A GAME-THEORETIC FRAMEWORK TO COMPETITIVE INDIVIDUAL TARGETING

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A GAME-THEORETIC FRAMEWORK TO COMPETITIVE INDIVIDUAL TARGETING

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Thesis

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ABSTRACT

Individual targeting is the process whereby a firm offers promotional incentives to individuals that the firm deems potential customers. With today’s information-intensive marketing environments, most firms have considerable information about consumers in their databases, allowing them to determine those that are loyal to them and those that are potential customers. Firms are taking advantage of this new found ability to target individuals with promotions in order to increase both patronage and their customer loyalty base. We develop a game-theoretic model to investigate simultaneous price and quality competition where the firms are allowed to both manipulate the quality of their product and target individuals with promotional incentives. The firms play a two-stage game of price and quality competition and promotions in which regular prices are chosen in the first stage and then strategies for promotion are chosen in the second stage. We find that in an industry where a larger firm is promoting, customers who are highly sensitive to quality should be targeted to ensure increasing profit. The smaller firm should focus its sales and marketing activities on customers who are less price sensitive and should focus on building customer loyalty.
ACKNOWLEDGEMENTS

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CHAPTER I
INTRODUCTION

As technology advances and internet services improve, firms are able to identify individual customers with increasing accuracy, in particular with the aim of targeting them individually with promotional incentives. Targeting is the task of prioritizing the segments of the consumer population on which an organization should focus its sales and marketing activities [1]. As part of their marketing strategies, firms may offer promotional incentives directed towards individuals that the firm deems potential customers. Individual targeting, a marketing approach that seeks to target individual customers with tailored offers [2], is different from mass-market targeting, wherein a firm offers the same promotional incentives to the market as a whole [1]. Individual targeting is becoming increasingly popular, and some researchers contend [3, 4] that this form of marketing strategy will replace mass-market targeting.

The idea of storing customer data in electronic formats to use them for individual marketing purposes has been around for years (for more on the past, present and future of database marketing read [4]). The advancement in computer systems and the internet make it possible to gain a detailed history of customer information including name, address, history of purchases, demographics and others. Many firms are already taking advantage of this new found ability to segment the market for
the goal of customizing their prices and products. For instance, Time Warner Cable Inc. launched Promotions On Demand, a video-on-demand (VOD) technology that gives consumers the opportunity to request promotional offers in the comfort of their homes using only their remote controls. Shortly after using the VOD service, viewers receive coupons and various promotional packages in the mail. "When customers get marketing materials they really want and advertisers get customers who are interested in their offers, everybody wins," says Joan Gillman, president of Time Warner Cable Media Sales [5]. As another example, credit card issuers occasionally send targeting offers to individuals based on their credit history.

When a customer makes his buying decisions, he does so based not just on price or promotional incentives, but also based on personal perceptions or expectations of the quality of the product. The perception of quality is important in the marketplace. In Crosby’s book, *Quality is Free*, he argues that the unquality things, that is the actions that cause one not to do things right the first time, are what cost money [6]. For instance, "a study commissioned by the United States Department of Commerce’s National Institute of Standards and Technology (NIST) found that software defects cost the U.S. economy almost 60 billion USD annually" [7]. Consumers want superior products and are willing to pay reasonable prices for them. In the long run, quality leads to increased profitability and growth of the company [8]. Hence, the issue of quality of a firm’s product is key especially now that firms face intense competition from foreign companies. We see statements as "Quality is the most important issue facing top management in the 1990”s [9], "Quality has become
the business strategy of the 80’s and 90’s” [10], and advertisement that put emphasis on a firm’s focus on ensuring quality to its customers (eg. ”Quality is Job one”, used by Ford Motor Company, ”We deliver Quality” by Domino’s Pizza and ”The quality goes in before the name goes on”, by Zenith Corporation). Quality is defined in many different ways. In marketing and economics, quality refers to the performance level of the product and depends on the level of product attributes [11, 12, 13]. In manufacturing applications, a product is high quality when it conforms to specifications [14, 15]. In operations research, quality may be defined to depend on the fitness of use, reliability, performance, looks as well as other concepts [16, 17]. However, since quality can be perceived without actual consumption experience, we follow the definition of quality by Zeithaml ”as the customer’s judgement about a product’s overall excellence and superiority” [18].

If the price and the quality of a product affect a consumer’s decision to buy, it is important to understand the ramifications of price and quality competition on a firm’s profit and market share when the firm targets individuals with promotional incentives. Many marketing and economic investigations have modeled the effect of customized marketing on a firm’s profits as well as market shares under different competitive settings. Narasimhan [19] uses game-theory to analyze the equilibrium pricing strategies (choosing promotional price, the depth of discounts away from a regular price, and the frequency of deals) of competing brands engaged in a pricing game. Each firm has its loyal customers and competes for other customers known as brand switchers. He shows that the behavior of these switchers characterize equilib-
rium. In equilibrium, there is no pure Nash equilibrium, that is choosing either of the pricing strategies, but mixed strategies with firms choosing prices according to a distribution function over a range of prices.

Neslin [20] models the effect of coupon promotions on market shares and firms profits using an econometric market response model. His results indicate that the effect of coupons varies across brands with some brands enjoying an increase in sales while others do not enjoy an increase in sales when the coupons are redeemed. Raju et al. [21] examine the influence of customer loyalty on a firm’s decision to use price promotions in a product category by implementing a finitely repeated game theory approach. They show that in equilibrium, a market in which a brand with a large brand loyalty competes with a weaker brand, the stronger brand promotes less frequently than the weaker brand. As a result, the weaker brand gains more from price promotions. Shaffer and Zhang [22] model price and coupon distribution in a spatial setting. In their model, firms first choose regular prices and coupon face values and then choose how to distribute the coupons. They find that mass media and coupon targeting are complementary with the former associated with higher regular prices and profit and the latter leading to increased competition and decreased firm profits. Rao [23] views promotion competition as a three-stage game in which regular prices are chosen first, then the choice of promotion depths (i.e. how many cents should be taken from the regular price to obtain the promotional price) and frequencies (the decision to charge regular price or to promote with the already chosen depths). In his analysis, he compares competition between a national brand and a private label.
From his result, in equilibrium, the national brand promotes to ensure that consumers are not lured to buy from the private label and so the private label does not promote. Assuming asymmetry of firms, Shaffer and Zhang [3] study the effect of one-to-one promotions on firm’s profits and market shares utilizing non-cooperative game theory. In their model, firms are allowed to choose a regular price and then promotional strategies. They find that one-to-one promotion allows a firm to generate more sales without always offering a discount. As part of their results, one-to-one promotions intensifies competition as all consumers, including loyal consumers, are contestable.

Individual targeting is, however, effective and optimal when firms have the ability to contact customers individually [24] and the ability to predict the preferences and purchase behaviors of individual consumers [25]. Chen and Iyer [26] study the ramifications of consumer addressability on competition between firms engaged in individual/direct marketing. They find that perfect addressability does not occur in equilibrium as assumed by other researchers [22, 27]. Also, because of insufficient data or inaccurate statistical inferences, a firm’s targetability is imperfect. When individual marketing is feasible, but imperfect, improvements in targetability by either or both firms can lead to both firms benefiting even if both firms behave non-cooperatively and the market does not expand [25].

The question of the joint effect of price and quality competition on profitability and market share incorporating promotional strategies has not received much attention. References exist, for instance [28, 29], which focus on price and quality competition, but these do not consider promotional strategies.
In particular, the study of quality and/or price competition has received attention primarily in the economics and marketing literature. Some of these papers [30, 31] have been inspired by Hotelling [32], who studied price competition between two firms assumed to be symmetric and consumers also having heterogeneous taste preferences that lie on a continuum. In his model, two firms compete on store location and price. Hotelling argues that in equilibrium, the strategy is for each firm to choose a location at the center of the market. He argues that for any location of one firm, the best response of the other firm is to move toward its opponent in order to expand the territory under his control and thus enjoy increased profits.

Moorthy [30] adopts the model and methodology of Hotelling but in his model, the store location in Hotelling’s model is the product. He uses game theory to examine the effect of the preferences of consumers, costs and price on the competitive product strategy of a firm. There are two identical firms serving consumers who prefer a higher quality product to a lower quality product but differ in how much they are willing to pay for quality. These consumers can choose a substitute if they do not like the product and the price offered by the two firms. From his analysis, the product strategy of a firm is not only to choose a product similar to its competitor but also to differentiate its product from its competitor as this weakens price competition and raises profits. He also finds that the first entrant in a market can defend himself from future entrants with his product. Banker et al. [28] study the effect of intense competition on quality under three different competitive settings: asymmetric duopoly where a leading firm’s intrinsic demand decreases, a symmetric
duopoly with the firms not allowed to cooperate and a symmetric oligopoly with the number of firms increasing. They find that the relation between equilibrium quality and intense competition depends on how one understands increased competition and also on the values of the parameter that describe the cost and demand structure of the industry.

Matsubayashi and Yamada [29] explore the impact of asymmetry in consumer’s loyalty on price and quality competition where the firms only select regular prices. They analyze competition in terms of welfare of the competing firms and consumers from a microeconomic viewpoint under varying consumer’s price and quality sensitivity. They find that in both the moderately quality-sensitive and price-sensitive markets, higher consumer sensitivity as well as lower consumer loyalty to any firm leads to intense competition, that results in a decrease of both firms equilibrium profits. The quality decision in the marketing and management science literature, for instance [14, 33], have no direct implications for production costs. On the contrary, in Banker et al. [28] and Matsubayashi and Yamada [29], demand is modeled as a linear function of price and quality levels, and cost as a quadratic function of the quality level. Banker et al. [28] explore asymmetry in demand and cost structures and its effects on price and quality competition by allowing the fixed cost parameter to be different between the two firms. Firms do not only select regular price and quality level but also offer promotional incentives with the aim of inducing customers who otherwise would buy from its competitors to actually buy from them.
Matsubayashi et al. [29] do not study the effect of price and quality competition on profitability and market share where the firms are allowed to promote.

In this thesis, our goal is to model the effect of price and quality competition on firms profits and market shares with the larger firm promoting. Our model differs from Shaffer and Zhang, Banker, Matsubayashi and Yamada in several aspects. First, whereas Shaffer and Zhang [3] model only price competition with promotions, we assume that firms can also select their quality levels so as to increase their profits. This is different from Bertrand price competition where price is the only decision variable. Secondly, Banker et al. [28] assume that the fixed cost parameter changes but we assume that they are the same so that there would not be the argument that the differences we are observing could be due to differences in technology costs. Again, in Banker’s analysis, the firms play a sequential game where quality levels are selected first, followed by each firm observing the other’s quality and then selecting their product price. Also, Banker, Matsubayashi and Yamada do not incorporate the effect of promotions on a firm’s profit and market share. We do so in our model.

As stated earlier, quality can be perceived without actual consumption so firms could incorporate the perception of their product by customers in their pricing decision. We call this perceived price just as in Matsubayasi and Yamada [29]. Our model helps to answer the following managerial questions:

(i) Does individual targeting affect the profitability and market share of a company? If so, which firms are likely to benefit from individual targeting?
(ii) Does the market’s expectation on quality of a firm’s product affect profitability and market share? (iii) How is the average industry quality level affected as competition intensifies? (iv) How should firms position themselves best to take advantage of new technologies?

To answer these questions we develop a game-theoretic model following [3], in which each of the two firms has their own loyal customers and competes for potential switchers. We maintain the assumptions of asymmetry in size of the two firms and customers having heterogeneous brand loyalty. We assume that firms do not only select price but also quality. The firms play a two-stage game of price-quality competition and promotions in which regular prices are chosen in the first stage and then strategies for promotion in the second stage. To the best of our knowledge, this thesis provides the first rigorous analysis of individual targeting combining perceived quality and promotional strategies. The firms seek to obtain patronage from customers deemed potential switchers, who will buy from either firm, given price and promotional incentives.

For a real world application of our model, consider an example of two internet service providers, AT&T and Time Warner Cable Inc., deciding to launch a new service while possibly offering promotions to customers. Both ISPs already have their loyal customers via their existing products and thus know their brand loyalty almost perfectly. The demand for the new service is likely to be affected by the price of the service, the perceptions on quality formed by customers and other exogenous variables. The demand structure in our model considers this situation.
Also the decision to promote or not to promote is likely to be affected by variables such as the marginal cost and the cost of targeting the marginal customer. Our cost model also captures this situation. AT&T is the largest firm in our example as it is the No. 1 U.S.A provider of wireless, Wi-Fi, high speed Internet and voice services with 60 percent ownership interest in Cingular Wireless, the No.1 U.S.A wireless services provider with 55.8 million wireless customers [34]. Time Warner Cable Inc. is the second-largest cable operator in the U.S.A serving more than 14 million customers who subscribe to one or more of its services [5] and could be assumed to be the smallest firm in our model. Both firms offer promotions as part of their marketing strategies. In our model, the firms do not promote simultaneously. Future work could consider the case where both firms promote simultaneously under price and quality competition. Shaffer and Zhang [3] study simultaneous promotion but only under price competition.

The rest of the paper proceeds as follows. Chapter 2 develops the model and discusses its properties. Chapter 3 solves for the second-stage game in which regular prices are chosen in the first stage and promotional decisions in the second stage. In chapter 4, we analyze the model and generate some useful results. We conclude with a summary of our results and provide some suggestions for future work in chapter 5.
CHAPTER II
THE MATHEMATICAL MODEL

In this chapter, we set up the scenarios for our analysis. In every game, there are the players, the rules and the strategies used. We explain the rules of the game, which are given by the assumptions imposed, and the strategies used by the players.

2.1 The Basic Model

Consider a duopoly market consisting of firms A and B selling brand A and brand B respectively of a competing commodity of a consumer good with nonnegative price $P_i$ and quality $X_i$ for $i \in \{A, B\}$. $X_i = 0$ is the baseline quality or the acceptable quality level. Each firm produces at a constant marginal cost, $v \geq 0$. Consumers are able to observe a commodity’s price and quality before they buy so when they make their buying decisions, we assume each customer buys at most one unit of the good that gives them the largest consumer surplus, the difference between what they are willing to pay and what they actually pay. Each consumer has a reservation price $R$, the maximum price the customer is willing to pay for the commodity or the minimum price the seller is willing to offer for a commodity, which is assumed to be greater or equal to the marginal cost ($R \geq v$). If the maximum consumer surplus that is obtained from the two firms is less than zero, then the consumer will not
buy the product from either firm. On the demand side, we assume that customer
loyalty, \( l \), is uniformly distributed on the interval \([-l_B, l_A]\). This assumption means
that consumers are heterogeneous in commodity loyalty in that they differ in how
much they are willing to pay for their less preferred commodity. These are the price
and quality insensitive customers and the others are potential switchers and will buy
from either firm depending on the price and quality level of the firm. We assume that
the total number of consumers in the market is constant.

Prior to buying a commodity, consumers form some perceptions about the
quality of the commodity and we assume that pricing information is readily available.
Consumers value product \( i \) based on the perceived price, defined as

\[
\omega_i = \alpha P_i - \beta X_i, \tag{2.1}
\]

where \( \alpha \) and \( \beta \) are positive market parameters that measure the responsiveness to
price and quality level. Thus if customers feel that a product is of higher quality, that
is above the baseline quality, then the perceived price is less than the actual price, \( P \),
assuming \( \alpha \leq 1 \). If the quality level is zero, indicating the minimum feasible level of
quality, then the perceived price coincides with the actual price of the product, with
\( \alpha = 1 \).

Without loss of generality, we assume that \( l_A > l_B \) which means that firm A
has more loyal customers than firm B. If \( l_A \) is close to \( l_B \), firms A and B are relatively
horizontally differentiated and consumers have differences in opinion even if prices
are the same. If \( l_A \) and \( l_B \) are very different, the two products are more vertically
differentiated. Let $\omega_A$ and $\omega_B$ be the perceived price for firm A and B respectively. We define a consumer’s loyalty as the minimum perceived-price differential to induce her to purchase from her less preferred firm. Thus, a consumer who prefers commodity A by a loyalty level of $l$ will buy commodity B, without any promotional incentives, if and only if the perceived price of firm A exceeds the perceived price of firm B by a loyalty level more than $l$. In other words, the consumer will purchase brand B if and only if $\omega_B + l \leq \omega_A$.

The firms play a non-cooperative two stage game, simultaneously selecting price and quality level in the first stage and then promotional strategies in the second stage. In the promotion stage, each firm must decide which consumers to target and with what discounts. Let the cost of targeting consumers located at $l$ be $z \geq 0$. This cost is incurred whether the consumer buys from the firm or not. Let $d_i(l)$, $(i \in \{A, B\})$, be the discount firm $i$ gives to customers located at $l$. This $d_i(l)$ is not only limited to discounts but also the monetary value of any individual-specific promotional incentives, such as coupons or prizes, given by firm $i$ to gain patronage. Consumers are rational and will seek to maximize their surplus given each firms perceived price and promotional incentives. Therefore, a consumer located at $l$ who receives only promotional coupons from firm A will purchase from firm A if and only if the net perceived price is less than or equal to the loyalty level of the consumer. That is, such a consumer will buy from firm A if and only if $\omega_A - d_A(l) \leq \omega_B + l$. If a consumer who is located at $l$ only receives promotional incentives from firm B, then the customer will purchase from firm A if and only if $\omega_A - (\omega_B - d_B) < l$. 13
A consumer who receives promotional incentives from both firms will purchase from firm B if and only if
\[ l < (\omega_A - d_A (l)) - (\omega_B - d_B (l)). \]

2.1.1 The Demand and Cost Structures

As indicated above \( P_i \) and \( X_i \) are the price and quality levels of the product from firm \( i \in \{A, B\} \). Let \( \hat{l} \equiv \omega_A - \omega_B \). It follows that in the absence of a discount, consumers with \( -l_B < l \leq \hat{l} \) buy from firm B and consumers with \( \hat{l} < l \leq l_A \) buy from firm A. Then the demand quantities of firms A and B, which are denoted as \( q_A \) and \( q_B \) respectively, are linear in price and quality and are given by

\[
Q_i(P_i, X_i) = q_i \text{ for } i \in \{A, B\},
\]

where \( q_i \) is the demand for a firm.

Using the analysis in [28], we select the cost function, \( C_i \), to be

\[
C_i(q_i, X_i) = (\epsilon X_i + v + z) q_i + \phi X_i^2,
\]

where \( \phi \) and \( \epsilon \) are nonnegative fixed and variable cost parameters respectively. Hence investment in programs that seek to increase quality also increase fixed costs. In our analysis, contrary to [28], we do not allow the fixed cost parameter \( \phi \) to be different between the two firms. For the analysis that follows, we scale all price and cost variables \((P_A, P_B, l_A, l_B, l, v, z, \omega_A, \omega_B)\) by the quantities \( l_A + l_B \). For convenience, we do not change any notation when making this change of variables. Hence, from here forward, the price and cost variables are unitless quantities. Furthermore, the difference \( l_A - \hat{l} \) now represents the proportion of the market that is loyal to firm A.
Similarly, $l_B + \hat{l}$ represents the proportion loyal to firm B. These results follow since $l_A + l_B = 1$ in the scaled variables.

2.2 The Game

We follow Shaffer and Zhang and use a subgame perfect Nash Equilibrium as our solution. Our main game of price-quality competition could be broken into subgames that contain all the available choices in our main game. In using this solution concept, we assume that regular prices are observable by each firm when they choose their promotional strategies. The strategy used is to promote or not to promote given the cost of production and the cost of targeting. The strategy profile for each firm is $S_i = \{ \text{Promote (P) , Do Not Promote (N)} \}$. Thus, individual targeting depends on the relationship that exists between the cost of targeting and the net production cost. That is the relationship between $z$ and $P_i - \epsilon X_i - v$. This leads to four subgames. We concentrate our analysis on only three of the subgames. In our analysis, the two firms simultaneously select their price and quality levels. This is in contrast to [28] where the firm selects price and then selects the quality level given that price.

2.2.1 Subgame One: $P_A \leq (\epsilon X_A + v) + z$ and $P_B \leq (\epsilon X_B + v) + z$

This subgame considers the cost of targeting to be so high that both firms do not find it feasible to target. The price of a unit commodity is less than the cost of producing that unit plus the cost of targeting the marginal customer located at $l$. 

15
The demand for firm A is sales to customers in region I of figure 2.1

\[ q_A = l_A - \hat{l} = l_A - \alpha P_A + \alpha P_B + \beta X_A - \beta X_B, \]  

(2.4)

and that of firm B is sales to consumers located in region II of figure 2.1

\[ q_B = l_B + \hat{l} = l_B + \alpha P_A - \alpha P_B - \beta X_A + \beta X_B. \]

The profit functions for the two firms are

\begin{align*}
\pi_A &= (P_A - \epsilon X_A - v) q_A - \phi X_A^2, \\
\pi_B &= (P_B - \epsilon X_B - v) q_B - \phi X_B^2.
\end{align*}

(2.5) (2.6)

The payoff matrix when no firm promotes is shown in Table 2.1. From the payoff matrix in table 2.1, if firm B promotes and thus earns zero profit, then firm A can promote or not promote. If A promotes, then the expected payoff is zero but if firm A does not promote, A would earn \( \pi_A \). If firm B should not promote and A promotes, A’s profit will be zero while that of B will be \( \pi_B \). But if both should not promote, A and B will earn \( \pi_A \) and \( \pi_B \) respectively. The pure Nash equilibrium for the game is \( (\pi_A, \pi_B) \). The optimal profit functions are given in the next chapter.
Table 2.1: Payoff Matrix when no firm promotes

<table>
<thead>
<tr>
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<th>Firm B</th>
</tr>
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<tbody>
<tr>
<td>Firm A</td>
<td>P</td>
</tr>
<tr>
<td>P</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>N</td>
<td>(π_A, 0)</td>
</tr>
</tbody>
</table>

2.2.2 Subgame Two: \( P_A > (\epsilon X_A + v) + z \) and \( P_B \leq (\epsilon X_B + v) + z \)

This subgame considers the case where A finds it profitable to promote and B does not promote because the cost of promoting is too high. Even though A promotes, A will not promote to all customers in the loyalty spectrum because those with loyalty \( l \geq \hat{l} \) are so loyal to the firm that they will buy regardless of any promotion. Others are so loyal to B that the maximum discount that A could offer and still earn positive profit is not sufficient to draw them away from B. For consumers located at \( l < \hat{l} \) (refer to figure 2.2), the maximum discount that A could give any consumer and still earn positive profit is \( d_A^{max} = P_A - \epsilon X_A - v - z \). For all other consumers located at \( l < \hat{l}_B \), with \( \hat{l}_B = \omega_A - \omega_B - d_A(l) = \alpha P_A - \beta X_A - \alpha P_B + \beta X_B - d_A(l) \), the maximum discount A could give is not sufficient to induce them to switch over to A. These are so loyal to firm B that they would buy from firm B even if they received \( d_A^{max} \) from firm A. Thus, firm A will find it unprofitable to promote to such customers. So in equilibrium, firm A will target consumers located between \( \hat{l}_B \) and \( \hat{l} \) in the loyalty spectrum. We solve for firm A’s optimal discount schedule in the targeting area.
Figure 2.2: Loyalty region for firm A and firm B when firm A promotes

Let $\tau_{A,2} \equiv \{ l \mid \hat{l}_B < l < \hat{l}_A \}$ be the targeting area for firm A and this is region III in figure 2.2. A consumer located at $l \in \tau_{A,2}$ will buy from firm A if and only if the difference between the net perceived price of A and the perceived price of B is less than her loyalty. That is if $\omega_A - d_A(l) - \omega_B \leq l$. Hence, firm A will offer such a consumer the smallest discount satisfying this inequality. That is, $d_A(l) = \omega_A - \omega_B - l$.

The profit that will accrue to firm A will be $P_A - \epsilon X_A - d_A(l) - v - z = P_A - \alpha P_A + \beta X_A - \epsilon X_A + \alpha P_B - \beta X_B + l - v - z$. In equilibrium, firm A’s profit is the sum of the profit earned from sales to customers who buy from the firm at the regular price and the profit from sales to customers persuaded to purchase with a discount. Firm B’s profit is from sales to customers who are loyal to B.

The equilibrium payoffs are given below:

$$\pi_{A,2} = (P_A - \epsilon X_A - v) (l_A - \hat{l}) - \phi X_A^2$$
$$+ \int_{l_B}^{\hat{l}} (P_A - \alpha P_A - \epsilon X_A + \beta X_A + \alpha P_B - \beta X_B - v - z + l) dl$$

$$\pi_{B,2} = (P_B - \epsilon X_B - v) (l_B + \hat{l}_B) - \phi X_B^2$$

with

$$\hat{l}_B = \alpha P_A - P_A - \beta X_A + \epsilon X_A - \alpha P_B + \beta X_B + v + z,$$

$$\hat{l} = \alpha P_A - \beta X_A - \alpha P_B + \beta X_B.$$  

(2.7)
Notice that firm B’s demand decreases when firm A promotes. This is because A targets customers, who would have bought from B, with promotional discounts. The profit margin that A obtains from selling to customers in the targeting zone is lower than the profit earned from selling at regular price. This is due to the cost of targeting and the promotional incentives offered. However, firm A’s ability to target individuals with promotional incentives causes A’s sales to increase. The fixed cost remains the same because the decision to promote or not promote does not affect one’s fixed cost. The payoff matrix when only firm A promotes is given in table 2.2.

From the payoff matrix, given that firm B promotes and thus earns zero profit, firm A can promote or not promote. If A promotes, then the expected payoff is $\pi_{A,2}$ but if firm A does not promote, A would earn $\pi_A$. If firm B should not promote and A promotes, A’s profit will be $\pi_{A,2}$ while that of B will be $\pi_{B,2}$. But if both should not promote, A and B will earn $\pi_A$ and $\pi_B$ respectively. The pure Nash equilibrium for the game is $(\pi_{A,2}, \pi_{B,2})$. The optimal profit functions are given in the next chapter.

**Table 2.2: Payoff Matrix when firm A promotes**

<table>
<thead>
<tr>
<th></th>
<th>Firm B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P</td>
</tr>
<tr>
<td>Firm A</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>$(\pi_{A,2}, 0)$</td>
</tr>
<tr>
<td>N</td>
<td>$(\pi_A, 0)$</td>
</tr>
</tbody>
</table>

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2.2.3 Subgame Three: $P_A < (\epsilon X_A + v) + z$ and $P_B > (\epsilon X_B + v) + z$

This subgame considers the case where firm B finds it profitable to promote and A does not. This game is analogous to subgame 2. Figure 2.3 depicts the targeting regions for both firms. If a customer is located at $l \leq \hat{l}$, firm B will not target this customer with promotional incentives as this customer is so loyal to B that she will buy without any incentives. The maximum discount that firm B could give customers located between $\hat{l}$ and $l_A$ and still earn positive profit is $d_B^{max} = P_B - \epsilon X_B - v - z$.

For all other customers located at $l > \hat{l}_A$, with $\hat{l}_A = \omega_A - (\omega_B - d_B(l))$, even if B were to offer the maximum discount to them, they would not be induced to switch firms. These customers are very loyal to firm A. Firm B will find it unprofitable to target such consumers on the loyalty spectrum. Thus B’s targeting area is $\tau_{B, 3} \equiv \{l \mid \hat{l} < l < \hat{l}_A\}$, which is region III in figure 2.3. Consumers in this targeting zone will buy from B if and only if $\omega_B - d_B(l) - \omega_A \leq l$. It happens that B will offer the minimum discount, that is, $\omega_B - \omega_A - l = d_B(l)$, giving a profit margin of $P_B - \epsilon X_B - d_B(l) - v - z$. 

![Figure 2.3: Loyalty region for firm A and firm B when firm B promotes](image-url)
In equilibrium, the payoffs are given as:

\[
\pi_{B,3} = (P_B - \epsilon X_B - v)(\hat{l} + l_B) - \phi X_B^2 \\
+ \int_{l}^{l_A} (P_B - \epsilon X_B - d_B(l) - v - z) \, dl ,
\]

(2.8)

\[
\pi_{A,3} = (P_A - \epsilon X_A - v)(l_A - \hat{l}_A) - \phi X_A^2
\]

where

\[
\hat{l} = \omega_A - \omega_B = \alpha P_A - \beta X_A - \alpha P_B + \beta X_B ,
\]

\[
\hat{l}_A = \alpha P_A - \beta X_A - \alpha P_B + P_B - \epsilon X_B + \beta X_B - v - z .
\]

The payoff matrix when only firm B promotes is given in table 2.3. From the payoff matrix, given that firm A promotes and so earns zero profit, firm B can decide to promote or not promote. If B promotes, then the expected payoff is \(\pi_{B,3}\) but if firm B does not promote, B would earn \(\pi_B\). If firm A should not promote and B promotes, B’s profit will be \(\pi_{B,3}\) while that of A will be \(\pi_{A,3}\).

Table 2.3: Payoff Matrix when firm B promotes

<table>
<thead>
<tr>
<th>Firm A</th>
<th>Firm B</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>P</td>
</tr>
<tr>
<td></td>
<td>N</td>
</tr>
<tr>
<td>N</td>
<td>((\pi_{A,3}), (\pi_{B,3}))</td>
</tr>
</tbody>
</table>
But if both should not promote, A and B will earn $\pi_A$ and $\pi_B$ respectively. The pure Nash equilibrium for the game is $(\pi_{A,3}, \pi_{B,3})$. In the next chapter, we obtain the optimal profits for the optimal payoffs found in this chapter.
CHAPTER III
SOLUTION PROCEDURE

This chapter outlines the solution procedure of our game. Before we start with the computation of the price and quality equilibrium, we want to explain the equilibrium concepts used. Suppose we are in the simultaneous price-quality choice model. The decision to choose a regular price or promotional price will depend on the relationship between the cost of targeting and the net production cost. To capture all the available choices in our game, we use the subgame perfect Nash equilibrium as our solution concept. A strategy profile is a subgame perfect Nash equilibrium if it specifies a Nash equilibrium in every subgame of the original game [35]. In using this solution concept, we assume that regular prices are observable by each firm when they choose their promotional strategies. That is, there is complete information on prices prior to selecting the promotional strategies. This competition between the two firms will occur in two stages. In the first stage, the firms will choose regular prices and quality levels simultaneously with the other firm. In the second stage, the firms would choose the promotional price and quality level. Given this move structure, we define equilibrium in two steps, beginning with the regular price and quality equilibrium. The Nash Equilibrium in price and quality is the price and quality of firm A and price and quality of firm B such that no firm would choose a different price and quality level.
unilaterally. In the second step, the price and quality equilibrium will be functions of the cost of promoting.

In all the subgames, to obtain the optimal profits and market shares, we differentiate the profit functions over $P_A, P_B, X_A, X_B$, solve the resulting system of equations and find the critical points to obtain maximum prices and quality levels, and then obtain the maximum profits and market shares from these. In the spirit of the analysis, the equilibrium is characterized by $T \equiv \beta - \alpha \epsilon$, that is consumers valuation of quality level relative to price and is normalized by the variable cost of the quality level [29]. If $T = 0$, the consumers valuation of price and quality is at par. That is, consumers are as sensitive to price as they are to quality. When $T < 0$, consumers are more sensitive to the price of the commodity than to its quality. When $T > 0$, consumers are more quality sensitive. We call this region, the quality sensitive region and the region $T \leq 0$ the price-sensitive region [29].

3.1 Subgame one: $P_A \leq (\epsilon X_A + v) + z$ and $P_B \leq (\epsilon X_B + v) + z$

Recall subgame one is the case where the cost of targeting customers located in the loyalty spectrum is so high that both firms would not find it profitable to promote. From chapter II, we determined the profit functions

$$
\pi_A = (P_A - \epsilon X_A - v) q_A - \phi X_A^2,
$$

$$
\pi_B = (P_B - \epsilon X_B - v) q_B - \phi X_B^2,
$$

(3.1)
with
\[
q_A = l_A - \hat{l} = l_A - \alpha P_A + \alpha P_B + \beta X_A - \beta X_B,
\]
\[
q_B = l_B + \hat{l} = l_B + \alpha P_A - \alpha P_B - \beta X_A + \beta X_B. \quad (3.2)
\]

Equation 3.2 is the market share of the firms when no firm promotes. We differentiate the profit functions over the variables that each firm has control over to obtain the following reaction functions for firm \(i \in \{A, B\}\) that give the best response for firm \(i\) given that firm \(j \in \{A, B\}, (i \neq j)\) chooses \(P_j\) and \(X_j\):
\[
\begin{align*}
\frac{\partial}{\partial P_A} \pi_A &= l_A - \alpha P_A + \alpha P_B + \beta X_A - \beta X_B - (P_A - \epsilon X_A - v) \alpha, \\
\frac{\partial}{\partial X_A} \pi_A &= -\epsilon (l_A - \alpha P_A + \alpha P_B + \beta X_A - \beta X_B) + (P_A - \epsilon X_A - v) \beta - 2 \phi X_A, \\
\frac{\partial}{\partial P_B} \pi_A &= l_B + \alpha P_A - \alpha P_B - \beta X_A + \beta X_B - (P_B - \epsilon X_B - v) \alpha, \\
\frac{\partial}{\partial X_B} \pi_A &= -\epsilon (l_B + \alpha P_A - \alpha P_B - \beta X_A + \beta X_B) + (P_B - \epsilon X_B - v) \beta - 2 \phi X_B. \\
\end{align*}
\quad (3.3)
\]

From the first order condition of maximization, we set the differentials above to zero and solve the system of equations to obtain the maximum equilibrium prices and quality levels
\[
\begin{align*}
P_A^* &= \frac{(2 \phi + \epsilon T^2) \{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B\}}{4\alpha \phi (3 \alpha \phi - T^2)} + v, \\
P_B^* &= \frac{(2 \phi + \epsilon T^2) \{(2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B\}}{4\alpha \phi (3 \alpha \phi - T^2)} + v, \\
X_A^* &= \frac{T \{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B\}}{4\alpha \phi (3 \alpha \phi - T^2)}, \\
X_B^* &= \frac{T \{(2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B\}}{4\alpha \phi (3 \alpha \phi - T^2)}. \quad (3.4)
\end{align*}
\]

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One can easily check that the second order derivatives are all negative, ensuring a maximum at the equilibrium levels. Substituting the equilibrium prices and quality levels in equations 3.4 into equation 3.1, we obtain the equilibrium profits of firms A and B:

\[
\pi_A^* = \frac{(4 \alpha \phi - T^2) \left\{ (4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B \right\}}{16 \alpha^2 \phi (3 \alpha \phi - T^2)^2},
\]

(3.5)

\[
\pi_B^* = \frac{(4 \alpha \phi - T^2) \left\{ (2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B \right\}}{16 \alpha^2 \phi (3 \alpha \phi - T^2)^2}.
\]

(3.6)

To obtain the market shares \( S_A^\star \) and \( S_B^\star \) for firm A and firm B respectively, we substitute equation 3.4 into the market share functions given by equation 3.2. This gives

\[
S_A^\star = \frac{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B}{2 (3 \alpha \phi - T^2)},
\]

\[
S_B^\star = \frac{(2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B}{2 (3 \alpha \phi - T^2)}.
\]

Also, substituting equation 3.4 into equation 2.1, we obtain the perceived prices for firms A and B

\[
w_A^\star = \alpha v + \frac{(2 \alpha \phi - T^2) \left\{ (4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B \right\}}{4 \alpha \phi (3 \alpha \phi - T^2)},
\]

\[
w_B^\star = \alpha v + \frac{(2 \alpha \phi - T^2) \left\{ (2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B \right\}}{4 \alpha \phi (3 \alpha \phi - T^2)}.
\]

To distinguish between the price sensitive region and quality sensitive region, we find the following to be true.

Theorem 1.

1) If \( T \leq 0 \), the unique equilibrium is given as:

\[
P_A^\star = v + \frac{2 l_A + l_B}{3 \alpha}, \quad P_B^\star = v + \frac{l_A + 2 l_B}{3 \alpha}, \quad X_A^\star = 0, \quad X_B^\star = 0,
\]
\[
\begin{align*}
\pi_A^* &= \frac{(2l_A + l_B)^2}{9\alpha}, \quad \pi_B^* = \frac{(l_A + 2l_B)^2}{9\alpha}, \quad \omega_A^* = \alpha v + \frac{2l_A + l_B}{3}, \\
\omega_B^* &= \alpha v + \frac{l_A + 2l_B}{3}, \quad S_A^* = \frac{(3\alpha - 1)l_A + l_B}{3\alpha}, \quad S_B^* = \frac{l_A + (3\alpha - 1)l_B}{3\alpha}.
\end{align*}
\]

2) If \( T > 0 \), the equilibrium will exist if and only if \((4\alpha \phi - T^2) > 0 \) and \((2\alpha \phi - T^2) l_A + (4\alpha \phi - T^2) l_B > 0 \). The unique equilibrium prices and quality levels are

\[
\begin{align*}
P_A^* &= \frac{2\phi + \epsilon T^2}{4\alpha \phi} \left\{ (4\alpha \phi - T^2) l_A + (2\alpha \phi - T^2) l_B \right\} + v, \\
P_B^* &= \frac{2\phi + \epsilon T^2}{4\alpha \phi} \left\{ (2\alpha \phi - T^2) l_A + (4\alpha \phi - T^2) l_B \right\} + v, \\
X_A^* &= \frac{T \{ (4\alpha \phi - T^2) l_A + (2\alpha \phi - T^2) l_B \}}{4\alpha \phi (3\alpha \phi - T^2)}, \\
X_B^* &= \frac{T \{ (2\alpha \phi - T^2) l_A + (4\alpha \phi - T^2) l_B \}}{4\alpha \phi (3\alpha \phi - T^2)}.
\end{align*}
\]

Further, the equilibrium perceived prices, profits and market shares are given as

\[
\begin{align*}
\omega_A^* &= \frac{(2\alpha \phi - T^2)}{4\alpha \phi} \left\{ (4\alpha \phi - T^2) l_A + (2\alpha \phi - T^2) l_B \right\} + \alpha v, \\
\omega_B^* &= \frac{(2\alpha \phi - T^2)}{4\alpha \phi} \left\{ (2\alpha \phi - T^2) l_A + (4\alpha \phi - T^2) l_B \right\} + \alpha v, \\
\pi_A^* &= \frac{(4\alpha \phi - T^2)^2 \left\{ (4\alpha \phi - T^2) l_A + (2\alpha \phi - T^2) l_B \right\}^2}{16\alpha^2 \phi (3\alpha \phi - T^2)^2}, \\
\pi_B^* &= \frac{(4\alpha \phi - T^2)^2 \left\{ (2\alpha \phi - T^2) l_A + (4\alpha \phi - T^2) l_B \right\}^2}{16\alpha^2 \phi (3\alpha \phi - T^2)^2}, \\
S_A^* &= \frac{(4\alpha \phi - T^2) l_A + (2\alpha \phi - T^2) l_B}{2 (3\alpha \phi - T^2)}, \\
S_B^* &= \frac{(2\alpha \phi - T^2) l_A + (4\alpha \phi - T^2) l_B}{2 (3\alpha \phi - T^2)}.
\end{align*}
\]

Refer to appendix A for the proof.

One can quickly recognize that when

\[(2\alpha \phi - T^2) l_A + (4\alpha \phi - T^2) l_B < 0,\]

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the price of the commodity for the smaller firm is less than the marginal cost of production and the smaller firm will be driven out of business. Also, with this condition, the quality level of the smaller firm is negative.

Rewriting this condition, we obtain \( \sqrt{\frac{2l_A + 4l_B}{l_A + l_B}} \alpha \phi < T < \sqrt{4 \alpha \phi} \), which is the case of an extremely quality sensitive market. The necessary and sufficient conditions stated above for the existence of a pure Nash equilibrium imply that if the customer’s valuation of quality relative to price \( T \) is too severe \( (T^2 > 4 \alpha \phi) \), the equilibrium will not exist. So in the quality sensitive market, we require customers valuation of quality relative to price to be relatively low in order to ensure that equilibrium exists \( (T^2 < 4 \alpha \phi) \). Thus, we assume that \( T < \sqrt{\frac{2l_A + 4l_B}{l_A + l_B}} \alpha \phi < \sqrt{3 \alpha \phi} \) for positive profits.

3.2 Subgame Two: \( P_A > (\epsilon X_A + v) + z \) and \( P_B \leq (\epsilon X_B + v) + z \)

This subgame considers the case where firm A finds it profitable to promote and B does not promote because the cost of promoting is too high. Referring to chapter II equation 2.7, we have the profit functions as

\[
\pi_{A,2} = (P_A - \epsilon X_A - v) (l_A - \hat{l}) - \phi X_A^2 \\
+ \int_{\hat{l}}^{\hat{l}_B} (P_A - \alpha P_A - \epsilon X_A + \beta X_A + \alpha P_B - \beta X_B - v - z + l) dl, \\
\pi_{B,2} = (P_B - \epsilon X_B - v) (l_B + \hat{l}_B) - \phi X_B^2.
\]

with

\[
\hat{l}_B = \alpha P_A - P_B - \beta X_A + \epsilon X_A - \alpha P_B + \beta X_B + v + z, \\
\hat{l} = \alpha P_A - \beta X_A - \alpha P_B + \beta X_B.
\]
The market share for firm A and firm B in this subgame is therefore,

\[ S_{A,2}^* = (l_A - \hat{l}) + (\hat{l} - \hat{l}_B) = l_A - (\alpha P_A - P_A - \beta X_A + \epsilon X_A - \alpha P_B + \beta X_B + v + z), \]

\[ S_{B,2}^* = l_B + \hat{l}_B = l_B + \alpha P_A - P_A - \beta X_A + \epsilon X_A - \alpha P_B + \beta X_B + v + z. \quad (3.7) \]

We differentiate the above profit functions over the price and quality level of each firm to obtain the following

\[
\frac{\partial}{\partial P_A} \pi_{A,2} = -\alpha (P_A - \epsilon X_A - v) + l_A - 2\alpha P_A + 2\alpha P_B + 2\beta X_A
\]
\[-2\beta X_B + 2P_A - 2\epsilon X_A - 2v - 2z + (\alpha P_A - \beta X_A - \alpha P_B + \beta X_B)\alpha
\]
\[-(\alpha P_A - P_A - \alpha P_B - \beta X_A + \epsilon X_A + \beta X_B + v + z)(\alpha - 1),
\]

\[
\frac{\partial}{\partial X_A} \pi_{A,2} = \beta (P_A - \epsilon X_A - v) - (l_A - \alpha P_A + \alpha P_B + \beta X_A - \beta X_B)\epsilon
\]
\[-P_A\epsilon + \alpha P_A\epsilon - \epsilon (P_A - \epsilon X_A - v - z) + \epsilon^2 X_A + \beta (P_A - \epsilon X_A - v - z)
\]
\[-\beta X_A\epsilon - \beta X_B\epsilon + \epsilon X_A + \epsilon X_B + v + z\epsilon + (\alpha P_A - \beta X_A - \alpha P_B + \beta X_B)\beta
\]
\[-(\alpha P_A - P_A - \alpha P_B - \beta X_A + \epsilon X_A + \beta X_B + v + z)(-\beta + \epsilon),
\]

\[
\frac{\partial}{\partial P_B} \pi_{B,2} = l_B + \alpha P_A - P_A - \alpha P_B - \beta X_A + \epsilon X_A + \beta X_B + v + z
\]
\[-(P_B - \epsilon X_B - v)\alpha,
\]

\[
\frac{\partial}{\partial X_B} \pi_{B,2} = -\epsilon (l_B + \alpha P_A - P_A - \alpha P_B - \beta X_A + \epsilon X_A + \beta X_B + v + z)
\]
\[+ (P_B - \epsilon X_B - v)\beta - 2\phi X_B.
\]
Setting these equations to zero, we obtain the equilibrium prices and quality levels when firm A promotes:

\[
P_A^* = \frac{(2\phi + \epsilon T)(4\alpha \phi - T^2)l_A + (2\alpha \phi - T^2)l_B - 2\phi \alpha z}{4\phi \alpha (3\alpha \phi - T^2 - \phi)} + v, \\
P_B^* = \frac{(2\phi + \epsilon T)(2\alpha \phi - T^2 - 2\phi)l_B + (4\alpha \phi - T^2 - 2\phi)l_B + 2\phi \alpha z}{4\phi \alpha (3\alpha \phi - T^2 - \phi)} + v, \\
X_A^* = \frac{T(4\alpha \phi - T^2)l_A + T(2\alpha \phi - T^2)l_B - 2T\phi \alpha z}{4\phi \alpha (3\alpha \phi - T^2 - \phi)}, \\
X_B^* = \frac{T(2\alpha \phi - T^2 - 2\phi)l_A + T(4\alpha \phi - T^2 - 2\phi)l_B + 2T\phi \alpha z}{4\phi \alpha (3\alpha \phi - T^2 - \phi)}. \\
(3.8)
\]

We substitute equation 3.8 in equation 3.7 to determine the equilibrium market share values

\[
S_{A,2}^* = \frac{(4\alpha \phi - T^2)l_A + (2\alpha \phi - T^2)l_B - 2\alpha \phi z}{2(3\alpha \phi - T^2 - \phi)}, \\
S_{B,2}^* = \frac{(2\alpha \phi - T^2 - 2\phi)l_A + (4\alpha \phi - T^2 - 2\phi)l_B + 2\alpha \phi z}{2(3\alpha \phi - T^2 - \phi)}. \\
\]

If we substitute equation 3.8 in 2.1, we get the equilibrium perceived prices,

\[
\omega_{A,2}^* = \alpha v + \frac{(2\alpha \phi - T^2)\{(4\alpha \phi - T^2)l_A + (2\alpha \phi - T^2)l_B - 2\phi \alpha z\}}{4\alpha \phi (3\alpha \phi - T^2 - \phi)}, \\
\omega_{B,2}^* = \alpha v + \frac{(2\alpha \phi - T^2 - 2\phi)\{(4\alpha \phi - T^2)l_A + (4\alpha \phi - T^2 - 2\phi)l_B + 2\phi \alpha z\}}{4\alpha \phi (3\alpha \phi - T^2 - \phi)}, \\
\]

The profits are

\[
\pi_{A,2}^* = \frac{(4\alpha \phi - T^2 - 2\phi)\{(4\alpha \phi - T^2)l_A + (2\alpha \phi - T^2)l_B\}^2}{16\phi \alpha^2 (3\alpha \phi - T^2 - \phi)^2} - \frac{z(4\alpha \phi - T^2 - 2\phi)\{(4\alpha \phi - T^2)l_A + (2\alpha \phi - T^2)l_B\}}{4\alpha (3\alpha \phi - T^2 - \phi)^2} \\
\]

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\[
\frac{z^2 \left\{ \phi \left(3T^2 - 8\alpha \phi\right) + 2 \left(3\alpha \phi - T^2\right)^2 \right\}}{4 \left(3\alpha \phi - T^2 - \phi\right)^2},
\]

\[
\pi_{B,2}^* = \frac{\left\{ (2\alpha \phi - T^2 - 2\phi) l_A + (4\alpha \phi - T^2 - 2\phi) l_B \right\}^2}{\alpha \left(3\alpha \phi - T^2 - \phi\right)^2}
\]

\[
+ \frac{\phi z \left\{ (2\alpha \phi - 2\phi - T^2) l_A + (4\alpha \phi - T^2 - 2\phi) l_B + \alpha \phi z \right\}}{(3\alpha \phi - T^2 - \phi)^2},
\]

after we use equation 3.8 in equation 2.7.

To account for the relationship between consumer’s price valuation and quality valuation, we find that in equilibrium, the following result is true.

**Theorem 2.**

1) If \( T \leq 0 \), the unique equilibrium prices and quality levels are

\[
P_A^* = v + \frac{2 l_A + l_B - z}{3\alpha - 1}, \quad P_B^* = v + \frac{l_A(\alpha - 1) + l_B(2\alpha - 1) + \alpha z}{\alpha (3\alpha - 1)}, \quad X_A^* = 0, \quad X_B^* = 0.
\]

The equilibrium profits, perceived prices and market shares are given as

\[
\pi_{A,2}^* = \frac{(4\alpha - 2) \left\{ (4\alpha l_A + 2\alpha l_B)^2 - 4\alpha z (4\alpha l_A + 2\alpha l_B) \right\} + 4\alpha^2 z^2 (-8\alpha + 18\alpha^2)}{16\alpha^2 (3\alpha - 1)^2},
\]

\[
\pi_{B,2}^* = \frac{l_A(\alpha - 1) + l_B(2\alpha - 1))^2 + 2\alpha z \left\{ l_A(\alpha - 1) + l_B(2\alpha - 1) \right\} + z^2 \alpha^2}{\alpha (3\alpha - 1)^2},
\]

\[
\omega_{A,2}^* = v\alpha + \frac{2\alpha l_A + \alpha l_B - \alpha z}{3\alpha - 1},
\]

\[
\omega_{B,2}^* = v\alpha + \frac{l_A(\alpha - 1) + l_B(2\alpha - 1) + \alpha z}{3\alpha - 1},
\]

\[
S_{A,2}^* = \frac{\alpha (2 l_A + l_B - z)}{3\alpha - 1},
\]

\[
S_{B,2}^* = \frac{l_A(\alpha - 1) + l_B(2\alpha - 1) + \alpha z}{3\alpha - 1}.
\]
2) If $T > 0$, the equilibrium will exist if and only if $4 \alpha \phi - T^2 - 2 \phi > 0$ and
\[
\left\{ (2 \alpha \phi - T^2 - 2 \phi) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B \right\} + 2 \alpha \phi z > 0.
\]

Here, the unique equilibrium prices and quality levels are
\[
P_A^* = \frac{(2 \phi + \epsilon T) \left\{ (4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B - 2 \phi \alpha z \right\}}{4 \phi \alpha (3 \alpha \phi - T^2 - \phi)} + v,
\]
\[
P_B^* = \frac{(2 \phi + \epsilon T) \left\{ (2 \alpha \phi - T^2 - 2 \phi) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B + 2 \phi \alpha z \right\}}{4 \phi \alpha (3 \alpha \phi - T^2 - \phi)} + v,
\]
\[
X_A^* = \frac{T \left\{ (4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B - 2 \phi \alpha z \right\}}{4 \phi \alpha (3 \alpha \phi - T^2 - \phi)},
\]
\[
X_B^* = \frac{T \left\{ (2 \alpha \phi - T^2 - 2 \phi) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B + 2 \phi \alpha z \right\}}{4 \alpha \phi (3 \alpha \phi - T^2 - \phi)}.
\]

The equilibrium profits, perceived prices and market shares are
\[
\pi_{A,2}^* = \frac{(4 \alpha \phi - T^2 - 2 \phi) \left\{ (4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B \right\}^2}{16 \phi \alpha^2 (3 \alpha \phi - T^2 - \phi)^2}
\]
\[
- z (4 \alpha \phi - T^2 - 2 \phi) \left\{ (4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B \right\}
\]
\[
+ \frac{z^2 \left\{ \phi (3 T^2 - 8 \alpha \phi) + 2 (3 \alpha \phi - T^2)^2 \right\}}{4 (3 \alpha \phi - T^2 - \phi)^2},
\]
\[
\pi_{B,2}^* = \frac{\left\{ (2 \alpha \phi - T^2 - 2 \phi) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B \right\}^2}{\alpha (3 \alpha \phi - T^2 - \phi)^2}
\]
\[
+ \frac{\phi z \left\{ (2 \alpha \phi - 2 \phi - T^2) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B + \alpha \phi z \right\}}{(3 \alpha \phi - T^2 - \phi)^2}.
\]
\[
\omega_{A,2}^* = \alpha v + \frac{(2 \alpha \phi - T^2) \{4 \alpha \phi - T^2\} l_A + (2 \alpha \phi - T^2) l_B - 2 \phi \alpha z}{4 \alpha \phi (3 \alpha \phi - T^2 - \phi)},
\]
\[
\omega_{B,2}^* = \alpha v + \frac{(2 \alpha \phi - T^2) \{(2 \alpha \phi - T^2 - 2 \phi) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B + 2 \phi \alpha z\}}{4 \alpha \phi (3 \alpha \phi - T^2 - \phi)},
\]
\[
S_{A,2}^* = \frac{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B - 2 \phi \alpha z}{2 (3 \alpha \phi - T^2 - \phi)},
\]
\[
S_{B,2}^* = \frac{(2 \alpha \phi - T^2 - 2 \phi) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B + 2 \phi \alpha z}{2 (3 \alpha \phi - T^2 - \phi)}.
\]

Please refer to appendix A for the proof.

When
\[
\{(2 \alpha \phi - T^2 - 2 \phi) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B\} + 2 \alpha \phi z < 0, \quad (3.9)
\]
the smaller firm sells the product at a price which is below the cost incurred in producing an extra product, that is, \(v\). But one of our assumptions for the analysis is that the reservation price from the seller’s perspective is the minimum price the seller is willing to offer for the commodity and this is greater than or equal to the marginal cost (\(R \geq v\)). If this is violated and inequality 3.9 holds true, the smaller firm must be driven out of business. Also, the quality level in this case is negative, which makes no economic sense. Therefore we impose the condition that
\[
\{(2 \alpha \phi - T^2 - 2 \phi) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B\} + 2 \alpha \phi z > 0.
\]
Rewriting this, we have
\[
T < \sqrt{2} \sqrt{\frac{\phi (az + l_A \alpha - l_A + 2 l_B \alpha - l_B)}{l_A + l_B}},
\]
which is the condition for positive profits and positive quality levels.
We recognize that the equilibrium values depend on the cost of targeting. The price and quality levels as well as the equilibrium profits and market shares will increase or decrease depending on what is happening to the cost of targeting, \( z \).

Consider the bound on the targeting cost when no firm promotes. We have

\[
 z \geq P_A - \epsilon X_A - v, \tag{3.10}
\]

and

\[
 z \geq P_B - \epsilon X_B - v. \tag{3.11}
\]

Substituting the equilibrium price and quality levels from equation 3.4 in equations 3.10 and 3.11, we obtain

\[
 z \geq P_A - \epsilon X_A - v \\
 \geq \frac{(2 \phi + \epsilon T) \{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B\}}{4 \alpha \phi (3 \alpha \phi - T^2)} \\
 - \frac{\epsilon T \{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B\}}{4 \alpha \phi (3 \alpha \phi - T^2)} \\
 \geq \frac{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B}{2 \alpha (3 \alpha \phi - T^2)}, \tag{3.12}
\]

and

\[
 z \geq P_B - \epsilon X_B - v \\
 \geq \frac{(2 \phi + \epsilon T) \{(2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B\}}{4 \alpha \phi (3 \alpha \phi - T^2)} \\
 - \frac{\epsilon T \{(2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B\}}{4 \alpha \phi (3 \alpha \phi - T^2)} \\
 \geq \frac{(2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B}{2 \alpha (3 \alpha \phi - T^2)}. \tag{3.13}
\]
We can rewrite inequalities 3.12 and 3.13 respectively as

\[
z \geq \frac{(2\alpha \phi - T^2)(l_A + l_B) + 2\alpha \phi l_A}{2\alpha (3\alpha \phi - T^2)},
\]  
(3.14)

\[
z \geq \frac{(2\alpha \phi - T^2)(l_A + l_B) + 2\alpha \phi l_B}{2\alpha (3\alpha \phi - T^2)}.
\]
(3.15)

Note that if \((2\alpha \phi - T^2) > 0\) and \(l_A > l_B\), then inequality 3.14 implies inequality 3.15 holds true. Suppose \((2\alpha \phi - T^2) < 0\). Then \(T^2 = 2\alpha \phi + y, y > 0\).

Then inequality 3.14 is \(z \geq \frac{2\alpha \phi l_A - y}{2\alpha \phi - y}\) and inequality 3.15 is \(z \geq \frac{2\alpha \phi l_B - y}{2\alpha \phi - y}\). In either case, inequality 3.12 ensures that both 3.12 and 3.13 are satisfied. Let us now consider the bound on the targeting cost when only firm A promotes. We have

\[
z < P_A - \epsilon X_A - v
\]
(3.16)

and

\[
z \geq P_B - \epsilon X_B - v
\]
(3.17)

Substituting the equilibrium price and quality levels in equation 3.8 into inequalities 3.16 and 3.17, we have

\[
z < \frac{(2\phi + \epsilon T)((4\alpha \phi - T^2)l_A + (2\alpha \phi - T^2)l_B - 2\phi \alpha z)}{4\phi \alpha (3\alpha \phi - T^2 - \phi)} - \frac{\epsilon T((4\alpha \phi - T^2)l_A + (2\alpha \phi - T^2)l_B - 2\phi \alpha z)}{4\phi \alpha (3\alpha \phi - T^2 - \phi)}
\]

\[
< \frac{(4\alpha \phi - T^2)l_A + (2\alpha \phi - T^2)l_B - 2\phi \alpha z}{2\alpha (3\alpha \phi - T^2 - \phi)}
\]

\[
< \frac{(4\alpha \phi - T^2)l_A + (2\alpha \phi - T^2)l_B}{2\alpha (3\alpha \phi - T^2)}
\]

\[
< \frac{(2\alpha \phi - T^2)(l_A + l_B) + 2\alpha \phi l_A}{2\alpha (3\alpha \phi - T^2)}
\]
(3.18)

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and
\[
\begin{align*}
z & \geq \frac{(2 \phi + \epsilon T) ((2 \alpha \phi - T^2 - 2 \phi) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B + 2 \phi \alpha z)}{4 \alpha \phi (3 \alpha \phi - T^2 - \phi)} - \epsilon T \left( \frac{(2 \alpha \phi - T^2 - 2 \phi) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B + 2 \phi \alpha z}{4 \alpha \phi (3 \alpha \phi - T^2 - \phi)} \right) \\
& \geq \frac{(2 \alpha \phi - T^2 - 2 \phi) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B + 2 \phi \alpha z}{2 \alpha (3 \alpha \phi - T^2 - \phi)} \\
& \geq \frac{(2 \alpha \phi - T^2 - 2 \phi) (l_A + l_B) + 2 \alpha l_B}{2 \alpha (3 \alpha \phi - T^2 - 2 \phi)}.
\end{align*}
\] (3.19)

Comparing the two, inequality 3.19 is the minimum. So the intersection of the two yields
\[
\begin{align*}
\frac{(2 \alpha \phi - T^2 - 2 \phi) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B}{2 \alpha (3 \alpha \phi - T^2 - \phi)} & \leq z < \frac{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B}{2 \alpha (3 \alpha \phi - T^2)}.
\end{align*}
\] (3.20)

From the calculations above, we have this characterization for the marginal cost of targeting.

**Theorem 3.**

For all \( z \geq 0 \), there exists a unique subgame perfect equilibrium such that

(a) if \( z \geq z^* = \frac{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B}{2 \alpha (3 \alpha \phi - T^2)} \), then the equilibrium regular prices and quality levels are respectively

\[
\begin{align*}
P_A^* & = \frac{(2 \phi + \epsilon T^2) ((4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B)}{4 \alpha \phi (3 \alpha \phi - T^2)} + v, \\
P_B^* & = \frac{(2 \phi + \epsilon T^2) ((2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B)}{4 \alpha \phi (3 \alpha \phi - T^2)} + v, \\
X_A^* & = \frac{T ((4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B)}{4 \alpha \phi (3 \alpha \phi - T^2)}, \\
X_B^* & = \frac{T ((2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B)}{4 \alpha \phi (3 \alpha \phi - T^2)}.
\end{align*}
\]
In such case neither firm would target individuals with promotional incentives.

(b) If \( z^{**} \leq z < z^* \) where \( z^{**} = \frac{(2\alpha \phi - T^2 - 2\phi)l_A + (4\alpha \phi - T^2 - 2\phi)l_B}{2\alpha (3\alpha \phi - T^2 - 2\phi)} \), then the equilibrium regular prices and quality levels are

\[
P_A^* = \frac{(2\phi + \epsilon T) \{ (4\alpha \phi - T^2) l_A + (2\alpha \phi - T^2) l_B - 2\phi \alpha z \}}{4\phi \alpha (3\alpha \phi - T^2 - \phi)} + v,
\]

\[
P_B^* = \frac{(2\phi + \epsilon T) \{ (2\alpha \phi - T^2 - 2\phi) l_A + (4\alpha \phi - T^2 - 2\phi) l_B + 2\phi \alpha z \}}{4\phi \alpha (3\alpha \phi - T^2 - \phi)} + v,
\]

\[
X_A^* = \frac{T (4\alpha \phi - T^2) l_A + T (2\alpha \phi - T^2) l_B - 2 T \phi \alpha z}{4\phi \alpha (3\alpha \phi - T^2 - \phi)},
\]

\[
X_B^* = \frac{T (2\alpha \phi - T^2 - 2\phi) l_A + T (4\alpha \phi - T^2 - 2\phi) l_B + 2 T \phi \alpha z}{4\phi \alpha (3\alpha \phi - T^2 - \phi)},
\]

and only firm A would target individuals with promotional incentives.

We notice from the theorem that for \( z > z^* \), no firm finds individual targeting profitable. In the next chapter, we present our findings and results of the analysis done so far. We answer the questions posed in chapter I.
In this chapter, we seek to answer the questions posed in the introduction and even more. Some of the questions we seek to answer are, first how does individual targeting affect the prices paid by customers and the quality level set by the firm? Second, as competition increases, what would be the effect of individual targeting on a firm’s price, profits, quality levels and market shares? Which firm has more incentive to initiate individual targeting? The equilibrium is characterized by \( T \equiv \beta - \alpha \epsilon \), that is consumers valuation of quality level relative to price and is normalized by the variable cost of the quality level [29]. If \( T = 0 \), the consumers valuation of price and quality is at par. That is consumers are sensitive to both price and quality. When \( T < 0 \), consumers are more sensitive to the price of the commodity. When \( T > 0 \), consumers are more quality sensitive.

4.1 Subgame one: No firm offers individual promotions

This is the subgame Matsubayashi and Yamada [29] consider. We replicate the result of Matsubayshi and Yamada and add the effect of price and quality competition on the firms market shares. We compare their results to our result in the scenario where the larger firm is promoting.
We also add the effect of intense competition on the industry quality. Recalling theorem 1 in chapter III,

1) If \( T \leq 0 \), the unique equilibrium is given as:

\[
P_A^* = v + \frac{2l_A + l_B}{3\alpha} \quad \quad P_B^* = v + \frac{l_A + 2l_B}{3\alpha} \quad \quad X_A^* = 0 \quad \quad X_B^* = 0,
\]

\[
\pi_A^* = \frac{(2l_A + l_B)^2}{9\alpha} \quad \quad \pi_B^* = \frac{(l_A + 2l_B)^2}{9\alpha} \quad \quad \omega_A^* = \alpha v + \frac{2l_A + l_B}{3} \quad \quad \omega_B^* = \alpha v + \frac{l_A + 2l_B}{3}
\]

\[
\omega_A^* = \alpha v + \frac{l_A + 2l_B}{3} \quad \quad \omega_B^* = \alpha v + \frac{l_A + 2l_B}{3} \quad \quad S_A^* = \frac{(3\alpha - 1)l_A + l_B}{3\alpha} \quad \quad S_B^* = \frac{l_A + (3\alpha - 1)l_B}{3\alpha}.
\]

2) If \( T > 0 \), the equilibrium will exist if and only if \( (4\alpha \phi - T^2) > 0 \) and when \( (2\alpha \phi - T^2)l_A + (4\alpha \phi - T^2)l_B > 0 \), then the unique equilibrium prices and quality levels are

\[
P_A^* = \frac{(2\phi + \epsilon T^2) \{(4\alpha \phi - T^2)l_A + (2\alpha \phi - T^2)l_B\}}{4\alpha \phi (3\alpha \phi - T^2)} + v, \quad P_B^* = \frac{(2\phi + \epsilon T^2) \{(2\alpha \phi - T^2)l_A + (4\alpha \phi - T^2)l_B\}}{4\alpha \phi (3\alpha \phi - T^2)} + v,
\]

\[
X_A^* = \frac{T \{(4\alpha \phi - T^2)l_A + (2\alpha \phi - T^2)l_B\}}{4\alpha \phi (3\alpha \phi - T^2)}, \quad X_B^* = \frac{T \{(2\alpha \phi - T^2)l_A + (4\alpha \phi - T^2)l_B\}}{4\alpha \phi (3\alpha \phi - T^2)}.
\]

The equilibrium perceived prices, profits and market shares are

\[
\omega_A^* = \frac{(2\alpha \phi - T^2) \{(4\alpha \phi - T^2)l_A + (2\alpha \phi - T^2)l_B\}}{4\alpha \phi (3\alpha \phi - T^2)} + \alpha v, \quad \omega_B^* = \frac{(2\alpha \phi - T^2) \{(2\alpha \phi - T^2)l_A + (4\alpha \phi - T^2)l_B\}}{4\alpha \phi (3\alpha \phi - T^2)} + \alpha v,
\]

\[
\pi_A^* = \frac{(4\alpha \phi - T^2) \{(4\alpha \phi - T^2)l_A + (2\alpha \phi - T^2)l_B\}}{16\alpha^2 \phi (3\alpha \phi - T^2)^2}, \quad \pi_B^* = \frac{(4\alpha \phi - T^2) \{(2\alpha \phi - T^2)l_A + (4\alpha \phi - T^2)l_B\}}{16\alpha^2 \phi (3\alpha \phi - T^2)^2}.
\]

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\[ S^*_A = \frac{(4\alpha \phi - T^2)l_A + (2\alpha \phi - T^2)l_B}{2(3\alpha \phi - T^2)} \]
\[ S^*_B = \frac{(2\alpha \phi - T^2)l_A + (4\alpha \phi - T^2)l_B}{2(3\alpha \phi - T^2)}. \]

We obtain figures 4.1(a) - 4.1(e) by setting \( \alpha = 1 \) (only in the quality sensitive region), \( v = 1, \epsilon = 1, \phi = 1, l_A = 0.6 \) and \( l_B = 0.4 \) in theorem 1. In all the figures, \( T^* = \sqrt{\frac{2l_A + 4l_B}{l_A + l_B}} \alpha \phi. \)

To understand the graphs and the analysis done, when we say consumers valuation of price increases, we mean \( \alpha \) increases from 1 as one moves farther away to the left of \( T = 0 \). Put differently, \( \alpha \) decreases means that consumers valuation of price approaches \( T = 0 \) from the left. \( T = 0 \) is the situation where consumers value price equally as quality and this corresponds to \( \alpha = 1, \beta = 1 \) and \( \epsilon = 1 \). When we move away from \( T = 0 \) to the right, consumers valuation of quality intensifies, that is \( \beta \) increases with \( \alpha \) and \( \epsilon \) fixed. In figure 4.1(a), the quality level for both firms is the baseline quality in the price sensitive region. The quality level of the larger firm increases throughout in the quality sensitive region. This may be because of the firms ability to undertake projects that increase the quality level in order to meet the expectations of highly quality sensitive customers. The smaller firm experiences increasing quality initially but as consumers become more sensitive to quality, it is not able to meet expectations and therefore adopts a low quality strategy. As we move away to the left from \( T = 0 \) in figure 4.1(b), consumers become more sensitive to price and therefore both firms reduce their price. High sensitivity to price leads to intense price competition. With quality increasing in the moderately quality sensitive region,
the prices of both firms increase as it costs to improve quality. However, the smaller firm has to decrease price for consumers who are highly sensitive to quality because it adopts a low quality strategy. The larger firm on the other hand, continues to raise its price to offset the cost associated with improving quality. From figure 4.1(c), we expect the perceived price of both firms to increase with increasing sensitivity to price. From the definition of perceived price in equation 2.1, we find the perceived price increasing in the price sensitive region as $\alpha$, consumers valuation of price increases, and with $X_i = 0$.

In the quality sensitive region, as quality comes to play, the perceived price of firm A decreases because quality increases always. The smaller firm’s perceived price decreases with increasing quality and thereafter begins to rise. From figure 4.1(d), the larger firm has a greater market share than the smaller firm as consumers become price and quality sensitive. As consumers sensitivity to price and quality decreases, the market share of the smaller firm increases. This is seen as we approach $T = 0$ from the left and right respectively. With price increasing as we approach $T = 0$ from the left, the profits of both firms increase. Refer to figure 4.1(e). In the moderately quality sensitive region, both firms profit decreases. We see both firms quality and price increasing in this region, depicting not much differences in the approaches used by both firms. However, as the larger firm distinguishes itself by producing high quality goods, the profit of the larger firm increases with more highly quality sensitive customers buying the product of the larger firm. The smaller firm’s profit decreases due to decreasing quality, price and market shares. This leads
Matsubayashi and Yamada [29] to conclude that larger firms should focus their sales activities to customers who are highly quality sensitive. Smaller firms on the other hand, should focus on building customer loyalty.

One of our interests is to know the effect of customer loyalty on the average industry quality. In answering this, we let $X_Q = \tilde{S}_A X_A + \tilde{S}_B X_B$ denote the weighted average equilibrium industry level when no firm promotes with $\tilde{S}_i = \frac{S_i}{S_A + S_B}$. Then,

$$X_Q = \frac{T \{ (l_A + l_B) T^4 - 6 \alpha \phi (l_A + l_B)^2 T^2 + 2 \phi^2 \alpha^2 (5 l_A^2 + 8 l_A l_B + 5 l_B^2) \}}{4 \alpha \phi (3 \alpha \phi - T^2)^2 (l_A + l_B)^2}. \quad (4.1)$$

First of all, we differentiate equation 4.1 to determine the effect of a slight change in the loyalty level of the larger firm on the average industry quality level,

$$\frac{\partial}{\partial l_A} X_Q = \frac{T \{ (l_A + l_B) T^4 - 6 \alpha \phi (l_A + l_B)^2 T^2 + 2 \phi^2 \alpha^2 (3 l_B^2 + 5 l_A^2 + 10 l_A l_B) \}}{4 \alpha \phi (3 \alpha \phi - T^2)^2 (l_A + l_B)^2}. \quad (4.2)$$

But in the quality sensitive region, $T > 0$ so $\frac{\partial}{\partial l_A} X_Q = 0$ if

$$\{ (l_A + l_B) T^4 - 6 \alpha \phi (l_A + l_B)^2 T^2 + 2 \phi^2 \alpha^2 (3 l_B^2 + 5 l_A^2 + 10 l_A l_B) \} = 0. \quad (4.2)$$

We replace $T^2$ with $y$ in equation 4.2 and obtain a new function

$$g(y) = (l_A + l_B)^2 y^2 - 6 \alpha \phi (l_A + l_B)^2 y + 2 \phi^2 \alpha^2 (10 l_A l_B + 5 l_A^2 + 3 l_B^2) = 0.$$

The second derivative is positive and the critical point is $y^* = 3 \alpha \phi$, which is positive. Therefore, $g(y^*) = \phi^2 \alpha^2 (l_A + 3 l_B) (l_A - l_B)$ is positive. Hence, $g(y)$ is always positive. Therefore, $\frac{\partial}{\partial l_A} X_Q > 0$.

We now differentiate equation 4.1 with respect to $l_B$ to determine the effect of a slight change in the loyalty level of firm B on the average industry quality level.
This yields
\[
\frac{\partial}{\partial l_B} X_Q = \frac{T \left\{ (l_A + l_B)^2 T^4 - 6 \alpha \phi (l_A + l_B)^2 T^2 + 2 \phi^2 \alpha^2 (10 l_A l_B + 3 l_A^2 + 5 l_B^2) \right\}}{4 (3 \alpha \phi - T^2)^2 \alpha \phi (l_A + l_B)^2}.
\] (4.3)

Similarly, \( \frac{\partial}{\partial l_B} X_Q = 0 \) if
\[
(l_A + l_B)^2 T^4 - 6 \alpha \phi (l_A + l_B)^2 T^2 + 2 \phi^2 \alpha^2 (10 l_A l_B + 3 l_A^2 + 5 l_B^2) = 0. \] (4.4)

Therefore, let
\[
g(y) = (l_A + l_B)^2 y^2 - 6 \alpha \phi (l_A + l_B)^2 y + 2 \phi^2 \alpha^2 (10 l_A l_B + 3 l_A^2 + 5 l_B^2) = 0,
\]
where we have replaced \( T^2 \) with \( y \). The critical point for \( g(y) \) is \( y^* = 3 \alpha \phi \) and \( g(y^*) = -\alpha^2 \phi^2 (3 l_A + l_B) (l_A - l_B) \). From the sign of the second derivative, \( g(y^*) \) is a minimum and since \( g(y^*) < 0 \), \( g(y) \) increases and decreases on the given interval.

Hence, as consumers become highly sensitive to quality, intense competition reduces the average quality level. In this region, customers care about the quality of the product. As the loyalty base of firm A increases, the firm finds it profitable to improve quality to meet the expectations of these customers. On average, the industry quality improves. However, when competition intensifies, the smaller firm improves on the quality but the larger firm may reduce the level of quality due to the cost of maintaining and improving quality. This causes the average quality level to reduce.

**Theorem 4.**

For any \( T \) such that \( 0 < T < \sqrt{\frac{2l_A+4l_B}{l_A+l_B}} \alpha \phi \), and \( l_A > l_B \), the average industry quality level increases as the loyalty base of the larger firm increases. But there is a
quality level beyond which the average industry quality level decreases with intense competition.

4.1.1 Recovering Shaffer and Zhang [3]

As stated in chapter 1, our model follows Shaffer and Zhang [3] with the addition of quality. If we let $\beta$, which is the measure of consumer’s responsiveness to quality, be 1 and $\alpha$, the measure of price sensitivity be 1, and the fact that the equilibrium quality level in the price sensitive region is zero, we get $w_i$, which is the price in Shaffer and Zhang’s paper. We also let $\epsilon$, the cost parameter, be 1. For instance, consider the equilibrium profit for firm A and B in the quality sensitive region

\[
\pi_A^* = \frac{(4 \alpha \phi - T^2) \{ (4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B \}^2}{16 \alpha^2 \phi (3 \alpha \phi - T^2)^2}, \tag{4.5}
\]

\[
\pi_B^* = \frac{(4 \alpha \phi - T^2) \{ (2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B \}^2}{16 \alpha^2 \phi (3 \alpha \phi - T^2)^2}. \tag{4.6}
\]

Let $\alpha = 1$, $\beta = 1$, $\epsilon = 1$ and $T = (\beta - \alpha \epsilon)$. Substitute these in equation 4.5 and 4.6 and we find

\[
\pi_A^* = \frac{(4 \phi l_A + 2 \phi l_B)^2}{36 \phi^2} = \frac{(2 l_A + l_B)^2}{9}, \tag{4.7}
\]

and

\[
\pi_B^* = \frac{(2 \phi l_A + 4 \phi l_B)^2}{36 \phi^2} = \frac{(l_A + 2 l_B)^2}{9}, \tag{4.8}
\]

which are the profit functions under subgame one in Shaffer and Zhang’s paper [3].
4.2 Subgame Two: Only Firm A Promotes

Referring to theorem 2 in chapter 3,

1) If $T \leq 0$, the unique equilibrium prices and quality levels are

$$P_A^* = v + \frac{2l_A + l_B - z}{3\alpha - 1}, \quad P_B^* = v + \frac{l_A(\alpha - 1) + l_B(2\alpha - 1) + \alpha z}{\alpha (3\alpha - 1)}, \quad X_A^* = 0, \quad X_B^* = 0.$$

The equilibrium profits, perceived prices and market shares are

$$\pi_{A,2}^* = \frac{(4\alpha - 2)\left((4\alpha l_A + 2\alpha l_B)^2 - 4\alpha z(4\alpha l_A + 2\alpha l_B)\right) + 4\alpha^2 z^2(-8\alpha + 18\alpha^2)}{16\alpha^2(3\alpha - 1)^2},$$

$$\pi_{B,2}^* = \frac{\left(l_A(\alpha - 1) + l_B(2\alpha - 1)\right)^2 + 2\alpha z\left(l_A(\alpha - 1) + l_B(2\alpha - 1)\right) + z^2\alpha^2}{\alpha (3\alpha - 1)^2},$$

$$\omega_{A,2}^* = v\alpha + \frac{2\alpha l_A + \alpha l_B - \alpha z}{3\alpha - 1},$$

$$\omega_{B,2}^* = v\alpha + \frac{l_A(\alpha - 1) + l_B(2\alpha - 1) + \alpha z}{3\alpha - 1},$$

$$S_{A,2}^* = \frac{\alpha (2l_A + l_B - z)}{3\alpha - 1},$$

$$S_{B,2}^* = \frac{l_A(\alpha - 1) + l_B(2\alpha - 1) + \alpha z}{3\alpha - 1}.$$

2) If $T > 0$, the equilibrium will exist if and only if $4\alpha \phi - T^2 - 2\phi > 0$ and if

$$\{2\alpha \phi - T^2 - 2\phi\} l_A + (4\alpha \phi - T^2 - 2\phi) l_B\} + 2\alpha \phi z > 0.$$

The unique equilibrium prices and quality levels are given as

$$P_A^* = \frac{\left(2\phi + \epsilon T\right)\{4\alpha \phi - T^2\} l_A + (2\alpha \phi - T^2) l_B - 2\phi \alpha z\}}{4\phi \alpha (3\alpha \phi - T^2 - \phi)} + v,$$

$$P_B^* = \frac{\left(2\phi + \epsilon T\right)\{(2\alpha \phi - T^2 - 2\phi) l_A + (4\alpha \phi - T^2 - 2\phi) l_B + 2\phi \alpha z\}}{4\phi \alpha (3\alpha \phi - T^2 - \phi)} + v,$$

$$X_A^* = \frac{\left(4\alpha \phi - T^2\right) l_A + (2\alpha \phi - T^2) l_B - 2\phi \alpha z\}}{4\phi \phi (3\alpha \phi - T^2 - \phi)},$$

$$X_B^* = \frac{T\{(2\alpha \phi - T^2 - 2\phi) l_A + (4\alpha \phi - T^2 - 2\phi) l_B + 2\phi \alpha z\}}{4\phi \phi (3\alpha \phi - T^2 - \phi)}.$$
And the equilibrium profits, perceived prices and market shares are

\[
\pi_{A,2}^* = \frac{(4 \alpha \phi - T^2 - 2 \phi) \{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B\}^2}{16 \phi \alpha^2 (3 \alpha \phi - T^2 - \phi)^2}
\]

\[- \frac{z (4 \alpha \phi - T^2 - 2 \phi) \{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B\}}{4 \alpha (3 \alpha \phi - T^2 - \phi)^2}
\]

\[+ \frac{z^2 \left\{ \phi (3 T^2 - 8 \alpha \phi) + 2 (3 \alpha \phi - T^2)^2 \right\}}{4 (3 \alpha \phi - T^2 - \phi)^2},\]

\[
\pi_{B,2}^* = \frac{\{(2 \alpha \phi - T^2 - 2 \phi) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B\}^2}{\alpha (3 \alpha \phi - T^2 - \phi)^2}
\]

\[+ \frac{\phi z \{(2 \alpha \phi - 2 \phi - T^2) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B + \alpha \phi z\}}{(3 \alpha \phi - T^2 - \phi)^2},\]

\[
\omega_{A,2}^* = \alpha v + \frac{(2 \alpha \phi - T^2) \{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B - 2 \phi \alpha z\}}{4 \alpha \phi (3 \alpha \phi - T^2 - \phi)}
\]

\[
\omega_{B,2}^* = \alpha v + \frac{(2 \alpha \phi - T^2) \{(2 \alpha \phi - T^2 - 2 \phi) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B + 2 \phi \alpha z\}}{4 \alpha \phi (3 \alpha \phi - T^2 - \phi)}
\]

\[
S_{A,2}^* = \frac{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B - 2 \phi \alpha z}{2 (3 \alpha \phi - T^2 - \phi)}
\]

\[
S_{B,2}^* = \frac{(2 \alpha \phi - T^2 - 2 \phi) l_A + (4 \alpha \phi - T^2 - 2 \phi) l_B + 2 \phi \alpha z}{2 (3 \alpha \phi - T^2 - \phi)}.
\]
4.2.1 Choosing the targeting cost

In our analysis, we generate graphs for the cases where $l_A = 0.6, l_B = 0.4$ and $l_A = 0.9, l_B = 0.1$. We have to ensure that the targeting cost we choose is true for both the price sensitive region and quality sensitive region. Suppose that $l_A = 0.6, l_B = 0.4$.

Then, the targeting cost in the price sensitive region must satisfy

$$\frac{2.8 \alpha - 1.0 (1 - \alpha)^2 - 2.0}{2\alpha (3\alpha - (1 - \alpha)^2 - 2)} \leq z < \frac{3.2 \alpha - 1.0 (1 - \alpha)^2}{2\alpha (3\alpha - (1 - \alpha)^2)}$$

(4.9)

and

$$\frac{-1.0 \beta - 1)^2 + 0.8}{2 (1 - (\beta - 1)^2)} \leq z < \frac{3.2 - 1.0 (\beta - 1)^2}{2 (3 - (\beta - 1)^2)}$$

(4.10)

in the quality sensitive region. If we substitute $T = (\beta - \alpha \epsilon), \beta = 1, \epsilon = 1, l_A = 0.6$ and $l_B = 0.4$ into the bound on $z$ when firm A promotes which is given by inequality 3.20 in chapter III, we obtain 4.9. Similarly, substituting $T = (\beta - \alpha \epsilon), \alpha = 1, \epsilon = 1, l_A = 0.6$ and $l_B = 0.4$ into 3.20, we obtain condition 4.10. Suppose that $l_A = 0.9$ and $l_B = 0.1$. Then substituting $T = (\beta - \alpha \epsilon), \beta = 1, \epsilon = 1$ and $\alpha = 1$ into inequality 3.20, we obtain the targeting cost in the price and quality sensitive regions

$$\frac{2.2 \alpha - 1.0 (1 - \alpha)^2 - 2.0}{2\alpha (3\alpha - (1 - \alpha)^2 - 2)} \leq z < \frac{3.8 \alpha - 1.0 (1 - \alpha)^2}{2\alpha (3\alpha - (1 - \alpha)^2)},$$

(4.11)

$$\frac{-1.0 (\beta - 1)^2 + 0.2}{2 (1 - (\beta - 1)^2)} \leq z < \frac{3.8 - 1.0 (\beta - 1)^2}{2 (3 - (\beta - 1)^2)},$$

(4.12)

respectively. We plot the targeting cost against $T$, customers valuation of price and quality, to determine the valid area for our analysis. Figure 4.2(a) is a plot of the targeting cost conditions in 4.9 and 4.10 against $T$. We also have plotted the line $z = 0.4$ and $z = 0.52$. The line $z = 0.4$ is an example of a targeting cost approaching
$z^{**}$, the minimum bound on $z$. The targeting cost $z = 0.52$ is an example of a targeting cost that approaches $z^*$, the maximum bound on $z$ when firm A promotes.

From the graph, if we suppose $z = 0.4$, then the feasible area for our analysis is the area where $z^{**} \leq 0.4 < z^*$. Otherwise, the condition on the targeting cost, which is $z^{**} \leq z < z^*$, for firm A promoting is violated. If the targeting cost increases, say $z = 0.52$, the feasible region for our analysis again is the area where $z^{**} \leq 0.52 < z^*$. We notice from the intersection of line $z = 0.52$ and $z^*$ that only a few customers in the price sensitive region will be targeted with individual promotions. In figure 4.2(b), we have plotted the targeting cost conditions in 4.11 and 4.12 against $T$. The line $z = 0.18$ is an example of a targeting cost which approaches the minimum targeting cost, $z^{**}$. Line $z = 0.62$ approaches the maximum targeting cost $z^*$ and $z = 0.18$ is in the targeting cost range. From the graph, if the promoting firm has a greater market share, $l_A \gg l_B$ and $z \rightarrow z^{**}$ (in this example, $z = 0.18$), the firm promotes to all individuals in the targeting zone. This is not the case when $l_A > l_B$ as seen in figure 4.2(a). However, as the targeting cost increases, only a few people in the price sensitive region are targeted with individual promotions. This is shown by the intersections of lines $z = 0.3$, $z = 0.62$ and $z = z^*$.

As part of our interests, we determine the relationship between the targeting cost and the equilibrium profits, market shares, prices and quality.
Theorem 5.

For any $T < \sqrt{2} \sqrt{\frac{\phi (\alpha z + l_A \alpha - l_A + 2 l_B \alpha - l_B)}{l_A + l_B}}$ and $z^{**} \leq z < z^*$,

$$\frac{\partial}{\partial z} \pi_{B,2}^* > 0, \quad \frac{\partial}{\partial z} S_{A,2}^* < 0, \quad \frac{\partial}{\partial z} S_{B,2}^* > 0, \quad \frac{\partial}{\partial z} P_A^* < 0,$$

$$\frac{\partial}{\partial z} P_B^* > 0, \quad \frac{\partial}{\partial z} X_A^* \leq 0, \quad \frac{\partial}{\partial z} X_B^* \geq 0.$$

The profit of firm A may increase or decrease with promotion.

In this subgame where the larger firm is able to promote, both firms react to a decrease (increase) in the marginal cost of targeting differently. In the quality sensitive region, firm A, the larger firm, reacts by increasing (reducing) the quality level while the smaller firm reacts by lowering (increasing) its quality level. This is depicted in figure 4.3(a). With quality increasing (decreasing), the larger firm, increases (reduces) the price of its product while the smaller firm lowers (increases) the price of the product. This can be seen from figure 4.3(b). As it becomes profitable for firm A to promote, the smaller firm finds it worthwhile to reduce the price of the product so as to maintain some share of the market. From the derivative of the market share of firm B with respect to the targeting cost, $z$, we notice that the smaller firm’s market share reduces but that of the larger firm increases with a decrease in marginal cost of targeting. This is also evidenced from figure 4.3(d). A smaller market share and a reduced price imply that the smaller firm is made worse off as the only decision variable in this region is price. However, this does not mean that the larger firm is necessarily better off. With marginal cost of targeting decreasing, more people buy on promotion. A similar situation arises in the quality sensitive region. Even though firm
B could increase profit by improving on quality, firm B will not improve quality due to the cost of improving quality. With a lower quality, lower price and lower market share, the smaller firm’s profit decreases with decreasing marginal cost of targeting. Please refer to figure 4.3(e). However, when the cost of targeting individuals with promotional incentives increases, the larger firm adopts a low quality strategy and therefore reduces the quality of its product. With quality decreasing, the larger firm reduces price to make up for the relatively low quality goods. The market share of this firm decreases as a result. Consumers leave to patronize the product of the smaller firm who has improved its quality. Hence the market share of firm B increases. An increase in quality coupled with an increase in price and market share leads to an increase in the profit of the smaller firm.

What happens to the equilibrium values as consumers become more sensitive to price and quality?

Theorem 6.

For any $T$ such that $0 \leq T < \sqrt{2} \sqrt{\frac{\phi(\alpha z+lA_\alpha-lA_2+2lB_\alpha-lB)}{lA+2lB}}$ and $z^{**} \leq z < z^*$,

i) $\frac{\partial \pi^*_{A,2}}{\partial \alpha} < 0$, $\frac{\partial \omega^*_{i,2}}{\partial \alpha} > 0$, $\frac{\partial S^*_{A,2}}{\partial \alpha} < 0$, $\frac{\partial P^*_A}{\partial \alpha} < 0$.

ii) $\frac{\partial S^*_{B,2}}{\partial \alpha} > 0$

iii) There exists $\alpha_1 < \alpha < \alpha_2$ such that $\frac{\partial \pi^*_{B,2}}{\partial \alpha} > 0$.

Otherwise the derivative is negative.
In the quality sensitive region, as consumers become more sensitive to quality, the following is always true,

$$\frac{\partial \pi^*_B}{\partial \beta} < 0, \quad \frac{\partial \omega^*_i}{\partial \beta} < 0, \quad \frac{\partial S^*_B}{\partial \beta} < 0, \quad \frac{\partial S^*_A}{\partial \beta} > 0.$$ 

But there exists a $\beta > \beta^{**}$ such that

$$\frac{\partial \pi^*_A}{\partial \beta} > 0.$$ 

Figures 4.4(a) - 4.4(e) illustrate the results above. In all the figures,

$$T^{**} = \sqrt{2} \sqrt{\frac{\phi (\alpha z + l_A \alpha - l_A + 2l_B \alpha - l_B)}{l_A + l_B}}.$$ 

In the quality sensitive region, as $\beta$, consumers valuation of quality increases, both firm’s improve the quality of their product and so we see an increasing quality function. However, as consumers become more sensitive to the quality of the product, the smaller firm is not able to meet expectations of highly sensitive quality customers and therefore adopts a low quality strategy. The larger firm takes advantage of its ability to produce high quality goods to meet these customers expectations. As a result, the larger firm keeps increasing its price to make up for the high quality goods it is producing. The smaller firm reduces its price for customers who are highly quality sensitive because it produces goods that are of low quality. As customers become sensitive to the price of the firms product, both firms are forced to reduce the price of their product. From figure 4.4(b), we see the price of firm A increasing always in the quality sensitive region but decreasing with intense price sensitivity. Firm B’s price increases with increasing quality and decreases with decreasing quality. With $\alpha$ increasing, consumers perception about the price of the product worsens. From figure 4.4(c), the perceived price of both firms increases as
consumers valuation of price intensifies. When quality comes to play in the quality sensitive region, the perceived prices of both firms decrease. The market share of the smaller firm increases in the price sensitive region but it is never more than the larger firm. That of the larger firm decreases because consumers believe that the price of the product is high.

The situation is different in the quality sensitive region. In this region, the market share of the larger firm increases always. We find the quality of this firm’s product increases and since consumers care about the quality of the product, they buy from the larger firm. This affects the market share of the smaller firm. For moderately sensitive customers, both firms improve on the quality so there is not much difference between the two firms. The profits of both firms therefore decrease. The profit of the smaller firm decreases till it breaks even and this is seen from figure 4.4(e) where the profit of the smaller firm is zero. But for the larger firm, its ability to improve on the quality of its product coupled with increasing price, results in increasing profit. However, when consumers are moderately price sensitive, the profit of the smaller firm increases. Both firms profits decrease with intense price sensitivity. We can therefore characterize competition in the price sensitive region also as moderately price sensitive and highly price sensitive. We have a similar situation when the cost of targeting is high (refer to figure 4.5 for $z = 0.52$). Nevertheless, we find the price sensitive region approaching $T = 0$, when customers are neutral with regards to price and quality. When the cost of targeting increases and consumers become more sensitive to price, they believe the price of the product of the promoting firm is very
No amount of promotions is enough to induce them to purchase from that firm. As was seen in figures 4.2(a) and 4.2(b), the intersection of the targeting cost and the lines $z = 0.52$ and $z = 0.62$ approaches $T = 0$. Therefore, with high targeting cost, the promoting firm can only promote to less price sensitive customers. This is also observed from the narrow price sensitive region in figures 4.5(a)- 4.5(e).

We seek to compare the equilibrium values of the firms when none of the firms finds it profitable to promote with the case when the larger firm promotes.

Theorem 7.

Suppose $z \to z^*$. For $l_A > l_B$,

a) The profits of firms A and B are higher in the price sensitive region when firm A promotes.

b) In the quality sensitive region,

i) Firm A’s profit when its promotes is lower than when no firm promotes.

ii) There exists $\beta_1 < \beta < \beta_2$ such that the profit of firm B with A promoting is higher than when none of the firms promotes. Otherwise, firm B’s profit is lower when A promotes.

For $l_A \gg l_B$, the profit of firm A is lower (for the most part) in the quality sensitive region when firm A promotes. Firm B’s profit is mostly higher when no firm promotes.

Please refer to figures 4.6(a) and 4.6(b) to confirm the above results.
Theorem 8.

For $0 \leq T < \sqrt{2} \sqrt{\frac{\phi(a z + l_A - l_A + 2 l_B - l_B) - l_A - l_B}{l_A + l_B}}$ we have the following,

a) For $l_A > l_B$ and $z \to z^{**}

i) The profit of firm A when A promotes is higher than when no firm promotes.

ii) The profit of firm B is higher when both firms do not promote.

b) For $l_A \gg l_B$ and $z^{**} \leq z < z^{*}$.

i) The profit of firm A when no firm promotes is higher always than when firm A promotes in the price sensitive region.

ii) There is a quality level beyond which the profit of firm A is higher when it promotes.

iii) The profit of firm B is higher when no firm promotes.

c) For $l_A \gg l_B$ and $z \to z^{**}

i) The profits of firms A and B are larger in the price sensitive region when no firm promotes.

ii) For $T > 0$, there exists a quality level beyond which the profit of firm A is higher when it promotes.

iii) For $T > 0$, the profit of the smaller firm is higher when none of the firms promotes.

Figures 4.7(a), 4.7(b) and 4.8 show this result. From the results above, we notice that the profit of the promoting firm increases when the cost of targeting is low. Comparing the results from promoting and not promoting, one might say that the larger firm is better off by not promoting.
We cannot quickly conclude that. The profits accruing to the firms when the larger firm promotes are the optimal of the optimal payoffs. From the payoff matrix in chapter II, we saw that in subgame 2 where the cost of promoting is profitable for firm A, the best decision is to promote. This was seen in the Nash Equilibrium. Deviating unilaterally will make the firm worse off.
Figure 4.1: Graph of firm A and firm B when no firm promotes (a) Quality; (b) Price; (c) Perceived price (d) Market share (e) Profit. Here, $v = 1$, $\epsilon = 1$, $\phi = 1$, $l_A = 0.6$ and $l_B = 0.4$.
Figure 4.2: Graph of (a) targeting cost with $l_A = 0.6$ and $l_B = 0.4$; (b) targeting cost with $l_A = 0.9$ and $l_B = 0.1$. Here, $v = 1$, $\epsilon = 1$, $\phi = 1$
Figure 4.3: Graph of firm A and firm B when firm A promotes (a) Quality; (b) Price; (c) Perceived price (d) Market share (e) Profit. Here, \( v = 1, \epsilon = 1, \phi = 1, l_A = 0.6 \) and \( l_B = 0.4 \).
Figure 4.4: Graph of firm A and firm B when firm A promotes (a) Quality; (b) Price; (c) Perceived price; (d) Market share; (e) Profit. Here, $v = 1$, $\epsilon = 1$, $\phi = 1$, $z = 0.4$, $l_A = 0.6$ and $l_B = 0.4$.
Figure 4.5: Graph of firm A and firm B when firm A promotes (a) Quality; (b) Price; (c) Perceived price (d) Market share (e) Profit. Here, $v = 1$, $\epsilon = 1$, $\phi = 1$, $z = 0.52$, $l_A = 0.6$ and $l_B = 0.4$. 
Figure 4.6: Profit of firm A and firm B under different promotional settings
(a) $z = 0.52, l_A = 0.6$ and $l_B = 0.4$; (b) $z = 0.62, l_A = 0.9$ and $l_B = 0.1$. Here, $v = 1, \epsilon = 1, \phi = 1$

Figure 4.7: Profit of firm A and firm B under different promotional settings
a) $z = 0.4, l_A = 0.6$ and $l_B = 0.4$; (b) $z = 0.3, l_A = 0.9$ and $l_B = 0.1$. Here $v = 1, \epsilon = 1, \phi = 1$
Figure 4.8: Profit of firm A and firm B under different promotional settings $v = 1$, $\epsilon = 1$, $\phi = 1$, $z = 0.18$, $l_A = 0.9$ and $l_B = 0.1$
CHAPTER V

CONCLUSION

In this paper, using game theory, we studied the effects of price and quality competition on the profits and market shares of two competing firms with a larger firm promoting. The firms were assumed to be asymmetric in customer loyalty. We followed the work of Shaffer and Zhang and Matsubayashi and Yamada. We incorporated quality in the model of Shaffer and Zhang and promotional strategy in the model of Matsubayashi and Yamada. We replicated the results of Matsubayashi and Yamada, added the effect of price and quality competition on market shares and compared their results to the results from our model (one firm promotes).

When no firm in the industry finds it profitable to target individuals with promotional incentives, and consumers are sensitive to price, the profits of both firms decrease. This is the usual observation for Bertrand price competition where the only decision parameter is price. The larger firm enjoys increasing market share. When customers are moderately quality sensitive, both firms profits decrease but the market share of the larger firm increases. Both firms are adopting high quality strategy so there is not much difference between them. However, as customers become more quality sensitive, the larger firm is able to improve on the quality of its product to meet expectations of these customers and therefore experiences increasing profit.
The smaller firm adopts a low quality strategy by reducing the quality of the product in order to stay in business as the price also decreases. This leads Matsubayashi and Yamada to conclude that larger firms should focus their sales activities to highly sensitive customers. The smaller firm should however focus on building customer loyalty. Adding the effect of price and quality competition on the average industry quality, we find that the average industry quality increases when the larger firm is able to acquire more customers. When competition sets in, the average quality industry eventually decreases as the larger firm adopts a low quality strategy.

Incorporating individual promotions in our model, we obtain slightly different results. When the market share of the promoting firm is far larger than the non-promoting firm, the promoting firm is able to target almost all individuals in the price sensitive region with promotional incentives only when the cost of targeting is very low. However, with high cost of targeting, the promoting firm sells to only customers who are less price sensitive. Customers who are price sensitive believe that the price of the firm’s product is very high. No amount of individual promotions is enough to obtain patronage from most of these customers. The marketshare and profit of this firm decreases when it sells to customers who are price sensitive. The smaller firm, on the other hand, enjoys increasing marketshare and therefore increasing profit by selling to less price sensitive customers. When customers sensitivity to price intensifies, the profit of this firm decreases. On the contrary, the promoting firm sells only to a few customers in the price sensitive region when its market share is only a little larger than the non-promoting firm. Intuitively, the smaller firm
also has a lot of the market share so the promoting firm has to offer enough promotions to lure some customers, especially those who are very price sensitive, away from the smaller firm. This might be expensive so even if the cost of targeting is low, it offers incentives to only a few customers who are less price sensitive. In the moderately quality sensitive region, both firms profits decrease but the market share of the promoting firm increases. This is because none of the firms has any advantage over its competitor in terms of quality strategy. By adopting a high quality strategy and therefore distinguishing itself, the larger firm’s profit increases as highly quality sensitive customers increase their purchase. If the smaller firm should also adopt a high quality strategy, it would be forced out of business. We can conclude that larger firms should invest in projects that seek to improve the quality of the firms product and target customers who are highly sensitive to quality. By so doing, they will enjoy increasing profit. Smaller firms, however, should focus on increasing their customer loyalty and also selling to less price sensitive customers.

In our analysis, we did not consider the scenario where the larger firm and smaller firm are both promoting. Future work could consider this case.
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APPENDICES
APPENDIX A

PROOF OF EQUILIBRIUM RESULTS

We first prove the equilibrium result given by theorem 1 in chapter II.

From the first order conditions when no firm promotes we have the equilibrium prices and quality levels:

\[ P_A^* = \frac{(2 \phi + \epsilon T)\{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B\}}{4\alpha \phi (3 \alpha \phi - T^2)} + v, \]

\[ P_B^* = \frac{(2 \phi + \epsilon T)\{(2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B\}}{4\alpha \phi (3 \alpha \phi - T^2)} + v, \]

\[ X_A^* = \frac{T\{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B\}}{4\alpha \phi (3 \alpha \phi - T^2)}, \]

\[ X_B^* = \frac{1}{4} \frac{T\{(2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B\}}{4\alpha \phi (3 \alpha \phi - T^2)}. \]

Consider the case that \( T \leq 0 \) and assume that \( X_A^* > 0 \) and \( X_B^* > 0 \). Then for the price, quality level and profit to be positive the following conditions must hold simultaneously:

\[ (4 \alpha \phi - T^2) \geq 0 \quad (A.1) \]

\[ \frac{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B}{(3 \alpha \phi - T^2)} < 0 \quad (A.2) \]

\[ \frac{(2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B}{(3 \alpha \phi - T^2)} < 0 \quad (A.3) \]

\[ (2 \phi + \epsilon T) < 0. \quad (A.4) \]
If A.1 is true, then firm \( i \) can increase profit indefinitely by increasing price. Suppose that \((3 \alpha \phi - T^2) > 0\). Let \( T^2 = (3 \alpha \phi - y) \) where \( y > 0 \). Then

\[
\frac{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B}{(3 \alpha \phi - T^2)} = \frac{(\alpha \phi + y) l_A + (-\alpha \phi + y) l_B}{y} = \frac{y (l_A + l_B) + \alpha \phi (l_A - l_B)}{y} = 1 + \frac{\alpha \phi (l_A - l_B)}{y} > 0,
\]

because \((l_A + l_B)\) is assumed to be one and \(l_A > l_B\). This means that \(X_A\) is negative, a contradiction. Also,

\[
\frac{(2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B}{(3 \alpha \phi - T^2)} = 1 - \frac{\alpha \phi (l_A - l_B)}{y} > 0.
\]

This implies that \(X_B\) is negative giving a contradiction. Similarly, for \((3 \alpha \phi - T^2) < 0\), and letting \( T^2 = (3 \alpha \phi + y)\), \( y > 0 \), we have

\[
\frac{(4 \alpha \phi - T^2) l_A + (2 \alpha \phi - T^2) l_B}{(3 \alpha \phi - T^2)} = 1 - \frac{\alpha \phi (l_A - l_B)}{y} > 0
\]

\[
\frac{(2 \alpha \phi - T^2) l_A + (4 \alpha \phi - T^2) l_B}{(3 \alpha \phi - T^2)} = 1 + \frac{\alpha \phi (l_A - l_B)}{y} > 0,
\]

which gives \(X_A < 0\) and \(X_B < 0\). Therefore, it is not possible that A.1, A.2, A.3 and A.4 will be satisfied simultaneously when \( T < 0 \). Thus, we must have \(X_A = 0 = X_B\) in equilibrium. With the adjusted qualities zero, we substitute these into the profit
function for each firm, differentiate over the variables that each firm has control over and solve the system of equations which yield the equilibrium price, and hence the first part of theorem 1.

Now consider the case with $T > 0$. Firm $i$ can increase profit by altering the price and or quality of its product. Suppose $4 \alpha \phi - T^2 < 0$ and $3 \alpha \phi - T^2 < 0$. Then firm $i$ can increase profit 3.1 indefinitely by taking $X_i$ very large and charging $P_i$. This implies that no equilibrium will exist. We will have a similar case if $4 \alpha \phi - T^2 < 0$ and $3 \alpha \phi - T^2 > 0$. So we assume that $4 \alpha \phi - T^2 > 0$. If $3 \alpha \phi - T^2 < 0$, $X_A < 0$, a contradiction. So we assume that $3 \alpha \phi - T^2 > 0$. With this, $X_A > 0$ as $l_A > l_B$. If $(2 \alpha \phi - T^2)l_A + (4 \alpha \phi - T^2)l_B < 0$, the price of firm $B$ is less than the marginal cost of production and the firm must be driven out of business. Also, $X_A < 0$, which is a contradiction. Therefore to ensure that we have positive equilibrium, $(2 \alpha \phi - T^2)l_A + (4 \alpha \phi - T^2)l_B > 0$ must hold.

We next prove the equilibrium result when firm A promotes. This is the result under theorem 2. Suppose $T \leq 0$ and assume that $X_A^* > 0$ and $X_B^* > 0$. Then for the price, quality level and profit in theorem 2 to be positive, the following conditions must be true

\[
(4 \alpha \phi - T^2 - 2 \phi) \geq 0 \quad \text{(A.5)}
\]

\[
\frac{\{(4 \alpha \phi - T^2)l_A + (2 \alpha \phi - T^2)l_B - 2 \phi \alpha z\}}{(3 \alpha \phi - T^2 - \phi)} < 0 \quad \text{(A.6)}
\]
\[
\frac{(2\alpha \phi - T^2 - 2\phi) l_A + (4\alpha \phi - T^2 - 2\phi) l_B + 2\phi \alpha z}{(3\alpha \phi - T^2 - \phi)} < 0 \quad \text{(A.7)}
\]

\[(2\phi + \epsilon T) < 0. \quad \text{(A.8)}
\]

If A.5 is true, then firm \( i \) can increase profit indefinitely by increasing price. No equilibrium will exist. Suppose that \((3\alpha \phi - T^2 - \phi) > 0\). Let \( T^2 = (3\alpha \phi - \phi - y) \) where \( y \) is a positive number and cannot be too large. We can impose a restriction \( y \leq (3\alpha \phi - \phi) \) to ensure that \( T^2 \) is positive.

Then A.6 becomes

\[
\frac{(4\alpha \phi - T^2) l_A + (2\alpha \phi - T^2) l_B - 2\phi \alpha z}{(3\alpha \phi - T^2 - \phi)}
\]

\[= \frac{(\alpha \phi + \phi + y) l_A + (\phi + y - \alpha \phi) l_B - 2\phi \alpha z}{y}
\]

\[= \frac{y(l_A + l_B) + \alpha \phi(l_A - l_B) + \phi(l_A + l_B) - 2\alpha \phi z}{y}
\]

\[= 1 - \frac{2\alpha \phi z}{y} + \frac{\phi[(\alpha(l_A - l_B) + 1]}{y}
\]

If \( \alpha \to \infty \) and we impose the restriction on \( y \) that makes \( T^2 \) positive, \( \frac{2\alpha \phi z}{y} \to \frac{2z}{3} \), \( \frac{\phi[(\alpha(l_A - l_B) + 1]}{y} \to \frac{(l_A - l_B)}{3} \) and equation A.9 becomes

\[1 - \frac{2 z}{3} + \frac{(l_A - l_B)}{3} > 0,
\]

because \((l_A + l_B)\) is assumed to be one, \( l_A > l_B \) and \( z \) is bounded in this subgame and thus small. This means that \( X_A \) is negative, a contradiction. If \( \alpha \to 0 \), \( \frac{2\alpha \phi z}{y} \to 0 \), \( \frac{\phi[(\alpha(l_A - l_B) + 1]}{y} \to -1 \) and A.9 = 0. Also,

\[
\frac{(2\alpha \phi - T^2 - 2\phi) l_A + (4\alpha \phi - T^2 - 2\phi) l_B + 2\phi \alpha z}{(3\alpha \phi - T^2 - \phi)}
\]

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\[
\begin{align*}
&= (y - \alpha \phi - \phi) l_A + (y + \alpha \phi - \phi) l_B + 2\phi \alpha z \\
&= y l_A + \alpha \phi (l_A - l_B) - \phi (l_A + l_B) + 2\phi \alpha z \\
&= 1 + \frac{2\alpha \phi z}{y} - \phi [\alpha (l_A - l_B) + 1]. \quad (A.11)
\end{align*}
\]

Using the same analysis, as \( \alpha \to \infty \), equation A.11 becomes
\[
1 + \frac{2z}{3} - \frac{(l_A - l_B)}{3} > 0 \quad \text{because (}l_A + l_B) = 1.
\]

Also, if we let \( \alpha \to 0 \), equation A.11 > 0. This implies that \( X_B \) is negative giving a contradiction. Similarly, for \((3\alpha \phi - \phi - T^2) < 0\), and letting \( T^2 = (3\alpha \phi - \phi + y) \), \( y > 0 \), we have
\[
\begin{align*}
\frac{(4\alpha \phi - T^2) l_A + (2\alpha \phi - T^2) l_B - 2\phi \alpha z}{(3\alpha \phi - T^2 - \phi)} \\
= \frac{(\alpha \phi + \phi - y) l_A + (\phi - y - \alpha \phi) l_B - 2\phi \alpha z}{-y} \\
&= -y (l_A + l_B) + \alpha \phi (l_A - l_B) + \phi (l_A + l_B) - 2\phi \alpha z \\
&= 1 + \frac{2\alpha \phi z}{y} - \phi \left[\alpha (l_A - l_B) + 1\right].
\end{align*}
\]

For \( T^2 > 0 \), \( y > \phi - 3\alpha \phi \) which is equivalent to \(-y \leq 3\alpha \phi - \phi\). If we follow the same thought process as above, this equation is positive. Also,
\[
\begin{align*}
\frac{(2\alpha \phi - T^2 - 2\phi) l_A + (4\alpha \phi - T^2 - 2\phi) l_B + 2\phi \alpha z}{(3\alpha \phi - T^2 - \phi)} \\
= \frac{(-\alpha \phi - \phi - y) l_A + (\alpha \phi - \phi - y) l_B + 2\phi \alpha z}{-y} \\
&= -y (l_A + l_B) - \alpha \phi (l_A - l_B) - \phi (l_A + l_B) - 2\phi \alpha z \\
&= 1 - \frac{2\alpha \phi z}{y} + \frac{\phi [\alpha (l_A - l_B) + 1]}{y}
\end{align*}
\]

> 0
from the same thought process giving $X_A < 0$ and $X_B < 0$. Therefore, it is not possible for A.5, A.6, A.7 and A.8 to be satisfied simultaneously when $T < 0$.

Thus, we must have $X_A = 0 = X_B$ in equilibrium. With the adjusted quality zero, we substitute these into the profit function for each firm, differentiate over the variables that each firm has control over and solve the system of equations which yield the equilibrium result in part 1 of theorem 2.

Now consider the case when $T > 0$. Suppose $4\alpha \phi - T^2 - 2\phi < 0$ and $3\alpha \phi - T^2 - \phi < 0$. Then firm $i$ can increase profit indefinitely by taking $X_i$ very large and charging $P_i$. This implies that no equilibrium will exist. We will have a similar case if $4\alpha \phi - T^2 - 2\phi < 0$ and $3\alpha \phi - T^2 - \phi > 0$ is true. So we assume that $4\alpha \phi - T^2 - 2\phi > 0$. If $3\alpha \phi - T^2 - \phi < 0$, \[ \frac{((4\alpha \phi - T^2)l_A + (2\alpha \phi - T^2)l_B - 2\phi \alpha z)}{(3\alpha \phi - T^2 - \phi)} < 0 \]

because $3\alpha \phi - \phi < T^2 < 4\alpha \phi - 2\phi$. This gives $X_A < 0$, a contradiction. So we assume that $3\alpha \phi - T^2 - \phi > 0$. With this, $X_A > 0$ as $l_A > l_B$. If $(2\alpha \phi - T^2 - 2\phi)l_A + (4\alpha \phi - T^2 - 2\phi)l_B - 2\alpha \phi z < 0$, the price of firm $B$ is less than the marginal cost of production and the firm must be driven out of business. Also, $X_A < 0$, which is a contradiction. Therefore to ensure that we have positive equilibrium, $(2\alpha \phi - T^2 - 2\phi)l_A + (4\alpha \phi - T^2 - 2\phi)l_B + 2\alpha \phi z > 0$ must hold.
APPENDIX B

PROOF OF RESULTS UNDER SUBGAME TWO - FIRM A PROMOTES

We prove the result in theorem 5. Consider the profit of firm A,

\[ \pi^*_{A,2} = \frac{(4\alpha - 2) \left( -4\alpha l_A - 2\alpha l_B \right)^2 + 4\alpha z \left( -4\alpha l_A - 2\alpha l_B \right) + 4\alpha^2 z^2 (-8\alpha + 18\alpha^2)}{16\alpha^2 (3\alpha - 1)^2}. \]  

(B.1)

This is a quadratic equation in \( z \) so we differentiate with respect to \( z \) and find the nature of the turning point.

\[ \frac{\partial}{\partial z} \pi^*_{A,2} = \frac{4 \left( 4\alpha - 2 \right) \alpha \left( -4\alpha l_A - 2\alpha l_B \right) + 8\alpha^2 z \left( -8\alpha + 18\alpha^2 \right)}{16\alpha^2 (3\alpha - 1)^2}. \]  

(B.2)

We now determine whether equation B.2 is always positive or negative. As an example, suppose \( l_A = .8, l_B = .2 \) and \( z = .3 \). Then, equation B.2 is

\[ \frac{\partial}{\partial z} \pi^*_{A,2} = \frac{-14.4 \left( 4\alpha - 2 \right) \alpha^2 + 2.4 \alpha^2 (-8\alpha + 18\alpha^2)}{16\alpha^2 (3\alpha - 1)^2}. \]  

(B.3)

A plot of equation B.3 reveals that this derivative function changes sign so that the profit function B.1 decreases and increases on the range of \( T \). Similarly,

\[ \frac{\partial}{\partial z} \pi^*_{B,2} = \frac{2\alpha \left( l_A (\alpha - 1) + l_B (2\alpha - 1) \right) + 2z\alpha^2}{\alpha (3\alpha - 1)^2} \]

\[ = \frac{\{l_A (\alpha - 1) + l_B (2\alpha - 1) + z\alpha\}}{\alpha (3\alpha - 1)^2} \]

\[ > 0, \]

\[ \frac{\partial}{\partial z} P^*_B = \frac{1}{3\alpha - 1} > 0, \quad \frac{\partial}{\partial z} X_A = 0, \quad \frac{\partial}{\partial z} X_B = 0, \]  

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\[ \frac{\partial}{\partial z} S_{A,2}^* = -\frac{\alpha}{3\alpha - 1} < 0, \quad \frac{\partial}{\partial z} S_{B,2}^* = \frac{\alpha}{3\alpha - 1} > 0. \]

Suppose \( 0 < T < \sqrt{2} \sqrt{\frac{\phi(\alpha z + l_A \alpha - l_A + 2l_B \alpha - l_B)}{l_A + l_B}} \) and \( z^{**} \leq z < z^* \). Then,

\[ \frac{\partial}{\partial z} \pi_{B,2}^* = 4\phi \alpha^2 \left( (2\alpha \phi - T^2 - 2\phi) l_A + (4\alpha \phi - T^2 - 2\phi) l_B + \alpha \phi z \right) + 4\phi^2 z\alpha^3 > 0. \]

The condition \( 0 < T < \sqrt{2} \sqrt{\frac{\phi(\alpha z + l_A \alpha - l_A + 2l_B \alpha - l_B)}{l_A + l_B}} \) could be rewritten as

\[
(2\alpha \phi - T^2 - 2\phi) l_A + (4\alpha \phi - T^2 - 2\phi) l_B + \alpha z\phi > 0,
\]

making the above inequality true. Also,

\[ \frac{\partial}{\partial z} S_{A,2}^* = -\frac{\alpha \phi}{3\alpha \phi - T^2 - \phi} < 0, \]

because \( 3\alpha \phi - T^2 - \phi > 0 \).

\[
\frac{\partial}{\partial z} S_{B,2}^* = \frac{\alpha \phi}{3\alpha \phi - T^2 - \phi} > 0, \quad \frac{\partial}{\partial z} P_A^* = -\frac{2(\phi + \epsilon T)}{2(3\alpha \phi - T^2 - \phi)} < 0,
\]

\[
\frac{\partial}{\partial z} P_B^* = \frac{2(\phi + \epsilon T)}{2(3\alpha \phi - T^2 - \phi)} > 0, \quad \frac{\partial}{\partial z} X_A^* = -\frac{T}{2(3\alpha \phi - T^2 - \phi)} < 0,
\]

\[
\frac{\partial}{\partial z} X_B^* = \frac{T}{2(3\alpha \phi - T^2 - \phi)} < 0.
\]

We next prove the result of theorem 6.

Let’s suppose that \( z = z^* - y, \ y > 0 \). We choose \( y \) such that \( z^* - y > z^{**} \). Then

\[ S_{A,2}^* = \frac{6\alpha l_A + 3\alpha l_B - 2l_A - l_B + 3y\alpha}{3(3\alpha - 1)}. \]

Differentiating with respect to \( \alpha \), we obtain

\[ \frac{\partial S_{A,2}^*}{\partial \alpha} = -\frac{y}{(3\alpha - 1)^2} < 0. \]
Similarly, if we suppose that \( z = z^* - y \) and use this in the market share of firm B in the price sensitive region, we have

\[
S^*_B,2 = \frac{3\alpha l_A - l_A + 6\alpha l_B - 2l_B - 3y\alpha}{3(3\alpha - 1)}.
\]

Differentiating with respect to \( \alpha \), we obtain

\[
\frac{\partial S^*_B,2}{\partial \alpha} = \frac{y}{(3\alpha - 1)^2} > 0,
\]

and this completes the proof of (ii). To prove that of the perceived prices, we follow the same thought process and obtain

\[
\omega^*_A,2 = v\alpha + \frac{2\alpha l_A + \alpha l_B - \alpha z}{3\alpha - 1} = \frac{6\alpha l_A + 3\alpha l_B - 2l_A - l_B + 3y\alpha + 9v\alpha^2 - 3v\alpha}{3(3\alpha - 1)}.
\]

Thus,

\[
\frac{\partial}{\partial \alpha} \omega^*_A,2 = \frac{v(3\alpha - 1)^2 - y}{(3\alpha - 1)^2} > 0,
\]

because \( y \) is very small and \( v \) the targeting cost is positive and greater than \( y \).

Similarly, for firm B, we have

\[
\omega^*_B,2 = \frac{l_A(\alpha - 1) + l_B(2\alpha - 1) + \alpha z}{3\alpha - 1} + v\alpha = \frac{3\alpha l_A - l_A + 6\alpha l_B - 2l_B - 3y\alpha + 9v\alpha^2 - 3v\alpha}{3(3\alpha - 1)}.
\]

Thus,

\[
\frac{\partial}{\partial \alpha} \omega^*_B,2 = \frac{v(3\alpha - 1)^2 + y}{(3\alpha - 1)^2} > 0.
\]
We now prove the result for firms’ profits. If we substitute \(z = z^*-y\) in \(\pi^*_A\), we get
\[
\pi^*_A = \frac{(4 \alpha - 2) ((4 \alpha l_A + 2 \alpha l_B)^2 - 4 \alpha z (4 \alpha l_A + 2 \alpha l_B)) + 4 \alpha^2 z^2 (-8 \alpha + 18 \alpha^2)}{16 \alpha^2 (3 \alpha - 1)^2}
\]
\[
= \frac{\{(81 \alpha^3 - 36 \alpha^2) y^2 - 6 \alpha (2 l_A + l_B) (3 \alpha - 1) y + 2 (2 l_A + l_B)^2 (3 \alpha - 1)^2\}}{18 \alpha (3 \alpha - 1)^2}
\]
Differentiating, we find
\[
\frac{\partial}{\partial \alpha} \pi^*_A = -\frac{(27 \alpha^3 - 18 \alpha^2) y^2 - 9 \alpha^2 (2 l_A + l_B) (3 \alpha - 1) y + (2 l_A + l_B)^2 (3 \alpha - 1)^3}{9 \alpha^2 (3 \alpha - 1)^3}
\]
But
\[
(27 \alpha^3 - 18 \alpha^2) y^2 - 9 \alpha^2 (2 l_A + l_B) (3 \alpha - 1) y + (2 l_A + l_B)^2 (3 \alpha - 1)^3 \quad (B.4)
\]
is quadratic in \(y\). Differentiating with respect to \(y\) and solving for \(y\) yields the critical point \(y^* = \frac{(2 l_A + l_B)(3 \alpha - 1)}{2(3 \alpha - 2)}\), which is positive. The second derivative suggests that \(y^*\) is a minimum. Substituting \(y^*\) into \(B.4\), we find
\[
\frac{(2 l_A + l_B)^2 (27 \alpha^2 - 36 \alpha + 8) (3 \alpha - 1)^2}{4(3 \alpha - 2)} > 0
\]
for \(\alpha > 1\) (this is true because \(\alpha\) varies in the price sensitive region). Therefore, equation \(B.4\) is positive. Hence, \(\frac{\partial}{\partial \alpha} \pi^*_A < 0\).

Also, if we substitute \(z = z^*-y\) in \(\pi^*_B\), we have,
\[
\pi^*_B = \frac{(l_A + 2 l_B - 6 \alpha l_B - 3 \alpha l_A + 3 y \alpha)^2}{9 \alpha (3 \alpha - 1)^2}
\]
Now,
\[
\frac{\partial}{\partial \alpha} \pi^*_B = -\frac{\{(9 \alpha^2 + 27 \alpha^3) y^2 - 18 \alpha^2 (l_A + 2 l_B) (3 \alpha - 1) y + (3 \alpha - 1)^3 (l_A + 2 l_B)^2\}}{9 \alpha^2 (3 \alpha - 1)^3}
\]  
(B.5)
The derivative of $\pi_{B,2}^{*}$ with respect to $\alpha$ will be zero only if

$$(9 \alpha^2 + 27 \alpha^3) y^2 - 18 \alpha^2 (l_A + 2 l_B) (3 \alpha - 1) y + (3 \alpha - 1)^3 (l_A + 2 l_B)^2) = 0. \quad (B.6)$$

We notice that equation B.6 is quadratic in $y$ with critical point $y^* = \frac{\{(3 \alpha - 1) l_A + (6 \alpha - 2) l_B\}}{1 + 3 \alpha}$, which is positive. Evaluation of the second derivative of equation B.6 suggests that this critical point is a minimum. The value of B.6 at the critical point is $-\frac{(3\alpha-1)^2(l_A+2l_B)^2}{1+3\alpha}$. This implies that the function given by equation B.6 decreases and increases on the given interval. Therefore, B.5 increases and then decreases as we vary $\alpha$. Hence, there exists $\alpha_1 < \alpha$ where $\pi_{B,2}^{*}$ is increasing and $\alpha < \alpha_2$ where $\pi_{B,2}^{*}$ is decreasing.
## APPENDIX C

### LIST OF VARIABLES AND DEFINITIONS

Table C.1: List of variables and definitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_i$</td>
<td>$P_i \geq 0$</td>
<td>Price of the product sold by the firm</td>
</tr>
<tr>
<td>$X_i$</td>
<td>$X_i \geq 0$</td>
<td>Quality level of the product sold by the firm</td>
</tr>
<tr>
<td>$R$</td>
<td>$R \geq v$</td>
<td>Reservation price of consumers</td>
</tr>
<tr>
<td>$v$</td>
<td>$v \geq 0$</td>
<td>Marginal cost of production</td>
</tr>
<tr>
<td>$z$</td>
<td>$z \geq 0$</td>
<td>Cost of targeting</td>
</tr>
<tr>
<td>$l$</td>
<td>$l \in [-l_B, l_A]$</td>
<td>Customer loyalty</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$\alpha \geq 0$</td>
<td>A measure of consumer’s responsiveness to price</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\beta \geq 0$</td>
<td>A measure of consumer’s responsiveness to quality</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$\phi \geq 0$</td>
<td>Fixed cost parameter</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>$\epsilon \geq 0$</td>
<td>Variable cost parameter</td>
</tr>
<tr>
<td>$d_i(l)$</td>
<td>$d_i(l) \geq 0$</td>
<td>Discount firm $i$ gives to customers located at $l$</td>
</tr>
</tbody>
</table>
Table C.1: List of variables and definitions (continued)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_i = \alpha P_i - \beta X_i$</td>
<td>$\omega_i \geq 0$</td>
<td>Perceived price</td>
</tr>
<tr>
<td>$\hat{l} \equiv \omega_A - \omega_B$</td>
<td>$l \in [\ell_B, \ell_A]$</td>
<td>Customer loyalty - the minimum perceived-price differential to induce a customer to buy from her less preferred firm</td>
</tr>
<tr>
<td>$T = \beta - \alpha \epsilon$</td>
<td>$T \in (-\infty, \infty)$</td>
<td>Consumers valuation of quality relative to price</td>
</tr>
<tr>
<td>$S_i$</td>
<td>$S_i \geq 0$</td>
<td>Market share of firm $i$ in subgame one</td>
</tr>
<tr>
<td>$S_{i,2}$</td>
<td>$S_{i,2} \geq 0$</td>
<td>Market share of firm $i$ in subgame two</td>
</tr>
<tr>
<td>$\pi_i$</td>
<td>$\pi_i &gt; 0$</td>
<td>Profit of firm $i$ in subgame one</td>
</tr>
<tr>
<td>$\pi_{i,2}$</td>
<td>$\pi_{i,2} &gt; 0$</td>
<td>Profit of firm $i$ in subgame two</td>
</tr>
<tr>
<td>$z^*$</td>
<td>$z^* &lt; z$</td>
<td>Maximum targeting cost for firm A to target individuals</td>
</tr>
<tr>
<td>$z^{**}$</td>
<td>$z^{**} \geq z$</td>
<td>Minimum targeting cost for firm A to target individuals</td>
</tr>
</tbody>
</table>