DEVELOPMENT OF NUMERICAL APPROACHES TO PREDICT
DUCTILE AND CLEAVAGE FRACTURE OF STRUCTURAL MATERIALS

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DEVELOPMENT OF NUMERICAL APPROACHES TO PREDICT
DUCTILE AND CLEAVAGE FRACTURE OF STRUCTURAL MATERIALS

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Dissertation

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ABSTRACT

Numerical simulations mainly using finite element method are playing a more and more important role in prediction of fracture-induced failure for high performance structure. This thesis seeks to develop numerical approaches to predict ductile and cleavage fracture in structural materials.

For ductile fracture, the discrete void approach reveals the failure mechanisms explicitly and is used to study the trends of fracture toughness. The porous continuum approach provides an effective means to predict extensive crack propagation. We consider the occurrence of material failure (void coalescence) as when localization of plastic flow takes place in the inter-void ligament and obtain the failure criterion as a function of the stress triaxiality ratio and the Lode angle. The Gologanu-Leblond Devaux (GLD) model, which accounts for the evolution of both void volume and void shape, is used to describe the porous plasticity behavior and is implemented into ABAQUS via a user subroutine. Numerical simulations are performed to predict extensive crack growth in ductile solids for a thin aluminum 2024-T3 plate and verified by successful predictions of crack extension in various specimens, including the multiple site damage specimens.

The effect of stress triaxiality and Lode angle is further analyzed and the Xue-Wierzbicki fracture locus is employed as a criterion for void coalescence. Combination of GLD model and X-W fracture locus is then applied to a DH-36 steel with specimens experiencing a wide range of stress triaxiality and Lode angles at failure. The numerical simulation results agree very well with the experimental results.
For cleavage fracture, a modified three parameter Weibull stress model is proposed and used to predict the fracture of A508 steel at three different temperatures. By integrating the Weibull stress model over the plastic process zone, the failure probability can be obtained and comparison is made with the experiment result. Issues addressed include calibration of the model parameters, introduction of a threshold parameter, dependencies of the model parameters on temperature, plastic strain effect and crack tip triaxiality effect, etc.
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CHAPTER I

INTRODUCTION

Most, if not all, engineering structures contain microscopic defects or crack-like flaws arising from the manufacture and operation processes. These defects and flaws can grow and propagate in the structure under combined loading and environmental conditions, leading to catastrophic failure. Fracture mechanics, which connects material science and solid mechanics, seeks to provide quantitative methodologies to evaluate how microscopic defects and structural flaws affect the behavior and integrity of structural components. The current and future significance of a known or postulated population of defects is determined by comparing the material’s resistance to further damage with the driving force for crack propagation caused by the operating conditions (temperature, loading mode, loading rate, environment, etc.). In a typical analysis, the driving force is estimated using analytical tools while the material resistance to fracture (“toughness”) is measured using laboratory tests on standard (small) specimens [1]. However, the complex material separation process depends strongly on the local stress and deformation fields, and therefore, differences in geometry, size, crack configuration, loading condition, etc. between actual structural components and laboratory test specimens complicate greatly the notions of crack driving force and material resistance to crack propagation. The
success of fracture mechanics lies in its ability to combine a theoretical framework with experimentally measured critical quantities [2].

1.1 Mechanism-Based Concept for Modeling Ductile Fracture

Mechanism-based concepts provide key insights for development of “transferable” fracture mechanics models for damage assessment. Ductile fracture in metallic alloys usually follows a multistep failure process involving several interacting, simultaneous mechanisms [3, 4]: 1) nucleation of microvoids by fracture or decohesion of second-phase inclusions, 2) growth of voids induced by plastic straining, 3) localization of plastic flow between the enlarged voids, and 4) final tearing of the ligaments between enlarged voids. Nucleation of voids from large inclusions generally occurs at relatively low stress levels, and therefore, voids are often assumed to present in the material at the onset of loading. The final material separation process usually proceeds very rapidly and is often facilitated by nucleation and growth of secondary microvoids. Based on the fracture mechanism, a straightforward approach to simulate ductile failure process is to model individual voids explicitly using refined finite elements, e.g., Aravas and McMeeking (1985) [5,6], Hom and McMeeking (1989) [7], Tvergaard and Hutchinson (2002) [8], Kim et al. (2003) [9] and Gao et al. (2005) [10]. A distinct advantage of this approach is the exact implementation of the void growth behavior. It provides an effective method to study the mechanisms of ductile fracture and to analyze the trends of fracture toughness [10, 11]. However, due to sizeable difference between the characteristic length scales
involved in the material failure process and the dimensions of the actual structural component, it is impractical to model every void in detail in structure failure analysis, especially for situations involving extensive crack propagation. For this reason, various forms of porous material models have been developed to describe void growth and the associated macroscopic softening during the fracture process. Calibration of these porous material models requires the predicted macroscopic stress-strain response and void growth behavior of the representative material volume (RMV) to match the results obtained from detailed finite element models with explicit void representation [12,13].

The Gurson-Tvergaard (GT) porous plasticity model [14, 15,16], which assumes voids are spherical in materials and remain spherical in the growth process, has been widely used in modeling ductile failure process and ductile crack extension [17,18,19,20]. But many processed materials, such as rolled plates, have non-spherical voids. And even for materials having initially spherical voids, the voids will change to prolate or oblate shape after deformation, depending on the state of the applied stress. In order to overcome these difficulties, Gologanu, Leblond and Devaux [21, 22, 23] derived a yield function for materials containing spheroidal voids. During plastic deformation, both the volume fraction and the shape of voids evolve as deformation increases. Since non-spherical voids are considered in the constitutive model, preferred material orientation exists and the plastic behavior becomes anisotropic. The Gologanu-Leblond-Devaux (GLD) model provides an important improvement to the widely adopted GT model in describing void growth and the corresponding material behavior during the ductile fracture process [11, 24, 25, 26, 27].
Modeling the void growth process has received considerable attention. However, the final void coalescence process has been considered for a long time as not more than just the end of the growth process. Consequently a critical void volume fraction ($f_c$) is often used to designate material failure, e.g., Needleman and Tvergaard [28], Xia et al. [17], and Gao et al. [20]. However, further studies show that $f_c$ depends strongly on factors such as void volume fraction, void shape, void spacing, stress triaxiality, strain hardening, etc. [13, 24, 29, 30]. Inspired by the void coalescence model proposed by Thomason [31, 32], Benzerga et al. [25, 26] and Pardoen and Hutchinson [11] developed different forms of flow potential for the void coalescence process. The flow potential together with the evolution equations of the state variables determines the behavior of the material during the coalescence process.

Since the time of Bridgman [33], interest of the fracture mechanics community has been focused on the effect of the hydrostatic stress on fracture. In literature, the stress triaxiality ratio, defined as the ratio of the hydrostatic stress to the equivalent stress, is often used as the sole parameter to characterize the effect of the triaxial stress state on ductile fracture. However, multiple stress states with different principal stress values can result in the same stress triaxiality ratio. Recent studies by Kim et al. [9, 13] and Gao et al. [10, 27] found that the macroscopic stress-strain response and the void growth and coalescence behavior of the voided RMV are different for each stress state even though the stress triaxiality ratio remains the same. Therefore, another parameter, e.g., the Lode parameter, should be introduced to distinguish the stress states having the same triaxiality ratio. Kim et al. [9, 13] and Gao et al. [10, 27] have demonstrated the significant effects of the Lode parameter on the ductile fracture process. Wierzbicki and Xue [34] address
this issue with a similar approach by proposing a ductile failure criterion as a function of both the first and third stress invariants.

We consider the occurrence of material failure (void coalescence) as when localization of plastic flow takes place in the inter-void ligament [35] and obtain the failure criterion as a function of the stress triaxiality ratio and the Lode angle by conducting systematic finite element analyses of the void-containing RMV subjected to different macroscopic stress states. Using the small scale yielding (SSY), boundary layer model [36] with discrete voids represented in the crack tip region by refined finite element mesh, two void growth mechanisms, void-by-void growth and multiple void interaction, are studied. With the material failure criterion obtained from unit cell analyses, the effects of the initial relative void spacing, void pattern, void shape and void volume fraction on ductile fracture toughness are investigated. In order to implement the porous plasticity models (often taking complicated forms) into finite element code, a generalized method is developed to formulate the consistent tangent moduli, which is crucial for preserving the quadratic convergence rate of the global Newton iterations in the solution of the incremental problem. Using this method, the GT model and GLD model are implemented into ABAQUS via user subroutines. A numerical approach is proposed to predict extensive crack growth in thin panels of a 2024-T3 aluminum alloy, where the GLD model is used to describe the void growth process and the material failure criterion is calibrated using experimental data. The calibrated computational model is then applied to predict crack extension in specimens having various initial crack configurations and the numerical predictions agree very well with experimental measurements.
1.2 Weakest Link Statistics and the Weibull Stress Model for Cleavage Fracture

For engineering structures constructed of ferritic steels, cleavage fracture in the ductile-to-brittle transition (DBT) region represents a potentially catastrophic failure mode. Cleavage fracture in structural steels can be simplified as a two-step process [37, 38]. At first microscopic cracks are formed due to slip-induced cracking of carbides, generally located on grain boundaries, followed by unstable propagation of these microcracks into the surrounding ferrite matrix. In the transition region, the process becomes strongly driven by a (often single) critical cleavage event at the metallurgical scale which triggers macroscopic brittle fracture. Therefore, cleavage fracture in the DBT region exhibits a “weakest link” phenomenon [39, 40, 41]. The carbide particles, which are “eligible” for initiating cleavage fracture, are those under sufficient tensile stress, with orientations favorable for nucleating microscopic cracks, and favorable for producing high enough energy release rate. Due to the highly localized nature of the failure mechanism coupled with microstructural inhomogeneity of the material, cleavage fracture toughness data often exhibits a considerable amount of scatter, a dependence on length of the crack front, and a strong sensitivity to local stress and deformation fields. Furthermore, cleavage fracture toughness also displays strong dependencies on temperature, loading rate, pre-straining, etc. All these complications greatly increase the difficulty to apply the laboratory measured toughness values in reliability assessments of structural components.

The significance of cleavage fracture behavior has stimulated an increasing amount of research in the past few decades. These research efforts have led to a quantitative understanding of the scatter and temperature dependence of macroscopic fracture
toughness (in terms of $J_c$ or $K_{jc}$) under high constraint, SSY conditions. Scatter of the SSY toughness data can be described by a three-parameter Weibull distribution, where the Weibull modulus for $K_{jc}$ distribution is 4 and the minimum fracture toughness for common ferritic steels is $K_{min} \approx 20\text{MPa}\sqrt{\text{m}}$ [1, 42]. This three-parameter Weibull distribution has been adopted in ASTM E-1921 (1998). E-1921 also adopts a so-called “master curve”, empirically derived by Wallin and others [42] to describe the dependence of cleavage fracture toughness on temperature for ferritic steels in the DBT region. The master curve defines the median fracture toughness at any temperature in the DBT region under SSY conditions while in engineering applications the crack front often experiences constraint loss. This motivates development of micromechanics-based models to address the transferability of cleavage fracture toughness across varying levels of crack-front constraint. The Beremin model (or Weibull stress model), developed based on weakest link statistics [41], provides a framework for quantifying the relationship between macro-scale and micro-scale driving forces for cleavage fracture. The introduction of the so-called Weibull stress, calculated by integrating a weighted value of the maximum principal stress over the fracture process zone, provides the basis for generalizing the concept of a probabilistic fracture parameter and supports the development of procedures that adjust toughness values across different crack configurations and loading modes (tension vs. bending) [43, 44, 45]. The Beremin model has two material parameters, the Weibull modulus ($m$) and the scale parameter ($\sigma_u$). Gao, et al. [46, 47] proposed a procedure to calibrate parameters of the Weibull stress model using fracture toughness data obtained from two sets of test specimens that exhibit different constraint levels at fracture. They also introduced a threshold Weibull stress value ($\sigma_{w-min}$) corresponding to
\( K_{\text{min}} \). Using the three-parameter Weibull stress model with parameters calibrated according to the proposed procedure, Gao, et al. [48] predicted the distributions of measured fracture toughness values in various specimens of an A515-70 pressure vessel steel, including surface crack specimens subject to different combinations of bending and tension. However, cleavage fracture involves a complex interaction between mechanical and metallurgical process while the Weibull stress model only employs two (or three) parameters to capture the fracture event. Potential dependencies of model parameters on temperature, loading rate, irradiation, pre-straining, etc. play a crucial role in applications [49, 50, 51]. In this thesis, we study the effects of loading rate and temperature on the Weibull stress model parameters and modify the model to include the effect of plastic strain on microcrack nucleation.
 CHAPTER II

DUCTILE FRACTURE IN MODELS FOR STRUCTURAL MATERIALS

Fracture has cost tremendous lost to the world in the form of both property and human lives. From time to time we have disasters of airplane, bridges, ships, and etc. due to either brittle fracture or ductile fracture. The traditional method which is based on linear elastic fracture mechanics or elastic-plastic fracture mechanics has been widely used by using a single parameter such as $K_{IC}$ or $J_{IC}$ or CTOD or CTOA as a measurement of the fracture resistance. In its recent development a second parameter (T and Q stress) has been taken account of by O’Dowd and Shih [52, 53].

The traditional fracture mechanics has been quite successful in many engineering applications under small deformation; however, structure integrity assessment requires the ability to predict fracture failure effectively not only for simple structure under small deformation, but also for complex structure subject to complicated loading conditions. Approaches to modeling crack growth relying on traditional fracture mechanics concepts ($K_{IC}$ or $J_{IC}$ or CTOD or CTOA, etc) are not applicable to crack growth in ductile materials because the plastic zone is large compared to the dimensions of the components. In cases such as these, the energy dissipated with crack growth is a strong function of the geometry and nature of the loading, and therefore, cannot be characterized by a single failure parameter.
So far, numeral fracture models have been developed to quantify the damage associated with material deformation and to predict the fracture initiation [14, 21-23, 54-60]. They can generally be divided into groups. One is the macroscopic approach in which the material is represented by its aggregate response. Sometimes it’s also called continuum mechanics model. Another is the micro mechanical approach in which the evolution of microscopic structure of the material is used to model the global behavior.

2.1 Continuum mechanics models

The conventional continuum mechanics fracture models fall in the category of classical plasticity theory. It doesn’t consider the joint effect of damage and plasticity and assumes the plasticity properties don’t change despite of the damage accumulated due to the plastic deformation. In these models, a separate damage variable which is independent of the material strength is used as an indicator for prediction of crack initiation. Among them Johnson-Cook model [60] is the one that gets most widely used.

For the classical J2 plasticity theory there is no criteria for damage as well as fracture, consequentially it can’t predict fracture initiation. The Johnson-Cook model adopts a damage variable which is a function of the unrecoverable plastic strain. The fracture onset takes place when the damage variable reaches a critical value. This approach is often called cumulative strain method.

The Johnson-Cook model assumes that the damage toward eventual fracture is due to the plastic deformation history of the material. The damage variable $D$ is defined as the integral of a weighted function with respect to equivalent plastic strain over the whole loading history.
\[ D = \int_{0}^{\epsilon} f(\text{field variables})d\varepsilon_p \]  

(2.1)

For a given material, when the accumulated damage exceeds a critical value, \( D_c \), i.e. \( D > D_c \), the material is considered to fail. The damage variable \( D \) can be further normalized with respect to \( D_c \) so that it has a unit value when failure happens. The integrand, the weighting function \( f \) is a function of field variables such as the stress, strain, temperature, strain rate, etc. The cumulative damage model can be written as

\[ D = \int_{0}^{\epsilon} f(\sigma, \varepsilon, T, \dot{\varepsilon}, ...)d\varepsilon_p = 1 \]  

(2.2)

The Johnson-Cook model is a widely used representative material constitutive model which has both strength function and failure function. Its strength function considers the effect of plastic hardening, strain rate sensitivity and temperature weakening in the uncoupled form. The equivalent stress \( \sigma_{eq} \) is defined as:

\[
\sigma_{eq} = [A + B\varepsilon_p^p] \left[ 1 + C \ln \left( \frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0} \right) \right] \left[ 1 - \left( \frac{T - T_0}{T_{melt} - T_0} \right)^q \right]
\]  

(2.3)

where \( \dot{\varepsilon}_p \) is the equivalent plastic strain rate, \( \dot{\varepsilon}_0 \) is the reference plastic strain rate, \( T_0 \) and \( T_{melt} \) are the room temperature and the material melting temperature, respectively, \( A, B, n, C \) and \( q \) are five material constants. The first term in Eq. (2.3) represents the quasi-static stress-strain relation at room temperature, the second term specifies the strain-rate hardening, and the third one deals with the temperature dependence of the stress-strain relation.

The fracture criterion is a weighted integral with respect to the effective strain [61], as defined below:
\[ D = \int_{\varepsilon_0}^{\varepsilon_f} \frac{d\varepsilon_p}{\varepsilon_f \left( \frac{\sigma_m}{\sigma_{eq}}, \dot{\varepsilon}_p, T \right)} , \quad (2.4) \]

where \( \sigma_m \) is the hydrostatic stress and is defined as \( \sigma_m = (\sigma_{11} + \sigma_{22} + \sigma_{33})/3.0 \), \( \frac{\sigma_m}{\sigma_{eq}} \) is the stress triaxiality. The effective strain is defined as a function of the stress triaxiality status, the strain rate and the temperature, as shown below:

\[ \varepsilon_f = \left[ D_1 + D_2 \exp \left( D_3 \frac{\sigma_m}{\sigma_{eq}} \right) \right] \left[ 1 + D_4 \log \frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0} \right] \left[ 1 + D_5 \frac{T - T_0}{T_{melt} - T_0} \right] , \quad (2.5) \]

where \( D_1, D_2, D_3, D_4 \) and \( D_5 \) are materials constants. The constants have to be determined by specimen tests under different triaxialities.

The conventional continuum mechanics models don’t consider the coupling effects of plasticity and damage. However, when the plastic deformation is beyond the moderate range, as many cases in ductile fracture, the damage effect on the material can no longer be neglected. Modifications to the continuum mechanics fracture has been made to include the coupling effect of damage and plasticity. The idea was firstly proposed by Kachanov [62] in his work of ductile creep and further developed by Rabotnov [63] and Lemaitre [64] etc. An example of the modification is that for J2 material, the uncoupled model has the elasticity law as \( \sigma = C_0 \varepsilon^\varepsilon \), while for the coupled model gives

\[ \sigma = C_0 \varepsilon^\varepsilon (1 - D) , \] where \( C_0 \) is the elasticity stiffness tensor. Borvik et al [65] introduced a weakening factor to the strength equation of Johnson-Cook model,

\[ \sigma_{eq} = (1 - \beta_0 D) [A + B \varepsilon_p^\varepsilon] \left[ 1 + C \ln \left( \frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0} \right) \right] \left[ 1 - \left( \frac{T - T_0}{T_{melt} - T_0} \right)^q \right], \quad (2.6) \]
where $D$ is the same as the original Johnson-Cook damage model and $\beta_h$ is a material constant.

2.2 Micro mechanical models

Fracture in ductile materials usually starts with nucleation, growth and coalescence of micro voids [66, 67, 68]. The void mainly nucleate at second phase particles, either by decohesion of particle-matrix interface or by particle fracture and final rupture involves the growth of neighboring voids to coalescence. The accumulation process leading to fracture is very complex. The material is considered to be porous media.

The pioneering work of the application of micro mechanical to ductile fracture is due to McClintock[54] and Rice and Tracy[57]. McClintock analyzed a long cylindrical cavity in a non-hardening material pulled in the axis direction while subjected to transverse tensile stresses [17]. Rice and Tracey [57] considered the growth of a spherical void in non-hardening material. The spherical void subjected to remote uniaxial tension strain rate field not only grows in radial direction, but also the shape changes. The void growth rate and the failure strain were then derived. A common feature of the research by McClintock and Rice and Tracey is that they didn’t consider the interaction between adjacent voids.

Gurson [14] approximated a porous plastic solid by a thick walled spherical shell and proposed a constitutive relation for a progressively cavitating solid by limit analysis. In the Gurson model, the matrix material is idealized as rigid perfectly plastic and obeying the von Mises yield criterion. Strain hardening is not taken into account. The void in the porous material is considered by a single parameter $f$, the void volume fraction. The
The presence of the voids leads to macroscopic dilatancy and pressure sensitivity of plastic flow.

The Gurson material model has received tremendous attention since its advent. Various modifications had been made. Yamamoto [69] introduced an average flow stress to replace the yield stress of the matrix material in the Gurson model to include indirectly the effect of strain hardening. Duva and Hutchinson [70] suggested further improvement concerning the strain hardening effects. Tvergaard attempted to improve the accuracy of the Gurson model by adjusting certain of its numerical coefficients so that the model more accurately reproduces detailed calculations for a voided material. One of the most important modifications is by Tvergaard and Needleman [56] which associated the model with complete loss of stress carrying capacity. Although the flow potential proposed by Gurson does permit a complete loss of stress carrying capacity at a critical void volume fraction, this critical void volume fraction is unrealistically high [56, 71].

The widely used GT (Gurson-Tvergaard) model has the yield function:

$$\Phi = \frac{\sigma_{eq}^2}{\bar{\sigma}^2} + 2q_1 f^* \cosh\left\{\frac{3q_2 \sigma_h}{2\bar{\sigma}}\right\} - \left[1 + (q_1 f^*)^2\right] = 0$$

(2.7)

where $\sigma_e$ is the Mises effective stress, $\sigma_h$ is the hydrostatic stress and $\bar{\sigma}$ is the yield stress of the undamaged material. Parameters $q_1$ and $q_2$ were introduced by Tvergaard to bring shear band bifurcation predictions of the Gurson constitutive relation into closer agreement with corresponding results of full numerical analyses of periodic array of voids.

Function $f^*$ was proposed by Tvergaard and Needleman [56] to account for the effects of rapid void coalescence at failure.
where \( f_c \) and \( f_f \) are the critical void fraction and the void volume fraction at fraction respectively. The void coalescence is considered to take place when the void volume fraction reaches a critical value \( f_c \).

By assuming associative flow rule, the plastic flow function is the same as the yield function. The macroscopic plastic strain can be derived as:

\[
\dot{\varepsilon}_p = \frac{\partial \Phi}{\partial \sigma}
\]

(2.9)

where \( \varepsilon_p \) and \( \sigma \) are the macroscopic strain and stress tensor respectively and \( (\cdot)' \) denotes the time derivative. The matrix plastic strain is linked to the macroscopic strain by the equivalence of the rate of the plastic work, i.e.

\[
\varepsilon_{pl} : \sigma = (1 - f)\bar{\sigma} \varepsilon_p
\]

(2.10)

Most engineering materials contain more than one population of inclusions and/or second phase particles. During the void coalescence process, secondary voids nucleate in the ligament between enlarged primary voids and rapid growth and coalescence of these secondary voids accelerates the final ligament separation. To take this into consideration, the evolution of the void volume fraction can be characterized as:

\[
\dot{f} = (\dot{f})_{\text{growth}} + (\dot{f})_{\text{nucleation}}
\]

(2.11)

where \((\dot{f})_{\text{growth}}\) and \((\dot{f})_{\text{nucleation}}\) represent the rate of increase of void volume fraction due to growth and nucleation respectively. When the void volume fraction reaches a critical
value \( f_c \), the voids begin to coalesce and the void containing volume will lost its load carrying capacity shortly.

In ductile metals, the void nucleation often occurs at non-deformable second-phase particles, such as oxides and carbides. The nucleation of voids can be considered in a strain based or a stress based fashion [72]. An analysis by Needleman and Tvergaard [28] suggests the localization will occur at very early stages of deformation. However, due to the physical complexity of the microstructure and the inclusion, the void nucleation particles are not homogenous within the solid. Chu and Needleman [73] suggested a normal distribution for the strain controlled void nucleation.

\[
\dot{f}_{\text{nucleation}} = D \ddot{e}, \quad (2.12)
\]

and

\[
D = \frac{f_N}{s_N \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\ddot{e} - \varepsilon_N}{s_N}\right)^2\right] \quad (2.13)
\]

where \( s_N \) and \( \varepsilon_N \) are the standard deviation and the mean value of the distribution of the plastic strain, \( F_N \) is the total void volume fraction that can be nucleated.

The matrix material is assumed to be plastically incompressible. The void growth rate is thus

\[
(\dot{f})_{\text{growth}} = (1 - f') \dot{\varepsilon}_{kk} \quad (2.14)
\]

where \( \dot{\varepsilon}_{kk} \) represents the plastic rate of volume change.

The GT model has been applied successfully in many engineering applications and it’s implemented into some leading commercial finite element analysis software like ABAQUS. However, it assumes the voids are spherical in materials and remain spherical
in the growth process, which is an assumption not true in many materials, such as rolled plates. And, even for materials having initially spherical voids, the void shape may change to prolate or oblate depending on the state of the applied stress. To overcome the limitation of the GT model, Gologanu et al. [21, 22, 23] derived a yield function for materials containing initially spheroidal voids. In the so call GLD model, both void volume fraction and the void shape evolve as deformation increases. Since non-spherical voids are considered in the constitutive model, preferred material orientation exists and the macroscopic plastic behavior becomes anisotropic. The GLD model returns to the Gurson model when voids become spherical. The yield function of the void-containing material is expressed as

\[
\Phi = \frac{C}{\bar{\sigma}^2} \| \Sigma^\prime + \eta \Sigma^h X \|^2 + 2q(g + 1)(g + f) \cosh \left( \kappa \frac{\Sigma^h}{\bar{\sigma}} \right) - (g + 1)^2 - q^2(g + f) = 0 \quad (2.15)
\]

where \( \Sigma^j \) are the macroscopic stress components, \( f \) represents the void volume fraction, \( S \) is the shape parameter defined as \( S = \ln(W) \) with \( W=R_y/R_x \), and \( \bar{\sigma} \) is the yield stress of the matrix material. In Eq. (2.14), \( \| \| \) denotes the von Mises norm, \( \Sigma^\prime \) is the deviatoric stress tensor, \( \Sigma^h \) is the generalized hydrostatic stress defined as

\[
\Sigma^h = \alpha_2 (\Sigma_{xx} + \Sigma_{zz}) + (1 - \alpha_2) \Sigma_{yy}, \quad X = (2/3) e_y \otimes e_y - (1/3) e_x \otimes e_x - (1/3) e_z \otimes e_z,
\]

where \((e_x, e_y, e_z)\) is an orthogonal basis with \( e_y \) parallel to the axisymmetric axis of the void, and \( \otimes \) denotes tensor product. The parameters \( C, \eta, g, \kappa \) and \( \alpha_2 \) are functions of \( f \) and \( S \) and the heuristic parameter \( q \) depends on initial void volume fraction, strain hardening exponent of the matrix material, \( S \) and the macroscopic stress triaxiality factor \( T \).
The evolution law for $f$ due to void growth is determined by requiring the matrix material to be plastically incompressible and derivations of the evolution equation for $S$ can be found in Gologanu et al. [21, 22, 23]. Pardoen and Hutchinson [11, 24] and Benzeraga et al. [26] provide detailed descriptions and formulation about the GLD model. Kim and Gao [74] developed a generalized approach to formulate the consistent tangent stiffness for complicated plasticity models. Using the approach developed by Kim and Gao, the GLD model was implemented into ABAQUS via a user subroutine.
3.1 Introduction

Engineering structures inevitably contain microscopic defects or crack-like flaws arising from the manufacture and operation processes. These defects and flaws can grow and propagate in the structure under combined loading and environmental conditions, leading to catastrophic failure. Fracture mechanics, which connects material science and solid mechanics, seeks to provide quantitative methodologies to evaluate how microscopic defects and structural flaws affect the behavior and integrity of structural components. The current and future significance of a known or postulated population of defects is determined by comparing the material’s resistance to further damage with the driving force for crack propagation caused by the operating conditions (temperature, loading mode, loading rate, environment, etc.). In a typical analysis, the driving force is estimated using analytical tools while the material resistance to fracture (“toughness”) is measured using laboratory tests on standard (small) specimens [1]. However, the complex material separation process depends strongly on the local stress and deformation fields, and therefore, differences in geometry, size, and crack configuration
loading condition, etc. between actual structural components and laboratory test specimens complicate greatly the notions of crack driving force and material resistance to crack propagation. The success of fracture mechanics lies in its ability to combine a theoretical framework with experimentally measured critical quantities [2].

Mechanism-based concepts provide key insights for development of “transferable” fracture mechanics models for damage assessment. Ductile fracture in metallic alloys usually follows a multistep failure process involving several interacting, simultaneous mechanisms [3, 4]: 1) nucleation of microvoids by fracture or decohesion of second-phase inclusions, 2) growth of voids induced by plastic straining, 3) localization of plastic flow between the enlarged voids, and 4) final tearing of the ligaments between enlarged voids. Nucleation of voids from large inclusions generally occurs at relatively low stress levels, and therefore, voids are often assumed to present in the material at the onset of loading. The final material separation process usually proceeds very rapidly and is often facilitated by nucleation and growth of secondary microvoids. Based on the fracture mechanism, a straight-forward approach to simulate ductile failure process is to model individual voids explicitly using refined finite elements, e.g., [5-10]. A distinct advantage of this approach is the exact implementation of the void growth behavior. It provides an effective method to study the mechanisms of ductile fracture and to analyze the trends of fracture toughness [10, 11].

The idea of the discrete void approach is very simple. However, it would require a huge number of elements to model crack extension in a structure component. Such a finite element model is far beyond the current computational capability. As a practical alternative, the porous continuum approach provides an effective means to predict
extensive crack propagation. Various forms of porous material models have been
developed to describe void growth and the associated macroscopic softening during the
fracture process. The Gurson-Tvergaard (GT) porous plasticity model [14, 15, 16], which
assumes voids are spherical in materials and remain spherical in the growth process, has
been widely used in modeling ductile failure process and ductile crack extension [17, 18,
19, 20]. But many processed materials, such as rolled plates, have non-spherical voids.
And even for materials having initially spherical voids, the voids will change to prolate or
oblate shape after deformation, depending on the state of the applied stress. In order to
overcome these difficulties, Gologanu, Leblond and Devaux [21, 22, 23] derived a yield
function for materials containing spheroidal voids. During plastic deformation, both the
volume fraction and the shape of voids evolve as deformation increases. Since non-
spherical voids are considered in the constitutive model, preferred material orientation
exists and the plastic behavior becomes anisotropic. Kim and Gao [74] recently proposed
a method to formulate the consistent tangent moduli for general plasticity models, which
is required for preserving the quadratic convergence rate of the global Newton iterations
in the solution of the incremental problem. Using this method, the Gologanu-Leblond-
Devaux (GLD) model, although having a very complicated form, can be easily
implemented into the finite element code. The GLD model provides an important
improvement to the widely adopted GT model in describing void growth and the
corresponding material behavior during the ductile fracture process [11, 24, 25, 26, 27].

Since the time of Bridgman [33], interest of the fracture mechanics community has
been focused on the effect of the hydrostatic stress on fracture. In literature, the stress
triaxiality ratio, defined as the ratio of the hydrostatic stress to the equivalent stress, is
often used as the sole parameter to characterize the effect of the triaxial stress state on ductile fracture. However, multiple stress states with different principal stress values can result in the same stress triaxiality ratio. Recent studies by Kim et al. [9, 13] and Gao et al. [10] found that the macroscopic stress-strain response and the void growth and coalescence behavior of the voided RMV are different for each stress state even though the stress triaxiality ratio remains the same. Therefore, another parameter, e.g., the Lode parameter, should be introduced to distinguish the stress states having the same triaxiality ratio. Kim et al. [9, 13] and Gao et al. [10, 27] have demonstrated the significant effects of the Lode parameter on the ductile fracture process. Wierzbicki and Xue [34] address this issue with a similar approach by proposing a ductile failure criterion as a function of both the first and third stress invariants.

This chapter discusses both the discrete void approach and the porous continuum approach to model ductile fracture. We consider the occurrence of material failure (void coalescence) as when localization of plastic flow takes place in the inter-void ligament [35] and obtain the failure criterion as a function of the stress triaxiality ratio and the Lode angle by conducting systematic finite element analyses of the void-containing RMV subjected to different macroscopic stress states. Using the small scale yielding (SSY), boundary layer model [36] with discrete voids represented in the crack tip region by refined finite element mesh, two void growth mechanisms, void-by-void growth and multiple void interaction, are studied. With the material failure criterion obtained from unit cell analyses, the effects of the initial relative void spacing, void pattern, void shape and void volume fraction on ductile fracture toughness are investigated. Finally, a numerical approach is proposed to predict ductile crack growth in thin panels of a 2024-
T3 aluminum alloy, where the GLD model is used to describe the void growth process and the material failure criterion is calibrated using experimental data. The calibrated computational model is applied to predict crack extension in specimens having various initial crack configurations and the numerical predictions agree very well with experimental measurements.

3.2 Modeling Ductile Fracture Using Explicit Void Representation

This Section presents a computational approach to model ductile fracture using explicit void representation. An array of voids ahead of the crack front is explicitly modeled using refined finite elements to understand the nature of void growth and different ductile failure mechanisms. A crack initiation and growth criterion is established based on the results of extensive unit cell analyses. And finally, the effects of the initial relative void spacing, void pattern, void shape and void volume fraction on ductile fracture toughness are discussed.

3.2.1 The SSY boundary layer model

The small scale yielding (SSY), boundary layer model simplifies the generation of numerical solutions for crack problems. Here it assumes that the plastic zone size is small comparing to the geometric dimensions of the specimen. Fig. 3.1(a) shows a periodical distribution of voids in the plane of crack propagation. In an attempt to rationalize fracture behavior, a local coordinate system is set up such that the \( x \)-axis represents the crack propagation direction, \( y \)-axis represents the crack opening direction and \( z \)-axis
represents the thickness direction. Considering the existence of symmetry about the crack plane, only half of the region needs to be modeled. Except near free surfaces the deformation in the thickness direction can be assumed periodically symmetric. Neglecting the free surface effect allows us to apply the periodic boundary conditions and consider half of the void spacing distance in the thickness direction only, Fig. 3.1(b).

Fig. 3.1 (a) Periodical distribution of voids in the plane of crack propagation. (b) Domain of the boundary value problem.

Fig. 3.2 (a) shows a typical finite element mesh of the SSY model. Close-up of the crack tip region is shown in Figs 3.2(b) and 3.2(c) for finite element modeling containing one row and two rows of voids respectively. A typical mesh containing two rows of voids consists of 18,000 twenty-node, isoparametric, brick elements (86,000 nodes) with reduced integration. With the assumption of the periodic void distribution, it is natural to consider the material ahead of the crack front as an array of unit cells with each unit cell containing a void at its center. The ratio of the void volume to the volume of the cell (including the void) defines the void volume fraction of the material. To resolve the crack tip deformation field and enhance convergence of the nonlinear iterations, the finite element mesh contains an initial root radius at the crack tip. Numerical solutions are
generated by imposing displacements of the elastic, asymptotic mode I field (plane strain) on the outer circular boundary.

Fig. 3.2 (a) A typical finite element mesh of the SSY model. (b) Close-up of the crack tip region containing a row of five voids. (c) Close-up of the crack tip region containing two rows of voids.

Three types of initial void shapes, spherical shape, prolate shape, and oblate shape, are considered. Fig. 3.3 shows the geometrical representation of the voids. The prolate and oblate voids are assumed to be axisymmetric about the y-axis and an initial aspect ratio is defined as $W_0 = R_0y/R_0x$. Therefore, $W_0=1$ corresponds to the spherical shape, $W_0>1$ corresponds to the prolate shape, and $W_0<1$ corresponds to the oblate shape.
Fig. 3.3 Geometric representation of voids: (a) spherical void, (b) prolate void, and (c) oblate void.

Several initial void arrangements are considered in this study. The spacing between adjacent voids in the crack propagation direction is denoted as $X_0$, which is the same as the distance from the first void to the crack tip. The distance between two adjacent rows of voids is denoted as $Y_0$. A parameter $\lambda_0$ is defined as the ratio of the void spacing in $y$-direction to the void spacing in $x$-direction, $\lambda_0 = Y_0/X_0$, measuring the relative void spacing.

3.2.2 Two void growth mechanisms

Our analyses confirm the exist of two distinct void growth mechanisms, i.e., void-by-void growth mechanism for materials containing small initial void volume fractions and multiple void interaction mechanism for materials containing large initial void volume fractions. As an example, Fig. 3.4 shows the deformed meshes for two models each containing one row of initially spherical voids. The initial void volume fraction
values are $f_0 = 0.0013$ and 0.014 respectively. The material chosen for this analysis has a Young’s modulus ($E$) of 200 GPa, Poisson's ratio ($\nu$) of 0.3 and yield stress ($\sigma_0$) of 600 MPa and obeys a power-law hardening (true) stress-strain relation with the hardening exponent $N=0.1$. The deformed mesh of the $f_0 = 0.0013$ model at an applied load level of $J/(X_0\sigma_0) = 2.73$ is shown in Fig. 3.4(a) and the deformed mesh of the $f_0 = 0.014$ model at an applied load level of $J/(X_0\sigma_0) = 0.98$ is shown in Fig. 3.4(b), where $J$ is Rice’s $J$-integral. In Fig. 4(a), only the first void from the crack tip has significant volume increase while in Fig. 4(b), the volumes of several voids increase simultaneously.

![Fig. 3.4 Deformed finite element meshes showing two distinct void growth mechanisms.](image)

Our further analyses reveal that, besides the initial void volume fraction, other factors also affect the void growth mechanism when the initial void volume fraction is large. Voids deviated from the crack growth plane reduce the interaction among voids on the crack growth plane and delay the transition from void by void growth mechanism to multiple voids interaction mechanism. Increase of $\lambda_0$ (relative void spacing) intensifies the interaction among neighboring voids and facilitates the transition from void-by-void
growth mechanism to multiple void interaction mechanism. Change of the void distribution pattern by shifting the positions of second row voids does not affect the growth rates of voids in the plane of crack propagation. Our results also show that when other parameters are the same, the oblate void grows faster than the spherical void and the spherical void grows faster than the prolate void.

3.2.3 Void coalescence criterion

In order to simulate crack formation and propagation, a criterion for void coalescence is required. After the onset of void coalescence, material loses load carrying capacity rapidly. The criterion for onset of void coalescence can be obtained by conducting systematic finite element analyses of the void-containing RMV subjected to different macroscopic stress states. Consider a RMV containing a void at its center and subjected to the macroscopic stresses $\Sigma_1$, $\Sigma_2$ and $\Sigma_3$. The stress triaxiality ratio is defined as

$$T = \frac{\Sigma_h}{\Sigma_e},$$  \hspace{1cm} (3.1)

where $\Sigma_h$ and $\Sigma_e$ represent the hydrostatic stress and the equivalent stress respectively, and the Lode angle is defined by

$$\tan \theta = \frac{2\Sigma_3 - \Sigma_2 - \Sigma_1}{\sqrt{3(\Sigma_2 - \Sigma_1)}}.$$

Kim et al. [9, 13] and Gao et al. [10, 27] have demonstrated that both the stress triaxiality ratio and the Lode parameter have significant effects on the ductile failure
process. Therefore, boundary conditions on the outer surfaces of the RMV are prescribed such that the macroscopic parameters $T$ and $\theta$ are kept constant during the entire deformation history. Faleskog et al. [12] and Kim et al. [13] provide detailed procedures of how to prescribe this kind of boundary conditions. In the calculations performed in this Section, it is assumed that $\Sigma_2$ is the major loading and $\Sigma_2 \geq \Sigma_3 \geq \Sigma_1$. This assumption results in the Lode angle in the range of $-30^\circ \leq \theta \leq 30^\circ$.

Fig. 3.5 shows a 1/8-symmetric finite element model for a cubic RMV containing a spherical void. An axisymmetric loading is considered first, where $\Sigma_2 = \Sigma_1 = \Sigma_3$ ($\theta = -30^\circ$). The material flow properties used in the finite element analyses represent a typical aluminum alloy, $E=70.4$ GPa, $\nu=0.3$, $\sigma_0=345$ MPa and $N=0.14$. The initial void volume fraction is taken as $f_0 = 0.02$.

Fig. 3.5 A one-eighth symmetric finite element mesh for a RMV containing a centered, spherical void.
Let $X_0$ be the initial width of the RMV in the $x$-direction and $X$ be the deformed width. Fig. 3.6(a) shows the variation of $X$ with the macroscopic effective strain ($E_e$) of the RMV. As loading continues, $X$ gradually decreases. But when the deformation reaches a certain level, $X$ stops decreasing and remains at a constant value. This implies that further deformation takes place in a uniaxial straining mode, which corresponds to flow localization in the ligament between adjacent voids. The shift to a macroscopic uniaxial strain state indicates the onset of void coalescence. We use $E_c$ to denote the macroscopic effective strain at the onset of void coalescence.

![Graphs](image)

(a) Variation of the deformed cell width in $x$-direction with the macroscopic effective strain of the cell revealing the shift to uniaxial straining. (b) Macroscopic effective stress versus effective strain of the void-containing RMV displaying the macroscopic softening.

The macroscopic effective stress versus effective strain curve, Fig. 3.6(b), provides an overview of the competition between matrix material strain hardening and porosity
induced softening. As deformation progresses, a maximum effective stress is reached (indicated by the filled circle), and then $\Sigma_e$ decreases as strain-hardening of matrix material is insufficient to compensate for the reduction in ligament area caused by void growth. As the macroscopic effective strain reaches $E_c$ (indicated by the open circle), a rapid drop in macroscopic effective stress occurs. As expected, both the peak stress value and the value of $E_c$ decrease with the stress triaxiality ratio $T$, reflecting the decrease of ductility.

Most engineering materials contain more than one population of inclusions and/or second phase particles. During the void coalescence process, secondary voids nucleate in the ligament between enlarged primary voids and rapid growth and coalescence of these secondary voids accelerates the final ligament separation. Faleskog and Shih [12] conducted 2D analysis of void coalescence where both primary and secondary voids are represented using refined finite element mesh. Here we assume nucleation of the secondary voids is plastic strain controlled and the nucleated voids are smeared in the material. It is further assumed that void nucleation follows a normal distribution as suggested by Chu and Needleman [73]. The GT porous plasticity model [14, 15, 16] is used to describe the material behavior after void nucleation and the $f^*$ function introduced by Tvergaard and Needleman [56] is used to account for the coalescence of secondary voids and its effect on material failure. Parameters similar to those used by Tvergaard and Needleman [56] are employed to describe the nucleation, growth and coalescence of secondary voids. These parameters are chosen just for the purpose of
demonstrating the effect of secondary voids on material failure. No attempt is made to represent the actual physical values.

Fig. 3.7 compares the macroscopic effective stress versus effective strain curves between models including and not including secondary voids. Here several values of stress triaxiality ratio, $T = 1/3, 2/3, 1, 1.5$ and 2, are considered. The open circles denote the onset of coalescence for models where secondary voids are not taken into account. The filled circles represent the onset of coalescence for models where nucleation, growth and coalescence of secondary voids are accounted for. It is clear that secondary voids significantly accelerate the void coalescence process. It is worth noting that, for cases having very low stress triaxiality, e.g., $T=1/3$, coalescence cannot occur without secondary voids.

![Fig. 3.7 Comparison of the macroscopic effective stress versus effective strain curves between models including and not including secondary voids.](image)

Fig. 3.7 Comparison of the macroscopic effective stress versus effective strain curves between models including and not including secondary voids.
The calculations presented above only consider the case where the macroscopic stress state subjected by the RMV is axisymmetric, i.e., \( \Sigma_2 \geq \Sigma_1 = \Sigma_3 \) (\( \theta = -30^\circ \)). However, as shown in Kim et al. [13], the Lode angle has significant effect on void growth and coalescence and material failure. Fig. 3.8 shows the variation of \( E_c \) as a function of \( T \) and \( \theta \): \( E_c \) decreases with \( T \) but increases with \( \theta \). It is interesting to note that the curves for \( \theta = -30^\circ, -15^\circ \) and \( 0^\circ \) are very close, which suggests that \( E_c \) can be approximated as a function of only \( T \) when the Lode angle takes values in this range.

After conducting a series of parameter studies, the following general conclusions can be made about \( E_c \): 1) \( E_c \) decreases as \( T \) increases, and the dependency of \( E_c \) on \( T \) is more pronounced in the low stress triaxiality range but saturates as \( T \) increases to higher level; 2) \( E_c \) increases with \( \theta \) in the range \(-30^\circ \leq \theta \leq 30^\circ \), but the change in \( E_c \) becomes less sensitive to \( \theta \) when \(-30^\circ \leq \theta \leq 0^\circ \); 3) \( E_c \) decreases with \( f_0 \), the initial volume fraction of the primary void; 4) Nucleation, growth and coalescence of secondary voids accelerate the ligament failure process and reduce \( E_c \); and 5) \( E_c \) increases with \( W_0 \), the aspect ratio of the primary void. For a given material, the failure criterion can also be expressed as the critical void volume fraction \( (f_c) \) or the critical ligament reduction ratio \( (\chi_c) \) as a function of \( T \) and \( \theta \).
Results displayed in Fig. 3.8 can be represented as 3D plots as shown in Fig. 3.9. The surface representing function $E_c(T, \theta)$ is referred to as the failure surface. Figs 3.9(a) and (b) display the failure surfaces obtained for the two cases, $f_0 = 0.02$ with and without secondary voids, respectively. Therefore, a ductile failure criterion can be established as

$$E_c = E_c(T, \theta)$$

(3.3)

where $E_c$ denote the macroscopic effective strain of the RMV. The RMV fails when $E_c$ reaches a critical value dependent of its stress state characterized by $T$ and $\theta$. 

Fig. 3.8 Variation of $E_c$ as a function of $T$ and $\theta$. 

![Graph showing variation of $E_c$ as a function of $T$ and $\theta$.](image_url)
Fig. 3.9 Material failure surface in terms of $E_c$ as a function of $T$ and $\theta$. (a) $f_0 = 0.02$ and no secondary voids, (b) $f_0 = 0.02$ with secondary voids.

With $\Sigma_2$ being the major stress component, Fig. 3.9 shows only a portion of the failure surface for the range of $-30^\circ \leq \theta \leq 30^\circ$, i.e., $\Sigma_2 \geq \Sigma_3 \geq \Sigma_1$. For the case of $\Sigma_2 \geq \Sigma_1 \geq \Sigma_3$ (corresponding to the range of Lode angle $-90^\circ \leq \theta \leq -30^\circ$), the failure surface is just a mirror image of the portion shown in Fig. 3.9. Fig. 3.10(a) shows a sketch of the failure surface for the range of $-90^\circ \leq \theta \leq 30^\circ$ where $\theta$ covers a 120° span on the $\pi$-plane. This includes all possible stress states of which $\Sigma_2$ is the major stress component.
Fig. 3.10 (a) A sketch of the failure surface $E_c(T, \theta)$ when $\Sigma_2$ is the major stress component. (b) A postulated relationship between $T$ and $\theta$.

The stress state for every RMV in a specimen can be different, each corresponding to a set of $(T, \theta)$ values. Even for the same RMV, the values of $(T, \theta)$ may change during the loading history. Fig. 3.10(b) shows a postulated relationship between $T$ and $\theta$. For each point on the $T$ versus $\theta$ curve, there is a corresponding point on the failure surface indicating the failure strain $E_c$ for this stress state. A locus of failure strain is shown in Fig. 3.10(a).

Bao and Wierzbicki [77] conducted a series of experiments including upsetting tests, shear tests and tensile tests of different specimens. The test results reveal a non-monotonic relationship between the failure strain and the stress triaxiality. The $E_c$ versus $T$ curve has cusps. This phenomenon has been noticed previously, e.g., Werner and Gese [34] reported that fracture strain under equi-biaxial tension ($T = 2/3$) is larger that the fracture strain under plane stress ($T = 1/\sqrt{3}$). These results cannot be explained by
conventional wisdom of failure strain monotonically decreasing with the increase of stress triaxiality. Using the idea of failure strain dependent of both the stress triaxiality and the Lode angle and with the aid of the failure surface as sketched in Fig. 3.10, these experimental results can be easily explained.

A similar failure surface was proposed by Wierzbicki and Xue [34], expressing the fracture strain as a function of both the first and third stress invariants. The features of Wierzbicki and Xue’s failure surface are similar to what was described above because one can easily relate the third stress invariant to the Lode angle.

3.2.4 Fracture toughness

Macroscopic crack initiation is said to have occurred upon coalescence of the growing voids with the crack tip. For the discrete void model described in Section 3.1, it is easier to use the critical ligament reduction ratio ($\chi_c$) as the failure criterion [10, 13]. Similar to $E_c$, $\chi_c$ also depends on the stress triaxiality and the Lode parameter. Construction of the function $\chi_c(T, \Theta)$ for various initial relative void spacing, void shape and void volume fraction follows the same analysis procedures as described in Section 3.2.3. The fracture initiation toughness ($J_{Ic}$) is determined as the applied $J$-value when the reduction of the ligament length between the first void and the crack tip reaches the critical ratio $\chi_c$. 37
Fig. 3.11 Predicted effects of the initial relative void spacing, void pattern, void shape and void volume fraction on the fracture initiation toughness of a structural steel having an intermediate strength and moderate strain hardening.

Our results reveal that $J_{IC}$ decreases with $f_0$. For the same value of $f_0$, $J_{IC}$ is highest when the initial void shape is prolate and lowest when the initial void shape is oblate. Existence of the second row voids and change of void pattern do not result in noticeable difference in $J_{IC}$. However, the initial relative void spacing has significant effect on $J_{IC}$.
Fig. 3.11 shows the effects of the initial relative void spacing, void pattern, void shape and void volume fraction on the fracture initiation toughness of a structural steel having an intermediate strength and moderate strain hardening (Here the material properties used in finite element analyses are $E=200$ GPa, $\nu=0.3$, $\sigma_0=600$ MPa and $N=0.1$). These results can be used to explain why various degrees of fracture toughness anisotropy are observed in industrial alloys.

To model crack growth, the ligament nodes on the symmetry plane are released when the ligament reduction ratio reaches the critical value $\chi_c$. A sudden release of the reaction forces at the ligament nodes causes numerical instability and therefore, several increments are needed to step down the reaction forces to zero. The deformed mesh immediately following the release of the nodal reaction forces of the first ligament is shown in Fig. 3.12. Since the reaction forces, which ensure plastic deformation of the ligament, are removed, a large portion of the released ligament experiences elastic unloading with only the weakest region undergoing severe deformation. As the applied load ($J$) increases, the next ligament reaches the critical reduction ratio and the ligament nodes on the symmetry plane are released. The process continues and the crack front moves forward. The crack growth resistance curve can be obtained by plotting the value of $J$ at which each ligament reaches the failure criterion versus the amount of crack length increase.
3.3 A Numerical Approach to Predict Extensive Ductile Crack Growth

The explicit approach described above provides an effective method for parameter studies to reveal the trends of fracture toughness. However, this approach cannot be applied to predict extensive crack extension in structural components due to the limitation of the current computational power. In this section, we describe a computational approach based on a porous plasticity model and apply it to predict crack propagation in thin panels of a 2024-T3 aluminum alloy. The idea is similar to the computational cell approach used by Xia et al. [17] and Gao et al. [20]. The fracture process zone is modeled using a layer of void-containing cell elements and the GLD model is used to describe the void growth process. The onset of coalescence is defined using the criterion developed in Section 2.3, i.e., $E_e = E_e(T, \theta)$, where $E_e$ represents the effect strain of the cell. The post-coalescence behavior is modeled using an approach similar to that of Tvergaard and Needleman [56]. Finally, the numerical approach is applied to predict crack growth in thin panels of a 2024-T3 aluminum alloy. The expression for $E_e$ is calibrated using experimental data and the calibrated computational model is applied to predict crack extension in fracture specimens having various geometries and subjected to different
loading conditions. The numerical predictions are compared with experimental
measurements.

3.3.1 Modeling the Void Growth Process

Because the fracture specimens contain non-spherical voids, we adopt the GLD
porous plasticity model (Gologanu et al., [21, 22, 23]) to describe the void growth
behavior and the macroscopic plastic response of the RMV. The void geometry is
illustrated in Fig. 3.3. The yield function of the void-containing material is expressed as

\[
\phi = \frac{C}{\sigma^2} \left[ \Sigma^\prime + \eta \Sigma_h n \right]^2 + 2q(g + 1)(g + f) \cosh \left( \kappa \frac{\Sigma_h}{\sigma} \right) - (g + 1)^2 - q^2(g + f) = 0 \tag{4}
\]

where \(\Sigma^\prime\) are the macroscopic stress components, \(f\) represents the void volume fraction, \(S\) is the shape parameter defined as \(S = \ln(W)\) with \(W=R_y/R_\sigma\), and \(\sigma\) is the yield stress of the
matrix material. In Eq. (4), \(\|\|\) denotes the von Mises norm, \(\Sigma^\prime\) is the deviatoric stress
tensor, \(\Sigma_h\) is the generalized hydrostatic stress defined

as \(\Sigma_h = \alpha_2 (\Sigma_{xx} + \Sigma_{zz}) + (1 - \alpha_2) \Sigma_{yy}\), \(\mathbf{X} = (2/3)e_y \otimes e_y - (1/3)e_x \otimes e_x - (1/3)e_z \otimes e_z\),

where \((e_x, e_y, e_z)\) is an orthogonal basis with \(e_y\) parallel to the axisymmetric axis of the
void, and \(\otimes\) denotes tensor product. The parameters \(C, \eta, g, \kappa\) and \(\alpha_2\) are functions of \(f\) and
\(S\) and the heuristic parameter \(q\) depends on initial void volume fraction, strain hardening
exponent of the matrix material, \(S\) and the macroscopic stress triaxiality factor \(T\).

The evolution law for \(f\) due to void growth is determined by requiring the matrix
material to be plastically incompressible and derivations of the evolution equation for \(S\)
can be found in Gologanu et al. [21, 22, 23]. Pardoen and Hutchinson [24] and Benzerga et al. [25] provide detailed descriptions and formulation about the GLD model. Kim and Gao [74] developed a generalized approach to formulate the consistent tangent stiffness for complicated plasticity models. Using this approach, we implemented the GLD model in ABAQUS via a user subroutine. See Kim and Gao [74] for details of the numerical implementation of the GLD model.

3.3.2 Onset of Coalescence

The GLD model provides a constitutive relation to describe void growth and the associated macroscopic softening of the cell element. It accounts for the evolution of both void shape and void volume fraction. However, the GLD model does not supply the stress-strain relation during the void coalescence process. Therefore it is necessary to introduce a criterion for the onset of coalescence and an equation to govern the cell behavior in the post-coalescence regime.

In developing engineering models for ductile crack growth, a critical void volume fraction ($f_c$) is often used to predict the onset of the final void coalescence process and a linear traction versus cell elongation relationship to describe the post-coalescence regime [17-20]. This approximation of the coalescence process greatly simplifies the numerical modeling of ductile fracture. Unfortunately, as shown by Kim et al. [13], $f_c$ is not a material constant. It varies sensitively with material flow properties, initial void volume fraction, as well as the triaxial stress state of the cell. Moreover, in cases where the stress triaxiality is very low, e.g., near the free surface of the specimen or in a very thin
specimen, the void volume fraction does not tend to increase much. Sometimes it never reaches the critical void volume fraction even in an unrealistically large deformation. The (constant) critical void volume fraction criterion cannot be applied in this situation.

In this study, we adopt the coalescence criterion developed in Section 3.2, e.g., coalescence initiates when the macroscopic effective strain \((E_e)\) of the cell reaches the triaxiality and Lode angle dependent critical value \((E_c)\).

3.3.3 A computational approach for the post-coalescence process

We adopt the \(f^*\) function, introduced by Tvergaard and Needleman [56], to account for the effects of rapid void coalescence at failure. After \(E_e\) reaches \(E_c\), \(f\) is replaced by \(f^*\) in the GLD model, where

\[
f^* = \begin{cases} 
    f, & f \leq f_c \\
    f_c + K(f - f_c), & f > f_c
\end{cases}
\]

(3.5)

In Eq. (3.5), \(f_c\) is the void volume fraction at \(E_e = E_c\), \(K = (f_u - f_c) / f_c\), and \(f_u\) is the \(f^*\) value at zero stress. For prolate void, \(f_u = 1/q\), and for oblate void, \(f_u = (1 + g - gq) / q\). Since AQAQUA/Standard does not provide an element removal procedure, a modification to Eq. (5) is needed for numerical stability. Eq. (5) is employed until \(f^* = 0.99 f_u\). Then an exponential function is used such that \(f^*\) gradually approaches to \(f_u\) (but can never reach \(f_u\)).
3.3.4 Simulation of crack growth in thin panels of a 2024-T3 aluminum alloy

The numerical approach described above is applied to predict ductile crack growth in thin aluminum panels. Dawicke and Newman [75, 76], and Werner et al. [79] performed extensive fracture tests on thin panels of a 2024-T3 aluminum alloy including tests of C(T), M(T), and multi-site damage (MSD) specimens with crack planes in both the LT and TL orientations. Fig. 3.13 shows the fracture specimens. The reported data for these tests are mostly for LT specimens and some of them have been analyzed by Gullerud et al. [78] using a Crack Tip Opening Angle (CTOA) criterion to govern crack growth and by Arun et al. [80] and Roychowdhury et al. [81] using cohesive elements to model crack propagation. The test data of our interest are from LT specimens with a sheet thickness of 2.3 mm. The specimens have very stiff guide plates (coated with Teflon tape) to constrain out-of-plane (buckling) displacements. In the L orientation, the 2024-T3 sheet material used in the experiments has a yield stress of 345 MPa, Young’s Modulus of 71.3 GPa, and Poisson’s ratio of 0.3. Quantitative metallographic analyses were performed to determine the inclusion volume fraction, shape and average spacing. It is found that the inclusion volume fraction \( f_0 \) is approximately 0.02, the average spacing between inclusions in the LT plane is about 50 \( \mu \)m, and in LT specimens, the inclusions can be approximated as prolate spheroids with \( W_0 = 4 \). Our finite element analyses use the measured, unaxial true stress versus logarithmic strain curve.
3.3.4.1 Model calibration

To predict crack growth, the function $E_c(T, \theta)$ needs to be determined. The results presented in Section 2 suggest that $E_c$ is not sensitive to $\theta$ when $\theta$ is in the range $-30^\circ \leq \theta \leq 0^\circ$. We perform finite element analyses of the fracture specimens considered in this study and find the $\theta$-values of the representative material volumes ahead of the crack front are in the range of $-15^\circ \leq \theta \leq 0^\circ$. Therefore, we treat $E_c$ as a function of $T$.
only. This greatly simplifies the calibration process. Based on the results presented in Section 3.2, we assume

\[ E_c = \alpha e^{\beta T} \]  

(3.6)

where \( \alpha \) and \( \beta \) are two parameters need to be calibrated using experimental data. A recent study by Bao [82] supports the power-law form of \( E_c(T) \) function defined by Eq. (3.6).

Bao conducted an experimental and numerical study of ductile failure of a 2024-T351 aluminum alloy using different tensile specimens including flat specimens, smooth round bars, notched bars and flat-grooved plates and found that the equivalent strain at failure versus the average stress triaxiality can be characterized by a function in the form of Eq. (3.6).

Two data points are needed to determine \( \alpha \) and \( \beta \). The tensile test provides one point. Fig. 3.14 shows the 1/4 model for the tensile specimen. Fig. 3.15 shows its experimental load-displacement curve. A sudden drop of the load-displacement curve suggests the onset of crack initiation. Both finite element analysis and analytical solution shows the critical element which has the maximum effective is located at the geometry center of the specimen. The stress and strain states for critical element at the crack initiation are obtained through finite element analysis. The triaxiality \( T \) and strain \( \varepsilon_f \) are calculated as 0.45 and 0.5 respectively. Substitution of these values into equation (3.6) yields \( 0.45 = \alpha e^{0.5\beta} \). The next step of the calibration process seeks to match the model predicted load versus crack propagation curve with the experimental measurements for the C(T) specimen. This step entails several finite element crack growth analyses of the C(T) specimen using different values of \( \beta \).
The C(T) specimen has a width of 150 mm with $a/W = 0.33$, where $a$ represents the initial crack length and $W$ represents the specimen width. Due to symmetry, only $\frac{1}{4}$ of the
specimen needs to be modeled. Fig. 3.16 shows the quarter-symmetric finite element mesh of the C(T) specimen having 27,400 eight-node, isoparametric solid elements (with reduced integration). The mesh near the crack front has six layers with varying thickness to capture the stress gradient in the thickness direction, where the thickest elements are at the symmetry plane. The elements directly ahead of the crack front have uniform in-plane dimensions ($L_e = 50 \mu m$) and are governed by the GLD model. All other elements follow $J_2$ flow plasticity. Loading of the C(T) specimen is controlled by prescribing a displacement on a rigid pin through the hole.

Fig. 3.16 A quarter-symmetric finite element mesh for the C(T) specimen. The mesh near the crack front has six layers with varying thickness to capture the stress gradient in the thickness direction. The elements directly ahead of the crack front have uniform in-plane dimensions ($L_e = 50 \mu m$) and are governed by the GLD model.

Fig. 3.17 shows the comparison between the model predicted load versus crack growth curve with the experimental measurements (two sets of experimental data) for
different choices of $\alpha$ and $\beta$, where the lines represent model predictions and the symbols denote experimental measurements. Here $\Delta a$ represent the amount of crack growth measured at the free surface. In the numerical model, the propagating crack front is defined by the elements which have reached the failure strain $E_c$. From Fig. 3.17, it can be seen that the choice of $\alpha = 0.93$ and $\beta = -1.45$ (solid line) results in the best fit to the experimental data. Therefore, these values are the calibrated values for $\alpha$ and $\beta$ and will be used to predict crack growth in other fracture specimens.

![Graph showing model predictions and experimental data](image)

Fig. 3.17 Comparison of the model predicted load versus crack growth curve with the experimental measured data (symbols) showing the choice of $\alpha = 0.1$ and $\beta = -0.5451$ (solid line) results in a best fit to the experimental data.
3.3.4.2 Prediction of crack growth in M(T) specimens

The calibrated computational model is employed to predict the crack extension behavior of M(T) specimens. Three M(T) specimens with $a/W$ ratios of 0.33, 0.42 and 0.5 are analyzed. The element size and arrangement in the region near the crack front are kept the same as used in the C(T) specimen. The nominal remote stress, $\sigma_R$, characterizes the loading for these specimens. Fig. 3.18 compares the computed load versus crack extension responses with experimental measurements, showing very good agreement for all three cases.

3.3.4.3 Prediction of crack growth in MSD specimens

We now apply the calibrated computational model to predict crack growth in MSD specimens. The presence of MSD can significantly influence the residual strength of a
Fig. 3.19 compares the computed load versus crack extension responses with experimental measurements for a MSD specimen containing three cracks as shown in Fig. 3.13(d). This specimen has the same width as the M(T) specimens considered above, $W = 300$ mm. The center-crack length is $a_2 = 100$ mm. The two lead cracks have the same length $a_1 = a_3 = 12.5$ mm. The tip-to-tip distance between the lead crack and the center crack is $b = 12.5$ mm. The model prediction captures accurately the load versus crack extension curve. The cusp on the predicted load versus crack extension curve corresponds to the point when the lead crack and the center crack link up. The center crack in this specimen starts to grow at about the same applied stress level as the M(T) specimen with $a/W = 0.33$. But in the MSD specimen, the center crack and lead cracks link up and fail at a significantly lower load than the single crack specimen.

![Fig. 3.19](image)

**Fig. 3.19** Comparison of the model predicted load versus crack extension responses (line) with experimental measurements (symbols) for a MSD specimen containing three cracks.

The computational model is also applied to predict crack growth in a series of MSD specimens containing three cracks but with various values of $b$. Fig. 3.20 compares the
predicted and measured loads at which the lead cracks and the center crack link up. The computed values and experimental data agree very well.

Fig. 3.20 Comparison of the predicted and measured loads at which the lead cracks and the center crack link up for a series of MSD specimens containing three cracks.

Fig. 3.21 compares the computed load versus crack extension curve with experimental records for a MSD specimen containing two cracks as shown in Fig. 3.13(c). For this specimen, $W = 300$ mm, $a = b = 50$ mm. Fig. 3.22 compares the computed and measured crack link-up loads for a series of two-crack specimens with various $b$-values. Again very good agreements are found between the model predictions and the experimental measurements. This further verifies the proposed computational approach.
Fig. 3.21 Comparison of the model predicted load versus crack extension responses (line) with experimental measurements (symbols) for a MSD specimen containing two cracks.

Fig. 3.22 Comparison of the predicted and measured crack link-up loads for a series of MSD specimens containing two cracks.
3.3.4.4 Prediction of crack growth in a pressurized cylindrical shell structure

To study the influence of a crack on the structural response of the transport fuselage structure, Starnes and Rose [83] conducted a series of pressurized cylindrical shell tests. The cylindrical shells were fabricated from 1mm-thick 2024-T3 aluminum alloy sheet, with the rolling direction orientated circumferentially. A longitudinal crack is located at the mid-length of the cylindrical shell. Here we apply the above calibrated computational model to predict the crack growth behavior in the pressurized cylindrical shell structure. Although the shell thickness is different from the C(T), M(T) and MSD specimens analyzed above, we assume that the microstructure and flow properties of the material remain the same. The specimen under our consideration has a radius of 230 mm and
length of 914 mm. The initial crack length is \( a = 25 \) mm. Fig. 3.23(a) shows the finite element mesh of the cylindrical shell. Due to symmetry of the geometry and loading conditions, only a quarter of the shell is modeled. Ten layers of eight-node brick elements are employed in the wall thickness direction. The element dimensions in the crack opening direction and crack growth direction are kept the same as used in the C(T), M(T) and MSD specimens in the crack front region. The pressure load is applied on the inner surface of the shell. Fig. 3.23(b) shows that the predicted load versus crack growth response agree well with the experimental measurements.

3.4 Concluding Remarks

Two approaches, one uses refined finite elements to represent microvoids in the material and the other uses porous plasticity models to describe the behavior of the void-containing material, are discussed in this chapter. A distinct advantage of the first approach is the exact implementation of the void growth behavior. It provides an effective method to study the mechanisms of ductile fracture and to analyze the trends of fracture toughness. With model parameters being properly calibrated, the second approach can be used to predict extensive crack extension in fracture specimens.

Our analyses re-affirm the two distinct void growth mechanisms put forth by Tvergaard and Hutchinson [8], i.e., void-by-void growth mechanism for materials containing small initial void volume fractions and multiple void interaction mechanism for materials containing large initial void volume fractions. Effects of the initial relative
void spacing, void pattern, void shape and void volume fraction are investigated in this study.

In order to simulate crack formation and propagation, a criterion for void coalescence is required. The critical strain at the onset of void coalescence depends on material flow properties and microstructural properties. It also depends on the stress state. Two stress parameters, the stress triaxiality ratio ($T$) and the Lode angle ($\theta$) can be used to characterize the effect of the macroscopic stress state on the void growth and coalescence process in the representative material volume (RMV). We obtain the failure criterion for the RMV in terms of the macroscopic equivalent strain ($E_c$) as a function of $T$ and $\theta$ by conducting systematic finite element analyses of the void-containing RMV subjected to different macroscopic stress states. A series of parameter studies are conducted to examine the effects of the shape and initial volume fraction of the primary void and nucleation, growth and coalescence of the secondary voids on the predicted failure surface $E_c(T, \theta)$.

Using the small scale yielding, boundary layer model with discrete voids represented in the crack tip region by refined finite element mesh, the effects of the initial relative void spacing, void pattern, void shape and void volume fraction fracture initiation toughness are investigated. The results can be used to explain the observed fracture toughness anisotropy in industrial alloys. With the aid of a node-release technique, crack growth and fracture resistance curve can be predicted.

Finally a numerical approach is proposed to predict extensive ductile crack growth in structural materials. The computational model is applied to predict crack growth in thin
panels of a 2024-T3 aluminum alloy, where the GLD porous plasticity model is used to describe the void growth process and a \( f^* \) function is employed to account for rapid material failure in the post-coalescence process. For the specimens considered in this study, it is found that the Lode angle is in the range of \(-15^\circ \leq \theta \leq 0^\circ\). The explicit void modeling analysis shows that critical strain is insensitive to Lode angle in this range. Consequently, the critical strain at the onset of void coalescence can be approximated as a function of the stress triaxiality ratio only and a procedure to calibrate \( E_c(T) \) is presented. The calibrated computational model accurately predicts crack extension in a series of fracture specimens having various initial crack configurations, including the compact tension specimen, middle crack tension specimens, multi-site damage specimens and the pressurized cylindrical shell specimen.
CHAPTER IV

EFFECT OF THE STRAIN STATE ON THE CRITICAL STRAIN TO DUCTILE FAILURE

4.1 Material failure criterion

Macroscopic crack initiation is said to have occurred upon coalescence of the growing voids with the crack tip. Several mechanistic observations have been put forth to explain void coalescence. Coalescence can occur through shear band formation, or through formation of “void sheets”, or through impingement of neighboring voids. However, it is very difficult to implement these coalescence mechanisms directly to the numerical model. Alternatively, several criteria based on the length of the ligament relative to the size of the void have been proposed.

Probably the most widely used coalescence criterion is the critical void volume fraction. This criterion assumes coalescence stages is reached when the volume fraction of the cell reaches a certain value, \( f_c \), no matter what the stress state condition is on a unit voided cell. This criterion was incorporated in the GT model and was successfully used in the simulation of crack growth in thick specimens. Because of its simplicity, it is widely applied in both theoretical analyses and practical applications [20, 46]. But there is a distinct disadvantage in this criterion. In the case of very low stress triaxiality state,
which usually occurs in a very thin plate or near the surface of a thick specimen, void volume fraction in the material does not increase much. Sometimes it never reaches the critical void volume fraction even in an unrealistically large deformation. Therefore, critical void volume fraction criterion cannot be applied in this kind of problems.

Rice and Johnson [84] suggested coalescence to occur when

\[ \gamma = \frac{L_x}{V_y} \]  

(reaches a critical value, where \( L_x \) is the length of the deformed ligament between the void and crack tip, and \( V_y \) is dimension of the void in the crack opening direction. Brown and Embury [85] proposed void coalescence to occur through the conjoint influences of shear band development and void impingement when the ratio of ligament length to the void dimension reached a critical value. In a companion study, Le Roy et al. [86] proposed an empirical relationship for void coalescence to occur only when

\[ \phi = \frac{2R_3}{L_x} \]  

(reaches a critical value, where \( 2R_3 \) is dimension of the void along the longest axis (major axis). Based on experimental data the value of \( \phi \) was suggested as 0.83 for the case of spherical voids. In their recent paper, Tvergaard and Hutchinson [8] used the ligament reduction ratio

\[ \chi = \frac{L_x}{L_0} \]  

as the parameter for the failure criterion. Here \( L_0 \) is the size of the initial ligament between the growing void and the advancing crack tip and \( L_x \) is the size of the deformed
ligament. In their study, a critical ligament reduction ratio \((\chi_c)\) of either 1/2 or 1/3 was used.

These simple geometry-based criteria can be easily incorporated into finite element analysis. However, recent micromechanics analyses by Kim et al. [9] show that neither \(\gamma\), or \(\phi\), or \(\chi\) can be taken as a constant.

Much attention has been paid to the effect of the hydrostatic pressure on fracture since the time of Bridgman [33]. Several types of empirical fracture criteria were proposed based on tests conducted in a limited range of stress triaxiality. Some of the most cited criteria are due to Johnson and Cook [60], Chauuadi et al. [87], Hancock and Mackenzie [88], Fischer et al. [89], Ganser et al. [90] and Norris [91] etc. Many investigators offered various plasticity based theoretical models which provided unique functional dependence of the fracture strain on the stress triaxiality (McClintock [54], Rice and Tracy [57]). A common feature of almost all the above mentioned papers is that the material ductility is a monotonically decreasing function of the stress triaxiality. Further more, most of the tests were performed under tensile loading of unnotched bar, notched bars and flat dog-bone specimens, all in the range of high stress triaxiality. While these criteria may give good prediction for some limited triaxiality range, it clearly lacks generality. Test results by Bao and Wierzbicki [77] reveal a non-monotonic relationship between the failure strain and the stress triaxiality. Werner and Gese [79] reported that fracture strain under equi-biaxial tension \((T = 2/3)\) is larger that the fracture strain under plane stress \((T = 1/\sqrt{3})\). These results suggest that the failure strain cannot be generalized as a monotonic function of stress triaxiality.
Kim et al. [9, 13] and Gao et al. [10, 27] have demonstrated that both the stress triaxiality ratio and the Lode parameter have significant effects on the ductile failure process. Wierzbicki and Xue [34] proposed a similar failure surface expressing the fracture strain as a function of both the first and third stress invariants. The features of Wierzbicki and Xue’s failure surface are similar to what was described by Kim and Gao because one can easily relate the third stress invariant to the Lode angle. A much earlier effort to make the fracture locus depend on the deviatoric state was by Wilkins et al [58]. The failure locus proposed by and Xue [34] postulates that the critical fracture strain depends on a joint action of the stress triaxiality and the deviatoric state according to

\[ \bar{\varepsilon}_f (\eta, \xi) = (C_1 e^{-C_2 \eta}) - [(C_1 e^{-C_2 \eta}) - (C_3 e^{-C_4 \eta})](1 - \xi^2) \]  

(4.4)

where \( C_1, C_2, C_3, C_4 \) are constants. Some concepts and definitions are as follows:

\[ I_1 = \sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \]  

(4.5)

where \( \sigma_1, \sigma_2, \sigma_3 \) are principal stresses.

The invariants of the deviatoric stress space are defined as:

\[ J_1 = \frac{1}{3}(s_1 + s_2 + s_3) = 0; \]

\[ J_2 = \frac{1}{2}(s_1^2 + s_2^2 + s_3^2); \]  

(4.6)

\[ J_3 = s_1 s_2 s_3 \]

Equivalent strain is: \( \bar{\varepsilon} = \sqrt{\frac{2}{3}} \sqrt{\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2} \), where \( \varepsilon_1, \varepsilon_2, \varepsilon_3 \) are principal stress. The equivalent stress (von-Mises stress) \( \bar{\sigma} = \sqrt{3J_2} \).
The dimensionless invariants are defined by \( \eta = \frac{\sigma_m}{\sigma} \) and \( \xi = \frac{27}{2} \frac{J_3}{\sigma^3} \). The variable \( \xi \) is related to the azimuth angle \( \theta_A \) which is a shifted Lode angle \( \theta_L \) by \( \pi/6 \) through

\[
\xi = \cos(3\theta_A) = \frac{3\sqrt{3}}{2} \frac{J_3}{J_2^{3/2}} = \frac{27}{2} \frac{J_3}{\sigma^3}
\]

The four parameters \( C_1, C_2, C_3, C_4 \) can be calibrated using experiment data. For example, \( C_1, C_2 \) can be decided from axisymmetric tests in which there is no loss of ductility due to the deviatoric state and \( \xi = 1 \). While \( C_3, C_4 \) can be obtained from test of the pure shear and transverse plane strain specimens of which \( \xi = 0 \).

A typical failure surface, or the so-called fracture locus as called by Xue and Wierzibicki, which is a function of both \( \eta \) and \( \xi \), has the shape as shown in Fig. 4.1.

Fig. 4.1 A Typical Xue- Wierzibicki fracture locus
4.2 Numerical simulation using the new failure criteria.

Xue and Wierzibicki [34] conducted fifteen representative tests and performed finite element analysis using X-W criterion. The match between prediction and experiment is surprisingly good. The addition of one more variable $\xi$ to the X-W fracture locus explained all experimentally observed features that the existing pressure dependent formulation of ductile fracture was unable to interpret [77, 92]. For example, the X-W fracture criterion predicts low failure in plane strain than in equibiaxial tension, which is consistent with tests [93]. The conventional pressure dependent ductility curve predicts the opposite.

As indicated in Chapter III, our numerical analysis suggests a exponential variation of the critical strain with stress triaxiality which coincides with the X-W fracture criterion. In this chapter we attempt to explore the effect of Lode angle on failure strain. We’ll adopt the function form proposed by Xue and Wierzibicki to describe the dependence of failure strain on the stress state and implement it into ABAQUS. We will use the GLD model to describe the material behavior damage evolution due to void growth and the stress state dependent failure criterion to predict the onset of ductile fracture. We follow the exact procedure of local approach to fracture method [94]: Firstly the model is calibrated using experimental results, and then the calibrated model will be used to predict fracture behavior for other specimens with the same material.
4.2.1 Experiment details

All the tests were done by NSWCCD (Naval Surface Warfare Center, Carderock Division). Test specimens include notched round bar with different notch radius, notched plane strain specimen with different notch radius, plane strain specimen, plane stress specimen, plane strain specimen with a hole at its center, plane stress specimen with two openings.

Fig. 4.2 shows the sketch of the notched round bar. All the notched round bars have the diameter of 0.176 inch at their notched cross section. The notch radii are 0.088 inch, 0.176 inch, 0.044 inch, 0.352 inch for specimen A, B, D, E respectively. Fig.4.3 is for notched plain strain specimens. The overall block has the dimension of 3.93”×1.18”×0.197”. The thickness of all the plain strain specimens at their thinnest cross section is 0.063 inch. Three notch radiuses are 0.063 inch, 0.157 inch, and 0.5 inch respectively.
Fig. 4.2 Geometries for notched round bars with different notch radii

Fig. 4.3 Notched Plane strain specimens with different notch radii

Fig. 4.4 (a) shows the geometry for the plane stress specimens. Fig. 4.4 (b) shows a modified plane stress specimen having two parallel notches. The depth of the opening is
0.483 inch. The distance between the two notches is 0.25 inch. Thickness for both specimens at the middle part is 0.08 inch.

Fig. 4.4 (a) Plane stress specimen. (b) Modified plane stress specimen

Experiment data for a plane strain specimen and a plane strain with a hole at its center are also available. Fig. 4.5 shows the geometry of both specimens. The two specimens have exactly the same size except that one has a hole at the center and another one does not. The global block has the dimension of \(6" \times 1.75" \times 0.5"\).

Fig. 4.5 (a) Plane strain specimen. (b) modified plain strain specimen.
Fig. 4.6 is the dimension for the torsion specimen. Strain gages were applied on opposite sides of the diameter in the gage section and were used to measure shear strains at early loadings. At higher loads, strain is calculated based on the measured angular displacement.

The material is a DH 36 steel. The micro-structural properties are obtained by analyzing micrograph of polished and etched surface using image processing program. The initial void volume fraction $f_0$ is 0.02%, the void has the oblate shape with the aspect 0.6. The average void spacing is around 80 μm.

All the tests are performed at room temperature and are considered to be quasistatic tests due to the approximate strain rates of $10^3 \text{s}^{-1}$. The stress strain curve is obtained by the standard round bar specimen. The material parameters are $E=2.9E7 \text{ psi}$, $\nu=0.3$. The true stress-strain response is listed in table 4-1.
Table 4-1 The true stress-plastic strain curve of a DH 36 steel

<table>
<thead>
<tr>
<th>True stress $\sigma$ (psi)</th>
<th>Plastic strain $\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>46000</td>
<td>0</td>
</tr>
<tr>
<td>61000</td>
<td>0.01491</td>
</tr>
<tr>
<td>66793</td>
<td>0.0209</td>
</tr>
<tr>
<td>72572</td>
<td>0.03057</td>
</tr>
<tr>
<td>77138</td>
<td>0.04148</td>
</tr>
<tr>
<td>80711</td>
<td>0.05343</td>
</tr>
<tr>
<td>83331</td>
<td>0.06464</td>
</tr>
<tr>
<td>87155</td>
<td>0.08789</td>
</tr>
<tr>
<td>89821</td>
<td>0.11073</td>
</tr>
</tbody>
</table>

4.2.2 Model calibration

The micro-structural analysis shows the inclusions in the material are non-spherical. Therefore, the GLD porous plasticity model is adopted to describe the void growth behavior and the macroscopic plastic response. The X-W failure criterion and the GLD model are implemented in ABAQUS through a user subroutine UMAT.

The experimental true stress-strain response is further fitted into a power-law curve as follows

$$
\varepsilon = \frac{\sigma}{E} \quad \sigma \leq \sigma_0
$$

$$
\varepsilon = \left(\frac{\sigma}{\sigma_0}\right)^{1/N} \quad \sigma > \sigma_0
$$

(4.8)

The material constants are taken to be $E=2.9E7$ psi, $\sigma_0=46000$ psi, $\nu=0.3$ and $N=0.148$.

$C_1$, $C_2$, $C_3$ and $C_4$ are calibrated as follows. We firstly perform a 3D finite element analysis taking no consideration of void coalescence for all the specimens needed for
calibration. The load-displacement curve from the experiment and the one from finite element analysis show good agreement from beginning of loading to until an obvious drop down occurs at the experimental load-displacement curve, at which point the crack is believed to initiate. The stress and strain states at the critical local location are then obtained at that particular load level. The effective strain, Lode angle and triaxiality are subsequently calculated and applied to the X-W failure criterion. We use the notched round bar specimens to calibrate parameters $C_1$ and $C_2$. As for the notched round bar, $\xi=1$. The X-W failure criterion changes into $\bar{\varepsilon}_f(\eta, \xi) = C_1 e^{-C_2 \eta}$. Only two data points are needed to calibrated $C_1$ and $C_2$. The notched plane strain specimens and the torsion specimens are used to calibrate $C_3$ and $C_4$ since $\xi=0$ applies and the failure criterion is $\bar{\varepsilon}_f(\eta, \xi) = C_3 e^{-C_4 \eta}$. Another two data sets are needed to obtain $C_3$ and $C_4$. We have four specimens for the notched round bar and three notched plane strain specimens which is more than what is required to get the parameters $C_1$, $C_2$, $C_3$ and $C_4$. To take into consideration of the scatter and variation of the experiment result, a lease square return method is used to decide the four constants.

Table 4-2 shows the stress and strain states for the notched round bar specimens from finite element analysis.

<table>
<thead>
<tr>
<th>Notched Round Bar</th>
<th>$\xi$</th>
<th>$\eta$</th>
<th>$\varepsilon_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R=0.044&quot;</td>
<td>1</td>
<td>1.319</td>
<td>0.34</td>
</tr>
<tr>
<td>R=0.088&quot;</td>
<td>1</td>
<td>1.06</td>
<td>0.55</td>
</tr>
<tr>
<td>R=0.176&quot;</td>
<td>1</td>
<td>0.933</td>
<td>0.6938</td>
</tr>
<tr>
<td>R=0.352&quot;</td>
<td>1</td>
<td>0.8995</td>
<td>0.881</td>
</tr>
</tbody>
</table>
The least square return method gives $C_1=5.29$, $C_2=2.1$.

Table 4-3 shows the stress and strain states for the notched plane strain specimens and torsion specimen. For the torsion specimen, the experiment result gives the relation of torsion and twist angle which is converted to stress and strain relation by the method described by Lindholm [95]. The shear stress is calculated as $\tau=T/AR$ and shear strain is $\gamma=R\theta/L$. $T$ is the applied torque, $A$ and $R$ are the cross-section area and mean radius. $L$ is the fixed axial distance between the ends of the specimen rotated through the angle of twist $\theta$. $A$, $R$ and $L$ are constant during the deformation.

Table 4-3. Stress and strain states for notched plane strain and torsion specimens

<table>
<thead>
<tr>
<th></th>
<th>$\xi$</th>
<th>$\eta$</th>
<th>$\epsilon_\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Notched Plane Strain Specimens</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R=0.063''$</td>
<td>0</td>
<td>1.03</td>
<td>2.4</td>
</tr>
<tr>
<td>$R=0.157''$</td>
<td>0</td>
<td>0.97</td>
<td>0.96</td>
</tr>
<tr>
<td>$R=0.5''$</td>
<td>0</td>
<td>0.9</td>
<td>0.85</td>
</tr>
<tr>
<td>Torsion Specimen</td>
<td>0</td>
<td>0</td>
<td>0.68</td>
</tr>
</tbody>
</table>

The least square return method gives $C_3=2.43$, $C_4=1.13$. The X-W failure locus is

$$\bar{\epsilon}_f(\eta,\xi) = (5.29e^{-2.1\eta}) - [(5.29e^{-2.1\eta}) - (2.43e^{-1.13\eta})](1 - \xi^2)$$ (4.9)

Fig 4.7 shows the failure surface generated for Eq. (4.9)
4.3 Model prediction

Once $C_1$, $C_2$, $C_3$ and $C_4$ are calibrated, the failure criterion is implemented into ABAQUS through user subroutine UAMT. Detailed nonlinear finite element analyses for all other specimens are performed. The comparison between the experiment and the finite element analysis result is made.

All the specimens contain one layer of GLD elements at the plane of its geometric center where crack will mostly likely initiate. The GLD element has a size of 80 μm. Due to the sizable difference of the size of GLD element and the specimen, lots of work is required in modeling the specimen. Numerical simulations for all the specimens under either tensile loading or torsion are performed using ABAQUS STANDARD.
displacement control is applied for all specimens. At each time increment the failure criteria is used to evaluate failure status of the GLD element. If the effective strain of the GLD element is greater than the one from the X-W failure criteria, the procedure same as discussed in section 3.3.3 will be followed.

Fig. 4.8 shows mesh for notched round bar specimens. Axisymmetry is considered and a 2D element CAX4R is used. For torsion specimen, the element type CGAX4R which has an additional degree of freedom to handle twist is used. Element type C3D8R is applied to all the other specimens. For the specimens modeled with 3D elements, the symmetry is considered and usually only 1/8 or 1/4 of the specimen is meshed. A typical model contains 30000 - 50000 nodes and 20000 - 40000 elements.

![Fig. 4.8 finite element meshes for notched round bar specimens.](image)

For consideration of accuracy and computational efficiency, the element type in all the analysis is first-order, reduced integration (CAX4R, CGAX4R and C3D8R). Since
the elements have only one integration point, it is possible for them to distort in such a way that the strains calculated at the integration point are zero, which in turn, leads to uncontrolled distortion of the mesh. This is the so called hourglass modes [96] in ABAQUS. An hourglass control applied used by using *HOURGLASS STIFFNESS in ABAQUS input file to provide increased resistance to hourglassing for all the analyses considered.

4.4 Result and discussions

The finite element analysis results are carefully examined. The energy balance is checked for every case. Fig. 4.9 shows the Mises stress distribution at the initiation of crack. The maximum Mises stress is slightly more than 2 times of the yield stress. Fig. 4.10 show the comparison of the deformed and undeformed shape for the GLD element located at the center of the bottom plane of notched round bar specimen A.

Fig. 4.9  Mises stress distribution for notched round bar specimen A (R=0.088")
Fig. 4.10 Deformed and undeformed shape of GLD element at onset of coalescence

Fig. 4.11 shows the comparison of the elemental Mises stress-effective plastic strain curve with the experimental true stress-plastic strain curve. The two curves match each other until coalescence occurs. And the element lost its load carrying capacity shortly after its coalescence, as can be seen from the Mises-effective strain curve. This is realized by artificially increase its void volume fracture in ABAQUS user subroutine UMAT.
The energy output is specified and energy balance is checked for each case. The energy variable involved includes plastic strain energy, elastic strain energy, external work, damage dissipation, energy to control spurious modes (hourglass control). The total energy balance is calculated as:

$$E_{\text{total}} = E_S + E_C - E_W$$  \hspace{1cm} (4.10)

where $E_{\text{total}}$ is the total energy balance, and $E_S$ is the strain energy, $E_C$ is the energy used to control spurious modes and $E_W$ is external work.

For all the cases the energy balance is less than 0.3% of the external work. Considering the round off error during computations, we think energy balance is achieved.

The load-displacement curves from both numerical and the experimental result are plotted for each specimen. Fig. 4.12 - Fig. 4.18 show the correlation of the experimental and numerical results in terms of the load-displacement relation for all the cases studied. As we can see that for all the cases, the numerical prediction gives almost a perfect match. In addition to the load-displacement curve, the finite element analysis predicts the crack initiation location which is also observed from the experiment.

It’s worth mentioning that when we calibrate $C_1$, $C_2$, $C_3$ and $C_4$, we take it for granted that the crack initiated when there is a significant load drop in the experimental load-displacement curve. Under certain circumstances this might not be true since the load can still increase even though the crack has initiated in the structure due to material hardening. However, for all the specimens we considered the crack grows very rapidly during the test. The finite element analysis also shows a rapid crack growth. Therefore, it
is reasonable to take the beginning of the drop as an approximate indication of the onset of fracture since the two stages are very close to each other.

Fig 4.12 Finite element prediction and experimental load-displacement curve for notched round bar specimens

Fig. 4.13 Finite element prediction and experimental load-displacement curve for plane strain specimens
Fig. 4.14 Finite element prediction and experimental load-displacement curve for torsion specimen

Fig. 4.15 Finite element prediction and experimental load-displacement curve for plane stress specimen
Fig. 4.16 Finite element prediction and experimental load-displacement curve for notched plane stress specimen

Fig. 4.17 Finite element prediction and experimental load-displacement curve for Plane strain specimen
4.5 Conclusion

In this chapter, we focus on development of a material failure criterion for ductile fracture model. The weak point of the existing criteria such as the critical void volume fraction $f_c$, critical ligament reduction ratio $\chi_c$ et. al is that they can’t be taken as constants as they proposed. The role of stress triaxiality and Lode angle in a failure criterion is considered. Our numerical analysis suggests the failure strain is a function of triaxiality in its exponential form. Similar conclusion is reached by Xue and Wierzbicki. We then take the X-W criterion to describe the dependence of the failure strain on stress triaxiality and the third invariant of the deviatoric stress. We use the GLD model to describe the plasticity behavior and damage evolution. The failure criterion and the GLD model are
implemented into a through ABAQUS user subroutine UMAT. Four constants $C_1$, $C_2$, $C_3$ and $C_4$ are calibrated using notched round bar specimens, notched plane strain specimens and torsion specimen. The calibrated failure model is then used to predict load-displacement response for plane strain specimen, plane strain specimen with a hole at its center, plane stress specimen, and plane stress specimen with two parallel notches. The finite element result agrees surprisingly well with the experiment result, which suggests the potential of application GLD model with the combination of the failure criteria in real engineering problems.
5.1. Introduction

The potential for catastrophic failure initiated by cleavage fracture remains a key element in fitness-for-purpose assessments of high-performance structures constructed of ferritic steels. Cleavage fracture in ferritic steels has been attributed primarily to slip-induced cracking of grain boundary carbides, followed by unstable propagation of the microscopic cracks into the surrounding ferritic matrix [37, 38]. Experimental studies reveal that the process becomes strongly driven by a (often single) critical event at the metallurgical scale that triggers macroscopic brittle fracture. Therefore, cleavage fracture in the ductile-to-brittle transition (DBT) region exhibits a “weakest link” phenomenon [39, 40, 41]. The carbide particles, which are “eligible” for initiating cleavage fracture, are those under sufficient tensile stress, with orientations favorable for nucleating microscopic cracks, and favorable for producing high enough energy release rate. Due to the highly localized nature of the failure mechanism coupled with micro-structural inhomogeneity of the material, cleavage fracture toughness data often exhibits a considerable amount of scatter, a dependence on length of the crack front, and a strong sensitivity to local stress and deformation fields. Furthermore, cleavage fracture toughness also displays strong dependencies on temperature, loading rate, pre-straining,
etc. All these complications greatly increase the difficulty to apply the laboratory measured toughness values in reliability assessments of structural components.

The significance of cleavage fracture behavior has stimulated an increasing amount of research in the past few decades. These research efforts have led to a quantitative understanding of the scatter and temperature dependence of macroscopic fracture toughness (in terms of $J_c$ or $K_{Jc}$) under high constraint, small scale yielding (SSY) conditions. Scatter of the SSY toughness data can be described by a three-parameter Weibull distribution, where the Weibull modulus for $K_{Jc}$ distribution is 4 and the minimum fracture toughness for common ferritic steels is $K_{min} \approx 20 \text{MPa}\sqrt{\text{m}}$ [42, 97]. This three-parameter Weibull distribution has been adopted in ASTM E1921 [98]. E1921 also adopts a so-called “master curve”, empirically derived by Wallin and others [99, 100], to describe the dependence of cleavage fracture toughness on temperature for ferritic steels in the DBT region. The master curve defines the median fracture toughness at any temperature in the DBT region under SSY conditions while in engineering applications the crack front often experiences constraint loss. This motivates development of micromechanics-based models to address the transferability of cleavage fracture toughness across varying levels of crack-front constraint. The Weibull stress model, originally proposed by the Beremin group [41] based on weakest link statistics, provides a framework for quantifying the relationship between macro-scale and micro-scale driving forces for cleavage fracture. The introduction of the so-called Weibull stress, calculated by integrating a weighted value of the maximum principal stress over the fracture process zone, provides the basis for generalizing the concept of a
probabilistic fracture parameter and supports the development of procedures that adjust toughness values across different crack configurations and loading modes (tension vs. bending). The Beremin model has two material parameters, the Weibull modulus ($m$) and the scale parameter ($\sigma_u$). Gao, et al. [47] proposed a procedure to calibrate parameters of the Weibull stress model using fracture toughness data obtained from two sets of test specimens that exhibit different constraint levels at fracture. They also introduced a threshold Weibull stress value ($\sigma_{w-min}$) corresponding to $K_{min}$. Using the three-parameter Weibull stress model with parameters calibrated according to the proposed procedure, Gao, et al. [48] predicted the distributions of measured fracture toughness values in various specimens of an A515-70 pressure vessel steel, including surface crack specimens subject to different combinations of bending and tension.

Recent efforts on the Weibull stress model have focused on effects of temperature, loading rate, etc. on the model parameters. Gao, et al. [49, 101] studied effects of loading rate on the Weibull stress model for the A515-70 steel and found that $m$ is rate independent over the range of low-to-moderate loading rates. The dependency of $\sigma_u$ on loading rate characterizes the effect of loading rate on fracture toughness. Petti and Dodds [50] argued that $m$ is independent of temperature and $\sigma_u$ increases with temperature to reflect the increase of microscale toughness of ferritic steels. They proposed a procedure to calibrate the variation of $\sigma_u$ with temperature using the master curve. Recently Faleskog, et al. [102] proposed a probabilistic model similar to the Weibull stress model and studied the effects of temperature and constraint on fracture toughness. In this model, a material length-scale was introduced and the effect of plastic deformation was considered. Gao, et al. [103] modified the Weibull stress model to
include the influences of plastic strain and stress triaxiality in the crack tip region. Using the new model, Gao, et al. [103] reanalyzed the experiments presented in [48] and found the predictions were improved, especially for specimens having low constraint levels.

This chapter presents a further study of the modified Weibull stress model where the roles of plastic strain and stress triaxiality at the crack tip are considered. We first review the Beremin model [41] and discuss why and how it is modified to account for the effects of plastic deformation and stress triaxiality. Next we introduce the threshold Weibull stress value ($\sigma_{w-min}$) and outline the procedures to calibrate the Weibull stress model parameters. After a brief summary of the numerical procedures and the finite element models, we calibrate the model parameters at different temperatures using the fracture toughness data of an A508 pressure vessel steel obtained by Faleskog, et al. [102] from deep-cracked ($a/W=0.5$) and shallow-cracked ($a/W=0.1$) SE(B) specimens. The calibrated model is verified by comparing the predicted cumulative failure probabilities with the rank probabilities of experimental data for not only the $a/W=0.5$ and $a/W=0.1$ specimens but also the $a/W=0.25$ specimens. Our results suggest that the modified Weibull stress model with a temperature independent Weibull modulus ($m$) provides an effective tool to predict cleavage fracture. The other two parameters are functions of temperature — $\sigma_u$ increases with temperature while $\sigma_{w-min}$ decreases with temperature.

5.2 Probabilistic treatment of cleavage fracture

Cleavage fracture can be simplified as a two-step process. At first the fine microscopic cracks are formed due to the inhomogeneous distribution of plastic
deformation. In mild steels, the grain boundary carbide particles are the potential sites for the initiation of microscopic cracks [37, 38, 39]. The second step is the propagation of these fine microscopic cracks. The critical stress required to propagate a microscopic crack can be related to the size of the microscopic crack through the Griffith criterion. Assuming the probability density function for the micro-cracks having size \( a \) has the form \( f(a) = c / a^\gamma \), where \( c \) is a constant and \( \gamma \) defines the shape of the micro crack distribution, Beremin [5] derived a two-parameter Weibull distribution to describe the cumulative failure probability based on the weakest link statistics

\[
P_f(\sigma_u) = 1 - \exp \left( - \left( \frac{\sigma_w}{\sigma_u} \right)^m \right) \quad \text{(5.1)}
\]

In this expression \( \sigma_u \) represents the Weibull stress defined as the integral of a weighted value of the maximum principal (tensile) stress \( (\sigma_1) \) over the process zone for cleavage fracture (i.e., plastic zone at the crack tip)

\[
\sigma_w = \left[ \frac{1}{V_0} \int_{V_p} \sigma_1^m dV \right]^{1/m} \quad \text{(5.2)}
\]

In Eq. (5.2), \( V_p \) represents the volume of the cleavage fracture process zone, \( V_0 \) is a reference volume and \( m \) denotes the Weibull modulus which defines the shape of the probability density function for microscopic cracks in the fracture process zone, \( m = 2\gamma - 2 \). In Eq. (5.1), \( \sigma_u \) represents the scale parameter of the Weibull distribution and defines the micro scale material toughness when cumulative failure probability is 63.2%.
In the context of probabilistic fracture mechanics, the Weibull stress, $\sigma_w$, thus emerges as a near-tip fracture parameter to describe the coupling of remote loading with a micromechanics model which incorporates the statistics of microcracks (weakest link philosophy). The Weibull stress model has been widely employed to predict cleavage fracture in ferritic steels. Despite the apparent success, Gao, et al. [48] noticed that the predicted failure probabilities for surface-cracked specimens under tension-dominated loading are less accurate than for other crack configurations. The stress level ahead of the crack front in these low constraint configurations is low but the plastic zone is large. This motivates us to consider the effect of plastic strain on cleavage failure.

In derivations of Eq. (5.1) and Eq. (5.2), it was assumed that microcracks nucleate at carbide particles once plasticity develops, i.e., all carbides become microcracks in the plastic zone. Experiments by Lindley, et al. [104] and Gurland [68] have shown that the number of microscopic cracks initiated at carbide particles increases with an increase in plastic strain. Thus, the probability density function for size of the microscopic crack assumed by Beremin [41] should be modified to include the effect of plastic strain

$$f(a) = g(\varepsilon_p) a^{-\gamma}$$

(5.3)

where $\varepsilon_p$ represent the local plastic strain. Specific forms of $g(\varepsilon_p)$ require comprehensive experimental research. From the work of Lindley [104] and Gurland [68], we make the following simple approximation

$$g(\varepsilon_p) = c \varepsilon_p$$

(5.4)

where $c$ is the coefficient of the approximated linear function.
Following the same procedure outlined by Beremin [41] in developing the original Weibull stress model, we obtain a modified expression for the Weibull stress as

\[
\sigma_w = \left[ \frac{1}{V_0} \int_{V_p} \epsilon_p \sigma_i^m \, dV \right]^{1/m},
\]

where the coefficient \(c\) in Eq. (4) is combined into \(\sigma_u\).

For the microscopic crack to propagate, the stress triaxiality (\(\sigma_m / \bar{\sigma}\)) at the crack tip must remain above a certain level [105], where \(\sigma_m\) represents the hydrostatic stress and \(\bar{\sigma}\) represents the current yield stress. In Fig. 5.1 is shown the distributions of (a) maximum principal stress, (b) effective plastic strain, and (c) stress triaxiality ahead of the crack tip under the plane strain, SSY conditions. The material properties used in the computation are those of the A508 steel. From Fig. 1 is seen there exists a large deformation zone at the crack tip region and inside the large deformation zone the stress triaxiality becomes very low. As a result, the microscopic cracks in this region tend to blunt and incapable of causing cleavage fracture. Therefore, this region should be excluded in the calculation of Weibull stress. We set a threshold value for \(\sigma_m / \bar{\sigma}\) and exclude the region in the large deformation zone where \(\sigma_m / \bar{\sigma}\) is below the threshold value. Our numerical analyses suggest that an appropriate threshold value for \(\sigma_m / \bar{\sigma}\) is around 1.5 [103].
The Weibull stress originally defined by Eq. (5.2) scales with $J^{m}$ under plane strain, SSY conditions, where $J$ represents the $J$-integral [47, 106]. It can be easily shown that the modified Weibull stress defined by Eq. (5.5) does not change this scaling between $\sigma_w$ and $J$. Fig. 5.2 shows the numerical results of $\sigma_w$ vs. $J$ for a plane strain, SSY model. Here
\( \sigma_w^m \) is normalized by \( CJ^2 \) where \( C \) is a constant. The finite element model for the SSY configuration and the material properties for the A508 steel are described in the coming sections. In calculating \( \sigma_w \), the \( m \)-value is taken as 10. Fig. 5.2 shows clearly that \( \sigma_w^m \) is proportional to \( J^2 \). The discrepancy in the early stage of loading is due to the effect of the small notch root radius used in the finite element model.

![Graph showing \( \sigma_w^m / CJ^2 \) vs. \( J \) (kJ/m²)](image)

Fig. 5.2 Computed \( \sigma_w^m \) vs. \( J \) relation for a plane strain, SSY model showing that \( \sigma_w^m \) is proportional to \( J^2 \).

The two-parameter model represents a pure weakest link description of the fracture event, which implies that a small \( K_f \) (stress intensity factor due to applied load) leads to a finite failure probability. However, the newly formed microscopic cracks cannot propagate in polycrystalline metals unless sufficient energy exists to break the atomic bonds, to drive the crack across grain boundaries and to perform plastic work. Consequently, there exists a minimum toughness value \( K_{min} \) below which the cracks are arrested. A simplified, three-parameter Weibull distribution describing the
macroscopic fracture toughness under small scale yielding (SSY) conditions, proposed by Anderson, et al. [97] and adopted by ASTM E1921 [98], has the form

\[
P_f(K_I) = 1 - \exp \left[ -\left( \frac{K_I - K_{\min}}{K_0 - K_{\min}} \right)^4 \right] \tag{5.6}
\]

Here \( K_0 \) represents the fracture toughness value at 63.2% failure probability, and \( K_{\min} \) represents the threshold toughness for the material. For the common ferritic steels, \( K_{\min} \) has an empirical value of 20 MPa√m (independent of temperature and loading rate) [98]. A maximum likelihood estimate for \( K_0 \) is given by

\[
K_0 = \left[ \frac{N}{\sum_{i=1}^{N} \frac{(K_{Jc(i)} - K_{\min})^4}{r - 0.3068}} \right]^{1/4} + K_{\min}, \tag{5.7}
\]

where \( N \) denotes the total number of specimens tested (both censored and uncensored) while \( r \) represents the number of uncensored tests (six minimum) [98].

Similar to Eq. (5.6), Gao, et al. [11, 12] introduced a \( \sigma_{w-min} \) in the Weibull stress model for the cumulative failure probability. The three-parameter Weibull stress model that is consistence with (6) under SSY conditions has the form [45]

\[
P_f(\sigma_w) = 1 - \exp \left[ -\left( \frac{\sigma_w - \sigma_{w-min}}{\sigma_{m/4} - \sigma_{w-min}} \right)^4 \right] \tag{5.8}
\]

In Eq. (5.8) \( \sigma_{w-min} \) is defined as the value of \( \sigma_w \) when \( K_I = K_{\min} \). Cleavage fracture cannot occur when \( \sigma_w < \sigma_{w-min} \).

In summary, the modified Weibull stress model adopts a three-parameter Weibull distribution to describe cumulative failure probability, Eq. (5.8), with the Weibull stress defined as Eq. (5.5). In computing the Weibull stress, the fracture process zone is defined
as the plastic zone at the crack tip, excluding the region closest to the crack tip experiencing large deformation and having a stress triaxiality ratio \( \frac{\sigma_m}{\sigma} \) to be less than 1.5.

5.3. Toughness scaling and calibration of the Weibull stress model parameters

The Weibull stress model allows for scaling of the toughness based on equal failure probability [43]. For fixed values of \( m \) and \( \sigma_u \), toughness scaling enables transferability of toughness values for specimens of different types, sizes and loading conditions. According to Eq. (5.8), the model to scale toughness values between two specimens having different constraint conditions should be constructed at identical values of \( \sigma_w^* \), where

\[
\sigma_w^* = \sigma_w^{m/4} - \sigma_w^{m/4}_{\min}.
\]

Consider two specimens, e.g., a shallow-cracked SE (B) specimen and a deep-cracked C (T) specimen. The \( \sigma_w^* \) versus \( J \) curve for both specimens can be computed by performing detailed, 3D finite element analyses. Since these two specimens have different constraint levels, the same \( \sigma_w^* \) value will correspond to different \( J \) values on the two \( \sigma_w^* \) versus \( J \) curves. This pair of \( J \) values defines a point on the toughness scaling diagram.

Gao et al. [47] proposed a procedure to calibrate the Weibull stress parameters using fracture toughness data obtained from two sets of specimens that exhibit different constraint levels at fracture. By applying the Weibull stress-based toughness scaling model, the calibration process seeks the \( m \)-value which “constraint corrects” the two sets
of fracture toughness data to have the same statistical properties (i.e., $K_0$) under SSY conditions. After $m$ is determined, $\sigma_u$ and $\sigma_w$ are found from the $\sigma_u$ vs. $K_J$ history for the SSY configuration: $\sigma_u$ equals the $\sigma_w$ -value at $K_J = K_0$ and $\sigma_w$ equals the $\sigma_w$ -value at $K_J = K_{\text{min}}$. This new procedure eliminates the non-uniqueness that arises in previous approaches to calibrate $(m, \sigma_u)$ using only fracture toughness data measured under high-constraint, SSY conditions where one parameter ($J$) describes the crack front fields. The procedures to calibrate $m$ can be summarized as follows:

1) Test two sets of specimens (A and B) with different crack configurations, e.g., deep-cracked SE(B) specimens and shallow-cracked SE(B) specimens, in the DBT region to generate two distributions of fracture toughness data. The specimen geometries and test temperature should be selected to ensure different evolutions of constraint levels with no ductile tearing prior to cleavage.

2) Perform detailed, 3D finite element analyses for the tested specimen geometries and the plane strain, SSY boundary layer model ($T$-stress = 0). The mesh refinement must be sufficient to ensure converged $\sigma_w$ vs. $J$ histories over the range of loading levels.

3) Assume an $m$-value and compute the $\sigma_w$ vs. $J$ history for A, B and SSY configurations respectively. Determine $\sigma_{w-\text{min}}$ and constraint-correct toughness distributions A and B to SSY configuration using the toughness scaling model based on equal $\sigma_w^*$-values.
4) Estimate the $K_0$ values for the two constraint-corrected toughness distributions (denoted as $K^A_0$ and $K^B_0$) and define an error function as $R(m) = (K^B_0 - K^A_0) / K^A_0$.

If $R(m) \neq 0$, repeat procedures (3) to (4) for additional values of $m$. The calibrated Weibull modulus makes $R(m) = 0$ within a small tolerance.

5.4 Experimental results, finite element models and numerical procedures

Faleskog, et al. [102] describe an extensive fracture testing program on a modified A508 steel. The modification consists of heat treatment to move the DBT temperature to an interval at which fracture tests can be performed easily. They tested a large number of SE(B) specimens at three different temperatures, -30, 25 and 55ºC. The specimens are all plane-sided with $B=20$ mm, $W=40$ mm and $S=4W$. All specimens fail by cleavage fracture without prior ductile tearing. The data set contains 20 specimens for $a/W=0.5$, and 12 specimens each for $a/W=0.1$ and 0.25 at each temperature. Standard treatment of the deep-notch toughness values following the procedures specified in ASTM E1921 [98] yields three estimates for $T_0$ (reference temperature for indexing fracture toughness) of 34, 42 and 46ºC using data (independently) at each of the test temperatures. The test temperature of -30ºC thus violates a restriction in the E1921 procedures that data must be obtained at temperatures no lower than $T_0 - 50$ºC. This suggests that test temperature of −30ºC is in the lower shelf region. The material has a Young’s modulus of 208 GPa, Poisson’s ration of 0.3 and yield stress ($\sigma_0$) of 597 MPa at -30ºC, 557 MPa at 25ºC and 552 MPa at 55ºC. Fig. 5.3 shows the measured, uniaxial stress-strain curves (true stress versus logarithmic strain) for this material at test temperatures of -30, 25 and 55ºC.
Readers are referred to Faleskog, et al. [102] for experimental details and lists of fracture toughness data.

Fig. 5.3 Uniaxial, tensile stress-strain curves for an A508 steel at -30, 25 and 55°C.

We perform nonlinear analyses on very detailed models of the fracture specimens using a research code, WARP3D [24]. The code provides a Mises constitutive model in a finite-strain framework and uses a continuously updated formulation naturally suited for solid elements having only translational displacement at the nodes. Using the equilibrium equations linearized in the current configuration, the global solution proceeds in an incremental-iterative (implicit) manner with nodal equilibrium enforced at the increment $n+1$. To advance the solution from $n$ to $n+1$, each Newton iteration utilizes the tangent stiffness computed for current estimate of the solution at $n+1$. Final increments of logarithmic strain from $n$ to $n+1$ are evaluated using the linear strain-displacement matrix that is evaluated on the converged mid-increment configuration $x_{n+1/2}$. 
The fracture models were constructed using 3D, eight-node, tri-linear, hexahedral elements. The so-called \( \bar{B} \) formulation is used to prevent mesh lock-ups as the deformation progresses into a fully plastic, incompressible mode. Fig. 5.4 shows a typical finite element mesh for the SE(B) specimen. A conventional mesh configuration having 44 focused rings of elements in the radial direction surround the crack front. The crack tip has an initial small root radius for the purpose of enhancing convergence of the finite-strain solutions. To limit the effects of initial root radius on Weibull stress calculations, the analyses makes use of finite element models having root radii of 0.25 \( \mu \text{m} \) and 2.5 \( \mu \text{m} \). The model with a 0.25 \( \mu \text{m} \) radius provides Weibull stress results early in the loading
history for relatively low values of $J$. At large values of $J$, element distortions along the crack front reach unacceptable levels using this model. The second model, which has an initial root radius of 2.5 μm, permits loading too much higher values of $J$. At intermediate values of $J$, both models generate essentially identical results, providing a verification of our approach. The typical mesh contains 34,200 nodes and 30,500 elements, with 20 variable thickness layers defined over the half-thickness. The thickest layer lies at $Z=0$ while the thinner layers are defined near the free surface ($Z=B/2$) to accommodate $Z$ variations in stress distribution. The $J$-integral values are evaluated using the domain integral procedure with the domains defined outside the material and having highly non-proportional histories at the crack front. The computed $J$-values exhibit strong domain (path) independence. The through-thickness (average) $J$-values provide a convenient parameter to characterize the average intensity of far field loading on the crack front. They also agree with $J$-values derived from estimation schemes based upon $η$-factors for deformation plasticity, which supports a direct connection of analysis values with experimentally measured $J_c$-values.

The plane strain, SSY ($T$-stress = 0) solution establishes the reference crack-front conditions to quantify the evolution of constraint loss for SE(B) specimens. Fig. 5.5 shows the finite element mesh for the SSY, boundary layer model. The SSY model (half of the circular region) has one layer of 2,800 eight-node, 3-D elements with plane-strain constraints ($w=0$) imposed at all of the nodes. Numerical solutions were generated by imposing displacements of the elastic, Mode I singular field on the outer circular boundary, which encloses the crack.
Fig. 5.5 (a) Small scale yielding (SSY) model. (b) Near-tip mesh for the SSY model and the fracture specimens.

Numerical evaluation of the Weibull stress using the finite element results proceeds as follows. Let $|J|$ denote the determinant of the coordinate Jacobian between the deformed Cartesian coordinates $(x_i)$ and the parametric coordinates $(\eta_i)$. Then, using standard procedures for integration over the volume element, the Weibull stress takes the form

$$\sigma_w = \left[ \frac{1}{V_0} \sum_{n_e} \int \int \int \epsilon_p \sigma_{w}^{m} |J| \eta_1 \eta_2 \eta_3 \right]^{1/m}$$

(5.9)

where $n_e$ is the total number of elements inside the fracture process zone. The reference volume ($V_0$) is taken as 1 mm$^3$ for purpose of convenience in all calculations. A change of the reference volume from $V_0$ to $\overline{V}_0$ results in a change of $\sigma_w$ to $\overline{\sigma}_w$ such that $V_0 \sigma_w^{m} = \overline{V}_0 \overline{\sigma}_w^{m}$. Consequently, values of $\sigma_u$ and $\sigma_{w-min}$ should vary in the same way to maintain identical failure probabilities.
5.5 Results and discussions

The calibration procedure proposed by Gao et al. [47] requires testing two sets of fracture specimens that exhibit different constraint levels at the same temperature. The test program of Faleskog, et al. [102] contains SE(B) specimens with three $a/W$ ratios at each temperature. We use fracture toughness data of the $a/W=0.5$ and $a/W=0.1$ specimens to calibrate parameters of the proposed Weibull stress model. The toughness data obtained from the $a/W=0.25$ specimens are used to verify the calibrated model.

At 25ºC, the calibration process using toughness data of the $a/W=0.5$ and $a/W=0.1$ specimens leads to $m=11.8$ and $\sigma_u = 4,522$ MPa. The $\sigma_{w-min}$ value corresponding to the SSY model having the same thickness as the SE(B) specimens is 2,730 MPa. Figs 6(a) and 6(b) show the evolution of the cumulative probability of cleavage fracture with increased loading ($J$) for the $a/W=0.5$ and $a/W=0.1$ specimens respectively. The solid lines represent predictions made using the modified Weibull stress model with the calibrated parameters. The symbols indicate median rank probabilities for the measured $J_c$-values. These are computed using $P_i = (i - 0.3)/(N + 0.4)$, where $i$ denotes the rank number and $N$ represents the total number of fracture tests. The dashed lines denote the 90% confidence limits for the estimates of rank probability of the experimental data. To compute these confidence limits, we assume that the (continuous) $P_i$-values from Eq. (8) provide the expected median rank probabilities for an experimental data set containing the number of measured values ($N$). Details of the procedures to compute these 90% confidence limits for the rank probabilities can be found in [12]. Although the calibration process only uses the $K_0$ shift between the two measured sets of toughness values due to
constraint loss, the calibrated Weibull stress model predicts the shape of the toughness distributions \((P_f \text{ vs. } J)\) for both sets of data. The model predictions agree very well with the experimental data and capture accurately the toughness change due to different levels associated with two specimen geometries.

The experimental data obtained using \(a/W=0.25\) specimens enable an independent validation of the calibrated model. Fig. 5.6(c) compares the predicted cumulative cleavage probabilities using the Weibull stress model and the parameters calibrated above with the median rank probabilities for the measured \(J_c\)-values of the \(a/W=0.25\) specimens. Good agreement between the model prediction and the experimental data verifies the proposed Weibull stress model and the parameters calibrated.

At 55ºC, the calibration process using toughness data of the \(a/W=0.5\) and \(a/W=0.1\) specimens leads to \(m=11.4\). This value is essentially the same as the \(m\)-value calibrated at 25ºC. The small difference in \(m\) may be attributed to the limited number of measured toughness values available for the calibrations. At -30ºC, the toughness values are in the lower shelf region and cannot be used to calibrate the Weibull stress model. Nonetheless, based on the results for 25ºC and 55ºC, we argue that \(m\) is independent of temperature. This argument can also be rationalized as follows. In derivation of the Weibull stress model, \(m\) is related to the size distribution of carbide particles (shape parameter). If the temperature change does not result in a change of the microstructure of the material, the carbide size distribution will not change, and consequently, the \(m\)-value will not change. Here we assume that \(m=11.8\) is applicable in the entire DBT region.
Fig. 5.6 Comparison of the predicted cumulative cleavage probabilities (solid lines) with rank probabilities of the measured $J_c$-values at 25°C. The dashed lines represent the 90% confidence limits for the estimates of the rank probabilities. (a) $a/W=0.5$, (b) $a/W=0.1$, (c) $a/W=0.25$. 
Fig. 5.7 Calibrated variations of $\sigma_u$ and $\sigma_{w\text{--}min}$ with temperature.

With the value of the Weibull modulus fixed at $m = 11.8$, the values of $\sigma_u$ and $\sigma_{w\text{--}min}$ at 55°C and -30°C can be easily determined. We use the $a/W=0.5$ data to determine $\sigma_u$ at each temperature. The $\sigma_u$ value is obtained by constraint-correcting the measured toughness data to the SSY configuration using the Weibull stress based toughness scaling model with $m=11.8$ and then evaluating the $\sigma_w\text{--}value of the SSY model at $K_I$ equal to $K_0$ [estimated from the constraint-corrected toughness values using Eq. (5.7)]. Fig. 5.7 shows the variations of $\sigma_u$ and $\sigma_{w\text{--}min}$ with temperature: $\sigma_u$ increases and $\sigma_{w\text{--}min}$ decreases with temperature. The variation of $\sigma_{w\text{--}min}$ follows simply from the temperature-dependent material flow properties. The variation of $\sigma_u$ reflects the combined effects of temperature on material flow properties and toughness.
Fig. 5.8 Comparison of the predicted cumulative cleavage probabilities (solid lines) with rank probabilities of the measured $J_c$-values at 55°C. The dashed lines represent the 90% confidence limits for the estimates of the rank probabilities. (a) $a/W=0.1$, (b) $a/W=0.25$, (c) $a/W=0.5$.

After the values of $\sigma_u$ and $\sigma_{w_{\text{min}}}$ are determined for 55°C and -30°C, we can check how well the proposed Weibull stress model with $m=11.8$ predicts cleavage fracture at these two temperatures. Fig. 5.8 compares the predicted cumulative cleavage probabilities with the median rank probabilities for the three sets of measured $J_c$-values at 55°C. Fig. 5.9 compares the predicted cumulative cleavage probabilities with the median
rank probabilities of experimental data at -30°C. In general, good agreements are observed for all cases. These results suggest that the constant value of $m (=11.8)$ remains applicable over the temperature range considered here.

Fig. 5.9. Comparison of the predicted cumulative cleavage probabilities (solid lines) with rank probabilities of the measured $J_c$-values at -30°C. The dashed lines represent the 90% confidence limits for the estimates of the rank probabilities. (a) $a/W=0.1$, (b) $a/W=0.25$, (c) $a/W=0.5$. 
Petti and Dodds [15] recently described an approach to calibrate the temperature dependence of $\sigma_u$ using the master curve approach with the reference temperature ($T_0$) estimated by testing standard fracture specimens at one temperature. They assume $m$ is independent of temperature and select $\sigma_u$ values that force the Weibull stress model to predict the Master Curve temperature dependence of $K_{Ic}$ values for the material. Since we have demonstrated in this study that $m$ can indeed be considered independent of temperature, the Petti-Dodds approach can also be applied here to obtain the dependence of $\sigma_u$ on temperature. However, it is worth noting that in Petti and Dodds [13], the original Weibull stress definition, Eq. (5.2), was adopted. Our recent study [17] shows that the modified Weibull stress model, Eq. (5.5), improves the prediction for cleavage fracture, especially for specimens having low constraint levels.

5.6 Concluding remarks

This chapter presents a modified Weibull stress model which includes the effects of plastic strain and stress triaxiality at the crack tip region and employs it to predict cleavage fracture in a modified A508 pressure vessel steel. The fracture toughness data obtained by Faleskog, et al. [102] using SE(B) specimens with $a/W=0.1, 0.25,$ and $0.5$ tested at -30, 25, and 55°C are used to calibrate and verify the proposed model. The proposed Weibull stress model accurately predicts the scatter of the measured fracture toughness data and the strong effects of constraint and temperature on cleavage fracture. Most experimental data fall within the 90% confidence bounds. It is demonstrated that the Weibull modulus ($m$) remains a constant in the temperature range considered. The
threshold value for the Weibull stress model, $\sigma_{w-min}$, decreases with temperature due to decrease of the yield stress with temperature. But $\sigma_{w-min}$ remains a parameter computable using the SSY boundary layer model once the calibrated value for $m$ becomes known. The Weibull stress scale parameter, $\sigma_u$, increases with temperature reflecting the combined effects of temperature on material flow properties and toughness. The master curve can be used to estimate the $\sigma_u$ dependence on temperature once the $m$-value is determined.
6.1 Conclusions on ductile fracture

Mechanism-based concepts provide key insights for development of transferable fracture mechanics models for prediction of ductile fracture and continuously expanding computational capability makes it possible to model the fracture process at metallurgical length-scale.

We use two approaches to model the behavior of void-containing materials. One is the explicit finite element representation by which the microvoid is modeled by very refined finite elements. The other one uses the porous plasticity model, e.g. the Gurson type of material model. The first approach provides insights to the mechanisms of ductile fracture by its exact implementation of the void growth behavior and can predict trends of fracture toughness. However, it’s not practicable to use it in engineering applications since it’s not possible to model every void in real structure, at least with today’s computational capability. The porous model can be used to predict extensive crack extension in fracture specimens.

The explicit void modeling approach shows a void-by-void growth mechanism for materials containing small initial void volume fractions and a multiple void interaction mechanism for materials containing large initial void volume fractions. Effects of the
initial relative void spacing, void pattern, void shape and void volume fraction are also investigated.

The GLD model, which accounts for both void shape and void volume fraction, is used to simulate crack formation and propagation for 2024-T3 aluminum alloy specimens. A criterion for void coalescence is required in order for the crack to initiate and grow. We take the critical strain as a measurement to predict void coalescence. The critical strain at the onset of void coalescence depends on material flow properties and microstructural properties as well as the stress state. We obtain the failure criterion for the RMV in terms of the macroscopic equivalent strain ($E_c$) as a function of $T$ and $\theta$ by conducting systematic finite element analyses of the void-containing RMV subjected to different macroscopic stress states. A series of parameter studies are conducted to examine the effects of the shape and initial volume fraction of the primary void and nucleation, growth and coalescence of the secondary voids on the predicted failure surface $E_c(T, \theta)$.

Our explicit void modeling shows that $E_c(T, \theta)$ is insensitive to $\theta$ when $\theta$ is in the range of $-15^\circ \leq \theta \leq 0^\circ$. For all the specimens we have for the 2024-T3 aluminum alloy, finite element analysis shows the Lode angle are between $-15^\circ$ and $0^\circ$. Consequently we took $E_c$ as a function of $T$ only for the 2024-T3 aluminum alloy specimens. The calibrated computational model accurately predicts crack extension in a series of fracture specimens having various initial crack configurations.

We extend our ductile fracture model to take consideration of angle effect. Experimental data of a DH 36 steel specimens covering a wide range of Lode angle
obtained from Naval Surface Warfare Center allow us to study the role of Lode angle. Our conclusion coincides with what Xue and Wierzbicki has found[34]. We then implement the X-W criterion into ABAQUS through ABAQUS user subroutine UMAT. Four constants $C_1$, $C_2$, $C_3$ and $C_4$ are calibrated using notched round bar specimens, notched plane strain specimens and torsion specimen. The calibrated failure model is then used to predict load-displacement response for other specimens with different geometries. All the predictions make a very good match with the experimental result.

6.2 Conclusion on cleavage fracture

A three parameter modified Weibull stress model is presented which includes the effects of plastic strain and stress triaxiality at the crack tip region. The model is employed to predict cleavage fracture in a modified A508 pressure vessel steel. Data sets for SE(B) specimen with different $a/W$ ratios are available at temperatures -30 ºC, 25 ºC, and 55ºC. Method to calibrate the model parameters is described. The scatter of the measured fracture toughness data and the strong effects of constraint and temperature on cleavage fracture are accurately predicted by the calibrated model. Most experimental data fall within the 90% confidence bounds. The Weibull modulus ($m$) remains a constant in the temperature range considered. The threshold value for the Weibull stress model, $\sigma_{n-min}$, decreases with temperature due to decrease of the yield stress with temperature. The Weibull stress scale parameter, $\sigma_u$, increases with temperature reflecting the combined effects of temperature on material flow properties and toughness.
6.3 Future work

Although prediction of fracture behavior for both ductile fracture and cleavage fracture has been successful in this thesis, improvements are still necessary and can be made in the following directions.

1) ductile fracture

1. *More experiment work is needed to study the Lode angle dependency*

So far the experimental work in this direction is still scarce. We calibrate the constants $C_1$, $C_2$, $C_3$ and $C_4$ using notched round bar specimens and notched plain strain specimens plus torsion specimens corresponding to $\xi=0$ and $\xi=1$ respectively. Though the prediction using the calibrated values agrees well with experiment result, it may lack generality.

Another issue in our calibration procedure is that we assume void coalescence takes place when there is an obvious load drop in the experimental load-displacement curve. This might apply for certain material but obviously can’t apply to all materials since for some material the material hardening can compensate the load carrying capacity due to crack initiation. Acoustic emission technique and other experimental techniques such as X-ray tomography are highly recommended to for obtaining the exact coalescence point. More detailed work needs to be done to obtain the necessary parameters.

2. *More research work is required for material failure criteria for GLD model*

Our current work shows the bright future of fracture prediction using micro-mechanic approach. By combining X-W failure locus and GLD model, finite element
analysis shows almost perfect match for the load-displacement curve between numerical simulation and experimental. However, this doesn’t mean that the failure criterion for void coalescence is perfect. Clearly more research is needed for deciding the void coalescence criterion regarding different initial void volume fraction, void shape, average void distance, etc.

3. More finite element analysis to verify and expand the application of GLD model combined with the proposed failure criterion

The static or quasi-static finite element analysis for structure is used in this thesis. However, when the loading rate is high, the static analysis is no longer valid since the inertia effect can no longer be ignored. So far, to the author’s best knowledge, no work has been conducted using the micro-mechanics for high speed fracture analysis, such as impact, collision and explosion. For the dynamic analysis, the explicit finite element analysis is desired and it’s of great value to implement the GLD model as well as the failure criteria into explicit finite element code and to broaden its applications in structure analysis.

2) Cleavage fracture

The temperature effects, the plastic strain and crack tip triaxiality effects have been considered in this thesis. However, for real engineering structures, complicated loading conditions and various service conditions combined together make prediction of cleavage fracture a difficult task. Challenges exist include but not limit to considerations of strain-rate effects, corrosion effects, irradiation effects, welding inhomogeneities effects, etc.
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