MODELING OF A THREE LAYER COATED NANOWIRE TRANSISTOR

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Master of Science

Naga Swathi Kucherlapati
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ABSTRACT

This thesis presents the analysis of nanowire transistors with 50nm or less base width. It assumes that the device structure is cylindrical in shape. The method of analysis in nanoscale is adopted from basic macroscale from a classical paper by Rittner. This work is an extension to Rittner’s linear analysis; it numerically solves the governing nonlinear equation for minority carrier distribution (holes) in a base region of a bipolar junction transistor. The work also considers a cylindrical model for nanowire transistor which differs from Rittner’s paper. It develops the linear and nonlinear equations for minority carrier distribution in a nanowire transistor, based on the mathematical formulations of BJTs. The nonlinear solutions derived for the minority carrier distribution matches the linear solutions at low level injections for both conventional BJT and nanowire transistors. These linear and nonlinear solutions for the nanowire model derived in this thesis have similar variations as in conventional BJTs, and thus provides a starting point for future nanowire transistor studies. Finally the JV characteristics of a BJT and a nanowire transistor are plotted and compared. These comparisons show that the current characteristics vary similarly for both conventional BJT and nanowire transistors.
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CHAPTER I
INTRODUCTION

Over the past century new developments emerged in the field of solid state devices [6]. The invention of the point contact transistor by John Bardeen and Walter Brattain [3] in 1947 and the junction transistor by William Shockley [25, 26] (as shown in Figure 1.1) revolutionized the field of electronics. Transistors are the basic building blocks in any electronic circuit. Transistors have allowed cheap production of small-scale electronic devices and were successful in replacing vacuum tubes.

Figure 1.1: First transistor invented at Bell Labs by Bardeen, Brattain, and Shockley in 1947 [6].

The size of the devices got smaller and led to the invention of integrated circuits (IC) by Jack Kilby of Texas Instruments and Robert Noyce of Fairchild
Semiconductor [19]. An IC is a small electronic circuit made of a thin semiconductor wafer consisting of transistors and other passive devices. It is used in almost every electronic device. The decrease in size of integral elements along with an increase in integration of a larger number of transistors per IC played a vital role in the development of new IC devices.

1.1 Transistors

A transistor is an electronic device used to amplify and switch electronic signals. A transistor is a three terminal device with emitter, base and collector terminals. It consists of two $p$-$n$ junctions. Figure 1.2 shows two different types of bipolar junction transistors.

The device is called bipolar since its operation involves both types of mobile carriers, electrons and holes. An $n$-type material contains excess electrons and $p$-type
material contains excess holes. The direction of the emitter arrow defines the type of transistor. Biasing and power supply polarity are positive for \textit{NPN} and negative for \textit{PNP} transistors. Figures 1.3 and 1.4 show the geometries of \textit{PNP} and \textit{NPN} transistors.

The main bipolar junction transistor (BJT) design limitations [18] include speed, current gain and emitter efficiency. One possible solution to these limitations is reducing the basewidth. This can be achieved by reducing the size of the devices to nanoscales. By reducing the size the device, speed can be increased as well as integration density.

One particular topic of research in nanoscale structures [4] such as carbon nanotubes (CNTs), silicon nanowires (SiNWs) and silicon nanodots (SiNDs) is the electronic properties and applications of nanowire materials. Different types of nanowires exist such as metallic (e.g., Ni, Pt, Au), semiconducting (e.g., Si [16], InP, GaN, etc.) and insulating (e.g., SiO$_2$, TiO$_2$).

Nanowires can be used in different applications such as in building the next
generation of computing devices. Research has been done on properties of coated nanowires, especially in polymer nanofibers where they can be coated [7, 8] with different metals. Nanowires, when coated with required layers of Si or Ge, can possibly become the required nanowire transistor and can be used as basic building blocks in many applications. This thesis gives a brief idea about the design of coated nanowire transistors.

1.2 Silicon Nanowires

Silicon nanowires [16, 17] as shown in Figure 1.5 can be used as basic conducting structures, because these structures replace carbon nanotubes and can be used to build next generation computing devices [9]. Semiconductor nanowires [15, 10] are good at electron and hole transportation similar to normal semiconductors. They can be used as the basic building blocks in any nanoscale electronic device without using any complex fabrication process. Much research has been done in nanowire FET devices [11] while little research has been performed in nanowire BJT devices.
Figure 1.5: Silicon nanowires: (a) low resolution images of a nanowire array, (b) high resolution images of a nanowire array [13].

Thus, this research considers only the $PNP$ BJT configuration, where holes are the minority carriers in the $n$-type base region.

1.3 Coated Nanowires

Hybrid nanostructures consisting of nanowires are coated with nanoparticles (as shown Figure 1.6) which are of interest for a variety of potential applications, including catalysis, sensors, electronic devices, magnetics, optics and others [13, 7, 8]. On the nanowires, coating of nanoparticles is done by various particle reactions on the surface achieving discrete islands as shown in Figure 1.6.

Layers of silicon can be coated on the nanowire to create a either a $PNP$ or an $NPN$ junction transistor. For a $PNP$ configuration, $p$-type Si nanoparticles are nucleated on the nanowire. Then, on the $p$-type coating, the next $n$-type Si layer is grown forming a $p-n$ junction. Finally to form the required nanowire transistor another layer of $p$-type Si is nucleated on the $p-n$ junction surface as shown in Figure 1.7.
Nanowires are roughly cylindrical in shape, and when coated with semiconductor materials, form a cylindrical structure as shown in Figure 1.7. Thus, cylindrical symmetry is considered in this work. Consequently, a cylindrical model of the transistor as an extension of the basic one-dimensional model transistor equations, is developed in this research.

1.4 Background Information

Rittner’s paper [21] describes the basic concept involved in deriving the linear equation from the governing nonlinear equation for minority carrier distribution in the base region and derives a solution to linear equation. It considers the traditional transistor with linear structure (shown in Figure 1.7 a) i.e., vertical bipolar junction transistor [24]. This thesis is an extension to Rittner’s paper where nanowire transistor with cylindrical structure (as shown in Figure 1.7 b) is studied.

Many research topics in lateral transistors [2, 30, 20] include cylindrical configuration. The cylindrical structure in these papers consists of concentric squares.
The analysis of a cylindrical mesa transistor [14] also considers a cylindrical structure; however, it uses different boundary conditions as compared to this thesis, and assumes current flow in axial direction.

1.5 Present Work

The goal of the present work is to make a realistic and consistent mathematical model for a nanowire transistor by following the old principles of Shockley [25, 26]. The physical parameters of nanowire transistor are chosen from the normal BJT parameters for the purposes of deriving the equations and boundary conditions. These parameters include the values of equilibrium hole and current densities, donor and electron densities, etc.

The analysis done in this thesis includes the derivation of both linear and nonlinear differential equations for minority carrier distribution (i.e., hole distribu-
Steady state solutions for minority carriers in the base region are derived with low level injection, neglecting recombination. For boundary conditions, it is assumed that (a) the hole concentration at the emitter junction is raised by Boltzmann factor above its equilibrium value because the applied voltage $V_e$ appears across the space charge layer and (b) the hole concentration at the collector junction is lowered by Boltzmann factor below its equilibrium value because the applied reverse voltage $V_c$ appears across the space charge layer. Shockley’s assumptions are used for problem of injection of minority carriers. It should be noted that exact solutions of the nonlinear equations have not been studied in the literature; only approximations are considered. Finally, the current characteristics of conventional BJT along with current characteristics of nanowire transistor are predicted which is completely novel from previous works. The linear and nonlinear solutions along with the JV characteristics of nanowire transistor are then related to the work of Rittner [21] and by comparing the results it is found that the nanowire transistor model has...
similar tendencies and provides a starting point for future nanowire bipolar junction transistor studies.

This research work is categorized in six chapters. The current chapter explains a brief introduction to the thesis and objective of the research.

Chapter II provides the background of the research. It reexamines the equations for minority carrier distribution of a BJT as introduced by Rittner [21].

In Chapter III, the mathematical model of the conventional BJT structure is transformed to the coated nanowire configuration shown in Figure 1.7.

In Chapter IV, numerical calculations of the linear and nonlinear solutions to minority carrier distribution in conventional BJT are presented. Specifically, the JV characteristics of BJT for linear and nonlinear cases are compared.

In Chapter V, numerical simplifications of the nanowire configuration as modeled in Chapter III are presented. Again linear and nonlinear solutions are analyzed via numerical simplifications, and the JV characteristics of nanowire configuration are plotted.

In Chapter VI, the conclusion and future work are detailed. In this chapter the linear and nonlinear solutions to the minority carrier distribution as well as the JV characteristics of conventional and nanowire transistors are compared.

Appendix A contains mathematical derivations of current density of holes and electrons, the drift-diffusion equations. Appendix B contains the concept of nondimensionalization of the equations, Matlab functions used in programming, derivations of the solution to the Emden-Fowler equation.
CHAPTER II
CONVENTIONAL BJT: MODEL DERIVATION FOR LINEAR GEOMETRY

2.1 Continuity equation

The continuity equation is the basic equation required for treating many problems involving the flow of matter. This equation states the principle of conservation of matter. It is the basic equation used to treat the behavior of holes and electrons under conditions in which the concentrations are functions of time and space. The continuity equation states that the rate of change of the carrier density is equal to the difference between the incoming carrier flux and the outgoing carrier flux, plus the generation minus the recombination. For the holes in a given region, the continuity equation is written as

\[
\frac{\partial p}{\partial t} = (g - r) - \frac{1}{q} \nabla \cdot J_p, \tag{2.1}
\]

where \( p(x, y, z, t) \) is carrier density at position \((x,y,z)\) and time \( t \), \( r \) is recombination rate, \( g \) is generation rate, \( q \) is the absolute value of electron charge and \( J_p \) is current density of holes.

To derive Equation (2.1), consider a small volume element \( dxdydz \) centered at \((x,y,z)\) as shown in the Figure 2.1. Each term from Equation (2.1) is derived separately. The first term in Equation (2.1) corresponds to the rate of change of
holes in $dxdydz$ which is given as

$$\frac{\partial p}{\partial t} \cdot dx \cdot dy \cdot dz.$$  

The change in the number of carriers in the volume results from carrier generation and recombination in the volume along with any net carrier flow across the surfaces. The generation and recombination process is represented by the second term as

$$(g - r)dxdydz.$$  

The third term containing $\nabla \cdot J_p$ represents hole flow across the boundaries. Figure 2.2 shows the current density of holes in the $x$-direction $J_{px}$, where the red and blue dots are electrons and holes respectively. At the midpoint of the $dydz$ face located at $x - \frac{dx}{2}$, the current density is

$$J_{px}(x - \frac{dx}{2}, y, z) = J_{px}(x, y, z) - \frac{\partial J_{px}(x, y, z)}{\partial x} \frac{dx}{2}.$$  

The midpoint value derived above gives the average rate of holes flowing through
the face $dydz$ into $dxdydz$, which can be written as

$$\frac{1}{q}[J_{px} - \frac{\partial J_{px}}{\partial x} \frac{dx}{2}]dydz.$$ 

The flow out of $dxdydz$ across the $dydz$ face located at $x + \frac{dx}{2}$ differs only by replacing the (-) with (+). Thus, the net flow entering the $dxdydz$ through the two $dydz$ faces is

$$\frac{1}{q}[J_{px} - \frac{\partial J_{px}}{\partial x} \frac{dx}{2}]dydz - \frac{1}{q}[J_{px} + \frac{\partial J_{px}}{\partial x} \frac{dx}{2}]dydz = -\frac{1}{q} \frac{\partial J_{px}}{\partial x} dxdydz.$$ 

Proceeding similarly for the $dxdy$ and $dxdz$ faces leads to net flow into $dxdydz$

$$-\frac{1}{q} \left[ \frac{\partial J_{px}}{\partial x} + \frac{\partial J_{py}}{\partial y} + \frac{\partial J_{pz}}{\partial z} \right]dxdydz = -\frac{1}{q} \nabla \cdot \mathbf{J}_p dxdydz.$$ 

Equating the net rate of change of holes in $dxdydz$ to the net generation plus the net flow across the surfaces leads to the continuity equation (2.1). In the absence
of external exciting energy, the recombination and generation term is given by

\[(g - r) = \frac{p_0 - p}{\tau_p},\]

where \(p_0\) is the equilibrium hole density in the \(n\)-type region and \(\tau_p\) is the lifetime for arbitrary injection of holes into \(n\)-type material. The continuity equation for the holes in the \(n\)-type material is given by

\[
\frac{\partial p}{\partial t} = -\frac{p - p_0}{\tau_p} - \frac{1}{q} \nabla \cdot J_p.
\]  

(2.2)

By a similar argument the continuity equation for electrons for an \(n\)-type material can be derived as

\[
\frac{\partial n}{\partial t} = -\frac{n - n_0}{\tau_n} + \frac{1}{q} \nabla \cdot J_n
\]  

(2.3)

where \(n\) is the free electron density and \(J_n\) is current density of electrons.

2.2 Current density

Current density is defined as the rate of flow of charge per unit area. The electron current density \(J_n\) and the hole current density \(J_p\) may be related to the electric field \(E\) and carrier concentration gradient by the drift-diffusion constitutive relations. The current density of holes is given by

\[
J_p = q \mu_p E - q D_p \nabla p,
\]  

(2.4)
and the current density of electrons is given by

\[ J_n = qn\mu_n E + qD_n \nabla n, \quad (2.5) \]

where \( E \) is electric field, \( \mu_p \) is the mobility of holes, \( \mu_n \) is the mobility of electrons, \( D_p \) is the diffusion constant of holes and \( D_n \) is the diffusion constant of electrons.

By definition the total current density \( J \) is the summation of the current density of holes (2.4) and electrons (2.5); that is,

\[ J = J_p + J_n. \quad (2.6) \]

Equations (2.2) through (2.6), with associated boundary conditions, are sufficient to determine the carrier distribution in any region of BJT.

In order to find the values of \( J_p, J_n, E \) Shockley’s assumptions are used [21, 22].
a) Electrons and holes disappear by mutual recombination at identical rates: i.e,

\[ \frac{p - p_0}{\tau_p} = \frac{n - n_0}{\tau_n}. \quad (2.7) \]

The electron and hole lifetimes are assumed equal.
b) Space charge neutrality is preserved at every point as

\[ p - n + N_d - N_a = 0. \quad (2.8) \]

Here \( N_d \) is density of donors and \( N_a \) is density of acceptors. From the above equation
it can be derived as

\[ \frac{\partial p}{\partial t} = \frac{\partial n}{\partial t}. \]  

(2.9)

The value of total current density is derived by replacing Equations (2.7), (2.9) in the continuity equations as

\[ \nabla \cdot J = 0. \]  

(2.10)

In the one-dimensional case, the value of \( J \) is

\[ J = J\hat{i}, \]

where \( \hat{i} \) is the unit vector in the \( x \)-direction. In this case, Equation (2.10) implies

\[ \frac{\partial J}{\partial x} = 0. \]

Thus, the value of \( J \) does not vary with position.

2.3 Calculation of \( E \) and \( J_p \)

\( E \) is the applied electric field, which can be calculated from the total current density \( J \). The \( J \) is the sum of the current density of holes \( J_p \) from Equation (2.4) and current density of electrons \( J_n \) from Equation (2.5),

\[ J = qp\mu_p E - qD_p \nabla p + qn\mu_n E + qD_n \nabla n. \]  

(2.11)
By rearranging terms, the value of the electric field strength can then be given as

\[
\mathbf{E} = \mathbf{J} + q(D_p \nabla p - D_n \nabla n) \frac{q(p \mu_p + n \mu_n)}{q(p \mu_p + n \mu_n)},
\]  

(2.12)

From space charge neutrality, the value of electron concentration can be calculated for an \( n \)-type semiconductor (where \( N_a << N_d \), i.e., the acceptor density is much lower than donor density),

\[
n = p + N_d.
\]  

(2.13)

Using (2.13) in (2.12), and assuming that \( N_d \) is uniform, the value of the electric field strength is given as

\[
\mathbf{E} = \mathbf{J} + q \nabla p(D_p - D_n) \frac{q(p \mu_p + (p + N_d)b)}{q(p \mu_p + (p + N_d)b)},
\]  

(2.14)

where \( \frac{\mu_n}{\mu_p} = b \). Factoring \( D_p \) gives

\[
\mathbf{E} = \mathbf{J} - q \nabla p D_p(\frac{D_n}{D_p} - 1) \frac{q(p \mu_p + (p + N_d)b)}{q(p \mu_p + (p + N_d)b)},
\]  

(2.15)

but \( \frac{D_n}{D_p} = b \). The value of \( b \) is substituted to get the required electric field strength

\[
\mathbf{E} = \mathbf{J} - qD_p(b - 1) \nabla p \frac{q(p \mu_p + b N_d)}{q(p \mu_p + b N_d + b N_d)}.
\]  

(2.16)

Now the hole current density is derived by substituting Equation (2.16) in Equation (2.4)

\[
\mathbf{J}_p = q p \mu_p \mathbf{J} - q D_p(b - 1) \nabla p \frac{q p \mu_p}{q p (b + 1) + b N_d} - q D_p \nabla p.
\]  

(2.17)
By rearranging the terms, the hole current density in an $n$-type region is found as

$$ J_p = \frac{pJ - \nabla p(2bpqD_p + qD_pbN_d)}{(p(b + 1) + bN_d)}. $$ (2.18)

### 2.4 Nonlinear equation formulation

In the following calculations only hole current density is considered in order to calculate minority carrier (hole) distribution. The majority carrier distribution could then also be calculated using Equation (2.13). Substituting (2.18) into the continuity equation for holes (2.2) gives

$$ -\left( \frac{\partial p}{\partial t} + \frac{p - p_0}{\tau_p} \right) = \frac{1}{q} \nabla \cdot \left( pJ - qD_p b(2p + N_d) \nabla p \right). $$ (2.19)

The general form of nonlinear equation is given as

$$ -\left( \frac{\partial p}{\partial t} + \frac{p - p_0}{\tau_p} \right) = \frac{\nabla p \cdot \mathbf{J} - qD_p N_d (1 + \frac{p(b+1)}{bN_d}) (1 + \frac{2p}{N_d}) \nabla^2 p - qD_p (b - 1) (\nabla p \cdot \nabla p)}{qbN_d [1 + \frac{p(b+1)}{bN_d}]^2}. $$ (2.20)

The one-dimensional nonlinear equation, shown in Appendix A.2, yields

$$ -\left( \frac{\partial p}{\partial t} + \frac{p - p_0}{\tau_p} \right) = \frac{1}{q} \left( J\left( \frac{\partial p}{\partial x} \right) - qD_p (b - 1)(\frac{\partial p}{\partial x})^2 - qD_p bN_d (1 + \frac{2p}{N_d}) (1 + \frac{p(b+1)}{bN_d}) (\frac{\partial^2 p}{\partial x^2}) \right). $$ (2.21)

Equation (2.21) represents the nonlinear equation for minority carrier (hole) distribution to be solved where $J$ is the forcing term which is independent on position.
The boundary conditions from Rittner’s paper [21] at $x = 0$ and $x = w$ are given as

$$p_{|x=0} = p_1 = p_0 \exp\left(\frac{qV_e}{kT}\right) \quad (2.22)$$

$$p_{|x=w} = p_2 = p_0 \exp\left(-\frac{qV_c}{kT}\right) \quad (2.23)$$

where $V_e$ is the DC bias potential on emitter relative to base, $V_c$ is the DC bias potential on collector relative to base, $k$ is the Boltzmann constant, $T$ is the temperature in Kelvin, $p_0$ is equilibrium hole density in an $n$-type base region. The boundary conditions may be considered as the effect of the injection of holes from the neighboring regions. The solution to this nonlinear equation is given in later chapters. The hole current density Equation (2.18), Equation (2.16) and the solution of $p$ of Equation (2.21) are also used in calculating $J_p$ and current characteristics.

2.5 Linear equation formulation

Shockley [21] simplified the problem of calculating the minority carrier distribution by assuming the conduction current is negligible when compared to the diffusion current ($E = 0$) and taking only Equations (2.2) and (2.4) into consideration for the one-dimensional flow of carriers. Shockley’s simplification yields the equation

$$\left(\frac{\partial p}{\partial t}\right) + \frac{p - p_0}{\tau_p} = D_p \frac{\partial^2 p}{\partial x^2}, \quad (2.24)$$

where $\tau_p$ is lifetime for holes in $n$-type material. This is equivalent to a linear form of the nonlinear Equation (2.21) by assuming $p/N_d << 1$ (low level injection) and neglecting the terms involving $\frac{\partial p}{\partial x}$ [21]. Equation (2.24) represents the linear equation
for minority carrier distribution with boundary conditions at \( x = 0 \) and \( x = w \) as shown in Equations (2.22) and (2.23).

The steady state solution of equation (2.24) assuming low-level injection and finite base width \( w \) is derived from

\[
\frac{\partial^2 p}{\partial x^2} - \frac{(p - p_0)}{\tau_p D_p} = 0.
\]

Define the diffusion length \( L_p \) as \( L_p = \sqrt{\tau_p D_p} \). Then the above equation reduces to

\[
\frac{\partial^2 p}{\partial x^2} - \frac{(p - p_0)}{L_p^2} = 0. \tag{2.25}
\]

Since

\[
\frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 (p - p_0)}{\partial x^2},
\]

Equation (2.25) changes to

\[
\frac{\partial^2 (p - p_0)}{\partial x^2} - \frac{(p - p_0)}{L_p^2} = 0. \tag{2.26}
\]

Boundary conditions are given by \( p(0) = p_1 \) and \( p(w) = p_2 \). Using the change of variables \( p - p_0 = \bar{p} \) in Equation (2.26),

\[
\frac{\partial^2 \bar{p}}{\partial x^2} - \frac{1}{L_p^2} \bar{p} = 0, \tag{2.27}
\]

where the boundary conditions also change to \( \bar{p}(0) = p_1 - p_0 \) and \( \bar{p}(w) = p_2 - p_0 \).

The characteristic equation of Equation (2.27) is of the form \( \lambda^2 - \frac{1}{L_p^2} = 0 \) with roots
\( \lambda = \pm \frac{1}{L_p} \). Then, the solution of Equation (2.27) is of the form:

\[
p = A \exp \frac{1}{L_p} x + B \exp \frac{-1}{L_p} x
\]  

(2.28)

where \( A \) and \( B \) are determined using boundary conditions. Applying boundary conditions and substituting the values of \( A \) and \( B \) (derived in Appendix A.3) into Equation (2.28) gives

\[
p - p_0 = (p_2 - p_0) \sinh(\frac{x}{L_p}) + (p_1 - p_0) \sinh(\frac{w-x}{L_p}) \frac{\sinh(\frac{w}{L_p})}{\sinh(\frac{w}{L_p})}.
\]  

(2.29)

Equation (2.29) is an approximate steady state solution to Equation (2.21) only when \( p/N_d \ll 1 \), i.e., for low injection levels. Thus, the steady state solution derived above is applicable only for low-level injection in the base region. This principle applies to the emitter and collector regions in all conditions, while it applies only for small currents in the base region [21].
Nanowires, as discussed earlier, are roughly cylindrical in shape and can be coated with semiconductor materials. Here 3 concentric coatings on a nanowire are considered so that there can be base, collector and emitter regions. Nanowires are long compared to their width. Thus, it is possible to analyze conduction in this configuration by using the one-dimensional continuity equation having radial symmetry.

Figure 3.1: Cylindrical model of a nanowire bipolar junction transistor.
3.1 Nonlinear equation

Shockley theory will be applied to the configuration of Figure 3.1. The principle of the nonlinear equation for minority carrier (hole) distribution is adapted from the principle of basic transistor operation as in Rittner’s paper. Referring to Equation (2.19) and beginning with

$$\frac{\partial p}{\partial t} = -\frac{p - p_0}{\tau_p} - \frac{1}{q} \nabla \cdot J_p$$

(3.1)

yields the nonlinear equation from previous Chapter as

$$-\left[\left(\frac{\partial p}{\partial t}\right) + \frac{p - p_0}{\tau_p}\right] = \frac{\nabla p \cdot J - qD_pN_d(1 + \frac{p(b+1)}{bN_d})(1 + \frac{2p}{N_d})\nabla^2 p - qD_p(b - 1)(\nabla p \cdot \nabla p)}{qbN_d[1 + \frac{b(b+1)}{bN_d}]^2}.$$

(3.2)

3.2 Transistor geometry

We transform this equation to cylindrical coordinates and assume that $p$ is independent of angular coordinate $\theta$ and of $z$. Thus, the nonlinear equation for minority carrier distribution reduces to

$$-\left[\left(\frac{\partial p}{\partial t}\right) + \frac{p - p_0}{\tau_p}\right] = \frac{J\left(\frac{\partial p}{\partial r}\right) - qD_p(b - 1)(\frac{\partial p}{\partial r})^2 - qD_p b N_d(1 + \frac{2p}{N_d})(1 + \frac{p(b+1)}{bN_d})(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r})}{qbN_d(1 + \frac{b(b+1)}{bN_d})^2}.$$

(3.3)
where \( J \) is a forcing term and in this geometry \( J \) depends on \( r \) as \( J(r) = aJ(a)/r \)

The boundary conditions at \( r = a \) and \( r = b \) are given by

\[
p(a) = p_1 = p_0 \exp\left(\frac{qV_c}{kT}\right) \quad (3.4)
\]
\[
p(b) = p_2 = p_0 \exp\left(-\frac{qV_c}{kT}\right). \quad (3.5)
\]

The above nonlinear equation for hole distribution can be solved only by numerical
techniques. The solution procedure is explained in next chapters.

3.3 Linear solution

The linear equation for distribution of holes in base region for nanowire transistor is
obtained by linearizing Equation (3.3). The same assumptions used by Shockley for
deriving the linear theory for a conventional BJT are used here. The assumptions
are low level injection and radial flow of carriers which lead to the linearized version.

In cylindrical coordinates the linearized version of (3.3) is

\[
\frac{\partial p}{\partial t} + \frac{p - p_0}{\tau_p} = D_p \left[ \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \right]. \quad (3.6)
\]

Further the steady state equation reduces to

\[
\left[ \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \right] = \frac{p - p_0}{\tau_p D_p}. \quad (3.7)
\]

Using the diffusion length \( L_p = \sqrt{\tau_p D_p} \), the equation above takes the form

\[
\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} - \frac{p - p_0}{L_p^2} = 0 \quad (3.8)
\]
and the boundary conditions remain

\[ p(a) = p_1 = p_0 \exp \left( \frac{qV_e}{kT} \right) \]
\[ p(b) = p_2 = p_0 \exp \left( -\frac{qV_c}{kT} \right). \]

Again for simplification purposes define \( p - p_0 = \bar{p} \). Then the equation changes to

\[
\frac{\partial^2 \bar{p}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{p}}{\partial r} - \frac{\bar{p}}{L_p^2} = 0 \tag{3.9}
\]

and the boundary conditions at \( r = a \) and \( r = b \) are given as

\[ \bar{p}(a) = \bar{p}_1 = p_0 \exp \left( \frac{qV_e}{kT} \right) - p_0 \tag{3.10} \]

and

\[ \bar{p}(b) = \bar{p}_2 = p_0 \exp \left( -\frac{qV_c}{kT} \right) - p_0. \tag{3.11} \]

The linear equation is written in the form

\[
r^2 \bar{p}_{rr} + r \bar{p}_r - \frac{r^2}{L_p^2} \bar{p} = 0 \tag{3.12}
\]

and the general solution to this problem is given by

\[ \bar{p}(r) = c_1 I_0(r/L_p) + c_2 K_0(r/L_p) \tag{3.13} \]

where \( I_0 \) and \( K_0 \) are modified Bessel functions \([1]\) and \( c_1, c_2 \) are constants to be evaluated using boundary conditions. The evaluation of the solutions at the boundaries
yield

\[ \bar{p}(a) = c_1 I_0(a/L_p) + c_2 K_0(a/L_p) \]  

(3.14)

and

\[ \bar{p}(b) = c_1 I_0(b/L_p) + c_2 K_0(b/L_p). \]  

(3.15)

The values of \( c_1 \) and \( c_2 \) are given by Cramer’s rule [5] as

\[ c_1 = \frac{K_0(b/L_p)\bar{p}_1 - K_0(a/L_p)\bar{p}_2}{\Delta} \]  

(3.16)

\[ c_2 = \frac{I_0(a/L_p)\bar{p}_2 - I_0(b/L_p)\bar{p}_1}{\Delta} \]  

(3.17)

where

\[ \Delta = I_0(a/L_p)K_0(b/L_p) - I_0(b/L_p)K_0(a/L_p). \]  

(3.18)
CHAPTER IV
NUMERICAL SOLUTIONS FOR CONVENTIONAL BJT

The ultimate goal of this research is to solve Equation (3.3) and to get a solution $p$. This solution is used to obtain the value of $J_p$ and finally to obtain JV characteristics. To solve this equation a numerical solution procedure is used. The function used in this solver [23] is a two-point boundary value problem solver which is defined in Appendix B.1. To verify that the numerical solutions are good approximate solutions, a nonlinear variation of the Emden-Fowler equation is used as a test candidate. This equation is similar in structure to Equation (3.3). The usefulness of considering this equation is that there are exact solutions available [28]. The exact solution formulation is shown in Appendix B.2. This provides a procedure to verify the tools used in Matlab.

4.1 Solver tester

Consider the Emden-Fowler differential equation

$$\frac{\partial^2 p}{\partial x^2} + \frac{1}{p + b} \left( \frac{\partial p}{\partial x} \right)^2 = 0. \quad (4.1)$$

The exact solution to the Emden-Fowler equation with boundary conditions $p(0) = p_1$ to $p(w) = p_2$ is

$$p(x) = \left[ \frac{(p_2 + w)^2 - (p_1 + w)^2}{w} x + (p_1 + w)^2 \right]^{1/2} - w. \quad (4.2)$$
where $p_1$ and $p_2$ are boundary values at $x = 0$ and $x = w$. The exact solution is derived in Appendix B.2.

Figure 4.1 shows that there is very little difference between the numerical and the exact solutions. Thus, the chosen numerical scheme indeed provides reasonable solutions to nonlinear equations such as Equation (3.3).

4.2 Linear solution

The linear equation for minority carrier distribution (hole distribution) in base region for conventional transistor is solved using the same procedure as the nonlinear equation. The linear equation at steady state is given by

$$\frac{p - p_0}{\tau_p} = D_p \frac{\partial^2 p}{\partial x^2} \quad (4.3)$$
and the boundary conditions at $x = 0$ and $x = w$ are given as

\begin{align}
    p(0) &= p_1 = p_0 \exp\left(\frac{qV_c}{kT}\right) \\
    p(w) &= p_2 = p_0 \exp\left(-\frac{qV_c}{kT}\right).
\end{align}

Using state variables

\begin{align}
    y_1 &= \tilde{p} \\
    y_2 &= \frac{\partial \tilde{p}}{\partial x}
\end{align}

where $p = N_d \tilde{p}$, the state equations can be written as

\begin{equation}
    y'_1 = y_2
\end{equation}
and
\[ y''_2 = \frac{N_d y_1 - p_0}{N_d L_p^2}. \] (4.9)

The boundary conditions are given by
\[ y_1(0) = \tilde{p}_1 = \frac{p_0}{N_d} \exp(\frac{qV_e}{kT}) \] (4.10)
\[ y_1(w) = \tilde{p}_2 = \frac{p_0}{N_d} \exp(-\frac{qV_c}{kT}) \] (4.11)

The concentration of holes in an n-type base region as a function of position, is plotted in Figure 4.2, where \( p \) is the linear solution solved from the above two state Equations (4.8), (4.9) and boundary conditions (4.10), (4.11) using Matlab two-point boundary value problem solver. In solving the problem the values 0.2V and 0.3333V are used for \( V_e \) and \( V_c \) respectively. The dependence of \( p \) on \( x \) is approximately linear \( (\frac{\partial^2 p}{\partial x^2} \) is considerably small) showing the carrier density varies linearly between boundary conditions assuming no significant recombination. Thus, the minority carriers without recombination end up as majority carriers in collector region.

4.3 Nonlinear solution

This section discusses the solution procedure for Equation (3.3) with boundary conditions (3.4) and (3.5). Since the equation contains several parameters of varying sizes it should be nondimensionalized. The importance of nondimensionalization [12] is to solve equations using a reduced number of constants. In nondimensionalization rescaling is done so that the parameters and variables can be computed without any size restrictions. The main goal in nondimensionalization is to reduce the number of constants while rescaling very large and small values to relatively similar values. The
nonlinear equation at steady state is given as

\[-\left(\frac{p - p_0}{\tau_p}\right) = \frac{J(\frac{\partial p}{\partial x}) - qD_p(b - 1)(\frac{\partial p}{\partial x})^2 - qD_p b N_d (1 + \frac{2p}{N_d})(1 + \frac{p(b+1)}{bN_d})(\frac{\partial^2 p}{\partial x^2})}{qbN_d(1 + \frac{p(b+1)}{bN_d})^2}.\]  \(4.12\)

and the boundary conditions at \(x = 0\) and \(x = w\) are given as

\[p_1 = p_0 \exp\left(\frac{qV_e}{kT}\right)\]  \(4.13\)

\[p_2 = p_0 \exp\left(-\frac{qV_c}{kT}\right).\]  \(4.14\)

Using the state variables

\[y_1 = \tilde{p}\]  \(4.15\)

\[y_2 = \frac{\partial \tilde{p}}{\partial x}\]  \(4.16\)

where \(p = N_d \tilde{p}, \; x = w\tilde{x}\) writing the nonlinear equation in state equation form as

\[\frac{\partial}{\partial x}y_1 = y_2\]  \(4.17\)

\[\frac{\partial}{\partial x}y_2 = \frac{(J/w)y_2 + con1(\frac{N_d \tilde{p} - p_0}{\tau_p}) - con3(y_2)^2}{con2},\]  \(4.18\)

and the values of \(con1, con2, con3\) are given by

\[con1 = qb(1 + \tilde{p}(b + 1))^2\]  \(4.19\)

\[con2 = qD_p b N_d (1 + 2\tilde{p})(1 + \tilde{p}(b + 1)/b)/(w^2)\]  \(4.20\)

\[con3 = qD_p N_d (b - 1)/(w^2)\]  \(4.21\)
\[ J = 100 \cdot q b D_p N_d / L_p. \]

\[ J = 50 \cdot q b D_p N_d / L_p. \]

\[ J = 10 \cdot q b D_p N_d / L_p. \]

\[ J = q b D_p N_d / L_p. \]

Figure 4.3: Dependence of \( p \) on \( x \) (a) \( J = 100 \cdot q b D_p N_d / L_p \), (b) \( J = 50 \cdot q b D_p N_d / L_p \), (c) \( J = 10 \cdot q b D_p N_d / L_p \), (d) \( J = q b D_p N_d / L_p \).

\[ \tilde{p}_1 = \left( \frac{p_0}{N_d} \right) \exp \left( \frac{q V_e}{kT} \right) \]

\[ \tilde{p}_2 = \left( \frac{p_0}{N_d} \right) \exp \left( -\frac{q V_e}{kT} \right). \]

State equations (4.17) and (4.18) along with the boundary conditions and values from Table A.2 are used in the Matlab program to obtain the required results. The Matlab code listing is given in Appendix B.3.

The variation of the concentration of holes in an \( n \)-type base region as a function of position was calculated for four different values of total current density, and is shown in Figure 4.3. For the lowest value of \( J \), the hole density profile is approximately linear with position. As current density increases, the profile becomes
4.4 Comparing linear and nonlinear solutions

Figure 4.4 compares the linear solution with all nonlinear solutions for holes distribution in the base region. The nonlinearity varies as the value of $J$ varies, where $J$ is a constant. As total current density increases, the nonlinearity of current density of holes increases. For $J << qbN_d/pL_p$ the nonlinear solution matches the linear solution as shown in Figure 4.4d, which is detailed in Rittner’s paper [21], i.e., at lower current densities both linear and nonlinear solutions coincide. This shows that the simplification of the nonlinear equation to obtain the linear equation with the approximations used by Shockley is valid.

Figure 4.4: Comparing linear and nonlinear solutions for a BJT (a) $J=100 \cdot qbD_pN_d/L_p$, (b) $J=50 \cdot qbD_pN_d/L_p$, (c) $J=10 \cdot qbD_pN_d/L_p$, (d) $J=qbD_pN_d/L_p$.

more nonlinear. This nonlinear profile of hole density increases as the $J$ increases, which is clearly depicted in Figure 4.3a, b.

4.4 Comparing linear and nonlinear solutions

Figure 4.4 compares the linear solution with all nonlinear solutions for holes distribution in the base region. The nonlinearity varies as the value of $J$ varies, where $J$ is a constant. As total current density increases, the nonlinearity of current density of holes increases. For $J << qbN_d/pL_p$ the nonlinear solution matches the linear solution as shown in Figure 4.4d, which is detailed in Rittner’s paper [21], i.e., at lower current densities both linear and nonlinear solutions coincide. This shows that the simplification of the nonlinear equation to obtain the linear equation with the approximations used by Shockley is valid.
4.5 Dependence of $J_p$ on $x$

The minority carriers in the emitter and collector regions contribute only small components to the emitter and collector currents [27]. The design of the transistor is to achieve high current gain, which is based on the emitter current consisting almost entirely of minority current flow from base region. The component of collector current due to minority carriers is a part of reverse current of the collector base junction and is very small. Thus, it can be said that the collector and emitter currents are arising solely from minority carrier flow in the base region. The plots shown in Figure 4.5 show how the total current density and the minority carrier current density vary. From Figure 4.5 it can be observed that for smaller values of $J$, $J_p > J$, which implies $J_n < 0$. This means that diffusion is the dominant mode of conduction for electrons.
in this case. For larger values of $J$, $J_p < J$, which implies $J_n > 0$, indicating $J_n$ is dominated by a drift component.

4.6 JV characteristics

The JV characteristics are plotted from the basic equation of hole current density (2.18) and this equation uses the solution $p$ obtained from previous sections. The value of $J$ is fixed to $q b N_d D_p / L_p$ and the value of $J_p$ is taken at $x = 0$. The emitter-base junction is similar to the junction in a $p$-$n$ junction diode. Therefore, the current characteristics look similar to that of the diode in the base region, as shown in Figure 4.6.
CHAPTER V
NUMERICAL SOLUTIONS FOR COATED NANOWIRE TRANSISTOR

This chapter introduces the method of solving the linear and nonlinear equations for the distribution of minority carriers in the base region for the nanowire configuration. It uses the same technique for solving the linear and nonlinear equations as in Chapter IV.

5.1 Linear solution

The steady state linear equation for minority carrier distribution (hole distribution) can be derived to be

\[
\frac{p - p_0}{\tau_p} = D_p \left( \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \right) 
\]

(5.1)

with the boundary conditions at \(r = a\) and \(r = b\) given as

\[
p(a) = p_1 = p_0 \exp\left(\frac{qV_e}{kT}\right) \]

(5.2)

and

\[
p(b) = p_2 = p_0 \exp\left(-\frac{qV_e}{kT}\right) 
\]

(5.3)
Figure 5.1: Dependence of $p$ on $r$ for small injection levels for a nanowire transistor.

where $p = N_a \tilde{p}$. The state equation form can be written from state variables

$$y_1 = \tilde{p}$$

$$y_2 = \frac{\partial \tilde{p}}{\partial r},$$

as

$$y'_1 = y_2$$

$$y'_2 = \frac{N_a y_1 - p_0}{N_a L_p^2} - \frac{1}{r} y_2.$$  

Figure 5.1 shows the dependence of $p$ on $r$, the radial coordinate in the base region. Due to the cylindrical structure the solution to the linear equation is a
nonlinear curve as shown in Figure 5.1.

5.2 Nonlinear solution

The steady state nonlinear equation for hole distribution in coated nanowire configuration from Chapter III is further simplified by considering radial symmetry. The simplified nonlinear equation is given as

\[-\left(\frac{p - p_0}{\tau_p}\right) = \frac{J(\frac{\partial p}{\partial r}) - q D_p (b - 1)(\frac{\partial p}{\partial r})^2 - q D_p b N_d (1 + \frac{2p}{N_d}) (1 + \frac{p(b+1)}{b N_d})(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r})}{q b N_d (1 + \frac{p(b+1)}{b N_d})^2},\]

(5.8)

with the boundary conditions at \(r = a\) and \(r = b\) given as

\[p_1 = p_0 \exp\left(\frac{q V_e}{kT}\right),\]

(5.9)

\[p_2 = p_0 \exp\left(-\frac{q V_c}{kT}\right),\]

(5.10)

The state equation form of the above nonlinear equation can be written as

\[y_1 = \tilde{p},\]

(5.11)

\[y_2 = \frac{\partial \tilde{p}}{\partial \tilde{r}},\]

(5.12)

where \(p = N_d \tilde{p}\) and \(r = a \tilde{r}\).

\[\frac{\partial}{\partial \tilde{r}} y_1 = y_2\]

(5.13)
\[
\frac{\partial}{\partial \tilde{r}} y_2 = \left( \frac{(J)/a}{y_2} + \text{con}1\left( \frac{N_{d}y_{1}-p_{0}}{\tau_{p}} \right) - \text{con}3(y_2)^2 \right) \text{con}2 - \frac{1}{\tilde{r}} y_2.
\]  
(5.14)

and the \(\text{con}1, \text{con}2, \text{con}3\) given as

\[\text{con}1 = qb(1 + \tilde{p}(b + 1)/b)^2\]  
(5.15)

\[\text{con}2 = qD_{p}bN_{d}(1 + \tilde{p})(1 + \tilde{p}(b + 1)/b)/(a^2)\]  
(5.16)

\[\text{con}3 = qD_{p}N_{d}(b - 1)/(a^2)\]  
(5.17)

The value of \(J\) in nanowire transistor varies with the radial position. The value of \(J\) is given as

\[J = \frac{aJ_0}{r}\]  
(5.18)

where the \(J_0\) is the current density at radius \(r = a\). Now, by nondimensionlizing w.r.t. \(r\) we get

\[J = \frac{J_0}{r}\]  
(5.19)

The above state equations (5.13), (5.14) along the boundary conditions are used to obtain the nonlinear solution for the nanowire transistor. These equations are used in matlab code in Appendix B.3. The variation of hole concentration as a function of radial position was calculated for four different values of total current densities \(J\) and is shown in Figure 5.2.
5.2 a: \( J_0 = 10 \cdot q b N_d D_p / L_p \).

5.2 b: \( J_0 = q b N_d D_p / L_p \).

5.2 c: \( J_0 = 0.4 \cdot q b N_d D_p / L_p \).

5.2 d: \( J_0 = 0.01 \cdot q b N_d D_p / L_p \).

Figure 5.2: Dependence of \( p \) on \( r \) for a nanowire transistor (a) \( J_0 = 10 \cdot q b N_d D_p / L_p \), (b) \( J_0 = q b N_d D_p / L_p \), (c) \( J_0 = 0.4 \cdot q b N_d D_p / L_p \), (d) \( J_0 = 0.01 \cdot q b N_d D_p / L_p \).

5.3 Comparing linear and nonlinear solutions

The linear and nonlinear solutions for nanowire configuration are compared in Figure 5.3. When the current density at \( r = a \) is \( J_0 << q b N_d D_p / L_p \) (low injection levels) the nonlinear and linear solutions have significant difference as shown in Figure 5.3 d as compared to conventional transistor. At significantly low current densities linearity persists in nanowire transistor. These values are much lower than the conventional transistor. For larger current densities, the deviation of the nonlinear solution from the linear solution becomes more pronounced.
5.3 a: \( J_0 = 10 \cdot q b D_p N_d / L_p \).

5.3 b: \( J_0 = 1 \cdot q b D_p N_d / L_p \).

5.3 c: \( J_0 = 0.4 \cdot q b D_p N_d / L_p \).

5.3 d: \( J_0 = 0.01 \cdot q b D_p N_d / L_p \).

Figure 5.3: Comparing linear and nonlinear solutions for the nanowire case (a) \( J_0 = 10 \cdot q b D_p N_d / L_p \), (b) \( J_0 = 1 \cdot q b D_p N_d / L_p \), (c) \( J_0 = 0.4 \cdot q b D_p N_d / L_p \), (d) \( J_0 = 0.01 \cdot q b D_p N_d / L_p \).

5.4 Dependence of \( J_p \) on \( r \)

As mentioned in the previous chapter the minority carrier current flow in the base region plays an important role as both collector and emitter minority currents are negligible. It is important to know how the minority carrier current density varies with position in the base region of a nanowire transistor. From Figure 5.4 it can be observed that for small values of \( J \), \( J_p > J \), which implies \( J_n < 0 \). For rest of the cases \( J_p < J \). This indicates that \( J_n \) is dominated by a drift component. Thus, the behaviour of nanowire transistor is similar to that of conventional BJT.
5.4 a: $J_0 = 10 \cdot q b D_p N_d / L_p = 4.527 \text{ A cm}^{-2}$.

5.4 b: $J_0 = q b D_p N_d / L_p = 0.4527 \text{ A cm}^{-2}$.

5.4 c: $J_0 = 0.4 \cdot q b D_p N_d / L_p = 0.1811 \text{ A cm}^{-2}$.

5.4 d: $J_0 = 0.01 \cdot q b D_p N_d / L_p = 0.0045 \text{ A cm}^{-2}$.

Figure 5.4: Variation of minority carrier current density w.r.t. base width for a nanowire transistor (a) $J_0 = 10 \cdot q b D_p N_d / L_p$, (b) $J_0 = q b D_p N_d / L_p$, (c) $J_0 = 0.4 \cdot q b D_p N_d / L_p$, (d) $J_0 = 0.01 \cdot q b D_p N_d / L_p$.

5.5JV characteristics

Using the same procedure as for the conventional BJT discussed in Chapter IV, the JV characteristics of the nanowire transistor are plotted here. The current density of holes is plotted against emitter voltage to obtain the required characteristics of the nanowire transistor. The value of $J_0$ is fixed to $q b N_d D_p / L_p$ and the value of $J_p$ is taken at $r = a$. The current characteristics in the base region of the nanowire transistor shown in Figure 5.5 look like diode characteristics.
Figure 5.5: Nanowire JV characteristics in the base region.
6.1 Conclusions

This study focuses on deriving a mathematical model for a nanowire transistor. For the primary analysis it considers the previous work by Rittner, where a simple physical theory based on the junction transistor of Shockley was presented. It explains the limitations of Shockley’s theory, such as low level injection, one-dimensional flow of carriers as well as the procedure used to solve the linear equation for minority carrier distribution (hole distribution) in the base region after imposing various limitations on the nonlinear differential equation for minority carrier distribution.

This thesis extends the procedure given by Rittner to solve the nonlinear equation for distribution of holes using two-point boundary value problem solvers. The nonlinear solutions are derived successfully and compared to the linear solution for various current densities. For a specific current density at $J << qbN_d D_p / L_p$ both linear and nonlinear solutions coincide, verifying the nonlinear solution as well as the linearization procedure. It also extends the theory by plotting the minority carrier current density based on base width. It explains the behavior of current density in the base region.

The same procedure is applied in finding the solution of both linear and nonlinear equations for hole distribution of a nanowire transistor. To preserve the physical basis of the model transistor, BJT parameter values from Table A.2 are used in this model. Linear and nonlinear solutions of the nanowire transistor are derived.
and compared.

The results of the model are justified in light of the results from standard BJT. The results for a nanowire transistor have similarities with the BJT. Figure 6.1 shows the comparison of linear solutions of conventional and nanowire transistor configurations. Due to the cylindrical structure and an extra nonlinear term, the linear plot is an nonlinear curve.

Solutions of the nonlinear equation in Figure 6.2 is a major achievement of this work. Figure 6.2 compares all the nonlinear solutions for the conventional and the nanowire transistor configurations. This work on the nanowire transistor provides a novel approach for the future technologies. These are the major contributions in this thesis.

Previous work is based on the linear approaches, but this work gives insight by solving and obtaining the nonlinear solution. This work plots the conventional JV
Figure 6.2: Comparing conventional BJT and nanowire transistor nonlinear solutions.

characteristics along with nanowire JV characteristics and compares them. Figures 6.3 show how the current characteristics vary for conventional and nanowire transistors. The magnitude of the current increases in nanowire transistor.

Based on the results there may be advantages as well as disadvantages to the nanowire transistor. The advantages include the high integration density, device speed and current gain. Due to reduced base size in the nanowire transistor, it works faster than normal bipolar junction transistors and the current gain is higher.

6.2 Future work

All previous works were based on the linear theory. This thesis presents the results of the full nonlinear continuity equations.

The proposed model can have more accurate results with better values of the nanowire model. These values can be researched so that the exact analysis of
linear and nonlinear solutions in nanoscale can be made. This can lead to notable inventions in the future.

In this thesis the concept of radial symmetry was used to derive the mathematical model, with the dependence of $\theta$ and $z$ neglected. Calculations based on the effect of $\theta$ and $z$ can be a significant continuation of this work.
APPENDICES
APPENDIX A

A.1 Diffusion equations

The drift-diffusion equations are developed from the Maxwell equations of electromagnetic theory. They are given as

\[ \nabla \cdot \mathbf{D} = \rho \]  
(A.1)

\[ \nabla \cdot \mathbf{B} = 0 \]  
(A.2)

\[ \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} = \nabla \times \mathbf{H} = \text{curl} \mathbf{H} \]  
(A.3)

\[ \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = -\text{curl} \mathbf{E} \]  
(A.4)

\[ \mathbf{D} = \varepsilon \mathbf{E}, \]  
(A.5)

\[ \mathbf{B} = \mu \mathbf{H}, \]  
(A.6)
where $\epsilon$ is permittivity, $\mu$ is permeability. The permittivity of free space and relative permittivity of silicon are given by

$$\epsilon_0 = 8.8541878176 \times 10^{12} \text{F/m}$$

and

$$\epsilon_r = 11.7$$

The permeability of free space is given by

$$\mu_0 = 4\pi \times 10^{-7} \text{N/A}^2.$$ 

In Equation (A.3)

$$\frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} = \nabla \times \mathbf{H}. \quad \text{(A.7)}$$

Applying the divergence operator

$$\frac{\partial}{\partial t} (\nabla \cdot \mathbf{D}) + \nabla \cdot \mathbf{J} = \nabla \cdot \nabla \times \mathbf{H}. \quad \text{(A.8)}$$

In the above equation $\nabla \cdot \nabla \times \mathbf{H} = 0$ which implies

$$\frac{\partial}{\partial t} (\nabla \cdot \mathbf{D}) + \nabla \cdot \mathbf{J} = 0, \quad \text{(A.9)}$$

but, from Equation (A.1)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0. \quad \text{(A.10)}$$
The total current density is given by

\[ J = J_p + J_n. \quad (A.11) \]

But \( \rho = q(p - n + C) \), where \( q \) is electron charge, and \( C \) is the concentration difference due to ionized impurities which is a constant. Hence,

\[ q \left( \frac{\partial p}{\partial t} - \frac{\partial n}{\partial t} \right) + \nabla \cdot J_p + \nabla \cdot J_n = 0. \quad (A.12) \]

where \( \frac{\partial C}{\partial t} = 0 \) since \( C \) is a constant. From Shockley’s assumptions \( \frac{\partial p}{\partial t} = \frac{\partial n}{\partial t} \). Therefore from above \( \nabla \cdot (J_p + J_n) = 0 \), which implies

\[ \nabla \cdot J = 0. \quad (A.13) \]

In one-dimension, the value of \( J \) is

\[ J = J\hat{i} \]

where \( \hat{i} \) is the unit vector in the \( x \)-direction and Equation (A.13) reduces to

\[ \frac{\partial J}{\partial x} = 0. \]

Thus, the value of \( J \) does not vary with position.
A.2 Derivation of nonlinear equation for the conventional BJT

The continuity equation for holes after substitutions gives

\[-\left(\frac{\partial p}{\partial t} + \frac{p - p_0}{\tau_p}\right) = \nabla \cdot \frac{pJ - qD_pb(2p + N_d)\nabla p}{q(p(b + 1) + bN_d)}.\]  \(\text{(A.14)}\)

The right-hand side will split into two terms. Applying the divergence operator to the first term gives

\[\nabla \cdot \frac{pJ}{q(p(b + 1) + bN_d)}\]

and using the formula for divergence \(\nabla \cdot \frac{A}{\phi} = \frac{1}{\phi} \nabla \cdot A + A \cdot \nabla \left(\frac{1}{\phi}\right)\), the first term is

\[\frac{[Jq(p(b + 1) + bN_d)(\frac{\partial p}{\partial x}) - Jq(p(b + 1)(\frac{\partial p}{\partial x}))]}{(q(p(b + 1) + bN_d))^2}\]

where \(J\) is independent of position in one-dimension. Rearranging, the first term becomes

\[\frac{J(\frac{\partial p}{\partial x})}{qbN_d(1 + \frac{p(b+1)}{bN_d})^2}.\]  \(\text{(A.15)}\)

Applying the divergence operator to the second term gives

\[\nabla \cdot \frac{qD_pb(2p + N_d)(\frac{\partial p}{\partial x})}{q(p(b + 1) + bN_d)}.\]
Using the divergence formula as before, gives
\[
\frac{[q(p(b + 1) + bN_d)][2qD_p b(\frac{\partial p}{\partial x})^2 + qD_p b(2p + N_d)(\frac{\partial^2 p}{\partial x^2})]}{q(p(b + 1) + bN_d)^2} - \frac{qD_p b(2p + N_d)(\frac{\partial}{\partial x})(\frac{\partial p}{\partial x})}{q(p(b + 1) + bN_d)^2}.
\]

Rearranging, the second term becomes
\[
\frac{[q(p(b + 1) + bN_d)][2qD_p b(\frac{\partial p}{\partial x})^2 + qD_p b(2p + N_d)(\frac{\partial^2 p}{\partial x^2})]}{q(p(b + 1) + bN_d)^2} - \frac{q^2 D_p b(2p + N_d)(b + 1)(\frac{\partial}{\partial x})(\frac{\partial p}{\partial x})}{q(p(b + 1) + bN_d)^2}.
\]

Expanding the first term of this equation gives
\[
\frac{2q^2 pD_p b(b + 1)(\frac{\partial p}{\partial x})^2 + 2N_d b^2 q^2 D_p(\frac{\partial}{\partial x})^2 + q^2 D_p b(2p + N_d)(p(b + 1) + bN_d)(\frac{\partial^2 p}{\partial x^2})}{q(p(b + 1) + bN_d)^2} - \frac{2q^2 pD_p b(b + 1)(\frac{\partial}{\partial x})(\frac{\partial p}{\partial x})^2 - q^2 pD_p b N_d (b + 1)(\frac{\partial}{\partial x})^2}{q(p(b + 1) + bN_d)^2}.
\]

Canceling the terms \(2q^2 pD_p b(b + 1)(\frac{\partial p}{\partial x})^2\) gives
\[
\frac{2N_d b^2 q^2 D_p(\frac{\partial}{\partial x})^2 - q^2 pD_p b N_d (b + 1)(\frac{\partial}{\partial x})^2 + q^2 D_p b(2p + N_d)(p(b + 1) + bN_d)(\frac{\partial^2 p}{\partial x^2})}{q(p(b + 1) + bN_d)^2}.
\]

Expanding the coefficient of \((\frac{\partial p}{\partial x})^2\) gives
\[
\frac{2N_d b^2 q^2 D_p(\frac{\partial}{\partial x})^2 - q^2 pD_p b N_d (\frac{\partial}{\partial x})^2 - q^2 pD_p b N_d (\frac{\partial}{\partial x})^2 - q^2 pD_p b N_d (\frac{\partial}{\partial x})^2}{q(p(b + 1) + bN_d)^2} + \frac{q^2 D_p b(2p + N_d)(p(b + 1) + bN_d)(\frac{\partial^2 p}{\partial x^2})}{q(p(b + 1) + bN_d)^2}.
\]
Canceling the terms $q^2 D_p b^2 N_d \left( \frac{\partial p}{\partial x} \right)^2$ gives

$$q^2 D_p b^2 N_d \left( \frac{\partial p}{\partial x} \right)^2 - q^2 p D_p b N_d \left( \frac{\partial p}{\partial x} \right)^2 + q^2 D_p b (2p + N_d) (p + 1 + b N_d) \left( \frac{\partial^2 p}{\partial x^2} \right)$$

$$q (p(b + 1) + b N_d)^2.$$  

Factoring $b$ gives

$$q^2 D_p b(b - 1) N_d \left( \frac{\partial p}{\partial x} \right)^2 + q^2 D_p b (2p + N_d) (p + 1 + b N_d) \left( \frac{\partial^2 p}{\partial x^2} \right)$$

$$q (p(b + 1) + b N_d)^2.$$  

Factoring $q b N_d$ gives

$$q D_p (b - 1) \left( \frac{\partial p}{\partial x} \right)^2 + q D_p N_d b \left( 1 + \frac{2p}{N_d} \right) \left( 1 + \frac{p(b + 1)}{b N_d} \right) \left( \frac{\partial^2 p}{\partial x^2} \right)$$

$$q b N_d \left( 1 + \frac{p(b + 1)}{b N_d} \right)^2.$$

(A.16)

Now substituting this back into (A.14) yields

$$-(\frac{\partial p}{\partial t} + \frac{p - p_0}{\tau_p}) = J \left( \frac{\partial p}{\partial x} \right) - q D_p (b - 1) \left( \frac{\partial p}{\partial x} \right)^2 - q D_p b N_d \left( 1 + \frac{2p}{N_d} \right) \left( 1 + \frac{p(b + 1)}{b N_d} \right) \left( \frac{\partial^2 p}{\partial x^2} \right)$$

$$q b N_d \left( 1 + \frac{p(b + 1)}{b N_d} \right)^2.$$  

(A.17)

A.3 Solution of the linear equation

The steady state linear equation can be written as

$$\frac{\partial^2 \bar{p}}{\partial x^2} - \frac{1}{L_p^2} \bar{p} = 0,$$

(A.18)

with boundary conditions $\bar{p}_1 = p_1 - p_0$ and $\bar{p}_2 = p_2 - p_0$,

$$\bar{p}_1 = A \exp \frac{1}{L_p} x + B \exp -\frac{1}{L_p} x$$

(A.19)

53
\[ \bar{p}_2 = A \exp\left(\frac{w}{L_p}\right) + B \exp\left(\frac{-w}{L_p}\right). \]

At \( x=0, \)

\[ B = \bar{p}_1 - A. \]

At \( x=w, \)

\[ \bar{p}_2 = A \exp\left(\frac{w}{L_p}\right) + (\bar{p}_1 - A) \exp\left(\frac{-w}{L_p}\right) \]

\[ \bar{p}_2 = A\left(\exp\left(\frac{w}{L_p}\right) - \exp\left(\frac{-w}{L_p}\right)\right) + \bar{p}_1 \exp\left(\frac{-w}{L_p}\right). \]

But, using \( (\exp\left(\frac{w}{L_p}\right) - \exp\left(\frac{-w}{L_p}\right)) = 2 \sinh\left(\frac{w}{L_p}\right) \) and substituting the value into the above equation, gives

\[ \bar{p}_2 - \bar{p}_1 \exp\left(\frac{-w}{L_p}\right) = 2A \sinh\left(\frac{w}{L_p}\right), \]

and by rearranging, the values of \( A \) and \( B \) are found as

\[ A = \frac{\bar{p}_2 - \bar{p}_1 \exp\left(\frac{-w}{L_p}\right)}{2 \sinh\left(\frac{w}{L_p}\right)} \quad (A.20) \]

\[ B = \frac{\bar{p}_1 \exp\left(\frac{-w}{L_p}\right) - \bar{p}_2}{2 \sinh\left(\frac{w}{L_p}\right)}. \quad (A.21) \]
A.4 Nonlinear equation in the radially symmetric case for the coated nanowire configuration

Refering to Equation (2.19) and begining with

$$\frac{\partial p}{\partial t} = -\frac{p - p_0}{\tau_p} - \frac{1}{q} \nabla \cdot J_p$$  \hspace{1cm} (A.22)

the applied electric field in gradient operator form is given as

$$E = \frac{J - qD_p(b-1)\nabla p}{q\mu_p(p(b+1) + bN_d)}$$  \hspace{1cm} (A.23)

and the current density of holes is given by

$$J_p = \frac{pJ - \nabla p(2bpqD_p - qD_pbN_d)}{q\mu_p(p(b+1) + bN_d)}.$$  \hspace{1cm} (A.24)

By substituting, the continuity equation of holes becomes

$$-(\frac{\partial p}{\partial t} + \frac{p - p_0}{\tau_p}) = \nabla \cdot \frac{pJ - qD_p b(2p + N_d)\nabla p}{q(p(b+1) + bN_d)}$$  \hspace{1cm} (A.25)

and using the divergence formula $\nabla \cdot \frac{\varphi}{\varphi} = \frac{1}{\varphi} \nabla \cdot A + A \cdot \nabla(\frac{1}{\varphi}),$

$$-\left[ (\frac{\partial p}{\partial t} + \frac{p - p_0}{\tau_p} \right] = \frac{1}{q(p(b+1) + bN_d)} \nabla \cdot [pJ - qD_p b(2p + N_d)\nabla p] +$$

$$[pJ - qD_p b(2p + N_d)\nabla p] \cdot \nabla \left[ \frac{1}{q(p(b+1) + bN_d)} \right].$$

Now solving the terms on right hand side separately,

$$\frac{1}{q(p(b+1) + bN_d)} \nabla \cdot [(pJ - qD_p b(2p + N_d)\nabla p)]
and using the divergence formula $\nabla \cdot p \mathbf{J} = p \nabla \cdot \mathbf{J} + \nabla p \cdot \mathbf{J}$, term 1 is

$$\nabla p \cdot \mathbf{J} - qD_p b[(2p + N_d)\nabla^2 p + 2(\nabla p \cdot \nabla p)] \over q(p(b + 1) + bN_d).$$

Now, solving term 2

$$[p \mathbf{J} - qD_p b(2p + N_d)\nabla p] \cdot \nabla \left[{1 \over q(p(b + 1) + bN_d)}\right]$$

using the formula $\nabla(\frac{1}{x}) = -\frac{\nabla x}{x^2}$. Now using the two solutions and putting them back in gives

$$\nabla p \cdot \mathbf{J} q p(b + 1) + \nabla p \cdot \mathbf{J} q b N_d \over [q(p(b + 1) + bN_d)]^2$$

$$- \left[{q^2 D_p b(b + 1) + q^2 D_p b^2 N_d}\over [q(p(b + 1) + bN_d)]^2\right](2p + N_d)\nabla^2 p + 2(\nabla p \cdot \nabla p)$$

$$+ \left[-\nabla p \cdot \mathbf{J} q p(b + 1) + q^2 D_p b(b + 1)(2p + N_d)(\nabla p \cdot \nabla p)\over [q(p(b + 1) + bN_d)]^2\right].$$

Canceling $\nabla p \cdot \mathbf{J} q p(b + 1)$ and $2q^2 D_p b b(b + 1)(\nabla p \cdot \nabla p)$ gives

$$\nabla p \cdot \mathbf{J} q b N_d - q^2 D_p b b(b + 1)(2p + N_d)\nabla^2 p - q^2 D_p b^2 N_d(2p + N_d)\nabla^2 p \over [q^2(p(b + 1) + bN_d)]^2$$

$$+ \left[-2q^2 D_p b^2 N_d(\nabla p \cdot \nabla p) + q^2 D_p b N_d b(b + 1)(\nabla p \cdot \nabla p)\over [q(p(b + 1) + bN_d)]^2\right].$$

Grouping the coefficients of $\nabla^2 p$ and $(\nabla p \cdot \nabla p)$ gives

$$\nabla p \cdot \mathbf{J} q b N_d - q^2 D_p b(b + 1) + bN_d)(2p + N_d)\nabla^2 p - q^2 D_p b N_d b(b + 1)(\nabla p \cdot \nabla p) \over [q(p(b + 1) + bN_d)]^2.$$
Thus, the nonlinear equation is reduced to

\[-\left(\frac{\partial p}{\partial t}\right) + \frac{p - p_0}{\tau_p} = \nabla p \cdot J - qD_p b N_d (1 + \frac{p(b+1)}{b N_d}) (1 + \frac{2p}{N_d}) \nabla^2 p - q D_p (b - 1) (\nabla p \cdot \nabla p) \frac{qb N_d}{1 + \frac{p(b+1)}{b N_d}^2}.\]

(A.26)

The nonlinear equation of a nanowire transistor is obtained when the values below are inserted in Equation (3.2)

\[\nabla p = \frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial p}{\partial \theta} + \frac{\partial p}{\partial z},\]

(A.27)

\[\nabla^2 p = \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial z^2}.\]

(A.28)

But further simplification is done because of the complexity of the problem.

A.5 Obtaining the state equation form of nonlinear equation for a conventional BJT

The nonlinear equation is given as

\[-\left(\frac{\partial p}{\partial t}\right) + \frac{p - p_0}{\tau_p} = \frac{J(\frac{\partial p}{\partial x}) - qD_p (b - 1)(\frac{\partial p}{\partial x})^2 - q D_p b N_d (1 + \frac{2p}{N_d}) (1 + \frac{p(b+1)}{b N_d}) (\frac{\partial^2 p}{\partial x^2})}{qb N_d (1 + \frac{p(b+1)}{b N_d})^2}.\]

(A.29)

At steady state \(\frac{\partial p}{\partial t}\) tends to 0, therefore the equation changes to

\[-\frac{p - p_0}{\tau_p} = \frac{J(\frac{\partial p}{\partial x}) - qD_p (b - 1)(\frac{\partial p}{\partial x})^2 - q D_p b N_d (1 + \frac{2p}{N_d}) (1 + \frac{p(b+1)}{b N_d}) (\frac{\partial^2 p}{\partial x^2})}{qb N_d (1 + \frac{p(b+1)}{b N_d})^2}.\]

(A.30)
and the boundary conditions at \( x = 0 \) and \( x = w \) are given as

\[
p_1 = p_0 \exp\left(\frac{qV_e}{kT}\right)
\]

(A.31)

\[
p_2 = p_0 \exp\left(-\frac{qV_c}{kT}\right).
\]

(A.32)

Nondimensionalizing the nonlinear equation using \( p = N_d\tilde{p} \), \( x = w\tilde{x} \) gives

\[
-N_d\tilde{p} - p_0 \tau_p = \frac{J(N_d \frac{\partial\tilde{p}}{\partial\tilde{x}}) - qD_p N_d^2 (b - 1)(\frac{\partial\tilde{p}}{\partial\tilde{x}})^2 - qD_p b N_d^2 (1 + \frac{2N_d\tilde{p}}{bN_d})(1 + \frac{N_d\tilde{p}(b+1)}{bN_d})(\frac{\partial^2\tilde{p}}{\partial\tilde{x}^2})}{qb N_d (1 + \frac{N_d\tilde{p}(b+1)}{bN_d})^2}.
\]

(A.33)

Canceling \( N_d \) on right hand side gives

\[
-N_d\tilde{p} - p_0 \tau_p = \frac{J(\frac{\partial\tilde{p}}{\partial\tilde{x}}) - qD_p N_d (b - 1)(\frac{\partial\tilde{p}}{\partial\tilde{x}})^2 - qD_p b N_d (1 + 2\tilde{p})(1 + \tilde{p}(b + 1)/b)(\frac{\partial^2\tilde{p}}{\partial\tilde{x}^2})}{qb (1 + \frac{\tilde{p}(b+1)}{b})^2}.
\]

(A.34)

The boundary conditions change to

\[
\tilde{p}_1 = \left(\frac{p_0}{N_d}\right) \exp\left(\frac{qV_e}{kT}\right)
\]

(A.35)

\[
\tilde{p}_2 = \left(\frac{p_0}{N_d}\right) \exp\left(-\frac{qV_c}{kT}\right).
\]

(A.36)

and by considering

\[
\text{con1} = q b \left(1 + \frac{\tilde{p}(b+1)}{b}\right)^2
\]

(A.37)

\[
\text{con2} = qD_p b N_d (1 + 2\tilde{p})(1 + \tilde{p}(b + 1)/b)/(w^2)
\]

(A.38)

\[
\text{con3} = qD_p N_d (b - 1)/(w^2)
\]

(A.39)
the equation changes to

\[-\text{con}1 \frac{N_d \bar{p} - p_0}{\tau_p} = (J/w) \left( \frac{\partial \bar{p}}{\partial x} \right) - \text{con}2 \left( \frac{\partial^2 \bar{p}}{\partial x^2} \right) - \text{con}3 \left( \frac{\partial \bar{p}}{\partial x} \right)^2. \tag{A.40} \]

Rearranging the above equation gives

\[\frac{\partial^2 \bar{p}}{\partial x^2} = J \left( \frac{\partial \bar{p}}{\partial x} \right) + \text{con}1 \left( \frac{N_d \bar{p} - p_0}{\tau_p} \right) - \text{con}3 \left( \frac{\partial \bar{p}}{\partial x} \right)^2. \tag{A.41} \]

Considering state variable form

\[y_1 = \bar{p} \tag{A.42} \]
\[y_2 = \frac{\partial \bar{p}}{\partial x}, \tag{A.43} \]

the above nonlinear equation can be written in state equation form as

\[\frac{\partial}{\partial x} y_1 = y_2, \tag{A.44} \]
\[\frac{\partial}{\partial x} y_2 = J y_2 + \text{con}1 \left( \frac{N_d \bar{p} - p_0}{\tau_p} \right) - \text{con}3 y_2^2. \tag{A.45} \]

A.6 Obtaining the state equation form of the linear equation for the conventional BJT

The linear equation is given by

\[\frac{\partial p}{\partial t} + \frac{p - p_0}{\tau_p} = D_p \frac{\partial^2 p}{\partial x^2}. \tag{A.46} \]
and at steady state $\frac{\partial p}{\partial t}$ is 0 which changes the equation to

$$\frac{p - p_0}{\tau_p} = D_p \frac{\partial^2 p}{\partial x^2}, \quad (A.47)$$

and finally,

$$\frac{p - p_0}{\tau_p D_p} = \frac{\partial^2 p}{\partial x^2}, \quad (A.48)$$

Using $\sqrt{\tau_p D_p} = L_p$ gives

$$\frac{\partial^2 p}{\partial x^2} - \frac{p - p_0}{L_p^2} = 0. \quad (A.49)$$

The boundary conditions at $x = 0$ and $x = w$ are given as

$$p_1 = p_0 \exp \left( \frac{qV_e}{kT} \right), \quad (A.50)$$

$$p_2 = p_0 \exp \left( - \frac{qV_c}{kT} \right). \quad (A.51)$$

Nondimensionlizing the linear equation using $p = N_d \tilde{p}$ and rearranging the terms gives

$$\frac{\partial^2 \tilde{p}}{\partial x^2} - \frac{N_d \tilde{p} - p_0}{N_d L_p^2} = 0. \quad (A.52)$$

The boundary conditions change to

$$\tilde{p}_1 = \left( \frac{p_0}{N_d} \right) \exp \left( \frac{qV_e}{kT} \right), \quad (A.53)$$

$$\tilde{p}_2 = \left( \frac{p_0}{N_d} \right) \exp \left( - \frac{qV_c}{kT} \right). \quad (A.54)$$
Defining the state variables as

\[ y_1 = \tilde{p} \quad \text{(A.55)} \]
\[ y_2 = \frac{\partial \tilde{p}}{\partial x} \quad \text{(A.56)} \]

allows the state equations to be written as

\[ y_1' = y_2 \quad \text{(A.57)} \]

and

\[ y_2' = \frac{N_d y_1 - p_0}{N_d L_p^2} \quad \text{(A.58)} \]

### A.7 Obtaining the state equation form of the nonlinear equation for the coated nanowire configuration

The nonlinear equation from Chapter coated nanowire configuration given as

\[
- \left[ \left( \frac{\partial p}{\partial t} \right) + \frac{p - p_0}{\tau_p} \right] = \frac{J \left( \frac{\partial p}{\partial r} \right) - q \nabla_p (b - 1) \left( \frac{\partial p}{\partial r} \right)^2}{q b \nabla_d (1 + \frac{p(b+1)}{b \nabla_d})^2} + q D_p b \nabla_d (1 + \frac{2p}{\nabla_d}) \left( \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \right) \frac{q b \nabla_d (1 + \frac{p(b+1)}{b \nabla_d})^2}{q b \nabla_d (1 + \frac{p(b+1)}{b \nabla_d})^2}.
\]

At steady state \( \frac{\partial p}{\partial t} \) is 0, therefore the nonlinear equation is given as

\[
- \frac{p - p_0}{\tau_p} = \frac{J \left( \frac{\partial p}{\partial r} \right) - q \nabla_p (b - 1) \left( \frac{\partial p}{\partial r} \right)^2 - q D_p b \nabla_d (1 + \frac{2p}{\nabla_d}) \left( \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \right)}{q b \nabla_d (1 + \frac{p(b+1)}{b \nabla_d})^2}.
\]

\[
\text{(A.59)}
\]
The boundary conditions at \( r = a \) and \( r = b \) are

\[
p_1 = p_0 \exp\left( \frac{qV_e}{kT} \right), \tag{A.60}
\]

\[
p_2 = p_0 \exp\left( -\frac{qV_e}{kT} \right). \tag{A.61}
\]

Nondimensionlizing the nonlinear equation using \( p = N_d\tilde{p} \) and \( r = a\tilde{r} \) rearranging the terms yields

\[
-N_d\tilde{p} - p_0 = \frac{(J/a)(\frac{\partial \tilde{p}}{\partial \tilde{r}}) - \frac{qD_p N_d (b-1)}{a^2} (\frac{\partial \tilde{p}}{\partial \tilde{r}})^2}{qb(1 + \tilde{p}(b + 1)/b)^2} - \frac{qD_p b N_d (1 + 2\tilde{p})(1 + \tilde{p}(b + 1)/b)(\frac{\partial^2 \tilde{p}}{\partial \tilde{r}^2} + \frac{1}{\tilde{r}} \frac{\partial \tilde{p}}{\partial \tilde{r}})}{qb(1 + \tilde{p}(b + 1)/b)^2}. \tag{A.62}
\]

The boundary conditions change to

\[
\tilde{p}_1 = \left( \frac{p_0}{N_d} \right) \exp\left( \frac{qV_e}{kT} \right) \tag{A.63}
\]

and

\[
\tilde{p}_2 = \left( \frac{p_0}{N_d} \right) \exp\left( -\frac{qV_e}{kT} \right). \tag{A.64}
\]

Substituting these definitions

\[
\text{con}1 = qb(1 + \tilde{p}(b + 1)/b)^2 \tag{A.65}
\]

\[
\text{con}2 = \left( qD_p b N_d (1 + \tilde{p})(1 + \tilde{p}(b + 1)/b) \right) / (a^2) \tag{A.66}
\]
\[ con3 = \left( qD_p N_d (b - 1) \right) / a^2. \]  \hspace{1cm} (A.67)

into the above equation gives

\[ -con1 \frac{N_d \tilde{p} - p_0}{\tau_p} = (J/a) \left( \frac{\partial \tilde{p}}{\partial r} \right) - con2 \left( \frac{\partial^2 \tilde{p}}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{p}}{\partial r} \right) - con3 \left( \frac{\partial \tilde{p}}{\partial r} \right)^2. \]  \hspace{1cm} (A.68)

Rearranging this equation gives

\[ \frac{\partial^2 \tilde{p}}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{p}}{\partial r} = \frac{(J/a) \left( \frac{\partial \tilde{p}}{\partial r} \right) + con1 \left( \frac{N_d \tilde{p} - p_0}{\tau_p} \right) - con3 \left( \frac{\partial \tilde{p}}{\partial r} \right)^2}{con2}. \]  \hspace{1cm} (A.69)

Defining the state variables as

\[ y_1 = \tilde{p}, \]  \hspace{1cm} (A.70)
\[ y_2 = \frac{\partial \tilde{p}}{\partial r}. \]  \hspace{1cm} (A.71)

Using these, the state equations for Equation (A.69) can be written as

\[ \frac{\partial}{\partial r} y_1 = y_2 \]  \hspace{1cm} (A.72)

\[ \frac{\partial}{\partial r} y_2 = \frac{(J/a) y_2 + con1 \left( \frac{N_d \tilde{p} - p_0}{\tau_p} \right) - con3 (y_2)^2}{con2} - \frac{1}{r} y_2. \]  \hspace{1cm} (A.73)

A.8 Obtaining the state equation form of the linear equation for the coated nanowire configuration

The linear equation is

\[ \frac{\partial p}{\partial t} + \frac{p - p_0}{\tau_p} = D_p \left[ \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \right]. \]  \hspace{1cm} (A.74)
At steady state $\frac{\partial p}{\partial t}$ is 0 giving

$$\frac{p - p_0}{\tau_p} = D_p \left[ \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \right]$$

(A.75)

$$\frac{p - p_0}{\tau_p D_p} = \left[ \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \right].$$

(A.76)

Replacing $\sqrt{\tau_p D_p} = L_p$ gives

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} - \frac{p - p_0}{L_p^2} = 0$$

(A.77)

with boundary conditions at $r=a$ and $r=b$ given as

$$p_1 = p_0 \exp\left(\frac{q V_e}{kT}\right)$$

(A.78)

and

$$p_2 = p_0 \exp\left(-\frac{q V_e}{kT}\right).$$

(A.79)

Nondimensionlizing the linear equation using $p = N_d \tilde{p}$ yields

$$\frac{\partial^2 \tilde{p}}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{p}}{\partial r} - \frac{N_d \tilde{p} - p_0}{N_d L_p^2} = 0.$$  

(A.80)

Defining the state variables as

$$y_1 = \tilde{p}$$

(A.81)

$$y_2 = \frac{\partial \tilde{p}}{\partial r},$$

(A.82)
gives the state equations in this case as

\[ y'_1 = y_2 \]  \hspace{1cm} (A.83)

\[ y'_2 = \frac{N_d y_1 - p_0}{N_d L_p^2} - \frac{1}{r} y_2. \]  \hspace{1cm} (A.84)

Table A.2 details the values used in calculations for finding solutions in Matlab.
Table A.1: Symbols.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p, n )</td>
<td>Concentration of holes and electrons</td>
</tr>
<tr>
<td>( p_0, n_0 )</td>
<td>Equilibrium hole density, electron density</td>
</tr>
<tr>
<td>( J_p, J_n )</td>
<td>Hole and electron current density</td>
</tr>
<tr>
<td>( \tau_p, \tau_n )</td>
<td>Volume lifetime recombination of holes, recombination of electrons</td>
</tr>
<tr>
<td>( g )</td>
<td>Net rate of generation of holes</td>
</tr>
<tr>
<td>( r )</td>
<td>Net rate of decay by recombination of holes</td>
</tr>
<tr>
<td>( q )</td>
<td>Charge of electrons</td>
</tr>
<tr>
<td>( D_p, D_n )</td>
<td>Diffusion constants of holes and electrons</td>
</tr>
<tr>
<td>( \mu_p, \mu_n )</td>
<td>Mobility of holes and electrons</td>
</tr>
<tr>
<td>( b )</td>
<td>Ratio of electron to hole mobility</td>
</tr>
<tr>
<td>( J )</td>
<td>Total current density</td>
</tr>
<tr>
<td>( N_d, N_a )</td>
<td>Density of donors and acceptors</td>
</tr>
<tr>
<td>( E )</td>
<td>Electric field strength</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Permittivity of a material</td>
</tr>
<tr>
<td>( L_p, L_n )</td>
<td>Diffusion length of holes and electrons</td>
</tr>
<tr>
<td>( w )</td>
<td>Base width</td>
</tr>
<tr>
<td>( V_e )</td>
<td>DC bias potential relative to base on emitter</td>
</tr>
<tr>
<td>( V_c )</td>
<td>DC bias potential relative to base on collector</td>
</tr>
<tr>
<td>( y_1, y_2 )</td>
<td>State variables</td>
</tr>
<tr>
<td>( p_1, p_2 )</td>
<td>boundary conditions</td>
</tr>
</tbody>
</table>
Table A.2: Parameter values assumed in simulations.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>0.005 cm</td>
<td>width of base</td>
</tr>
<tr>
<td>$p_0$</td>
<td>$1.25 \cdot 10^{12}/cm^3$</td>
<td>equilibrium hole density in an $n$-type base region</td>
</tr>
<tr>
<td>$D_p$</td>
<td>44 $cm^2/sec$</td>
<td>diffusion constant of holes</td>
</tr>
<tr>
<td>$L_p$</td>
<td>$25.6905 \cdot 10^{-3} cm$</td>
<td>diffusion length of holes in an $n$-type base region</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>$15 \cdot 10^{-6} s$</td>
<td>volume lifetime for arbitrary injection of holes into $n$-type material</td>
</tr>
<tr>
<td>$V_e$</td>
<td>0.2 V</td>
<td>DC bias potential on emitter relative to base</td>
</tr>
<tr>
<td>$V_c$</td>
<td>0.3333 V</td>
<td>DC bias potential on collector relative to base</td>
</tr>
<tr>
<td>$q$</td>
<td>$1.60210 \cdot 10^{-19} C$</td>
<td>charge of electron</td>
</tr>
<tr>
<td>$N_d$</td>
<td>$1.1 \cdot 10^{15} cm^{-3}$</td>
<td>density of donors</td>
</tr>
<tr>
<td>$k$</td>
<td>$1.38 \cdot 10^{-23} J/K$</td>
<td>Boltzmann constant</td>
</tr>
<tr>
<td>$T$</td>
<td>300 K</td>
<td>room temperature</td>
</tr>
<tr>
<td>$\mu_p$</td>
<td>1700 $cm^2/(Vs)$</td>
<td>mobility of holes</td>
</tr>
<tr>
<td>$b$</td>
<td>1.5</td>
<td>ratio of electron to hole mobility</td>
</tr>
</tbody>
</table>
APPENDIX B

B.1 Matlab functions

**BVP4C [23]:**
This function is used to solve the boundary value problems for partial differential equations.
syntax:
```
sol = bvp4c(odefun,bcfun,solinit)
sol = bvp4c(odefun,bcfun,solinit,options)
solinit = bvpinit(x, yinit, params).
```
It uses the following function handles for evaluation.
`odefun`: odefun function handler evaluates the differential equations. It can have the form:
```
dydx = odefun(x,y)
```
or
```
dydx = odefun(x,y,parameters)
```
where `x` is a scalar corresponding to `x`, `y` is a column vector corresponding to `y` and parameters is a vector of unknown parameters. The output `dydx` is a column vector.
`bcfun`: A function handle that computes the residual in the boundary conditions. For two-point boundary value conditions of the form , bcfun can have the form
```
res = bcfun(ya,yb)
```
or
```
res = bcfun(ya,yb,parameters)
```
where `ya` and `yb` are column vectors corresponding to `y(a)` and `y(b)`. parameters is a vector of unknown parameters. The output `res` is a column vector.
solinit: A structure containing the initial guess for a solution. You can create solinit using the function bvpinit. solinit has the fields $x$ and $y$, where $x$ is ordered nodes of the initial mesh. $y$ is the initial guess of the solution.

B.2 Solution to Emden-Fowler equation

The Emden-Fowler equation is given as

$$\frac{\partial^2 p}{\partial x^2} + \frac{1}{p + b} \left( \frac{\partial p}{\partial x} \right)^2 = 0. \quad (B.1)$$

Consider the solution to the above as

$$p(x) = (\alpha x + \beta)^{1/2} - b \quad (B.2)$$

where $\alpha$ and $\beta$ are given by

$$\alpha = \frac{(p_b + b)^2 - (p_a + b)^2}{b} \quad (B.3)$$

$$\beta = (p_b + b)^2. \quad (B.4)$$

The the first and second order derivatives of $p(x)$ are given as

$$p_x = \frac{1}{2} (\alpha x + \beta)^{-1/2} \cdot \alpha \quad (B.5)$$

$$p_{xx} = -\frac{1}{4} (\alpha x + \beta)^{-3/2} \cdot \alpha^2. \quad (B.6)$$
Substituting the values in the given Emden-Fowler equation gives

\[
p_{xx} + \frac{1}{p+b} (p_x)^2 = -\frac{1}{4} (\alpha x + \beta)^{-3/2} \cdot \alpha^2 + \left( \frac{1}{2} (\alpha x + \beta)^{-1/2} \cdot \alpha^2 \right) = 0 \quad (B.7)
\]

which shows that \( p(x) \) is an exact solution of Emden-Fowler equation.

B.3 Matlab programs

Plotting linear solution for conventional transistor

```matlab
function linear_x_program_test1
a=5*10^(-3); % (cm)
b=10*10^(-3); % (cm)
w=b-a;
solinit=bvpinit(linspace(a,b,1000),[-0.1,0]);
sol=bvp4c(@state_eq_x_linear_test1,@linear_bvp_x_test1,solinit);
x=linspace(a,b);
y=deval(sol,x);
hold all
plot((x)/w-1,((y(1,:))),'r');
end
```

```matlab
function dydx=state_eq_x_linear_test1(x,y)
Dp=44;% (cm^2/sec)
Nd=1.1*10^15;% (1/cm^3)
tau=15*10^(-6); % (sec)
p0=1.25*10^-12;% (1/cm^3)
dydx=[y(2);(Nd*y(1)-p0)/(Nd*tau*Dp)];
end
```

```matlab
function val=linear_bvp_x_test1(ya,yb)
Nd=1.1*10^15;% (1/cm^3)
end
```
\begin{verbatim}
\texttt{p0=1.25*10^{-12};\%(1/cm^{3})}
q=1;\%(eV)
ve=0.2;\%(V)
vc=0.333;\%(V)
k=8.617*10^{-5};\%(eV/K)
T1=300;\%(K)
p1=(p0/Nd)*exp((q*ve)/(k*T1));
p2=(p0/Nd)*exp(-(q*vc)/(k*T1));
val=[ya(1)-p1;yb(1)-p2];

Plotting nonlinear solution for conventional transistor.

function nonlinear_x_program_tests1
clc
clear
a=5*10^{-3};\%(cm)
b=10*10^{-3};\%(cm)
w=b-a;
solinit=bvpinit(linspace(a/w,b/w,1000),[0,0.000001]);
sol=bvp4c(@state_eq_x_tests1,@nonlinear_bvp_x_tests1,solinit);
x=linspace(a/w,b/w);
y=deval(sol,x);
hold all
plot((x)-1,(y(1,:)),'b');
function dydx=state_eq_x_tests1(x,y)
a=5*10^{-3};\%(cm)
b=10*10^{-3};\%(cm)
Dp=44;\%(cm^{2}/sec)
\end{verbatim}
\[ \text{Nd} = 1.1 \times 10^{15} \text{/(cm}^3\text{)} \]

\[ p0 = 1.25 \times 10^{12} \text{/(cm}^3\text{)} \]

\[ \tau = 15 \times 10^{-6} \text{/(sec)} \]

\[ B = 1.5 \text{;/constant} \]

\[ q = 1 \text{;/eV} \]

\[ c = b - a \]

\[ L_p = (\sqrt{\tau D_p}) \]

\[ J = 100 q N_d B D_p / L_p \]

\[ \text{con1} = q B (1 + (y(1) (B + 1)) / B)^2 \]

\[ \text{con2} = q B D_p N_d (1 + (y(1) (B + 1)) / B) (1 + (2 y(1)) / c^2) \]

\[ \text{con3} = q D_p (B - 1) N_d / c^2 \]

\[ dy/dx = [y(2); ((J/c) y(2) + \text{con1} (y(1) N_d - p0) / \tau - \text{con3} (y(2)^2) / \text{con2})] \]

\[function\ val = \text{nonlinear}\_bvp\_x\_tests1(ya, yb)\]

\[ q = 1 \text{;/eV} \]

\[ \text{ve} = 0.2 \text{;/V} \]

\[ \text{vc} = 0.333 \text{;/V} \]

\[ k = 8.617 \times 10^{-5} \text{;/eV/K} \]

\[ Nd = 1.1 \times 10^{15} \text{/(cm}^3\text{)} \]

\[ p0 = 1.25 \times 10^{12} \text{/(cm}^3\text{)} \]

\[ T_1 = 300 \text{;/K} \]

\[ p1 = (p0 / Nd) \times \exp((q \times \text{ve}) / (k \times T_1)) \]

\[ p2 = (p0 / Nd) \times \exp(-(q \times \text{vc}) / (k \times T_1)) \]

\[ \text{val} = [ya(1) - p1; yb(1) - p2] \]

Plotting linear solution for nanowire transistor

\[function\ linear\_r\_program\_test1\]

72
clc

clear all

a = 5e-6; \%(cm)
b = 10e-6; \%(cm)
w = b - a;
solinit = bvpinit(linspace(a, b, 1000), [-0.01, 0.000001]);
sol = bvp4c(@state_eq_r_linear_test1, @linear_bvp_r_test1, solinit);
x = linspace(a, b);
y = deval(sol, x);
hold all
plot(x/w-1, y(1,:), 'b');

function dydx = state_eq_r_linear_test1(x, y)
    Dp = 44; \%(cm^2/sec)
    Nd = 1.1e15; \%(1/cm^3)
    tau = 15e-6;
    p0 = 1.25e12; \%(1/cm^3)
    dydx = [y(2); (Nd*y(1)-p0)/(Nd*tau*Dp)-(1/(x))*y(2)];

function val = linear_bvp_r_test1(ya, yb)
    Nd = 1.1e15; \%(1/cm^3)
p0 = 1.25e12; \%(1/cm^3)
q = 1; \%(eV)
ve = 0.2; \%(V)
vc = 0.333; \%(V)
k = 8.617e-5; \%(eV/K)
T1 = 300; \%(K)
p1 = (p0/Nd)*exp((q*ve)/(k*T1));
\[ p_2 = (p_0/N_d) \exp\left(-\frac{q v c}{k T_1}\right) \]
\[
val = [ya(1) - p_1; yb(1) - p_2];
\]

Plotting nonlinear solution for nanowire transistor

```matlab
function nonlinear_r_program_test4
clc
clear all
Nd=1.1*10^(15);%(1/cm^3)
a=5*10^(-6);%(cm)
b=10*10^(-6);%(cm)
w=b-a;
solinit=bvpinit(linspace(a/w,b/w,1000),[-0.01,0.000001]);
sol=bvp4c(@state_eq_r_test4,@nonlinear_bvp_r_test4,solinit);
x=linspace(a/w,b/w);
y=deval(sol,x);
hold all
plot((x)-1,(y(1,:)),'b');
function dydx=state_eq_r_test4(x,y)
a=5*10^(-6);%(cm)=50nm
b=10*10^(-6);%(cm)=100nm
Dp=44;%(cm^2/sec)
Nd=1.1*10^(15);%(1/cm^3)
p0=1.25*10^(-12);%(1/cm^3)
B=1.5;%constant
c=b-a ;
q=1;%(eV)
tau=15*10^(-6);%(sec)
```
Lp=(sqrt(tau*Dp));
J=0.1*q*B*Nd*Dp/((x*c)*Lp);
con1=q*B*(1+(y(1)*(B+1)))/(B))^{-2};
con2=(q*B*Dp*Nd*(1+(y(1)*(B+1)))/(B))*(1+(2*y(1)))/(c^{-2});
con3=q*Dp*(B-1)*Nd/(c^{-2});
dydx=[y(2);((J/c)*y(2)+(con1)*(y(1)*Nd-p0)/tau -con3*(y(2))^2)/(con2)-(1/x)*y(2)];

function val=nonlinear_bvp_r_test4(ya,yb)
    q=1;%(eV)
    ve=0.2;%(V)
    vc=0.3333;%(V)
    k=8.617*10^{-5};%(eV/K)
    Nd=1.1*10^{-15};%(1/cm^{-3})
    p0=1.25*10^{-12};%(1/cm^{-3})
    T1=300;%(K)
    p1=(p0/Nd)*exp((q*ve)/(k*T1));
    p2=(p0/Nd)*exp(-(q*vc)/(k*T1));
    val=[ya(1)-p1;yb(1)-p2];

Plotting dependence of J_p on x

function nonlinear_x_program_test10
    clc
    clear
    a=5*10^{-3};%(cm)
    b=10*10^{-3};%(cm)
    w=b-a;
    solinit=bvpinit(linspace(a,b,1000),[-0.1,0]);
sol=bvp4c(@state_eq_x_test10,@nonlinear_bvp_x_test10,solinit);
x=linspace(a,b);
y=deval(sol,x);

Dp=44;%cm^2/sec
B=1.5;%constant
mup=1700;
tau=15*10^-6;
q=1.60210*10^-19; % q is changed to coulombs
Nd=1.1*10^15;
Lp=(sqrt(tau*Dp));
J=50*q*B*Dp/Lp;
E=(J-q*Dp*(B-1).*y(2,:))/(q*mup*(y(1,:)*(B+1)+B));
Jp=(q*mup*y(1,:).*E-q*Dp.*y(2,:));
plot(x/w-1,Jp*Nd);

function dydx=state_eq_x_test10(x,y)
Dp=44;%cm^2/sec
Nd=1.1*10^15;%c/m
p0=1.25*10^12;%c/m
tau=15*10^-6;
B=1.5;%constant
q=1;%eV
a=5*10^-3;%cm
b=1*10^-3;%cm
c=b-a;
Lp=(sqrt(tau*Dp));
J=50*q*B*Nd*Dp/Lp;
\[ \text{con1} = q \cdot B \cdot (1 + (y(1) \cdot (B+1)) / (B))^2; \]
\[ \text{con2} = q \cdot B \cdot Dp \cdot Nd \cdot (1 + (y(1) \cdot (B+1)) / (B)) \cdot (1 + (2 \cdot y(1))) / (c^2); \]
\[ \text{con3} = q \cdot Dp \cdot (B-1) \cdot Nd / (c^2); \]
\[ \text{dydx} = [y(2); ((J/c) \cdot y(2) + \text{con1} \cdot (y(1) \cdot Nd - p0) / \tau \text{ - \text{con3} \cdot (y(2))^2}) / \text{con2}]; \]

\[
\begin{align*}
q &= 1; \%(\text{eV}) \\
\text{ve} &= 0.2; \%(\text{V}) \\
\text{vc} &= 0.333; \%(\text{V}) \\
k &= 8.617 \cdot 10^{-5}; \%(\text{eV/K}) \\
Nd &= 1.1 \cdot 10^{15}; \%(1/\text{cm}^3) \\
p0 &= 1.25 \cdot 10^{12}; \%(1/\text{cm}^3) \\
T1 &= 300; \%(\text{K}) \\
p1 &= (p0 / Nd) \cdot \exp((q \cdot \text{ve}) / (k \cdot T1)); \\
p2 &= (p0 / Nd) \cdot \exp(-(q \cdot \text{vc}) / (k \cdot T1)); \\
\text{val} &= [\text{ya}(1) - p1; \text{yb}(1) - p2];
\end{align*}
\]

Plotting dependence of \( J_p \) on \( r \)

\[
\begin{align*}
\text{function nonlinear_r_program_test40} \\
\text{clc} \\
\text{clear all} \\
a &= 50 \cdot 10^{-6}; \%(\text{cm}) \\
b &= 100 \cdot 10^{-6}; \%(\text{cm}) \\
c &= 50 \cdot 10^{-6}; \%(\text{cm}) \\
\text{solinit} &= \text{bvpinit(linspace(a/c,b/c,1000),[0,0.000001]);} \\
\text{sol} &= \text{bvp4c(@state_eq_r_test40,@nonlinear_bvp_r_test40,solinit);} \\
x &= \text{linspace(a/c,b/c);} \\
\end{align*}
\]
y = deval(sol, x);

Dp = 44;

B = 1.5; % constant
mup = 1700;
q = 1.60210*10^(-19); % q is changed to coulombs
Nd = 1.1*10^15; % (1/cm^3)
tau = 15*10^(-6); % (sec)
Lp = (sqrt(tau*Dp));

J = 0.4*q*B*Dp./(x*Lp);
E = (J-q*Dp.*(B-1).*y(2,:))./(q*mup*(y(1,:)*(B+1)+B));
Jp = (q*mup*y(1,:).*E-q*Dp.*y(2,:));
plot(x-1, Jp*Nd);

function dydx = state_eq_r_test40(x, y)

Dp = 44; % (cm^2/sec)
Nd = 1.1*10^15; % (1/cm^3)
p0 = 1.25*10^12; % (1/cm^3)

q = 1; % (eV)
tau = 15*10^(-6);
B = 1.5; % constant
c = 50*10^(-6); % (cm)

con1 = q*B*(1+y(1)*(B+1)) / (B)^2;
con2 = q*B*Dp*Nd*(1+y(1)*(B+1)) / (B)*(1+(2*y(1)))/(c^2);
con3=q*Dp*(B-1)*(Nd/(c^2));

dydx=[y(2) ;((J/c)*y(2)+(con1)*(y(1)*Nd-p0)/tau
-con3*(y(2))^2)/(con2/(c^2))-(1/x)*y(2)];

function val=nonlinear_bvp_r_test40(ya,yb)
    q=1;%eV
    ve=0.2;%V
    vc=0.3333;%V
    k=8.617*10^(-5);%eV/K
    Nd=1.1*10^(15);%(1/cm^(3))
    p0=1.25*10^(12);%(1/cm^(3))
    T1=300;%K
    p1=(p0/Nd)*exp((q*ve)/(k*T1));
    p2=(p0/Nd)*exp(-(q*vc)/(k*T1));
    val=[ya(1)-p1;yb(1)-p2];
BIBLIOGRAPHY


