FINITE ELEMENT MODELING AND SIMULATION ON THE QUENCHING EFFECT FOR SPUR GEAR DESIGN OPTIMIZATION

A Thesis

Presented to

The Graduate Faculty of The University of Akron

In Partial Fulfillment

of the Requirement for the Degree

Master of Science

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August, 2008
ABSTRACT

Developing an analytical approach and modeling procedure to evaluate stress
distribution and quenching process under velocity and moment would provide a useful
tool to improve spur gear design with high efficiency and low cost. Based on the theories
of gear engagement, contact analysis and friction, a three dimensional finite element
model of the spur gear system was established to investigate stress distribution and
analyze the advantage of quenching process. A full-scale deformable-body model and a
simplified discrete model were both shown to be accurate through extensive comparisons to the
theoretical database generated in this study. The major work is summarized as follows.

- Applying Finite Element Method, contact stress analysis of meshing spur gears
  was conducted.

- Applying the relation equation in Pro/Engineer, an accurate three dimensional
  spur gear model was developed. This model can be used to analyze many
  similar spur gears for design optimization.

- Three dimensional finite element models of spur gear system were established to
  investigate stress distributions over operating speeds with consideration of
  lubrication conditions. The three-dimensional FEA program developed can be a
  useful tool in investigating design parameters for spur gears.

- A theoretical finite element model of spur gear system was developed. The
  research result shows that the theoretical methods presented in this thesis have
good simulation accuracy. This method could also be applied to many other engineering problems.

➢ Finite Element simulation of spur gear was developed and used to predict distributions of stress and other material properties. Thermo-elastic–plastic constitutive equation coupled with the mechanical strain, thermal strain, phase transformation strain, and transformation induced plasticity is described in detail. The quenching result in the simulation proved the theory and ensured product quality.
ACKNOWLEDGEMENTS

I would like to express sincerely appreciation to my advisor: Dr. Yueh-Jaw Lin. He has contribution so much through this study. Also, I am grateful to thank Dr. Xiaosheng Gao to provide the finite element analysis software.

I profoundly thank Dr. Jon S. Gerhardt, Dr. Xiaosheng Gao for serving on the committee and providing valuable suggestions and advices on my work. I also wish my sincere thanks to the department chair, Dr. Celal Batur, for extending all the necessary help.

I am also grateful for the opportunity to study in the Mechanical Engineering Department, University of Akron. Thanking for the professors and people who ever taught and helped me.

Thanks to my wife Ying Wang. Without her encouragement, patience and sacrifice I cannot think I can finish my study.
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NOMENCLATURE

\( \alpha \)  
thermal expansion coefficient  
\( \alpha_n \)  
Pressure angle

\( A_s \)  
the function of the stress dependency of the TTT curves  
\( A_c \)  
Functions of the carbon content.

\( b \)  
Face width  
\( c \)  
specific heat

\( [c] \)  
damping matrix  
\( C_{ijkl} \)  
elasticity matrix

\( C \)  
the carbon content  
\( d \)  
Pitch diameter

\( d_a \)  
Addendum circle  
\( d_b \)  
Basic Circle

\( d_d \)  
Dedendum circle  
\( E \)  
Young's modulus

\( F \)  
force  
\( \{F\} \)  
Strength vector

\( \bar{f}_{\cdot \tau(T)} \)  
a function of temperature  
\( \bar{f}_{c(C)} \)  
the function of the carbon content

\( h_a \)  
Addendum height  
\( h_g \)  
Dedendum height

\( h_s \)  
non-work tooth face convective heat transfer coefficient  
\( h_t \)  
meshing tooth face

\( m \)  
normal modulus  
\( [m] \)  
Quality matrix

\( n \)  
Avrami number  
\( P_h \)  
Circular pith

\( \rho \)  
density  
\( Q_F \)  
spur gear mating surface heat flux by surface friction

\( K \)  
the function of the temperature  
\( k \)  
thermal conductivity

\( K_J \)  
Specific heat  
\( [k] \)  
rigidity matrix,

\( \tau \)  
tangential frictional stress  
\( T_0 \)  
initial material temperature
\( T_a \)  air temperature \( T_w \)  the coolant temperature

\( \nu \)  the Poisson ratio \( W \)  Conductivity

\( Z_{di} \)  Teeth number of driving gear \( \bar{\sigma} \)  the effective stress

\( Z_{dn} \)  Teeth number of driving gear \( \sigma_{ij} \)  the stress

\( \sigma_m \)  the mean stress, \( \dot{\varepsilon}_{ii} \)  the strain rate

\( \dot{\varepsilon}_{ij}^{m} \)  the dilatation because of the structural changes 

\( \frac{1 + \nu}{E} \)  function of the carbon content 

\( 1 \ + \ \nu \)  function of the volume fraction 

\( 1 + \frac{\nu}{E} \)  

\( ^\circ \text{C} \)  Temperature \( \zeta \)  the volume fraction of the \( i \) th phase

\( \Phi \)  gear rotation
CHAPTER I

INTRODUCTION

1.1 Background

Along with modern high speed manufacturing industry development, gears are used widely in many applications ranging from automotive transmission to robot and aerospace engines. Therefore, the dynamic analysis is more and more important in the modern manufacture. Different kinds of metallic gears are currently being manufactured for various industrial purposes. Seventy-four percent of them are spur gears, fifteen percent helical, five percent worm, four percent bevel, and the others are either epicyclical or internal gears. [1] In this study, the research of spur gears is the basic step in gear systems. Other special gears can be studied in the same way. A Gear system is an effective power transmission element. High torque to weight ratios, large speed reductions in compact room, co-axial shaft arrangements, super reliability and high efficiency are required. [2] In the gear design, the gear performance forecast is one of the main analytical steps in the production. It is very difficult to obtain accurate data to examine. The theoretical results can only be obtained through the theoretical calculation. In order to improve mechanical properties of spur gears, such as strength, hardness, and wear/fatigue resistances, it is necessary to analyze mechanical properties by the Finite Element Analysis method.

In this thesis, the parameterization precise gear model is built though Pro/Engineer software. The gear outline with the relation equation forms the gear accurate surface. Its
parameterization characteristic implements the gear fast modification; thus it implements the fast establishment standardization model. The general files style: IGES transfer from Pro/E to Finite Element Analysis software Abaqus 6.7. The dynamic analysis and the correlation simulation are carried out in the next step. This study provides both theoretical and Finite Element Analysis of gear dynamics stress influence. The predicted results were successfully validated with analytical results. In order to reduce the gear size, strengthen its surface strength and optimize service life, quenching plays a very important role in the gear production process. In the last section, a theoretical FE model suitable for the spur gear system and analysis of quenching problems is described.

1.2 Literature review

A number of published studies in the gear kinematics focused on kinematic analysis formulations. [2, 3] The two-dimensional finite element model was developed by Parker. [4] This model shows a unique finite element-contract analysis solver specialized for gear dynamics. It is match very well between the analytical and finite element model’s response across a range of complicated behaviors.

A recent study by Kahraman considers each gear as a deformable body and meshes them to predict load, stress and deformations. Based on these results presented, a deformable body analysis with a thin rim is necessary. [5]

Pourboghrat studies several material problems resulting from the production of steel worm shafts. Finite Element Analysis model of composite worm gear was developed and compared with a steel gear. The work life of composite worm gear was found to be better than steel gear. [6]
3D static and dynamic contract/impact analysis of gear drives were developed by Rufang Li. The tooth load allocation and result are derived under the static load. She analyzes the stress distribution of gear system under dynamic loading conditions and simulates the stress of gears under conditions of initial speed and a sudden load being applied. [7]

Deng used tooth contract analysis, loaded tooth contact analysis and finite element method to analyze the meshing behavior, tooth surface contract stress, maximum tensile bending stress and maximum compressive bending stress. The modified pith cone method is first presented and verified in the gear research center of Dong Feng vehicle-bridge Co., Ltd. [8]

Cheung developed a new type of element consistent with this specific finite element theory. In his study, he showed the deformations of the spatial motions of a gear pair and found some errors induced by manufacturing and assembly processes. All new findings reported on the simulation studies. [9]

Yoo defined the deformation overlap for numerical investigation of elastic deflection in the design of a gear pair. He compared the solution of elastic contact problem and FEM. The tooth tip relief calculation and the design of a profile shifted gear pair demonstrated the deformation overlap. [10]

Mao developed an advanced non-linear finite element method, which has been successfully used to accurately simulate gear contact behavior under real load conditions. This method was proved to be successful in automotive transmission gear surface fatigue wear reduction. [11]
Stress analysis of a helical gear set with localized bearing contact was developed by Tsay. It derived on the theory of gearing for the mathematical models of the complete tooth geometry of the pinion and the gear. Though FEA package, ABAQUS/Stand show the gear stress distribution. Furthermore, it presents some examples, which demonstrate the influences of the gear’s design parameters. [12]

A gear tooth fault model is developed to simulate the effects of pitting and wear on the vibration signal under normal operating conditions. The Wigner-Ville Distribution (WVD) was applied to calculate the theory. The theory was compared with the experiment result later. The study verified that the WVD method can successfully detect a faulty gear system. [13]

Moslehy shows a method for reducing the stresses in a gear tooth. By introducing a circular hole in the tooth, stress redistribution in the highly stressed region (root of the tooth) will result, which significantly reduces the stress concentration. [14] They carry out the experiment and finite element analysis software-I-DEAS to solve the above problem. Photoelastic analysis on these physical models, experimental and FEA results were compared with each other.

1.3 Thesis outline

Development of an analysis approach and modeling procedure to evaluate stress distribution and quenching process under the speed operation condition would provide a useful tool to improve spur gear design with high efficiency and low costs. Based on the theories of gear engagement, contact analysis and friction, a three dimensional finite element model of spur gear system was established to investigate stress distribution,
sensitivity analysis the advantage of quenching process. A full-scale deformable-body model and a simplified discrete model were both shown to be accurate through extensive comparisons to the theoretical database generated in this study. The major work is summarized as follows.

- Applying Formulate of Hertz Contact and Finite Element Method, contact stress analysis of meshing spur gear was conducted.
- Applying the relation equation in Pro/Engineer to develop the accurate three dimensional spur gear models. It is easy to change the parameters of spur gear to arrive different model to analysis.
- Three dimensional finite element models of spur gear system were established to investigate stress distribution over operating speed with consideration of lubrication conditions. Three-dimensional FEA program developed can be a useful tool in investigating design parameters for the spur gear.
- A theoretical finite element model of spur gear system was developed. The research result shows that the theoretical methods presented in this thesis have fine simulation precision. This method could be applied to analyze engineering problem.
- Compare the difference between the excision spur gear systems with spur gears. The result shows that the strength of excision spur gear becomes small. In order to keep the same strength and improve the serve life, quenching process should be applied.
- Finite Element simulation of spur gear was developed and used to predict distributions of stress and other material properties. Thermo-elastic–plastic constitutive equation coupled with the mechanical strain, thermal strain, phase
transformation strain, and transformation induced plasticity is described in detail.

The result of quenching proved the theory and ensured product quality.

1.4 Layout of Thesis

Chapter I presents a general introduction, and the objectives to be achieved. It also gives the newest research about spur gear systems. Finally the layout of the thesis is described. The new mathematical model was developed in chapter II through formulation of Hertz Contact and Finite Element Method. The three dimension software Pro/Engineer was applied to design the spur gear system. A detailed FE-based computational model was developed in Chapter III and a simplified discrete model of spur gear designed to predict the stress distribution under the same speed and moment. At the end, the predictions of both models are validated through comparisons to extensive theory calculation data. Chapter IV focuses on FEA model of excision spur gear and compared the difference with FE-based deformable-body model developed in Chapter III. Chapter V presents the base quenching theory, applies the FEA to simulate quenching and improves the strength of spur gear. Chapter VI summaries the conclusions from this thesis and states its contributions to gear design. A number of recommendations for future work are listed in Chapter VII.
CHAPTER II
BUILDING 3D SPUR GEAR MODEL

2.1 Basic theory about spur gear

Gears can be divided into three major classes: parallel-axis gears, nonparallel but coplanar gears, and nonparallel and non-coplanar gears. [15] Parallel-axis gears are the simplest and the most universal type of gear. They may transfer very much power and the high efficiency in this classification, while spur gear is the main kind of gears. From AGMA and others books, the basic dynamic motion theory of spur gear developed. [16-17]

The location of gear element in three dimensional spaces (Figure 2.1) may be represented in vector form:

\[ r_n = x_n i_n + y_n j_n + z_n k_n \]  

Where \((i_n, j_n, k_n)\) are the unit vectors of coordinate axes as the column matrix,

\[ r_* = \begin{bmatrix} x_* \\ y_* \\ z_* \end{bmatrix} \]  

The subscript “n” presents as the position in the coordinate system \(S_n(X_n, Y_n, Z_n)\). This Matrix also can be written as a row matrix:

\[ r_* = [X_n, Y_n, Z_n]^T \]  

The superscript “\(T\)” indicates that \(r_n^T\) is a transpose matrix with respect to \(r_n\).
Generally, point \( n \) is determined in Cartesian coordinates with three numbers: \( x, y, z \).

Matrix operations of coordinate transformation should derive from multiplication and addition of matrices operations. Application of such coordinates for coordinate transformation was introduced by Litvin’s theory. \[18\] Homogenous coordinates of a point in 3D space are present as four numbers \((x', y', z', t')\), if these four numbers are not as zero simultaneously and of which only three are independent.

\[
X = \frac{x'}{t'} \quad y = \frac{y'}{t'} \quad z = \frac{z'}{t'} \quad (2.4)
\]

With \( t' = 1 \), a point may be specified by homogenous coordinates \((x, y, z, 1)\), and a position vector can be listed as following

\[
r_n = \begin{bmatrix}
x_n \\
y_n \\
z_n \\
1
\end{bmatrix} \quad \text{or} \quad r_n = [X, Y, Z, 1]^T
\]
Two coordinate systems $S_n (X_n, Y_n, Z_n)$ and $S_m (X_m, Y_m, Z_m)$ are shown in figure 2.1.

Point N is listed by the position vector in coordinate system $S_n$.

$$ r_n = [X_n, Y_n, Z_n, 1]^T $$  \hspace{1cm} (2.5)

The same point N can be transferred in coordinate system $S_m$ by the position vector.

$$ r_m = [X_m, Y_m, Z_m, 1]^T $$  \hspace{1cm} (2.6)

So the matrix equation

$$ r_m = M_{mn} r_n $$  \hspace{1cm} (2.7)

Matrix $M_{mn}$ is represented by

$$ M_{mn} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$
(i_n, j_n, k_n) are the unite vectors of the axis of the “old” coordinate system, (i_m, j_m, k_m) are the unite vectors of the axis of the “new” coordinate system, O_m and O_n are the origins point in the two coordinate systems. M_{mn} is the transform vector form S_n to S_m.

(2.8)

\[
\begin{bmatrix}
  i_m, i_n & i_m, j_n & i_m, k_n & O_mO_n, i_m \\
  j_m, i_n & j_m, j_n & j_m, k_n & O_mO_n, j_m \\
  k_m, i_n & k_m, j_n & k_m, k_n & O_mO_n, k_m \\
  0 & 0 & 0 & 1
\end{bmatrix}
\]

The rotation is performed about an axis in the coordinate system. Consider the initial position is designated as OA = \( \rho \). After rotation though an angle \( \Phi \), vector OA will be a new position designated as OA* = \( \rho^* \). These two vectors are considered in the same coordinate system S_a. Matrix equation relating components \( \rho_a \) and \( \rho_a^* \) developed

\[
\rho_a^* = L_a \rho_a
\] (2.9)
The other coordinate system $S_b$ represents the same position vector as the matrix equation

$$\rho_b = L_{ab} \rho_a$$  \hspace{2cm} (2.10)

$\rho_a$ and $\rho_b$ designate them as

$$\rho_a = a_1 i_a + a_2 j_a + a_3 k_a$$  \hspace{2cm} (2.11)

$$\rho_b = a_1 i_b + a_2 j_b + a_3 k_b$$  \hspace{2cm} (2.12)

The derivations process is as follows. \[19\]

$$\rho_a^* = \rho_a + \rho_a^* - \rho_a$$  \hspace{2cm} (2.13)

Where $c_a$ is the unite vector of the axis of rotation represented in $S_a$.

$$\rho_a^* = (c_a \cdot \rho_a^*) c_a$$  \hspace{2cm} (2.14)

$$\rho_a^* = [\rho_a - (c_a \cdot \rho_a) c_a] \cos \Phi$$  \hspace{2cm} (2.15)

$$\rho_a^* = \rho_a \sin \alpha$$  \hspace{2cm} (2.16)

Equations (2.13), (2.14), (2.15), (2.16) yield

$$\rho_b^* = \rho_b + (1 - \cos \phi) [c_a \times (c_a \times \rho_a)] + \sin \phi (c_a \times \rho_a)$$  \hspace{2cm} (2.17)

Derivation of coordinate transformation by rotation is listed:

$$L_{ba} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$  \hspace{2cm} (2.18)

Where

$$a_{11} = \cos \Phi (1 - c_1^2) + c_1^2$$
\[ a_{12} = (1 - \cos \Phi) c_1 c_2 \pm \sin \Phi c_3 \]

\[ a_{13} = (1 - \cos \Phi) c_1 c_3 \pm \sin \Phi c_2 \]

\[ a_{21} = (1 - \cos \Phi) c_1 c_2 \pm \sin \Phi c_3 \]

\[ a_{22} = \cos \Phi (1 - c_2) \pm c_2^2 \]

\[ a_{23} = (1 - \cos \Phi) c_2 c_3 \pm \sin \Phi c_1 \]

\[ a_{31} = (1 - \cos \Phi) c_1 c_3 \pm \sin \Phi c_2 \]

\[ a_{32} = (1 - \cos \Phi) c_2 c_3 \pm \sin \Phi c_1 \]

\[ a_{33} = \cos \Phi (1 - c_3^2) \pm c_3^2 \]

Figure 2.4 Spur gear in rotation motions
Two spur gears rotate between parallel axes in two coordinate systems S1 \((x_1, y_1, z_1)\) and S2 \((x_2, y_2, z_2)\). Based on the rotation theory, angles of gear rotation \(\Phi_1\) and \(\Phi_2\) relate with the diameter.

\[
\frac{\phi_1}{\phi_2} = \frac{\rho_1}{\rho_2} \quad (2.19)
\]

The matrix equation based on the coordinate transformation from S1 to S2

\[
r_1 = M_{12} r_2 = M_{1f} M_{fp} M_{p2} r_2 \quad (2.20)
\]

Where

\[
r_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix}, \quad M_{p2} = \begin{bmatrix} \cos \phi_2 & \sin \phi_2 & 0 & 0 \\ -\sin \phi_2 & \cos \phi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
r_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}, \quad M_{1f} = \begin{bmatrix} \cos \phi_1 & \sin \phi_1 & 0 & 0 \\ -\sin \phi_1 & \cos \phi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
M_{fp} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & E \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.21)
\]

From the equation (2.21) obtain
\[ M_{12} = \begin{bmatrix} \cos(\phi_1 + \phi_2) & \sin(\phi_1 + \phi_2) & 0 & E \sin \phi_1 \\ -\sin(\phi_1 + \phi_2) & \cos(\phi_1 + \phi_2) & 0 & E \sin \phi_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]  

(2.22)

Use these above equation

\[ x_1 = x_2 \cos(\Phi_1 + \Phi_2) + y_2 \sin(\Phi_1 + \Phi_2) + E \sin \Phi_1 \]

\[ y_1 = -x_2 \sin(\Phi_1 + \Phi_2) + y_2 \cos(\Phi_1 + \Phi_2) + E \cos \Phi_1 \]  

(2.23)

\[ z_1 = z_2 \]

Relative velocity should be considered for the friction force

\[ V^{12}_f = [-y_f (\omega^1 + \omega^2)] t_f + [x_f (\omega^1 + \omega^2) + E \omega^2] j_f \]  

(2.24)

Kinematic analysis

At present, the international gear theory on the stress basis of calculation (ISO international gear standard (1978), AGMA American gear standard (1982), BS436 English gear standard (1986), DIN3990 German gear standard (1987) and so on) [20-23] as Lewis equation and Hertz theory have formulated the gear root bending strength and the gear surface anti-pitting capacity rating method. In this thesis, Hertz theory is used to analyze the gear system.

Spur gear is 3D model, each gear has 5 degrees of freedom (three transfer motions, two rotate motions), and its equation of motion may represent as

\[ [m] \{\ddot{s}\} + [c]\{\dot{s}\} + [k]\{s\} = \{F\} \]  

(2.25)
Where $\{\delta\}$ transfer vector,

$$\{\delta\} = \{x_1, y_1, z_1, y_{10z}, x_2, y_2, z_2, y_{20y}, y_{20z}\}^T$$  \hspace{1cm} (2.26)

[m] quality matrix, [c] damping matrix, [k] rigidity matrix, \{F\} strength vector.

If the gear meshing the cross section rigidity and surface damping are known, the equations are shown as follows:

$$K_{mx} = k_t \tan \alpha_n, \quad K_{my} = k_t, \quad K_{mc} = 0$$  \hspace{1cm} (2.27)

$$C_{mx} = c_t \tan \alpha_n, \quad C_{my} = c_t, \quad C_{mc} = 0$$  \hspace{1cm} (2.28)

Figure 2.5 Kinematic analysis of Spur gear in rotation motions
So, the corresponding dynamic meshing strength is

\[ F_x = k_{mx}(x_1-x_2) + c_{mc}(\dot{x}_1\dot{x}_2) \]

\[ = k_1 \tan \alpha_n [x_2 \cos(\Phi_1 + \Phi_2) + y_2 \sin(\Phi_1 + \Phi_2) + E \sin(\Phi_1-x_2)] + c_1 \tan \alpha_n [\dot{x}_2 \cos(\Phi_1 + \Phi_2) - x_2 \sin(\Phi_1 + \Phi_2)(\dot{\Phi}_1 + \dot{\Phi}_2) + \ddot{y}_2 \sin(\Phi_1 + \Phi_2) + y_2 \cos(\Phi_1 + \Phi_2)(\dot{\Phi}_1 + \dot{\Phi}_2) + E \sin(\Phi_1-x_2)] \]

\[ = k_1 \tan \alpha_n [x_2 (\cos(\Phi_1 + \Phi_2)-1) + y_2 \sin(\Phi_1 + \Phi_2) + E \sin(\Phi_1)] + c_1 \tan \alpha_n [\dot{x}_2 (\cos(\Phi_1 + \Phi_2)-1) + (\dot{\Phi}_1 + \dot{\Phi}_2)(y_2 \cos(\Phi_1 + \Phi_2) - x_2 \sin(\Phi_1 + \Phi_2)) + \ddot{y}_2 \sin(\Phi_1 + \Phi_2) + E \sin(\Phi_1)] \quad (2.29) \]

\[ F_y = k_{my}(y_1-y_2) + c_{yc}(\dot{y}_1\dot{y}_2) \]

\[ = k_1 [-x_2 \sin(\Phi_1 + \Phi_2) + y_2 \cos(\Phi_1 + \Phi_2) + E \cos(\Phi_1-y_2)] + c_1 [-x_2 \sin(\Phi_1 + \Phi_2) - x_2 \cos(\Phi_1 + \Phi_2)(\dot{\Phi}_1 + \dot{\Phi}_2) + \ddot{y}_2 \sin(\Phi_1 + \Phi_2) - y_2 \cos(\Phi_1 + \Phi_2)(\dot{\Phi}_1 + \dot{\Phi}_2) - E \sin(\Phi_1-y_2)] \]

\[ = k_1[y_2 \cos(\Phi_1 + \Phi_2)-1] - x_2 \sin(\Phi_1 + \Phi_2) + E \cos(\Phi_1) - c_1 [(\dot{\Phi}_1 + \dot{\Phi}_2)(x_2 \cos(\Phi_1 + \Phi_2) + y_2 \sin(\Phi_1 + \Phi_2)) + \ddot{y}_2 \sin(\Phi_1 + \Phi_2) + \dot{y}_2 (1-\cos(\Phi_1 + \Phi_2)) + E \sin(\Phi_1)] \quad (2.30) \]

\[ F_z = k_{mz}(z_1-z_2) + c_{zc}(\dot{z}_1\dot{z}_2) \]

\[ = 0 \quad (2.31) \]

May obtain the spur gear system analysis model

\[ M_{1x} \ddot{x}_1 + c_{1x} \dot{x}_1 + k_{1x} x_1 = -F_x \]

\[ M_{1y} \ddot{y}_1 + c_{1y} \dot{y}_1 + k_{1y} y_1 = -F_y \]
\begin{align*}
M_{1x} \dddot{x}_1 + c_{1x} \dot{x}_1 + k_{1x} x_1 &= F_x = 0 \\
J_{1x} \dddot{\Phi}_{1x} + c_{1\Phi x} \dot{\Phi}_{1x} + k_{1\Phi x} \Phi_{1x} &= -F_z R_1 \\
I_{1z} \dddot{\Phi}_{1z} &= -T_1 - F_x R_1 \\
M_{2x} \dddot{x}_2 + c_{2x} \dot{x}_2 + k_{2x} x_2 &= F_x \\
M_{2y} \dddot{y}_2 + c_{2y} \dot{y}_2 + k_{2y} y_2 &= F_y \\
M_{2z} \dddot{z}_2 + c_{2z} \dot{z}_2 + k_{2z} z_2 &= -F_z = 0 \\
J_{2x} \dddot{\Phi}_{2x} + c_{2\Phi x} \dot{\Phi}_{2x} + k_{2\Phi x} \Phi_{2x} &= -F_z R_2 \\
I_{2z} \dddot{\Phi}_{2z} &= -T_2 - F_x R_2 \quad (2.32) \\
\end{align*}

Each dynamic meshing strength substitution in above formula

\begin{align*}
M_{1x} \dddot{x}_1 + c_{1x} \dot{x}_1 + k_{1x} x_1 &= -k_1 \tan(\alpha_n) [x_2 \cos(\Phi_1 + \Phi_2) - 1] + y_2 \sin(\Phi_1 + \Phi_2) + E \sin \Phi_1 + \\
&\quad + c_1 \tan(\alpha_n) [x_2 \cos(\Phi_1 + \Phi_2) - 1] + (\dot{\Phi}_1 + \dot{\Phi}_2) [y_2 \cos(\Phi_1 + \Phi_2) - \\
&\quad - x_2 \sin(\Phi_1 + \Phi_2)] + \dddot{x}_2 \sin(\Phi_1 + \Phi_2) + E \sin \Phi_1 \Phi_{1z} \quad (2.33) \\
M_{1y} \dddot{y}_1 + c_{1y} \dot{y}_1 + k_{1y} y_1 &= -k_1 [y_2 \cos(\Phi_1 + \Phi_2) - 1] - x_2 \sin(\Phi_1 + \Phi_2) + E \cos \Phi_1 + \\
&\quad + c_1 [(\dot{\Phi}_1 + \dot{\Phi}_2) (x_2 \cos(\Phi_1 + \Phi_2) + y_2 \sin(\Phi_1 + \Phi_2)) + \\
&\quad + \dddot{x}_2 \sin(\Phi_1 + \Phi_2) + \dddot{y}_2 (1 - \cos(\Phi_1 + \Phi_2)) + E \sin \Phi_1] \quad (2.34) \\
M_{1z} \dddot{z}_1 + c_{1z} \dot{z}_1 + k_{1z} z_1 &= 0 \quad (2.35) 
\end{align*}
\[
\begin{align*}
J_{1x} \Phi_{1x} + c_{1\Phi x} \Phi_{1x} + k_{1\Phi x} \Phi_{1x} = - F_z R_1 & = 0 \quad \text{(2.36)} \\
I_{1z} \Phi_{1z} = - T_1 - k_1 R_1 \tan \alpha \left[ x_2 \cos(\Phi_1 + \Phi_2) - 1 \right] + y_2 \sin(\Phi_1 + \Phi_2) + E \sin \Phi_1 + \\
& + c_t \tan \alpha \left[ x_2 \cos(\Phi_1 + \Phi_2) - 1 \right] + (\Phi_1 + \Phi_2) \left( y_2 \cos(\Phi_1 + \Phi_2) - x_2 \sin(\Phi_1 + \Phi_2) \right) \\
& + y_2 \sin(\Phi_1 + \Phi_2) + E \sin \Phi_1 \Phi_1 \quad \text{(2.37)} \\
M_{2x} \Phi_{2x} + c_{2x} \Phi_{2x} + k_{2x} \Phi_{2x} = k_1 \tan \alpha \left[ x_2 \cos(\Phi_1 + \Phi_2) - 1 \right] + y_2 \sin(\Phi_1 + \Phi_2) + E \sin \Phi_1 + \\
& + c_t \tan \alpha \left[ x_2 \cos(\Phi_1 + \Phi_2) - 1 \right] + (\Phi_1 + \Phi_2) \left( y_2 \cos(\Phi_1 + \Phi_2) - \\
& x_2 \sin(\Phi_1 + \Phi_2) \right) & + y_2 \sin(\Phi_1 + \Phi_2) + E \sin \Phi_1 \Phi_1 \quad \text{(2.38)} \\
M_{2y} \Phi_{2y} + c_{2y} \Phi_{2y} + k_{2y} \Phi_{2y} = k_2 \left[ y_2 \cos(\Phi_1 + \Phi_2) - 1 \right] - x_2 \sin(\Phi_1 + \Phi_2) + E \cos \Phi_1 + \\
& + c_t \left[ (\Phi_1 + \Phi_2) \left( x_2 \cos(\Phi_1 + \Phi_2) + y_2 \sin(\Phi_1 + \Phi_2) \right) + \\
& x_2 \sin(\Phi_1 + \Phi_2) \right] + y_2 \left( 1 - \cos(\Phi_1 + \Phi_2) \right) + E \sin \Phi_1 \quad \text{(2.39)} \\
M_{2z} \Phi_{2z} + c_{2z} \Phi_{2z} + k_{2z} \Phi_{2z} = - F_z & = 0 \quad \text{(2.40)} \\
J_{2x} \Phi_{2x} + c_{2\Phi x} \Phi_{2x} + k_{2\Phi x} \Phi_{2x} = - F_z R_2 & = 0 \quad \text{(2.41)} \\
I_{2z} \Phi_{2z} = - T_2 - k_2 R_2 \tan \alpha \left[ x_2 \cos(\Phi_1 + \Phi_2) - 1 \right] + y_2 \sin(\Phi_1 + \Phi_2) + E \sin \Phi_1 + \\
& + c_t \tan \alpha \left[ x_2 \cos(\Phi_1 + \Phi_2) - 1 \right] + (\Phi_1 + \Phi_2) \left( y_2 \cos(\Phi_1 + \Phi_2) \right) \\
& - x_2 \sin(\Phi_1 + \Phi_2) \right) & + y_2 \sin(\Phi_1 + \Phi_2) + E \sin \Phi_1 \Phi_1 \quad \text{(2.42)}
\end{align*}
\]
2.2 Discuss the parameterization modeling based on Pro/Engineer

1) Introduction of Pro/Engineer

In recent years, the computer has become a powerful tool in design and manufacturing. CAD/CAM systems (Computer Aided Designing and Computer Aided Manufacturing) can increase design accuracy, reduce lead times and improve overall engineering productivity in the design and manufacturing industry. [24] Pro/Engineer is one of new CAD/CAE/CAM software in the world, featuring the best operation in the design. The advantages are listed as the following: [25]

➢ Formidable, parameter design function permission, superior product differentiated and manufacturability.

➢ Integrates the application to develop out from the concept to the manufacture within one kind of application.

➢ The design change system allows you to float variable.

➢ Completes the virtual simulation function to enable you to improve the product performance and to surpass the product quality goal.

2) Basic parameterization process of Spur Gear

➢ Base circle building

In this study, we apply the relationship equation to design the spur gear. At first, the basic circle was built in the drawing. Secondly, parameter name and type were set. The relationships between each kind of parameter of these steps were added. The pitch line, the base circle, the addendum circle and the root circle...
basic circle size of gear are determined. The Geometry of gear set is listed as following:

Table 2.1 Geometry of the gear set

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teeth number of driving gear</td>
<td>( z_{di} )</td>
<td>20</td>
</tr>
<tr>
<td>Teeth number of driven gear</td>
<td>( z_{dn} )</td>
<td>20</td>
</tr>
<tr>
<td>Pressure angle</td>
<td>( \alpha_n )</td>
<td>20’</td>
</tr>
<tr>
<td>Module (mm)</td>
<td>( m )</td>
<td>4</td>
</tr>
<tr>
<td>Circular pith (mm)</td>
<td>( P_h = \pi \times m )</td>
<td>12.5636</td>
</tr>
<tr>
<td>Pitch diameter (mm)</td>
<td>( d = m \times z )</td>
<td>80</td>
</tr>
<tr>
<td>Addendum height (mm)</td>
<td>( h_a = 1 \times m )</td>
<td>4</td>
</tr>
<tr>
<td>Dedendum height (mm)</td>
<td>( h_g = 1.25 \times m )</td>
<td>5</td>
</tr>
<tr>
<td>Addendum circle (mm)</td>
<td>( d_a = d + 2 \times m )</td>
<td>88</td>
</tr>
<tr>
<td>Dedendum circle (mm)</td>
<td>( d_d = d - (2 + \pi/z) \times m )</td>
<td>70</td>
</tr>
<tr>
<td>Base Circle (mm)</td>
<td>( d_b = d \times \cos \alpha_n )</td>
<td>75.12754</td>
</tr>
<tr>
<td>Face width (mm)</td>
<td>( b )</td>
<td>20</td>
</tr>
</tbody>
</table>
Tooth’s drawing

In this design, the gear outline basic establishment and the gear crown contour line must be consistent with the addendum circle position; the gear base contour line should be super consistent with the root circle position. Then, we set their parameter name in turn, give the corresponding relational equation. Lastly, the final gear contour line was regenerated.
Figure 2.7 the final gear contour line

The main relations are the equation tabulation is as follows.

Table 2.2 Equation tabulation of tooth

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tooth radius (mm)</td>
<td>Tooth_rad= d/8</td>
<td>10</td>
</tr>
<tr>
<td>Half tooth thick (mm)</td>
<td>Half_tooth_tk=3.1415*m/4</td>
<td>3.1415</td>
</tr>
<tr>
<td>Tip radius (mm)</td>
<td>Tip_rad=3.1415*m/8</td>
<td>1.57</td>
</tr>
</tbody>
</table>
Building tooth in Pro/engineer

In order to establish the final 3D gear precise model, first, we should stretch the tooth as face width. Secondly, we take the base circle center of a circle as the center of rotation, the degrees rotation is 360/number of gear. The elemental operation process is as follows:

Copy-move/select/ independent/done

Make the tooth curve – done

Rotate

Csys

Select PRT_CSYS_DEF

Z axis

Flip/Okey

Input the number : 360/no_of_teeth

Done move

Done

Ok

At last through arraying operation, all gear outline formation is implemented. With stretching operating procedure, final 3D gear precise model is established.
Figure 2.8 the teeth of spur gear

- Gear internal entity establishment and axis hole excision

  Stretch along the root circle path to the same length of the tooth. Cut a hole in the center section so that we install an axis and analyze next step.
From the above design process, its outline character is completed with the relational equation. This method guarantees the gear accuracy and provides the reliable design for the model analysis and the processing.
CHAPTER III

FINITE ELEMENT ANALYSIS OF SPUR GEAR

3.1 Introduction of finite element software-Abaqus

Abaqus is the product by Hibbitt, Karlsson & Scorensen, Inc, which is established in 1978. Abaqus/CAE (complete ABAQUS Environment) is a complete Abaqus environment that provides a simple, consistent interface for creating, submitting, monitoring, and evaluating the result from Abaqus/Stand and Abaqus/Explicit simulations. Relations among modules are shown as follows:

![Figure 3.1 relations among modules](image)

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3.2 Three dimension FEA deformable gear model

1) Contact function introduction

Assuming two objects have the contact, we usually define the name of object contacts. Moreover, one is the goal body. One of the contact surfaces is called the main contact face. The two contact objects must satisfy the following condition.

The law mainly satisfies the non-penetration restraint to the contact condition; regarding the point \( p \) of \( S^A \) surface as well as on the point \( q \) of the \( S^B \) surface should have the following relationship:

\[
g_n = g(x_p^A, t) = (x_p^A - x_q^B) \cdot n_q^B \tag{3.1}
\]

\( g_n \) represents contact two distances: \( x_p^A, x_q^B \) represents two point \( (p, q) \) coordinates separately. \( n_q^B \) represents \( p \) to \( q \) direction vector.

![Figure 3.2 Non-penetration contact restraint](image)

The Abaqus contact algorithm basic flow is as follows
Figure 3.3 The Abaqus contact algorithm basic flow
The gear drive organization may move back and forth, change the direction of velocity and other functions is extremely broad in each transmission system application. In the gear transmission process, the strength produces through the gear surface between contact transmissions, the complex contact relations as well as from this the stress which produces between the tooth face, the impact and so on. It often causes failure to a gear, such as the gear breaks off, the tooth face plastic flow, the tooth face wears, agglutination and so on. Studying the gear design is important content during the gear failing. The contact simulation can prove that theory. The method can be used to calculate the contact position, the area, the contact strength, the stress distribution as well as the plastic flow rule in the gear drive process. It can help to study the failure of gears, improve the gear design and enhance the efficiency and the service life of the gear drive system.

2) Modeling solution

In this modeling solution process, the gear contact definition and the gear grid partition are the key steps. The gear finite element stress analysis flow diagram list is as follows:
Input the 3D gear model and built the axis part

Define the material property for the gear

Assembles all parts

Define analysis process and output parameter

Define interaction

Define finite element boundary condition and load condition

Partition mesh

Analyze FEA model

Output the report and analyze gear system

Figure 3.4 spur gear finite element stress analysis flow diagram
1) Create the part

The gear was transferred by the exchange standards as IGES from Pro/engineer to finite element software. The entire gear is defined as the geometry set. Other surface except end surface and axis hole, key slot is defined as Surf-gear- contact. The surface of the end surface and axis hole, key slot is defined as Surf-gear- inner.

The model is focused on the stress of the gear, not the spindle, so this study defines the spindle as a discrete part and neglects the character of the spindle in dynamic analysis. The zero point is defined as the reference point. Gear outside surface is selected as Surf-axis.

The gear and axis copy to gear2 and axis2. They also build the same parameters: surface, reference point and define different name.

2) Define material property

The material property is input to the gear set.

Table 3.1 Material properties of gear set

<table>
<thead>
<tr>
<th>Description</th>
<th>Material property</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material of driving gear</td>
<td>Steel</td>
</tr>
<tr>
<td>Material of driven gear</td>
<td>Steel</td>
</tr>
<tr>
<td>Density of driving gear (Kg/m³)</td>
<td>7800</td>
</tr>
<tr>
<td>Density of driven gear (Kg/m³)</td>
<td>7800</td>
</tr>
<tr>
<td>Young’s module of driving gear (Mpa)</td>
<td>210X10³</td>
</tr>
</tbody>
</table>
Table 3.1 Material properties of gear set (continued)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s module of driven gear (Mpa)</td>
<td>$210 \times 10^3$</td>
</tr>
<tr>
<td>Poisson ratio of driving gear (Mpa)</td>
<td>0.3</td>
</tr>
<tr>
<td>Poisson ratio of driven gear (Mpa)</td>
<td>0.3</td>
</tr>
</tbody>
</table>

3) Assembly

Gear, axis, gear2 and axis2 are selected to assemble environment as a dependent part. The gear2 and axis2 move to the appropriate position. The adjustment contact relations between the spur gear set are shown in figure 3.5. The cylindrical coordinate system is defined as Assem-CSYS-CYL.

Figure 3.5 the assembly of spur gear
4) Define the analysis step

The analysis step is defined as Dynamic Explicit with the time period as 2s. The geometry non-linear button is clicked at the same time. A 3D FE-based deformable gear model can be used in predicting the characteristics of gear dynamic accurately. Even though a 3D FE-based deformable gear model can get the perfect result, it has a major disadvantage. It would take a long time and a great computational effort to calculate. For example, this system has the two same gears and has (8145) mesh required more than a week on the PC 3GHz processor and 1G memory. In the guarantee computation precision situation, the selection quality magnification factor defines as 10000. It can speed up the computation speed and save time. The contacted area and the contact strength output of the gear are defined in this step.

5) Interaction

The friction is one kind of complex phenomenon, with contact surface degree of hardness, humidity, normal stress and relative interaction speed, but its mechanism was still the research difficult problem. When two surfaces have the contact and the transmission produces between the contact face, in ordinary circumstances, parts resolve into the tangential force and the normal force; friction force must be considered between the contact face in the analysis. ABAQUS software has these friction models: coulomb friction, penalty friction, Lagrange friction, explicit friction model. In this FEA model, the coulomb friction was used. Coulomb friction between the contact surfaces affect mutual friction model; it also broadly applies in many processing analysis and in the other existence friction problems.

Coulomb friction model list as follow:
\[ \tau \leq -\mu p t \]  \hspace{1cm} (3.2)

\( \tau \) is tangential frictional stress; \( \mu \) as friction coefficient; \( p \) as Contact node-pair method to stress; \( t \) as In sliding speed direction tangential unit vector.

Friction has a tremendous influence in FEA model including the friction. The friction coefficient is defined as 0.1 in this study.

When FEA software calculates between two contact surfaces and the mesh size of two contact surfaces is different, the computation time has a very big difference. According to two contact surfaces of the displacement size and the unit size, Abaqus describes two kinds of different contact formulas between two contact faces: small slipping and finite slipping. In this study, finite slipping was applied in FEA model.

In the analysis process, Abaqus needs to judge which part has the contact between the surface node and main surface node. The computation time is longer with small slipping, special for the deformable body. The two-dimensional structure and the three dimensional structure both can apply the finite slipping. It requires that the main surface should be smoother than the other surface; otherwise, it cannot get the perfect result.

The slipping and the friction contact were the two most important attributes in the simulation. In order to get quite precise contact simulation, it is insufficient to consider only these two contact factors. Other contact parameters should be considered: law functional and restraint relations between contact faces.
The distance between two contract surfaces was named as clearance. When the clearance becomes zero, two surfaces begin to contact and exert the contact restraint in the corresponding node. As a result the contact pressure is generated between the contact face. When contact pressure becomes the zero or the negative value, two contacts faces separate, simultaneously the corresponding contact node and the contact restraint separate, too. This kind of contact behavior is called hard contact in Abaqus. Hard contact was applied in this FEA model.

We modify field variable output frequency as 100 so that the animation continuity becomes better. The output variable: CFN, CFT, CAREA, which outputs the contact force, contact strength and the contacted area between the gears is added. In this study, we do not consider the stress distribution of axis. So Axis tie with spur gear so that axis and spur gear have the same movement. Tie restrained all stable motion degrees of freedom and two surfaces are fettered together in the analysis process. Each node of the second surface has the same movement with the node of the master surface, so it is helpful to eliminate rigid body displacement and reduce greatly the computation time. However, the tie model requires that the master surface must be harder rather than the other surface. Applying rotate velocity as -3.0 m/s and the moment as 1500N/m with driving gear at the same time. Thus, under the contact action, driven gear is possible to revolve with its axle. Then the stress distribution and the contacted area between the two gears can be studied.
6) Mesh

How to mesh the gear teeth part is the key research area during contact analysis of the spur gear, especially the gear tooth root part is easiest to appear in the stress concentration. For saving the simulation time and ensuring the calculation accuracy, FEM mesh in the region of the contact area is small and FEM mesh in the region apart from the contact area is large.

The hexahedron has the higher computation precision than the tetrahedron, so this model applies the hexahedron as the mesh element. However, this model cannot apply the hexahedron naturally. Therefore, gear must be separated along the root circle. The high seed number in the gear contact face and the axis is defined. Element type as 3D stress is assigned at the same time. (C3D8R)

<table>
<thead>
<tr>
<th>Table 3.2 Data of FE model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driving Gear</td>
</tr>
<tr>
<td>Node</td>
</tr>
<tr>
<td>Element</td>
</tr>
</tbody>
</table>
7) Result

The model task is submitted to analyze, if it cannot obtain the result, the corresponding modification based on the software showing should be changed. If this model runs successfully and obtains the corresponding analysis result at last, we can examine the final output and carry on the analysis.

- Check the stress distribution of spur gear system at 2s. Also show the sometime radial, tangential, axial direction stress distribution in the cylindrical coordinate system. (S11, S22, S33)
Figure 3.7 the stress distribution at 2s

Figure 3.8 X, Y, Z direction stress distribution at 2s
The driving spur gear and the driven spur gear contact stress are equal, but the vector direction is reverse. When the tooth of the spur gear contact gradually, the contact pressure also increases gradually. Then, the tooth of spur gear separates gradually, the contact pressure decreases gradually to zero. It will periodically change.
The contract area

![Figure 3.10 Contacted areas during 2s](image)

From the spur gear dynamic contacted area, the contacted area gradually achieves a stable value as the time grows gradually. The spur gear contact completely, then separated completely, in the periodic variation situation, caused the contacted area to have the same change. However, each spur gear contacted area is basically equal. This proved basically that the finite element model is correct and feasible.
The kinetic energy

Figure 3.11 the kinetic energy for whole model

The kinetic energy starts after the contact, carries on the periodic change under the velocity and moment. However, the lowest number of kinetic energy increases with time, so this gear system belongs to the middle shock system. It explains the spur gear contacts discontinuity.
Intern energy for whole model

Figure 3.12 Internal energy for whole model

The internal energy starts after the contact, as a result of the impact influence, can have a very great value, then periodically shake change.

The stress report of the driving gear is listed as follows:


Source 1

ODB: d:/Temp/gear-load-contract.odb

Step: Apply-BCs-and-forces

Frame: Increment 38623: Step Time = 2.000
Loc 1: Element nodal values from source 1 (Average criteria = 75%, Not averaged across region boundaries )

Output sorted by column "Element Label".

Field Output reported at element nodes for part: GEAR-1

Computation algorithm: EXTRAPOLATE_COMPUTE_AVERAGE

Averaged at nodes

Averaging regions: ODB_REGIONS

<table>
<thead>
<tr>
<th>Element</th>
<th>Node</th>
<th>S.Mises</th>
<th>S.S11</th>
<th>S.S22</th>
<th>S.S33</th>
</tr>
</thead>
<tbody>
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<td>@Loc 1</td>
<td>@Loc 1</td>
<td>@Loc 1</td>
<td>@Loc 1</td>
</tr>
<tr>
<td>---------</td>
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<tr>
<td>1</td>
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<td>324.782E+06</td>
<td>30.6172E+06</td>
<td>275.781E+06</td>
<td>94.5856E+06</td>
</tr>
<tr>
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<td>1</td>
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<td>109.921E+06</td>
<td>568.533E+06</td>
<td>205.497E+06</td>
</tr>
<tr>
<td>12475</td>
<td>8182</td>
<td>347.588E+06</td>
<td>-171.262E+06</td>
<td>139.918E+06</td>
<td>-7.87065E+06</td>
</tr>
<tr>
<td>12475</td>
<td>8175</td>
<td>407.413E+06</td>
<td>-300.377E+06</td>
<td>99.5222E+06</td>
<td>-58.4967E+06</td>
</tr>
<tr>
<td>Minimum</td>
<td></td>
<td>13.7555E+06</td>
<td>-3.27893E+09</td>
<td>-2.07998E+09</td>
<td>-1.40249E+09</td>
</tr>
<tr>
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<td></td>
<td>10687</td>
<td>11932</td>
<td>6479</td>
<td>11932</td>
</tr>
<tr>
<td>Node</td>
<td></td>
<td>190</td>
<td>352</td>
<td>605</td>
<td>4833</td>
</tr>
</tbody>
</table>
From the report, the biggest stress appears in 7827 and unit 11738 nodes place, the greatest value is $3.10973 \times 10^9$. It is observed in Figure 3.13:

![Figure 3.13 Maximum stress positions](image)

From the above result, the stress value at the tooth root section and the gear contact section is higher than other places in the transmission process; therefore the root of the spur gear is easy to break off, the tooth face easy to appear the attrition, the agglutination
as well as plastic flow. It is consistent with the project application in an actual situation.

The gear contacted area and the contact stress change unceasingly. After the gear has the
constant speed, the kinetic energy of the gear also tends to be a fixed value.

The deformable-body model presents accurate results, but it has a major disadvantage:
it requires significant computational effort. For instance, a spur gear system (include 2
spur gears) analysis requires more than a week on a PC with a 3 GHz processor. This
makes the model as an analysis tool rather than being a design tool.

3.3 Spur Gear theory dynamic analysis

This study uses the discrete rigid gear under no friction as theory dynamic model. A
rigid body represents a theoretic approach for spur gear systems. A gear can be modeled
as a rigid body, including the mass, inertia and external influences. [27] In rigid - to -
flexible contact problems, one or more of the contacting surfaces are treated as being
rigid material, which has a much higher stiffness relative to the deformable body it
contacts. Many metal forming problems fall into this category. [28] The proposed of a
discrete model is computational efficiency and reasonable accurate when compared to the
prediction of the deformed model and theory calculation data. [29]

As an analysis rigid body, the gear neglects itself stress and obtains the theory stress
value in the final. The FE-based deformable-body models were presented to be
computationally demanding. However, being the statistically based on the capabilities of
the manufacturing processes and machine tools should give the manufacturing errors. For
the deformable body model analyses, considerable computational time would have to be
performed. In order to address this problem, based on the behavior observed from the
experiments and the deformable body model predictions, a simplified discrete model of a spur gear with speed sharing will be developed as the main objective of this chapter. The accuracy of the discrete model is quantified by comparing its predictions to the predictions of the computational model. [29] This condition is schematically illustrated in Figure 3.14.

Figure 3.14 the stress distribution at 2s in theory model

The model reduced the plastic deformation and the frictional influence on the driving gear, between the stresses of spur gear increases.
Contrasting the figure of contacted area between the deform model and the discrete rigid model, the contacted area graphics maximum value and tends to the stable value is nearly same. It basically explains the rigid body model is credible.

3.4 Theory calculation analysis

Contact Ratio

The circular pitch is

$$ p_c = \frac{\pi}{d} = \frac{\pi}{80} = 0.0393 \text{ mm} $$

The center distance is

$$ c_c = \frac{z_d}{2} + \frac{Z_{ds}}{d} = \frac{20}{2} + \frac{20}{80} = \frac{1}{4} = 0.25 \text{ mm} $$
The pitch circle radii for the driving gear and driven gear are

\[ r_{di} = \frac{z_d}{2d} = \frac{20}{2 \times 80} = 0.125 \text{ mm} \]

\[ r_{do} = \frac{z_d}{2d} = \frac{20}{2 \times 80} = 0.125 \text{ mm} \]

The base circle for the driving gear and driven gear are

\[ r_{bsi} = r_{di} \cos \phi = \frac{1}{8} \cos 20^\circ = 0.1175 \text{ mm} \]

\[ r_{bsn} = r_{do} \cos \phi = \frac{1}{8} \cos 20^\circ = 0.1175 \text{ mm} \]

The outside radii for the driving gear and driven gear are

\[ a = \frac{1}{d} = \frac{1}{80} = 0.0125 \text{ mm} \]

\[ r_{odi} = r_{di} + a = 0.125 + 0.0125 = 0.1375 \text{ mm} \]

\[ r_{odn} = r_{do} + a = 0.125 + 0.0125 = 0.1375 \text{ mm} \]

So the contact ratio is

\[
C_r = \frac{1}{p_c \cos \phi} \left[ \sqrt{r_{odi}^2 - r_{bsi}^2} + \sqrt{r_{odn}^2 - r_{bsn}^2} \right] - \frac{C_a \tan \phi}{p_c}
\]

\[
= \frac{1}{0.0393 \cos 20^\circ} \left[ 2 \sqrt{0.1375^2 - 0.1175^2} \right] - \frac{0.25 \tan 20^\circ}{0.0393}
\]

\[
= 1.5573
\]
Tooth load equations express as in SI units

\[ W_t = \frac{T}{
\hat{a}/2} \]  \hspace{1cm} (3.3)

\[ W_r = W_t \tan \alpha_n \]  \hspace{1cm} (3.4)

\[ W = \frac{W_t}{\cos \alpha_n} \]  \hspace{1cm} (3.5)

Figure 3.16 Forces acting on individual gear tooth.
Lewis equation was first developed by Wilfred Lewis in 1892 and has universal acceptance in the world. The bending stress can be expressed as

\[ \sigma = \frac{T}{I/c} \]  
(3.6)

\[ T = w_t L \]  
(3.7)

\[ I/c = \frac{1}{12} b t^3/(t/2) = bt^2/6 \]  
(3.8)

So equation 3.6 can becomes

\[ \sigma = \frac{6w_tL}{bt^2} \]  
(3.9)
From the triangles

\[
\tan \alpha = \frac{x}{t/2} = \frac{t/2}{L} \quad (3.10)
\]

So we can get from equation 3.10

\[
L = \frac{t^2}{4x} \quad (3.11)
\]

Substituting equation 3.11 to equation 3.9, the equation can be gotten

\[
\sigma = \frac{6wL}{b2} = \frac{w}{b} \frac{1}{t/2} = \frac{w}{b} \frac{1}{t/2} \frac{1}{4L} \frac{1}{4/6} = \frac{w}{b} \frac{1}{2} \frac{1}{3} \frac{1}{d} \quad (3.12)
\]

Where \(d\) is pitch diameter,

\[
Y = \frac{2w}{3d} = \text{Lewis form factor}
\]

So equation 3.9 changes as

\[
\sigma = \frac{w_r}{bdy} \quad (3.13)
\]

The Lewis form factors independent of tooth geometry. The number of Lewis form factors shows in the table 3.3
Table 3.3 Lewis form factors for various numbers of teeth (pressure angle 20°, full depth involute). [15]

<table>
<thead>
<tr>
<th>Number of Teeth</th>
<th>Lewis form factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.176</td>
</tr>
<tr>
<td>11</td>
<td>0.192</td>
</tr>
<tr>
<td>12</td>
<td>0.210</td>
</tr>
<tr>
<td>13</td>
<td>0.223</td>
</tr>
<tr>
<td>14</td>
<td>0.236</td>
</tr>
<tr>
<td>15</td>
<td>0.245</td>
</tr>
<tr>
<td>16</td>
<td>0.256</td>
</tr>
<tr>
<td>17</td>
<td>0.264</td>
</tr>
<tr>
<td>18</td>
<td>0.270</td>
</tr>
<tr>
<td>19</td>
<td>0.277</td>
</tr>
<tr>
<td>20</td>
<td>0.283</td>
</tr>
<tr>
<td>22</td>
<td>0.292</td>
</tr>
<tr>
<td>24</td>
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<tr>
<td>26</td>
<td>0.308</td>
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<tr>
<td>28</td>
<td>0.314</td>
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<tr>
<td>30</td>
<td>0.318</td>
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<tr>
<td>32</td>
<td>0.322</td>
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<td>34</td>
<td>0.325</td>
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<td>36</td>
<td>0.329</td>
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<tr>
<td>38</td>
<td>0.332</td>
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<tr>
<td>40</td>
<td>0.336</td>
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<tr>
<td>45</td>
<td>0.340</td>
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<tr>
<td>50</td>
<td>0.346</td>
</tr>
<tr>
<td>55</td>
<td>0.352</td>
</tr>
<tr>
<td>60</td>
<td>0.355</td>
</tr>
<tr>
<td>65</td>
<td>0.358</td>
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<tr>
<td>70</td>
<td>0.360</td>
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<tr>
<td>75</td>
<td>0.361</td>
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<tr>
<td>80</td>
<td>0.363</td>
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<tr>
<td>90</td>
<td>0.366</td>
</tr>
<tr>
<td>100</td>
<td>0.368</td>
</tr>
<tr>
<td>150</td>
<td>0.375</td>
</tr>
<tr>
<td>200</td>
<td>0.378</td>
</tr>
<tr>
<td>300</td>
<td>0.382</td>
</tr>
</tbody>
</table>

Equation 3.13 is known as the modified Lewis equation. Motte developed a more accurate model in 1992. [31]

\[
\sigma = \frac{wK_KK_v}{bdyK_v}
\]  

(3.14)

Where \(k_a\) is the application factor, \(k_s\) is the size factor, \(k_m\) is the load distribution factor, \(K_v\) is the dynamic factor. These numbers can be gotten from these tables and figures.
Table 3.4 Application factor $K_a$ as a function of driving power source and driven machine.

<table>
<thead>
<tr>
<th>Power Source</th>
<th>Uniform</th>
<th>Light shock</th>
<th>Moderate shock</th>
<th>Heavy shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>1.00</td>
<td>1.25</td>
<td>1.50</td>
<td>1.75</td>
</tr>
<tr>
<td>Light shock</td>
<td>1.20</td>
<td>1.40</td>
<td>1.75</td>
<td>2.25</td>
</tr>
<tr>
<td>Moderate shock</td>
<td>1.30</td>
<td>1.70</td>
<td>2.00</td>
<td>2.75</td>
</tr>
</tbody>
</table>

Table 3.5 size factor as a function of diametric pitch or module. [15]

<table>
<thead>
<tr>
<th>Diametral pitch $p_d$, in.-1</th>
<th>Module, $m$, mm</th>
<th>Size factor, $K_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq 5$</td>
<td>$\leq 5$</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>1.05</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>1.15</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>1.25</td>
</tr>
<tr>
<td>1.25</td>
<td>20</td>
<td>1.40</td>
</tr>
</tbody>
</table>

Figure 3.18 Load distribution factor as function of face width and ratio of face width to pitch diameters. [31]
The Hertzian contract stress of gear teeth is based on some assumptions:

- Assumes it has the maximum bending load at the tips. All of the torques applies only one tooth and maximum load occurs near the pitch circle.
- Only bending component of the force applying on the tooth and the compressive stress over the base cross section caused by the radial force.
- No considering contact stresses
- Assumes the loads are static.

The maximum Hertzian pressure in the contact can be expressed as

$$P_h = E' \left( \frac{W}{2\pi} \right)^{1/2}$$  \hspace{1cm} (3.15)

Where

$$E' = \frac{2}{\frac{1-V_r^2}{E_r} + \frac{1-V_b^2}{E_b}} = \text{effective modulus of elasticity}$$
\[ W = \frac{w'}{E' R_x} \text{ = dimensionless} \]

\[ W' = \frac{P_z}{B_w} \text{ = load per unit width} \]

\[
\frac{1}{R_x} = \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \frac{1}{\sin \psi} = \left( \frac{1}{d_1} + \frac{1}{d_2} \right) \frac{2}{\sin \psi}
\]  \hspace{1cm} (3.16)

Rx is the effective radius and a function of pitch diameter and pressure angle.

Equation (3.14) uses modification factors to describe the bending stress; there, contact stress also can use these modification factors. Incorporating the modification factors from Equation (3.14) gives, the equation (3.15) may modify as

\[
\sigma_c = E' \left( \frac{W K_a K_e K_m}{bdy K_v} \right)^{\frac{1}{2}} = P_h \left( \frac{K_a K_e K_m}{K_v} \right)^{\frac{1}{2}}
\]  \hspace{1cm} (3.17)

From equation 3.14 the maximum Hertzian stress is

\[
c = P_h = E' \left( \frac{W}{2 \pi} \right)^{\frac{1}{2}} = 2.208 \text{ Gpa}
\]

From figure3.11, ka should be considered as middle shock from table 3.4, the application factor should be defined as 1.5, from table 3.5 for m-4mm is \( k_s = 1.0 \), from figure 3.18 for \( b=20 \text{mm} \) and \( b/d=0.25 \), the load distribution factor \( k_m \) is 1.12.

\[
N_{ap} = 60 \left( \frac{\omega}{2 \pi} \right) = 28.6624 \text{ rpm}
\]
So the pitch-line velocity is

\[ v_t = \frac{\pi \, dN \, \omega_p}{12} \approx 23.621 \text{ ft} / \text{in} \]

The transmitted horsepower is

\[ h_p = \frac{dN \, \omega_w}{126050} \approx 6.032 \text{ hp} \]

From Figure 3.19, for \( v_t = 23.621 \text{ ft/in} \) and suppose the \( Q_v = 9 \) the dynamic factor \( k_v = 0.97 \).

Thus

\[ \sigma = P_h \left( \frac{K_s K_c K_m}{K_v} \right)^{\frac{1}{2}} = 2.208 \times 10^5 \left[ \frac{(1.5)(1.0)(1.12)}{0.97} \right]^{\frac{1}{2}} = 3.01167 \text{ Gpa} \]

We substitute the real number to the equations and get the result to compare the numbers from FEA model.

<table>
<thead>
<tr>
<th>Table 3.6 Maximum stress among models</th>
<th>stress (Gpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Herzian pressure</td>
<td>2.208</td>
</tr>
<tr>
<td>Maximum contact pressure</td>
<td>3.01167</td>
</tr>
<tr>
<td>Two spur gears FEA model under friction</td>
<td>3.1</td>
</tr>
<tr>
<td>Two spur gears FEA discrete model</td>
<td>5.888</td>
</tr>
</tbody>
</table>

The theory calculation data developed here provide an accurate and repeatable measurement of a spur gear under realistic load conditions. From these data, the theory
data match very well with FEA results. It can draw a conclusion that the FEA model is correct.
CHAPTER IV

EXCISION GEAR FINITE ELEMENT ANALYSIS

4.1 Introduction

In order to pursue the biggest profit value for the company, they always hope to reduce the quality and hold the appropriate strength. Therefore, the middle section of the gear has to be cut to reduce its size. However in order to hold the same strength value and enhance the service life of a gear system, quenching should be carried out to enhance the gear intensity. If the searcher carries on the full-scale test operation, it is very difficult to obtain the precise value. All of these will waste much money and time. The finite element software can carry on this simulation to confirm crafts feasibility. The specific parameter and the finite element analysis result are presented in the following figure. The obtained data from the FEA model are compared with the perfect gear FEA model. The intensity and the kinetic energy are reduced due to small mass.

4.2 Finite element analysis of Excision gear

Table 4.1 Excision gear FE model

<table>
<thead>
<tr>
<th></th>
<th>Driving Gear</th>
<th>Driven Gear</th>
</tr>
</thead>
<tbody>
<tr>
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<td>8182</td>
</tr>
<tr>
<td>Element</td>
<td>15557</td>
<td>12475</td>
</tr>
</tbody>
</table>
Figure 4.1 Excision gear stress diagram

Under the same conditions, the maximum stress value of an excision spur gear system reduces; this basically proved the quality is a dynamic attribute.

Figure 4.2 Excision gear contacted area
Because the quality reduces, under the same speed and moment conditions, the contacted area also reduces correspondingly.

According to Newton’s Law of Conservation of Energy, kinetic energy corresponds in reducing to a certain value because the quality of the spur gear system reduces.

Figure 4.3 the kinetic energy for excision spur gear

Figure 4.4 internal energy for spur gear system
The stable internal energy reduces because of quality reducing.

Generates on the driving gear the stress report:

Field Output Report, written Mon May 19 21:44:03 2008

Source 1

ODB: d:/Temp/gear-cut-load.odb

Step: Apply-BCs-and-forces

Frame: Increment 88616: Step Time = 2.000

Loc 1: Element nodal values from source 1 (Average criteria = 75%, Not averaged across region boundaries)

Output sorted by column "Element Label".

Field Output reported at element nodes for part: GEAR-1

Computation algorithm: EXTRAPOLATE_COMPUTE_AVERAGE

Averaged at nodes

Averaging regions: ODB_REGIONS

<table>
<thead>
<tr>
<th>Element Label</th>
<th>Node</th>
<th>S.Mises</th>
<th>S.S11</th>
<th>S.S22</th>
<th>S.S33</th>
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<td>2.88752E+06</td>
<td>10.4156E+06</td>
<td>4.02217E+06</td>
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<tr>
<td>4.34375E+06</td>
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</tr>
</tbody>
</table>

<table>
<thead>
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<th>Element</th>
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</tr>
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<td>1.23489E+12</td>
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<td></td>
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<td>872.506E+09</td>
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<td></td>
</tr>
<tr>
<td>40.560E+09</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Compared with the excision gear and the perfect gear’s stress value under the same speed and moment, the excision gear causes the stress value reduction due to the reducing quality. The number of total kinetic energy changes small and the intensity to reduce as small mass. The requirement for material performance and reliability is increasing with the development and advancement of science and technology.
CHAPTER V
QUENCHING PROCESS SIMULATION

5.1 Introduction of quenching

Quenching is one of the most important manufacturing methods used to improve material performance and reliability in the manufacturing industry. [32] There are many quenching process: flame heating surface quenching, induction heating case hardening and electrical pick-off heating surface quenching. The surface quenching enables the work piece surface layer to obtain a high degree of hardness cell-martensite, but the inner section still holds the original toughness. Temperature variations, cooling rates, and corresponding microstructure evolution during the quenching process have important effects on the dimensional quality and mechanical property of the spur gear.

- Flame heating surface quenching
  Heats up the work piece surface rapidly, after hardening temperature fast cooling (for example spraying of water) quenching craft. The hard level depth is generally 2~6mm. This kind of quenching method device is simple, the ease of operation, the cost is inexpensive, is suitable especially for the large-scale work piece, the single unit, the small serial production.

- Induction heating case hardening
  The work piece is put in the electric current coil with (intermediate frequency, power frequency) high frequency. Then the thermal reaction use the induced current through the work piece (kelvin effect) to cause the work piece surface layer (or
local) to heat up and carry on cooling rapidly the quenching craft at last. These steps are called the induction heating case hardening. If the heat up time is short and the quenching surface layer organization is thin, the performance is good. There are many advantages to induction heating case hardening. 1) The production efficiency of induction heating case hardening is high. 2) The work piece surface oxidation and the decarburization are extremely few. 3) The distortion is also small. 4) The hard level depth is easy to check and implement the automation. However the device is expensive. It is suitable to be used in the shape simple work piece mass production.

When the spur gear receives the alternation curving loads, surface layer has higher stress than heart section. The surface layer is worn slowly under friction. Therefore, we proposes to the surface layer which requires high strength, high degree of hardness, high resistance to wear and high limit of fatigue, only the surface quenching can satisfy the above requirement. Surface quenching applies in the production widely. So in This study, surface quenching was used to improve the hardness of the surface on the gear.

5.2 Quenching process simulation

Quenching is an important method to improve metallic product properties. Large thermal gradients, volumetric changes and phase transformations will affect the residual stresses and distortion of the spur gear. In the past, studies of the residual stresses and distortion were primarily experimental, but experimental approaches consume considerable time, labor, and cost. Furthermore, such result cannot be easily extended to predict the effects caused by changes in materials, dimensions, cooling rates, etc. [33]
Therefore, finite element simulation of the spur gear grows as the effective study of quenching.

During the quenching process of steel components, how to achieve the desired microstructure, physical properties, surface properties and residual stresses with minimum distortion is an essential task to achieve production goals and reliable service performance. The quenching process is a complex process. In order to get the desirable combination of microstructure, material properties, residual stress and dimensional accuracy in the final product, a heat treatment process may involve several heating, cooling steps and include carburizing and diffusion processes. [34] Many research groups have examined the causes of distortion and found that the phase transformations as well as thermal stresses play an important role during the heat treatment. [35] All of these phenomena are illustrated in figure 5.1.

Figure 5.1 the main change relations of quenches
Process 1 represents heat generated due to the deformation; process 2 shows deformation caused by the heat conduction; process 3: thermal stress; Process 4 means transformation plasticity; Process 5 represents the latent heat; the last process means phase dependence on temperature. Mathematical modeling and numerical simulation techniques are often used to study the mechanical and thermal behavior of metallic alloys. They can lead to significant reductions in the cost/efficiency relation of both the design and the analysis of heat treatment processes. [36]

5.2.1 Energy equation

According to Fourier’s law, the Fourier heat conduction equation of transient problem with the phase transformation latent heat generated by using conservation of energy in a rectangular coordinates system. The equation can be described as

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \sigma_{ij} \dot{\varepsilon}_{ii} - \sum l_i l \right)$$  \hspace{1cm} (5.1)

Where $\rho$, $c$, and $k$ denote density, specific heat, and thermal conductivity, respectively; $l_i$ is the latent heat produced by the $i$-th constituent. $\dot{\varepsilon}_{ii}$ is the strain rate and $\sigma_{ij}$ is the stress.

The boundary condition of quenching is the mixed heat exchange boundary of convection and radiation belongs to the third type of condition.

$$-k \frac{\partial T}{\partial x} n_m = h_t (T-T_w) + Q_F \hspace{1cm} (5.2)$$

$$-k \frac{\partial T}{\partial x} n_t = h_t (T-T_w) \hspace{1cm} (5.3)$$

$$-k \frac{\partial T}{\partial x} n_r = h_a (T-T_a) \hspace{1cm} (5.4)$$
Where \( \frac{\partial T}{\partial x_k} \), \( \frac{\partial T}{\partial x_l} \), and \( \frac{\partial T}{\partial x_k} \) respectively is the temperature along the meshing tooth face, non-work tooth face and gear end surface change gradient. \( h_e \) and \( h_s \) is meshing tooth face, non-work tooth face convective heat transfer coefficient. \( Q_F \) is spur gear mating surface heat flux. \( T_w \) and \( T_a \) respectively are the coolant temperature and air temperature.

5.2.2 Constitutive equation

The total strain rate \( \dot{\varepsilon} \) includes elastic, plastic and thermal strain rates with other strain caused by the phase transformation,

\[
\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^e + \dot{\varepsilon}_{ij}^p + \dot{\varepsilon}_{ij}^t + \dot{\varepsilon}_{ij}^m + \dot{\varepsilon}_{ij}^{tp}
\]  

(5.5)

Where elastic strain is

\[
\varepsilon_{ij}^e = \frac{1}{E} (\sigma_{ij} - \sigma_{kk} \delta_{ij})
\]  

(5.6)

Where \( E \) is the Young modulus, \( v \) is the Poisson ratio. \( \varepsilon_{ij}^c \) call function of the carbon content, \( \varepsilon_{ij}^v \) the function of the volume fraction.

The thermal strain can be calculated by the initial material temperature \( T_0 \) and the thermal expansion coefficient \( \alpha \) which is a function of the carbon content and volume fraction,

\[
\varepsilon_{ij}^t = \frac{1}{\alpha} (T-T_0) \delta_{ij}
\]  

(5.7)

According to the general Hooke’s law, the stress \( \sigma_{ij} \) represent as function of elastic strain \( \varepsilon_{ij}^c \) and elasticity matrix \( C_{ijkl}^e \) as follows:

\[
\sigma_{ij} = C_{ijkl}^e \varepsilon_{kl}^e
\]  

(5.8)
The plastic strain rate represents a function of temperature-dependent material properties:

$$\dot{e}_{ij}^p = \lambda \frac{\partial F}{\partial \epsilon_{ij}}$$

(5.9)

$$\lambda = G \left( -\frac{\partial F}{\partial \epsilon_{ij}} + \frac{\partial F}{\partial T} \right) + \sum_{i=1}^{n} \frac{\partial F}{\partial \xi_i} + \frac{\partial F}{\partial \xi}$$

(5.10)

$$\frac{1}{G} = -\left( \frac{\partial F}{\partial \epsilon_{ij}} + \frac{\partial F}{\partial \xi} \right)$$

(5.11)

$$F = F(T, C, \sigma_{ij}, \epsilon_p, \xi_i, k)$$

(5.12)

Here k is the hardening parameter.

In this thesis, the following transformation induced plasticity increment defined as $$\dot{e}_{ij}^m$$ and $$\dot{e}_{ij}^{tp}$$ was used. [37]

$$\dot{e}_{ij}^m = \sum_{i=1}^{n} \beta_i \xi_i \dot{e}_{ij}$$

$$\dot{e}_{ij}^{tp} = \frac{1}{2} \sum_{i=1}^{n} K_i h(\xi_i) \xi_i S_{ij}$$

(5.13)

Where $$\dot{e}_{ij}^m$$ is the dilatation because of the structural changes and $$\dot{e}_{ij}^{tp}$$ is the I-th constituent of the transformation plasticity.

5.2.3 Kinetics of phase transformation

The conditions and manner from one phase to another classify the kinetics model as adiffusion-type transformation and diffusionless-type transformation. For example, as steel, the diffusion type transformation depending on the temperature, stress history and carbon content governs the austenite-ferrite and austenite-pearlite structure changes. The
diffusionless transformation from austenite to martensite shows a shear process depending on the same condition as the diffusion-type transformation.

➢ The diffusion-type transformation

Inoue et al. [38, 39] presented a method for analyzing temperature, metallic structure, stress and strain, and carbon content, and a modified Johnson–Mehl–Avrami equation with considering the effect of stress, temperature and carbon content can be written as

\[ \xi = 1 - \exp(kt^n) \]  

(5.14)

Where \( \xi \) indicates the volume fraction of the \( i \) th phase during the phase transformation, \( t \) is time and \( n \) is Avrami number depending on the kinds of transformation. \( K \) is the function of the temperature, stress and carbon content: [40]

\[ K = - f_T(T) f_s(\sigma_m) f_c(C) \]  

(5.15)

Where \( f_T(T) \) a function of temperature, \( f_c(C) \) the function of the carbon content, \( \sigma_m \) is the mean stress, \( C \) the carbon content, \( f_T(T) \) can be represent as [41]

\[ f_T(T) = A_T \left( \frac{T_0 - T}{T_0} \right)^{A_{T1}} \left( \frac{T_{TT} - T}{T_{TT}} \right)^{A_{T7}} \]  

(5.16)

\[ f_s(\sigma_m) = \exp(A_s\sigma_m) \]  

(5.17)

\[ f_c(C) = \exp(A_{c1}(C-A_{c2}) \]  

(5.18)

Where \( A_s \) is the function of the stress dependency of the TTT curves, \( A_{c1} \)and \( A_{c2} \) are functions of the carbon content.
Diffusionless-type (martensitic) transformation

The Mathematics of diffusionless-type (martensitic) transformation depends on the temperature, stress and carbon content.

\[
\xi_M = 1 - \exp(\psi_1 T + \psi_2 (C-C_0) + \psi_{31}\sigma_m + \psi_{31}\bar{\sigma} + \psi_4 )
\]  

(5.19)

Where \(\bar{\sigma}\) is the effective stress. \(\psi_2/\psi_1, \psi_{31}/\psi_1\) and \(\psi_{32}/\psi_1\) should be determined by the given carburized conditions and applied stress. \(\psi_1\) and \(\psi_4\) derived from the start temperature (\(\xi_M=0\)), \(T_M\) and for 50% martensite (\(\xi_M=0.5\)), \(T_{M50}\).

Since the quenching process is simulated by finite element software, it will be different from the spur gear simulation. Therefore, the analysis step should be changed to the dynamic, temp-disp, explicit. The analysis step is shown in figure 5.2.

![Figure 5.2 Analysis step of quenching](image)

At first, the temperature of the initial condition stays at 20°C; secondly the spur gear is kept in a high temperature environment; thirdly decrease the temperature to room temperature: 20°C; this step is the quenching working procedure, which we called in
In general, the last step, the spur gear system applied the speed under room temperature 20°C.

Because each parameter of steel can change with the temperature, in order to obtain the resolution precise result, the parameter tabulation is as follows:

Table 5.1 Main physical parameters for simulating calculation of gear steel [42, 43]

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>Conductivity W. (m.K)(^{-1})</th>
<th>Specific heat KJ. (Kg.K)(^{-1})</th>
<th>Expansion (10^{-6} \degree \text{C}^{-1}) Reference temperature 29°C</th>
<th>Density (Kg.m(^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>45.26</td>
<td>0.51</td>
<td>-</td>
<td>7800</td>
</tr>
<tr>
<td>100</td>
<td>46.39</td>
<td>0.55</td>
<td>12.10</td>
<td>7800</td>
</tr>
<tr>
<td>200</td>
<td>46.73</td>
<td>0.60</td>
<td>12.55</td>
<td>7800</td>
</tr>
<tr>
<td>300</td>
<td>45.80</td>
<td>0.65</td>
<td>13.05</td>
<td>7800</td>
</tr>
<tr>
<td>400</td>
<td>45.71</td>
<td>0.73</td>
<td>13.65</td>
<td>7800</td>
</tr>
<tr>
<td>500</td>
<td>40.11</td>
<td>0.75</td>
<td>14.25</td>
<td>7800</td>
</tr>
<tr>
<td>600</td>
<td>35.04</td>
<td>0.80</td>
<td>14.70</td>
<td>7800</td>
</tr>
<tr>
<td>700</td>
<td>28.30</td>
<td>0.85</td>
<td>15.12</td>
<td>7800</td>
</tr>
<tr>
<td>800</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7800</td>
</tr>
<tr>
<td>880</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7800</td>
</tr>
</tbody>
</table>
Figure 5.3 Excision gear after quenching stress diagram

Field Output Report, written Mon May 19 21:33:26 2008

Source 1

ODB: d:/Temp/gear-cut-load-temp.odb

Step: Step-3

Frame: Increment 86854: Step Time = 2.000

Loc 1 : Element nodal values from source 1 ( Average criteria = 75%, Not averaged across region boundaries )
Output sorted by column "Element Label".

Field Output reported at element nodes for part: GEAR-1

Computation algorithm: EXTRAPOLATE_COMPUTE_AVERAGE

Averaged at nodes

Averaging regions: ODB_REGIONS

<table>
<thead>
<tr>
<th>Element</th>
<th>Node</th>
<th>S.Mises</th>
<th>S.S11</th>
<th>S.S22</th>
<th>S.S33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Label</td>
<td>Label</td>
<td>@Loc 1</td>
<td>@Loc 1</td>
<td>@Loc 1</td>
<td>@Loc 1</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>17.6935E+06</td>
<td>14.5674E+06</td>
<td>-2.36138E+06</td>
<td>3.68131E+00</td>
</tr>
<tr>
<td>1</td>
<td>263</td>
<td>12.6384E+06</td>
<td>11.0507E+06</td>
<td>-237.089E+03</td>
<td>3.26175E+06</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td>-------------</td>
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<td>-------------</td>
<td>-------------</td>
</tr>
<tr>
<td>10944</td>
<td>8504</td>
<td>58.1898E+06</td>
<td>-21.815E+06</td>
<td>-60.0186E+06</td>
<td>-24.5263E+06</td>
</tr>
<tr>
<td>10944</td>
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<td>53.0236E+06</td>
<td>-26.3677E+06</td>
<td>-40.686E+06</td>
<td>-20.091E+06</td>
</tr>
<tr>
<td>Minimum</td>
<td></td>
<td>782.273E+03</td>
<td>-186.458E+06</td>
<td>-287.927E+06</td>
<td>-114.493E+06</td>
</tr>
<tr>
<td>At Element</td>
<td></td>
<td>6055</td>
<td>10587</td>
<td>10470</td>
<td>10539</td>
</tr>
<tr>
<td>Node</td>
<td>115</td>
<td>8399</td>
<td>8414</td>
<td>8414</td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>295.45E+06</td>
<td>164.316E+06</td>
<td>145.484E+06</td>
<td>117.9E+06</td>
<td></td>
</tr>
</tbody>
</table>
From these data, the stress number improve a lot of after quenching (the maximum stress from $1.2.726E+08$ to $2.9555E+08$), so that quenching process can uniform mechanical properties, minimize residual stresses, and avoid wear for the gear system.
6.1 Summary

In this study, theoretical investigations on the gear characteristic were conducted to quantify their influence on gear stress and deflections. The deformable-body model of the spur gear system with the same condition was developed. The deformable-body model matches very well with the measured result from theoretical calculation and discrete-body model. The spur gear system in the present thesis carries on the basic analysis and compare with exsiccation gear. However, in order to enhance the degree of hardness, the strength and the resistance, the spur gear must carry on the quenching process. Due to the problems mentioned above, a method has been identified to improve the situation by incorporating finite element analysis. One of the goals of our development effort was to make the analysis procedure available to a design and process engineer. [44]

6.2 Conclusion and contributions

In this study, a 3D deformable-body model of spur gears was developed. The result is checked with theoretical calculation data. The simulation results have good agreement with the theoretical results, which implies that the deformable-body model is correct. This study provides a sound foundation for future studies on the other gear series: Helical gear, annular gear, turbine wheel and so on. The model was applied onto commercial
FEA software Abaqus. Simulation results were compared and confirmed by the theoretical calculation data. According to these results, we can draw the following conclusions:

1) Used the relational equation in Pro/Engineer, the accurate three dimensional spur gear models are developed. The parametrical process can increase design accuracy, reduce lead times and improve overall engineering productivity.

2) A discrete model of the spur gear was proposed. The FE simulation result matched well with the theoretical calculation and deformable-body model results. The discrete-parameter model is a new contribution to the literature as it provides a fast method of computing stress problems of the spur gear system.

3) Compared the difference between the excision gear and the spur gear system in Chapter III. The result indicated that, the excision spur gear strength becomes small. In order to preserve the identical strength and the improvement service life, the quenching process should be applied.

4) It was found out that the numerically obtained values of stress distributions were in good agreement with the theoretical results. Therefore, the three-dimensional thermo-elastic-plastic FE program developed in this study can be useful in investigating the processing parameters for the quenching process of spur gears.

Meanwhile, this study provides a good way to improve the material property of spur gears. Although, it necessary to comprehend that the quenching process can improve the material property of gears in manufacturing, the real data prove the material improvement is right. In this study, the quenching process of 3D deformable-body model was developed, which provided accurate data for the gear manufacture. It also provides a tool
to help understand the research and development for spur gear design and quenching process. With the results obtained in this study, it is possible to estimate accurately the materials properties of spur gears after the quenching process, thus, to be able to trace the improvement stages of spur gears.
CHAPTER VII

FUTURE WORKS

The spur gear simulation should be considered each kind of factors; the multitudinous factors should perform to study in the future. Below is a direction which the future will possibly need to study.

- The temperature condition in the model will affect the stress and other conditions. After we compare the result between low temperature (20°C) and high temperature (800°C), we may find some theory about the temperature.

- Velocity also is one of important factor for the gear dynamics analysis. Compare material propriety of gear system under low velocity and high velocity.

- Along with the modern manufacturing industry high speed development, many materials were to deign the gear system, such as plastic and polymer material. Many kinds of material of gear system should be compared each other.

- In the ordinary circumstances, the gear always has some flaw, such as crack and slight defect. These flaws can play the significant role in the dynamic analysis. It has the significant influence on the gear stress and the serve life.

- If the gear dynamic simulation existence common tolerance, such as the eccentric common difference, should be able to affect the gear the stress and their service life.

- The temperature and time have an import effect on the material property in quenching process. Apply this FEA model to investigate the effect of temperature
variation and phase transformation on the dimensional change and stress distribution.
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20. AGMA, “Standard for Rating Pitting Resistance and Bending Strength for Spur and Helical Involute Gear Teeth”, AGMA 218.01, 1965


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