A GAME THEORETIC APPROACH TO THE PROBLEM OF DETERMINING
THE OPTIMAL NUMBER OF YEARS OF EDUCATION

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A GAME THEORETIC APPROACH TO THE PROBLEM OF DETERMINING

THE OPTIMAL NUMBER OF YEARS OF EDUCATION

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Thesis

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ABSTRACT

Higher education leads to higher productivity and thus to higher income. The issue of the cost of higher education versus the economic return is addressed in this paper using a game theoretic approach. The game has two players: the worker and the government. Each has two choices: the worker can choose to get higher education or not and the government can choose to subsidize some portion of the schooling or not. Both players have the same goal which is to maximize their income. We find that if the taxation rate imposed by the government exceeds the ratio of the increment of net loss of income due to subsidy to the increment of net gain of income due to that subsidy, then the government should subsidize some portion of the education. We also confirm that the individual should continue education if the extra income in a lifetime is greater than the cost of schooling.
ACKNOWLEDGEMENTS

First I thank God for the strength and determination that He gave me to finish this thesis. I would also like to thank my advisors for their great input and help, their time, patience, and understanding. Lastly, I would like to thank my family for all the help and support. I want to dedicate this paper to my grandfather who recently passed away but who I know is watching over me and is really proud right now.
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CHAPTER I
INTRODUCTION

The knowledge accumulated over time is a major source of income growth for a worker. In general, knowledge can be acquired through different ways, such as personal research or study, life experience, learning by doing, or on-the-job training, but the most important and valuable method is through schooling. In 1960, Schultz introduced the notion of human capital [1]. In his article and also in studies made by Becker [2] it is emphasized that human capital acquired through education has a great influence on workers’ productivity. Thus, as a consequence, it affects their income.

In the U.S., higher education requires a great deal of resources such as the money the student could have earned while being in school and the tuition paid to the school. These two components play a major role in a worker’s decision to attend college or not. Another important role in this decision may be played by the government. Through taxation, the government accumulates income according to workers’ wages. Since wage is correlated to higher education, it might well be in the interest of the government to subsidize a worker’s education. There are two main questions this paper is attempting to answer using a game theory approach. First, how much education would maximize a worker’s lifetime income and second, should
the government subsidize any of the education in order to maximize its own income.

The problem of maximizing income was addressed in many papers since this was and still is an important issue. Correa’s study [3] constitutes the starting point of this paper. In his article he considers two players: the worker whom he calls the individual, and the government, in a scenario where the worker can control the number of years of education he gets and the government controls the number of people who get higher education. Correa provides the payoff functions of the individual, depending on the number of years of education, and of the government, whose payoff function has two components. The first component is the income of workers with high school education only, and the second component represents the income of workers with higher education. The next step in his analysis is to find the Nash-Cournot equilibrium which is based on the assumption that the worker and the government are acting independently of each other. He is also considering the situation when the decision made by the government depends upon the worker’s decision. This is called the Nash-Stackelberg equilibrium.

Fender and Wang [4] analyse the idea that education needs to be financed by borrowing and they focus on credit market imperfections. From their point of view people live for two different periods. In the first period, the educated persons acquire knowledge and skills and after that they apply them for a higher wage whereas the uneducated people work at a constant low wage. In the second period, both the educated and the uneducated retire and consume. Fender and Wang consider the case when the education is financed by taxing the uneducated, and also the case of
education subsidy. Their research showed that credit constraints reduce the amount of
human capital acquired, thus it may result in a lower education level for the workers.

Laing, Palivos, and Wang [5, 6] look at a matching model. That is, educated
workers look for jobs that match their skills. In these papers, the worker wants to
acquire the optimal level of schooling in order to increase his wage and his probability
of employment.

Another matching model is presented by Charlot et al. [7]. After schooling,
the workers enter the labour market. The workers have marketable characteristics
such as skills, adaptability, and productivity. According to this study, an increase in
the number of years of higher education reduces the number of unemployed workers.

A similar study was made by Decreuse and Granier [8]. They focus on the
power of the employers of setting the wage and the influence of this power on the
level of education of the workforce. The paper suggests that education can reduce
unemployment and can also reduce the power of setting the wage by allowing job-to-
job mobility.

Another group of papers focus on the general skills training provided by the
employer. Shintoyo [9] presents a matching model with companies that are willing
to provide training to fill their vacancies. He demonstrates that when firms agree to
train workers, more job vacancies are created, more skilled employees result, and the
unemployment rate decreases. In his model, Shintoyo also includes the class of firms
that are not able to provide the training and thus have to hire skilled workers.

Acemoglu [10] studies the adoption of new technology by firms. He considers
the investment in the inevitable training but he also includes the idea that future employers who embrace the same innovations may benefit from the already skilled worker in case of termination from a previous firm. The paper emphasizes the pairing of worker and firm. The purpose is to maximize the output of their partnership.

Brunello and Medio [11] compare the differences in education and training in the United States, Germany, and Japan. This study is based on workers’ decisions to invest in education and firms’ decisions to invest in training. The two authors develop a model which is based on three features. First, workers’ investment in education reduces training costs for firms. Second, job vacancies can be filled by skilled workers or by individuals who require additional training. And third, training positions are filled by educated workers because they are cheaper to train.

Stadler and Wapler [12] extend Acemoglu’s model [10]. They look at the effect of adopting new technology which requires high-skilled workers. Their work suggests that a higher supply of skilled job vacancies lowers the high-skilled wage. This fact may cause firms to create more jobs for the high-skilled worker since the firm would now pay him less than before.

Coles and Masters [13] introduce a model based on what they call ”unlearning by not doin”. They explain how being unemployed for a long period of time, leads to out of date skills and employers avoid hiring such individuals. Also, if the skill level drops too much, the worker becomes unemployable. The authors’ conclusion is that the best thing to do is to subsidize job creation instead of subsidizing retraining and eliminating long-term unemployment as much as possible.
Pitchik [14] makes an interesting point by presenting a model which predicts that even in the presence of high skills or high education, wage decline with seniority may occur under certain conditions.

A unique approach to job searching is considered by Coulson et al. [15]. They introduce the idea of spatial mismatch which by their definition is "inner-city unemployment, in the heart of an otherwise strong metropolitan economy". They argue that this phenomenon occurs because of the lack of information available to inner-city workers regarding open suburban jobs.

In this thesis, our goal is to model both the individual’s and the government’s income in the presence or the absence of tuition subsidy and when the individual decides to go to school or not. Our model differs from Correa’s in several ways. First, we introduce the possibility of a government subsidy which increases the number of educated people. Secondly, we take into consideration other parameters such as the taxation rate, the cost of schooling, and the benefits of being educated. One of the benefits of getting a higher education and obtaining a better job is health insurance [16]. Higher-education leads to better health which allows an individual to work for a longer period of time. In this paper we do not differentiate between disciplines. We assume they weigh equally in the society and they bring the individual the same lifetime income.

After we present the model with all the payoff functions, we analyse the dominant strategy for the government and for the worker. Then we find the equilibrium of each game by optimizing the payoff functions. This way we obtain the optimal
number of years of education, the optimal portion of education subsidized by the
government, and maximum lifetime income for each player. Lastly, we present our
conclusions.
CHAPTER II
THE MATHEMATICAL MODEL

In this chapter we present the game. We set up the payoff matrix and analyse the dominant strategies for both the government and the individual. We also set the stage for optimization.

2.1 The Model

Following Correa [3], we assume we have two players: the individual and the government. The objective of the individual is to find out how many years of education would maximize his lifetime income. That of the government is to maximize its total income obtained through taxation.

We consider that each player has two choices. The individual has to decide whether or not to go to school, and the government’s options are to subsidize or not subsidize part of the higher education. We consider higher education to be anything beyond high school.

In his model, Correa took into consideration the base salary of a person that has only a high school education, the increment of income due to schooling past high school, and the loss of income due to competition. To these variables, we add the cost of schooling, the taxation rate, and the portion of schooling subsidized by the
government. We assume that a more educated population brings the government a greater benefit. As mentioned before, studies show that higher education leads to a rise in income. An individual with a higher income pays more taxes to the government and contributes more to the wealth of the society. This idea is also presented in [17].

Next we set up the game by building the payoff functions of the individual and of the government. We have four different subgames. The first subgame is when the government is subsidizing the tuition and the individual goes to school, the second one is when the government is subsidizing and the individual does not go to school, the third subgame is when the government is not subsidizing and the individual goes to school, and the fourth game is when the government is not subsidizing and the individual does not go to school. The variables that we use in these subgames are presented in tables 2.1-2.3.

Table 2.1: List of variables with units and definitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>years</td>
<td>Average number of years worked after the age of 18</td>
</tr>
<tr>
<td>$t_1$</td>
<td>years</td>
<td>Number of years of education beyond high school</td>
</tr>
<tr>
<td>$\bar{t} = t + t^*$</td>
<td>years</td>
<td>Average number of years worked after the age of 18 by individuals who went to school beyond high school</td>
</tr>
</tbody>
</table>
Table 2.2: List of variables with units and definitions (continued)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t^* = t_1 )</td>
<td>years</td>
<td>Extra number of years worked by people who went to school beyond high school</td>
</tr>
<tr>
<td>( n )</td>
<td>person</td>
<td>Number of high school graduates</td>
</tr>
<tr>
<td>( n_e = \beta n )</td>
<td>person</td>
<td>Number of high school graduates that continue education</td>
</tr>
<tr>
<td>( \tilde{n}_e = n_e + (n - n_e)\frac{\gamma}{2} )</td>
<td>person</td>
<td>Number of high school graduates that continue education if the government offers a subsidy</td>
</tr>
<tr>
<td>( a_0 )</td>
<td>( \text{dollar (year)(person)} )</td>
<td>Annual income of an individual with a high school degree</td>
</tr>
<tr>
<td>( a_1 = c_0 - c_1 t_1 )</td>
<td>( \text{dollar (year)^2(person)} )</td>
<td>Increment of income due to schooling past high school</td>
</tr>
<tr>
<td>( c_0 )</td>
<td>( \text{dollar (year)^2(person)} )</td>
<td>Largest base increment of a worker’s income due to schooling</td>
</tr>
</tbody>
</table>
Table 2.3: List of variables with units and definitions (continued)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>dollar (year)(^2)(person)</td>
<td>Detrimental fact to a worker’s income for staying in school for too long</td>
</tr>
<tr>
<td>$b_0$</td>
<td>dollar (year)(person)(^2)</td>
<td>Loss of lifetime income of a worker with a high school degree due to competition</td>
</tr>
<tr>
<td>$b_1$</td>
<td>dollar (year)(person)(^2)</td>
<td>Loss of lifetime income of a worker with education beyond high school due to competition</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$0 &lt; \tau &lt; 1$</td>
<td>Taxation rate</td>
</tr>
<tr>
<td>$P$</td>
<td>dollar (year)(person)</td>
<td>Total cost of education per year</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$0 &lt; \alpha &lt; 1$</td>
<td>Interest rate</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$0 &lt; \gamma &lt; 1$</td>
<td>Proportion of schooling subsidized by the government</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$0 &lt; \beta &lt; 1$</td>
<td>Proportion of high school graduates that went to school beyond high school</td>
</tr>
<tr>
<td>$k$</td>
<td>$0 &lt; k &lt; 1$</td>
<td>Benefit to the government for a more educated population</td>
</tr>
</tbody>
</table>
2.1.1 Subgame One

In this subgame we consider the case where the government will subsidize part of the education, and the individual decides to go to school. Under these circumstances, the individual’s payoff function is

\[ W_1 = [a_0 + a_1 t_1 - b_0(n - \tilde{n}_e) - b_1 \tilde{n}_e](\tilde{t} - t_1)(1 - \tau) - [(1 - \gamma)Pt_1 + \alpha P(t_1)]. \] (2.1)

This payoff function has two main components. The first component is the lifetime income of a worker after he finishes educating himself and after he pays taxes to the government at a rate of \( \tau \). The second component is the cost of schooling that we subtract from the lifetime income. We assume that an individual is not working while getting educated. We consider the base salary for a worker with high school degree to be \( a_0 \) and, for each year of education beyond high school, the increment of income is \( a_1 t_1 \), where \( t_1 \) is the number of years of higher education. The variable \( t_1 \) is controlled by the individual. He decides how many years to go to school. The purpose of this paper is to determine the optimal number of years of education, \( t_1 \), such that the worker’s lifetime income is maximized. The next two terms in the expression of \( W_1 \) represent the loss of income due to competition. According to Correa’s model, \( b_0 \) is the loss of income due to the competition between individuals with a high school degree, and \( b_1 \) is the loss of income due to competition between individuals with higher degrees. We assume there is a total number of \( n \) high school graduates from which \( \tilde{n}_e \) decide to continue their education when the government is offering a subsidy.
The expression for $\tilde{n}_e$ is

$$\tilde{n}_e = n_e + (n - n_e)\frac{\gamma}{2}, \quad (2.2)$$

where $n_e$ is the total number of high school graduates that continue their education even without a subsidy. The parameter $b_0$ is a function of the number of high school graduates $n$. This loss of income due to competition varies inversely as the number of high school graduates since we do not want to have too much competition in a large population, competition that would drive salaries to zero. The proportion of schooling subsidized by the government is represented by $\gamma$, with $0 < \gamma < 1$. We are assuming that under these circumstances another $(n - n_e)\frac{\gamma}{2}$ individuals will pursue higher education. Observe that if $\gamma = 1$, that is, if the government subsidizes the entire college education, then we assume that half of the workers who would not go to school if they had to pay for their education would go if there exists a subsidy.

In this payoff function we also incorporate the benefits of being educated. We assume that an individual with higher education has access to better health insurance and preventive health care, things that lead to better health and life expectancy and thus he would work for a longer period of time. This idea is presented in [16]. In our model we include this idea in the expression for $\tilde{t}$,

$$\tilde{t} = t + t^*, \quad (2.3)$$

where $t$ is the average number of years worked and $t^*$ is the extra number of years worked due to better health.
The second component of the payoff function of the worker is the cost of schooling when the government is subsidizing some proportion of the education,

\[ \tilde{\theta} = (1 - \gamma)Pt_1 + \alpha Pt_1. \]  
(2.4)

Here \( P \) represents the total cost of education per year and \( \alpha \) is the interest rate for the loan.

The next component of this subgame is the payoff function of the government,

\[ G_1 = \{[a_0 + a_1 t_1 - b_0(n - \tilde{n}_e) - b_1 \tilde{n}_e](\tilde{t} - t_1)\tau\} \tilde{n}_e + kPt_1 \tilde{n}_e - \gamma Pt_1 \tilde{n}_e. \]  
(2.5)

This payoff function represents income from the educated workers and does not include income from the uneducated workers. The payoff function (2.5) has three main components. The first one is the amount that results from taxing the workers’ income at a rate of \( \tau \), the second component, \( kPt_1 \tilde{n}_e \), is the benefit from a more educated population, and the third component, \( \gamma Pt_1 \tilde{n}_e \), is the cost of the subsidy. According to [16], the major benefit is higher income, and thus higher taxes paid to the government. The author also mentions multiple other ”social benefits”. These include: better health of the educated individuals, which is beneficial to the government, as well as the reduced spending on social programs, such as unemployment compensation, Medicare and Medicaid, food, and welfare programs. Also according to [16], other benefits are a reduced incarceration rate for college graduates, smaller chance of illegitimate births, and intangible benefits such as, more people volunteering for community service and an increased voting rate.
The goal of this work is to determine the portion of school subsidized by the government so as to maximize the government’s income. The workers then determine the amount of schooling they will undertake so as to maximize their income.

2.1.2 Subgame Two

In this subgame we assume that the government subsidizes a portion of the higher education cost, but the individual chooses not go to school. In this case, the expression for the payoff function for the high school educated worker is

\[ W_2 = [a_0 - b_0(n - \tilde{n}_e) - b_1\tilde{n}_e]t(1 - \tau). \] (2.6)

The payoff function of the government is

\[ G_2 = \{[a_0 - b_0(n - \tilde{n}_e) - b_1\tilde{n}_e]t\tau\}(n - \tilde{n}_e), \] (2.7)

where, as before,

\[ \tilde{n}_e = n_e + (n - n_e)\frac{\gamma}{2}. \] (2.8)

Here, \( G_2 \) represents the government’s income generated by taxing the uneducated workers. It does not include the income from the educated workers. We observe that in this situation, only the government can optimize on \( \gamma \). The individual has no options. Also notice that in this case subsidizing means less uneducated workers, by (2.8).

2.1.3 Subgame Three

In this subgame we suppose that the government does not offer a subsidy, but the worker decides to educate himself. Under these circumstances, the payoff function for
the individual is

\[ W_3 = [a_0 + a_1 t_1 - b_0(n - n_e) - b_1 n_e](\bar{t} - t_1)(1 - \tau) - [P t_1 + \alpha P t_1]. \] (2.9)

Observe that in this situation, \((n - n_e)^2\) fewer workers will go to school. Note that the cost of schooling for the worker is going to be higher since there is no subsidy.

The payoff function of the government is

\[ G_3 = \{[a_0 + a_1 t_1 - b_0(n - n_e) - b_1 n_e](\bar{t} - t_1)\tau\} n_e + k P t_1 n_e. \] (2.10)

Notice that if the government decides not to subsidize, the income generated from taxing educated people’s wages will be smaller, due to lower wages earned by the workers. Also, the benefit from a more educated population will be reduced. \(G_3\) does not include the income obtained from the uneducated workers. In this subgame, only the individual can optimize by choosing \(t_1\). The government has no control in this situation.

2.1.4 Subgame Four

In this last subgame we presume that the government does not offer a subsidy for higher education and the individual does not go to school. In this case, the payoff function of the worker is

\[ W_4 = [a_0 - b_0(n - n_e) - b_1 n_e] t (1 - \tau). \] (2.11)

The payoff function of the government is

\[ G_4 = \{[a_0 - b_0(n - n_e) - b_1 n_e] t \tau\} (n - n_e). \] (2.12)
Here $G_4$ represents the income from the uneducated individuals. In this case, neither of the two players can optimize their incomes.

2.1.5 The Payoff Matrix

The payoff matrix for the four games is given in table 2.4.

Table 2.4: Payoff Matrix

<table>
<thead>
<tr>
<th>Government</th>
<th>Individual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Go To School</td>
</tr>
<tr>
<td>Subsidize</td>
<td>$(G_1, W_1)$</td>
</tr>
<tr>
<td>Do Not Subsidize</td>
<td>$(G_3, W_3)$</td>
</tr>
</tbody>
</table>

2.2 Dominant Strategies

The first thing that we examine is when it would be more beneficial for the government to subsidize the education and it would be more beneficial for the individual to go to college. This means we have to find the dominant strategy for each player.

2.2.1 Dominant Strategy for the Government

For the government, we want to determine under which circumstances its total income is greater when it decides to subsidize. The government collects income from both educated and uneducated workers. $G_1 + G_2$ represents this sum when the government
subsidizes. $G_3 + G_4$ represents this sum when the government does not subsidize. Thus, the governments will only subsidize when $G_1 + G_2 > G_3 + G_4$.

We assume that $n_e = \beta n$, where $0 < \beta < 1$, is the proportion of high school graduates who continue their education. Another important assumption that we make is that the additional health benefit will allow the educated individual to live $t_1$ years longer and can hence work that amount of time. So we say that $t^* = t_1$. Under these assumptions, the government will subsidize the higher education if

\[(b_0 - b_1)(1 - \beta)n\frac{\gamma}{2}t_1 + a_1t_1t\gamma(1 - \beta)n\frac{\gamma}{2} + (k - \gamma)P\frac{\gamma}{2}(1 - \beta)n\gamma - \gamma P_t$ $\beta n > 0. \tag{2.13}\]

We divide both sides of the inequality by the positive variable $t$ and regroup to obtain

\[(1 - \beta)\frac{n\gamma}{2}t[a_1t_1 + (b_0 - b_1)n] + (1 - \beta)\frac{n\gamma}{2}P\frac{\gamma}{t}[(k - \gamma) - \frac{2\beta}{1 - \beta}] > 0. \tag{2.14}\]

The first component of (2.14) represents the government’s incremental gain to income by subsidizing and the second component is the net loss due to subsidizing. We assume that $b_0 > b_1$ because otherwise we would be saying that there is a greater competition for the educated workers than for the uneducated ones. We observe that if $\beta$ is too big, meaning too close to 1, then almost every high school graduate would attend higher education, so that the component $\frac{2\beta}{1 - \beta}$ approaches infinity and the entire expression (2.14) will be negative instead of positive. This means that it is not in the government’s best interest to subsidize a big portion of the education because more people will go to school and the losses would be greater than the benefits. On the other hand, if the benefit from a more educated population, $k$ is large, then the second component will be positive and inequality (2.14) is always true. This would
imply that the government should always subsidize. It is most likely that $k$ is not too large and that the net loss to the government due to subsidy is a negative quantity. So if we solve for $\tau$ in (2.14), we obtain
\[
\tau > -(1-\beta)\frac{\eta\nu}{2}\frac{P_{t1}}{k}[\frac{(k-\gamma) - \frac{2\beta}{1-\beta}}{(1-\beta)\frac{\eta\nu}{2}[a_1t_1 + (b_0 - b_1)n]}].
\] (2.15)

The interpretation of this condition is that if the taxation rate satisfies
\[
\tau > \frac{\text{increment of net loss of the government due to subsidizing}}{\text{increment of net gain of the government due to subsidizing}},
\] (2.16)
then the government will subsidize the education.

2.2.2 Dominant Strategy for the Individual

For the individual, we want to determine under which circumstances going to school result in a maximum income. Here we consider two different cases, when the government offers a subsidy, and when it does not. In the first case we want to compare the lifetime income $W_1$ with $W_2$. The worker has a greater lifetime income when they go to school than when they do not if
\[
[a_0 + a_1t_1 - b_0(n - \bar{n}_e) - b_1\bar{n}_e](\bar{t} - t_1)(1 - \tau) - [(1 - \gamma)Pt_1 + \alpha Pt_1]
> [a_0 - b_0(n - \bar{n}_e) - b_1\bar{n}_e]t(1 - \tau). \quad (2.17)
\]

We assume again that everybody works the same number of years, i.e. $t^* = t_1$. After we simplify by combining like terms, we obtain
\[
a_1t_1t(1 - \tau) > [(1 - \gamma)Pt_1 + \alpha Pt_1]. \quad (2.18)
\]
In other words, if the extra income in a lifetime of working after taxation is greater than the cost of schooling, then an individual should go to school. Solving for \( t_1 \) we find

\[
t_1 > \frac{\alpha P t_1}{a_1 t (1 - \tau) - (1 - \gamma) P}.
\] (2.19)

For this condition to be satisfied, \( a_1 \) needs to be large due to the fact that interest an individual pays for the money borrowed for schooling, can be a very large amount. At the same time, it is important that a person does not go to school for a long period of time because then the costs are going to be greater than the lifetime gain. This is a good example of diminishing returns, when increasing one input will diminish the output. So using this principle, we choose \( a_1 = c_0 - c_1 t_1 \), where \( c_0 \) is the largest base increment possible in the individual’s income due to schooling and \( c_1 \) is an economic penalty term for not finishing school or a detrimental fact to a worker’s wage for staying in school for too long. Substituting the new expression \( a_1 = c_0 - c_1 t_1 \) in (2.18) we obtain

\[
-c_1 t (1 - \tau) t_1^2 + (c_0 t (1 - \tau) - (1 - \gamma) P - \alpha P) t_1 > 0,
\] (2.20)

which is a quadratic function in \( t_1 \) with a negative leading coefficient. This means the graph is a concave down parabola. Hence, going to school for a long period of time can be a detriment to an individual’s lifetime income.

The second case, when the government does not offer an education subsidy is very similar. It is desired that the lifetime income of an educated worker, \( W_3 \), is
larger than the lifetime income of an uneducated worker, \( W_4 \); thus

\[
[a_0 + a_1 t_1 - b_0(n - n_e) - b_1 n_e](\tilde{t} - t_1)(1 - \tau) - (Pt_1 + \alpha Pt_1) > [a_0 - b_0(n - n_e) - b_1 n_e]t(1 - \tau). \tag{2.21}
\]

After simplifying and assuming that \( a_1 = c_0 - c_1 t_1 \), we find

\[
a_1 t_1 t(1 - \tau) > Pt_1 + \alpha Pt_1. \tag{2.22}
\]

For the extra lifetime income, after taxation, to be greater than the cost of schooling it is required that

\[
t_1 > \frac{\alpha Pt_1}{a_1 t(1 - \tau) - P}. \tag{2.23}
\]

For (2.23) to be true, the gain in extra income due to schooling should be large.

In the next chapter we present the optimization process. We want to determine the optimal number of years an individual should go to school and the optimal proportion of schooling subsidized by the government. We then present the expressions for the maximized income of both players.
In this chapter we present the optimal payoff functions for both players. The common goal of the two players is to maximize their income. The government can do that by controlling $\gamma$, the proportion of schooling subsidized, and the individual can obtain a maximum income by controlling the number of years of higher education, $t_1$. Following Correa’s example in [3] we study both Nash-Cournot and Nash-Stackelberg static equilibria. These two types of equilibria are also presented in [18].

The Nash-Cournot equilibrium implies that the government’s and the individual’s decisions are independent of each other. The two players make their decisions simultaneously. To compute this type of equilibrium we optimize the payoff functions $W$ and $G$ by choosing $t_1$ and $\gamma$ simultaneously. This is done by setting the partial derivative of $W$ with respect to $t_1$ and the partial derivative of $G$ with respect to $\gamma$, equal to zero and solve for $t_1$ and $\gamma$ simultaneously. The results are substituted into the corresponding payoff functions to obtain the maximum incomes.

The Nash-Stackelberg equilibrium assumes that the decision of one player, in our case the government, depends upon the decision of the other player, the individual. The process is to let the worker optimize first by choosing $t_1$. That is, we have to set the partial derivative of $W$ with respect to $t_1$ equal to zero and solve for $t_1$. 

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This way we obtain an expression for $t_1$ in terms of $\gamma$. We plug this expression into $G$. We then optimize this government’s payoff function by choosing $\gamma$.

3.1 Subgame One, Nash-Cournot Equilibrium

In this scenario, both players optimize simultaneously, independent of each other’s decision. The government optimizes by choosing $\gamma$ and the worker optimizes by choosing $t_1$. We use the same assumptions as before:

$$n_e = \beta n,$$

$$t^* = t_1,$$

$$a_1 = c_0 - c_1 t_1.$$

After we substitute these in the expression of $G_1$ and simplify, we obtain

$$G_1 = \left\{ [a_0 + a_1 t_1 - b_0(1 - \beta)n - b_1 \beta n + (b_0 - b_1)(1 - \beta)n \frac{\gamma}{2} | t \tau} \right\}$$

$$\cdot [\beta n + (1 - \beta)n \frac{\gamma}{2}] + k P t_1 [\beta n + (1 - \beta)n \frac{\gamma}{2}]$$

$$- \gamma P t_1 [\beta n + (1 - \beta)n \frac{\gamma}{2}]. \quad (3.1)$$

The partial derivative of $G_1$ with respect to $\gamma$ is
\[
\frac{\partial G_1}{\partial \gamma} = (b_0 - b_1)(1 - \beta)\frac{n^2}{2}t\tau \beta + (b_0 - b_1)(1 - \beta)^2n^2t\tau \frac{\gamma}{2} \\
+ \left\{[a_0 + a_1 t_1 - b_0(1 - \beta)n - b_1 \beta n]t\tau \right\}(1 - \beta)\frac{n}{2} \\
-P t_1 \beta n + kP t_1 (1 - \beta)\frac{n}{2} - P t_1 (1 - \beta)n \gamma. \quad (3.2)
\]

We now set this expression equal to zero and solve for \( \gamma \), which represents the optimal proportion of education subsidized by the government. We find

\[
\gamma = [2P t_1 \beta n - kP t_1 (1 - \beta)n - [a_0 + a_1 t_1 - b_0(1 - \beta)n - b_1 \beta n + (b_0 - b_1)\beta n]t\tau (1 - \beta)n] \\
/[(b_0 - b_1)(1 - \beta)^2n^2t\tau - 2P t_1 (1 - \beta)n]. \quad (3.3)
\]

We want \( \gamma \geq 0. \) Under our assumptions the denominator is positive. After dividing the numerator by \( n \) and rearranging we find \( \gamma \geq 0 \) provided

\[
a_1 t_1 t\tau (1 - \beta) \leq P t_1 (2\beta - k(1 - \beta)) - [a_0 - b_0(1 - \beta)n - b_1 \beta n + (b_0 - b_1)\beta n]t\tau (1 - \beta). \quad (3.4)
\]

This condition means that the government’s extra income due to a more educated population multiplied by the the percentage of workers who did not go to school has to be less than or equal to the adjustment to the tuition due to the fact that people did not go to school. In other words, the government needs to subsidize the education if the tuition is too high.

To see how \( \gamma \) varies with \( t_1 \) and with \( t \), we make a plot. In order to be able to do that we need some data for all the variables used in our model. The information was obtained for years 2008 – 2009 from the sources [19, 20, 21, 22, 23, 24, 25].
find that an individual can retire at the age of 67 which leads to 49 years in the workforce if he starts working when he is 18 years old. We average that to 45 years worked in a lifetime. We consider that an individual goes to school for four years. We note that in 2008 – 2009, there were 3.3 million high school graduates of which 69% continued their education. From the 3.3 million high school graduates we consider a sample set of 3300. In 2008 – 2009, the minimum wage was $6.55 per hour which results in $13,624 per year if the person works 40 hours per week for 52 weeks. Next we find that the mean annual wage in 2008 was $42,270 which represents a $7,162 increase for each of the four years of higher education. We consider that the largest base increment due to education is $10,000 per year and the economic penalty for staying in school is $709.50 per year. Next we assume that the loss due to competition for uneducated workers is 12% and for educated workers is 5%. We also found that in 2008 – 2009 the income tax rate was between 15% and 25% so we average that to 20%. In the academic year 2008 – 2009, the average tuition for a four-year public college was $6,585. This amount does not include room and board. The money borrowed for tuition is paid back at a 6.8% interest rate. Lastly, we assume that the government’s benefit from a more educated population is 10%. All this data is presented in table 3.1.
Table 3.1: List of variables with assigned values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>45</td>
<td>[24]</td>
</tr>
<tr>
<td>$t_1$</td>
<td>4</td>
<td>[20]</td>
</tr>
<tr>
<td>$n$</td>
<td>3300</td>
<td>[20]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.69</td>
<td>[20]</td>
</tr>
<tr>
<td>$a_0$</td>
<td>13,624</td>
<td>[19]</td>
</tr>
<tr>
<td>$a_1$</td>
<td>7162</td>
<td>[19]</td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.12</td>
<td>[25]</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.05</td>
<td>[25]</td>
</tr>
<tr>
<td>$c_0$</td>
<td>10,000</td>
<td>[19]</td>
</tr>
<tr>
<td>$c_1$</td>
<td>709.5</td>
<td>[19]</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.2</td>
<td>[22]</td>
</tr>
<tr>
<td>$P$</td>
<td>6585</td>
<td>[21]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.068</td>
<td>[23]</td>
</tr>
<tr>
<td>$k$</td>
<td>0.1</td>
<td>[25]</td>
</tr>
</tbody>
</table>
Using this data, we plot (3.3) as a function of $t_1$.

![Figure 3.1: Optimal $\gamma$, equation (3.3), as a function of $t_1$.](a) $P = $6,585/year  (b) $P = $16,585/year

Observe in Figure 3.1 part (a) that in order to obtain $0 < \gamma < 1$ an individual should go to school for approximately $10-11$ years. If the tuition is higher, observe in Figure 3.1 part (b), that the individual should go to school for $3-6$ years in order to obtain $0 < \gamma < 1$. Thus, if the tuition is too high, the government needs to subsidize.

We define $\gamma$ to be a piecewise function. We choose is to be 0 if it is a negative percentage and we choose it to be 1 if it is a percentage greater than 1. We make these assumptions because in the American society a worker does not pay the government to go to school nor does the government pay an individual to go to further education.

Next we plot the same function (3.3) in terms of $t$. 
Figure 3.2: Optimal $\gamma$, equation (3.3), as a function of $t$.  
(a) $P = \$6,585$/year  
(b) $P = \$16,585$/year

Observe in Figure 3.2 how the tuition again influences $\gamma$. For a lower tuition, the optimal value for $\gamma$ is between 0 and 1 when the individual works for 13 to 20 years. For a higher tuition, $0 < \gamma < 1$ when the individual works for approximately 34 – 45 years. Notice in Figure 3.2 (b) that if an individual works for 45 years, then the government should offer a 70% subsidy for education.

To find the optimal payoff function $G_1$ we substitute (3.3) into (3.1). After simplifying we obtain

$$G_1 = \frac{1}{4} \frac{n[P_t(2\beta + k(1 - \beta)) + t\tau(1 - \beta)(a_0 + a_1t_1 - b_0n)]^2}{[\tau t_n(1 - \beta)(b_0 - b_1) - 2P_t](\beta - 1)}. \quad (3.5)$$

Observe that the numerator of (3.5) is always positive. Since we want the optimal payoff function of the government to be a positive quantity, we need the denominator of (3.5) to also be positive. Note that $\beta < 1$, so $\beta - 1 < 0$. This observation leads to
the conclusion that $G_1 > 0$ if

$$
\tau tn(1 - \beta)(b_0 - b_1) < 2P t_1.
$$

(3.6)

In other words, the government will obtain a maximum income if the product of the taxation rate, the number of years worked by an individual, the number of uneducated workers, and the loss of income of a worker due to competition, is less than twice the tuition paid by one individual.

Next we want to analyse how the optimal income of the government (3.5) changes with $t_1$ and $t$ in this subgame.

![Figure 3.3: Optimal income of the government, $G_1$ equation (3.5), in terms of $t_1$.](image)

In Figure 3.3 we observe that the government obtains a maximum income when the tuition is lower. In that case, the optimal payoff function is obtained if the individual goes to school for approximately 5 years. We also observe that the higher
the tuition, the more the workers need to go to school such that the government earns a maximum income from the educated workers.

Next we graph the optimal $G_1$ (3.5) versus $t$.

![Graph of optimal income of the government, $G_1$ equation (3.5), in terms of $t$ for $t_1 = 5$ and $P = 6,585$.](image)

To obtain Figure 3.4 we used the variable values stated in Table 3.1. We changed the number of years of education, $t_1$, from 4 to 5 years since in Figure 3.4 we obtained $t_1 = 5$ to be the optimal number of years of education for a tuition of $P = 6,585$. Here we observe that the more the individual works, the more the government earns in a lifetime. If each individual who went to school for 5 years works for 45 years then the government’s lifetime income is approximately 1.1 billion dollars from this sample set of individuals.
Now that we have found the optimal payoff function of the government, we want to use the same procedure to determine the optimal payoff function of the individual. For this we optimize \( W_1 \) by choosing \( t_1 \), the variable controlled by the worker. Observe that

\[
\frac{\partial W_1}{\partial t_1} = c_0 t(1 - \tau) - 2c_1 t_1 (1 - \tau) - (1 - \gamma + \alpha) P. \tag{3.7}
\]

Setting (3.7) equal to zero and solving for \( t_1 \), we obtain

\[
t_1 = \frac{c_0 t(1 - \tau) - (1 - \gamma + \alpha) P}{2c_1 t(1 - \tau)}. \tag{3.8}
\]

We want \( t_1 \geq 0 \). Since the denominator of (3.8) is positive we need

\[
c_0 t(1 - \tau) \geq (1 - \gamma) P + \alpha P. \tag{3.9}
\]

So \( t_1 \) is positive if the largest base increment to a worker’s salary, after taxation, is greater than the tuition per year, after the subsidy plus the interest per year. If this condition is satisfied, than it is worthwhile for a worker to educate himself.

To obtain the optimal payoff function of the individual we substitute (3.8) into \( W_1 \) (2.1),

\[
W_1 = [a_0 + \frac{c_0^2 t(1 - \tau) - c_0 (1 - \gamma + \alpha) P}{2c_1 t(1 - \tau)} - \frac{[c_0 t(1 - \tau) - (1 - \gamma + \alpha) P]^2}{4c_1 t(1 - \tau)^2} 
- b_0 (1 - \beta)n - b_1 \beta n + (b_0 - b_1)(1 - \beta) n \frac{\gamma}{2} t(1 - \tau) 
- [(1 - \gamma + \alpha) P \frac{c_0 t(1 - \tau) - (1 - \gamma + \alpha) P}{2c_1 t(1 - \tau)} 
+ \frac{c_0 t(1 - \tau) \alpha P - (1 - \gamma + \alpha) \alpha P^2}{2c_1 t(1 - \tau)}]. \tag{3.10}
\]
Due to the algebraic complexity of this expression we decided to graph it as a function of $\gamma$. For comparison purposes, we plot the base wage of an individual on the same graph.

![Graph showing the portion of education subsidized, $\gamma$, and lifetime income of a worker, $W$.](image)

*Figure 3.5: Optimal lifetime income of an educated worker, $W_1$, equation (3.10), compared to the base wage of an uneducated worker, $W_4$, equation (2.11), for optimal $t_1$, equation (3.8), and $P = $6,585.*

Observe in Figure 3.5 that if an individual decides to go to school for $t_1$ years and the government subsidizes a proportion $\gamma$ of the education, then the individual gains approximately 1.2 million dollars extra in a lifetime. We also note that $\gamma$ does not have a big influence on the optimal income.

Next, we consider the number of years of schooling (3.8) as a function of government subsidy (3.3).
Figure 3.6: Optimal number of years of education, $t_1$ equation (3.8), in terms of $\gamma$, for $P = $6,585.

In Figure 3.6 we observe that to obtain a maximum lifetime income when the government is subsidizing the education, an individual should go to school for approximately 7 years. The number of years of education that maximizes the lifetime income of a worker does not change much with $\gamma$.

Considering (3.8) together with (3.3), we notice that we have a system of two equations with two unknowns, $t_1$ and $\gamma$. To solve it we substitute (3.8) into (3.3). We obtain a quadratic equation in $\gamma$

$$a \gamma^2 + b \gamma + c = 0, \quad (3.11)$$

where,

$$a = \frac{P^2(1 - \beta)n(4 - 3\tau)}{4c_1t(1 - \tau)^2}, \quad (3.12)$$
\[ b = \frac{\beta n P^2}{c_1 t(1 - \tau)} - \frac{P^2(1 - \beta)n(k + 2 + 2\alpha)}{2c_1 t(1 - \tau)} + \frac{(1 - c_0)\tau(1 - \beta)nP}{2c_1(1 - \tau)} - \frac{(1 + \alpha)P^2\tau(1 - \beta)n}{2c_1(1 - \tau)^2} + \frac{P(1 - \beta)c_0}{c_1} - (b_0 - b_1)(1 - \beta)^2n^2t\tau, \tag{3.13} \]

\[ c = \frac{(2\beta - k + k\beta)Pn(c_0 t - c_0 t\tau - P - \alpha P)}{2c_1 t(1 - \tau)} + \frac{\tau(1 - \beta)n(1 + \alpha)^2P^2 - c_0^2t^2(1 - \tau)^2}{4c_1 t(1 - \tau)^2} - [a_0 - b_0(1 - \beta)n - b_1\beta n + (b_0 - b_1)\beta n]t\tau(1 - \beta)n. \tag{3.14} \]

To solve (3.11), we use the quadratic formula

\[ \gamma_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{3.15} \]

Due to the algebraic complexity of the solution, we graph it. We use the values of the variables from Table 3.1. To obtain a value for \( \gamma \), with \( 0 < \gamma < 1 \), that satisfies (3.11) with tuition at \$13,170 per year, we consider Figure 3.7.
Observe in Figure 3.7 that we restricted the domain of the quadratic function to $\gamma \in [0, 1]$. In this interval we have one solution for $\gamma$, $\gamma_1 = 0.3$. That is

$$\gamma_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}. \quad (3.16)$$

This means that in this subgame, under these circumstances, the government should subsidize 30% of the education to obtain a maximum lifetime income. We substitute this value of $\gamma$ in (3.8) to find $t_1$, the optimal number of years an individual should go to school to earn a maximum lifetime income when the government is subsidizing the education. We find $t_1 = 6.8$. Thus, to obtain a maximum lifetime income when the government offers a 30% tuition subsidy, an individual should go to school for approximately seven years.
Now that we know what the optimal $\gamma$ and $t_1$ are in this subgame, we use them to determine how the maximal incomes of the government and worker vary with $t$, the number of years worked by an individual. For that we plot (3.5) and (3.10) as functions of $t$.

![Graph showing the optimal income of the government as a function of the number of years worked by individuals.]

Figure 3.8: Optimal income of the government, $G_1$ equation (3.5), in terms of the number of years worked by individuals, $t$, for optimal $\gamma$ equation (3.16), optimal $t_1$ equation (3.8), for $P = $13,170, $\tau = 0.2$, and the rest of the variable values stated in Table 3.1.

Observe in Figure 3.8 that if the government subsidizes at optimal $\gamma$ and the individual goes to school for an optimal number of years $t_1$, then the income of the government collected from the educated workers is increasing with the number of years worked by the individuals. That amount reaches a maximum of one billion dollars when everybody works for 45 years. Also notice that the government starts earning income from the workers only after they worked for more than seven years.
Figure 3.9: Maximum lifetime income of an educated worker, $W_1$ equation (3.10), in terms of the number of years worked, $t$, for optimal $\gamma$ equation (3.16), optimal $t_1$ equation (3.8), for $P =$13,170, $\tau = 0.2$, and the rest of the variable values stated in Table 3.1.

Notice in Figure 3.9 that if the individual goes to school for the optimal number of years of education, $t_1$ (3.8), and the government subsidizes at optimal $\gamma$ (3.16), then the worker’s lifetime income is increasing with the number of years worked reaching a maximum of approximately 1.7 million dollars. To obtain real values for $\gamma$ we need to consider that an individual works for at least eight years.

Next we analyse how $t_1$ in (3.8), the optimal number of years of education, varies with $\tau$ and with $t$. We start with $t_1$ versus $\tau$. 

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Figure 3.10: Optimal number of years of education, $t_1$ equation (3.8), in terms of the income tax rate, $\tau$, for optimal $\gamma$ equation (3.16), for different values of $P$, and the rest of the variable values stated in Table 3.1.

Observe in Figure 3.10 that the optimal number of years of education does not change too much with the income tax rate. If the government subsidizes at optimal $\gamma$ then an individual should go to school for approximately seven years. If the tuition is higher then the government should subsidize if the income tax rate is higher. Notice in Figure 3.10 that the graph has kinks due to the piecewise nature of $\gamma$. On the first piece of the graph $\gamma = 0$, on the second piece $0 < \gamma < 1$, and on the third piece $\gamma > 1$.

Now we examine $t_1$ versus $t$. 
Figure 3.11: Optimal number of years of education, $t_1$ equation (3.8), in terms of the number of years worked, $t$, for optimal $\gamma$ equation (3.16), for different values of $P$, and the rest of the variable values stated in Table 3.1.

In Figure 3.11 we notice that for a tuition of $P = $6,585 an individual should work for at least 5 years to gain financial advantage by going for further education. For a higher tuition, an individual needs to work for longer. In either case, if an individual works for 45 years, then the optimal number of years of education is approximately seven. The kinks in Figure 3.11 are due to the piecewise nature of $\gamma$. For $P = $6,585 and for $t \in [4, 20)$, $\gamma < 0$ thus it is set to 0. For the same tuition and for $t \in [29, 45]$, $\gamma > 1$ thus it is set to 1.

Next we want to analyse how $\gamma$, the optimal portion of education subsidized by the government varies with $\tau$ and $t$. We start with $\gamma$ versus $\tau$. 
Figure 3.12: Optimal percentage of tuition subsidy, $\gamma$ equation (3.16), in terms of the income tax rate, $\tau$, for different values of $P$, and for the variable values stated in Table 3.1.

In Figure 3.12 we notice that the slope of the lines are very large. This indicates a high rate of change of $\gamma$ with respect to $\tau$. Also notice that for an optimal $\gamma$, a higher tuition leads to a higher income tax rate.
We observe in Figure 3.13 that for a lower tuition, if the individuals are in the work force for 20 to 28 years then the optimal $\gamma$ has a high rate of change from 0 to 1. For a higher tuition, if they work for 40 to 45 years, then the optimal $\gamma$ is between 0 and 0.3. Thus, note that if an individual pays yearly tuition of $P = $13,170 and works for 45 years then the government should subsidize 30% of the tuition. Also observe that the kinks in Figure 3.13 are due to the piecewise nature of $\gamma$ and they match the kink locations in Figure 3.11. The value of $\gamma$ is between 0 and 1 when $t$ is between 20 and 28.

Next we examine how $W_1$ varies with $\tau$ for different values of $P$, the yearly tuition.
Figure 3.14: Optimal lifetime income of a worker, $W_1$ equation (3.10), in terms of the income tax rate, $\tau$, for optimal $\gamma$ equation (3.16), and for the variable values stated in Table 3.1 and different tuitions.

We notice in Figure 3.14 that a high income tax rate leads to a decrease in lifetime income of an individual. This should be expected. If the government is taxing more, then the worker keeps less of his income. We also observe that a higher tuition does not influence the lifetime income of a worker very much. If there is no taxation rate, then the educated individual earns approximately 2.1 million dollars after working for 45 years. For a 20% taxation rate, he earns approximately 1.7 million dollars after working for 45 years. Note that in Figure 3.14 the kinks that appear due to the piecewise nature of $\gamma$ are consistent with the kinks in Figure 3.10.

We have seen how the individual’s lifetime wage varies with the income tax rate, $\tau$, for different tuitions. Next we want to examine how their wage varies with
the number of years worked, \( t \), for different \( \tau \).

Figure 3.15: Optimal lifetime income of a worker, \( W_1 \) equation (3.10), in terms of the number of years worked, \( t \), for optimal \( \gamma \) equation (3.16), for \( P = 13,170 \), and for the variable values stated in Table 3.1.

Observe in Figure 3.15 that the lifetime income of a worker increases with the number of years worked reaching a maximum value after 45 years in the work force. We also notice that a higher income tax rate leads to a lower lifetime income. For a 20% tax rate an individual earns a total of approximately 1.7 million dollars if he works for 45 years.

Next we analyse the way the government’s income varies with the number of years worked by individuals and also how it depends on the income tax rate.
Figure 3.16: Optimal government income, $G_1$ equation (3.5), in terms of the number of years worked by individuals, $t$, for optimal $\gamma$ equation (3.16), optimal $t_1$ equation (3.8), for $F =$13,170, different values of $\tau$, and the rest of the variable values stated in Table 3.1.

In Figure 3.16 we notice that the government starts earning money after the individuals worked for 8 years, for a lower income tax rate, and 10 years for a higher $\tau$. After that, the government’s lifetime income from the educated workers is increasing with the number of years worked by the individuals. This income also increases with the taxation rate. Observe that if the taxation rate is 40%, compared to 20%, then the lifetime income of the government doubles.

3.2 Subgame One, Nash-Stackelberg Equilibrium

In this scenario, the government’s decision depends upon the individual’s decision. So the two players will not act simultaneously as in the previous case. The individual
will optimize first by choosing $t_1$, then the government will optimize by choosing $\gamma$.

This procedure requires that after we find the optimal $t_1$, which will be a function of $\gamma$, we plug it into $G_1$ and then optimize. Since we already computed the optimal $t_1$, we use (3.8) to plug into (2.5). After we simplify the resulting expression we set it equal to zero and obtain

$$
- \gamma^2 \left[ \frac{3P^2(1-\beta)n\tau}{8c_1t(1-\tau)^2} + \frac{3P^2(1-\beta)n}{4c_1t(1-\tau)} \right]
+ \gamma \left[ \frac{(1+\alpha)P^2(1-\beta)n\tau - P^2\beta n\tau}{2c_1t(1-\tau)^2} + (b_0 - b_1)(1-\beta)^2n^2t\tau \right]
+ \left( \frac{1+\alpha+k)P^2(1-\beta)n - 2P^2\beta n}{2c_1t(1-\tau)} - \frac{c_0P(1-\beta)n}{2c_1} \right]
+ \left[ \frac{4(1+\alpha)P^2\beta n\tau - (1+\alpha)^2P^2(1-\beta)n\tau}{8c_1t(1-\tau)^2} \right]
+ \frac{2(1+\alpha+k)P^2\beta n - (1+\alpha)kP^2(1-\beta)n}{4c_1t(1-\tau)}
+ \frac{c_0^2t\tau(1-\beta)n}{8c_1} - \frac{c_0P\beta n}{2c_1} + \frac{k\beta P(1-\beta)n}{4c_1}
+ a_0t\tau(1-\beta)\frac{n^2}{2} + (1-\beta)\frac{n^2}{2}t\tau(2\beta(b_0 - b_1) - b_0) \right] = 0. \quad (3.17)
$$

Observe that (3.17) is a quadratic equation in $\gamma$, and the coefficient of $\gamma^2$ is negative. This implies that the graph of (3.17) is a concave down parabola that achieves the maximum value at its vertex. Thus, we can easily find the maximum
value of \( \gamma \) by finding the location of the vertex,

\[
\gamma_{\text{max}} = \frac{4}{3P^2(1-\beta)(2-\tau)} [Pc_0 t(1-\tau)^2(1-\beta) \\
- (b_0 - b_1)(1-\beta)^2nt^2\tau c_1(1-\tau)^2 \\
+ P^2\beta(2-\tau) - P^2(1-\beta)(1+\alpha+k-k\tau)].
\] (3.18)

To interpret this expression, we graph it in terms of \( \tau \). Using the same values for parameters as before, see Table 3.1, we obtain a value for \( \gamma_{\text{max}} > 1 \). Since we are interested in \( 0 < \gamma < 1 \) we suppose that \( \beta = 0.50 \), that is, 50\% of the high school graduates continue their education rather than 69\%.

Figure 3.17: Optimal proportion of education subsidized in terms of the taxation rate (a) \( \gamma_{\text{max}} > 1 \) (b) \( 0 < \gamma_{\text{max}} < 1 \)

In Figure 3.17 part (a) we observe that to obtain \( 0 < \gamma_{\text{max}} < 1 \), \( \tau \) needs to be high. Careful examination shows that \( \gamma_{\text{max}} = 1 \) when the tax rate \( \tau \) is slightly less than 90\%, and decreases to 0 as the tax rate approaches 100\%. This seems rather
unrealistic, because the government cannot tax 90% of an individual's income. So it appears that the government will never operate at $\gamma_{\text{max}}$.

If we substitute the value of $\gamma_{\text{max}}$ into $G_1$, we obtain the optimal payoff function for the government. Again, since this process is algebraically complex, we choose to graph it.

In Figure 3.18 we notice that the maximum income is obtained when the taxation rate is close to 90%. From our previous plot, Figure 3.17, this value of $\tau$ yields $\gamma_{\text{max}} = 1$. The maximum lifetime income in this case is approximately 4.5 billion dollars but is obtained for an unrealistic income tax rate, $\tau$.

This game makes no economic sense. We conclude that the only reasonable approximation to reality is to have the players make their decisions simultaneously.
3.3 Subgame Two, Nash-Cournot Equilibrium

In this subgame, since the government is offering a subsidy but the individual does not go to school, then the government is the only player that can optimize. Also, since only one player can optimize, then what we will find is the Nash-Cournot equilibrium. For this purpose we take the partial derivative of $G_2$ with respect to $\gamma$, set it equal to zero, and solve for the optimal $\gamma$,

$$\frac{\partial G_2}{\partial \gamma} = (n - n_e)^2 t \tau (b_1 - b_0) \gamma + \frac{1}{2} t \tau (n - n_e)(b_1 n_e - a_0)$$

$$+ (n - n_e)^2 t \tau(b_0 - \frac{b_1}{2}). \quad (3.19)$$

Under the assumption that $n_e = \beta n$, where as before $\beta$ is the proportion of high school graduates who continue education, we solve for $\gamma$ in (3.19),

$$\gamma = \frac{2 \beta n(b_1 - b_0) + n(2b_0 - b_1) - a_0}{(b_0 - b_1)(1 - \beta)n}. \quad (3.20)$$

We need $\gamma$ to be a positive quantity, so this leads to the condition

$$a_0 < b_1 \beta n + 2b_0(1 - \beta)n - b_1(1 - \beta)n. \quad (3.21)$$

Hence the base salary of a worker with a high school diploma needs to be less than the loss in income due to the competition for both educated and uneducated individuals minus the gain from people not competing.

To obtain the optimal payoff function for the government, we need to substitute (3.20) into $G_2$. Due to the algebraic complexity of this process we will graph the optimal $G_2$ as a function of $\beta$, the proportion of workers that go to higher education.
For \( b_0 = 0.12 \) and \( b_1 = 0.05 \) and the variable values stated in Table 3.1 we obtain a negative value for \( \gamma \). Since this is unrealistic, we set \( \gamma = 0 \). This means that if the competition between the uneducated workers is greater than the competition between educated workers then the government does not want people to go to school. On the other hand, if \( b_0 = 0.05 \) and \( b_1 = 0.12 \), which means that there is a greater competition among the educated workers, we obtain \( \gamma > 1 \) which we set equal to 1. Thus in this situation, the government stimulates the individuals to go to school. The government’s income in the two situations is presented next.

Figure 3.19: Optimal payoff function for the government \( G_2 \), equation (2.7), in terms of the proportion of high school graduates that go to school, \( \beta \).

(a) \( b_0 = 0.12, \ b_1 = 0.05, \ \gamma = 0 \)  
(b) \( b_0 = 0.05, \ b_1 = 0.12, \ \gamma = 1 \)
$G_2$ represents the government’s income from the uneducated workers. Observe in Figure 3.19 (a), that if the government does not subsidize and the individuals do not go to school then its maximum income is approximately 400 million dollars. On the other hand, if the government subsidizes 100% of the tuition then the maximum income is 210 million dollars. Also observe that if everybody goes to school then the government is not earning money from the uneducated workers but is earning from the educated ones.

So far in the first two subgames, from the government’s perspective, we examined how the total number of high school graduates that continue education influences the total income of the government. Next we want to examine the average net income from an educated worker and from an uneducated worker depending on the number of years worked.
Observe in Figure 3.20 that the government starts earning income from an educated individual only after he works for approximately 8 years. Also notice that the optimal income of the government from an educated worker increases with the number of years worked by the individual, reaching a maximum of approximately $420,000 if the individual works for 45 years. From an uneducated worker, the government earns $150,000 with no subsidy and $75,000 with a 100% subsidy.

Our final analysis considers the conditions under which the government breaks even. We look at the extra income from an educated worker due to subsidy and the government’s cost of education per person.

Figure 3.20: The government’s optimal income from an educated worker for optimal $\gamma$ equation (3.16), optimal $t_1$ equation (3.8), for $P = $13,170, $\tau = 0.2$, and the rest of the variable values stated in Table 3.1, and the government’s optimal income from an uneducated worker
(a) $b_0 = 0.12$, $b_1 = 0.05$, $\gamma = 0$  (b) $b_0 = 0.05$, $b_1 = 0.12$, $\gamma = 1$. 
Observe in Figure 3.21 that in either case, the government is paying $25,000 for the education of one individual but if the individual works for 8 years, then the government is earning $100,000. Also notice that if an individual works for 45 years then the government earns an extra $300,000 from an educated worker. Thus we conclude that is it at the government’s best interest to encourage people to go to school.

3.4 Subgame Three, Nash-Cournot Equilibrium

In this subgame, the government is not subsidizing any of the higher education but the individual decides to go to school. Thus, in this situation, only the worker can optimize their income. Similar to subgame two, we optimize $W_3$ by choosing $t_1$, set...
the expression equal to zero, solve to find the optimal $t_1$ and then substitute it into $W_3$ to determine the optimal payoff function for the individual namely,

$$\frac{\partial W_3}{\partial t_1} = c_0 t (1 - \tau) - 2 c_1 t_1 t (1 - \tau) - (1 + \alpha) P,$$

(3.22)

$$t_1 = \frac{c_0 t (1 - \tau) - P (1 + \alpha)}{2 c_1 t (1 - \tau)}.$$

(3.23)

The expression (3.23) represents the number of years an individual should go to school to maximize his lifetime income. We want this variable to be positive, so by imposing this condition, we obtain

$$c_0 t (1 - \tau) \geq P (1 + \alpha).$$

(3.24)

This means that the largest base increment of income has to be greater than the tuition per year plus interest, in order that it makes economic sense to go on to higher education.

Next we want to see how $t_1$ in (3.23) varies with $t$, the number of years worked by an individual.
In Figure 3.22 observe that the optimal number of years of education is approximately 7 as long as the individual is in the work force for at least eight years. If we compare the optimal $t_1$ from both subgames where the individual optimized, that is compare (3.23) to (3.8), we notice that if the government is subsidizing the tuition then the worker goes to school more, since in (3.8) there is an extra positive term in the numerator, $\gamma P$.

Now we plug (3.23) in $W_3$ to obtain the optimal payoff function of the worker,

$$W_3 = [a_0 - b_0(1 - \beta)n - b_1 \beta n]t(1 - \tau) + \frac{[c_0 t(1 - \tau) - P(1 + \alpha)]^2}{4c_1 t(1 - \tau)}.$$  \hspace{0.5cm} (3.25)

In (3.25) we notice that the second component is always positive. The first component of (3.25) is the lifetime income of a worker with high school degree which we assume is a positive quantity. Thus in this situation an individual always wins if
he educates himself. Also notice that increasing the base salary of a worker with a high school diploma would increase the maximum lifetime income of a worker with a higher degree and thus encourage more people to go to school. To examine how $W_3$ varies with $t$ we make a plot. For purposes of comparison, we graph $W_3$ together with $W_1$.

![Graph of $W_3$ and $W_1$](image)

**Figure 3.23:** Optimal income of an individual in the presence or absence of tuition subsidy.

Observe in Figure 3.23 that the lifetime income of an educated individual increases with the number of years worked, reaching a maximum of 1.65 million dollars if an individual works for 45 years. Also notice that the presence of a tuition subsidy does not influence the lifetime income much. In fact, a worker earns only an extra $20,000 in a lifetime if the government subsidizes the education.
3.5 Subgame Four, No Equilibrium

In this subgame, neither player can optimize because they don’t make any choices.

The government does not offer a subsidy and the worker does not go to school.
CHAPTER IV

CONCLUSIONS

In this paper, we used a game theoretic approach to study the issue of the cost of higher education versus the economic return. In our game we have two players each of whom has two choices. The first player is the government who has to choose whether to subsidize some portion of the education to maximize its income. The second player is the individual who has to choose between continuing education after high school or going into the work force, in order to maximize lifetime income. These strategies created four different subgames.

To start with, we followed the model of Correa [3] and added some new parameters such as the taxation rate imposed by the government, the portion of the cost of higher education subsidized, the benefit of the government from a more educated population, and the total cost of schooling.

Our assumptions were:

- an individual who is obtaining higher education is not working during this time,

- an individual who has a higher degree can obtain a better paid job with greater benefits,
• an individual with a better job has good health due to good health insurance and access to preventive health care,

• a worker with better health lives longer,

• the government realizes benefits from a more educated population. These benefits are higher incomes to tax, less money spent on unemployment programs and on the health care system, and other social benefits,

• if the government subsidizes the entire education then half of the people who would not go to higher education otherwise will do so,

• the individual can choose the number of years of education he wants to complete,

• the government can choose the proportion of education to subsidize,

• there is greater competition for uneducated workers than for educated ones,

• every individual, with or without education, works the same number of years.

Incorporating these assumptions in our model we obtained the following results:

• in the scenario when both players optimize simultaneously, the government should subsidize if the tuition is too high,

• for a higher tuition, the government should subsidize if the individual works for at least 34 years,

• if the government is operating at the optimal point then the higher the tuition, the lower the government’s income,
• in the same scenario, it is worth it for the individual to go to school if the largest base increment to his salary is greater than the tuition per year plus the interest paid,

• using the data from the years 2008 – 2009, if an individual would have went to school for approximately 7 years then he would have earned an extra 1.2 million dollars in a lifetime,

• in this subgame we find that an individual should go to school for approximately 7 years to maximize his lifetime income and the government should subsidize 30% of education to obtain a maximum income from the educated workers,

• if a worker goes to school for 7 years and the government subsidizes education at an optimal percentage then his maximum lifetime income is approximately 1.7 million dollars,

• if the government subsidizes 30% of education then its maximum income obtained from the educated workers is one billion dollars,

• the government is earning an extra $420,000 from each educated individual who works for 45 years,

• the government starts earning income from an educated worker after the individual worked for 8 years,

• an individual should go to school if he plans to work for at least 8 years,
• in the scenario when the government’s decision depends upon the individual’s
decision we found that for the same years as above the government would obtain
a maximum lifetime income if the taxation rate were around 90% and in this
case the lifetime gain would be 4.5 billion dollars,

• the Nash-Stackelberg equilibrium, where the government makes the decision
after the individual, does not make economic sense thus the players should
avoid it,

• in the subgame when the government is subsidizing education and the individual
does not go to school we found that if the base salary of a worker with no higher
education is less than the loss of income due to competition for both educated
and uneducated individuals then the government should subsidize,

• in the same subgame we found that more individuals attending school leads to
reduced income from the uneducated workers,

• in the scenario where there is no subsidy but the individual goes to school, we
found that it would be worth doing so if the largest base increment is greater
than the tuition per year. In this situation, the worker always wins,

• in this subgame the optimal number of years of education is also approximately
seven,

• the lifetime income of an educated worker is only $20,000 higher when the
government subsidizes education than when it does not,
• the government’s cost of education for one worker is $25,000 but if the individual works for at least 8 years then the government is making an extra $100,000 from an educated worker.

Some possible modifications to this paper would be considering the importance of attending mid-level schools versus ivy league schools which could result in large differences in wages. Another important addition to this paper’s analysis would be differentiating between disciplines. One could also consider a different percentage of individuals who would go to school when the government subsidizes education. Another modification to our model could be to introduce a yearly percentage salary raise.
BIBLIOGRAPHY


