LANDSLIDE STABILIZATION USING A SINGLE ROW OF ROCK-SOCKETED DRILLED SHAFTS AND ANALYSIS OF LATERALLY LOADED DRILLED SHAFTS USING SHAFT DEFLECTION DATA

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ABSTRACT

An accurate and practical methodology for stability analysis and design of drilled shafts reinforced slopes was developed utilizing limiting equilibrium method of slices. Complex soil stratifications and general failure slip surfaces can be handled in the developed method. The effect of soil arching due to the presence of the drilled shafts was accounted for by using a load transfer factor. The numerical values of the load transfer factor were developed based on 3-D FEM parametric study results. Many of the design variables controlling the slope/shaft systems, such: drilled shafts size, shafts location, shaft fixity (the necessary rock-socket length), and the required spacing between the drilled shafts to prevent soil from flowing around the shafts can be successfully determined from the developed method. The optimum location where the drilled shafts could be placed within the sliding soil mass so that the cost associated with the landslide repair using the drilled shafts is minimized can be searched for and determined from the developed methodology. From geotechnical point of view, the global factor of safety for slope/shaft systems can be determined. From structural point of view, the forces acting on the stabilizing drilled shafts due to the moving ground can be successfully estimated.

In addition to the developed design methodology, Real-time instrumentation and monitoring were carried out for three landslide sites in the Southern part of Ohio. Various types of instruments were extensively installed inside the stabilizing shafts and the
surrounding soils to monitor and better understand the behavior of slope/shaft systems. The UA Slope program developed by Dr. Robert Liang in corporation with ODOT and FHWA has been used in designing these landslides. The field instrumentation and monitoring processes have provided excellent and unique information on the lateral responses of shafts undergoing slope movements. Also, the results of the instrumented cases have provided that the structural design (moments, shear, lateral deflection, and shaft tip fixity) of the shafts are overestimated (i.e., estimated forces acting on the shafts are high), and the geotechnical design (FS of slope/shaft system: movement and rate of movement) is achieved in two case studies but not fully achieved for the third case.

On the other hand, in an effort to develop an efficient analytical method for analysis of laterally loaded drilled shafts using only lateral shaft deflection data, numerical procedures were proposed based on the principle of superposition. The lateral shaft deflections along the shaft length due to superposition of the lateral applied load to the drilled shaft were added together to establish the compatibility equations that govern the lateral behavior of the drilled shaft system. The compatibility equations allow for the determination of the net applied loads to the drilled shaft responsible for specific amount of shaft deflections. Once the loads were determined, basic equilibrium equations were applied to calculate shear forces and bending moments along the shaft length. A computer program was developed implementing the proposed numerical procedures to facilitate numerical computations. Many laterally loaded drilled shaft examples were described and used to verify the validity of the developed method. Included in the cases for validation were two actual full-scale drilled shafts at Jefferson County: (1) landslide repair using drilled shafts; and (2) lateral load test, were demonstrated.
DEDICATION

To anyone who would read this dissertation, use this work or build on it to add something to the state of knowledge.
ACKNOWLEDGEMENTS

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1.1 Overview

An increased popularity of using drilled shafts to stabilize an unstable slope in highway applications could be attributed to several factors: (1) various construction techniques are available for installing drilled shafts in almost any type of soil and rock conditions; (2) lateral load test can be performed to verify the lateral load-resistance capacity of the drilled shafts; (3) the use of drilled shafts avoids the need to address the right-of-way issues that may be needed for other types of slope stabilization methods; (4) the drilled shafts sometimes offer a reliable and economical solution compared to other slope stabilization methods; and (5) the drilled shafts are typically structurally capable of resisting long-term environmental effects. The most fundamental causes of slope instability are reduction of shear strength of the soil or increase in shear stress. Installing drilled shafts in a slope reduces the shear stress required for equilibrium, which in turn, leads to satisfactory stabilization of a slope. There have been numerous documentations in the literature regarding the successful utilization of drilled shafts to stabilize a slope (e.g., Fukumoto, 1972 and 1973; Sommer, 1977; Ito et al., 1981 and 1982; Nethero,
Landslide stabilization methods are varied and dependent upon site situation. Installing drilled shafts in a moving soil mass is considered one of the effective and reliable techniques in landslide stabilization. Arresting an unstable slope using a single row of rock-socketed drilled shafts as shown in Figure 1.1 requires the soil engineers to determine the following important key points: (1) drilled shafts diameter; (2) spacing between the shafts to ensure development of soil arching; (3) the necessary socket length of the drilled shafts in the non-yielding strata (e.g., rock) so that the shafts act as a cantilever against the moving soil; (4) location of the drilled shafts within the slope body so that the global factor of safety of the stabilized slope is optimized for the most economical configuration of the drilled shafts; (5) the forces imparted on the drilled shafts due to sliding mass. However, the available methods that deal with drilled shafts stabilized slopes do not provide enough information on how to stabilize landslides using drilled shafts especially because of the many idealized assumptions made by several investigators trying to overcome the complexity and difficulties encountered. In addition, these idealized assumptions have sometimes led to over designing the drilled shafts stabilized slopes with respect to geotechnical and structural aspects, which in turn, would increase the cost associated with the construction process of the landslide repair. So, for these reasons, there is a compelling need to (a) develop a step-by-step design methodology that allows the engineers perform a complete design for landslides
stabilization using a single row of rock-socketed drilled shafts; (b) perform real-time field instrumentation and monitoring to better understand the behavior of the drilled shafts and the overall stability of the slope/shaft system; and (c) combine the theoretical and the actual findings to ensure an economical and safe design.

Figure 1.1: Statement of the problem

1.3 Objectives

The main objective of this research study is to develop a pertinent design methodology of a single row of rock-socketed drilled shafts to stabilize slopes. The developed method utilizes the general procedure of slices for composite slip surfaces of any shape and incorporates the effect of soil arching due to the moving soil through the opening between the drilled shafts. Such integrated approach would allow not only the
determination of the global safety factor of the stabilized slope, but also, the net forces acting on the drilled shafts due to the relative soil movements and lateral pressures transfer. A step-by-step design procedure needs to be developed to enable soil engineers to design drilled shafts for landslide stabilization. These objectives can be achieved when the following two issues are addressed:

a) Geotechnical Design Issues.

The geotechnical design requirements are said to be satisfied when the global factor of safety for the new slope/shaft system is determined. In other words, the design entails evaluating the enhancement in the stability of the slope when a single row of drilled shafts is installed with specific drilled shafts design configurations. Usually, a target factor of safety is applied based on the importance of the site, adjacent properties, and the budget available for slide repair.

b) Structural Design Issues.

Structurally, the installed drilled shafts will start to act similar to cantilever beams if drilled shafts fixity was provided; therefore, shafts will require steel reinforcement in order to resist the shear and bending stresses developed in the shafts due to lateral earth pressures acting on the shafts. For that reason, the forces imparted on the drilled shafts need to be determined. After determination of shaft forces (i.e., shear forces and bending moments) and based on drilled shafts configurations and the surrounding soil materials; the shaft section is designed structurally to withstand these forces and prevent any excessive movement.
1.4 General Work Plan

From the above discussion, it is obvious that the use of ultimate state of stresses or two-dimensional type of analysis would not be sufficient to evaluate the soil arching. On the other hand, friction circle method and ordinary method of slices (Fellenius 1936) were too simplified to consider general failure slip surface and nonhomogeneous soil stratification. In addition, there is a lack of supporting actual full-scale field measured data that can be used to validate the design method.

In order to achieve the proposed objectives to overcome the stated problem statement: landslide stabilization using a single row of rock-socketed drilled shafts; the general framework to accomplish these objectives will consist of three-dimensional type of analyses, the general procedure of limiting equilibrium approach, and the use of real-time monitoring of large-scale drilled shafts stabilized landslide sites.

On the theoretical basis, limiting equilibrium method of slices is considered the most conventional and widely-used method that deals with slope stability applications (i.e., determination of safety factor) because of its simplicity; but, the method is not yet ready to account for drilled shafts. Therefore, limiting equilibrium method of slices needs to be re-derived and extended to accomplish the following: (1) to consider the drilled shaft element; (2) to derive a limiting equilibrium equation for the global factor of safety for the whole slope; (3) to consider the external loads in the limiting equilibrium equation; and (4) to develop a numerical closed-form solution that directly relates the amount of the reduction in the driving forces, due to the presence of the drilled shafts, to the factor of safety. Extension of the limiting equilibrium method (LEM) alone is not enough to resolve the problem because the problem in hand is considered to be a three-
dimensional problem rather than the typical simple two-dimensional slope stability application. Therefore, three-dimensional finite element modeling needs to be conducted to simulate the real situation in the field (i.e., to give an idea on how the lateral earth pressures are transferred between soil and shaft due to soil movement. This three-dimensional finite element simulation is conducted to contribute in developing the design method for landslide stabilization using drilled shafts considering the following aspects: (1) three-dimensional state of stresses (i.e., real situation) rather than two-dimensional plane strain condition; (2) shaft modulus, total length, and location within the slope; (3) rock modulus and socket length; (4) depth of slip surface; (5) soil cohesion and friction angle; (6) the composite form of relative soil movements. In order to assess the load transfer mechanism of slope stabilization caused by the presence of drilled shafts; the three-dimensional finite element modeling was performed considering elastic behavior of drilled shafts, nonlinear plastic behavior of soil, and elastic behavior of the soil layer where the drilled shafts will be socketed into (i.e., rock). Real simulation of the complex relative displacement field is utilized. Frictional interaction forces between the soil and the rock with the shafts are implicitly considered. These finite element simulations seek to gain better understanding and insight on the behavior of drilled shafts stabilized slopes.

Combining the theoretical findings from limiting equilibrium method and finite element method, a pertinent design methodology is developed to allow soil engineers to analyze and design unstable slopes using a single row of rock-socketed drilled shafts. The many idealized assumptions made by other investigators have led to an inaccurate and costly design of the problem in hand as mentioned earlier. On the contrary, in this study,
these assumptions will be taken care of in order to lead to an accurate rigorous design approach.

On the other hand, practical experience would be very helpful in confirming theoretical findings. Four actual full-scale landslide sites in the southern part of Ohio were stabilized using a single row of drilled shafts. Drilled shafts and the surrounding soil were accurately monitored by means of installing extensive instruments such as: piezometers, inclinometers, earth pressure cells, concrete cells, and strain gages. All these sensors are multiplexed with a timely-automated data acquisition system to observe the slope movement, shaft movement, shear and bending stresses, and the overall stability of the stabilized slope.

Figure 1.2 illustrates the general work plan to achieve the objectives discussed in this section. Detailed description of each subdivision in this work plan is explained herein.

Figure 1.2: Flowchart explains the general work plan
1.5 Dissertation Outlines

Chapter II discusses the work done by many researchers and reviews the previous work with respect to theoretical and practical basis. In this chapter, the literature review is provided for the landslide stabilization using drilled shafts.

Chapter III presents the mathematical formulation for the extension of the limiting equilibrium method of slices to account for drilled shafts.

Chapter IV presents the three-dimensional finite element modeling performed using ABAQUS/CAE computer program to simulate the slope/shaft system.

Chapter V introduces the developed pertinent design methodology of a single row of rock-socketed drilled shafts.

Chapter VI presents the field instrumentation and monitoring for the four actual full-scale landslide sites in the Southern part of Ohio.

Chapter VII introduces the new developed analytical method for the analysis of laterally loaded drilled shafts based on the principle of superposition using only shaft deflection data.

Chapter VIII presents conclusions and recommendations.
Drilled shafts as a mean to stabilize moving slopes have the advantage of being installed without significantly decreasing slope stability (Wang and Yen, 1974). The structural capability of drilled shafts in arresting unstable slope movement is significant compared with other slope stabilization methods. Nowadays, construction techniques of drilled shafts are available in almost any type of soil and rock conditions. Installing large reinforced concrete cylinders into active or potential failure slopes act as pins or cantilever wall can arrest slope failure (Gould, 1970). Reduction in shear strength of the soil and increase in shear stress are the basic causes of slope failure. Installing a row of drilled shafts, socketed enough into a stable soil and spaced properly apart so that soil cannot flow around the shafts, would reduce the shear stresses; this in turn, would lead to satisfactory stabilization of slope. The reduction in the shear stresses occurs when soil is forced to squeeze between the shafts, at that time, shafts start to transfer some of the driving forces through its reinforced concrete section from the upper unstable soil zone to the lower more stable soil zone. This transferring process of forces is called soil arching. Soil arching is a localized phenomenon (Terzaghi, 1936; Bosscher et al., 1986) in which the stresses in the yielding soil are transferred to the unyielding portion of the soil.
Previous works on soil arching can be generally categorized into: (a) the assumed shear plane method as by (Terzaghi 1936; 1943) which may not reflect the actual failure planes; (b) the elastic approach as examined by (Finn 1963; Chelapati 1960) which is only valid for small deformations and strains whereas soil arching is usually accompanied by large deformations (Bosscher and Gray, 1986); (c) the approach of soil-structured model studies as found by (Getzler at al., 1968; Whitman and Luscher, 1962; Wang and Yen, 1974) which appears to be powerful tool in understanding the mechanism of soil arching. One of the requirements to practicing soil engineers is to fully understand the factors influencing the development for the soil arching. Incorporating the arching mechanism into slope stability analysis and thereafter the stabilization design, however, requires a comprehensive investigation of the conditions of soil arching to develop. At present, the lack of an adequate information leads to design error, and to place the shafts in an inefficient way.

Wang and Yen (1974) studied the soil arching in slopes assuming infinite slope analysis, rigid-plastic soil behavior, and predefined failure planes. Therefore, stress-strain relationship is not involved in the analysis and the assumed failure planes may not be reasonable approximations of actual failure surfaces. Bosscher and Gray (1986) conducted a laboratory test to experimentally model soil arching for sandy slopes when piles are introduced. It was found that the spacing between the piles was very important and the proportion of the load will be imparted to the piles increased as spacing between the shafts increased. Typically, when clear spacing was three times of pile diameter, the percent of transferred load was found to reach the value of around 30% for the specific case investigated. In other experimental studies (Adachi, 1989; Low et al., 1994; Chen et
al., 1997), results indicated that the pile and soil properties are important factors assisting in the development of soil arching, which in turn, would lead to a significant enhancement in slope stability. Liang and Zeng (2002) investigated soil arching mechanism in drilled shafts for slope stabilization using 2-dimensional finite element approach as shown in Figure 2.1 assuming rigid-plastic soil behavior, plain-strain conditions, and constant soil movement with respect to shaft length. The formulation of soil arching was implemented by applying a triangular displacement field occurring in the soil between the drilled shafts. It was found that soil arching is highly dependent on soil movement, soil properties, and drilled shafts configurations.

Methods used to analyze response of drilled shafts installed in unstable slopes due to relative lateral movements are generally classified into stress-based or displacement-based approaches. In the stress-based approach, some researchers make use of the ultimate soil pressure to be applied to the shafts in order to estimate the shaft response (e.g., Reese, 1992; Hassiotis et al., 1997) utilizing several procedures (e.g., Broms, 1964, Viggiani, 1981; Ito and Matsui, 1975). The theory of plasticity by Ito and Matsui provides an estimate of the lateral force per unit thickness of soil layer acting on the stabilizing shafts when soil is forced to squeeze between the shafts (see Figure 2.2). In this theoretical approach, shafts were assumed rigid and soils just around the shafts were assumed in a plastic state of equilibrium, which in turn, tends to underestimate the lateral force in the shaft. In addition to that, the frictional forces between the soil and the shafts were neglected and effect of sloping ground was not considered.
Figure 2.1: Finite element model for slope/shaft system (after Liang and Zeng, 2002)
The assumptions made by Ito and Matsui can be summarized as follows:

1. When the soil layer deforms, two sliding surfaces occur along the lines AEB and A'E'B', in which the lines EB and E'B' make an angle of \( \left( \frac{\pi}{4} + \frac{\phi}{2} \right) \) with the x-axis.

2. The soil layer becomes plastic only in the soil AEBB'E'A' just around piles, in which the Mohr-Coulomb's yield criterion is applied. Then, the soil layer is represented as plastic solids with an angle of internal friction \( \phi \) and a cohesion \( c \).

3. The soil layer is in a plane-strain condition in the direction of depth.

4. Even if frictional forces act on the surfaces AEB and A'E'B', the stress distribution in the soil AEBB'E'A' is almost the same as that in the case of no frictional forces on those surfaces.

5. Piles are rigid

Figure 2.2: State of plastic deformation in the ground just around piles (after Ito and Matsui, 1975)
Displacement-based approaches, however, estimate the shafts response based on using relative lateral displacements between the shaft and the soil which simulates the real situation in the field. Displacement-based approaches are considered more complicated than the stress-based approaches because the relationship between the movements of the shafts and the soil are strongly interrelated. Displacement-based approach is utilized in this study, and detailed analyses and discussions are introduced herein. Generally speaking, the displacement-based method is superior to the earth pressure method, because it can better capture the mechanisms of soil-shaft interaction under the moving soil mass, and give good agreement when compared with experimental results (Liang and Zeng, 2002).

Slope stability applications have been widely performed assuming 2-dimensional analysis, in which, this type of analysis has been found and considered to be appropriate because it yields conservative estimate when compared with 3-dimensional analysis. However, when drilled shafts are installed as a row in an unstable slope, 2-dimensional analysis is no longer applicable because drilled shafts diameter and length (i.e., shaft rigidity), shafts spacing, and soil properties are important factors influencing the development of soil arching which directly affects the stability of the slope. Therefore, three-dimensional analysis is highly recommended to evaluate and better understand the arching mechanism and the transfer of loads through drilled shafts as the soil tends to move.

The successful use of drilled shafts in moving soil to improve the stability of slopes has been described by several investigators as mentioned earlier. In spite of these
applications, the approaches used vary widely and the validity of some of the approaches appears to be subject to doubt.

Ito et al. (1981) presented an approach to evaluate the stability for a slope using a single row of piles, by considering that, piles would provide an additional force to the available resisting forces provided by the slope. The definition of factor of safety is, then, the total resisting forces divided by the total driving forces. These forces can be estimated by using the Ordinary method of slices. The additional resistance force provided by the piles are estimated using the theory of plasticity by Ito and Matsui (1975) which assumes rigid piles, soils just around the piles are in a plastic state of equilibrium, and frictional forces are ignored as mentioned earlier.

Hassiotis et al (1997) extended the friction circle method to incorporate for the piles as illustrated in Figure 2.3 by considering the piles to exert an additional resistance force on the slope; this additional force is estimated utilizing the theory of plasticity by Ito and Matsui (1975). The friction circle method used by Hassiotis et al. is capable of dealing with only homogeneous soil profile and circular failure slip surfaces.

Reese et al. (1992) has presented a procedure to evaluate the stability of slopes reinforced with drilled shafts. The factor of safety is determined by adding the force provided by the drilled shafts to the resistant forces, this force is obtained considering the ultimate state of soil failure which requires large soil movements as shown in Figure 2.4. This procedure overestimates the force applied to the shaft which overdesigns the shaft structurally. Poulos (1995, 1999) assumed two-layer soil system with horizontal sliding plane, the upper soil layer is the unstable one, and the lower is the stable soil layer. The
determination of lateral response of drilled shafts is assessed utilizing either the theory of plasticity or Brom's method (1964).

Figure 2.3: Forces on slope reinforced with piles (after Hassiotis et al., 1997)
Figure 2.4: Soil failure behind the shaft (after Reese, 1992)
Zeng and Liang (2002) developed a design methodology based on fundamental understanding of the stabilization mechanism provided by the drilled shafts: arching mechanism. The effect of arching mechanism was quantified by a series of 2-dimensional finite element numerical simulations (Liang and Zeng, 2002) considering rigid shaft behavior and plain-strain condition, with soil strength parameters, drilled shaft diameters and center-to-center spacings varied in the parametric study. The methodology uses the limiting equilibrium method of slices by satisfying force equilibrium for each slice. Many idealized assumptions were involved in this approach which has led to errors. These deficiencies can be mainly attributed to the fact that the authors ignored the effect of soil arching in many aspects such as: shafts location within the slope, degree of shaft tip fixity, total shaft length, depth of the slip surface, material properties of the drilled shafts and the unyielding strata where the shafts need to be socketed into. In addition to that, the range of the drilled shafts diameter that was considered in the study was from 1 ft to 3 ft, whereas, drilled shafts diameter in some cases in practical applications are larger than 3 ft which limits the validity of this approach for large shafts diameter. Although the range of drilled shafts diameter is small, the effect of arching appears to be not very much sensitive for shafts diameter of 1 ft to 3 ft.

The development of suitable analysis and design methods for the drilled shafts to stabilize a slope involves two fundamental issues: (a) to determine the factor of safety, FS, of the slope with the installed drilled shafts; and (b) to determine the design loads for structural design of the drilled shafts. The definition of FS of a slope with the stabilizing drilled shafts within the framework of limiting equilibrium slope stability analysis technique has not been well established. Limit equilibrium analysis in conjunction with
the method of slices is the most widely used method for evaluating stability of slopes.

The techniques can accommodate complex geometry and variable soil properties and water pressure conditions. The limit equilibrium analysis method can provide a global safety factor. Numerous limit equilibrium methods for slope stability analysis have been proposed by several investigators, including the celebrated pioneers Fellenius (1936), Bishop (1955), Janbu (1954), Morgenstern and Price (1965), Spencer (1967), and Sarma (1973). These efforts, however, were related to a slope without drilled shafts. The analysis of a slope stabilized with the drilled shafts requires a development of an approach to account for the contribution of drilled shafts.

Furthermore, the earth pressures applied to the drilled shafts are highly dependent upon the relative movement of the soil and the drilled shafts, which in fact is an indeterminate problem as the structural response of the shaft depends on the earth pressure applied, which in turn, relies on the structural response (deflection) of the shaft. The movement of soil is an important factor in designing the soil-shaft system, since as the soil movement increases, the force imparted on the shaft increases as well (see Figure 2.5), up to the point where the structural capacity of the shafts is reached. The drilled shafts used for stabilizing a slope have often been referred to as passive shafts due to the fact that the lateral pressure acting on the shaft is dependent upon the movement of the slope and the interaction between the shaft and the surrounding soils. Ideally, this complex soil structure interaction mechanism should be studied using three-dimensional FEM simulation techniques, taking into account the nonlinear and plastic nature of soil constitutive behavior as well as the soil-shaft interactions.
Figure 2.5: Soil movement vs. shaft load
CHAPTER III

FORMULATION OF THE LIMITING EQUILIBRIUM METHOD FOR SLOPE/SHAFT SYSTEMS

3.1 Overview

In this chapter, a limiting equilibrium method of slices based solution for calculating global factor of safety (FS) of a slope with or without the presence of a row of drilled shafts is developed. Force equilibrium was applied for each individual slice to solve for the unknown variables in a system of slope and drilled shafts. Assumptions regarding the inclination of the interslice forces and location of the thrust line were made to render the problem determinate. A mathematical relationship between the load transfer factor (amount of the reduction in the forces due to the presence of the drilled shafts) and the global FS of the slope/shaft system is introduced herein. Effects of shaft location on the load transfer factor and FS of the slope/shaft system were investigated utilizing the developed numerical closed-form solution of the load transfer factor. A detailed mathematical formulation with its derivations and assumptions are presented for solving FS of a slope stabilized with drilled shafts. An application example involving a hypothetical homogeneous slope is analyzed using the developed mathematical formulas. The effects of shaft location on the global factor of safety and the amount of reduction in
the forces are investigated. A further analysis regarding the determination of suitable design configurations in order to achieve a specific numeric load transfer factor is discussed at the end of this chapter.

3.2 General Statement

A single row of drilled shafts with diameter D and spaced S apart on centers are installed in an unstable slope and socketed deep enough into an unyielding stratum (i.e., rock) to provide fixity for the drilled shaft element with respect to the moving soil mass. Slope stability analysis is usually performed for cross-sections (i.e., 2-dimensional analysis); however, when drilled shafts are installed in a slope, the problem becomes more complicated and two-dimensional type of analysis is no longer applicable. The presence of a row of drilled shafts would improve the stability of the slope due to arching mechanism. The arching mechanism occurs when soil starts to move through the opening between the drilled shafts, particularly when the opening is small and the drilled shafts are fixed deep enough into a stable stratum, the soil starts to squeeze through the openings and earth pressures start to transfer to the drilled shafts. The earth pressure transferred to the drilled shafts are eventually resisted by portion of the shaft in the unyielding stratum (i.e., rock). Eventually, soil movements start to slow down between the drilled shafts when it reaches its minimum value at the location of the drilled shafts. From this scenario, it is obvious that arching help reduce the driving forces and displacements.

In order to achieve the desired reduction in the driving force in a slope with installation of drilled shafts, specific shaft configurations are required. In other words, the
problem can be categorized into: (a) the effect of the reduction in the driving forces on the global stability (i.e., FS); and (b) the required configurations of a single row of drilled shafts to achieve the necessary reduction in the driving forces.

The first issue is addressed in this chapter by extending the conventional limiting equilibrium method of slices to account for the presence of the drilled shafts, while satisfying force equilibrium conditions. The second issue is addressed in the next chapter utilizing three-dimensional finite element parametric study to generate design charts for estimating the load transfer factor.

3.3 Mathematical Formulation

Figure 3.1 shows one row of drilled shafts installed within a slope to arrest the moving unstable soil. The mathematical formulation can be incorporated in any method of slices limiting equilibrium approach that has the capability to determine the interslice forces. The conventional method of slices can be readily used to accommodate for complex geometries, variable soil conditions, general failure slip surface, and the influence of external boundary loads. Utilizing the method of slices for slope stability analysis, the slide mass is divided into n smaller vertical slices as shown in Figure 3.1 (starting with slice 1 at the top of the slope and ending with slice n at the bottom of the slope), each slice is acting as unique sliding block and affected by a general system of forces as depicted in Figure 3.2.

If each individual slice is assumed to be in a state of equilibrium, then there are (7n – 3) unknowns as listed in Table 3.1. Also, since three equations can be written for the limit equilibrium for the system (i.e., only forces equilibrium will be applied for each
individual slice), the solution is statically indeterminate. However, a solution is possible providing the number of unknowns can be reduced by making some simplifying assumptions. The following assumptions are made in order to render the problem determinate:

(1) FS is assumed to be the same along the failure slip surface, thus reducing the total number of unknowns by \((n - 1)\) to \((6n - 2)\).

(2) The normal force on the base of the slice acts at the midpoint of the slice base, thus reducing the total number of unknowns by \((n)\) to \((5n - 2)\).

(3) The location of the thrust line can be placed generally at one-third of the average interslice height \((h_i)\) above the failure surface as in Janbu (1973). The thrust line as indicated in Figure 3.2 connects the points of application of the right- and left-interslice forces. This assumption reduces the total number of unknowns by \((n - 1)\) to \((4n - 1)\).

(4) The assumption by Zeng and Liang (2002) regarding the inclination of the interslice forces was adopted. The right-interslice force \((R_i)\) is assumed to be parallel to the inclination of the preceding slice base (i.e., \(\alpha_{i-1}\)), and the left-interslice force is assumed to be parallel to the slice base (i.e., \(\alpha_i\)). Thus, the total number of unknowns is reduced by \((n - 1)\) to \((3n)\).

Therefore, one can formulate the governing limiting equilibrium equations for the sliding soil mass as the number of unknowns is equal to the number of equations.

Generally speaking, the safety factor (FS) in slope stability analysis is defined as the ratio of the available shear strength along the failure surface at failure \(\tau_f\) to the shear stress, \(\tau\),
required to bring the soil mass into state of equilibrium. The factor of safety, FS, can be written as:

\[ FS = \frac{\tau_f}{\tau} \]  
Where \( FS \leq 1.0 \) at failure  \hspace{1cm} (3.1)

Table 3.1: Equations and unknowns associated with the method of slices

<table>
<thead>
<tr>
<th>Equations</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>2n</td>
<td>Force equilibrium in two directions for each slice</td>
</tr>
<tr>
<td>n</td>
<td>Mohr-Coulomb relationship between shear strength and normal effective stress</td>
</tr>
</tbody>
</table>

| 3n        | Total number of equations |

<table>
<thead>
<tr>
<th>Unknowns</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>FS at base of each slice</td>
</tr>
<tr>
<td>n</td>
<td>Normal force at base of each slice (N)</td>
</tr>
<tr>
<td>n</td>
<td>Location of normal force (N)</td>
</tr>
<tr>
<td>n</td>
<td>Shear force at base of each slice (T)</td>
</tr>
<tr>
<td>n - 1</td>
<td>Interslice forces</td>
</tr>
<tr>
<td>n - 1</td>
<td>Inclination of interslice forces</td>
</tr>
<tr>
<td>n - 1</td>
<td>Location of interslice forces</td>
</tr>
</tbody>
</table>

| 7n - 3    | Total number of unknowns |

The available shear strength along the failure surface is assumed to follow the Mohr-Coulomb failure criterion, which can be expressed in terms of effective stress as:

\[ \tau_f = C + \sigma' \tan(\phi) = C + (\sigma - u) \tan(\phi) \] \hspace{1cm} (3.2)
Where $\tau$ is the failure shear strength, $C$ is the cohesion, $\phi$ is the internal friction angle, $\sigma'$ is the effective stress, $\sigma$ is the total stress, and $u$ is the pore water pressure.

Applying equations of force equilibrium for a typical slice $i$ in both perpendicular and parallel directions ($\alpha_i$) to the base of each slice, one obtains the following equations:

\[
\begin{align*}
N_i &= W_i \cos(\alpha_i) + Q_i \cos(\beta_i - \alpha_i) + R_i \sin(\alpha_{i-1} - \alpha_i) \quad (3.3) \\
T_i &= W_i \sin(\alpha_i) + Q_i \sin(\beta_i - \alpha_i) + R_i \cos(\alpha_{i-1} - \alpha_i) - L_i \quad (3.4)
\end{align*}
\]

Where,

$W_i =$ weight of slice $i$

$N_i =$ force normal to the base of slice $i$

$T_i =$ force parallel to the base of slice $i$

$Q_i =$ external surcharge applied at slice $i$

$R_i =$ right-interslice force of slice $i$

$L_i =$ left-interslice force of slice $i$

$\alpha_i =$ inclination of slice $i$ base

$\alpha_{i-1} =$ inclination of slice $i-1$ base

$\beta_i =$ inclination of the external surcharge applied at slice $i$

Substituting Mohr-Coulomb strength criterion into Equation 3.4 and combining with Equation 3.3, then the left-interslice force ($L_i$) can be related to the right-interslice force ($R_i$) for a typical slice $i$ as follows:

\[
L_i = A_i + B_i R_i \quad (3.5)
\]

Where,
\[ A_i = \left[ W_i \sin(\alpha_i) + Q_i \sin(\beta_i - \alpha_i) - \frac{C_i b_i \sec(\alpha_i)}{FS} \right] \frac{\tan(\phi_i)}{FS} - \left[ W_i \cos(\alpha_i) + Q_i \cos(\beta_i - \alpha_i) - U_i \right] \frac{\tan(\phi_i)}{FS} \]  

\[ B_i = \left[ \cos(\alpha_{i-1} - \alpha_i) - \sin(\alpha_{i-1} - \alpha_i) \frac{\tan(\phi_i)}{FS} \right] \]  

and,

\[ C_i = \text{soil cohesion at the base of slice } i \]
\[ \phi_i = \text{soil friction angle at the base of slice } i \]

Slope/Shaft System:

If a single row of drilled shafts is installed directly after slice \( m \) as shown in Figure 3.1. Also, assume there is a load transfer factor between two consecutive slices (e.g., \( i \) and \( i-1 \)) to transfer the interslice force from one slice to another (i.e., \( L_{i-1} \) to \( R_i \)).

For convenience, the load transfer factor is denoted as \( \eta \). Based on this definition,

\[ R_i = \eta_{i-1} L_{i-1} \; ; \; \eta_{i-1} = 1 \; \forall \; i \in \{2, 3, 4, ..., n\} - \{m\} \]  

Where,

\[ \eta_{i-1} = \text{the load transfer factor between slice } i-1 \text{ and } i, \text{ and is equal to 1 for all slices except for the interface between slices } m \text{ and } m+1 \text{ where the row of drilled shafts is installed.} \]

Hence,

Substitute 3.8 into Equation 3.5, one gets

\[ L_i = A_i + B_i \eta_{i-1} L_{i-1} \]  

(3.9)
Equation 3.8 is an expression that can be used to relate the left- and right-interslice forces for a slice $i$. Further, Equation 3.9 is an expression that can be used to relate the left-interslice forces for two consecutive slices (e.g., $L_i = f(L_{i-1})$ for slices $i$ and $i-1$). Thus, Equations 3.8 and 3.9 can be applied and expanded for the $n$ vertical slices (i.e., the whole sliding soil mass) as follows:

\begin{align}
  i = 1 & \rightarrow L_1 = A_1 + B_1 R_1 \\
  i = 2 & \rightarrow L_2 = A_2 + B_2 R_2 = A_2 + B_2 \eta_1 L_1 = A_2 + B_2 \eta_1 (A_1 + B_1 R_1) \quad (3.10a) \\
  i = 3 & \rightarrow L_3 = A_3 + B_3 R_3 = A_3 + B_3 \eta_2 L_2 = A_3 + B_3 \eta_2 (A_2 + B_2 \eta_1 (A_1 + B_1 R_1)) \quad (3.10b) \\
  i = 4 & \rightarrow L_4 = A_4 + B_4 R_4 = A_4 + B_4 \eta_3 L_3 \\
 & = A_4 + B_4 \eta_3 (A_3 + B_3 \eta_2 (A_2 + B_2 \eta_1 (A_1 + B_1 R_1))) \quad (3.10c) \\
  \vdots \\
  i = n & \rightarrow L_n = A_n + B_n R_n = A_n + B_n \eta_{n-1} L_{n-1} \quad (3.10d)
\end{align}

The above illustrative equations (3.10a–e) are to show how each slice depends on all the preceding slices. To determine the left-interslice force, for example for slice 3, then the interslice forces for slices 1 and 2 are required, as well as the factor of safety $FS$. It should be noted that the right-interslice force for slice 1 ($R_1$) and the left-interslice force for slice $n$ ($L_n$) are boundary conditions which are the external boundary loads. Simplifying these recursive equations into one simple form that relates $R_1$ with $L_n$ would yield the following equation that governs the entire sliding soil mass:

\begin{align}
  L_n = A_n + \left[ \sum_{i=2}^{n} A_{i-1} \prod_{j=i}^{n} B_j \eta_{j-1} \right] + \left[ B_n \prod_{k=1}^{n-1} B_k \eta_k \right] R_1 
\end{align}

(3.11)
Where $\sum$ denotes the summation and $\prod$ denotes the multiplication. FS for a slope with or without drilled shafts can be determined using Equation 3.11, since it is one equation with only one unknown (i.e., FS is the only unknown in the A's and B's coefficients).

In order to determine the global factor of safety for the entire sliding soil mass when no drilled shafts are involved, Equation 3.11 can be simplified by setting all load transfer factors $\eta$ for all interfaces to 1.0. Thus,

$$L_n = A_n + \left[ \sum_{i=2}^{n} A_{i-1} \prod_{j=1}^{n} B_j \right] + \left[ B_n \prod_{k=1}^{n-1} B_k \right] R_j \tag{3.12}$$

The only unknown in Equation 3.12 is the global factor of safety which can be determined in an iterative procedure to satisfy the boundary conditions as described earlier.

The second term in the right-hand side of Equation 3.11 can be simplified and broken down into the following:

$$\left[ \sum_{i=2}^{n} A_{i-1} \prod_{j=1}^{n} B_j \eta_{j-1} \right] = B_{m+1} \eta_m \left[ \sum_{i=2}^{m+1} A_{i-1} \prod_{j=i}^{n} B_j \eta_{j-1} \right] + \left[ \sum_{i=m+2}^{n} A_{i-1} \prod_{j=i}^{n} B_j \eta_{j-1} \right] \tag{3.13}$$

Thus, with further mathematical simplification, $L_n$ can be re-written as:

$$L_n = A_n + \left[ B_{m+1} \eta_m \sum_{i=2}^{m+1} A_{i-1} \prod_{j=i}^{n} B_j \eta_{j-1} \right] + \left[ \sum_{i=m+2}^{n} A_{i-1} \prod_{j=i}^{n} B_j \eta_{j-1} \right] \tag{3.14}$$

Therefore, $\eta_m$ can be determined using the following equation:
For a feasible solution and acceptable FS in Equation 3.15, the value of $\eta_m$ should be positive and less than or equal to 1 (i.e., $0 < \eta_m \leq 1.0$). Thus, $\eta_m$ calculated using Equation 3.15 is the amount of the reduction in the interslice forces due to the presence of the drilled shafts that is required to achieve the FS used in the equation. Since the load transfer factor is equal to 1 for all interfaces between all soil slices except for interface of slices $m$ and $m+1$, Equation 3.15 can be simplified into the following closed-form solution:

$$
\eta_m = \frac{L_n - A_n - \left( \sum_{i=m+2}^{n} A_{i-1} \prod_{j=i}^{n} B_{j} \eta_{j-1} \right)}{\left[ B_{m+1} \sum_{i=2}^{m+1} A_{i-1} \prod_{j=i}^{n} B_{j} \eta_{j-1} \right] + \left[ B_n B_m \sum_{i=1; \ i \neq m}^{n-1} B_{i} \eta_{i} \right] R_1} ; \ 0 < \eta_m \leq 1.0
$$

Equation 3.16 is a very useful closed-form solution that enables us to determine the required reduction, $\eta_m$, in the forces to improve the stability of the slope/shaft system to a desired factor of safety of FS. Especially, one can plot $\eta_m$ versus FS for several possible values of FS to determine the minimum and maximum values of FS that can be achieved when a drilled shaft with diameter D is installed at a specific.

Equation 3.16 provides us with an understanding on the effects of shaft location within the slope on the load transfer factor and the global factor of safety. The optimum shaft location, where the drilled shafts need to be installed so that the final design
configurations of the slope/shaft system are economical, can be searched for using Equation 3.16. Advantages and applications of this closed-form solution will be discussed in the next section using an illustrative example.

Figure 3.1: Slope with sliding soil mass divided into n vertical slices
Figure 3.2: General system of forces acting on a typical slice i
3.4 Application Example

Consider a 2:1 homogeneous slope with a pre-defined circular slip surface as shown in Figure 3.3. Analyses of this slope with or without a single row of drilled shafts will be conducted utilizing the procedure discussed earlier in this chapter. The circular sliding soil mass is divided into 10 vertical slices with equal width of the slice bases \( b_i = 5.11 \text{ m} \). Equation 3.12 can be used to determine the factor of safety for the circular failure slip surface. The soil cohesion and internal friction angle are 20 kPa and 8°, respectively. The unit weight of the soil is taken as 16 kN/m\(^3\). The factor of safety was iteratively found to be 0.95. After the determination of the factor of safety, the interslice forces can be calculated by solving Equation 3.5 for slice 1, then slice 2, and so on. The right interslice force for the first slice is set to zero (i.e., \( R_1 = 0 \)), from which \( L_1 \) can be calculated. After that, \( L_1 \) is transferred to the next slice (slice 2) using Equation 3.8; in such a way, \( R_2 \) is equal to \( L_1 \) with a load transfer factor of 1. This procedure can be similarly applied to determine \( R_i \)'s and \( L_i \)'s for all slices. Table 3.2 provides a summary for the calculations of safety factor and the interslice forces for the 2:1 slope when no drilled shafts are considered.

Installing to investigate the use of a row of drilled shafts in the slope to improve the stability, the possible shaft location is considered to be between 30 m and 60 m, and a shaft diameter of 1.0 m. \( \eta \) versus FS diagram for various shaft locations can be developed using Equation 3.16 as plotted in Figure 3.4. The right-interslice force for the first slice \((R_1)\) and the left-interslice force for the tenth slice \((L_{10})\) were set to zero in Equation 3.16. Figure 3.4 provides the soil engineers with important information regarding the possible
range of FS for the slope/shaft system when drilled shafts are installed at the pre-defined specific locations.

Some comments on the $\eta$-FS Diagram in Figure 3.4 can be drawn as follows:

- At a specific drilled shaft location, the load transfer factor decreases (the amount of the reduction in the forces increases) the factor of safety for the slope/shaft system increases.

- A wide range of global factor of safety can be obtained depending upon the locations of the drilled shaft. Particularly, when drilled shafts are installed at locations between $x = 30$ m and $x = 45$ m, the FS varied a lot.

- The highest factor of safety of 1.15 can be obtained when a row of drilled shafts is installed at $x = 60$ m, regardless the design configurations of the slope/shaft system. However, the highest FS is 1.75 when the shafts are at $x = 55$ m.

- For the same factor of safety, the shaft location that gives the highest load transfer factor ($\eta_{\text{max}}$) is at $x = 40$ m. This shaft location can be considered as the optimum location, where the design configuration of the slope/shaft system is economized.
Figure 3.3: Slope geometry and slip surface specifications
Table 3.2: Summary for calculations of load safety factor and interslice forces (FS\(^{(1)}\) = 0.95); slices width = 5.112 m.

<table>
<thead>
<tr>
<th>Slice</th>
<th>(\alpha_i) (rad.)</th>
<th>(h_i) (m)</th>
<th>(W_i) (kN)</th>
<th>(A_i^{(2)})</th>
<th>(B_i^{(3)})</th>
<th>(\prod_{j=1}^{n} B_j)</th>
<th>(A_{i-1} \prod_{j=1}^{n} B_j)</th>
<th>(L_i^{(4)}) (kN)</th>
<th>(R_i^{(5)}) (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.03</td>
<td>2.6</td>
<td>218.5</td>
<td>-35.0</td>
<td>0.0000</td>
<td>---</td>
<td>---</td>
<td>11.3</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>0.80</td>
<td>10.2</td>
<td>836.6</td>
<td>366.7</td>
<td>0.9430</td>
<td>0.7330</td>
<td>-25.6</td>
<td>333.7</td>
<td>11.3</td>
</tr>
<tr>
<td>3</td>
<td>0.63</td>
<td>14.2</td>
<td>1163.1</td>
<td>417.9</td>
<td>0.9589</td>
<td>0.7772</td>
<td>285.0</td>
<td>737.9</td>
<td>333.7</td>
</tr>
<tr>
<td>4</td>
<td>0.47</td>
<td>15.0</td>
<td>1228.3</td>
<td>284.0</td>
<td>0.9654</td>
<td>0.8105</td>
<td>338.7</td>
<td>996.5</td>
<td>737.9</td>
</tr>
<tr>
<td>5</td>
<td>0.33</td>
<td>14.7</td>
<td>1208.4</td>
<td>115.6</td>
<td>0.9689</td>
<td>0.8396</td>
<td>238.5</td>
<td>1081.1</td>
<td>996.5</td>
</tr>
<tr>
<td>6</td>
<td>0.19</td>
<td>13.6</td>
<td>1119.7</td>
<td>-50.1</td>
<td>0.9709</td>
<td>0.8665</td>
<td>100.1</td>
<td>999.5</td>
<td>1081.1</td>
</tr>
<tr>
<td>7</td>
<td>0.06</td>
<td>11.8</td>
<td>970.3</td>
<td>-186.1</td>
<td>0.9719</td>
<td>0.8925</td>
<td>-44.7</td>
<td>785.3</td>
<td>999.5</td>
</tr>
<tr>
<td>8</td>
<td>-0.06</td>
<td>9.3</td>
<td>763.8</td>
<td>-269.5</td>
<td>0.9722</td>
<td>0.9183</td>
<td>-170.9</td>
<td>493.9</td>
<td>785.3</td>
</tr>
<tr>
<td>9</td>
<td>-0.20</td>
<td>6.1</td>
<td>500.2</td>
<td>-279.6</td>
<td>0.9719</td>
<td>0.9445</td>
<td>-254.6</td>
<td>200.4</td>
<td>493.9</td>
</tr>
<tr>
<td>10</td>
<td>-0.33</td>
<td>2.1</td>
<td>177.1</td>
<td>-194.7</td>
<td>0.9718</td>
<td>0.9718</td>
<td>-271.7</td>
<td>0.0</td>
<td>200.4</td>
</tr>
</tbody>
</table>

\[ \sum_{i=2}^{n} A_{i-1} \prod_{j=1}^{n} B_j = 194.7 \]

Notes on Table 3.2:

1. Satisfying Equation 3.12: \(A_{10} = -194.7\); \(\sum_{i=2}^{n} A_{i-1} \prod_{j=1}^{n} B_j = 194.7\); and \(R_1 = L_{10} = 0\).


4. Using Equation 3.5.

5. Using Equation 3.8; \(\eta_i = 1.0 \ \forall \ i \in \{2, 3, ..., n\}\)
Figure 3.4: $\eta$-FS diagram for the 2:1 simple slope example at various shaft locations

Effect of Shaft Location:

From the $\eta$-FS diagram shown in Figure 3.4, it can be seen that the selection of the shaft location is very important as it directly affects the behavior load transfer factor vs. the global factor of safety for the slope/shaft system. Furthermore, from the definition of the load transfer factor, for a selected FS, as $\eta$ increases the amount of the reduction needed in the interslice force decreases. This means that the selection of the optimum shaft location should correspond to the highest possible value of $\eta$. Tables 3.3 and 3.4 provide calculation summary for the load transfer factors and the left-interslice forces at various shaft locations to achieve a pre-determined factor of safety (FS) of 1.5 and 2.0, respectively. These calculation results are plotted in Figures 3.5 and 3.6, respectively.
The load transfer factor needed to achieve a factor of safety of 1.5 increases as the shaft location moves away from the toe of the slip surface up to a point, where $\eta$ reaches its maximum value. Thereafter, $\eta$ starts to decrease as the shaft location approaches the highest point of contact between the displaced material and the main scarp. Based on observations from Figure 3.5 and 3.6, the optimum shaft location is at $x \simeq 40$ m, if there are no constructability-related issues at that location. This optimum shaft location is constant and remains unchanged for this example. In other words, for all possible values of FS for this specific example, the optimum shaft location is at $x \simeq 40$ m. In addition, the smallest value of $\eta$ (high amount of reduction) is found, for a specific factor of safety, at the location where the drilled shaft is close to the top of the slope. This is attributed to the fact that the interslice forces at that location are the smallest. From Figure 3.6, one can see that FS of 1.5 for the slope/shaft system in this example can not be achieved when the shaft location denoted by $x$ is larger than 55 m. Similarly, as shown in Figure 3.6, the FS of 2.0 can not be achieved when $x$ is larger than 50 m. One can notice a very similar shape in the relationship between the load transfer factor and the interslice force vs. drilled shaft locations, as shown in Figures 3.5 and 3.6.

The distribution of the interslice forces vs. the $x$-coordinate along the slip surface is plotted in Figure 3.7 for the following conditions: (a) no drilled shafts in the slope; (b) drilled shafts are installed at the optimum location ($x = 40$ m) with an improved FS of 1.5; and (c) drilled shafts are installed at $x = 40$ m with an improved FS of 2.0. The reduction in the interslice forces at $x = 40$ m is obvious in the figure due to the presence of the drilled shafts at that location. The interslice forces for the upper portion of the slope (below the drilled shafts) increase as the FS for the slope/shaft system increases as
illustrated in Figure 3.7, the interslice forces reach a value of 1,795.8 kN when FS is equal to 2.0. The interslice forces reach a value of 1,544.1 kN when FS is equal to 1.5. Furthermore, as can be seen in Figure 3.8 for a given factor of safety (e.g., FS = 1.5), the amount of the reduction in the interslice forces when the drilled shaft is installed at the optimum location (x = 40 m) is less than that when the drilled shaft is at x = 30 m.

Table 3.3: Calculated load transfer factors and left-interslice forces to obtain FS of 1.5 at various shaft locations

<table>
<thead>
<tr>
<th>FS = 1.5</th>
<th>X (m)</th>
<th>( \eta_m )</th>
<th>( L_m ) (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.245187</td>
<td>1355.855</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.38319</td>
<td>1675.758</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>0.419222</td>
<td>1590.767</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.351457</td>
<td>1409.37</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>0.139781</td>
<td>1058.084</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.4: Calculated load transfer factors and left-interslice forces to obtain FS of 2.0 at various shaft locations

<table>
<thead>
<tr>
<th>X (m)</th>
<th>η_m</th>
<th>L_m (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.168747</td>
<td>1679.504</td>
</tr>
<tr>
<td>40</td>
<td>0.266845</td>
<td>1926.174</td>
</tr>
<tr>
<td>45</td>
<td>0.273158</td>
<td>1797.349</td>
</tr>
<tr>
<td>50</td>
<td>0.171617</td>
<td>1570.691</td>
</tr>
<tr>
<td>55</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>60</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Figure 3.5: Load transfer factor and left-interslice force at various shaft locations to obtain FS of 1.5
Figure 3.6: Load transfer factor and left-interslice force at various shaft locations to obtain FS of 2.0
Figure 3.7: Comparison of distributions of the interslice forces with no drilled shafts and with drilled shafts at the optimum location (x = 40 m)
Estimation of Load Transfer Factor ($\eta$)

The parameters influencing the load transfer factor $\eta$ can be classified into three main categories as follows:

1. Material Properties:
   - Soil Properties: Cohesion and internal friction angle.
   - Drilled shaft properties: modulus of elasticity of shaft.
   - Rock properties: modulus of elasticity of rock.

2. Geometric Properties:
   - Total shaft length.
• Shaft diameter.
• Center-to-center spacing between the drilled shafts.
• Shaft location within the sliding soil mass.
• Depth of slip surface.
• Rock-socket length.

3. Soil movement: the soil movement is essential for the arching mechanism to develop and for the transferring process of forces from the soil to the drilled shafts to take place. It is evident and reported in the literature that as the soil movement increases the load transfer factor and the load imparted to the drilled shafts increases as well. It is recommended to use the highest value of $\eta$ at the ultimate state (i.e., maximum soil movement) for both geotechnical and structural design, since it will provide the engineer with the most economical design configurations of the slope/shaft system.

A study carried out by Liang and Zeng (2002) provides a simplified equation for the load transfer $\eta$ as in the following equation:

$$\eta = \frac{1}{S/D} + \left(1 - \frac{1}{S/D}\right)R_p$$

(3.17)

Where,

$S$: the center-to-center spacing between drilled shafts;

$D$: drilled shaft diameter;

$R_p$: Percent of residual load acting on soil mass between the drilled shafts.
R_p in Equation 3.17 can be estimated utilizing the two-dimensional parametric finite element study by Liang and Zeng (2002) which was performed for the following sets of parameters: S/D = 2, 3, and 4; D = 30.48 cm, 60.96 cm, and 91.44 cm; C = 0 kPa to 41 kPa; and \(\phi = 0^\circ\) to \(40^\circ\). Equation 3.17 can be solved for the necessary spacing S and the corresponding shaft diameter D for the required load transfer factor \(\eta\). The equation by Liang and Zeng (2002) is too simplified and based on many idealized assumptions as contrast to the current study.

From the above discussion, the behavior of the many factors influencing the load transfer factor \(\eta\) is considered complicated and nonlinearly inter-related. For more accurate and comprehensive estimation of the load transfer factor, there is a compelling need for a three-dimensional FE simulation for the combined slope/shaft system. Chapter IV presents the FEM simulations for estimating \(\eta\).

3.6 Summary and Conclusions

Limiting equilibrium solution, considering the external applied loads, to determine the global factor of safety for the whole slope including drilled shaft effect was derived into a relatively simple equation. This solution is fundamentally valid and applicable to general failure slip surfaces while accommodating complex soil stratifications. The equations of force equilibrium were satisfied for all slices. Some key assumptions were adopted to ensure formulation of the limiting equilibrium method in a determinate problem. A numerical closed-form solution was derived for the slope/shaft system to provide a better understanding of the behavior of the transfer of forces vs. the global factor of safety due to the presence of the drilled shafts at a specific location.
within the slope body. An application example was analyzed using the derived mathematical expressions. The effects of shaft location on the effectiveness of improving FS of the slope/shaft system were thoroughly investigated. The optimum location of the drilled shafts and the necessary reduction in the forces to obtain a specific factor of safety can be searched for and determined using the discussed procedure. The distribution of the interslice forces along the sliding soil mass with or without drilled shafts were analyzed and compared.
CHAPTER IV

THREE-DIMENSIONAL FINITE ELEMENT SIMULATION FOR SLOPE/SHAFT SYSTEMS

4.1 Overview

Slope stability analysis has been widely performed assuming a two-dimensional case, in which, a representative cross-section of a slope is used in the limiting equilibrium analysis. The 2-D representation in the slope stability analysis has been found and considered to be appropriate because it yields conservative estimate of FS when compared with three-dimensional analysis. However, when drilled shafts are installed in a row in an unstable slope, two-dimensional analysis is no longer applicable due to the effect that many of the influential factors would be ignored, as enumerated: (1) the rigidity of the drilled shafts as influenced by its diameter, modulus of elasticity, and total length is not accounted for; (2) shafts spacing and location; (3) the material properties of rock and the socket length of shaft; and (4) the soil movement and strength parameters. Therefore, three-dimensional analysis is needed to evaluate influencing factors mentioned in the above. Furthermore, only true 3-D analysis can capture the arching mechanism and the load transfer phenomenon discussed in the previous chapter.
In order to assess the load transfer mechanism of a slope stabilized by a row of drilled shafts, the three-dimensional finite element modeling technique was used, where the nonlinear and plastic behavior of soil and the elastic behavior of the drilled shafts were adopted. The non-yielding (or rock stratum) where the drilled shafts will be socketed into was modeled as elastic material. The interface between the shaft and the surrounding soil and the rock is implicitly considered as a frictional model. Sensitivity analysis of the importance of the effects of several controlling factors on the load transfer behavior was thoroughly investigated. Evidences of soil arching and reduction in the stresses and displacements through the load transfer mechanisms due to the presence of the drilled shafts were highlighted through the FEM simulation results. Based on a comprehensive sensitivity and parametric study and careful regression analysis, design charts were created to obtain a numerical value of the load transfer factor at the failure state of the slope/shaft system. Validity of the finite element simulation and model technique were presented by comparing results using two different computer programs based on two different solution procedures. This chapter presents a summary and assessment of FEM simulation results, which in turn are used to provide an insight on the behavior of drilled shafts stabilized slopes.

4.2 General Statements

The slope/shaft model studied and presented herein is analyzed using the three-dimensional finite element program ABAQUS (1998). This program is capable of handling nonlinear finite element analysis. Figure 4.1 shows the side view, plan view, and boundary conditions of the conceptual model used in the present finite element...
simulations. Only one drilled shaft is modeled due to symmetric nature of the problem. The boundary conditions used in the model simulate the real situation in the field. As illustrated in Figure 4.1, both upper and lower sides as depicted in the plan view are restrained in the z-direction but free to move in the x- and y-directions. The right and left sides as depicted in both views are allowed to move freely in the y- and z-directions, but not in the x-direction. At the bottom plane of the model, all movements are restrained (i.e., \( u_{X,Y,Z} = 0 \)). Figure 4.2 shows the corresponding 3D assembly of the slope/shaft model. Figure 4.3 shows the 3D view of the finite element mesh generated by ABAQUS program. The FEM mesh is well refined at and near the region where the drilled shaft is installed to enable a closer examination of the state of stresses and displacements at that region. The mesh sensitivity and solution convergence are studied by comparing FEM results due to the use of different number of elements. Therefore, it is believed that the mesh size used in the model to be small enough so that the FEM simulation results are not sensitive to mesh density.

The constitutive behavior of soil continuum is modeled using Mohr-Coulomb model. The Mohr-Coulomb strength criterion has been widely used for geotechnical materials.

The Mohr-Coulomb yield surface used by ABAQUS (1998), as illustrated in Figure 4.4, can be written as:

\[
F = R_{mc} q - p \tan \phi - C = 0
\]

(4.1)

Where

\[
p = \text{equivalent pressure stress} = \frac{1}{3} \sigma'_{kk}
\]

(4.2)

\[
q = \text{the Mise’s equivalent stress} = \sqrt{\frac{1}{2} S_{ij} \delta_{ij}}
\]

(4.3)
\( S_{ij} = \) stress deviator = \( \sigma'_{ij} + p \delta_{ij} \) \( (4.4) \)

\( \sigma'_{ij} = \) effective stress tensor

\( C = \) the cohesion of the material

\( \phi = \) the friction angle of the material

\( R_{mc} = \) the Mohr-Coulomb deviatoric stress measure determined by in the following equation:

\[
R_{mc}(\Theta,\phi) = \frac{1}{\sqrt{3} \cos \phi} \sin \left( \Theta + \frac{\pi}{3} \right) + \frac{1}{3} \cos \left( \Theta + \frac{\pi}{3} \right) \tan \phi
\]

\( (4.5) \)

Where \( \Theta = \) the deviatoric polar angle (Chen and Han, 1988) and can be determined as follows:

\[
\cos(3\Theta) = \frac{9}{2} \frac{S_{ij}S_{jk}S_{ki}}{q^3}
\]

\( (4.6) \)

A hyperbolic function in the meridional stress plane and a smooth elliptic function in the deviatoric stress plane are used by ABAQUS (1998) to describe the flow potential \( G \) as follows:

\[
G = \sqrt{(\varepsilon C + \tan \psi)^2 + (R_{mw} q)^2} - p \tan \psi
\]

\( (4.7) \)

Where \( \psi \) is the dilation angle, \( \varepsilon \) is a parameter referred to as the eccentricity that defines the rate at which the function approaches its asymptote, and \( R_{mw}(\Theta,\varepsilon) \) is the deviatoric elliptic function used by Menétrey and William (1995) as in the following equation:

\[
R_{mw}(\Theta,\varepsilon) = \frac{4(1 - e^2) \cos^2 \Theta + (2e - 1)^2}{2(1 - e^2) \cos \Theta + (2e - 1)\sqrt{4(1 - e^2) \cos^2 \Theta + 5e^2 - 4e}} R_{mc}\left(\frac{\pi}{3},\phi\right)
\]

\( (4.8) \)
Where \( e \) = the out-of-roundness parameter calculated as follows:

\[
e = \frac{3 - \sin \phi}{3 + \sin \phi}
\]  

(4.9)

Clay soils tend to show little dilatancy (\( \psi \approx 0 \)). The dilatancy of sand depends on both the density and the friction angle. For \( \phi > 30^\circ \), dilatancy angle \( \psi \) can be taken as \( \psi \approx \phi - 30^\circ \), for the \( \phi \)-values less than 30°; otherwise, the angle of dilatancy is mostly zero. For further information about the link between the friction angle and dilatancy, the reader can refer to Bolton (1986).

In addition to the elasto-plastic soil continuum, both drilled shaft and rock continuum were modeled as a linear-elastic material with modulus of elasticity and Poisson’s ratio as input parameters. Slippage and gapping were allowed at the soil or rock interface with the drilled shaft by using the interface elements with Coulomb’s friction theory where the coefficient of friction (\( \mu \)) is the input parameter.
Figure 4.1: Conceptual model and boundary conditions
Figure 4.2: Three-dimensional finite element modeling
Figure 4.3: Three-dimensional finite element modeling
4.3 Model Parameters

The range of the values for various parameters used in the finite element sensitivity study is discussed in this section. Parameters used to define the soil properties are: cohesion (C), friction angle (φ), and unit weight (γ). The elastic behavior of the drilled shaft and the rock layer properties are defined using modulus of elasticity and Poisson's ratio (E_p, ν_p) and (E_r, ν_r) for the shaft and the rock, respectively. Four geometric parameters are introduced to represent different geometric dimensions in the system of rock-socketed drilled shafts and the slope. The notation ξ with different subscript is used to represent the various geometric parameters depicted in Figure 4.5. They are: (1) ξ_x =
\( x_i / X \), where \( x_i \) is the horizontal distance between the toe of the slope and the shaft, and \( X \) is the horizontal distance of the sloping ground. \( \xi_x \) is used to define the shaft location within the slope body; (2) \( \xi_y = y_i / Y \), where \( y_i \) is the vertical distance from the shaft top to the slip surface, and \( Y \) is the vertical distance from the shaft top to the top of rock. \( \xi_y \) is used to define the depth ratio along the drilled shaft with respect to the portion of the shaft length above the rock layer; (3) \( \xi_r = L_r / L_p \), where \( L_r \) is length of the portion of the shaft inside the rock layer, and \( L_p \) is total length of the shaft. \( \xi_r \) is used to define the rock-socket length with respect to the total shaft length; and (4) \( \xi_s = S / D \), where \( S \) is center-to-center spacing between two adjacent drilled shafts, and \( D \) is shaft diameter. \( \xi_s \) is used to define the spacing-to-diameter ratio. In addition, the interface interaction at the surfaces between the drilled shafts with the rock and the soil are defined in accordance with Coulomb's friction theory with the friction coefficients \( \mu_{s-p} \) and \( \mu_{r-p} \) for soil and rock, respectively. At the top of the slope, a load intensity \( q_f \) with a unit of \([F/L^2]\) is applied to generate soil movement similar to that may occur in the field. Model parameters discussed herein were thoroughly investigated in a parametric study to assess their influences on the behavior of the drilled shafts stabilized slope, in particular the soil arching and load transferring behavior is quantified by way of comprehensive regression.
Figure 4.5: Model parameters
4.4 Load Transfer Mechanism

The phenomenon of stress transfer from a yielding part of soil to adjacent non-yielding soil through shear transfer is usually referred to as arching. Based on a review of literature (Terzaghi, 1936, 1943; Bosscher et al., 1986), several properties controlling the soil arching are summarized in Table 4.1.

Table 4.1: List of properties influencing soil arching mechanism

<table>
<thead>
<tr>
<th>Soil Properties</th>
<th>Cohesion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Friction angle</td>
</tr>
<tr>
<td></td>
<td>Unit weight</td>
</tr>
<tr>
<td></td>
<td>Soil movement</td>
</tr>
<tr>
<td>Drilled Shaft Properties</td>
<td>Modulus of elasticity</td>
</tr>
<tr>
<td></td>
<td>Poisson's ratio</td>
</tr>
<tr>
<td>Geometric Properties</td>
<td>Slope height</td>
</tr>
<tr>
<td></td>
<td>Rock socket length</td>
</tr>
<tr>
<td></td>
<td>Drilled shafts spacing</td>
</tr>
<tr>
<td></td>
<td>Drilled shafts diameter</td>
</tr>
<tr>
<td></td>
<td>Drilled shaft location</td>
</tr>
<tr>
<td></td>
<td>Total drilled shaft length</td>
</tr>
<tr>
<td>Interaction Properties</td>
<td>Soil-shaft friction</td>
</tr>
<tr>
<td></td>
<td>Soil-rock friction</td>
</tr>
<tr>
<td></td>
<td>Rock-shaft friction</td>
</tr>
</tbody>
</table>

Three-dimensional finite element modeling was performed to determine the load transfer factor for drilled shafts installed in one row within the slope body and socketed deep enough into a stable soil (i.e., rock). The load transfer factor is denoted as $\eta$. Factors
that would affect $\eta$ as discussed earlier in the model parameters section are thoroughly investigated in this finite element parametric study. The numerical values of $\eta$ express the reduction in the total driving forces within the slope body due to the existence of drilled shafts. In other words, $\eta$ defines the ratio of the forces that the drilled shaft will exert on the soil on the downslope side to the force that the soil will exert on the shaft on the upslope side. Figure 4.6 provides a schematic illustration of $\eta$. Therefore, $\eta$ reflects the amount of forces that will be transmitted to the downslope soils directly behind the drilled shafts. $\eta$ can be written as:

$$\eta = \frac{F'}{F}$$  \hspace{1cm} (4.10)

Where $F'$ is the total force acting on the vertical plane at the interface between the drilled shaft and the soil just on the downslope side and can be determined by double integrating the downslope stresses in the $x$-direction ($\sigma'_x$) one time over the model thickness ($S$) and the second time over the portion of the shaft length above the slip surface, and $F$ is similarly obtained but at the interface on the upslope side. Mathematical expressions for calculating $F'$ and $F$ are as follows:

$$F' = \int \int (\sigma'_x) \, dz \, dy$$  \hspace{1cm} (4.11)

$$F = \int \int (\sigma_x) \, dz \, dy$$  \hspace{1cm} (4.12)
One of the important factors that could affect $\eta$ is the relative lateral displacement between the shaft and the surrounding soil. The forces that are acting on the drilled shafts are very small when no soil movement is occurring, while the forces start to develop gradually as the relative soil movement increases. Therefore, in order to generate a three-dimensional real-time lateral displacement field, load intensity ($q_0$) as explained earlier in the model parameters section is applied at the top of the slope. For the purposes of
determining η at the ultimate state, which is defined as the state where the lateral soil movement is reached, beyond a complete failure of the soil-shaft system occurs. In other words, the relative lateral displacement between the shaft and the surrounding soil exceeded the limits that the shaft-soil can withstand.

In this study, the maximum allowable movement the drilled shaft can withstand in the displaced ground is used, rather than the soil movement to represent the state of failure of the soil-shaft system. This is due to the variability of the soil movement along the shaft length, as well as the variability direction of the spacing between the drilled shafts. For convenience, lateral shaft movement is denoted as δ.

The displacement contours obtained from the finite element simulation of a typical slope with drilled shafts are shown in Figure 4.7a. The simulation results showed that the lateral soil movements are complex and non-uniform, especially along the shaft depth as well as in the direction of the spacing between the drilled shafts as shown in the close-up view in Figure 4.7b. It is also obvious from Figure 4.7b the occurrence of soil arching; by the effect that the displacement contours illustrate the maximum lateral displacement occurs at the location between the drilled shafts, while the relative displacements is minimum at the location of drilled shaft.

Another evidence for the occurrence of soil arching can be seen in Figure 4.8a, where, the stress contours illustrate a maximum stress value at the location of the drilled shaft compared with a minimum stress value between the drilled shafts. Transfer of stresses and the reduction of these stresses from upslope to downslope sides due to the drilled shaft can also be seen in Figure 4.8b. The FEM simulation results of both displacement and stress contours as shown in Figures 4.7 and 4.8 agree with the behavior
of arching asserted by other researchers (e.g., Wang and Yen, 1974; Bosscher and Gray, 1986; Adachi et al. 1989; Liang and Zeng, 2002).

The relationship between $\eta$ and $\delta$ as well as the load acting on the drilled shafts are plotted in Figure 4.9 for five loading stages (i.e., five load intensity values), starting from no load intensity as stage 1 (i.e., $q_f = 0$), and ending with maximum load intensity as in stage 5 (i.e., $q_f$). In all stages, the gravity weight of the slope is included in the analysis. Load transfer factor and the load acting on the shafts reach their respective maximum values when $q_f$ is at the maximum. In order for the drilled shafts to work as an earth retaining structural elements for the soil slope, a minimum lateral soil movement to cause a shaft movement is required.

For the purpose of geotechnical design of the drilled shafts stabilized slope, as well as, the structural design of the shaft section; $\eta$ at failure should be considered and used due to the following considerations: (1) the value of $\eta$ is maximum (i.e., the reduction in the transferred forces to the downslope soil is minimum); and (2) the applied load to the shaft is maximum (i.e., safe structural design).
Figure 4.7a: Displacement contours for the whole model
Figure 4.7b: Displacement contours for the vertical plane just before the drilled shaft
Figure 4.8a: Stress contours for the vertical plane just before the drilled shaft
Figure 4.8b: Stress contours for the horizontal plane at some depth
Figure 4.9: Effect of loading stages used in the FE simulations on $\eta$, $\delta$, and shaft load.

4.5 Finite Element Parametric Study

An extensive parametric study using ABAQUS FEM program was carried out to examine systematically the effects of several influencing factors on the load transfer factor ($\eta$) and the lateral shaft movement ($\delta$). The influencing parameters discussed earlier were varied systematically in a series of three-dimensional finite element parametric study. This FEM parametric study will better quantify the importance and effect on soil arching for slope/shaft system.

The FEM parametric study is carried out in two stages. The range of parameters studied is listed in Table 4.2. The two stages of parameters study are as follows: (a) Stage
1: by studying the effect of each parameter individually by changing one parameter and fixing the other parameters in order to determine the most and least important parameters that can affect $\eta$ and $\delta$. The number of successful runs performed in this stage is 68. (b) Stage 2: by randomly selecting many combinations of the shaft-soil-rock parameters so that the effects of variation of parameters on $\eta$ and $\delta$ are captured. The number of successful runs in this stage is 60.

Tables 4.3 and 4.4 summarize the sensitivity analysis results for stage 1 for the calculated results of $\eta$ and $\delta$, respectively. The same results are plotted in a pie chart in Figures 4.10 and 4.11. The order of importance of the parameters affecting $\eta$ can be ranked from high to low as follows: ($\xi_y$, $D$, $\xi_x$, $L_p$, $C$, $\xi_r$, $\xi_s$, $E_r$, $E_p$, $\mu_{s-p}$, $\gamma$, $\nu_p$, and $\nu_r$). The relative order of importance on $\delta$ is as follows: ($C$, $D$, $L_p$, $\xi_y$, $\xi_x$, $\xi_s$, $\xi_r$, $E_r$, $E_p$, $\phi$, $\gamma$, $\nu_l$, $\nu_p$, and $\mu_{s-p}$). It can be observed that $\mu_{s-p}$, $\gamma$, $\nu_p$, and $\nu_r$ have exerted relatively small effects on $\eta$ and $\delta$. These parameters are therefore held constant throughout the finite element simulations and excluded from the parametric study.

The results of finite element simulations are further analyzed by using SPSS (2003) program (i.e., Statistical Package for the Social Sciences). It was assumed, for the purposes of nonlinear regression, that $\eta$ and $\delta$ are equal to separate functions multiplied by each other, and each function expresses the effect of one parameter on $\eta$ and $\delta$ as in the following equations:

$$\eta = \eta(C)\eta(\phi)\eta(D)\eta(\xi_s)\eta(E_r)\eta(\xi_r)\eta(L_p)\eta(E_p)\eta(\xi_x)\eta(\xi_y)$$

$$\delta = \delta(C)\delta(\phi)\delta(D)\delta(\xi_s)\delta(E_r)\delta(\xi_r)\delta(L_p)\delta(E_p)\delta(\xi_x)\delta(\xi_y)$$
The built-in Levenberg-Marquardt algorithm (LMA) in the SPSS is used to perform the curve fitting to find $\eta(C)$, $\eta(\phi)$, etc... and $\delta(C)$, $\delta(\phi)$, etc... functions. The LMA is an iterative technique that provides a generic curve-fitting. Essentially, it provides a numerical solution to the mathematical problem of minimizing a sum of squares of nonlinear functions that depend on a common set of parameters. The solution obtained from the SPSS is presented for convenience in five design charts given in Figures 4.12 (a) through (e). These charts are used in accordance with the following equations:

$$
\eta = \eta(C, \phi) \eta(D, \xi_x) \eta(E_r, \xi_r) \eta(L_p, E_p) \eta(\xi_x, \xi_y)
$$

$$
\delta = K \delta(C, \phi) \delta(D, \xi_x) \delta(E_r, \xi_r) \delta(L_p, E_p) \delta(\xi_x, \xi_y)
$$

Where the constant $K$ in Equation 33 is equal to 1.0 for $\delta$ in centimeters and 0.3937 for $\delta$ in inches.

Goodness of the nonlinear regression analysis presented in the design charts in Figures 4.12 (a) through (e) was statistically examined using R-squared (i.e., statistical measure of how well a regression line approximates real data points, in which, an R-squared of 1.0 indicates a perfect fit). R-squared was found to be 0.88 for the regression line between $\eta_{\text{regression}}$ and $\eta_{\text{FEM}}$ as shown in Figure 4.13, and 0.96 between $\delta_{\text{regression}}$ and $\delta_{\text{FEM}}$ as shown in Figure 4.14. These R-squared values are considered to be appropriate, especially for such a highly nonlinear and complex combination of parameters.
Table 4.2: List of parameters and ranges used in the finite element parametric study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion (C)</td>
<td>0 – 100 kPa (0-2088.5 psf)</td>
</tr>
<tr>
<td>Friction Angle ($\phi$)</td>
<td>0 – 45$^\circ$</td>
</tr>
<tr>
<td>Unit Weight ($\gamma$)</td>
<td>16.5 – 21 kN/m$^3$ (103 – 131 pcf)</td>
</tr>
<tr>
<td>Shaft Diameter (D)</td>
<td>0.6 – 3.6 m (2 – 11.8 ft)</td>
</tr>
<tr>
<td>Spacing-to-Diameter Ratio ($\xi_s$)</td>
<td>2 - 5</td>
</tr>
<tr>
<td>Rock Socket Length Ratio ($\xi_r$)</td>
<td>0.05 – 0.45</td>
</tr>
<tr>
<td>Rock Modulus (E$_r$)</td>
<td>3.5 – 35 GPa (500 – 5000 ksi)</td>
</tr>
<tr>
<td>Rock Poisson’s Ration ($\nu_r$)</td>
<td>0.20 – 0.25</td>
</tr>
<tr>
<td>Shaft Length (L$_p$)</td>
<td>5 – 13.5 m (16.4 – 80.7 ft)</td>
</tr>
<tr>
<td>Shaft Modulus (E$_p$)</td>
<td>18 – 30 GPa (2610 – 4350 ksi)</td>
</tr>
<tr>
<td>Shaft Poisson’s Ration ($\nu_p$)</td>
<td>0.20 – 0.25</td>
</tr>
<tr>
<td>Friction coefficient between soil and shaft ($\mu_{s-p}$)</td>
<td>0.0 – 0.5</td>
</tr>
<tr>
<td>Friction coefficient between rock and shaft ($\mu_{r-p}$)</td>
<td>0.8</td>
</tr>
</tbody>
</table>
Table 4.3: Sensitivity analysis of $\eta$ (Stage I)

| Parameter | $\eta_{\text{min}}$ | $\eta_{\text{max}}$ | $|\eta_{\text{max}} - \eta_{\text{min}}|$ | $\frac{|\eta_{\text{max}} - \eta_{\text{min}}|}{\sum |\eta_{\text{max}} - \eta_{\text{min}}|} \times 100\%$ |
|-----------|----------------|----------------|-------------------------------|------------------------------------------------------|
| $\xi_y$   | 0.00000       | 100000.        | 10000.                        | 25.09379                                             |
| C         | 0.611894      | 100000.        | 0.388106                      | 9.739052                                             |
| $\phi$    | 0.214321      | 0.331299       | 0.116978                      | 2.935422                                             |
| $\gamma$  | 0.767616      | 0.781599       | 0.013983                      | 0.350887                                             |
| $\xi_s$   | 0.713406      | 0.775647       | 0.062241                      | 1.561863                                             |
| D         | 0.038003      | 0.815843       | 0.77784                       | 19.51895                                             |
| $E_r$     | 0.764489      | 0.828818       | 0.064329                      | 1.614259                                             |
| $\xi_r$   | 0.3901        | 0.730177       | 0.340077                      | 8.533822                                             |
| $E_p$     | 0.742864      | 0.802385       | 0.059521                      | 1.493608                                             |
| $L_p$     | 0.370595      | 0.820000       | 0.449405                      | 11.27728                                             |
| $\xi_x$   | 0.043593      | 0.706279       | 0.662686                      | 16.62931                                             |
| $\mu_{s-p}$ | 0.721807     | 0.754420       | 0.032613                      | 0.818384                                             |
| $\nu_p$   | 0.82124       | 0.838216       | 0.016976                      | 0.425992                                             |
| $\nu_r$   | 0.775444      | 0.775738       | 0.000294                      | 0.007378                                             |

$\Rightarrow \sum 3.985049 = 100$
Table 4.4: Sensitivity analysis of $\delta$ (Stage I)

| Parameter | $\delta_{\text{min}}$ | $\delta_{\text{max}}$ | $|\delta_{\text{max}} - \delta_{\text{min}}|$ | $\frac{|\delta_{\text{max}} - \delta_{\text{min}}|}{\sum |\delta_{\text{max}} - \delta_{\text{min}}|} \times 100\%$ |
|-----------|-----------------|-----------------|------------------|----------------------------------|
| $\xi_y$   | 0.000162        | 0.01283         | 0.012668         | 6.950585                         |
| C         | 0.017730        | 0.073318        | 0.055588         | 30.49862                         |
| $\phi$    | 0.003743        | 0.006257        | 0.002514         | 1.379171                         |
| $\gamma$  | 0.01049         | 0.012688        | 0.002198         | 1.205884                         |
| $\xi_s$   | 0.008999        | 0.015776        | 0.006777         | 3.718275                         |
| D         | 0.001226        | 0.051542        | 0.050315         | 27.60566                         |
| $E_r$     | 0.01112         | 0.013828        | 0.002708         | 1.485587                         |
| $\xi_r$   | 0.010794        | 0.016014        | 0.005221         | 2.864462                         |
| $E_p$     | 0.009986        | 0.013697        | 0.003711         | 2.035924                         |
| $L_p$     | 0.006067        | 0.03726         | 0.031193         | 17.11393                         |
| $\xi_x$   | 0.000599        | 0.009732        | 0.009133         | 5.010932                         |
| $\mu_{s-p}$ | 0.011278     | 0.011459        | 0.000181         | 0.099416                         |
| $\nu_p$   | 0.011416        | 0.011469        | 5.31E-05         | 0.029133                         |
| $\nu_r$   | 0.011437        | 0.011441        | 4.4E-06          | 0.002414                         |

$\Rightarrow \sum 0.182265 \quad 100$
Figure 4.10: Sensitivity analysis of $\eta$ (Stage I)
Figure 4.11: Sensitivity analysis of $\delta$ (Stage I)
Figure 4.12a: Design charts to estimate η and δ for the effects of cohesion and friction angle
Figure 4.12b: Design charts to estimate $\eta$ and $\delta$ for the effects of shaft diameter and spacing-to-diameter ratio.
Figure 4.12c: Design charts to estimate $\eta$ and $\delta$ for the effects of rock-socket length ratio and rock modulus.
Figure 4.12d: Design charts to estimate $\eta$ and $\delta$ for the effects of shaft length and shaft modulus.
Figure 4.12e: Design charts to estimate $\eta$ and $\delta$ for the effects of shaft location and depth of slip surface
Figure 4.13: Regression analysis of $\eta$
Figure 4.14: Regression analysis of $\delta$
4.6 Observations Based on FE Simulation Results

Some observations can be drawn from finite element simulation results and the developed design charts regarding the characteristics of $\eta$ and $\delta$ for a single row of rock-socketed drilled shafts in a moving soil slope.

- With no or small soil movements, the loads impart on the drilled shafts are very small.
- As soil movement increases, the loads impart on the drilled shafts grow rapidly.
- As soil movement increases, $\eta$ and $\delta$ increase as well.
- The profile of lateral soil movements in the vertical direction and in the transverse direction is complex and non-uniform.
- $\eta$ and $\delta$ increase as the drilled shaft rock-socket length ratio decreases. This means that drilled shafts must be socketed deep enough into a firm soil in order to fully take advantage of the soil arching mechanism as well as to help the forces to be transferred to the drilled shafts.
- $\eta$ increases with an increase of the total length of the drilled shafts up to a length of 7.5 m (25 ft). Further increase of shaft length beyond 7.5 m (25 ft) does not affect the $\eta$ value. $\delta$ increases with an increase in $L_p$.
- The shaft location within the slope is an important factor affecting $\eta$ and $\delta$.
- The deeper the slip surface, the higher $\eta$ value, if all other parameters are kept constant.
- Shaft diameter is an important factor affecting $\eta$ and $\delta$. For shaft diameter less than 1 m (3.2 ft), $\eta$ is more or less a constant. However, for shaft diameter
larger than 1 m (3.2 ft), \( \eta \) decreases as the shaft diameter increases. On the other hand, \( \delta \) increases as the shaft diameter increases up to a diameter of 1.6 m (5.3 ft); thereafter, \( \delta \) remains as a constant.

- \( \eta \) increases as the soil cohesion increases. However, \( \eta \) decreases with an increasing as the friction angle of the soil.
- \( \eta \) and \( \delta \) increase as the spacing-to-diameter ratio of the drilled shafts increases.
- As the shaft rigidity increases (i.e., increasing \( E_p \) and decreasing \( L_p \)), \( \delta \) and \( \eta \) decrease. Therefore, a more rigid drilled shaft provides more reduction in the forces compared to a less rigid shaft.
- The rank of influencing factors affecting \( \eta \) is as follows: depth of slip surface (25%), drilled shaft diameter (20%), drilled shaft location (17%), total shaft length (11%), and soil cohesion (10%).
- The rank of influencing factors affecting \( \delta \) is as follows: soil cohesion (30%), drilled shaft diameter (28%), and total shaft length (17%).

4.7 Finite Element Validation and Model Test

Finite element simulations were compared with two hypothetical cases solved using two different computer programs that are established on the basis of different solution approaches. The first case used to validate the FEM is a 2:1 homogeneous slope reported by Abramson et al. (1996) as shown in Figure 4.15. The slope stability analysis was performed using the limiting equilibrium based XSTABL program. Spencer's method of slices was selected for the analysis. This same model was also analyzed using
ABAQUS. The vertical stresses along an assumed circular failure slip surface were compared for both programs and the comparison shown in Figure 4.16 are very good.

To validate the FE simulations for a slope with drilled shafts, the lateral shaft movements for the hypothetical case shown in Figure 4.17 are computed by ABAQUS. LPILE (Reese et al., 2004) program used to analyze an equivalent model shown in Figure 4.18, by using silt-cemented c-φ soil to model the soil and weak rock-Reese model to represent the rock. Zero shear and moment at the shaft head are the boundary conditions. The loading is applied by a trapezoidal load along the shaft as shown in Figure 4.18. This loading condition is obtained from the finite element simulation and applied as an input in the LPILE for the following reasons: (a) to generate an equivalent lateral relative displacement field to those generated by the FEM; (b) to account for the shaft location within the slope; and (c) to account for the spacing between the drilled shafts, since, the LPILE program deals only with a two-dimensional representation. The lateral shaft deflections obtained from both programs are plotted in Figure 4.19. As can be seen, these two sets of computed shaft deflections are about the same.
Figure 4.15: Slope geometry and slip surface specifications modeled in LEM and FEM

Circular slip surface

Note: dimensions in meters

R = 38.1

C = 20 kPa
ϕ = 20°
γ = 16 kN/m³
Figure 4.16: Comparison of vertical stresses obtained from both LEM and FEM
A row of drilled shafts spaced 3.6 m on centers:

- $E_p = 24$ GPa;
- $S = 3.6$ m.

Rock:
- $E_r = 21$ GPa

Soil:
- $C = 40$ kPa;
- $\phi = 10^\circ$;
- $\gamma = 18.5$ kN/m$^3$

Figure 4.17: Slope geometry and drilled shaft configurations (illustrative example)
Figure 4.18: Model configurations used in the LPILE program

Soil (silt – cemented c- φ soil)
- C = 40 kPa
- φ = 10°
- γ = 18.5 kN/m³

Rock (weak rock - Reese)
- Er = 21 GPa

Drilled Shaft
- Ep = 24 GPa
Figure 4.19: Comparison of lateral shaft deflection results obtained from LPILE and model developed
4.8 Summary and Conclusions

In this chapter, a three-dimensional finite element model of the slope/shaft system was developed to provide parametric study results for better understanding the behavior of soil arching and transfer of forces when one row of drilled shafts are installed in a slope and socketed into a stable soil (i.e., rock). The load transfer factor and the shaft movement in a slope/shaft system undergoing soil movement were thoroughly investigated through a series of FEM simulations. The essential features of the FE modeling were described in details in this chapter, including model parameters and material constitutive models. Three-dimensional state of stresses and displacements were closely investigated in the FE simulation. Frictional forces in the interfaces of soil, the rock, and the drilled shaft were implicitly considered. The finite element modeling analysis results were compared favorably for two cases with the results obtained by two different computer programs that utilize different solution procedures. The load transfer factor $\eta$ and the shaft deflection $\delta$ were presented in a chart form. Useful observations from the parametric finite element simulations and the developed design charts were made regarding the behavior of the drilled shafts installed in a moving slope. Design charts may be useful for preliminary design purpose. Conclusions from the FE simulation can be summarized as below.

1) Load transfer factor can be readily employed in the conventional methods of slope stability analysis to assess the improved factor of safety in a slope/shaft system.

2) FE simulations have shown numerical evidences for the reduction in the stresses and displacements from the upslope side of the drilled shaft to the downslope side of the shaft, so that significant improvement in the factor of safety of a slope is
achieved when a row of drilled shafts are installed and spaced two to four times the shaft diameter and socketed enough in rock.

3) Since the load transfer factor and the load acting on the stabilizing drilled shafts increase as the soil movement increase, geotechnical and structural stability of the slope/shaft systems need to be evaluated and examined at the ultimate state of soil failure.

4) The factors listed below affect greatly the behavior of drilled shafts stabilized slopes:

- Drilled shaft: modulus of elasticity, total length, location within the slope body, shaft diameter, and rock-socket length.
- Rock: modulus of elasticity.
- Soil: cohesion, friction angle, and displacement fields.
- Depth of slip surface.
CHAPTER V

DEVELOPMENT OF PERTINENT DESIGN METHOD FOR SLOPE/SHAFT SYSTEMS

5.1 Overview

An accurate and practical methodology for stability analysis and design of drilled shafts reinforced slopes was developed utilizing limiting equilibrium method of slices. Complex soil stratifications and general failure slip surfaces can be handled in the developed method. The effect of soil arching due to the presence of the drilled shafts was accounted for by using a load transfer factor. The numerical values of the load transfer factor were developed based on 3-D FEM parametric study results. Many of the design variables controlling the slope/shaft systems, such: drilled shafts size, shafts location, shaft fixity (the necessary rock-socket length), and the required spacing between the drilled shafts to prevent soil from flowing around the shafts can be successfully determined from the developed method. The optimum location where the drilled shafts could be placed within the sliding soil mass so that the cost associated with the landslide repair using the drilled shafts is minimized can be searched for and determined from the developed methodology. From geotechnical point of view, the global factor of safety for slope/shaft systems can be determined. From structural point of view, the forces acting on
the stabilizing drilled shafts due to the moving ground can be successfully estimated. Detailed mathematical expressions and useful design charts used in the design of slope/shaft systems were presented herein. For convenience, a relatively simple step-by-step numerical procedure to be followed by the soil engineers was prepared in this context and presented in this chapter. A complete illustrative design example employing the proposed numerical procedures was introduced. Many solutions were obtained and presented in a graphical form. Five different possible design alternatives were suggested for the illustrative design example that can satisfy a particular target factor of safety when drilled shafts are installed at a specific location within the slope body.

5.2 LEM For Slope/Shaft Systems

As explained earlier in Chapter III regarding the formulation of the limiting equilibrium method (LEM) for a slope/shaft system, the global factor of safety for a slope was developed in a relatively simple mathematical expression governing the entire sliding soil mass. Force equilibrium was satisfied for each individual slice, as well as the Mohr-Coulomb failure criterion at the base of each slice. Some assumptions as described previously in details were adopted in order to make the problem statically determinate. The global factor of safety (FS) and the left-interslice forces for any slice (L_i's) can be determined using Equation 5.1. For the calculations of FS, k in Equation 5.1 is set to n and the boundary loads at the first and last slices (R_1 and L_n) are applied. Equation 5.1 can be used iteratively to solve for the factor of safety (FS) which is the only unknown in the equation. Additionally, Equation 5.1 can be directly used to determine the interslice forces for any slice k.
\[
L_k = A_k + \left[ \sum_{i=2}^{k} A_{i-1} \prod_{j=1}^{k} B_j \eta_{j-1} \right] + \left[ B_k \prod_{i=1}^{k-1} B_i \eta_i \right] R_1
\] (5.1)

\( \eta \) in Equation 5.1 is the load transfer factor between two consecutive slices, which is used to quantify the transfer of the interslice forces from one slice to another. This load transfer factor (\( \eta \)) is set to 1.0 for all the interfaces between two consecutive slices when there is no drilled shaft exists. When the drilled shafts exist in a slope, this load transfer factor reflects the amount of the reduction in the forces due to the presence of the drilled shafts and can be mathematically determined as in Equation 5.2. The forces \( F \) and \( F' \) in Equation 5.2 are as conceptually illustrated in Figure 5.1.

\[
\eta = \frac{F'}{F}
\] (5.2)

The optimum location of the drilled shafts can be searched for and determined using the derived closed-form solution for the load transfer factor (\( \eta \)) and the global factor of safety for the slope/shaft system (FS) according to the following equation:

\[
\eta_m = \frac{L_n - A_n - \left( \sum_{i=2}^{n} A_{i-1} \prod_{j=1}^{n} B_j \right)}{\left[ B_{m+1} \sum_{i=2}^{m+1} A_{i-1} \prod_{j=i+1}^{m+1} B_j \right] + \left( B_n B_m \prod_{i=m+1}^{n} B_i \right) R_1}
\] ; \( 0 < \eta_m \leq 1.0 \) (5.3)

Equation 5.3 can be used to calculate the load transfer factor (\( \eta_m \)) needed to obtain a derived factor of safety (FS) for the new slopes/shaft system when the drilled shafts are installed at a specific location. Detailed derivations of Equation 5.3 can be found in Chapter III.
5.3 Estimation of Load Transfer (\( \eta \)) Based on 3D-FEM Simulation Results

In Chapter IV, three-dimensional finite element simulations were established to model the slope/shaft system, considering the elasto-plastic nature of soil and the elastic behavior of both the drilled shafts and the firm rock layer where the shafts will be socketed into. The FEM simulations have provided the soil engineers with a better understanding of soil arching mechanism and the lateral earth pressures transfer process in a slope/shaft system where soil is moving. The reduction in the stresses and displacements due to the development of soil arching from the upslope side directly in front of the drilled shafts to the downslope side directly behind the drilled shafts was observed and found to be significant. An extensive finite element parametric study was
carried out and a nonlinear statistical regression analysis was performed on the FE results to provide engineers with numerical values of the load transfer factor for slope/shaft systems. Equations 5.4 and 5.5 can be used to estimate the load transfer factor ($\eta$) and shaft deflection ($\delta$) at the failure state of the interaction between the soil and the drilled shafts, respectively.

\[ \eta_{\text{target}} = \eta(C, \phi) \eta(D, \xi_x) \eta(E_r, \xi_r) \eta(L_p, E_p) \eta(\xi_x, \xi_y) \] (5.4)

\[ \delta = K \delta(C, \phi) \delta(D, \xi_x) \delta(E_r, \xi_r) \delta(L_p, E_p) \delta(\xi_x, \xi_y) \] (5.5)

Where the constant $K$ in Equation 5.5 is equal to 1.0 for $\delta$ in centimeters and 0.3937 for $\delta$ in inches. $\eta(C, \phi)$, $\eta(D, \xi_x)$, ..., etc, and $\delta(C, \phi)$, $\delta(D, \xi_x)$, ..., etc are load transfer factor functions and shaft deflection functions which can be obtained using the developed design charts in Chapter IV.

5.4 Structural Design of Drilled Shafts

From the conceptual model of the load transfer factor shown in Figure 5.1, if the difference between the total forces at the vertical planes directly in front of and behind the drilled shaft is assumed to be the net force taken by the drilled shaft for a width of $S$, then the total force imparted to the drilled shafts ($F_{\text{shaft}}$) can be determined using the following equation:

\[ F_{\text{shaft}} = F - F' \] (5.6)

Obtaining $F_{\text{shaft}}$ based on the conceptual approach presented herein is somehow conservative because some of this force will be taken by the soil on both sides of the drilled shafts for an effective zone width $S$. 
Combining Equation 5.2 with Equation 5.6, one can calculate $F_{shaft}$ as follows:

$$F_{shaft} = (1 - \eta)F$$  \hspace{1cm} (5.7)

Comparing the forces obtained from limiting equilibrium with the finite element, then, the total force $F$ acting on the vertical plane directly before the drilled shafts on the upslope side for an area bounded by a distance $S$ and a length from the shaft head to the slip surface is equal to the left-interslice force for the slice $m$ (i.e., the slice just before the drilled shaft), $L_m$, multiplied by the center-to-center spacing between the stabilizing drilled shafts $S$.

Thus,

$$F = L_m S$$  \hspace{1cm} (5.8)

Substituting Equation 5.8 into Equation 5.7, one can estimate $F_{shaft}$ as follows:

$$F_{shaft} = (1 - \eta)L_m S$$  \hspace{1cm} (5.9)

As $\eta$ in Equation 5.9 increases, less force will be imparted to the drilled shaft with respect to the left-interslice force ($L_m$). When $\eta$ is equal to 1 then, no transfer in the forces would occur (i.e., all the interslice forces are transferred to the downslope soils without reduction). As a result, $F_{shaft}$ is equal to zero. As $\eta$ gets smaller or approaches to zero, the shaft force increases up to a maximum value of the left-interslice force multiplied by the center-to-center spacing between the drilled shafts.

Once the net shaft force is estimated, the structural design of a drilled shaft can be carried out in a straightforward procedure as explained by Chris et al (2007). As shown in Figure 5.2, the portion of drilled shaft below the slip surface is subjected to shear force $V$ and bending moment $M$ at the location of slip surface. The shear force $V$ is equal to the
net shaft force $F_{\text{shaft}}$ obtained from Equation 5.9. The bending moment $M$ is calculated by multiplying the net shaft force ($F_{\text{shaft}}$) by the one-third of the distance from the slip surface to the shaft head as depicted in Figure 5.2.

Figure 5.2: Structural design principles for drilled shafts stabilized slopes
5.5 Development of Pertinent Design Method

Development of pertinent design method to stabilize unstable slopes using a single row of rock-socketed drilled shafts can be achieved when geotechnical and structural related-design issues are successfully addressed. For geotechnical design, the engineer is ultimately required to determine the final design configuration for the new slope/shaft system so that the new factor of safety of the slope is improved to the target FS (FS\textsubscript{target}). These geotechnical-related design configurations include the following shaft-related items: diameter, spacing, location, total length, and the necessary socket length to ensure shaft fixity so that the enhanced slope/shaft system is working as a single unit. As shown in Figure 5.3, LEM, utilizing the derived mathematical expressions, has helped determining the amount of the reduction in the forces (\(\eta_m\)) to obtain a target factor of safety FS\textsubscript{target} at a specific shaft location, and also, has helped searching for the optimum location of the drilled shafts, while, FEM has helped determining the necessary design configurations to cause that much of a reduction in the forces (\(\eta_m\)).

On the other hand, the designer is required to estimate the forces imparted on the stabilizing drilled shafts (F\textsubscript{shaft}) in order to design the drilled shafts structurally as illustrated on the right-hand side of the flow chart in Figure 5.3, this includes determining the necessary components of the design: overall lateral drilled shaft movements, transverse reinforcement, and flexural reinforcement; in such away, these components are able to resist the lateral earth pressures acting on the shafts and the lateral shaft movements do not exceed the permissible movement specified by the engineer. LEM has helped determining the load transfer factor (\(\eta\)) and the interslice forces, while, the FEM has helped determining the necessary spacing between the drilled shafts. The
The aforementioned discussion has led to a complete and practical geotechnical and structural designs for slope/shaft systems based on limiting equilibrium and finite element methods as shown in Figure 5.3 which is established and prepared as a step-by-step procedure in the next section.

Figure 5.3: Development of pertinent design method for slope/shaft systems
5.6 Proposed Analytical Step-by-step Procedure for Slope/Shaft Systems

From the previous discussion, a pertinent design methodology was developed to enable the engineer to design a single row of rock-socketed drilled shafts to stabilize an unstable slope. The design methodology has led to a complete geotechnical and structural designs as discussed in the previous section. For convenience, a step-by-step design procedure for slope/shaft systems is established and enumerated as below. The numerical procedures below are also summarized in a flow chart as illustrated in Figure 5.4.

1. Determine $F_{S0}$ of the existing slope where drilled shafts have not been installed. Equation 5.1 can be used for this purpose.

2. Specify a target factor of safety to achieve with the installation of drilled shafts. ($F_{Starget}$)

3. Specify the possible locations of drilled shafts where it can be placed within the slope.

4. Arbitrarily, choose an initial shaft diameter $D$ and shaft location defined by $\xi_x$.

5. Calculate $\eta_m$ using Equation 5.3 for several possible values of safety factor by gradually increasing $FS$, starting from $FS_{min}$ corresponding to $\eta_{max}$ (i.e., the minimum factor of safety that can be obtained when drilled shafts with diameter $D$ are placed at $\xi_x$) and ending with $FS_{max}$ corresponding to $\eta_{min}$ (i.e., the maximum factor of safety that can be obtained when drilled shafts are placed at that location).

6. Determine the left-interslice force ($L_m$) for slice $m$ (i.e., the slice just before the drilled shaft on the upslope side) using Equation 5.1 for shafts with diameter $D$ and placed at that specific location.
7. Repeat Steps 4 – 6 for several shaft locations.

8. For \( \xi_x \) and \( D \) selected in Step 4, and results obtained from Step 5. Create \( \eta_{m}\)-FS diagram for several possible drilled shaft locations.

9. Determine the target load transfer factor (\( \eta_{\text{target}} \)) from \( \eta_{m}\)-FS diagram for several shaft locations. \( \eta_{\text{target}} \) is the load transfer factor corresponding to \( FS_{\text{target}} \).

10. Plot in one or two graph(s) the target load transfer factors (\( \eta_{\text{target}} \)) and Left-interslice forces (\( L_m \)) versus shaft locations (\( \xi_x \)).

11. From \( \eta_{\text{target}}-\xi_x \) diagram, specify the optimum location of the drilled shafts or some other locations if desired (\( \xi_x^* \)). Usually, an optimum shaft location is corresponding to the maximum target load transfer factor.

12. Assume spacing-to-diameter ratio \( \xi_s \). Usually, \( \xi_s \) can be taken between two and four. (start with \( \xi_s = 2 \))

13. To satisfy \( \eta_{\text{target}} \) at the specific spacing-to-diameter ratio assumed in Step 12, calculate the rock-socket length ratio \( \xi_r \) using Equation 5.4, and lateral shaft deflection \( \delta \) using Equation 5.5.

14. Calculate the shaft force, \( F_{\text{shaft}} \), using Equation 5.9.

15. Repeat Steps 13 - 14 for other selected \( \xi_s \). Do not increase \( \xi_s \) to be more than 4.

16. Create \( \xi_r-\xi_s \) diagram for the selected drilled shaft location \( \xi_x^* \) and shaft diameter \( D \).

17. Create \( F_{\text{shaft}}-\xi_s \) diagram for selected drilled shaft location \( \xi_x^* \) and shaft diameter \( D \).
18. Repeat Step 4 for different shaft diameters (D), usually, the selected drilled shaft location \( \xi^* \) in Step 11 is used. There is no need to try different shaft locations at this stage.

19. From the diagrams obtained from the previous steps, one can choose the appropriate \( (D, \xi_x, \xi_r, \xi_s, \text{and} \ F_{shaft}) \) for several combinations of design variables.

20. Perform structural analysis to design the shaft for transverse shear, flexural moment, and fixity. The computer program LPILE can be used to establish lateral shaft movement, shear, and moment diagrams.

21. If structural adequacy is not satisfied, another combination of design variables can be chosen as in Step 19.
Figure 5.4: Proposed design procedure for slope/shaft systems
5.7 General Remarks on Selection of Design Variables

In order to facilitate the selection of design variables and expedite the solution convergence, the design engineer should take the following general remarks into consideration:

- Shaft Location ($\xi_x$): the best location that provides an economical design to obtain the target factor of safety ($FS_{\text{target}}$) is found at the location where $\eta_{\text{target}}$ is highest from the $\eta_m$-FS diagram. At that location, interslice forces are usually high, which in turn, would lead to high shaft force $F_{\text{shaft}}$. Although geotechnical design can be fully satisfied, structural design may not be successfully or easily satisfied due to the high forces imparted to the drilled shafts ($F_{\text{shaft}}$). If this is the case, the shafts can be placed at locations corresponding to lower target load transfer factor (i.e., lower interslice forces). Furthermore, constructability issues sometimes may dedicate the location of the drilled shafts. Poulos (1995) indicated that the optimum location of stabilizing shafts is near the center of the sliding mass. This may not necessarily always true.

- Shaft Diameter (D): as a starting point, drilled shafts diameter can be initially taken as 1.2 m (4 ft), after that, the designer can increase or decrease the shaft size as needed and the results can be compared. ODOT has shown preference for 3 ft diameter shaft. The economic benefits of 3 ft diameter shafts need to be established on case by case basis. Usually, both structural- and geotechnical-related design issues control the selection of shafts diameter. Poulos (1995) suggested that a smaller number of large-diameter shafts generally can result in more effective stabilization than a larger number of small-diameter drilled shafts.
- Spacing-to-diameter ratio ($\xi_s$): usually this ratio can be taken between 2 and 4 to ensure the development of soil arching. In other words, this range of $\xi_s$ values would allow for the row of drilled shafts to work as a single unit. Poulos (1995) also indicated the usual spacing-to-diameter ratio is usually between two to four.

- Shaft Force ($F_{\text{shaft}}$): usually controls the structural design of the drilled shafts. If the structural design fails to meet capacity requirements, the engineer can resolve this issue by one of the following: (a) select another shaft location where the interslice forces at that location are less; (b) decrease the spacing-to-diameter ratio; and (c) increase the shaft diameter.

- Rock-socket length ratio ($\xi_r$): the selected $\xi_r$ from the $\xi_r$-$\xi_s$ diagram would successfully satisfy the geotechnical design requirements. However, lateral shaft movements and shaft fixity for structural design requirements may not be successfully satisfied. Rock-socket length ratio may be increased to reduce lateral shaft deflections and provide additional drilled shafts fixity.

5.8 Illustrative Design Example

In this section, an illustrative design example is discussed in detail to elucidate the analytical design methodology explained earlier. The geometry for a hypothetical soil slope consisting of three soil layers and a stiff rock layer underneath is illustrated in Figure 5.5. The location of the slip surface specification is also shown in the figure. According to the proposed step-by-step numerical procedure in Section 5.6,

1. The stability of the existing condition of the slope was examined using Equation 5.1 and the factor of safety $FS_0$ was found to be 1.07.
2. Assume $F_{\text{Stargt}}$ is equal to 1.5.

3. For illustration purposes, assume no constructability issues and the possible drilled shaft locations are between $X = 16$ m and $X = 24$.

4. Assume a shaft diameter $D = 1.0$ m and shaft location of $X = 16$ m, then, $\xi_x$ is calculated as follows:

   $\xi_x = \frac{X - 15}{25 - 15} = \frac{X - 15}{10}$

5. Calculate $\eta_m$ using Equation 5.3 for several possible values of FS.

6. Calculate the left-interslice force ($L_m$) using Equation 5.1.

7. Repeat Steps 4 – 6 for several drilled shaft locations. Consider an increment of 1.5 m for drilled shaft locations.

8. For $\xi_x$ and D selected in Step 4, and results obtained from Step 5. Create $\eta_m$-FS diagram as shown in Figure 5.6 for several drilled shaft locations.

9. Determine the target load transfer factor ($\eta_{\text{target}}$) from $\eta_m$-FS diagram for several shaft locations corresponding to $F_{\text{Starget}}$.

10. Plot the target load transfer factors ($\eta_{\text{target}}$) and Left-interslice forces ($L_m$) versus shaft locations ($X$) as shown in Figure 5.7.

11. From Figure 5.7, the optimum location of the drilled shafts is at $X = 19$ m (i.e., $\xi_x = 0.4$) where the maximum target load transfer factor is found 0.416.

12. Assume $\xi_s = 2$.

13. Calculate the rock-socket length ratio $\xi_r$. $\eta_{\text{target}}$ in Equation 5.4 is set to 0.416, the cohesion and friction angle of the soils along the shaft to the slip surface can be taken as the weighted average as follows:
\[ C_{\text{avg}} = \frac{(0.5 \text{ m})(25 \text{ kPa}) + (3.35 \text{ m})(13 \text{ kPa})}{0.5 \text{ m} + 3.35 \text{ m}} = 14.56 \text{ kPa} \]

\[ \varphi_{\text{avg}} = \tan^{-1}\left[ \frac{(0.5 \text{ m}) \tan(16^\circ) + (3.35 \text{ m}) \tan(8^\circ)}{0.5 \text{ m} + 3.35 \text{ m}} \right] = 9.06^\circ \]

Assume the modulus of elasticity for the drilled shafts \( E_p \) is equal to 24 Gpa.

At \( X = 19 \text{ m} \), \( y_i = 4.85 \text{ m} \) and \( Y = 6.5 \text{ m} \)

Then,

\[ \xi_y = \frac{4.85}{6.5} = 0.746 \]

And,

\[ L_p = L_r + Y \Rightarrow \text{(divide by } L_p) \Rightarrow L_p = \frac{Y}{1 - \xi_r} = \frac{6.5}{1 - \xi_r} \]

Therefore, Equation 5.4 can be solved for \( \xi_r \) for several assumed values of \( \xi_s \).

14. Calculate the shaft force \( F_{\text{shaft}} \) using Equation 5.9 for different spacing-to-diameter ratios as follows:

At \( X = 19 \text{ m} \); \( L_m = 297.06 \text{ kN/m} \).

Then,

\[ F_{\text{shaft}} = (1 - 0.416)(297.06)(S) = 173.5(S) = 173.5(\xi_y)(D) = 173.5(\xi_y) \text{ kN} \]

15. Repeat Steps 13 – 14 for several spacing-to-diameter ratios. Consider an increment of 0.1.

16. Create \( L_r-\xi_s \) diagram for drilled shaft diameter \( D = 1.0 \text{ m} \) at \( X = 19 \text{ m} \) as shown in Figure 5.8.

17. Create \( F_{\text{shaft}}-\xi_s \) diagram for drilled shafts diameter \( D = 1.0 \text{ m} \) at \( X = 19 \text{ m} \) as shown in Figure 5.8.
18. Repeat Step 4 for different drilled shafts diameter of 0.8 m and 1.2 m, and use the optimum location of the drilled shafts found in step 11 at X = 19 m. The solution charts for the rock-socket length and net shaft force for D = 0.8 m and 1.2 m are as shown in Figures 5.9 and 5.10, respectively.

19. From the solution charts (Figures 5.8, 5.9, and 5.10) obtained from the previous steps, one can choose the appropriate (D, ξ_x, ξ_r, ξ_s, and F_{shaft}) for several combinations of design variables. Table 5.1 provides a summary for five possible design alternatives to obtain a target factor of safety of 1.5 when drilled shafts are installed at X = 19 m.

20. Perform structural analysis to design the shaft for transverse shear, flexural moment, and fixity. The computer program LPILE can be used to establish lateral shaft movement, shear, and moment diagrams. For example, the loading boundary conditions for the design alternative No. 2 as listed in Table 5.1 can be taken as V = 345.2 kN and M = (345.2 kN) (4.45 m)/3 = 512 kN.m.
Figure 5.5: Slope geometry and slip surface specifications (Illustrative design example)
Figure 5.6: $\eta_m$-FS diagram for the illustrative example at various drilled shaft locations

(D = 1.0 m)
Figure 5.7: $\eta_{\text{target}}$ and $L_m$ vs. shaft location (X) for the illustrative example ($D = 1.0 \text{ m}$ and $F_{S_{\text{target}}} = 1.5$)
Figure 5.8: Solution chart for the rock-socket length and net shaft force vs. spacing-to-diameter ratio (D = 1.0 m; X = 19 m; and F_{Starget} = 1.5)
Figure 5.9: Solution chart for the rock-socket length and net shaft force vs. spacing-to-diameter ratio ($D = 0.8$ m; $X = 19$ m; and $FS_{\text{target}} = 1.5$)
Figure 5.10: Solution chart for the rock-socket length and net shaft force vs. spacing-to-diameter ratio ($D = 1.2$ m; $X = 19$ m; and $FS_{\text{target}} = 1.5$)
Table 5.1: List of possible design alternatives for the illustrative slope example (FS_{target} = 1.5 and X = 19 m)

<table>
<thead>
<tr>
<th>Design Alternative</th>
<th>D (m)</th>
<th>ξ_s</th>
<th>L_r(1) (m)</th>
<th>L_p(2) (m)</th>
<th>F_{shaft} (kN)</th>
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</thead>
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<tr>
<td>1</td>
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<td>2</td>
<td>4.6</td>
<td>11.1</td>
<td>288.7</td>
</tr>
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<td>2.6</td>
<td>9.1</td>
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</tr>
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<td>3</td>
<td>4.9</td>
<td>11.4</td>
<td>517.8</td>
</tr>
<tr>
<td>4</td>
<td>1.2</td>
<td>2</td>
<td>1.1</td>
<td>7.6</td>
<td>416.0</td>
</tr>
<tr>
<td>5</td>
<td>1.2</td>
<td>3</td>
<td>3.8</td>
<td>10.3</td>
<td>623.9</td>
</tr>
</tbody>
</table>

Notes:

(1) L_r = Minimum rock-socket length.
(2) L_p = L_r + 6.5 m, where 6.5 m is the length of shaft above the slip surface at X = 19 m.

Comments on the Illustrative Design Example:

From the results obtained for the illustrative example, one should notice the following:

- At shaft location X = 24 m, it was not possible to obtain a target factor of safety of 1.5 as shown in Figure 5.6. FS_{max} that may be obtained is 1.3, because the driving interslice force at that location is very small (i.e., L_m = 80 kN/m).
- For shaft diameter D = 0.8 m; spacing-to-diameter ratio can not be more than 2.2 as shown in Figure 5.9. In other words, if ξ_s > 2.2, then FS_{target} of 1.5 is not satisfied.
- For shaft diameter \( D = 1.0 \text{ m} \); the behavior of \( L_r \) vs. \( \xi_s \) as shown in Figure 5.8 starts to stabilize and reach a plateau at \( \xi_s \approx 3.25 \). While, for \( \xi_s > 3.25 \), the behavior of drilled shafts can be described as single shafts. In addition, for shaft diameter \( D = 1.2 \text{ m} \), a similar behavior is observed at \( \xi_s \approx 3.5 \) as shown in Figure 5.10.

- For a specific drilled shaft diameter at a specific location, many possible design alternatives were obtained which can satisfy \( \text{FS}_{\text{target}} \).

5.9 Summary and Conclusions

In this chapter, a step-by-step analytical design procedure for drilled shafts to stabilize an unstable slope was proposed. The limiting equilibrium based slope stability analysis algorithm, together with semi-empirical design charts from FEM parametric study, were used to provide a comprehensive design approach for slope/shaft systems. Force equilibrium and Mohr-Coulomb were applied and satisfied for each individual slice in the method of slices approach. Additionally, elasto-plastic nature of soil and elastic nature of the drilled shafts and the rock layer were incorporated in the finite element study. Many of the assumptions that were previously imposed by several investigators to simplify and reduce the number of design variables controlling slope/shaft systems were found in this study to significantly affect the design of slope/shaft systems. Examples and comparison of assumptions in the present method and other existing methods are listed in Table 5.2.
Table 5.2: A comparison between several authors with respect to the assumptions involved in the design of slope/shaft systems

<table>
<thead>
<tr>
<th>Assumptions</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement field along the shaft depth</td>
<td>Uniform</td>
</tr>
<tr>
<td>Displacement field along the spacing direction</td>
<td>Complex</td>
</tr>
<tr>
<td>Rock modulus</td>
<td>Not considered</td>
</tr>
<tr>
<td>Shaft modulus</td>
<td>Not considered</td>
</tr>
<tr>
<td>Rock-socket length</td>
<td>Not considered</td>
</tr>
<tr>
<td>Soil arching</td>
<td>Not considered</td>
</tr>
<tr>
<td>Shaft location</td>
<td>Not considered</td>
</tr>
<tr>
<td>Complex soil geometries</td>
<td>Considered</td>
</tr>
<tr>
<td>General failure slip surface</td>
<td>Considered</td>
</tr>
<tr>
<td>Depth of slip surface</td>
<td>Not considered</td>
</tr>
<tr>
<td>Frictional forces between the shafts and the soil</td>
<td>Ignored</td>
</tr>
</tbody>
</table>
In conclusion, the work presented in this chapter can be summarized as follows:

- A complete analytical, practical, relatively simple, and yet accurate design methodology has been developed based on the limiting equilibrium approach and FEM generated empirical design charts for solving the stabilization of unstable slopes using a single row of rock-socketed drilled shafts.
- The developed method includes solutions for both geotechnical and structural design issues.
- The developed method, utilizing the semi-empirical design charts, is not only capable of determining the variables necessary to design drilled shafts (i.e., shafts diameter, spacing between shafts, and rock-socket length), but also providing the designers with a set of possible design alternatives to choose from.
- The developed method, utilizing the limiting equilibrium method, is capable of handling complex soil geometries, general failure slip surfaces, and different locations of the drilled shafts.
- The methodology is capable of estimating the forces imparted on the drilled shafts due to soil movements.
- The soil arching is accounted for, thus yielding the best approximation of the strength and loads in the field.
- Many useful guidelines regarding the selection of design variables were proposed to help the designer choose the most effective and economical design configurations of the drilled shafts.
Among the factors that can be controlled by the designer; drilled shafts diameter, shaft location, and rock-socket length were the most important that need to be carefully selected.

An illustrative design example was presented using the developed method. Several possible design alternatives were suggested for the hypothetical slope example.
6.1 Overview

Real-time instrumentation and monitoring were carried out for three landslide sites in the Southern part of Ohio. A single row of rock-socketed drilled shafts as a stabilization mean was used in these sites to arrest the slope movement. Various types of instruments were extensively installed inside the stabilizing shafts and the surrounding soils to monitor and better understand the behavior of slope/shaft systems. The UA Slope program developed by Dr. Robert Liang in corporation with ODOT and FHWA has been used in designing these landslides. The field instrumentation and monitoring processes have provided excellent and unique information on the lateral responses of shafts in slopes. Also, the results of the instrumented cases have shown that the structural design (moments, shear, lateral deflection, and shaft tip fixity) of the shafts are safe (i.e., estimated forces acting on the shafts are high), and the geotechnical design (FS of slope/shaft system: movement and rate of movement) is satisfactory in two cases but still under observation for the third case.
6.2 Statement of the Problem

Landslides of natural slopes and slope failures of man-made embankment fills or slope cuts are one type of geo-hazards that have frequently threatened the safety of the highway systems in Ohio. The Geotechnical Engineering Office in the Ohio Department of Transportation (ODOT) has been very proactive in the development of a comprehensive approach in dealing with this particular type of geo-hazards, ranging from the development of internet-based landslide database together with the landslide hazards (risks) rating matrix to the development of landslide remediation cost database as well as the remediation decision trees. These concerted efforts embarked by the Geotechnical Engineering Office certainly will result in the establishment of an excellent management tool for decision-making on policies pertinent to the landslide geo-hazards.

In conjunction with the development of the landslide management tools as described in the above, the Geotechnical Engineering Office has also devoted considerable research efforts in the development of appropriate design/analysis methods for using the drilled shafts to stabilize the landslides, unstable slopes of embankment fills as well as slope cuts. The research project conducted by Liang (2002) on the "Drilled Shaft Foundation for Noise Barrier Walls and Slope Stabilization" provided a proposed design and analysis method, which was further coded into a PC based, user-friendly computer program for design applications. Subsequently, ODOT Geotechnical Engineering Office has carefully reviewed the developed method and decided to adopt the method in their in-house practice. In the meantime, ODOT also recognizes that consultants in Ohio tend to design drilled shafts for slope stabilization based on a highly ad-hoc approach due to a lack of well developed and accepted method in the textbook or
open literature. As a result, many ODOT landslide remediation projects, as designed by consultants, are either too conservative and costly, or deficient in theoretical basis. As an interim policy, ODOT Geotechnical Engineering Office has distributed the Liang's computer program to consultants for their applications to ODOT projects.

6.3 Objectives

Plan and carry out field instrumentation and long-term monitoring program at ODOT landslide stabilization project sites to collect long-term field data on the structural responses (i.e., forces, bending moments, and deflections) of the drilled shafts, the earth pressure thrusting upon the drilled shafts, and ground movement.

6.4 Geotechnical Instrumentation

Instrumentation process is usually performed by installing the sensors in the desired location (e.g., inside the drilled shaft – mounted on the steel cage, or in the soil), then, all sensors were multiplexed with a 16-channel or 32-channel multiplexer, then, all multiplexers were connected with a data acquisition system to manage and operate the reading of the sensors in a timely automated manner. Data acquisition system has a limited hard drive storage which requires downloading the data from the hard drive from time to time to allow space for a new set of data collection. The downloading process was usually performed using SRS-323 interface between the data acquisition system and the laptop. This whole process is illustrated in Figure 6.1. In this existing project, many sensors were used for the purpose of monitoring. The following is a list of instrumentation used in the project:
- Jackout Pressure Cells. (On the drilled shafts)
- Earth Stress Cells. (In the soil)
- Vibrating Wire Piezometer. (In the soil)
- Strain Gages and Sister Bars. (inside the drilled shafts)
- Glue-Snap ABS Inclinometer Casing. (in the soil and the drilled shafts)
- Multiplexers. (data acquisition system)
- Dataloggers. (data acquisition system)
- Readout Device. (reading sensors before setting up data acquisition system)

Figure 6.1: Geotechnical instrumentation (data management)
6.5 Instrumented and Monitored Landslide Sites

Three actual full-scale sites were instrumented and monitored with high technology timely-automated geotechnical sensors. The landslide sites are located in the southern part of Ohio in Jefferson County, Washington County, and Morgan County as shown in the Ohio map in Figure 6.2. The instrumented landslide sites were designed with different design configurations to cover a wide range of design variables and to better understand the real behavior of the slope/shaft systems. Table 6.1 provides a summary for the design configurations for the instrumented and monitored landslide sites in this project. More details about the geotechnical information, laboratory and field tests, design process, and field data monitoring results are explained in this chapter.

Figure 6.2. Ohio map - Location of instrumented and monitored landslide sites
Table 6.1: Summary of drilled shafts design configurations for the instrumented landslide sites.

<table>
<thead>
<tr>
<th>Site</th>
<th>D (ft)</th>
<th>S/D</th>
<th>L_p (ft)</th>
<th>L_r (ft)</th>
<th>F_{shaft} (kips)</th>
<th>Total number of shafts</th>
<th>Offset (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>JEF-152-1.30</td>
<td>3.5</td>
<td>2</td>
<td>45</td>
<td>20</td>
<td>100</td>
<td>42</td>
<td>40</td>
</tr>
<tr>
<td>WAS-7-48.0</td>
<td>4</td>
<td>2, 3</td>
<td>40</td>
<td>10</td>
<td>250</td>
<td>128</td>
<td>90</td>
</tr>
<tr>
<td>MRG-376-1.10</td>
<td>4</td>
<td>2</td>
<td>40</td>
<td>18</td>
<td>165</td>
<td>23</td>
<td>20</td>
</tr>
</tbody>
</table>

6.5.1 JEF-152-1.30 Landslide Repair Site

This section contains general description of JEF-152-1.3 landslide repair site, field and laboratory tests involved, instrumentation details, and the field data monitoring results.

6.5.1.1 General Site Description

The landslide at this site is occurring along the Westside of State Route 152, approximately 800 feet north of the intersection of SR 152 and TR 125 in Jefferson County. The area exhibiting instability is approximately 280 feet long, from station 54 + 40 to 57 + 20. The landslide is considered to be translational-type movement, sliding along the top of bedrock. The sliding mass appears to be "hinged" at the southern end near station 54 + 40. Movement direction is near perpendicular to the slope, at a skew with the road. Other observations are presented as follows:
All movements measured in inclinometers occur at or just above the top of soft mudstone bedrock.

Bedrock slopes down steeply at a 0.9 (H): 1.0 (V) slope, from north to south, between stations 57 + 50 and 55 + 75. No borings were drilled north of 57 + 75 to identify bedrock gradient down station. Borings drilled north of 57 + 50 indicate bedrock parallels the ground surface up station. This bedrock depression creates a trough for groundwater flow.

Bedrock slopes down at an estimated 2.5:1 slope from 20 feet right to 55 feet left, when it flattens to a 12:1 slope.

Groundwater is clearly a contributing factor to the instability. Groundwater is believed to be flowing along the top of and through the bedrock and hydraulically connected to upland abandoned underground mines.

Given the instability to completely control the groundwater and the sliding action along a relatively flat bedrock surface, a restraining system at the ground surface, such as a counterberm or flatter slope will not be sufficient to arrest the movement. Installing drilled shafts is recommended to arrest slide movement along this section of road. Ohio DOT engineers designed the slope/shaft system at this site and estimated the forces imparted on the drilled shafts using UA Slope program. Table 6.2 provides a summary of the general information and design configurations performed at JEF-152-1.3 landslide site.
Table 6.2: Site specific information and design recommendations (JEF-152-1.3)

<table>
<thead>
<tr>
<th>District</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitude/Latitude</td>
<td>80° 52' 40&quot; / 40°13'30&quot;</td>
</tr>
<tr>
<td>Contractor</td>
<td>Kokosing</td>
</tr>
<tr>
<td>Construction Start Date</td>
<td>February 28, 2006</td>
</tr>
<tr>
<td>Backfilling and Grading</td>
<td>April 11 – May 01, 2006</td>
</tr>
<tr>
<td>Construction End Date</td>
<td>May 07, 2006</td>
</tr>
<tr>
<td>Area Exhibiting Instability</td>
<td>54+30.0 – 57+10.0 (280 ft)</td>
</tr>
<tr>
<td>Total Number of Drilled Shafts</td>
<td>42</td>
</tr>
<tr>
<td>Instrumented Drilled shafts</td>
<td>Shaft #20 (55+80.0)</td>
</tr>
<tr>
<td></td>
<td>Shaft #21 (55+87.0)</td>
</tr>
<tr>
<td>Offset (from centerline of pavement)</td>
<td>40 ft</td>
</tr>
<tr>
<td>Total Shaft Length (Lp)</td>
<td>45 ft</td>
</tr>
<tr>
<td>Rock-socket Length (Lr)</td>
<td>20 ft</td>
</tr>
<tr>
<td>Drilled Shaft Diameter (D)</td>
<td>3.5 ft</td>
</tr>
<tr>
<td>Spacing-to-Diameter Ratio (S/D)</td>
<td>2</td>
</tr>
<tr>
<td>F&lt;sub&gt;Starget&lt;/sub&gt; – UA Slope Analysis</td>
<td>1.36</td>
</tr>
<tr>
<td>F&lt;sub&gt;shaft&lt;/sub&gt; – UA Slope Analysis</td>
<td>151 kips</td>
</tr>
<tr>
<td>Nominal Moment Capacity (M&lt;sub&gt;n&lt;/sub&gt;) – LPILE Analysis</td>
<td>2,824 ft-kips (26#11 Bars)</td>
</tr>
<tr>
<td>Shaft Head Deflection – LPILE Analysis</td>
<td>3.2 in</td>
</tr>
</tbody>
</table>
6.5.1.2 Laboratory and Field Tests

Field pressuremeter test was performed in July 2005 on the rock at a depth of 26.5 ft. The high pressure calibration curve for the dilatometer probe is shown in Figure 6.3. The average modulus of elasticity for the intermediate geomaterials encountered at JEF-152 site is 14 ksi and an unconfined compressive strength of 100 psi. Detailed test results can be found in Table 6.3. The p-y curves at a depth of 26.5 ft and 31.5 ft were deduced using Briaud's method (The pressuremeter, 1992) as shown in Figure 6.4. All weathered rock core samples were visually inspected and selected samples were chosen for laboratory testing. The tests were conducted at the University of Akron laboratories to determine the unconfined compressive strength and Poisson's ratio. Table 6.4 provides a summary for the test results. Onsite pictures for the pressuremeter test, core samples for the sandstone, and sample testing at UA laboratory can be found in Appendix A. (see Figures A.1 – A.3)
Figure 6.3: Pressure vs. volume during pressuremeter test at JEF-152 site (Depth = 26.5 ft)

Table 6.3: Pressuremeter test results at JEF-152 site

<table>
<thead>
<tr>
<th>Sample</th>
<th>Depth (ft)</th>
<th>Limit Pressure (psi)</th>
<th>$Q_u$ (psi)</th>
<th>$E_m$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26.5</td>
<td>710</td>
<td>100</td>
<td>15140</td>
</tr>
<tr>
<td>2</td>
<td>31.5</td>
<td>905</td>
<td>125</td>
<td>13300</td>
</tr>
</tbody>
</table>
Figure 6.4: P-y curves for JEF-152 deduced using Briaud's method

Table 6.4: Laboratory test results at JEF-152 site

<table>
<thead>
<tr>
<th>Sample</th>
<th>Top (ft)</th>
<th>Bottom (ft)</th>
<th>$Q_u$ (psi)</th>
<th>$E_i$ (psi)</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26.5</td>
<td>27</td>
<td>39</td>
<td>16700</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>27</td>
<td>27.5</td>
<td>21</td>
<td>4200</td>
<td>0.39</td>
</tr>
<tr>
<td>3</td>
<td>28</td>
<td>28.5</td>
<td>57</td>
<td>4460</td>
<td>0.43</td>
</tr>
<tr>
<td>4</td>
<td>29</td>
<td>29.5</td>
<td>56</td>
<td>4690</td>
<td>N/A</td>
</tr>
<tr>
<td>5</td>
<td>31.5</td>
<td>32</td>
<td>15</td>
<td>550</td>
<td>N/A</td>
</tr>
<tr>
<td>6</td>
<td>32</td>
<td>32.5</td>
<td>16</td>
<td>580</td>
<td>0.38</td>
</tr>
</tbody>
</table>
6.5.1.3 Instrumentation Plans

The slope/shaft system at JEF-152-1.3 site was extensively instrumented and monitored. Two drilled shafts (shafts #20 and #21) as well as the surrounding soil mass were instrumented and monitored (see the general plan view in Figure 6.5). Instruments inside each drilled shaft include nine vibrating wire pressure cells at 3 different levels (i.e., depth from shaft top = 10 ft, 16 ft, and 22 ft), three pressure cells at each level (i.e., upslope side, downslope side, and 45°). In addition to the pressure cells, sixteen vibrating wire strain gages were installed on each shaft at 8 different levels (i.e., depth from shaft top = 13 ft, 16 ft, 19 ft, 22 ft, 25 ft, 27 ft, 29 ft, and 31 ft), two strain gages at each level (i.e., upslope side and downslope side). Also, two in-place inclinometers were installed inside each drilled shaft to measure the shaft tilt due to slope movement. The tilt is usually converted into deflections. On the other hand, instruments inside the soil include three in-place inclinometers (i.e., upslope, between the shafts, and downslope), three earth pressure cells installed between the drilled shafts at 3 levels (i.e., depth from ground surface = 10 ft, 16 ft, and 22 ft), and three vibrating wire piezometers installed at 3 locations across the slope (i.e., upslope – depth = 20 ft, between the drilled shafts – depth = 22 ft, and downslope – depth = 23 ft). The aforementioned discussion is summarized in Table 6.5. Figure 6.6 shows a cross-section at station 56 + 00. The slip surface is shown in the figure, as well as, the water table, shaft location, piezometers, and inclinometers. Figure 6.7 shows a schematic of the instrumentation details along the shafts length for shafts #20 and #21. Locations of the pressure cells mounted on the drilled shafts and installed between the shafts are shown in the plan view in Figure 6.8. All instruments except the inclinometers were multiplexed with four 16-channel multiplexers. These four
multiplexers were connected with an automated datalogger installed in a traffic enclosure box onsite. The data acquisition system at this site is managed as summarized in Table 6.6. Some onsite pictures during instrumentation, construction, and data acquisition setup at JEF-152-1.3 can be found in Appendix A. (see Figures A.4 – A.12)

Figure 6.5: General Plan View – instrumentation details (JEF-152 site)
Table 6.5: Instrumentation details at JEF-152-1.3 site

<table>
<thead>
<tr>
<th>Inclinometers</th>
<th>Depth (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inclinometer No.</strong></td>
<td></td>
</tr>
<tr>
<td>INC-1</td>
<td>45</td>
</tr>
<tr>
<td>INC-2</td>
<td>45</td>
</tr>
<tr>
<td>INC-3</td>
<td>45</td>
</tr>
<tr>
<td>INC-4</td>
<td>42</td>
</tr>
<tr>
<td>INC-5</td>
<td>45 (Reaction Shaft – Lateral Load Test)</td>
</tr>
<tr>
<td>INC-6</td>
<td>45 (Test Shaft – lateral Load Test)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Piezometers</th>
<th>Depth (ft)/Location</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Piezometer No.</strong></td>
<td></td>
</tr>
<tr>
<td>PZ-1</td>
<td>20/upslope</td>
</tr>
<tr>
<td>PZ-2</td>
<td>22/between the drilled shafts</td>
</tr>
<tr>
<td>PZ-3</td>
<td>23/downslope</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pressure cells</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>9 Pressure cells on shaft #20</td>
<td>3 levels (10 ft, 16 ft, and 22 ft)/(upslope, downslope, and 45°)</td>
</tr>
<tr>
<td>9 Pressure cells on shaft #21</td>
<td>3 levels (10 ft, 16 ft, and 22 ft)/(upslope, downslope, and 45°)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Earth Pressure Cells</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3 earth pressure cells between the drilled shafts</td>
<td>3 levels (10 ft, 16 ft, and 22 ft)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Strain Gages</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>16 strain gages on shaft #20</td>
<td>8 levels (13 ft, 16 ft, 19 ft, 22 ft, 25 ft, 27 ft, 29 ft, and 31 ft)/(upslope and downslope)</td>
</tr>
<tr>
<td>16 strain gages on shaft #21</td>
<td>8 levels (13 ft, 16 ft, 19 ft, 22 ft, 25 ft, 27 ft, 29 ft, and 31 ft)/(upslope and downslope)</td>
</tr>
</tbody>
</table>
Figure 6.6: Instrumentation Plans (Cross-section 1) – JEF-152
Figure 6.7: Instrumentation Plans (Cross-section 2) – JEF-152
Figure 6.8: Instrumentation Plan (Plan view) – JEF-152
Table 6.6: Management of data acquisition system (JEF-152 site)

<table>
<thead>
<tr>
<th>Dataloggers</th>
<th>Multiplexers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 datalogger</td>
<td>4 multiplexers</td>
</tr>
<tr>
<td></td>
<td><strong>Multiplexers</strong></td>
</tr>
<tr>
<td>Mux-1</td>
<td>16 strain gages on shaft #20</td>
</tr>
<tr>
<td>Mux-2</td>
<td>16 strain gages on shaft #21</td>
</tr>
<tr>
<td>Mux-3</td>
<td>9 pressure cells on shaft #20 + 7 pressure cells on shaft #21</td>
</tr>
<tr>
<td>Mux-4</td>
<td>2 pressure cells on shaft #21 + 3 earth pressure cells between the shafts + 3 piezometers</td>
</tr>
</tbody>
</table>

6.5.1.4 Field Data Monitoring Results

All field monitoring results were recorded on an hourly basis at this site starting on March 2006 for strains, water level fluctuation, and earth pressures thrusting on the drilled shafts and between the shafts. Furthermore, inclinometer readings for the instrumented shafts and for the surrounding soils were measured on a quarterly basis. All monitoring results can be found in Appendix A. (see Figures A.13 – A.14 for bending moment diagrams, Figures A.15 – A.21 for pressure distribution diagrams, Figure A.22 for water level changes, and Figures A.22 – A34 for deflection profiles)

Further analysis was performed on the deflection and bending moment profiles in order to ensure success of the stabilizing shafts at this site. Figure 6.9 shows the
cumulative movement of shaft 1, the movement values are at a depth of 3 ft from the ground surface. About 50% of the shaft movement happened during backfilling and grading of the slope processes, which ended on May 2006. Figure 6.10 shows the rate of movement of shaft 1, high rate of movement occurred during construction (March – May, 2006). The current rate of movement for shaft 1 is 0.013 in/yr which can considered very small.

For shaft 2, the maximum shaft movement occurred is about 0.85 in from the beginning of the construction as shown in Figure 6.11. The same behavior was observed during the construction which indicates high rate of movement as shown in Figure 6.12. The current rate of shaft movement is 0.266 in/yr which is high rate compared to shaft 1. On the other hand, maximum soil movement measured between the drilled shafts is about 0.78 in as shown in Figure 6.13. The rate of movement of that soil is 0.18 in/yr. More than 50% of the total soil movement occurred during construction.

Measured strain readings were used to calculate the bending moments by multiplying the curvature (strain/distance c) by the flexural rigidity of the shaft (EI). Figure 6.15 shows the development of the measured bending moment in shaft 1 with time, the maximum measured moment is 480 ft-kips, while for shaft 2, the maximum moment is 710 ft-kips as shown in Figure 6.16. The shaft at this site were designed to withstand a nominal flexural capacity of 2,824 ft-kips, which is a factor of four when compared with the measured values. Goodness of the measured moments was checked and verified by comparing the calculated bending moments obtained from strain measurements (i.e., strain gages) and from tilt measurements (i.e., inclinometers). The comparison was performed on two sets of readings as indicated in Figure 6.17. The
method of calculating the moment from the shaft deflection data is based on the principle of superposition and it is introduced and explained in details in the next chapter.

Unfortunately, current pressure cells mounted on the drilled shafts to measure the thrusting pressure and earth pressure cells between the drilled shafts were recording very small or nonsense readings. The behavior is not understood.

Figure 6.19: Cumulative movement for shaft 1
Figure 6.10: Rate of movement for shaft 1

JEF-152
Shaft #1 (at 3 ft below the ground level)

Rate of Movement (in/Day)

Date

0.000
0.002
0.004
0.006
0.008
0.010
0.012
0.014


0.001 in/Month
0.013 in/Year
Figure 6.11: Cumulative movement for shaft 2
Figure 6.12: Rate of movement for shaft 2

<table>
<thead>
<tr>
<th>Date</th>
<th>Rate of Movement (in/Day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/01/2006</td>
<td>0.00074</td>
</tr>
<tr>
<td>04/01/2006</td>
<td>0.02214</td>
</tr>
<tr>
<td>06/01/2006</td>
<td>0.2656</td>
</tr>
</tbody>
</table>

JEF-152
Shaft #2 (at 3 ft below the ground level)
Figure 6.13: Cumulative soil movement between the drilled shafts
Figure 6.14: Rate of soil movement between the drilled shafts
Figure 6.15: Cumulative measured moment for shaft 1
Figure 6.16: Cumulative measured moment for shaft 2
6.5.2 WAS-7-48.00 Landslide Repair Site

This section contains general description of WAS-7-48.0 landslide repair site, subsurface profile, slope stability analysis, field tests, instrumentation details, and the field data monitoring results.
6.5.2.1 General Site Description

The landslide is occurring between SR 7 and the Ohio River, located just north of Beavertown, Ohio, along SR 7. The area exhibiting instability is approximately 1100 feet long, from station 2528 + 00 to station 2539 + 00.

The site lies in the unglaciated portion of Ohio along a gradual outside bend of the Ohio River. The site is approximately Mile 146 of the river, just upstream of the old Lock and Dam Number 16. The soils underlying the road and forming the upper river bank are colluvial and residual soils, weathered off of the parent sandstone and shale bedrock, while the lower river bank is composed of alluvial (water-laid) soils.

According to the soil borings made along the road and river bank, the bedrock surface is near elevation 591.4 to 596.0 ft under the road, and slopes downward at an average 11(H):1(V) grade towards the river. In all of the project borings, bedrock was encountered between elevations 586.4 and 596.0 ft, with an average elevation of approximately 591.6 ft. The underlying bedrock is Pennsylvanian age, of the Monongahela formation.

The edge of pavement is between 100 and 165 ft from river's edge, lying atop a low soil embankment at the crest of the river bank. The ground line between the road and river has a fairly gentle and consistent slope. The lower river bank has a slope of approximately 2.5(H):1(V) across the entire site, while the upper embankment has a slope varying between 2.5(H):1(V). A nearly level bench, approximately 35 to 50 ft wide, lies between the lower river bank and upper embankment for most of the length of the site. The average slope between the road and the edge of the river is approximately 3.8(H):1(V). The bank is covered with grass, weeds, brush, and small trees. A ditch runs
along the side of the road away from the river, while the side towards the river is lined by
guardrail. Several small culverts cross under the road to outlet on the bank. Utility poles
are located along the edge of the road away from the river.

The Ohio River typical pool elevation at this location is at elevation 602 ft. The
roadway surface varies in elevation between approximately 632 ft to 638 ft, rising to the
north-east with an approximately 0.5% slope. The slope uphill of the road rises steeply
several hundred feet, to the top of the ridge line, at approximate elevation 1100 ft. No
evidence of instability was noted on the uphill slope. Evidence of landslide movement
along SR 7 consists of dropped and bowed sections of guardrail and cracks and drop-off
in the pavement.

6.5.2.2 Subsurface Profile

For this investigation, seven borings were drilled along the roadway and lower
slope as indicated in Figure 6.18. The first four borings, B-1 through B-4, were drilled
along the right edge of the road in late March, 2005. The last three borings, B-5 through
B-7 were drilled along mid-slope bench, below the roadway, in late July, 2005. Ten older
borings, B-1 through B-10 were also drilled at this location in February and March of
1993, as part of the project WAS-7-48.00.

Borings along the roadway encountered from 10 ft to 11 ft of fill above the native
soils, upon which the roadway was constructed. These soils appear to be of local native
origin, containing many fragments of weathered sandstone and shale. This fill is
composed of soils generally described as soft to stiff clayey gravel (A-2-6), sandy silt (A-
The average soil in the fill is medium stiff silt and clay (A-6a).

A layer of colluvium, ranging in thickness from 7.5 ft to 27.5 ft was encountered next in the borings. The colluvium was encountered below the fill in the borings along the roadway, but was encountered just below the surface in the borings further down the slope. Colluvium is a loose deposit of rock debris accumulated through the action of gravity at the base of a cliff or slope. The colluvium is composed of soils generally described as stiff to hard silty and clayey gravel (A-2-4 and A-2-6), silty clays (A-6a and A-6b), and clay (A-7-6), with many fragments of weathered sandstone and shale. The average soil in the colluvium is very stiff silty clay (A-6b).

A layer of alluvium, ranging in thickness form 16.7 ft to 18 ft was encountered below the colluvium in borings B-1 and B-5. Alluvium is a water deposited soil consisting of sediments which have fallen to the bottom of the river. These soils make up the original river bank, over which the colluvium flowed from further uphill. The alluvium is composed of soils generally described as medium stiff to stiff sandy silt (A-4a), silt (A-4b), and silt and clay (A-6a), with very little to no coarse sand or gravel. The average soil in the alluvium is medium stiff silt and clay (A-6a).

A layer of residual soils, weathered directly off of the parent bedrock, was encountered beneath the alluvium or colluvium. This layer ranges in thickness from 3 to 16 ft, and consists of residual soils made up of highly decomposed bedrock at the top to soft, highly weathered, but intact at the bottom. The residual soils are composed of material generally described as medium stiff to stiff sandy silt (A-4a) and silty clays (A-
6a and A-6b). SPT blow-count refusal was encountered at the bottom of this layer, in materials composed of highly weathered sandstone and shale bedrock.

Bedrock was encountered in all borings between elevations 586.4 ft and 596.0 ft, with an average elevation of approximately 591.6 ft. Bedrock is generally described as being composed of interbedded layers of sandstone, shale, siltstone, and mudstone. The RQD in the bedrock varied between 0% and 91%. The RQD of recovered shale, siltstone, and mudstone was 0%. One recovered sample of sandstone had an RQD of 91%, however, most sandstone samples had an RQD varying between 0% and 33%. No rock unconfined compression tests were performed.

Inclinometers were installed in six of the borings for this project. Borings B-1 and B-3 through B-7 contain inclinometers, B-1, B-3, and B-4 at the crest of the upper embankment and B-5 through B-7 along the mid-slope bench, in line with the upper embankment inclinometers. Three stations, 2529+00, 2535+50, and 2537+75 were instrumented in this way.

All inclinometers except B-5 are showing substantial movement, at varying rates and depths as shown in Figures 6.19 and 6.20 for B-1 and B-3, respectively. Inclinometer B-5 has shown only a small amount of movement: approximately 0.1 inch since the baseline reading on August 4, 2005. In general, less movement is being shown in the lower (mid-slope) inclinometers as compared to the upper inclinometers. This may be due to the upper soil mass translating and sliding over the lower soil mass. However, the general pattern of movement displayed in the inclinometers is consistent with a large, deep block failure, sliding along the bedrock surface, and extending out into the river. For this reason, relatively shallow restraining systems, such as benched reconstruction,
counter-berm, dump rock, or flatter slope are not sufficient to arrest the deep-set movement.

Figure 6.18: Plan view – location of borings (WAS-7 site)
Figure 6.19: Cumulative displacement profile for B-1 (before landslide repair)
Figure 6.20: Cumulative displacement profile for B-3 (before landslide repair)
6.5.2.3 Slope Stability Analysis

Based on the above discussion, the slope movement is considered to be predominantly attributed to a combination of a periodic high water level and a weak layer of residual soils present directly above the bedrock. The failure was aggravated by the rapid drawdown event which occurred at the end of January and in early February, 2005, when runway barges were lodged in the Belleville Lock Dam. This event rapidly dropped the water level at this site about 27 ft, from approximate elevation 629 ft to approximate elevation 602 ft. The failure, as stated above, is a large deep-set block failure, which extends down to the top of bedrock, travels along the bedrock surface, and reemerges in the river bottom. Similar failures were observed at several locations along the Ohio River in this area. Based on that, deep restraining system, consisting of drilled shafts socketed into bedrock, is required to arrest the movement.

A joint analysis of the subsurface conditions was undertaken by the ODOT Office of Geotechnical Engineering and the University of Akron (UA). As a result of the analysis, installation of drilled shafts to arrest slide movement along this section of road is recommended. The analysis considered varying drilled shaft sizes at varying spacings between shafts and at varying offsets from the centerline of the roadway, in order to optimize the designed shaft size and placement.

Residual soil strengths were back-calculated using PCSTABL5M. Assuming a failure surface that extends down to the top of bedrock, the loads on laterally loaded drilled shafts were determined using the methodology developed in the research document "Drilled Shaft Foundations for Noise Barrier Walls and Slope Stabilization". This document was developed by Dr. Robert Liang (UA) in cooperation with ODOT and
FHWA. Table 6.7 provides more details about the design recommendations and site specific information.

A typical cross-section of WAS-7 site is shown in Figure 6.21. Soil stratifications, pool elevation, slip surface, and shaft offset from centerline of SR 7 are also shown in the figure. Table 6.8 provides a summary for the soil properties used in the analysis. Trying to optimize the design, two shaft sizes were considered in the analysis (i.e., 4 ft and 5 ft). Figures 6.22 and 6.23 show the results obtained from UA Slope program for the safety factor and shaft force vs. the shaft offset for shaft diameter of 4 ft and 5 ft, respectively. LPILE analysis was performed for many selected design configurations to ensure shaft fixity and satisfy shaft design requirements structurally (i.e., determine flexural and transverse reinforcements needed) to withstand the applied loads. LPILE analysis has shown at least the lower 6 ft portion of the shaft is experiencing no translational movement when 4 ft shaft diameter is used as shown in Figure 6.24.

Also, the maximum lateral shaft movement at the shaft top is about 2.6 in. LPILE analysis has shown, for the flexural analysis of the shaft, a maximum bending moment of 2,750 in-kips as shown Figure 6.25 and it occurs at depth of 34 ft from the ground surface. The maximum shear force obtained from the LPILE analysis was about 600 kips at a depth of 38 ft from the ground surface as shown in the shear force diagram in Figure 6.26. Final arrangement of the reinforcement distribution is shown in Figure 6.27.
Table 6.7: Site specific information and design recommendations (WAS-7 site)

<table>
<thead>
<tr>
<th>District</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitude/Latitude</td>
<td>81° 06' 42&quot; / 39°28'48&quot;</td>
</tr>
<tr>
<td>Contractor</td>
<td>Shelly &amp; Sands</td>
</tr>
<tr>
<td>Construction Start Date</td>
<td>September 01, 2006</td>
</tr>
<tr>
<td>Construction End Date</td>
<td>November 10, 2006</td>
</tr>
<tr>
<td>Area Exhibiting Instability</td>
<td>2528+03.94 – 2538+96.91 (1093 ft)</td>
</tr>
<tr>
<td>Total Number of Drilled Shafts</td>
<td>128</td>
</tr>
<tr>
<td>Instrumented Drilled shafts</td>
<td>Shaft #53 (2532+72.97) Shaft #54 (2532+84.97)</td>
</tr>
<tr>
<td>Offset (from centerline of pavement)</td>
<td>100 ft</td>
</tr>
<tr>
<td>Total Shaft Length ($L_p$)</td>
<td>40 ft</td>
</tr>
<tr>
<td>Rock-Socket Length ($L_r$)</td>
<td>10 ft</td>
</tr>
<tr>
<td>Drilled Shaft Diameter ($D$)</td>
<td>4 ft</td>
</tr>
<tr>
<td>Spacing-to-Diameter Ratio (S/D)</td>
<td>2 &amp; 3 (instruments at S/D = 3)</td>
</tr>
<tr>
<td>$F_{\text{S, target}}$ – UA Slope Analysis</td>
<td>1.4</td>
</tr>
<tr>
<td>$F_{\text{S, shaft}}$ – UA Slope Analysis</td>
<td>290 kips</td>
</tr>
<tr>
<td>Nominal Moment Capacity ($M_n$) – LPILE Analysis</td>
<td>4,918 ft-kips (32#14 Bars)</td>
</tr>
<tr>
<td>Shaft Head Deflection – LPILE Analysis</td>
<td>2.15 in</td>
</tr>
</tbody>
</table>
Figure 6.21: Cross-section of WAS-7-47.90 landslide

Table 6.8: Soil parameters (WAS-7 site)

<table>
<thead>
<tr>
<th>Layer No.</th>
<th>Description</th>
<th>$\phi$ (Deg.)</th>
<th>C (lb/ft$^2$)</th>
<th>$\gamma$ (lb/ft$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Colluvium</td>
<td>22</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>Alluvium</td>
<td>24</td>
<td>100</td>
<td>110</td>
</tr>
<tr>
<td>3</td>
<td>Residuum</td>
<td>26</td>
<td>100</td>
<td>120</td>
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<tr>
<td>4</td>
<td>Soft Rock</td>
<td>12</td>
<td>0</td>
<td>125</td>
</tr>
</tbody>
</table>
Figure 6.22: Variation of safety factor and shaft force at different offsets and S/d ratios for shaft diameter of 4 ft
Figure 6.23: Variation of safety factor and shaft force at different offsets and S/d ratios for shaft diameter of 5 ft
Figure 6.24: Lateral shaft deflection profile – LPILE analysis (D = 4 ft)
Figure 6.25: Bending moment diagram – LPILE analysis (D = 4 ft)
Figure 6.26: Shear force diagram – LPILE analysis (D = 4 ft)
Figure 6.27: Distribution of reinforcement (D = 4 ft)
6.5.2.4 Field Tests

Field pressuremeter test was performed at this site in July 2005 on the rock at a depth of 32 ft. The high pressure calibration curve for the dilatometer probe is shown in Figure 6.28. For the calculation of young's modulus, corrected volume versus corrected pressure was prepared as shown in Figure 6.29. The modulus of elasticity for the sandstone encountered at this site is 707.066 ksi and the undrained shear strength is 578.4 psi. The p-y curve at a depth of 32 ft was deduced using Briaud's method (the pressuremeter, 1992) as shown in Figure 6.30. Some onsite pictures for the dilatometer test setup and rock core samples can be found in Appendix B (see Figures B.1 – B.2).
Figure 6.29: Corrected volume vs. corrected pressure – dilatometer test (WAS-7 site)
Figure 6.30: p-y curve at depth of 32 ft (WAS-7-48.0 site)
6.5.2.5 Instrumentation Plans

The slope/shaft system at WAS-7-48.0 site was extensively instrumented and monitored. Two drilled shafts (shafts #53 and #54) as well as the surrounding soil mass were instrumented and monitored. Instruments inside each drilled shaft include nine vibrating wire pressure cells at 3 different levels (i.e., depth from shaft top = 12.5 ft, 18.5 ft, and 24.5 ft), three pressure cells at each level (i.e., upslope side, downslope side, and 45°). In addition to the pressure cells, sixteen vibrating wire strain gages were installed on each shaft at 8 different levels (i.e., depth from shaft top = 11 ft, 14 ft, 17 ft, 20 ft, 23 ft, 26 ft, 29 ft, and 32 ft), two strain gages at each level (i.e., upslope side and downslope side). Also, two in-place inclinometers were installed inside each drilled shaft to measure the shaft tilt due to slope movement. The tilt is usually converted into deflections. On the other hand, instruments inside the soil include three in-place inclinometers (i.e., upslope, between the shafts, and downslope), three earth pressure cells installed between the drilled shafts at 3 levels (i.e., depth from ground surface = 12 ft, 18 ft, and 24 ft), and three vibrating wire piezometers installed at 3 locations across the slope (i.e., upslope – depth = 28 ft, between the drilled shafts – depth = 27 ft, and downslope – depth = 25 ft).

The aforementioned discussion is summarized in Table 6.9. Figure 6.31 shows a cross-section at station 2532 + 75.0. The piezometers, inclinometers, and shaft offset are shown in the figure. Figure 6.32 shows a schematic of the instrumentation details along the shafts length for shafts #53 and #54. Locations of the pressure cells mounted on the drilled shafts and installed between the shafts are shown in the plan view in Figure 6.33. All instruments except the inclinometers were multiplexed with four 16-channel multiplexers. These four multiplexers were connected with an automated datalogger.
installed in a traffic enclosure box onsite. The data acquisition system at this site is managed as summarized in Table 6.10. Some onsite pictures during instrumentation, construction, and data acquisition setup at WAS-7-48.0 site can be found in Appendix B. (see Figures B.3 – B.7)
Table 6.9: Instrumentation details at WAS-7-48.0 site

<table>
<thead>
<tr>
<th>Inclinometers</th>
<th>Depth (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>INC-1</td>
<td>63</td>
</tr>
<tr>
<td>INC-2</td>
<td>63</td>
</tr>
<tr>
<td>INC-3</td>
<td>42</td>
</tr>
<tr>
<td>INC-4</td>
<td>42</td>
</tr>
<tr>
<td>INC-5</td>
<td>63</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Piezometers</th>
<th>Depth (ft)/Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>PZ-1</td>
<td>28/upslope</td>
</tr>
<tr>
<td>PZ-2</td>
<td>27/between the drilled shafts</td>
</tr>
<tr>
<td>PZ-3</td>
<td>25/downslope</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pressure cells</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>9 Pressure cells on shaft #20</td>
<td>3 levels (12.5 ft, 18.5 ft, and 24.5 ft)/(upslope, downslope, and 45°)</td>
</tr>
<tr>
<td>9 Pressure cells on shaft #21</td>
<td>3 levels (12.5 ft, 18.5 ft, and 24.5 ft)/(upslope, downslope, and 45°)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Earth Pressure Cells</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3 earth pressure cells between the drilled shafts</td>
<td>3 levels (12 ft, 18 ft, and 24 ft)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Strain Gages</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>16 strain gages on shaft #20</td>
<td>8 levels (11 ft, 14 ft, 17 ft, 20 ft, 23 ft, 26 ft, 29 ft, and 32 ft)/(upslope and downslope)</td>
</tr>
<tr>
<td>16 strain gages on shaft #21</td>
<td>8 levels (11 ft, 14 ft, 17 ft, 20 ft, 23 ft, 26 ft, 29 ft, and 32 ft)/(upslope and downslope)</td>
</tr>
</tbody>
</table>
Table 6.10: Data acquisition system at WAS-7-48.0 site

<table>
<thead>
<tr>
<th>Dataloggers</th>
<th>Multiplexers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 datalogger</td>
<td>four 16-channel multiplexers</td>
</tr>
<tr>
<td>Mux-1</td>
<td>16 strain gages on shaft #53</td>
</tr>
<tr>
<td>Mux-2</td>
<td>16 strain gages on shaft #54</td>
</tr>
<tr>
<td>Mux-3</td>
<td>9 pressure cells on shaft #53 + 7 pressure cells on shaft #54</td>
</tr>
<tr>
<td>Mux-4</td>
<td>2 pressure cells on shaft #54 + 3 earth pressure cells between the shafts + 3 piezometers</td>
</tr>
</tbody>
</table>
Figure 6.31: WAS-7 - Instrumentation Plans (Cross-section 1)
Figure 6.32: WAS-7 - Instrumentation Plans (Cross-section 2)
Figure 6.33: WAS-7 - Instrumentation Plans (Plan view)
6.5.2.6 Field Data Monitoring Results

All field monitoring results at WAS-7 site were recorded on an hourly basis starting on November 2006 for strains, water level fluctuation, and earth pressures thrusting on the shafts and between the shafts. Furthermore, inclinometer readings for the two instrumented shafts and the surrounding soil were measured on a quarterly basis. Plots for all monitoring results were prepared and can be found in Appendix B (see Figures B.8 – B.9 for bending moment diagrams, Figures B.10 – B.16 for pressure distribution diagrams, Figure B.17 for water level changes, and Figures B.18 – B.27 for deflection profiles).

No further analyses were performed on the soil/shaft movements and developed bending moments because the monitored values are considerably small. The maximum lateral deflection of shaft #53 is 0.2 in which lies within the error of the device. The same deflection profile was observed for shaft #54 and for the slope itself (upslope and downslope sides). Strain measurements were used to determine the developed bending moments along the shaft length. The maximum bending moment is very small because no soil movements were developed at and around the shafts. In other words, the forces imparted on the drilled shafts are very small. Slope and shaft movements at this site were efficiently controlled due to the installation of drilled shafts at least for the last year of monitoring.
6.5.3 MRG-376-1.10 Landslide Repair Site

This section contains general description of MRG-376-1.1 landslide repair site, subsurface profile, slope stability analysis, laboratory tests, instrumentation details, and the field data monitoring results.

6.5.3.1 General Site Description

State Route 376 is aligned parallel with the Muskingum River. The pavement distress is near the centerline of the roadway and is described as progressively opening cracks with some vertical displacement. The slope immediately below the roadway generally ranges from 1.5(H):1(V) to 4(H):1(V). It appears that some erosion is occurring at the toe of the slope from the river. Cross-section of the landslide at MRG-376-1.1 site is shown in Figure 6.34. The affected area is approximately 150 ft long. Based on the site observations, the subsurface conditions and slope geometry, the landslide appears to be rotational in nature, with the toe near the river. Onsite picture for the pavement distress near the centerline of the roadway is shown in Appendix C (see Figure C.1).
6.5.3.2 Subsurface Profile

The preliminary bedrock geology map of the Stockport, Ohio (Ohio Division of Geological Surveys, 1997) indicates that the site is underlain by bedrock of the Monongahela Group representing the Pennsylvanian geologic period. This formation is typically identified as shale, siltstone, limestone, sandstone, and coal. Abandoned mine mapping available on the Ohio Department of Natural Resource's website indicates that no mining was performed in the immediate site vicinity.

Four borings were advanced along the affected alignment as shown in Figure 6.35; three along the downhill edge of the roadway (B-1 through B-3) and one near the
uphill edge (B-4). The subsurface elevations of the borings varied from 666.7 ft (B-2) to
667.7 ft (B-1) as determined by Canter Surveying. Asphalt pavement was observed at the
surface of all four borings ranging in thickness from 0.7 ft (B-1, 3, and 4) to 1.0 ft (B-2).
Groundwater was observed in all of the borings ranging in depth from 17.8 ft (elevation
649.5 ft) in B-3 to 34.0 ft (elevation 633.7 ft) in B-1.

Fill material was observed below the pavement in all of the borings to depth
ranging from 7 ft in B-2 and B-3 to 7.3 ft in B-1. The fill was typically identified as clay
(A-7-6), gravel with sand, silt and clay (A-2-6), gravel with sand (A-1-b) or gravel (A-1-
a). Water contents of 19 % - 21% and SPT N-values of 7 – 12 blows per foot were
recorded in the clay fill. Water contents of 3 % - 16% and SPT N-values of 6 – over 50
blows per foot were recorded in the gravelly fill.

Below the fill, soils identified as silty clay (A-6b) or clay (A-7-6) were observed
to depth ranging from 17 ft (B-3) to 24.5 ft (B-4) and described as moist and stiff to very
stiff. Water contents ranged from 16% - 26% and SPT N-values varied from 10 – 24
blows per foot.

Softer cohesive soils were encountered below the aforementioned cohesive layer
and identified as silt and clay (A-6a) or sandy silt (A-4a). These soils were described as
moist to wet and soft to medium stiff. Water contents varied from 22% - 32% and SPT N-
values varied from 3 – 7 blows per foot.

Bedrock was encountered at depths ranging from 29.2 ft (B-3, elevation 638.0 ft)
to 34.5 ft (B-1, elevation 633.2 ft). Approximately 10 ft of rock coring was performed in
B-2 and the bedrock was identified as shale and described as gray, soft to moderately
hard and thin to medium bedded with silty to sandy and weathered zones. No cores loss
and a Rock Quality Designation (RQD) value of 40% were recorded in this coring interval.

Figure 6.35: Plan view – location of borings (MRG-376 site)
6.5.3.3 Slope Stability Analysis

Analysis was performed along a cross-section through B-2 and B-4. The UA Slope Computer program was used to estimate the critical failure surface. Existing conditions were modeled by using the topographic information, the results of borings, assumed soil parameters and an assumed failure surface. The assumed soil parameters were adjusted until a factor of safety approaching 1.0 was achieved, indicating marginal stability. It was determined that a rotational surface extending through the soft cohesive layer to the top of bedrock with a toe within the river was the critical surface. The same conditions and parameters were then used with the addition of a row of drilled shafts at varying offsets. Iterations performed with the program indicated that optimal row positions with offsets of 48 ft and 75 ft provided factors of safety of at least 1.5 and reasonable loading. Loads of approximately 80 to 100 kips (triangular distribution above the failure surface) were estimated at these row positions. Table 6.11 provides more details regarding the design of the slope/shaft system at MRG-376 site.

The computer program LPILE was used to design the appropriate reinforcing steel for the drilled shafts. Analyses were performed using the unfactored loads to estimate the pile head deflections. Pile head deflections on the order of 0.5 to 1.5 inches were estimated. An analysis was then performed using a factored load (the unfactored load multiplied by 1.69) to check if the moment capacity of the drilled shaft was greater than the maximum moment from loading. Iterative analyses were used to determine the required reinforcing steel to resist the maximum anticipated moment. Shaft diameters of 48 inches did provide adequate moment capacity. For the instrumentation purposes, only two drilled shafts were reinforced with steel cages, while, the rest of the shafts were
reinforced with W10x22 steel beams. Final arrangement for the shaft reinforcement is shown in Figure 6.36. Drilled shafts layout at this site is shown in Figure 6.37. The placement of riprap at the toe of the slope is recommended to prevent further erosion. The riprap should be approximately 5 ft thick and extend from the bottom of the river to the top of the river bank at an approximate elevation of 645 to 650 ft. Some onsite pictures for the riprap at the downslope and the pavement after landslide repairing can be found in Appendix C (see Figures C.2 – C.3).
Table 6.11: Site specific information and design recommendation (MRG-376 landslide)

<table>
<thead>
<tr>
<th>District</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitude/Latitude</td>
<td>81° 46' 27&quot; / 39°33'30&quot;</td>
</tr>
<tr>
<td>Contractor</td>
<td>Troy Wagner</td>
</tr>
<tr>
<td>Construction Start Date</td>
<td>August 25, 2006</td>
</tr>
<tr>
<td>Construction End Date</td>
<td>October 01, 2006</td>
</tr>
<tr>
<td>Area Exhibiting Instability</td>
<td>53+69.6 – 55+21.6 (152 ft)</td>
</tr>
<tr>
<td>Total Number of Drilled Shafts</td>
<td>20</td>
</tr>
</tbody>
</table>
| Instrumented Drilled shafts | Shaft #10 (54+41.6)  
|                       | Shaft #11 (54+49.6) |
| Offset (from centerline of pavement) | 20 ft |
| Total Shaft Length (Lₚ) | 43.6 ft |
| Rock-Socket Length (Lₑ) | 20 ft |
| Drilled Shaft Diameter (D) | 4 ft |
| Spacing-to-Diameter Ratio (S/D) | 2 |
| FS_{target} – UA Slope Analysis | 1.8 |
| F_{shaft} – UA Slope Analysis | 97 kips |
| Nominal Moment Capacity (Mₙ) – LPILE Analysis | 2,820 ft-kips (28#14 Bars) |
| Shaft Head Deflection – LPILE Analysis | 2.44 in |
Figure 6.36: Distribution of reinforcement for drilled shafts at MRG-376 site (D = 4 ft)
Figure 6.37: Plan view, shaft locations and elevations (MRG-376 site)
6.5.3.4 Laboratory Tests

An unconfined compressive strength of cohesive soil test (ASTM D-2166) was performed on a sample taken from B-3, at a depth of 10 ft – 11 ft. The visual description indicates clay with some sand and a little gravel, reddish-brown, moist, very stiff. About 0.9 inches was recovered from the sample. The stress-strain curve during the test is shown in Figure 6.38. The unconfined compressive strength was 2.28 tsf and undrained shear strength of 1.14 tsf. The pocket penetrometer reading was 2.25 tsf. The strain at maximum stress was 15% for a strain rate to failure of 0.99 %/min. The test was performed by FMSM Engineers.

Figure 6.38: Stress-strain curve for concrete samples at MRG-376 site
6.5.3.5 Instrumentation Plans

Only inclinometers were installed at this site because the area exhibiting instability is about 150 ft and the expected lateral earth pressures are considered small. A total of 20 4 ft-drilled shafts were placed with an offset of 20 ft from the centerline of the roadway of SR 376, and the shafts were spaced 8 ft on centers. A total of four inclinometers are installed at this site. Two inclinometers are inside two drilled shafts (shafts #10 and #11) with a total length of 45 ft as shown in Figure 6.39; and two inclinometers are between the drilled shafts (one on the upslope side and one on the downslope side) with a total length of 45 ft as shown in Figure 6.40. The inclinometers layout at this site is designed in such away to capture the displacement fields in and around the drilled shafts. Onsite picture for the inclinometers at MRG-376 landslide site while measuring the tilts along the shaft length and the surrounding soils can be found in Appendix C (see Figure C.4).
Figure 6.39: Instrumentation details – cross section (MRG-376 site)
Drilled Shaft No. 11

Drilled Shaft No. 10

Inclinometers (River side)

Figure 6.40: Instrumentation details – plan view (MRG-376 site)
6.5.3.6 Field Data Monitoring Results

All tilt readings for the instrumented shafts (i.e., shafts #10 and #11) and the surrounding soil (upslope and downslope) were measured on a quarterly basis starting on October 2006. All tilt measurement results were converted into deflections since the distance between two successive tilt sensors is fixed and is equal to 2 ft and by imposing zero displacement at the shaft tip as a boundary condition. All developed deflection profiles can be found in Appendix C (see Figures C.5 – C.8 for the deflection profiles for the shafts and Figures C.9 – C.12 for the deflection profiles for the surrounding soils).

Based on about a year of monitoring of the slope and shafts movements, the monitoring results indicate a stabilized slope/shaft system at this site. The deflection profiles for shafts #10 and #11 show no movement along the shafts length as indicated in the deflection profiles in Appendix C. Similarly, deflection profiles for the soil on the upslope side and the downslope side show no incremental movement. Developing bending moment diagrams from the shaft deflection profiles are not applicable because shaft deflections are very small. No or minimal bending moments are assumed to be developed along the shafts length.

6.6 Summary and Conclusions

Three actual full-scale landslide sites (JEF-152-1.3, WAS-7-48.0, and MRG-376-1.1) stabilized with drilled shafts were fully instrumented and monitored. Onsite automated data acquisition systems were installed onsite. Inclinometers were installed inside and outside the drilled shafts to capture and better understand the displacement fields for the slope/shaft systems. Strain gages were installed inside four drilled shafts
along the depth to measure the developed bending moments along the shaft length. Bending moment distributions obtained from both strain readings and inclinometer readings were verified. Table 6.12 provides a side by side comparison for the three landslide sites in terms of the design parameters, geotechnical response, and structural response. The shafts and soil movements at WAS-7 and MRG-376 are way less than the movements at JEF-152. The rate of shaft movement at JEF-152 is considered critical if it keeps the same rate, while, the rates of movement at WAS-7 and MRG-376 are very small. The measured moments at WAS-7 and MRG-376 are one-twentieth of the design moments, while, it is just one-third for JEF-152.

Table 6.12: Summary of final design parameters, geotechnical design, and structural design for the instrumented landslide cases

<table>
<thead>
<tr>
<th>Landslide Site</th>
<th>JEF-152</th>
<th>WAS-7</th>
<th>MRG-376</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Parameter</td>
<td>D (ft)</td>
<td>S/D</td>
<td>L_r/L_p</td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>2</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2</td>
<td>0.45</td>
</tr>
<tr>
<td>Geotechnical Design</td>
<td>Shaft movement (in)</td>
<td>0.9</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>Slope movement (in)</td>
<td>0.8 (0.5 @ slip surf.)</td>
<td>&lt; 0.1</td>
</tr>
<tr>
<td></td>
<td>Rate of movement</td>
<td>0.26 in/yr</td>
<td>Very Slow</td>
</tr>
<tr>
<td>Structural Design</td>
<td>FS_{min} = M_{design}/M_{measured}</td>
<td>3.5</td>
<td>20</td>
</tr>
</tbody>
</table>
Detailed conclusions can be summarized as below:

- Drilled shafts can be a practical and effective mean of stabilizing landslides.
- Geotechnical factor of safety was successfully enhanced for WAS-7 and MRG-376, but not fully enhanced for JEF-152.
- Structural factor of safety was highly overestimated for the studied landslide sites.
- Using smaller number of large-diameter shafts can be more effective than larger number of small-diameter shafts.
- For all studied landslide sites, drilled shafts fixity was successfully achieved. (rock socket length >10% Lp)
- The typical range for S/D is 2 – 4. There is no much difference in the design between S/D of 2 and 3. (S/D = 3 can be effectively used)
- Backfilling and grading of the upper portion of the slope behind the shafts has, to some extent, an adverse effect on the overall stability of slope/shaft systems.
7.1 Overview

In an effort to develop an efficient analytical method for analysis of laterally loaded drilled shafts using only lateral shaft deflection data, numerical procedures were proposed based on the principle of superposition. The lateral shaft deflections along the shaft length due to superposition of the lateral applied load to the drilled shaft were added together to establish the compatibility equations that govern the lateral behavior of the drilled shaft system. The compatibility equations allow for the determination of the net applied loads to the drilled shaft responsible for specific amount of shaft deflections. Once the loads were determined, basic equilibrium equations were applied to calculate shear forces and bending moments along the shaft length. Detailed mathematical derivation and assumptions associated with the problem formulation are discussed in this chapter. A computer program was developed implementing the proposed numerical procedures to facilitate numerical computations. Five different laterally loaded drilled shaft examples were described and used to verify the validity of the developed method.
Included in the cases for validation were two actual full-scale drilled shafts at Jefferson County: (1) landslide repair using drilled shafts; and (2) lateral load test.

7.2 Statement of the Problem

The method of p-y curves for analysis of laterally loaded drilled shafts has been widely accepted because of its ability to capture the essence of nonlinear soil/shaft behavior. In this method, the soil/shaft interaction mechanism is basically idealized as a beam on a Winkler spring medium to represent the soil response to lateral shaft deflection. The adoption of the beam theory, together with the use of p-y curves to represent Winkler springs, provides a relatively simple means of computing bending stresses, shear stresses, and deflections along the shaft length. The development of pertinent p-y curves are generally based on well-instrumented lateral load tests, including the use of strain gages. The cost of embedding a large number of strain gages in the instrumented shafts in lateral load tests can be considered high. Furthermore, the number of tests performed and documented is relatively few, and the methods of analysis for obtaining p-y curves from such a test are largely diverse and some of them may actually lead to inaccurate solution. Because of these reasons, there is a compelling need to develop a simple, accurate, and cost-effective approach to derive pertinent p-y curves from laterally loaded drilled shafts where strain gages need not be used.

7.3 Objectives

The objective of this study is to develop a new yet simple analytical numerical procedure for the analysis of drilled shafts subjected to lateral loading based on the
principle of superposition when only shaft deflection data is obtained. The principle of superposition is a general procedure which can be used for analyzing linear elastic structural elements. The main objectives from this study are to enable soil engineers utilizing only lateral shaft deflection data to determine (a) the actual applied load acting on drilled shaft that causes the measured shaft deflection; (b) the developed bending moments and shear forces in the shaft; and (c) the resulting soil response under this loading condition. The proposed method can be applicable to many geotechnical engineering applications. Two important applications involve primarily drilled shafts subjected to lateral loading. First, the method can be used to interpret the deflection data of the drilled shafts used for stabilizing unstable slopes, where long-term monitoring of these slope stabilizing drilled shaft deflections are generally difficult to interpret due to a lack of data interpretation technique as illustrated in Figure 7.1. Second, the derivation of p-y curves and interpreting the resulting internal forces (shear and bending moment) from lateral load tests (see Figure 7.2) involving the use of only deflection measurement devices.
Figure 7.1: Landslide repair using drilled shafts
Figure 7.2: Laterally loaded drilled shaft
7.4 Literature Review

Although many methods are available for determining soil/shaft behavior from the shaft deflection data, there is a lack of consistency in these methods when compared with each other. Furthermore, some of these methods yield results that are quite divergent from the actual measured responses in the field. Broadly, the most commonly used methods can be categorized into: (1) methods involving fitting a series of curves (i.e., piecewise curve or global high order polynomial) to represent the deflected shape of the shaft, then the fitted deflection function is differentiated two times to obtain bending moments as in Equation 7.1, three times to obtain shear forces as in Equation 7.2, and four times to obtain p-y curves as in Equation 7.3; (2) methods utilizing the concept of energy conservation of a pile-soil system; and (3) methods using a least-square regression technique.

\[
EI \frac{d^2y}{dz^2} = M \tag{7.1}
\]

\[
EI \frac{d^3y}{dz^3} = \frac{dM}{dz} = V \tag{7.2}
\]

\[
EI \frac{d^4y}{dz^4} = \frac{d^2M}{dz^2} = \frac{dV}{dz} = -p \tag{7.3}
\]

According to Tang (1984), uncertainty in estimating the bending moments by twice differentiating the deflection function may be attributed to several sources of error, such as: (a) model error, (b) systematic error, (c) inherent spatial variability, and (d) statistical uncertainty due to insufficient measurement frequency.

Soares (1983) back-calculated the bending moments along the depth for a diaphragm wall in Rio de Janeiro, Brazil after fitting a ninth-order polynomial to
represent the profile of the deflected shape. Similarly, Poh et al. (1997) back-calculated the moments for a diaphragm wall in Singapore considering a seventh-order polynomial to the deflected shape. Price et al. (1987) determined the moments on shafts in a laterally loaded test in Plancoet, France by using a sixth-order polynomial deflection function to represent the deflection shape. Miller et al. (2000) established a method based on statistical models methods to determine the appropriate degree of the global polynomial for representing the deflection shape of a laterally loaded shaft. As can be seen, there is no consensus regarding the appropriate degree of polynomial to be used to represent the deflection shape.

Unlike global high-order polynomial curve fitting techniques, cubic splines methods provide smooth transition from point-to-point. The main advantage of cubic splines over the global polynomials is that the resulting curve can pass through all the specified data points. For this local point control, piecewise cubic curves are generally preferred. Wilson (1969) fitted a circular arc through two and three adjacent points to represent the deflection function. Ooi and Ramsey (2003) reviewed twelve methods for estimating curvature. These methods were applied to sixty sets of inclinometer readings; whereas, six of them with both inclinometers and strain gages. Ooi and Ramsey compared both curvatures from inclinometers and strain gages. The comparison has shown a good agreement for a piecewise cubic polynomial curve fitting a moving window of five data points. The various method of analysis yielded curvature values that are widely divergent.

Lin and Liao (2006) developed a relatively simple method for interpreting lateral single-pile load test results based on measured inclinometer data. Fourier series functions
were used to represent the deflection of a pile. Convergence of the series after differentiation is guaranteed by applying the Cesaro sum technique to obtain shear, moment, and soil reaction. Three full-scale lateral load tests were used to verify the feasibility of the developed method. Many assumptions were involved in the derivation of the developed method; they are: (1) soil response is idealized as one-dimensional; (2) pile material behavior is assumed nonlinear elastic; (3) lateral loading is applied only at pile head; and (4) long pile which is defined with the ratio of the pile length versus the relative stiffness factor larger than 4 (Prakash and Sharma, 1990)

Brown et al. (1994) proposed a method to determine p-y curves from simple inclinometer data in a lateral load test that converges to a solution after minimizing the error between the predicted and measured deflections along the pile. However, the analysis requires use of a computer code, COM624 (Wang and Reese, 1991), to predict deflection. In addition, the shape of the p-y curve must be assumed and iteration over two or more soil parameters is needed for calibration of p-y curves.

7.5 The principle of Superposition

In general, superposition is an act of superposing or the state of being superposed. In linear algebra, the principle of superposition states that, for a linear system, a linear combination of solutions to the system is also a solution to the same linear system. In other words, the principle holds that two or more solutions to a linear equation or set of linear equations can be added together so that their sum is also a solution to the system. The superposition principle applies to linear systems of algebraic equations, linear differential equations, or systems of linear differential equations. The principle of
superposition is widely used in engineering applications because many of these applications may be modeled as a linear system. In civil engineering, the principle of superposition is often used for analyzing linear elastic structural elements. For the laterally loaded drilled shaft, the superposition principle may be applied by stating that the shaft deflection at any depth due to the superposition of the applied lateral load is equal to the sum of the shaft deflections due to the individual lateral applied load at that depth.

Since the drilled shaft is assumed to behave elastically, the resulted lateral shaft deflection due to the net applied load to the drilled shaft (i.e., both applied load and soil reactions) can be superimposed to account for the combined effects of all these loads. Once the shaft deflection is determined at each depth along the shaft length, the net applied load responsible for the deflection can be determined by establishing a set of compatibility equations, which is equal in number to the number of available shaft deflections along the shaft length. In satisfying compatibility requirements, the deflected shape of the drilled shaft must be consistent with the constraints imposed. Once the net applied load is obtained, further analyses based on beam theory and a set of equilibrium equations can be performed on the net applied load in order to determine shear forces, bending moments, and soil reactions. It should be noted here that the application of the principle of superposition for the laterally loaded drilled shafts is limited to the shaft response that is within the linear elastic region.
7.6 General Statements

Consider a distributed lateral load $W$ is applied to a drilled shaft with a total shaft length $L$ as shown in Figure 7.3. Under this loading condition, the shaft experiences lateral deflections $Y$. Consequently, the soil reaction to the shaft deflections is denoted as $P$. The lateral shaft deflections can be measured using servoaccelerometer probes (ASTM 1998), which can be installed inside the drilled shafts during the construction process. Basically, these probes provide a measure of slope or tilt at the point of measurement. This measured tilt is converted to a lateral deflection. If an imaginary distributed load $R$ is applied to a drilled shaft in lieu of both the applied load $W$ and the net soil reactions $P$ (i.e., $R = W + P$), as illustrated in Figure 7.4, that would result in shaft deflection $Y$ along the shaft length that is the same as that would be produced by applying $W$ and $P$ on the drilled shaft. Establishing the compatibility equations for the equivalent laterally loaded drilled shaft based on the principle of superposition, the unknown net load $R$ corresponding to shaft deflection $Y$ can be determined. Furthermore, the equations of equilibrium can be applied along the shaft length to determine the shear forces and bending moment.
Figure 7.3: Conceptual model of a laterally loaded drilled shaft
Figure 7.4: Equivalent conceptual model of a laterally loaded drilled shaft
7.7 Mathematical Formulation

Consider a drilled shaft that is divided into \((n-1)\) intervals, such that there are \(n\) imaginary nodes along the shaft length (i.e., node \(i = 1\) at shaft top and node \(i = n\) at shaft tip). Besides, shaft deflections are assumed to be known at each node \(i\); that is, there are \(n\) shaft deflections \(Y\) available along the shaft length. Applying the principle of superposition, the load \(R\) is superimposed as shown in Figure 7.5 into individual loads applied to the drilled shaft. The superposition of \(R\) yielded the deflected shapes \(\Delta_{1j}, \Delta_{2j}, \ldots, \Delta_{ij}, \ldots, \Delta_{nj}\) as a result of load application of \(R_1, R_2, \ldots, R_i, \ldots, R_n\) at locations \(Z_1, Z_2, \ldots, Z_i, \ldots, Z_n\), respectively. Note that the deflection along the drilled shaft length due to an individual load \(R_i\) applied at \(Z_i\) is denoted as \(\Delta_{ij}\) where the first subscript \(i\) indicates that the applied load \(R_i\) is applied at \(Z_i\) while the second subscript \(j\) indicates that the shaft deflection at location \(Z_j\). Therefore, the net shaft deflections \(Y\) must be consistent with the \(n\) deflected shapes \(\Delta\) due to an application of \(n\) set of \(R_i\) along the drilled shaft length.

In other words, all the individual deflected shapes \(\Delta\) can be added together so that their sum is equal to the total shaft deflections \(Y\). In a simple mathematical form:

\[
Y_j = \sum_{i=1}^{n} \Delta_{ij} \; ; \text{for } j = 1 \text{ to } n \tag{7.4}
\]

Expanding the summation series in Equation 7.4 for \(n\) nodes along the shaft length, the following compatibility equations can be established:

\[
Y_1 = \Delta_{11} + \Delta_{21} + \cdots + \Delta_{1n} + \cdots + \Delta_{n1} \tag{7.5a}
\]

\[
Y_2 = \Delta_{12} + \Delta_{22} + \cdots + \Delta_{12} + \cdots + \Delta_{n2} \tag{7.5b}
\]

\[
\vdots
\]

\[
Y_i = \Delta_{1i} + \Delta_{2i} + \cdots + \Delta_{1i} + \cdots + \Delta_{ni} \tag{7.5c}
\]
The above set of compatibility equations describes the consistency of a deflected drilled shaft, in which, lateral shaft deflection at any specific location along the shaft length is consistent with the summation of all lateral shaft deflections at that specific location due to the application of an individual net load \( R_i \) at locations \( Z_i \) for all nodes.

Additionally, in order to determine the rotations (\( \theta \)) and deflections (\( \Delta \)) of a drilled shaft due to the application of an individual load \( R_i \) at \( Z_i \) as illustrated in Figure 7.6, a closed-form solution is derived utilizing the basic differential equation of the elastic curve of a beam element, which can be written as follows:

\[
\frac{d^2 \Delta}{dZ_j^2} = \frac{M(Z_j)}{EI} \quad (7.6)
\]

Where

\( Z_j \) = the depth from shaft top to node \( j \).

\( E \) = modulus of elasticity of the drilled shaft;

\( I \) = moment of inertia of the drilled shaft; and

\( M \) = bending moment.

By imposing fixed boundary conditions at the shaft tip (i.e., rotation \( \theta_n \) and deflection \( \Delta_n \) are zeros), the bending moment \( M(Z_j) \) in Equation 7.7 along the shaft length can be expressed as:

\[
M(Z_j) = \begin{cases} 
0 & ; 0 \leq Z_j \leq Z_i \\
R_i(Z_j - Z_i) & ; Z_i \leq Z_j \leq L 
\end{cases} \quad (7.7)
\]
Substituting $M(Z_j)$ into Equation 7.6 and integrating $M(Z_j)/EI$ twice to establish the equations for rotation $\theta$ and deflection $\Delta$ along the shaft length as in Equations 7.8 and 7.9, respectively.

$$
\theta_j = \theta\left(R_i, Z_j\right) = \begin{cases}
C_1 & ; 0 \leq Z_j \leq Z_i \\
\frac{R_i}{EI} \left(\frac{Z_j^2}{2} - Z_i Z_j\right) + C_2 & ; Z_i \leq Z_j \leq L
\end{cases} \quad (7.8)
$$

$$
\Delta_j = \Delta\left(R_i, Z_j\right) = \begin{cases}
C_1 Z_j + C_3 & ; 0 \leq Z_j \leq Z_i \\
\frac{R_i}{EI} \left(\frac{Z_j^3}{6} - \frac{Z_i Z_j^2}{2}\right) + C_2 Z_j + C_4 & ; Z_i \leq Z_j \leq L
\end{cases} \quad (7.9)
$$

Where

$\theta_j$ = shaft rotation at $Z_j$ due to $R_i$ applied at $Z_i$; and

$\Delta_j$ = shaft deflection at $Z_j$ due to $R_i$ applied at $Z_i$.

In order to evaluate the constants of integration $C_1$, $C_2$, $C_3$, and $C_4$ in Equations 7.8 and 7.9, the following four boundary conditions were assumed based on the boundary condition imposed at the shaft tip:

1. At $Z_j = L$; $\theta(R_i, L) = 0$.
2. At $Z_j = L$; $\Delta(R_i, L) = 0$.
3. At $Z_j = Z_i$; $\theta(R_i, Z_i^+) = -\theta(R_i, Z_i^-)$.
4. At $Z_j = Z_i$; $\Delta(R_i, Z_i^+) = \Delta(R_i, Z_i^-)$.

Hence, the constants of integration in Equations 7.8 and 7.9 can be determined as follows:
\[ C_1 = \frac{R_i}{EI} \left( \frac{Z_i^2}{2} - Z_iL + \frac{L^2}{2} \right) \]  
\[ (7.10a) \]

\[ C_2 = \frac{R_i}{EI} \left( Z_iL - \frac{L^2}{2} \right) \]  
\[ (7.10b) \]

\[ C_3 = \frac{R_i}{EI} \left( \frac{-5Z_i^3}{6} + 3Z_i^2L - 3Z_iL^2 + \frac{L^3}{3} \right) \]  
\[ (7.10c) \]

\[ C_4 = \frac{R_i}{EI} \left( \frac{L^3}{3} - \frac{Z_iL^2}{2} \right) \]  
\[ (7.10d) \]

Substituting the constants of integration \( (C_1, C_2, C_3, \text{and } C_4) \) into Equation 7.9 would lead to the following simple flexibility equation:

\[ \Delta_{ij} = f_{ij} R_i \]  
\[ (7.11) \]

Where

\[ f_{ij} = \frac{1}{EI} \left\{ \frac{(L - Z_j)^2}{2} \left( \frac{2L}{3} + \frac{Z_i}{3} - Z_j \right) \right\}; \quad 0 \leq Z_j \leq Z_i \]
\[ (7.12) \]

\[ f_{ij} = \frac{1}{EI} \left\{ \frac{(L - Z_j)^2}{2} \left( \frac{2L}{3} + \frac{Z_i}{3} - Z_j \right) + \frac{(Z_j - Z_i)^3}{6} \right\}; \quad Z_i \leq Z_j \leq L \]

Equation 7.11 can be called the flexibility relation of the drilled shaft because the shaft deflection \( \Delta_{ij} \) is equal to a coefficient multiplied by the net applied load \( R_i \). This coefficient \( f_{ij} \) is usually called the flexibility coefficient. Substituting the flexibility relation into Equations 7.5a through 7.5d, the following compatibility equations are obtained:

\[ Y_1 = f_{11}R_1 + f_{12}R_2 + \cdots + f_{1n}R_n \]  
\[ (7.13a) \]

\[ Y_2 = f_{12}R_1 + f_{22}R_2 + \cdots + f_{2n}R_n \]  
\[ (7.13b) \]
Re-writing Equations 7.13a through 7.13d in a matrix form as follows:

\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
\vdots \\
Y_n
\end{bmatrix} =
\begin{bmatrix}
f_{11} & f_{12} & \cdots & f_{1n} \\
f_{21} & f_{22} & \cdots & f_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
f_{n1} & f_{n2} & \cdots & f_{nn}
\end{bmatrix}
\begin{bmatrix}
R_1 \\
R_2 \\
\vdots \\
R_n
\end{bmatrix}
\]

(7.14)

Hence, the flexibility relation of a laterally loaded drilled shaft can be written in a compact-form matrix as follows:

\[
[Y]_{n \times 1} = [f]_{n \times n} [R]_{n \times 1}
\]

(7.15)

Equation 7.15 can be used to solve for the net applied load \( R \). Thus, shear force \( V \), bending moment \( M \), and soil reaction \( P \) at any location \( m \) along the shaft length can be determined by applying the equations of equilibrium as follows:

\[
V_m = \sum_{i=1}^{n} R_i
\]

(7.16)

\[
M_m = \sum_{i=1}^{n} R_i (Z_m - Z_i)
\]

(7.17)

\[
P_m = R_m - W_m
\]

(7.18)

The resultant forces \([R]\), soil reactions \([P]\), and applied loads \([W]\) in Equation 7.18 should be divided by the effective widths \([b]\) to obtain the unit of forces per unit length \([F/L]\). The effective width \( b_i \) of a load \( R \) applied at \( Z_i \) is obtained as follows:

\[
b_i = \frac{Z_{i+1} - Z_{i-1}}{2}
\]

(7.19)
Figure 7.5: Principle of superposition

\[ R = R(Z) + \cdots + R(Z) \]

\[ V = V(R(Z)) \]

\[ Z = Z(Z) \]

\[ L = L \]
Figure 7.6 Deflected shape of drilled shaft due to a single load $R$ applied at $Z_i$
7.8 Proposed Numerical Procedures

The foregoing discussion for the analysis of laterally loaded drilled shafts using only shaft deflection data based on the principle of superposition is summarized in the following step-by-step numerical procedures as below:

1. Establish the lateral shaft deflection matrix \([Y_i]_{nx1}\) for \(i = 1\) to \(n\), where \(n\) is the total number of known lateral shaft deflections along the shaft, \(Y_1 =\) lateral shaft deflection at shaft top, and \(Y_n =\) lateral shaft deflection at shaft tip.

2. Establish the depth matrix \([Z_i]_{nx1}\) for \(i = 1\) to \(n\), where \(Z_1 = 0\) and \(Z_n =\) total shaft length (L).

3. Establish the flexibility matrix \([f_{ij}]_{nxn}\) using Equation 7.12, for \(i = 1\) to \(n\) and \(j = 1\) to \(n\).

4. Calculate the resultant forces matrix \([R_i]_{nx1}\) by solving Equation 7.15, for \(i = 1\) to \(n\).

5. Calculate the shear forces matrix \([V_i]_{nx1}\) using Equation 7.16, for \(i = 1\) to \(n\).

6. Calculate the bending moments matrix \([M_i]_{nx1}\) using Equation 7.17, for \(i = 1\) to \(n\).

7. Establish the effective width matrix \([b_i]_{nx1}\) using Equation 7.19, for \(i = 1\) to \(n\).

8. Divide the resultant forces matrix \([R_i]_{nx1}\) by the effective width matrix \([b_i]_{nx1}\) in order to obtain the resultant forces per unit length along the drilled shaft.

9. Establish the load applied on the drilled shaft matrix \([W_i]_{nx1}\) for \(i = 1\) to \(n\). If \([W_i]_{nx1}\) is unknown, then, an estimation of \(W\) can be determined by taking the first derivative of the shear forces distribution along the shaft length after finding a suitable curve that fits the shear forces matrix obtained in Step 6.

10. Calculate soil reactions \([P_i]_{nx1}\) using Equation 7.18, for \(i = 1\) to \(n\).
7.9 Verification of the Proposed Method

The application of the proposed technique described above is demonstrated and verified using several examples. Three theoretical examples were analyzed using LPILE program (Reese et al., 2004), which has been confidently used by practicing engineers. The three theoretical examples were selected for different applied loading conditions, configurations of the drilled shafts (i.e., shaft diameter, length, and modulus of elasticity), and soil stratifications. In addition, two actual full-scale load tests data with measurements of deflection and bending strains were analyzed.

A computer program is developed using Microsoft Visual Basic to perform the numerical analysis based on the proposed mathematical procedures outlined in the previous section. The program consists of one screen as shown in Figure 7.7, to allow the user to browse the location of the input file that needs to be processed. The input file is basically a Microsoft Excel file of the deflection data. The user needs to input the depth along the shaft versus shaft deflection in the first sheet in the Excel file as shown in Figure 7.8. The program will perform the analysis and process the input data following the numerical procedures outlined in Section 7.8, by assembling the flexibility matrix of the laterally loaded drilled shaft, solving the linear system of equations (i.e., using Gauss elimination technique) to solve for the net applied force R, and computing the developed shear forces and bending moments. The output results will be smoothened and saved in the same Excel file in three additional sheets (sheet 2, sheet 3, and sheet 4). The net load R, shear force V, and bending moment M will be automatically generated and saved in the Excel file. The programming code used in the P-Y Program is listed in Appendix B.
Figure 7.7: p-y analysis program main screen

Figure 7.8: Microsoft Excel Input file for P-Y Analysis program
7.9.1 Theoretical Example 1: Horizontal Force at Shaft Head

This example as shown in Figure 7.9 consists of a 15.2 m long shaft with 0.373 m in diameter. The modulus of elasticity of shaft is 200 GPa. The shaft is subjected to 400 kN lateral load applied at the shaft head. Furthermore, the ground surface is assumed to be inclined 10° from the horizontal (i.e., sloping ground). The soil profile is modeled using two soil layers; the upper 5 m soil layer is modeled as stiff clay with cohesion of 96.5 kN/m², friction angle of 0°, and effective unit weight of 19 kN/m³. The lower soil layer is modeled as sand with subgrade modulus k of 16,300 kN/m³, friction angle of 35°, and effective unit weight of 19.9 kN/m³. The predicted shaft deflections from LPILE using constant EI option, as shown in Figure 7.10, is used as known shaft deflections in the proposed analytical method to determine the net load R, actual applied load W, soil response P, shear force V, and bending moment M along the shaft length. The predicted results using the proposed method and those obtained from LPILE program match favorably for the net applied load, shear force, and bending moment as shown in Figures 7.11 through 7.13, respectively. The predicted applied load W as shown in Figure 7.11 is zero along the shaft length except at the shaft head which represents the horizontal load applied at the shaft head. The 400 kN lateral load applied at the shaft head is also predicted in the shear forces distribution diagram as shown in Figure 7.12.
Figure 7.9: Configurations of soil layers and drilled shaft (Theoretical Example 1)

- **Stiff Clay**
  - $C = 96.5 \text{ kN/m}^2$
  - $\phi = 0^\circ$
  - $\gamma = 19 \text{ kN/m}^3$

- **Sand**
  - $k = 16,300 \text{ kN/m}^3$
  - $\phi = 35^\circ$
  - $\gamma = 19.9 \text{ kN/m}^3$

- $H = 400 \text{ kN}$
- $D = 0.373 \text{ m}$
- $5 \text{ m}$
- $10.2 \text{ m}$
- $10^\circ$
Figure 7.10: Lateral shaft deflection diagram - LPILE results (Theoretical Example 1)
Figure 7.11: Analysis of net load, soil response, and applied load results (Theoretical Example 1)
Figure 7.12: A comparison of shear force results along the shaft between LPILE and the developed method (Theoretical Example 1)
Figure 7.13: A comparison of bending moment results along the shaft between LPILE and the developed method (Theoretical Example 1)
7.9.2 Theoretical Example 2: Horizontal Force and Moment at Shaft Head

Another theoretical example using the LPILE program is discussed herein. In this example, the shaft is subjected to a lateral load of 88.9 kN and a bending moment of 81.3 kN-m applied at the shaft head. As shown in Figure 7.14, the shaft is 13.4 m long with 0.914 m in diameter. The modulus of elasticity is assumed to be 27.6 GPa. The upper 8.4 m of the shaft is standing free in the air (i.e., no soil surrounding the shaft). Furthermore, the soil profile was modeled using two soil layers; a 0.5 m thin weak rock layer with cohesion of 0.7 MPa, modulus of elasticity of 104 MPa, effective unit weight of 16 kN/m$^3$, and RQD of 65%, followed by another weak rock layer with cohesion of 0.86 MPa, modulus of elasticity of 91 MPa, effective unit weight of 13.6 kN/m$^3$, and RQD of 60%. The soil/shaft system described above was analyzed utilizing the proposed method, and the deflection data under this loading condition was obtained from the LPILE and the results were plotted in Figure 7.15. Comparisons of the net loads, shear forces, and bending moments results obtained from the developed method and LPILE program are in good agreement as shown in Figures 7.16 through 7.18. The 88.9 kN lateral load applied at the shaft head was predicted in the shear force diagram as in Figure 7.17. In addition to that, the 81.3 kN-m concentrated moment was predicted in the bending moment diagram as in Figure 7.18. On the other hand, the soil reaction in the upper 8.4 m was predicted to be zero and the shear force to be constant, which totally agree with the soil boundary condition in that the upper portion of the shaft is standing free in the air.
Figure 7.14: Configurations of soil layers and drilled shaft (Theoretical Example 2)
Figure 7.15: Lateral shaft deflection diagram - LPILE results (Theoretical Example 2)
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Figure 7.17: A comparison of shear force results along the shaft between LPILE and the developed method (Theoretical Example 2)
Figure 7.18: A comparison of bending moment results along the shaft between LPILE and the developed method (Theoretical Example 2)
7.9.3 Theoretical Example 3: Triangular Load Along the Shaft Length

Unlike the previous two theoretical examples, the boundary load in this example is a distributed triangular load applied along the shaft as shown in Figure 7.19 (i.e., zero at the shaft top and increases linearly to 525 kN/m at a depth of 6.35 m). The shaft has a modulus of elasticity of 24.85 GPa, a total length of 12.2 m, and a diameter of 1.07 m. Two soil layers were considered: the upper 7.62 m soil layer is modeled as silt and clay with cohesion of 4.8 kN/m² and friction angle of 20°. The lower 4.58 m is modeled as weak rock with RQD of 60% and modulus of elasticity of 101 MPa. The shaft deflection results obtained from LPILE under this distributed triangular load are plotted in Figure 7.20. The results are used as inputs in the developed method to predict the net loads, shear forces, and bending moments along the shaft length. The results of the net load R and soil response P as shown in Figure 7.21 are in good agreement for the portion of the shaft below 6.35 m because there is no load applied at that portion. The difference between R and P for the upper 6.35 m portion yielded the applied distributed triangular load. Shear force and bending moment diagrams have shown perfect agreement for both LPILE and the developed method as shown in Figures 7.22 and 7.23, respectively.
Figure 7.19: Configurations of soil layers and drilled shaft (Theoretical Example 3)

- **Silt and Clay**
  - $C = 4.8 \text{ kN/m}^2$
  - $\phi = 20^\circ$
  - $\gamma = 19.8 \text{ kN/m}^3$

- **Weak Rock**
  - $E = 101 \text{ MPa}$
  - $\gamma = 12.2 \text{ kN/m}^3$
  - RQD = 60%

- $q = 525 \text{ kN/m}$
- $D = 1.07 \text{ m}$
Figure 7.20: Lateral shaft deflection diagram - LPILE results (Theoretical Example 3)
Figure 7.21: Analysis of net load, soil response, and applied load results (Theoretical Example 3)
Figure 7.22: A comparison of shear force results along the shaft between LPILE and the developed method (Theoretical Example 3)
Figure 7.23: A comparison of bending moment results along the shaft between LPILE and the developed method (Theoretical Example 3)
7.9.4 Actual Example 1: Ohio DOT Landslide Repair (JEF-152)

A single row of 42 drilled shafts was installed with an offset of 12.2 m from the centerline of State Route 152 at Jefferson County, Ohio as depicted in Figure 7.24 to arrest the sliding soil mass. The shafts are 1.07 m in diameter with a total length of 12.2 m. The modulus of elasticity for shaft is 28.06 GPa. The drilled shafts were reinforced with 24#11 steel bars. The drilled shafts were placed 2.13 m apart on centers. Two drilled shafts (shaft #20 & #21) were extensively instrumented and monitored with strain gages distributed at several depths along the shaft length and in-place inclinometer to monitor the lateral response of the shafts. The drilled shafts were installed on March 2006. Two sets of readings for one instrumented shaft were used in this example for the purposes of illustration and comparison; the first set was collected on November 2006 and the second set was collected on March 2007. It is noted that the construction on site was completed on May 2006. The measured inclinometer readings as shown in Figure 7.25 for both sets were employed and used as input in the proposed method to determine the net resultant forces applied on the drilled shafts R (see Figure 7.26), shear forces V (see Figure 7.27), and bending moments M (see Figure 7.28) along the shaft length. Strain results were used to determine the actual bending moments along the shaft length due to the ground movement. The predicted bending moments are in good agreement with the measured ones, as shown in Figure 7.28 for both sets of data.
Figure 7.24: Ohio DOT landslide repair at Jefferson County – plan view (Actual Example 1)
Figure 7.25: Measured lateral shaft deflections along the shaft. (Actual Example 1)
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Figure 7.27: Predicted shear force V along the shaft. (Actual Example 1)
Figure 7.28: A comparison of measured and predicted bending moment results along the shaft. (Actual Example 1)
7.9.5  Actual Example 2: Ohio DOT Lateral Load Test (JEF-152)

A full-scale lateral load test was conducted on March 2006 at the same site described in the previous example (see Figure 7.24 for the location of the lateral load test shafts). Two 12.2 m long drilled shafts were installed. The compressive strength of concrete is 39.3 MPa. The first shaft is the reaction shaft which also consists of 1.83 m steel casing with 1.3 cm thickness. The second shaft is the test shaft which is about 1.07 m in diameter. The two shafts were 4.3 m away from each other and embedded deep into a deposit of mudstone at 7.6 m from the ground surface. The top 7.6 m of the test shaft was isolated from the surrounding clay soil with a 1.83 m steel casing. This is to ensure that only the response of the mudstone at the site was involved. Extensive instruments were installed inside both reaction and test shafts to monitor soil/shaft interaction response. These instruments include inclinometers, strain gages, and tiltmeters. The shaft head was subjected to lateral load with a load increment of 88.9 kN, 266.9 kN, 400 kN, 533.8 kN, and 734 kN. The lateral load was applied using a hydraulic jack with 2224 kN capacity laid horizontally and attached to a 15.2 cm long load cell as shown in Figure 7.29. The schematic of the instrumentation plan for the lateral load test at this site is shown in Figure 7.30. At each load increment, tilt measurements at different depths obtained from the inclinometer were converted to lateral shaft deflections. Furthermore strain measurements obtained from the strain gages were used to determine bending moments along the shaft length. The proposed method was used, together with the deflection data, to perform the analysis at each load increment so that the mudstone response, shear force, and bending moment along the depth are obtained. The results obtained are plotted in Figures 7.31 through 7.34. Predicted and measured bending
moments have shown perfect agreement using constant EI. The load vs. shaft deflection at top of rock is plotted in Figure 7.35. Comparing the behavior of the load-deflection for the loading and unloading stage, the slopes for both stages appear to be about the same which indicates that the drilled shaft response is still in the linear-elastic range. The p-y curves describing the upper 2.7 m of the mudstone is also developed as shown in Figure 7.36.

Figure 7.29: Ohio DOT lateral load test setup at Jefferson County (March 2006)
Figure 7.30: Instrumentation plans for lateral load test at JEF-152
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Figure 7.36: p-y curves of JEF-152-1.3 test shaft
7.10 Summary and Conclusions

An accurate and yet simple analytical method for analyzing laterally loaded drilled shafts using only shaft deflection data was developed based on the principle of superposition. The developed method together with known shaft deflection profile can be successfully used to determine lateral behavior of a drilled shaft with boundary conditions free at the shaft top and fixed at the shaft tip. The use of the principle of superposition limits the developed method to the drilled shaft material in a linear-elastic range. Theories behind the developed method, as well as detailed mathematical formulation were established in this study. The developed method can successfully provide soil engineers with soil responses, net applied loads, shear stresses, and bending stresses along the shaft length for any given deflection profile. For computational purpose, a step-by-step numerical procedure was developed to determine applied loads, soil reactions, shear forces, and bending moments along the shaft length, based on input of shaft deflection data. A computer program was developed to implement the proposed numerical procedure to expedite the numerical computation. Furthermore, the developed method can be directly used to estimate the shaft deflections in the case when the net loads applied to the drilled shafts are known by satisfying the derived flexibility relation.

Five examples were used to verify the accuracy of the developed method. Included were two actual full-scale drilled shafts, one relates to lateral load test data and the second one relates to deflections due to moving slope. The predicted results using the developed method have shown perfect agreement when compared with the measured. Also included in the validation examples were three artificially generated cases where LPILE was used to generate deflection data. For drilled shafts with large strains, the developed method
overestimates the shear and bending stresses because the actual flexural rigidity constant is less than the linear-elastic flexural rigidity constant due to the nonlinear behavior of the drilled shafts. However, the developed method can be successfully used for analyzing drilled shafts used in landslide repair applications since the strains in the drilled shafts would be normally small; while, careful attention is advised when used for lateral load tests especially at high loads. Further study is needed to develop an analytical model that can consider the nonlinear response of the drilled shafts and different boundary conditions at the shaft top.
8.1 Overview

This chapter contains overall summaries, conclusions, and recommendations for the two main end-products achieved in this dissertation: (1) Design methodology for landslide stabilization using drilled shafts; and (2) Analytical method for response of laterally loaded drilled shafts using shaft deflection data.

8.2 Design Methodology for Landslide Stabilization using Drilled Shafts

In this part, a step-by-step analytical design procedure to improve the stability of an unstable slope using a single row of rock-socketed drilled shafts as a mean of stabilization was proposed. The limiting equilibrium based slope stability analysis algorithm, together with semi-empirical design charts from FEM parametric study, were used to provide a comprehensive design approach for slope/shaft systems. Force equilibrium and Mohr-Coulomb strength criterion were applied and satisfied for each individual slice in the method of slices approach. Additionally, elasto-plastic nature of soil and elastic nature of the drilled shafts and the rock layer were incorporated in the finite element study. Many of the assumptions that were previously imposed by several
investigators to simplify and reduce the number of design variables controlling slope/shaft systems were found in this study to significantly affect the design of slope/shaft systems. Illustrative design examples were introduced and explained in details. In addition to the theoretical work, field instrumentation and monitoring work were carried out at three actual landslide sites where a single row of rock-socketed drilled shafts were used. Various types of instruments were extensively installed inside the stabilizing shafts and the surrounding soils to provide better understanding with respect to slope/shaft systems.

The following conclusions can be summarized from this research effort:

- A complete analytical, practical, relatively simple, and yet accurate design methodology has been developed based on the limiting equilibrium approach and FEM generated empirical design charts for solving the stabilization of unstable slopes using a single row of rock-socketed drilled shafts.
- The developed method includes solutions for both geotechnical and structural design issues.
- The developed method, utilizing the semi-empirical design charts and the derived closed-form solutions, is not only capable of determining the variables necessary to design drilled shafts (i.e., shafts diameter, spacing between shafts, and rock-socket length), but also providing the designers with a set of possible design alternatives to choose from.
The developed method, utilizing the limiting equilibrium method, is capable of handling complex soil geometries, general failure slip surfaces, and different locations of the drilled shafts.

The methodology is capable of estimating the forces imparted on the drilled shafts due to soil movements.

The soil arching is accounted for, thus yielding the best approximation of the loads in the field.

Many useful guidelines regarding the selection of design variables were proposed to help the designer choose the most effective and economical design configurations of the drilled shafts.

Among the factors that can be controlled by the designer; drilled shafts diameter, shaft location, and rock-socket length were the most important that need to be carefully selected.

At a specific shaft location in a slope/shaft system, the load transfer factor $\eta$ decreases as the factor of safety $FS$ increases.

Selection of the shaft location within the slope is an important factor influencing greatly the $FS$ and $\eta$ for the slope/shaft systems. At a specific $FS$, the optimum shaft location is found to be at the location where the $\eta$ value is the highest.

FE simulations have shown numerical evidences for the reduction in the stresses and displacements from the upslope side of the drilled shaft to the downslope side of the shaft, so that significant improvement in the factor of safety of a slope is achieved when a row of drilled shafts are installed and spaced two to four times the shaft diameter and socketed enough in rock.
Since the load transfer factor and the load acting on the stabilizing drilled shafts increase as the soil movement increase, geotechnical and structural stability of the slope/shaft systems need to be evaluated and examined at the ultimate state of soil failure.

- Drilled shafts can be a practical and effective mean of stabilizing landslides.
- Geotechnical factor of safety was successfully enhanced for many full-scale landslide repair sites using drilled shafts.
- Structural factor of safety was highly overestimated for the studied landslide sites.
- Using smaller number of large-diameter shafts can be more effective than larger number of small-diameter shafts.
- For all instrumented landslide sites, drilled shafts fixity was successfully achieved.
- The typical range for S/D is 2 – 4. There is not much difference in the design between S/D of 2 and 3. (S/D = 3 can be effectively used)
- Backfilling and grading of the upper portion of the slope behind the shafts has, to some extent, adverse effect on the overall stability of slope/shaft systems.

The author would like to consider the following as recommendations for additional studies to enhance this work:

- Study the effect of multiple rows of drilled shafts on the load transfer factors and the overall stability.
- Study the effect of the various boundary conditions (hinged and fixed) at the shafts head.
- Study the effect of ground water table on the forces and load transfer factors for slope/shaft systems.
- Study the effect of the reduced shaft length (stub piers) on the stabilization of slope/shaft systems compared to full length shafts.
- Study the effect of regeneration of the critical slip surface after shaft installation.
- Verify and test the developed method before applying to practical landslide repair cases.

8.3 Analytical Method for Response of Laterally Loaded Drilled Shafts using Shaft Deflection Data

An accurate and yet simple analytical method for analyzing laterally loaded drilled shafts using only shaft deflection data was developed based on the principle of superposition. For computational purpose, a step-by-step numerical procedure was developed to simplify the estimation of shaft lateral responses. A computer program was developed to implement the proposed numerical procedure to expedite the numerical computation. The feasibility of the developed method was verified through many theoretical and actual examples.

The following conclusions can be summarized from this research effort:

- The developed method together with known shaft deflection profile can be successfully used to investigate lateral behavior of a drilled shaft with boundary conditions free at the shaft top and fixed at the shaft tip, which includes the
The determination of soil responses, net applied loads, shear stresses, and bending stresses.

- The use of the principle of superposition limits the developed method to the drilled shaft material in a linear-elastic range.

- The developed method can be directly used to estimate the shaft deflections in the case when the net loads applied to the drilled shafts are known by satisfying the derived flexibility relation.

- The predicted results using the developed method have shown good agreement when compared to measured data for many cases.

- The developed method can be successfully used for analyzing drilled shafts used in landslide repair applications since the strains in the drilled shafts would be normally small, while, careful attention is advised when used for lateral load tests especially at high loads.

The authors would like to consider the following as recommendations for additional studies to enhance this work:

- Develop an analytical model that can consider the nonlinear response of the drilled shafts
- Develop an analytical model for different boundary conditions at the shaft top.
- The developed method should be checked for more practical cases to verify the accuracy and usability of the method.
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APPENDIX C

MRG-376-1.10 LANDSLIDE REPAIR

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APPENDIX D

P-Y COMPUTER PROGRAM

FORM 1

Private Sub Command1_Click()
    On Error Resume Next
    If File_Name = "" Then
        MsgBox "Select your input file first!!!"
        Exit Sub
    End If
    aaa = MsgBox("Start running the file ===> [" & File_Name & "]", vbOKCancel, "P-Y")
    If aaa = 2 Then Exit Sub
    MainFrm.WindowState = vbNormal
    Set ExlObj = CreateObject("excel.application")
    ExlObj.Workbooks.Open (File_Name)
    ExlObj.Visible = True
    ExlObj.WindowState = xlMinimized
    With ExlObj.Worksheets(1)
        nPoints = ExlObj.Worksheets(1).Cells(2, 4).Value
        EI = .Cells(3, 4).Value
        nP = .Cells(4, 4).Value
        nM = .Cells(5, 4).Value
        nV = .Cells(6, 4).Value
    End With
End Sub
ReDim Y(nPoints)
ReDim z(nPoints)
ReDim z0(nPoints)
ReDim Delta_H(nPoints)
ReDim Delta_1(nPoints, nPoints)

For I = 1 To nPoints
    z(I) = .Cells(9 + I, 2).Value
    Y(I) = .Cells(9 + I, 3).Value
    z0(I) = z(I)
Next I
End With

L = z(nPoints) - z(1)
For I = 1 To nPoints
    If z(I) <= Z0_H Then
        Delta_H(I) = (H * (L - Z0_H) ^ 2 * (2 * L / 3 + Z0_H / 3 - z(I))) / (2 * EI)
    Else
        Delta_H(I) = (H * (L - Z0_H) ^ 2 * (2 * L / 3 + Z0_H / 3 - z(I))) / (2 * EI) + (H * (z(I) - Z0_H) ^ 3) / (6 * EI)
    End If
Next I

For J = 1 To nPoints
    If z(I) <= z0(J) Then
        Delta_1(I, J) = (L - z0(J)) ^ 2 * (2 * L / 3 + z0(J) / 3 - z(I)) / (2 * EI)
    Else
        Delta_1(I, J) = (L - z0(J)) ^ 2 * (2 * L / 3 + z0(J) / 3 - z(I)) / (2 * EI) + (z(I) - z0(J)) ^ 3 / (6 * EI)
    End If
Next J
Next I

Number = nPoints - 1
Define
Kkk = 0
For I = 1 To nPoints - 1
    For J = 1 To nPoints - 1
        Kkk = Kkk + 1
    Next J
Next I
LeftX(Kkk) = Delta_1(I, J)
LeftXX(Kkk) = Delta_1(I, J)

Next J
Next I

For I = 1 To nPoints - 1
Right(I) = Y(I)
RightX(I) = Y(I)
Next I

Ordering
Eliminating
Substitution

ReDim Pp(nPoints - 1)
ReDim Pp(nPoints - 1)
ReDim P(nPoints - 1)
ReDim Zz(nPoints - 1)

For I = 1 To nPoints - 1
P(I) = UnKnowns(I)
Pp(I) = P(I)
Zz(I) = z(I)
Next I

Dim Pp1 As Double
Dim Pp2 As Double
Dim Kf As Double

If nP = 0 Then
    With ExlObj.Worksheets(2)
    For I = 1 To 400
        .Cells(I + 1, 1).Value = Null
        .Cells(I + 1, 2).Value = Null
    Next I
    For I = 1 To nPoints - 1
        Select Case I
        Select Case I
Case 1
\[ K_f = \frac{(Z_z(2) - Z_z(1))}{2} \]
Case nPoints - 1
\[ K_f = \frac{(Z_z(nPoints - 1) - Z_z(nPoints - 2))}{2} \]
Case Else
\[ K_f = \frac{(Z_z(I + 1) - Z_z(I - 1))}{2} \]
End Select
.Cells(I + 1, 1).Value = P(I) / K_f
.Cells(I + 1, 2).Value = Zz(I)
Next I
End With
GoTo NoSmoothing_P
End If

For J = 1 To nP
For I = 1 To nPoints - 2 * J - 1
\[ P_{pp}(I) = \frac{(P(I) + 2 * P(I + 1) + P(I + 2))}{4} \]
Next I
For I = 1 To nPoints - 2 * J - 1
Pp(I) = Ppp(I)
Next I
Next J

With ExlObj.Worksheets(2)
For I = 1 To 400
.Cells(I + 1, 1).Value = Null
.Cells(I + 1, 2).Value = Null
Next I
For I = 1 To nPoints - 2 * nP - 1
.Cells(I + 1, 1).Value = Ppp(I) / (Zz(2) - Zz(1))
.Cells(I + 1, 2).Value = Zz(I + nP)
Next I
End With
NoSmoothing_P:
ReDim M(nPoints - 1)
ReDim V(nPoints - 1)

For I = 1 To nPoints - 1
M(I) = 0
For $J = I - 1$ To 1 Step -1
$M(I) = M(I) + P(J) \times (z(I) - z(J))$
Next $J$
$M(I) = M(I) + H \times (z(I) - Z0_H)$
Next $I$

For $I = 1$ To $nPoints - 1$
$Pp(I) = M(I)$
Next $I$

If $nM = 0$ Then
  With ExlObj.Worksheets(3)
    For $I = 1$ To 400
      .Cells($I + 1$, 1).Value = Null
      .Cells($I + 1$, 2).Value = Null
    Next $I$
    For $I = 1$ To $nPoints - 1$
      .Cells($I + 1$, 1).Value = $Pp(I)$
      .Cells($I + 1$, 2).Value = $Zz(I)$
    Next $I$
  End With
  GoTo NoSmoothing_M
End If

For $J = 1$ To $nM$
For $I = 1$ To $nPoints - 2 \times J - 1$
$Ppp(I) = (Pp(I) + 2 \times Pp(I + 1) + Pp(I + 2)) / 4$
Next $I$
For $I = 1$ To $nPoints - 2 \times J - 1$
$Pp(I) = Ppp(I)$
Next $I$
Next $J$
With ExlObj.Worksheets(3)
For $I = 1$ To 400
  .Cells($I + 1$, 1).Value = Null
  .Cells($I + 1$, 2).Value = Null
Next $I$
For $I = 1$ To $nPoints - 2 \times nM - 1$
  .Cells($I + 1$, 1).Value = $Pp(I)$
  .Cells($I + 1$, 2).Value = $Zz(I + nM)$
Next $I$
End With
NoSmoothing_M:

For I = 1 To nPoints - 1
V(I) = 0
For J = I - 1 To 1 Step -1
V(I) = V(I) + P(J)
Next J
V(I) = V(I) + H
Next I

For I = 1 To nPoints - 1
Pp(I) = V(I)
Next I

If nV = 0 Then
   With ExlObj.Worksheets(4)
   For I = 1 To 400
      .Cells(I + 1, 1).Value = Null
      .Cells(I + 1, 2).Value = Null
   Next I
   For I = 1 To nPoints - 1
      .Cells(I + 1, 1).Value = Pp(I)
      .Cells(I + 1, 2).Value = Zz(I)
   Next I
   End With
   GoTo NoSmoothing_V
End If

For J = 1 To nV
   For I = 1 To nPoints - 2 * J - 1
      Ppp(I) = (Pp(I) + 2 * Pp(I + 1) + Pp(I + 2)) / 4
   Next I
   For I = 1 To nPoints - 2 * J - 1
      Pp(I) = Ppp(I)
   Next I
Next J

With ExlObj.Worksheets(4)
   For I = 1 To 400
      .Cells(I + 1, 1).Value = Null
      .Cells(I + 1, 2).Value = Null
   Next I
For I = 1 To nPoints - 2 * nV - 1
    .Cells(I + 1, 1).Value = Ppp(I)
    .Cells(I + 1, 2).Value = Zz(I + nV)
Next I

End With

NoSmoothing_V:
ExlObj.Workbooks(1).Save
"ExlObj.Workbooks(1).Close
ReDim Y(0)
ReDim z(0)
ReDim z0(0)
ReDim Delta_H(0)
ReDim Delta_1(0, 0)
ReDim Ppp(0)
ReDim Pp(0)
ReDim P(0)
ReDim Zz(0)
ReDim M(0)
ReDim V(0)
Beep
Beep
Beep
MainFrm.WindowState = vbMinimized
ExlObj.WindowState = xlMaximized
Set ExlObj = Nothing

Exit Sub

ErrSub:
MsgBox "Error occurred."

End Sub

Public Sub Define()
ReDim Right(Number)
ReDim RightX(Number)
ReDim UnKnowns(Number)
ReDim Order(Number)
ReDim ScalX(Number)
ReDim LeftX(Number * Number)
ReDim LeftXX(Number * Number)
ReDim Cal(Number)
Private Sub Form_Load()
Label1.Caption = "No File Open."
End Sub

Private Sub mExit_Click()
End
End Sub

Private Sub mOpen_Click()
On Error Resume Next
FileX.DialogTitle = "Open File"
FileX.InitDir = App.Path
FileX.Filter = "*.xls"
FileX.ShowOpen
File_Name = FileX.FileName
If File_Name = "" Then Label1.Caption = "No File Open."
Label1.Caption = File_Name
End Sub

MODULE 1
Public Delta_H() As Double
Public Delta_1() As Double

Public nPoints As Long
Public Y() As Double
Public z() As Double
Public Zz() As Double
Public z0() As Double
Public EI As Double

Public H As Double
Public Z0_H As Double
Public L As Double
Public nP As Integer
Public nM As Integer
Public nV As Integer

Public Ppp() As Double
Public P() As Double
Public Pp() As Double

Public M() As Double
Public V() As Double

Public ExlObj As Excel.Application

Public Number As Long
Public Right() As Double
Public RightX() As Double
Public UnKnowns() As Double
Public Order() As Long
Public ScalX() As Double
Public LeftX() As Double
Public LeftXX() As Double
Public Cal() As Double
Public LL As Double
Public Cont As Boolean
Public File_Name As String

Public I As Long
Public J As Long
Public K As Long
Public Kkk As Long
Public Sub Ordering()
For I = 1 To Number
    Order(I) = I
    ScalX(I) = Abs(LeftX(Number * (I - 1) + 1))
For J = 2 To Number
If (Abs(LeftX(Number * (I - 1) + J)) > ScalX(I)) Then
    ScalX(I) = (Abs(LeftX(Number * (I - 1) + J)))
End If
Next J
Next I
End Sub

Public Sub Eliminating()
Dim Factor As Double
For K = 1 To Number - 1
    Pivoting
    For I = K + 1 To Number - 1
        Factor = LeftX(Number * (Order(I) - 1) + K) / LeftX(Number * (Order(K) - 1) + K)
        For J = K + 1 To Number
            LeftX(Number * (Order(I) - 1) + J) = LeftX(Number * (Order(I) - 1) + J) - Factor * LeftX(Number * (Order(K) - 1) + J)
        Next J
    Next I
Next K
Right(Order(I)) = Right(Order(I)) - Factor * Right(Order(K))
Next I
Next K
End Sub

Public Sub Pivoting()
Dim Pivit As Long
Dim II As Long
Dim iDUM As Long
Dim Big
Dim Dummy
Pivit = K
Big = Abs(LeftX(Number * (Order(K) - 1) + K) / ScalX(Order(K)))
For II = K + 1 To Number
Dummy = Abs(LeftX(Number * (Order(II) - 1) + K) / ScalX(Order(II)))
If (Dummy > Big) Then
Big = Dummy
Pivit = II
End If
Next II
iDUM = Order(Pivit)
Order(Pivit) = Order(K)
Order(K) = iDUM
End Sub

Public Sub Substitution()
Dim Sum As Double
UnKnowns(Number) = Right(Order(Number)) / LeftX(Number * (Order(Number) - 1) + Number)
For I = Number - 1 To 1 Step -1
Sum = 0
For J = I + 1 To Number
Sum = Sum + LeftX(Number * (Order(I) - 1) + J) * UnKnowns(J)
Next J
UnKnowns(I) = (Right(Order(I)) - Sum) / LeftX(Number * (Order(I) - 1) + I)
Next I
End Sub