COMPREHENSIVE MODELING OF SHAPE MEMORY ALLOYS FOR
APPLICATIONS IN LARGE-SCALE STRUCTURES

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Dissertation

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Inspired by universe, the design of smart systems and structures that can repair themselves have always been one of the ambitions of engineers and researchers. Throughout the history of science, metals and alloys have played an important role in the advancement of engineering, science & technology. With the emerging new class of materials, Shape Memory Alloys (SMAs) are being viewed as an alloy for a new era. The good performance of SMAs in commercial applications (e.g. Boeing adaptive chevron, Mars Sojourner Rover Actuator, Biomedical Stents etc.) has been well established.

SMAs are primarily known for their shape recovery characteristics that occur as a result of the transformation between two phases: Austenite and Martensite. These materials undergo a diffusionless, thermoelastic, martensitic phase transformation, and can recover strains as large as 8%. However, the true potential of these materials has not yet been realized in practical applications. A lack of sufficient experimental information and unified approaches to constitutive modeling has made it difficult for designers to incorporate these vastly unique materials in practical ways. In general, the constitutive modeling approaches utilized by various research groups around the world have been based on the classical plasticity theories. Also, most of the models reported in the literature are either one dimensional, and/or extended in ad-hoc ways to capture three dimensional aspects. These models are prepared with an eye towards fitting a specific set
of experimental data (i.e. a specific set of SMA features). The most common feature of SMAs of *practical* importance is the *evolution* with cycles that has almost always been neglected by the researchers.

In the current research, a new, fully general, completely three dimensional, multimechanism based, viscoelastoplastic, unified SMA constitutive model has been developed as an extension of previously formulated constitutive theory by Saleeb et al. for high temperature Ti-alloys. The success of the present unified model in describing all the salient features of SMAs (uniaxial, multiaxial, coupled thermomechanical, evolution with cycles, rate effects, etc.) is attributed to the careful partitioning of energy (into storage and dissipation) through multiplicity of viscoelastoplastic mechanisms, and to the strict adherence to the well established mathematical and thermodynamical requirements of convexity, associativity, normality, etc. It will be shown that the present thermodynamical framework is able to capture all the known features, as well as the evolutionary characteristics, of SMAs.

The present unified SMA constitutive model has been fully integrated with a commercial, large-scale, finite element package. For the demonstration of its capability and efficiency for boundary value problems, an application of SMAs as actuators for wing morphing (shape control) of a conceptual aircraft wing will be presented. Unlike other SMA actuation mechanisms, where only uniaxial response and one – way shape memory effects have been utilized, the current design concept focuses on the complete *three dimensional* description of actuation under biased, *thermal – cycling* conditions, and has led to an interesting concept for future aircraft wings.
DEDICATION

I dedicate this dissertation to all my teachers, friends, family and individuals who have helped, guided and supported me during my education in the past and present. I have been a part of an educational society who strives for the growth and bring best in you. I am obliged to encounter one of the best teachers & scientists in my life. Their constant support and guidance led me to be a scientist and I dedicate my dissertation to all of them.
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CHAPTER I

INTRODUCTION

1.1 Motivation

For centuries, metals have played an important role as structural materials. The human ability to understand material behavior (mechanical, thermal, electrical, etc.) from microstructure, and to engineer different material properties for a variety of applications has enabled the development of new alloys and composites. With advancements in science and technology, new classes of multifunctional materials have emerged, among which Shape Memory Alloys (SMAs) have a particular significance in advanced engineering applications. SMAs are known primarily for one fundamental and unique property – the ability to remember and recover from large strains without permanent deformation. Due to their ability to recover large deformations/strains upon mechanical unloading or heating, SMAs are suitable for innovative application in almost all engineering fields including civil, mechanical, aerospace and bio-medical engineering.

SMAs are a unique class of shape memory materials with the ability to recover their shape when the temperature is increased (even under high applied loads therefore resulting in high actuation energy densities). In addition, under specific conditions, when subjected to applied mechanical/thermal cyclic loading, SMAs can absorb and dissipate
mechanical/thermal energies by undergoing a reversible hysteretic shape change. These unique characteristics of SMAs have made them popular for their use for sensor and actuator, impact absorption and vibration damping applications.

SMAs have attracted a lot of attention, and have motivated a variety of constitutive models to facilitate their efficient utilization in advanced engineering applications. Over the years, researchers presented many constitutive models to describe the complex behavior exhibited by SMAs, following one of the two approaches; (i) micromechanical, by considering the granular microstructure of SMA, and (ii) phenomenological, by isolating different energies associated with phase transformation through internal state variables. These material models require either independent implementation, or in conjunction with commercially available large scale finite element (FE) programs.

Researchers have been mainly focusing on the mathematical developments and physical relevancy of SMA material models to capture various experimentally observed phenomena. Very limited studies are conducted on implementation and practical utilization of the developed models in the available large scale FE codes such as ABAQUS, ANSYS, and LSDYNA. Furthermore, independent developments of numerical schemes/algorithms are usually employed depending upon the model and the type of application. Thus, engineering users/practitioners are in an urgent need of isolating actual implementation of these material models in available FE codes, and their application in real physical problems.
1.2 Objective and Scope of the Study

The main aim of the present study is the utilization of SMAs in advanced engineering applications such as sensor/actuator/vibration absorber. It can be subdivided into three main objectives:

(1) Development of a SMA material constitutive material model.

(2) Implementation in large scale FE program, and study of capabilities of the developed model.

(3) Analysis of a boundary value engineering problem, dealing with the application of SMA in advanced engineering designs such as actuator in wing morphing technique.

Firstly, a SMA material constitutive model will be arrived at as an extension of development from the generalized class of viscoelastic-, viscoplastic-, and multimechanism- based material models developed by Saleeb et al. (see (Arnold and Saleeb 1994; Saleeb and Arnold 2001; Saleeb and Arnold 2004; Saleeb et al. 2001; Saleeb and Wilt 1993; Saleeb et al. 1998; Saleeb et al. 2000; Saleeb et al. 2002)). A successful implementation of this particular model in describing the constitutive behavior of high temperature Ti alloys can be found in the mentioned articles (Arnold et al. 2001; Lissenden et al. 2007; Saleeb and Arnold 2001). More specifically, the work reported here in conjunction of items (i) and (ii) above, is a part of an overall research project dealing with the developments of comprehensive material model to describe the
evolutionary response of different SMA alloys under extended cycles of thermomechanical loads, as recently reported by (Saleeb et al.)

This will be utilized with a commercial large scale FE program, ABAQUS/Standard®. In order to assess the generality and capabilities of material model, a qualitative study needs to be performed for different experimentally observed behaviors, exhibited by SMAs, under mechanical and/or thermo – mechanical loading conditions. Furthermore, efficiency and robustness of different numerical schemes must be studied before its implementation in real physical problems.

Finally, equipped with a general constitutive material model, application of SMAs in the field of actuation will be implemented in a large scale FE code. Note that the scope of above mentioned three tasks are independent of each other; i.e. SMAs can be utilized without considering its specific application (e.g. in control/actuation/vibration reduction systems in as diverse fields as of civil, aerospace and biomedical applications). Characterization of SMA under conditions of varying temperature and training cycles require availability of extensive independent test results. Therefore, it will prove to be very useful to a practicing engineer.

1.3 General Outline

The introductory chapter is followed by background and review on the literature in chapter two. The literature survey is separated into subsections, specific to the two major tasks of this study; i.e. micromechanical study of the general properties of SMAs, and the critical study of available constitutive material models to describe the properties
of SMAs. This is followed by a description of developed material model in chapter three. Then, material characterizations and numerical simulations pertinent to the study of capabilities of developed model under the various experimental test conditions (as highlighted in the literature) will be discussed in chapter four. These are then combined in the later chapter five to a “conceptual”, boundary value structural problem in the field of SMA actuators, dealing with shape morphing of aircraft wings. Finally, summary, conclusions, and direction of the future research are presented in chapter six.
2.1 A Brief History of SMAs

Inspired by the universe, design of smart systems and structures that can repair themselves have always been one of the ambitions of engineers and researchers. With the advancement in science and technology, SMAs are now emerging as “smart materials of the new age”. Shape memory effect (SME) was being utilized in engineering, since its first finding in 1930 (Maji and Negret 1998; Peultier et al. 2006a; Schwartz 2002). The breakthrough for engineering applications took place with the discovery of “Nickel – Titanium” (NiTi), commonly known as “NiTiNOL”, by Buehler and his co-workers in 1963. Improvements in the thermo-mechanical properties of NiTi, by addition of alloying element such as Cobalt (Co) or Iron (Fe), were observed in 1965, which led to its first commercial application in the F-14 fighter aircraft. This was followed by the development of NiTiNb alloy in 1989 (Lagoudas 2008). Several other types of SMAs were developed, such as NiTiCu, TiAu, NiAl, FeMnSi and TiPd. Among other SMAs, NiTi is more popular due to its strong shape memory effect (Patoor et al. 2006). Some of the SMAs and their compositions are shown in Table 2.1 (Janke et al. 2005; Lagoudas 2008; Patoor et al. 2006).
2.2 Phase Transformation in SMAs: A Micro-structural Phenomenon

SMAs are metallic alloys that are capable of recovering permanent strains when heated above certain temperature due to martensitic phase transformations. Unlike conventional metals that recover less than 1% strain before plastic deformation, SMAs undergo a diffusionless, thermoelastic martensitic phase transformation as a result of a twinning process and allows complete recovery of strains as large as 8% (Schwartz 2002). The key to this characteristic is the existence of high crystallographic symmetry in austenite phase than the low temperature, low symmetry in martensite phase.

Table 2.1: Some Shape Memory Alloys and Their Composition (table adapted from (Janke et al. 2005)).

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Composition (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni-Ti</td>
<td>49 – 51 Ni</td>
</tr>
<tr>
<td>Ni-Ti-Cu</td>
<td>8 – 20 Cu</td>
</tr>
<tr>
<td>Ni-Al</td>
<td>36 – 38 Al</td>
</tr>
<tr>
<td>Fe-Ni-Nb</td>
<td>31 Ni, 7 Nb</td>
</tr>
<tr>
<td>Fe-Ni-Co-Ti</td>
<td>33 Ni, 10 Co, 4 Ti 31 Ni, 10 Co, 3 Ti</td>
</tr>
<tr>
<td>Cu – Al – Ni</td>
<td>28 – 29 Al, 3 – 4.5 Ni</td>
</tr>
<tr>
<td>Cu – Zn</td>
<td>38.5 – 41.5 Zn</td>
</tr>
<tr>
<td>Cu – Zn – X</td>
<td>X = Al, Si, Sn, Ga</td>
</tr>
</tbody>
</table>
Within an operating temperature, SMAs have two different crystal lattice structure and, hence, different properties. The high symmetry cubic phase that exists at high temperature is called *Austenite* (*A*), whereas the low temperature phase and less symmetric, tetragonal, orthorhombic or monoclinic (Firstov et al. 2008), phase is called *Martensite* (*M*). The transformation from *A* to *M* (or vice versa) occurs by means of a lattice shearing mechanism, and is known as *martensitic transformation* (Bhattacharya 2003; Lagoudas 2008). It is *first order* (abrupt change in lattice parameter), *diffusionless* (no rearrangement of atoms), *solid to solid* phase transformation (Bekker and Brinson 1997; Bhattacharya 2003). Since, the product *M* crystal has low symmetry than parent *A*; it can have a different orientation (called *variants*) direction in crystal lattice structure. The number of variants depends upon the change in symmetry during transformation (Bhattacharya 2003).

An illustrative schematic of the transformation between *A* (parent phase at high temperature) and *M* (product phase at low temperature) is shown in the Fig. 2.1. Initially, the material is in *A* phase at temperature above a critical temperature, named as *austenite finish temperature* (*A_f*). Upon cooling under isobaric (constant stress) condition, *A* starts transformation into *M* at *martensite start temperature* (*M_s*), and completes transformation when *martensite finish temperature* (*M_f*) is reached. Throughout the transformation process, both phases coexist. During this process, different martensite variants arrange themselves in a manner such that there is no microscopic change in the shape of crystal. This phase is called *twinned* martensite (*M_t*). With the application of mechanical load, different variants reorient themselves to a single variant (*detwinned martensite, M_d*),
resulting large inelastic microscopic strains (Patoor et al. 2006), and retain its deformed shape when the loads are removed. The detwinning process is limited by stress levels: \textit{detwinning start stress}, \(\sigma_s\), and \textit{detwinning finish stress}, \(\sigma_f\) (Lagoudas 2008).

\textbf{Figure 2.1:} Phase transformation in SMAs as a result of changes in the thermomechanical loading conditions.

When the material is heated, transformation from \(M\) to \(A\) starts at \textit{austenite start temperature} \((A_s)\), and finishes at \textit{austenite finish temperature} \((A_f)\). Since there is only one variant in \(A\), the crystal structure completely returns to its original high symmetry (cubic) configuration. Upon cooling, it returns back to detwinned martensite phase. This
phenomenon is termed as SME. The phase transformation from $A$ to $M$ ($A \rightarrow M$) is called as *forward transformation*, whereas from $M$ to $A$ ($M \rightarrow A$) is termed as *reverse transformation*.

From an energetics point of view, a unique low energy state exists at high temperature (greater than $A_f$), and multiple low energy states at low temperatures (lower than $M_f$). During forward transformation, the latent heat associated with transformation is released (exothermic process), whereas heat is absorbed in the reverse transformation. The amount of heat is proportional to the volume fraction of the transformed material (Patoor et al. 2006). The energy supplied from mechanical loading, is dissipated in reorientation of martensite variants. Therefore, if sufficiently high stress is applied in $A$ at temperature above $A_f$, a forward transformation takes place, leading to fully detwinned martensite, $M_d$ (see Fig. 2.2). Upon release of stress, the crystals reorient themselves to the parent high symmetry shape with some remanent (or untransformed) martensite crystals; thus recovering nearly all strain. Since there is very little permanent strain at the end of cycle, this behavior is termed as *pseudoelasticity* or *superelasticity*. Note that the transformation takes place without inducing plastic flow, and dissipation of energy (although very small) leads to hysteretic behavior of SMA. The energy dissipations are due to interfacial friction, defects, and growth of martensite faces and their interactions during transformation (Bekker and Brinson 1997).
Figure 2.2: An idealized phase diagram for SMAs in stress – temperature subspace.

The transformation temperatures are not only dependent on the composition, processing, micro-structural defects, and grain sizes, but also on the applied stress, $\sigma$. An idealized stress temperature phase diagram (although real SMAs have nonlinear phase diagrams) is shown in Fig. 2.2. At any stress state, the transformation temperatures are higher than that at zero stress level ($M_s$, $M_f$, $A_s$, and $A_f$) due to the increased energy of the lattice. At a constant temperature, $A\rightarrow M$ transformation starts at a critical stress, $\sigma_{M_s}$, and completes at stress, $\sigma_{M_f}$. During stress removal process, $M$ unloads elastically to a stress level, $\sigma_{A_s}$, after which reverse $M\rightarrow A$ transformation takes place until stress level, $\sigma_{A_f}$ is reached, where complete $M\rightarrow A$ transformation finishes. The material behaves elastically once the transformation is completed (Piedboeuf et al. 1998). Furthermore, the stress-strain response of SMA is highly dependent upon the stress state (tension or
compression). It is experimentally verified that the flow stress in compression is higher than that in tension, and this tension – compression asymmetry vary with temperature (Adharapurapu et al. 2006; Gall et al. 2002; Gall et al. 2001; Liu et al. 1998). The reasons for this behavior and related microstructure variation under compression have not yet been understood (Liu et al. 1998). Under any thermomechanical loading path, the material behavior becomes more complicated. Collectively, unlike conventional materials, the SMA responses are intricately coupled in the stress – strain – temperature space. These complex microscopic properties are the main motivation for their innovative application in engineering.

2.3 General Properties and Applications of SMAs

The solid to solid, diffusionless, first order phase transformations forms the basis for the complex nonlinear properties (such as SME, pseudoelasticity, and damping) exhibited by SMAs. The SME described in the previous section is also called as one way shape memory effect (OWSME), where an external stress is necessary to induce detwinning/reorientation, and to transform the material in a new deformed configuration that can be brought back to original shape upon heating above $A_f$. Note that one way shape memory effect does not induce significant transformation strains during cooling (Patoor et al. 2006). The OWSME is utilized in industries ranging from aerospace to emerging bio-medical applications. The amount of force generated during the transformation process of shape memory alloys provides an alternative means of actuation. For example, TiNi Alloy Company developed Frangibolt® Release Bolts to replace exploding bolt devices in aerospace release mechanism (Schwartz 2002).
Commonly, SMA wires are used for actuation because of their capability of delivering high force and displacements (Shu et al. 1997); e.g. fixed wing aircraft, variable geometry chevron (Lagoudas 2008), Mars Sojourner Rover Actuator (Lagoudas 2008; Schwartz 2002). Researchers are also trying to develop thin films for micro-electro-mechanical systems such as micromanipulator and microrelays (Huang et al. 2008). Furthermore, due to biocompatibility, high strength and toughness, and shape memory behavior, SMAs are widely being utilized in biomedical industry (Bansiddhi et al. 2008; Chu et al. 2004; Duerig 1990; Hartl et al. 2010; Hartl and Lagoudas 2009; Savi et al. 2008).

Diffusion processes during transformations are suppressed at high temperatures that lead to high reversibility during martensitic transformation (Firstov et al. 2004). This property is termed as superelasticity or pseudoelasticity; i.e. the ability to undergo large deformation under mechanical/thermal load cycles without accumulating significant irrecoverable strain. This property makes them suitable for several medical applications, ranging from orthodontic appliances to blood vessel occlusion treatment (Auricchio and Sacco 2001). Eyeglass frames, rice cooker and cellular phone antennas are examples of versatile application of superelastic behavior of SMAs in consumer products (Duerig 1990; Lagoudas 2008; Schwartz 2002).

Another SME that is commonly observed with SMAs is two way shape memory effect (TWSME) and stress assisted two way memory effect (SATWME) (Wada and Liu, 2008). Both of these properties are acquired properties and not intrinsic; i.e. after application of a number of thermal and/or mechanical cycles. Repeated
thermomechanical load cycles induce permanent changes in the microstructure (i.e. formation of preferred $M$ variants), and only partial recovery of transformation induced strains takes place. In TWSME, a trained shape in the austenitic phase returns to a second trained shape in martensitic phase after forward transformation, allowing the material to cycle between two different shapes. This has been successfully utilized in engineering; e.g. as stable TWSME extension spring (Wang et al. 2003), as helical SMA coils in cold weather insulating jackets (Yoo et al. 2008), to adjust the clearance of taper roller bearings in bearing technology (Predki et al. 2008).

Hysteretic and superelastic behaviors exhibited by SMAs make them capable of being used in vibration isolation in civil, automobile and aerospace industries. Reduction in stiffness during transformation (and hence the system frequencies) allows them to be used for damping/vibration isolation in civil structures (Collet et al. 2001; Duffy et al. 2008; Ghomshei et al. 2005; Hashemi and Khadem 2006; Ibrahim 2008; Janke et al. 2005). NiTi SMA shows strong amplitude-dependent internal friction in the martensitic condition. Increase in load amplitude increases the damping/vibration reduction capabilities of SMAs (Hashemi and Khadem 2006). Two large research projects, “Memory Alloys for New Seismic Isolation and Energy Dissipation Devices” and “Shape Memory Alloy Devices for Seismic Protection of Cultural Heritage Structures”, had been carried out between 1995 – 1999 to study and develop SMA devices for vibration and energy dissipation systems (Janke et al. 2005).

Apart from thermomechanical properties, the electrical resistance of the SMAs is used as a feedback signal in neural – fuzzy algorithm (Lee and Lee 2000). It has been shown
that electrical resistance based motion controlled SMA actuator is very precise (Lind et al. 2000). A model based position control and a neural-fuzzy logic based position control for a SMA wire actuator has successfully been developed (Lei and Yam 2000).

In summary, SMA applications continue to gain acceptance in a wide variety of industries throughout the world. Unlike conventional materials, different properties and design techniques for SMAs are not standardized. Hence, attempts are being made by researchers around the globe to study complete characteristics of SMAs, and to develop acceptable material models in order to predict/simulate the response under desired application conditions. In the next section, a comprehensive review of different modeling approaches will be presented.

2.4 Constitutive Modeling of SMAs

The good performances of SMAs in commercial applications are presently well understood in as many diverse fields as structural, aerospace, and biomedical engineering. Furthermore, the emerging new classes of high-temperature SMA (HTSMA) material systems offer great potential in many innovative technologies; e.g. flow control issues such as adaptive inlets, actively-controlled airfoils, shape-changing varies and blades, variable area exhaust nozzles, and adaptive chevrons (DeCastro et al. 2007; Hartl et al. 2010; Hartl and Lagoudas 2009; Lagoudas 2008; Schwartz 2002). In order to utilize these alloys in modern engineering design and applications, there is a need for the development of a complete and an efficient material model. The constitutive model should be able to capture complex behavior exhibited by SMAs due to phase transformation. It should also lead to numerically robust and efficient schemes in order to
be implemented in large scale finite element software to analyze various complex design problems.

Over the years, researchers presented many alternative approaches for modeling the complex responses exhibited by SMAs, following one of the two approaches; (i) micromechanical, by considering the granular microstructure of SMA, and (ii) phenomenological, by isolating different energies associated with phase transformation through internal state variables. The current status and features of these models are discussed in details below.

2.4.1 Micromechanical Models

Micromechanical models mainly focus on the description of micro-scale features, such as nucleation, interface motion, twin growth, etc (Abeyaratne and Kim 1997). Micromechanics based macroscopic models use thermodynamics laws to describe the transformation, and utilize micromechanics to develop macro-scale state equations and kinetics (Gao et al. 2000; Huang et al. 2000; Zaki and Moumni 2007a; Zaki and Moumni 2007b). For polycrystalline SMA models, micromechanics methods are utilized to assemble single crystal response based on variant formation and to formulate polycrystalline response based on an assembly of single crystal grains (Huang et al. 2000; McDowell and Lim 2002; Sittner and Novak 2000). Macroscopic stress–strain response depends on the microstructure of $M$ and crystallographic texture of the material (Thamburaja 2005). Most of these models utilize the martensite transformation strains, and some, also, consider self-accommodation, reorientation, and interface of variants,
along with other martensitic transformation characteristics (Auricchio et al. 2003; Auricchio and Sacco 1997; Auricchio and Taylor 1997; Brocca et al. 2002; Peultier et al. 2006a; Peultier et al. 2006b; Sadjadpour and Bhattacharya 2007a; Sadjadpour and Bhattacharya 2007b; Siredey et al. 1999; Thamburaja 2005). A summary of some micromechanics based SMA model is given in the Table 2.2 at the end of this chapter.

A sense of the complexity involved in modeling of SMA response aspects, starting on the very small scale of fundamental crystallography and micromechanics, can be appreciated from the schematics shown in Fig. 2.3. For example, in a typical system such as Nickel–Titanium (NiTi) (with corresponding highlighted numbers in Fig. 2.3), there is a possibility of co–existence (depending on temperature and stresses) of 192 habit planes and 12 product variants in each of the grains that constitute the polycrystalline specimen or macrostructure. In a textured polycrystalline structure, thousands of grains with the previously mentioned habit planes and product variants are present, rendering it impossible to be analyzed using the present day computational resources (Saint-Sulpice et al. 2009). Furthermore, for the minimization problem, numerous sets of constraint equations (e.g. positivity of phase fractions, conservation of mass, grain boundary conditions, grain interaction, etc.) are employed in the micromechanics based models. Depending upon the approach, these sometimes lead to a lack of convergence, and/or non-unique solution (Bhattacharya 2003). In order to reduce the intricacy of the problem, several nonphysical assumptions or averaging schemes are employed, which further restricts the scope of the model. Bhattacharya has concluded that the behavior of polycrystalline SMA material can be significantly different from that
of a single crystal, rendering it impossible to study the micromechanics of polycrystals (Bhattacharya 2003). In summary, these Models have good predictive capabilities, but require a large number of internal variables (Bouvet et al. 2004), extensive experimentation, and associated complicated parameterization procedures.

**Figure 2.3: Landscape of energetics for microstructure of polycrystalline SMA system (highlighted numbers are typical for NiTi materials).**

### 2.4.2 Phenomenological Models

The other and more popular type of models assume macroscopic energy function that depends on internal state variables to describe global mechanical response while all the microscopic details are ignored. The evolution equations are derived in conjunction with the second law of thermodynamics. These models are categorized as
phenomenological, since they seek solutions to boundary value problems on the structural level by energy minimization, as in classical plasticity.

Most of the early constitutive models (Boyd and Lagoudas 1994; Boyd and Lagoudas 1996a; Boyd and Lagoudas 1996b; Brinson 1993) were based on a thermodynamic structure, and the martensitic volume fraction as an internal state variable in order to account for the influence of the microstructure. The one-dimensional model of (Brinson 1993), based on phase diagram kinetics and phenomenological constitutive law with martensite fraction as an internal variable, was one of the first to include modeling of detwinning of $M$. This work was further refined by Bekker and Brinson (Bekker and Brinson 1997; Bekker and Brinson 1998). A unified thermodynamic constitutive model for Shape Memory Alloy (SMA) materials, based on these early thermodynamic frameworks was presented by Lagoudas et al. (Lagoudas et al. 1996). A rate independent one dimensional (1D) model based on Gibbs free energy, by forming increments in elastic potential energy and Gibbs energy with respect to increments of martensite, was developed by Bo and Lagoudas (Bo and Lagoudas 1999a; Bo and Lagoudas 1999b; Bo and Lagoudas 1999c; Lagoudas and Bo 1999). The evolution of internal state variables (martensitic volume fraction, macro-transformation strain, and back and drag stresses) during phase transformation was proposed based on the micromechanical analysis over the representative volume element. The energy balance equation for the heat exchange during phase transformation was derived using the first law of thermodynamics. The stress – induced transformation accounted for both transformation and plastic strains to develop simultaneously as a result of the applied
load. The evolution equations for plastic strains and plasticity related back and drag stresses were described by a methodology similar to viscoplasticity models. The model was very good in predicting the experimentally observed SMA behaviors under cyclic loading. It was further extended to three dimension by (Entchev and Lagoudas 2004; Lagoudas and Entchev 2004). Qidwai and Lagoudas (Qidwai and Lagoudas 2000b) studied the different transformation functions and their effect on the material response, and proposed a transformation function, based on the second and third invariants of deviatoric stress tensor, and first invariant of stress tensor, to account for the experimentally observed tension–compression asymmetry.

Study of SMAs behavior under proportional and non proportional, multi-axial loading conditions has also been an active area of research. Helm and Haupt (Helm and Haupt 2003) proposed a three dimensional (3D) material model to describe the behavior of SMAs in multi-axial loading conditions, assuming small deformation and elastic behavior to be isotropic. The model was developed on the basis of a free energy function (which was partitioned into several storage and dissipative components) based on mixture theory, and by satisfying the Clausius–Duhem inequality (which is a special form of the second law of thermodynamics). Bouvet et al. (Bouvet et al. 2004) presented a model (limited to pseudoelastic response only) by introducing a novel concept of two different yield surfaces; one for forward phase transformation, and the other one for the reverse phase transformation. These surfaces were defined as function of the second and third invariants of stress tensor, in order to take into account the well-known tension–compression asymmetry, and an equivalent transformation strain from the transformation
strain tensor to determine the martensite volume fraction. Recently, an extension of the unified model of Leclerq and Lexcellent (Leclercq and Lexcellent 1996) had been proposed by Panico and Brinson (Panico and Brinson 2007), accounting for the effects of multi-axial stress states and non-proportional loading histories. This model was able to account for the evolutions of both twinned and detwinned martensite, and reorientation of martensite according to loading direction by separating inelastic strain into two contributions from creation of detwinned martensite and reorientation of previously existing martensite variants. The novel feature of this model is that it treats parent phase transformation and martensite variant reorientation as two different physical processes and has associated two distinct evolution laws. It is able to capture the main features of SMA behavior under multi-axial loading conditions, but lacks tension compression asymmetry and response under cyclic loadings.

The numerical implementation of phenomenological models has also been an active area of research. Similar to rate independent plasticity, return mapping algorithms have mostly been utilized in literature (Lagoudas et al. 2006). In return mapping algorithm, an elastic solution is computed as trial stress state. If it violates the flow laws of constitutive model, an inelastic correction is computed based on trial stress state. For the use in SMA constitutive modeling, several variations of return mapping algorithms based on the backward Euler integration scheme for the transformation correction have been proposed in the literature (e.g. see (Auricchio 2001; Auricchio and Taylor 1997)). Qidwai and Lagoudas (Qidwai and Lagoudas 2000a) implemented return-mapping algorithms for the family of SMAs models having a single martensite variant. Govindjee
and Miehe (Govindjee and Miehe 2001) implemented return mapping algorithms on model with multiple martensitic variants and internal variables.

A summary of the features of some more commonly used SMA models is given in Table 2.2*. For more elaborate review of current state of art in constitutive modeling of SMA, the readers are encouraged to refer to these references (Birman 1997; Kan and Kang 2010; Lagoudas 2008; Lagoudas et al. 2006; Patoor et al. 2006).

2.5 Summary of Literature Review

From the engineering applicability standpoint, two major conclusions are reached through the extensive review of the constitutive modeling of SMAs presented in the current literature:

(1) Micromechanical models are good in understanding the microscopic phenomena, such as nucleation, interface growth, grain boundary formation, etc. These models will certainly help the scientific & engineering community in understanding the material, which will lead to proper characterization and optimal utilization of these materials. However, as discussed in section 2.4.1, the exact micromechanical details are often unavailable to the researcher, hence rendering it difficult to develop a mathematical model to describe the complete micro-structural information. In addition, a typical engineering application model consists of millions of grains, requiring large number of solution variables, and

* ζ is used to denote the volume fraction of all martensite.
constraint conditions, which in turn translates into computation resource requirements. Therefore, they are not suitable from the viewpoint of modern engineering applications.

(2) The phenomenological models have high prospect for the use in “smart” engineering design applications. Also, in comparison to the micromechanical approach, the current state of art in phenomenological models is significantly ahead in terms of applicability, as well as the number of features that can be described by the constitutive model.

However, the current literature reveals some serious common assumptions/treatments in the modeling approaches. Few examples and consequences of some common assumptions utilized in literature are given below:

(1) In general, the use of martensite volume fraction, \(\xi\), implies that only one variant (or a specific set of variants) is (are) forming in transformations under all thermo – mechanical loading conditions.

(2) Linear dependence of material properties with respect to, \(\xi\), limits the model to proportional multi-axial loading cases only.

(3) Contrary to the experimental observation, the elastic properties of austenite and martensite are sometimes assumed to be the same.

(4) Utilization of an idealized phase diagram (i.e. distinct and separate critical stress/temperature defining the boundary of transformation region, see Fig. 2.2) leads to the sharp corners in the resulting stress – strain (or
temperature – strain) curves, which is not observed experimentally. This further negates the possibility of overlapping regions in phase diagram.

(5) Separate flow rules for the transformation during loading and unloading (forward and reverse transformation) are utilized, implying the existence of different transformation surface. In the numerical applications, this leads to convergence difficulties in the associated models.

(6) Pseudoplasticity, which is observed at temperatures below \( M_f \), is mostly neglected in the presented formulations. Also, despite rate dependent behavior of SMAs (Hartl et al. 2010; Hartl and Lagoudas 2009; Helm and Haupt 2001; Nemat-Nasser et al. 2005a; Nemat-Nasser et al. 2005b), rate independency is often considered in the constitutive formulation.

(7) The plastic strains are often used to represent the residual strain during thermomechanical cycles. However, experimentally it has not been verified.

(8) Hydrostatic pressure (volumetric stress) is sometimes accounted for in order to model ATC; but from a micro – mechanical point of view, transformation from the cubic to the monoclinic crystal does not lead to volume change (see Table 9.1 in (Bhattacharya 2003)).

The assumptions utilized in mathematical modeling sometimes lead deviation from the actual physics of phase transformation. As a corollary to item (8) above, let us consider the effective elastic modulus in terms of volume fraction, \( \zeta \), elastic modulus of
austenite \((E^A)\) and martensite \((E^M)\), using rule of mixtures (e.g. see (Hartl et al. 2010; Lagoudas 2008; Lagoudas and Entchev 2004; Qidwai and Lagoudas 2000b)) as:

\[
\frac{1}{E} = \frac{1}{E^A} + \xi \left( \frac{1}{E^M} - \frac{1}{E^A} \right), \tag{2.1}
\]

The Poisson’s ratio, \(\nu\), is always assumed to be constant (in range 0.25 – 0.3), which implies that the bulk modulus of the material changes with the evolution of martensite. In other words, the transformation will also be dependent on hydrostatic pressure component. To show this, let us consider the first term of the expression of driving energy for transformation (thermodynamic force, \(\pi\), conjugate to, \(\xi\), in eq. (6) in (Lagoudas and Entchev 2004)),

\[
\pi = \frac{1}{2} \sigma_{ij} \Delta S_{ijkl} \sigma_{kl} + \cdots, \tag{2.2}
\]

where, \(\sigma_{ij}\) , and \(\Delta S_{ijkl}\) are the stress tensor, and compliance tensor, respectively. The compliance tensor, \(\Delta S_{ij}\), in the above mentioned reference was defined as the difference of compliance tensors for \(M\) and \(A\) phases. The Eq. (2.2) can be written in terms of \(I_1\) (first invariant of stress tensor), \(J_2\) (second invariant of deviatoric stress tensor), shear modulus, \(G^*\), and the bulk modulus, \(K^*\), as (Eq. 4.47b in (Chen and Saleeb 1994))

\[
\pi = \frac{J_2}{G^*} + \frac{I_1^2}{18K^*} + \cdots, \tag{2.3}
\]
where,

\[ G^* = \left( \frac{1}{E^M} - \frac{1}{E^A} \right)^{-1} \frac{1}{2(1+\nu)}, \]

\[ K^* = \left( \frac{1}{E^M} - \frac{1}{E^A} \right)^{-1} \frac{1}{3(1-2\nu)}. \]  

(2.4)

In case of pure hydrostatic pressure loading condition (i.e. equal pressure, \( p \), along all the three axes), the \( J_2 \) will be zero, leading to the reduced form of Eq. (2.2) as,

\[ \pi = \frac{3}{2} \left( \frac{1}{E^M} - \frac{1}{E^A} \right) (1-2\nu) p^2 \]  

(2.5)

The Eq. (2.5) that is the driving force for the transformation is clearly dependent on the hydrostatic stress state; i.e. transformation under pressure. This is in contradiction with the findings of experiments, as well as the micromechanics of transformation (see Table 9.1 in (Bhattacharya 2003)).

It is clear that the most common approach in the literature has been to adopt classical plasticity and corresponding thermodynamic formulations in terms of conventional phase fractions and transformation strains as the internal state variables (Saint-Sulpice et al. 2009). The Fig. 2.4 shows the common difficulties that can arise from classical plasticity treatment for the phase transformation phenomena. However, note that some of the above mentioned approaches (in section 2.4.2) constitute a radical deviation from the well established plasticity theories, which, in turn, may lead to possible problematic mathematical treatments that require careful consideration.
In terms of mathematical requirements of constitutive modeling, the classical plasticity theory rests on three fundamental principles: (i) normality, (ii) convexity and single transformation surface for both forward and reverse loading conditions, and (iii) associativity of the corresponding flow law (Chen and Saleeb 1994). These well established concepts are essential in any extension to the comprehensive modeling of SMA behavior. Unless future experimental programs suggest otherwise, the deviation from these theories is undesirable.
Another important aspect of the SMA constitutive modeling literature is the evolution/extension of the proposed formulation. All leading research groups either started with or extended the 1D model to account for different features of SMA responses. This is generally followed by ad-hoc generalization to 3D. A brief summary of the work done by some of the leading research group over the years is given in Table 2.4, where the evolution of their respective models can clearly be seen. These models are developed mainly to fit experimental data under the superelastic and strain controlled thermomechanical cyclic conditions thus necessitating a newer model or further development of the existing models to account for some other behaviors such as ratcheting with cycles. A brief review of such models have been provided in the Table 1 of Ref. (Kan and Kang 2010).

Clearly, there is a need for the development of an efficient, numerically robust constitutive model to assess the SMA response characteristics accurately, that can be integrated into a commercial, large scale finite element packages for the efficient utilization of SMAs. Therefore, the prime motivation of the work reported is to develop a sufficiently general 3D constitutive model, to describe the important effects of the phase transformations in SMA. For this purpose, a multimechanism based, viscoelastoplastic framework with underlying mathematical constraints was adopted to develop a fully general, 3D SMA constitutive model. The presented work is an extension of the previously developed work by Saleeb et al. (Arnold et al. 2001; Saleeb and Arnold 2001; Saleeb et al. 2001). The proposed multimechanism based model will be described in the next chapter.
Table 2.2: A Summary of Micromechanics based SMA Constitutive Model (table adapted from (Patoor et al. 2006)).

<table>
<thead>
<tr>
<th>Model</th>
<th>Features</th>
</tr>
</thead>
</table>
| (Patoor et al. 1996) | • Multivariant model.  
• Increasing interaction force with increasing volume fraction.  
• Continuous strain hardening effect.  
• Good qualitative agreement with experimental uniaxial results. |
| (Vivet and Lexcellent 1998) | • Single variant in each grain.  
• Interaction force among grains decreased with increased volume fraction.  
• Very good agreement with uniaxial experimental results. |
| (Huang and Brinson 1998) | • Multivariant model.  
• Based on the habit plane and transformation directions for martensite variants.  
• Self accommodating groups of variants without interaction among themselves.  
• Slow calculation, but good qualitative agreement to uni- and multi-axial experimental results. |
• Multivariant model.  
• Anisotropy of martensite and austenite were not considered.  
• Good qualitative agreement with multi-axial experimental results. |
| (Govindjee and Miehe 2001) | • Multivariant model  
• Based on lattice correspondence variants.  
• Able to account for detwinning and martensite reorientation effects.  
• Good agreement with experimental results, however limited only to isotropic material, and pseudoelastic effects. |
| (Gao and Brinson 2002) | • Multivariant model.  
• Based on invariant nature of martensitic transformation.  
• Accounts for anisotropy in single crystal  
• Good agreement to uni- and multi-axial experimental results |
| (Levitas and Ozsoy 2009a; Levitas and Ozsoy 2009b) | • Multivariant model.  
• A novel, explicit expression for the driving forces for interface reorientation, considering homogeneous stresses and strains in phases, was derived.  
• Good agreement with experimental observations. |
Table 2.3: A Summary of Phenomenological Constitutive Model (table adapted from (Birman 1997; Lagoudas 2008; Lagoudas et al. 2006))

<table>
<thead>
<tr>
<th>Model</th>
<th>Formulation</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Tanaka et al. 1986)</td>
<td>Helmholtz free energy</td>
<td>• 1D model.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Used an exponential hardening rule for the phase transformation.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Material properties remain constant during phase transformation.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Restricted to the stress-induced martensite phase transformation.</td>
</tr>
<tr>
<td>(Liang and Rogers 1990; Liang and Rogers 1992)</td>
<td>Helmholtz free energy</td>
<td>• Extension of Tanaka model.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• A cosine relationship to describe the ( \xi ) as a function of the stress and temperature.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• ( J_2 )-type transformation surface</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• This model can not describe shape memory effect due to martensite detwinning at temperature below ( M_f ).</td>
</tr>
<tr>
<td>(Brinson 1993)</td>
<td>Helmholtz free energy</td>
<td>• Two internal variables to allow modeling of both stress induced and temperature induced martensite.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Used a cosine hardening law and nonconstant elastic stiffness during phase.</td>
</tr>
<tr>
<td>(Boyd and Lagoudas 1994)</td>
<td>Gibbs free energy, quadratic in ( \xi )</td>
<td>• Used the volume fraction of stress induced martensitic internal variable and associated flow rule for transformation strain.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• ( J_2 )-type transformation surface for the forward phase transformation and a non-associative flow rule during reverse transformation to account for non-proportional loading paths</td>
</tr>
<tr>
<td>(Boyd and Lagoudas 1996a; Boyd and Lagoudas 1996b)</td>
<td>Gibbs free energy, quadratic in ( \xi )</td>
<td>• Provided a unified framework and generalizes the earlier models of Tanaka (1986), Liang and Rogers (1990) and Boyd and Lagoudas (1994a)</td>
</tr>
<tr>
<td>(Auricchio and Sacco 1997)</td>
<td>Phase diagram based</td>
<td>• 1D model</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Only pseudoelasticity was considered.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Elastic modulus as a function of ( \xi ).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Plasticity based model, utilizing different critical stresses for transformation.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Very good agreement with experimental results.</td>
</tr>
<tr>
<td>(Auricchio and Taylor 1997)</td>
<td>Phase diagram based</td>
<td>• 3D model.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Rate independent model accounting for austenite – martensite transformation, and martensite reorientation.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Utilized transformation rule similar to Drucker-</td>
</tr>
<tr>
<td>Reference</td>
<td>Methodology</td>
<td>Description</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>-----------------------------------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>(Bekker and Brinson 1997; Bekker and Brinson 1998)</td>
<td>Phase diagram based</td>
<td>Formulated in terms of possible thermomechanical paths in stress-temperature space.</td>
</tr>
<tr>
<td>(Bo and Lagoudas 1999a; Bo and Lagoudas 1999b; Bo and Lagoudas 1999c; Lagoudas and Bo 1999)</td>
<td>Gibbs free energy</td>
<td>Extension of the work of Boyd and Lagoudas (1994a) to include modeling of transformation induced plasticity by connecting state variables’ (transformation strain, plastic strain, drag stress, back stress) evolution to the evolution of the $\zeta$.</td>
</tr>
<tr>
<td>(Lagoudas and Shu 1999)</td>
<td>Gibbs free energy</td>
<td>Only a one-dimensional implementation was provided.</td>
</tr>
<tr>
<td>(Qidwai and Lagoudas 2000a; Qidwai and Lagoudas 2000b)</td>
<td>Gibbs free energy, quadratic in $\zeta$</td>
<td>An extension of the works by Boyd and Lagoudas (1994a, 1996a).</td>
</tr>
<tr>
<td>(Brocca et al. 2002)</td>
<td>Microplane based</td>
<td>Employed the model by Brinson (1993) on each microplane.</td>
</tr>
<tr>
<td>(Helm and Haupt 2003)</td>
<td>Free energy</td>
<td>Free energy function (according to a mixture theory) as a function of temperature and internal state variables (martensite fraction, inelastic strain tensor, internal stress field).</td>
</tr>
</tbody>
</table>

Prager criteria in plasticity.
- Exponential hardening law.
- Currently implemented in commercial, large scale finite element packages ABAQUS® (ABAQUS 2008) and ANSYS® (ANSYS 2008).

Cosine hardening rule for the phase transformation.

Gibbs free energy, quadratic in $\zeta$.

Microplane based
- Employed the model by Brinson (1993) on each microplane.
- Thermal expansion is neglected and the elastic moduli are assumed constant for the two phases.
- Able to account for non-proportional loading paths.

Free energy
- Free energy function (according to a mixture theory) as a function of temperature and internal state variables (martensite fraction, inelastic strain tensor, internal stress field).
- Able to account for non-proportional loading.
| (Entchev and Lagoudas 2004; Lagoudas and Entchev 2004) | Gibbs free energy | • Extension of Lagoudas (1999a,b,c) and Lagoudas and Bo (1999) in 3-D.  
• Accounts for the simultaneous development of transformation and plastic strains during stress-induced phase transformation. |
|---|---|---|
• Separate evolution laws of parent phase transformation and martensite variant reorientation.  
• Not capable of capturing tension-compression asymmetry and irreversible plastic strains generated by cyclic loading |
| (Sadjadpour and Bhattacharya 2007a; Sadjadpour and Bhattacharya 2007b) | Helmoltz free energy | • Both 1D and 3D models were presented.  
• Assumes that the austenite and the martensite have the same elastic moduli.  
• Evolution of variants is described in terms of effective transformation strain that depends on crystallography of the material. |
| (Auricchio et al. 2007) | | • 3D model.  
• Inelastic and transformation strain tensors were utilized to capture the accumulation of residual strain under mechanical cyclic loading conditions.  
• No distinction between austenite and the twinned martensite.  
• Limited to pseudoelastic regime. |
| (Arghavani et al. 2010) | Helmoltz free energy | • 3D model, based on scalar stress induced martensite volume fraction and tonsorial preferred variant direction.  
• Not able to account for asymmetry in tension and compression. |
Table 2.4: A Summary of Leading Researchers and Their Work over the Years in the Area of Constitutive Modeling of SMAs.

<table>
<thead>
<tr>
<th>Group</th>
<th>Year</th>
<th>Description</th>
</tr>
</thead>
</table>
|         | 1997 (Auricchio and Sacco 1997; Auricchio and Taylor 1997) | • 1D superelastic phenomenological model under isothermal conditions, utilizing volume fraction of single martensite variant, accounting for reorientation of martensite.  
• One dimensional model utilizing single martensite volume fraction, accounting for different elastic properties of austenite and martensite. |
<p>|         | 1999 (Auricchio and Sacco 1999)  | • 1D phenomenological model, using two martensite fractions (single variant and multiple variant), accounting for superelastic response, OWSME, and ATC, and reorientation of single variant. Different elastic properties of martensite and austenite had been considered. |
|         | 2001 (Auricchio and Sacco 2001)  | • 1D phenomenological model accounting for superelasticity, ATC, and cyclic load effects.                                                                 |
|         | 2003 (Auricchio et al. 2003)     | • 1D phenomenological model, based on, accounting for superelasticity, ATC, different properties of austenite and martensite, permanent residual strain, cyclic training effect and TWSME under mechanical loading and isothermal conditions. |
|         | 2007 (Auricchio et al. 2007)     | • 3D phenomenological model, using inelastic strain tensor, describing pseudoelasticity, OWSME, nonproportional multi-axial loading response, and degradation and permanent strain under mechanical cyclic loading and isothermal conditions. |
| Brinson | 1993 (Brinson 1993)              | • 1D phenomenological model, utilizing stress induced and temperature induce martensite fraction, accounting for pseudoelasticity and OWSME. |
|         | 1997, 1998 (Bekker and Brinson 1997; Bekker and Brinson 1998) | • 1D phenomenological model utilizing possible thermomechanical paths in phase diagram, accounting for pseudoelastic response, major and minor loops in isothermal and isobaric conditions. |
|         | 2000 (Gao et al. 2000; Huang et al. 2000) | • 3D micromechanical (single crystal and polycrystalline) model, accounting for grouping of variants, anisotropy, and pseudoelastic response under multi-axial and thermal loading. |
|         | 2002 (Brocca et al. 2002)        | • 3D phenomenological model based on microplane theory, describing pseudoelasticity, ATC, OWSME, and major and minor loops. Most of the presented results are of uniaxial type. |</p>
<table>
<thead>
<tr>
<th>Year</th>
<th>Authors and Reference</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>Panico and Brinson</td>
<td>• 3D phenomenological model, by introducing twinned and detwinned martensite fractions, accounting for reorientation of martensite, pseudoelasticity, OWSME, and responses under non-proportional multi-axial loading. However, this model does not consider ATC.</td>
</tr>
<tr>
<td>1996</td>
<td>Boyd and Lagoudas</td>
<td>• 3D phenomenological model, using martensite volume fraction, accounting for pseudoelasticity and responses under non-proportional loading. This model is generalization of earlier work of Tanaka (1986), and Liang and Rogers (1990).</td>
</tr>
<tr>
<td>1999</td>
<td>Bo and Lagoudas</td>
<td>• 3D phenomenological model accounting for pseudoelasticity, TWSME, and major and minor loops. This model is extension of Boyd and Lagoudas (1996). Transformation induced plasticity is accounted for by introducing transformation strain, plastic, strain, drag stress, back stress as internal variables that depend upon the evolution of martensite.</td>
</tr>
<tr>
<td>2000</td>
<td>Qidwai and Lagoudas</td>
<td>• Extension of Boyd and Lagoudas (1996) to account for ATC. Three different transformation functions (based on $J_2$, $J_2 - I_1$, and $J_2 - J_3 - I_1$ transformation functions) and their effect on ATC was presented.</td>
</tr>
<tr>
<td>2004</td>
<td>Entchev and Lagoudas</td>
<td>• Extension of 3-D the model by Bo and Lagoudas (1999) to account for the simultaneous development of transformation and plastic strains during stress-induced phase transformation. The model is able to reproduce pseudoelasticity, ATC, and major and minor loops.</td>
</tr>
</tbody>
</table>
CHAPTER III

A MULTIMECHANISM BASED VISCOELASTIC-VISCOPLASTIC UNIFIED SMA CONSTITUTIVE MATERIAL MODEL

In this chapter, a summary of the unified viscoelastoplastic, multimechanism based SMA constitutive model will be presented. This particular model is the extension of the earlier work done by Saleeb et al. (see (Arnold et al. 2001; Saleeb and Arnold 2001; Saleeb and Arnold 2004; Saleeb et al. 2001)). This model had already been proved to be very successful in describing the constitutive behavior of high temperature Ti alloys (Arnold et al. 2001; Lissenden et al. 2007; Saleeb and Arnold 2001). For convenience in further discussion on the present constitutive model, a number of definitions and illustration will be provided in the following sections and subsections.

3.1 Forward/Reverse Phase Transformations and Residual State

For convenience in later discussions, we use in this and in the following sections, a number of simple illustrations to introduce several definitions and terminologies. For instance, Fig. 3.1 shows an idealized “flag – like” stress – strain curve for pseudoelastic response of SMA under uniaxial isothermal loading conditions. This is generally observed at a temperature, $T$, much higher than the finish temperature, $A_f$, of the transformation to the parent $A$ phase. The material can be loaded from zero stress (point
“0” in Fig. 3.1) in the \( A \) (parent) phase, and will follow essentially a purely elastic response until a critical stress level is reached (point “1” in Fig. 3.1). At this point, the start of the stress – induced, \textit{forward phase transformation} from \( A \) to \( M \) occurs. In the idealized case, region 1–2 is limited by a nearly constant stress (small hardening). This stress limitation in the present 1D representation corresponds to the \textit{limit stress} transformation surface (states) in the 3D case. The end of the rapid transformation region is signified by the appearance of a \textit{rehardening} region “2–3”. State “3” corresponds to a \textit{fully – transformed} material according to the classical viewpoint of the idealized material systems (i.e. a fully martensitic structure). Note that this latter region is bounded by the transformation–strain “magnitude”, which is a key material property in SMA modeling (signifying the maximum volume fraction of all produced “\( M \)” phases at the end of transformation). This corresponds to a \textit{transformation strain bounding/limit} surface in the 3D case. The part “3–4–5–6” represents the unloading, where region “4–5” corresponds to a rapid \textit{reverse} transformation from martensite to austenite. Note that in real experiments, the final state “6” (complete unloading to zero stress) typically indicates the existence of “small” amounts of residual strains, which may be attributed to \textit{residual} (retained) martensite. Also, note that many existing models in the SMA literature typically discard this feature; i.e., the idealization of purely austenite for state “6”. Finally, the differences in critical stress levels for the onset of phase transformations (forward versus reverse), as well as the maximum amount of residual strain, gives rise to a “flag” shaped stress – strain curve for these materials.
Figure 3.1: The 1D, idealized stress strain curve for pseudoelastic response of SMA: (0) parent zero stress state; (1) beginning of forward phase transformation; (2) end of forward phase transformation; (3) the material reorientation phase; (4) start of reverse phase transformation; (5) end of reverse phase transformation; (6) residual state. Note that the critical states marked in 1D correspond to limiting surfaces in 3D.

3.2 Scope and Outline of Formulation

For the purposes of this dissertation, we limited ourselves to the range of small deformations. In our mathematical formulation and the associated numerical implementation schemes, we followed the details given in Ref. (Saleeb et al. 2001). A summary of some key points is given below.

We adopted a complete potential – based framework in terms of strain energy (Gibb’s type, $\Phi$) and dissipation ($\Omega$) functions (Arnold and Saleeb 1994; Saleeb et al. 2001; Saleeb and Wilt 1993). These are expressed in terms of a number of internal state
variables of the tensorial type that allow us to account for the unique features of the SMA response. With reference to the recent literature on SMA modeling, we mention Ref. (Luig and Bruhns 2008), where the importance of the use of such internal variables with directional properties is emphasized (in contrast to other “scalar” type quantities such as phase fractions that prevailed in the early developments of SMA models).

In particular, we utilize the following internal state variables (where “$a$” and “$b$” are counters denoting specific mechanisms):

1. A decomposed total strain tensor, $\varepsilon_{ij} = \varepsilon_{ij}^{ve} + \varepsilon_{ij}^{I} + \varepsilon_{ij}^{th}$, where $\varepsilon_{ij}^{ve}$ accounts for the possible rate dependency of the individual phases ($A$ and $M$) within the material; $\varepsilon_{ij}^{I}$ stands for the transformation induced deformations; and $\varepsilon_{ij}^{th}$ denotes the thermal strain. For simplicity, and also because of their insignificant magnitudes (relative to the major part of transformation – induced strain, $\varepsilon_{ij}^{I}$), we will discard thermal strains in all subsequent discussions.

2. A set of stress – like tensors, $q_{ij}^{(a)}$, and their associated strain – like conjugates (fluxes), $p_{ij}^{(a)}$, accounting for the rate dependencies of $\varepsilon_{ij}^{ve}$.

3. A set of nonlinear kinematic hardening stress tensors, $\alpha_{ij}^{(b)}$, and their conjugates, $\gamma_{ij}^{(b)}$, accounting for the partitioned energy storage/dissipation in conjunction with $\varepsilon_{ij}^{I}$.
Note that items (2) and (3) above will play a major role in the future treatment of high transformation temperature materials (e.g. NiTiPt, NiTiPdX, and other ternary or quaternary SMA alloys). More specifically, regarding item (3) above, we have elected to partition the various SMA mechanisms as follows (see the schematic in Fig. 3.2):

1. The first \((b = 1)\) accounts for the rapid developments of transformation strains near the critical state (state “1” depicted in Fig. 3.1) and is used to control the amounts and evolution of residual transformation strains (see part “5–6” in Fig. 3.1).

2. The second \((b = 2)\) accounts for the possible gradual hardening in the transformation regime (i.e. the “flag” region of pseudoelastic response depicted in Fig. 3.1). Together with (1) above, this mechanism is also used to control the amount of thermal energy for transformation in superthermal effects.

3. The third \((b = 3)\) and final internal mechanism in the SMA block of Fig. 3.2 provides for an ever-increasing hardening function (as a novel extension of Saleeb et al. (Saleeb and Arnold 2004; Saleeb et al. 2001)) that naturally leads to a limiting internal force demarking the completion of all phase transformations; see regions “2–3–4” in Fig. 3.1). This also enables us to bypass any complications in the “traditional” treatment of the condition of “a unit value for the sum of all volume – fractions – of – M – phases”, as utilized by others in the literature; e.g. Refs. (Bekker and
Brinson 1997; Bekker and Brinson 1998; Popov and Lagoudas 2007; Qidwai and Lagoudas 2000a; Qidwai and Lagoudas 2000b).

(4) The additional hardening-recovery mechanisms (b = 4 to N in Fig. 3.2) are reserved for the evolutionary character of SMA deformation response under cyclic thermomechanical load effects, both in pseudoelastic and pseudoplastic regions.

![Figure 3.2: A brief 1D schematic of multi-mechanism based model. Note that F is inelastic (transformation) potential.](image)

It is also important to note that the same evolution (rate) equations for the inelastic strains, $\varepsilon^I$, are used in both the forward and the reverse transformations (see Eq. (3.27) in section 3.3.7). This will completely bypass the need for any non–associative flow rules (e.g. see Refs. (Lagoudas et al. 2006; Patoor et al. 2006; Qidwai and Lagoudas...
These evolutionary equations were developed through the use of transformation “yield” functions with curved meridians and a *distorted* (non-circular) shape in the deviatoric plane (along the lines described in Chapters 5 and 7 of Chen and Saleeb, 1994, Ref. (Chen and Saleeb 1994)). This provides limiting conditions for phase transformation (i.e., from austenite to martensite, as well as possible reorientations of martensite variants). This will also enable the modeling of asymmetry in tension/shear/compression, as often observed experimentally for SMA materials. We avoid the *explicit* use of phases (and the ensuing cumbersome computations as in Refs. (Auricchio et al. 2003; Govindjee and Kasper 1999; Lagoudas et al. 2006; Wang and Dai 2010), and specially recently review on some advantages and some limitations on the use of multiple phase fractions to differentiate between twinned, detwinned, reoriented phase fractions under general loading (Popov and Lagoudas 2007)). Instead, we account, in \( \varepsilon^I \), for all deformation – producing transformations; i.e. direct \( A \rightarrow M \), detwinning of \( M \) – variants, reorientations of any \( M \) – variant under non-proportional loading.

For the numerical integration scheme, we utilize the *implicit backward* Euler method for its unconditional *stability* and *robustness* (Saleeb and Wilt 1993; Saleeb et al. 1998b; Saleeb et al. 2000). A concise form is developed by exploiting the mathematical structures of the model equations, leading to a very efficient implementation of the update and its *algorithmic* (consistent), (material) *tangent* stiffness. In particular, the *closed-form*, expressions for the tangent stiffness arrays, whose dimensions are *independent* of the number of state variables employed, are derived (i.e. the stress tensor, tensorial viscoelastic internal parameters, and tensorial viscoplastic state variables).
Especially, the dimension of the resulting arrays is only determined by the underlying problem dimensions (e.g. six for three dimensional problems, four for plane strain, three for plane stress problems etc.). These expressions and tangent stiffness have been proved effective in implementing the Newton iterative scheme utilized in the integration.

A brief outline of the formulation is given in section 3.3. Finally, we refer the reader to Refs. (Saleeb and Arnold 2004; Saleeb et al. 2000; Saleeb et al. 2002) for many details of the derivations and algorithmic developments alluded to above.

3.3 Outline of the Multi – mechanism based Unified SMA Material Model

We begin by summarizing the basic equations governing the behavior of a SMA material element. The following discussion is limited to the case of small deformations under isothermal conditions and stress free initial (virgin) state. A Cartesian frame of reference is utilized, along with indicial notation (wherein summation is implied for repeated “subscripts”). We also utilize “superscript” letters placed between parentheses as indices to identify sets of internal state parameters and when needed the summation over these will be indicated explicitly by the summation symbol.

Furthermore, it is noted that we do not consider here any heat transfer aspects (e.g. release/absorption of latent heat, etc.) for a formal thermodynamical treatment of the coupled thermomechanical problem. However, we give below a number of supporting arguments to justify our choice.

Firstly, under different rates of loading, it is often observed that the change in response of an “ordinary” SMA is due to the generation and absorption of latent heat.
This causes local change in the temperature, leading to the “apparent” rate dependency in response. These were the main points emphasized in (Grabe and Bruhns 2008; Hartl et al. 2010; Lim and McDowell 1999; Peyroux et al. 1998; Shaw and Kyriakides 1995; Tobushi et al. 1998) considering the pseudoelastic response with significant latent heat generation during the $A \rightarrow M$ transformation. On the other hand, there are other reported cases where ordinary transformation of NiTi SMAs exhibit significant relaxation and creep in the pseudoplastic regime, which was attributed to genuine viscoplasticity (Helm and Haupt 2001; Helm and Haupt 2003). Note that in these later cases the main strain producing mechanism is detwinning/orientation of $M$ phase product (unlike the $A \rightarrow M$ transformation alluded to above). Furthermore, significant variations in the SMA response are observed under very high rate of loading (Nemat-Nasser et al. 2005a; Nemat-Nasser et al. 2005b; Nemat-Nasser and Guo 2006), which cannot be explained solely on the basis of latent heat exchanges. Clearly, there is a debate in the literature on the heat transfer and rate dependency aspects in SMA response that needs to be concluded by systematic and more elaborate experimental programs.

In summary, the important points regarding the consideration of heat transfer and rate effect in our modeling approach are as follows:

(1) In the application of HTSMA, viscoplasticity plays an important role in the rate dependency effects at elevated temperatures (Hartl et al. 2010; Kumar and Lagoudas 2010; Lexcellent et al. 2005; Mukherjee 1968). Therefore, it must be included in the constitutive modeling approaches targeting to capture the HTSMA response. We reemphasize that our
intention is to provide a unified treatment for both ordinary SMAs (the focus in the present work), as well as the HTSMA response.

(2) Under “strict” isothermal conditions and slow rates of loading, the effect of latent heat absorption/dissipation is minimal. On the other hand, significant latent heat exchange leads to changes in specimen’s temperature; hence the testing condition can then be looked as non–isothermal (or variable thermal – mechanical loading). This can easily be accounted for in our approach by simply providing the varying temperature history, along with the other mechanical controls (stress/strain), as applied “load controls” (i.e., input) during the analysis.

With the special emphasis on isothermal conditions and/or moderate or slow rate of temperature change, heat transfer effects will be minimized in the SMA response. Therefore, all the nonisothermal effects are simply reduced to a mere temperature – dependency of few material coefficients in the developed equations (e.g. see also the last part in Table 4.1 in Chapter IV). Note, however, that this does not affect in any “significant” way the capability of the resulting model (e.g., we refer to Chapter IV with an extensive set of numerical results demonstrating the many desirable phenomena exhibited by the present SMA model). Finally, because of their rather small values (relative to the very significant magnitudes of transformation – induced strains), we here elected to discard any thermal strains in the subsequent discussions.

All equations are written for the general 3D case; but the forms are also directly applicable in subspaces (e.g., two dimensional, 2D, plane – stress or plane – strain, etc.),
see (Saleeb et al. 1998a; Saleeb and Wilt 1993; Saleeb et al. 1998b). For conciseness, we define the total number of dissipative viscoelastic state variables (each of the second–order tensor type) as \( M \); i.e.; the corresponding non–equilibrium stress tensor (Arnold et al. 2001; Saleeb and Arnold 2001) are \( q_{ij}^{(a)} \) \((a = 1, 2, \ldots, M)\) and their associated conjugate (strain–like) tensors are \( p_{ij}^{(a)} \). In addition, we define the equilibrium stress tensor \((\sigma_{\epsilon})_{ij}\) and strain conjugate \(\varepsilon_{ij}^{ve}\). Similarly, for the viscoplastic mechanisms, we use the notation \(\alpha_{ij}^{(b)} \) \((b = 1, 2, \ldots, N)\) for the back stresses (kinematic hardening) and \(\gamma_{ij}^{(b)}\) for their conjugate or dual (strain–like) variables, for a total of \(N\) viscoplastic (second order) tensorial state variables.

3.3.1 Potential and State Equations

The total strain tensor, \(\varepsilon_{ij}\), is assumed to be decomposed into two components; i.e. a reversible (i.e., elastic/viscoelastic), \(\varepsilon_{ij}^{ve}\); and irreversible (i.e. transformation–induced/viscoplastic), \(\varepsilon_{ij}^{I}\), components;

\[
\varepsilon_{ij} = \varepsilon_{ij}^{ve} + \varepsilon_{ij}^{I}. \tag{3.1}
\]

Recall that we utilize the tensor \(\varepsilon_{ij}^{I}\) to implicitly account for all transformation–induced deformations; i.e., forward/reverse transformations between \(A\) and \(M\) phases at higher temperature, the detwinning of \(M\)–phase variants at lower temperature, as well as reorientations, variant coalescence and other allied effects under non–proportional states of stresses and strains. We also intend to include part of the loading rate– and time– dependency for treating HTSMA materials in the evolution equations of \(\varepsilon_{ij}^{I}\) (see Eq. (3.9) below)
The two fundamental energy potentials; that is, the Gibbs's, complementary function, \( \Phi \), and dissipation function, \( \Omega \), are assumed to be decomposed into:

\[
\Phi \left( \sigma_{ij}, \alpha_{ij}^{(b)}, q_{ij}^{(a)} \right) = \Phi_{R} \left( \sigma_{ij}, q_{ij}^{(a)} \right) + \Phi_{IR} \left( \alpha_{ij}^{(b)} \right),
\]

\( (3.2) \)

\[
\Omega \left( \sigma_{ij}, \alpha_{ij}^{(b)}, q_{ij}^{(a)} \right) = \Omega_{R} \left( q_{ij}^{(a)} \right) + \Omega_{IR} \left( \left( \sigma_{ij} - \alpha_{ij} \right), \alpha_{ij}^{(b)} \right).
\]

\( (3.3) \)

where the functional dependencies on the external \( (\sigma_{ij}) \) and internal \( (q_{ij}^{(a)} \) and \( \alpha_{ij}^{(b)} \) state variables, with \( (a = 1, 2, \ldots, M) \) and \( (b = 1, 2, \ldots, N) \), and the conjugate variables \( \varepsilon_{ij} \) (and also \( \varepsilon_{ij}^{ve} \)), \( e_{ij}^{ve} \) (also \( p_{ij}^{(a)} \)), and \( \gamma_{ij}^{(b)} \), respectively, are given by the following equation of state:

\[
\varepsilon_{ij} - \varepsilon_{ij}^{ve} = \frac{\partial \Phi_{R}}{\partial (\sigma_{ij})},
\]

\( (3.4) \)

\[
\varepsilon_{ij}^{ve} - p_{ij}^{(a)} = \frac{\partial \Phi_{R}}{\partial q_{ij}^{(a)}},
\]

\( (3.5) \)

\[
\gamma_{ij}^{(b)} = \frac{\partial \Phi_{IR}}{\partial \alpha_{ij}^{(b)}}.
\]

\( (3.6) \)

The specific functional form of the potential functions will be described later.

Note that in Eq. (3.5) only a single viscoelastic strain component is associated with all viscoelastic stress components. Also, for the viscoelastic response, the total stress is decomposed into an equilibrium stress, \( (\sigma_{s})_{ij} \), and a non-equilibrium (dissipative) reversible stress, \( q_{ij} \), i.e.,

\[
\sigma_{ij} = (\sigma_{s})_{ij} + q_{ij}, \text{ where } q_{ij} = \sum_{a=1}^{M} q_{ij}^{(a)}.
\]

\( (3.7) \)
Similarly, for the viscoplastic response, the total stress is decomposed again into an equilibrium, \((\sigma_{ij} - a_{ij})\), and non-equilibrium, \(a_{ij}\), components, where the total back stress tensor is defined as,

\[
\alpha_j = \sum_{k=1}^{N} \alpha_{ij}^{(b)}.
\]  

From Eqs. (3.4) - (3.6), the corresponding rate forms are then,

\[
\dot{\epsilon}_{ij}^{(b)} - \dot{\epsilon}_{ij}^{(s)} = \frac{d}{dt} \left( \frac{\partial \Phi_{R}}{\partial (\sigma_{ij})} \right) = \frac{\partial^2 \Phi_{R}}{\partial (\sigma_{ij}) \partial (\sigma_{kl})} (\dot{\sigma}_{kl}), \tag{3.9}
\]

\[
\dot{\epsilon}_{ij}^{(s)} - \dot{\nu}_{ij}^{(a)} = \frac{d}{dt} \left( \frac{\partial \Phi_{R}}{\partial q_{ij}^{(a)}} \right) = \frac{\partial^2 \Phi_{R}}{\partial q_{ij}^{(a)} \partial q_{kl}^{(a)}} \dot{q}_{kl}^{(a)}, \tag{3.10}
\]

\[
\dot{\gamma}_{ij}^{(b)} = \frac{d}{dt} \left( \frac{\partial \Phi_{R}}{\partial \alpha_{ij}^{(b)}} \right) = \frac{\partial^2 \Phi_{R}}{\partial \alpha_{ij}^{(b)} \partial \alpha_{kl}^{(b)}} \alpha_{kl}^{(b)} \quad \text{or} \quad \dot{\alpha}_{kl}^{(b)} = \left[ \frac{\partial^2 \Phi_{R}}{\partial \alpha_{ij}^{(b)} \partial \alpha_{kl}^{(b)}} \right]^{-1} \dot{\gamma}_{ij}^{(b)} \tag{3.11}
\]

where the over dot represents the time derivatives (i.e., \(\dot{\cdot} = \frac{d}{dt}(\cdot)\)). Eqs. (3.10) and (3.11) above constitute the rate equations for governing all of the (non-equilibrium) state variable for each inelastic mechanism.

Similarly, from the dissipation function (Eqs. (3.3) and (3.8)), the corresponding flow and evolution (rate) equations in terms of the “conjugate” internal state variables are obtained as follows:

\[
\dot{\epsilon}_{ij}^{(s)} = \frac{\partial \Omega_{IR}}{\partial \sigma_{ij}}, \tag{3.12}
\]
\[ \dot{\gamma}_{ij}^{(b)} = -\frac{\partial \Omega_{ij}}{\partial \alpha_{ij}^{(b)}}, \quad b = 1,2,\ldots,N, \quad (3.13) \]

\[ \Phi_{ij}^{(a)} = \frac{\partial \Omega_{ij}}{\partial q_{ij}^{(a)}}, \quad a = 1,2,\ldots,M. \quad (3.14) \]

### 3.3.2 Specific Potential Functional Forms

For completeness, we review here the specified functional forms that have been utilized in the previous work by Saleeb et al. (Arnold et al. 1995; Arnold et al. 2001; Saleeb and Arnold 2001; Saleeb and Arnold 2004; Saleeb et al. 2001; Saleeb and Wilt 1993) to motivate the several “significant” extensions needed for SMA materials. We have utilized quadratic form for the viscoelastic contribution and the same “power – type” nonlinear forms for the viscoplasticity functions \( \tilde{H}_{(b), \Omega}, \) and \( \tilde{\Omega}^{(b)}_{2} \) in terms of the invariants of their respective arguments \( (\alpha_{ij}^{(b)}, (\sigma_{ij} - \alpha_{ij}^{(b)}) \) and \( \alpha_{ij}^{(b)}, \) respectively). However, a few, carefully – selected, functional forms for the hardening function, \( \tilde{h} \left( g^{(b)} \right) \) for the three dedicated mechanisms of energy storage (with no recovery contribution acting in any of them) in the SMA block \((b = 1, 2, 3), \) and \( h(G^{(b)}) \) for \( b \geq 4 \) (with possible thermal recovery acting in each for the HTSMA cases) in the remaining dissipative mechanisms (see Fig. 3.2) will be investigated.

The specific forms for the reversible and irreversible potentials and driving functions are as follows:

\[ \Phi_{R} \left( \sigma_{ij}, q_{ij}^{(a)} \right) = \frac{1}{2} \left( \sigma_{ij} \right) E_{ijkl}^{-1} \left( \sigma_{kl} \right) + \frac{1}{2} \sum_{a=1}^{M} q_{ij}^{(a)} \left[ M_{ijkl}^{(a)} \right]^{-1} q_{ij}^{(a)} + \sum_{a=1}^{M} q_{ij}^{(a)} p_{ij}^{(a)}, \quad (3.15) \]
\[ \Phi_{IR} (\sigma_{ij}, \alpha_{ij}^{(b)}) = \sigma_{ij} \varepsilon_{ij}' + \sum_{b=1}^{N} \tilde{H}_{(b)} \left( G^{(b)} \right), \quad (3.16) \]

and,

\[ \Omega_{R} \left( q_{ij}^{(a)} \right) = \frac{1}{2} \sum_{a=1}^{M} q_{ij}^{(a)} \left[ \eta_{ijl}^{(a)} \right]^{-1} q_{ij}^{(a)}, \quad (3.17) \]

\[ \Omega_{IR} \left( (\sigma_{ij} - \alpha_{ij}), \alpha_{ij}^{(b)} \right) = \Omega \left( F \left( \sigma_{ij} - \alpha_{ij} \right) \right) + \sum_{b=1}^{N} \Omega_{2}^{(b)} \left( G^{(b)} \left( \alpha_{ij}^{(b)} \right) \right), \quad (3.18) \]

where,

\[ F \left( \sigma_{ij} - \alpha_{ij} \right) = \frac{1}{\kappa^2} \left[ \frac{1}{2\rho^2} \left( \sigma_{ij} - \alpha_{ij} \right) M_{ijkl} \left( \sigma_{ij} - \alpha_{ij} \right) + a_1 \left( \sigma_{kk} - \alpha_{kk} \right) + a_2 \left( \sigma_{kk} - \alpha_{kk} \right)^2 \right]^{-1}, \quad (3.19) \]

\[ G^{(b)} \left( \alpha_{ij}^{(b)} \right) = \frac{1}{2\kappa_{(b)}^2} \left( \alpha_{ij}^{(b)} M_{ijkl} \alpha_{ij}^{(b)} \right), \quad (3.20.a) \]

\[ g^{(b)} \left( I_{ij}^{(b)} \right) = I_{ij}^{(b)} \left( Z_m \right)_{ijkl} I_{kl}^{(b)}, \quad (3.20.b) \]

\[ \rho = \frac{1 + c\sqrt{d}}{1 + c\sqrt{d} + k}, \quad (3.20.c) \]

and where the specific Gibb’s functions entering the expressions for dissipation (Eq. (3.18)) and Gibb’s energies (Eq. (3.16)) are:

\[ \Omega_{1} (F') = \int \frac{\kappa^2 F'^n}{2\mu} dF, \quad \Omega_{2}^{(b)} \left( G^{(b)} \right) = \kappa_{(b)}^2 \int \frac{r \left( G^{(b)} \right)}{h \left( G^{(b)} \right)} dG^{(b)}, \quad (3.21.a) \]
\[
\bar{H}_{(b)} = \begin{cases} 
\kappa_{(b)}^2 \int \frac{1}{h(G^{(b)})} \, dG^{(b)}, & \text{for } b = 1, 2, 3 \\
\kappa_{(b)}^2 \int \frac{1}{h(G^{(b)})} \, dG^{(b)}, & \text{for } b \geq 4.
\end{cases}
\] (3.21.b)

Note that in the above, \(E_{ijkl}\) and \(M_{ijkl}\) are fourth order tensors of viscoelastic stiffness moduli (with the corresponding compliance tensors \([E_{ijkl}]^{-1}\) and \([M_{ijkl}]^{-1}\), respectively); \(\eta_{ijkl}^{(a)}\) \((a = 1, 2, \ldots, M)\) are the fourth order tensorial viscosity coefficients associated with the \(a\)th dissipative viscoelastic mechanisms (e.g. for the use in applications with HTSMA). All the latter tensors are taken to correspond to isotropic behavior in the present applications reported in Chapter IV (with corresponding material constants \(E_s\) and \(v\), in \(E_{ijkl}\); \(E_m^{(a)}\) and \(v\) in \(M_{ijkl}\); and \(\bar{\rho}^{(a)}\) in \(\eta_{ijkl}^{(a)}\)). Typically, in applications, one takes \(\eta_{ijkl}^{(a)} = \bar{\rho}^{(a)} M_{ijkl}^{(a)}\), where \(\bar{\rho}^{(a)}\) is the characteristic relaxation time for each of the \(a\)th viscoelastic mechanism. As alluded to earlier no considerations are made here for HTSMA, and the “viscoelasticity” will simply be reduced to an “elastic” part (see Table 4.1 in Chapter IV).

The \(\Omega_1\) and \(\Omega_2^{(b)}\) are the inelastic dissipations due to transformation – induced strain and associated hardening, and static (thermal) recovery (in HTSMA materials), respectively. Each of the functions \(\bar{H}_{(b)}\) is taken to be nonlinear in terms of the internal state tensor \(\alpha_{ij}^{(b)}\), and/or its conjugate internal strain \(\gamma_{ij}^{(b)}\) for each of the hardening mechanisms. To emphasize the function dependency in the potentials, e.g. \(\Phi_R\), \(\Phi_{IR}\) and \(\bar{H}_{(b)}\), the corresponding arguments are shown in parentheses.
Furthermore, in compliance with the experimental fact that inelastic deformation of most SMAs are “essentially” *independent* of hydrostatic stress (incompressible), we selected the fourth order tensor, $\mathcal{M}_{ijkl}$, to be purely deviatoric (see Refs. (Saleeb and Wilt 1993; Saleeb et al. 2002) and Eq. (3.33) below). The limited amount of pressure sensitivity and associated volumetric dilatancies (if any) will be treated separately in our future work through the terms containing $a_1$ and $a_2$ given in Eq. (3.19). Note that $\rho$ is defined in Fig. (3.2) in terms of the Lode’s angle associated with the *effective* stresses, $\sigma_{ij} - \alpha_{ij}$, thus leading to significant dependency on the *directions* of both external and internal stress tensors, $\sigma_{ij}$ and $\alpha_{ij}$, respectively. In particular, the evolution of the internal stats in $\alpha_{ij}$ will entail a strong “strain – induced” *anisotropy* of the SMA materials’ response (even if we assume an *initially* – isotropic behavior with a random orientations of many grains in the polycrystals).

The remaining functions that are needed to be assumed are those defining the hardening and recovery processes within the SMA material. For the thermal recovery, we take here a “power type” function (as before), that is, for $b \geq 4$:

$$r\left(G^{(b)}\right) = R_{(b)}\left[G^{(b)}\right]^{m_{(b)}}. \tag{3.22}$$

Note that Eq. (3.22) is shown here only for completeness since all applications in Chapter IV were restricted to “ordinary” SMA material systems (not the HTSMA class operating at high temperatures). We have therefore, suppressed the recovery terms of all mechanisms here.
For the hardening functions, we separately treat in the sequel the individual energy–storage mechanisms \((b = 1, 2, 3)\) in the SMA block of Fig. 3.2 compared to the remaining mechanisms (i.e. \(b \geq 4\)) in the counterpart dissipative block.

### 3.3.3 SMA Energy Storage Block

The driving forces for the evolution of hardening in every one of these dedicated energy storage mechanisms are given by function “\(h\)” whose arguments are internal strains \(\gamma_{ij}^{(b)}\).

\[
h\left(g^{(b)}\right) = \frac{\rho^{(b)}_{\beta} H^{(b)} \left(\sqrt{g^{(b)}}\right)^{(\beta(b))^{-1}}}{\kappa^{(b)} + H^{(b)} \left(\sqrt{g^{(b)}}\right)^{\beta(b)}} \text{ for } b = 1, 2, \quad (3.23.a)
\]

\[
h\left(g^{(b)}\right) = \rho^{(b)} H^{(b)} \left[1 + \left(\frac{\sqrt{g^{(b)}}}{\kappa^{(b)} / H^{(b)}}\right)^{\beta(b)}\right] \text{ for } b = 3, \quad (3.23.b)
\]

where,

\[
\rho^{(b)} = \frac{1 + c^{(b)} \sqrt{d^{(b)}}}{1 + c^{(b)} \sqrt{d^{(b)}} + k^{(b)}}, \quad (3.23.c)
\]

\[
k^{(b)} = \cos 3\theta^{(b)}. \quad (3.23.d)
\]

with Lode’s angle \(\theta^{(b)}\) being evaluated from the invariants of internal state tensor \(a_{ij}^{(b)}\) as per the well–known relations (e.g. Ref. (Chen and Saleeb 1994)).
3.3.4 SMA Energy Dissipation Block

On the other hand, for each of the dissipative mechanisms, we use functions with internal stress arguments that explicitly exhibit limit/bounding/saturation states (see Ref. (Saleeb and Arnold 2004)). To this end, we make use of the Heaviside function \( \hat{h}(L) \) with the loading index \( L = \alpha_j^{(b)} \Gamma_{ij} \), where \( \Gamma_{ij} \) is defined in Eq. (3.30) below, to account for the effects of cyclic and non–proportional loadings:

\[
h\left(G^{(b)}\right) = H_{(b)} \left[ 1 - \left( \frac{\sqrt{G^{(b)}}}{\rho_{(b)}} \right)^{\beta_{(b)}} \hat{h}(L) \right], \text{ for } b \geq 4. \tag{3.24}
\]

Note that \( \rho_{(b)} \) is defined as before. Note also that \( k_3 \) in Fig. 3.2 and every \( k_j^{(b)} \) in Eq. (3.23.d) above is always bounded in the range -1 to +1; \( c \), and \( c_{(b)} \) are positive material constants; and each of the material constants \( d \) and \( d_{(b)} \) in Eqs. (3.20.c) and (3.23.c) satisfy the convexity conditions of being positive and greater that one in magnitude (see Table 1 for a sampling of the “convexity” constraints connecting \( d \) and \( c \), or any of the pairs \( d_{(b)} \) and \( c_{(b)} \) for \( b = 1, 2, \ldots, N \). It is also remarked here that all forms in Eqs. (3.23.a), (3.23.b) and (3.24) contain the same number of material constants.

Table 3.1: Convexity Constraints for Different Degrees of ATC.

<table>
<thead>
<tr>
<th>Values of ( d ) (or ( d_{(b)} )) ≥ 1</th>
<th>Values of ( \bar{c}<em>u ) in ( c ) (or ( c</em>{(b)} )) ≤ ( \bar{c}_u )</th>
<th>Degree of ATC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.565 (extreme case)</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>0.790</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>0.154</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>1.870</td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td>4.000</td>
<td></td>
</tr>
</tbody>
</table>

\( \uparrow \) (increasing)
Several important remarks are in order here regarding the above selected forms:

(1) There are several convex functions and a number of generalized normality rules employed here; i.e., “F”, “G”, and \( g^{(b)} \) as given in Eqs. (3.19) and (3.20); and Eqs. (3.4) – (3.6) and (3.12) – (3.14), respectively. In particular, note that with Eqs. (3.12), (3.19), (3.27), and (3.30) the normality of \( \dot{\epsilon}_{ij} \) is now in terms of generalized/effective stresses \( (\sigma_{ij} - \alpha_{ij}) \), and not only the stress \( \sigma_{ij} \). This latter property will prove essential in capturing some of the important observations in recent SMA experiments on non-proportional multi-axial loading.

(2) The dual nature of the arguments in the “h” functions will enable us to provide critical/limit states simultaneously for both internal stresses and transformation strains. While the function “h” for \( b = 1, 2 \) tends to reduce rapidly and become bounded when internal “transformation” strains, \( \gamma_{ij}^{(b)} \), tend to grow unboundedly, the corresponding state variables for \( b = 3 \) (under the same conditions) will increase gradually to become unbounded. On the other hand, for every mechanism \( b \geq 4 \), any tendency for the internal stresses to increase will eventually render all corresponding “h” bounded. Indeed, it is this “orchestrated” partitioning of evolutions in the tensorial hardening mechanisms under stress– and “transformation” strain– controlled conditions that enabled the SMA model to produce the
“favorable” test cases of Figs. 4.10 and 4.12 in Chapter IV (see also LD = Limits on the Duals in Fig. 2.4 in Chapter II).

(3) As a corollary to (1) above, the appropriate use of “$h$” functions will automatically bypass any need for such alternative “specialized” treatment (often with further complications) as the Lagrange multipliers (Lagoudas et al. 2006; Patoor et al. 2006; Siredey et al. 1999), “polyconvex” constraints on the phase fractions (Govindjee and Kasper 1999; Govindjee and Miehe 2001), or use of indicator (singular) functions (Auricchio et al. 2007; Lim and McDowell 1999) in order to introduce some of the “critical” states mentioned above.

(4) As a result of the specific generalized normality rule in Eqs. (3.12) – (3.14), the thermodynamic admissibility condition in the form of dissipation inequality (2nd law of thermodynamics) is now guaranteed by the non negativity of the dissipation rate function ($\Omega, \Omega_R, \Omega_{IR}$), which is trivially satisfied by the convexity condition alluded to the item 1 above; i.e. a convex dissipation function $F$ in Eq. (3.18) and $\Omega_R$ and $\Omega_{IR}$ (see Eqs. (3.3), and (3.17) – (20)). For further details on these mathematical aspects of the presented model, we refer readers to Refs. (Saleeb and Arnold 2001; Saleeb and Wilt 1993).

3.3.6 Material Constants and Temperature Effects

Finally, in the above, $\kappa, \mu, n, c, d$ represent inelastic flow material constants, whereas the $H(b), \beta(b), c(b), d(b)$ are hardening material constants. The $R_{(b)}$ and $m_{(b)}$ are
recovery material constants; and the constants $\kappa_{(b)}$ are hardening threshold stresses for the individual viscoplastic mechanisms $(b)$, where $b = 1, 2, \ldots, N$, depending upon the form of the hardening function assumed in Eqs. (3.23) and (3.24).

From the theoretical standpoint, it is possible for any of these material parameters to be made dependent on temperature; however, if done “arbitrarily”, this will often require further complications in model characterization from experiments. Consequently, and also to indicate the great potential of the presented SMA model in the simplest setting, we have elected to restrict temperature – dependency to only a few inelastic hardening mechanisms; i.e., $b = 1, 2, \text{and } 4$. Furthermore, among the many available parameters governing these three mechanisms, we have chosen one single parameter; i.e., the threshold $\kappa_{(b)}$ for this purpose (see also the last part of Table 4.1 in Chapter IV).

3.3.7 Resulting Multi–axial Flow and Evolution Equations

The following expressions, along with the particular functional forms above, constitute the governing associated flow and evolution equations in the present viscoelastoplastic model,

$$\sigma_{ij} = E_{ijkl} (\dot{e}_{kl} - \dot{e}^l_{kl}) + \sum_{a=1}^{M} q_{ij}^{(a)}, \quad (3.25)$$

$$q_{ij}^{(a)} = M_{ijkl} (\dot{e}_{kl} - \dot{e}^l_{kl}) - M_{ijkl} n_{klrs} q_{rs}^{(a)}, \quad (3.26)$$

$$\dot{e}_{ij}^l = \begin{cases} f (F) \Gamma_{ij} & \text{if } F \geq 0, \\ 0 & \text{otherwise,} \end{cases}, \quad f (F) = \frac{F^\mu}{2\mu} \quad (3.27)$$
\[
\dot{\alpha}_{ij}^{(b)} = Q_{ijkl}^{(b)} \left[ \dot{\varepsilon}_{ki}^{(b)} - \frac{r(G^{(b)})}{h(G^{(b)})} \pi_{kl}^{(b)} \right],
\]  
(3.28)

where the nonlinear fourth order \( Q_{ijkl} \) operator for each hardening mechanisms can be straightforwardly derived from Eqs. (3.20) to (3.24) above. For example, we show below the case corresponding to \( b \geq 4 \):

\[
Q_{ijkl}^{(b)} = \left[ \frac{\partial^2 H^{(b)}}{\partial \alpha_{ij}^{(b)} \partial \alpha_{kl}^{(b)}} \right]^{-1} h(G^{(b)}) \left[ (Z_m)_{ijkl} + \frac{h'(G^{(b)})}{h(G^{(b)})} \left( 1 - \frac{h'(G^{(b)})}{2\kappa^2(\beta)G^{(b)}} \right) \alpha_{ij}^{(b)} \alpha_{kl}^{(b)} \right],
\]  
(3.29)

\[
\Gamma_j = \frac{\partial F}{\partial (\sigma_j - \alpha_j)}, \quad \pi_{kl}^{(b)} = M_{ijkl} \alpha_{ij}^{(b)},
\]  
(3.30)

\[
q_j = \sum_{a=1}^{M} q_{ij}^{(a)}, \quad \alpha_j = \sum_{b=1}^{N} \alpha_{ij}^{(b)}.
\]  
(3.31)

In the above, we have made use of the following (recalling the earlier form of function \( \hat{h}(L) \) in the part preceding Eq. (3.24) above):

\[
h'(\cdot) = \frac{\hat{h}(L) \partial h(\cdot)}{\kappa^2(\beta) \partial G^{(b)}}, \text{ for } b = 4, 5, \ldots, N
\]  
(3.32)

Note \((Z_m)_{ijkl}\) is the “generalized” inverse of \( M_{ijkl} \) (see Ref. (Saleeb and Wilt 1993) for further elaboration on this). Details regarding intermediate steps, extension to cases involving different degrees of initial anisotropy, which is important in considering...
“texture” effects (e.g. see Ref. (Khan and Huang 1995)) in SMA materials, and the numerical solution of these equations have been previously discussed (see Refs. (Saleeb et al. 2001; Saleeb and Wilt 1993; Saleeb et al. 2003)). In particular, for a simpler case of anisotropy; i.e. with transverse isotropy (as in thin – walled cylinders) we have:

\[
\mathcal{M}_{ijkl} = \overline{p}_{ijkl} - \xi A_{ijkl} - \frac{1}{2} \zeta \overline{S}_{ijkl}, \quad (3.33.a)
\]

where,

\[
\overline{p}_{ijkl} = \frac{1}{2} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) - \frac{1}{3} \delta_{ij} \delta_{kl}, \quad (3.33.b)
\]

\[
A_{ijkl} = \frac{1}{2} (\overline{D}_{ik} \delta_{jl} + \overline{D}_{jl} \delta_{ik} + \overline{D}_{il} \delta_{jk} + \overline{D}_{jk} \delta_{il}) - 2 \overline{D}_{ij} \overline{D}_{kl}, \quad (3.33.c)
\]

\[
\overline{S}_{ijkl} = 3 \overline{D}_{ij} \overline{D}_{kl} - (\overline{D}_{ij} \delta_{kl} + \overline{D}_{kl} \delta_{ij}) + \frac{1}{3} \delta_{ij} \delta_{kl}, \quad (3.33.d)
\]

\[
\overline{D}_{ij} = \hat{u}_i \hat{u}_j. \quad (3.33.e)
\]

In the above equations, \(0 \leq \xi \leq 1\) and \(0 \leq \zeta \leq 1\) are the non – dimensional material strength ratios for anisotropy. In addition, \(\delta_{ij}\) is the Kronecker delta, and \(\hat{u}_i\) represents the unit vector along the material fiber direction. Note that, when \(\xi = \zeta = 0\) (i.e. isotropic case), \(\mathcal{M}_{ijkl}\) simply reduces to \(\overline{p}_{ijkl}\), leading to the classical von Mises – type forms (e.g. in some terms of function \(G(b)\)). Note also that \(\xi\) and \(\zeta\) will require experimental measurements of the transformation onset critical stresses with loading on the specimen aligned in different directions relative to the anisotropy direction \(\hat{u}_i\) of the SMA material (i.e. four measurements involving two shear and two tension/compression components in
the transverse plane normal to \( \hat{v}_i \) as well as longitudinally along \( \hat{v}_i \); see Ref. (Saleeb and Wilt 1993) for further details). Data from experiments of this latter type are presently lacking in the literature. Therefore, all applications in Chapter IV are restricted to the initially isotropic case.

3.4 Material Parameters and Their Physical Significance

In order to describe the role played by various material parameters in the presented unified SMA material model (see Fig. 3.2), some simple guidelines are given below.

In the Fig. 3.2, \( \kappa, \mu, n, c, d \) represent inelastic flow material constants, whereas the \( H_{(b)}, \beta_{(b)}, c_{(b)}, d_{(b)} \) are hardening material constants. The \( R_{(b)}, \) and \( m_{(b)} \) are recovery material constants; and the constants \( \kappa_{(b)} \) are hardening threshold stresses for the individual viscoplastic mechanisms \( (b) \), where \( b = 1, 2, \ldots, N \). To account for the temperature effects, the temperature – dependency was restricted to only a few inelastic hardening mechanisms; i.e., \( b = 1, 2, \) and 4. More specifically, only the threshold \( \kappa_{(b)} \) was chosen for this purpose.

The various material parameters involved can be decided based on some simple guidelines listed below. However, automation of material parameters characterization will be left for future work.

1) \( E_s \) and \( \nu \) are taken of the order of SMA elastic modulus and Poisson’s ratio, which can vary from 50 GPa to 100 GPa for typical SMA materials elastic modulus and between 0.2 – 0.35 for Poisson’s ratio.
(2) $\kappa$, $n$, and $\mu$ are inelastic, rate dependent response governing parameters. In the current research work, “nearly” rate–independent responses in SMA was targeted.

a) Typically, $\kappa$ is taken as small critical transformation (“yield”) stress at the lowest temperature of interest (below $M_f$). $\sqrt[3]{3}\kappa$ gives a rough estimation of beginning of inelastic response in the uniaxial tension test.

b) $(2n + 1)$ shows the severity of rate effect.

c) The material viscosity (units MPa·s) $\mu$ (ranging from $10^5$ to $10^{11}$) would give rate independent behavior in most of the cases with the range of $n$ above.

(1) Mechanisms 1, 2, and 3 are used as energy storage mechanisms. Mechanisms 1 & 2 are mainly account for temperature – dependency on thermal loading. These two mechanisms lead to transitional responses ranging from pseudoelastic to pseudoplastic responses as temperature decreases.

(2) The ratio of $\kappa(b)$ to $H(b)$ (where $b = 1 \rightarrow 6$) determines the transformation strain ranges after which each hardening mechanism will reach its limiting values governed by $\kappa(b)$. For example, $\kappa(3)/H(3)$ gives a good estimate for the beginning of rehardening strain (i.e. completion of forward transformation. See the region “2 – 3” in Fig. 3.1).
(3) $\beta_{(b)}$ (where $b = 1 \rightarrow 6$) dictates how fast each mechanism will show its maximum effect (i.e. degree of roundness in stress – strain or strain – temperature curves).

(4) Mechanisms 4, 5, and 6 are dissipative mechanisms, leading to hysteretic behavior in SMA responses. Mechanism 4 and 5 will sense the hysteresis width and depth. Mechanism 6 works as an “ever evolving” mechanism to provide evolution under sustained cycles, and in determining the “long term” slopes of stress – transformation strain curves well into the highly developed phase transformation regime (see the 2% – 8% strain transformation region in the pseudoplastic case reported in Fig. 4.1 in Chapter IV).

(5) The $c$ and $d$ for flow and hardening mechanisms determines the ATC. The limiting values of these parameters are given in the Table 3.1.
CHAPTER IV

MODELING CAPABILITIES OF THE MULTIMECHANISM BASED
VISCOELASTOPLASTIC UNIFIED SMA MODEL

4.1 Introduction

In the field of engineering, the simulation of the deformation behavior of various components is an essential task during the design process. This is achieved via two means: (i) by conducting experiments on the prototype under real physical conditions, and (ii) by computer assisted simulation. The later procedure can be more economical in terms of cost and time, since various conditions and design parameters can be studied in less time. This becomes especially important in order to assess the potential of SMAs in advanced engineering applications. However, for the efficient and optimal utilization of SMAs, the constitutive model must demonstrate the essential features of transformation related phenomena.

In this chapter, numerical simulations will be presented to demonstrate the modeling capabilities of the unified SMA model (described in the Chapter III). The simulations are performed using a UMAT routine developed for use with a commercial, large scale, finite element package ABAQUS®(ABAQUS 2008). The test cases considered were inspired by a number of recent experimental results reported in the
literature. These include a wide range of variations of both stress, and temperature and include monotonic, cyclic, as well as non-proportional, multi-axial loading conditions.

The study of the *qualitative* predictions of the responses from the unified SMA model can be subdivided into three categories, which will be discussed in detail separately:

1. Supermechanical aspects.
2. Superthermal aspects.
3. Evolutionary characteristics under extended thermomechanical cycles.

A single set of material parameters, listed in Table 4.1 (at the end of this chapter), were utilized in all the numerical cases. Currently, a complete set of experimental results, pertinent to one specific SMA system (composition, heat treatment, etc.), that describes all the known features of transformation (e.g. proportional/non-proportional loading, isobaric cycles, constrained recovery, shear vs. tensile vs. compressive loading modes, sustained cycles to attraction/saturation states, extended thermomechanical cycles etc.) does not exist. As a result, it is impossible to attain a complete characterization of a specific SMA system. Therefore, the material parameters listed in Table 4.1 were utilized in order to demonstrate that a number of features can be captured by the multimechanism based viscoelastoplastic unified SMA model in an average sense; i.e. the response characteristics would demonstrate the major aspects of transformation related phenomena for a typical SMA system. Note that the critical parameters were selected such that the present SMA system would have the transformation temperatures, critical stresses for
transformation, and transformation induced strain of magnitude similar to those presented in the literature. However, as mentioned earlier in Chapter III, a more systematic and automated characterization procedure will be necessary once a complete set of experimental data (uniaxial, multi-axial, coupled thermomechanical loadings, evolutionary cycles, etc.) are available.

With the above mentioned information, and the simple guidelines provided in Chapter III, a number of experimental data from the literature were utilized to design a single set of material parameters (Table 4.1) to be utilized in the numerical study of the unified model capabilities. Note that the model predictions were compared with the experimental data that were reported in the literature for different compositions of NiTi based SMA, and some non-NiTi based SMAs. Note also that, the distortion constants ($c$, $d$, $c_{(b)}$, and $d_{(b)}$) were used only for the results presented in Sections 4.2.1, 4.2.5, 4.2.6, 4.3 and 4.4. Furthermore, a constant strain-, stress-, and temperature-rate of $10^{-4}$ per seconds ($s^{-1}$), 0.125 MPa/s, and 0.012 °C/s, respectively, were utilized in all the cases. Furthermore, a virgin (“as received”) material state was assumed. In all subsequent plots, stresses in megapascal (MPa), strains in percentage (%), and temperatures in degree Celsius (°C) are reported. Furthermore, normal stress, shear stress, normal strain, shear strain, and temperature are denoted by $\sigma$, $\tau$, $\varepsilon$, $\gamma$, and $T$, respectively, in all the figures.

4.2 Supermechanical Aspects

The numerical simulation cases that were considered utilized either a stress or strain controlled loading scheme under “strict” isothermal conditions. In addition to
uniaxial cases, a number of proportional and non-proportional, multi-axial loading paths were also considered. These were compared with the similar experimental results reported in the literature to demonstrate the validity of the obtained responses.

4.2.1. Pseudoelasticity, Pseudoplasticity, Asymmetry in Tension and Compression, and Rate Dependency

Although transformation in SMAs is not accompanied by a change in the volume (Bhattacharya 2003), there are distinct differences in the obtained response when loaded in tension and compression. To demonstrate this aspect, the material parameters listed in Table 4.1, with active distortion constants, were utilized. The test cases involved uniaxial, strain control (magnitude 10%) at two extreme temperatures, i.e. at 150 °C (T > A_f) for pseudoelastic, and at 30 °C (T < M_f) for pseudoplastic responses.

The obtained results are shown in Fig. 4.1 for both the pseudoelastic and pseudoplastic cases. The maximum ratio of peak stress reached at the end of loading in tension vs. compression was 1.417 for the pseudoelastic case, whereas this same ratio was observed to be 1.218 in the pseudoplastic case. The degree of asymmetry was not only observed for the peak stresses but was apparent at every important point during the loading and unloading, such as at the critical transformation stresses, residual strains, etc. Note that a small amount of stress remained after unloading to zero strain at high temperature (Fig. 4.1a), signifying the development of residual martensite. The above obtained responses were in agreement with experimental observation that have been
reported in the literature (Adharapurapu et al. 2006; Lim and McDowell 1999; Liu et al. 1998).

![Graphs showing asymmetry in tension and compression in pseudoelasticity and psedoplasticity](image)

**Figure 4.1:** Asymmetry in tension and compression in (a) pseudoelasticity at $T = 150$ °C, and (b) psedoplasticity at $T = 30$ °C, (c) experimental results for nearly equiatomic NiTi in pseudoelastic regime from (Lim and McDowell 1999). Note that in all the figures dashed curves were obtained at higher strain rate.

The rate dependency of SMA is another important property that has been an issue of debate in the literature. Some groups attribute the rate dependency to the latent heat generation/absorption (Grabe and Bruhns 2008; Hartl et al. 2010a; Lim and McDowell 1999; Peyroux et al. 1998; Shaw and Kyriakides 1995; Tobushi et al. 1998), whereas
others believe that it is due to genuine viscoplasticity in the material (Hartl et al. 2010a; Helm and Haupt 2001; Helm and Haupt 2003; Kumar and Lagoudas 2010; Lexcellent et al. 2005; Mukherjee 1968). Without getting further into the debate of the rate dependency aspects, it can be shown that the unified model is able to account for both the rate dependent as well as the rate independent behaviors of SMAs. In this work, a nearly rate independent behavior was targeted. However, to demonstrate such capability, the same tension/compression tests were conducted with a higher (two orders of magnitude higher) strain rate of $10^{-2}$ s$^{-1}$. As can be seen in Fig. 4.1, increasing the strain rate by two orders of magnitude led to observable changes in the response, albeit relatively minor (validating the “nearly” rate independent behavior); however, all aspects of pseudoelasticity and pseudoplasticity are well exhibited. This is in agreement with the experimental results reported in the literature (Lim and McDowell 1999). Note that a strain rate of $10^{-4}$ s$^{-1}$ would be used in the remainder of the simulated test cases. The test cases here clearly demonstrated the unified model’s ability to capture the pseudoelastic, pseudoplastic, ATC, and rate dependent aspects associated with SMA response.

4.2.2. Strain Controlled Multi-axial Deformation Path Tests: Proportional and Non-proportional Loadings

Several multi-axial (proportional and non-proportional) deformation path tests in the pseudoelastic regime ($T = 150 ^\circ C$) were analyzed to demonstrate the 3D response capability of the unified model. These strain controlled tests were inspired by the work done by (Helm and Haupt 2003).
First, the responses under proportional loading were considered. The five deformation paths were defined in terms of applied shear, $\gamma$, and normal, $\varepsilon$, strains as:

$$\gamma = \sqrt{3} \varepsilon \cdot \tan \varphi. \quad (4.1)$$

The proportionality angle $\varphi$ was varied from 0° (pure tension) to 90° (pure shear), with intermediate values of 22.5°, 45°, and 67.5°, as shown in Fig. 4.2(a). Note that one loading – unloading cycle, starting from the virgin state, was performed, and a constant resultant strain magnitude of 4% was maintained for each path. The obtained response in stress subspace is shown in Fig. 4.2(b). Qualitatively, the model’s prediction was found to be similar to that reported in the reference (see Fig. 3b in (Helm and Haupt 2003)). Also, the stress-strain plots in the normal and shear stress-strain subspace are shown in Figs. 4.2(c) and (d). Note that despite the apparent narrower loops in either strain or stress subspace, significantly wider hysteresis stress-strain loops were captured by the unified model. This clearly emphasizes that studying SMA response in one subspace can mislead the observer as to the true nature of the deformation response.

Next, a non-proportional loading condition was considered such that the resulting path in the strain subspace was square in form (path ABCDA in Fig. 4.3a). Here, the material was loaded first in tension to produce incomplete transformation, followed by subsequent shear loading while maintaining the constant normal strain. The obtained response in conjugate stress subspace is shown in Fig. 4.3(b) (path ABCDA’), along with the experimental observation on NiTi based SMA under a similar state of loading in Fig. 4.3(c). Here also, the unified model’s prediction closely matched with the experimental
counterpart in a manner such that all the major response characteristics were well demonstrated.

Figure 4.2: Proportional, radial deformation path test at $T = 150 \, ^\circ C$; (a) loading path in strain subspace, (b) response path in stress subspace, (c) normal stress vs. normal strain, and (d) equivalent shear stress vs. equivalent shear strain.
Figure 4.3: Non-proportional, square deformation path test at $T = 150 \, ^\circ C$; (a) loading path in strain subspace, (b) response path in stress subspace, (c) experimental observation on NiTi based SMA (Helm and Haupt 2003).

The equivalent stress vs. equivalent strain plot for the square path test is shown in Fig. 4.4. The unified model was able to predict the formation of new $M$ variants during shear loading, which resulted in the sharp change in the equivalent stress (along path BC). Similarly, as observed experimentally (Fig. 4.4b), it can be seen that there is an
increase in the equivalent stress with decreasing applied equivalent strain (near point ‘D’). The micromechanical aspects associated with this behavior are not yet clear, however, it has been suggested that such behavior is due to the SME (McNaney et al. 2003).

![Figure 4.4: Equivalent stress vs. equivalent strain for the non-proportional square path test. (a) Unified model’s prediction, and (b) experimental results (McNaney et al. 2003).](image)

To further investigate the effect that the degree of loading has on the observed behavior, the same material was subjected to a similar square loading path with the initial tension loading corresponded to the complete phase transformation, as shown in Fig. 4.5(a). Unlike the previous case, where the subsequent shear loading led to the formation of new $M$ variants, the material reorientation effect in the subsequent shear loading shown in Fig. 4.5b was observed to be different. In this case, a significant increase in the both the normal and shear stresses were observed during shear loading (path BC in Fig. 4.5b). During the tension unload, a monotonic drop (without forming a hysteretic loop...
along path BCD) in the equivalent stress with decreasing equivalent strain was observed (Fig. 4.5c).

Finally, a strain controlled, non-proportional deformation path in the form of a butterfly in the strain subspace was considered (path ABCDA in Fig. 4.6a). Similar to the
earlier cases presented in this subsection, the applied strain magnitude was 4% (incomplete forward transformation). The obtained response in the stress subspace (ABCDA’ Fig. 4.6b) was found to be very similar to that observed experimentally in Fig. 4.6(c) (see also Fig. 8 in (Grabe and Bruhns 2009)). Note that the pseudoelastic response was maintained (by recovering nearly all the applied strain, compare point A’ with A) in all the tests presented in this subsection. Hence, in a qualitative sense, the unified model was able to capture all the characteristics of proportional and non-proportional, strain controlled loading path tests.

4.2.3. Stress Controlled Multi-axial Deformation Path Tests: Proportional and Non-proportional Loadings

Two multi-axial, non-proportional, stress controlled (in form of rectangular path) tests at T = 150 °C were performed. First, the material was loaded in tension to a stress level of 300 MPa (less than the stress required for forward transformation) along path AB in Fig. 4.7(a), followed by a subsequent loading in shear to a stress magnitude of 250 MPa while maintaining the normal stress at 300 MPa (see path BC in Fig. 4.7a). After this, the material was unloaded along path CDA (first in tension followed by in shear). Note that the magnitudes of the applied stresses, individually (either tension or shear alone), do not initiate transformation (see Fig. 4.1a). However, the combined loading led to the formation of stress induced $M$ variants. At the end of the initial tension loading (path AB), 1.17% normal strain was developed (path AB in Fig. 4.7b). During the shear loading, both normal and shear strains increased to 2.31% and 5.84%, respectively. Similar to experimental observation (as shown in Fig. 4.7c), the transformation followed
the direction of loading path BC due to the combined state of stress. The qualitative
differences along path CD were due to the fact that the experimental result in Fig. 4.7(c)
were pertinent to Cu-Al-Zn-Mn SMA, whereas the same averaged SMA material
parameters (as in Table 4.1, determined from the data of several SMA compositions)
were utilized to obtain the results for this complex multi-axial loading path test.
However, it is important to mention that, qualitatively, all the essential experimentally
measured response characters of Fig. 4.7 (c) were demonstrated. For instance, during path
CD, significant reorientation effects were observed in the experiment (Fig. 4.7c), which
was also predicted by the unified model (Fig. 4.7b). Similarly, upon the completion of
reverse transformation, the model was able to recover almost all the transformation
induced strain, as denoted by point A’ in Fig. 4.7(b).
Figure 4.6: Non-proportional butterfly deformation path test at $T = 150^\circ$; (a) loading path in strain subspace, (b) response path in stress subspace, (c) experimental observation on NiTi based SMA (Helm and Haupt 2003)
As a second case of stress controlled multi-axial deformation path, the material was subjected to the path ABCDEFBA shown in the Fig. 4.8(a). Here, the initial tensile loading stress led to the completion of the forward phase transformation. Therefore, during the shear loading (path BC), material reorientation effects were anticipated. This
was clearly demonstrated in the obtained result by the decrease in normal strain and increase in shear strain as shown Fig. 4.8(b). Note that the ATC was not observed due to the deactivation of distortion constants $c, d, c_{(b)}$, and $d_{(b)}$ ($b = 1, 2, \ldots, 6$).

*Figure 4.8: Stress controlled square path test at $T = 150 \, ^\circ C$; (a) applied loading path in stress subspace, (b) response in strain subspace, and (c) numerical results from (Panico and Brinson 2007).*
4.2.4. **Determination of Limiting Stress Surfaces under Strain Controlled Tests**

Similar to classical plasticity, the determination of the limiting stress surface for the initiation of phase transformation (\(A \rightarrow M\) or \(M \rightarrow A\) or between twinned and detwinned \(M\)) has always been an area of interest to researchers. Several experimental as well as theoretical procedures have been outlined in the literature to predict the limiting transformation surface (see (Bouvet et al. 2004; Huang 1999; Lexcellent and Blanc 2004; Lexcellent et al. 2006; Lexcellent et al. 2002; Lim and McDowell 1999; Taillard et al. 2006)). This necessitates the multi-axial response analysis of SMA.

The general shape of the transformation surface can best be described by its *cross–sectional* shapes in the deviatoric plane and its *meridians* on the meridian planes. These cross sections are the intersection curves between the transformation surface and a deviatoric plane (which is perpendicular to the hydrostatic axis). The meridians of the transformation surface are the intersection curves between the transformation surface and the plane containing the hydrostatic axis with constant angle of similarity, \(\theta\) (also known as Lode’s angle) (Chen and Saleeb 1994). For an isotropic material, the cross sectional shape of transformation surface possesses six fold symmetry as shown in Fig. 4.9. Therefore, the whole trace can be obtained by the symmetry of one sector \(0^\circ \leq \theta \leq 60^\circ\) (see part ABC in Fig. 4.9). Note that the meridian planes corresponding to \(\theta = 0^\circ, 30^\circ,\) and \(60^\circ\) are called *tensile* (T), *shear* (S), and *compressive* (C) meridians, respectively.
Figure 4.9: General character of the trace of transformation surface in the deviatoric plane.

Since the phase transformation is essentially governed by shear, the most appropriate plane to plot the trace of the transformation surface would be the π-plane (a deviatoric plane with zero hydrostatic stress/volumetric strain). Let $\sigma'_i$ be the projection of the $\sigma_i$ axis ($i = 1, 2, 3$ for three principal stress/strain axes as shown in Fig. 4.9) on the deviatoric plane. Then, the projection of the deviatoric stress (strain) vector on the $\sigma'_1$ axis can be written as,

$$\rho \cos \theta = \frac{1}{\sqrt{6}} (2s_1 - s_2 - s_3) \quad (4.2)$$

where, $\rho$ is the length of the deviatoric stress (strain) vector, and is defined as $\sqrt{2J_2}$. The $s_i$ (for $i = 1, 2, 3$) are the principal deviatoric stresses (strains). The readers are referred to the section 2.11 in (Chen and Saleeb 1994) for further details.
It can be shown that

\[ \rho \sin \theta = \frac{1}{\sqrt{2}} (s_2 - s_3) \]  

(4.3)

In order to obtain one sector of the trace in the \( \pi \) – plane for different stress states corresponding to the different values of \( \theta \) (between 0\(^\circ\) and 60\(^\circ\)), Eqs. (4.2) and (4.3) are plotted as ordinate and abscissa, respectively. Note that for stress controlled tests, deviatoric stresses can be replaced with deviatoric strain in Eqs. (4.2) and (4.3) to obtain the trace of the transformation surface in strain subspace.

To plot the trace of the transformation surface of the current material, tri-axial strain – controlled tests, with proportional normal, principal strains (yielding constant magnitude for \( \rho \)) along each of the three axes were considered. If \( \varepsilon_1, \varepsilon_2 \) and \( \varepsilon_3 \) represent the applied strains along each axis (see Fig. 4.9), then their values can conveniently be parameterized by the variable \( B \) as defined below:

\[
\frac{\varepsilon_2 - \varepsilon_3}{\varepsilon_1 - \varepsilon_3} = B \\
\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0 \\
\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 = \rho^2
\]  

(4.4)

Note that \( \rho \) is the magnitude of the strain vector. Furthermore, it can be shown that the direction of this strain vector is given by,

\[ B = \frac{2 \tan \theta}{\sqrt{3} + \tan \theta} \]  

(4.5)
It is clear from Eq. (4.5) that the factor, $B$, will increase, monotonically, from 0, to 0.5, to 1 as $\theta$ varies from 0° (tension path “T”), to 30° (shear path “S”), to 60° (compression path “C”), respectively.

The trace of six – fold symmetric projection (ABC in Fig. 4.9) of the transformation surface on the deviatoric plane is plotted following the procedure described below, in which the relationships given in Eqs. (4.2) – (4.5) above were utilized.

According to Eqs. (4.4) and (4.5), the experimental setup was guided by the two key parameters; the magnitude ($\rho$) of the control (stress or strain), and directionality ($B$). For demonstration purposes, a controlled strain of magnitude (Fig. 4.10a), $\rho = 3.5\%$, at temperature, $T = 30 \, ^\circ C$ ($T < M_f$), was used in preparing the results plotted in Fig. 4.10(b). Note that similar results would be obtained at any other temperature. When a strain – control condition was used for the projection of transformation surface, the magnitude of the transformation stress vector monotonically increased from a minimum in mode 1 (“T” mode) to a maximum in mode 3 (“C” mode) with an intermediate value being obtained in mode 2 (“S” mode).

On the other hand, when the same material was subjected to a stress – controlled condition (Fig. 4.10c) that allowed for the complete conversion to detwinned martensite ($\rho = 800 \, MPa$), a reversal in the nature of the response was observed (compare Fig. 4.10d to Fig. 4.10b). In particular, the limiting value of the resulting transformation strain magnitude now changed to a maximum in the “T” mode, with intermediate in the “S” mode, and a minimum in the “C” mode. Furthermore, the “S” mode response here was
shifted more towards the “T” mode, which was opposite to what was observed earlier in the strain – controlled condition in Fig. 4.10(b). This logical, complete, and neat duality between the stress and strain spaces resulted naturally from the energy partitioning and the associated mathematical constructs that form the basis of the present unified model. Note that in both cases, the linear controls paths resulted into nonlinearity in the predicted response paths.

For the purpose of comparison with experimental observations, the projection of the surface in Fig. 4.10(b) on the plane – stress plane (i.e. $\sigma_3 = 0$), along with the transformation strain rate vector, is shown in Fig. 4.11(a). The transformation strain rate vector can be obtained by two alternatives: (i) numerical differentiation of transformation strain histories, and (ii) analytically by manipulation of the data stored for global and internal stress state variables. Here, the “five point stencil” numerical differentiation method was utilized to obtain the strain rate vectors.

It can be seen that the obtained surface in plane – stress subspace was found to be very close to that obtained from a series of bi-axial experiments conducted on CuAlBe SMA, thin-walled tube specimens (Lexcellent et al. 2002), as shown in Fig. 4.11(b). The normality of transformation strain rate vector in the plane-stress subspace had been well captured by the unified model.
Figure 4.10: Trace of transformation surface on $\pi -$ plane at $T = 30 ^\circ C$; (a) controlling strain surface, (b) resulting limiting transformation stress surface, (c) controlling stress surface, (d) resulting limiting transformation strain surface.
In order to view the shape of transformation surface at different stages of detwinning, a set of proportional, biaxial stress controlled tests in plane stress space were performed. The loading paths along two principal axes (1 and 2) were defined in terms of applied stresses as:

\[ \sigma_1 = \rho \cos \theta, \quad \sigma_2 = \rho \sin \theta. \] (4.6)

The magnitude, \( \rho \), of each loading path was 1000 MPa to allow the complete detwinning of \( M \) variants for any value of \( \theta \). The proportionality angle, \( \theta \), was varied from 0º to 360º, with increments of 4º. The resulting transformation surfaces in plane stress subspace, along with the transformation strain rate vectors, are shown in Fig. 4.12 for the transformation strain magnitudes of 0.125% (initiation of detwinning), 5% (during transformation), and 10% (end of \( M \) reorientation). As evident from Figs. 4.12(a),...
(c), and (e), the shape of transformation surfaces changed significantly during the detwinning and reorientation of $M$ variants. For example, contrast the smooth corners in Fig. 4.12(a) with the sharp corners in Fig. 4.12(e). Furthermore, the strain rate vectors were nearly normal to the stress surface at the beginning of $M$ detwinning (Fig. 4.12a), followed by a slight deviation from normality during the detwinning process (Fig. 4.12c), and finally the normality condition in the global stress space was no longer satisfied at the end of detwinning (Fig. 4.12e). However, the elliptic shape of transformation strain surface was maintained throughout the transformation (see Figs. 4.12b, d, and f). Note that the linear, proportional stress control paths (see Eq. (4.6)) led to nonlinearity in the resulting transformation strain (hence global strain) responses. Note further that the “equal” intervals covered by the control paths in stress space ($\theta$ values in Eq. (4.6)), do not correspond to equal interval in angles covered by the resulting response in both transformation as well as the global strain space (see Fig. 4.12b & f).
Figure 4.12: Transformation surface in plane stress space at different instance of transformation in (a), (c), and (e); and corresponding transformation strain paths in (b), (d), and (f). Note the change in the critical transformation surface in plane stress subspace as predicted by the unified model for (a) $\varepsilon^I = 0.125\%$ (beginning of reorientation) and (e) $\varepsilon^I = 10\%$ (end of reorientation). Note also that red, blue, magenta, and green colored segment in (b), (d) and (f) corresponds to the values of $\theta$ in (0º-90º), (90º-180º), (180º-270º), and (270º-360º).
4.2.5. On the Proper Treatment and Interpretation of Apparent Deviation from Normality

In one of the important fundamental studies dealing with deformational aspects of SMA responses, Lim & McDowell (Lim and McDowell, 1999) had presented the results of a comprehensive biaxial loading (tension – torsion), together with the mapping of the limiting stress transformation surfaces and associated strain rate vectors. The main objective in this study was to investigate the validity of the assumptions in classical plasticity theories when extended for the treatment of SMA materials. In particular, the issue of normality of the transformation strain rate vector to the limit surface in the stress space, and the related concepts of the associative flow rules were examined. From this work, three main conclusions were reached;

(1) There is a significant deviation between the strain rate vector direction and the normal to the limit transformation surface plotted in the space of applied (global) stress components ($\sigma$ and $\tau$)

(2) Simple classical plastic models, purely formulated in terms of the “global” stress tensor components (e.g. $J_2$ – $J_3$ type), and associative flow rules are not applicable for the observed SMA response

(3) There is a phase – angle shift between the vectors defining the stress path compared to the strain – path counterpart vectors (see also angles $\varphi_i$ in Fig. 4.13 below).
Since that time, these conclusions have been repeatedly misinterpreted by several researchers in the field (Auricchio et al. 2003; Auricchio et al. 2007; Boyd and Lagoudas 1994; Entchev and Lagoudas 2004; Helm and Haupt 2003; Lagoudas and Entchev 2004) as requiring non–associative flow rules. Such issues are very delicate in the field of constitutive modeling, and require careful consideration. On the other hand, it will be shown that when interpreted differently in a more generalized sense involving internal state parameters (to account for the microstructural changes in the SMA material systems), one is indeed directly able to capture both of the above observed phenomena without violating normality nor associativity conditions.

To demonstrate such “apparent” deviation from normality, a strain controlled circular path test (Fig. 4.13a) was carried out. The resulting transformation strain rate vectors are shown in Fig. 4.13(b), along with the response of the material in the conjugate stress space. Note that Lim & McDowell performed numerical differentiation to obtain the transformation strain rate vector (see (Lim and McDowell 1999) for the details of derivation of transformation strain rate vectors).

The similarity between the unified model’s prediction in Fig. 4.13(b) and the experimental counterpart (Fig. 4.13c) can clearly be seen. Note that, despite the similarity in obtained response in stress subspace (see also Fig. 5b in Ref. (Grabe and Bruhns 2009)), and the smooth and convex transformation surfaces of the unified SMA model (Figs. 4.10, 4.11), the apparent deviation from normality was observed, which was the same as exhibited in the actual results reported by Lim & McDowell (Lim and McDowell 1999).
On the other hand, when the material was subjected to similar nonproportional circular loading path with radius 0.175% strain, the resulting transformation strain rate vectors showed the normality in the stress subspace (Fig. 4.13d). Note that 0.175% normal strain corresponded to the beginning/onset of the forward transformation in the uniaxial tension test. This indicates that, during evolution, the transformation strain rate vectors would continuously change direction, moving from the extreme condition of nearly normal to the other extreme condition of nearly tangential relative to the trace of the transformation surface in the concerned stress subspace, which was also demonstrated by Figs. 4.13(b) & (d).

The normality conditions in the unified model are defined in terms of generalized “effective” stresses, involving the internal variables, and not merely in terms of the applied (“global” or “overall”) stresses. That is, individual internal mechanism components exhibit normality with respect to their respective surfaces, whereas the total (global) components, which are byproducts (resultant) of the internal mechanisms, may or may not respect these constraint conditions.
Figure 4.13: Transformation strain rate vector on resulting stress path in (b) from applied circular strain path test in (a) at $T = 150 \, ^\circ C$ leading to significant deformations toward the end of the transformation region (near state point “2” in Fig. 3.1), along with (c) experimental observation on NiTi SMA (Lim and McDowell 1999), and (d) transformation strain rate vector resulting from a strain path similar to (a) but with radius corresponding to the beginning of forward transformation. Note that stress path is shifted by phase angle $\varphi_i$ ($i = 1 \rightarrow 4$) = (11.75°, 6.31°, 4.81°, 11.27°) from respective strain path. Note also that the transformation strain vectors are nearly normal to the transformation surface at the onset of transformation in (d).
4.2.6. *Evolutionary Characteristics under Isothermal Conditions (Minor Loops)*

In order to capture the existence of minor loops during sustained cycling, two separate cases of uniaxial, stress-controlled, partial loading and unloading cycles between 525 MPa (upper stress level) and 350 MPa (lower stress level) were performed. In the first case, the material was subjected to partial loading-unloading cycles during the initial segment of forward transformation (Fig. 4.14a), whereas in the second case the material was loaded to the completion of forward transformation, followed by partial loading-unloading cycles during the reverse transformation (Fig. 4.14b). In the first case (minor loops from left, Fig. 4.14a), the mean strain (average of strains at 350 MPa and 525 MPa) evolved from 3.644% in the first cycle to a nearly saturated value of 5.350% in 35 cycles (as shown by the dotted datum line). In the case second case (i.e. minor loops from right, Fig. 4.14b), the mean strain moved from a starting value of 7.088% and approached 5.738% near saturation. The loops from each case were attracted towards each other (as shown by the arrows in Fig. 4.14). In this type of complex loading condition, the predicted response from the unified model was in good qualitative agreement with the experimental counterpart presented in the literature (Lim and McDowell 1995). In this reference, the minor loop cycles were performed after a number of major loop training cycles on a NiTi SMA alloy. It was observed that four minor loop cycles were needed to reach “saturation” under stress control between 150 MPa and 280 MPa (see Fig. 4.14d, with cycles approaching from the left), as opposed to only one cycle to reach the stabilized inner loop cycle (approached from the right Fig.4.13d) under the same range of stress cycling.
Figure 4.14: Minor attractor loops at $T = 150 ^\circ C$; (a) approach from left, (b) approach from right, (c) attracted minor loops from left and right, and experimental observation on a NiTi based SMA (Lim and McDowell 1995).

4.3 Superthermal Aspects

In the following subsections, the capability of the multimechanism based unified SMA model in capturing the superthermal effects using the same unified material parameters (see Table 4.1) will be discussed. It is important to remember that these
parameters were obtained in an average sense for the binary NiTi SMAs. Note that, the
distortion constants were deactivated while preparing the results presented in the
following subsections.

4.3.1. The One Way Shape Memory Effect (OWSME)

The most common superthermal effect that has been successfully utilized in the
modern engineering applications is the one way shape memory effect (OWSME).
Therefore, any comprehensive SMA constitutive model must be able to capture this
aspect of the SMA response. To evaluate the present unified SMA model, the material
was subjected to one stress controlled load-unload cycle at the low temperature ($T = 30$
°C $< M_f$), followed by heating to high temperature ($T = 150 ^\circ C > A_f$) at zero stress, and
cooling back to the low temperature. The obtained OWSME in 3D stress-strain-
temperature space is shown in the Fig. 4.15.

At $30 ^\circ C$, the material had accumulated a total residual strain of 3.25% at the end
loading-unloading (path ABC) cycle. During the subsequent heating to $T = 150 ^\circ C$, this
residual strain gradually decreased and reached a residual strain of 0.111% at the point
‘D’. During the subsequent cooling to the low temperature, this residual strain increased
slightly to 0.152%. This slight increase in residual strain was due to the fact that the
OWSME test was performed on the virgin specimen, and not on the trained one. This
transient behavior will be explored in the next sections. However, the predicted OWSME
behavior was very similar to the experimental counterpart shown in Fig. 4.15(b). Note
that, the experimental results were obtained for a high temperature NiTiPd SMA (Noebe et al. 2005).

![Figure 4.15: The one way shape memory effect; (a) unified model prediction, (b) experimental observation on NiTiPd HTSMA (Noebe et al. 2005).]

4.3.2. Generalized OWSME under Non-proportional Multi-axial Loading Conditions

A series of biaxial (tension/torsion) thermomechanical tests were performed by Grabe & Bruhns (Grabe and Bruhns 2009) to study the effect of the interaction between loading path and the thermal state of the specimen on the response characteristics of SMA. Out of several thermomechanical tests, two innovative cases were considered here. In the experimental procedure, the low temperature M phase material was loaded in tension. Subsequently, a shear deformation was imposed at the same temperature. For the unloading sequence, two different procedures were considered:

1. The material was unloaded in the M phase
2. The material was unloaded in the A phase.
Both of these load control paths are shown in Fig. 4.16(a) in the stress-temperature subspace. Note that the loading in tension and shear (path ABC in Fig. 4.16a) for both of the cases were identical; i.e. loading occurred in the $M$ phase. The main conclusion from these experiments was that the recovery of residual strains (related to OWSME) in the normal and shear directions were decoupled, whereas the shift in the transformation temperature of specific $M$ variants was dependent on the stress state of the material as well as the prior thermomechanical history.

![Figure 4.16](image)

Figure 4.16: The generalized OWSME under multi-axial, non-proportional loading; (a) stress controlled loading paths (i) path ABCDAME and (ii) path ABCKLME, (b) response from unified model in stress-strain-temperature space, (c) 2D projection of the response on strain plane, and (d) experimental observation on NiTi SMA (from Tables A.1, A.2 in Ref. (Grabe and Bruhns 2009)).

Following the guidelines provided in the reference (Grabe and Bruhns 2009), the two cases of biaxial OWSME were prepared. In case (1), a stress controlled, non-
proportional, square loading path was considered, and the whole loading-unloading cycle was performed in the $M$ phase (i.e. at $T = 30$ °C). After that, the material was heated to the $A$ phase ($T = 150$ °C), followed by cooling to -20 °C (see path ABCDAME in Fig. 4.16a). In case (2), after the loading along path ABC, the material was brought into the $A$ phase (heating to $T = 150$ °C) at the existing stress state at point C. Thereafter, the material was unloaded in tension followed by shear along path KLM (dashed path in Fig. 4.16a) in the $A$ phase, and finally cooled to the same -20 °C under zero stress state (path ABCKLME in Fig. 4.16a). Note that in both cases, the magnitudes of the applied stress, individually, do not initiate martensite detwinning process (see Fig. 4.1b).

The obtained results from these two tests are shown in Figs. 4.16(b) and (c) in the strain-temperature and normal-shear strain subspaces, respectively. Since, the applied normal stress was less than the critical stress for the initiation of transformation, the subsequent shear loading led to forward transformation, as signified by the increase in both normal as well as shear strains along path BC. In case (1), the shear strain increased during the unloading in tension. At the end of shear unloading, 0.47% and 1.08% residual normal and shear strains, respectively, were obtained. The residual strains at ‘E’ diminished during the strain recovery process (i.e. heating followed by cooling) to 0.06% and 0.1% in the normal and shear directions, respectively.

During the heating process for case (2) (where unloading was done in the $A$ phase), path CK showed a slight increase followed by reduction in both strain components. This transient behavior of SMA is observed only during the first heating in cases where the material is prestressed in the $M$ phase. This will be elaborated on further
in the next subsection. Since the unloading process was done in the $A$ phase for case (2), the strains dropped almost linearly during the tension and shear unload. Furthermore, unloading in the $A$ phase resulted in a narrower loop compared to that in the first case (i.e. unloading in $M$ phase). Overall, the unified model was able to demonstrate the experimentally observed behaviors under multi – axial, thermomechanical loading conditions. Note that the transformation temperature region had been shifted from 60 °C – 100 °C in the first case to 80 °C – 123 °C in the second case due to the biased stress state. Note also that the experimental curves in Fig. 4.16(d) were obtained directly from the measurements’ tables provided in the appendix of the reference (Grabe and Bruhns 2009). However, this latter reference did not provide explanations as to the “significant” differences in the measured response along the part of path $AB_1C_1$ and $AB_2C_2$ (identical temperature and stresses, $\sigma$ and $\tau$, for each of the controls of the coincident portions $ABC$ in both cases (1) and (2) above).

4.3.3. Evolution under Thermal Cycling at Constant Stress/Strain (Load Bias and Constrained Recovery)

To demonstrate the transient and evolutionary responses of an untrained SMA under thermal cycles, two thermomechanical loading cases were considered. In the first case (load bias test), the material was stressed to 250 MPa isothermally at 30 °C in the fully martensitic state, while in the second case of constrained recovery, the material was loaded to 2.25% normal strain (conjugate to 250 MPa) at the same low temperature. After that, 5 thermal cycles between 30 °C and 150 °C were performed in each case. The
obtained results are shown in Fig. 4.17, along with the corresponding experimental results reported in the literature.

Figure 4.17: Evolutionary character of SMA under (a) biased load, and (b) constrained recovery condition, (c) experimental observed evolution on 55NiTi at 200 MPa (Padula II et al. 2008b), and (d) experimental constrained recovery tests performed at 2% strain on Ti$_{50}$Ni$_{45}$Cu$_{5}$ SMA (Šittner et al. 2000).
In the case of load bias test (Fig. 4.17a), the normal strain increased from 2.243% (at the end of isothermal loading) to 3.218% at 65 °C, and then remained nearly constant until 80.7 °C. This gradually decreased to a final value of 2.561% at 117.3 °C, followed by a relatively sharp drop to 2.352% at 122.4 °C, and finally reached 2.180% at 150 °C. Note that this strain was higher than the strain level if the material was loaded at 150 °C (see Fig. 4.14). As mentioned in the previous subsection (section 4.3.2), this transient behavior (see also 4.16c) is observed only when the initial loading is performed on the virgin material in the $M$ state (Padula II et al. 2008b). This further validates the unified viscoelastoplastic model’s capability to demonstrate the physics of SMA transformation. During the thermal cycles, the load bias (isobaric) case showed consistent evolution of strain at both the low as well as high temperature ends.

On the other hand, the constrained recovery (isostrain or constant strain) case showed response characteristics opposite to that in load bias; e.g. complementary to the slight increase in strain observed for the load bias case, the constrained recovery case showed a decrease in stress (237 MPa at 64.8 °C), followed by an increase in stress to a value of 338 MPa at 150 °C. Similarly, the load bias case showed a gradual increase in strain with cycling at both temperature extremes, whereas the constrained recovery case showed a decrease in the stress (see Figs. 4.17a & b). Furthermore, the isobaric case showed no sign of saturation in the first five thermal cycles, whereas the isostrain case saturated in three cycles. These observations further emphasize the “duality” of stress and strain commonly observed in SMA materials. These unique characters of SMA are very important as they are intimately linked to the determination of optimal conditions for the
economical training of these materials. Note that in both cases, the obtained response from the unified model was in close qualitative agreement with the experimental observation, as shown in Figs. 4.17(c) and (d).

It is important to mention that the coefficient of thermal expansion was not considered in any of the cases presented so far. Typically, the thermal expansion coefficient for NiTi based SMAs is of order of $10^{-5}$ /°C. In all the presented cases under superthermal aspects (obviously the thermal expansion coefficient would have no effect in the isothermal cases), the highest temperature difference was of order of 100 °C that would produce thermal strains of order of 0.1%. This will have very little effect on the obtained solution. However, the thermal expansion coefficient would produce a more prominent effect in the constrained recovery case.

To validate the above argument, the two thermomechanical cycle tests were performed again, but this time with the effect of “linear” thermal expansion. The corresponding results (dashed curves) are also shown in the Figs 4.16(a) & (b). As mentioned earlier, the thermal expansion coefficient had more of an effect in the constrained recovery case. However, the main character of the evolution remained the same (in particular, note the attainment of “quick” saturation of cycles in the present constrained recovery case for both with or without thermal expansion effects).

4.3.4. Evolutionary characteristics under Isobaric Conditions (Minor Loops)

For completeness, the superthermal aspects, under conditions where minor loops occur, are considered here. Very few studies on the thermomechanical response under
conditions that generate minor hysteresis loops have been presented in the literature (Bo and Lagoudas 1999c). In order to assess the behavior under minor loop conditions, two uniaxial, load – bias tests with partially varying temperature (in the transformation region) were simulated. Similar to the load – bias test in the previous section, the material was loaded to 250 MPa (stress control), followed by heating to 150 ºC. After that, the material was partially cooled to 74 ºC before it was subjected to thermal cycles between T₀ = 74 ºC and T₁ = 109 ºC (case – 1). In an alternate case (case – 2), the material was cooled to 30 ºC after the first heating to 150 ºC, followed by reheating to 109 ºC. Then the material was subjected to partial heating-cooling cycles between the same temperature range (T₀ and T₁). The response (Fig. 4.19a and b) in the strain – temperature space showed the appearance of minor loops. As was the case for the isothermal minor loops (Fig. 4.14), case – 1 (approach from below in Fig. 4.18a) continuously evolved upward toward the attraction point. However, case – 2 (approach from above in Fig. 4.18a) exhibited a reversal in direction; i.e. it initially evolved downward for five cycles (see Fig. 4.18b), reversed direction and then evolved upward. Note that neither of these two cases reached the saturation state in the 50 minor thermal cycles imposed.

Next the same tests were performed again with the T₀ was shifted to 85 ºC and T₁ remaining the same; i.e. minor isobaric loops between 85 ºC and 109 ºC. In this test, a different response (Fig. 4.18c and d) was observed. Unlike the previous case, no reversal in the directions was observed. Also, the two minor loops approached the attraction point from opposite directions. Note that the minor loops approaching from above reached the attraction point earlier. The results obtained in this section further emphasize the fact that
the response of SMAs are intricately linked with the loading condition as well as prior thermomechanical states.

**Figure 4.18:** Isobaric attractor loops at $\sigma = 250$ MPa: (a) minor loop cycles between $T_0 = 74$ °C and $T_1 = 169$ °C, (b) evolution of strain at $T_1$ for the cycles in (a), minor loop cycles between $T_0 = 85$ °C and $T_1 = 109$ °C, (b) evolution of strain at $T_1$ for the cycles in (b). Note that just by shifting $T_0$, a complete change in the attractor loop response was observed.

In the last two sections (sections 4.2 and 4.3), the unified model’s predictions under various types of loading conditions were found to be in good qualitative conformity with the experimental observations. It is important to remember that all of
these results were obtained from a single material parameter set. To the best of author’s knowledge, no SMA constitutive model in the literature has been able to demonstrate this many responses utilizing only one single parameter set (for examples of the cases, where different sets of parameters were utilized, see (Bo and Lagoudas 1999b; Bo and Lagoudas 1999c; Bouvet et al. 2004; Entchev and Lagoudas 2004; Lagoudas and Bo 1999; Lagoudas and Entchev 2004; Paiva et al. 2005)). In the next section, the evolutionary character of SMA under sustained cycling will be presented. The results presented will prove to be useful in understanding the evolutionary behavior of SMAs, and will shed light on how to efficiently utilize these materials under cyclic loading conditions.

4.4 Evolutionary Character of SMAs under Sustained Cycles

In the typical application as an actuator or vibration isolation system (Auricchio and Sacco 2001; Collet et al. 2001; Hashemi and Khadem 2006; Janke et al. 2005; Motahari et al. 2007; Perreux and Lexcellent 1999), SMAs are subjected to cyclic thermomechanical conditions. Under these conditions, the effect of cycles on the recovery forces and deformations becomes very important. Understanding of these cyclic characteristics is a key if successful design and implementation into real world components is to be achieved.

The cyclic, nonlinear behavior of SMA response is dependent on the stress state, temperature, stress/strain rate, and the prior thermomechanical loading history. Earlier experimental investigations showed the existence of distinct transformation initiation stress lines for forward as well as reverse transformations (Lexcellent and Tobushi 1995).
Furthermore, there was an observed monotonic increase in transformation stresses with temperature (Shaw and Kyriakides 1995; Tobushi et al. 1991); however, the transformation stress decreased with the number of thermomechanical cycles (Gall et al. 2001; Tobushi et al. 1991). It was concluded that these effects were mainly due to the internal microscopic mechanisms (e.g. slip, nucleation, interface boundaries, etc.) involved in the transformation, which led to the permanent changes in the underlying microstructure (Adharapurapu et al. 2006; Bo and Lagoudas 1999a; Gall et al. 2002; Gall et al. 2001; Lagoudas and Bo 1999; Liu et al. 1998; Nemat-Nasser and Guo 2006; Shaw and Kyriakides 1995). The interaction between the external mechanisms and relevant internal mechanisms becomes the key to the “smart” experimental training program.

In the last two decades, a number of SMA constitutive models have been established to predict the SMA response characteristics. Among these, some cyclic constitutive models utilized internal variables, representing the residual and transformation strains. The evolution laws in these models were inspired by the classical plasticity theories (Abeyaratne and Kim 1997; Auricchio et al. 2003; Auricchio et al. 2007; Auricchio and Sacco 2001; Bo and Lagoudas 1995; Bo and Lagoudas 1999a; Bo and Lagoudas 1999b; Entchev and Lagoudas 2004; Kan and Kang 2010; Lagoudas and Bo 1999; Lagoudas and Entchev 2004; Paiva et al. 2005; Saint-Sulpice et al. 2009; Zaki and Moumni 2007). These internal variables were introduced to account for the existence of residual (permanent) inelastic deformation and functional degradation effects during the thermomechanical cyclic loading. However, these models were developed mainly to fit experimental data under the superelastic and strain controlled thermomechanical cyclic
conditions (Kan and Kang 2010); thus necessitating a newer model or further developments of the existing models to account for some other behaviors such as ratcheting. A brief review of such models have been provided in the Table 1 of Ref. (Kan and Kang 2010).

In this section, a number of numerical results, obtained by using the 3D, unified viscoelastoplastic SMA model, under sustained cyclic conditions, will be presented. In the earlier sections of this chapter, it was shown that the selected material parameters were able to capture a wide variety of SMA responses. Therefore, with the help of the same single set of material parameters (as listed in Table 4.1 at the end of this chapter), the extended cyclic capability under thermomechanical loading conditions will be discussed in the following subsections. Note that the present “averaged” SMA material has austenite and martensite finish temperatures ($A_f$ and $M_f$, respectively) to be 65 °C and 120 °C, respectively.

In order to simplify the presentation in later subsections, some nomenclature will be introduced here. The Stress, strain, and temperature will be represented by symbols $\sigma$, $\varepsilon$, and $T$ respectively. Their subscripts “$u$, $m$, $l$” denote upper, mean (average of upper and lower values), and lower values, respectively. Similarly, the superscript will denote the temperature at which the analysis had been performed. For example, $\varepsilon_u^{(T)}$, denotes the upper (peak) value of strain at temperature, $T$.

The control load history is shown in Fig. 4.19, where the stress (strain) is varied linearly between the upper (peak) stress (strain) level, $\sigma_u^{(T)}$ ($\varepsilon_u^{(T)}$), and the lower (valley) stress (strain) level, $\sigma_l^{(T)}$ ($\varepsilon_l^{(T)}$), at a constant stress (strain) rate of 0.125 MPa·s$^{-1}$ ($10^{-4}$ s$^{-1}$).
Note that in all the figures in the later subsections, stresses are shown in megapascals (MPa), strains in percentage (%), and temperature in Degree Celsius (°C). Furthermore, in every case 100 isothermal stress (strain) cycles were conducted on a virgin specimen.

![Figure 4.19: Loading time history for all the isothermal numerical simulation cases.](image)

Note that the “distortion” material parameters \( c, d, c_{(b)}, d_{(b)} \), \( b = 1, 2, \ldots, 6 \) (see Table 4.1) were activated in all the cases presented here. Furthermore, every test has been performed assuming an initially virgin specimen.

4.4.1. Isothermal, Stress Controlled, Extended Cycles in the Pseudoplastic Regime

A set of isothermal, uniaxial, stress controlled cycles were conducted to study the effect of the mean stress on the evolutionary characteristics of SMA response. These numerical simulations were performed at 30 °C (\( T < M_f \)). The \( \sigma_u^{(30 \degree C)} \) was varied over a range from the beginning of transformation (150 MPa) to the completion of martensite
reorientation (550 MPa), with an intermediate stress level of 300 MPa. All the tests were conducted with $\sigma_l^{(30 \degree C)} = 0$, and for 100 cycles in each test.

The extended cyclic evolutions in the stress-strain response, together with the rate of strain saturation, are shown in Figs. 4.20. In the simulated cases, the net movements (strain accumulations) at the lower and upper stress levels were found to be nearly equal (Figs. 4.20a, c and e). Furthermore, the number of cycles required to reach the saturation state were lower for the higher mean stresses (Figs. 4.20b, d and f). Furthermore, similar to experimental observations reported in the literature (e.g. (Kan and Kang 2010; Nemat-Nasser and Guo 2006) ), the mechanical hysteresis loop width diminished gradually with cycles, becoming insignificant near saturation. Finally, a typical, experimentally obtained, stress-strain response on a 55NiTi SMA material, tested isothermally at room temperature (RT) of 25 ºC between the stress levels of $\sigma_u^{(25 \degree C)} = 490$ MPa and $\sigma_l^{(25 \degree C)} = 0$ MPa is shown in Fig. 4.21. Note that the upper stress level corresponds to a state of nearly complete detwinning/reorientation of the martensite ($M$) phase. One can also clearly observe the close similarity in the cyclic evolutions for the experimental test results (Fig. 4.21) and its numerical simulations counterpart depicted in Fig. 4.20(e).
Figure 4.20: Extended stress controlled cycles at $T = 30 \, ^\circ C$; stress vs. strain for $\sigma_u^{(30 \, ^\circ C)} = 150 \, MPa$, $300 \, MPa$ and $550 \, MPa$ ($\sigma_l^{(30 \, ^\circ C)} = 0$) in (a) (c) and (e), respectively, with their corresponding evolution of the conjugate strains in (b), (d) and (f).
As can be seen in Figs. 4.20(a) and (b), the time to reach the saturation state was notably long for the cycles performed at $\sigma_{u}^{(30 \degree C)} = 150$ MPa (beginning of transformation). To investigate these cyclic aspects further, the case at 150 MPa was further cycled until 1000 cycles. To the best of this author’s knowledge, simulation under such very large number of cycles has not been presented in the literature; despite the fact that some SMAs require an even greater number of cycles than this in order to reach saturation. The evolution in this extended cycling case is shown in Fig. 4.22(a). Here, the loops continued to evolve throughout the 1000 cycles with no indication that saturation was imminent in the strain vs. cycle plot (Fig. 4.22b). This further indicates the importance of training parameters.
Figure 4.22: 1000 cycles at $T = 30 \, ^\circ\text{C}$ for $\sigma_u^{(30 \, ^\circ\text{C})} = 150 \, \text{MPa}$ ($\sigma_l^{(30 \, ^\circ\text{C})} = 0$); (a) stress vs. strain plot, and (b) evolution of the conjugate strains.

To investigate the training characteristics at low temperature further, the internal stress components ($a_{11}^{(b)}$, $b = 3, 5$) for the most active mechanisms ($b = 5$) at low temperature, along with the mechanisms responsible for the completion of transformation ($b = 3$), are shown in Fig. 4.23. It is important to remember that mechanisms 1 & 2 would have apparently no effect at low temperature, whereas mechanisms 4 & 5 would behave similarly. Mechanism 6 is independent of temperature; therefore, its evolution will be shown in the next subsection (as part of the study of cyclic training at high temperature). In order to better visualize the evolution of the internal variables, they have been plotted vs. normal strain. When the evolutions of $a_{11}^{(3)}$ for the three stress levels (Figs. 4.23a, c, and e) were compared, it was found that this internal stress variable did not reach the threshold value for $\sigma_u^{(30 \, ^\circ\text{C})} = 150 \, \text{MPa}$ in 1000 cycles, leading to an ever evolving character during the 1000 cycles. Similar behavior was observed for the $a_{11}^{(5)}$ (Figs. 4.23b, d, and f) for which the threshold value was 30 MPa. These observations have proven to be helpful in understanding the evolution characteristics of the unified model,
as well as the specific roles played by the individual mechanisms in organizing the cyclic evolution of the SMA material response.

In order to look into the effect of mean stress on the training of SMA, the material was subjected to 100 cycles between three different $\sigma_l^{(30 °C)}$ (270 MPa, 200 MPa, and 100 MPa) and a constant $\sigma_u^{(30 °C)}$ of 300 MPa (giving mean stresses of 285 MPa, 250 MPa, and 200 MPa, respectively). The obtained results from these conditions are shown in Fig. 4.24. As observed before, the stabilized state was reached fastest by the highest mean stress (285 MPa), followed by the intermediate (250 MPa), and lastly by the lowest mean stress (200 MPa). Furthermore, the evolution of strain with cycles for the lowest mean stress (Fig. 4.24f) was very close to the case with $\sigma_l^{(30 °C)} = 0$ MPa in Fig. 4.20(d), whereas that in the cases with $\sigma_l^{(30 °C)} = 270$ MPa and 200 MPa were found to be similar.
Figure 4.23: Internal stress variables \( (\alpha_{ij}^{(b)}, b = 3, 5) \) for the three extended cycles cases at low temperature: \( \alpha_{ij}^{(3)} \) in (a), (c), and (e), \( \alpha_{ij}^{(5)} \) in (b), (d), and (f) for \( \sigma_u^{(30 \degree C)} = 150 \) MPa, \( 300 \) MPa, and \( 550 \) MPa \( (\sigma_l^{(30 \degree C)} = 0) \), respectively.
Figure 4.24: Extended stress controlled cycles at $T = 30 \, ^\circ C$; stress vs. strain for $\sigma_{(30 \, ^\circ C)}^{l} = 370 \, MPa$, $200 \, MPa$ and $100 \, MPa$ ($\sigma_{u}(30 \, ^\circ C) = 300 \, MPa$) in (a) (c) and (e), respectively, with their corresponding evolution of the conjugate strains in (b), (d) and (f).
4.4.2. Isothermal, Stress Controlled, Extended Cycles in the Pseudoelastic Regime

In a similar way as was previously done at the low temperature, the virgin material was subjected to stress controlled mechanical cycles in austenite ($A$) phase ($T = 160 \, ^\circ\text{C} > A_f$); i.e. 100 isothermal stress cycles were applied in the pseudoelastic regime. More specifically, the peak stress levels considered were chosen to be a direct analog to what was done at low temperature, with 400 MPa (beginning of transformation), 500 MPa, and 900 MPa (nearly the completion of phase transformation from $A$ to detwinned $M$ phase detwinning region) being the values imposed.

The obtained results are shown in Fig. 4.25. As observed experimentally, the unified model was able to demonstrate the gradual reduction in the critical stress for transformation during mechanical cycling, as well as a reduction in the hysteresis loop width. Furthermore, when these results were compared to the results obtained in the previous case (i.e. extended cycles at low temperature), the effect of the mechanism of transformation was clearly visible. In contrast to the martensitic detwinning process (the transformation mechanism at low temperature), forward transformation led to obvious asymmetry in the accumulation of strains (lower as well as upper levels, $\varepsilon_l$ and $\varepsilon_u$, respectively) with cycles. While in the low temperature case (extended cycles in the pseudoplastic regime), a nearly equal amount of strain accumulation took place at both the lower and upper stress levels, here we can see the transition from a nearly equal evolution in Fig. 4.25(a), to more evolution at the upper bound, $\sigma_u^{(160 \, ^\circ\text{C})}$, in Figs. 4.25(c), and finally more evolution at the lower bound, $\sigma_l^{(160 \, ^\circ\text{C})}$, in Fig. 4.25(e). Furthermore, the hysteresis loops stabilized quickly in the pseudoelastic regime (Fig. 4.25) compared to
what was observed in the pseudoplastic regime (Fig. 4.20). Also, cycling at lower mean stress produced saturation much faster than cycling at higher mean stresses. Moreover, at the highest peak stress state case ($\sigma_u^{(160\,\degree C)} = 900$ MPa), a negative slope in the strain vs. cycle plot (Fig. 23f) was predicted by the model. This observation has been observed in some experiments in the literature (see Fig. 6c in (Kan and Kang 2010)).

The evolution plots of the internal stress components for the remaining mechanisms (i.e $b = 1$ and 6) are shown the Fig. 4.26. Here, mechanism 1 and 6 reached their respective saturation states quickly, leading to the fast convergence on the attraction state in the global stress strain diagram. The unified model not only showed evolution in the global components, but also in the internal mechanisms. As mentioned earlier, every evolutionary limit condition, as well as the mathematical constraint conditions, was defined in terms of the internal variables. Therefore, the global response would be the resultant of these internal responses; i.e. saturation in these internal mechanisms would translate into stabilization in the global response quantities.

Similar to the previous section, three cases of extended cycles with varying lower stress, $\sigma_l^{(160\,\degree C)} = 450$ MPa, 350 MPa, and 150 MPa, but fixed upper stress ($\sigma_u^{(160\,\degree C)} = 500$ MPa) were prepared. The obtained responses in the stress – strain subspace, as well as the evolution of strains with cycling are shown in Fig. 4.27. It was observed that the number of cycles required to reach saturation was lowest for the highest mean stress condition (i.e. for the case of $\sigma_l^{(160\,\degree C)} = 450$ MPa), whereas the lowest mean stress condition (i.e. $\sigma_l^{(160\,\degree C)} = 150$ MPa) required more cycles before reaching the attraction state.
Figure 4.25: Extended stress controlled cycles at $T = 160 \, ^\circ\text{C}$; stress vs. strain for $\sigma_{u}^{(160 \, ^\circ\text{C})} = 400 \, \text{MPa}$, $500 \, \text{MPa}$ and $900 \, \text{MPa}$ ($\sigma_{l}^{(160 \, ^\circ\text{C})} = 0 \, \text{MPa}$) in (a) (c) and (e), respectively, with their corresponding evolution of the conjugate strains in (b), (d) and (f).
Figure 4.26: Internal stress variables ($\alpha_{11}^{(b)}$, $b = 1, 6$) for the three extended cycles cases at low temperature: $\alpha_{11}^{(1)}$ in (a), (c), and (e), $\alpha_{11}^{(6)}$ in (b), (d), and (f) for $\sigma_u^{(160 \, ^\circ C)} = 400$ MPa, $500$ MPa, and $900$ MPa ($\sigma_l^{(160 \, ^\circ C)} = 0$ MPa), respectively.
Figure 4.27: Extended stress controlled cycles at $T = 160 \, ^\circ\text{C}$; stress vs. strain for $\sigma_l^{(160 \, ^\circ\text{C})}$ = 450 MPa, 350 MPa and 150 MPa ($\sigma_u^{(160 \, ^\circ\text{C})} = 500$ MPa) in (a) (c) and (e), respectively, with their corresponding evolution of the conjugate strains in (b), (d) and (f).
In the first two subsections, the extended cyclic capabilities under different peak/valley stresses in the pseudoplastic and pseudoelastic regimes were demonstrated. The results clearly demonstrated the relationship between the attraction state and the mean stress. In the next two subsections, extended cyclic capabilities under strain-controlled conditions, with different magnitudes of the applied peak strains, will be presented.

4.4.3. Isothermal, Strain Controlled, Extended Cycles in the Pseudoplastic Regime

Similar to the stress controlled tests at low temperature, the same virgin material was subjected to strain controlled extended cycles in the $M$ phase. Here also, three levels of $\varepsilon_a^{(30 \, ^{\circ}C)}$ were considered. The peak strain levels were nearly conjugate to the peak stresses utilized in section 4.4.2; i.e. 1% (at the beginning of transformation region), 6% for the intermediate strain level, and 10% (from the “rehardening” or end of transformation region).

The model was able to demonstrate an accumulated amount of permanent stress that saturated on a stable value after a certain number of cycles, as well as the degradation effects in the pseudoplastic hysteresis loops (Fig. 4.28). Note also the analogy between stress and strain in the evolutionary character of SMAs. For example, the mean strains in the case of stress assisted training at low temperature were higher than those in the first cycle (see Figs. 4.20b, and d), and were lower for training in the detwinned martensite state (Fig. 4.20f), whereas opposite behaviors were observed for the conjugate mean stresses (Figs. 4.28 b, d and f) in the strain controlled training at the same
temperature. Also, fewer numbers of cycles were required to reach the attraction state. Once more, this also speaks for the rather neat and complete duality that exists between stress- and strain- response in the present model, as manifested also frequently by reported experiments on SMA materials.

4.4.4. Isothermal, Strain Controlled, Extended Cycles in the Pseudoelastic Regime

Here, the virgin material was subjected to strain controlled, extended cycles at 160 °C. The same levels of peak strains as were used in the previous case at low temperature were considered. The obtained evolutions with cycling are shown in Fig. 4.29. In this case, the model was able to show the degradation of stress with cycles, as well as reduction in the hysteretic loop width, as observed experimentally. Note the similarity between experimentally observed behaviors reported in the literature (Fig. 4 in (Auricchio et al. 2003), Fig. 1 in (Auricchio et al. 2007), Fig. 3 in (Saint-Sulpice et al. 2009), and Fig. 10 in (Hornbogen 2004)), and the response predicted by the unified model. All the observations made in the strain controlled training at low temperature were also exhibited at the high temperature by the unified model.
Figure 4.28: Extended strain controlled cycles at $T = 30 \, ^\circ \text{C}$; stress vs. strain for $\varepsilon_u^{(30 \, ^\circ \text{C})} = 1\%$, 6\% and 10\% in (a) (c) and (e), respectively, with their corresponding evolution of the conjugate stresses in (b), (d) and (f).
Figure 4.29: Extended strain controlled cycles at $T = 160$ °C; stress vs. strain for $\varepsilon^{(160 \, ^{\circ}\text{C})}_{u} = 1\%$, 6\% and 10\% in (a) (c) and (e), respectively, with their corresponding evolution of the conjugate stresses in (b), (d) and (f).
4.4.5. Load Bias, Variable Temperature Extended Cycles

Almost all of the practical engineering applications of SMAs require a number of thermomechanical cycles during the function phase of the device, and a stable SME (or TWSME) is looked for from these materials (Kockar et al. 2008; Paiva et al. 2005; Piedboeuf et al. 1998; Predki et al. 2008; Tobushi et al. 1991). Therefore, cyclic evolutionary behaviors of SMAs under thermal cycling conditions become very important in the study of SMAs. Some of the applications of SMAs also require training under thermal cycles (Hartl et al. 2010b; Hartl et al. 2010c).

As a necessary step towards the complete characterization of the cyclic thermomechanical response of the multimechanism based, viscoelastoplastic unified SMA model, three extended thermal cycle test cases under different bias stresses were considered. The test protocol involved the following two step procedure:

1. Firstly, the virgin material was loaded in stress – control to the desired bias stress state in $M$ phase at temperature $T_0$.

2. The stress state of the material was held constant, and the temperature was varied between $T_0$ and $T_1$.

This test protocol is the same as was utilized in section 4.16(a). Here, three bias stress levels were considered: 150 MPa, 200 MPa, and 300 MPa. Note that, as in the other extended cycle cases in this section, distortion constants, $c, d, c^{(b)}, d^{(b)}, b = 1, 2, \ldots, 6$ (see Table 4.1), were active.
The model predictions for the three bias stress levels are shown in Fig. 4.30. The plots include the conjugate strain response vs. temperature in the left column (Fig. 4.30 a, c, and e), and the evolutions of strains at the two temperature extremes vs. cycles in the right column (Figs. 4.30 b, d, and f).

As experimentally observed, the bias stress had a very significant effect on the thermomechanical behavior of the SMA (Bigelow et al. 2007; Padula II et al. 2008a; Padula II et al. 2007; Padula II et al. 2008b). As opposed to the thermal cycles at $\sigma = 200$ MPa, and 300 MPa, the case at $\sigma = 150$ MPa showed the “ever” evolving character. This was found to be very similar to the results obtained by experiments on 55NiTi (Fig. 4.31), where, the continuous evolution took place until 14% of strain at a bias stress of 200 MPa. Recall here that this behavior was similar to that observed in the counterpart isothermal test case with the $\sigma_u = 150$ MPa, as shown in Fig. 4.22(a).
Figure 4.30: Extended thermal cycles under bias stress; (a), (c), and (e) $\sigma = 150$ MPa, 200 MPa, and 300 MPa, respectively; and their respective evolution at low and high temperature extremes vs. cycles in (b), (d), and (f).
The internal stress components for the six internal mechanisms, as shown in Fig. 4.32, also showed the “ever evolving” character in some of their respective evolutions. It is important to mention that the mechanisms 1 & 2 behave somewhat differently initially for the transient period of the first few cycles, but later behave very similarly in reaching their saturated state, as seen in the plots in Figs. 4.32(a) and (b). On the other hand, mechanisms 4 and 5 showed the fastest approach to the state of saturation with cycles (Figs. 4.32d & e), but with very different stabilized values on the two extreme sides of the temperature range. In particular, notice the nearly zero saturation value on the “A” (high temperature) side for the mechanism 4, as opposed to the large compressive counterpart value in mechanism 5, as opposed to the situation on the “M” (low temperature) side, where both of these mechanisms saturated with (nonzero) tensile
values. Finally, it was noted that the mechanisms ultimately responsible for achieving saturation in the SMA strain response are mechanisms 3 & 6 in Figs. 4.32(c) & (f). These were found to be ever evolving throughout the 100 temperature cycles completed here. This eventually led to the non-saturating behavior at 150 MPa, as evident by the relatively high but nearly constant rate of strain accumulation with cycles in Fig. 4.30(b).

In contrast, compare Fig. 4.30(b) to the case of $\sigma = 300$ MPa in Fig. 30(f) which reached true saturation rather quickly.
Figure 4.32: Evolution of internal active stress components vs. temperature during 100 thermal extended cycles at $\sigma = 150$ MPa; (a) mechanism – 1, (b) mechanism – 2, (c) mechanism – 3, (d) mechanism – 4, (e) mechanism – 5, and (f) mechanism – 6. Note that mechanisms – 3 & 6 are responsible of saturation with cycles, which were still evolving at the end of 100 cycles.
Table 4.1: Set of Material Parameters Used for Simulated Test Cases.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Viscoelastic Mechanisms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>“Deflated” Elastic stiffness modulus, $E_s$</td>
<td>MPa</td>
<td>35000</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu$</td>
<td>–</td>
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</tr>
<tr>
<td>$E_m$</td>
<td>MPa</td>
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</tr>
<tr>
<td>$\bar{\rho}$</td>
<td>s</td>
<td>$10^{21}$</td>
</tr>
<tr>
<td><strong>Viscoplastic Mechanisms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of viscoplastic mechanisms</td>
<td>–</td>
<td>6</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>MPa</td>
<td>55</td>
</tr>
<tr>
<td>$n$</td>
<td>–</td>
<td>5</td>
</tr>
<tr>
<td>$\mu$</td>
<td>MPa·s</td>
<td>$10^5$</td>
</tr>
<tr>
<td>$\kappa_{(b)}$, $b = 3, 5, 6$</td>
<td>MPa</td>
<td>8, 30, 350,</td>
</tr>
<tr>
<td>$m_{(b)}$, $b = 1, 2, \ldots, 6$</td>
<td>–</td>
<td>2, 1, 1, 1.1, 1.1, 1.1</td>
</tr>
<tr>
<td>$\beta_{(b)}$, $b = 1, 2, \ldots, 6$</td>
<td>–</td>
<td>14, 10, 10, 1, 1, 1</td>
</tr>
<tr>
<td>$R_{(b)}$, $b = 1, 2, \ldots, 6$</td>
<td>s$^{-1}$</td>
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</tr>
<tr>
<td>$H_{(b)}$, $b = 1, 2, \ldots, 6$</td>
<td>MPa</td>
<td>$50\times10^3$, $30\times10^3$, 100, $10\times10^3$, $15\times10^3$, 1150</td>
</tr>
<tr>
<td>Distortion Constants (used only for tension compression asymmetry, and yielding surface/stress transformation)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>–</td>
<td>0.56</td>
</tr>
<tr>
<td>$d$</td>
<td>–</td>
<td>1.01</td>
</tr>
<tr>
<td>$c_{(b)}$, $b = 1, 2, \ldots, 6$</td>
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<tr>
<td>$d_{(b)}$, $b = 1, 2, \ldots, 6$</td>
<td>–</td>
<td>1.01, 1.01, 1.01, 1.01, 1.01, 1.01</td>
</tr>
<tr>
<td>Temperature variation of Strength Parameters $\kappa_{(b)}$ (interpolated linearly between temperatures, $T_1$ and $T_2$)</td>
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<td></td>
</tr>
<tr>
<td>Mechanisms</td>
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<td>$T_2$</td>
</tr>
<tr>
<td>$b = 1, 2$</td>
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<td>120 °C</td>
</tr>
<tr>
<td>$b = 4$</td>
<td>30 °C</td>
<td>78 °C</td>
</tr>
</tbody>
</table>
 CHAPTER V

AN APPLICATION OF SMA AS AN ACTUATOR IN WING MORPHING TECHNOLOGY

5.1 Introduction

Actuators are devices that perform a specific set of tasks either on demand or in response to certain changes in their surroundings, in order to manipulate the motion of the working components. The reversible transformation between two phases and the associated changes in the mechanical/electrical properties have made SMAs an ideal candidate for ingenious actuation mechanisms in many engineering fields. SMAs are primarily known for their high actuation capacity (i.e. actuation energy to volume ratio), silent operation, solid state design, and miniaturization potential (Ganilova and Cartmell 2010; Georges et al. 2009; Hadi et al. 2010; Torra et al. 2007).

Over the past few decades, researchers have been utilizing SMAs for actuation in many engineering applications. The first commercial application of SMAs was in a F-14 fighter aircraft in 1969 (Schwartz 2002). Since then the applications of SMAs have been steadily growing in many diverse fields. Some examples of applications involving SMAs, including conceptual designs, can be found in the works of various authors (Chu et al.
5.2 SMAs in Wing Morphing Technology

All engineering disciplines are aimed towards the development of new systems and/or improvement of existing systems to get optimal performance during service. In the aerospace industry, one part of the design challenge is to optimize the aerodynamics of the wing structure to get maximum lift with minimal drag across the entire flight envelop. This has a direct effect on the efficiency of the aircraft. “Wing Morphing” is a concept to achieve such a design in the aerospace industry. Wing morphing is defined as the automated modification of the aerodynamics of the wing through computer control.

The history of wing morphing started with the first airplane, built by the Wright brothers in 1903, in which flexible wingtips were twisted with an arrangement of cables and pulleys (Thill et al. 2008). After the first US patent for a variable camber wing in 1916, no ideas were implemented in the actual airplane production until the end of WW2, after which significant advancement in the research on wing morphing technology took place (Thill et al. 2008). While the earlier wing morphing designs involved mechanical actuation, the need for smooth, solid, aeroelastic control surfaces led to an innovation in aerospace materials, as well as in actuation techniques. With the advancement in science and technology, the modern morphing designs target seamless, aerodynamically efficient, lightweight aircrafts capable of not only changing the shape but also orienting wing elements relative to each other (Thill et al. 2008). A novel bio inspired concept of the
**adaptive aerospace vehicle of the future** (see Fig. 5.1) was introduced by NASA, where not only the wings but also the engine inlets and nozzles would be able to shape morph (NASA 2007; Thill et al. 2008). This conceptual design relies heavily on the use of smart materials and embedded actuators.

![NASA's concept of bio-inspired adaptive aerospace vehicle](image)

**Figure 5.1: NASA’s concept of bio-inspired adaptive aerospace vehicle (NASA 2007).**

The large energy density to volume ratio of SMAs has made them suitable for embedded actuators in the aerospace industry. Embedded actuators imply the use of
actuation materials as an integral part of the main structure. These actuators have many advantages over conventional external actuation mechanisms such as few moving parts, high energy efficiency, fewer control and support systems, etc. (Barbarino et al. 2009a; Loewy 1997). Several design concepts focused on embedded actuation to achieve the desired control of the aircraft have been proposed in the literature (Barbarino et al. 2009b; Cho and Kim 2005; Georges et al. 2009; Hartl et al. 2010a; Hartl et al. 2010b; Jardine et al. 1997; Seelecke and Muller 2004; Terriault et al. 2006).

Wing morphing is classified into three broad categories (see Fig. 5.2): (i) planform alternation (Fig. 5.3), (ii) airfoil adjustments (Fig. 5.4), and (iii) out-of-plane transformation (Sofla et al. 2010). The span length and chord length change are done to increase the aspect ratio (ratio of span length and mean chord length), which in turn increases the lift to drag ratio. For steady (cruise) flight, high lift to drag ratio leads to higher aerodynamic efficiency (Monner 2001). Furthermore, sweep angle (measured along 25% chord line) change is used to reduce shock wave induced drag when the airplane speed is close to the speed of sound waves. SMAs have rarely been used to accomplish this type of morphing (Sofla et al. 2010). Mostly, telescopic structures have been the choice of researchers to achieve planform change in wing morphing.

The airfoil adjustment refers to the changes in the airfoil profile without affecting the mean camber line, as shown in Fig. 5.4. Unlike planform alteration, SMAs have been utilized for this type of morphing. For example, in a design suggested by researchers (Georges et al. 2009), the upper camber was shifted with the help of a crank-slider
mechanism, which was being driven by a SMA wire, working in parallel with a bias compression spring.

Figure 5.2: Classification of wing shape morphing (adapted from (Sofla et al. 2010)). The highlighted types of wing morphing under out-of-plane transformation will be the targeted actuation technique.

Figure 5.3: Planform alteration in wing morphing technology; (a) span change, (b) chord length change, and (c) sweep angle change adapted from (Sofla et al. 2010)).
To increase the aerodynamic efficiency during different mission conditions, the wing can be reconfigured in one of the three configurations as shown in Fig. 5.5. The use of SMAs in this third type of wing morphing (i.e. out-of-plane transformation) has attracted a lot of attention. The well known “Smart Wing program” by DARPA involved the use of SMA torque tubes to initiate spanwise twist in the wings (Jardine et al. 1997). In the work presented by Terriault et al., a prestressed SMA wire was connected to the wing skin prototype, and was actuated by means of heating and cooling to achieve chordwise bending (Terriault et al. 2006). Similarly, Barbarino et al. presented experimental and numerical results for a conceptual design to change the upper wing camber (Barbarino et al. 2009a). In the actuator design, a prestretched SMA ribbon was connected parallel to an elastic plate, and the actuation was obtained by heating and cooling the SMA ribbon. In the same year, another design was proposed by the same group, in which the actuation was done on the level of wing rib (Barbarino et al. 2009b). The prototype rib was made out of five separate panels, coupled together by lamina plates, and a SMA wire running through the lower end of the rib skeleton. Upon heating, the eccentricity of the SMA wire with respect to the center of rotation of the connection
plates resulted in a relative angular movement of the bulk plates. Another SMA actuated prototype wing design employed counter rotating, concentric torsion SMA tubes to control the wing camber, and levers in conjunction with SMA wires for local shape control (Icardi and Ferrero 2009). Elzey et al. proposed a multifunctional sandwich panel, made out of SMA face sheets, and an elastic truss core (Elzey et al. 2005) to achieve chordwise bending as well as local shape change. The proposed model relied upon the OWSME and an antagonistic arrangement of SMA face sheets. There are several other proposed (SMA/non-SMA based) actuator designs on wing morphing techniques reported in the literature. Interested readers are referred to (Loewy 1997; Seelecke and Muller 2004; Sofla et al. 2010; Thill et al. 2008) for more information on wing morphing techniques.

Figure 5.5: Out-of-plane morphing; (a) wing twisting, (b) chordwise bending, and (c) spanwise bending adapted from (Sofla et al. 2010)).
In the wing actuation technologies, SMAs are mostly used in form of wires or springs that requires only one dimensional response characteristics of the SMA. However the true potential of SMAs can be utilized only when the complete 3D behavior of material is appreciated. In the next sections, a conceptual wing rib with an embedded SMA actuator will be described, and the complete 3D SMA behavior will be demonstrated by numerical simulation of the actuation mechanism.

5.3 Finite Element Analysis of the Proposed SMA Actuated Aircraft Wing Rib

The present day aircraft wing is characterized by a fixed airfoil geometry that is optimized for single operating point conditions though movable appendages such as flaps and slats. Typically, the aircraft wings are made of prestressed aluminium alloy panels or fiber reinforced polymer composites as the outer skin. The skin is supported by metallic ribs in the chordwise direction, and by stringers and spars along the wing span. The spars are also used to provide support to the ribs. A sandwich substructure in the form of a honey comb core is often installed to mount the engines. The ribs, stringers, and spars withstand most of the axial and bending loads due to gravity and wind pressure, whereas the skin is subjected to a combination of membrane and shear forces.

Due to the reasonably stiff nature of the wing structure (to counteract the lift and drag forces), it is difficult to come up with flexible wing morphing techniques. While most of the SMA actuated wing morphing techniques employ the elementary “wire/spring” type actuator, leading to a local change in the wing camber, a novel SMA embedded rib is suggested in this research. The advantages of such an approach are that
not only the local chordwise bending is possible, but also spanwise twisting can be obtained by applying different degrees of chordwise bending. The proposed wing rib is shown in the Fig. 5.6 below. Note that the dimensions are normalized with respect to the wing rib chord length.

![Figure 5.6: The proposed wing rib architecture. Note that the distances are shown as percentage of wing rib length.](image)

The elastic wing rib consists of two vertical members located at ⅓ of the rib chord length from either end (i.e. leading and trailing end). The two SMA members of length 11.8% of the rib length are attached to the wing rib. The SMA members’ mid point locations are at 50% and 74% from the leading edge. Note that the SMA members are welded to the rib frame at the rib openings; therefore a perfect bonding between SMA and rib material is assumed. The whole installation and actuation procedure is described below:

1. The SMA member is placed and welded to the rib in the $M$ phase.
2. A prestressing force is applied at the ends of the SMA members to initiate the prestressing.
(3) While maintaining the force, SMA members are brought into the \( A \) phase by heating.

(4) Subsequent cooling and heating would cause change in the length of the members, leading to changes in the mean chord line of the rib.

A 3D finite element (FE) model (Fig. 5.7) was prepared to validate the proposed concept of chordwise bending of the wing structure. The rib length was chosen to be 846.6 mm, whereas, the SMA members were of 100 mm length. The thickness of the wing rib was assumed to be 10 mm. A linear elastic material with elastic modulus of 35 GPa, and Poisson ratio of 0.3 was assigned to the rib, whereas the same material parameters that were used in the model characterization (Table 4.1) were utilized for the SMA members.

\[ \text{Figure 5.7: The 3D finite element model of SMA actuated rib.} \]
The actuated rib model was subdivided into 1448 3D continuum, eight noded brick elements: 1288 incompatible modes brick (C3D8I) elements for the elastic rib, and 80 reduced integration (C3D8R) elements for each of the SMA members. The elements were selected in order to reduce the computation cost, and to increase the accuracy of the solution. According to the ABAQUS reference manual, the “incompatible modes” element give more accurate results compared to regular full integration 3D brick elements in the models governed by bending modes (ABAQUS 2008). Since linear elastic material properties were assigned to the C3D8I elements, the required computational costs would not be excessive. However, since a user defined material was utilized for the SMA response, the 160 elements in the SMA members would require heavy computational time (due to input-output communication between the ABAQUS solver and UMAT). Therefore, the choice of reduced integration elements (with one integration point for internal stresses/strains/solution variables calculations) for the SMA members was justified. To support the above argument, results from a separate, relatively cruder mesh (982 C3D8R elements for the elastic rib and 60 C3D8R elements for each of the SMA members) will also be presented. It is important to mention that the UMAT utilized in this work gives identical results in cases involving a single element, irrespective of the number of integration points inside the element.

The wing rib was supported at four points ‘A’ (the interior nodes of leading edge), ‘B’ (mid-point of first vertical member), ‘C’ and ‘D’ (the nodes on the interior edge of the first SMA member) as shown in the Fig. 5.7. The translations along the x- and y- axes were restrained at the point ‘A’, whereas the points ‘B’, ‘C’, and ‘D’ were restrained
along the y-axis. Furthermore, all the nodes on the front face of the rib (xz plane) were not allowed to move along the z-axis. Note that in the model shown in Fig. 5.7, the wing skin was included for demonstration purposes.

Following the steps mentioned earlier, three scenarios were considered in the computer assisted simulation:

(1) Both SMA members were simultaneously subjected to identical temperature cycles.

(2) Only the 1st SMA member was subjected to temperature cycles.

(3) Only the 2nd SMA member was subjected to temperature cycles.

In all cases, 55 °C was used for the $M$ phase temperature, and 130 °C was used for the $A$ phase temperature. Recall that the $M_f$ and $A_f$ for the current SMA material were 65 °C and 120 °C, respectively. The initial prestressing was done in 1000 s, whereas each temperature cycle was performed in 13500 s. In each case, five thermal cycles were considered, starting from the virgin state.

The deformed shapes (scale 1:1) of the wing at high temperature (130 °C) and low temperature (55 °C) during the first temperature cycle are shown in Fig. 5.8. The deformed shape clearly showed significant amounts of camber change (or chordwise bending) at these two temperatures.
To view the exact amount of actuation, the wing tip deflections, $\delta$, during thermal cycling for the three cases, along with the difference between the displacements at the two temperatures extremes (defined as stroke = $\delta^{(30^\circ C)} - \delta^{(160^\circ C)}$) with cycles, are shown in Fig. 5.9. Note that a positive value of $\delta$ was defined along the *negative* global y-axis. During the prestressing stage, the wing tip was deflected by 38.9 mm (Fig. 5.9a). In case – 1, the transient behavior of the SMA response in the first heating (as explained in section 4.3.3, Chapter IV) increased the $\delta$ to 48.3 mm, which reduced to 38.7 mm at the end of the first heating. Following a pattern similar to the strain response under isobaric conditions (section 4.3.3 in Chapter IV), the $\delta$ increased to 55.7 mm, giving an approximate stroke of 16 mm that reduced monotonically to 12.7 mm in five cycles (Fig. 5.9b). The stroke appeared to be stabilizing near 12.7 in this case. In other scenarios, the total stroke reduced from 7.8 mm to 5.4 mm in case – 2 (Fig. 5.9d), and from 9.0 mm to 6.6 mm in case - 3 (Fig. 5.9f). In the two later cases the total stroke tended to reach a stabilized state in five cycles. It is interesting to note that the total stroke at the end of five cycles in case - 1 was more than the sum of the strokes obtained in the later cases -2 & -3. This implies that any stroke between 5 mm and 12 mm can be obtained with this conceptual SMA actuator system. This can be achieved by different combinations of heating cooling sequence, as well as by controlling the temperature gradients.
Figure 5.9: Evolution of wing-tip vertical deflection during temperature cycles for the three cases in (a), (c) and (e); Evolution of actuation stroke (defined as the difference between the deflections at the two temperature ends) with cycles in (b), (d) and (f). Note that the crude mesh in the first case gave nearly same amount of stroke.
As mentioned earlier, the results from a relatively cruder mesh for case – 1 are also shown in Figs. 5.9(a) & (b). At any instance of time, the wing tip deflection from the crude mesh was always less than that of the refined mesh by approximately 2.5%. However, the stroke was very close to that obtained from the refined mesh. The results from the crude mesh reinforced the accuracy of the solution, and indicate that a proper FE mesh had been selected.

To support the multi-axial aspects of the problem, the contours of three major stress components (\(\sigma_{11}\), \(\sigma_{22}\), and \(\sigma_{33}\)) in the two SMA members at the end of the prestressing stage are shown in Fig. 5.10. Note that all three cases were identical up until the prestressing stage. As exhibited in the contours, the \(\sigma_{22}\) and \(\sigma_{33}\) attained respectable values (near the ends of each member) compared to the applied \(\sigma_{11}\). Also, the transformation led to different states of stress inside the different regions of the SMA members. This distribution of stresses result in deformation induced inhomogeneity within the material. This aspect of SMA response in boundary value problems is commonly overlooked in the current literature.

The stress vs. temperature and strain vs. temperature for each of the elements, as labeled in Fig. 5.10, are shown for the three cases in the Figs. 5.11 and 5.12. Although, the members were subjected to a prestressing force that was kept constant throughout the analyses, the obtained response showed development of hysteretic loops in \(\sigma_{11}\) (Fig. 5.11a and b). These stress loops were more prominent in the other two components (\(\sigma_{22}\) and \(\sigma_{33}\) in Figs 5.11c to f). Note that, the stress/strain state (hence the principle axes) at any material point would continuously change during the heating/cooling phases.
Therefore, the uniaxial response assumption is not valid here. This further emphasizes the need for a complete 3D description of SMA response. Note that the hysteretic loops in Figs. 5.11 and 5.12 indicated the “mixed control” nature of the boundary value problem.

As reported in the literature, for most of the SMA actuation concepts under bias load conditions, the prestressing force is removed during the thermal cycle. Therefore, a good amount of the energy, supplied by the thermal cycles, is dissipated in the form of strain, as well as stress hysteresis loops, which leads to a loss of efficiency in the actuation system. In the present model, the prestressing force was maintained constant during the cycles, thus minimizing the energy loss in the hysteresis loops. The numerical analysis of the conceptual SMA based wing rib system demonstrated that a significant amount of chordwise bending in the wing can be obtained with the help of the multi-axial aspect of the SMA response. Also, with the application of different degree of chordwise bending in the different ribs of the wing, spanwise wing twisting can also be achieved. Furthermore, similar design concepts can be utilized at the spar level to obtain other types of wing shape morphing, such as spanwise bending.
Figure 5.10: Contours of (a) $\sigma_{11}$, (b) $\sigma_{22}$, and (c) $\sigma_{33}$ in the two SMA members at the end of prestressing stage. Notice non-uniform stress state in the members.
Figure 5.11: Evolution of stresses with thermal cycles in the three test cases for the two elements shown in Fig 5.10; (a) & (b) $\sigma_{11}$ vs. temperature, (c) & (d) $\sigma_{22}$ vs. temperature, and (e) & (f) $\sigma_{33}$ vs. temperature.
Figure 5.12: Evolution of strains with thermal cycles in the two elements shown in Fig 5.10; (a) & (b) $\varepsilon_{11}$ vs. temperature, (c) & (d) $\varepsilon_{22}$ vs. temperature, and (e) & (f) $\varepsilon_{33}$ vs. temperature for the three test cases.
CHAPTER VI

CONCLUSION AND FUTURE RESEARCH

6.1 Conclusion

In this research work, the 3D, generalized viscoelastoplastic, multimechanism based framework was extended to develop a unified, comprehensive SMA constitutive model that can describe every aspect of the deformation behavior of regular, as well as high temperature, SMAs. With special emphasis on the multi-axial nature of SMA deformations, the present unified model can also account for the true evolutionary response quantities under different thermomechanical loading conditions. The conclusions drawn from the work presented in this dissertation are summarized below:

(1) The deformation response in SMAs is strongly linked to the internal microstructural evolution of the material during the thermomechanical loading. In an analogous manner, the thermomechanical energy was partitioned into storage and dissipation through the notion of multiplicity of tensorial hardening/dissipation, viscoelastoplastic mechanisms.

(2) The individual mechanisms were defined in the dual set of stress and conjugate strain tensorial variables, with their own distinct evolutionary characteristics. This proved to be very successful in capturing many of the
SMA behaviors categorized into supermechanical and superthermal responses.

(3) It was concluded from the test cases presented in sections 4.2.4 and 4.2.5 in Chapter IV that when the strict mathematical requirements of normality and convexity are interpreted in the generalized space of “global” and “internal” state variables, and a single flow rule for both forward and reverse transformation is used, it is possible to capture the “apparent” deviation from normality (when transformation strain rate vectors are plotted onto the external stress surface, see Figs. 4.12 and 4.13 in Chapter IV), as observed experimentally. From the set of numerical test cases, it was shown that the projection of the transformation surface in different stress spaces continuously changed during the transformation/reorientation, and the classical notion of normality in the global response spaces might not be respected.

(4) Similar to the microstructural changes in the real material under cyclic thermomechanical (stress/strain/temperature) loading conditions, the evolution of internal mechanisms was found to be directly linked to the loading procedure as well as the conditions of training, which in turn affected the speed by which the material achieved the saturation states.

(5) To capture a host of experimentally observed behaviors of SMAs, it is not necessary to use a specialized formulation and/or independent sets of material parameters. It was shown in Chapter IV that the present unified
model with a carefully selected \textit{single} set of material parameters (see Table 4.1 in Chapter IV) can demonstrate every transformation/reorientation induced phenomena in the generalized stress-strain-temperature space as is commonly exhibited by SMAs.

(6) Finally, the results from the numerical simulation of the conceptual SMA based wing rib (Chapter V) emphasize the fact that utilization of the multi-axial nature of SMA deformation can provide significant insight into the use of SMAs in wing morphing techniques. In particular, with the help of multi-axial deformations, a substantial change the geometry of the structure can be achieved. Furthermore, the use of SMAs in different structural components, and their combination during actuation can prove effective in achieving different types of wing morphing.

6.2 \textit{Future Research}

On the basis of the present accomplishment in this dissertation, the prospects of the future research work are summarized below:

(1) There is no general consensus in the research community regarding the experimental protocol to evaluate SMA/HTSMA materials. A more comprehensive, systematic experimental program is necessary to guide the further development of the presented unified model.

(2) The established experimental protocol must be standardized for the characterization of SMAs. With the help of a consistent experimental data
set, a proper, automated characterization protocol for SMA/HTSMA materials can then be used to treat the specifics of the evolutionary character of internal parameters, and to determine the appropriate material parameters for the specific SMA system.

(3) As evident from the test cases presented in the section 4.4 (Chapter IV), the training under cyclic conditions is linked intrinsically to the thermomechanical state/history of the material. Therefore, the study of cyclic evolutionary behavior of SMAs in the generalized stress-strain-temperature space must be recognized in order to train these materials (by exploiting their multi-axial deformation behavior) quickly and efficiently. This will open new doors for the many innovative applications of SMAs in the “smart” structures of the future.
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