CONTINUUM TRAFFIC FLOW AT A HIGHWAY INTERCHANGE

A Thesis

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Master of Science

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ABSTRACT

A second order, viscous continuum traffic flow model is studied and implemented numerically using a fourth order Runge Kutta adaptive time step algorithm. This model is applied to a two section one lane highway with an entrance ramp and an exit ramp. This method is also applied to a four section highway network comprised of two one lane roads. A ‘look ahead’ feature is built into the numerical solution of the model so that future traffic flow conditions can be predicted given information from the past traffic conditions.
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CHAPTER I
INTRODUCTION

Traffic flow is a physical phenomenon that is extremely complicated to model mathematically. The most basic traffic scenario is a one dimensional pipeline road with one-way traffic. This is a typical scenario for engineers to model due to its simplicity. This one dimensional pipeline model is used to model highway traffic flow. A slightly more complicated scenario is a highway interchange that is comprised of two one-way pipeline models. Today, highway traffic flow is currently analyzed by traffic engineers using sensors and discrete mathematical models. This study focuses on traffic flow through a highway interchange with the underlying goal of developing and implementing a model that will reduce the number of traffic sensors used in practice. It also provides the first numerical implementation of Zhang’s model [1] and introduces a new approach - through boundary conditions - to modeling a highway interchange.

There are two main approaches taken to model traffic flow; microscopic approaches and macroscopic approaches. Microscopic models map traffic flow as a set of individual vehicles, while macroscopic models map traffic flow as fluid flow where each vehicle is analogous to a molecule of fluid. Macroscopic models place more emphasis on traffic flow as a continuum versus a collection of individual vehicles. The movements of vehicles are studied collectively through combinations of basic waves
such as decelerating (shock) waves and accelerating (rarefaction) waves. Continuum traffic flow modeling generally uses a macroscopic perspective, although microscopic principles can be incorporated into continuum models. For our purposes, there are three types of continuum models: simple models, higher order models without viscosity, and higher order models with viscosity. There are advantages and disadvantages to each type of model.

The Lighthill, Whitham, and Richards (LWR) model \[2\],

\[
\rho_t + (\rho v)_x = 0,
\]

where \( \rho \) is the density of traffic measured in vehicles per meter and \( v \) is velocity measured in meters per second, is the primary simple continuum traffic model. The LWR model is based on the premise that all vehicles travel at equilibrium velocity. Equilibrium velocity is defined as the ‘ideal’ velocity vehicles will attain given the current traffic density. This means that \( v \) in (1.1) is at equilibrium, which is modeled mathematically by an equilibrium velocity model \( v = V_\ast(\rho) \). Using the LWR model, simple traffic flow problems can be solved analytically by the method of characteristics and numerically by finite differences. Thus, the LWR model is mainly used for its simplicity since the density is the only dependent variable. However, the model can not explain the amplifications of small disturbances on heavy traffic. This is due to the fact that stop and go characteristics of traffic would require the relative vehicle velocities to allow some fluctuation. Realistically, not all vehicles travel at equilibrium velocity. LWR theory is also based on the premise that a traffic disturbance is
propagated by waves. When disturbances or linear perturbations occur, LWR theory suggests that the perturbations neither grow nor die out as time goes on; they only shift their locations at some speed. Realistically, platoons of traffic tend to disperse over time. This phenomenon is not captured by the LWR theory. The LWR model is capable of simulating shock and rarefaction waves in traffic, but fails to explain instabilities in traffic flow, such as vehicle clustering from initially homogeneous traffic conditions. These problems all stem from the fact that all vehicles in this model travel at equilibrium velocity. Therefore, non-equilibrium models are needed.

The first non-equilibrium model was the Payne-Whitham (PW) model [3]:

\[ \rho_t + (\rho v)_x = 0, \]  
\[ v_t + vv_x + \frac{c^2(\rho)}{\rho} \rho_x = \frac{V_*(\rho) - v}{\tau}, \]

where \( V_* \) is the equilibrium velocity, \( \tau \) is the relaxation time, and \( c \) is the traffic sound speed. The relaxation time is the time it takes drivers to adjust their speed due to frontal stimuli. The PW model is an improvement over the LWR model because it can describe the amplification of small disturbances in heavy traffic and allow fluctuations of speed around the equilibrium values. The forcing term of the model, \( \frac{V_*(\rho) - v}{\tau} \), is an acceleration term that forces vehicle velocity towards equilibrium. This results because when the traffic flow velocity is greater than the equilibrium velocity the forcing term forces the traffic flow to decelerate. Likewise, when the traffic flow velocity is less than the equilibrium velocity the forcing term forces the flow to accelerate. Thus the PW model is capable of modeling the formation of vehicle
clusters. However, in the PW model there exists a characteristic speed that is greater than the macroscopic flow velocity. This means that the future traffic conditions of a traffic flow will be affected by the traffic conditions behind the flow. This violates a fundamental principle of traffic flow, in that vehicles only travel in one direction and respond only to frontal stimuli. This problem is commonly termed the ‘wrong-way’ travel problem. The PW model improves upon the deficiencies of the LWR model, however, the ‘wrong way’ travel problem is a major flaw in this model. Therefore, another model must be explored.

A new higher-order continuum model has been developed by Zhang [1]:

\[ \begin{align*}
\rho_t + (\rho v)_x &= 0, \\
v_t + (v + 2\beta c(\rho)) v_x + \frac{c^2(\rho)}{\rho} \rho_x &= \frac{V_*(\rho) - v}{\tau} + \mu(\rho) v_{xx},
\end{align*} \tag{1.4, 1.5}\]

where

\[ \begin{align*}
\mu(\rho) &= 2\beta \tau c^2(\rho), \\
c(\rho) &= \rho V'_*(\rho).
\end{align*} \tag{1.6, 1.7}\]

Here \( \mu \) is the viscosity coefficient and \( \beta \) is a dimensionless parameter. Different models are used for the equilibrium velocity.
The equilibrium velocity model used in this study, proposed by Kerner and Konhaeuser [4], is illustrated in Figure 1.1 and is given by

\[
V_* (\rho) = v_f \left[ 1 + \exp \left( \frac{\rho}{\rho_{\text{max}}} - 0.25 \right) \right]^{-1} - 3.72E - 6, \tag{1.8}
\]

Here, \(v_f\) is the maximum speed, \(\rho_{\text{max}}\) is the maximum density (\(\rho_{\text{max}} = 0.2\) veh/m assuming bumper to bumper traffic and an average vehicle length of 5 m).

Zhang’s continuum model was developed by taking into account drivers’ car-following behavior. The goal of this new model is to improve both the LWR model and the higher order models with a model that can handle vehicle clusters, and eliminate the wrong-way travel flaw. The derivation of this model was based on the assumption that drivers adjust their vehicle speeds according to traffic conditions.
ahead of them. The new model includes the LWR model and PW model as special cases and removes many of the deficiencies of these models without introducing the undesirable property of wrong-way travel. It was found that driver memory in car-following leads to viscous effects in continuum traffic flow dynamics. ‘Driver memory’ means that a driver’s speed partially depends on the driver’s speed at a previous time. The idea is that a driver’s speed is affected by both the speed of the vehicles ahead and their own past travel speed (inertia). As a result, a second order continuum model was developed with viscosity. The model contains traffic viscosity, which is linked to driver memory, and is shown to be bi-directional. The bi-directional behavior dies out over time thus the model becomes fully uni-directional. It was also shown that the general viscous model has a stable wave hierarchy of first and higher-order traffic sound waves. Due to the improvements over the LWR model and the PW model, this model seems to be a reasonable choice for modeling traffic flow at a highway interchange.

1.1 Literature Review

Although Zhang’s model was chosen for this study, other models have also been developed for the study of traffic flow.

The frozen wave model [5] was introduced to attempt to improve upon the previous models. From numerical simulations, it was found that the model behaves in a manner similar to LWR and PW models when it is stable. However, when traffic is unstable, numerical solution methods fail to produce physical solutions. The frozen-
wave model has serious limitations; for example, it can not simulate vehicle clusters. Thus this model is not an improvement over the LWR and PW models.

A new continuum model was created by Jiang, Wu, and Zhu [6]. It was developed by introducing an improved car-following theory and applying the connection between macro-micro variables. This model considers the effects of both the distance and the relative speed of two successive vehicles. In the new model, the velocity gradient replaces the density gradient as the anticipation term. This enables the new model to overcome the characteristic speed problem, so the ‘wrong way’ travel issue does not exist. Numerical tests verify that the model is able to simulate complex traffic phenomena such as shock waves, rarefaction waves, stop-and-go waves, and local cluster effects.

A new high-order continuum model was developed by Liu, Lyrintzis, and Michalopoulos [7]. The development of this model was based on hyperbolic conservation laws with relaxation, linear stability analysis, and more realistic considerations of traffic flow than previous models. This new model exhibits smooth solutions rather than discontinuities, and is able to explain the amplifications of small disturbances on heavy traffic. The model also allows fluctuations of speed around the equilibrium and does not result in a ‘wrong-way’ travel problem. The model is extended to interrupted geometries (i.e., merging, diverging, and weaving areas). A Riemann problem-based numerical method based on the Harten-Lax-van Leer (HLL) flux difference splitting method is employed for the solution of the new high-order model. The HLL method creates a clear picture of interactions involved and automatically handles shock waves.
and interfaces with different waves. The new model and proposed numerical method were tested against the LWR model and the Lax numerical method using field data. The results suggest that the new model is an improvement over the LWR model, and that the HLL method is better for implementation of the new model over the Lax method.

Two new high-order models were developed by Liu, Lyrintzis, and Michalopoulos [8]. Both models are suitable for congested flows and complex geometrics. The first formulation introduces a viscosity term to address traffic friction due to lane changing at freeway entrance and exit ramps. The first formulation is a semi-viscous model. The equilibrium speed is replaced by free flow speed. The upwind scheme is used to implement this model because it worked better than other finite difference methods. The second formulation was developed by treating traffic flow as a viscous compressible mass. This formulation does not include an equilibrium relationship. The viscous model was implemented by the simple Euler method. The two high order continuum models are more practical to implement in field applications since they do not require an equilibrium speed-density relationship. Field testing shows that in congested situations the new models produce lower error while maintaining relatively high computational efficiency. The new models both seem to be significantly better than the simple LWR continuum model. Also, the numerical method (upwind vs. Lax) seems to have an impact on accuracy. When entrance and exit ramps are present, the error in general was small. As traffic becomes highly congested, the error increases.
Many other traffic models have also been developed and studied [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23].

1.2 Goals and Contributions

The main goals of this research are as follows:

1: Implement Zhang’s model numerically (Chapter II)

2: Model an interchange between two sections of road via boundary conditions (Chapter III)

3: Model an interchange between four sections of road via boundary conditions (Chapter IV)

4: Implement the Two Section and Four Section models numerically (Chapters III and IV)

The motivation for achieving these goals is that it would be extremely advantageous to accurately model highway traffic flow while reducing the number of sensors used in practice by civil engineers. Successful implementation of Zhang’s model numerically, although never before accomplished, may possibly improve upon traffic flow models used in the past. Using boundary conditions to model the interchange between two sections of road is necessary to insure that this is a feasible technique that can be applied to a four section interchange. Using boundary conditions to model the interchange between four sections of road in combination with successful numerical
simulation of the four section model will reduce the number of sensors needed for traffic flow prediction at an interchange. Thus, all of the goals must be accomplished in order to reduce the number of sensors used in practice.

The standard approach of modeling a highway interchange comprised of two one-way roads is to treat the interchange as four individual pipeline models and physically measure the traffic density at each boundary of each section using sensors. This would require four sensors at the interchange and four sensors at the boundaries. Alternatively, the approach used in this study incorporates expressions of vehicle flow rate and momentum flux at the interchange which give boundary conditions at the interchange. Therefore, the density at the interchange can be predicted by the model rather than measured; thus the need for sensors at the interchange is eliminated.
CHAPTER II

THE ONE SECTION MODEL

In this chapter, the goal is to model one section of highway using Zhang’s model to highlight the issues of numerical implementation. The formal problem statement is presented, the equations are discretized, and numerical results are presented and discussed.

2.1 Problem Statement

The section of highway used in this study is a one lane roadway with no entrance or exit ramps with length $0 \leq x \leq L$. The model is comprised of two equations in density, $\rho(x,t)$, and velocity $v(x,t)$. The governing equations are

$$\rho_t + (\rho v)_x = 0, \quad (2.1)$$

$$v_t + (v + 2\beta c(\rho))v_x + \frac{c^2(\rho)}{\rho} \rho_x = \frac{V_*(\rho) - v}{\tau} + \mu(\rho) v_{xx}, \quad (2.2)$$

where

$$\mu(\rho) = 2\beta \tau c^2(\rho), \quad (2.3)$$

$$c(\rho) = \rho V_*(\rho). \quad (2.4)$$
The initial conditions are

\[ \rho(x, 0) = \rho_I(x), \quad (2.5) \]
\[ v(x, 0) = V_s(\rho_I(x)), \quad (2.6) \]

and the boundary conditions are

\[ \rho(0, t) = \bar{\rho}(t), \quad (2.7) \]
\[ v(0, t) = V_s(\bar{\rho}(t)), \quad (2.8) \]
\[ \rho_{xx}(L, t) = 0, \quad (2.9) \]
\[ v_{xx}(L, t) = 0. \quad (2.10) \]

Nomenclature is listed in Table 2.2. Equation (2.1) is the conservation of mass equation. Equation (2.2) is conservation of momentum. Density is measured in units of vehicle/meter, with a maximum of \( \rho_{\text{max}} = 0.2 \text{ veh/m} \). (This is based on bumper to bumper traffic with an average vehicle length of 5 m). Velocity is measured in units of meters/second, with a max of 30 m/s (equivalent to 65 mph).

The initial condition function \( \rho_I(x) \) may be set arbitrarily, but should be continuous or at least piecewise differentiable to avoid introducing numerical instabilities. Boundary conditions (2.7, 2.8) correspond to a specified input \( \bar{\rho} \) onto the roadway; it is expressly assumed that the input density arrives in equilibrium. At the outgoing boundary, \( x = L \), \( \rho_{xx} \) and \( v_{xx} \) are set equal to zero to allow the traveling wave solution to pass through the boundary without creating any artificial reflections.
2.2 Discretization

The model equations are discretized as

\[
\begin{align*}
\rho_i^{k+1} - \rho_i^k &= -\rho_i^k v_i^k - \rho_{i-1}^k v_{i-1}^k, \\
v_i^{k+1} - v_i^k &= -(v_i^k + 2\beta c(\rho_i^k))(v_x)_i^k - \frac{c^2(\rho_i^k)}{\rho_i^k} \left[ \frac{\rho_i^k - \rho_{i-1}^k}{dx} \right] \\
&+ \frac{V_s(\rho_i^k) - v_i^k}{\tau} + \mu(\rho_i^k) \left[ \frac{v_{i+1}^k - 2v_i^k + v_{i-1}^k}{dx^2} \right],
\end{align*}
\]

where the velocity \((v_x)_i^k\) in (2.12) is determined by upwinding. The convective wave speed is given by

\[a = (v_i^k + 2\beta c(\rho_i^k)),\]

so that

\[
(v_x)_i^k = \begin{cases} 
\left[ \frac{v_i^k - v_{i-1}^k}{dx} \right] & \text{if } a \geq 0, \\
\left[ \frac{v_{i+1}^k - v_i^k}{dx} \right] & \text{if } a < 0.
\end{cases}
\]

At the right boundary, a second order backwards difference is used to discretize (2.9) and (2.10), yielding the discrete conditions:

\[
\begin{align*}
\rho_n^k &= \frac{5\rho_{n-1}^k - 4\rho_{n-2}^k + \rho_{n-3}^k}{2}, \\
v_n^k &= \frac{5v_{n-1}^k - 4v_{n-2}^k + v_{n-3}^k}{2}.
\end{align*}
\]
The overall algorithm is simple:

1. Obtain $\rho_{k+1}^1$ and $v_{k+1}^1$ from the boundary conditions (2.7) and (2.8)

2. Obtain $\rho_{i+1}^k$ and $v_{i+1}^k$ for $i = 2, ..., n$ from (2.11, 2.12)

3. Obtain $\rho_{n+1}^k$ and $v_{n+1}^k$ from (2.15, 2.16)

2.3 Numerical Results

To test the model, three cases are considered. In each case the section of roadway is $L = 500$ m long, and is discretized into $n = 2001$ sections so that $dx = 0.25$.

The initial density profile is given by

$$\rho_I(x) = 0.03 + 0.01 \left( e^{-0.001(x-150)^2} \right). \quad (2.17)$$

This simulates a concentrated mass of high density traffic. The input densities are given by

Case 1: $\tilde{\rho}(t) = 0.03 - 0.02 \sin(t)$, (periodic input)

Case 2: $\tilde{\rho}(t) = 0.03 - 0.02(e^{-(t-4)^2})$, (concentrated area of low density)

Case 3: $\tilde{\rho}(t) = 0.03 + 0.02(e^{-(t-4)^2})$. (concentrated area of high density)

The goal of these simulations is to identify the basic features encountered when a section of high density traffic or a section of low density traffic enters the roadway. The values of the input parameters are the same for each case; they are listed in Table 2.1.
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<td>( \beta )</td>
<td>1</td>
<td>dimensionless</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.1</td>
<td>s</td>
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**Case 1.** The early time behavior for Case 1 is shown in Figure 2.1 (density) and Figure 2.2 (velocity). The initial cluster of traffic centered at \( x = 150 \) m begins to spread. The traffic ahead of the hump sees clearer road ahead and hence travels more quickly and spreads out. The traffic behind the hump (to the left), sees higher density ahead, slowing down and bunching up. The basic feature is that the hump tightens on the left (upstream) and spreads on the right (downstream). The input density is low density traffic and simply propagates for early time. As time passes, the low density traffic begins to bunch up because the low density traffic is travelling faster than the initial cluster of traffic and eventually begins to catch up to the cluster. The late time behavior is shown in Figure 2.3 (density) and Figure 2.4 (velocity). After 20 seconds have passed the initial cluster spreads and disperses considerably. As the initial cluster spreads, the bunched input density is allowed to begin to spread and speed up as seen at time \( t = 20 \) sec.

**Case 2.** The early time behavior for Case 2 is shown in Figure 2.5 (density) and Figure 2.6 (velocity). The initial cluster of traffic centered at \( x = 150 \) m spreads in the same fashion as in Case 1. As in Case 1, the input density is also a low density traffic. As a result, the low density input behaves in the same fashion as in Case 1.
The late time behavior is shown in Figure 2.7 (density) and Figure 2.8 (velocity). The late time behavior has the same properties observed in Case 1. Case 2 demonstrates that a low density exponential input has virtually the same effects as a low density sinusoidal input.

**Case 3.** The early time behavior for Case 3 is shown in Figure 2.9 (density) and Figure 2.10 (velocity). The initial cluster of traffic centered at \( x = 150 \) m spreads in the same fashion as in Case 1 and Case 2. The effect of the input density is hard to see on these figures, so Figures 2.11 and 2.12 zoom in on these features. The main effect is shown in Figure 2.9 in the density at time \( t = 4 \) sec. The incoming density is increasing, and this leads to a clustering effect near the leading edge of the input. For relatively low densities, this cluster eventually disperses, as seen in the long time plots in Figures 2.13 and 2.14 - the cluster is evident at time \( t = 5 \) sec but has dissipated by time \( t = 10 \) sec. Both humps (the initial hump at \( x = 150 \) m and the input hump at \( x = 0 \) m) disperse and spread in a similar fashion.

**Numerical Convergence.** The RK4 algorithm used in this study is an adaptive time step algorithm. At each spatial grid location the accepted solution value must be within a certain tolerance in order for each time step value to be acceptable. The spatial grid sized used in this study, \( dx = 0.25 \), is used to obtain accurate solutions in a timely fashion. In order to test the numerical convergence of the algorithm, the simulation was performed with smaller spatial grid (\( dx \)) sizes and the results were compared. Decreasing the spatial grid size had minimal effects on the numerical solution thus the numerical solution converges.
Figure 2.1: Case 1 early time

Figure 2.2: Case 1 early time
Figure 2.3: Case 1 late time

Figure 2.4: Case 1 late time
Figure 2.5: Case 2 early time

Figure 2.6: Case 2 early time
Figure 2.7: Case 2 late time

Figure 2.8: Case 2 late time
Figure 2.9: Case 3 early time

Figure 2.10: Case 3 early time
Figure 2.11: Case 3 early time zoom

Figure 2.12: Case 3 early time zoom
Figure 2.13: Case 3 late time

Figure 2.14: Case 3 late time
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<td>Velocity</td>
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<td>$\rho(x,t)$</td>
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<td>$c(\rho)$</td>
<td>Traffic Sound Speed</td>
<td>m/s</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Parameter</td>
<td>dimensionless</td>
</tr>
</tbody>
</table>
CHAPTER III

THE TWO SECTION MODEL

In this chapter, two one-dimensional sections of road are linked together. The fundamental idea of modeling exit and entrance ramps through boundary conditions is introduced. A driver on Section 1 now has the option to exit or to continue driving onto Section 2. The governing equations are exactly the same as the one section model for both roads. The only new feature is that traffic may enter or exit at the junction. Traffic enters and travels along Section 1. As the traffic approaches the junction, some percentage of the traffic will exit while the rest of the traffic will continue onto Section 2. Some traffic from another source will enter section 2 at the junction along with the traffic from Section 1. Figure 3.1 illustrates this scenario.

Figure 3.1: Two Section Model

3.1 Model Equations

The model equations for both sections of road are almost identical to the one section model equations. The model is comprised of two sets of equations in density, $\rho(x,t)$,
and velocity \( v(x, t) \). Velocity is assumed to be continuous at the interchange, but the density is not necessarily continuous due to the exit and entrance ramps. Thus, density must be treated as two separate variables in the model. The two density variables are labeled \( \rho_1(x, t) \) and \( \rho_2(x, t) \) respectively.

The governing equations are

\[
\begin{align*}
(\rho_1)_t + (\rho_1 v)_x &= 0, \\
(\rho_2)_t + (\rho_2 v)_x &= 0,
\end{align*}
\]

\[
\begin{align*}
v_t + (v + 2\beta c(\rho_1))v_x + \frac{c^2(\rho_1)}{\rho_1}(\rho_1)_x &= \frac{V_\ast(\rho_1) - v}{\tau} + \mu(\rho_1)v_{xx}, \\
v_t + (v + 2\beta c(\rho_2))v_x + \frac{c^2(\rho_2)}{\rho_2}(\rho_2)_x &= \frac{V_\ast(\rho_2) - v}{\tau} + \mu(\rho_2)v_{xx}.
\end{align*}
\]

Section 1 is defined for \( 0 \leq x \leq L \) and Section 2 is defined on \( L \leq x \leq 2L \). Hence, (3.1) and (3.3) are valid on \( 0 \leq x \leq L \) and (3.2) and (3.4) are valid on \( L \leq x \leq 2L \).

As for the boundary conditions, the left boundary of section 1 and the right boundary of section 2 are treated exactly as the boundaries in the one section model. However, there is a linkage between section 1 and section 2. The linkage is comprised of the right boundary of section 1, the left boundary of section 2, the entrance ramp, and the exit ramp. The entrance and exit ramps at the juncture are modeled not as source and sink terms, but rather through a pair of boundary conditions that link the two sections together. The ramps are therefore assumed to exist at the single point \( x = L \), and the flow of traffic on the ramps is not explicitly modeled. The first boundary condition is an expression of the conservation of vehicle flow rate at the
\[ \rho_2(L, t)v_2(L, t) = \rho_1(L, t)v_1(L, t)A_{12} + \tilde{\rho}_2 V_\ast(\rho_2) \]  

(3.5)

where \( \tilde{\rho}_2 \) is the traffic density entering at the juncture and \( A_{ij} \) is the probability that a vehicle travels from section \( i \) to section \( j \). The probability definitions are also listed in Table 3.1. The interpretation is that the vehicle flow rate just before the juncture equals the vehicle flow rate just past the juncture (which includes the traffic passing through the juncture plus the traffic entering from the ramp). The second boundary condition is an expression of momentum flux at the juncture:

\[ \nu_i(v_1(L, t))_x = \nu_2(v_2(L, t))_x A_{12} + \nu_{exit} v_{exit} \]  

(3.6)

where \( \nu_i \) is the viscosity coefficient on section \( i \), \( \nu_{exit} \) is the viscosity coefficient on the exit ramp, and \( v_{exit} \) is the velocity of the traffic exiting the juncture. The interpretation is that the momentum flux just before the juncture equals the momentum flux just past the juncture. Another way to interpret the momentum flux expression at the juncture is to consider the inertia of the density that flows into the juncture. Equation (3.6) expresses that the inertia of the density that flows into the juncture is equal to the sum of the inertia of the density that flows onto section 2 and the inertia of the density that exits. In fluids, this phenomena is interpreted as a normal force balance. In terms of traffic flow, this phenomena can also be interpreted as a memory effect. Equation (3.6) requires that vehicles which flow onto section 2 along with vehicles that exit ‘remember’ their velocities just before they flow into the juncture:
ture. This means that drivers have knowledge of their past travel speed and that this knowledge has some effect on their future travel speed.

### 3.2 Discretization

Discretization and numerical computation of the interior grid points for both individual sections of road ($i = 1, \ldots, n-1$ and $i = n+1, \ldots, 2n$) are exactly the same as in the one section model. Figure 3.2 illustrates the discretized grid; each point on the grid represents the density and velocity at location $x_i$, where $i = 1, \ldots, 2n$, at time step $k$. However, calculating the density and velocity at the linkage point ($i = n$) requires discretization of (3.1), (3.5), and (3.6) and orderly calculation of the density and velocity on each section at the juncture.

As observed in Figure 3.2, the linkage point consisting of $\rho_{1(n)}^{k+1}$, $\rho_{2(n)}^{k+1}$, $v_{1(n)}^{k+1}$, and $v_{2(n)}^{k+1}$, is unknown. The linkage point is determined by first using the RK4 algorithm to compute the other points on each section of road ($i = 1, \ldots, n-1$ and $i = n+1, \ldots, 2n$) at the next time step. This is possible since all points are known at the previous time step as illustrated in Figure 3.2. Next, the governing equation for density, (3.1), is
discretized at the linkage point using a central difference.

$$\rho_{n+1} = \rho_n - \left[ \frac{dt}{2dx} \right] \left( \rho_{n+1} v_{n+1}^{k+1} - \rho_{n-1} v_{n-1}^{k+1} \right).$$ (3.7)

This gives the density on section 1 ($\rho_{1(n)}^{k+1}$) at the linkage point. Next, the conservation of vehicle flow rate equation (3.5) and momentum flux equation (3.6) are discretized. Since the points before the juncture are known on section 1 and the points after the juncture are known on section 2, a backwards difference is used to compute ($v_1(L, t))_x$ in (3.6) while a forwards difference is used to compute ($v_2(L, t))_x$ in (3.6). Discretizing (3.5) and (3.6) give

$$\rho_{2(n)}^{k+1} v_{2(n)}^{k+1} = \rho_{1(n)}^{k+1} v_{1(n)}^{k+1} A_{12} + \rho_{2(n)} V_s(\rho_{2(n)}^{k+1}).$$ (3.8)

$$\nu_1 \left[ \frac{v_{1(n)}^{k+1} - v_{1(n-1)}^{k+1}}{dx} \right] = \nu_2 \left[ \frac{v_{2(n+1)}^{k+1} - v_{2(n)}^{k+1}}{dx} \right] A_{12} + \nu_{exit} v_{exit(n)}.$$ (3.9)

Now there are two equations and three unknowns, $\rho_{2(n)}^{k+1}$, $v_{1(n)}^{k+1}$, and $v_{2(n)}^{k+1}$. Since it is assumed that the vehicle flow rate and the momentum flux across the juncture remain the same, it is logical to assume that the velocity on both sections is also the same across the juncture. Thus, $v_{1(n)}^{k+1} = v_{2(n)}^{k+1} = v_n^{k+1}$. Now the system of equations comprised of (3.8) and (3.9) has two unknowns and is solvable for $\rho_{2(n)}^{k+1}$ and $v_n^{k+1}$.

The idea behind this method of computing the linkage point is that it will give the model a 'look ahead' feature. This means that in the numerical simulation future traffic density and velocity should be affected by past traffic density and velocity.
3.3 Numerical Results

To test the two section model, three cases are considered. In each case both sections of roadway are \( L = 500 \) m long, and are discretized in the same fashion as the one section model. Since two sections are being linked together at a junction, the total length of the road containing both sections is 1000 m. The initial density profile \( \rho_I(x) \) is given by

\[
\rho_I(x) = 0.03 + \lambda \left( e^{-0.001(x-150)^2} \right),
\]

where \( \lambda = 0.01 \) in case 1 and \( \lambda = 0.02 \) in case 2 and case 3. The input densities are given by

Case 1: \( \bar{\rho}(t) = 0.03 \),

Case 2: \( \bar{\rho}(t) = 0.03 + 0.02(e^{-(t-4)^2}) \),

Case 3: \( \bar{\rho}(t) = 0.03 + 0.02(e^{-(t-4)^2}) \).

The goal of these simulations is to identify the basic features encountered when two sections of roadway are linked together given an input density and density entering and exiting at the junction. The probability input varies for each case while all other input parameter values are fixed; they are listed in Table 3.2.

**Case 1.** Case 1 is simply a test case to validate that the two section code with entrance and exit ramps turned off \((A_{12} = 0)\) is essentially the same as the one section code with \( L = 1000 \) m. This result is illustrated in Figure 3.3 (density) and Figure 3.4 (velocity).
Case 2. In case 2, a small percentage (20%) of existing traffic exits at the junction. The entrance ramp in this case is turned off. The early time behavior for Case 2 is shown in Figure 3.5 (density) and Figure 3.6 (velocity). The initial cluster of traffic centered at \( x = 150 \) m spreads in the same fashion as in the one section model. However, the focus of this simulation is the effect that the exiting traffic has on the density and corresponding velocity of the traffic flow. For early time, the density at \( x = 500 \) m slightly decreases due to the exiting traffic. This allows a slight increase in the velocity. As time passes, the initial cluster of traffic is allowed to spread and disperse more quickly than in the one section case because the low density traffic ahead created by the exit ramp allows for faster traffic flow. The middle time behavior is shown in Figure 3.7 (density) and Figure 3.8 (velocity). The low density ‘valley’ is sustained for longer distances as time passes as a result of the continual exiting of traffic. This can be observed at \( t = 25 \) sec in Figure 3.7. The late time behavior is shown in Figure 3.9 (density) and Figure 3.10 (velocity). At time \( t = 50 \) sec the initial cluster of traffic and input density have completely passed through the junction.

Case 3. In case 3, a small constant density of traffic \( (\tilde{\rho}_{2(n)} = 4.5E - 3 \) veh/m) will enter at the junction. The exit ramp in this case is turned off. The early time behavior for Case 3 is shown in Figure 3.11 (density) and Figure 3.12 (velocity). The initial cluster of traffic centered at \( x = 150 \) m spreads in the same fashion as in the one section model. The focus of this simulation is the effect that the entering traffic has on the density and corresponding velocity of the traffic flow.
For early time, the density at $x = 500$ m slightly increases due to the entering traffic. This promotes a slight decrease in the velocity. As time passes, the initial cluster of traffic spreads and causes the density at the junction to increase which leads to a decrease in velocity. The high density traffic at the junction sees lower density traffic ahead and speeds up, therefore, the traffic density after the junction decreases. This phenomenon can be observed in Figure 3.13 (density) and Figure 3.14 (velocity). The late time behavior is shown in Figure 3.15 (density) and Figure 3.16 (velocity). At time $t = 50$ sec the initial cluster of traffic and input density have completely passed through the junction. The traffic following the junction continues to speed up in the same manner and therefore become less dense.

**Numerical Convergence.** Numerical convergence was tested in the same manner as in the One Section model. The numerical solution converges.
Figure 3.3: Case 1

Figure 3.4: Case 1
Figure 3.5: Case 2 early time

Figure 3.6: Case 2 early time
Figure 3.7: Case 2 middle time

Figure 3.8: Case 2 middle time
Figure 3.9: Case 2 late time

Figure 3.10: Case 2 late time
Figure 3.11: Case 3 early time

Figure 3.12: Case 3 early time
Figure 3.13: Case 3 middle time

Figure 3.14: Case 3 middle time
Figure 3.15: Case 3 late time

Figure 3.16: Case 3 late time
Table 3.1: Definition of Probability and Exit Ramps

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ij}$</td>
<td>Probability that a vehicle travels from Section $i$ to Section $j$</td>
</tr>
<tr>
<td>$\nu_i$</td>
<td>Viscosity coefficient on road $i$</td>
</tr>
</tbody>
</table>

Table 3.2: Input Parameters

<table>
<thead>
<tr>
<th>Input</th>
<th>Value (C1/C2/C3)</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{12}$</td>
<td>0 / 0.8 / 1</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.1</td>
<td>s</td>
</tr>
<tr>
<td>$\nu_i$</td>
<td>1</td>
<td>(veh-m)/s</td>
</tr>
<tr>
<td>$\nu_{exit}$</td>
<td>1</td>
<td>veh/s</td>
</tr>
</tbody>
</table>
CHAPTER IV
THE FOUR SECTION MODEL

In this chapter four one-dimensional sections of road are linked together at a single node. At the node, a driver has the option to either continue driving onto the same path or exit onto the other road. The governing equations are the same as the one section model on each of the four sections of road. The set up is very similar to the two section model except the input density at the node will come from another section of road. Likewise, the exiting density will exit onto another section of road. The methodology for solving the model is exactly the same as in the two section model. Figure 4.1 illustrates the four section model.

Figure 4.1: Four Section Model
4.1 Model Equations

The model equations for all four sections of road are almost identical to the two section model equations. The model is comprised of four sets of equations in density $\rho(x,t)$ and velocity $v(x,t)$. Velocity is assumed to be continuous across the junction between Section 1 and Section 2, and across the junction between Section 3 and Section 4. However, the density is not necessarily continuous due to the exiting and entering of traffic onto different sections. Thus, density must be treated as four separate variables in the model. The four density variables are labeled $\rho_1(x,t)$, $\rho_2(x,t)$, $\rho_3(x,t)$, and $\rho_4(x,t)$ respectively.

The governing equations are

\begin{align*}
(\rho_1)_t + (\rho_1 v)_x &= 0, \\
(\rho_2)_t + (\rho_2 v)_x &= 0, \\
(\rho_3)_t + (\rho_3 v)_x &= 0, \\
(\rho_4)_t + (\rho_4 v)_x &= 0,
\end{align*}

\begin{align*}
v_t + (v + 2\beta c(\rho_1))v_x + \frac{c^2(\rho_1)}{\rho_1}(\rho_1)_x &= \frac{V_*(\rho_1) - v}{\tau} + \mu(\rho_1)v_{xx}, \\
v_t + (v + 2\beta c(\rho_2))v_x + \frac{c^2(\rho_2)}{\rho_2}(\rho_2)_x &= \frac{V_*(\rho_2) - v}{\tau} + \mu(\rho_2)v_{xx}, \\
v_t + (v + 2\beta c(\rho_3))v_x + \frac{c^2(\rho_3)}{\rho_3}(\rho_3)_x &= \frac{V_*(\rho_3) - v}{\tau} + \mu(\rho_1)v_{xx}, \\
v_t + (v + 2\beta c(\rho_4))v_x + \frac{c^2(\rho_4)}{\rho_4}(\rho_4)_x &= \frac{V_*(\rho_4) - v}{\tau} + \mu(\rho_2)v_{xx}.
\end{align*}

Section 1 and Section 3 are defined for $0 \leq x \leq L$ and Section 2 and Section 4 are defined on $L \leq x \leq 2L$. Hence, (4.1), (4.3), (4.5) and (4.7) are valid on $0 \leq x \leq L$.
and (4.2), (4.4) (4.6), and (4.8) are valid on $L \leq x \leq 2L$.

As for the boundary conditions, the left boundary of Section 1 and the right boundary of Section 2 are treated exactly as the boundaries in the two section model. Likewise, the lower boundary of Section 3 and the upper boundary of Section 4 are treated the same (refer to Figure 4.1). However, there is a linkage between Section 1, Section 2, Section 3, and Section 4. The linkage is comprised of the right boundary of Section 1, the left boundary of Section 2, the upper boundary of Section 3, and the lower boundary of Section 4. This linkage is handled in a similar fashion as in the two section case. The equations are listed below.

\[
\begin{align*}
\rho_2(L, t)v_2(L, t) &= \rho_1(L, t)v_1(L, t)A_{12} + \rho_3(L, t)v_3(L, t)A_{34}, \quad (4.9) \\
\rho_4(L, t)v_4(L, t) &= \rho_3(L, t)v_3(L, t)A_{34} + \rho_1(L, t)v_1(L, t)A_{14}, \quad (4.10) \\
\nu_1(v_1(L, t))_x &= \nu_2(v_2(L, t))_xA_{12} + \nu_4(v_4(L, t))xA_{14}, \quad (4.11) \\
\nu_3(v_3(L, t))_x &= \nu_4(v_4(L, t))xA_{34} + \nu_2(v_2(L, t))xA_{32}. \quad (4.12)
\end{align*}
\]

4.2 Discretization

The discretization of the four section model follows in the same manner as the two section model. The density on Section 1 ($\rho_{1(n)}^{k+1}$) and the density on Section 3 ($\rho_{3(n)}^{k+1}$) are obtained by discretizing the governing equations (4.1) and (4.3) using a central difference. However, now there are two conservation of momentum equations (4.9) and (4.10) and two flux equations (4.11) and (4.12). A backwards difference is used to compute $(v_1(L, t))_x$ and $(v_3(L, t))_x$ in (4.11) and (4.12); while a forwards difference
is used to compute \((v_2(L,t))_x\) and \((v_4(L,t))_x\) in (4.11) and (4.12). As a result, two unknowns are eliminated which enables the system of equations to be solvable for \(\rho_{2(n)}^{k+1}, \rho_{4(n)}^{k+1}, v_{2(n)}^{k+1},\) and \(v_{4(n)}^{k+1}\). It is assumed that \(v_{1(n)}^{k+1} = v_{2(n)}^{k+1}\) and \(v_{3(n)}^{k+1} = v_{4(n)}^{k+1}\).

4.3 Numerical Results

To test the four section model, one case is considered here. Two roads each comprised of two sections of roadway \(L = 500\) m long are linked together at a common node. The sections of roadway are discretized in the same fashion as in the two section model. Since two sections on each of two roads are being linked together at a common node, there are a total of four sections of roadway which make up two intersecting roads of length 1000 m, as illustrated in Figure 4.2. In this simulation, the probability that vehicles travel from Section 1 onto Section 2 is 80 percent \((A_{12} = 0.8)\), while the probability that vehicles travel from Section 3 onto Section 4 is 40 percent \((A_{34} = 0.4)\).

The initial density profile on each road \(\rho_I(x)\) is given by

Road 1: \(\rho_I(x) = .02,\)

Road 2: \(\rho_I(x) = .024.\)

The input densities are given by

Road 1: \(\tilde{\rho}(t) = .03 + .02(e^{-(t-4)^2}),\)

Road 2: \(\tilde{\rho}(t) = .03 + .015(e^{-(t-7)^2}).\)
The goal of this simulation is to identify the basic features encountered when two roads are linked together with different input densities and different initial conditions one each road.

**Case 1.** In case 1, the initial conditions on both roads are a constant density. For early time the input density on each road acts in the same manner as in the two section case. More interestingly, the focus of this simulation is the effect that the entering and exiting traffic at the node has on the density and corresponding velocity of the traffic flow of each road. Since 80 percent of the traffic from Section 1 continues to travel onto Section 2 and 40 percent of the traffic from Section 3 continues to travel onto Section 4, it is expected that Road 1 will experience an increase in density at the node. This is expected because the initial conditions on each road are relatively close and 80 percent of traffic from Section 1 will combine with 60 percent of the traffic from Section 3 at the node to form the traffic flow onto Section 2. This is exactly what happens in the simulation. Figure 4.2 (density) and Figure 4.3 (velocity) show that for early time the density increases at the node ($x = 500$ m). The high density traffic sees lower density ahead and begins to speed up. As time passes, this behavior continues. The input density eventually passes completely through the node. This can be observed in Figure 4.6 (density) and Figure 4.7 (velocity). Likewise, it is expected that Road 2 will realize a decrease in density at the node. This also results as observed in Figure 4.4 (density) and Figure 4.5 (velocity). The decrease in density allows for the traffic to speed up and display traffic characteristics that are very similar to Case 2 of the two section simulation. Eventually the input density
Table 4.1: Input Parameters

<table>
<thead>
<tr>
<th>Input Case 1</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{12}$</td>
<td>0.8</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$A_{34}$</td>
<td>0.4</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.1</td>
<td>s</td>
</tr>
<tr>
<td>$\nu_i$</td>
<td>1</td>
<td>(veh-m)/s</td>
</tr>
</tbody>
</table>

completely passes through the node. This may be observed in Figure 4.8 (density) and Figure 4.9 (velocity). The input values used in case 1 are listed in Table 4.1.

**Numerical Convergence.** Numerical convergence was tested in the same manner as in the Two Section model. The numerical solution converges.
Figure 4.2: Case 1: Road 1 early time

Figure 4.3: Case 1: Road 1 early time
Figure 4.4: Case 1: Road 2 early time

Figure 4.5: Case 1: Road 2 early time
Figure 4.6: Case 1: Road 1 late time

Figure 4.7: Case 1: Road 1 late time

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Figure 4.8: Case 1: Road 2 late time

Figure 4.9: Case 1: Road 2 late time
CHAPTER V

CONCLUSION

A second order, viscous continuum traffic flow model developed by Zhang [1] is studied and implemented numerically using a fourth order Runge Kutta adaptive time step algorithm. This model is applied to a two section one lane highway with an entrance ramp and an exit ramp, and to a four section highway network comprised of two one lane roads. This is the first numerical implementation of Zhang’s model using boundary conditions to model the traffic interchange. The underlying goal of this research is to effectively reduce the number sensors used by civil engineers for traffic flow prediction. The traffic simulations show this method to be extremely promising in that the results resemble hypothesized real world results. Each simulation has similar behavior to real traffic characteristics given a similar input traffic cluster. The main deficiency with this method is the modeling of the entrance and exit ramps. In the current model, traffic instantaneously enters and exits at the node. This leads to numerical difficulties when both the input traffic density at the node and the traffic density on the road are relatively high. More sophisticated ramp modeling must be introduced to avoid an instantaneous influx or dispersement of traffic density at the node. A time delay is necessary to more accurately model entrance and exit ramps.
As for future work, ramp modeling of the interchange is the next logical focus of this research. After the ramps are modeled more accurately, two highways that interchange will be considered and data will be taken from traffic sensors at the input of each highway. Initial conditions on each highway will also be measured using sensors. This data will then be used to run simulations that will predict the future traffic conditions. The future traffic conditions will be measured and compared against the simulation results to determine the accuracy of the model.
BIBLIOGRAPHY


