FLEXURAL BEHAVIOR OF BASALT FRP BAR REINFORCED CONCRETE MEMBERS WITH AND WITHOUT POLYPROPYLENE FIBER

A Thesis

Presented to

The Graduate Faculty of The University of Akron

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

Subhashini Neela

December, 2010
FLEXURAL BEHAVIOR OF BASALT FRP BAR REINFORCED CONCRETE MEMBERS WITH AND WITHOUT POLYPROPYLENE FIBER

Subhashini Neela

Thesis

Approved:  

Advisor  
Dr. Anil Patnaik

Department Chair  
Dr. Wieslaw K. Binienda

Department Chair  
Dr. Wieslaw K. Binienda

Committee Member  
Dr. William Schneider, IV

Dean of the College  
Dr. George K. Haritos

Committee Member  
Dr. Kallol Sett

Dean of the Graduate School  
Dr. George R. Newkome

Date
The thesis presents the results of an experimental investigation of the performance characteristics of concrete members reinforced with basalt fiber reinforced polymer (BFRP) bars along with polypropylene fibers. The primary objective of the research is the identification of the stress-strain relationship which ensues the determination of the load-strain behavior and maximum load capacity of the basalt FRP reinforced slabs reinforced with or without polypropylene fiber. The slab tests were designed to determine the influence of concrete strength and percentage volume of fiber on the maximum load capacity, shear strength, deflections and ductility. One of the objectives of the slab tests is also the study of the load-deflection behavior of the basalt FRP reinforced beams with and without polypropylene fiber. Another objective of the research is to check the validity of the code defined design methods for the calculation of shear strength for FRP reinforced beams made from fiber reinforced concrete.

The secondary objective of this research was to study the effect of polypropylene fiber on the post-cracking strengths of beams and round determinate panels and to find the correlation between the beam and panel specimens.

To achieve the objectives of this study, large number of plain and fiber reinforced concrete slab elements, and cylinders were cast with two different fiber dosages (1.0% and 0.5% volume fraction). The type of fiber used was Ferro (2.25”). All the slabs were tested under four-point bending to determine the maximum load capacity
of slabs. Six fiber reinforced concrete beams and two round panels with 0.5% volume fraction of fiber were cast to determine the average residual strength (ARS) and toughness properties respectively. The standard test methods ASTM C1399 was used for testing the beams and ASTM C1550 was used for testing the round panels.

The cylinder compression tests revealed that compressive strength decreased marginally with the increase in fiber dosage. The load carrying capacity of the slabs particularly in shear strength mode is found to increase with the addition of polypropylene fiber to the concrete in spite of the lower concrete strength. The concrete compressive strains and the tensile bar strains were found to increase with the addition of fiber. The deflections were decreased with the addition of fiber to the concrete.

For the polypropylene fiber reinforced concrete slabs, an average of 8% difference was observed in the predicted values of maximum load obtained using the proposed model, an average of 9% difference using the Desayi and Krishnan curve for plain concrete, an average of 8% difference using the Hognestad’s Model and an average of 20% difference using the ACI 440.1R method with failure loads being greater than the predicted strengths. For the slabs without polypropylene fiber, an average of 16% difference was observed in the predicted values of maximum load obtained using Desayi and Krishnan curve and an average of 18% difference was observed using Hognestad’s Model and 12% difference using the ACI method with predicted strengths being much greater than the corresponding failure loads obtained from tests. The theoretical deflections determined using the ACI 400.1R method was reasonably close to the experimental deflections obtained from tests. A need for the improvement of shear
strength equations given by ACI 440.1R is determined based on the comparison of experimental shear strength to the shear strength equation given by ACI 440.1R.

The amount of energy stored in concrete with respect to that stored in BFRP bars is determined using the Proposed method and Hognestad’s model. The evaluations show that in spite of the lower concrete strengths of the polypropylene fiber-reinforced concrete slabs compared to the plain slabs, the percentage of energy stored in concrete for the polypropylene-fiber reinforced concrete slabs is found almost more or less equal to the percentage of energy stored in concrete for the slabs without fiber. For the ductility of the slabs, the ductility index is found to decrease with increasing reinforcement ratio. With the addition of polypropylene fiber to the slabs, the ductility of the slabs was found to be less than that for the slabs without fiber due to the lower concrete strength of the polypropylene fiber-reinforced concrete slabs.

For the study of post-cracking strength, five beams and two round panels were tested. From the beam tests, the average residual strength of the polypropylene fiber reinforced concrete beams were found to be greater than the average residual strength of the beam observed from literature. From the round panel tests, the toughness of the polypropylene fiber reinforced concrete panels was found to be greater than toughness of the panels observed from the published literature. From these tests, the correlation between the flexural toughness of beam and panel specimens was also studied and compared with the published literature. It was found that the linear correlation suggested in literature for other types of fiber is equally valid for polypropylene fiber.
ACKNOWLEDGMENTS

I would like to thank my advisor Dr. Anil Patnaik, for his constant cooperation and guidance throughout my research and for the entire period of my study.

I am also thankful to my committee members, Dr. William Schneider, IV, Dr. Kallol Sett for their valuable suggestions and also for being on my graduate committee.

I would also like to thank my fellow graduate students Nicholas Novak, Pouyan BaniBayat, Sudeep Adhikari, Udaykar Bathini, Amir Pasha, Paul Robertson, and David Mc Vaney for their valuable help in conducting of tests during the research period of this project.

This project was partially funded by ReforceTech, AS. I thank one and all for their positive help which accounted for the grand success of this project.

Subhashini Neela
# TABLE OF CONTENTS

| LIST OF TABLES | ............................................................ | x |
| LIST OF FIGURES | ............................................................ | xii |

## CHAPTER

### I. INTRODUCTION

1.1 Research Significance .................................................................3

1.2 Research Objectives ..................................................................5

1.3 Research Methodology ...............................................................6

1.4 Thesis Outline ...........................................................................7

### II. LITERATURE REVIEW

2.1 Currently Used Fiber Types ..........................................................9

2.2 Literature on ASTM C1550 .........................................................14

2.3 Literature on Polypropylene Fiber-Reinforced Concrete ..............22

2.4 Literature on Various Test Methods Available for Measuring the Post-Crack Performance of FRC Members .............................................38

### III. MATERIALS

3.1 Fibers ....................................................................................44

3.2 Basalt Fiber-Reinforced Polymer Bars ........................................45
3.3 Cement.................................................................................................45
3.4 Coarse Aggregates .............................................................................46
3.5 Fine Aggregate.....................................................................................46
3.6 Water ....................................................................................................46

IV. EXPERIMENTAL PROGRAM..............................................................47
4.1 Slab Test.................................................................................................47
4.2 Test Results............................................................................................56
4.3 ASTM C-1399 Third-Point Loaded beam Tests..................................77
4.4 ASTM C-1550 Round Determinate Panel Tests................................82

V. ANALYTICAL PROCEDURE.................................................................88
5.1 Theoretical Approach for Polypropylene Fiber-Reinforced Concrete  
  Slabs based on Proposed Method...............................................................88
5.2 Theoretical Approach using Hognestad’s Model for Polypropylene  
  Fiber-Reinforced Concrete Slabs.................................................................97
5.3 Theoretical Approach for Plain Slabs (Slabs without Polypropylene Fiber  
  Reinforcement) ..........................................................................................99
5.4 Calculation of Maximum Load Capacity using ACI Method for  
  Slabs reinforced with and without Polypropylene Fiber Reinforcement........101
5.5 Calculation of Shear Strength for Slabs reinforced with and without  
  Polypropylene Fiber Reinforcement..........................................................104
5.6 Theoretical Approach of finding the Deflections for Slabs with and without  
  Polypropylene Fiber Reinforcement..........................................................108
5.7 Analytical Results..................................................................................112
5.8 Theoretical and Experimental Stress – Strain Curves for Polypropylene Fiber  
  Reinforced Concrete Cylinders and Plain Cylinders.................................126
5.9 Energy Dissipation for Slab tests based on different approaches...........133
5.10 Elastic energy versus In-elastic energy dissipation based on Test curve.....145
**LIST OF TABLES**

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Mix Proportions for Polypropylene Fiber Reinforced Concrete Slabs</td>
<td>48</td>
</tr>
<tr>
<td>2: Mix Proportions for Plain Slabs</td>
<td>48</td>
</tr>
<tr>
<td>3: Cylinder Compression Test Results</td>
<td>56</td>
</tr>
<tr>
<td>4: Summary of Test Results of Series1 Slabs</td>
<td>77</td>
</tr>
<tr>
<td>5: Summary of Test Results of Series2 Slabs</td>
<td>77</td>
</tr>
<tr>
<td>6: ARS Beam Test Results</td>
<td>82</td>
</tr>
<tr>
<td>7: Round Panel Test Results</td>
<td>87</td>
</tr>
<tr>
<td>8: Strain Compatibility Spreadsheet for 5 mm PP1 slab</td>
<td>95</td>
</tr>
<tr>
<td>9: Shear Capacity of Slabs reinforced with and without Polypropylene fiber</td>
<td>105</td>
</tr>
<tr>
<td>10: Experimental Shear Strength and Shear Factor for Slabs with and without Polypropylene fiber</td>
<td>106</td>
</tr>
<tr>
<td>11: Spreadsheet for the Calculation of Theoretical Deflection for 5 mm PP1 Slab</td>
<td>111</td>
</tr>
<tr>
<td>12: Energy stored in Concrete and FRP bars based on Hognestad’s model</td>
<td>134</td>
</tr>
<tr>
<td>13: Energy stored in Concrete and FRP bars based on Proposed model</td>
<td>140</td>
</tr>
<tr>
<td>14: Elastic and Inelastic Energy released at Failure based on Experimental Load-Deflection Curve</td>
<td>147</td>
</tr>
<tr>
<td>15: Percentage Increase in Load Capacity of Slabs with addition of Polypropylene fiber</td>
<td>150</td>
</tr>
<tr>
<td>16: Percentage Increase in Maximum Load Capacity of Slabs with increase in Compressive Strength</td>
<td>151</td>
</tr>
</tbody>
</table>
17: Prediction of Theoretical Maximum Load Capacity for Polypropylene FRC Slabs using Proposed Model……………………………………………………..151

18: Prediction of Theoretical Maximum Load Capacity for Polypropylene FRC Slabs using Desayi and Krishnan Curve for Plain Concrete…………………………..153

19: Prediction of Theoretical Maximum Load Capacity for Polypropylene FRC Slabs using Hognestad’s Model……………………………………………………………154

20: Prediction of Theoretical Maximum Load Capacity for Polypropylene FRC Slabs using ACI 440.1R Method……………………………………………………………155

21: Prediction of Theoretical Maximum Load Capacity for Plain Slabs using Desayi and Krishnan Curve……………………………………………………………………157

22: Prediction of Theoretical Maximum Load Capacity for Plain Slabs using Hognestad’s Model…………………………………………………………………………………158

23: Prediction of Theoretical Maximum Load Capacity for Plain Slabs using ACI 440.1R Method…………………………………………………………………………………159

24: Comparison of Different Methods used for predicting Maximum load for Polypropylene FRC Slabs………………………………………………………………………………160

25: Comparison of Different Methods used for predicting Maximum Load for Plain Slabs…………………………………………………………………………………160
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Hooked end fibers</td>
<td>11</td>
</tr>
<tr>
<td>2: Crimped steel fibers</td>
<td>11</td>
</tr>
<tr>
<td>3: Overwrap Configurations</td>
<td>23</td>
</tr>
<tr>
<td>4: Elastic and Inelastic energy from Orozco and Maji (2004)</td>
<td>27</td>
</tr>
<tr>
<td>5: Analytical Model of Naaman and Fanella (1985)</td>
<td>31</td>
</tr>
<tr>
<td>6: Flexural Test Apparatus Used for applying Third-Point Loading</td>
<td>42</td>
</tr>
<tr>
<td>7: Typical Deflection Measurement Apparatus</td>
<td>42</td>
</tr>
<tr>
<td>8: Flexural Toughness</td>
<td>43</td>
</tr>
<tr>
<td>9: Plan View of the Reinforcement Detail</td>
<td>50</td>
</tr>
<tr>
<td>10: Reinforcement Cage with Strain Gages</td>
<td>51</td>
</tr>
<tr>
<td>11: Concrete Mix</td>
<td>52</td>
</tr>
<tr>
<td>12: Cylinders for Compression test</td>
<td>53</td>
</tr>
<tr>
<td>13: Strain Gages Mounted on the Surface of a Concrete Slab Specimen</td>
<td>54</td>
</tr>
<tr>
<td>14: Four Point Loading Arrangement for the Slab Specimens</td>
<td>55</td>
</tr>
<tr>
<td>15: Test Setup for determining Cylinder Compressive Strength</td>
<td>55</td>
</tr>
<tr>
<td>16: Concrete Compressive Strain for Polypropylene Fiber-reinforced Concrete Slab with 5 mm Basalt FRP bars of Series1</td>
<td>57</td>
</tr>
<tr>
<td>17: Failure mode of Polypropylene Fiber-reinforced concrete Slab with 5 mm Basalt reinforcing bars of Series1</td>
<td>58</td>
</tr>
</tbody>
</table>
18: Concrete Compressive Strain for Plain concrete slab with 5 mm Basalt FRP bars of Series1

19: Failure mode of Plain Concrete Slab with 5 mm Basalt FRP bars of Series1

20: Concrete Compressive Strain for Polypropylene Fiber-reinforced concrete Slab with 7 mm Basalt FRP bars of Series1

21: Failure mode of Polypropylene Fiber-reinforced Concrete Slab with 7 mm Basalt FRP bars of Series1

22: Concrete Compressive Strain for Plain concrete Slab with 7 mm Basalt FRP bars of Series1

23: Failure mode of Plain Concrete Slab with 7 mm Basalt FRP bars of Series1

24: Average Concrete Compressive Strain for Plain and Polypropylene FRC slabs with 5 mm Basalt FRP reinforcing bars

25: Average Concrete Compressive Strain for Plain and Polypropylene FRC slabs with 7 mm Basalt FRP reinforcing bars

26: Average Tensile Bar Strain for Plain and Polypropylene FRC slabs with 5 mm Basalt FRP reinforcing bars

27: Average Tensile Bar Strain for Plain and Polypropylene FRC slabs with 7 mm Basalt FRP reinforcing bars

28: Average Deflection for Plain and PPN FRC slabs with 5 mm Basalt FRP reinforcing bars

29: Average Deflection for Plain and PPN FRC slabs with 7 mm Basalt FRP reinforcing bars

30: Concrete Compressive Strain for Polypropylene Fiber-reinforced concrete Slab with 5 mm Basalt FRP bars of Series2

31: Failure mode of Polypropylene Fiber-reinforced Concrete Slab with 5 mm Basalt FRP bars of Series2

32: Concrete Compressive Strain for Plain concrete Slab with 5 mm Basalt FRP bars of Series2

33: Failure mode of Plain Concrete Slab with 5 mm Basalt FRP bars of Series2
34: Concrete Compressive Strain for Polypropylene Fiber-reinforced concrete Slab with 7 mm Basalt FRP bars of Series2

35: Failure mode of Polypropylene Fiber-Reinforced Concrete Slab with 7 mm Basalt FRP bars of Series2

36: Concrete Compressive Strain for Plain concrete Slab with 7 mm Basalt FRP bars of Series2

37: Failure mode of Plain Concrete Slab with 7 mm Basalt FRP bars of Series2

38: Average Concrete Compressive Strain for Plain and Polypropylene FRC slabs with 5 mm basalt reinforcing bars – Series 2

39: Average Concrete Compressive Strain for Plain and Polypropylene FRC slabs with 7 mm basalt reinforcing bars – Series 2

40: Average Tensile Bar Strain for Plain and Polypropylene FRC slabs with 5 mm basalt reinforcing bars – Series2

41: Average Tensile Bar Strain for Plain and Polypropylene FRC slabs with 7 mm basalt reinforcing bars – Series2

42: Average Deflection for Plain and Polypropylene FRC slabs with 5 mm basalt reinforcing bars – Series2

43: Average Deflection for Plain and Polypropylene FRC slabs with 7 mm basalt reinforcing bars – Series2

44: Beams and Round Determinate Panels casted with Polypropylene Fiber reinforced Concrete

45(a): Arrangement of LVDT at the bottom surface of the Beam for Deflection measurement

45(b): Test Setup for determining the Average Residual-Strength of Fiber-reinforced Concrete

46: Load Vs Deflection Curve for Polypropylene Fiber-reinforced Concrete Beam

47(a): Making of Round Panel (Consolidation Using a Vibrating Table)
47(b): Screeding and Finishing the Top Surface of Test Specimen..........................83

47(c): Fiber-Reinforced Concrete Round Determinate Panel...............................83

48: Test Setup used for determining the Flexural Toughness of the Round Determinate Panels .................................................................85

49: Failure mode of the Round Determinate Panels..........................................86

50: Load Vs Deflection Curve for the Round Determinate Panel 1 .....................86

51: Load Vs Deflection Curve for the Round Determinate Panel 2 .....................87

52: Influence of the Volume fraction of fibers on the Compressive stress-strain curve…90

53: Theoretical Stress-Strain Curve for Polypropylene Fiber-Reinforced Concrete Slab with 5 mm Basalt FRP bars- Series1 ......................................................91

54: Hognestad’s Model..................................................................................98

55: Typical Stress-Strain Curve using Different Equations for $f_c' = 3,580$ psi ........99

56: Typical Stress-Strain Curve using Different Equations for $f_c' = 4,000$ psi ....101

57: Shear Factor for Basalt FRP Reinforced Slabs With and Without Polypropylene Fiber .........................................................................................107

58(a): Experimental and Theoretical load-concrete strain curve for 5 mm series1 slab with polypropylene fiber .........................................................112

58(b): Experimental and Theoretical load-bar strain curve for 5 mm series1 slab with polypropylene fiber .........................................................113

58(c): Experimental and Theoretical load-deflection curve for 5 mm series1 slab with polypropylene fiber .........................................................113

59(a): Experimental and Theoretical load-concrete strain curve for 5 mm series2 slab with polypropylene fiber .........................................................114

59(b): Experimental and Theoretical load-bar strain curve for 5 mm series2 slab with polypropylene fiber .........................................................115

59(c): Experimental and Theoretical load-deflection curve for 5 mm series2 slab with polypropylene fiber .........................................................115
60(a): Experimental and Theoretical load-concrete strain curve for 7 mm series1 slab with polypropylene fiber

60(b): Experimental and Theoretical load-bar strain curve for 7 mm series1 slab with polypropylene fiber

60(c): Experimental and Theoretical load-deflection curve for 7 mm series1 slab with polypropylene fiber

61(a): Experimental and Theoretical load-concrete strain curve for 7 mm series2 slab with polypropylene fiber

61(b): Experimental and Theoretical load-bar strain curve for 7 mm series2 slab with polypropylene fiber

61(c): Experimental and Theoretical load-deflection curve for 7 mm series2 slab with polypropylene fiber

62(a): Experimental and Theoretical load-concrete strain curve for 5 mm series1 slab without fiber

62(b): Experimental and Theoretical load-bar strain curve for 5 mm series1 slab without fiber

62(c): Experimental and Theoretical load-deflection curve for 5 mm series1 slab without fiber

63(a): Experimental and Theoretical load-concrete strain curve for 5 mm series2 slab without fiber

63(b): Experimental and Theoretical load-bar strain curve for 5 mm series2 slab without fiber

63(c): Experimental and Theoretical load-deflection curve for 5 mm series2 slab without fiber

64(a): Experimental and Theoretical load-concrete strain curve for 7 mm series1 slab without fiber

64(b): Experimental and Theoretical load-bar strain curve for 7 mm series1 slab without fiber

64(c): Experimental and Theoretical load-deflection curve for 7 mm series1 slab without fiber

65(a): Experimental and Theoretical load-concrete strain curve for 7 mm series2 slab without fiber
65(b): Experimental and Theoretical load-bar strain curve for 7 mm series2 slab without fiber

65(c): Experimental and Theoretical load-deflection curve for 7 mm series2 slab without fiber

66: Polypropylene Fiber reinforced Concrete Cylinders tested under Compression

67: Plain Cylinders tested under Compression

68: Experimental and Theoretical Compressive Stress-Strain Curve for 5 mm PP1 Cylinder

69: Experimental and Theoretical Compressive Stress-Strain Curve for 5 mm PP2 Cylinder

70: Experimental and Theoretical Compressive Stress-Strain Curve for 5 mm PP3 Cylinder

71: Experimental and Theoretical Compressive Stress-Strain Curve for 5 mm Plain1 Cylinder

72: Experimental and Theoretical Compressive Stress-Strain Curve for 5 mm Plain2 Cylinder

73: Experimental and Theoretical Compressive Stress-Strain Curve for 5 mm Plain3 Cylinder

74: Energy stored in Concrete and FRP bars for 5 mm PP1 Slab

75: Energy stored in Concrete and FRP bars for 5 mm PP2 Slab

76: Energy stored in Concrete and FRP bars for 7 mm PP1 Slab

77: Energy stored in Concrete and FRP bars for 7 mm PP2 Slab

78: Energy stored in Concrete and FRP bars for 5 mm Plain1 Slab

79: Energy stored in Concrete and FRP bars for 5 mm Plain2 Slab

80: Energy stored in Concrete and FRP bars for 7 mm Plain1 Slab

81: Energy stored in Concrete and FRP bars for 7 mm Plain2 Slab

82: Energy stored in Concrete and FRP bars for 5 mm PP1 Slab
83: Energy stored in Concrete and FRP bars for 5 mm PP2 Slab.........................141
84: Energy stored in Concrete and FRP bars for 5 mm Plain1 Slab.......................142
85: Energy stored in Concrete and FRP bars for 5 mm Plain2 Slab.......................142
86: Energy stored in Concrete and FRP bars for 7 mm PP1 Slab.......................143
87: Energy stored in Concrete and FRP bars for 7 mm PP2 Slab.......................143
88: Energy stored in Concrete and FRP bars for 7 mm Plain1 Slab.......................144
89: Energy stored in Concrete and FRP bars for 7 mm Plain2 Slab.......................144
90: Elastic and Inelastic energy from Orozco and Maji (2004)..........................145
91: Correlation between the Beam and Panel Specimens obtained from the present study plotted against Bernard’s results (2002).............................................................149
92: Test Load Vs Predicted Load for Polypropylene FRC Slabs using Proposed Method..........................................................152
93: Test Load Vs Predicted Load for Polypropylene FRC Slabs using Desayi and Krishnan Method..........................................................154
94: Test Load Vs Predicted Load for Polypropylene FRC Slabs using Hognestad Method..........................................................155
95: Test Load Vs Predicted Load for Polypropylene FRC Slabs using ACI Method……156
96: Test Load Vs Predicted Load for Plain Slabs using Desayi and Krishnan Method..157
97: Test Load Vs Predicted Load for Plain Slabs using Hognestad Method.............158
98: Test Load Vs Predicted Load for Plain Slabs using ACI Method......................159
CHAPTER I
INTRODUCTION

Fiber-reinforced concrete (FRC) has become an important material for the construction of buildings, and production of linings for tunnels in the recent years. Its ability to support load after cracking and to reduce the brittleness of concrete has positive effect on structural performance of concrete. This makes it attractive for its use in almost all the major constructions in the world. Fiber reinforced concrete is mostly used for slabs on grade and pavements, but is considered for a wide range of structural members such as beams, slabs, walls, etc. Steel fiber reinforced concrete is more ductile than any other fiber reinforced concrete but it can only be used in application where one can avoid corrosion and rust stains. Thus, synthetic fiber reinforced concrete, such as polypropylene fiber reinforced concrete, is gaining popularity due to its low cost and non-corrosive nature. Basalt fiber can also be used in fiber reinforced concrete. Basalt fiber is strong and inexpensive but historically has not resisted the alkaline environment of concrete. Fiber reinforced concrete increases the structure’s service life and can also reduce the thickness of the structural section by reducing the concrete cover used over the reinforcement.
In order to predict the in-situ behavior of fiber reinforced concrete structures, several tests are available which determine the post-crack behavior of fiber reinforced concrete structural members. There are ASTM test methods to determine the toughness and load carrying capacity of the fiber reinforced concrete after cracking. ASTM C1018 is a continuation of standard test method for flexural strength ASTM C78. ASTM C1018 is not active anymore and thus ASTM C1609 is used which is similar to ASTM C1018. Load - deflection curves are produced from a closed-loop testing system in ASTM C1018 and ASTM C1609 test methods. Other test methods, including ASTM C1550, need specialized equipment to perform the tests in a laboratory. Unlike these, ASTM C1399 test method does not require any specialized equipment and can be simply performed in an open loop machine. Another test method, called JSCE-SF4 proposed by Japanese Society of Civil Engineers, is also used to determine the toughness of fiber reinforced concrete. This test method is found to have its own constraints and is implied as an approximate method to determine toughness. There are no exclusive methods proposed by ASTM for testing the cylinders made of fiber reinforced concrete. The standard test method, ASTM C39 that is also used for the plain concrete is used for determining the compressive strength of fiber reinforced concrete cylinders.

The ASTM C1399 test method uses a standard flexural beam to measure the ability of fibers to carry the load after the cracking of concrete at its ultimate flexural strength. From the studies on ASTM C1018, it was found that there is an initial shock load to the fibers at the first crack. This constitutes a release of the energy stored in the test frame during loading which affects the data recovery and the results produced. ASTM C1399 test method was developed in such a way as to minimize the affect of shock load by
providing a stainless steel plate under the concrete beam thereby limiting the deflection and providing stiffness soon after the first crack is formed.

Studies indicate that the post-crack behavior of plate-like, fiber-reinforced concrete structural members is better represented by a centrally loaded round panel test as described in ASTM C1550. The failure mode of the test specimen when subjected to a central point load is related to the in-situ behavior of structures such as shotcrete tunnel linings and concrete slabs-on-grade. The post-crack performance is represented by the energy absorbed by the panel up to a specified deflection. This is calculated as the area under the load-deflection curve up to the specified deflection.

The primary structural elements where the fiber-reinforced polymer (FRP) bars can be used are slabs and beams. Structural concrete slabs or beams that are reinforced with steel or FRP can be designed as under-reinforced or over-reinforced sections owing to the amount of reinforcement used in it. An under-reinforced section is one which typically fails by yielding rupture of the reinforcement and over-reinforced is one which typically fails by crushing of concrete. In the design of FRP reinforcement, we prefer the section to be over-reinforced, which is because the fibers that carry the tensile stresses in the FRP bar do not have well defined yield zone before complete failure thus making the failure to be sudden and catastrophic which is not desirable.

1.1 Research Significance

During the past two decades, the use of fiber-reinforced concrete in structural applications like buildings, bridges, tunnels and mine drives has increased. This requires
the evaluation of design methods that establish sufficient safety against catastrophic failure. The significance of the characteristics of stress-strain properties of fiber-reinforced concrete in compression is needed when new FRP materials are used. This constitutes for the experimental and analytical evaluation of the fiber-reinforced concrete in compression. The enhancement of ACI 440.1R design method defined for the calculation of shear strength for beams with FRP bars as flexural reinforcement is needed for the present day applications of FRP in structures as they can be in the form of both reinforcing bars and small length fibers added to concrete mix. As the FRP materials are brittle in nature, the energy absorbed by fiber-reinforced concrete beams through the straining in the reinforcing rebars is limited unlike the traditional steel reinforced concrete beams. Therefore, the definition of ductility based on the energy absorption using FRP rebars and fiber as reinforcement are needed. In applications such as tunnel linings, the structural performance of the FRP material is reliant on its post-crack flexural capacity. Both beam and panel tests are available for the measure of post-crack flexural capacity but the relation between the performance data produced in these was found to be unclear. An analysis was needed to analyze the desirable correlations between the beam and panel specimens and to find the more applicable test for a given application. This analysis was studied by Bernard [1] and the accuracy of this analysis was verified as a part of this research.
1.2 Research Objectives

The primary objective of the research is the determination of the stress-strain relationship of polypropylene fiber reinforced concrete so as to be used for understanding of the flexural behavior of concrete members reinforced with basalt fiber-reinforcing bars as flexural reinforcement. The identification of the stress-strain relationship thus ensues the determination of the load-strain behavior and maximum load capacity of the polypropylene fiber reinforced concrete slabs reinforced with basalt FRP bars. The slab tests were designed to determine the influence of concrete strength and percentage volume of fiber on the maximum load capacity, shear strength, deflections and ductility. One of the objectives of the slab tests is also the study of the load-deflection behavior of the basalt FRP reinforced beams with and without polypropylene fiber. Another objective of the research is to check the validity of the code defined design methods for the calculation of shear strength for FRP reinforced beams made from polypropylene fiber reinforced concrete.

The secondary objective of this research was to derive a correlation between the post-cracking strengths of fiber reinforced concrete beams and round determinate panels. The study also includes the determination of the energy absorbed by the polypropylene fiber-reinforced concrete round determinate panels and average residual strength of polypropylene fiber-reinforced concrete beams.
1.3 Research Methodology

The study includes the determination of the stress-strain relationship of polypropylene fiber-reinforced concrete. From this relationship, the load strain curve of the slab elements is predicted using the strain compatibility method. Another analytical load-strain curve was developed using the stress-strain relation obtained from literature that follows the strain compatibility method. These two analytical load-strain curves are compared to the experimental curves. Along with the above two approaches, the maximum load capacity of the slab elements is also predicted using ACI 440.1R method and strain compatibility method using parabolic stress-strain curve for concrete. The predicted maximum loads using these four different stress-strain relations are tabulated and compared to the experimental maximum load.

For the slabs reinforced with basalt FRP bars without polypropylene fiber, two analytical stress-strain relations taken from the literature are used to obtain the load-strain data that follows strain compatibility method. These analytical load-strain curves are plotted and compared to the experimental curve. The maximum load capacity of the slab is predicted using the ACI 440.1R method along with the other two methods. The predicted maximum load using the three different approaches is tabulated and compared to the experimental maximum load.

For the load-deflection analysis of the slab elements, the relation for the effective moment of inertia for the cracked section given by ACI 440.1R is considered. The analytical load-deflection curves were plotted and compared to the experimental curve. Based on the experimental load-deflection curve, the effect of addition of polypropylene
fibers to the concrete on the ductility of the slabs is studied based on a method described by other researchers. The fracture of energy stored and dissipated in both concrete and FRP bars of the slab is analyzed. All the slab elements were tested under four-point bending. For the determination of shear strength of the slabs, the ACI 440.1R method is used. Based on the comparison of the analytical shear strength to the experimental shear strength, the need for the improvement of shear strength factor described by ACI is recognized.

For the post-cracking strength analysis of the polypropylene fiber reinforced concrete beams and round determinate panels, the relevant test-method conforming to ASTM standards was followed. The consistency of the correlations found by earlier researchers between the behavior of beam and panel specimens was checked for the polypropylene fiber material used in this study.

1.4 Thesis Outline

This thesis comprises both analytical and experimental research components that were performed at The University of Akron. This thesis is organized into the following chapters.

Literature Review is presented in chapter 2. The characteristics of the materials used in this study are outlined in chapter 3. The details of the experimental program are described in chapter 4. Complete treatment and description of the analytical procedures are given in chapter 5. The test results are analyzed in chapter 6 and the conclusions are listed in chapter 7.
Concrete is one of the most widely used building materials in the world due to its low cost, ease in production and longevity. Iron rods were used initially to reinforce concrete since 19th century and steel is used as reinforcement at the present time. One of the biggest problems with concrete reinforced with steel is its durability and corrosion. In harsh environments, the concrete matrix around the embedded steel bars is insufficient for protection. When steel bars corrode, the surrounding concrete will degrade causing the need for very expensive and time consuming repair works or replacement. The major drawback with concrete is its low tensile strength. When tension develops in concrete, it undergoes cracking leading to brittle failures which can be catastrophic. The use of FRP as reinforcement in the concrete structures has been gaining importance over the past two decades. The major advantages of FRP over steel are that the material can be tailored to the loads acting on the system, improved resistance to corrosion, an increase in service life and durability [2].

An FRP reinforcing bar is a structural reinforcing bar made from filaments or fibers held in a polymeric resin matrix binder. The FRP Bar can be made from various types of fibers such as Glass (GFRP), Aramid (AFRP) or Carbon (CFRP). FRP bars can have a surface treatment that facilitates a bond between the finished bar and the concrete into which they are placed. FRP bars are intended for use as concrete reinforcing in areas
where steel reinforcing has a limited life span due to the effects of corrosion. They are also used in situations where electrical or magnetic transparency is needed. In addition to reinforcing for new concrete construction, FRP bars can also be used to structurally strengthen existing masonry, concrete or wood members.

Short length fibers are used in concrete because they improve the structural integrity of the members. Materials like horsehair were used in ancient times as fiber in mortar for concrete construction. In 1900s, use of asbestos fibers in concrete came into existence. By 1960’s, fiber reinforced concretes such as steel, glass and carbon fiber reinforced concretes were used in construction. Based on the type of fiber used in the concrete and its orientation, aspect ratio and density of fibers influence the characteristic properties of concrete.

2.1 Currently Used Fiber Types

This section discusses about the currently used fibers with their advantages and disadvantages when used in structural elements.

2.1.1 Carbon Fibers

Carbon fibers are extremely thin fibers which are only 0.005-0.010 mm in diameter and are generally used in shorter lengths. The density of carbon fibers is very low when compared to steel, (one-fifth that of steel) thus making it a lightweight material. Apart from the low density, carbon fibers have high tensile strength, low
thermal expansion, good abrasion resistance and stability at high temperatures with relatively high stiffness which makes it a popular material in the industries like aerospace, civil engineering, military, etc. However, the carbon fibers are relatively costly when compared to materials like fiberglass and synthetic fiber.

2.1.2 Glass fibers

Short length glass fiber is formed from the extremely thin fibers of glass. The types of glass fibers include E-glass, A-glass, E-CR-glass, C-glass, D-glass, R-glass and S-glass. The elastic modulus of glass fiber is found to be approximately one third of steel but when compared to carbon fiber, it elongates much more before failure [3]. But the main disadvantage of glass fibers is that small diameter fibers have large surface area which makes it more sensitive towards alkali environment. In recent years, attempts have been made to improve the alkali resistance properties of fiberglass.

2.1.3 Steel Fibers

Steel fibers are commonly used in concrete to increase the toughness of concrete. Initially, steel fibers were used primarily for crack control and to replace the secondary reinforcements in pavements and tunnel linings. Today, while steel fibers are still used extensively for repair applications, they are also used increasingly to partially replace conventional steel reinforcements. The introduction of steel fibers in concrete can increase the resistance to fatigue, impact, blast or seismic events but the strength
improvement is very little for low fiber volumes. The main advantage of using steel fibers in concrete is to increase the post-peak load carrying capacity of concrete after initial cracking. Types of steel fibers used in concrete are [3]:

- Smooth cold-drawn wire;
- Deformed cold-drawn wire;
- Smooth or deformed cut sheet;
- Melt-extracted fibers;
- Mill-cut or modified cold-drawn wire.

The tensile strengths of steel fibers are in the range of 345-2100 MPa and the ultimate elongations of 0.5-35%. The main disadvantage of high strength fibers is that it causes severe spalling around the fiber ends. Straight and smooth fibers have poor bond with the matrix. Thus the surfaces of fibers were modified in the recent years to provide measures to increase the bond with matrix. These include hooked end, crimped, deformed, and enlarged end fibers.

Fig. 1: Hooked end fibers [4]  
Fig. 2: Crimped steel fibers [5]

Round fibers are produced by cutting or chopping wires, with diameters typically in the range of 0.25-1.0 mm and flat fibers are produced by shearing sheets or flattening wire. The dimensions of flat fibers are in the range of 0.15-0.40 mm thick and 0.25-0.90 mm wide. Fibers are also produced by the hot melt extraction process.
2.1.4 Synthetic Fibers

The synthetic polymeric fibers in use in the construction industry include acrylic, aramid, carbon, nylon, polyester, polyethylene, and polypropylene fibers. The concept of using polymeric fibers was tried in 1965 [6]. In 1970s, the large scale use of polymeric fibers in concrete had started. All these fibers have a very high tensile strength, but most of these fibers (except aramids) have a relatively low modulus of elasticity. As the diameters of the polymeric fibers are in the order of micrometers, their very high length-to-diameter ratios are useful in fiber reinforced concrete. The major disadvantages of polymeric fibers are low modulus of elasticity, poor bond with cement matrix, and low melting point. Their bond to the cement matrix is improved by twisting several fibers together or by treating the fiber surface.

Polypropylene:

Polypropylene has been the most successful commercial application so far among the synthetic polymeric fibers discussed above. Polypropylene fibers are used mainly to control cracking in the initial setting stages [2]. These fibers are produced from homopolymer polypropylene resin in a variety of shapes and sizes, and with different properties. The main advantages of polypropylene fibers are their alkali resistance, relatively high melting point (165°C) and the low cost of the material. Their disadvantages are poor fire resistance, sensitivity to sunlight and oxygen, a low modulus of elasticity (1-8 GPa) and poor bond with matrix. These apparent disadvantages are not necessarily critical. Embedment in the matrix provides a protective cover, helping to minimize sensitivity to fire and other environmental effects.
Polypropylene FRC in Compression:

According to Balaguru (1988), the uniaxial compression test is normally used to evaluate the behavior of concrete in compression. This produces a combination of shear failure near the ends of the specimen with lateral swelling of the unconfined central section accompanied by cracking parallel to the loading axis when the lateral strain exceeds the matrix cracking strain in tension. Fibers can affect these facets of uniaxial compressive behavior that involve shear stress and tensile strain. This can be seen from the increased strain capacity and also from the increased toughness (area under the curve) in the post-crack portion of the stress-strain curve. For hardened concretes with low fiber volume fractions (<2%), the introduction of fibers hardly affects any property including the modulus of elasticity (Balaguru, 1988). The addition of fibers up to a volume fraction of 0.1% does not affect the compressive strength. When tested under compression, failure occurs at or soon after the peak load providing very little toughness. It is found that the fibers have very little effect on compressive strength calculated from the peak load, and both slight increase and decrease in strength have been reported with increase in fiber content. The decrease in strength is mostly reasoned due to incomplete consolidation (Johnston and Skarendahl, 1992). In some instances, if more water is added to fiber concrete to improve its workability, a reduction in compressive strength can occur. This reduction should be attributed to additional water or due to an increase in entrapped air, not fiber addition (Khajuria and Balaguru, 1989). The reduction does not occur in all cases.
2.2 Literature on ASTM C1550

The need for a reliable and economical estimation for the post-crack performance of fiber-reinforced concrete material has led to the development of a round determinate panel test. Over the last 20 years, the fibers are being used as reinforcement in concrete in many sectors of the underground construction industry. The addition of fibers to concrete provides ductility to the concrete.

Post-crack performance in fiber reinforced concrete was assessed using beams. In the past years, ASTM C-1018 beam test was used to assess the post crack performance. In Australia, a similar method to C-1018 called EFNARC beam test was used for the measurement of post crack performance. EFNARC beam test [7] is a third-point loaded beam test used for the assessment of FRS (Fiber Reinforced Shotcrete) performance. The test specimen measures 75 x 125 x 550 mm tested over a span of 450 mm in such a way that the upper surface of the beam conforms to the plane of the sprayed surface. The EFNARC specification does not clearly mention how to test the beam in order to obtain central deflections excluding the extraneous deformation. Therefore Bernard [1] had used the deflection measurement procedure described in ASTM C1018 [8]. But these methods often caused problems for the designers and contractors as the results obtained from the beam tests had high with-in batch variability (Bernard, April 2002). Thus toughness assessment based on beam tests provided poor reliability. Thus the author Bernard [9] had performed several tests to examine the quality assurance of EFNARC beam test.

In the EFNARC beam test, the within-batch variability was found to exceed 20% for residual strength at 3.0mm central deflection and for the ASTM $I_{50}$ index; the
coefficient of variation in post-crack performance has averaged about 15%. Therefore the result is that the mean performance of many batches of apparently adequate FRS fails to pass minimum requirements. The area of the concrete that experiences failure in a beam is very small compared to the volume of the concrete contained within an FRS structure. The performance of the beam is not representative of the majority of concrete in a structure because the performance is strongly dependent on the number of fibers that happen to occur at the crack.

As the EFNARC beam tests produced poor results with low quality assurance, Bernard [9] has developed EFNARC panel tests in Australia. The results of these tests are influenced by the flatness of the specimen base. A truly flat specimen rests evenly on the flat support fixture during testing and results in a single peak in the load-deflection curve, whereas a slightly distorted specimen suffers uneven stressing and multiple peaks in the load-deflection curve as it progressively cracks and redistributes load. In order to minimize these discrepancies and to improve the test methods for better quality assurance, Bernard developed the round determinate panel test in 1998. The following sections provide a brief review of the relevant work as described in each paper.


The influence of support conditions on structural behavior in FRC panels is investigated in 1997 by testing round panels which was similar to C 1550 configuration. Bernard (2000) in his paper “Behavior of round steel fiber reinforced concrete panels under point loads”, has examined the possible differences in behavior between
continuous in situ tunnel linings and specimen panels representing portions of a lining, four modes of edge support were used to restrain round specimens that varied in both thickness and diameter. Determinate and simple support conditions were examined as possible modes of panel support for routine laboratory testing. Fully clamped and quasi-continuous edge restrained panels were used as approximations of an in situ lining. Performance was measured in terms of peak load capacity and energy absorption. Over the several tests performed, the author concluded that the determinate mode of support gave more consistent results than all the other alternatives including the insensitivity to flatness of the specimen and a consistent mode of failure. The peak load carrying capacity and total energy absorption were found to be lower than other alternative types of support. In fully clamped and quasi-continuous edge conditions, the mode of failure was by shear and the degree of shearing was inconsistent for identical specimens whereas for the determinate and simply supported conditions, flexural mode of failure was observed. Moreover, the pattern of failure was found to be consistent in case of determinate support thus making it reliable for the assessment of post crack performance.

Later, an extended comparative study is made on the post-crack performance based on both beam and panel tests which resulted in performance correlations for several types of toughness tests. This study established the superior repeatability of the round panel test compared with the alternatives and aroused the interest of ASTM Subcommittee as to its possible development as a standard test method.

Round Determinate Panel typically consists of a round plywood base to which a sheet steel strip is nailed, producing a dish. The width of the steel strip is chosen to produce a final depth of 75mm inside the dish. Timber runners are fastened to the base of
the form for easier handling of the specimen. This form can be inclined against any convenient object while production. Once the form is full, the surface must be screeded to produce a flat specimen of uniform thickness. The thickness of the specimen plays a major role in the variability of the performance data. Therefore, the specimens are produced with uniform thickness to obtain lower variability in the performance data.

A round panel measuring 800 mm in diameter and 75 mm in thickness is supported on three symmetrically arranged pivots placed on a 750 mm pitch circle diameter. A central point load is imposed at a controlled rate of displacement using a screw-driven loading mechanism. The failure pattern is formed by three radial cracks that run from the center and bisect each unsupported sector of the panel. This pattern of failure occurs regardless of the tolerances on the flatness of the specimen base, and the total length of the crack amounts to 1200 mm. Performance is measured by calculating the cumulative energy absorption up to selected values of central deflection (5 mm and 40 mm central deflection) and the load causing first crack of the concrete matrix. Following a test, the thickness of the specimen is measured at a number of points along each radial crack and a fiber count is carried out at 100 mm interval along the crack. Thus the mean fiber density is estimated by counting the fibers protruding from 10×100 mm long portions of the crack. The steel FRS exhibits higher residual load capacity immediately after cracking while the macro synthetic FRC exhibits higher residual capacity when the cracks have opened significantly. The average crack rotation angle $\Phi$ suffered by the three radial cracks in a C-1550 panel at a given central deflection $\delta$ is determined as

$$\Phi = \sqrt{3} \frac{\delta}{r}$$
where \( r \) is the support radius, normally equal to 375 mm. The coefficient of variation in the post crack performance parameters (based on energy absorption) is about 6%. Thus round panel test provides a greater reliability than any other test for the assessment of post-crack performance. Moreover, round determinate panels do not require saw-cutting as we do for the beams. Therefore, in cases of large tunneling projects, the cost for the quality assurance can significantly be reduced and the time spent in saw-cutting can also be saved.


In this paper, the author studied the possible correlations in the behavior between the beam and panel specimens and determined which will be the most appropriate type of test for a given FRS application. The mean Modulus of Rupture derived from both the third-point and centrally loaded beam tests produced almost the same result. The MOR derived from centrally loaded beam test is slightly higher in magnitude.

The post crack performance parameter (Japanese Toughness T) of the third-point loaded beams were plotted against energy absorption in the centrally loaded beams. The correlation is weakly linear. This may be due to the fact that the energy absorption in centrally loaded beams is measured at relatively high degree of deformation. However, the data produced in the centrally loaded beam test is of much greater structural relevance because it directly describes the relation between rotation at a crack in a FRS lining and moment capacity.
The correlation in energy absorption between the two types of panels (EFNARC and Round panel) is strongly linear. This indicates that the performance of both panels is largely interchangeable. The post-cracking performance of the third-point loaded beams has been plotted as a function of energy absorption in the round determinate panels. For the beam performance parameters derived for higher levels of deformation are strongly correlated to energy absorption in panels. However, for low levels of deformation, it is less strongly correlated. At comparable levels of deformation between beams and panels, the correlations were relatively good.

2.2.3 Khaloo and Afshari, “Flexural behavior of small steel fiber reinforced concrete slabs,” 2005.

In this paper, “Flexural behavior of small steel fiber reinforced concrete slabs” was studied to determine the influence of length, volume percentage of fibers, and concrete strength on energy absorption of small steel fiber reinforced concrete slabs under flexure. The authors proposed a design method according to allowable deflection for SFRC slabs for the range of fiber volumetric percentages used in the study. Flexural test results of slabs were shown as load-deflection and absorbed energy-deflection curves. The fibers did not considerably influence flexural characteristics of slabs prior to cracking. The main effect of fibers is on energy absorption capacity. Increase in fiber content decreased the growth rate of total energy absorption. Fibers with high aspect ratio provided higher energy absorption capacity. Increase in concrete strength also increased the energy absorption capacity.
Mode of failure of plain slabs and SFRC slabs was also discussed. Plain slabs failed suddenly at cracking load without any appreciable deflection warning where as SFRC slabs maintained integrity to relatively high deflection after ultimate strength. Theoretical behavior of SFRC after cracking is discussed. The theoretical load-deflection curves are obtained for the SFRC slabs using the flexural analysis with yield line theory. These curves are compared to the experimental results. Theoretical predictions show higher energy absorption than actual behavior.

The authors divided the load-deflection curve in to three regions for comparison. First region starts from the beginning of the loading until maximum load (cracking). Second region corresponds to transition state i.e., it begins at ultimate load and ends at a point where the tensile stress is resisted totally by bond between fibers and concrete. Theoretical results correlate well in the third region (almost parallel) with the experimental results. Design recommendation based on allowable deflection was presented. This method covers volumetric percentage of steel fibers in the range of 0.75-1.75.

The authors recommend using fiber volumetric percentages in the range of 0.75-1.75. In slabs with 0.5% fiber volume, the resisting load after cracking was relatively small. Longer fibers provided higher energy absorption. Increase in strength of FRC enhances the energy absorption. Addition of fibers does not significantly increase the ultimate flexural strength of SFRC slabs, but improves the energy absorption capacity. The rate of improvement in energy absorption reduced with increase in fiber content.

In this paper, “Influence of Toughness on the Apparent Cracking load of Fiber-reinforced Concrete Slabs”, the author demonstrated that the peak in load resistance associated with cracking of a FRC slab in bending cannot be determined as a simple function of the MOR alone. Experimental results have indicated that plasticity in the immediate post-crack range can influence the peak in load resistance associated with cracking, even if the MOR is unchanged.

For lightly reinforced concrete, the load at first crack approaches approximately 0.7 of the load capacity predicted by the yield line theory $P_{YLT}$. The results indicate that the cracking load of a C-1550 panel can be predicted quite accurately using yield line theory provided the material comprising the panel exhibits high residual load capacity immediately after cracking compared to the load resistance at first crack. The strength of the relation between toughness and the accuracy of yield line predictions deteriorates when toughness is assessed at higher levels of deformation.

The results revealed through this investigation suggest that the method of structural analysis used to design conventionally reinforced and fiber-reinforced slabs should be governed by the degree of strain softening or plasticity exhibited by the material in the immediate post-crack range. The heavily reinforced concrete slabs with high steel fiber dosage rate in the tensile zone of the slab can be designed using plastic theory. Lightly reinforced slabs or plain concrete slabs should be designed using elastic theory or have a significant capacity reduction factor applied to capacities developed using plastic theory. Yield line theory can be applied effectively to the design of all
concrete slabs-on-grade if a variable capacity reduction factor incorporating some measure of toughness in the immediate post-crack range is applied to predicted load capacities. The reduction factor proposed as a linear function varies from 1.5 to 1.2.

2.2.5 Chao, Cho, Karki, and Yazdani, "FRC Performance Comparison: Direct Tensile Test, Beam-Type Bending Test, and Round Panel Test," November 2008.

The FRC performance comparisons are done by doing direct tensile test, beam-type bending test and round panel test. High variation in the post crack behavior is observed for uni-axial direct tensile test and third-point loading test (ASTM C1609). This is due to the lack of control over the location of cracks, therefore, a need to test more number of specimens for reliability. Degraded Strengths after cracking were 15%, 45%, and 75% for direct-tensile test, third-point load bending test and round panel test respectively. Both the third point load test and round panel test showed deflection softening response. The three major types of material test methods used for evaluating the FRC performance resulted in significant different performance results. The author concluded that the round panel test provided higher level of reliability in post-cracking performance.

2.3 Literature on Polypropylene Fiber-Reinforced Concrete

This section includes literature on various research papers that include tests and conclusions on various properties of polypropylene fiber-reinforced concrete structural elements.

A set of 30 concrete beams were cast with carbon FRP reinforcement with included variations in overwrap configuration as shown in Fig. 3, addition of steel stirrups, addition of polypropylene fibers. An overwrap configuration is defined as a single tow of carbon fiber wrapped around the reinforcing rods with different configurations, using a room temperature cure epoxy. These beams are compared with four other beams reinforced with equivalent steel reinforcement. Due to the use of carbon fiber overwrap on the smooth pultruded FRP rods, adequate bond is obtained which in turn helped in the increase of crushing strain of fiber reinforced concrete beams indicating ductility behavior. The fracture energy estimated analytically in this research showed that there is ductility due to the large fraction of strain energy that is absorbed in the concrete.

Fig. 3: Overwrap Configurations (Orozco and Maji, 2004).
The experimental program consists of testing of thirty concrete beams and the corresponding cylinders. The compressive strength of concrete is determined from the cylinder tests performed at 11 days after casting. The beams were tested to study the influence of the reinforcement ratio, overwrap configurations, addition of steel stirrups, and the addition of 1% by volume of polypropylene fibers to the concrete mix, and were also compared with steel reinforced beams.

The main purpose of adding the polypropylene fibers was to check the increase in ductility of the matrix. The overwraps which were placed periodically were to check the effect of bond-slippage on overall ductility. There were two types of overwraps used in this study, one of them had alternating 101.6 mm at 12.7 mm pitch and 101.6 mm with no wrapping as shown in Fig. 3(b) and the other one had alternating 152.4 mm at 12.7 mm pitch and 76.2 mm with no wrapping as shown in Fig. 3(c). The beams were cast in wooden forms to produce dimensions of 106.7 X 10.2 cm. The mix proportions of the concrete used were in the ratio of 2:1.8:1:0.40 corresponding to coarse, fine aggregate, cement, and water respectively along with fly ash and super plasticizer. The reinforcement ratios used were 0.0035, 0.007, 0.0105, and 0.021. The yield strength of the reinforcing bars was 448 MPa and was placed at 3.8 cm (1.5 in) spacing along the length of the beam. The diameter of the reinforcing bars was 3.18 mm (1/8 in). The cylinder strength at 11 days after casting was found to be 46.54 MPa and the 28 day strength was 55.16 MPa.

The beams were supported on a simply supported span of 101.6 cm. The beams are tested under three-point bending condition conducted on a Baldwin universal testing machine. The load cell applied was of 10 kips and the beam was supported on a 182.9 cm
(6 ft) long W8X8 steel beam so that the loading frame can be extended. Displacements were measured with a 0.025 mm (0.001 in) sensitivity dial gauge. In addition to this test, in order to test the effect of loading condition, two beams with two FRP reinforcing bars were tested under four-point bending. The strain gauges used were CEA-06-250UW-350 Option P2, from Measurements Group, Inc. Strain gauges were installed on the surface of the beam at ½ and ¼ of the span.

For beams reinforced with uniform overwrap, cracking pattern was observed to be distributed with crack spacing around 76.2 and 114.3 mm and for beams with alternating overwraps, the failure was caused by a single crack located at mid span. This was due to the inadequate stress transfer to the FRP. For the beams reinforced with six FRP bars had lower deflections than the beams reinforced with one FRP bar signifying that as the reinforcement ratio increased, the deflection capability decreased.

There were two types of overwrap configurations used in the tests to check the effect of overwrap modifications on the capability of the rebars to develop adequate bonding to the concrete. In case of the two types of non-uniform overwrap configurations, both of them had shown similar strengths and had failed much before than the beam having uniform overwrap configuration. This was due to the disbanding of overwraps from the rebar.

By adding the stirrups, the strength of the beams was found to decrease but the ductility or the deforming capability had increased by 23%. 1% by volume of polypropylene fibers was added to the concrete mix to see the effect of ductility behavior of the beams. The tests revealed a decrease in strength of 23% and an increase in the maximum deflection by 40%. The beams having FRP bars had shown higher strength
than the beams with the same number of steel reinforcement. The steel reinforced beams had shown more ductility than FRP reinforced beams.

The theoretical curve was obtained using the Hognestad curve (Hognestad et al. 1955). This analytical procedure includes the use of non-linear stress-strain behavior of concrete and linear behavior of the FRP. The theoretical strains were found using the strain compatibility procedure satisfying the equilibrium condition. These strains were compared with the strain data obtained from the experiments for validation of their proposed model. The predicted strains were in good agreement with the experimental strains up to strain of about 0.002. Later, the predicted strains were found slightly higher than the experimental strains by about 10%. The total strain energy stored in the beam is calculated as the area under the load-deflection curve. The energy stored in the FRP rebars is calculated as the area under load-deflection curve up to the load corresponding to a strain of 0.002. The remaining energy is calculated as the energy stored in the concrete. The evaluations showed that the fraction of energy going to the concrete is decreased with increase in reinforcement ratio. The energy fraction in concrete increased with the addition of stirrups and with the addition of polypropylene fibers. The fraction of energy in concrete for the FRP reinforced beams varies from 50.3 to 75.3%. Whereas for the steel beams, it varies from 4.2 to 21.8% only, since the yielding of steel is responsible for most of the energy dissipation.

The elastic energy is calculated by the method proposed by Naaman and Jeong (1995) as shown in Fig. 4. It is calculated as the area of the triangle formed at the failure load by the line having the weighted average slope of the two initial straight lines of the
curve. The analysis revealed that the elastic energy calculated has been over estimated by 20% compared to the tests.

![Image of energy calculations](image)

**Fig. 4**: Elastic and Inelastic energy from Orozco and Maji (2004).

The analytical model which was used to find the fraction of energy stored and dissipated in concrete and FRP is checked by comparing the measured strains from the test with those predicted using strain compatibility method. From the tests carried out on the 30 FRP reinforced beams with various configurations, the results showed that their ductility is less than that of steel reinforced beams but the energy dissipation from concrete cracking shows that there is considerable ductility in FRP reinforced beams.

2.3.2 Fanella and Naaman, “*Stress-Strain Properties of Fiber Reinforced Mortar in Compression*”, 1985.

The main objective of this paper was to find the stress-strain behavior of fiber reinforced concrete in compression. Experiments were conducted with different types of fibers i.e., steel, glass, and polypropylene with various fiber contents to study the effects
of these parameters on the peak stress and strain, and shape of the stress-strain curve after
the peak point. The descending part of the curve is used to measure the toughness
resistance of the material. In case of structures that have to be analyzed for impact,
earthquakes or fatigue loading, it is essential to know the toughness or the energy
absorption capacity of the material used in the structure. Thus, in this paper an analytical
model is developed to predict the complete stress-strain curve of steel fiber reinforced
mortar for the better prediction of behavior of structures using fiber reinforced mortar.

Three mixes were prepared with mix proportions of 1:3:0.5 (cement: sand: water),
1:2:0.5 and 1:1:0.35 having three different volume fractions 1, 2, 3 % of fiber in each of
the mix. For steel fibers, three different aspect ratios 47, 83 and 100 were used. The
cement used in the mixes was Type III Portland cement satisfying ASTM C150
requirements. The sand which passed through the No.4 ASTM Standard sieve was used
in the mixes. All the three mixes contained super plasticizer. Three different kinds of
fibers i.e., steel, glass and polypropylene were used. The steel fibers were smooth and
brass coated with a specific gravity of 7.8. The glass and the polypropylene fibers used
were of 1 in (25mm) length with specific gravities 2.6 and 7.8 respectively.

Cylinders were cast to obtain a size of 3 x 6 in and cured for 7 days. The cylinders
were capped with sulfur at one end and hydrostone at the other to obtain a uniform length
of 6.25 in for all the cylinders. The cylinders are tested in a closed-loop, servocontrolled
hydraulic testing machine for compression tests. The load capacity of the testing machine
was 200 kips and is applied at a rate of 0.0661 in/min. The specimen deformations were
recorded with the help of two LVDTs provided.
Similar to what other researchers found, it was found in this study that the compressive strength of the concrete did not increase with the addition of fibers. The addition of steel fibers showed little increase about 0-15% in the compressive strength of concrete. The toughness of the concrete was found to increase with the addition of fibers up to 3 percent. Above this, due to high fiber content, there was difficulty in mixing the concrete leading to a harsh concrete mix.

As the volume fraction of fibers was increased, the slope of the descending curve was found to increase. For steel fibers, as the aspect ratio is increased with constant volume fraction, the slope of the descending curve is increased improving the toughness of the composite. Keeping all other factors constant, improvements due to addition of fiber was comparatively more significant at low compressive strengths of matrix.

$$RI = \frac{V_f l}{\phi}$$

Where, $V_f$ – Volume fraction of fibers;

$l$ – Length of the fiber;

$\phi$ – Diameter of the fiber;

Reinforcing index influences the slope of descending branch in such a way that as reinforcing index is increased, the slope of the descending curve is decreased there by increasing the area under the curve. This trend is seen only up to a reinforcing index of approximately 2, above which the properties start to deteriorate which indicates that an optimum has been reached.

An attempt was made to derive the analytical expression for the stress-strain curve of fiber reinforced concrete. The expression developed by Wang (1978) and Ahmad (1981) was used in this study. The expression consists of normalized stress, strain and
four constants A, B, C, and D that are derived for both the ascending branch and descending branch of the stress-strain curve with the respective boundary conditions developed. The expression is represented as

\[ Y = \frac{(AX + BX^2)}{(1 + CX + DX^2)} \]

Where \( X \) = normalized strain \((\epsilon/\epsilon_p)\)

\[ Y = \text{normalized stress} (\sigma/f'_{cf}) \]

\( \epsilon \) = strain in general

\( \epsilon_p \) = strain at peak stress

\( \sigma \) = stress in general

\( f'_{cf} \) = peak stress of fiber reinforced matrix

Four points are defined on the stress strain curve which can be seen in the graph in Fig. 5. They are namely \((x_1, y_1)\), \((x_i, y_i)\), and \((x_f, y_f)\). The first point \((x_1, y_1)\) refers to the point on the curve at 45% of the peak stress and \((x_i, y_i)\) refers to the point on the curve where curve is zero and \((x_f, y_f)\) represents the far point on the stress – strain curve. With the help of defined points, the constants are derived using the following boundary conditions for ascending curve and

- Slope is equal to initial modulus at origin i.e., \( dY/dX = A = E_c/E_o \) at \( X = 0 \), \( Y = 0 \).
- Curve passes through 45% of the peak stress \( f'_{cf} \) at \( X = 0.45 \), \( Y = 0.45 \).
- Curve passes through the peak at \( X = 1 \), \( Y = 1 \), and
- Slope \( dY/dX = 0 \) at peak point \( X = 1 \), \( Y = 1 \).

following boundary conditions for descending part of the curve.

- Slope is equal to zero at the peak and passes through the peak point.
- Curve passes through the inflection point \((x_i, y_i)\)
- Curve is zero i.e., \(d^2Y/dX^2 = 0\) at inflection point and finally,
- Curve passes through the far point \((x_f, y_f)\).

![Analytical Model of Naaman and Fanella (1985)](image)

**Fig. 5:** Analytical Model of Naaman and Fanella (1985)

With the first four boundary conditions, the constants \(A_1, B_1, C_1, D_1\) are determined for the ascending branch and with the second four boundary conditions the constants \(A_2, B_2, C_2, D_2\) are evaluated for descending branch of the stress-strain curve. Prediction equations were further developed assuming square fitting lines for the key parameters of the compressive stress-strain curves of fiber reinforced mortar. Thus linear equations in the form \(y = ax + b\) were developed. For example, the prediction equations developed for the twisted polypropylene are as mentioned below:

\[
f_{cf}' = f_c' \\
\epsilon_p = 0.0462V_f + 0.0033
\]
\[ f_i = 0.58f_{cf} + 75000V_f \]
\[ \epsilon_i = 0.0318V_f + 0.0043 \]
\[ f_f = 84000V_f + 0.1f_{cf} \]
\[ E_{cf} = 162f_{cf}' + 1.2 \times 10^6 \]
\[ \epsilon_f = 0.0154 \]

The Young’s modulus found using the above equation given for \( E_{cf} \) is almost half of the young’s modulus of concrete i.e., \( 57000\sqrt{f_{cf}'} \). In literature, it is found that the young’s modulus of concrete doesn’t change much with the addition of polypropylene fiber (Balaguru, 1992).

The addition of fibers to the concrete matrix had changed the basic characteristics of stress-strain curve. The descending portion of the curve had major change compared to the ascending branch of the curve. The authors used two set of constants one for the ascending branch and the other for the descending branch to determine the effects of fibers in concrete. As the fiber content was increased, it was observed that the slope of descending portion of the curve is decreased thus increasing the ductility and toughness of the material. The fiber content was found to vary linearly with the strain at the peak stress of fiber reinforced concrete.

2.3.3 Ezeldin, Balaguru, “Normal- and High Strength Fiber-Reinforced Concrete under Compression”, 1992.

This paper presents an analytical approach to predict the complete stress-strain curve of steel fiber reinforced concrete under compression. The predicted stress-strain
curves are compared with the experimental stress-strain curve. The experimental tests consisted of a total of 18 concrete mixes. The compressive strength of fiber reinforced concrete was in the range of 5 ksi to 12 ksi. The parameters include three fiber volumes and three aspect ratios. Two 4x8 in cylinders were tested under compression according to ASTM C-39 standards after 28 days. The cylinders were tested in a universal testing machine which has 1000 kip load capacity. The LVDT’s were used to measure the deformation of the middle half of the cylinder. The strains and corresponding loads were recorded by an electronic data acquisition system. The effect of the addition of steel fibers to concrete with and without silica fume was studied.

The experimental results indicate that the addition of steel fibers to the concrete mix increase the secant modulus of elasticity and strain capacity in the prefailure zone. Also, the strain capacity was found to increase with the increase in either fiber content or length of the fiber or both. Thus toughness was more for the concrete mix with higher fiber volumes. But however, in cases where fiber volume is very high i.e., 100 lb/cu yd and the aspect ratio is 100, the toughness of the concrete could decrease due to difficulty in mixing which causes non-uniform distribution of fibers. Naaman and Fanella (1985) had used a strain limit of 0.015 which is five times the ultimate concrete strain 0.003 (ACI Code). By using such adequate stress limit, they were able to show the effect of fibers on the post crack behavior of concrete. Similar to them, this paper presents a combined effect of both the weight fraction \( W_f \) and aspect ratio \( l/\phi \) in the form of a fiber reinforcing parameter, reinforcing index \( RI = W_f \frac{l}{\phi} \). Where \( W_f \) = weight fraction; \( l = \) length of fiber; \( \phi = \) diameter of fiber; The toughness measured as the area under the curve was found to increase with the increase in reinforcing index.
Toughness Ratio = \frac{\text{Area under the curve}}{f'_{cf} \times 0.015}

Where, \( f'_{cf} \) = peak stress corresponding to \( \varepsilon_{of} \)

Square fitting line analysis was performed to determine the possible relationship between steel fiber reinforcing index and the major parameters of stress strain curve. Equations were developed to determine these parameters namely \( f'_{cf}, \varepsilon_{of}, E_{cf}, TR_f \) by regression analysis performed using all data points where

\( f'_{cf} \) = Compressive Strength of fiber reinforced concrete

\( \varepsilon_{of} \) = Strain corresponding to maximum stress \( f'_{cf} \)

\( E_{cf} \) = Secant modulus of elasticity of fiber reinforced concrete

\( TR_f \) = Toughness ratio for fiber reinforced concrete.

These equations were determined as a first approximation in design. The correlation coefficients were in the range of 0.6-0.8. The higher order regression analysis did not yield better correlation coefficients. The authors concluded that more data points were needed to obtain a better correlation.

For concrete with silica fume, the compressive strength is high. These high strength concretes were found to have higher strain at the peak compressive stress compared to lower strength concretes. But, the ultimate strain or the strain before failure is lower compared to lower strength concretes. Thus, addition of steel fibers to such concrete mixes would increase the strain capacity of the concrete mix in the post-crack region improving the ductility of the material. As the percentage of silica fume is increased, the slope of descending branch of the stress-strain curve was found to increase.
indicating a decrease in toughness of the concrete. When steel fiber reinforcing index was increased to 1.88 or higher, the negative effect of silica fume on the brittleness of concrete may be avoided.

Analytical equations for fiber reinforced concrete were developed based on the earlier empirical equation given by Carreira and Chu (1985) for plain concrete. The equation is shown below consists of the major parameters i.e., \( f'_{cf}, \epsilon_{of}, E_{cf}, \text{ and } RI \):

\[
\frac{f_c}{f'_{cf}} = \frac{\beta \epsilon_c}{\epsilon_{of}} \frac{1}{\beta - 1 + \left(\frac{\epsilon_c}{\epsilon_{of}}\right)^\beta}
\]

Where, \( f_c, \epsilon_c = \) stress and strain at a given point on the stress-strain curve;

\( f'_{cf} = \) compressive strength of fiber concrete;

\( \epsilon_{of} = \) strain corresponding to peak compressive stress, \( f'_{cf} \);

\( \beta = \) material parameter.

The relationship between material parameter \( \beta \) and RI was developed performing a best fitting statistical analysis on the experimental data. This relation is obtained for hooked end fibers with RI value ranging from 0.75 to 2.5. Similarly based on the experimental data obtained from Fanella and Naaman (1985), the relation is obtained for straight end fibers. The value of \( \beta \) given by Carreira and Chu (1985) was:

\[
\beta = \left(\frac{f'_{cf}}{4.7}\right)^3 + 1.55
\]

The strain corresponding to peak stress is taken as 0.002 for plain concrete (International Recommendations 1970). Further, the author used 0.002 as the strain corresponding to the peak stress for fiber reinforced concrete also as there was no reliable
experimental data. The proposed analytical expression provided good correlation with the experimental data.

2.3.4 Hughes and Fattuhi, “Stress-Strain Curves for Fiber Reinforced Concrete in Compression”, 1977.

This paper includes experimental tests for ultimate compressive strength of cubes, equivalent cubes and prism specimens. The fracture toughness of fiber reinforced specimens is calculated. The specimens were cast with different types of fibers including fibrillated polypropylene, round straight, duoform, crimped and hooked steel fibers. The effect of fibers in concrete on the compressive stress-strain behavior and toughness are studied.

The authors used 1.5 % fiber volume for all types of fibers when added to concrete. A decrease in compressive strength of concrete, density and dynamic modulus of elasticity was observed for the specimens reinforced with polypropylene fibers while there was no much change noted for specimens reinforced with steel fibers. A decrease in compressive strength was observed for specimens reinforced with crimped steel fibers and maximum increase in compressive strength was noted for the duoform steel fiber reinforced specimens. The results indicate that the fracture toughness was considerably increased for all fiber mixes. The cube tests showed much gradual and ductile type of failure compared to prism tests. The strength and the initial slope of the stress strain curves for all fiber mixes was found to slightly increase with age.
2.3.5 Ramakrishnan, “Materials and Properties of Fiber Reinforced Concrete”, 1987

Ramakrishnan (1987) had studied the toughness properties of fiber reinforced concrete in his research and estimated that the toughness is established on the total energy absorbed before complete failure. This is computed as the area under the load-deflection curve up to complete failure. Properties that effect the maximum load capacity and toughness of fiber reinforced concrete include the type and shape of fiber, percentage volume, aspect ratio, and orientation of fibers in the matrix.


This paper provides a critical review on the ASTM C1399-04 test method and discusses its validity with respect to broader implementation. Recommendations were made from the thorough observations of test results, variability, precision and bias. Relating to the number of specimens, the review states that the test method requires the preparation of 5 test specimens from each sample of fresh or hardened concrete. The authors state that a clear comment or recommendation should be made to increase the number of test specimens from 3 to 5. In this test method, the initial loading is terminated when the beam deflects to a value of 0.02 inch with the stainless steel plate under the beam. This arrangement had resulted in a smooth curve with no sudden fall in the strength after the peak stress. This means that the test beam had allowed additional cracking and therefore a lower average residual strength is obtained compared to the beam which has its initial load stopped at the first crack. For the beams which have been
tested with its initial load stopped at first crack, the peak load in the reloading curve is found to be higher than the other beam tested according to ASTM C1399-04.

ASTM C1399-04 is based on deflection measurements at the mid span of test beam. This indicates that the deflection is related to crack opening at the tension face. When the crack occurs under the point loads for the test beam, depending on the geometry of the beam deflection shows that the crack opening has to be 33.3% greater than the amount of crack opening that forms at mid span in order to record identical deflections for the two crack locations. In the determination of ARS, the loads corresponding to the four standard deflections will usually be smaller than the loads if crack formed at the mid span. This paper also provides a discussion of the importance of crack location on the ARS values.

2.4 Literature on Various Test Methods Available for Measuring the Post-Crack Performance of FRC Members

Fiber reinforced concrete has become one of the major developments in construction industry in the recent years. The key reason behind this major development is the withstanding capacity of the fiber reinforced concrete structure after cracking of concrete. Many researchers had done tests on various types of fiber reinforced concrete mixtures to predict its survivability. Greater survivability provides a safeguard for rescue workers and survivors of the initial cause of building damage.

To provide safer structures, many code officials are considering the call for additional provisions that require manufacturers and producers of construction materials to provide a measure of toughness. For years, the engineering community has primarily
judged in-place concrete based on its compressive strength. But as building code research focuses once again on seismic design and residual strength, the engineering community is recognizing dire need for quantitative measurement of additional concrete properties, including toughness. As engineers consider design improvements to meet these proposed provisions, they will rely on approved testing methods to measure and quantify such concepts.

American Society of Testing Materials, one of the major societies to publish specifications and provisions, has test methods namely ASTM C 1399, ASTM C 1609, ASTM C 1550 for the evaluation of Fiber Reinforced Concrete. These test methods are discussed below along with the Japanese Test Method JSCE-SF4 used in evaluating toughness properties of fiber reinforced concrete.

2.4.1 ASTM C 1399-07 Standard Test Method for Obtaining Average Residual Strength of Fiber-Reinforced Concrete:

The average residual strength is computed using specified beam deflections that are obtained from a beam that has cracked in a standard manner. This test method provides a measure of post-cracking strength, useful in the evaluation of the performance of fiber-reinforced concrete. This test method can be used for comparing beams with different fiber types, dimensions and different fiber contents, and evaluates the better proportions of fiber-reinforced concrete mixtures for the provided construction specifications. It is also used to evaluate fiber-reinforced concrete which has been in service. Banthia and Dubey [10] compared results using this test method with residual strengths at the same net deflections using a test protocol that is similar to that described
in Test Method C 1609 on 45 beams with a single fiber configuration at proportions of 0.1, 0.3, and 0.5% by volume. The results by this test method were on average 6.4% lower than by the procedure of Test Method C 1609.

2.4.2 ASTM C 1609 Standard Test Method for Flexural Performance of Fiber-Reinforced Concrete (Using Beam with Third-Point Loading)

This test method evaluates the flexural performance of fiber-reinforced concrete using parameters derived from the load-deflection curve obtained by testing a simply supported beam under third-point loading. The first-peak strength, peak strength, and residual strengths determined by this test method reflect the behavior of fiber-reinforced concrete under static flexural loading. The energy absorption capacity is the measure of the area under the load-deflection curve which is of little direct relevance to the performance of fiber-reinforced concrete structures since the absolute values of energy absorption depend directly on the size and shape of the specimen. The results of this test method may be used for comparing the performance of various fiber-reinforced concrete mixtures.

2.4.3 ASTM C 1550 Standard Test Method for Flexural Toughness of Fiber Reinforced Concrete (Using Centrally Loaded Round Panel)

This test method evaluates the flexural toughness of fiber-reinforced concrete expressed as energy absorption calculated as the area under the load-deflection curve up to a specified central deflection. This represents the ability of the fiber-reinforced concrete to redistribute stresses following cracking. The round panels subjected to central
point load that is simply supported on three symmetrically arranged pivots experiences bi-axial bending and exhibits a mode of failure similar to the in-situ behavior of structures such as shotcrete tunnel linings, concrete slabs-on-grade, and thus it will represent the post-crack behavior of such plate-like, fiber-reinforced concrete structural members.

The consistency of the failure mode arises through the use of three symmetrically arranged support pivots compared to the other boundary conditions resulting in low within-batch variability (Bernard, April 2002).

2.4.4 JSCE- SF4 Method of Tests for Flexural Strength and Flexural Toughness of Steel Fiber Reinforced Concrete

This test is used for measuring flexural strength and flexural toughness of steel fiber reinforced concrete by third-point loading. The flexural test apparatus is used for applying the third point load, and the deflection measurement apparatus consisting of electrical linear variable differential transformers is used for measuring the load-deflection curve of the specimen. The examples of the both the apparatus is shown in the following figures.
The deflection is measured at the middle of the span. The flexural toughness factor is calculated by the equation shown below and is determined to three significant digits.

\[
\sigma_b = \frac{T_b}{\delta_{tb}} \cdot \frac{l}{bh^2}
\]
Where, $\sigma_b = \text{flexural toughness factor} \left( \frac{\text{Kgf}}{\text{cm}^2} \right)$

$T_b = \text{flexural toughness (Kgf. cm)}$

$\delta_{tb} = \text{deflection of } \frac{1}{150} \text{ of span (l) (cm)}$

Fig. 8: Flexural Toughness

Flexural toughness $T_b$ as shown in Fig. 8 is calculated from the area below the load-deflection curve until measured deflection becomes 1/150 of the span.
CHAPTER III
MATERIALS

3.1 Fibers

Fibers were supplied by Forta® Corp. The fiber type was chopped Ferro polypropylene. The length of fiber used in the slab tests was 2.25 in (54 mm) and its specific gravity is 0.91. The dosage levels were 7.5, 15 Pounds per cubic yard. Forta Ferro is a high density high modulus polymeric fiber which can be used to reinforce plain concrete mixes. It is used to improve the resistance to spalling and edge damage. The addition of Forta Ferro has shown the ability to improve tensile strength, residual strength, fire resistance, plastic shrinkage crack reduction, non-corrosive nature and also used to reduce the thickness of the section. Forta Ferro meets the requirements of ASTM C-1116 Standard Specification for Fiber Reinforced Concrete and Shotcrete.

Physical Properties

Materials: Virgin Copolymer/Polypropylene

Color: Grey

Form: Monofilament/ Fibrillated Fiber System

Acid/Alkali Resistance: Excellent

Specific Gravity: 0.91
Absorption: Nil

Tensile Strength: 90-110 ksi (620-758 MPa)

Compliance: A.S.T.M. C-1116

Length: 2.25” (54mm)

3.2 Basalt Fiber Reinforced Polymer Bars

The Basalt FRP bars used in this research were provided by ReforceTech, AS (Norway). The bars were of net fiber diameters 5 mm and 7 mm. These bars are embedded in the concrete slabs at 0.75 in clear cover from bottom of the slab. Basalt fiber is similar to carbon fiber and fiberglass, but basalt has better mechanical properties than fiberglass and is cheaper in cost than carbon fiber. It is used as a fireproof textile in the aerospace and automotive industries and can also be used as a composite to produce a wide range of products. Some of the main features of Basalt fiber are it is high strength and high modulus fiber, ease of handling and processing, environmentally friendly, higher temperature resistance, low cost alternative and better chemical resistance than e-glass.

3.3 Cement

Type I/II Normal Portland Cement, which is commonly available in the world market satisfying ASTM C150 requirements are used.
3.4 Coarse Aggregates

The course aggregates used for making concrete for cylinders and slabs was crushed lime stone aggregates with maximum size of 0.75 in (19 mm). For concrete slabs without fiber, in addition to the 0.75 in coarse aggregate, small coarse aggregates i.e. P-Gravel was also added to the concrete mix. The coarse aggregate was angular and free from clay and other impurities.

3.5 Fine Aggregate

The fine aggregate used in the making of concrete for cylinders and slabs was purchased from a local supplier. The sand was free from clay and other impurities.

3.6 Water

The water used for making concrete mix was the tap water in the University of Akron supplied by the city of Akron.
CHAPTER IV
EXPERIMENTAL PROGRAM

In this chapter, the experimental program carried out in the present research project is discussed. The slab tests that were done to study the effect of concrete compressive strain, tensile BFRP bar strain on maximum load capacity of slabs with the addition of polypropylene fiber to the concrete are described. This includes the discussion of preparation of the test specimens for Series1 and Series2 slabs, which differ by the amount of fiber added to the concrete. The amount of fiber used in Series1 slabs is 15 pound per cubic yard (1% by volume) and Series2 slabs is 7.5 pound per cubic yard (0.5% by volume). The cylinder compression test results, slab test results and the comparison of test results for both the series slabs are included. The beam and round panel tests that were done conforming to the ASTM C1399 and ASTM C1550 standards respectively are discussed. The results obtained from these tests are also included in this program.

4.1 Slab Tests

This section includes the mix design, preparation of test specimens and test procedure for the slab test.
4.1.1 Preparation of Test Specimens

The Table 1 and Table 2 provide the information on the mix proportions used in the slab tests. Table 1 includes the details of the mix proportions used for making the concrete mix for polypropylene fiber reinforced slabs of Series 1 and Series 2. Table 2 includes the details of the mix proportions used for making concrete mix for slabs without fiber.

Table 1: Mix Proportions for Polypropylene Fiber Reinforced Concrete Slabs

<table>
<thead>
<tr>
<th>Material</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portland Cement</td>
<td>675 lb/yd³</td>
</tr>
<tr>
<td>Water</td>
<td>351 lb/yd³</td>
</tr>
<tr>
<td>0.75” Coarse Aggregate</td>
<td>1424 lb/yd³</td>
</tr>
<tr>
<td>Fiber</td>
<td>15 lb/yd³</td>
</tr>
<tr>
<td>Fiber</td>
<td>7.5 lb/yd³</td>
</tr>
<tr>
<td>Sand</td>
<td>1424 lb/yd³</td>
</tr>
<tr>
<td>High Range Water Reducer</td>
<td>As required</td>
</tr>
<tr>
<td>Batch Size</td>
<td>4 cubic ft</td>
</tr>
</tbody>
</table>

Table 2: Mix Proportions for Plain Slabs

<table>
<thead>
<tr>
<th>Material</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portland Cement</td>
<td>675 lb/yd³</td>
</tr>
<tr>
<td>Water</td>
<td>351 lb/yd³</td>
</tr>
<tr>
<td>0.75” Coarse Aggregate</td>
<td>1068 lb/yd³</td>
</tr>
<tr>
<td>P-Gravel Course Aggregate</td>
<td>356 lb/yd³</td>
</tr>
<tr>
<td>Fiber</td>
<td>0 lb/yd³</td>
</tr>
<tr>
<td>Sand</td>
<td>1424 lb/yd³</td>
</tr>
<tr>
<td>High Range Water Reducer</td>
<td>As required</td>
</tr>
<tr>
<td>Batch Size</td>
<td>2.1 cubic ft</td>
</tr>
</tbody>
</table>
Eight slabs were prepared in order to address the primary objective of this research. The first four slabs correspond to a dosage of 15 lb/yd$^3$, named as Series 1 slabs, and the other four slabs correspond to 7.5 lb/yd$^3$ dosage of fiber, named as Series 2 slabs. The experimental program for the Series 1-four slabs which is grouped into two sets is discussed. The same procedure is followed for the Series 2 slabs except with a change in the dosage of fiber. The first four slabs were grouped into sets of two by the diameter of the FRP reinforcing bars used. Two slabs contained 7 mm diameter basalt FRP bars, while the other two contained 5 mm diameter basalt FRP bars. Each set of slabs had a control slab, which was a slab made from concrete without polypropylene fiber.

Firstly, the molds were prepared with plywood to make an 8” wide x 4” tall x 52” long mold to hold the wet concrete in this desired shape until the concrete had cured to a desired hardness so that the mold may be removed.

Secondly, basalt FRP bars were cut to 50” lengths and filed and sanded at the mid-point of the bar to prepare the surface for application of the strain gauges. The basalt reinforcing bars, once filed, and sanded with 220 grit, 320 grit, and 400 grit sandpaper to achieve an extremely smooth surface were conditioned with a water-based acidic surface cleaner to remove any loose material that may prevent adequate bonding of the strain gauges to the reinforcing bars. The surface was then neutralized using a water-based alkaline surface cleaner to remove any further unwanted loose material that may prevent adequate adhesion of the strain gauge and to neutralize the acidic surface caused by the conditioner. After the surface was clean, the strain gauges were applied. Catalyst was spread over the prepared surface of the basalt rebar and the strain gauge was pressed and held against the catalyst for one minute. Adhesive was applied to the catalyst, and the
strain gauges were again pressed and held against the surface for 1 minute to allow for the hardening reaction between the catalyst and adhesive.

After the strain gauges were sufficiently bonded to the rebar, wire leads were soldered to the terminals of the gauge for connection to the acquisition machine. The strain gauges, wire leads, and terminals that were applied to the basalt fiber reinforcing bars were then coated in wax to protect them from the water in concrete during the pour and casting of the slabs.

Next, the reinforcement cages were assembled as shown in Fig. 9.

![Fig. 9: Plan View of the Reinforcement Detail.](image)

The bars were tied together with rebar wire to hold them in place. Two cages were put together using 7 mm diameter bars as the reinforcement, one for a control slab and one for FRC slab. Two cages were also put together using 5 mm diameter reinforcement, again one for a control slab and one for the FRC slab. All cages were constructed using four cross member ties and all ties were 5 mm diameter basalt fiber reinforcing bars.
After the reinforcement cages with strain gauges were assembled, the cages were lowered into the molds. Form oil was sprayed on the inside of all of the forms and cylinders to enable an easy release of the slabs and cylinders once the concrete has cured for 24 hours. The cages were placed in the plywood molds and set on top of ¾” chairs placed on the bottom of the mold to insure a constant clear distance from the bottom of the slab.

Two separate batches of concrete using the same mix design for each batch were prepared. The first batch was used for the slabs containing the 7 mm diameter reinforcement, and the second batch for the 5 mm diameter reinforcement. To insure there would not be a loss of cement or moisture in the bottom of the dry mixer, a butter mix was prepared. A 10% of the total mix was measured separate from the batch for
butter mix. The butter mix was mixed, and the mix was removed from the mixer with the some residue as the batch mix. The sand, coarse aggregate and pea gravel were all added to the mixer along with cement and mixed suitably to provide a well graded aggregate with minimal voids, similar to what is shown in Fig. 11.

Fig. 11: Concrete Mix

Approximately two thirds of the water was first mixed with the aggregate (fine and coarse) to obtain an even distribution and mixed for two minutes in the mixing machine. The Portland cement and the remaining water was then added to the batch and mixed for three minutes. The mix was then allowed to sit for two minutes, followed by three additional minutes of mixing at the end of which the concrete was ready to pour.

The plywood mold containing one of the 7 mm cages was placed on top of a vibrating table and concrete was added. The vibrations from the table cause the concrete to consolidate in the mold. Three 8" tall x 4" diameter cylinders were also filled and vibrated to be used for compression tests later to determine the compressive strength of the concrete.
The required amount of polypropylene fiber was then added to the remaining concrete in the mixer and then mixed for an additional three minutes. The above steps were followed to pour the second set of specimens containing the second set of 7 mm cage and cylinders. After both 7 mm reinforced slabs were complete, the mixer was cleaned and steps began with the butter mix for the 5 mm reinforcement.

The concrete slabs were towed smooth, then covered with a plastic sheet to help prevent moisture loss, and allowed to cure for 24 hours. The following day, the concrete slabs and cylinders were taken out of their molds and placed in a moisture room that maintains 100% humidity in order to prevent moisture loss that would slow the reaction, for the remaining curing time.

The slabs and cylinders were removed from the moisture room after 21 days and placed in the laboratory at room temperature to dry prior to the application of strain gauges. The surface of the top face at the midpoint of the concrete slabs was ground smooth using a series of increasingly fine grinder pads, similar to the sanding of the rebar, in preparation for strain gauge attachment. A process similar to the one used for

Fig. 12: Cylinders for Compression test
attaching the strain gauges to the reinforcement was used to attach strain gauges to the concrete slabs. The surface was conditioned and neutralized, catalyst was mixed with resin to activate the adhesive, and the strain gauges were placed on the prepped areas and glued. Once the gauge was placed, it was covered with a rag, clamped, and left overnight to cure. The following day, the clamps and rags were removed, and wire leads were soldered to the strain gauge terminals.

Fig. 13: Strain Gages Mounted on the Surface of a Concrete Slab Specimen

Same procedure is followed for the second series of slabs, containing two 5 mm and two 7 mm basalt FRP reinforcing bars.

The concrete slabs were tested under a four point loading condition as shown in Fig. 14.
The beams were loaded to failure and beyond to possibly achieve the maximum concrete crushing strain which was recorded and retrieved from the acquisition machine. The cylinders were then tested to determine the compressive strength of the concrete. Three cylinders were tested for each set of slabs and the average compressive strength was calculated. The average compressive strengths for all the eight slabs in series 1 and 2 are given in Table 3.
4.2 Test Results

This section includes the test results from cylinder compression tests, series 1 slab tests and series 2 slab tests.

Table 3: Cylinder Compression Test Results

<table>
<thead>
<tr>
<th></th>
<th>Average Compressive Strength $f_c$ (psi) - Series 1</th>
<th>Average Compressive Strength $f_c$ (psi) - Series 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 mm Plain</td>
<td>3315</td>
<td>5009</td>
</tr>
<tr>
<td>5 mm PP</td>
<td>3580</td>
<td>4251</td>
</tr>
<tr>
<td>7 mm Plain</td>
<td>3058</td>
<td>4506</td>
</tr>
<tr>
<td>7 mm PP</td>
<td>2551</td>
<td>3950</td>
</tr>
</tbody>
</table>

4.2.1 Series 1 Slabs Test Results

Series 1 comprises of four slab specimens of which two contain polypropylene fiber with basalt FRP reinforcing bars of 5mm and 7mm diameter respectively. The fiber dosage used in these slabs was 15 pound per cubic yard. The other two slabs are reinforced with basalt FRP reinforcing bars without polypropylene fiber.
Fig. 16: Concrete Compressive Strain for Polypropylene Fiber-reinforced concrete Slab with 5 mm Basalt FRP bars of Series1

5 mm Reinforced Slab With Fiber:

For test beam with 5 mm FRP bars with fiber, the compressive strain of concrete not only reached what is widely understood to be the nominal crushing strain of concrete, but surpassed it. The beam was able to carry a maximum load of 4382 lbs at a strain of 0.004166 in/in. By adding polypropylene fiber to the mix, the compressive strain of the concrete increased by 39% compared to the theoretical crushing strain of concrete, 0.003 in/in.

Failure occurred by rupture of basalt fiber reinforcement because of the under-reinforced nature of the beam once the limit state of the crushing strain was reached. Fig. 17 shows a long horizontal crack down the right side of the concrete slab at the level of the reinforcement which indicates that the failure was through debonding of the reinforcement.
Fig 17: Failure mode of Polypropylene Fiber-reinforced concrete Slab with 5 mm Basalt reinforcing bars of Series1

Fig 18: Concrete Compressive Strain for Plain concrete slab with 5 mm Basalt FRP bars of Series1

5 mm Reinforced Slab Without Fiber:

For the test beam without fiber, the concrete compression strain never reached the point of concrete crushing. The member failed in shear before the strain could reach
0.003 in/in. Only one strain gauge recorded data due to strain gauge malfunction. The beam failed in shear at a failure load of 4,464 pounds which yielded a compressive strain of 0.0017 in/in.

Fig. 19: Failure mode of Plain Concrete Slab with 5 mm Basalt FRP bars of Series1.

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Strain (in/in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>1000</td>
<td>0.002</td>
</tr>
<tr>
<td>2000</td>
<td>0.004</td>
</tr>
<tr>
<td>3000</td>
<td>0.006</td>
</tr>
<tr>
<td>4000</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td></td>
</tr>
<tr>
<td>6000</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 20: Concrete Compressive Strain for Polypropylene Fiber-reinforced concrete Slab with 7 mm Basalt FRP bars of Series1
**7 mm Reinforced Slab With Fiber:**

The 7 mm FRP reinforced concrete slab with the addition of the polypropylene fiber was the more interesting of the two experiments as it produced a higher increase in crushing strain of concrete. The average of the two strains just prior to peak loading was 0.0047 in/in which is a 57% increase in the compressive strain carrying capacity of concrete without fiber i.e., 0.003. The maximum applied load was approximately 5,610 lbs with a shear failure initiating at approximately at 5,000 lbs. Fig. 21 shows the failure mode of the test specimen.

![Image of a reinforced concrete slab](image)

**Fig. 21:** Failure mode of Polypropylene Fiber-reinforced Concrete Slab with 7 mm Basalt FRP bars of Series1.
Fig. 22: Concrete Compressive Strain for Plain concrete Slab with 7 mm Basalt FRP bars of Series 1

7 mm Reinforced Slab Without Fiber:

Similar to the 5 mm reinforced slab without polypropylene fiber, the 7 mm reinforced slab without polypropylene fiber resulted in a shear failure mode before the compressive strain of the concrete reached the critical strain of 0.003 in/in. The maximum load as well as the failure load was 4,262 lbs with an average compressive strain of 0.0011 in/in indicated in the above graph depicting the test data (Fig. 22). This slab also had a similar mode of failure as the 5 mm reinforced slab without the polypropylene fiber since both of the slabs were not provided with shear reinforcement.
4.2.2 Comparisons of Test data for Series 1 slabs

The load-strain data obtained from the experiment for both the plain concrete slabs and the slabs with polypropylene fibers are plotted in Fig. 24 to Fig. 29.
Fig. 24: Average Concrete Compressive Strain for Plain and Polypropylene FRC slabs with 5 mm Basalt FRP reinforcing bars.

Fig. 25: Average Concrete Compressive Strain for Plain and Polypropylene FRC slabs with 7 mm Basalt FRP reinforcing bars.
Fig. 26: Average Tensile Bar Strain for Plain and Polypropylene FRC slabs with 5 mm Basalt FRP reinforcing bars.

Fig. 27: Average Tensile Bar Strain for Plain and Polypropylene FRC slabs with 7 mm Basalt FRP reinforcing bars.
From the above graphs, it is clear that the addition of polypropylene fiber has increased the crushing strain of concrete. The tensile strain in the BFRP bars seems to be nearly equal, as are
the deflections. Generally, there seems to be marginal improvement of the overall behavior of the concrete slabs. All the slabs with and without fiber failed in a shear failure mode because shear reinforcement was not provided. The 5 mm reinforced slab with fiber was the only one to have failed by the failure through reinforcement indicating that the failure was in under reinforced mode. The 7 mm reinforced slab with polypropylene fiber had a similar mode of failure as 7 mm without polypropylene fiber. There was 39% increase in the compressive strain capacity for 5 mm concrete slab when fiber is added to the mix and 57% increase for 7 mm concrete slab when polypropylene fiber is added to the concrete mix. Even though the compressive strength had decreased due to addition of fiber, there was a significant increase in the compressive strain capacity and the maximum load capacity of the slabs. And also, the deflection curves indicate that the deflections are either decreased or remained almost the same with the addition of fiber, thus improving the overall structural integrity of the slabs.

4.2.3 Series 2 Slabs Test Results:

Series 2 comprises four slabs of which two are reinforced with polypropylene fiber with a fiber dosage of 7.5 pound per cubic yard and basalt FRP bars of 5 mm and 7 mm diameter respectively. The other two slabs are reinforced with basalt FRP bars without polypropylene fiber.
5mm - PP2 - Concrete Compressive Strain

Fig. 30: Concrete Compressive Strain for Polypropylene Fiber-reinforced concrete Slab with 5 mm Basalt FRP bars of Series2

5 mm Reinforced Slab With Fiber:

The slab was able to carry a maximum load of 5747 lbs which we can see is greater than that obtained for the 5 mm PP – series 1 slab i.e., 4382 lbs. Unlike other polypropylene fiber reinforced concrete slabs, this slab had shown a very low compressive strain capacity. This may be due to malfunction of strain gages during the experiment. Strains were recorded only up to a concrete compressive strain of 0.002815 in/in and the strains after that were not captured due to debonding of the strain gage. The failure of the test slab was due to debonding of FRP bars within the specimen as seen in Fig. 31.
Fig. 31: Failure mode of Polypropylene Fiber-reinforced Concrete Slab with 5 mm Basalt FRP bars of Series2.

Fig. 32: Concrete Compressive Strain for Plain Concrete Slab With 5 mm Basalt FRP bars of Series2
5 mm Reinforced Slab Without Fiber:

During the test, the concrete compression strain never reached concrete crushing same as for the 5 mm plain slab in series1. The member failed in shear before the strain could reach 0.003 in/in. The slab was able to carry a maximum load of 4742 pounds with an average concrete compressive strain of 0.0013 in/in and was able to withstand until load ultimate concrete strain of 0.0017 in/in was reached. The maximum load carried by this slab i.e., 4742 pounds, is found to be greater than that obtained for the 5 mm plain slab with basalt rebar in series 1 i.e., 4464 lbs.

Fig. 33: Failure mode of Plain Concrete Slab with 5 mm Basalt FRP bars of Series2.
7 mm Reinforced Slab With Fiber:

Similar to the specimen in series 1, the 7 mm reinforced concrete slab with the addition of the chopped fiber was the most interesting of all the experiments. The maximum applied load was 6,670 lbs. The slab failed with transverse cracks on one side and the final strains were not captured due to the debonding of strain gages. The maximum load carried by this slab is greater than the 7 mm reinforced concrete slab with fiber in series 1 i.e., 5500 lbs.
Fig. 35: Failure mode of Polypropylene Fiber-Reinforced Concrete Slab with 7 mm Basalt FRP bars of Series2.

Fig. 36: Concrete Compressive Strain for Plain concrete Slab with 7 mm Basalt FRP bars of Series2
7 mm Reinforced Slab Without Fiber:

Similar to other reinforced slabs without fiber, the 7 mm reinforced slab in series 2 also resulted in a shear failure mode before the compressive strain of the concrete achieved the critical strain of 0.003 in/in. Diagonal cracks appeared from both sides of the slab. The maximum load was 5258 lbs. The maximum load carried by the slab is greater than that carried by the 7 mm reinforced slab without fiber in series 1. All the plain reinforced slabs in both the series failed in shear as all of them were not provided with shear reinforcement.

Fig. 37: Failure mode of Plain Concrete Slab with 7 mm Basalt FRP bars of Series 2.
4.2.4 Comparisons of test data for Series 2 slabs

The load strain curves for slabs with fiber and without fiber of different basalt diameter reinforcements are plotted in Fig. 38 to Fig. 43.

Fig. 38: Average Concrete Compressive Strain for Plain and Polypropylene FRC slabs with 5 mm basalt reinforcing bars – Series 2
Fig. 39: Average Concrete Compressive Strain for Plain and Polypropylene FRC slabs with 7 mm basalt reinforcing bars – Series 2

Fig. 40: Average Tensile Bar Strain for Plain and Polypropylene FRC slabs with 5 mm basalt reinforcing bars – Series 2.
Fig. 41: Average Tensile Bar Strain for Plain and Polypropylene FRC slabs with 7 mm basalt reinforcing bars – Series2.

Fig. 42: Average Deflection for Plain and Polypropylene FRC slabs with 5 mm basalt reinforcing bars – Series2.
Fig. 43: Average Deflection for Plain and Polypropylene FRC slabs with 7 mm basalt reinforcing bars – Series2.

Similar to Series 1 data, after adding the polypropylene fiber to the concrete mix, the compressive strain capacity of the slabs were found to increase. Therefore, ductility of the concrete can be increased with the addition of fiber. There was not much increase in the maximum load capacity of 5 mm slab of series1 when fiber is added. But for 7 mm slabs, there was an increase of 27% in the maximum load capacity with the addition of fiber. Similarly, an increase of 19% and 24% was found in the maximum load capacity of 5 mm and 7 mm slabs of Series2 with the addition of fiber. The deflections of the polypropylene fiber-reinforced concrete slabs of Series1 were found to be less than the deflections of the polypropylene fiber-reinforced concrete slabs of Series2 indicating that more volume of fiber in Series1 had resulted in the reduction of deflections of the slabs.
Table 4: Summary of Test Results of Series1 Slabs

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Beam Dimensions, in</th>
<th>Concrete Strength, psi</th>
<th>No. of bars</th>
<th>Diameter of bars, mm (inch)</th>
<th>Failure Load, lbs</th>
<th>Failure Mode</th>
<th>Maximum Average Deflection, in</th>
<th>Average Concrete Strain at Max. Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mm PP1</td>
<td>7.625x4x52</td>
<td>3580</td>
<td>2</td>
<td>7.1 (0.28)</td>
<td>4382</td>
<td>Shear</td>
<td>0.869</td>
<td>0.004166</td>
</tr>
<tr>
<td>5mm Plain1</td>
<td>7.625x4x52</td>
<td>3315</td>
<td>2</td>
<td>7.1 (0.28)</td>
<td>4464</td>
<td>Flexure/Tensile</td>
<td>0.909</td>
<td>0.0017</td>
</tr>
<tr>
<td>7mm PP1</td>
<td>8x4x52</td>
<td>2551</td>
<td>2</td>
<td>9.8 (0.386)</td>
<td>5610</td>
<td>Shear</td>
<td>0.829</td>
<td>0.004694</td>
</tr>
<tr>
<td>7mm Plain1</td>
<td>8x4x52</td>
<td>3058</td>
<td>2</td>
<td>9.8 (0.386)</td>
<td>4262</td>
<td>Shear</td>
<td>0.8238</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

Table 5: Summary of Test Results of Series2 Slabs

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Beam Dimensions, in</th>
<th>Concrete Strength, psi</th>
<th>No. of bars</th>
<th>Diameter of bars, mm (inch)</th>
<th>Failure Load, lbs</th>
<th>Failure Mode</th>
<th>Maximum Average Deflection, in</th>
<th>Average Concrete Strain at Max. Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mm PP2</td>
<td>7.625x4x52</td>
<td>4251</td>
<td>2</td>
<td>7.1 (0.28)</td>
<td>5747</td>
<td>Debonding</td>
<td>1.071</td>
<td>0.0027</td>
</tr>
<tr>
<td>5mm Plain2</td>
<td>7.625x4x52</td>
<td>5009</td>
<td>2</td>
<td>7.1(0.28)</td>
<td>4742</td>
<td>Shear</td>
<td>0.891</td>
<td>0.0013</td>
</tr>
<tr>
<td>7mm PP2</td>
<td>8x4x52</td>
<td>3950</td>
<td>2</td>
<td>9.8 (0.386)</td>
<td>6670</td>
<td>Flexure/Longitudinal</td>
<td>0.998</td>
<td>*</td>
</tr>
<tr>
<td>7mm Plain2</td>
<td>8x4x52</td>
<td>4506</td>
<td>2</td>
<td>9.8 (0.386)</td>
<td>5258</td>
<td>Shear</td>
<td>0.92</td>
<td>0.00263</td>
</tr>
</tbody>
</table>

*indicates data not available

4.3 ASTM C-1399 Third-Point Loaded Beam Tests

This section includes the discussion of specimen preparation, test procedure and test results for the third-point loaded beam test performed on polypropylene fiber-reinforced concrete.
4.3.1 Specimen Preparation

Set of five beam specimens were cast in the steel formwork producing a final dimension of 100×100×350 mm (4 in by 4 in by 14 in). The concrete mix was prepared with Ferro 2.25" type of fiber and with a fiber dosage of 7.5 lb/yd$^3$. The concrete strength was obtained from cylinder tests and was found to be 5910 psi. The steel rectangular formwork was cleaned from dust and form oil was applied with a brush to avoid adhesion of concrete to the surface of formwork and also to get a smooth finishing after the beam concrete set. The concrete mix was poured into the formwork and was placed on a vibrating table for consolidation. The surface of the beam was trowelled smooth and the beams are cured for 7 days.

![Fig. 44: Beams and Round Determinate Panels casted with Polypropylene Fiber Reinforced Concrete](image-url)
4.3.2 Test Procedure (Average Residual Strength Test Per C1399)

The beam was centered on the steel plate with its top surface positioned for loading. The steel plate was placed at the bottom of the beam during the initial loading cycle in order to control the deflection of the beam upon cracking. The beam and the steel plate were placed on the supporting apparatus. The specimens were tested in a hydraulic testing machine which has a loading cell with 50 kN capacity. The rate of loading was controlled by servo hydraulics which is set to a displacement rate of 0.65 ± 0.15 mm/min (0.025 ± 0.005 in/min). The beam was loaded at this rate until the mid-span deflection was 0.20 mm (0.008 in). After the first crack has appeared, the steel plate was removed and the cracked beam was centered on the lower bearing blocks similar to the initial loading cycle. The transducer was reset to zero and adjusted to lightly contact the beam. The data acquisition system was used to record the load-deflection data and the X-Y plotter records the load-deflection curve. The beam was reloaded at 0.65 mm/min rate of loading until it deflects to 1.25 mm (0.05 in) measured from the beginning of reloading.
4.3.3 Test Results

This test method was used for obtaining the average residual-strength of fiber-reinforced concrete test beam. The load values at 0.5, 0.75, 1.00 and 1.25 mm (0.02, 0.03, 0.04, and 0.05 in) deflections on the reloading curve are averaged and are used to calculate the average-residual strength. The ARS (average residual strength) can be formulated as:

$$ARS = \frac{(P_A + P_B + P_C + P_D)/4L}{bd^2}$$

Where,
L is length of the beam, mm (in),

b is average width of the beam, mm (in),

d is average depth of the beam, mm (in), and

$P_A + P_B + P_C + P_D$ is sum of recorded loads at specified deflections on reloading curve, N (lbf).

A typical load-deflection curve obtained for one of the beams is shown in Fig. 18. The curve closer to the y-axis represents the initial loading curve and the curve closer to x-axis represents the reloading curve.

![Load Deflection Curve - Beam # PF2](image)

**Fig. 46:** Typical Load Vs Deflection Curve for Polypropylene Fiber-reinforced Concrete Beam
Table 6: ARS Beam Test Results

<table>
<thead>
<tr>
<th></th>
<th>Concrete Strength, psi</th>
<th>Fiber Type</th>
<th>Dosage</th>
<th>ARS, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam PF2</td>
<td>5910</td>
<td>Ferro 2.25&quot;</td>
<td>7.5 lb/yd³</td>
<td>253</td>
</tr>
<tr>
<td>Beam PF4</td>
<td></td>
<td></td>
<td></td>
<td>281</td>
</tr>
<tr>
<td>Beam PF5</td>
<td></td>
<td></td>
<td></td>
<td>305</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td>280</td>
</tr>
</tbody>
</table>

The average residual strength obtained from the beam tests shown in Table 6 is found to be greater than the average residual strength of 234 psi determined by Patnaik, et al. (South Dakota, May 2007).

4.4 ASTM C-1550 Round Determinate Panel Tests

This section includes the discussion of specimen preparation, test procedure and test results for the round panel test performed on polypropylene fiber-reinforced concrete.

4.4.1 Specimen Preparation

A sample consisting of two specimens were prepared with the exact dimensions required for testing. The formwork consists of a plywood base with a sheet steel strip attached to it which provides a thickness of round panel of 75 mm and a diameter of 800 mm. Form oil was provided to the formwork so that the hardened specimen can be removed without distorting or causing any damage to the specimen. The fiber reinforced
concrete mix was filled into the formwork and the surface of the specimen was screeded to obtain a uniform thickness. Regardless of the size of the aggregate and length of fiber used in the concrete, the dimensions are maintained to be constant. The concrete mix for the round panel tests consists of a polypropylene fiber called Ferro 2.25" and a fiber dosage of 7.5 lb/yd$^3$ (4.45 kg/m$^3$). The concrete strength is found to be 3046 psi. A successful test involves a failure that includes at least three radial cracks.

47(a)
Making of Round Panel
(Consolidation Using a Vibrating Table)

47(b)
Screeding and Finishing the Top Surface of Test Specimen

47(c)
Fiber-reinforced Concrete Round Determinate Panel
The test apparatus consists of a load actuator which can control the rate of increase in displacement and the load sensing device has the resolution sufficient to record load to ±50 N. From the tests performed by Bernard (April 2002), the specimens supported on three symmetrically arranged pivots have resulted in low within-batch variability and consistent failure modes. The test specimen was mounted on the test fixture with its molded face on the top placed over three transfer plates resting on the pivots. The three pivots are arranged symmetrically on a pitch circle diameter of 750 mm. The three supports are capable of supporting a load of up to 100 kN applied vertically at the center of the specimen. For a test involving a specimen displaying a peak load capacity of 100 kN, the three supports should not displace more than 0.5 mm in both radial and circumferential directions between the onset of loading and 40 mm central deflection. The contact between the round panel and each pivot shall comprise a steel transfer plate with dimensions of approximately 40 ×50 mm with a spherical seat of about 4mm depth machined into one surface to accept the ball pivot. The round panel is centered with respect to both the supports and loading piston. The loading is started on to the specimen at a rate of 4 mm/min. The test is carried out until the specimen deflects to 40 mm at the center. The central deflection is measured by LVDT which can record deflections to ±0.05 mm. The deflections and the corresponding load applied on the round panel are recorded in a data logging system up to a central deflection of 40 mm. Two round panels were tested in the study. The two specimens failed successfully with three radial cracks starting from the center to the perimeter. The post-crack performance
of the round determinate panels subjected to a central point load is represented by calculating the energy absorbed by the panel up to a specified central deflection. The energy absorbed (toughness) by the panels is calculated by finding the area under the load deflection up to the specified central deflection. In the current tests, the toughness in Nm or kNm (Joules) is calculated at point of cracking, 5 mm, 10 mm, and 40 mm central deflections. This toughness indicates the ability of fiber-reinforced concrete to absorb energy along the cracks. The toughness obtained from the two panels in this study was averaged and is compared with the toughness obtained for a similar panel which has polypropylene fiber and slightly higher dosage of fiber 8.43 lb/yd³, from the tests performed by Bernard (April 2002). The toughness obtained from this study is found to be greater than the values in the published literature.

Fig. 48(a) & (b): Test Setup used for determining the Flexural Toughness of the Round Determinate Panels
4.4.3 Test Results

This section shows the load-deflection curves obtained from the round panel test and the toughness values calculated from these graphs.

Fig. 50: Load Vs Deflection Curve for the Round Determinate Panel 1.
**Table 7: Round Panel Test Results**

<table>
<thead>
<tr>
<th></th>
<th>Concrete Strength</th>
<th>PP Fiber</th>
<th>Dosage</th>
<th>Energy (Toughness), Joules</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MPA</td>
<td></td>
<td>lb/yd²</td>
<td>At Crackling / At 5 mm / At 10 mm / At 40 mm</td>
</tr>
<tr>
<td>Round Panel 1</td>
<td>21*</td>
<td>Ferro 2.25</td>
<td>7.5 (4.45)</td>
<td>18 / 57 / 106 / NA</td>
</tr>
<tr>
<td>Round Panel 2</td>
<td>21*</td>
<td>Ferro 2.25</td>
<td>7.5 (4.45)</td>
<td>2 / 46 / 90 / 256</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>21</strong></td>
<td><strong>Ferro 2.25</strong></td>
<td><strong>7.5 (4.45)</strong></td>
<td><strong>11 / 51 / 100 / 256</strong></td>
</tr>
<tr>
<td>Published literature*</td>
<td>54.5</td>
<td>Straight PP</td>
<td>8.43 (5.0)</td>
<td>15 / 42 / 54 / 137</td>
</tr>
</tbody>
</table>

*Bernard, E. S., "Correlations in the Behavior of Fiber Reinforced Shotcrete Beam and Panel Specimens," Materials and Structures, RILEM, Vol. 35, April 2002, pp. 156-164. **Concrete was 14 days old, and therefore has lower strength.

In spite of the very low concrete strength of the round panels tested in this study, the toughness of these panels is found to be higher than the toughness of the panel reinforced with similar material that was published in Bernard (April 2002).
CHAPTER V
ANALYTICAL PROCEDURE

Fanella and Naaman [11] developed a useful stress-strain curve for fiber reinforced concrete. Their approach was used in this study. Their expression consisted of normalized stress, strain and four constants A, B, C, and D that are derived for both the ascending branch and descending branch of the stress-strain curve with suitable boundary conditions. The boundary conditions used in deriving these constants were explained in section 2.3.2 of this thesis in literature review on polypropylene fiber reinforced concrete. The approach used in the present study for the development of analytical stress-strain curve for polypropylene fiber reinforced concrete based on Fanella and Naaman [11] is referred to as the Proposed Method. The section 5.1 describes the Proposed method in detail.

5.1 Theoretical Approach for Polypropylene Fiber-Reinforced Concrete Slabs using Proposed Method

Proposed method describes the theoretical approach used in this study for the development of stress-strain curve of polypropylene fiber reinforced concrete. In order to develop the theoretical stress-strain curve, the results of the slab tests and cylinder tests
done in accordance to the ASTM standards in this study are being used. These experimental tests and its results were discussed in chapter 4. The experimental data recorded in the slab tests were until failure loads. The stress-strain values beyond the peak load point were not recorded in the tests. The approach given in Fanella and Naaman [11] uses the complete stress-strain curve as shown in Fig. 5 on which three points namely \((x_1, y_1)\), \((x_i, y_i)\), and \((x_f, y_f)\) are defined. The points \((x_i, y_i)\), and \((x_f, y_f)\) fall in the descending branch of the stress-strain curve i.e., after the peak stress is reached. Since the experimental stresses and strains in the slab tests are recorded only until the peak stress point in the present study, in order to develop the complete analytical stress-strain curve, the stress-strain values beyond the peak stress have been taken from the experimental stress-strain curve as shown in Fig. 52 from Fanella and Naaman [11] for twisted polypropylene fibers with a percentage volume of 1%. Similar to Naaman and Fanella, six points have been defined on the stress-strain curve namely \((x_1, y_1)\), \((x_p, y_p)\), \((x_i, y_i)\), \((x_m, y_m)\), \((x_n, y_n)\), and \((x_f, y_f)\) as shown in Fig. 53 dividing the curve in to five parts named as Curve 1, Curve 2, Curve 3, Curve 4 and Curve 5. The x coordinate represents strain and y coordinate represents stress. \((x_1, y_1)\) indicate the strain and stress at 45% of the peak stress respectively. \((x_p, y_p)\) indicate the stress at the peak point corresponding to a strain of 0.002. This value of 0.002 is taken from the literature (Ezeldin and Balaguru, 1992; Orozco and Maji, 2004). \((x_i, y_i)\) indicate the strain and stress at the inflection point. \((x_m, y_m)\), \((x_n, y_n)\) indicate the points on the descending branch after the inflection point and \((x_f, y_f)\) refers to the final point on the curve. These points beyond the peak point were taken from the reference curve (Fig. 52) multiplied with a factor of \(f'_c/f'_{cn}\) where \(f'_c\) is the compressive strength of the cylinder from the current study and \(f'_{cn}\) is the maximum
compressive strength of the twisted polypropylene FRC from Fanella and Naaman [11] as shown in Fig. 52. Once after the stress-strain relationship is determined, the corresponding loads are determined using the strain compatibility method for each slab specimen.

![Diagram](image)

Fig. 52: Influence the volume fraction of fibers on the compressive stress-strain curve (reproduced from Fanella and Naaman [11]).
Fig. 53: Theoretical Stress-Strain Curve for Polypropylene Fiber-Reinforced Concrete Slab with 5 mm Basalt FRP bars- Series1

The young’s modulus doesn’t change much due to the addition of polypropylene fiber [12]. Therefore a slope equal to \(57000\sqrt{f_c}\) was used to generate the first part of the analytical curve starting from origin to the point \((x_1, y_1)\). A series of simultaneous equations are developed based on the assumed boundary conditions and the unknown coefficients were determined by solving the simultaneous equations. Thus, the relation between the stress and strain is determined. Based on the defined ranges of strain i.e., \(x_1\), \(x_p\), \(x_i\), \(x_m\), \(x_f\), the stresses can be calculated using the obtained relation. The set of boundary conditions assumed for each part of the stress-strain curve are listed below:

**Boundary Conditions for Curve 1:**

The general form of the equation is \(y = b_1x\)

*The curve passes through origin \((0, 0)\)

*The curve passes through 45% of the peak stress \((x_1, y_1)\)
**Boundary Conditions for Curve 2:**

The general form of curve 2 is a parabolic equation, \( y = a_2 x^2 + b_2 x + c_2 \)

*The slope of the curve at a peak point i.e., \((x_p, y_p)\) is equal to zero.

*The curve passes through the peak point \((x_p, y_p)\)

*The curve passes through 0.45 of the peak point, \((x_1, y_1)\)

**Boundary Conditions for Curve 3:**

The general form of the equation is \( y = a_3 x^2 + b_3 x + c_3 \)

*The slope of the curve 3 is at the peak point i.e., \((x_p, y_p)\) is equal to zero.

*The curve passes through the peak point \((x_p, y_p)\)

*The curve passes through the inflection point \((x_i, y_i)\)

**Boundary Conditions for Curve 4:**

The general form of the equation is \( y = a_4 x^3 + b_4 x^2 + c_4 x + d_4 \)

*The curve passes through the inflection point \((x_i, y_i)\)

*The slope of the curve 4 is equal to the slope of the line 5 at the point \((x_n, y_n)\)

*The curve passes through the point \((x_n, y_n)\)

* The curve passes through the point \((x_m, y_m)\)

**Boundary Conditions for line 5:**

The general form of the equation is \( y = b_5 x + c_5 \)

*The line passes through the tail point \((x_t, y_t)\)

*The line passes through the point \((x_n, y_n)\)

The general form of the simultaneous equation matrix is shown below:
Equation 1: General Matrix Form developed for Proposed Model

\[
\begin{bmatrix}
X_1 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
1.00 & 2X_p & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
1.00 & X_p^2 & X_p & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
1.00 & X_1^2 & X_1 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & X_p^2 & X_p & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 2X_p & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & X_1^2 & X_1 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 3X_n^2 & 2X_n & 1.00 & 0.00 & 0.00 & -1.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & X_i^3 & X_i^2 & X_i & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & X_m^3 & X_m^2 & X_m & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & X_n^3 & X_n^2 & X_n & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & X_f & 1.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & X_f & 1.00 \\
\end{bmatrix}
\begin{bmatrix}
b_1 \\
a_2 \\
b_2 \\
c_2 \\
a_3 \\
b_3 \\
c_3 \\
a_4 \\
b_4 \\
c_4 \\
a_5 \\
b_5 \\
c_5 \\
a_6 \\
b_6 \\
\end{bmatrix}
= \begin{bmatrix}
y_1 \\
y_p \\
y_1 \\
y_p \\
o \\
y_i \\
0 \\
y_i \\
y_m \\
y_n \\
y_n \\
y_n \\
y_f \\
y_f \\
\end{bmatrix}

93
The above matrices can be denoted by the equation 
\[ [A] [B] = [C] \]. Using the simultaneous equations developed from the boundary conditions, the inverse of matrix 
\([A]\) is multiplied with matrix \([C]\) to give the unknown coefficients. From the above equations, the complete theoretical stress-strain curve starting from origin until 0.0154 strains is established for the polypropylene fiber reinforced concrete under compression. A typical stress-strain curve developed for 5 mm - PP1 slab is shown in Fig. 53.

The next step was to determine the loads corresponding to a particular stress. This was done using the strain compatibility method. The depth to the neutral axis was divided into 20 equal elements of height \(\Delta h = h/20\). The strains are calculated at the center of every element and the corresponding stresses are calculated using the equations obtained from the matrix representation shown in Equation (1). The resultant compressive force is equated to tensile force to maintain the equilibrium condition. All this process was done using spreadsheets developed for each slab. Separate spreadsheets were maintained for developing stress-strain relationship and for calculating loads using strain compatibility. The corresponding moment-strength is also calculated in the spreadsheet.
### Table 8: Strain Compatibility Spreadsheet for 5 mm PP1 Slab

<table>
<thead>
<tr>
<th>Data</th>
<th>Beam Length</th>
<th>Lt</th>
<th>4.33333 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crushing strain of concrete</td>
<td>ε_{cu}</td>
<td>0.002 in/in</td>
<td>(assumed)</td>
</tr>
<tr>
<td>Width of the beam</td>
<td>b</td>
<td>7.625 in</td>
<td></td>
</tr>
<tr>
<td>Overall height of the beam</td>
<td>h</td>
<td>4 in</td>
<td></td>
</tr>
<tr>
<td>Effective concrete cover</td>
<td>dc</td>
<td>0.75 in</td>
<td></td>
</tr>
<tr>
<td>Tensile strength of FRP</td>
<td>f_{f,ave}</td>
<td>157.2 ksi</td>
<td></td>
</tr>
<tr>
<td>Modulus of elasticity of FRP</td>
<td>E_f</td>
<td>6032 ksi</td>
<td></td>
</tr>
<tr>
<td>Area of FRP bars</td>
<td>A_f</td>
<td>0.12907 in²</td>
<td></td>
</tr>
<tr>
<td>Concrete Compressive Strength</td>
<td>f’c</td>
<td>3580 psi</td>
<td></td>
</tr>
<tr>
<td>Effective depth</td>
<td>d</td>
<td>3.25 in</td>
<td></td>
</tr>
<tr>
<td>Depth to neutral axis</td>
<td>c</td>
<td>0.483 in</td>
<td></td>
</tr>
<tr>
<td>Number of divisions - comp block</td>
<td>n</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Height of each element</td>
<td>Δh</td>
<td>0.024140743</td>
<td></td>
</tr>
</tbody>
</table>

#### Calculations

<table>
<thead>
<tr>
<th>Layer</th>
<th>a (effective depth to the layer from top fiber), inches</th>
<th>Strain ε</th>
<th>Stress Δσ, ksi</th>
<th>Force ΔF, kips</th>
<th>Moment ΔM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.012</td>
<td>0.00195</td>
<td>3.58</td>
<td>0.659</td>
<td>0.0007</td>
</tr>
<tr>
<td>2</td>
<td>0.036</td>
<td>0.00185</td>
<td>3.56</td>
<td>0.656</td>
<td>0.0020</td>
</tr>
<tr>
<td>3</td>
<td>0.060</td>
<td>0.00175</td>
<td>3.53</td>
<td>0.649</td>
<td>0.0033</td>
</tr>
<tr>
<td>4</td>
<td>0.084</td>
<td>0.00165</td>
<td>3.48</td>
<td>0.640</td>
<td>0.0045</td>
</tr>
<tr>
<td>5</td>
<td>0.109</td>
<td>0.00155</td>
<td>3.41</td>
<td>0.628</td>
<td>0.0057</td>
</tr>
<tr>
<td>6</td>
<td>0.133</td>
<td>0.00145</td>
<td>3.32</td>
<td>0.612</td>
<td>0.0068</td>
</tr>
<tr>
<td>7</td>
<td>0.157</td>
<td>0.00135</td>
<td>3.22</td>
<td>0.593</td>
<td>0.0078</td>
</tr>
<tr>
<td>8</td>
<td>0.181</td>
<td>0.00125</td>
<td>3.11</td>
<td>0.572</td>
<td>0.0086</td>
</tr>
<tr>
<td>9</td>
<td>0.205</td>
<td>0.00115</td>
<td>2.97</td>
<td>0.547</td>
<td>0.0093</td>
</tr>
<tr>
<td>10</td>
<td>0.229</td>
<td>0.00105</td>
<td>2.82</td>
<td>0.519</td>
<td>0.0099</td>
</tr>
<tr>
<td>11</td>
<td>0.253</td>
<td>0.00095</td>
<td>2.65</td>
<td>0.488</td>
<td>0.0103</td>
</tr>
<tr>
<td>12</td>
<td>0.278</td>
<td>0.00085</td>
<td>2.46</td>
<td>0.454</td>
<td>0.0105</td>
</tr>
<tr>
<td>13</td>
<td>0.302</td>
<td>0.00075</td>
<td>2.26</td>
<td>0.416</td>
<td>0.0105</td>
</tr>
<tr>
<td>14</td>
<td>0.326</td>
<td>0.00065</td>
<td>2.04</td>
<td>0.376</td>
<td>0.0102</td>
</tr>
<tr>
<td>15</td>
<td>0.350</td>
<td>0.00055</td>
<td>1.81</td>
<td>0.332</td>
<td>0.0097</td>
</tr>
<tr>
<td>16</td>
<td>0.374</td>
<td>0.00045</td>
<td>1.53</td>
<td>0.282</td>
<td>0.0088</td>
</tr>
<tr>
<td>17</td>
<td>0.398</td>
<td>0.00035</td>
<td>1.19</td>
<td>0.220</td>
<td>0.0073</td>
</tr>
<tr>
<td>18</td>
<td>0.422</td>
<td>0.00025</td>
<td>0.85</td>
<td>0.157</td>
<td>0.0055</td>
</tr>
<tr>
<td>19</td>
<td>0.447</td>
<td>0.00015</td>
<td>0.51</td>
<td>0.094</td>
<td>0.0035</td>
</tr>
<tr>
<td>20</td>
<td>0.471</td>
<td>0.00005</td>
<td>0.17</td>
<td>0.031</td>
<td>0.0012</td>
</tr>
<tr>
<td></td>
<td>Total Comp.</td>
<td>8.924</td>
<td>0.1360</td>
<td>8.924</td>
<td>2.4170</td>
</tr>
</tbody>
</table>

| FRP   | 3.25                                                 | 0.01146  | 69.14          | 0.01146        |

| Equation to establish equilibrium | 0.000 |

| Moment Strength | 2.28 kip-ft |

Gross-section properties were used until cracking and strain compatibility method after cracking. The strains until cracking moment, M_{cr} are found using gross section properties of the slab.

\[
M_{cr} = \frac{I}{y_t} f_r, \quad \text{Equation (2)}
\]
Where,

Modulus of rupture, $f_r = 7.5 \sqrt{f'_c}$.  

$\text{Equation (3)}$

$I$ – Moment of inertia of gross section

$y_t$ – Depth of neutral axis from extreme tension fiber.

The load corresponding to the cracking moment, i.e., cracking load is found as shown.

$$
P_{cr} = \frac{4(M_{cr} - M_D)}{(L^2 - b^2)/12}$$  

$\text{Equation (4)}$

Where,

$M_{cr}$ – Cracking moment, kip-ft.

$M_D$ – Moment due to dead load, kip-ft.

$L$ - Span of the slab, in.

$b$ - Width of spreader, in.

Once the cracking load is found for a particular slab, moment is found for $P_{cr}$. Using elastic bending theory, the stresses corresponding to the moments are calculated and then the strains were determined from stress-strain relationship.

$$
\sigma = \frac{M}{I} y$$  

$\text{Equation (5)}$

Where,

$M$ – Moment at the load $P$, ($P \leq P_{cr}$).

$I$ – Moment of inertia of gross section.

$y$ – Distance to the neutral axis from bottom tension fiber.

$$
\epsilon = \frac{\sigma}{E_c}$$  

$\text{Equation (6)}$

Where,

$\epsilon$ – Strain at a Stress $\sigma$.

$E_c$ – Young’s modulus of concrete (i.e., $57000 \sqrt{f'_c}$)
$f_c'$ – Average compressive strength of cylinders.

The average $\varepsilon_u$, i.e., ultimate strain was considered to be in the range of 0.0034 to 0.0038 irrespective of the compressive strength (Hognestad, 1955 and Orozco and Maji, 2004). The loads are calculated until a strain value of 0.0038. Thus, Load-Strain data is predicted for all the slab tests using the proposed method. The theoretical and experimental graphs are shown in Fig. 58 to Fig. 61 and the maximum load capacity of the slabs found using this method is tabulated in Chapter 6.

5.2 Theoretical Approach using Hognestad’s model for Polypropylene FRC slabs:

According to Hognestad’s model, the stress-strain relation for plain concrete was given as follows (Hognestad, 1955 and Orozco and Maji. 2004):

For $0 \leq \varepsilon_c \leq \varepsilon_o$: $f_c = f_c' (\varepsilon_c/\varepsilon_o)(2 - \varepsilon_c/\varepsilon_o)$; 

Equation (7)

For $\varepsilon_o \leq \varepsilon_c \leq \varepsilon_u$: $f_c = f_c' [1 - 0.15(\varepsilon_c - \varepsilon_o)/(\varepsilon_u - \varepsilon_o)]$; 

Equation (8)

$\varepsilon_o = 1.8 f_c'/E_c$; 

Equation (9)

Where,

$\varepsilon_c$ = Strain at any point on the stress-strain curve;

$f_c$ = Stress in concrete at a concrete strain $\varepsilon_c$;

$\varepsilon_o$ = Concrete ultimate strain in Hognestad’s curve;

$\varepsilon_u$ = Ultimate strain i.e., 0.0038;

$f_c'^*$ = Concrete ultimate strength in Hognestad’s curve;
The above equations given for finding stresses are incorporated in the spreadsheet with a suitable boundary condition for strain. Similar to the approach proposed in Section 5.1, the strain compatibility method is used to determine the loads corresponding to different strains. The load-strain graphs are plotted along with the experimental data to see the correlation. The theoretical load-strain graphs using this approach are also shown in Fig. 58 to Fig. 61. The maximum load capacity of the polypropylene fiber-reinforced concrete slabs found using this approach are tabulated in Chapter 6.

\[
\begin{align*}
\text{For } 0 & \leq \varepsilon \leq \varepsilon_0: \\
f_c &= f'c(\varepsilon / \varepsilon_0)(2 - \varepsilon / \varepsilon_0) \\
\text{For } \varepsilon_0 & \leq \varepsilon \leq \varepsilon_u: \\
f_c &= f'c \left[ 1 - 0.15(\varepsilon - \varepsilon_0)/(-\varepsilon_u - \varepsilon_0) \right]
\end{align*}
\]

Fig. 54: Hognestad’s Model
5.3 Theoretical Approach for Plain slabs (Slabs without Polypropylene Fiber Reinforcement)

This section discusses the theoretical approach used for predicting stress-strain relationship of plain concrete using two different equations.

5.3.1 Approach using Hognestad’s model

The equation discussed in the Section 5.2 was used for determining the stresses corresponding to a particular strain in the strain compatibility spreadsheet developed for plain slab. The load-strain data is plotted against the experimental load-strain curve as

![Stress-Strain Curve](image)

Fig. 55: Typical Stress-Strain Curve using Different Equations for $f'_c = 3,580$ psi
shown in Fig. 62 to Fig. 65 and the maximum load capacity of the plain slabs found using this approach are tabulated in Chapter 6.

5.3.2 Approach using Desayi and Krishnan equation of stress-strain for Plain concrete

Desayi and Krishnan (1964) proposed a simple parabolic model describing stress strain of concrete as:

\[ f_c = \frac{2f'_c \varepsilon}{\varepsilon_0 \left[1 + \left(\frac{\varepsilon}{\varepsilon_0}\right)^2\right]} \]  
Equation (10)

Where,

\( f_c \) = Stress in concrete at a concrete strain of \( \varepsilon \);

\( f'_c \) = Peak compressive strength of concrete obtained from a cylinder test;

\( \varepsilon_0 \) = Concrete strain at the peak compressive stress, assumed to be 0.002;

To predict the theoretical loads, for every strain value, the corresponding theoretical stress is found using Desayi and Krishnan equation for stress-strain relationship. After finding the set of stresses for strains, the loads are determined using strain compatibility method similar to all the above approaches. The obtained loads are plotted against strains and are compared with the experimental load-strain curves. The maximum load capacity is found at a strain that is slightly less than or equal to the maximum allowable strain \( \varepsilon_{cu} \) defined below. The maximum load capacity of the plain slabs found using this approach are tabulated in Chapter 6.

\[ \varepsilon_{cu} = 0.00421 \left( \frac{f_c - 1450}{5800} \right) \times 0.00138 \]
5.4 Calculation of Maximum Load Capacity using ACI Method for Slabs reinforced with and without Polypropylene Fiber Reinforcement

In the present study, slabs that are reinforced with basalt FRP bars with and without the addition of polypropylene fiber were tested. The flexural capacity if these slabs is determined according to the standard ACI 440.1R. In the FRC members, the failure due to concrete crushing is preferred for flexural members reinforced with FRP bars. If the failure occurs due to FRP bar rupture, the failure mode is sudden and catastrophic. In order to compensate the lack of ductility in FRP-reinforced concrete members, the member should possess a higher strength. This will allow the usage of high-strength properties of FRP bars and will increase the stiffness of the cracked section. The important assumptions made in the calculation of flexural strength according to ACI
440.1R are that plane sections before loading remain plane after loading and the maximum usable compressive strain in the concrete is 0.003. The tensile behavior of the FRP bars is considered linearly elastic until failure and that the bond between concrete and FRP reinforcement is perfect.

The nominal flexural strength of the member multiplied by a strength reduction factor $\phi$, is referred as the design flexural strength. The nominal flexural strength of a fiber reinforced concrete member is based on the principles of strain compatibility, internal force equilibrium and the controlling mode of failure. If the FRP reinforcement ratio $\rho_f$, is greater than the balanced reinforcement ratio $\rho_{fb}$, the concrete crushing failure mode governs. Otherwise, FRP rupture failure mode governs. All the slabs in the present study are designed as over-reinforced sections which assume failure through concrete crushing.

$$\rho_f = \frac{A_f}{b d}$$

$$\rho_{fb} = 0.85 \beta_1 \frac{f'_c}{f_{fu}} \frac{E_f \varepsilon_{cu}}{E_f \varepsilon_{cu} + f_{fu}}$$

In case of the member that fails due to the crushing of concrete ($\rho_f > \rho_{fb}$), the stress distribution in the concrete is approximated with the ACI rectangular stress block and the nominal flexural strength $M_n$ is derived based on the equilibrium condition and strain compatibility as

$$M_n = A_f f_f \left(d - \frac{a}{2}\right)$$

Where,
According to ACI 318-05, the strength reduction factor $\Phi$, for compression controlled member is 0.65 and 0.55 for FRP reinforcing bar rupture failure. In transition between these two values, the factor $\Phi$ is defined linearly.

\[
\Phi = \begin{cases} 
0.55 & \text{for } \rho_f \leq \rho_{fb} \\
0.3 + 0.25 \frac{\rho_f}{\rho_{fb}} & \text{for } \rho_{fb} < \rho_f < 1.4 \rho_{fb} \\
0.65 & \text{for } \rho_f \geq 1.4 \rho_{fb}
\end{cases}
\]

Thus the maximum load capacity of the slabs is calculated from the nominal moment capacity found with an assumed concrete crushing strain equal to 0.003 according to ACI 440.1R. The maximum load capacity of the slabs calculated using ACI method is tabulated in Chapter 6. The difference between maximum load capacity found by ACI method and from the slab tests with and without polypropylene fiber are also tabulated in Chapter 6.
5.5 Calculation of Shear Strength for Slabs reinforced with and without polypropylene FRP reinforcement

The shear strength of flexural members that are not provided with shear reinforcement has indicated that the concrete shear strength is influenced by the stiffness of the flexural reinforcement as reported by many researchers (Nagasaka, 1993; Zhao, 1995; JSCE 1997b; Sonobe, 1997; Michaluk, 1998;). The shear capacity of concrete $V_c$ of flexural members using FRP as main reinforcement is calculated according to the Equation 11 given by ACI 440.1R

$$V_c = 5k \sqrt{f_{ct}} b_w d$$  \hspace{1cm} \text{Equation (11)}

Where, $b_w$ = width of the web, in.

$$k = \sqrt{2\rho_f n_f + (\rho_f n_f)^2} - \rho_f n_f$$

$$\rho_f = \frac{A_f}{b_w d}, \quad n_f = \frac{E_f}{E_c}$$

Where,

$E_f$ - Modulus of FRP bars;

$E_c$ - Modulus of Concrete;

$A_f$ - Area of FRP bars;

The Equation 11 has been shown to provide an acceptable factor of safety for FRP-reinforced specimens over the range of concrete strengths and reinforcement ratios tested to date. In the present study, the slabs are provided with basalt FRP bars as flexural reinforcement. The shear reinforcement i.e., the FRP stirrups are not provided in the
slabs. Thus the shear strength of the slabs is derived from the shear capacity of concrete alone. The shear capacity of the slabs with and without polypropylene fiber is calculated and tabulated in Table 9.

Table 9: Shear Capacity of slabs reinforced with and without polypropylene fiber:

<table>
<thead>
<tr>
<th>Slab</th>
<th>Shear Strength, $V_c$, lbs</th>
<th>Slab</th>
<th>Shear Strength, $V_c$, lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mmPlain1</td>
<td>921</td>
<td>7mm Plain1</td>
<td>1262</td>
</tr>
<tr>
<td>5mm Plain2</td>
<td>1028</td>
<td>7mm Plain2</td>
<td>1402</td>
</tr>
<tr>
<td>5mm PP1</td>
<td>940</td>
<td>7mm PP1</td>
<td>1200</td>
</tr>
<tr>
<td>5 mm PP2</td>
<td>984</td>
<td>7mm PP2</td>
<td>1353</td>
</tr>
</tbody>
</table>

Including the flexural reinforcement i.e., the basalt FRP bars, the slabs are also reinforced with the polypropylene fiber. The shear capacity tabulated in Table 9 is found to be very low compared to the shear strength obtained from the experiment. This raised question on whether the polypropylene fiber added to the concrete would also provide any shear strength. In order to determine if the shear strength increases due to the addition of polypropylene fiber, the shear capacity of the slabs obtained from the experiment including the shear capacity due to self weight is divided by $k \sqrt{f'_c b_w d}$ when correlated with Equation 11 to obtain the shear factor ‘s’. This is explained as follows:

$$V_u = \frac{P_{max}}{2} + w_d \frac{L_t}{2}$$  \hspace{1cm} \text{Equation (12)}$$

where

$V_u$ - Shear strength of the slab obtained from test, lbs;

$P_{max}$ - Maximum load capacity of the slab obtained from test, lbs;
$w_d$ - Self weight of the slab, lb/ft;

$L_t$ – Total length of the slab, ft;

The shear capacity due to concrete is written in the general form of Equation 11 as shown in Equation 13. The shear capacity obtained from test using Equation 12 is equated to Equation 13 to obtain the shear factor ‘s’. The shear factors calculated for the slabs with and without polypropylene fibers are tabulated in Table 10. The Fig. 57 shows the shear factors plotted for all the slabs.

$$V_u = sk\sqrt{f'_c b_w d}$$

Equation (13)

Table 10: Experimental Shear Strength and Shear Factor for slabs with and without Polypropylene Fiber

<table>
<thead>
<tr>
<th>Slab</th>
<th>Exp. Max Load, lbs</th>
<th>Slab Dimensions, in</th>
<th>Shear Strength $V_u$, lbs</th>
<th>Concrete Strength $f'_c$, psi</th>
<th>Shear factor, s</th>
<th>% Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 mmPP1</td>
<td>4382</td>
<td>7.625x4x52</td>
<td>2260</td>
<td>0.1268</td>
<td>3580</td>
<td>12</td>
</tr>
<tr>
<td>5 mmPlain1</td>
<td>4464</td>
<td>7.625x4x52</td>
<td>2301</td>
<td>0.1291</td>
<td>3315</td>
<td>12</td>
</tr>
<tr>
<td>5 mm PP2</td>
<td>5747</td>
<td>7.625x4x52</td>
<td>2942</td>
<td>0.1218</td>
<td>4251</td>
<td>15</td>
</tr>
<tr>
<td>5 mmPlain2</td>
<td>4742</td>
<td>7.625x4x52</td>
<td>2440</td>
<td>0.1173</td>
<td>5009</td>
<td>12</td>
</tr>
<tr>
<td>7 mmPP1</td>
<td>5610</td>
<td>8x4x52</td>
<td>2877</td>
<td>0.1828</td>
<td>2551</td>
<td>12</td>
</tr>
<tr>
<td>7 mmPlain1</td>
<td>4262</td>
<td>8x4x52</td>
<td>2203</td>
<td>0.1755</td>
<td>3058</td>
<td>9</td>
</tr>
<tr>
<td>7 mm PP2</td>
<td>6670</td>
<td>8x4x52</td>
<td>3407</td>
<td>0.1656</td>
<td>3950</td>
<td>13</td>
</tr>
<tr>
<td>7 mmPlain2</td>
<td>5258</td>
<td>8x4x52</td>
<td>2701</td>
<td>0.1607</td>
<td>4506</td>
<td>10</td>
</tr>
</tbody>
</table>

106
The factor of 5 is given in Equation 11 by ACI for members reinforced with flexural FRP reinforcement. In the present study, for slabs without polypropylene fiber which are reinforced with basalt FRP bars, the shear factor had increased and is in the range of 9 to 12. This increase from ACI 440.1R recommended factor of 5 could be due to the use of basalt FRP reinforcement or due to conservative approach adopted by ACI 440.1R. Furthermore, the shear factor had increased much more for the slabs that are reinforced with polypropylene fiber and basalt FRP bars and is in the range of 12 to 15. This suggests that the addition of polypropylene fiber to the concrete increases the shear strength. Except for the 5 mm PP1 slab, for each slab reinforced with polypropylene fiber, the shear factor increased by 3 compared to the corresponding slab without polypropylene fiber. And also, the shear strength of the polypropylene fiber reinforced concrete slabs is found to be greater than the corresponding plain slabs even though their concrete strengths were higher.
5.6 Theoretical Approach of finding the Deflections for Slabs with and without Polypropylene Fiber Reinforcement

The slabs are subjected to four-point loading on a simply supported span with a spreader beam of 6 in width. The slab is subjected to dead load deflection and deflection due to four-point loading. The sum of these two deflections is used to determine the theoretical deflections. While calculating the deflections, gross moment of inertia is used until the slab first cracks and effective moment of inertia is used after the first crack appears on the slab.

**Dead load deflection:**
\[ \Delta = \frac{5W_D l^4}{384EI} \]  
Equation (14)

Where \( W_D \) - Self weight of the slab, lb/in.
L – Effective length of the slab, in.
E – Young’s modulus of concrete, psi.
I – Moment of Inertia, \( \text{in}^4 \)

**Point load deflection:**
\[ \Delta = \frac{Wa}{24EI} (3l^2 - 4a^2) \]  
Equation (15)(AISC Manual, 13\(^{th}\) Ed.)

Where \( W \) – Point load applied on the slab, lbs.
a – Distance from support to the point where load is applied, inches.
l – Effective length of the slab, in.
E – Young’s modulus of the concrete, psi.
I – Moment of inertia of the slab, \( \text{in}^4 \)

When a section is uncracked, its moment of inertia is equal to the gross moment of inertia, \( I_g \). For a rectangular section, the gross moment of inertia is calculated as
By substituting the values of $E$, $I_g$, $l$, $a$, $W$ in the Equations (14) and (15), the summation of deflections is the final deflection for $W \leq P_{cr}$. 

When the applied moment, $M_a$, exceeds the cracking moment, $M_{cr}$, cracking occurs, which causes a reduction in the stiffness; and the moment of inertia is based on the cracked section, $I_{cr}$. $I_{cr}$ can be calculated using an elastic analysis for FRP reinforced concrete which is similar to the analysis used for steel reinforced concrete (concrete in tension is neglected).

$$I_{cr} = \frac{bd^4}{3}k^3 + n_f A_f d^2 (1 - k)^2$$  \hspace{1cm} \text{Equation (17)}

Where $k = \sqrt{2\rho_f n_f + (\rho_f n_f)^2 - \rho_f n_f}$ and

$$n_f = \frac{E_f}{E_c}$$ is the modular ratio between the FRP reinforcement and the concrete.

The overall stiffness of a flexural member, after it had experienced cracking, varies between $E_c I_g$ and $E_c I_{cr}$, depending on the magnitude of the applied moment. This transition is derived as an equation by Branson (1977) and is followed by ACI 318-05. The Branson’s equation given for effective moment of inertia, $I_e$ is shown in Equation 18.

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr}$$  \hspace{1cm} \text{Equation (18)}

This equation was based on the behavior of steel-reinforced beams at service loads. Many researchers (Benmokrane et al. 1996a; Brown and Bartholomew 1996; Zhao at al. 1997; Yost at al. 2003; Rasheed et al. 2004) worked on the deflection of FRP-
reinforced beams reported that on a plot of load-versus-deflection of simply supported beams, the experimental curves are approximately parallel to those predicted by Branson’s equation. The effective moment of inertia of FRP-reinforced beams has been found to be overestimated by Branson’s equation. The reduced tension stiffening in FRP-reinforced members may be attributed to the lower modulus of elasticity and different bond stress levels for the FRP reinforcement as compared with those of steel. Gao et al. (1998a) had given a modified expression shown in Equation 19 for the effective moment of inertia which accounts for reduced tension stiffening in FRP-reinforced members.

\[ I_e = \left( \frac{M_{cr}}{M_a} \right)^3 \beta_d I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \]  
Equation (19)

Where \( \beta_d \) - Reduced Coefficient Factor;

\[ \beta_d = \frac{1}{5} \left( \frac{\rho_f}{\rho_{fb}} \right) \leq 1.0 \]

\[ M_{cr} = \frac{I_g f_r}{y_t} \]

\[ M_a = \frac{p(1 - b_{sp})}{48} + M_D \]  
Equation (20)

In other words, when the superimposed load is greater than the first cracking load, effective moment of inertia \( I_e \) is used in the above Equations (14) and (15) given for dead load and live load deflections to calculate the final deflection. The spreadsheet developed for calculation of theoretical deflection for 5 mm PP1 slab is shown in Table 11.
Table 11: Spreadsheet for the Calculation of Theoretical Deflection for 5 mm PP1 Slab

<table>
<thead>
<tr>
<th>Data</th>
<th>Description</th>
<th>Value</th>
<th>Load, lbs</th>
<th>Ma, kip-ft</th>
<th>Ie, in4</th>
<th>Ther. Def, in</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>ft</td>
<td>3.33333</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f’c</td>
<td>psi</td>
<td>3580</td>
<td>0</td>
<td>0.0402</td>
<td>0.0006</td>
<td></td>
</tr>
<tr>
<td>Ec</td>
<td>psi</td>
<td>3410487</td>
<td>500</td>
<td>0.3944</td>
<td>0.0099</td>
<td></td>
</tr>
<tr>
<td>Ef</td>
<td>ksi</td>
<td>6032</td>
<td>1000</td>
<td>0.7485</td>
<td>0.0192</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>in</td>
<td>7.625</td>
<td>1017</td>
<td>0.7606</td>
<td>0.0196</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>in</td>
<td>17</td>
<td>1017</td>
<td>0.7606</td>
<td>24.3285</td>
<td>0.0327</td>
</tr>
<tr>
<td>bsp</td>
<td>in</td>
<td>6</td>
<td>1500</td>
<td>1.1027</td>
<td>9.3378</td>
<td>0.1243</td>
</tr>
<tr>
<td>h</td>
<td>in</td>
<td>4</td>
<td>1600</td>
<td>1.1735</td>
<td>8.0905</td>
<td>0.1529</td>
</tr>
<tr>
<td>dc</td>
<td>in</td>
<td>0.75</td>
<td>2000</td>
<td>1.4569</td>
<td>5.1911</td>
<td>0.2966</td>
</tr>
<tr>
<td>deff</td>
<td>in</td>
<td>3.25</td>
<td>2500</td>
<td>1.8110</td>
<td>3.6691</td>
<td>0.5227</td>
</tr>
<tr>
<td>MD</td>
<td>kip-ft</td>
<td>0.0402</td>
<td>3000</td>
<td>2.1652</td>
<td>2.9835</td>
<td>0.7697</td>
</tr>
<tr>
<td>Mcr</td>
<td>kip-ft</td>
<td>0.760379</td>
<td>3500</td>
<td>2.5194</td>
<td>2.6303</td>
<td>1.0169</td>
</tr>
<tr>
<td>Pcr</td>
<td>lbs</td>
<td>1016.724</td>
<td>4000</td>
<td>2.8735</td>
<td>2.4301</td>
<td>1.2564</td>
</tr>
<tr>
<td>n</td>
<td>in</td>
<td>1.768662</td>
<td>4382</td>
<td>3.1441</td>
<td>2.3322</td>
<td>1.4331</td>
</tr>
<tr>
<td>Af</td>
<td>in2</td>
<td>0.12907</td>
<td>4533</td>
<td>3.2511</td>
<td>2.3021</td>
<td>1.5015</td>
</tr>
<tr>
<td>Af,tr</td>
<td>in2</td>
<td>0.099211</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ac</td>
<td>in2</td>
<td>30.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>in2</td>
<td>30.59921</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y_bar</td>
<td>in</td>
<td>2.004053</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yt</td>
<td>in</td>
<td>1.995947</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Icr</td>
<td>in4</td>
<td>2.0164</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ig</td>
<td>in4</td>
<td>40.6667</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wd</td>
<td>klf</td>
<td>0.0318</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ_f</td>
<td>klf</td>
<td>0.005208</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k</td>
<td></td>
<td>0.126834</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ_fb</td>
<td></td>
<td>0.00174</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>β_d</td>
<td>Ok</td>
<td>0.598664</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.7 Analytical Results

This section includes the comparison of predicted concrete strains, bar strains and deflections with the experimental values for slab tests.

5.7.1 Comparison of Theoretical and Experimental Curves for Polypropylene fiber reinforced concrete slabs.

This section includes the theoretical and experimental curves for basalt FRP bar reinforced slabs with polypropylene fiber.

*Theoretical and Experimental Curves for 5 mm PP1 Slab:*

![Graph](image.png)

Fig. 58(a): Experimental and Theoretical load-concrete strain curve for 5 mm series1 slab with polypropylene fiber.
Fig. 58(b): Experimental and Theoretical load-bar strain curve for 5 mm series1 slab with polypropylene fiber.

Fig. 58(c): Experimental and Theoretical load-deflection curve for 5 mm series1 slab with polypropylene fiber.
From Fig. 58(a, b, and c) we can see that the predicted curves are matching well with the test curves. Gao’s equation (Gao et al., 1998a) for calculating deflections had also given good approximation of deflections in the slabs. The theoretical strain in the Basalt FRP bars is very well correlated with the strains from the experiment. The Proposed theory predicts a maximum load of 4552 lbs and Hognestad’s theory predicts a maximum load of 4533 lbs at a strain of 0.0038 in/in. When compared to experimental maximum load capacity, both the methods show a percentage difference less than 5% i.e, Proposed method with a percentage difference of 3.8% and Hognestad’s theory with a percentage difference of 3.4% which can be acceptable for design purposes.

*Theoretical and Experimental Curves for 5mm PP2 Slab:*

![Graph showing comparison of theoretical and experimental concrete compressive strain](image)

**Fig. 59(a):** Experimental and Theoretical load-concrete strain curve for 5 mm series2 slab with polypropylene fiber.
Fig. 59(b): Experimental and Theoretical load-bar strain curve for 5 mm series2 slab with polypropylene fiber.

Fig. 59(c): Experimental and Theoretical load-deflection curve for 5 mm series2 slab with polypropylene fiber.
A maximum load of 5017 lbs and 4982 lbs by Proposed theory and Hognestad’s theory respectively. These results show a percentage difference of 14% when compared to the maximum load capacity from the load test (5747 lbs). The nature of the test curves is matching with the theoretical curves but there seems to be noticeable difference in the strains and deflections predicted. There may have been a malfunction during this particular experiment.

*Theoretical and Experimental Curves for 7 mm PP1 Slab:*

![Graph showing comparison of theoretical and experimental concrete compressive strain](image)

Fig. 60(a): Experimental and Theoretical load-concrete strain curve for 7 mm series1 slab with polypropylene fiber.
Fig. 60(b): Experimental and Theoretical load-bar strain curve for 7 mm series1 slab with polypropylene fiber.

Fig. 60(c): Experimental and Theoretical load-deflection curve for 7 mm series1 slab with polypropylene fiber.
A maximum load of 4984 lbs and 4986 lbs are predicted by Proposed and Hognestad’s theories respectively. Both these results show a percentage difference of 12% when compared with the experimental maximum load capacity of 5610 lbs. The concrete compressive strains and bar strains are under-predicted by both the theories. So these theories would be conservative in the design of structural elements or in other words is a safer design for structural members using polypropylene fiber.

*Theoretical and Experimental Curves for 7 mm PP2 Slab*

![7 mm PP2 - Comparison of Theoretical Concrete Compressive Strain](image)

Fig. 61(a): Experimental and Theoretical load-concrete strain curve for 7 mm series2 slab with polypropylene fiber.
From Fig. 61(a, b, and c), we can see that the theoretical curves had correlated to some extent with the experimental curves. The concrete compressive strains have been
underestimated in all the slab tests except for 5 mm PP2 meaning that the theoretical method is conservative. Even the strains at the level of basalt reinforcing bar are correlating well the experimental data. The predicted curves for Series2 slabs (5 mm PP2 and 7 mm PP2) are little way off from the test curves even though the nature of the curve is predicted well. This may be due to the low fiber volume content i.e., 7.5 lb/yd³. So, we can say that a factor should be used for these theory curves when used for designing the structural members with volume percentages of 0.5% (<1%).

5.7.2 Comparison of Theoretical and Experimental Curves for Concrete Slabs without Polypropylene Fiber.

This section includes the theoretical and experimental curves for basalt FRP bar reinforced slabs without polypropylene fiber.

![Diagram](image_url)

**Fig. 62(a):** Experimental and Theoretical load-concrete strain curve for 5 mm series1 slab without fiber.
Fig. 62(b): Experimental and Theoretical load-bar strain curve for 5 mm series1 slab without fiber.

Fig. 62(c): Experimental and Theoretical load-deflection curve for 5 mm series1 slab without fiber.
Fig. 63(a): Experimental and Theoretical load-concrete strain curve for 5 mm series2 slab without fiber.

Fig. 63(b): Experimental and Theoretical load-bar strain curve for 5 mm series2 slab without fiber.
Fig. 63(c): Experimental and Theoretical load-deflection curve for 5 mm series2 slab without fiber.

Fig. 64(a): Experimental and Theoretical load-concrete strain curve for 7 mm series1 slab without fiber.
Fig. 64(b): Experimental and Theoretical load-bar strain curve for 7 mm series1 slab without fiber.

Fig. 64(c): Experimental and Theoretical load-deflection curve for 7 mm series1 slab without fiber.
Fig. 65(a): Experimental and Theoretical load-concrete strain curve for 7 mm series2 slab without fiber.

Fig. 65(b): Experimental and Theoretical load-bar strain curve for 7 mm series2 slab without fiber.
Fig. 65(c): Experimental and Theoretical load-deflection curve for 7 mm series2 slab without fiber.

From Fig. 62 to Fig. 65, we can see that both the Hognestad’s theory, Desayi and Krishnan theory gave similar theoretical concrete compressive strains and bar strains. The theoretical deflection is correlating well with the deflections recorded from the test. We can see that the bar strains of all the slabs are predicted well by the theoretical method. A factor may be used for the theoretical curves in case of concrete compressive strains in beams.

5.8 Theoretical and Experimental Stress – Strain Curves for Polypropylene Fiber Reinforced Concrete Cylinders and Plain Cylinders:

Apart from the cylinders tested to determine the compressive strength for slabs, six other cylinders of size 6" diameter × 12" height were tested to establish stress-strain curves of polypropylene fiber reinforced concrete. Among the six cylinders, three of the specimens were reinforced with polypropylene fiber and the other three were plain. Strain
gages were mounted on diametrically opposite side surfaces of cylinders at mid height. The load was imposed at increments of 5 kips. The corresponding strains were collected in the data acquisition system. The stresses were calculated by using the load over actual area of the cylinder \( (P_i / A_{cylinder}) \). The polypropylene reinforced cylinders went up to a maximum load of 110 kips where as the plain cylinders went up to a maximum load of 125 kips.

Fig. 66(a), (b), (c): Polypropylene Fiber reinforced Concrete Cylinders tested under Compression
The maximum compressive strength $f'_c$ is determined by dividing the maximum load by the area of cylinder. Then using the proposed approach, the points needed for determining the stress-strain curve is followed. Young’s modulus $= 57000\sqrt{f'_c}$ is used which was found to be almost equal to the experimental young’s modulus derived from the test points. The procedure discussed in Section 5.1 (proposed model) is followed to get the unknown coefficients of the stress-strain relation equations. The analytical stress-strain curve using the Proposed approach for polypropylene reinforced cylinders are plotted along with the experimental stress-strain curve in Fig. 68 to Fig. 70.

Hognestad’s model representing the stress-strain relationship of concrete is also derived for the cylinder tests and plotted along with the experimental and proposed model stress-strain curves in Fig. 68 to Fig. 70.

For $0 \leq \varepsilon_c \leq \varepsilon_o$: $f_c = f'_c (\varepsilon_c/\varepsilon_o)(2 - \varepsilon_c/\varepsilon_o)$;

For $\varepsilon_o \leq \varepsilon_c \leq \varepsilon_u$: $f_c = f'_c [1 - 0.15(\varepsilon_c - \varepsilon_o)/(\varepsilon_u - \varepsilon_o)]$;
\[ \varepsilon_o = 1.8 \frac{f_c^u}{E_c}; \]

Where,

\( \varepsilon_c \) = Strain at any point on the stress-strain curve;

\( f_c \) = Stress in concrete at a concrete strain \( \varepsilon_c; \)

\( \varepsilon_o \) = Concrete ultimate strain in Hognestad’s curve;

\( \varepsilon_u \) = Ultimate strain i.e., 0.0038;

\( f_c^u \) = Concrete ultimate strength in Hognestad’s curve/ Cylinder Compressive Strength;

---

Fig. 68: Experimental and Theoretical Compressive Stress-Strain Curve for 5 mm PP1 Cylinder.
Fig. 69: Experimental and Theoretical Compressive Stress-Strain Curve for 5 mm PP2 Cylinder.

Fig. 70: Experimental and Theoretical Compressive Stress-Strain Curve for 5 mm PP3 Cylinder.
From Figs. 68 to 70, the proposed model and Hognestad’s model are seen to be in good correlation and also correlate well with the experimental data up to the peak point. The correlation beyond the peak point is unknown as the test data was not recorded. The experimental and theoretical strains corresponding to the stresses are in good agreement up to a strain of about 0.002; beyond this, the predicted strain is slightly higher by about 10% (Orozco, 2004).

Similar to PP cylinders, two theoretical curves comprising Hognestad’s model and Desayi and Krishnan model for Plain cylinders are plotted in Fig. 71 to Fig. 73. Hognestad’s model is same as explained above where as Desayi and Krishnan equation used was:

\[ f_c = \frac{2f'_c \varepsilon}{\varepsilon_0 \left[ 1 + \left( \frac{\varepsilon}{\varepsilon_0} \right)^2 \right]} \]

Where,

- \( f_c \) = Stress in concrete at a concrete strain of \( \varepsilon \);
- \( f'_c \) = Peak compressive strength of concrete obtained from a cylinder test;
- \( \varepsilon_0 \) = Concrete strain at the peak compressive stress, assumed to be 0.002;

The experimental and theoretical stress-strain curves for Plain cylinders in compression are shown in Fig. 71 to Fig. 73. Both the theoretical models are in good correlation with the test data.
Fig. 71: Experimental and Theoretical Compressive Stress-Strain Curve for 5 mm Plain1 Cylinder.

Fig. 72: Experimental and Theoretical Compressive Stress-Strain Curve for 5 mm Plain2 Cylinder.
Similar to the polypropylene fiber reinforced concrete cylinders, the Hognestad’s model, Desayi and Krishnan curve correlate very well with the experimental results up to the peak point. Both the theoretical curves show that the predicted strains are in good agreement up to 0.002; beyond this, the test data was not recorded in this study.

5.9 Energy Dissipation for the slab tests based on different approaches

This section includes the discussion on the energy dissipation values calculated for the slab tests based on two different approaches.
5.9.1 Energy under Load-Deflection curve for Slabs based on Hognestad’s Model

The energy stored in concrete and FRP bars is calculated for the slab tests performed in this research study based on the Hognestad’s curve.

Table 12: Energy stored in Concrete and FRP bars based on Hognestad’s model

<table>
<thead>
<tr>
<th>Beam</th>
<th>$f_t$ ksi/Mpa</th>
<th>$E'_T$</th>
<th>$E'_f,Nm$</th>
<th>$E'_c,Nm$</th>
<th>$E'_c/E'_T$%</th>
<th>$E'_c/E'_f$</th>
<th>Concrete Strength, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 mm PP1</td>
<td>157.2/1083.9</td>
<td>494.55</td>
<td>215.55</td>
<td>279.00</td>
<td>56.41</td>
<td>1.29</td>
<td>3580</td>
</tr>
<tr>
<td>5 mm PP2</td>
<td>157.2/1083.9</td>
<td>592.00</td>
<td>247.50</td>
<td>344.50</td>
<td>58.19</td>
<td>1.39</td>
<td>4251</td>
</tr>
<tr>
<td>7 mm PP1</td>
<td>154.7/1066.62</td>
<td>329.12</td>
<td>177.99</td>
<td>151.13</td>
<td>45.92</td>
<td>0.85</td>
<td>2551</td>
</tr>
<tr>
<td>7 mm PP2</td>
<td>154.7/1066.62</td>
<td>577.16</td>
<td>269.42</td>
<td>307.74</td>
<td>53.32</td>
<td>1.14</td>
<td>3950</td>
</tr>
<tr>
<td>5 mm Plain1</td>
<td>157.2/1083.9</td>
<td>452.00</td>
<td>198.00</td>
<td>254.00</td>
<td>56.19</td>
<td>1.28</td>
<td>3315</td>
</tr>
<tr>
<td>5 mm Plain2</td>
<td>157.2/1083.9</td>
<td>707.50</td>
<td>286.00</td>
<td>421.50</td>
<td>59.58</td>
<td>1.47</td>
<td>5009</td>
</tr>
<tr>
<td>7 mm Plain1</td>
<td>154.7/1066.62</td>
<td>410.31</td>
<td>215.08</td>
<td>195.23</td>
<td>47.58</td>
<td>0.91</td>
<td>3058</td>
</tr>
<tr>
<td>7 mm Plain2</td>
<td>154.7/1066.62</td>
<td>627.50</td>
<td>301.50</td>
<td>326.00</td>
<td>51.95</td>
<td>1.08</td>
<td>4506</td>
</tr>
</tbody>
</table>

The strain energy stored in the basalt reinforcing bars is calculated assuming the Hognestad non-linear stress strain curve for concrete (Hognestad, 1955). The energy under the load-deflection curve up to a strain of 0.002 is considered as the energy in rebars, $E'_f$, and the remaining as the energy stored in concrete, $E'_c$. To do this, say for the slab test 5 mm PP1, the load corresponding to the strain 0.002 is calculated from the spreadsheet that was already available based on Hognestad’s approach. After knowing the load $P$, the deflection corresponding to load $P$ is pointed from the load-deflection curve. A line is drawn from load $P$ to the deflection point on x-axis. The area under the load-deflection curve up to this line is calculated as the energy in bars and the remaining area is attributed to the energy in concrete. The sum of the two energies is the total
energy stored in the slab. The ratio of the percentage of energy stored in concrete to that of the total energy and to that of the FRP rebars is calculated and shown in Table 12. This gives the relative amount of energy dissipated and stored in the concrete. The fraction of energy stored in concrete increases with the addition of polypropylene fibers and with the addition of reinforcing bars (Orozco, 2004). In the tests of the current project, except in the case of 5 mm PP1 slab and 7 mm PP2 slab, the energy in concrete is found to be little more than the energy stored in concrete for 5 mm Plain1 and 7 mm Plain1 respectively. In all the remaining cases, the plain slabs had showed more energy stored in concrete than the ones with polypropylene fiber. This shows that there is no significant difference with the addition of polypropylene fiber on the fraction of energy stored in concrete. This may be due to the use of very low fiber volumes. On the other side, even though the concrete strengths of the PP slabs are lower than the plain slabs, the percentage energy stored in concrete for all these slabs is found to be almost equal. The energy stored in FRP bars and in concrete based on Hognestad’s approach are shown in Figs. 74 to 81 of the load-deflection curves.
Fig. 74: Energy stored in Concrete and FRP bars for 5 mm PP1 Slab

Fig. 75: Energy stored in Concrete and FRP bars for 5 mm PP2 Slab
Fig. 76: Energy stored in Concrete and FRP bars for 7 mm PP1 Slab

Fig. 77: Energy stored in Concrete and FRP bars for 7 mm PP2 Slab
Fig. 78: Energy stored in Concrete and FRP bars for 5 mm Plain1 Slab

Fig. 79: Energy stored in Concrete and FRP bars for 5 mm Plain2 Slab
Fig. 80: Energy stored in Concrete and FRP bars for 7 mm Plain1 Slab

Fig. 81: Energy stored in Concrete and FRP bars for 7 mm Plain2 Slab
5.9.2 Energy under Load-Deflection curve for Slabs based on Proposed Model

The energy stored in concrete and FRP bars is calculated for the slab tests performed in this research study based on the proposed curve.

Table 13: Energy stored in Concrete and FRP bars based on Proposed model

<table>
<thead>
<tr>
<th>Beam</th>
<th>$f_i$ ksi/Mpa</th>
<th>$E'_f$</th>
<th>$E'_f$Nm</th>
<th>$E'_c$Nm</th>
<th>$E'_c$/E'_t, %</th>
<th>$E'_c$/E'_f</th>
<th>%diff b/w Proposed&amp;Hognestad Model</th>
<th>Cylinder Strength, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mmPP1</td>
<td>157.2/1083.9</td>
<td>498.97</td>
<td>211.67</td>
<td>287.30</td>
<td>57.58</td>
<td>1.36</td>
<td>0.9</td>
<td>3580</td>
</tr>
<tr>
<td>5mmPP2</td>
<td>157.2/1083.9</td>
<td>600.79</td>
<td>251.67</td>
<td>349.12</td>
<td>58.11</td>
<td>1.39</td>
<td>1.5</td>
<td>4251</td>
</tr>
<tr>
<td>7mmPP1</td>
<td>154.7/1066.62</td>
<td>331.22</td>
<td>170.03</td>
<td>161.19</td>
<td>48.67</td>
<td>0.95</td>
<td>0.6</td>
<td>2551</td>
</tr>
<tr>
<td>7mmPP2</td>
<td>154.7/1066.62</td>
<td>550.90</td>
<td>269.83</td>
<td>281.07</td>
<td>51.02</td>
<td>1.04</td>
<td>4.7</td>
<td>3950</td>
</tr>
<tr>
<td>5mmPlain1</td>
<td>157.2/1083.9</td>
<td>474.15</td>
<td>196.67</td>
<td>277.48</td>
<td>58.52</td>
<td>1.41</td>
<td>4.8</td>
<td>3315</td>
</tr>
<tr>
<td>5mmPlain2</td>
<td>157.2/1083.9</td>
<td>770.77</td>
<td>328.77</td>
<td>442.00</td>
<td>57.35</td>
<td>1.34</td>
<td>8.6</td>
<td>5009</td>
</tr>
<tr>
<td>7mmPlain1</td>
<td>154.7/1066.62</td>
<td>424.38</td>
<td>209.65</td>
<td>214.73</td>
<td>50.60</td>
<td>1.02</td>
<td>3.4</td>
<td>3058</td>
</tr>
<tr>
<td>7mmPlain2</td>
<td>154.7/1066.62</td>
<td>671.22</td>
<td>327.96</td>
<td>343.26</td>
<td>51.14</td>
<td>1.05</td>
<td>6.7</td>
<td>4506</td>
</tr>
</tbody>
</table>

The same procedure discussed in the Section 5.9.1 is repeated with the proposed approach. The energy under the load-deflection curve was based on the stress-strain curve developed by proposed approach. The energies stored in basalt reinforcing bars and in concrete are calculated and shown in Table 13. The ratio of energy stored in concrete to that of rebars and total energy is computed. Similar to the results shown by Hognestad’s model in Section 5.9.1, in spite of their lower concrete strengths, the percentage energy stored in concrete for the slabs with fiber is almost more or less equal to the percentage energy stored in concrete for the slabs without fiber. The percentage difference between the two methods is less than 10%. Further tests are needed to evaluate the influence of increase in fiber volume on the energy stored in the concrete.
Fig. 82: Energy stored in Concrete and FRP bars for 5 mm PP1 Slab

Fig. 83: Energy stored in Concrete and FRP bars for 5 mm PP2 Slab
Fig. 84: Energy stored in Concrete and FRP bars for 5 mm Plain1 Slab

Fig. 85: Energy stored in Concrete and FRP bars for 5 mm Plain2 Slab
Fig. 86: Energy stored in Concrete and FRP bars for 7 mm PP1 Slab

Fig. 87: Energy stored in Concrete and FRP bars for 7 mm PP2 Slab
Fig. 88: Energy stored in Concrete and FRP bars for 7 mm Plain1 Slab

Fig. 89: Energy stored in Concrete and FRP bars for 7 mm Plain2 Slab
5.10 Elastic energy versus In-elastic energy dissipation based on test curve

Naaman and Jeong (1995) had proposed a method to determine the elastic energy based on experimental load-deflection curve. The slopes of the two initial straight lines S1 and S2 of the load-deflection curve is noted. The weighted average of these slopes is S given by

\[ S = \frac{P_1S_1 + (P_2 - P_1)S_2}{P_2} \]

Fig. 90: Elastic and Inelastic energy (reproduced from Orozco and Maji, 2004).

Where \( P_1, \delta_1 \) are the load and deflection of the end point of the line having slope \( S_1 \); and \( P_2, \delta_2 \) are the load and deflection corresponding to the endpoint of the line having slope \( S_2 \).

Elastic energy is defined as the area of triangle formed by drawing a line from the failure load \( P_u \) on to the deflection axis with a slope \( S \) defined above. This area is considered as the elastic energy released at failure and the remaining area as the inelastic
energy consumed prior to failure. In the present study, the deflections are not noted until the point of failure for the slab test. Therefore, the test load-deflection curves are not available until the ultimate load $P_u$. So, an approximation of the curve is made to the experimental load-deflection curve from the last data point available on the curve till the ultimate load. This is done by extending the curve to the ultimate point on the theoretical load-deflection curve. The elastic and inelastic energies thus computed are listed below in Table 14. The sum of these energies is computed as total energy $E_T$. The total energy under the load-deflection curve obtained by this method is compared with the total energy calculated as the sum of energy in concrete and energy in rebars in the above section by Hognestad’s model and Proposed model. The percentage difference is listed in Table 14. The percentage difference is in between 8-50% for Hognestad’s model and 2-56% for proposed model when compared with Naaman and Jeong (1995).

Further, the ductility index $\mu$ is computed based on the general equation proposed by Naaman and Jeong (1995) which can be applied when FRP reinforcements are used. 

Ductility index is defined as:

$$\mu = \frac{1}{2} \left( \frac{E_{Total}}{E_{elastic}} + 1 \right)$$

Similar to the results shown in Table 12 and Table 13 corresponding to the energy stored in concrete to that of rebars and total energy, we can see that the ductility index is showing a decrease when fiber is added to the concrete mix as shown in Table 14. This is not in agreement with the conclusions made by Naaman and Jeong (1995) that the ductility index can be increased either with the addition of conventional reinforcing bars or by confinement or by addition of fibers to the concrete mix. This could be due to the
difference in concrete strength of the slabs. From Table 14, it can be seen that the average value of ductility index for slabs without fiber is higher than the average value of ductility index for corresponding slabs with fiber due to its high concrete strength. Orozco (2004) stated that the fraction of energy going to the concrete decreases with increasing number of rebars, leading to decreasing ductility with increasing reinforcement ratio. This can be observed in the ductility index values tabulated in Table 14. For example, the ductility index value for 7 mm PP1 slab is lesser than the ductility index value for 5 mm PP1 due to higher reinforcement ratio in 7 mm PP1 slab. A similar behavior is observed in all the slabs when compared with the corresponding slab of higher reinforcement ratio.

Table 14: Elastic and Inelastic Energy released at failure based on experimental load-deflection curve

<table>
<thead>
<tr>
<th>Beam</th>
<th>f&lt;sub&gt;b&lt;/sub&gt;, ksi/Mpa</th>
<th>E&lt;sub&gt;elastic,Nm&lt;/sub&gt;</th>
<th>E&lt;sub&gt;elastic,Nm&lt;/sub&gt;</th>
<th>E&lt;sub&gt;r, Nm&lt;/sub&gt;</th>
<th>%diff b/w Hognestad&amp; Test</th>
<th>%diff b/w Proposed&amp; Test</th>
<th>Ductility Index,μ</th>
<th>Average Ductility Index of Identical Beams</th>
<th>Cylinder Strength, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mmPP1</td>
<td>157.2/1083.9</td>
<td>122.14</td>
<td>342.45</td>
<td>464.59</td>
<td>-6.2</td>
<td>-7.1</td>
<td>2.40</td>
<td>3580</td>
<td></td>
</tr>
<tr>
<td>5mmPP2</td>
<td>157.2/1083.9</td>
<td>78.47</td>
<td>676.67</td>
<td>755.14</td>
<td>24.2</td>
<td>22.8</td>
<td>5.31</td>
<td>4251</td>
<td></td>
</tr>
<tr>
<td>7mmPP1</td>
<td>154.7/1066.62</td>
<td>169.37</td>
<td>288.52</td>
<td>457.89</td>
<td>32.7</td>
<td>32.1</td>
<td>1.85</td>
<td>2551</td>
<td></td>
</tr>
<tr>
<td>7mmPP2</td>
<td>154.7/1066.62</td>
<td>221.56</td>
<td>455.89</td>
<td>677.45</td>
<td>16.0</td>
<td>20.6</td>
<td>2.03</td>
<td>3950</td>
<td></td>
</tr>
<tr>
<td>5mmPlain1</td>
<td>157.2/1083.9</td>
<td>50.59</td>
<td>477.69</td>
<td>528.28</td>
<td>15.6</td>
<td>10.0</td>
<td>5.72</td>
<td>3315</td>
<td></td>
</tr>
<tr>
<td>5mmPlain2</td>
<td>157.2/1083.9</td>
<td>125.26</td>
<td>462.00</td>
<td>587.26</td>
<td>-18.6</td>
<td>-27.0</td>
<td>2.84</td>
<td>3315</td>
<td></td>
</tr>
<tr>
<td>7mmPlain1</td>
<td>154.7/1066.62</td>
<td>85.24</td>
<td>169.39</td>
<td>254.63</td>
<td>-46.8</td>
<td>-50.0</td>
<td>1.99</td>
<td>3058</td>
<td></td>
</tr>
<tr>
<td>7mmPlain2</td>
<td>154.7/1066.62</td>
<td>103.32</td>
<td>271.29</td>
<td>374.61</td>
<td>-50.5</td>
<td>-56.7</td>
<td>2.31</td>
<td>2.15</td>
<td>4506</td>
</tr>
</tbody>
</table>

5.11 Correlation in Behavior of the Beam and Round Determinate Panel Specimens

The relativity of the correlations found by Bernard (April 2002) in between the beam and panel specimens was studied for the new type of fiber (Ferro 2.25") used in this study. Bernard (April 2002) plotted the residual strength obtained from the beam tests at
0.5 mm central deflection with the energy absorbed in the panels up to a central deflection of 5 mm. The residual strength is calculated by using residual load capacity of the beam at specified deflection. The energy absorbed by the panel is obtained by calculating the area under the load-deflection curve up to the specified central deflection. The graph obtained by Bernard shows a good correlation between the beam and panel specimens. He concluded that the applications requiring shotcrete that exhibits good performance at small crack widths should therefore be assessed using beams or panels deflected to a low deformation (crack width of around 0.5 mm).

Similarly, in this study the results obtained from the ASTM C-1399 test and the ASTM C1550 round panel test are plotted along with the results of Bernard in the same graph. The average residual strength obtained from the three beams tested is taken average and plotted against the energy absorbed by the round panels 1 and 2. The graph obtained is shown in Fig. 91. From the figure, it is clear that the results obtained from this study correlate well with the published literature (Bernard, 2002).
Fig. 91: Correlation between the Beam and Panel Specimens obtained from the present study plotted against Bernard’s results (2002).
CHAPTER VI
ANALYSIS OF TEST RESULTS

The results obtained from the experimental program discussed in Chapter 4 are analyzed in this chapter comparing the differences between the results due to difference in concrete strength and the percentage volume of fiber. The maximum load capacity of slabs predicted using different analytical approaches are tabulated in this chapter and compared to the maximum load capacity obtained from test.

Table 15: Percentage Increase in Load Capacity of Slabs with addition of Polypropylene Fiber

<table>
<thead>
<tr>
<th>Fiber Type</th>
<th>5mmPP1</th>
<th>7mmPP1</th>
<th>5mmPP2</th>
<th>7mmPP2</th>
<th>5mmPlain1</th>
<th>7mmPlain1</th>
<th>5mmPlain2</th>
<th>7mmPlain2</th>
<th>% Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mmPP1</td>
<td>4382</td>
<td>5610</td>
<td></td>
<td></td>
<td>5mmPlain1</td>
<td>4262</td>
<td></td>
<td></td>
<td>-1.85</td>
</tr>
<tr>
<td>5mmPlain1</td>
<td>4464</td>
<td>4262</td>
<td></td>
<td></td>
<td>7mmPlain1</td>
<td>27.31</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Increase</td>
<td>-1.85</td>
<td>27.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5mmPP2</td>
<td>5747</td>
<td>6670</td>
<td></td>
<td></td>
<td>7mmPlain2</td>
<td>5258</td>
<td></td>
<td></td>
<td>19.16</td>
</tr>
<tr>
<td>5mmPlain2</td>
<td>4742</td>
<td>5258</td>
<td></td>
<td></td>
<td>7mmPlain2</td>
<td></td>
<td></td>
<td></td>
<td>23.68</td>
</tr>
<tr>
<td>%Increase</td>
<td>19.16</td>
<td>23.68</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In spite of the lower concrete strengths of the polypropylene fiber-reinforced concrete slabs compared to the plain slabs, there was increase in load capacity of about 20-30% with the addition of fiber to the plain concrete mix except for the 5 mm Series1.
Table 16: Percentage increase in Maximum Load Capacity of Slabs with increase in Compressive Strength

<table>
<thead>
<tr>
<th>Slab</th>
<th>Experimental Max. Load, lbs</th>
<th>Concrete Strength, psi</th>
<th>Slab</th>
<th>Experimental Max. Load, lbs</th>
<th>Concrete Strength, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mmPP2</td>
<td>5747</td>
<td>4251</td>
<td>7mmPP2</td>
<td>6670</td>
<td>3950</td>
</tr>
<tr>
<td>5mmPP1</td>
<td>4382</td>
<td>3580</td>
<td>7mmPP1</td>
<td>5610</td>
<td>2550.5</td>
</tr>
<tr>
<td>%Increase</td>
<td>26.95</td>
<td>%Increase</td>
<td>17.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5mmPlain2</td>
<td>4742</td>
<td>5009</td>
<td>7mmPlain2</td>
<td>5258</td>
<td>4506</td>
</tr>
<tr>
<td>5mmPlain1</td>
<td>4464</td>
<td>3315</td>
<td>7mmPlain1</td>
<td>4262</td>
<td>3058</td>
</tr>
<tr>
<td>%Increase</td>
<td>6.04</td>
<td>%Increase</td>
<td>20.92</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Even though the fiber volume for Series1 slabs is double that of the fiber volume for Series2 slabs, the maximum load capacity of Series2 slabs is high due to its higher compressive cylinder strength.

Table 17: Prediction of Theoretical Maximum Load Capacity for Polypropylene FRC Slabs using Parabolic Curve Method/ Proposed Model

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>Beam Dimensions, in</th>
<th>Cylinder Strength, psi</th>
<th>Area of Bars, in²</th>
<th>Dosage of fiber(PCY)</th>
<th>Experimental Max Load, lbs</th>
<th>Theoretical Max Load, lbs</th>
<th>Concrete Compressive Strain</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mmPP1</td>
<td>7.625x4x52</td>
<td>3580</td>
<td>0.129</td>
<td>15</td>
<td>4382</td>
<td>4552</td>
<td>0.0038</td>
<td>-3.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4710</td>
<td>0.004166</td>
<td>-7.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4869</td>
<td>0.004688</td>
<td>-10.5</td>
</tr>
<tr>
<td>Average % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.2</td>
</tr>
<tr>
<td>5mmPP2</td>
<td>7.625x4x52</td>
<td>4251</td>
<td>0.129</td>
<td>7.5</td>
<td>5747</td>
<td>5017</td>
<td>0.0038</td>
<td>13.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4212</td>
<td>0.0027</td>
<td>30.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4316</td>
<td>0.002815</td>
<td>28.4</td>
</tr>
<tr>
<td>Average % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>24.3</td>
</tr>
<tr>
<td>7mmPP1</td>
<td>8x4x52</td>
<td>2550.5</td>
<td>0.234</td>
<td>15</td>
<td>5610</td>
<td>4984</td>
<td>0.0038</td>
<td>11.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5242</td>
<td>0.004694</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5292</td>
<td>0.005784</td>
<td>5.8</td>
</tr>
<tr>
<td>Average % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.0</td>
</tr>
<tr>
<td>7mmPP2</td>
<td>8x4x52</td>
<td>3950</td>
<td>0.234</td>
<td>7.5</td>
<td>6670</td>
<td>6512</td>
<td>0.0038</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6260</td>
<td>0.00345</td>
<td>6.3</td>
</tr>
<tr>
<td>Average % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.4</td>
</tr>
<tr>
<td>Overall % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.9</td>
</tr>
</tbody>
</table>

*Strain corresponding to Experimental peak load is not available.
From Table 17, we can see that the predicted maximum loads show a percentage difference in between 4-8%, except for 5 mm PP2. The overall percentage difference in predicting the loads is 7.9% with test results being greater than the predicted values. Fig. 92 shows a plot of test load versus predicted load by proposed method at a strain of 0.0038 for all the polypropylene fiber-reinforced concrete slabs.

Fig. 92: Test Load Vs Predicted Load for Polypropylene FRC Slabs using Proposed Method
Table 18: Prediction of Theoretical Maximum Load Capacity for Polypropylene FRC Slabs using Desayi and Krishnan Curve for Plain Concrete

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>Beam Dimensions, in</th>
<th>Cylinder Strength, psi</th>
<th>Area of Bars, in²</th>
<th>Dosage of fiber (PCY)</th>
<th>Experimental Max Load, lbs</th>
<th>Theoretical Max Load, lbs</th>
<th>Concrete Comp. Strain</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mm PP1</td>
<td>7.625x4 x52</td>
<td>3580</td>
<td>0.12907</td>
<td>15</td>
<td>4382</td>
<td>4092/4504</td>
<td>0.003/0.0037</td>
<td>6.84/2.75</td>
</tr>
<tr>
<td></td>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.8</td>
</tr>
<tr>
<td>5mm PP2</td>
<td>7.625x4 x52</td>
<td>4251</td>
<td>0.12907</td>
<td>7.5</td>
<td>5747</td>
<td>4517/4890</td>
<td>0.003/0.00354</td>
<td>23.97/16.11</td>
</tr>
<tr>
<td></td>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>20.0</td>
</tr>
<tr>
<td>7mm PP1</td>
<td>8x4x52</td>
<td>2550.5</td>
<td>0.23436</td>
<td>15</td>
<td>5610</td>
<td>4518/5011</td>
<td>0.003/0.00395</td>
<td>21.56/11.28</td>
</tr>
<tr>
<td></td>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16.4</td>
</tr>
<tr>
<td>7mm PP2</td>
<td>8x4x52</td>
<td>3950</td>
<td>0.23436</td>
<td>7.5</td>
<td>6670</td>
<td>5896/6404</td>
<td>0.003/0.00362</td>
<td>12.32/4.07</td>
</tr>
<tr>
<td></td>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.2</td>
</tr>
<tr>
<td>Overall</td>
<td>% Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.6</td>
</tr>
</tbody>
</table>

From Table 18, we can see that the maximum load capacity predicted by Desayi and Krishnan curve for Polypropylene fiber reinforced concrete slabs is around 5-20% lower compared to experimental maximum load capacities. The overall percentage difference was found to be 8.6%. All the slabs failed at a larger load than predicted which can be seen in Fig. 93 plotted below.
Table 19: Prediction of Theoretical Maximum Load Capacity for Polypropylene FRC Slabs using Hognestad’s Model

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>Beam Dimensions, in</th>
<th>Cylinder Strength, psi</th>
<th>Area of Bars, in²</th>
<th>Dosage of fiber(PCY)</th>
<th>Experimental Max Load, lbs</th>
<th>Theoretical Max Load, lbs</th>
<th>Concrete Compressive Strain</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mm PP1</td>
<td>7.625x4x52</td>
<td>3580</td>
<td>0.1290</td>
<td>15</td>
<td>4382</td>
<td>4052</td>
<td>4533</td>
<td>0.003</td>
</tr>
<tr>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.6</td>
</tr>
<tr>
<td>5mm PP2</td>
<td>7.625x4x52</td>
<td>4251</td>
<td>0.1290</td>
<td>7.5</td>
<td>5747</td>
<td>4433</td>
<td>4982</td>
<td>0.003</td>
</tr>
<tr>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>20.0</td>
</tr>
<tr>
<td>7mm PP1</td>
<td>8x4x52</td>
<td>2550.5</td>
<td>0.2343</td>
<td>15</td>
<td>5610</td>
<td>4536</td>
<td>4986</td>
<td>0.003</td>
</tr>
<tr>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16.5</td>
</tr>
<tr>
<td>7mm PP2</td>
<td>8x4x52</td>
<td>3950</td>
<td>0.2343</td>
<td>7.5</td>
<td>6670</td>
<td>5813</td>
<td>6471</td>
<td>0.003</td>
</tr>
<tr>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.4</td>
</tr>
<tr>
<td>Overall % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.1</td>
</tr>
</tbody>
</table>
From Table 19, we can see that the overall percentage difference in predicting the loads by Hognestad’s model is found to be 8.1% (same as Proposed theory). The failure loads obtained from the tests are greater than those predicted. Fig. 94 shows a bar graph with test load versus predicted load by Hognestad’s model.

![Fig. 94: Test Load Vs Predicted Load for Polypropylene FRC Slabs using Hognestad Method.](image)

Table 20: Prediction of Theoretical Maximum Load Capacity for Polypropylene FRC Slabs using ACI 440.1R Method

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Beam Dimensions, in</th>
<th>Cylinder Strength, psi</th>
<th>Area of Bars, in²</th>
<th>Dosage of Fiber (PC Y)</th>
<th>Experimental Max Load, lbs</th>
<th>Max Load by ACI Method, lbs</th>
<th>Concrete Compressive Strain</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mm PP1</td>
<td>7.625x4x5 2</td>
<td>3580</td>
<td>0.12907</td>
<td>15</td>
<td>4382</td>
<td>3927</td>
<td>0.003</td>
<td>11.0</td>
</tr>
<tr>
<td>5mm PP2</td>
<td>7.625x4x5 2</td>
<td>4251</td>
<td>0.12907</td>
<td>7.5</td>
<td>5747</td>
<td>4259</td>
<td>0.003</td>
<td>29.7</td>
</tr>
<tr>
<td>7mm PP1</td>
<td>8x4x52</td>
<td>2550.5</td>
<td>0.23436</td>
<td>15</td>
<td>5610</td>
<td>4429</td>
<td>0.003</td>
<td>23.5</td>
</tr>
<tr>
<td>7mm PP2</td>
<td>8x4x52</td>
<td>3950</td>
<td>0.23436</td>
<td>7.5</td>
<td>6670</td>
<td>5596</td>
<td>0.003</td>
<td>17.5</td>
</tr>
<tr>
<td>Overall % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>20.4</td>
</tr>
</tbody>
</table>
The overall percentage difference in predicting the maximum loads using ACI 440.1R method for polypropylene fiber reinforced concrete slabs is slightly higher. It is around 20% (from Table 20) with failure loads obtained from tests much greater than ACI 440.1R predicted strengths as shown in Fig. 95.

Fig. 95: Test Load Vs Predicted Load for Polypropylene FRC Slabs using ACI Method
Table 21: Prediction of Theoretical Maximum Load Capacity for Plain Slabs using Desayi and Krishnan Curve:

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Beam Dimensions, in</th>
<th>Cylinder Strength, psi</th>
<th>Area of Bars, in²</th>
<th>Experimental Max Load, lbs</th>
<th>Theoretical Max Load, lbs</th>
<th>Concrete Compressive Strain</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mm Plain1</td>
<td>7.625x4x52</td>
<td>3315</td>
<td>0.12907</td>
<td>4464</td>
<td>3904 4334</td>
<td>0.003 0.003761</td>
<td>13.38 2.96</td>
</tr>
<tr>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.2</td>
</tr>
<tr>
<td>5mm Plain2</td>
<td>7.625x4x52</td>
<td>5009</td>
<td>0.12907</td>
<td>4742</td>
<td>4966 5250</td>
<td>0.003 0.003356</td>
<td>-4.61 -10.17</td>
</tr>
<tr>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.4</td>
</tr>
<tr>
<td>7mm Plain1</td>
<td>8x4x52</td>
<td>3058</td>
<td>0.23436</td>
<td>4262</td>
<td>5040 5572</td>
<td>0.003 0.003826</td>
<td>-16.73 -26.64</td>
</tr>
<tr>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>21.7</td>
</tr>
<tr>
<td>7mm Plain2</td>
<td>8x4x52</td>
<td>4506</td>
<td>0.23436</td>
<td>5258</td>
<td>6388 6823</td>
<td>0.003 0.003465</td>
<td>-19.41 -25.91</td>
</tr>
<tr>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>22.7</td>
</tr>
<tr>
<td>Overall % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16.4</td>
</tr>
</tbody>
</table>

All beams are over reinforced.

Fig. 96: Test Load Vs Predicted Load for Plain Slabs using Desayi and Krishnan Method
Table 22: Prediction of Theoretical Maximum Load Capacity for Plain Slabs using Hognestad’s Model

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Beam Dimensions, in</th>
<th>Cylinder Strength, psi</th>
<th>Area of Bars, in²</th>
<th>Experimental Max Load, lbs</th>
<th>Theoretical Max Load, lbs</th>
<th>Concrete Compressive Strain</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mm Plain1</td>
<td>7.625x4x52</td>
<td>3315</td>
<td>0.12907</td>
<td>4464</td>
<td>3889 4342</td>
<td>0.003 0.0038</td>
<td>13.77 2.77</td>
</tr>
<tr>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.3</td>
</tr>
<tr>
<td>5mm Plain2</td>
<td>7.625x4x52</td>
<td>5009</td>
<td>0.12907</td>
<td>4742</td>
<td>4820 5443</td>
<td>0.003 0.0038</td>
<td>-1.63 -13.77</td>
</tr>
<tr>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.7</td>
</tr>
<tr>
<td>7mm Plain1</td>
<td>8x4x52</td>
<td>3058</td>
<td>0.23436</td>
<td>4262</td>
<td>5038 5566</td>
<td>0.003 0.0038</td>
<td>-16.69 -26.54</td>
</tr>
<tr>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>21.6</td>
</tr>
<tr>
<td>7mm Plain2</td>
<td>8x4x52</td>
<td>4506</td>
<td>0.23436</td>
<td>5258</td>
<td>6242 6977</td>
<td>0.003 0.0038</td>
<td>-17.11 -28.10</td>
</tr>
<tr>
<td>Avg % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>22.6</td>
</tr>
<tr>
<td>Overall %</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>17.8</td>
</tr>
</tbody>
</table>

All beams are over reinforced.

Fig. 97: Test Load Vs Predicted Load for Plain Slabs using Hognestad Method
Table 23: Prediction of Theoretical Maximum Load Capacity for Plain Slabs using ACI 440.1R Method

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Beam Dimensions, in</th>
<th>Cylinder Strength, psi</th>
<th>Area of Bars, in²</th>
<th>Experimental Max Load, lbs</th>
<th>Max Load by ACI Method, lbs</th>
<th>Concrete Compressive Strain</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mm Plain1</td>
<td>7.625x4x52</td>
<td>3315</td>
<td>0.12907</td>
<td>4464</td>
<td>3781</td>
<td>0.003</td>
<td>16.6</td>
</tr>
<tr>
<td>5mm Plain2</td>
<td>7.625x4x52</td>
<td>5009</td>
<td>0.12907</td>
<td>4742</td>
<td>4576</td>
<td>0.003</td>
<td>3.6</td>
</tr>
<tr>
<td>7mm Plain1</td>
<td>8x4x52</td>
<td>3058</td>
<td>0.23436</td>
<td>4262</td>
<td>4899</td>
<td>0.003</td>
<td>-13.9</td>
</tr>
<tr>
<td>7mm Plain2</td>
<td>8x4x52</td>
<td>4506</td>
<td>0.23436</td>
<td>5258</td>
<td>5962</td>
<td>0.003</td>
<td>-12.5</td>
</tr>
<tr>
<td>Overall % Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11.6</td>
</tr>
</tbody>
</table>

All beams are over reinforced.

Fig. 98: Test Load Vs Predicted Load for Plain Slabs using ACI Method
From Tables 21, 22 and 23, we can see that the lowest percentage difference was found to be by ACI 440.1R method i.e., 11.6% in predicting the maximum load capacity of plain concrete slabs. The other two methods Desayi & Krishnan curve, Hognestad model yielded about the same percentage difference, but predicted larger load than the corresponding failure load obtained from tests.

Table 24: Comparison of Different Methods used for predicting maximum load for Polypropylene FRC slabs

<table>
<thead>
<tr>
<th>Beam</th>
<th>f, Ksi/Mpa</th>
<th>Max Load by Proposed Method, lbs</th>
<th>Max Load by Desayi &amp; Krishnan, lbs</th>
<th>Max Load by Hognestad, lbs</th>
<th>Max Load by ACI method, lbs</th>
<th>Exp. Max Load, lbs</th>
<th>%diff using Proposed method</th>
<th>%diff using D&amp;K</th>
<th>%diff using Hognestad</th>
<th>%diff using ACI</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mmPP1</td>
<td>157.2/1084</td>
<td>4552</td>
<td>4504</td>
<td>4533</td>
<td>3927</td>
<td>4382</td>
<td>-3.8</td>
<td>-2.7</td>
<td>-3.4</td>
<td>11.0</td>
</tr>
<tr>
<td>5mmPP2</td>
<td>157.2/1084</td>
<td>5017</td>
<td>4862</td>
<td>4986</td>
<td>4259</td>
<td>5747</td>
<td>13.6</td>
<td>16.1</td>
<td>14.3</td>
<td>29.7</td>
</tr>
<tr>
<td>7mmPP1</td>
<td>154.7/1067</td>
<td>4904</td>
<td>5011</td>
<td>4986</td>
<td>4429</td>
<td>5610</td>
<td>11.8</td>
<td>11.3</td>
<td>11.8</td>
<td>23.5</td>
</tr>
<tr>
<td>7mmPP2</td>
<td>154.7/1067</td>
<td>6512</td>
<td>6404</td>
<td>6471</td>
<td>5596</td>
<td>6670</td>
<td>2.4</td>
<td>4.1</td>
<td>3.0</td>
<td>17.5</td>
</tr>
<tr>
<td>Average %diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.9</td>
<td>8.6</td>
<td>8.1</td>
<td>20.4</td>
</tr>
</tbody>
</table>

From Table 24, we can see that the lowest average percentage difference between the predicted maximum load and experimental maximum load is by using Proposed method. Desayi and Krishnan curve (D&K), Hognestad’s curve also determined almost same maximum load capacities but are unconservative. ACI method predictions were slightly below compared to the experimental loads, but are safe predictions.

Table 25: Comparison of Different Methods used for predicting maximum load for Plain slabs

<table>
<thead>
<tr>
<th>Beam</th>
<th>f, Ksi/Mpa</th>
<th>Max Load by Desayi &amp; Krishnan, lbs</th>
<th>Max Load by Hognestad, lbs</th>
<th>Max Load by ACI method, lbs</th>
<th>Exp. Max Load, lbs</th>
<th>%diff using D&amp;K</th>
<th>%diff using Hognestad</th>
<th>%diff using ACI</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mmPlain1</td>
<td>157.2/1083.9</td>
<td>4334</td>
<td>4342</td>
<td>3781</td>
<td>4464</td>
<td>3.0</td>
<td>2.8</td>
<td>16.6</td>
</tr>
<tr>
<td>5mmPlain2</td>
<td>157.2/1083.9</td>
<td>5250</td>
<td>5443</td>
<td>4576</td>
<td>4742</td>
<td>-10.2</td>
<td>-13.8</td>
<td>3.6</td>
</tr>
<tr>
<td>7mmPlain1</td>
<td>154.7/1066.62</td>
<td>5572</td>
<td>5566</td>
<td>4899</td>
<td>4262</td>
<td>-26.6</td>
<td>-26.5</td>
<td>-13.9</td>
</tr>
<tr>
<td>7mmPlain2</td>
<td>154.7/1066.62</td>
<td>6823</td>
<td>6977</td>
<td>5962</td>
<td>5258</td>
<td>-25.9</td>
<td>-28.1</td>
<td>-12.5</td>
</tr>
<tr>
<td>Avg %Diff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16.4</td>
<td>17.8</td>
<td>11.6</td>
</tr>
</tbody>
</table>
From Table 25, we can see that the lowest average percentage difference between the predicted maximum load and experimental maximum load is by using ACI method i.e., 11.6%. The other two methods Desayi & Krishnan curve, Hognestad’s model predictions were slightly above the experimental loads. However, the test results and the comparisons need to be understood in the context of shear strength because all the test slabs failed by shear.
CHAPTER VII
CONCLUSIONS

This section includes the conclusions that have been drawn from the slab tests conducted on basalt FRP reinforced slabs reinforced with and without polypropylene fiber. It also includes the conclusions on polypropylene fiber-reinforced concrete beam and panel tests.

1. From the slab test results, it has been concluded that the load carrying capacity, concrete compressive strain and tensile bar strain of slabs are increased with the addition of fiber. The deflections of the slabs were decreased with the addition of fiber to the concrete.

2. The prediction of load-strain behavior of slabs by different methods studied in this report was reasonably close to the experimental load-strain behavior. In the case of slabs with polypropylene fiber, the Proposed, Hognestad, Desayi and Krishnan methods predicted load-carrying capacities of the slabs very close to failure loads obtained from the tests but are unconservative because of failures initiated in shear mode. The ACI 440.1R method predictions for the load-carrying capacity were found to be safer predictions. In case of slabs without polypropylene fiber, the ACI 440.1R method predicted the maximum load carrying capacity of the
slabs very close to the failure load from the test. The other two methods, Desayi and Krishnan and Hognestad methods predicted larger load than the corresponding failure load obtained from tests. However, the slab specimens failed mostly in shear mode.

3. The load-deflection behavior predicted using the ACI 440.1R method was found to be reasonably close to the experimental results. The shear strength prediction using the ACI 440.1R method was found to be low compared to the experimental shear strength. For the basalt FRP reinforced beams without fiber, the shear factor was found to be greater by about 40% compared to the shear factor given by ACI 440.1R. For the basalt FRP reinforced beams with polypropylene fiber, the shear factor was found to be even higher when compared to the beams without fiber and the difference in shear factor for the corresponding beams was found to be 3 (about 25 to 33%). Based on the analysis in this study, it can be concluded that an enhancement in the present shear strength equation given by ACI 440.1R may be possible for beams reinforced with FRP bars and polypropylene fiber.

4. The analysis of fracture energy of the slabs revealed that in spite of the lower concrete strengths of the slabs with fiber, percentage of energy absorbed in concrete for the basalt FRP reinforced slabs with polypropylene fiber is more or less equal to the percentage energy stored in concrete for the basalt FRP reinforced slabs without fiber. Elastic and inelastic energy dissipation analysis revealed that the ductility of the slab specimens decreases with increasing reinforcement ratio. With the addition of polypropylene fiber, the ductility of the slabs was found to be decreased when compared to the slabs without fiber. This
could be due to the lower concrete strength of the polypropylene fiber reinforced concrete slab specimens. Further tests are needed to evaluate the influence of increase in fiber volume on the ductility of the slabs.

5. The effect of the use of polypropylene fiber on the post-cracking strengths of beams and round panels was studied and the average residual strength obtained from the beam test and the toughness obtained from the round panel test were found to be greater than that compared to the results obtained from literature. From these tests, it was concluded that the correlation between the beams and panels conducted in the present study was consistent with earlier results reported by others.
REFERENCES


33. ACI Committee 318, 2005, “Building Code Requirements for Structural Concrete (ACI 318-05) and Commetary (318R-05),” American Concrete Institute, Farmington Hills, Mich., 430 pp.


APPENDIX A

LIST OF NOTATIONS

d - Effective depth of the slab
h - Overall depth of the slab
b - Width of the slab
d_c - Effective concrete cover
f_f - Tensile stress in the FRP bar
Φ - Average crack rotation angle
δ - Central deflection
r - Support radius
RI - Reinforcing Index
V_f - Volume fraction of fibers
l - Length of the fiber
Ø - Diameter of the fiber
ε - Strain in general
ε_p - Strain at peak stress
σ - Stress in general
f'_{cf} - Peak stress of fiber reinforced matrix
W_f - Weight fraction of fibers
\( \sigma_b \) — Flexural toughness factor

\( T_b \) — Flexural toughness

\( \delta_{tb} \) — Deflection of \( \frac{1}{150} \) of span

\( M_{cr} \) - Cracking moment

\( f_c \) - Modulus of rupture

\( I \) - Moment of inertia of gross section

\( y_t \) - Depth of neutral axis from extreme tension fiber

\( M_D \) - Moment due to dead load

\( y \) - Distance to the neutral axis from bottom tension fiber

\( \sigma \) - Stress in general

\( \varepsilon \) - Strain at a stress \( \sigma \)

\( E_c \) - Young’s modulus of concrete

\( f_c' \) - Peak compressive strength of concrete obtained from a cylinder test

\( \varepsilon_u \) - Ultimate strain

\( \rho_f \) - FRP reinforcement ratio

\( \rho_{fb} \) - Balanced FRP reinforcement ratio

\( M_n \) - Nominal flexural strength

\( \Phi \) - Strength reduction factor

\( E_f \) - Modulus of FRP bars

\( A_f \) - Area of FRP bars

\( s \) - Shear factor

\( n_f \) - Modular ratio between the FRP reinforcement and the concrete

\( I_{cr} \) - Moment of inertia of the cracked section

\( I_e \) - Effective moment of inertia of the section
$E'_f$ - Energy in FRP rebar

$E'_c$ - Energy stored in concrete

$\mu$ Ductility index
**Table B1:** Spreadsheet for Predicting Deflections for 5 mm Plain1 Slab

<table>
<thead>
<tr>
<th>Data</th>
<th>Value</th>
<th>Load, lbs</th>
<th>Ma, kip-ft</th>
<th>Ie, in4</th>
<th>Ther. Def, in</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>3.3333 ft</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f'_c</td>
<td>3315 psi</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ec</td>
<td>3281834 psi</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ef</td>
<td>6032 ksi</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>7.625 in</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>17.0 in</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bsp</td>
<td>6 in</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>4 in</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dc</td>
<td>0.75 in</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>deff</td>
<td>3.25 in</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MD</td>
<td>0.0402 kip-ft</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mcr</td>
<td>0.731695 kip-ft</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pcr</td>
<td>976.23 lbs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n_f</td>
<td>1.837997</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Af</td>
<td>0.12907 in²</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I_cr</td>
<td>2.0883 in⁴</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I_g</td>
<td>40.6667 in⁴</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W_D</td>
<td>0.0318 klf</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ_f</td>
<td>0.005208</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>0.129127</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ_fb</td>
<td>0.00164</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>β_d</td>
<td>0.635167</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shear Capacity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V_c</td>
<td>921.1935 lbs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>( L = 3.3333 \text{ ft} )</td>
<td>load, lbs</td>
<td>( M_a, \text{kip-ft} )</td>
<td>( I_e, \text{in}^4 )</td>
<td>Ther. Def, in</td>
</tr>
<tr>
<td>------</td>
<td>-------------------------</td>
<td>-----------</td>
<td>---------------------------</td>
<td>---------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>( f'c )</td>
<td>5009 psi</td>
<td>0</td>
<td>0.0402</td>
<td>0.0005</td>
<td></td>
</tr>
<tr>
<td>( E_c )</td>
<td>4034134 psi</td>
<td>500</td>
<td>0.3944</td>
<td>0.0084</td>
<td></td>
</tr>
<tr>
<td>( E_f )</td>
<td>6032 ksi</td>
<td>1000</td>
<td>0.7485</td>
<td>0.0163</td>
<td></td>
</tr>
<tr>
<td>( b )</td>
<td>7.625 in</td>
<td>1213</td>
<td>0.8994</td>
<td>0.0196</td>
<td></td>
</tr>
<tr>
<td>( a )</td>
<td>17 in</td>
<td>1213</td>
<td>0.8994</td>
<td>18.9123</td>
<td>0.0422</td>
</tr>
<tr>
<td>( bsp )</td>
<td>6 in</td>
<td>1500</td>
<td>1.1027</td>
<td>11.0531</td>
<td>0.0888</td>
</tr>
<tr>
<td>( h )</td>
<td>4 in</td>
<td>1700</td>
<td>1.2444</td>
<td>8.2174</td>
<td>0.1350</td>
</tr>
<tr>
<td>( dc )</td>
<td>0.75 in</td>
<td>2000</td>
<td>1.4569</td>
<td>5.7722</td>
<td>0.2255</td>
</tr>
<tr>
<td>( deff )</td>
<td>3.25 in</td>
<td>2500</td>
<td>1.8110</td>
<td>3.8338</td>
<td>0.4229</td>
</tr>
<tr>
<td>( M_D )</td>
<td>0.0402 kip-ft</td>
<td>3000</td>
<td>2.1652</td>
<td>2.9607</td>
<td>0.6557</td>
</tr>
<tr>
<td>( Mcr )</td>
<td>0.899423 kip-ft</td>
<td>3500</td>
<td>2.5194</td>
<td>2.5109</td>
<td>0.9006</td>
</tr>
<tr>
<td>( Pcr )</td>
<td>1213.022 lbs</td>
<td>4000</td>
<td>2.8735</td>
<td>2.2560</td>
<td>1.1442</td>
</tr>
<tr>
<td>( n )</td>
<td>1.49524 in</td>
<td>4500</td>
<td>3.2277</td>
<td>2.1009</td>
<td>1.3809</td>
</tr>
<tr>
<td>( A_f )</td>
<td>0.12907 in²</td>
<td>4742</td>
<td>3.3991</td>
<td>2.0474</td>
<td>1.4926</td>
</tr>
<tr>
<td>( I_{cr} )</td>
<td>1.7291 in⁴</td>
<td>5443</td>
<td>3.8957</td>
<td>1.9406</td>
<td>1.80591</td>
</tr>
<tr>
<td>( I_g )</td>
<td>40.6667 in⁴</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w_d )</td>
<td>0.0318 klf</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_f )</td>
<td>0.005208</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k )</td>
<td>0.117257</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_{fb} )</td>
<td>0.00224</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta_d )</td>
<td>0.465033 Ok</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_c )</td>
<td>1028.271 lbs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B3: Spreadsheet for Predicting Deflections for 5 mm PP1 Slab

<table>
<thead>
<tr>
<th>Data</th>
<th>ft, lbs, kip-ft, in, in4, lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>3.3333 3580 500 1000 1017 1017 1500 1600 2000 2500 4000</td>
</tr>
<tr>
<td>f'c</td>
<td>0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>Ec</td>
<td>0.0402 0.3944 0.7485 0.7606 0.7606 1.1027 1.1735 1.4569 1.8110 2.1652</td>
</tr>
<tr>
<td>Ef</td>
<td>0.7603 0.3944 0.7485 0.7606 0.7606 1.1027 1.1735 1.4569 1.8110 2.1652</td>
</tr>
<tr>
<td>b</td>
<td>6 17 7.625 0.75 3.25 0.402 0.3944 0.7485 0.7606 0.0402</td>
</tr>
<tr>
<td>a</td>
<td>6 17 7.625 0.75 3.25 0.402 0.3944 0.7485 0.7606 0.0402</td>
</tr>
<tr>
<td>bsp</td>
<td>6 17 7.625 0.75 3.25 0.402 0.3944 0.7485 0.7606 0.0402</td>
</tr>
<tr>
<td>h</td>
<td>4 4 4 4 4 4 4 4 4 4</td>
</tr>
<tr>
<td>dc</td>
<td>0.75 0.75 0.75 0.75 0.75 0.75 0.75 0.75 0.75 0.75</td>
</tr>
<tr>
<td>MD</td>
<td>0.0402 0.3944 0.7485 0.7606 0.7606 1.1027 1.1735 1.4569 1.8110 2.1652</td>
</tr>
<tr>
<td>Mcr</td>
<td>0.760379 0.760379 0.760379 0.760379 0.760379 0.760379 0.760379 0.760379 0.760379</td>
</tr>
<tr>
<td>n</td>
<td>1.768662 1.768662 1.768662 1.768662 1.768662 1.768662 1.768662 1.768662 1.768662</td>
</tr>
<tr>
<td>Af</td>
<td>0.12907 0.12907 0.12907 0.12907 0.12907 0.12907 0.12907 0.12907 0.12907</td>
</tr>
<tr>
<td>Ac</td>
<td>30.5 30.5 30.5 30.5 30.5 30.5 30.5 30.5 30.5</td>
</tr>
<tr>
<td>y_bar</td>
<td>2.004053 2.004053 2.004053 2.004053 2.004053 2.004053 2.004053 2.004053 2.004053</td>
</tr>
<tr>
<td>yt</td>
<td>1.995947 1.995947 1.995947 1.995947 1.995947 1.995947 1.995947 1.995947 1.995947</td>
</tr>
<tr>
<td>Icr</td>
<td>2.0164 2.0164 2.0164 2.0164 2.0164 2.0164 2.0164 2.0164 2.0164</td>
</tr>
<tr>
<td>Ig</td>
<td>40.6667 40.6667 40.6667 40.6667 40.6667 40.6667 40.6667 40.6667 40.6667</td>
</tr>
<tr>
<td>wd</td>
<td>0.0318 0.0318 0.0318 0.0318 0.0318 0.0318 0.0318 0.0318 0.0318</td>
</tr>
<tr>
<td>ρ_f</td>
<td>0.005208 0.005208 0.005208 0.005208 0.005208 0.005208 0.005208 0.005208 0.005208</td>
</tr>
<tr>
<td>k</td>
<td>0.126834 0.126834 0.126834 0.126834 0.126834 0.126834 0.126834 0.126834</td>
</tr>
<tr>
<td>ρ_f</td>
<td>0.00174 0.00174 0.00174 0.00174 0.00174 0.00174 0.00174 0.00174</td>
</tr>
<tr>
<td>β_d</td>
<td>0.598664 0.598664 0.598664 0.598664 0.598664 0.598664 0.598664 0.598664</td>
</tr>
<tr>
<td>Vc</td>
<td>940.3111 940.3111 940.3111 940.3111 940.3111 940.3111 940.3111 940.3111</td>
</tr>
</tbody>
</table>

176
**Table B4**: Spreadsheet for Predicting Deflections for 5 mm PP2 Slab

<table>
<thead>
<tr>
<th>Data</th>
<th></th>
<th>load, lbs</th>
<th>Ma,kip-ft</th>
<th>le, in4</th>
<th>Ther. Def, in</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>3.3333</td>
<td>ft</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f'c</td>
<td>4251</td>
<td>psi</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ec</td>
<td>3716383</td>
<td>psi</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ef</td>
<td>6032</td>
<td>ksi</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>7.625</td>
<td>in</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>17</td>
<td>in</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bsp</td>
<td>6</td>
<td>in</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>4</td>
<td>in</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dc</td>
<td>0.75</td>
<td>in</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>deff</td>
<td>3.25</td>
<td>in</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MD</td>
<td>0.0402</td>
<td>kip-ft</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mcr</td>
<td>0.828579</td>
<td>kip-ft</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pcr</td>
<td>1113.008</td>
<td>lbs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>1.623084</td>
<td>in2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Af</td>
<td>0.12907</td>
<td>in2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Af,tr</td>
<td>0.080421</td>
<td>in2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ac</td>
<td>30.5</td>
<td>in2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>30.58042</td>
<td>in2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y_bar</td>
<td>2.003287</td>
<td>in</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yt</td>
<td>1.996713</td>
<td>in</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Icr</td>
<td>1.8642</td>
<td>in4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ig</td>
<td>40.6667</td>
<td>in4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wd</td>
<td>0.0318</td>
<td>klf</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρf</td>
<td>0.005208</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>0.121849</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρfb</td>
<td>0.00199</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>βd</td>
<td>0.523455</td>
<td></td>
<td>Ok</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vc</td>
<td>984.3735</td>
<td>lbs</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table B5: Spreadsheet for Predicting Deflections for 7 mm Plain1 Slab

<table>
<thead>
<tr>
<th>Data</th>
<th>load, lbs</th>
<th>Ma,kip-ft</th>
<th>le, in4</th>
<th>Ther. Def, in</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>3.3333 ft</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f'c</td>
<td>3058 psi</td>
<td>0</td>
<td>0.0421</td>
<td>0.0007</td>
</tr>
<tr>
<td>Ec</td>
<td>3152054 psi</td>
<td>500</td>
<td>0.3963</td>
<td>0.0103</td>
</tr>
<tr>
<td>Ef</td>
<td>6530 ksi</td>
<td>981.5</td>
<td>0.7373</td>
<td>0.0195</td>
</tr>
<tr>
<td>b</td>
<td>8 in</td>
<td>981.5</td>
<td>0.7373</td>
<td>42.6657</td>
</tr>
<tr>
<td>a</td>
<td>17 in</td>
<td>1000</td>
<td>0.7504</td>
<td>40.6744</td>
</tr>
<tr>
<td>bsp</td>
<td>6 in</td>
<td>1500</td>
<td>1.1046</td>
<td>15.4866</td>
</tr>
<tr>
<td>h</td>
<td>4 in</td>
<td>2000</td>
<td>1.4588</td>
<td>8.9764</td>
</tr>
<tr>
<td>dc</td>
<td>0.75 in</td>
<td>2500</td>
<td>1.8129</td>
<td>6.5834</td>
</tr>
<tr>
<td>deff</td>
<td>3.25 in</td>
<td>3000</td>
<td>2.1671</td>
<td>5.5047</td>
</tr>
<tr>
<td>MD</td>
<td>0.0421 kip-ft</td>
<td>3500</td>
<td>2.5213</td>
<td>4.9486</td>
</tr>
<tr>
<td>Mcr</td>
<td>0.737322 kip-ft</td>
<td>4000</td>
<td>2.8754</td>
<td>4.6333</td>
</tr>
<tr>
<td>Pcr</td>
<td>981.4917 lbs</td>
<td>4200</td>
<td>3.0171</td>
<td>4.5456</td>
</tr>
<tr>
<td>n</td>
<td>2.071665 in</td>
<td>4262</td>
<td>3.0610</td>
<td>4.5217</td>
</tr>
<tr>
<td>Af</td>
<td>0.23436 in2</td>
<td>5566</td>
<td>3.9847</td>
<td>4.2261</td>
</tr>
<tr>
<td>Af,tr</td>
<td>0.251155 in2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ac</td>
<td>32 in2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>32.25116 in2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y_bar</td>
<td>2.009734 in</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yt</td>
<td>1.990266 in</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lcr</td>
<td>3.9810 in4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lg</td>
<td>42.6667 in4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wd</td>
<td>0.0333 klf</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρf</td>
<td>0.00901</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>0.175481</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρfb</td>
<td>0.00169</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>β_d</td>
<td>1.066727 Not Ok - Use 1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vc</td>
<td>1261.515 lbs</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B6: Spreadsheet for Predicting Deflections for 7 mm Plain2 Slab

<table>
<thead>
<tr>
<th>Data</th>
<th>3.33333 ft</th>
<th>load, lbs</th>
<th>Ma,kip-ft</th>
<th>le,in4</th>
<th>Ther. Def, in</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>3.33333 ft</td>
<td>3.33333 ft</td>
<td>3.33333 ft</td>
<td>3.33333 ft</td>
<td>3.33333 ft</td>
</tr>
<tr>
<td>f’c</td>
<td>4506 psi</td>
<td>0</td>
<td>0.0421</td>
<td>0.0006</td>
<td></td>
</tr>
<tr>
<td>Ec</td>
<td>3826225 psi</td>
<td>500</td>
<td>0.3963</td>
<td>0.0085</td>
<td></td>
</tr>
<tr>
<td>Ef</td>
<td>6530 ksi</td>
<td>1000</td>
<td>0.7504</td>
<td>0.0164</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>8 in</td>
<td>1204</td>
<td>0.8949</td>
<td>0.0196</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>17 in</td>
<td>1204</td>
<td>0.8949</td>
<td>33.5979</td>
<td>0.0249</td>
</tr>
<tr>
<td>bsp</td>
<td>6 in</td>
<td>1500</td>
<td>1.1046</td>
<td>19.4389</td>
<td>0.0533</td>
</tr>
<tr>
<td>h</td>
<td>4 in</td>
<td>2000</td>
<td>1.4588</td>
<td>10.3386</td>
<td>0.1328</td>
</tr>
<tr>
<td>dc</td>
<td>0.75 in</td>
<td>2500</td>
<td>1.8129</td>
<td>6.9937</td>
<td>0.2446</td>
</tr>
<tr>
<td>deff</td>
<td>3.25 in</td>
<td>3000</td>
<td>2.1671</td>
<td>5.4857</td>
<td>0.3733</td>
</tr>
<tr>
<td>MD</td>
<td>0.0421 kip-ft</td>
<td>3500</td>
<td>2.5213</td>
<td>4.7083</td>
<td>0.5066</td>
</tr>
<tr>
<td>Mcr</td>
<td>0.895023 kip-ft</td>
<td>4000</td>
<td>2.8754</td>
<td>4.2676</td>
<td>0.6380</td>
</tr>
<tr>
<td>Pcr</td>
<td>1204.128 lbs</td>
<td>4500</td>
<td>3.2296</td>
<td>3.9993</td>
<td>0.7651</td>
</tr>
<tr>
<td>n</td>
<td>1.706643</td>
<td>5000</td>
<td>3.5838</td>
<td>3.8268</td>
<td>0.8877</td>
</tr>
<tr>
<td>Af</td>
<td>0.23436 in2</td>
<td>5200</td>
<td>3.7254</td>
<td>3.7751</td>
<td>0.9356</td>
</tr>
<tr>
<td>Af,tr</td>
<td>0.165609 in2</td>
<td>5258</td>
<td>3.7665</td>
<td>3.7615</td>
<td>0.9494</td>
</tr>
<tr>
<td>Ac</td>
<td>32 in2</td>
<td>6977</td>
<td>4.9841</td>
<td>3.5309</td>
<td>1.33983</td>
</tr>
<tr>
<td>A</td>
<td>32.16561 in2</td>
<td>32</td>
<td>4.9841</td>
<td>3.5309</td>
<td>1.33983</td>
</tr>
<tr>
<td>y_bar</td>
<td>2.006436 in</td>
<td>1.993564 in</td>
<td>1.993564 in</td>
<td>1.993564 in</td>
<td>1.993564 in</td>
</tr>
<tr>
<td>yt</td>
<td>1.993564 in</td>
<td>3.3559 in4</td>
<td>3.3559 in4</td>
<td>3.3559 in4</td>
<td>3.3559 in4</td>
</tr>
<tr>
<td>Icr</td>
<td>3.3559 in4</td>
<td>42.6667 in4</td>
<td>42.6667 in4</td>
<td>42.6667 in4</td>
<td>42.6667 in4</td>
</tr>
<tr>
<td>Ig</td>
<td>42.6667 in4</td>
<td>0.0333 klf</td>
<td>0.0333 klf</td>
<td>0.0333 klf</td>
<td>0.0333 klf</td>
</tr>
<tr>
<td>wd</td>
<td>0.0333 klf</td>
<td>0.009014</td>
<td>0.009014</td>
<td>0.009014</td>
<td>0.009014</td>
</tr>
<tr>
<td>ρ_f</td>
<td>0.160695</td>
<td>0.00229</td>
<td>0.00229</td>
<td>0.00229</td>
<td>0.00229</td>
</tr>
<tr>
<td>ρ_fb</td>
<td>0.787235 Ok</td>
<td>0.787235 Ok</td>
<td>0.787235 Ok</td>
<td>0.787235 Ok</td>
<td>0.787235 Ok</td>
</tr>
<tr>
<td>Vc</td>
<td>1402.298 lbs</td>
<td>1402.298 lbs</td>
<td>1402.298 lbs</td>
<td>1402.298 lbs</td>
<td>1402.298 lbs</td>
</tr>
</tbody>
</table>
Table B7: Spreadsheet for Predicting Deflections for 7 mm PP1 Slab

<table>
<thead>
<tr>
<th>L</th>
<th>3.33333 ft</th>
<th>load, lbs</th>
<th>Ma,kip-ft</th>
<th>le, in4</th>
<th>Ther. Def, in</th>
</tr>
</thead>
<tbody>
<tr>
<td>f'c</td>
<td>2551 psi</td>
<td>0</td>
<td>0.0421</td>
<td>0.0008</td>
<td></td>
</tr>
<tr>
<td>Ec</td>
<td>2878923 psi</td>
<td>500</td>
<td>0.3963</td>
<td>0.0113</td>
<td></td>
</tr>
<tr>
<td>Ef</td>
<td>6530 ksi</td>
<td>891</td>
<td>0.6732</td>
<td>0.0195</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>8 in</td>
<td>891</td>
<td>0.6732</td>
<td>42.7022</td>
<td>0.0195</td>
</tr>
<tr>
<td>a</td>
<td>17 in</td>
<td>1000</td>
<td>0.7504</td>
<td>32.0293</td>
<td>0.0290</td>
</tr>
<tr>
<td>bsp</td>
<td>6 in</td>
<td>1500</td>
<td>1.1046</td>
<td>13.0009</td>
<td>0.1059</td>
</tr>
<tr>
<td>h</td>
<td>4 in</td>
<td>2000</td>
<td>1.4588</td>
<td>8.0826</td>
<td>0.2258</td>
</tr>
<tr>
<td>dc</td>
<td>0.75 in</td>
<td>2400</td>
<td>1.7421</td>
<td>6.5246</td>
<td>0.3347</td>
</tr>
<tr>
<td>deff</td>
<td>3.25 in</td>
<td>2500</td>
<td>1.8129</td>
<td>6.2749</td>
<td>0.3623</td>
</tr>
<tr>
<td>MD</td>
<td>0.0421 kip-ft</td>
<td>3000</td>
<td>2.1671</td>
<td>5.4599</td>
<td>0.4985</td>
</tr>
<tr>
<td>Mcr</td>
<td>0.673432 kip-ft</td>
<td>3500</td>
<td>2.5213</td>
<td>5.0398</td>
<td>0.6290</td>
</tr>
<tr>
<td>Pcr</td>
<td>891.2938 lbs</td>
<td>4000</td>
<td>2.8754</td>
<td>4.8016</td>
<td>0.7536</td>
</tr>
<tr>
<td>n</td>
<td>2.268209</td>
<td>4500</td>
<td>3.2296</td>
<td>4.6566</td>
<td>0.8733</td>
</tr>
<tr>
<td>Af</td>
<td>0.23436 in$^2$</td>
<td>5000</td>
<td>3.5838</td>
<td>4.5633</td>
<td>0.9894</td>
</tr>
<tr>
<td>Af,tr</td>
<td>0.297217 in$^2$</td>
<td>5500</td>
<td>3.9379</td>
<td>4.5007</td>
<td>1.1028</td>
</tr>
<tr>
<td>Ac</td>
<td>32 in$^2$</td>
<td>5610</td>
<td>4.0158</td>
<td>4.4897</td>
<td>1.1274</td>
</tr>
<tr>
<td>A</td>
<td>32.29722 in$^2$</td>
<td>5610</td>
<td>4.0158</td>
<td>4.4897</td>
<td>1.1274</td>
</tr>
<tr>
<td>y_bar</td>
<td>2.011503 in</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yt</td>
<td>1.988497 in</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Icr</td>
<td>4.3088 in$^4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ig</td>
<td>42.6667 in$^4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wd</td>
<td>0.0333 klf</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>0.009014</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>0.1828</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{fb}$</td>
<td>0.00145</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_d$</td>
<td>1.243289</td>
<td>Not Ok - Use 1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vc</td>
<td>1200.257 lbs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B8: Spreadsheet for Predicting Deflections for 7 mm PP2 Slab

<table>
<thead>
<tr>
<th>Data</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>L 3.33333 ft</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f'c 3950 psi</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ec 3582394 psi</td>
<td>500</td>
<td>0.3963</td>
<td>0.0090</td>
<td></td>
</tr>
<tr>
<td>Ef 6530 ksi</td>
<td>1000</td>
<td>0.7504</td>
<td>0.0175</td>
<td></td>
</tr>
<tr>
<td>b 8 in</td>
<td>1124</td>
<td>0.8383</td>
<td>0.0196</td>
<td></td>
</tr>
<tr>
<td>a 17 in</td>
<td>1124</td>
<td>0.8383</td>
<td>36.9466</td>
<td>0.0226</td>
</tr>
<tr>
<td>bsp 6 in</td>
<td>1500</td>
<td>1.1046</td>
<td>18.1500</td>
<td>0.0610</td>
</tr>
<tr>
<td>h 4 in</td>
<td>2000</td>
<td>1.4588</td>
<td>9.8930</td>
<td>0.1483</td>
</tr>
<tr>
<td>dc 0.75 in</td>
<td>2500</td>
<td>1.8129</td>
<td>6.8580</td>
<td>0.2664</td>
</tr>
<tr>
<td>deff 3.25 in</td>
<td>3000</td>
<td>2.1671</td>
<td>5.4897</td>
<td>0.3984</td>
</tr>
<tr>
<td>MD 0.0421 kip-ft</td>
<td>3500</td>
<td>2.5213</td>
<td>4.7844</td>
<td>0.5325</td>
</tr>
<tr>
<td>Mcr 0.837987 kip-ft</td>
<td>4000</td>
<td>2.8754</td>
<td>4.3845</td>
<td>0.6632</td>
</tr>
<tr>
<td>Pcr 1123.607 lbs</td>
<td>4500</td>
<td>3.2296</td>
<td>4.1411</td>
<td>0.7892</td>
</tr>
<tr>
<td>n 1.822803</td>
<td>5000</td>
<td>3.5838</td>
<td>3.9845</td>
<td>0.9106</td>
</tr>
<tr>
<td>Af 0.23436 in²</td>
<td>5500</td>
<td>3.9379</td>
<td>3.8793</td>
<td>1.0282</td>
</tr>
<tr>
<td>Af,tr 0.192832 in²</td>
<td>6000</td>
<td>4.2921</td>
<td>3.8060</td>
<td>1.1427</td>
</tr>
<tr>
<td>Ac 32 in²</td>
<td>6500</td>
<td>4.6463</td>
<td>3.7533</td>
<td>1.2547</td>
</tr>
<tr>
<td>A 32.19283 in²</td>
<td>6670</td>
<td>4.7667</td>
<td>3.7388</td>
<td>1.2923</td>
</tr>
<tr>
<td>y_bar 2.007487 in</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yt 1.992513 in</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Icr 3.5572 in⁴</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ig 42.6667 in⁴</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wd 0.0333 klf</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρf 0.009014</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k 0.165588</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρfb 0.00208</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>βd 0.866716 Ok</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vc 1352.92 lbs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B9: Load-Strain data for 5 mm Plain1 Slab based on Hognestad’s Model

<table>
<thead>
<tr>
<th>load (lbs)</th>
<th>Conc (in/in)</th>
<th>Bar (in/in)</th>
<th>Moment (kip-ft)</th>
<th>Stress (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.228E-06</td>
<td>4.518E-06</td>
<td>0.0402</td>
<td>23.723</td>
</tr>
<tr>
<td>100</td>
<td>1.997E-05</td>
<td>1.248E-05</td>
<td>0.1110</td>
<td>65.522</td>
</tr>
<tr>
<td>500</td>
<td>7.091E-05</td>
<td>4.432E-05</td>
<td>0.3944</td>
<td>232.721</td>
</tr>
<tr>
<td>800</td>
<td>1.091E-04</td>
<td>6.820E-05</td>
<td>0.6069</td>
<td>358.120</td>
</tr>
<tr>
<td>976</td>
<td>1.315E-04</td>
<td>8.221E-05</td>
<td>0.7315</td>
<td>431.687</td>
</tr>
<tr>
<td>976</td>
<td>0.0005379</td>
<td>0.00363</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>0.000551</td>
<td>0.00371</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1200</td>
<td>0.000664</td>
<td>0.00442</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1400</td>
<td>0.000781</td>
<td>0.00513</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>0.000841</td>
<td>0.00549</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.001156</td>
<td>0.00726</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.001511</td>
<td>0.00906</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3012</td>
<td>0.00194</td>
<td>0.01093</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3500</td>
<td>0.002465</td>
<td>0.01277</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3889</td>
<td>0.003</td>
<td>0.01429</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>0.003175</td>
<td>0.01473</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4323</td>
<td>0.003761</td>
<td>0.01605</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>4342</strong></td>
<td><strong>0.0038</strong></td>
<td><strong>0.01613</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4464</td>
<td>0.00406</td>
<td>0.01664</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Theoretical Strain
<table>
<thead>
<tr>
<th>Load (lbs)</th>
<th>Conc. Strain</th>
<th>Bar. Strain</th>
<th>Moment (k-ft)</th>
<th>Stress (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.88048E-06</td>
<td>3.6753E-06</td>
<td>0.0402</td>
<td>23.72</td>
</tr>
<tr>
<td>500</td>
<td>5.76879E-05</td>
<td>3.6055E-05</td>
<td>0.3944</td>
<td>232.72</td>
</tr>
<tr>
<td>1000</td>
<td>0.000109495</td>
<td>6.8435E-05</td>
<td>0.7485</td>
<td>441.72</td>
</tr>
<tr>
<td>1213</td>
<td>0.000131565</td>
<td>8.2228E-05</td>
<td>0.8994</td>
<td>530.75</td>
</tr>
</tbody>
</table>

Theoretical Strain, in/in

Table B10: Load-Strain data for 5 mm Plain2 Slab based on Hogestad’s Model
Table B11: Load-Strain data for 5 mm PP1 Slab based on Hognestad’s Model

<table>
<thead>
<tr>
<th>Load (conc strain)</th>
<th>Bar. Strain</th>
<th>Conc. Strain</th>
<th>Theoretical Strain by Strain Comp</th>
<th>Theor. Strains based on gross properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>500</td>
<td>0.0000682</td>
<td>0.0000682</td>
<td>0.000042625</td>
<td>500</td>
</tr>
<tr>
<td>1000</td>
<td>0.0001295</td>
<td>0.0001295</td>
<td>8.09375E-05</td>
<td>1000</td>
</tr>
<tr>
<td>1017</td>
<td>0.0005467</td>
<td>0.00377</td>
<td>0.000132</td>
<td>1017</td>
</tr>
<tr>
<td>1500</td>
<td>0.000817</td>
<td>0.00548</td>
<td>0.000825</td>
<td></td>
</tr>
<tr>
<td>1600</td>
<td>0.000875</td>
<td>0.00583</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.00112</td>
<td>0.00725</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.001455</td>
<td>0.00904</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>0.00184</td>
<td>0.01086</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3500</td>
<td>0.002313</td>
<td>0.01271</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4052</td>
<td>0.003</td>
<td>0.01484</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4382</td>
<td>0.003526</td>
<td>0.01616</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4480</td>
<td>0.0037</td>
<td>0.01656</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4533</td>
<td>0.0038</td>
<td>0.01677</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B12: Load-Strain data for 5 mm PP2 Slab based on Hognestad’s Model

<table>
<thead>
<tr>
<th>load, lbs</th>
<th>Conc Bar conc. Strain</th>
<th>Theoretical Strain based on gross prop load, lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 0 0 0</td>
<td>0 0 0 0</td>
</tr>
<tr>
<td>500</td>
<td></td>
<td>0.0006263 500 3.91438E-05</td>
</tr>
<tr>
<td>1000</td>
<td></td>
<td>0.001189 1000 7.43125E-05</td>
</tr>
<tr>
<td>1113</td>
<td>0.000566 0.0041</td>
<td>0.0001316 1113 0.00008225</td>
</tr>
<tr>
<td>1500</td>
<td>0.000769 0.00546</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.001047 0.00723</td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.001347 0.00901</td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>0.001677 0.0108</td>
<td></td>
</tr>
<tr>
<td>3500</td>
<td>0.002052 0.01261</td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>0.002508 0.01446</td>
<td></td>
</tr>
<tr>
<td>4180</td>
<td>0.0027 0.01514</td>
<td></td>
</tr>
<tr>
<td>4281</td>
<td>0.002815 0.01553</td>
<td></td>
</tr>
<tr>
<td>4433</td>
<td>0.003 0.01611</td>
<td></td>
</tr>
<tr>
<td>4795</td>
<td>0.0035 0.01753</td>
<td></td>
</tr>
<tr>
<td>4922</td>
<td>0.0037 0.01804</td>
<td></td>
</tr>
<tr>
<td>4982</td>
<td>0.0038 0.01829</td>
<td></td>
</tr>
</tbody>
</table>
Table B13: Load-Strain data for 7 mm Plain1 Slab based on Hogestad’s Model

<table>
<thead>
<tr>
<th>load (lbs)</th>
<th>Conc.Strain</th>
<th>Bar.strain</th>
<th>Moment (kip-ft)</th>
<th>Stress (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.51237E-06</td>
<td>4.69523E-06</td>
<td>0.0421</td>
<td>23.6794</td>
</tr>
<tr>
<td>500</td>
<td>7.071E-05</td>
<td>4.41937E-05</td>
<td>0.396265</td>
<td>222.88165</td>
</tr>
<tr>
<td>982</td>
<td>0.000131632</td>
<td>8.22703E-05</td>
<td>0.73768006</td>
<td>414.91262</td>
</tr>
<tr>
<td>982</td>
<td>0.0003966</td>
<td>0.00189</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>0.000404</td>
<td>0.00192</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>0.00061</td>
<td>0.00284</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.00083</td>
<td>0.00376</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2065</td>
<td>0.000859</td>
<td>0.00388</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.001066</td>
<td>0.00468</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2866</td>
<td>0.001252</td>
<td>0.00536</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>0.001324</td>
<td>0.00562</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3500</td>
<td>0.001615</td>
<td>0.00656</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>0.00196</td>
<td>0.00753</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4200</td>
<td>0.002122</td>
<td>0.00793</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4262</td>
<td>0.002175</td>
<td>0.00806</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4500</td>
<td>0.002395</td>
<td>0.00854</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5038</td>
<td>0.003</td>
<td>0.00968</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5388</td>
<td>0.0035</td>
<td>0.01045</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5566</td>
<td>0.0038</td>
<td>0.01086</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B14: Load-Strain data for 7 mm Plain2 Slab based on Hognestad’s Model

<table>
<thead>
<tr>
<th>Load (lbs)</th>
<th>Conc. Strain</th>
<th>Bar. Strain</th>
<th>Moment (kip-ft)</th>
<th>Stress (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0421</td>
<td>23.68</td>
</tr>
<tr>
<td>500</td>
<td>6.18871E-06</td>
<td>3.86794E-06</td>
<td>0.3963</td>
<td>222.88</td>
</tr>
<tr>
<td>700</td>
<td>7.9076E-05</td>
<td>4.94225E-05</td>
<td>0.5379</td>
<td>302.56</td>
</tr>
<tr>
<td>1000</td>
<td>0.000110313</td>
<td>6.89459E-05</td>
<td>0.7504</td>
<td>422.08</td>
</tr>
<tr>
<td>1204</td>
<td>0.000131555</td>
<td>8.22218E-05</td>
<td>0.8949</td>
<td>503.36</td>
</tr>
<tr>
<td>1204</td>
<td>0.0004284</td>
<td>0.00228</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>0.000535</td>
<td>0.00282</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1863</td>
<td>0.000668</td>
<td>0.00348</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.00072</td>
<td>0.00373</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.000913</td>
<td>0.00464</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>0.001116</td>
<td>0.00556</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3500</td>
<td>0.00133</td>
<td>0.00647</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>0.00156</td>
<td>0.0074</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4500</td>
<td>0.001809</td>
<td>0.00834</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>0.002085</td>
<td>0.00928</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5200</td>
<td>0.002206</td>
<td>0.00967</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5258</td>
<td>0.002242</td>
<td>0.00978</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5634</td>
<td>0.0025</td>
<td>0.01051</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6242</td>
<td>0.003</td>
<td>0.01174</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6699</td>
<td>0.003465</td>
<td>0.01269</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6977</td>
<td>0.0038</td>
<td>0.01329</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B15: Load-Strain data for 7 mm PP1 Slab based on Hognestad’s Model

<table>
<thead>
<tr>
<th>load</th>
<th>Conc. Strain</th>
<th>Bar. Strain</th>
<th>Theoretical Strain</th>
<th>Theoretical Strain based on Gross prop.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>500</td>
<td>0.0000774</td>
<td>0.00048375</td>
<td>500</td>
<td>0.000048375</td>
</tr>
<tr>
<td>891</td>
<td>0.0003825</td>
<td>0.001188</td>
<td>891</td>
<td>0.00007425</td>
</tr>
<tr>
<td>1000</td>
<td>0.00043</td>
<td>0.001315</td>
<td>800</td>
<td>0.0001188</td>
</tr>
<tr>
<td>1500</td>
<td>0.000652</td>
<td>0.00285</td>
<td>891</td>
<td>8.21875E-05</td>
</tr>
<tr>
<td>2000</td>
<td>0.000893</td>
<td>0.00378</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2400</td>
<td>0.001103</td>
<td>0.00452</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.001158</td>
<td>0.00471</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>0.001461</td>
<td>0.00566</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3500</td>
<td>0.00183</td>
<td>0.00664</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>0.00231</td>
<td>0.00766</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4500</td>
<td>0.002946</td>
<td>0.00873</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4536</td>
<td>0.003</td>
<td>0.00881</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4986</td>
<td>0.0038</td>
<td>0.00986</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load</td>
<td>Conc. Strain</td>
<td>Bar. Strain</td>
<td>Conc. Strain</td>
<td>Load</td>
</tr>
<tr>
<td>------</td>
<td>--------------</td>
<td>-------------</td>
<td>--------------</td>
<td>------</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>500</td>
</tr>
<tr>
<td>500</td>
<td>0.000118</td>
<td>1000</td>
<td>0.00007375</td>
<td></td>
</tr>
<tr>
<td>1124</td>
<td>0.0001316</td>
<td>1124</td>
<td>0.00008225</td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>0.000559</td>
<td>0.00283</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.000754</td>
<td>0.00374</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.00096</td>
<td>0.00466</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>0.001178</td>
<td>0.00557</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3500</td>
<td>0.001412</td>
<td>0.0065</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>0.001668</td>
<td>0.00744</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4500</td>
<td>0.001953</td>
<td>0.00839</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>0.002288</td>
<td>0.00936</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5500</td>
<td>0.002697</td>
<td>0.01036</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5813</td>
<td>0.003</td>
<td>0.01101</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6000</td>
<td>0.0032</td>
<td>0.0114</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6210</td>
<td>0.00345</td>
<td>0.01185</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6326</td>
<td>0.0036</td>
<td>0.0121</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6400</td>
<td>0.0037</td>
<td>0.01227</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6471</td>
<td>0.0038</td>
<td>0.01243</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Table B17**: Load-Strain data for 5 mm Plain1 Slab based on Desayi & Krishnan Curve

<table>
<thead>
<tr>
<th>load (lbs)</th>
<th>Conc.Strain</th>
<th>Bar.Strain</th>
<th>Moment (k-ft)</th>
<th>Stress (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.228E-06</td>
<td>4.518E-06</td>
<td>0.0402</td>
<td>23.723</td>
</tr>
<tr>
<td>100</td>
<td>1.997E-05</td>
<td>1.248E-05</td>
<td>0.1110</td>
<td>65.522</td>
</tr>
<tr>
<td>500</td>
<td>7.091E-05</td>
<td>4.432E-05</td>
<td>0.3944</td>
<td>232.721</td>
</tr>
<tr>
<td>800</td>
<td>1.091E-04</td>
<td>6.820E-05</td>
<td>0.6069</td>
<td>358.120</td>
</tr>
<tr>
<td>976</td>
<td>1.315E-04</td>
<td>8.221E-05</td>
<td>0.7315</td>
<td>431.687</td>
</tr>
<tr>
<td>976</td>
<td>0.000546</td>
<td>0.00363</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>0.00056</td>
<td>0.00372</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1200</td>
<td>0.00067</td>
<td>0.00441</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1400</td>
<td>0.00079</td>
<td>0.00515</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>0.00085</td>
<td>0.00551</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1600</td>
<td>0.00091</td>
<td>0.00586</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1700</td>
<td>0.00097</td>
<td>0.00621</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1800</td>
<td>0.00103</td>
<td>0.00655</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1900</td>
<td>0.00110</td>
<td>0.00694</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.00110</td>
<td>0.00726</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2100</td>
<td>0.00120</td>
<td>0.00764</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2200</td>
<td>0.00130</td>
<td>0.0081</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2400</td>
<td>0.00140</td>
<td>0.0087</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.00150</td>
<td>0.00908</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2669</td>
<td>0.00160</td>
<td>0.00968</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2800</td>
<td>0.00170</td>
<td>0.01016</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2900</td>
<td>0.00180</td>
<td>0.01054</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>0.00190</td>
<td>0.0109</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3100</td>
<td>0.00200</td>
<td>0.01125</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3200</td>
<td>0.00210</td>
<td>0.01163</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3400</td>
<td>0.00230</td>
<td>0.01241</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3500</td>
<td>0.00240</td>
<td>0.01277</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3600</td>
<td>0.00250</td>
<td>0.01315</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3800</td>
<td>0.00280</td>
<td>0.01394</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3900</td>
<td>0.00290</td>
<td>0.01429</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>0.00310</td>
<td>0.01473</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4100</td>
<td>0.00330</td>
<td>0.01514</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4200</td>
<td>0.00340</td>
<td>0.01552</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4300</td>
<td>0.00360</td>
<td>0.01594</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4334</td>
<td>0.00376</td>
<td>0.01608</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Theoretical Strain
**Table B18**: Load-Strain data for 5 mm Plain2 Slab based on Desayi & Krishnan Curve

<table>
<thead>
<tr>
<th>load (lbs)</th>
<th>Conc.Strain</th>
<th>Bar.Strain</th>
<th>Moment, (k-ft)</th>
<th>Stress (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.880E-06</td>
<td>3.675E-06</td>
<td>0.0402</td>
<td>23.72</td>
</tr>
<tr>
<td>500</td>
<td>5.769E-05</td>
<td>3.605E-05</td>
<td>0.3944</td>
<td>232.72</td>
</tr>
<tr>
<td>1000</td>
<td>1.095E-04</td>
<td>6.843E-05</td>
<td>0.7485</td>
<td>441.72</td>
</tr>
<tr>
<td>1213</td>
<td>1.316E-04</td>
<td>8.223E-05</td>
<td>0.8994</td>
<td>530.75</td>
</tr>
<tr>
<td>1213</td>
<td>0.0005341</td>
<td>0.00443</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>0.000661</td>
<td>0.00543</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1501</td>
<td>0.000662</td>
<td>0.00544</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1700</td>
<td>0.00075</td>
<td>0.00611</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.000893</td>
<td>0.00718</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.001144</td>
<td>0.00895</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2778</td>
<td>0.001294</td>
<td>0.00993</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>0.00142</td>
<td>0.01072</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3462</td>
<td>0.001705</td>
<td>0.01237</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3500</td>
<td>0.00173</td>
<td>0.01251</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>0.002089</td>
<td>0.01433</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4001</td>
<td>0.00209</td>
<td>0.01433</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4500</td>
<td>0.002514</td>
<td>0.01618</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4742</td>
<td>0.00275</td>
<td>0.01708</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4966</td>
<td>0.003</td>
<td>0.01795</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5250</td>
<td>0.003356</td>
<td>0.019053</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Table B19**: Load-Strain data for 7 mm Plain1 Slab based on Desayi & Krishnan Curve

<table>
<thead>
<tr>
<th>Load (lbs)</th>
<th>Conc. Strain</th>
<th>Bar. Strain</th>
<th>Moment (k-ft)</th>
<th>Stress (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.512E-06</td>
<td>4.695E-06</td>
<td>0.0421</td>
<td>23.68</td>
</tr>
<tr>
<td>500</td>
<td>7.071E-05</td>
<td>4.419E-05</td>
<td>0.3963</td>
<td>222.88</td>
</tr>
<tr>
<td>982</td>
<td>1.316E-04</td>
<td>8.227E-05</td>
<td>0.7377</td>
<td>414.91</td>
</tr>
<tr>
<td>982</td>
<td>0.000415</td>
<td>0.0019</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>0.000422</td>
<td>0.00193</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>0.00063</td>
<td>0.00284</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.000852</td>
<td>0.00376</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>0.000859</td>
<td>0.00379</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.00109</td>
<td>0.00468</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2813</td>
<td>0.001252</td>
<td>0.00527</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>0.001353</td>
<td>0.00562</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3500</td>
<td>0.00165</td>
<td>0.00657</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>0.001997</td>
<td>0.00754</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4200</td>
<td>0.00215</td>
<td>0.00792</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4262</td>
<td>0.00221</td>
<td>0.00807</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4500</td>
<td>0.00242</td>
<td>0.00855</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5040</td>
<td>0.003</td>
<td>0.00967</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5393</td>
<td>0.0035</td>
<td>0.01045</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5572</td>
<td>0.003826</td>
<td>0.010876</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B20: Load-Strain data for 7 mm Plain2 Slab based on Desayi & Krishnan Curve

<table>
<thead>
<tr>
<th>load (lbs)</th>
<th>Conc.Strain</th>
<th>Bar.Strain</th>
<th>Moment (k-ft)</th>
<th>Stress (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.189E-06</td>
<td>3.868E-06</td>
<td>0.0421</td>
<td>23.68</td>
</tr>
<tr>
<td>500</td>
<td>5.825E-05</td>
<td>3.641E-05</td>
<td>0.3963</td>
<td>222.88</td>
</tr>
<tr>
<td>700</td>
<td>7.908E-05</td>
<td>4.942E-05</td>
<td>0.5379</td>
<td>302.56</td>
</tr>
<tr>
<td>1000</td>
<td>1.103E-04</td>
<td>6.895E-05</td>
<td>0.7504</td>
<td>422.08</td>
</tr>
<tr>
<td>1204</td>
<td>1.316E-04</td>
<td>8.222E-05</td>
<td>0.8949</td>
<td>503.36</td>
</tr>
<tr>
<td>1204</td>
<td>0.000403</td>
<td>0.00228</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>0.0005</td>
<td>0.00281</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1995</td>
<td>0.000668</td>
<td>0.0037</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.00067</td>
<td>0.00371</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.000847</td>
<td>0.00462</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>0.001035</td>
<td>0.00553</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3500</td>
<td>0.00124</td>
<td>0.00646</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>0.00146</td>
<td>0.00739</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4500</td>
<td>0.0017</td>
<td>0.00832</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>0.00197</td>
<td>0.00925</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5200</td>
<td>0.00209</td>
<td>0.00964</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5258</td>
<td>0.00213</td>
<td>0.00976</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5798</td>
<td>0.0025</td>
<td>0.01082</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6388</td>
<td>0.003</td>
<td>0.01202</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6823</td>
<td>0.003465</td>
<td>0.01294</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B21: Load-Strain data for 5 mm PP1 Slab based on Proposed Model

<table>
<thead>
<tr>
<th>Load</th>
<th>Conc. Strain</th>
<th>Bar. Strain</th>
<th>Conc strain</th>
<th>Load</th>
<th>Bar strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>500</td>
<td>0.000682</td>
<td>500</td>
<td>0.00042625</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>0.001295</td>
<td>1000</td>
<td>8.09375E-05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1017</td>
<td>0.0005485</td>
<td>0.000132</td>
<td>0.0000825</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>0.00082</td>
<td>0.00548</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1600</td>
<td>0.000879</td>
<td>0.00583</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.00113</td>
<td>0.00727</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.00147</td>
<td>0.00905</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>0.00186</td>
<td>0.01087</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3500</td>
<td>0.002325</td>
<td>0.01272</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>0.00291</td>
<td>0.01463</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4382</td>
<td>0.00348</td>
<td>0.01613</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4393</td>
<td>0.0035</td>
<td>0.01617</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4552</td>
<td>0.0038</td>
<td>0.01683</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4643</td>
<td>0.004</td>
<td>0.01722</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4710</td>
<td>0.004166</td>
<td>0.01752</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4869</td>
<td>0.004688</td>
<td>0.01829</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B22: Load-Strain data for 5 mm PP2 Slab based on Proposed Model

<table>
<thead>
<tr>
<th>load (lbs)</th>
<th>Conc.Strain</th>
<th>Bar Strain</th>
<th>Conc.Strain</th>
<th>load</th>
<th>Bar strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>500</td>
<td>0.00006263</td>
<td>0.0006263</td>
<td>0.0006263</td>
<td>500</td>
<td>3.91438E-05</td>
</tr>
<tr>
<td>1000</td>
<td>0.0001189</td>
<td>0.0001189</td>
<td>0.0001189</td>
<td>1000</td>
<td>7.43125E-05</td>
</tr>
<tr>
<td>1113</td>
<td>0.00057</td>
<td>0.0041</td>
<td>0.0041</td>
<td>1113</td>
<td>0.00008225</td>
</tr>
<tr>
<td>1500</td>
<td>0.000765</td>
<td>0.00545</td>
<td>0.00545</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.00104</td>
<td>0.00723</td>
<td>0.00723</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.001333</td>
<td></td>
<td>0.009</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>0.00166</td>
<td>0.01079</td>
<td>0.01079</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3500</td>
<td>0.002034</td>
<td>0.0126</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>0.00248</td>
<td>0.01445</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4212</td>
<td>0.0027</td>
<td>0.01525</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4316</td>
<td>0.002815</td>
<td>0.01565</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4500</td>
<td>0.00303</td>
<td>0.01634</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4838</td>
<td>0.0035</td>
<td>0.01768</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4961</td>
<td>0.0037</td>
<td>0.01817</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5017</td>
<td>0.0038</td>
<td>0.01841</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Table B23**: Load-Strain data for 7 mm PP1 Slab based on Proposed Model

<table>
<thead>
<tr>
<th>load (lbs)</th>
<th>Conc. Strain</th>
<th>Bar Strain</th>
<th>conc. Strain load</th>
<th>bar strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>500</td>
<td>0.000387</td>
<td>0.00173</td>
<td>0.0001188</td>
<td>0.000048375</td>
</tr>
<tr>
<td>891</td>
<td>0.000432</td>
<td>0.00193</td>
<td>0.0001315</td>
<td>8.21875E-05</td>
</tr>
<tr>
<td>1000</td>
<td>0.000663</td>
<td>0.00285</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>0.000926</td>
<td>0.00379</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.001157</td>
<td>0.00454</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2400</td>
<td>0.00122</td>
<td>0.00474</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.001545</td>
<td>0.00569</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>0.00192</td>
<td>0.00666</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3500</td>
<td>0.00238</td>
<td>0.00767</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>0.00297</td>
<td>0.00872</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4500</td>
<td>0.0037</td>
<td>0.00972</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4939</td>
<td>0.0038</td>
<td>0.00983</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4984</td>
<td>0.0045</td>
<td>0.01046</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5207</td>
<td>0.004694</td>
<td>0.01059</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5242</td>
<td>0.005486</td>
<td>0.01077</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5281</td>
<td>0.005784</td>
<td>0.01108</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5292</td>
<td>0.0065</td>
<td>0.01124</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5241</td>
<td>0.0065</td>
<td>0.01124</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table B24: Load-Strain data for 7 mm PP2 Slab based on Proposed Model

<table>
<thead>
<tr>
<th>Load (lbs)</th>
<th>Conc. Strain</th>
<th>Bar Strain</th>
<th>Theoretical Strain (in/in)</th>
<th>Theoretical Strain based on gross prop load (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>500</td>
<td>0.0000622</td>
<td>500</td>
<td>0.000038875</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>0.000118</td>
<td>1000</td>
<td>0.00007375</td>
<td></td>
</tr>
<tr>
<td>1124</td>
<td>0.0001316</td>
<td>1124</td>
<td>0.00008225</td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>0.000425</td>
<td>1500</td>
<td>0.00214</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.000561</td>
<td>2000</td>
<td>0.00282</td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>0.000956</td>
<td>2500</td>
<td>0.00465</td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>0.001175</td>
<td>3000</td>
<td>0.00557</td>
<td></td>
</tr>
<tr>
<td>3500</td>
<td>0.00141</td>
<td>3500</td>
<td>0.0065</td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>0.001666</td>
<td>4000</td>
<td>0.00744</td>
<td></td>
</tr>
<tr>
<td>4500</td>
<td>0.001952</td>
<td>4500</td>
<td>0.00839</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>0.002282</td>
<td>5000</td>
<td>0.00935</td>
<td></td>
</tr>
<tr>
<td>5500</td>
<td>0.002672</td>
<td>5500</td>
<td>0.01035</td>
<td></td>
</tr>
<tr>
<td>5935</td>
<td>0.00308</td>
<td>5935</td>
<td>0.01124</td>
<td></td>
</tr>
<tr>
<td>6047</td>
<td>0.0032</td>
<td>6047</td>
<td>0.01147</td>
<td></td>
</tr>
<tr>
<td>6260</td>
<td>0.00345</td>
<td>6260</td>
<td>0.01193</td>
<td></td>
</tr>
<tr>
<td>6375</td>
<td>0.0036</td>
<td>6375</td>
<td>0.01218</td>
<td></td>
</tr>
<tr>
<td>6446</td>
<td>0.0037</td>
<td>6446</td>
<td>0.01234</td>
<td></td>
</tr>
<tr>
<td>6512</td>
<td>0.0038</td>
<td>6512</td>
<td>0.01249</td>
<td></td>
</tr>
</tbody>
</table>