e.g. the transition from cavitation to film)
BIBLIOGRAPHY


