SLOPE STABILITY ANALYSIS USING 2D AND 3D METHODS

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SLOPE STABILITY ANALYSIS USING 2D AND 3D METHODS

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Thesis

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ABSTRACT

The analysis and design of failing slopes and highway embankments requires an in-depth understanding of the failure mechanism in order to choose the right slope stability analysis method. The main difference between the limit equilibrium analysis methods is the consideration of the interslice forces and the overall equilibrium of the sliding mass. The effectiveness of any slope failure remediation method depends on the analysis method. Understanding the limitations of the limit equilibrium methods will help in designing more stable slopes and will help in developing more rigorous analysis methods that can accurately predict the slope behavior.

The objectives of this thesis are several fold: (1) Perform a literature review to study the theoretical background of the most widely used 2D and 3D slope stability methods, (2) perform comparison between 2D and 3D analysis methods, (3) evaluate the effect of ignoring the side forces in 2D and 3D analysis methods, (4) evaluate the effect of using advanced slip surface searching techniques on the selection of the most critical slip surface.

Theoretical limitations of the slope stability methods were discussed. In addition, the limitations were assessed using numerical examples. The studied slope stability
methods included 2D and 3D slope stability methods using limit as well as finite element analysis methods. Based on the results, more rigorous limit equilibrium slope stability methods should be used which should consider the side resistance of the sliding mass and the searching technique for the most critical slip surface.
DEDECATION

To my husband
ACKNOWLEDGMENT

I would like to thank all those people who made this thesis possible and an enjoyable experience for me.

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CHAPTER I
INTRODUCTION

In recent years, the Federal Highway Administration (FHWA) and the Federal Emergency Management Agency (FEMA) have seen an increasing number of projects in which slope remediation was the main solution to stabilize highway embankments and failing earth masses. However, due to the high cost associated with such projects, better engineering analysis tools should be used. Slope stability methods vary in their theoretical background and approach and hence the analysis results vary depending on the used theory. In addition, simplified assumptions in 2D slope stability methods have led to factors of safety that differ from the more rigorous 3D slope stability analysis methods. In practice, 3D analysis of slope stability is not performed unless the geometry of the slope is very complicated or the failure mechanism is complex.

At the present time, no single analysis method is preferred over the other by agencies and therefore, the reliability of any remediation solution to any slope failure is completely left to the engineer in charge. Most practitioners prefer one method over the other based on their familiarity with the method rather than on the conditions of the failing slope. As a result, the remediation methods are either underdesigned or overdesigned. In reality, the mechanism of the failing slope should be studied in detail by
gathering enough field data and observations in order to choose the most appropriate
analysis method.

It is essential that a research effort be devoted to gain better understanding of the
slope failure analysis methods and to understand the weakness and strength of the
methods and to point out practical aspects in the analysis procedures.

1.1 Objectives.

The specific objectives of this thesis are as follows:

1. Perform literature review to study the theoretical background of the most widely
   used 2D and 3D slope stability methods.
2. Perform comparison between 2D and 3D analysis methods.
3. Evaluate the effect of ignoring the side forces in 2D and 3D analysis methods.
4. Evaluate the effect of using advanced slip surface searching techniques on the
   selection of the most critical slip surface.

1.2 Outline of the thesis

In chapter I, the problem statement was made concerning the need for better
understanding of the slope stability analysis methods and the need for more accurate and
robust methods. The objectives of the thesis research and an outline of the organization of
the thesis were also given in chapter I. In chapter II, 2D limit equilibrium analysis
methods were reviewed in details and were compared based on their theoretical
background. Ten two dimensional analysis methods were reviewed including seven limit equilibrium analysis methods. In chapter III, a literature review was provided for various methods used to calculate the three dimensional factor of safety. The accuracy of these methods was discussed based on their theoretical background. In chapter IV, the accuracy of the location of the most critical slip surface was assessed by comparing different search techniques used in practice. Analysis was performed using different commercially available computer programs and fully discussed. In chapter V, the effect of the side resistance in slope stability analysis is studied theoretically, as well as analytically. Comparison between results using 2D and 3D limit equilibrium as well as finite element analysis was performed. Finally, conclusions drawn from the research results were summarized in chapter VI.
2.0 Introduction

Slope stability analysis using computers is an easy task for engineers when the slope configuration and the soil parameters are known. However, the selection of the slope stability analysis method is not an easy task and effort should be made to collect the field conditions and the failure observations in order to understand the failure mechanism, which determines the slope stability method that should be used in the analysis. Therefore, the theoretical background of each slope stability method should be investigated in order to properly analyze the slope failure and assess the reliability of the analysis results.

Two dimensional slope stability methods are the most common used methods among engineers due to their simplicity. However, these methods are based on simplifying assumptions to reduce the three-dimensional problem to a two-dimensional problem and therefore the accuracy of the analysis results vary between the different analysis methods.
Two dimensional slope stability methods using limit equilibrium technique can be divided into the method of slices, circular methods, noncircular methods. The method of slices is based on dividing the slope into different slices and analysis the stability of the failing mass taking into consideration the static equilibrium of the slices individually and the overall equilibrium of the failing mass as whole. The static equilibrium of the slices can be achieved by different assumptions including neglecting or considering the interslice forces and the moment equilibrium of the slices. On the other hand, circular and noncircular limit equilibrium methods consider the equilibrium of the whole failing mass only, and therefore the internal equilibrium of the sliding mass is not considered. Such methods may not be appropriate if the slope remediation method involves installing structural elements in the sliding mass such as slope stabilizing piles.

In this chapter, different slope stability analysis methods were fully reviewed and compared to address the theoretical background of each and the associated limitations that should be considered when reviewing the results of each method.
2.1 Swedish Circle / $\varnothing = 0$ Method

The simplest circular analysis used to analyze the short-term stability for both homogeneous and inhomogeneous slopes based on the assumptions that a rigid, cylindrical block will fail by rotation about its center and the friction angle is zero so the shear strength is assumed to be due to cohesion only. The factor of safety, defined as the ratio of the allowable shear strength to mobilized shear strength, can be calculated by summing moments about the center of the circular surface:

$$ F = \frac{\text{Resisting moment}}{\text{Driving moment}} $$

$$ F = \frac{c_u LR}{Wx} \quad (2.1) $$

where: $c_u =$ undrained shear strength

$R =$ radius of circular surface

$L =$ length of circular arc

$W =$ weight the soil mass above the circular slip surface

$x =$ horizontal distance between circle center, O, and the center of the gravity of the soil mass

Figure 2.1, Swedish Circle / $\varnothing = 0$ Method (Abramson et al., 1996)
The $\phi = 0$ Method achieves a statically determinate solution with respect to moment equilibrium by assuming that the normal stresses act through the center of the circle and the shear stresses act at the same distance from the center of the circle and therefore their moment arm is constant and independent of their solution.

In this method, the undrained shear strength mobilized around the slip surface was assumed to be independent of the stress level. In an effective stress analysis, the shear strength on the slip surface is related to the effective normal stress by the Mohr-Coulomb failure criterion and thus the variation of the normal stress around the failure surface must be determined. And this is achieved in the following methods by dividing the failure mass into a number of slices.

2.2 Log-Spiral procedure

In the logarithmic spiral procedure a statically determinant solution is achieved by assuming a specific logarithmic spiral shape for the slip surface. The radius of the spiral shape varies with the angle of rotation, $\theta$, about the center of the spiral according to the expression:

$$ r = r_0 e^{\theta \tan \phi_d} \tag{2.2} $$

where: $r =$ the radial distance from the center point to a point on the spiral, $r_0 =$ the initial radius, $\theta =$ angle between $r$ and $r_0$, and $\phi_d =$ developed friction angle depends on the friction angle of the soil and the factor of safety.
The stresses along the slip surface consist of the normal stress ($\sigma$) and the shear stress ($\tau$) that can be expressed by the following equations:

\[
\tau = \frac{C}{F} + \sigma \frac{\tan \phi}{F} \quad (2.3a)
\]
\[
\tau = C_{d} + \sigma \tan \phi_{d} \quad (2.3b)
\]

where, \((C, \phi)\) are the shear strength parameters, \((C_{d}, \phi_{d})\) are the developed shear strength parameters, and \(F\) is the factor of safety.

By assuming this shape, the resultant forces produced by the normal stresses and the frictional components of the shear stress ($\sigma \tan \phi_{d}$) pass through the center of the spiral. Therefore, they produce no net moment about the center of the spiral and the only forces that produce moment about the spiral center are the weight force and the developed cohesion that can be used to get the factor of safety.
Since the shear surface is defined by assuming the value of ($\phi_d$), the developed cohesion, which is calculated, may result in different factors of safety with respect to the cohesion that was assumed in calculating ($\phi_d$). Thus, several trials should be done to get a balanced factor of safety that satisfies:

$$F = \frac{C}{C_d} = \frac{\tan \phi}{\tan \phi_d}$$  \hspace{1cm} (2.4)

2.3 The Friction Circle Procedure

This method is suitable for total or effective stress types of analysis in homogeneous soils with $\phi > 0$. The direction of the resultant of the normal and frictional component of shear strength mobilizes along the failure surface forming a tangent to a circle, with a radius, $R_f = R \sin \phi_m$ called the friction circle. Where $R$ is the radius of failure circle and $\phi_m$ is the mobilized friction angle that can be calculated (Abramson et al., 1996):

$$\phi_m = \tan^{-1} \left( \tan \phi \frac{F \phi}{F_\phi} \right)$$  \hspace{1cm} (2.5)

where, $F_\phi$ is the factor of safety against the frictional resistance.

This is equivalent to assuming that all of the normal stresses are concentrated at a single point along the failure arc. This assumption is guaranteed to give a lower bound FOS value (Lambe and Whitman, 1969).
Figure 2.3, Friction circle procedure (Abramson et al., 1996)

The following friction circle procedure was suggested by Abramson et al. (1996):

1) Calculate weight of slide, W.

2) Calculate magnitude and direction of the resultant pore water force, U.

3) Calculate perpendicular distance to the line of action of C_m, which can be located using:

\[ R_c = \frac{L_{arc}}{L_{chord}} \cdot R \]  

where \( R_c \) is the perpendicular distance from the circle center to force, \( C_m \). The lengths, \( L_{arc} \) and \( L_{chord} \), are the lengths of the circular arc and chord defining the failure mass.

4) Find effective weight resultant, \( W' \), from forces W and U, and its intersection with the line of action of \( C_m \) at A.

5) Assume a value of \( F_\phi \).
6) Calculate the mobilized friction angle, $\phi_m$.

7) Draw the friction circle, with radius $R_f$.

8) Draw the force polygon with $W'$ appropriately inclined, and passing through point A.

9) Draw the direction of $(P)$, the resultant of the normal and frictional (shear) force, tangential to the friction circle.

10) Draw direction of $C_m$, according to the inclination of the chord linking the endpoints of the circular failure surface.

11) The closed polygon will then provide the value of $C_m$.

12) Using this value of $C_m$, calculate $F_c$:

$$F_c = \frac{cL_{chord}}{C_m}$$

(2.7)

13) Repeat steps 5 to 12 until $F_c \approx F_\theta$.

2.4 Methods of Slices

In the method of slices, the soil mass above the slip surface is divided into a number of vertical slices and the equilibrium of each of these slices is considered. The actual number of the slices depends on the slope geometry and soil profile. However, breaking the mass up into a series of vertical slices does not make the problem statically determinate. In order to get the factor of safety by using method of slices, it is necessary to make assumptions to remove the extra unknowns and these assumptions are the key roles of distinguishing the methods.
Most computer programs use the methods of slices, as they can handle complex slope geometries, variable soil and water conditions and the influence of external boundary loads. Therefore, they are the most commonly used methods in slope stability analysis. Some of the most popular and significant methods were described below.

2.4.1 Ordinary method of slices

This method is also referred to as "Fellenius' Method" and the "Swedish Circle Method"; it is the simplest method of slices to use. The method assumes that the resultant of the interslice forces acting on any slice is parallel to its base; therefore the interslice forces are neglected (Fellenius, 1936). Only moment equilibrium is satisfied. In this respect, factors of safety calculated by this method are typically conservative. Factors of safety calculated for flat slopes and/or slopes with high pore pressures can be on the conservative by as much as 60 percent, when compared with values from more exact solutions (Whitman and Baily, 1967). For this reason this method is not used much nowadays.

For the slice shown in figure 2.4 below, the Mohr-Coulomb failure criteria is:

\[ s = c' + (\sigma - u) \tan \phi' \]  \hspace{1cm} (2.8)

where, \( \sigma \) is the total normal stress, \( u \) is the pore pressure, \( c' \) is the effective cohesion intercept, \( \Phi' \) is the effective friction angle and \( s \) is the shear stress.
Using a factor of safety (F): \( \tau = s/F \), \( P = \sigma \cdot l \) and \( T = \tau \cdot l \), the equation will be:

\[
T = \frac{1}{F} (c' l + (p - ul) \tan \phi')
\]  

From neglecting the interslice forces, the normal force on the base of the slice can be expressed as:

\[
P = W \cos \alpha
\]  

where, \( W \) is the weight of the slice, and \( \alpha \) is the angle between the tangent of the center of the base of the slice and the horizontal.

The factor of safety is derived from the summation of moments about a common point, O and expressed as:

\[
\sum WR \sin \alpha = \sum TR
\]  

so,

\[
F = \frac{\sum (c' l + (W \cos \alpha - ul) \cdot \tan \phi'))}{\sum W \sin \alpha}
\]
2.4.2 Simplified Bishop Method

The simplified Bishop method also uses the method of slices to find the factor of safety for the soil mass. Several assumptions were made in this method:

1. The failure is assumed to occur by rotation of a mass of soil on a circular slip surface centered on a common point as shown in figure 2.5. Thus Bishop’s method should not be used to compute the factor of safety for non-circular surfaces unless a frictional center of rotation is used (Anderson and Richards, 1987).

2. The forces on the sides of the slice are assumed to be horizontal and thus there are no shear stresses between slices (Bishop, 1955).

3. The total normal force is assumed to act at the center of the base of each slice, and is derived by summing forces in a vertical direction. Substituting
the failure criteria, the normal force is then given by the expression:

\[
P = \frac{W - \frac{1}{F}(c'l \sin \alpha - ul \tan \phi' \sin \alpha)}{m_\alpha}
\]  

(2.13)

where,

\[
m_\alpha = \cos \alpha + \frac{(\sin \alpha \tan \phi')}{F}
\]  

(2.13a)

Taking moments about the center of the circle derives the factor of safety:

\[
F = \frac{\sum \left[ c'l \cos \alpha + (W - ul \cos \alpha) \tan \phi' \right]}{\sum W \sin \alpha}
\]  

(2.14)

As this equation contains \( F \) on both sides, it has to be solved iteratively. Convergence is usually quick and so the method is suitable for hand calculation (Bishop, 1955).

Figure 2.5, Bishop’s simplified method of slices (Anderson and Richards, 1987)
Although the simplified Bishop method does not satisfy complete static equilibrium, the procedure gives relatively accurate values for the factor of safety. Bishop (1955) showed that the Simplified Bishop method is more accurate than the Ordinary Method of Slices, especially for effective stress analysis with high pore water pressure. Also, Wright et al., (1973) have shown that the factor of safety calculated by the Simplified Bishop method agrees favorably (within about 5%) with the factor of safety calculated using finite element procedures. The primary limitation of the Simplified Bishop method is that it is limited to circular slip surface.

2.4.3 Janbu’s Simplified Method

The Janbu’s simplified method (1956) is applicable to non-circular slip surfaces as shown in figure 2.6. In this method, the interslice forces are assumed to be horizontal and thus the shear forces are zero. Therefore, the expression obtained for the total normal force on the base of each slice is the same as that obtained by Bishop’s method (1955):

\[
P = \frac{W - \frac{1}{F}(c' l \sin \alpha - u l \tan \phi' \sin \alpha)}{m_\alpha}
\]  

(2.15)
By examining overall horizontal force equilibrium, a value of the factor of safety $F_0$ is obtained:

$$F_0 = \frac{\sum(c' l + (P - ul) \tan \phi') \sec \alpha}{\sum W \tan \alpha}$$  \hspace{1cm} (2.16)

To take account of the interslice shear forces, Janbu et al., (1956) proposed the correction factor $f_0$ shown in Figure 2.7:

where,

$$F = f_0 \cdot F_0$$  \hspace{1cm} (2.17)

This correction factor is a function of the slide geometry and the strength parameters of the soil. The correction factor was presented by Janbu based on a number of slope stability computations using both the simplified and rigorous methods for the same slopes.
For convenience, this correction factor can also be calculated according to the following formula (Abramson et al., 1996):

$$f_0 = 1 + b_1 \left[ \frac{d}{L} - 1.4 \left( \frac{d}{L} \right)^2 \right]$$  \hfill (2.18)

where, $b_1$ varies according to the soil type:

- $\phi = 0$ soils: $b_1 = 0.69$
- $C = 0$ soils: $b_1 = 0.31$
- $C > 0$, $\phi > 0$ soils: $b_1 = 0.5$

![Figure 2.7, Janbu’s correction factor for the simplified method (Abramson et al., 1996)]
2.4.4 Janbu’s Generalized Procedure of Slices (GPS)

Janbu’s generalized method of slices (1957, 1973) is an iterative procedure using vertical slices and any shape slip-surface. The procedure, in its rigorous form, satisfies all conditions of equilibrium to include vertical and horizontal force equilibrium, moment equilibrium of the slices, and moment equilibrium of the entire slide mass but the last slice.

By considering the overall force equilibrium, an expression for the factor of safety $F_f$ is obtained (Anderson and Richards, 1987):

$$ T = \frac{1}{F} (c'l + (P-ul) \tan \phi') $$ \hspace{1cm} (2.19)

By resolving vertically:

$$ P \cos \alpha + T \sin \alpha = W - (X_R - X_L) $$ \hspace{1cm} (2.20)

By substituting for $T$:

$$ P = \frac{W - (X_R - X_L) - \frac{1}{F} (c'l \sin \alpha - ul \tan \phi' \sin \alpha)}{m_a} $$ \hspace{1cm} (2.21)

By resolving parallel to base of slice:

$$ T + (E_R - E_L) \cos \alpha = (W - (X_R - X_L)) \sin \alpha $$ \hspace{1cm} (2.22)
By rearranging and substituting for T and considering the absence of the surface loading:

\[
\sum (E_R - E_L) = \sum (W - (X_R - X_L)) \tan \alpha - \frac{1}{F_f} \sum (c'l + (P - ul) \tan \phi') \sec \alpha = 0 \tag{2.23}
\]

so,

\[
F_f = \frac{\sum (c'l + (P - ul) \tan \phi') \sec \alpha}{\sum (W - (X_R - X_L)) \tan \alpha} \tag{2.24}
\]

Figure 2.8, Janbu’s generalized procedure (Anderson and Richards, 1987)

The interslice forces were calculated by considering the moment equilibrium about the center of the base of each slice. For this Janbu (1973) assumed a position of the line of the thrust of the interslice forces as shown in figure 2.9, an imaginary line drawn through the points where the interslice forces act, to render the problem statically determinate and make the overall moment equilibrium implicitly satisfied.

\[
E_R b \tan \alpha_i - X_R b - (E_R - E_L) h_i = 0 \tag{2.25a}
\]

\[
\Rightarrow X_R = E_R \tan \alpha_i - (E_R - E_L) \frac{h_i}{b} \tag{2.25b}
\]

where, \( h_i \) is the height of the line of thrust above the slip surface, and \( \alpha_i \) is an angle measured from the horizontal and represents the slope of the line of thrust.
To solve for the factor of safety, first the shear forces \((X_R - X_L)\) can be assumed zero. Then the values of \(E\) and \(X\) are calculated based on the equations above. Next, the factor of safety is recalculated with these computed values of interslice forces and the iterations are stopped when successive values of \(F\) are nearly identical so both overall force equilibrium and moment equilibrium are satisfied.

![Figure 2.9, Line of thrust describing the locations of the interslice forces on the slice](Duncan and Wright, 2005)

2.4.5 Spencer’s Method

Spencer’s method was presented originally for the analysis of circular slip surfaces, but it is easily extended to non-circular slip surfaces by adopting a frictional center of rotation. This method based on the assumption that the interslice forces are parallel so they have the same inclination:

\[
\tan \theta = \frac{X_L}{E_L} = \frac{X_R}{E_R} \quad (2.26)
\]

where \(\theta\) is the angle of resultant interslice force from the horizontal.

Spencer (1967) summed forces perpendicular to the interslice forces to derive the normal force on the base of the slice:
\[ P = \frac{W - (E_K - E_L) \tan \theta - \frac{1}{F} (c' l \sin \alpha - ul \tan \phi' \sin \alpha)}{m_\alpha} \]  

(2.27)

where,

\[ m_\alpha = \cos \alpha (1 + \tan \alpha \frac{\tan \phi'}{F}) \]  

(2.27a)

By considering overall force equilibrium and overall moment, two values of factor of safety \( F_f \) and \( F_m \) are obtained. This is because a total of \( 2n-1 \) assumptions have been made and the problem is over specified.

The factor of safety \( (F_m) \) can be derived based on the overall moment equilibrium about a common point \( (O) \):

\[ \sum WR \sin \alpha = \sum TR \]  

(2.28a)

\[ T = \frac{1}{F} (c' l + (P - ul) \tan \phi') \]  

(2.28b)

so,

\[ F_m = \frac{\sum (c' l + (P - ul) \tan \phi')}{\sum W \sin \alpha} \]  

(2.29)

The factor of safety \( (F_f) \) can be derived based on the overall force equilibrium:

\[ \sum F_H = 0 \]
\[ T \cos \alpha - P \sin \alpha + E_R - E_L = 0 \]  
\[ (2.30a) \]

By rearranging and substituting for \( T \):

\[ \sum (E_R - E_L) = \sum P \sin \alpha - \frac{1}{F_f} \sum [c' l + (P - ul) \tan \phi'] \cos \alpha \]  
\[ (2.30b) \]

Using Spencer’s assumption, \( \frac{X}{E} = \tan \theta = \text{constant throughout the slope} \), and

\[ \sum (X_R - X_L) = 0 \] in absence of surface loading:

\[ F_f = \frac{\sum (c' l + (P - ul) \tan \phi') \sec \alpha}{\sum (W - (X_R - X_L)) \tan \alpha} \]  
\[ (2.31) \]

\[ \text{Figure 2.10, Spencer’s method (Anderson and Richards, 1987)} \]

Trial and error procedure is used to solve the equation. However, Spencer examined the relationship between \( F_f \) and \( F_m \) for a typical problem as shown in figure 2.11. At some angle of the interslice forces, the two factor of safety are equal and both moment and force equilibrium are satisfied.
Figure 2.11, Variation of $F_m$ and $F_f$ with $\theta$ (Spencer, 1967)
2.4.6 Morgenstern and Price’s Method

This method was developed by Morgenstern and Price (1965), which consider not only the normal and tangential equilibrium but also the moment equilibrium for each slice in circular and non-circular slip surfaces. In this method, a simplifying assumption is made regarding the relationship between the interslice shear forces \( X \) and the interslice normal forces \( E \) as:

\[
X = \lambda \cdot f(x) \cdot E
\]  

(2.32)

where, \( f(x) \) is an assumed function that varies continuously across the slip, and \( \lambda \) is an unknown scaling factor that is solved for as part of the unknowns.

The unknowns that are solved for in the Morgenstern and Price method are the factor of safety \( F \), the scaling factor \( \lambda \), the normal forces on the base of the slice \( P \), the horizontal interslice force \( E \), and the location of the interslice forces (line of thrust). Once the above unknowns are calculated using the equilibrium equations, the vertical component on the interslice forces \( X \) is calculated from the equation 2.32.

An alternative derivation for the Morgenstern-Price method was proposed by Fredlund and Krahn (1977). They had shown that almost identical results may be obtained using their general formulation of the equations of equilibrium (GLE) together with Morgenstern and Price’s assumption about the interslice shear forces (equation 2.32). The solution satisfies the same elements of static but the derivation is more consistent with that used in the other method of slices and also presents a
complete description of the variation of the factor of safety with respect to $\lambda$.

According to Fredlund and Krahn (1977) the normal force is derived from the vertical force equilibrium equation as shown in figure 2.12:

![Figure 2.12, General Method of slices (Fredlund and Krahn, 1977)](image)

\[ P = \frac{W - (X_R - X_L) - \frac{1}{F}(c'l \sin \alpha - ul \tan \phi' \sin \alpha)}{m_a} \]  
(2.33)

Two factor of safety equations are computed, one with respect to moment equilibrium ($F_m$) and the other with respect to force equilibrium ($F_f$). The moment equilibrium equation is taken with respect to a common point as:

\[ F_m = \frac{\sum [c'l + (P - ul) \tan \phi']R}{\sum (Wd - Pf)} \]  
(2.34)

For circular slip surfaces: $f = 0$, $d = R \sin \alpha$ and $R = \text{constant}$;
The factor of safety with respect to force equilibrium is:

\[
F_f = \frac{\sum (c' + (P - ul) \tan \phi) \cos \alpha}{\sum P \sin \alpha}
\]  

(2.36)

On the first iteration, the vertical shear forces \((X_L \text{ and } X_R)\) are set to zero. On subsequent iterations, the horizontal interslice forces are first computed from:

\[
(ER - EL) = P \sin \alpha - \frac{1}{F} [c' + (P - ul) \tan \phi] \cos \alpha
\]  

(2.37)

Then the vertical shear forces are computed using an assumed \(\lambda\) value and \(f(x)\). Once \(X_L\) and \(X_R\) are determined, the normal force \(P\) on the base of each slice is then calculated and the value of \(\lambda\) for which \(F_m = F_f\) can then be found iteratively as shown in figure 2.13.
2.4.7 Sarma’s Method

The Sarma method (1973) is a different approach to determining the safety factor for a slope because it considers the seismic coefficient ($k_c$) to be unknown, and the factor of safety is to be known. Usually a value for the factor of safety is assumed to be 1 and the seismic coefficient required to produce this factor of safety is solved for as an unknown. This coefficient represents the seismic coefficient required to cause sliding.

The Sarma method determines slope stability by applying a horizontal acceleration (as a fraction of the gravitational constant) to the material above the failure surface and calculating the resulting factor of safety.

Soil properties:
$C'/\gamma h = 0.02$
$\phi = 40^\circ$
$r_u = 0.5$
Geometry:
Slope = 26.5°
Height = 100 ft

Figure 2.13, Variation of the factor of safety with respect to moment and force equilibrium vs. $\lambda$ for the Morgenstern-Price method (Fredlund and Krahn, 1977).
This method was developed by Sarma as part of research done for the Corps of Engineers to predict deformation of earthen dams due to seismic loading. Sarma extended the research to calculating safety factors since he reasoned that as a mass of soil moves from no movement to failure during a seismic event, the mass must pass through acceleration where the safety factor is 1.0, that is, the point where the mass is at limiting equilibrium. He called this acceleration value the critical acceleration and labeled it \( (k_c) \).

Sarma and Bhave (1974) found through analysis that a linear relationship exists between the critical acceleration and the static factor of safety as shown in figure 2.14 and suggested a starting value of \( F \) given by:

\[
F = 1 + 3.33K_c
\]  

(2.38)
Figure 2.14. Relationship between $K_c$ and factor of safety in Sarma’s Method (Sarma and Bhave, 1974).
In Sarma’s method, the shear force between slices is related to shear strength by the relationship:

\[ X = \lambda f(x)S_v \]  \hspace{1cm} (2.39)

where, \( S_v \) is the available shear force on the slice boundary, \( \lambda \) is an unknown scaling parameter, and the \( f(x) \) is an assumed function with prescribed values at each vertical slice boundary. The shear force, \( S_v \), depends on the shear strength parameters for the soil along the slice boundary and for frictional materials \((\phi, \phi' > 0)\) on the normal (horizontal) interslice force, \( E \) (Duncan and Wright, 2005).

In Sarma’s method the seismic coefficient and other unknowns can be calculated directly; no iterative, trial-and-error procedure is required. The factor of safety is defined as the factor by which the strength of the material must be reduced to produce a state of limiting equilibrium. That is, the strength where the driving forces equal the resisting forces. This factor is derived by a series of trial and error reductions of the engineering properties to reach a \((k_c)\) value of 0.0 representing a safety factor of 1.0.

The Sarma method of analysis is normally used for complex slope stability problems where there is a requirement to have either a complex failure surface, non-vertical slices, to include faults, or where normal stresses on slice boundaries are likely to be critical.
2.5 Comparison of two-dimensional limit equilibrium methods of analysis

The various methods of two-dimensional limit equilibrium analysis differ from each other in two regards:

1. Different methods use different assumptions to make up the balance between the number of equations of equilibrium and the number of unknowns.

2. Different methods use different assumptions regarding the location and orientation of the internal forces between the assumed slices.

Some analysis methods do not satisfy all conditions of equilibrium or even the conditions of force equilibrium. A summary of some of the commonly used methods was provided in table 2.1 and 2.2.
Table 2.1, Characteristics of Commonly Used Methods of Limit Equilibrium Analysis  
(Modified after Duncan and Wright, 2005)

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Equilibrium Condition Satisfied</th>
<th>Shape of slip surface</th>
<th>Assumptions</th>
<th>Unknowns Solved for</th>
</tr>
</thead>
</table>
| Swedish Circle (Ø = 0) Method | Moment Equilibrium about center of circle | Circular | • The slip surface is circular.  
• The friction angle is zero. | 1 Factor of safety = 1 Total unknown |
| Logarithmic Spiral Method | Moment Equilibrium about center of spiral | Log-spiral | • The slip surface is a logarithmic spiral. | 1 Factor of safety = 1 Total unknown |
| Friction Circle Method | Moment and Force Equilibrium | Circular | • Resultant of the normal and frictional component of shear strength tangent to friction circle. | 1 Factor of safety = 1 Total unknown |
| Ordinary Method of slices | Moment Equilibrium about center of circle | Circular | • The forces on the sides of the slices are neglected.  
• The normal force on the base of slice is Wcosα and the shear force is Wsinα. | 1 Factor of safety = 1 Total unknown |
| Simplified Bishop Method | Vertical equilibrium and overall moment equilibrium | Circular | • The side forces are horizontal (i.e., all interslice shear forces are zero. | 1 Factor of safety  
Normal force on the base of slices (N) = n + 1 Total unknowns |
| Janbu’s Simplified Method | Force equilibrium (vertical and horizontal) | Any shape | • The side forces are horizontal. | 1 factor of safety  
Normal force on the base of slices (N), n-1 Resultant interslice forces (Z) =2n Total unknowns |
Table 2.1, Characteristics of Commonly Used Methods of Limit Equilibrium Analysis (Modified after Duncan and Wright, 2005) (Continued)

<table>
<thead>
<tr>
<th>Method</th>
<th>Conditions of Equilibrium</th>
<th>Shape</th>
<th>Assumptions</th>
<th>Equations / Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Janbu’s GPS Procedure</td>
<td>All conditions</td>
<td>Any</td>
<td>Assumed a position of the line of the thrust of the interslice forces.</td>
<td>1 factor of safety&lt;br&gt;1 interslice force inclination ($\theta$)&lt;br&gt;n Normal force on the base of slices (N), n-1&lt;br&gt;Resultant interslice forces (Z) = 2n</td>
</tr>
<tr>
<td>Spencer’s Method</td>
<td>All conditions</td>
<td>Any</td>
<td>Interslice forces are parallel (i.e., all have the same inclination).&lt;br&gt;The normal force (N) acts at the center of the base of the slice.</td>
<td>1 Factor of safety&lt;br&gt;1 interslice force inclination ($\theta$)&lt;br&gt;n Normal force on the base of slices (N), n-1&lt;br&gt;Resultant interslice forces (Z), n-1&lt;br&gt;Location of side forces (line of thrust) = 3n&lt;br&gt;Total unknowns</td>
</tr>
<tr>
<td>Morgenstern and Price’s Method</td>
<td>All conditions</td>
<td>Any</td>
<td>Interslice shear force is related to interslice normal force by:&lt;br&gt;$$ X = \lambda f (x) E $$&lt;br&gt;The normal force acts at the center of the base of the slice.</td>
<td>1 Factor of safety&lt;br&gt;1 interslice force inclination “scaling factor” ($\lambda$), n Normal force on the base of slices (N)&lt;br&gt;n-1 Horizontal interslice forces (E), n-1&lt;br&gt;Location of interslice forces (line of thrust) = 3n&lt;br&gt;Total unknowns</td>
</tr>
</tbody>
</table>
Table 2.1, Characteristics of Commonly Used Methods of Limit Equilibrium Analysis (Modified after Duncan and Wright, 2005) (Continued)

<table>
<thead>
<tr>
<th>Method</th>
<th>Conditions of Equilibrium</th>
<th>Shape</th>
<th>Characteristics</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sarma’s Method</td>
<td>All conditions of</td>
<td>Any</td>
<td>• Interslice shear force is related to the interslice shear strength, $S_v$, by:</td>
<td>1 Seismic coefficient ($k$) Interslice force scaling factor ($\lambda$)</td>
</tr>
<tr>
<td></td>
<td>equilibrium</td>
<td>shape</td>
<td>$X = \lambda f(x) S_v$</td>
<td>n Normal force on the base of slices ($N$), n-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Interslice shear strength depends on shear strength parameters, pore</td>
<td>Horizontal interslice forces ($E$), n-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>water pressure, and the horizontal component of interslice force.</td>
<td>Location of side forces (line of thrust) = 3n Total unknowns</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• The normal force acts at the center of the base of the slice.</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2, summary of Procedures for Limit Equilibrium Slope Stability analysis and their Usefulness (Modified after Duncan and Wright, 2005)

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swedish Circle ((\phi = 0)) Method</td>
<td>Applicable to slopes where (\phi = 0) (i.e., undrained analyses of slopes in saturated clays).</td>
</tr>
<tr>
<td>Logarithmic Spiral Method</td>
<td>Suitable for homogeneous slopes. Useful for developing slope stability charts and used some in software for design of reinforced slopes.</td>
</tr>
<tr>
<td>Friction Circle Method</td>
<td>Suitable for total or effective stress types of analysis in homogeneous soils with (\phi &gt; 0).</td>
</tr>
<tr>
<td>Ordinary Method of slices</td>
<td>Applicable to non-homogeneous slopes and c-(\phi) soils where slip surface can be approximated by a circle. Very convenient for hand calculations. Inaccurate for effective stress analyses with high pore water pressures.</td>
</tr>
</tbody>
</table>
Table 2.2, summary of Procedures for Limit Equilibrium Slope Stability analysis and their Usefulness (Modified after Duncan and Wright, 2005) (Continued)

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simplified Bishop Method</td>
<td>Applicable to non-homogeneous slopes and c-ø soils where slip surface can be approximated by a circle. More accurate than Ordinary Method of slices, especially for analyses with high pore water pressures. Calculations feasible by hand or spreadsheet.</td>
</tr>
<tr>
<td>Janbu’s Simplified Method</td>
<td>Applicable to non-circular slip surfaces. Also for shallow, long planar failure surfaces that are not parallel to the ground surface.</td>
</tr>
<tr>
<td>Janbu’s GPS Procedure</td>
<td>Suitable for any shape of slip surface and for the rigorous analysis.</td>
</tr>
<tr>
<td>Spencer’s Method</td>
<td>An accurate procedure applicable to virtually all slope geometries and soil profiles. The simplest complete equilibrium procedure for computing factor of safety.</td>
</tr>
<tr>
<td>Morgenstern and Price’s Method</td>
<td>An accurate procedure applicable to virtually all slope geometries and soil profiles. Rigorous, well established complete equilibrium procedure.</td>
</tr>
<tr>
<td>Sarma’s Method</td>
<td>Suitable for more complex problems particularly where non-vertical slice boundaries are significant. A convenient complete equilibrium procedure for computing the seismic coefficient required producing a given factor of safety.</td>
</tr>
</tbody>
</table>

2.5.1 Fredlund and Krahn’s study

Fredlund and Krahn (1977) had presented a good example comparing the different limit equilibrium methods. Both circular and composite failure surfaces in the slope shown in Figure 15 were analyzed. Various combinations of geometry, soil and groundwater conditions were considered and the results were presented in Table 2.3 and Figure 2.16 as
a function of \((\lambda)\), which can be defined as a ratio of the normal and shear forces acting along the vertical slice boundaries.

The results in Table 2.3 along with those from Figure 2.16 showed that the factor of safety with respect to moment equilibrium \((F_m)\) is relatively insensitive to the interslice force assumption. Therefore the factors of safety obtained by the Spencer and Morgenstern-Price methods are in good agreement with the simplified Bishop results, whereas the simplified and rigorous Janbu factors of safety values appeared to be slightly lower. In contrast the factor of safety \((F_f)\) determined by the satisfying force equilibrium was very sensitive to the side force assumption.

The two curves in Figure 2.16 labeled \((F_m)\) and \((F_f)\) represented the location of the points corresponding to the factor of safety and \((\lambda)\) values in satisfying static moment or force equilibrium, respectively. The intersection of the two curves means that a complete static equilibrium within the context of the applied assumptions is satisfied.
Figure 2.15, Example Problem using circular and non-circular sliding surfaces

(After Fredlund and Krahn, 1977)
Figure 2.16, Influence of interslice forces on factors of safety (after Fredlund and Krahn, 1977)
Table 2.3. Comparison of Factors of Safety for Example Problem (After Fredlund and Krahn, 1977)

<table>
<thead>
<tr>
<th>Case no.</th>
<th>Example problem</th>
<th>Ordinary method</th>
<th>Simplified Bishop method</th>
<th>Spencer's method</th>
<th>Janbu's simplified method</th>
<th>Janbu's rigorous method*</th>
<th>Morgenstern-Price method f(x) = constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Simple 2:1 slope, 40 ft (12 m) high, ( \phi' = 20^\circ ), ( c' = 600 ) psf (29 kPa)</td>
<td>1.928</td>
<td>2.080</td>
<td>2.073</td>
<td>14.81</td>
<td>0.237</td>
<td>2.041</td>
</tr>
<tr>
<td>2</td>
<td>Same as 1 with a thin, weak layer with ( \phi' = 16^\circ ), ( c' = 0 )</td>
<td>1.288</td>
<td>1.377</td>
<td>1.373</td>
<td>10.49</td>
<td>0.185</td>
<td>1.448</td>
</tr>
<tr>
<td>3</td>
<td>Same as 1 except with ( r_u = 0.25 )</td>
<td>1.607</td>
<td>1.766</td>
<td>1.761</td>
<td>14.33</td>
<td>0.255</td>
<td>1.735</td>
</tr>
<tr>
<td>4</td>
<td>Same as 2 except with ( r_u = 0.25 ) for both materials</td>
<td>1.029</td>
<td>1.124</td>
<td>1.118</td>
<td>7.93</td>
<td>0.139</td>
<td>1.191</td>
</tr>
<tr>
<td>5</td>
<td>Same as 1 except with a piezometric line</td>
<td>1.693</td>
<td>1.834</td>
<td>1.830</td>
<td>13.87</td>
<td>0.247</td>
<td>1.827</td>
</tr>
<tr>
<td>6</td>
<td>Same as 2 except with a piezometric line for both materials</td>
<td>1.171</td>
<td>1.248</td>
<td>1.245</td>
<td>6.88</td>
<td>0.121</td>
<td>1.333</td>
</tr>
</tbody>
</table>

* The line of thrust is assumed at 0.333 of height of each slice.
CHAPTER III
THREE-DIMENSIONAL ANALYSIS OF SLOPE STABILITY

3.0 Introduction

Three-dimensional analysis methods consider the 3-D shapes of slip surface. These methods, like 2-D methods, require assumptions to achieve a statically determinate definition of the problem. There are several ways to do that; some methods do it by decreasing the number of unknowns, and others by increasing the number of equations, or both, such that the two numbers can be equal.

The three-dimensional analysis becomes important in cases where the geometry is complex that makes it difficult to select a typical two-dimensional section to analyze, the geometry of the slope and slip surface varies significantly in the lateral direction, the material properties are highly inhomogeneous or anisotropic, the slope is locally surcharged, the slope with a complicated shear strength and/or pore-water pressure which requires combining the effects of slope geometry and shear strength to determine the direction of movement that leads to a minimum factor of safety, or to back calculate the shear strength of the failed slope. In these situations, a 3D analysis may be necessary.
A large number of three-dimensional slope stability analysis methods based on the limit equilibrium concept have been developed since 1960s. Many of them are valid only under certain conditions. These methods have been reviewed extensively to understand the limitations of each. A comprehensive comparison between these methods was shown in table 3.1. The methods were discussed below.

3.1 Anagnosti (1969)

Anagnosti (1969) developed a method for the determination of the factor of safety of the potential sliding mass of different shapes, which is an extension of the 2-D Morgenstern-Price method (1967), by setting equilibrium equations for the series of thin vertical slices assuming limit equilibrium conditions on sliding sides of each slice. The main assumption in this method is the distribution of the interslice shear forces that satisfies all equilibrium conditions. Comparison with some analyses assuming 2-D slices reveals that the actual factor of safety increases of over 50%. Sensitivity studies indicated that the calculated factors of safety appeared to be very insensitive to the interslice shear assumption.

Anagnosti showed that the analogous 3-D analysis required four times the number of statical assumptions in the 2-D analysis to satisfy all six equilibrium equations. However, the main limitation of Anagnosti’s extension is the 3-D slip surface; since the geometry of a slice is restricted to two sliding sides and the 3-D slip surface is left unspecified, the selection of the critical surface, in many cases, becomes virtually impossible.
3.2 Hovland (1977)

Hovland’s method is an extension of the assumptions associated with the two-dimensional ordinary method but instead of slices, columns were used. In Hovland’s method, all intercolumn forces acting on the sides of the columns are ignored. The normal and shear forces acting on the base of each column are derived as components of the weight of the column. Another assumption was that there was motion only in one direction and the equilibrium of the system was calculated for this direction.

Hovland (1977) defined the three-dimensional factor of safety as the ratio of the total available resistance along the failure surface to the total mobilized stress along it. In the two-dimensional case shown in figure 3.1, the factor of safety is:

\[
F_2 = \frac{\sum(cA_z + W_z \cos \alpha_{yz} \tan \phi)}{\sum W_z \sin \alpha_{yz}} = \frac{\sum \left( \frac{c\Delta y}{\cos \alpha_{yz}} + \gamma \Delta y \cos \alpha_{yz} \tan \phi \right)}{\sum \gamma \Delta y \sin \alpha_{yz}}
\]  

(3.1)

if cohesion \(c\), friction angle \(\phi\), density \(\gamma\) and \(\Delta y\) are constant, then:

\[
F_2 = \left( \frac{c}{\gamma} \right) \sum \sec \alpha_{yz} + (\tan \phi) \frac{\sum z \cos \alpha_{yz}}{\sum z \sin \alpha_{yz}}
\]  

(3.2)

or,

\[
F_2 = \left( \frac{c}{\gamma h} \right) G_{c2} + \tan \phi G_{\phi2}
\]  

(3.3)
The $G_{c2}$ and $G_{\phi2}$ are only functions of geometry. The $G_{c2}$ term determines how cohesional resistance is influenced by geometry for a 2-D case, the $G_{\phi2}$ term determines how frictional resistance is influenced by geometry for a 2-D case and h is the height of the slope.

![Figure 3.1, Section View of Two-Dimensional Slope Stability Analysis (Hovland, 1977)](image)

In the three-dimensional case shown in figure 3.2 and figure 3.3, the factor of safety was presented by dividing the soil mass above the failure surface into a number of vertical soil columns assuming the x- and y-coordinates are perpendicular and are in the horizontal plane, the z-coordinate is vertical and the y-coordinate is to be in the direction of down slope movement. The area of the soil column in the XY plane is defined by $\Delta x$ and $\Delta y$. By assuming that both $\Delta x$ and $\Delta y$ are constant for all columns, then the factor of safety can be expressed as:
\[
F_3 = \frac{\sum \sum [cA_3 + W_3 \cos(DIP) \tan \phi]}{\sum \sum W_3 \sin \alpha_{yz}}
\]

(3.4)

in which, \( \alpha_{xz} \) and \( \alpha_{yz} \) are the dip angles in the XZ and YZ planes respectively, and:

\[
A_3 = \Delta x \Delta y \left[ \frac{(1 - \sin^2 \alpha_{xz} \sin^2 \alpha_{yz})^{1/2}}{\cos \alpha_{xz} \cos \alpha_{yz}} \right]
\]

(3.5a)

\[
\cos(DIP) = \left(1 + \tan^2 \alpha_{xz} + \tan^2 \alpha_{yz}\right)^{-1/2}
\]

(3.5b)

\[
W_3 = \gamma \Delta x \Delta y
\]

(3.5c)

for \( \alpha_{xz} = 0 \):

\[
F_3 = \frac{\sum \sum \left[ \frac{c\Delta x \Delta y \sin \theta + \gamma \Delta x \Delta y \cos(DIP) \tan \phi}{\cos \alpha_{xz} \cos \alpha_{yz}} \right]}{\sum \sum \gamma \Delta x \Delta y \sin \alpha_{yz}}
\]

(3.6)

where,

\[
\sin \theta = (1 - \sin^2 \alpha_{xz} \sin^2 \alpha_{yz})^{1/2}
\]

(3.7)

if \( c, \phi, \gamma, \Delta x \) and \( \Delta y \) are constant:

\[
F_3 = \left( \frac{c}{\gamma} \right) \frac{\sum \sum \sec \alpha_{xz} \sec \alpha_{yz} \sin \theta}{\sum \sum \sin \alpha_{yz}} + \tan \phi \frac{\sum \sum \z \cos(DIP)}{\sum \sum \z \sin \alpha_{yz}}
\]

(3.8)

Hovland (1977) reported that the every c-\( \phi \) soil may have its own critical shear surface and geometry. His studies also showed that the \( F_3/F_2 \) ratio is quite sensitive to the soil parameters \( c \) and \( \phi \), and to the basic shape of the shear surface, but relatively
insensitive to the width of the shear surface. However, three-dimensional factors of safety are generally much higher than 2-D factors of safety, although in some situations in cohesionless soils the 3-D factor of safety can be lower than the 2-D factor of safety.

Figure 3.2, Plan, Section, and Three-Dimensional Views of one Soil Column

(Hovland, 1977)
3.3 Chen (1981), Chen and Chameau (1983)

Chen (1981) and Chen and Chameau (1983) developed a comprehensive study of the three-dimensional effects on the slope stability for a large variety of soil parameters. They suggested methods for the analysis of the three-dimensional block surfaces figure 3.4, as well as for the rotational surfaces figure 3.5. The method showed Hovland’s method (1977) to be conservative and derived an extension of Spencer’s (1967).
Figure 3.4, 3-D Block Type Failure (Chen, 1981)

Figure 3.5, Spoon Shape failure in the Embankment (Chen, 1981)
In studying the stability of translational slides, the computer program BLOCK3 was generated to perform the block analysis, which was divided into three parts as shown in figure 3.6:

1. Calculation of the total force acting on the central block from the active block. This force is a function of the factor of safety.
2. Calculation of the total force acting on the central block from the passive block. This force is also a function of the factor of safety.
3. Calculation of base, side, and end forces on the central block and of the factor of safety against failure.

The main assumptions of the block failure method are (Chen, 1981):

1. The problem is three-dimensional and symmetrical.
2. Three slopes and well-defined toe and crest define the ground surface.
3. The soil strata are laterally continuous.
4. The sliding surfaces are plane.
5. The boundaries between (1) active and central blocks, (2) passive and central blocks are vertical. No shear forces along these boundaries.
6. The bottom surfaces are at (45+\(\phi/2\)) and (45-\(\phi/2\)) angles with the horizontal for active and passive zones, respectively.
7. The factor of safety is the same throughout the whole failure surface.
8. The water surface is far below the ground surface.
Figure 3.6, Free Body Diagram of: (a) Active Case, (b) Passive Case, and (c) Central Block. (Chen, 1981)
The most important conclusions obtained from Chen’s study of translational slides are:

1. The 3-D factors of safety are usually larger than the 2-D factors of safety and this 3-D effect is more significant for cohesive soils than for cohesionless soils.
2. Wedge type of failure will result in the value of \( F_3/F_2 \) less than unity.
3. A steep weak soil layer always yields smaller \( F_3/F_2 \) ratios than a gently inclined layer.
4. The lower the strength in the weak soil stratum, the more profound the 3-D effect.
5. Reducing the inclination of the ends of the central block cause a higher factor of safety due to the increase in end areas.

In the rotational slides analysis the soil was assumed to be homogeneous and the 3-D failure surface was composed of a central cylinder attached by two semi-ellipsoids at the two ends. A computer program LEMIX using the limit equilibrium method was generated to achieve the required analysis.

The main assumptions of the rotational type of failure are (Chen, 1981):

1. The failure mass is symmetrical and divided into many vertical columns, a free body diagram of a column is shown in figure 3.7.
2. Direction of movement is along the X-Y plane only (no movement in Z-direction); therefore at the instant of failure the shear stresses along Y-Z plane are assumed to be zero.
3. The intercolumn shear forces are parallel to the base of the column and to be a function of their positions.
4. The intercolumn normal stress distribution is assumed to be linear with depth.

5. The inclination of the interslice forces is assumed the same throughout the whole failure mass.

Based on the previous assumptions, the problem is statically determinate and therefore, for a computed factor of safety, force and moment equilibrium are satisfied for each column as well as for the total mass.

Figure 3.7, Free Body Diagram of a Column (Chen, 1981).
Chen’s analysis of the rotational slides concluded that:

1. The 3-D effects are more significant at smaller lengths of the failure mass.
2. For gentle slopes, the 3-D effects are most significant for soils of high cohesion intercept and low friction angle.
3. For soils of low cohesion intercept and high friction angle, the 3-D factor of safety may be slightly less than that for the 2-D case.
4. Pore water pressures may cause the 3-D effects to be even greater.
5. The interslice angle influences the factor of safety.

3.4 Baligh and Azzouz (1975)

Baligh and Azzouz (1975), Azzouz and Baligh (1976, 1978, 1983) and Azzouz, et al., (1981) extended the concept of the two-dimensional circular arc method to evaluate the end effects of the three-dimensional slip surface developed in a cohesive slope. Rather than assume the revolution of an infinitely long cylinder, they considered a rigid body motion of a cylinder of finite length $l_c$ and either a cone of length $l_n$ or ellipsoid of length $l_e$ attached to it (figure 3.8). In addition, they assumed that all elemental shear resistance forces developed over the slip surface are perpendicular to the axis of revolution. Based on that, a numerical solution procedure was developed.
Since only cohesive soil was considered, Baligh and Azzouz (1975) determined the factor of safety using the moment equilibrium equation written about the axis of revolution for two approaches:

a) Vertical cut in clay ($\phi = 0$), where the axis of revolution was taken at the crest of the cut and the cylinder passed through the toe as shown in figure 3.8. Although not providing the minimum factor of safety, these conditions illustrate the
fundamental three-dimensional effects and lead to results that could be checked analytically. Figure 3.9 represents the ratio of the factor of safety as obtained using the 2-D approach to the factor of safety as obtained using the 3-D approach.

![Figure 3.9, Effect of shear Surface Geometry on the Factor of Safety for Vertical Cuts in Clay (Baligh and Azzouz, 1975)](image)

Based on the figure 3.9, Baligh and Azzouz (1975) concluded the following:

1. The 3-D effect tends to increase the factor of safety compared with 2-D.
2. As \( l_c / H \) increases, the value of \( F_3 / F_2 \) decreases. Failures having \( l_c / H \) in excess of four can be considered close enough to plane-strain.
3. For a fixed value of \( l_c / H \), the factor of safety ratio reaches a minimum at a critical value of \( l / H \) which indicates the most likely length of failure. However in the critical value of region the curves are quite flat, especially for small
values of $l_c / H$. therefore, even if the factor of safety can be predicted with reasonable accuracy, the length of failure, $2L$, is more difficult to predict.

4. When $l = 0$, the values of $F$ are the same for both plots in figure 3.8.

b) Toe failure of clay slopes having finite length, by using numerical techniques and following a specified procedure showed in Baligh and Azzouz (1975), the critical shear surface and the corresponding factor of safety were determined. The slip surface was imposed to pass through the toe for computational time-saving.

The limitations of Baligh and Azzouz (1975) are:

1. The method as presented can be applied only to cohesive slopes.
2. Only moment equilibrium is satisfied, and the shear forces acting on the slip surface are assumed to be perpendicular to the axis of revolution.
3. The geometry of slip surface in 3-D space is assumed.
4. More variation of parameters relative to the equivalent 2-D method is needed in order to find the minimal factor of safety.

Azzouz and Baligh (1978) attempted to deal with c-$\phi$ soils also. The same slip surface as in the cohesive soil case was assumed. Also, all elementary shear resistance forces were assumed to be perpendicular to the axis of revolution. In order to make the problem statically determinate, it is necessary to specify the normal stress distribution along the slip surface, which was estimated based on two assumptions: a) the OMS approach where all of the interslice forces were neglected and normal stresses were
obtained from equilibrium of each slice; b) the Massachusetts Institute of Technology (MIT) approach. For the 2-D case, (MIT) method assumes that at the failure surface (1) the vertical effective stress is a principal stress and is equal to the overburden pressure, and (2) the horizontal stress is a minor principal stress when the slope of the tangent to the slip surface is negative, and is a major principal stress when this slope is positive.

Baligh and Azzouz (1978) extended the 2-D approach by assuming a third principal stress, acting parallel to the axis of rotation. This longitudinal principal stress was defined as $K$ times the overburden pressure, where $K$ is an unspecified parameter. Using this principal stress tensor, the stress normal to the 3-D failure surface was calculated. The shear stress over the slip surface was determined by using Coulomb’s equation combined with the calculated normal stress. The results showed that the OMS approach underestimated the shear resistance on steep planes for frictional materials and that the MIT approach appeared to be more reasonable in prediction.

3.5 Leshchinsky et al. (1985)

Leshchinsky et al. (1985) proposed a 3-D mathematical approach to slope stability (figure 3.10), which is based on limit equilibrium and variational analysis introduced by Kopacsy (1957). They presented a formulation that quantified the margin of safety of a given slope relative to its available shear strength and therefore allowed the application of a limiting condition (i.e., the adjusted Coulomb’s failure criterion) to stable slopes. This margin of safety is a function of three unknown functions: the slip surface, the normal stress, and the shear stress direction over this surface. The mathematical problem of
the 3-D slope stability then is to obtain the above three functions which realize the minimum value of the factor of safety $F_s$ that satisfy all equations of limiting equilibrium.

Figure 3.10, Basic conventions and definitions for the three-dimensional analysis: (a) the vectors $r$ and $R$; (b) the direction $\theta$ of the elementary shear force (Leshchinsky et al., 1985)

From arranging the equilibrium equations as an isoperimetric problem and then carrying out variational extremization, Leshchinsky et al. (1985) got the following conclusions:

1. For external slip surface functions, the factor of safety is independent of the normal stress function.

2. The direction of the elementary shear force over the slip surface depends on the slip surface function, but not on the normal stress function.
3. The external slip surfaces are smooth, that is, they have continuous first derivatives.

They derived a first-order partial differential equation, relating the external slip surface and its derivatives. Based on the conclusions above, such a solution is sufficient to determine the minimal factor of safety. However, the solution for this equation is complicated. Therefore, the problem was restricted to symmetrical slip surfaces where the complexity of the problem is significantly reduced.

They found that the solution of the partial differential equation is controlled by unknown curve, which coincides with the slip surface on the symmetry plane y = 0 as shown in figure 3.11.

In terms of a spherical coordinate system, having its origin at \((x_c, 0, z_c)\), the differential equation is:

\[
\rho_\beta = -\psi_m \sin \alpha \sqrt{\rho^2 + \rho_\alpha^2} 
\]

where, the function \(\rho = \rho (\alpha, \beta)\) is the spherical representation of the slip surface \(z (x, y)\), \((\rho, \alpha, \beta)\) are spherical coordinates centered at the point \((x_c, 0, z_c)\), \(\rho_\alpha\) and \(\rho_\beta\) are the partial derivations of \(\rho\) with respect to \(\alpha\) and \(\beta\), respectively; and \(\psi_m = (\tan \phi)/F_s\) where \(\phi\) is the friction angle.
For the case of $\psi_m = \text{constant}$, they found that there are two potential slip surfaces representing two possible modes of failure; one of local nature and the second is a cylindrical mode of failure. Therefore, equation 3.9 has two fundamental solutions:

$$\rho = Ae^{-\psi_m \beta} \sin \alpha$$  \hspace{1cm} (3.10a)

$$\rho = Ae^{-\psi_m \beta} \sin \alpha$$  \hspace{1cm} (3.10b)

where, A is a constant of integration.
The existence of equation (3.10a) that is represented in figure (3.12a) is restricted to the following values:

\[3(\pi/4) \geq \alpha \geq \pi/4\]  
(3.11a)

\[3(\pi/2) - \tan^{-1}(\psi_m) \geq \beta \geq \pi/2\]  
(3.11b)

The second mode of failure (cylindrical) is defined by equation (3.10b), which is plotted in figure (3.12b). In this case there are no restrictions on \(\alpha\); however, \(\beta\) is still restricted by equation (3.11b).
It was found that for the two failure modes, the less cohesive the soil or the steeper the slope, the shallower the resulting slip surface. Generally, the factor of safety obtained for local limited failure surfaces is greater than the factor of safety obtained for long cylindrical failure surfaces.

Leshchinsky and Baker (1986) limited the study for just homogeneous-symmetrical problems, which resulted in a failure mechanism consisting of a cylindrical body with end caps attached to it. So, they used only one-half of the sliding mass (figure 3.13) with only three limiting equilibrium equations (force in x- and z-axis direction and moment about y-axis); because of the symmetry the other three equations are automatically satisfied.

Figure 3.13, The translated coordinate system for the assumed symmetrical problem

(Leshchinsky and Baker, 1986)
Leshchinsky and Baker (1986) presented the moment equation written about the y-axis, and they attained the following expression:

\[
M = \int \left[ \frac{A}{F_s} (c' - u \psi) M_{11} - A M_{12} \right] dx dy + \int \left[ \frac{A}{F_s} (c' - u \psi) M_{21} - A M_{22} \right] dx dy = 0 \tag{3.12}
\]

Where, \( D_1 \) and \( D_2 \) are the projections of the cap and cylindrical parts, respectively into the x-y plane, \( u \) is the pore pressure at \((\alpha, \beta)\), \( \gamma \) is the total unit weight of the soil, and \( \psi \) is the friction angle.

\[
M_{11} = \frac{e^{-\psi \alpha} \sin^2 \alpha}{(\cos^2 \alpha - \sin^2 \alpha) \cos \beta + \psi \sin \beta} \tag{3.13a}
\]

\[
M_{12} = \left[ A e^{-\psi \alpha} \sin^2 \alpha \sin \beta - (Z - z_c) \right] e^{-\psi \alpha} \sin^2 \alpha \sin \beta \tag{3.13b}
\]

\[
M_{21} = \frac{e^{-\psi \alpha}}{\psi \sin \beta - \cos \beta} \tag{3.13c}
\]

\[
M_{22} = \left[ A e^{-\psi \alpha} \cos \beta - (Z - z_c) \right] e^{-\psi \alpha} \sin \beta \tag{3.13d}
\]

where, \( Z \) is the slope surface elevation at \((\alpha, \beta)\).

Substituting the values of \( M_{11}, M_{12}, M_{21}, \) and \( M_{22} \) into equation 3.12 and solving for \( F_s \) gives:
\[
F_s = \frac{c \left( \int_{D_1} M_{11} \, dx \, dy + \int_{D_2} M_{21} \, dx \, dy \right) - \psi \left( \int_{D_1} uM_{11} \, dx \, dy + \int_{D_2} uM_{21} \, dx \, dy \right)}{\gamma \left( \int_{D_1} M_{12} \, dx \, dy + \int_{D_2} M_{22} \, dx \, dy \right)}
\] 

(3.14)

For a given problem (i.e., given slope surface \(Z(x, y)\), pore pressure \(u(x, y)\) and soil parameters \((\gamma, \phi, c)\)) and a given slip surface (i.e., given \((A, x_c, y_c, z_c)\)) the equation of \(F_s\) provides a non-linear relation for the determination the corresponding factor of safety \(F_s\). This non-linearity is due to the presence of the term \(\psi_m = \psi/F_s\) in the previous equations.

Leshchinsky and Baker (1986) concluded that:

1. For cohesionless soil \((c=0)\) the potential slip surface and slope surface coincide. Thus, in this case there are no end effects.

2. The most pronounced end effects are for cohesive soil.

3. Unlike some other 3-D methods, the results here consistently converge to known solutions when boundary cases are checked.

Leshchinsky and Huang (1992) presented a method, which is based on the variational limit-equilibrium approach. It is an extension to of Leshchinsky and Huang’s (1992) 2-D analysis and is rigorous in which all-global limiting-equilibrium equations are explicitly satisfied. This is obtained through a mathematical process in which the normal stress over the specified slip surface is part of the solution.
They proposed a numerical procedure that has been tested against several problems. It involves solving $n$ simultaneous linear equations and three non-linear ones. It was observed that there are several possible combinations of roots for these equations, all basically giving the same factor of safety. They presented an example that demonstrates the significant of 3-D back-analysis. And they concluded that if end-effects are ignored, the in-situ soil strength calculated through local analysis can be overestimated. And this may lead to unsafe conclusions regarding the adequacy of laboratory and field test results to be used in stability analyses.

3.6 Hungr (1987)

Hungr (1987) proposed a 3-D method that is a direct extension of the assumptions associated with Bishop’s (1954) 2-D simplified method using a microcomputer program (CLARA-3):

1. Vertical shear forces acting on both the longitudinal and the lateral vertical faces of each column can be neglected in the equilibrium equations.

2. The vertical force equilibrium equation of each column and the summary moment equilibrium equation of the entire assemblage of columns are sufficient conditions to determine all the unknown forces.

The total normal force $N$ acting on the base of a column (figure 3.14) can be derived from the vertical force equilibrium equation:
\[ N = \frac{W - cA \sin \alpha \gamma / F + uA \tan \phi \sin \alpha \gamma / F}{m_s} \]  

(3.15)

where, \( W \) is the total weight of the column, \( u \) is the pore pressure acting in the center of the column base, \( A \) is the true base area, \( c \) is the cohesion, \( \phi \) is the friction angle, \( F \) is the factor of safety, and

\[
m_s = \cos \gamma \left( 1 + \frac{\sin \alpha \gamma \tan \phi}{F \cos \gamma} \right) \]  

(3.16a)

\[
A = \Delta x \Delta y \left( 1 - \sin^2 \alpha_x \sin^2 \alpha_y \right)^{1/2} \cos \alpha_x \cos \alpha_y \]  

(3.16b)
From the moment equilibrium equation for an assemblage of $j$ columns, with the substituting of $N$, the factor of safety can be written as follows:

$$F = \sum_{i=1}^{j} \left[ (W - uA \cos \gamma_z) \tan \phi + cA \cos \gamma_z \right] / m_a \times \left( \sum_{i=1}^{j} W \sin \alpha_y \right)^{-1} \quad (3.17)$$

Hungr (1987) solved the example problem used by Chen and Chameau (1982)
and used it as a comparison between the two methods. Hungr’s results indicated that, for all cases, the ratio $F_3/F_2$ was greater than 1.0.

Hungr et al. (1989) used a three-dimensional method that was an extension of the assumptions in Bishop’s (1954) simplified and Janbu’s simplified two-dimensional models and they showed comparisons for a number of solutions.

From the results of the comparison, they found the following:

1. The Bishop simplified method gives accurate estimates of the 3-D factor of safety for rotational and symmetric sliding surfaces. It is therefore applicable to a wide range of practical problems, similarly to its 2-D equivalent.
2. The Bishop simplified method tends to be conservative when used for some non-rotational and asymmetric surfaces, because it neglects internal strength.
3. For 2-D surfaces resembling the bilinear geometry, the Janbu simplified methods is always more conservative than the Bishop method.
4. The Bishop method appears reasonably accurate in the important class of problems involving composite surfaces with weak basal planes.

3.7 Summary and Conclusion

Duncan (1996) summarized the studies of three-dimensional slope stability as shown in table 3.1. Based on these studies, Duncan (1996) reported the following conclusions:

1. The factor of safety for three-dimensional analysis is greater than the factor of safety for two-dimensional analyses. The only studies that
indicate otherwise are those by Hovland (1977), Chen and Chameau (1983), and Seed et al. (1990). Hovland’s analyses were based on an extension of the Ordinary Method of slices, which is inaccurate because it assumes zero normal stress on vertical surfaces. Azzouz and Baligh (1978) showed that results calculated using this method are illogical for some conditions, and that extension of the Ordinary Method of Slices is not an adequate approach to 3-D analysis. Hutchison and Sarma (1985), Cavounidius (1987), and Hungr (1987) questioned some of the assumptions used by Chen and Chameau, found that $F_3$ was greater than $F_2$ rather than smaller, as Chen and Chameau had found. Seed et al. (1990) compared results of 2D and 3D analyses that did not satisfy all conditions of equilibrium. The horizontal force imbalance in their approximate 3D analysis was 3.7% of the weight of the sliding mass. Because the friction angles along the slip surface they studied were so small (8° to 9°), this force imbalance could result in as much as a 25% difference in the calculated factor of safety. Thus, all of the cases where $F_3$ was found to be smaller than $F_2$ appear to involve significant potential inaccuracies.

2. Hutchinson and Sarma (1985) and Leshchinsky and baker (1986) pointed out that 2D and 3D analyses should give the same factor of safety for slopes in homogeneous cohesionless soils, because the critical slip surface is a shallow plane parallel to the surface of the slope.

3. Azzouz et al. (1981), and Leshchinsky and Huang (1992) noted that, if 3D effects are neglected in analyses to back calculate shear strengths; the back calculated strengths will be too high.
Table 3.1, Methods of analyzing 3D Slope Stability (Duncan, 1996)

<table>
<thead>
<tr>
<th>Authors (1)</th>
<th>Method (2)</th>
<th>Strength (3)</th>
<th>Geometry of slope/slip surface (4)</th>
<th>3D effects found (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anagnosti (1969)</td>
<td>Extended Morgenstern and Price</td>
<td>c, φ</td>
<td>Unrestricted/unrestricted</td>
<td>F₁ = 1.5 F₂ in one case</td>
</tr>
<tr>
<td>Baligh and Azzouz (1975)</td>
<td>Extended circular arc</td>
<td>φ = 0</td>
<td>Simple slopes/surfaces of revolution</td>
<td>F₁ &gt; F₂</td>
</tr>
<tr>
<td>Giger and Krizek (1975)</td>
<td>Upper bound theory of perfect plasticity</td>
<td>c, φ</td>
<td>Slopes with corners/log spiral</td>
<td>F₁ &gt; F₂</td>
</tr>
<tr>
<td>Giger and Krizek (1976)</td>
<td>Extended circular arc</td>
<td>φ = 0</td>
<td>Simple loaded slopes/surfaces of revolution</td>
<td>F₁ &gt; F₂</td>
</tr>
<tr>
<td>Baligh et al. (1977)</td>
<td>Upper bound theory of perfect plasticity</td>
<td>c, φ</td>
<td>Four real embankments/surfaces of revolution</td>
<td>F₁ &gt; F₂, for some cases</td>
</tr>
<tr>
<td>Hovland (1977)</td>
<td>Extended ordinary method of slices</td>
<td>c, φ</td>
<td>Unrestricted/unrestricted</td>
<td>F₁ &gt; F₂, for some cases</td>
</tr>
<tr>
<td>Azzouz et al. (1981)</td>
<td>Extended Swedish circle</td>
<td>ϕ = 0</td>
<td>Same as Baligh and Azzouz with loads on top</td>
<td>F₁ &gt; F₂</td>
</tr>
<tr>
<td>Chen and Chameau (1982)</td>
<td>Extended Spencer, and finite element</td>
<td>c, ϕ</td>
<td>Slopes with loads/unrestricted</td>
<td>F₁ &gt; F₂</td>
</tr>
<tr>
<td>Chen and Chameau (1983)</td>
<td>Extended Spencer</td>
<td>c, ϕ</td>
<td>Unrestricted/unrestricted</td>
<td>F₁ &gt; F₂, for some cases</td>
</tr>
<tr>
<td>Azzouz and Baligh (1983)</td>
<td>Extended Swedish circle</td>
<td>ϕ = 0</td>
<td>Unrestricted/unrestricted</td>
<td>F₁ &gt; F₂</td>
</tr>
<tr>
<td>Dennhardt and Forster (1985)</td>
<td>Assumed z on slip surface</td>
<td>c, ϕ</td>
<td>Vertical slopes/cylindrical</td>
<td>F₁ &gt; F₂</td>
</tr>
<tr>
<td>Leshchinsky et al. (1985)</td>
<td>Limit equilibrium and variational analysis</td>
<td>φ = 0</td>
<td>Slopes constrained in 3d dimension/unrestricted</td>
<td>F₁ &gt; F₂ for c &gt; 0, F₁ = F₂, c = 0</td>
</tr>
<tr>
<td>Ugas (1985)</td>
<td>Limit equilibrium and variational analysis</td>
<td>c, ϕ</td>
<td>Conical heaps/unrestricted</td>
<td>F₁ &gt; F₂, for c &gt; 0, F₁ = F₂, c = 0</td>
</tr>
<tr>
<td>Leshchinsky and Baker (1986)</td>
<td>Limit equilibrium and variational analysis</td>
<td>c, ϕ</td>
<td>Unrestricted/unrestricted</td>
<td>F₁ must be &gt; F₂</td>
</tr>
<tr>
<td>Baker and Leshchinsky (1987)</td>
<td>Limit equilibrium and variational analysis</td>
<td>c, ϕ</td>
<td>Unrestricted/unrestricted</td>
<td>F₁ &gt; F₂</td>
</tr>
<tr>
<td>Cavounidis (1987)</td>
<td>Limit equilibrium</td>
<td>c, φ</td>
<td>Unrestricted/unrestricted</td>
<td>F₁ &gt; F₂</td>
</tr>
<tr>
<td>Hugr (1987)</td>
<td>Extended Bishop's modified</td>
<td>c, φ</td>
<td>Unrestricted/unrestricted</td>
<td>F₁ &gt; F₂</td>
</tr>
<tr>
<td>Gens et al. (1988)</td>
<td>Extended Swedish circle</td>
<td>φ = 0</td>
<td>Simple slopes/surfaces of revolution</td>
<td>F₁ &gt; F₂</td>
</tr>
<tr>
<td>Leshchinsky and Mullet (1988)</td>
<td>Limit equilibrium and variational analysis</td>
<td>c, φ</td>
<td>Vertical slopes with corners/unrestricted</td>
<td>F₁ &gt; F₂</td>
</tr>
<tr>
<td>Ugas (1988)</td>
<td>Extended ordinary method of slices, Bishop's modified, Janbu, and Spencer</td>
<td>c, ϕ</td>
<td>Unrestricted/unrestricted</td>
<td>F₁ &gt; F₂, except for OMS</td>
</tr>
<tr>
<td>Xing (1988)</td>
<td>Limit equilibrium</td>
<td>c, ϕ</td>
<td>Unrestricted/unrestricted</td>
<td>F₁ &gt; F₂</td>
</tr>
<tr>
<td>Michalowski (1989)</td>
<td>Kinematical theorem of limit plasticity</td>
<td>c, ϕ</td>
<td>Unrestricted/unrestricted</td>
<td>F₁ &gt; F₂</td>
</tr>
<tr>
<td>Seed et al. (1990)</td>
<td>Ad hoc 2D and 3D</td>
<td>c, ϕ</td>
<td>One particular case, the Kettleman Hills failure</td>
<td>F₁ &lt; F₂</td>
</tr>
<tr>
<td>Leshchinsky and Huang (1991)</td>
<td>Limit equilibrium and variational analysis</td>
<td>c, ϕ</td>
<td>Unrestricted/unrestricted</td>
<td>F₁ &gt; F₂</td>
</tr>
</tbody>
</table>
CHAPTER IV

SLOPE STABILITY SEARCHING METHODS COMPARISON

4.0 Introduction

Slope stability is one of the most common problems in geotechnical engineering. The need for a better understanding of failure mechanism of both man-made and natural earthen slopes is necessary to better assist any remediation measure of potentially moving or failing slopes. Slope stability analysis methods developed over many years and continue to develop as a result of advancement in technology, instrumentations, soil behavior and theories, and new constitutive soil models.

Thousands of research articles had been published since the publication of the first method of analysis by Fellenius (1936) that were either related to slope stability or involved slope stability analysis subjects. Among the available analysis methods are the limit equilibrium methods of slices, boundary element methods (Jiang, 1990), finite element methods (Matsui and San, 1992), and neural network methods (Jaritngam et al, 2001).

Limit equilibrium methods of slices are the most common used methods among others since simplicity and ease of use are the main advantages. Those methods satisfy
either some or all of the equilibrium conditions. Satisfied equilibrium conditions include:

(1) some or all interslice forces (Fellenius 1936; Janbu 1954, 1968, 1973; Corps of Engineers Modified Swedish, 1970; Lowe and Karafiath, 1960), (2) moment and/or some forces (Taylor, 1937; Bishop, 1955), (3) moment and all forces (Morgensten and Price, 1965; Spencer, 1967, 1973; Sarma, 1973, 1979). Taylor and Fellenius methods can be used for circular slip surfaces while the others can be used for circular and non-circular slip surfaces.

Due to the large number of possible slip surfaces, computers are used to facilitate computations. Interestingly, factors of safety obtained from stability analysis methods that satisfy all limit equilibrium conditions are within 6% of each other (Duncan, 1996). These methods include friction circle methods; log spiral methods, rigorous limit equilibrium methods, and finite element methods. A possible reason of difference, if the same stability analysis method is used, can be attributed to numerical problems in the static simplified search techniques used in computations, where all the trial surfaces are pre-selected. More advanced dynamic search techniques had been proposed within the last two decades and led to more accurate and acceptable results. Dynamic search techniques include the pattern search algorithm (De Natale, 1991), alternating variable method (Celestino and Duncan, 1981; Li and White, 1987), dynamic programming method (Baker, 1980), random search method (Chen, 1992), simplex method (Nguyen, 1985; De Natale, 1991). Dynamic search methods are somewhat efficient since a large number of slip surfaces is generated leading
to a higher accuracy in locating the slip surface with the minimum factor of safety.

Recently, more advanced search techniques based on Monte Carlo techniques were proposed by Greco (1996), and Husein Malkawi et al (2000, 2001a,b, 2002, 2003). The Monte Carlo technique is a structured random searching and optimization technique where the slip surface is located using either random jumping or random walking methods. Random jumping methods involve the use of a large number of independent slip surfaces while the random walking methods involve the use of a large number of dependent slip surfaces to examine the most critical failure surface. Slip surfaces are called dependent when each new slip surface is created based on the result of a preceding slip surface. Independent slip surfaces are all generated regardless of analysis results.

4.1 Finite Element Method and Limit Equilibrium Methods

Finite element method is a very powerful computational tool in engineering. It gains its power from the ability to simulate physical behaviors using computational tools without the need to simplify the problem. Indeed, complex engineering problems need finite element methods to obtain more reliable and accurate results. Nowadays, new proposed analysis methods, in engineering, can be verified using finite element method as a benchmark.

Slope stability analysis using finite element uses a similar failure definition as the limit equilibrium method for the soil mass and does not need simplifying assumptions.
Limit equilibrium methods first define a proposed slip surface then the slip surface is examined to obtain the factor of safety, which is defined as the ratio between the available resisting moments and the driving moments along the surface. Many methods for slope stability analysis using finite elements have been proposed during the last two decades. Among those methods, gravity increase method (Swan and Seo, 1999) and strength reduction method (Matsui and San, 1992) are considered the most widely used methods. In the gravity increase method, gravity forces are increased gradually until the slope fails \((g_f)\) then the factor of safety is defined as the ratio between the gravitational acceleration at failure \((g_f)\) and the actual gravitational acceleration \((g)\). In the strength reduction method, soil strength parameters are reduced until the slope becomes unstable, therefore, the factor of safety is defined as the ratio between the initial strength parameter and the critical strength parameter. Therefore, strength reduction method has exactly the same definition as the limit equilibrium methods (Griffiths and Lane, 1999). The gravity increase method is used to study the stability of embankments during construction since it gives more reliable results while the strength reduction method is used to study the stability of existing slopes. In order to compare results of limit equilibrium methods with finite element analysis results, strength reduction method was selected in this study since it resembles the limit equilibrium approach more than the gravity increase method.

Selection of constitutive models in finite element analysis is vital and should be based on the physical behavior of the studied problem. For example, the soil-hardening model gives more reliable results than the Mohr-Coulomb model when modeling soil
excavations. Mohr-Coulomb model is a linearly elastic, perfectly plastic model where soil parameters are assumed to be constant during all stages of soil loading and unloading during the analysis. Soil hardening model is an elastoplastic model that simulates the behavior of soils in triaxial test (shear hardening) and in oedometer test under isotropic loading (compression hardening) where the yield cap is expanded due to plastic strains (Brinkgreve et al, 2001).

In finite element slope stability analysis, the use of the strength reduction method with advanced soil models leads to a behavior similar to the Mohr-Coulomb model since stress-dependent stiffness behavior and hardening effects are excluded (Brinkgreve et al, 2001). In advanced constitutive soil models, stiffness modulus is stress-dependent and changes based on step-size computation increments. When strength reduction method is used stiffness modulus from the previous step is used as a constant stiffness modulus during computations (Brinkgreve et al, 2001), and as a result, the advanced soil model behaves like the Mohr-Coulomb model where a constant stiffness modulus is used, as well.

The Mohr-Coulomb model needs six parameters as input; $\phi$ friction angle, $c$ cohesion, $\psi$ dilation angle, $E$ deformation modulus (Young’s modulus), $v$ Poisson’s ratio, and $\gamma$ unit weight. For slope stability analysis, $\psi = 0$ was adopted assuming a non-associated flow rule, based on the results by Griffiths and Lane (1999). Young’s modulus and Poisson’s ratio are of insignificant importance in slope stability analysis using strength reduction method, due to the nature and mathematical formulation of the method.
(Matsui and San, 1992). In this study, Young’s modulus and Poisson’s ratio were given arbitrary values of 1x10^6 kN/m^2 (or 1x10^6 psf) and 0.3, respectively. As a result, strength reduction method using Mohr-Coulomb method required only three soil parameters (φ, c, and γ), same as the limit equilibrium method. Therefore, factors of safety from both limit equilibrium methods and strength reduction method can be compared.

Triangular elements with 15 nodes were used to construct meshes in 2D analysis assuming plane strain conditions, while 15-node wedges were used to construct meshes in 3D analysis. In the 3D analysis, a width of 100m was used in all cases.

4.2 SAS-MCT4 and Monte Carlo Techniques

Monte Carlo techniques are very fast and easy-to-implement optimization techniques and can be used for most common slope stability methods. Unlike most slope stability analysis search techniques (Whitman and Baily, 1967; Baker, 1980; Chowdhury and Zhang, 1990), no slip surface is rejected to achieve convergence in computations (Husein Malkawi et al, 2001). Thus, limit equilibrium methods are used fully without the need to make any restraints in addition to the numerical simplifications that are already assumed in the analysis methods. On the other hand, the use of Monte Carlo techniques allows the increase of the number of slices used during slip surface analysis since computer analysis time can be optimized and more detailed and accurate results can be obtained.

Husien Malkawi and Hassan (2003) implemented new Monte Carlo search
techniques in a slope stability analysis program (SAS-MCT 4.0). SAS-MCT 4.0 is a limit equilibrium analysis program that uses the Monte-Carlo random jumping and two-point walking optimization techniques. Slope stability analysis, using SAS-MCT 4.0, can be carried out using conventional slope stability analysis methods coupled with Monte Carlo searching technique. Irregular slip surfaces can be generated with up to 12 vertices. Approximately, up to 10,000 slip surfaces can be generated for 50 slices during the analysis which makes SAS-MCT 4.0 one of the most powerful and rigorous limit equilibrium analysis tools. Another powerful feature of SAS-MCT 4.0 is the ability to estimate uncertainties in slope stability analyses, by generating a large number of expected soil parameters, and then estimating a reliability index and a probability of failure factor. This feature is very useful especially when the soil parameters are influenced by regional variations in soil behavior, technical quality, spatial variations, sampling and testing procedures, etc.

4.3 Monte Carlo Techniques Verification

In order to verify the power of implementing Monte-Carlo technique in slope stability analysis, a comparison was made between results from finite element analysis using PLAXIS 3D (Brinkgreve et al, 2001), limit equilibrium analysis using SAS-MCT 4.0 (Husein Malkawi and Hassan, 2003), and other commercially available slope stability programs. Some programs, for example UTEXAS3 (Wright, 1991), use the rectangular grid search technique while others, for example STABL5M, uses the linear grid search technique to generate the slip surfaces. The investigated programs are widely used by
Most programs have different options that influence the minimum factor of safety in each available stability method. These options are highly dependent on the judgments of the user and should be used by an experienced engineer. However, Bishop’s method is implemented in most programs and the human factor is almost negligible. Hence, Bishop’s method was used as a reference method for comparison between programs and search techniques.

Using the same stability method for comparison gave many advantages. Location of slip surfaces cannot be compared unless results from different programs that use the same stability method are compared, since different stability methods give different slip surfaces, and the only thing that can be compared is the factor of safety. Using the same stability method as a reference allowed comparing both the slip surface location and the associated factor of safety. As a result, efficiency of search techniques can be compared as well.

Additional comparisons were made using FEA (PLAXIS 2D and 3D) as a “benchmark”. Factors of safety from each method were compared with factors of safety from FEA to investigate the ability of the search techniques to locate the minimum slip surface.

4.4 Comparison Examples

In order to investigate the efficiency of the Monte Carlo technique, three examples
were used for comparison with other programs that use different search techniques. The investigated geometries and soil profiles ranged from simple to complex.

4.4.1 Example No.1

The geometry, strength parameters, and unit weight of the homogeneous slope of Example No.1 were shown in Figure 4.1 and Table 4.1. Factors of safety from 2D and 3D finite element analyses were considered as upper and lower bounds and were in the order of 1.70 and 1.80, respectively. Results from finite element analysis were shown in Figures 4.2 and 4.3. Analysis using simplified Bishop’s, SAS-MCT 4.0, STABL5M, and UTEXAS3 showed that the factors of safety and the critical slip surfaces were practically the same and all were in the order of 1.70. Limit equilibrium results were shown in Figure 4.1. A summary of the results was shown in Table 4.2.

Figure 4.1, Geometry of slope and slip surfaces for example 1.
### Table 4.1, Soil properties for the three examples used in this study

<table>
<thead>
<tr>
<th>Example No.</th>
<th>Layer No.</th>
<th>$\gamma$ (kN/m$^3$)</th>
<th>Cohesion (kPa)</th>
<th>$\phi$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example No.1</td>
<td>No.1</td>
<td>17</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>No.2</td>
<td>21.0</td>
<td>0.0</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>No.3</td>
<td>18.2</td>
<td>100.0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>No.4</td>
<td>18.0</td>
<td>40.0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>No.5</td>
<td>16.9</td>
<td>95.0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>No.6</td>
<td>18.3</td>
<td>95.0</td>
<td>0</td>
</tr>
<tr>
<td>Example No.2</td>
<td>No.1</td>
<td>18.2</td>
<td>80.0</td>
<td>0</td>
</tr>
<tr>
<td>Example No.3</td>
<td>No.1</td>
<td>120</td>
<td>300</td>
<td>30</td>
</tr>
</tbody>
</table>

### Table 4.2, Factors of safety using different slope stability softwares

<table>
<thead>
<tr>
<th>Example No.</th>
<th>SAS-MCT 4.0</th>
<th>UTEXAS3</th>
<th>STABL5M</th>
<th>PLAXIS 2D</th>
<th>PLAXIS 3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.70</td>
<td>1.70</td>
<td>1.713</td>
<td>1.70</td>
<td>1.80</td>
</tr>
<tr>
<td>2</td>
<td>1.179</td>
<td>1.22</td>
<td>1.214</td>
<td>1.14</td>
<td>1.22</td>
</tr>
</tbody>
</table>

Figure 4.2, Exaggerated deformed mesh for the finite element 2D model in example 1.
4.4.2 Example No.2

In this example, slope stability of Karameh Dam (Husein Malkawi and Hassan, 2003) was investigated. The geometry of the studied problem, as shown in Figure 4.4, is more complex than in Example 1. Soil parameters of the heterogeneous soil profile as reported by Malkawi and Hassan (2003) were shown in Table 4.1. Analysis results were shown in Figure 4.4 and Table 4.2. As it can be seen, SAS-MCT 4.0, STABL5M, and UTEXAS3 results were 1.179, 1.214, and 1.22, respectively. On the other hand, results from finite element analysis showed that the factor of safety was between 1.14 (2D analysis) and 1.22 (3D analysis). Finite element results from 2D and 3D analysis were shown in Figures 4.5 and 4.6.
Figure 4.4, Geometry of slope and slip surfaces for example 2.

Figure 4.5, Deformed mesh for the finite element 2D model in example 2.
4.4.3 Example No.3

This example was taken from Pockoski and Duncan (2000). In their work, they investigated and compared the performance of eight slope stability programs. In this example, another homogeneous slope, as shown in Figure 4.7 and Table 4.1, with a simple geometry was investigated. The difference in complexity between Example No.1 and this example is only the presence of water table. Results from SAS-MCT 4.0 analyses were shown in Table 4.3. Results from other programs as reported by Pockoski and Duncan (2000) were shown in Table 4.4. Results from finite element analysis showed that the factor of safety was between 1.268 (2D analysis) and 1.336 (3D analysis). Finite element results from 2D and 3D analysis were shown in Figures 4.8 and 4.9. Best estimate for the factor of safety was between 1.27 and 1.32 (Pockoski and Duncan, 2000).
Figure 4.7, Geometry of slope and slip surfaces using Bishop’s method in example 3.

Table 4.3: Factors of safety calculated for example 3 using SAS-MCT 4.0

<table>
<thead>
<tr>
<th>Method</th>
<th>FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bishop</td>
<td>1.289</td>
</tr>
<tr>
<td>Corrected Janbu</td>
<td>1.206</td>
</tr>
<tr>
<td>Ordinary</td>
<td>1.023</td>
</tr>
</tbody>
</table>
Table 4.4, Factors of safety calculated for Example 3.  
(Pockoski and Duncan, 2000)

<table>
<thead>
<tr>
<th>Program</th>
<th>Method</th>
<th>FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>UTEXAS4</td>
<td>Spencer</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>Bishop</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>Simplified Janbu</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>Lowe &amp; Karafiath</td>
<td>1.32</td>
</tr>
<tr>
<td>SLOPE/W</td>
<td>Spencer</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>Bishop</td>
<td>1.29</td>
</tr>
<tr>
<td></td>
<td>Simplified Janbu</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>Morgenstern-Price</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>Ordinary</td>
<td>1.04</td>
</tr>
<tr>
<td>SLIDE</td>
<td>Spencer</td>
<td>1.31</td>
</tr>
<tr>
<td></td>
<td>Bishop</td>
<td>1.29</td>
</tr>
<tr>
<td></td>
<td>Simplified Janbu</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>Lowe &amp; Karafiath</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td>Ordinary</td>
<td>1.05</td>
</tr>
<tr>
<td>XSTABL</td>
<td>Corrected Janbu</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>Bishop</td>
<td>1.29</td>
</tr>
<tr>
<td>WINSTABL</td>
<td>Spencer</td>
<td>1.34</td>
</tr>
<tr>
<td></td>
<td>Bishop</td>
<td>1.34</td>
</tr>
<tr>
<td></td>
<td>Simplified Janbu</td>
<td>1.20</td>
</tr>
<tr>
<td>RSS</td>
<td>Simplified Janbu</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>Bishop</td>
<td>1.29</td>
</tr>
<tr>
<td>SNAIL</td>
<td>Wedge method</td>
<td>1.22</td>
</tr>
<tr>
<td>GOLDNAIL</td>
<td>Circular Method</td>
<td>1.32</td>
</tr>
</tbody>
</table>
Figure 4.8, Deformed mesh for finite element 2D model in example 3.

Figure 4.9, Deformed mesh for finite element 3D model in example 3.
4.5 Discussion

The geometry of the investigated slopes ranged from simple to complex. In Example No.1 results from SAS-MCT 4.0, STABL5M and UTEXAS3 were all, practically, of the order 1.7 and showed good agreement with the 2D finite element analysis. Results from finite element 2D and 3D analysis were 1.7 and 1.8, respectively.

In Example No. 3, the upper and lower bounds of the factor of safety were 1.29 and 1.34, and 1.268 and 1.336 based on the analysis by Pockoski and Duncan (2000), and finite element analysis, respectively. Results from SAS-MCT 4.0 were in good agreement with the acceptable limits. Results from Example No.3 suggested that the obtained factors of safety using any limit equilibrium method, even for ordinary method of slices, coupled with Monte-Carlo technique can be enhanced since more optimized slip surfaces can be searched. A comparison using Bishop’s slip surfaces was shown in Figure 4.7. As it can be seen, all search techniques except Monte Carlo missed the most critical slip surface. The most critical slip surface obtained by SAS-MCT 4.0 was located between other slip surfaces obtained by the other programs, and even though, no other search method was able to locate it. The same observation was true for the comparison between slip surfaces obtained using Janbu’s method, as shown in Figure 4.8. As a result, it was evident that pre-determined step size searches can influence the location of the slip surface and hence the value of the factor of safety.

Example No.2 has more complex slope geometry than Examples No. 1 and No. 3.
In this example all factors of safety were in good agreement with the acceptable range (1.14 to 1.22). However, UTEXAS3 (FS=1.22) and STABL5M (FS=1.214) showed factors of safety that matched the upper bound limit while SAS-MCT 4.0 (FS=1.179) showed a value that was, relatively, more conservative. It can be seen that factors of safety using any limit equilibrium method coupled with Monte-Carlo technique can give more reasonable factors of safety. However, the critical slip surface generated by STABL5M was located far above the one generated by SAS-MCT 4.0 indicating a shallow slip surface while the one generated by UTEXAS3 was shifted to the right suggesting that the searched slip surfaces in STABL5M and UTEXAS3 were not influenced by the obtained results during search, and therefore, some slip surfaces were missed easily. Each generated slip surface in SAS-MCT 4.0 was generated based on the results from the preceding surfaces using Monte-Carlo optimization technique and thus the most critical slip surface was effectively located.

In all cases, the effort needed by the user is the least when SAS-MCT 4.0 is used. SAS-MCT 4.0 uses a fully automated search procedure while all other programs need engineering judgment to determine the search sequence and procedure.

4.6 Conclusions

In this study, comparisons were made between strength reduction technique and the effect of search techniques on the results obtained from limit equilibrium method using different commercially available programs. It was shown that reliability of using limit equilibrium methods in the analysis can be largely increased using Monte-Carlo
optimization technique as shown by results obtained using SAS-MCT 4.0.

It can be seen that SAS-MCT 4.0 is more powerful tool than any other commercially available slope stability program since Monte-Carlo technique can be used during the search for critical slip surfaces. Heterogeneous slopes and slopes with complex geometry should be investigated using more advanced search techniques since the generation of critical slip surfaces is not based on a pre-determined step-size and is based on a step-size that is influenced by the location of the preceding critical slip surface. This will insure that all possible critical slip surfaces are investigated and no potential critical slip surface is left behind.

Predicted failure zone can be influenced by the search technique. It was evident from Example No.2 that the use of a simple search technique can lead to a shallow slip surface while the most critical slip surface was deep. On the other hand, different search techniques can give almost the same factors of safety but with different slip surfaces. As it can be seen from Example No.2, SAS-MCT 4.0 proved that Monte Carlo technique should be used for complex soil profiles and geometries and when more reliable and accurate results are needed.

In this study it was shown that limit equilibrium methods are reliable and can be used with confidence to investigate the stability of slopes. Analysis using finite element methods can be expensive and not justified unless other complex soil boundary conditions exist. Hence, the development of advanced search and optimization techniques are vital to
enhance the limit equilibrium methods, which are adequate for regular slope stability problems.

In most commercially available slope stability programs, user’s judgment may influence the search technique and the obtained factor of safety. Automating the search techniques can be a cumbersome task if classic search techniques are used. Advanced optimization techniques showed promising potential for further developments and future applications in slope stability and in geotechnical engineering.
5.0 Introduction

Slope stability analysis techniques have been used for more than four decades assuming simplified conditions or time effective analysis techniques such as the two-dimensional limit equilibrium method. The 2D limit equilibrium method assumes mainly that the slope failure is dominated by plane-strain conditions and hence assumes that the slip surface is infinitely wide. This assumption is very important since it reduces other unknowns involved in the analysis such as the side forces and therefore significantly reduces the analysis time largely. However, all slopes in nature have finite widths and therefore this simplification affects the analysis by ignoring the width of the slide mass.

In practice, the two dimensional analysis of slopes is more accepted since lower factors of safety result from ignoring the side effects of the 3D slide masses in the limit equilibrium methods (Stark and Eid, 1998). This can be accepted for any design stages of slopes since more conservative designs can be obtained.

On the other hand, to account for more realistic analysis of failures in slopes the 3D limit equilibrium is recommended. The 3D analysis is used to back calculate the shear
strength of the slope at failure by including the end effects of the slope such as the finite
width of the slope (Duncan 1992, Stark and Eid 1998). However, such analysis can be
time and money consuming due to the number of resulting unknowns, which directly
affect the computation time especially using the limit equilibrium method.

A relation between the two dimensional analysis results and the three dimensional
analysis results can be obtained using Skempton’s correction factor (Skempton 1985).
Skempton (1985) suggested a relation between the backcalculated shear strengths from
two and three dimensional limit equilibrium analysis methods.

\[ S_u(3D) = S_u(2D) \frac{1}{1 + \frac{KD}{B}} \]  

(5.1)

where \( S_u(3D) \) = back calculated shear strength using 3D limit equilibrium analysis,
\( S_u(2D) \) = back calculated shear strength using 2D limit equilibrium analysis, K= earth
pressure coefficient at failure, D= depth of the failure mass, B= width of the failure mass.

Skempton (1985) reported that the above correction factor can produce a 5% increase in
the backcalculated shear strength which can be used to simplify the above equation to:

\[ S_u(3D) = 1.05 \ S_u(2D) \]  

(5.2)

However, the variation between the backcalculated 2D and 3D shear strengths at
failure was found to be as high as 30% (Stark and Eid, 1998) depending on the analyzed
slope configuration, geometry, boundary conditions, and the soil types. This variation suggested the necessity of 3D analysis and the limitations of such shear strength factor.

In this chapter, the performance of different slope stability methods will be compared using 2D and 3D methods. The comparison will be performed using well-documented case histories and analysis using limit equilibrium as well as finite element analysis methods.

5.1 3D Failure vs. 2D Failure

Analysis of slopes using 3D and 2D slope configurations has led to controversial results over the last four decades showing both the importance and the difficulty to handle the subject. The differences in the obtained results can be attributed to the limited amount of studies and to the limited cases handled in each study. Most of the studies were aimed at providing either example to validate the proposed method or to study certain limitations of the proposed method. In addition, the case studies that were addressed in most researches were limited to certain situations such as (Duncan, 1996): 1) slopes with curves or corners in plane; 2) slopes with loads of limited extent; and 3) slopes in which the potential failure surface is constrained by physical boundaries. However, none of the studies addressed the practical use and the limitations of the methods from the standpoint of the engineers and practitioners, and hence limited the use of such methods to the judgments of the user. Therefore, it is important to study the efficacy of such methods and to compare the available tools that are currently in use by engineers to help in bridging the gap between the research community and the
Since the early 1960’s, different studies led to different results regarding the difference between the 2D analysis and the 3D analysis of slopes. The results in some cases showed higher factors of safety for two-dimensional analysis when compared with 3D analysis (Baligh and Azzouz 1975, Giger and Krizek 1975, Leshchinsky et al. 1985, Gens et al. 1988, Leshchinsky and Huang 1992) while it showed the opposite in other studies (Hovland 1977, Chen and Chameau 1983, Seed et al. 1990). The differences in studies resulted from the difference of the methods used in each case. In general, the 3D analysis was carried out for the most critical section from the 2D analysis due to the simplicity of calculating the most critical slip surface using 2D analysis. Therefore, the generated critical slip surface did not necessarily match the most critical slip surface that can be generated from 3D analysis or field observations if a finite width of the slope is considered. On the other hand, extending the 2D version of the method and therefore carrying the limitations of the 2D method to the 3D method proposed most 3D methods. For example, Hovland’s method (1977) was based on the extension of the Ordinary Method of Slices and therefore led to erroneous results in 3D analysis because normal stresses on vertical surfaces were assumed zero and therefore the computed 3D factors of safety were less than the 2D factors of safety. On the other hand, the complexity of the extension of Spencer’s method by Chen and Chameau (1983) resulted in misleading results that suggested higher 2D factors of safety compared to 3D factors of safety (Ugai, 1988). Therefore, it can be said that, in general, slope stability methods that produce
factors of safety higher in 3D analysis than those in 2D analysis are more accurate while those that produce the opposite are inaccurate due to the simplifying assumptions or the erroneous derivation or extension of the method.

In addition, some studies showed a constant ratio between the 3D and 2D factors of safety. Azzouz et al. (1981) suggested that the ratio between 3D factors of safety and 2D factors of safety (F3/F2) for slopes in undrained cohesive soils using the Extended Swedish Circle was between 1.07 to 1.3. Such results should not be adapted to other general cases due to the limited number of cases that led to such conclusion, which in Azzouz et al.’s (1981) research was four.

On the other hand, the use of 3D analysis to back-calculate the shear strength of the slope would result in higher shear strengths when compared to 2D backcalculation (Azzouz et al. 1981, Leshchinsky and Huang 1991). Slopes in homogeneous cohesionless slopes are expected to have shallow failure surfaces that are parallel to the surface of the slope resulting in the same factor of safety regardless of the analysis dimension (3D or 2D) (Hutchinson and Sharma 1985, Leshchinsky and Baker 1986).

5.2 Slope Stability Analysis Practice

Slope stability analysis is performed using computers due to the lengthy computational procedures needed in limit equilibrium as well as finite element techniques. A large number of commercially available slope stability programs are in use by engineers all around the world. The discrepancy in the computed factors of safety has
led to many problems due to the accuracy of the method used, the numerical approach in programming, search technique for the most critical slip surface, the dependency on the user, simplified geometry of the slope, number of iterations in the analysis, and the computational accuracy and tolerance. Some agencies and engineers recommend the use of some programs over others due to the common and wide base of users rather than solid and coherent technical reasons. On the other hand, some programs are user-friendly than others making their use more accepted by engineers. However, since no general reference is available to select the most appropriate 2D or 3D analysis program, engineers have been left to their own judgments giving the accuracy and the validity of the results less concern.

It should be noted that limitations of slope stability analysis methods do not suggest that they have no value. Stability analysis methods should be used with a solid understanding of their backgrounds and theory and with the ability to evaluate the analysis results based on the cases and the used method.

In practice, different slope stability methods produce different slip surfaces and hence different minimum factors of safety (Duncan and Wright, 1980). Therefore, It is important to compare the analysis results with field measurements rather than comparing the factors of safety only. The most critical issue in evaluating different analysis methods is the ability of that method to capture the most critical slip surface. Under a certain critical slip surface, factors of safety from different methods can be different due to the
difference in the mathematical formulation and the simplifications used in developing the method.

On the other hand, limitations of slope stability analysis can be noted after extensive use of the method. For example, the presence of a hard cohesive layer at the top of the slope or the presence of a very dense granular layer at the bottom of the slope can produce slip surfaces that are almost vertical. In such cases, good engineering practice and experience suggest the use of tension cracks at the top of the slope to flatten the exit angle at the lower end of the slip surface (Duncan, 1996). Such cases are mostly ignored by most engineers due to the lack of solid engineering understanding of slope stability analysis limitations resulting in misleading factors of safety.

Performing the 3D analysis requires having enough soil data from field borings. However, increasing the number of borings will increase the needed budget for the project and therefore is not preferred. Additional borings can help in creating soil profiles at different cross sections within the sliding mass. Due to the associated high cost for 3D slope stability analysis, engineers limit the use to the following cases (Cornforth, 2004):

1. When the width (W) to the length (L) ratio of the landslide is greater than 2 (W/L>2).
2. When the depth of the slippage changes significantly between the center and the sides of the slide.
3. When the change in the surface geology or the groundwater conditions across
the width of the slide is significant.

Lambe and Whitman (1969) suggested the use of a pseudo 3-D approach to calculate the 3D factor of safety. In this approach the 2D factor of safety is calculated for several parallel cross-sections and then used to find the weighted average factor of safety of the slope in 3D. The area of each cross section should be used as the weighting factor. It is common to use two sections or three sections to perform the analysis. The two-section method is performed by making the sections at the quarter-point widths of the slope while the three-section method is performed by making the sections at the one-sixth, centerline, and one-sixth widths of the slope sections. More sections can be obtained by assuring that the sections are covering equal areas of the slip (Cornforth, 2004). The two-section method was shown in Figure 5.1. This method is a simplification of the 3D analysis and has been used by engineers for a long time. In addition, such assumption will always yield a 2D factor of safety close to the 3D factor of safety.

Figure 5.1, Cross Section Locations for Two-Section Analysis (Cornforth, 2004).
5.3 Side Resistance and Slope Stability Analysis

End effects in 2D limit equilibrium slope stability analysis methods are eliminated by assuming plane strain since the slip surface is assumed to be infinitely wide. This assumption neglects the normal and horizontal side resisting forces along the sides of the sliding mass (end effect), which in turn underestimate the factor of safety leading to more conservative results. On the other hand, backcalculated shear strengths ignoring the end effects tend to be high or unconservative.

Lefebvre and Duncan (1973) showed that neglecting the end effect can severely affect the factor of safety results especially in narrow slopes with slope surface angles steeper than 20 degrees. Baligh and Azzouz (1975) showed that the ignoring the end effect can reduce the factor of safety by 40%. These studies emphasize the importance of the side and end effects and demonstrate the need for powerful methods that can simulate such conditions accurately. However, commercially available three dimensional limit equilibrium analysis methods ignore the shear resistance along the sides of the failing mass (Stark and Eid, 1998). End effects in slope stability analysis can be observed by comparing the rotational and the translational slope failures as shown in Figure 5.2.

Side effect in slope stability analysis is most pronounced in translational failure modes. The translational modes are more common than other failure modes since it involves the failure of the soil mass where the soil layers that comprise the slope is inhomogeneous and more than one soil layer exist. The slip failure of the soil mass in the
translational mode runs through different soil types and hence the failure progress starting from the layer where the available shear strength is less than the applied load. It is common for the translational failure to start in the relatively weakest soil layer then progress to other layers in the failed soil mass.

Some of the main reasons that make the end effect in translational slope failure modes more pronounced and critical than other modes are summarized below (Stark and Eid 1998, Arellano and Stark, 2000):

1. The mobilized shear strength along the back scarp or the sides of the slope are

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Figure 5.2, Rotational and Translational Failure Modes for Slopes
(After Arellano and Stark, 2000)
higher or lower than that mobilized along the base of the slope in the translational modes due to the inhomogeneity of the soil mass in the translational mode failure.

2. Failures in translational failure modes occur due to weak nature of the underlying layer and hence are pronounced in relatively flat slopes. As the flatness of the slope increases the effect of the ignored shear stresses at the ends of the slope increases which in turn increases the difference between the 2D and 3D factors of safety.

3. Noncircular slip planes with a long and nearly horizontal sliding plane through the weak soil layer characterize the translational failure.

4. The end effect is more pronounced in slopes with cohesive soil layers than slopes with homogeneous granular layers.

The above reasons highly influence the calculated factors of safety and should be considered in any 3D slope stability analysis to obtain more realistic and accurate results. In addition, as the inclination of the side slopes in 3D slopes stability analysis increases (become vertical), the shear resistance along the sides (end effect) decreases and less effect is pronounced compared to cases where the end effect is ignored.

Among the many available 3D slope stability analysis programs, Arellano and Stark (2000) suggested the use of Janbu's (1956) simplified method for 2D and 3D translational failure modes. However, Janbu’s simplified method satisfies the horizontal and vertical force equilibrium while it does not satisfy the moment equilibrium. In
addition, the method assumes that the resultant interslice forces (2D analysis) are horizontal while a correction factor is applied to account for the vertical interslice forces. However, the correction factor is only available for 2D slope stability analysis and no correction is available for the intercolumn forces in the extended 3D slope stability methods. Therefore, using Janbu’s simplified method, in its current extended 3D formulation (Hungr 1988, 1989), is more appropriate for 2D translational failures and can be misleading for the 3D analysis and backcalculation of slopes.

5.4 Consideration of Side Forces in 3D Analysis

The effect of side shear forces along the sides of the slopes is overlooked in all of the commercially available 3D slope stability analysis softwares. This limitation was tackled by assuming external horizontal and vertical forces acting at the center of the two parallel vertical sides of the sliding mass (Arellano and Stark, 2000). The external forces were assumed to be equivalent to the resultant of the at rest earth pressures as shown in Figure 5.3. If the vertical force $S_z'$ is located within the plan area of the slide mass it should be added to the total weight of the column while it should be ignored if located outside the sliding area. The shear resistance can be calculated using ($S_y' = \sigma_s' \tan \phi'$) where $\phi'$ is the friction angle of the layer.
Furthermore, using commercially available 3D analysis softwares, the following can be used to consider the effect of shearing resistance along the sides of the sliding mass:

Assume an imaginary material layer surrounding only the sides of the sliding mass. The soil parameters of this layer should be:

a) The imaginary and the upper layer have equal unit weights, $\gamma_{imag}' = \gamma_{upper}'$

b) The imaginary layer is frictionless, $\phi_{imag}' = 0$

c) The cohesion of the imaginary layer is equal to the shear strength from the lateral pressure acting along the sides of the sliding mass, $c_{imag}' = K \sigma_z' \tan \phi_{upper}'$, where $\phi_{upper}'$ is the secant friction angle of the upper material corresponding to the approximated average effective normal stress on the sides of the sliding mass ($\sigma_z'$). The lateral earth pressure coefficient is given by $K_s = 1 - \phi_{upper}'$. 

Figure 5.3, Assumed External Pressure Acting on a Vertical Side in a 3D Slope Model (Arellano and Stark, 2000)
5.5 Finite Element Methods

The finite element method (FEM) is more powerful than the limit equilibrium method (LEM) in slope stability analysis since the FEM satisfies the equilibrium at any point in the soil mass (local domain) and the surrounding soil (global domain), while the LEM satisfies the global force or moment equilibrium for the sliding mass only. Therefore, the need for assumptions in the FEM will be only for the definition of the failure criteria rather than the derivation complication and assumptions in the LEM. The derived factor of safety in the FEM is believed to be more accurate than the LEM in 2D and 3D analysis as well.

Finite element methods can be used to simulate any geometry and topography of slopes. On the other hand, some of the 3D limit equilibrium methods have restrictions regarding the shape of the slip surface. For example, the 3D method proposed by Leshchinsky and Huang (1992) assumed symmetrical slip surface. Due to the irregular geometry of the sliding mass and the varying soil layer thicknesses and profile, symmetrical slip surfaces are limited only to homogeneous slopes of simple geometries. Adding more simplifications to the geometry of the slope will decrease the reliability of the calculated factor of safety due to the inherited limited reliability imposed by the simplified assumptions during the mathematical derivation of the method and hence should be avoided in practice.
5.6 PLAXIS Finite Element Program

PLAXIS is a finite element software for soil and rock that has been used by geotechnical engineers and researchers for more than two decades. The software was first developed by The Technical University of Delft in 1987 to analyze soft soils of the low lands of Holland (Brinkgreve and Vermeer, 2001). The software then was extended to cover all aspects and applications of geotechnical engineering simulation using a user-friendly interface with the power of finite element. The first version of PLAXIS was commercially available in 1998.

Different soil models are incorporated in PLAXIS with a versatile library of structural elements. The automated mesh generation tool in the program makes the creation of soil models easy and practical since 6-node as well as 15-node triangular elements are available. The program uses the $c - \phi$ method to calculate the factors of safety of slopes.

5.6.1 The $c - \phi$ Method in Slope Stability Analysis

The $c - \phi$ method is based on the reduction of the shear strength ($c$) and the tangent of the friction angle ($\tan \phi$) of the soil. The parameters are reduced in steps until the soil mass fails. PLAXIS uses a factor to relate the reduction in the parameters during the calculation at any stage with the input parameters according to the following Equation:
\[ RF = \frac{\tan \phi_{\text{input}}}{\tan \phi_{\text{reduced}}} = \frac{c_{\text{input}}}{c_{\text{reduced}}} \]  \hspace{1cm} (5.3)

where \( RF \) = the reduction factor at any stage during calculations, \( \tan \phi_{\text{input}} \) and \( c_{\text{input}} \) are the input parameters of the soil, \( \tan \phi_{\text{reduced}} \) and \( c_{\text{reduced}} \) are the reduced parameters calculated by the program.

At the failure stage of the slope, the factor of safety is given by:

\[ SF = \frac{\text{Available strength}}{\text{Strength at failure}} = RF_{\text{at failure}} \]  \hspace{1cm} (5.4)

It can be seen that the factor of safety in this case is independent of the stress level in this method and therefore, the modulus of elasticity and the Poisson’s ratio will have negligible effect on the obtained factors of safety. This method is suitable for our comparison since the same parameters that are used in the limit equilibrium method will be used in the finite element analysis method.

5.7 Comparison Example

The effectiveness of the Stark and Eid (1998) method to handle the side and end effects in 3D slope stability method was investigated in the following sections. Stark and Eid (1998) analyzed a 3D slope using limit equilibrium slope stability programs with modifications to overcome the shortcomings of the current analysis methods. The finite element method was used to study the accuracy and effectiveness of their model and to
study some critical issues in any finite element analysis for slopes using the \( c - \phi \) method.

Figure 5.4, Cross Section for Slope Model (Stark and Eid, 1998)

Figure 5.4 showed the geometry of the studied slope as given by Stark and Eid (1998). The drained friction angles of the upper and lower materials are 23° and 8°, respectively, while the width of the sliding mass was 200 meters. The model represented a translational failure and was designed using many field case histories considering the slide mass dimensions, ground surface, side and base slope inclinations, and material unit weights and shear strength. The used soil shear strength parameters represented the drained effective conditions. The same parameters and geometry were used by the finite element software (PLAXIS) to study the model in both 2D and 3D.

The 2D finite element analysis was carried out using 15-node triangular elements and assuming plane strain conditions. In all cases the mesh was generated using the automated mesh generation tool in PLAXIS. Three cases were analyzed in the 2D models. The first case was identical to the case shown in Figure 5.4 (depth=zero) while in the second and third cases, the boundary line at the bottom of the model was moved 38
and 19 meters below the ground surface, respectively. The original boundary line was located 0.2 meters below the ground surface.

The 3D finite element analysis was carried out using 15-node wedge elements and 3D parallel plane conditions. In all cases the mesh was generated using the automated mesh generation tool in PLAXIS. Three cases were analyzed in the 3D models. The first case was identical to the case shown in Figure 5.4 while in the second and third cases, the boundary line at the bottom of the model was moved 38 and 19 meters below the ground surface, respectively. The original boundary line was located 0.2 meters below the ground surface.

Stark and Eid (1998) used CLARA 2.31 software to carry out their analysis. The software was developed by Hungr (1987) and can be used to calculate 3D factors of

Figure 5.5, Intercolumn forces in CLARA 2.31 (Hungr, 1987)
safety using extended Bishop’s simplified method or extended Janbu’s simplified method. The soil mass in CLARA 2.31 is divided into columns. The intercolumn shear forces in the extended Bishop’s method are neglected while the vertical force and moment equilibrium equations are used to solve for the unknowns. The factor of safety is calculated by calculating the moment that is found about a common horizontal axis parallel to the x-axis. The intercolumn horizontal and vertical force equilibrium equations are used in Janbu’s method to solve the unknowns. The factor of safety is calculated by using the horizontal force equilibrium equation. The intercolumn forces as used in CLARA 2.31 were shown in Figure 5.5

5.7.1 Analyses Results and Discussion

The results of the Stark and Eid (1998) analysis are summarized in Table 5.1.

<table>
<thead>
<tr>
<th>Analysis Type</th>
<th>Side Condition</th>
<th>Method of Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bishop</td>
</tr>
<tr>
<td>3D</td>
<td>Ignoring side resistance</td>
<td>0.900</td>
</tr>
<tr>
<td>3D</td>
<td>Considering side resistance</td>
<td>1.014</td>
</tr>
</tbody>
</table>

It can be seen that the factor of safety in 3D limit equilibrium models is highly dependent on the analysis method. Janbu’s method produced lower factors of safety.
while Bishop’s method produced higher factors of safety. On the other hand, ignoring the side resistance in Janbu’s method and Bishop’s method reduced the factors of safety by 13% in both methods. This percent indicates that the effect of the side forces can be the same regardless of the analysis method for the same slope and case. However, it should be noted that the failure slip surface in both cases was the same, lending credibility to the conclusions.

The finite element analysis results were summarized in Table 5.2 and shown in Figures 5.6 through 5.11. This table showed the sensitivity of the results to the location of the boundary conditions. Increasing the depth of the boundary line below the ground surface reduced the factors of safety in both 2D analysis and 3D analysis. The boundary location between the ground surface in Stark and Eid (1998) example was located at 0.2 meters below the ground surface. Increasing the boundary location from 0.2 m to 38 m can reduce the factor of safety by 50% in the 2D analysis and by 43% in the 3D analysis. Stark and Eid’s (1998) example assumed the slip surface to run 0.2 m below the ground surface and hence did not include the half space below the 0.2 m elevation. Due to the loose consistency of the lower material, the depth of the lower layer in finite element analysis should be determined rather than ignored.
Table 5.2, Results of the 2D Finite Element Slope Stability Analysis

<table>
<thead>
<tr>
<th>Case</th>
<th>Depth of Boundary Below Ground Surface (m)</th>
<th>2D-FS</th>
<th>3D-FS</th>
<th>3D/2D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case1</td>
<td>38</td>
<td>0.6356</td>
<td>0.6993</td>
<td>1.1002</td>
</tr>
<tr>
<td>Case2</td>
<td>19</td>
<td>0.6692</td>
<td>0.7387</td>
<td>1.1039</td>
</tr>
<tr>
<td>Case3</td>
<td>0.2</td>
<td>0.9521</td>
<td>1.0032</td>
<td>1.0537</td>
</tr>
</tbody>
</table>

On the other hand, the 3D analysis using the finite element method included the side resistance effect since the factors of safety have been calculated locally at each point within the domain using the \( c - \phi \) method. Increasing the depth of the boundary below the ground surface increased the size of the sliding mass and hence reduced the factors of safety of both the 2D analysis and the 3D analysis.

In addition, the results indicated that the finite element 3D/2D factor of safety ratio is always lower than what has been found by Stark and Eid (1998). Their results indicated that the ratio is close to 13\% while that using the finite element for the same boundary location (bottom boundary at 0.2 m) is 5\%. This variation can be attributed to the assumption that the side stresses are equal to the soil at rest stresses. Based on triaxial tests, a horizontal strain as small as 0.5\% is required to develop the Rankine active stresses and to change the stresses from the at rest conditions to the active conditions (Lambe and Whitman, 1979).

According to Rankine’s theory, the active earth pressure coefficient can be given
by:

\[ K_a = \frac{1 - \sin \phi}{1 + \sin \phi} \]  

(5.5)

where, \( K_a, \phi \) are the coefficient of active earth pressure and the soil friction angle, respectively.

Due to the movement of the soil mass vertically and horizontally along the sides of the sliding mass, it is expected that the stresses acting on the sides are active and not at rest stresses. The at-rest stresses are always higher than the active stresses and hence can create higher confining forces leading to higher factors of safety in the 3D analysis using the limit equilibrium methods. In this current example, the at-rest earth pressure coefficient was equal to 0.61 based on the upper material friction angle, which is 23 and based on the suggested method by Stark and Eid (1998). Rankine’s active earth pressure coefficient of the upper material is 0.44. The above two coefficients indicate that the confining (side) stresses from the active earth pressure stresses can be 39\% lower than those from the at rest earth pressure coefficients. Therefore, it is anticipated that the 3D factors of safety using side forces based on active earth pressures should produce lower factors of safety than those obtained using Stark and Eid’s (1998) method.

Table 5.2 indicated that the 3D factors of safety are dependent on the area of slope sides. Increasing the area of the sides will increase the side forces and hence will increase the 3D factor of safety. This shows the importance of doing the 3D slope stability
analysis in large and deep slopes where the side forces can have significant effect on the results.

Figures 5.6 through 5.8 showed that changing the location of the boundary at the bottom of lower layer in 2D finite element analysis can be critical. The figures showed that the slip surface can change from semi circular slip surface when the bottom boundary is located at 38 feet below the surface to noncircular when the bottom boundary is located at 0.2 feet below the ground surface. Increasing the depth of the boundary in the 2D finite element analysis increases the depth of the slip surface and hence the concentration of the stresses within the sliding mass reduces as can be seen in the figures. In addition, the figures showed that the failure started at the toe of the slope and propagated toward the crest. The rate of the propagation of the failure stresses increases toward the crest as the depth of the bottom boundary decreases.

The 3D finite element analysis results were shown in Figures 5.9 through 5.11. It can be seen that the same observations as those found in the 2D analysis still valid. However, the depth and concentration of stress zones in the 3D analysis are shallower than those observed in the 2D analysis suggesting higher values for the factors of safety. This observation indicates that ignoring the side effects in the 2D analysis not only decreases the factor of safety but also it increases the concentration of the stresses in the slope and can largely affect the geometry and depth of the slip surface. This observation is critical in analysis where the location and the design of some structural elements within
the slope are inevitable such as using piles to stabilize slopes.

5.8 Conclusions

The consideration of the side effect in slope stability analysis is very important especially in back calculating shear strength parameters and when the geometry of the slope cannot be analyzed with only one section. The analysis showed that the factor of safety in the 3D analysis is always higher than that in the 2D analysis. In addition, the location and depth of the factor of safety is highly influenced by the analysis type (2D or 3D). On the other hand, the shape and depth of the slip surface is highly dependent on the depth of the boundary at the bottom of the model.

The analysis indicated that using the at-rest earth pressure coefficient as recommended by Stark and Eid (1998) in 2D limit equilibrium analysis, as a way to account for 3D side forces, can overestimate the factor of safety due to the state of the soil and the mobilization of the active stresses during the sliding of the soil mass. In reality, the slope will not move as one block at the same time but rather the failure will progress in the slope over time suggesting the existence of areas with active stresses as well as areas with at-rest stresses. These areas will result from the sides of the sliding mass moving away from the surrounding soil. However, at the moment of the sliding, the slope will be under the working conditions and hence, the stresses acting on the sides of the slopes will be higher or equal to the active earth stresses but lower than the at-rest earth stresses. Therefore, the active earth pressure coefficient can be used to calculate the
lower bound 3D factor of safety and the at-rest earth pressure coefficient can be used to calculate the upper bound factor of safety.
Figure 5.6, 2D Finite element analysis results for Case 1, a) Slope geometry, b) Displacement vectors, and c) Slip surfaces.
Figure 5.7, 2D Finite element analysis results for Case 2, a) Slope geometry, b) Displacement vectors, and c) Slip surfaces.
Figure 5.8, 2D Finite element analysis results for Case 3, a) Slope geometry, b) Displacement vectors, and c) Slip surfaces.
Figure 5.9, 3D Finite element analysis results for Case 1, a) Slope geometry, b) Displacement vectors, and c) Slip surfaces.
Figure 5.10, 3D Finite element analysis results for Case 2, a) Slope geometry, b) Displacement vectors, and c) Slip surfaces.
Figure 5.11, 3D Finite element analysis results for Case 3, a) Slope geometry, b) Displacement vectors, and c) Slip surfaces.
CHAPTER VI
SUMMARY AND CONCLUSION

The research described in this thesis has achieved the objectives outlined in chapter I. The accomplishments can be summarized as follows:

1. Identification of the limitations associated with each slope stability method.
2. Evaluation of 2D and 3D slope stability analysis methods based on their theoretical background.
3. Assessing the accuracy of the slope stability analysis in 2D and 3D considering the effect of ignoring the side resistance.
4. Assessing the accuracy of locating the most critical slip surface and the associated factor of safety considering the type of the searching technique.

6.1 Conclusions.

Comparison and limitations of limit equilibrium methods were fully discussed in this research. Detailed conclusions have been presented in preceding chapters; the major points may be summarized as follows:

1. The selection of the slope stability method is critical since the accuracy of the analysis results depends on the mechanism of the failure.
2. Two dimensional limit equilibrium methods will always estimate lower
factors of safety compared to three-dimensional methods.

3. Slope side effect should be considered in 3D limit equilibrium methods to produce more realistic simulation of the problem.

4. Side resistance using the at-rest earth pressure coefficient should be considered can produce lower bound 3D factors of safety while using the active earth pressure coefficient can produce upper bound factor of safety.

5. The location and depth of the factor of safety is highly influenced by the analysis type (2D or 3D).

6. Prediction of the most critical slip surface is influenced by the search technique. The Monte Carlo technique proved to be the most rigorous method in location the most critical slip surface and therefore more accurate factors of safety.
REFERENCES


