DYNAMICS BASED DAMAGE DETECTION OF PLATE-TYPE STRUCTURES

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ABSTRACT

There is a pressing need to develop effective techniques for structural health monitoring (SHM), so that the safety and integrity of the structures can be improved. The main objective of this study is to evaluate the dynamics-based damage detection techniques for the plate-type structures using smart piezoelectric materials and modern instrumentation like Scanning Laser Vibrometer (SLV). The study comprises of testing an E-glass/epoxy composite plate with an embedded delamination and an aluminum plate with a saw-cut crack using two different actuator-sensor systems: (1) SLV with PZT (lead-zirconate-titanate) actuators (PZT-SLV), and (2) Polyvinylidenefluoride (PVDF) sensors with PZT actuators (PZT-PVDF). The numerical finite element (FE) analysis is also performed to complement the damage detection. Three relatively new damage detection algorithms (i.e., Simplified Gapped Smoothing Method (GSM), Generalized Fractal Dimension (GFD), and Strain Energy Method (SEM)) are employed to analyze the experimental and numerical mode shape data and Uniform Load Surface (ULS). From the damage detection outcomes, it is observed that the PZT-SLV system proves to be more convenient and effective, and it is capable of scanning a large number of points over the entire plate specimens; while the PZT-PVDF system, in which the curvature mode shapes are directly acquired, exhibits good sensitivity to damage. The damage detection algorithms like the GSM, GFD and SEM based on the utilization of three
consecutive mode curvatures (modes 3 to 5) and resulting ULS curvature successfully identify the presence and location of delamination in the composite plate; however, they do not show much success in locating the saw-cut crack in the aluminum plate with the same mode curvatures. Using the transverse bending dominated modes (e.g., modes 6 and 12), the above damage detection algorithms are capable of locating the saw-cut crack in the aluminum plate. Due to refined analysis of FE approach, all the algorithms are viable of detecting the damage using the FE data. The add-on advantage of the above three damage detection algorithms is that the data from the healthy plates are not required as the reference, thus simplifying the detection process. The GSM and SEM seem to provide better damage detection than the GFD. The employment of the combined static-dynamic experimental technique apparently increases the effect of damage (e.g., the delamination), and the application of GSM, GFD and SEM further facilitate to quantify the magnifying effect.

In conclusion, dynamic response measurement using two proposed sensor systems (PZT-SLV and PZT-PVDF) and related implementation of damage detection algorithms for damage evaluation of plate-type structures are demonstrated in this study. The successful implementations of the damage detection techniques, such as GSM, GFD, SEM, ULS, and the combined static-dynamic experimental technique, validate the effectiveness and accuracy of the two sensor systems in damage detection of plate-type structures.
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DEDICATED

To my coming baby
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CHAPTER I

INTRODUCTION

1.1 Background

Damage can cause structural failure, and sudden failure during high load operation may lead to catastrophic consequences. Development of an early damage detection method for structural failure is one of the most important keys in maintaining the integrity and safety of structures.

Structural health monitoring (SHM) is defined in the literature as the “acquisition, validation and analysis of technical data to facilitate life-cycle management decisions” (Kessler et al. 2001). More generally, SHM denotes a reliable system with the ability to detect and interpret adverse “change” in a structure due to damage or normal operation. The greatest challenge in designing a SHM system is to identify the underline changes due to damage or defect.

Lots of damage detection techniques have been proposed for structural health monitoring. Some of the nondestructive evaluation approaches that utilize technologies such as X-ray imaging, ultrasonic scans, infrared thermograph, and eddy current can identify damages. However, they are somehow difficult to implement, and some of
them are impractical in many cases such as in service aircraft testing and in-site space structures. Almost all of the above techniques require that the vicinity of the damage is known in advance and the portion of the structure being inspected is readily accessible.

The dynamics-based damage detection techniques using smart materials are other kind of damage detection technique. The dynamic response of structures can offer unique information on defects that may be contained within the structures. Changes in the physical properties of the structures due to damage will alter the dynamic responses such as natural frequencies, damping and mode shapes. These physical parameter changes can be extracted to estimate damage information.

1.2 Objectives

To develop dynamics-based damage detection techniques, the plate-type structures are evaluated using two different actuator/sensor systems: one system (PZT-SLV) utilizes the PZT patches as actuators and the scanning laser vibrometer (SLV) as non-contact sensors to acquire the dynamic response; while the other system (PZT-PVDF) employs piezoelectric materials as both the actuator (PZT) and sensors (surface-bonded PVDF films) to obtain the curvature mode shapes and related response. The main goal of this work is to conduct an exploratory study on dynamics-based damage detection of plate-type structures using the above two measurement systems. The detailed objectives of this study include:

• Conduct dynamic tests and obtain the displacement and curvature mode shapes of plate specimens (the composite plate with delamination and the aluminum plate
with saw-cut damage) using two measurement systems (PZT-SLV and PZT-PVDF).

- Apply damage detection algorithms to the measured dynamic response data.
- Compare the effectiveness of different damage detection algorithms.
- Perform numerical finite element simulations of damaged plates and validate the damage detection algorithms.
- Implement a new experiment technique of static/dynamic test in damage detection of the delaminated composite plate under different compression load levels.

This thesis is organized as follows. A literature review of dynamics-based damage detection techniques is given first in this study, followed by introduction of different damage detection algorithms (Chapter 2). Experimental set-ups and procedures based on the two measurement systems (PZT-SLV and PZT-PVDF) are subsequently described in Chapter 3. Damage detection of an E-glass/epoxy composite plate with a delamination and an aluminum plate with a saw-cut damage is given in Chapters 4 and 5, respectively. The numerical finite element simulation is also conducted to verify the validity of the damage detection algorithms. A new experimental technique of combined static/dynamic test is implemented (also in Chapter 4) to obtain the dynamic response of the composite structures under different loading levels, and it intends to magnify the effect of damage to the dynamic response. Finally, the conclusions and findings of this study are summarized in Chapter 6.
1.3 Literature Review

The dynamics-based damage detection is an effective method due to its simplicity of implementation and ability of acquiring both the global and the local information of the structure. Significant efforts have already been spent to develop damage detection algorithms using dynamics-based approach. Some of the researches related to damage detection using dynamics-based approach are summarized as follows.

1.3.1 Frequency-based Damage Detection

Natural frequency-based method may be the easiest one of dynamics-based damage detection due to ease of measurement of the natural frequencies (only a single sensor and a single point of measurement are required in many applications). The foundation of this method is that damage produces a change (decrease in most of cases) in structural stiffness, which in turn, results in changes of natural frequencies compared to the healthy or intact structures. Cawley and Adams (1979) demonstrated that the ratio of the frequency changes is a function of damage location. Gomes and Silva (1999) showed that the local changes in stiffness produce shifts in natural frequencies, which affect each mode differently depending on the damage location. Therefore, the measurement of frequencies at different modes could be used to probe possible damage locations. Their study showed that the frequency-based method for damage identification was successfully applied to simple laboratory structures with only single or a few damage locations. Chinchalkar (2001) developed a numerical method for determining the crack location in a damaged beam. A rotation spring was used to model
the beam. The graphs of spring stiffness versus crack location were plotted for three natural frequencies, and the point of intersection of the three curves indicated the location of the crack. Lee and Chung (2000) used the first four frequencies of a simulated cantilever beam to locate a single crack and assess their corresponding effectiveness. The crack depth was then approximated iteratively to match the frequency as closely as possible before the location of the crack was finally defined.

However, there are some contradictions over the usefulness of natural frequency-based method in damage detection. Sanders et al. (1992) found that the natural frequency shifts alone may not be sufficient for unique identification of the location of structural damage. Kessler et al. (2001) further studied the effect of various kinds of damage (holes, delamination, impact, bending induced cracks and fatigue) in clamped composite plates on frequency response. The frequency-based method is usually reliable for detecting the presence of damage in a simple composite structure, whereas the important information about damage size and type, location, and orientation cannot be obtained using this simple method since several combinations of these variables can yield similar or identical frequency change.

A further application of this method is to predict the remaining service life of the structure. Surendra (2000) used the frequency shifts method to successfully predict the fatigue life of a concrete structure. The experimental results showed that the natural frequencies were sensitive to the presence of fatigue cracking, and the rate of the natural frequency was then correlated with the fatigue life.
1.3.2 Mode Shape-based Damage Detection

A more robust application of dynamics-based approach for damage detecting is based on mode shapes. This method has been developed to assess damage directly using the measured displacement mode shapes or curvatures mode shapes.

Rizos et al. (1991) were among the first to develop the idea of damage detection using displacement mode shapes. In their study, the transverse surface crack extending uniformly along the width of the beam was identified in a cantilever beam. The displacement mode shapes were determined experimentally using accelerometers and analytically using E-B beam theory, and their comparison showed that the crack location could be found and the crack depth was estimated with satisfactory accuracy. Salawu and Williams (1995) tested a reinforced concrete bridge before and after repair. Although the first seven natural frequencies shifted by less than 3%, the mode shapes showed substantial changes leading the authors to argue that comparing the mode shapes might be a more robust technique for damage detection than only measuring the changes of natural frequencies.

A potential disadvantage of mode shape-based method is that it needs to measure a large number of points. A technique using Scanning Laser Vibrometer (SLV), which allows measuring a great deal of points easily, becomes increasingly promising. The laser doppler vibrometer, used as a vibration transducer, has the advantage of being non-contacting, high spatial resolution and reduced work time. It has proved to be of general use for measuring structural vibration in modal testing. The first attempt of using SLV in a continuously scanning mode was explored in a steel structure by
Stanbridge et al. (1997), and the mode shapes of a structure were obtained directly by the phase-sensitive demodulation. Castellinia and Revel (1999) applied the SLV measuring technique to determine delamination size, depth, and location in composite materials. To determine the efficiency of postprocessing algorithms for experimental SLV data, a lumped parameter model was developed to describe the dynamic behavior of a delaminated composite plate. The root-mean-square (RMS) values computed in different frequency bands were used as indicators of the delamination information. Subsequently, an experiment using SLV was performed on five kinds of panels, (i.e., undetected panel, delamination of the first layer; delamination of the second layer, delamination of the third layer, and delamination of the fourth layer). Maps of the RMS values at different frequency bands were obtained and the characteristics of the defect were detectable from these maps. The accuracy of the RMS calculation depends on the total acquisition time and the width of the considered frequency band.

Further research was developed in the application of SLV (Khan et al. 2000). Three types of laboratory-test structures (i.e., a metal cantilever beam with a narrow slot cut through up to 80% of its thickness, thin metal cantilever plates with and without a through cut and a fatigue crack, and reinforced concrete beams with and without load-induced cracks) were evaluated. At each stage, the frequency response functions (FRFs) (up to 1 kHz) and mode shapes were measured and compared. A method of defect detection using a short linear scan at the crack location was developed. It was found that the through-cracks were easily detected in thin metal plates; but in the thick structures the defects were only detectable when they extend through more than half of the thickness. It was concluded that the SLV had considerable potential for defect
detection in the field. Further trials and improvements in the technique, particularly in controlling speckle noise interference, were necessary before it could be reliably applied in practice.

A more effective method of damage detection based on the mode shapes is the use of curvature mode shapes. Pandey et al. (1991) developed a new damage detection method using the curvature mode shapes. From Euler-Bernoulli beam theory, the curvature mode shape \( \kappa \) is related to the Young's Modulus of beam and the beam cross sectional geometric properties as

\[
\kappa = \frac{d^2 \phi}{dx^2} = \frac{M}{EI} \tag{1.1}
\]

where \( \phi \) is the transverse displacement, \( \frac{d^2 \phi}{dx^2} \) is the curvature, \( M \) is the bending moment, \( E \) is the modulus of elasticity, and \( I \) is the moment of inertia. The use of curvature mode shapes in damage identification is based on the assumptions that the curvature of an undamaged structure is smooth and continuous and the irregularity of the curvature can thus determine the location of the damaged for a homogeneous structure. The changes in the curvature mode shapes are highly localized to the region of damages, and they are more pronounced than the changes in the displacement mode shapes.

The curvature is often calculated from the measured displacement mode shapes by using a central difference approximation

\[
\phi_{ji}^{\prime\prime} = \frac{\phi_{i(j+1)j} - 2\phi_{ij} + \phi_{i(j-1)j}}{L^2} \tag{1.2}
\]
or a higher order approximation

\[ \phi_{j,i}'' = \frac{-\phi_{(j+2)i} + 16\phi_{(j+1)i} - 30\phi_{ji} + 16\phi_{(j-1)i} - \phi_{(j-2)i}}{12L^2} \]  

(1.3)

where \( i \) is the mode shape number; \( j \) is the node number; \( L \) is the distance between the nodes.

Ratcliffe (1998) used a gapped smoothing method to successfully locate a delamination in a composite beam. The method assumes that mode shapes of healthy or undamaged structures have smooth surface. In their work, the displacement mode shape of a damaged beam was first converted to the curvature shape using the central difference approximation. The curvature mode shape was then locally smoothed using a gapped polynomial at each point to generate the mode shapes representing the healthy beam. The damage index was defined as the difference between the curvature and the gapped polynomial at each point, from which the largest index indicated the location of the delamination. This method does not require an undamaged reference. The procedure operates solely on the measured mode shape obtained from the damaged structure. A unique advantage of this method is that it can be applied to an existing structure where there is no prior knowledge of its undamaged state.

Yoon et al. (2005) recently developed a two-dimensional gapped smoothing method to detect a damage location in the plate. This method was an extension of normal gapped smooth method in one dimension, and it was capable of locating regions with stiffness variability from either the curvature mode shape data, or the frequency dependent operating displacement shape data.
Wahab and De Roeck (1999) successfully applied a curvature-based method to the Z24 Bridge in Switzerland, and introduced a damage indicator named the curvature damage factor (CDF). The difference in curvature before and after damage averaged over a number of considered modes is defined as the CDF

\[ CDF = \frac{1}{N} \sum_{n=1}^{i} |\phi_{oi} - \phi_{di}| \]  

(1.4)

where \( N \) is the total number of mode to be considered, \( \phi_{oi} \) is the curvature of the undamaged beam for the \( i \)th mode, \( \phi_{di} \) is the curvature of the damaged beam for the \( i \)th mode. When this algorithm was applied to the bridge data, a clear set of peaks was observed in the CDF near one of the bridge piers where damage was first visually located during the experiment. They concluded that the use of modal curvature to locate damage in civil engineering structures seemed promising and further development of this method was needed to improve the quality of the measured mode shapes.

Hamey et al. (2004) recently evaluated four types of damage detection algorithms (i.e., Absolute Differences Method (ADM), Curvature Damage Factor (CDF), Damage Index Method (DIM), FRF Curvature Method (FCM)) in carbon/epoxy composite beams with several possible damage configurations. The results showed that all the methods of the curvature modes measured by the piezoelectric sensors could be used as a potential method in damage detection techniques and the Damage Index Method (DIM) detected and isolated the damage better than other methods studied.
1.3.3 Summary

In summary, the dynamics-based approach has been widely used to identify damage. The frequency-based method indicates its usefulness in probing the presence of the defect, but it is relatively difficult to implement this method to locate/size the damage. Most of the above studies showed that the mode shapes-based method could be applied to detect damage type, size, and location in either large or small structures with some success. The approach based on dynamic response is relatively straightforward and easy to use, and it can be used as a viable technique for damage detection. Thus, the dynamics-based damage detection approach is adopted in this study, and several newly developed damage detection algorithms are evaluated for plate-type structures.
CHAPTER II

DAMAGE DETECTION ALGORITHMS

2.1 Introduction

Experimentally measured data of dynamic response (e.g., frequency and mode shapes) are sometime not readily implementable for direct damage detection, and they are often coupled with damage detection algorithms, as part of data reduction procedures to extract the useful information. To achieve this aim, several damage detection algorithms and data processing techniques are introduced in this chapter.

2.2 Damage Detection Algorithms

Some of the damage detection algorithms discussed in the literature review (see Chapter 1) are briefly described, and a few newly developed algorithms (Wang and Qiao 2005) are introduced as well. The algorithms introduced include: (1) absolute difference method (ADM), (2) damage index method (DIM), (3) generalized fractal dimension (GFD), (4) simplified gapped-smoothing method (GSM), and (5) strain energy method (SEM). The above algorithms are later employed in this study to both the experimentally measured response and numerical finite element dynamic response data.
2.2.1  Absolute Difference Method

The absolute difference method (ADM) is a very simple approach for damage detection approach. Damage parameter in this method is defined as the absolute difference in the magnitudes of the mode shapes between the undamaged and damaged structure, and it is expressed as

$$\Delta \kappa_{ij} = \| \kappa_{ij} \| - \| \kappa_{ij,damaged} \|$$

(2.1)

where $\Delta \kappa_{ij}$ is the absolute difference of the damaged ($\kappa_{ij,damaged}$) and undamaged ($\kappa_{ij}$) mode shapes. $i$ is the node number and $j$ is the mode number. Results based on this method very depend on the mode shape selection, which leads to the development of other multiple mode methods.

2.2.2  Damage Index Method

The damage index method (DIM) is more complex than the aforementioned approach. The damage parameter formulation of this method is presented as (Farrar and Jaurequi 1998)

$$\beta_{ij} = \frac{\left( \{ \kappa_{ij,damaged} \}^2 + \sum_{i=1}^{\max} \{ \kappa_{ij,damaged} \}^2 \right) \sum_{i=1}^{\max} \{ \kappa_{ij} \}^2}{\left( \{ \kappa_{ij} \}^2 + \sum_{i=1}^{\max} \{ \kappa_{ij} \}^2 \right) \sum_{i=1}^{\max} \{ \kappa_{ij,damaged} \}^2}$$

(2.2)

where $\beta_{ij}$ is the damaged index at location $i$ for mode $j$. For a multiple mode approach, the summation of $\beta_{ij}$ is defined as
\[ \nu_i = \sum_j \beta_{ij} \quad (2.3) \]

where \( \nu_i \) is the summation of individual mode damage index at point \( i \).

**2.2.3 Simplified Gapped Smoothing Method**

Ratcliff and Bagaria (1998) proposed the gapped-smoothing method (GSM) for damage detection in a beam-type structure without prior knowledge of the healthy structure. In this study, the method is extended and applied to two dimension (2-D) structures like thin plate structures. The basic theory of the method is that the mode shape of a healthy plate has a smooth surface, and it can be approximated by a polynomial in two variables as

\[ U_{\text{fitting}}(x,y) = \sum_{i=0}^{m} \sum_{j=0}^{n} C_{ij} x^i y^j \quad (2.4) \]

where \( C_{ij} \) is the coefficients calculated by the surface-fitting, \( m \) and \( n \) are the order of the polynomial and are chosen equal to four in this study in the study. The damage parameter based on this method is simply defined as the square of the difference between the measured data and the smoothed fitted value, where the maximum value of \( GSM \) indicates the location of damage

\[ D_{\text{GSM}}(x,y) = (U_{\text{measured}}(x,y) - U_{\text{fitting}}(x,y))^2 \quad (2.5) \]
2.2.4 Generalized Fractal Dimension

The concept of fractal and its relevant mathematical model were developed by Mandelbrot (1968). When we reduce the ruler length by $1/r$ in a curve, its length would increase to $L = r^D$ multiplying the original one. $D$ is called the fractal dimension (FD) of a fractal curve, and it can be expressed as

$$FD(x) = \frac{\log(n)}{\log(n) + \log \left( \frac{d(x,M)}{L(x,M)} \right)}$$

(2.6)

$$L(x_i,M) = \sum_{j=1}^{M} \sqrt{(y(x_{i+j}) - y(x_{i+j-1}))^2 + (x_{i+j} - x_{i+j-1})^2}$$

(2.7)

$$d(x_i,M) = \sum_{i=\text{all}M} \sqrt{(y(x_{i+j}) - y(x_i))^2 + (x_{i+j} - x_i)^2}$$

(2.8)

where $x = \frac{1}{2}(x_i + x_{i+M})$, $n = 1/\alpha$ and $\alpha$ is the average distance between successive points, $x_i$ and $y_i$ are the coordinate values of curve. The term $M$ represents the sliding window dimension length. The dimension of a smooth curve is 1, and the more fractions the curve has, the larger the dimension of the curve is. The sharp peak of the FD curve indicates the location of the damage. Based on the principle of FD, it is feasible that it could be applied to probe the abnormality in the mode shapes caused by damage and be used as a viable technique for damage detection.

However, when the higher mode shape is considered, the above FD approach may give some misleading peak information in the location of maximum and minimum point.
in a curve. To overcome this shortcoming, a new generalized fractal dimension (GFD) is recently defined by Wang and Qiao (2005) as a modification of the FD method

\[
GFD_s(x) = \frac{\log(n)}{\log(n) + \log\left(\frac{d_s(x_i, M)}{L_s(x_i, M)}\right)}
\]  

\[
L_s(x_i, M) = \sum_{j=1}^{M} \sqrt{(y(x_{i+j}) - y(x_{i+j+1}))^2 + s^2(x_{i+j} - x_{i+j+1})^2}
\]  

\[
d_s(x_i, M) = \max_{1 \leq j \leq M} \sqrt{(y(x_{i+j}) - y(x_i))^2 + s^2(x_{i+j} - x_i)^2}
\]

where \(s\) is a scale parameter as inspired by the wavelet transformation. Compared to the mode shape itself, the irregularity caused by the damage on the deformation mode shape is local and smaller, and we can therefore filter the sharp peak value of FD introduced by the mode shape itself while keeping the one caused by the local damage through choosing a proper scale value \(s\). Figures 2.1~2.5 show the GFD of the 6th displacement mode shape of a damaged aluminum plate using the value of \(s\) from 1000 to 0.1. When \(s\) is equal to 1 (FD method) or less than 1, it is difficult to find where the damage location is due to the multiple peaks in the plots (see Figures 2.4 and 2.5). The damage location becomes clearer and more pronounced with the increment of \(s\). As shown in Figure 2.1 to 2.5, the proper number of \(s\) is around 100, and it identifies valid location of damage.
Figure 2.1  The GFD of the 6th mode shape of a damaged aluminum plate using $s = 1000$

Figure 2.2  The GFD of the 6th mode shape of a damaged aluminum plate using $s = 100$
Figure 2.3 The GFD of the 6th mode shape of a damaged aluminum plate using $s = 10$

Figure 2.4 The GFD of the 6th mode shape of a damaged aluminum plate using $s = 1$
2.2.5 Strain Energy Method

A particular vibration mode stores a certain amount of strain energy, and the frequency and the shape of the mode are highly sensitive to changes in the stiffness of the structure, which are associated with the strain energy. Thus, the changes in the modal strain energy may also be considered as a logical choice of indicator for the damage location.

The strain energy associated with a particular mode shape may be calculated from,

\[ U_i = \int_0^l EI \left( \frac{\partial^2 \phi_i}{\partial x^2} \right)^2 dy \] (2.12)
where $\square_i$ is the displacement mode shape. The curvature required for this calculation is either extracted from the displacement mode shapes measured by the PZT-SLV system using a central difference approximation or obtained directly from the measurement by the PZT–PVDF system. Equation (2.12) means that the value of strain energy ($U$) is proportional to the square of the curvature

$$U \propto \kappa^2 = \left( \frac{\partial^2 \phi_i}{\partial x^2} \right)^2$$  \hspace{1cm} (2.13)

The damage index based on this method is defined as the absolute difference between the square of measured data and square of the smoothed fitted curvature value

$$D_{sem}(x, y) = \left| \kappa_{measured}(x, y)^2 - \kappa_{fitting}(x, y)^2 \right|$$  \hspace{1cm} (2.14)

2.3 Uniform Load Surface (ULS)

The uniform load surface (ULS) was first studied by Zhang and Aktan (1998). For a structural system with $n$ degrees-of-freedom, the stiffness matrix $K$ and the flexibility matrix $F$ can be defined by the normalized modes $\phi_i$ using the mode expansion as

$$K = M\Phi\Omega\Phi^T M = M\left( \sum_{i=1}^{n} \omega_i^2 \phi_i\phi_i^T \right) M$$  \hspace{1cm} (2.15)

$$F = [f_{i,j}] = K^{-1} = \Phi\Omega\Phi^T = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \phi_i\phi_i^T$$  \hspace{1cm} (2.16)
where \( \Phi = [\phi_1, \phi_2, \phi_3, \ldots, \phi_n] \) is the mode shape matrix, \( \Omega = [\omega_1, \omega_2, \ldots, \omega_n] \) is the eigenvalue matrix, and \( \omega_i \) is the \( i^{th} \) natural frequency.

The ULS is defined as the deflection vector of the structure under the uniform load

\[
ULS = F \cdot L
\]  

(2.17)

where \( L = [1, \ldots, 1]^T \) is the unit vector representing the uniform load acting on the structure. From Equation (2.16), we can observe that the ULS converges very fast as the number of modes increases. Therefore, the ULS can be well approximated by the first few mode shapes. The ULS of a clamped-clamped E-glass/epoxy composite plate approximated by the first three mode shapes from the finite element analysis (FEA) is shown in Figure 2.6. The exact ULS of the clamped-clamped plate (Figure 2.7) is also obtained by FEA through solving a static problem under the uniform unit load. As shown in Figures 2.6 and 2.7, an excellent agreement between the approximate and exact solutions is achieved. Therefore, the ULS computed using the first few mode shapes (e.g., the first three) may be sufficient for application in damage detection.
Figure 2.6  Approximation of the ULS with the first three mode shapes of a clamped-clamped plate

Figure 2.7  Exact ULS of a clamped-clamped plate

The ULS of a healthy plate is a smooth surface or shape. When there is a damage or defect, the shape changes in the ULS, i.e., a peak or abrupt slope will appear at the damage location. Based on this principle, this method is generalized for damaged detection. In a recent study (Wu and Law 2004), the ULS was shown to be sensitive to
the local damage and could be used to locate damage in the structure instead of the individual displacement mode shapes.

The ULS is the combination of several modes of a structure, and this averaging-like property reduces the effect of noise due to the reason that noise and measurement error are random and really occur in the same location of the structure at each mode. As an illustration, the experimental data of a composite plate (detailed experimental results are given in Chapter 4) is analyzed using ULS. The results of the simplified gapped-smoothing method (GSM) using the first displacement mode shape and the ULS are shown in Figures 2.8 and 2.9, respectively. It is noted that the noise and measurement errors are reduced when the ULS data is used.

Figure 2.8  Application of simplified gapped-smoothing method using the first mode

![Figure 2.8](image-url)
2.4 Savitzky-Golay Filter

To reduce the effect of experimental noises and measurement error, the Savitzky-Golay filter is used to process the original experiment data.

This approach for smoothing the time series is to replace each value of the series with a new value which is obtained from a polynomial fit to \(2n+1\) neighboring points (including the point to be smoothed), with \(n\) being equal to or greater than the order of the polynomial (see Figure 2.10). Therefore, this method is also called digital smoothing polynomial filter or “least-squares smoothing filter”.

Figure 2.9  Application of simplified gapped-smoothing method using the ULS
Savitzky-Golay smoothing filters are typically used to "smooth out" a noisy signal whose frequency span (without noise) is large. In this type of application, Savitzky-Golay smoothing filters perform much better than the standard averaging FIR (Finite Impulse Response) filters, which tend to filter out a significant portion of the high frequency content in the signal along with the noise. Figure 2.11 illustrates the original measured data and smoothed results after using Savitzky-Golay filter.

(a) Original data

Figure 2.11 Application of Savitzky-Golay filter
2.5 Summary

In this chapter, five damage detection algorithms are discussed. The absolute difference method (ADM) and damage index method (DIM) need the prior knowledge of healthy structure as a benchmark, but the generalized fractal dimension (GFD), simplified gapped-smoothing method (GSM) and strain energy method (SEM) do not require the prior information of healthy state which in some cases are not available. In the following chapters, all the algorithms are applied to the same set of experimental data, and their validity is compared among each other. The uniform load surface (ULS) is also applied in the data analysis, and it is capable of minimizing ambient noise and random measurement error. Savitzky-Golay filter is further used in the data reduction process, and it filters out some noise and measurement error inherent in the tests.
CHAPTER III

THEORETICAL, EXPERIMENTAL AND NUMERICAL MODAL ANALYSES

3.1 Introduction

The basic principle of describing the dynamic behavior of a structure in terms of its vibration mode shapes was first introduced by Rayleigh over a century ago. Modal analysis based on force excitation has shown to be an effective means of studying structural dynamic characteristics. It includes the experimental, analytical and numerical modal analyses, and the most powerful of which are those involving both the theoretical and practical considerations. Smart piezoelectric materials are incorporated in the study, and they are used either as actuator or sensor. Lead Zirconate Titanate (PZT) ceramic patch is used as actuator; while polyvinylidene (PVDF) thin polymer film is adopted as sensor. Two sensor systems, the one based on the PVDF thin films as sensor to acquire the curvature mode shape and the PZT ceramic patch as actuator (PZT-PVDF) and the other using the scanning laser vibrometer (SLV) to obtain the displacement mode shape and the PZT as actuator (PZT-SLV), are employed in this study to conduct experimental modal analysis. This chapter describes the theoretical
background and the experimental and numerical methods used in this study to conduct modal analysis.

3.2 Theoretical Modeling of 2-D Thin Plates

Consider a thin uniform plate with length \( l_b \), width \( b_b \) and thickness \( t_b \) as shown in Figure 3.1, a PZT ceramic patch with dimension of \( l_c \times b_c \times t_c \) is used as the actuator. The conventional vibration governing equation of the isotropic plate excited by the PZT actuator can be obtained as follows (Tzou and Fu 1994)

\[
D \left( \frac{\partial^4 w(x,y,t)}{\partial x^4} + 2 \frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x,y,t)}{\partial y^4} \right) + \rho \frac{\partial^2 w(x,y,t)}{\partial t^2} + c \frac{\partial w(x,y,t)}{\partial t} + \nu \frac{\partial^2 w(x,y,t)}{\partial x^2} = \frac{\partial^2 M_{xx}^e}{\partial x^2} + \frac{\partial^2 M_{yy}^e}{\partial y^2} \quad (3.1)
\]

where
\[
D = \frac{Y h^3}{12(1-\mu^2)} \quad Y \quad \rho \quad h \quad c \quad \mu
\]

\( Y \) is the Young’s modulus, \( \rho \) is the density, \( h \) is the thickness of the plate (\( h = t_b \)), \( c \) is the damping ratio, and \( \mu \) is the Poisson’s ratio. A PZT patch is bonded to the plate, and the voltage \( V_c \) is applied to the PZT to generate the excitation force (Figure 3.1). The equivalent force induced by the PZT patch can be modeled as four distributed moments at all four edges of the PZT patch, and therefore, the force functions can be expressed using the unit step function \( u_t \) (Tzou and Fu 1994)

\[
M_{xx}^e = M_{eqx} \left[ u_t(x-x_{c_1}) - u_t(x-x_{c_2}) \right] \left[ u_t(y-y_{c_1}) - u_t(y-y_{c_2}) \right] e^{iat} \quad (3.2)
\]

\[
M_{yy}^e = M_{eqy} \left[ u_t(x-x_{c_1}) - u_t(x-x_{c_2}) \right] \left[ u_t(y-y_{c_1}) - u_t(y-y_{c_2}) \right] e^{iat} \quad (3.3)
\]

where
\[
M_{eqx} = C_0 \Lambda_x, \quad M_{eqy} = C_0 \Lambda_y, \quad \Lambda_x = \frac{d_{31}}{t_c} V_a, \quad \Lambda_y = \frac{d_{32}}{t_c} V_a, \quad c_0 = \frac{t_c^2 E_b}{6 + \Psi b_c}, \quad \Psi = \frac{t_c E_b}{t_c E_c}
\]
$V_a$ is the applied voltage on actuator; $d_{31}$ and $d_{32}$ are the piezoelectric dielectric strain constants; $E_c$ and $E_b$ are the Young’s modulus of the PZT patch and plate, respectively; $t_c$ and $t_b$ are the thickness of the PZT patch and plate, respectively. The displacement function $w(x,y,t)$ can be expressed as the product of two functions, one involving only the space coordinates $x$ and $y$, and the other involving the variable time $t$: 

$$w(x,y,t) = w(x,y)T(t)$$

Equation (3.4)

Substituting Equations (3.2), (3.3) and (3.4) into Equation (3.1) and solving the partial differential equation by applying the boundary conditions of the plate, the solution of $w(x,y)$ and $T(t)$ can be obtain, where $w(x,y)$ is the unit spatial part (mode shape function) and $T(t)$ is the time part - an amplitude factor called modal participation factor. As an example, the frequency response function (FRF) of a clamped-clamped plate is given in Figure 3.2. Using the product of $w(x,y)$ and $T(t)$, the theoretical time-domain response function of the plate is obtained.

![Figure 3.1 Schematic of a plate mounted with the PZT and PVDF](image_url)

Figure 3.1 Schematic of a plate mounted with the PZT and PVDF
The PVDF film has been successfully used as sensors for measuring structural vibration. The mathematical models of PVDF film are well developed and experimentally verified for simple structural applications, such as beams and plates. Consider a PVDF film with dimension of $l_p \times b_p \times t_p$ surface-bonded to a plate as a sensor as shown in Figure 3.1. The shape function of the PVDF sensor can be expressed as (Tzou and Fu 1994)

$$\Gamma(x,y)=[u_s(x-x_1)-u_s(x-x_2)][u_s(y-y_1)-u_s(y-y_2)]$$

(3.5)

where $u_s(x)$ is the unit step function. The sensing equation of the PVDF sensor is

$$q(t)=\frac{(t_p+t_b)}{2} b_p e_{31} \int_0^{l_p} \int_0^{b_p} \Gamma(x,y) \frac{\partial^2 w(x,y,t)}{\partial x^2} dx + \frac{(t_p+t_b)}{2} l_p e_{32} \int_0^{l_p} \int_0^{b_p} \Gamma(x,y) \frac{\partial^2 w(x,y,t)}{\partial y^2} dy$$

(3.6)

where $q(t)$ is the charge; $t_p$, $b_p$ and $l_p$ are the thickness, width and length of the PVDF sensor respectively; $e_{31}$ and $e_{32}$ are the piezoelectric field intensity constant. The measured voltage of sensor can be written as

$$V_{measured}(t)=\frac{q(t)}{\varepsilon A_p} t_p$$

(3.7)

where $A_p=l_p b_p$ is the area of sensor; $\varepsilon$ is the permittivity; Through applying the Fourier transform in time domain data $V_{measured}(t)$, the theoretical curvature FRFs of the PVDF sensor can be obtained, and one example is shown in Figure 3.2.
3.3 Experimental Modal Analysis

The procedure of experimental modal analysis includes the measurement of time-domain data, transformation of measurement data to FRFs by using Fourier transform, and data analysis of FRFs. The type of FRFs depends on the types of sensing device. The curvature FRFs and displacement FRFs are the two common types, and in this study they are obtained by using the PZT-PVDF and PZT-SLV measurement systems, respectively. The data analysis of FRFs includes extraction of the modal parameters, e.g., natural frequencies, damping ratios and mode shapes.

3.3.1 Curvature Modal Parameters Using PZT-PVDF System

The procedures of PZT-PVDF system are shown in Figure 3.3, and the experimental set-up and specimens are given in Figure 3.4.
Figure 3.3 PZT-PVDF experimental procedure
(a) System schematic

(b) Test equipment and dSPACE

Figure 3.4 PZT-PVDF system
In this study, to obtain the frequency response function (FRF), the following procedures are used: (1) The function generator yields a sweep harmonic signal running through the PZT actuator to excite the plate; (2) The dSPACE data acquisition system is used to record the time-domain response of a plate through the PVDF sensors under excitation of PZT ceramic actuator; and (3) After obtaining the time-domain data, a program based on the principle of Fourier transform is used to transfer the time-domain data to the frequency-domain data, from which the FRFs are obtained. The spectrum function 'SPECTRUM' provided in the signal processing toolbox of MatLab is used to carry out the Fourier transform. The FRFs are then plotted in MatLab using ‘SPECGRAPH’ function. As an example, the measured FRF of the clamped-clamped plat is shown in Figure 3.5.
Experimental modal parameters are commonly estimated by curve-fitting a set of FRFs. The curve fitting is a process of matching a parametric model of an FRF to experimental data. The unknown parameters of the parametric model include the modal frequency, damping and residues (mode shape components) for each mode. There are basically two classes of methods available: one is the Single-Degree-Of-Freedom (SDOF) method, and the other is the Multi-Degree-Of-Freedom (MDOF) method. The SDOF method estimates the modal parameters of one mode at a time, and it is used to estimate the modal parameters without incurring significant errors. The SDOF method can quickly obtain the mode shape estimates. On the other hand, the MDOF method simultaneously estimates the modal parameters of two or more modes from a set of FRFs, and it is more suitable for a large system or structure that has similar dimensions along
multiple axes. The MDOF method should be used in the case if the resonance peaks are very close to one another to distinguish.

In this study, the modal parameters are extracted using the software MEScope. Before using the MEScope, the input file must be first prepared. This file consists of all FRFs data from each measurement point and follows the format required by the MEScope. The MEScope includes both the SDOF and MDOF curve fitting methods to estimate the modal parameters. The CoQuad and Peak methods are the SDOF based techniques; while the Polynomial and Complex Exponential methods are the MDOF based techniques. In this study, the MDOF-based Polynomial method is used. The fitting curve and modal parameters after applying the polynomial curve fitting are shown in Figure 3.6.

![Figure 3.6 Modal parameters using polynomial curve fitting](image)

Figure 3.6  Modal parameters using polynomial curve fitting
Once a set of modal frequency and damping parameters is obtained, the modal residues (mode shapes) are estimated during a separate curve fitting step on the FRF data. During the residue curve fitting process, the coefficients of the numerator polynomial of each FRF are estimated by a least squared error curve fitting process. After the modal frequency, damping and residues are estimated for all the modes of interest, the modal parameters are saved in a Shape Table file (see Figure 3.7). The first mode of their corresponding mode shapes can be displayed in an animation of a 3-D view (Figure 3.8).

<table>
<thead>
<tr>
<th>Shape</th>
<th>Label</th>
<th>Frequency</th>
<th>Units</th>
<th>Damping (%)</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
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<td>GOP:SCP</td>
<td>10.7 Hz</td>
<td>Hz</td>
<td>2.89</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>GOP:SCP</td>
<td>30.3 Hz</td>
<td>Hz</td>
<td>1.44</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>GOP:SCP</td>
<td>56.4 Hz</td>
<td>Hz</td>
<td>1.17</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>GOP:SCP</td>
<td>108 Hz</td>
<td>Hz</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>GOP:SCP</td>
<td>193 Hz</td>
<td>Hz</td>
<td>0.295</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>GOP:SCP</td>
<td>223 Hz</td>
<td>Hz</td>
<td>0.347</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>GOP:SCP</td>
<td>353 Hz</td>
<td>Hz</td>
<td>0.537</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>GOP:SCP</td>
<td>365 Hz</td>
<td>Hz</td>
<td>0.506</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>GOP:SCP</td>
<td>398 Hz</td>
<td>Hz</td>
<td>0.666</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>GOP:SCP</td>
<td>436 Hz</td>
<td>Hz</td>
<td>0.384</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.7  Shape table in MEScope

Figure 3.8  Mode shape animation (3rd mode)
3.3.2 Displacement Modal Parameters Using PZT-SLV System

One of the drawbacks of the PZT-PVDF system introduced in the last section is that only a limited number of points are measured, due to the size of PVDF film and the intense labor involvement. An alternative approach is to use the non-contact SLV, in which a dense grid of measurement points can be easily applied. The basic procedure of using the PZT-SLV system to detect damage location is shown in Figure 3.9.

Figure 3.9 PZT-SLV experimental procedure
The SLV measures the out-of-plane surface motion utilizing the Doppler shift phenomenon in order to obtain the velocity or the displacement of the surface vibration. It is designed and widely used for vibration measurements in many engineering applications, including modal analysis and vibration-based damage detection. A set-up of PZT-SLV system is shown in Figure 3.10.

(a) System Schematic

(b) SLV system

Figure 3.10  PZT-SLV system
(c) Laser head

(d) Test specimen with PZT actuator

Figure 3.10  PZT-SLV system (continued)
Similar to the PZT-PVDF system, a sweep harmonic signal is applied in the PZT-SLV system to obtain the dynamic response of the structure. After exciting the structure by using the sweep harmonic signal through the PZT actuator (see Figure 3.10d), the SLV first measures the time domain response of each point in the structure. Then the measured time-domain data are transferred to the frequency domain using Fourier transform. As an example, an FRF curve obtained from the PZT-SLV system is shown in Figure 3.11.

![Figure 3.11 An FRF curve of a clamped-clamped plate obtained by the PZT-SLV system](image)

The modal parameters like the mode shapes and natural frequencies are obtained by analyzing all the measured points on the structure using the mode analysis software in SLV. The first mode of a clamped-clamped plate, obtained from the PZT-SLV system is shown in Figure 3.12.
Figure 3.12  The first mode shape obtained by the PZT-SLV system

3.4 Numerical Modal Analysis

Numerical modal analysis based on the finite element (FE) modeling is also performed for studying the dynamic response of a structure. The natural frequencies and mode shapes are important modal parameters in designing a structure under dynamic loading conditions. The numerical analysis is carried out by using the commercial finite element program ANSYS and mainly used to verify the validity and effectiveness of damage detection algorithms used in this study (see Chapter 2).

In this study, the specimens made of two types of materials are considered, one is aluminum, and the other is E-glass/epoxy composite (more details are given in Chapters 4 and 5). The FE modeling of an aluminum plate is carried out with the solid element (Solid45); while the composite plate is simulated with the layered element (Solid46). The composite plate consists of several orthotropic layers, and it is considered as
orthotropic material. The models of two plates are shown in Figures 3.13 and 3.14, for the composite and aluminum plates, respectively. The corresponding material properties of two plates are listed in Tables 3.1 and 3.2.

![Figure 3.13](image) FE modeling of the E-glass/epoxy composite plate

**Table 3.1** Material properties of the composite plate

<table>
<thead>
<tr>
<th></th>
<th>CSM</th>
<th>UM12081</th>
<th>C1800(0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x$</td>
<td>15.6 GPa</td>
<td>22 GPa</td>
<td>30 GPa</td>
</tr>
<tr>
<td>$E_y$</td>
<td>15.6 GPa</td>
<td>4.6 GPa</td>
<td>5.4 GPa</td>
</tr>
<tr>
<td>$G_{xy}$</td>
<td>5.54 GPa</td>
<td>2.1 GPa</td>
<td>2.62 GPa</td>
</tr>
<tr>
<td>$G_{yz}$</td>
<td>5.54 GPa</td>
<td>2.01 GPa</td>
<td>2.56 GPa</td>
</tr>
<tr>
<td>$\nu_{xy}$</td>
<td>0.41</td>
<td>0.41</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Element types:

- Solid 46 (16 layers): 0.27'' (x) ×0.27'' (y) ×0.19'' (z)
- Link 10 is used to simulate the delamination and it is generated automatically.
Three types of dynamic analysis of a structure are available in ANSYS: modal analysis, harmonic analysis and transient analysis. Modal analysis is used to obtain the natural frequencies and mode shapes of a structure in this study. A comparison of the natural frequencies of an aluminum plate (see Figure 3.14) obtained by analytical, numerical, and experimental studies are given in Table 3.3. As shown in Table 3.3, close correlations of the first three mode frequencies among the three approaches are achieved.
Table 3.3  The first three natural frequencies of an aluminum plate

<table>
<thead>
<tr>
<th>Mode</th>
<th>Analytical (Hz)</th>
<th>Numerical (Hz)</th>
<th>Experiment (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.47</td>
<td>7.44</td>
<td>7.25</td>
</tr>
<tr>
<td>2</td>
<td>29.12</td>
<td>28.91</td>
<td>31.22</td>
</tr>
<tr>
<td>3</td>
<td>47.05</td>
<td>46.83</td>
<td>48.47</td>
</tr>
</tbody>
</table>

A noticeable problem in the numerical modal analysis process is how to choose the Mode-Extraction Method. There are six approaches in ANSYS to obtain the mode shapes of a structure: Block Lanczos method, Subspace method, PowerDynamics method, Reduced (Householder) method, Unsymmetric method, Damped method, and QR Damped method. Block Lanczos method and Subspace method is used for large symmetric eigenvalue problems. The PowerDynamics method is used for very large models and especially useful to obtain a solution for the first several modes to learn how the model will behave. The reduced method is faster than the subspace method because it uses reduced (condensed) system matrices to calculate the solution. However, it is less accurate because the reduced mass matrix is approximated. The unsymmetric method is used for problems with unsymmetric matrices, such as fluid-structure interaction problems. The damped method and QR Damped method is used for problems where damping cannot be ignored. Block Lanczos method is adopted in this study. The first mode shapes of an E-glass/epoxy plate obtained by analytical, numerical and experimental method are shown in Figure 3.15, and similar mode shapes among the three approach are observed.
Figure 3.15  The first mode shapes of an E-glass/epoxy composite plate
Delamination is probably the most frequently occurring damage in composite materials. Delamination may develop during manufacturing or in service, such as impact by foreign objects. The presence of delamination in a composite material may significantly reduce its structural integrity as well as its mechanical properties, such as stiffness and strength. This change in the mechanical properties could result in changes of the natural frequencies and mode shapes of the composite structure. Delamination exhibits a local separation between two layers in composite, and it involves a nonlinear problem. Such a nonlinear phenomenon cannot be accounted for in modal analysis using ANSYS because any nonlinear characteristics are neglected. In the FE modeling of delamination in this study, a bilinear element (Link10) is used to model the delamination damage. LINK10 is a 3-D spar element with the unique feature of a bilinear stiffness matrix resulting in a tension-only or compression-only element. To model the delamination gap, the LINK 10 is designed as compression-only element, which means that the stiffness is zero if the element is under tension. The simulation of the delamination using Link10 element is shown in Figure 3.16. Through linking two nodes that locate at each side of the gap but at the same location with Link10 element, a delamination damage is successfully simulated.
Figure 3.16  Illustration of implementing Link10 element to simulate the delamination in composites

3.5  Summary

In summary, the analytical, numerical, and experimental techniques for modal analyses are introduced in this chapter. The numerical and experimental techniques are later used in the following two chapters to conduct modal analyses of E-glass/epoxy composite and aluminum plates. In combination with the damage detection algorithms introduced in Chapter 2, they are applied to form dynamic-based methodology for damage identification.
4.1. Introduction

In this chapter, experimental and numerical modal analyses are conducted, and the damage detection algorithms introduced in Chapter 2 are used to process the curvature mode shapes of cantilever composite plates to detect the delamination damage. A combined static/dynamic technique is later introduced to the clamped-clamped composite plate, and the different compression load levels are applied to enlarge the effect of delamination in damage detection.

4.2 Experimental Set-up

An E-glass/Epoxy composite plate was tested in this study for locating the damaged area by using dynamics-based Methods. The plate is made of E-glass fiber and epoxy resins and has a \([\text{CSM}/0/90/0/90/0/90/0]_s\) lay-up for a total of 16 layers. The thickness of each layer is 0.0254 cm (0.01 in) for the CSM, 0.0549 cm (0.0216 in) for the UM1208 (unidirectional), and 0.0297 cm (0.0117 in) for the C1800 (0/90). A delamination area about 10.16 cm (4 in) \(\times\) 10.16 cm (4 in) was created during the
fabrication process of the sample by inserting a piece of Teflon tape between the second and the third layer of the composite plate (see Figure 4.1).

During the experiment, two different experimental methods were employed, i.e., acquiring the displacement mode shapes using the PZT-SLV system and the curvature mode shapes using the PZT–PVDF system. The detail of experimental procedures for the PZT–SLV and PZT–PVDF systems are introduced in Chapter 3. To excite the plate, a sweep sine signal was run through the PZT ceramic actuators. The parameters of the sweep sine waves are given in Table 4.1. A signal generator integrated in the scanning laser vibrometer (SLV) was used to induce the sweep sine in the PZT-SLV system; while a Hewlett Packard 33120A waveform generator played as the signal source of the
PZT-PVDF system. All the applied sweep waves were linear. For the PZT-SLV system, the plate was divided into the measurement grid areas which are about 0.638 cm (0.25 in) × 0.638 cm (0.25 in), and for the PZT-PVDF system, the plate sample was divided into the grids about 2.54 cm (1.0 in) × 2.54 cm (1.0 in).

When the plate was clamped in the cantilever configuration, the specimen had a free span length of 50.8 cm (20 in). A 1.02 cm (0.4 in) × 1.52 cm (0.6 in) piece of PZT ceramic sensor was surface-bonded near the bottom of the plate as an actuator.

| Table 4.1  Parameters of sweep sine wave in the two systems |
|-----------------|-----------------|-----------------|
| Parameters      | SLV - PZT       | PVDF - PZT      |
| Frequency (Hz)  | 1~1000          | 1~450           |
| Time (s)        | 6               | 60              |

4.3. Experiment of an E-glass/Epoxy Cantilever Composite Plate

The experiment was conducted first for the “healthy” plate by keeping the Teflon tape in the plate. After finishing the “healthy” plate experiment, the Teflon tape was removed, and the composite plate became a damage one with the delamination in the former Teflon tape area. The two experiments (for the healthy plate and the damaged one) were implemented with the same boundary conditions and sweep sine excitation.

4.3.1 Experimental and Numerical Results

The results of the frequency measurements and mode shapes from the experiments and numerical analysis are presented in this section. The first five natural
frequencies of the damaged composite plate obtained from the FE simulation and the PZT-SLV and PZT-PVDF systems are compared with the undamaged one, and the results are presented in Table 4.2. It can be observed clearly that the three methods almost obtain the same natural frequencies, and the differences between the “healthy” plate and damaged one are very small with the maximum difference percentage is about 0.15%. Thus, it is difficult to use the natural frequencies comparison method to identify a delamination in a cantilever composite plate.

The mode shapes of both the healthy and damaged plates, which obtained from the FE simulation, PZT–SLV and PZT-PVDF are shown in Tables 4.3 and 4.4. The displacement mode shapes obtained from the experiments of both the damaged (Table 4.4) and healthy (Table 4.3) composite plates using the PZT-SLV system match relatively well with those of finite element simulation, and they thus indicate the validity of the experimental data. However, based on a visual inspection of the displacement mode shapes, the damage location of the composite plate could not be discerned. Thus, the damage detection algorithms (see Chapter 2) were implemented to evaluate the damage (delamination) of the composite plate as introduced in the next few sections.

For the experimental data from the PZT–SLV and PZT–PVDF measurement systems, only the three consecutive mode shapes from the 3rd to 5th mode are implemented to the damage detection algorithms; while there are much ambient and measurement noise in the low frequency range of the 1st and 2nd mode shapes, and they (i.e., the 1st and 2nd mode shapes) are thus not used in this study.

Table 4.2  Natural frequencies of the cantilever composite plates

52
Table 4.3  Mode shapes of the cantilever healthy composite plate

<table>
<thead>
<tr>
<th>Mode</th>
<th>FE (displacement)</th>
<th>PZT-SLV (displacement)</th>
<th>PZT-PVDF (curvature)</th>
</tr>
</thead>
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<tr>
<td>3</td>
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Table 4.4  Mode shapes of the cantilever damaged composite plate

<table>
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<th>Mode</th>
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<th>PZT-SLV (displacement)</th>
<th>PZT-PVDF (curvature)</th>
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<td><img src="image8" alt="Image" /></td>
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</table>

4.3.2 Damage Detection Using the PZT-SLV System

Based on the displacement mode shapes obtained from the PZT-SLV system, the central difference approximate method (Equations 4.1 and 4.2) is used to calculate the longitudinal ($\kappa_x$) and transverse ($\kappa_y$) curvature mode shapes, and the Savitzky-Golay filter (see Chapter 2) is used to filter the original data before the central difference approximate method is applied. $\kappa_x$ and $\kappa_y$ of both the healthy and damaged plates are listed in Tables 4.5 and 4.6, and they are computed using the following formulas.
\[ \kappa_x = \frac{\delta_{i+1,j} - \delta_{i,j} + \delta_{i-1,j}}{\Delta x^2} \]  \hspace{1cm} (4.1)

\[ \kappa_y = \frac{\delta_{i,j+1} - \delta_{i,j} + \delta_{i,j-1}}{\Delta y^2} \]  \hspace{1cm} (4.2)

Table 4.5  Curvature mode shapes of the cantilever healthy composite plate obtained from PZT-SLV

<table>
<thead>
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<th>Mode</th>
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<th>( \kappa_y )</th>
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Table 4.6  Curvature mode shapes of the cantilever damaged composite plate obtained from PZT-SLV

<table>
<thead>
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<th>$\kappa_y$</th>
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<td>5</td>
<td><img src="image5" alt="Graph" /></td>
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</tbody>
</table>

Then, the obtained curvature mode shapes are applied to the damage detection algorithms (i.e., ADM, DIM, GSM, GFD, and SEM) introduced in Chapter 2 for locating the damage.
4.3.2.1 Damage Detection Based on the Longitudinal Curvature Mode Shapes ($\kappa_x$)

This section presents the detection results based on the longitudinal curvature mode shapes of the cantilever composite plate.

Absolute difference method (ADM)

As shown in Figures 4.2~4.5 and Table 4.7, the absolute difference method (ADM), which is only applied to the $3^{rd}$, $4^{th}$, $5^{th}$ mode shapes and ULS of the cantilever composite plate, can approximate the delamination location of the plate. However, besides showing the peaks around the delamination area, they show peaks outside the delamination area as well. The ADM values around the delamination location are usually larger than elsewhere, leading to relative effectiveness of the ADM.

![Figure 4.2 Results of the ADM using the 3$^{rd}$ mode shape ($\kappa_x$)
Figure 4.3  Results of the ADM using the 4th mode shape ($\kappa_4$)

Figure 4.4  Results of the ADM using the 5th mode shape ($\kappa_5$)
Figure 4.5  Results of the ADM using the ULS ($\kappa_s$)

Table 4.7  Estimated damage location of the cantilever damaged composite plate based on ADM and $\kappa_s$

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Mode 3</th>
<th>Mode 4</th>
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<th>ULS</th>
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<td>0~10</td>
<td>0~9</td>
<td>0~9</td>
<td>0~9</td>
<td>0~9</td>
</tr>
<tr>
<td>L (cm)</td>
<td>25.4~36.5</td>
<td>20~35</td>
<td>20~35</td>
<td>18~35</td>
<td>20~35</td>
</tr>
</tbody>
</table>

Damage index method (DIM)

Based on the results obtained from applying the DIM to the 3\textsuperscript{rd}, 4\textsuperscript{th} and 5\textsuperscript{th} mode shapes of the cantilever composite plate (Figures 4.6), the damage location and area can be relatively detected. The detected damage location is at $L = 22$~38 cm and D
= 0–10 cm, and it matches closely with the actual damage area (L = 25.4–36.5 cm D = 1–10 cm).

Figure 4.6  Results of the DIM using the 3rd to 5th mode shapes ($\kappa_3$)

*Simplified gapped smoothing method (GSM)*

The results using the GSM are shown in Figures 4.7–4.10 and Table 4.8. The GSM is capable of locating the delamination in the plate, though there are still some discrepancies in the estimated location and size of damage.
Figure 4.7  Results of the GSM using the 3rd mode shape ($\kappa_3$)

Figure 4.8  Results of the GSM using the 4th mode shape ($\kappa_4$)
Figure 4.9 Results of the GSM using the 5th mode shape ($\kappa_{5}$)

Figure 4.10 Results of the GSM using the ULS ($\kappa_{\text{ULS}}$)
Table 4.8  Estimated damage location of the cantilever damaged composite plate based on GSM and $\kappa_x$

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</tr>
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<td>L (cm)</td>
<td>25.4–36.5</td>
<td>21–38</td>
<td>20–34</td>
<td>20–34</td>
</tr>
</tbody>
</table>

*Generalized fractal dimension (GFD)*

As shown in Figures 4.11–4.14 and Table 4.9, GFD is effective in detecting the delamination location by showing a distinct peak. However, the result from the 5th mode fails to estimate the location of the damage.

![Figure 4.11](image.jpg)

Figure 4.11  Results of the GFD using the 3rd mode shape ($\kappa_x$)
Figure 4.12  Results of the GFD using the 4th mode shape ($\kappa_x$)

Figure 4.13  Results of the GFD using the 5th mode shape ($\kappa_x$)
Table 4.9  Estimated damage location of the cantilever damaged composite plate based on GFD and $\kappa_x$

<table>
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<tr>
<td>L (cm)</td>
<td>25.4~36.5</td>
<td>25~38</td>
<td>22~30</td>
<td>-</td>
<td>24~30</td>
</tr>
</tbody>
</table>

*Strain energy method (SEM)*

As shown in Figures 4.15~4.18 and Table 4.10, the SEM is also relatively effective in detecting the delamination location and area when using the longitudinal curvature mode shapes 3, 4, 5 and ULS.
Figure 4.15  Results of the SEM using the 3rd mode shape ($\kappa_3$)

Figure 4.16  Results of the SEM using the 4th mode shape ($\kappa_4$)
Figure 4.17  Results of the SEM using the 5th mode shape ($\kappa_5$)

Figure 4.18  Results of the SEM using the ULS ($\kappa_\sigma$)
4.10 Estimated damage location of the cantilever damaged composite plate based on SEM and $\kappa_x$.

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<td>0~8</td>
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<td>L (cm)</td>
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<td>20~38</td>
<td>28~36</td>
<td>20~35</td>
<td>25~40</td>
</tr>
</tbody>
</table>

4.3.2.2 Damage Detection Based on the Transverse Curvature Mode Shapes ($\kappa_y$)

The transverse curvature mode shapes of the cantilever composite plate are also processed by the algorithms of ADM, DIM, GSM, GFD, and SEM to detect the delamination location and area in the plate. As shown in Figures 4.19 ~ 4.35, the results indicate that the algorithms in most cases fail to detect the damage location based on the transverse curvature mode shapes. Because the delamination is located at the edge of the composite plate in the transverse direction, the edge effect of the central difference approximation method can significantly diminish the peaks due to the damage. This can cause misleading information for the location of the damage (see Figures 4.23 and 4.28 to 4.31).
Absolute difference method (ADM)

Figure 4.19  Results of the ADM using the 3rd mode shape ($\kappa_3$)

Figure 4.20  Results of the ADM using the 4th mode shape ($\kappa_4$)
Figure 4.21  Results of the ADM using the 5th mode shape ($\kappa_5$)

Figure 4.22  Results of the ADM using the ULS ($\kappa_5$)
Damage index method (DIM)

Figure 4.23  Results of the DIM using the first three mode shapes ($\kappa_y$)

Simplified gapped smoothing method (GSM)

Figure 4.24   Results of the GSM using the 3rd mode shape ($\kappa_y$)
Figure 4.25  Results of the GSM using the 4th mode shape ($\kappa_y$)

Figure 4.26  Results of the GSM using the 5th mode shape ($\kappa_y$)
Figure 4.27  Results of the GSM using the ULS ($\kappa_y$)

*Generalized fractal dimension (GFD)*

Figure 4.28  Results of the GFD using the 3rd mode shape ($\kappa_y$)
Figure 4.29 Results of the GFD using the 4th mode shape ($\kappa_y$)

Figure 4.30 Results of the GFD using the 5th mode shape ($\kappa_y$)
Figure 4.31  Results of the GFD using the ULS ($\kappa_y$)

Strain energy method (SEM)

Figure 4.32  Results of the SEM using the 3rd mode shape ($\kappa_y$)
Figure 4.33  Results of the SEM using the 4th mode shape ($\kappa_y$)

Figure 4.34  Results of the SEM using the 5th mode shape ($\kappa_y$)
4.3.3 Damage Detection Using the PZT-PVDF System

By using the PZT-PVDF system, the longitudinal curvature mode shapes were first obtained. Then the experiment data were filtered by the Savitzky-Golay filter as introduced in Chapter 2. Finally, the damage detection algorithms of ADM, DIM, GSM, GFD, and SEM were applied to the filtered curvature mode shapes to locate the delamination in the plate. The PVDF sensor with an aspect ratio of length-to-width of 4:1 was surface-bonded to the composite plate, and the length of the PVDF sensor was aligned with the longitudinal direction of the plate; thus, the acquired curvature mode shapes are treated as the longitudinal ones in this study. Although the measured data might contain the components of the transverse ($\kappa_y$) and twisting ($\kappa_{xy}$) curvatures as well, these components are negligibly small compared to the longitudinal component.
**Absolute difference method (ADM)**

As shown in Figures 4.36–4.39 and Table 4.11, the ADM, which employed the 3rd, 4th, and ULS curvatures of the cantilever composite plate, is capable of detecting the delamination in the plate, except for the 5th curvature mode shape (see Figure 4.38). Thus, the ADM seems viable in locating the delamination.

![Graph showing results of the ADM using the 3rd mode shape (κ3)](image)

**Figure 4.36** Results of the ADM using the 3rd mode shape (κ3)
Figure 4.37  Results of the ADM using the 4\textsuperscript{th} mode shape ($\kappa_x$)

Figure 4.38  Results of the ADM using the 5\textsuperscript{th} mode shape ($\kappa_x$)
Table 4.11 Estimated damage location of the cantilever damaged composite plate based on ADM and $\kappa_x$.

<table>
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<td>0–9</td>
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<tr>
<td>L (cm)</td>
<td>25.4–36.5</td>
<td>25–36</td>
<td>20–31</td>
<td>-</td>
<td>25–35</td>
</tr>
</tbody>
</table>

 DAMAGE INDEX METHOD (DIM)

Based on the results obtained from applying the DIM using the mode shapes 3, 4 and 5 of the cantilever composite plate (Figures 4.40), the location and area of the delamination can be estimated at $L = 22–32$ cm and $D = 0–9$ cm. However, there are
some other peaks that can be mistakenly considered as another damage in the plate. It matches relatively well with the actual damage area (D = 1~10 cm, L = 25.4~36.5 cm).

![Figure 4.40](image)

Figure 4.40  Results of the DIM using the three mode shapes of 3rd to 5th ($\kappa_x$)

*Simplified gapped smoothing method (GSM)*

As shown in Figures 4.41~4.44 and Table 4.12, the GSM is relatively accurate in locating the delamination of the cantilever composite plate. Again, the GSM based on the 5th curvature mode shape (see Figure 4.43) is unable to locate the delamination.
Figure 4.41  Results of the GSM using the 3\textsuperscript{rd} mode shape ($\kappa_x$)

Figure 4.42  Results of the GSM using the 4\textsuperscript{th} mode shape ($\kappa_y$)
Figure 4.43  Results of the GSM using the 5\textsuperscript{th} mode shape ($\kappa_x$)

Figure 4.44  Results of the GSM using the ULS ($\kappa_x$)
Table 4.12  Estimated damage location of the cantilever damaged composite plate based on the GSM and $\kappa_x$

<table>
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<td>L (cm)</td>
<td>25.4–36.5</td>
<td>25–36</td>
<td>28–37</td>
<td>-</td>
<td>25–35</td>
</tr>
</tbody>
</table>

Generalized fractal dimension (GFD)

As shown in Figures 4.45–4.48 and Table 4.13, the GFD is very effective in detecting the delamination location and area in the cantilever composite plate. The GFD based on the 3rd and 4th mode shapes and ULS work well to detect the delamination. The result based on the 5th mode shape (see Figure 4.47) is inconclusive and not able to identify the damage.

Figure 4.45  Results of the GFD using the 3rd mode shape ($\kappa_x$)
Figure 4.46  Results of the GFD using the 4\textsuperscript{th} mode shape ($\kappa_4$)

Figure 4.47  Results of the GFD using the 5\textsuperscript{th} mode shape ($\kappa_5$)
Figure 4.48  Results of the GFD using the ULS ($\kappa_x$)

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<td>20~32</td>
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</tr>
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</table>

Table 4.13  Estimated damage location of the cantilever damaged composite plate based on GFD and $\kappa_x$

Strain energy method (SEM)

As shown in Figures 4.49~4.52 and Table 4.14, the SEM is also promising in detecting the delamination location and area in the cantilever composite plate. Except
of the SEM from the 5th curvature mode shape (see Figure 4.51), all the other SEMs perform well in the delamination detection.

Figure 4.49  Results of the SEM using the 3rd mode shape ($\kappa_x$)

Figure 4.50  Results of the SEM using the 4th mode shape ($\kappa_x$)
Figure 4.51  Results of the SEM using the 5th mode shape ($\kappa_5$)

Figure 4.52  Results of the SEM using the ULS ($\kappa_s$)
Table 4.14  Estimated damage location of the cantilever damaged composite plate based on SEM and $\kappa_x$

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<td>L (cm)</td>
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<td>27~36</td>
<td>28~40</td>
<td>-</td>
<td>28~31</td>
</tr>
</tbody>
</table>

4.3.4 Damage Detection Using the Numerical Finite Element Simulation

Based on the finite element (FE) simulation, the displacement mode shapes are obtained, and then the same method (i.e., central difference approximate method) as applied to the PZT-SLV system is used to compute the longitudinal ($\kappa_x$) and transverse ($\kappa_y$) curvature mode shapes of the cantilever composite plate. $\kappa_x$ and $\kappa_y$ of both the healthy and damaged composite plates are shown in Tables 4.15 and 4.16. The damage detection algorithms of ADM, DIM, GSM, GFD, and SEM are then applied to the curvature mode shapes of the plate for locating the location and area of the delamination in the plate. Again, corresponding to the curvature mode shapes used in the PZT-SLV and PZT-PVDF systems, the mode shapes from the 3rd to 5th modes are adopted in the FE-based damage detection. The purpose of the FE analysis is to verify the validity of the damage detection algorithms used in this study.
Table 4.15 Curvature mode shapes of the cantilever healthy composite plate (FE)

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<td>5</td>
<td><img src="image5" alt="Plot 5" /></td>
<td><img src="image6" alt="Plot 5" /></td>
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</tbody>
</table>
Table 4.16  Curvature mode shapes of the cantilever damaged composite plate (FE)

<table>
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<th>Mode</th>
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<th>$\kappa_y$</th>
</tr>
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<tr>
<td>5</td>
<td><img src="#" alt="Image 5" /></td>
<td><img src="#" alt="Image 6" /></td>
</tr>
</tbody>
</table>

4.3.4.1 Damage Detection Based on the Longitudinal Curvature Mode Shapes

The longitudinal curvature mode shapes ($\kappa_x$) of the cantilever composite plate obtained from FE are used in the algorithms of ADM, DIM, GSM, GFD, and SEM to detect the location and area of the delamination in the plate. The results of the damage
detect algorithms are shown in Figures 4.53~4.69, and the details are elaborated as follows.

**Absolute difference method (ADM)**

The results based on the ADM of the longitudinal curvature mode shapes are plotted in Figures 4.53 to 4.56. It is interesting to note that there are two distinct peaks at the edges of the delamination, indicating the beginning and ending of the delamination area. The comparison between the actual and ADM locations is given in Table 4.17. As shown in Figures 5.53 to 4.56 and Table 4.17, the damage detection based on the ADM could be used to accurately evaluate the delamination damage in the structures, thus verifying the effectiveness of the algorithm. However, be aware that the healthy information is needed when applying the ADM.

![Figure 4.53 Results of the ADM using the 3rd mode shape (κ₃)](image)
Figure 4.54  Results of the ADM using the 4th mode shape ($\kappa_4$)

Figure 4.55  Results of the ADM using the 5th mode shape ($\kappa_5$)
Figure 4.56 Results of the ADM using the ULS ($\kappa_x$)

Table 4.17 Estimated damage location of the cantilever damaged composite plate based on ADM and $\kappa_x$

<table>
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<td>21~30</td>
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<tr>
<td>L (cm)</td>
<td>13.5~24.6</td>
<td>15~25</td>
<td>14~25</td>
<td>14~25</td>
<td>14~25</td>
</tr>
</tbody>
</table>

Damage index method (DIM)

The DIM result is plotted in Figure 4.57, and two distinct peaks appear to indicate the beginning and ending of the delamination. The DIM estimated delamination matches very well with the actual location, thus verifying the usefulness of the algorithm.
Figure 4.57  Results of the DI using the mode shapes 3, 4, and 5 ($\kappa_x$)

*Simplified gapped smoothing method (GSM)*

Similarly, the analysis using GSM is plotted in Figures 4.58 to 4.61, and the results of the comparison between the GSM and actual delamination location are presented in Table 4.18. From the results, the GSM is very effective in detecting the location of the delamination as well.
Figure 4.58  Results of the GSM using the 3rd mode shape ($\kappa_3$)

Figure 4.59  Results of the GSM using the 4th mode shape ($\kappa_4$)
Figure 4.60  Results of the GSM using the 5th mode shape ($\kappa_5$)

Figure 4.61  Results of the GSM using the ULS ($\kappa_u$)
Table 4.18  Estimated damage location of the cantilever damaged composite plate based on GSM and $\kappa_x$:

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</table>

*Generalized fractal dimension (GFD)*

The analysis using GFD is shown in Figure 4.62 to 4.65. A major peak is demonstrated at the location of the delamination in most of case. However, an undulation along transverse direction is noticeable on the results from the 3rd and 4th modes; the large undulations are present in the results from the 5th mode and ULS. The comparison of the GFD detected locations to the actual one is given in Table 4.19.

![Figure 4.62 Results of the GFD using the 3rd mode shape ($\kappa_x$)](image-url)
Figure 4.63  Results of the GFD using the 4th mode shape ($\kappa_x$)

Figure 4.64  Results of the GFD using the 5th mode shape ($\kappa_x$)
Table 4.19  Estimation damage location of the cantilever damaged composite plate based on GFD and $\kappa_x$

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<td>15–25</td>
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*Strain energy method (SEM)*

The results based on SEM are shown in Figure 4.66 to 4.69 and Table 4.20. Although some peaks are generated at different locations, the large peaks are observed at the location of the delamination, while prompting the effectiveness of the method.
Along with GSM and GFD, the data of healthy information is not needed when these damage detection algorithms are applied.

Figure 4.66  Results of the SEM using the 3rd mode shape ($\kappa_x$)

Figure 4.67  Results of the SEM using the 4th mode shape ($\kappa_x$)
Figure 4.68  Results of the SEM using the 5\textsuperscript{th} mode shape ($\kappa_x$)

Figure 4.69  Results of the SEM using the ULS ($\kappa_x$)
Table 4.20  Estimated damage location of the cantilever damaged composite plate based on SEM and $\kappa_x$.

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4.3.4.2 Damage Detection Based on the Transverse Curvature Mode Shapes ($\kappa_y$)

The validity of the damage detection algorithms is also verified using the transverse curvature mode shapes ($\kappa_y$) of the cantilever composite plate obtained from FE in the algorithms (i.e., ADM, DIM, GSM, GFD, and SEM). The results of the damage detect algorithms are shown in Figures 4.70~4.86, and the comparisons of the detection results with the actual location of delamination are presented in Table 4.21. Compared to the results of the PZT-SLV based on the transverse curvature mode shapes ($\kappa_y$), the results using the FE are capable of locating the delamination. This is primarily due to the refined analysis of finite element analysis which provides more consistent valid data.
Absolute difference method (ADM) (see Figures 4.70 to 4.13)

Figure 4.70  Results of the ADM using the 3rd mode shape ($\kappa_3$)

Figure 4.71  Results of the ADM using the 4th mode shape ($\kappa_4$)
Figure 4.72  Results of the ADM using the 5th mode shape ($\kappa_5$)

Figure 4.73  Results of the ADM using the ULS ($\kappa_5$)
Damage index method (DI) (see Figures 4.74)

Figure 4.74  Results of the DI using the first three mode shapes ($\kappa_y$)

Simplified gapped smoothing method (GSM) (see Figures 4.75 to 4.78)

Figure 4.75  Results of the GSM using the 3$^{rd}$ mode shape ($\kappa_y$)
Figure 4.76  Results of the GSM using the 4th mode shape ($\kappa_4$)

Figure 4.77  Results of the GSM using the 5th mode shape ($\kappa_5$)
Figure 4.78  Results of the GSM using the ULS ($\kappa_y$)

Generalized fractal dimension (GFD)  (see Figures 4.79 to 4.82)

Figure 4.79  Results of the GFD using the 3rd mode shape ($\kappa_y$)
Figure 4.80  Results of the GFD using the 4\textsuperscript{th} mode shape ($\kappa_y$)

Figure 4.81  Results of the GFD using the 5\textsuperscript{th} mode shape ($\kappa_y$)
Figure 4.82  Results of the GFD using the ULS ($\kappa_y$)

*Strain energy method (SEM)*  (see Figures 4.83 to 4.86)

Figure 4.83  Results of the SEM using the 3rd mode shape ($\kappa_y$)
Figure 4.84  Results of the SEM using the 4th mode shape ($\kappa_y$)

Figure 4.85  Results of the SEM using the 5th mode shape ($\kappa_y$)
Figure 4.86  Results of the SEM using the ULS ($\kappa_y$)

Table 4.21  Estimated damage location of the cantilever damaged composite plate based on ADM, DIM, GSM, GFD, SEM and $\kappa_y$

<table>
<thead>
<tr>
<th>Mode</th>
<th>D (cm)</th>
<th>L (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>20–30</td>
<td>13.5–24.6</td>
</tr>
<tr>
<td>DIM</td>
<td>20–24</td>
<td>14–25</td>
</tr>
<tr>
<td>ADM</td>
<td>3 20–30</td>
<td>14–25</td>
</tr>
<tr>
<td></td>
<td>4 20–30</td>
<td>14–26</td>
</tr>
<tr>
<td></td>
<td>5 20–30</td>
<td>14–25</td>
</tr>
<tr>
<td>ULS</td>
<td>20–28</td>
<td>13–25</td>
</tr>
<tr>
<td>GSM</td>
<td>3 20–24</td>
<td>14–26</td>
</tr>
<tr>
<td></td>
<td>4 20–24</td>
<td>14–25</td>
</tr>
<tr>
<td></td>
<td>5 20–25</td>
<td>14–25</td>
</tr>
<tr>
<td>ULS</td>
<td>20–24</td>
<td>14–25</td>
</tr>
<tr>
<td>GFD</td>
<td>3 20–23</td>
<td>14–24</td>
</tr>
<tr>
<td></td>
<td>4 20–24</td>
<td>14–25</td>
</tr>
<tr>
<td></td>
<td>5 20–22</td>
<td>13–26</td>
</tr>
<tr>
<td>ULS</td>
<td>20–24</td>
<td>14–25</td>
</tr>
<tr>
<td>SEM</td>
<td>3 20–24</td>
<td>14–25</td>
</tr>
<tr>
<td></td>
<td>4 20–25</td>
<td>15–25</td>
</tr>
<tr>
<td></td>
<td>5 20–24</td>
<td>14–26</td>
</tr>
<tr>
<td>ULS</td>
<td>20–25</td>
<td>14–25</td>
</tr>
</tbody>
</table>
4.3.4.3 Summary on the FE Simulation

When the damage detection algorithms of ADM, DIM, GSM, GFD, and SEM are applied to both the longitudinal ($\kappa_x$) and transverse ($\kappa_y$) curvature mode shapes generated by numerical FE analysis, the damage location and area are correctly detected. Thus, it validates the viability of the damage detection algorithms used in this study. The accuracy and effectiveness of damage detection using the FE data shows the promise of the damage detection algorithms if the valid data (e.g., from the experiment) are available.

4.4 Static/Dynamic Approach

In many cases, the damage effects on the dynamic response are very small and hardly detectable. To enlarge these effects, a combined static/dynamic loading technique is proposed. Applying the static loading (e.g., compression to the composite plate with delamination) may enlarge the effect of the damage in the dynamic response, which in turn it may make the damage to be easily detected. The static/dynamic test under various static load levels was implemented. The results are compared with the ones without the static load to evaluate the performance of the proposed approach.

The experimental set-up of the combined static/dynamic loading is displayed in Figure 4.87. As shown in Figure 4.87, the composite plate was clamped along both ends using wooden blocks which were sitting on two flat steel plates. The span length of the clamped-clamped (c-c) composite plate is 50.8 cm (20 in). The PZT actuator is located near one clamped boundary. The compression force of 3,113 N (700 lbf) and
5,782 N (1300 lbf) were applied through the MTS. The dynamic responses under no compression force, 3,113 N (700 lbf) and 5,782 N (1300 lbf) were measured using the PZT-SLV and PZT-PVDF systems, respectively. Then, the mode shapes obtained from the respective two sensor systems (PZT-SLV and PZT-PVDF) and the numerical FE simulation were applied to the GSM, GFD, SEM to obtain damage information.

Figure 4.87 Static/dynamic test

(a) Specimen dimension

(b) Testing set-up
4.4.1 Experimental and Numerical Results

With the sweep sine as an excitation through the PZT actuator, the results of the experimental frequency measurement and mode shapes analysis under three different compression levels (i.e., 0 N, 3,113 N, and 5,782 N) are presented in Tables 4.22~4.27. The numerical analysis (frequencies and mode shapes) of the damaged composite plate obtained from the finite element (FE) simulation were also conducted. Based on a visual inspection of the unsmooth surface at the damage location, the location of the delamination in the composite plate could be approximately identified using either the displacement mode shapes (PZT-SLV) or curvature mode shapes (PZT-PVDF).

Table 4.22  Natural frequencies of the composite plate under the 0 N compression

<table>
<thead>
<tr>
<th>Mode</th>
<th>FE (Hz)</th>
<th>PZT-SLV (Hz)</th>
<th>PZT-PVDF (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70.25</td>
<td>69.67</td>
<td>69.29</td>
</tr>
<tr>
<td>2</td>
<td>87.65</td>
<td>88.64</td>
<td>88.36</td>
</tr>
<tr>
<td>3</td>
<td>174.35</td>
<td>171.89</td>
<td>172.56</td>
</tr>
</tbody>
</table>

Table 4.23  Natural frequencies of the composite plate under the 3,113 N compression

<table>
<thead>
<tr>
<th>Mode</th>
<th>FE (Hz)</th>
<th>PZT-SLV (Hz)</th>
<th>PZT-PVDF (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65.27</td>
<td>66.23</td>
<td>66.52</td>
</tr>
<tr>
<td>2</td>
<td>82.34</td>
<td>81.08</td>
<td>82.04</td>
</tr>
<tr>
<td>3</td>
<td>167.35</td>
<td>165.81</td>
<td>166.23</td>
</tr>
</tbody>
</table>
Table 4.24  Natural frequencies of the composite plate under the 5,782 N compression

<table>
<thead>
<tr>
<th>Mode</th>
<th>FE (Hz)</th>
<th>PZT-SLV (Hz)</th>
<th>PZT-PVDF (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>58.78</td>
<td>61.25</td>
<td>60.87</td>
</tr>
<tr>
<td>2</td>
<td>75.02</td>
<td>76.50</td>
<td>75.80</td>
</tr>
<tr>
<td>3</td>
<td>159.39</td>
<td>160.50</td>
<td>161.73</td>
</tr>
</tbody>
</table>

Table 4.25  Mode shapes of the composite plate under the 0 N compression

<table>
<thead>
<tr>
<th>Mode</th>
<th>FE (displacement)</th>
<th>PZT-SLV (displacement)</th>
<th>PVDF – PZT (curvature)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
<td><img src="image3" alt="Image" /></td>
</tr>
<tr>
<td>2</td>
<td><img src="image4" alt="Image" /></td>
<td><img src="image5" alt="Image" /></td>
<td><img src="image6" alt="Image" /></td>
</tr>
<tr>
<td>3</td>
<td><img src="image7" alt="Image" /></td>
<td><img src="image8" alt="Image" /></td>
<td><img src="image9" alt="Image" /></td>
</tr>
</tbody>
</table>
Table 4.26  Mode shapes of the composite plate under the 3,113 N compression

<table>
<thead>
<tr>
<th>Mode</th>
<th>FE (displacement)</th>
<th>PZT-SLV (displacement)</th>
<th>PVDF – PZT (curvature)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1" alt="FE Displacement" /></td>
<td><img src="image2" alt="PZT-SLV Displacement" /></td>
<td><img src="image3" alt="PVDF – PZT Curvature" /></td>
</tr>
<tr>
<td>2</td>
<td><img src="image4" alt="FE Displacement" /></td>
<td><img src="image5" alt="PZT-SLV Displacement" /></td>
<td><img src="image6" alt="PVDF – PZT Curvature" /></td>
</tr>
<tr>
<td>3</td>
<td><img src="image7" alt="FE Displacement" /></td>
<td><img src="image8" alt="PZT-SLV Displacement" /></td>
<td><img src="image9" alt="PVDF – PZT Curvature" /></td>
</tr>
</tbody>
</table>
Table 4.27  Mode shapes of the composite plate under the 5,782 N compression

<table>
<thead>
<tr>
<th>Mode</th>
<th>FE (displacement)</th>
<th>PZT-SLV (displacement)</th>
<th>PVDF – PZT (curvature)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td>2</td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
<tr>
<td>3</td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
</tr>
</tbody>
</table>

4.4.2 Damage Identification Analysis

4.4.2.1 Damage Detection Using the PZT-SLV System

Based on the measurement of the clamped-clamped composite plate under the three different load level from the PZT-SLV system, the first three displacement mode shapes are obtained. The ULS is then calculated using the obtained displacement mode shapes. The results from the GSM, GFD, SEM based on the ULS under the three different load conditions (0 N, 3,113 N, and 5,782 N) are shown in Figures 4.88~4.96.
It can be obviously observed that the larger the load was applied to the specimen, the more significant the maximum peak at the delamination location appears.

![Figure 4.88 Results of the GSM using the ULS with no compression load](image1)

![Figure 4.89 Results of the GSM using the ULS under the 3,113 N compression load](image2)
Figure 4.90  Results of the GSM using the ULS under the 5,782 N compression load

Figure 4.91  Results of the GFD using the ULS with no compression load
Figure 4.92  Results of the GFD using the ULS under the 3,113 N compression load

Figure 4.93  Results of the GFD using the ULS under the 5,782 N compression load
Figure 4.94  Results of the SEM using the ULS with no compression load

Figure 4.95  Results of the SEM using the ULS under the 3,113 N compression load
The comparison of the ratios of the maximum magnitude under two load conditions to the one without force case is listed in Table 4.28; while the comparison of the damage parameters along the longitudinal edge \((x = 1 \text{ cm})\) are shown in Figure 4.97. The increased ratio obtained from all the three algorithms indicates that the damage is much easier to be detected under the compression condition. From Figure 4.97, it indicates that increasing the compression load levels enlarges the dynamic responses, e.g., as in terms of damage parameters in this study. Thus, this technique may be viable to quantify the relative magnitude of the damage. As shown in Table 4.28 and Figure 4.97, all the three damage detection algorithms are effective in locating the delamination, though only the displacement mode shapes and resulting ULS are used in the data processing.
Table 4.28  The comparison of the damage parameter magnitudes under different compression load levels

<table>
<thead>
<tr>
<th>Force</th>
<th>Maximum magnitude ratio</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GSM</td>
<td>GFD</td>
<td>SEM</td>
</tr>
<tr>
<td>0 N</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>3,113 N</td>
<td>1.51</td>
<td>1.0009</td>
<td>1.35</td>
</tr>
<tr>
<td>5,782 N</td>
<td>2.07</td>
<td>1.0014</td>
<td>1.62</td>
</tr>
</tbody>
</table>

Figure 4.97  Comparison results along the edge based on the GSM, GFD, and SEM

(a) GSM
Figure 4.97  Comparison results along the edge based on the GSM, GFD, and SEM (continued)
4.4.2.2 Damage Detection Using the PZT-PVDF System

Based on the PZT-PVDF system, the first three curvature mode shapes were obtained, the ULS was then calculated. The results from the GSM, GFD, and SEM based on the ULS under three different load conditions (0 N, 3,113 N, and 5,782 N) are shown in Figures 4.98~4.106, respectively. It is also observed that the larger the compression load was applied to the specimen, the more significant the maximum peak at the damage location appears.

Figure 4.98  Results of the GSM using the ULS with no compression
Figure 4.99  Results of the GSM using the ULS under the 3,113 N compression load

Figure 4.100  Results of the GSM using the ULS under the 5,782 N compression load
Figure 4.101  Results of the GFD using the ULS with no compression

Figure 4.102  Results of the GFD using the ULS under the 3,113 N compression load
Figure 4.103  Results of the GFD using the ULS under the 5,782 N compression load

Figure 4.104  Results of the SEM using the ULS with no compression
Figure 4.105  Results of the SEM using the ULS under the 3,113 N compression load

Figure 4.106  Results of the SEM using the ULS under the 5,782 N compression load
The comparison of the ratios of the maximum magnitude under two load conditions to the case of no compression is presented in Table 4.29. Similarly, the damage parameters along the longitudinal edge of the plate is shown in Figure 4.107. The same conclusion can be made as those obtained from the PZT-SLV.

Table 4.29 The comparison of the damage parameter magnitude under different compression load levels

<table>
<thead>
<tr>
<th>Force</th>
<th>Maximum magnitude ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GSM</td>
</tr>
<tr>
<td>0 N</td>
<td>1.0</td>
</tr>
<tr>
<td>3,113 N</td>
<td>1.28</td>
</tr>
<tr>
<td>5,782 N</td>
<td>1.85</td>
</tr>
</tbody>
</table>

Figure 4.107 Comparison results along the edge based on the GSM, GFD, and SEM
Figure 4.107  Comparison results along the edge based on the GSM, GFD, and SEM

(continued)
4.4.2.3 Damage Detection Using the FE Simulation

Based on the finite element (FE) simulation, the displacement and curvature mode shapes are obtained under the three different load levels (i.e., 0 N, 3,113 N, and 5,782 N), and then the GSM, GFD, and SEM are applied to detect the damage. The results are shown in Tables 4.30 and 4.31 based on the displacement and longitudinal curvature mode shapes, respectively. As shown in Tables 4.30 and 4.31, the damage parameters based on either the displacement or curvature ULS are all capable of detecting the delamination, and as the compression load levels increase, the damage parameters correspondingly increase as well.

Table 4.30 Results using the ULS under different compression load levels ($\delta$)

<table>
<thead>
<tr>
<th>Force</th>
<th>GSM</th>
<th>GFD</th>
<th>SEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 N</td>
<td>![Graph]</td>
<td>![Graph]</td>
<td>![Graph]</td>
</tr>
<tr>
<td>3,113 N</td>
<td>![Graph]</td>
<td>![Graph]</td>
<td>![Graph]</td>
</tr>
<tr>
<td>5,782 N</td>
<td>![Graph]</td>
<td>![Graph]</td>
<td>![Graph]</td>
</tr>
</tbody>
</table>
Table 4.31  Results using the ULS under different compression load levels ($\kappa_x$)

<table>
<thead>
<tr>
<th>Force</th>
<th>GSM</th>
<th>GFD</th>
<th>SEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 N</td>
<td><img src="image1" alt="GSM Image" /></td>
<td><img src="image2" alt="GFD Image" /></td>
<td><img src="image3" alt="SEM Image" /></td>
</tr>
<tr>
<td>3,113 N</td>
<td><img src="image4" alt="GSM Image" /></td>
<td><img src="image5" alt="GFD Image" /></td>
<td><img src="image6" alt="SEM Image" /></td>
</tr>
<tr>
<td>5,782 N</td>
<td><img src="image7" alt="GSM Image" /></td>
<td><img src="image8" alt="GFD Image" /></td>
<td><img src="image9" alt="SEM Image" /></td>
</tr>
</tbody>
</table>

Similarly, the comparison of the ratios of the maximum damage parameters under the two load levels to the one without compression is listed in Table 4.32, and the damage parameters of GSM, GFD, and SEM along the longitudinal edge are shown in Table 4.33. From all the comparison results of the cases under three different compression load levels, it indicates that application of the additional static compression load could magnify the effect of damage in the dynamic response, and it makes the damage manifest. Further, the combined static/dynamic technique coupled with the damage detection algorithms may be used as a tool to quantify the damage effect in the structure.
Table 4.32  The comparison of the damage parameter magnitude under different compression conditions (FE)

<table>
<thead>
<tr>
<th>Force</th>
<th>Maximum magnitude ratio (δ)</th>
<th>Maximum magnitude ratio (κx)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GSM</td>
<td>GFD</td>
</tr>
<tr>
<td>0 N</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>3,113 N</td>
<td>1.43</td>
<td>1.0009</td>
</tr>
<tr>
<td>5,782 N</td>
<td>2.64</td>
<td>1.0020</td>
</tr>
</tbody>
</table>

Table 4.33  Comparison results along the edge based on the GSM, GFD, and SEM

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>δ</th>
<th>κx</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSM</td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
<tr>
<td>GFD</td>
<td><img src="image3.png" alt="Graph" /></td>
<td><img src="image4.png" alt="Graph" /></td>
</tr>
<tr>
<td>SEM</td>
<td><img src="image5.png" alt="Graph" /></td>
<td><img src="image6.png" alt="Graph" /></td>
</tr>
</tbody>
</table>
4.5 Concluding Remarks

The composite plate with the cantilever boundary conditions was first tested by the PZT-SLV and PZT-PVDF measurement systems. The natural frequencies and curvature mode shapes were obtained and processed using the damage detection algorithms (ADM, DIM, GSM, GFD, and SEM) to detect the delamination in the composite plate. In most cases, the delamination area of the composite plate is detected successfully using the algorithms of ADM, DIM, GSM, GFD, and SEM. However, in some cases, significant peaks outside the damage area occur, and they cause misleading identification of multiple damage location. In general, The GSM and SEM detect and isolate the delamination of composite plate better than the other algorithms.

The finite element (FE) simulation was implemented to further verify the validity of the damage detection algorithms. The displacement and curvature mode shapes obtained from the experiments using the two sensor systems (PZT-SLV and PZT-PVDF) match very well with those of FE simulation. The successful implementation of five damage detection algorithms using the FE data indicates that the damage detection methodologies adopted in this study are capable of locating the delamination in the composite plates.

The combined static/dynamic loading technique was applied to the clamped-clamped composite plate with intention to enlarge the damage effect through the additional static loading (in this study, the compression load was applied to magnify the effect of the delamination). The first three measured displacement (from PZT-SLV) or curvature (from PZT-PVDF) mode shapes are used to produce the ULSs. The
performance of the proposed static/dynamic technique was evaluated using the damage
detection algorithms of GSM, GFD, and SEM. The results show that as the
compression load increases, the delamination is much easier to be identified through the
enlarged damage parameters. Thus, the combine static/dynamic technique seems to be
viable to quantify the relative magnitude of the damage. On the other hand, by
comparing the damage detection results of composite plate under two different boundary
conditions, it is interesting to note that the delamination in the clamped-clamped
composite plate is easier to be detected than the one in the cantilever plate.
5.1 Introduction

Results of the experimental modal analyses and the damage detection algorithms of the aluminum specimens are presented in this chapter. The same approaches and procedures as in Chapter 4 are adopted. The aluminum plate has a saw-cut crack through the thickness. Both the experimental and numerical modal analyses are applied, and the mode shape data are further processed with the proposed damage detection algorithms introduced in Chapter 2.

5.2 Experimental Set-up

Two aluminum plates were tested using the aforementioned measurement systems (i.e., the PZT-SLV and PZT-PVDF). One plate was undamaged, and the other was damaged by saw-cutting a 7.62 cm (3 in) long crack along the longitudinal direction through the plate thickness (see Figure 5.1).
Figure 5.1  Illustration of the healthy and damaged aluminum plate specimens
Similar to the composite plate, two different experimental methods were employed, i.e., acquiring the displacement mode shapes using the PZT-SLV system and the curvature mode shapes using the PZT-PVDF system. A linear sweep sine excitation signal from a signal generator was run through the PZT ceramic actuator, and the parameters of the sweep sine waves are shown in Table 5.1. For the PZT-SLV system, the plate was divided into the measurement grids which are about 0.838 cm (0.33 in) × 0.838 cm (0.33 in); while for the PZT-PVDF system, the plate sample was divided into the grids about 2.54 cm (1.0 in) × 2.54 cm (1.0 in).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>PZT-SLV</th>
<th>PZT-PVDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (Hz)</td>
<td>1~1000</td>
<td>1~700</td>
</tr>
<tr>
<td>Time (s)</td>
<td>6</td>
<td>60</td>
</tr>
</tbody>
</table>

5.3. Experiment of a Cantilever Aluminum Plate

The experiment was conducted first for the healthy aluminum plate, followed by the damaged aluminum plate with a longitudinal saw-cut crack. The two experiments (for the healthy plate and the damaged one) were conducted under the same methods and conditions.

When the plate was clamped in the cantilever configuration, the specimen had a free span length of 50.8 cm (20 in). A piece of 1.02 cm (0.4 in) × 1.52 cm (0.6 in) PZT ceramic patch was attached at the bottom of the plate as an actuator.
5.3.1 Experimental and Numerical Results

The results of the frequency measurements and mode shapes from the experimental measurements and the numerical analysis are presented in this section. The first five natural frequencies of the damaged aluminum plate obtained from PZT-SLV system, PZT-PVDF system, and finite element (FE) simulation are compared with the undamaged one, which are presented in Table 5.2. It can be observed clearly that the three methods almost retain the same natural frequencies and the differences between the healthy plate and the damaged one are very small with the maximum difference percentage of about 1.6%. Thus, the frequency alone is not capable of detecting the damage.

The mode shapes of both the healthy and damaged plates are shown in Tables 5.3 and 5.4, respectively. Due to noise interference produced in the low frequency range (i.e., the 1st and 2nd), the three consecutive mode shapes from 3rd to 5th are plotted and thus used in this study.

Table 5.2 Natural frequencies of the cantilever aluminum plates

<table>
<thead>
<tr>
<th>Mode</th>
<th>FEA</th>
<th>SLV</th>
<th>PVDF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Healthy (Hz)</td>
<td>Damaged (Hz)</td>
<td>Healthy (Hz)</td>
</tr>
<tr>
<td>1</td>
<td>8.16</td>
<td>8.06</td>
<td>7.53</td>
</tr>
<tr>
<td>2</td>
<td>29.83</td>
<td>30.12</td>
<td>29.52</td>
</tr>
<tr>
<td>3</td>
<td>50.74</td>
<td>49.32</td>
<td>47.12</td>
</tr>
<tr>
<td>4</td>
<td>105.56</td>
<td>105.25</td>
<td>101.42</td>
</tr>
<tr>
<td>5</td>
<td>142.58</td>
<td>140.95</td>
<td>138.34</td>
</tr>
</tbody>
</table>
Table 5.3  Mode shapes of the cantilever healthy aluminum plates

<table>
<thead>
<tr>
<th>Mode</th>
<th>FE (displacement)</th>
<th>PZT-SLV (displacement)</th>
<th>PZT-PVDF (curvature)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td><img src="image1.png" alt="Image of mode 3" /></td>
<td><img src="image2.png" alt="Image of mode 3" /></td>
<td><img src="image3.png" alt="Image of mode 3" /></td>
</tr>
<tr>
<td>4</td>
<td><img src="image4.png" alt="Image of mode 4" /></td>
<td><img src="image5.png" alt="Image of mode 4" /></td>
<td><img src="image6.png" alt="Image of mode 4" /></td>
</tr>
<tr>
<td>5</td>
<td><img src="image7.png" alt="Image of mode 5" /></td>
<td><img src="image8.png" alt="Image of mode 5" /></td>
<td><img src="image9.png" alt="Image of mode 5" /></td>
</tr>
</tbody>
</table>
Table 5.4  Mode shapes of the cantilever damaged aluminum plates

<table>
<thead>
<tr>
<th>Mode</th>
<th>FE (displacement)</th>
<th>PZT-SLV (displacement)</th>
<th>PZT-PVDF (curvature)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td>4</td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
<tr>
<td>5</td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
</tr>
</tbody>
</table>

5.3.2 Damage Detection Using the PZT-SLV System

Based on the displacement mode shapes obtained from the PZT-SLV system, the Savitzky-Golay filter was applied to filter the experimental data, and the central difference approximate method was then used to calculate the longitudinal ($\kappa_x$) and transverse ($\kappa_y$) curvature mode shapes. $\kappa_x$ and $\kappa_y$ of both the healthy and damaged plates are listed in Tables 5.5 and 5.6.
Table 5.5  Curvature mode shapes of the healthy aluminum plate (PZT-SLV)

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\kappa_x$</th>
<th>$\kappa_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>![Mode 3 Diagram]</td>
<td>![Mode 3 Diagram]</td>
</tr>
<tr>
<td>4</td>
<td>![Mode 4 Diagram]</td>
<td>![Mode 4 Diagram]</td>
</tr>
<tr>
<td>5</td>
<td>![Mode 5 Diagram]</td>
<td>![Mode 5 Diagram]</td>
</tr>
</tbody>
</table>
Then, the damage detection algorithms (i.e., ADM, DIM, GSM, GFD, and SEM) are applied to the curvature mode shapes to locate the crack damage in the plate.

5.3.2.1 Damage Detection Based on the Longitudinal Curvature Mode Shapes ($\kappa_x$)

The longitudinal curvature mode shapes of the cantilever composite plate are processed using ADM, DIM, GSM, GFD, and SEM. As shown in Figures 5.2 ~ 5.18,
the results indicate that all the algorithms of ADM, DIM, GSM, GFD, and SEM fail to detect the damage location when the longitudinal curvature mode shapes and ULS are used. Because the saw-cut is along the longitudinal direction of the plate, the longitudinal bending (3rd and 5th) and twisting dominated (4th) modes hardly show any changes due to this type of damage.

*Absolute difference method (ADM)*

![Figure 5.2 Results of the ADM using the 3rd mode shape ($\xi_3$)](image)

**Figure 5.2** Results of the ADM using the 3rd mode shape ($\xi_3$)
Figure 5.3  Results of the ADM using the 4\textsuperscript{th} mode shape ($\kappa_x$)

Figure 5.4  Results of the ADM using the 5\textsuperscript{th} mode shape ($\kappa_x$)
Figure 5.5  Results of the ADM using the ULS ($\kappa_1$)

*Damage index method (DIM)*

Figure 5.6  Results of the DIM ($\kappa_3$)
**Simplified gapped smoothing method (GSM)**

![Graph 5.7](image1.png)

Figure 5.7  Results of the GSM using the 3\textsuperscript{rd} mode shape ($\kappa_3$)

![Graph 5.8](image2.png)

Figure 5.8  Results of the GSM using the 4\textsuperscript{th} mode shape ($\kappa_4$)
Figure 5.9  Results of the GSM using the 5th mode shape ($\kappa_5$)

Figure 5.10  Results of the GSM using the ULS ($\kappa_c$)
Generalized fractal dimension (GFD)

Figure 5.11  Results of the GFD using the 3\textsuperscript{rd} mode shape ($\kappa_3$)

Figure 5.12  Results of the GFD using the 4\textsuperscript{th} mode shape ($\kappa_4$)
Figure 5.13  Results of the GFD using the $5^{th}$ mode shape ($\kappa_x$)

Figure 5.14  Results of the GFD using the ULS ($\kappa_x$)
Strain energy method (SEM)

Figure 5.15  Results of the SEM using the 3rd mode shape ($\kappa_3$)

Figure 5.16  Results of the SEM using the 4th mode shape ($\kappa_4$)
Figure 5.17  Results of the SEM using the 5\textsuperscript{th} mode shape ($\kappa_x$)

Figure 5.18  Results of the SEM using the ULS ($\kappa_x$)
5.3.2.2 Damage Detection Based on the Transverse Curvature Mode Shapes ($\kappa_y$)

Similar to the results of the longitudinal curvature mode shape, when the transverse curvature mode shapes of the cantilever aluminum plate are applied, all the algorithms are not capable of detecting the damage location (see Figure 5.19–5.35). Again, the inability of the transverse curvature mode shapes to detect the longitudinal crack is due to the selected modes which are insensitive to the damage.

*Absolute difference method (ADM)*

![Figure 5.19](image)  
Results of the ADM using the 3rd mode shape ($\kappa_y$)
Figure 5.20  Results of the ADM using the 4th mode shape ($\kappa_y$)

Figure 5.21  Results of the ADM using the 5th mode shape ($\kappa_y$)
Figure 5.22 Results of the ADM using the ULS ($\kappa_\gamma$)

*Damage index method (DIM)*

Figure 5.23 Results of the DIM ($\kappa_\gamma$)
Simplified gapped smoothing method (GSM)

Figure 5.24 Results of the GSM using the 3\textsuperscript{rd} mode shape ($\kappa_y$)

Figure 5.25 Results of the GSM using the 4\textsuperscript{th} mode shape ($\kappa_z$)
Figure 5.26  Results of the GSM using the $5^{\text{th}}$ mode shape ($\kappa_y$)

Figure 5.27  Results of the GSM using the ULS ($\kappa_y$)
Generalized fractal dimension (GFD)

Figure 5.28  Results of the GFD using the 3rd mode shape ($\kappa_3$)

Figure 5.29  Results of the GFD using the 4th mode shape ($\kappa_4$)
Figure 5.30  Results of the GFD using the 5th mode shape (κ_y)

Figure 5.31  Results of the GFD using the ULS (κ_y)
Strain energy method (SEM)

Figure 5.32  Results of the SEM using the 3rd mode shape ($\kappa_3$)

Figure 5.33  Results of the SEM using the 4th mode shape ($\kappa_4$)
Figure 5.34  Results of the SEM using the $5^{th}$ mode shape ($\kappa_y$)

Figure 5.35  Results of the SEM using the ULS ($\kappa_y$)
5.3.3 Damage Detection Using the PZT-PVDF System

By using the PZT-PVDF system, the curvature mode shapes are directly obtained. After the experimental data are filtered using the Savitzky-Golay filter, the damage detection algorithms (i.e., ADM, DIM, GSM, GFD, and SEM) are applied to the filtered curvature mode shapes to locate the damage. The same conclusions as the PZT-SLV system are obtained for the PZT-PVDF system, and all the algorithms again fail to detect the saw-cut type damage (see Figures 5.36–5.52). Again, the insensitivity of the longitudinal curvature mode shapes to damage detection may be caused by the modes selected (i.e., modes 3 to 5 which are longitudinal dominated modes, and they may be sensitive to the transverse saw-cut crack).

*Absolute difference method (ADM)*

![Figure 5.36](image.png)  
**Figure 5.36** Results of the ADM using the 3rd mode shape ($\kappa_x$)
Figure 5.37  Results of the ADM using the 4\textsuperscript{th} mode shape (\(\kappa_4\))

Figure 5.38  Results of the ADM using the 5\textsuperscript{th} mode shape (\(\kappa_5\))
Figure 5.39  Results of the ADM using the ULS ($\kappa_x$)

*Damage index method (DIM)*

Figure 5.40  Results of the DIM ($\kappa_x$)
Simplified gapped smoothing method (GSM)

Figure 5.41 Results of the GSM using the 3\textsuperscript{rd} mode shape ($\kappa_3$)

Figure 5.42 Results of the GSM using the 4\textsuperscript{th} mode shape ($\kappa_4$)
Figure 5.43  Results of the GSM using the 5th mode shape ($\kappa_c$)

Figure 5.44  Results of the GSM using the ULS ($\kappa_c$)
Generalized fractal dimension (GFD)

Figure 5.45 Results of the GFD using the 3\textsuperscript{rd} mode shape ($\kappa_3$)

Figure 5.46 Results of the GFD using the 4\textsuperscript{th} mode shape ($\kappa_4$)
Figure 5.47  Results of the GFD using the 5\textsuperscript{th} mode shape ($\kappa_5$)

Figure 5.48  Results of the GFD using the ULS ($\kappa_x$)

170
Strain energy method (SEM)

Figure 5.49  Results of the SEM using the 3rd mode shape ($\kappa_3$)

Figure 5.50  Results of the SEM using the 4th mode shape ($\kappa_4$)
Figure 5.51  Results of the SEM using the 5th mode shape ($\kappa_5$)

Figure 5.52  Results of the SEM using the ULS ($\kappa_i$)
5.3.4 Damage Detection Using the Transverse Bending Dominated Mode Shapes

As shown in the above, all the algorithms fail to exclusively locate the longitudinal saw-cut damage by either using the individual lower mode shape (i.e., the 3rd, 4th, and 5th mode shapes) or using the ULS. No obvious peaks can be prompted around the damage location. Therefore, the transverse bending dominated mode shapes (i.e., the 6th and 12th mode shapes) which are sensitive to the longitudinal crack were examined. The promise of selecting the 6th and 12th modes is that they are transverse bending dominated and very sensitive to the longitudinal crack.

The natural frequencies and mode shapes of both the healthy and damaged plates are shown in Tables 5.7~5.9. As shown in Table 5.7, the frequencies of the healthy and damage plates exhibit apparent difference.

Table 5.7 Natural frequencies of the cantilever aluminum plates

<table>
<thead>
<tr>
<th>Mode</th>
<th>FE Healthy (Hz)</th>
<th>Damaged (Hz)</th>
<th>PZT-SLV Healthy (Hz)</th>
<th>Damaged (Hz)</th>
<th>PZT-PVDF Healthy (Hz)</th>
<th>Damaged (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>160.87</td>
<td>152.48</td>
<td>161.51</td>
<td>154.75</td>
<td>161.56</td>
<td>153.28</td>
</tr>
<tr>
<td>12</td>
<td>424.26</td>
<td>408.25</td>
<td>418.10</td>
<td>405.19</td>
<td>417.26</td>
<td>406.75</td>
</tr>
</tbody>
</table>
Table 5.8  Mode shapes of the cantilever healthy aluminum plate

<table>
<thead>
<tr>
<th>Mode</th>
<th>FE (displacement)</th>
<th>PZT-SLV (displacement)</th>
<th>PZT-PVDF (curvature)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
<td><img src="image3" alt="Image" /></td>
</tr>
<tr>
<td>12</td>
<td><img src="image4" alt="Image" /></td>
<td><img src="image5" alt="Image" /></td>
<td><img src="image6" alt="Image" /></td>
</tr>
</tbody>
</table>

Table 5.9  Mode shapes of the cantilever damaged aluminum plate

<table>
<thead>
<tr>
<th>Mode</th>
<th>FE (displacement)</th>
<th>PZT-SLV (displacement)</th>
<th>PZT-PVDF (curvature)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td><img src="image7" alt="Image" /></td>
<td><img src="image8" alt="Image" /></td>
<td><img src="image9" alt="Image" /></td>
</tr>
<tr>
<td>12</td>
<td><img src="image10" alt="Image" /></td>
<td><img src="image11" alt="Image" /></td>
<td><img src="image12" alt="Image" /></td>
</tr>
</tbody>
</table>
The displacement mode shapes obtained from the experiments of the both damaged and healthy composite plates using the PZT-SLV system match very well with those of finite element (FE) simulations. The curvature mode shapes computed by central difference approximation method are given in Tables 5.10 and 5.11.

Table 5.10 Curvature mode shapes of the cantilever healthy aluminum plate

(PZT-SLV)

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\kappa_x$</th>
<th>$\kappa_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>12</td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Table 5.11  Curvature mode shapes of the cantilever damaged aluminum plate

(PZT-SLV)

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\kappa_x$</th>
<th>$\kappa_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td><img src="image1.png" alt="3D plot" /></td>
<td><img src="image2.png" alt="3D plot" /></td>
</tr>
<tr>
<td>12</td>
<td><img src="image3.png" alt="3D plot" /></td>
<td><img src="image4.png" alt="3D plot" /></td>
</tr>
</tbody>
</table>

The damage detection algorithms of GSM, GFD, and SEM are applied to the obtained curvature mode shapes for detecting the damage location. The results based on the longitudinal and transverse curvature mode shapes from the PZT-SLV measurement are list in Tables 5.12~5.15; while the results based on the PZT-PVDF system are shown in Tables 5.16 and 5.17. The actual damage location is $L = 9.6\sim17.2$ cm, $D = 20.1\sim20.2$ cm. As shown in Tables 5.13, 5.15, and 5.17, both the curvature mode shapes of 6 and 12 obtained from the PZT-SLV and PZT-PVDF systems are relatively accurate to locate the longitudinal crack in the aluminum plate.
Table 5.12  Damage detection using the longitudinal curvature mode shapes (PZT-SLV)

<table>
<thead>
<tr>
<th>Mode</th>
<th>GSM</th>
<th>GFD</th>
<th>SEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Mode 6: 9.6~17.2 cm (L)</td>
<td>Mode 6: 9~19 cm (L)</td>
<td>Mode 6: 9~19 cm (L)</td>
</tr>
<tr>
<td></td>
<td>20.1~20.2 cm (W)</td>
<td>18~20 cm (L)</td>
<td>18~20 cm (L)</td>
</tr>
<tr>
<td>12</td>
<td>Mode 12: 9.6~17.2 cm (L)</td>
<td>Mode 12: 9~18 cm (L)</td>
<td>Mode 12: 6~19 cm (L)</td>
</tr>
<tr>
<td></td>
<td>20.1~20.2 cm (W)</td>
<td>18~20 cm (W)</td>
<td>18~22 cm (W)</td>
</tr>
</tbody>
</table>

Table 5.13  Estimated the damage location using the longitudinal curvature mode shapes (PZT-SLV)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Actual</th>
<th>GSM</th>
<th>GFD</th>
<th>SEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>9.6~17.2 cm (L)</td>
<td>8~20 cm (L)</td>
<td>9~19 cm (L)</td>
<td>8~20 cm (L)</td>
</tr>
<tr>
<td></td>
<td>20.1~20.2 cm (W)</td>
<td>18~20 cm (W)</td>
<td>18~21 cm (W)</td>
<td>17~22 cm (W)</td>
</tr>
<tr>
<td>12</td>
<td>9.6~17.2 cm (L)</td>
<td>9~18 cm (L)</td>
<td>6~19 cm (L)</td>
<td>9~22 cm (L)</td>
</tr>
<tr>
<td></td>
<td>20.1~20.2 cm (W)</td>
<td>18~20 cm (W)</td>
<td>18~22 cm (W)</td>
<td>17~22 cm (W)</td>
</tr>
</tbody>
</table>
Table 5.14  Damage detection using the transverse curvature mode shapes (PZT-SLV)

<table>
<thead>
<tr>
<th>Mode</th>
<th>GSM</th>
<th>GFD</th>
<th>SEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>9.6~17.2 cm (L)</td>
<td>11~20 cm (L)</td>
<td>12~18 cm (L)</td>
</tr>
<tr>
<td></td>
<td>20.1~20.2 cm (W)</td>
<td>19~21 cm (W)</td>
<td>10~19 cm (L)</td>
</tr>
<tr>
<td>12</td>
<td>9.6~17.2 cm (L)</td>
<td>11~18 cm (L)</td>
<td>12~18 cm (L)</td>
</tr>
<tr>
<td></td>
<td>20.1~20.2 cm (W)</td>
<td>19~21 cm (W)</td>
<td>11~19 cm (L)</td>
</tr>
</tbody>
</table>

Table 5.15  Estimated the damage location using the transverse curvature mode shapes (PZT-SLV)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Actual</th>
<th>GSM</th>
<th>GFD</th>
<th>SEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>9.6~17.2 cm (L)</td>
<td>11~20 cm (L)</td>
<td>12~18 cm (L)</td>
<td>10~19 cm (L)</td>
</tr>
<tr>
<td></td>
<td>20.1~20.2 cm (W)</td>
<td>19~21 cm (W)</td>
<td>19~21 cm (W)</td>
<td>19~21 cm (W)</td>
</tr>
<tr>
<td>12</td>
<td>9.6~17.2 cm (L)</td>
<td>11~18 cm (L)</td>
<td>12~18 cm (L)</td>
<td>11~19 cm (L)</td>
</tr>
<tr>
<td></td>
<td>20.1~20.2 cm (W)</td>
<td>19~21 cm (W)</td>
<td>19~21 cm (W)</td>
<td>18~21 cm (W)</td>
</tr>
</tbody>
</table>
Table 5.16 Damage detection using the longitudinal curvature mode shapes (PZT-PVDF)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Mode 6</th>
<th>Mode 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSM</td>
<td><img src="image1" alt="GSM Mode 6" /></td>
<td><img src="image2" alt="GSM Mode 12" /></td>
</tr>
<tr>
<td>GFD</td>
<td><img src="image3" alt="GFD Mode 6" /></td>
<td><img src="image4" alt="GFD Mode 12" /></td>
</tr>
<tr>
<td>SEM</td>
<td><img src="image5" alt="SEM Mode 6" /></td>
<td><img src="image6" alt="SEM Mode 12" /></td>
</tr>
</tbody>
</table>

Table 5.17 Estimated the damage location using the longitudinal curvature mode shapes (PZT-PVDF)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Actual</th>
<th>GSM</th>
<th>GFD</th>
<th>SEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>9.6~17.2 cm (L)</td>
<td>10~24 cm (L)</td>
<td>10~20 cm (L)</td>
<td>10~18 cm (L)</td>
</tr>
<tr>
<td></td>
<td>20.1~20.2 cm (W)</td>
<td>16~22 cm (W)</td>
<td>17~26 cm (W)</td>
<td>19~23 cm (W)</td>
</tr>
<tr>
<td>12</td>
<td>9.6~17.2 cm (L)</td>
<td>7~19 cm (L)</td>
<td>8~19 cm (L)</td>
<td>9~20 cm (L)</td>
</tr>
<tr>
<td></td>
<td>20.1~20.2 cm (W)</td>
<td>17~25 cm (W)</td>
<td>16~25 cm (W)</td>
<td>17~26 cm (W)</td>
</tr>
</tbody>
</table>
5.4. Concluding Remarks

When the lower mode shapes are used to detect the longitudinal saw-cut crack in the cantilever aluminum plate, all the methods fail to locate the damage area (either using the individual lower mode shape (i.e., the 3rd, 4th, and 5th mode shapes) or using ULS). The inability of the lower mode shapes to locate the damage is due to the longitudinal bending dominated nature of the selected modes and their insensitivity to the longitudinal crack. But when the transverse bending dominated mode shapes (i.e., the 6th, and 12th mode shapes of 6, and 12) were used, the damage parameters based on GSM, GFD, and SEM display a significant peak around the damage location. Thus, for a particular type of damage (e.g., the longitudinal crack), it may be imperative to use the dominated vibration response (e.g., transverse bending) perpendicular to the direction of crack.
6.1 Concluding Remarks

The present study focuses on the identification of the presence and location of damage in the plate-type specimens by extracting the modal parameters obtained from the results of the modal analysis on the test samples. The experimental program comprises of testing one E-glass/epoxy composite laminated plate with an embedded delamination and one aluminum plate with crack-type damage using two different actuator-sensor measurement systems, i.e., (1) PZT-PVDF: Polyvinylidenefluoride (PVDF) sensors and PZT (lead-zirconate-titanate) actuators, and (2) PZT-SLV: Scanning Laser Vibrometer (SLV) and PZT actuators. The numerical finite element (FE) analysis is also performed to complement the damage detection and verify the validity of the damage detection algorithms. Several different damage detection algorithms (i.e., Absolute Difference Method (ADM), Damage Index Method (DIM), Uniform Load Surface (ULS), Gapped Smoothing Method (GSM), Strain Energy Method (SEM) and Generalized Fractal Dimension (GFD)) are employed to analyze the experimental and numerical data. Out of the above algorithms, the GSM, SEM and GFD are newly developed, and they are
implemented using the ULS curvatures with no need of the healthy plate information as reference data. A combined static-dynamic approach is further proposed to magnify the effect of delamination in the composite plate. The conclusions arrived as an outcome of the study can be summarized as follows.

6.1.1 PZT-PVDF System vs. PZT-SLV System

The observation of the results indicates that the experimental approach using both the measurement systems proved to be successful in extracting the modal parameters (natural frequencies and mode shapes) and thus, helped in determining the existence and location of damage. Compared to the numerical finite element (FE) simulation, the PZT-SLV seems to provide better results than the PZT-PVDF system, due to refined scanning mesh used in the PZT-SLV system. The measurement from the PZT-PVDF system is successful in generating the curvature mode shapes directly, and it is capable of real time, on-board monitoring. On the other hand, the non-contact SLV system enjoy the advantages like simplicity in operation, ability to account for large number of nodes and no wiring management problems.

6.1.2 Modal Analysis – Displacement vs. Curvature Mode Shapes

The modal analysis is conducted, and the responses are acquired in the form of modal parameters, i.e., natural frequencies and mode shapes. The PZT-SLV system provides the output in the form of displacement mode shapes, from which the curvature mode shapes are calculated using the central difference method; while the PZT-PVDF system directly yields the curvature mode shapes. The results indicate that, unlike the
displacements, the second derivative (curvatures) of the displacements locate the damage more effectively and identify the presence and location of damage.

6.1.3 ULS Curvature Mode Shapes

The Uniform Load Surface (ULS) curvature based on the principle of flexibility indicates high sensitivity to damage and is found to be robust to the data truncation effects and measurement error. Hence, it is proved to be effective in detecting the damage. The ULS curvatures are further employed as the input data for the damage detection algorithms, like GSM, GFD and SEM.

6.1.4 Damage Detection Algorithms

Several damage detection algorithms are employed to the data extracted from the testing of the beam specimens. The Absolute Difference Method (ADM) and Damage Index Method (DIM) require both the healthy and damaged data information for detecting the damage. The relatively new damage detection schemes like Gapped Smoothing Method (GSM), Strain Energy Method (SEM) and Generalized Fractal Dimensional (GFD) method that utilized the Uniform Load Surface (ULS) curvatures for damage detection are proved to be effective means of efficient and accurate detection of the damage in the structure. GSM, GFD and SEM methods also enjoy the advantage of not requiring the data from the healthy beam to identify the damage, and hence have great potential to be used as an online health monitoring tool. The GFD method although shows sharp peaks identifying the damage location, and it also depicts extra peaks at other locations along the length of the beam. The GSM and SEM identify the presence
and location of damage, but they also depict several small peaks at locations other than where the actual damage is located.

6.1.5 Damage types

Two types of damage are evaluated in this study: delamination in composite laminated plate and crack type damage in aluminum plate by saw-cut. The damage detection algorithm seems more viable in detecting delamination in composite structures. Specific modes which are perpendicular to the saw-cut crack damage (e.g., the transverse bending dominated mode to the longitudinal saw-cut damage) are needed to be able to detect the presence of the crack damage in the aluminum plate.

6.1.6 Static-dynamic Approach

The static-dynamic modal analysis on the composite laminated plate specimens is carried out with the intention to magnify the effect of the damage thereby aiding in the process of damage identification. The results from the testing indicate that the new approach is fairly successful in intensifying the damage. However, it is also noticed that the method could be more effectively employed for magnifying damage types like delamination, where the application of the compressive loads results in the widening of the gap between the two sub-laminate layers. The relatively new damage detection approaches like GSM, GFD and SEM are employed to the static-dynamic technique to quantify the extent of magnification under the influence of the compressive loads. The results indicate that the magnification of damage increases with increase in compressive loads.
6.1.7 Boundary Conditions

The results of delamination detection in the composite plates also show the significant effect of boundary conditions on the damage evaluation outcomes. It seems that the delamination is much easier to be detected in the clamped-clamped boundary condition than in the cantilever one. The displacement mode shapes of the clamped-clamped composite plate directly obtained from the PZT-SLV system and numerical FE simulations are used in the damage detection algorithms, and they are effective of identifying the location of the delamination when no compression load is applied; while in the cantilever case, only the curvature mode shapes obtained from the second derivative of the displacement are effective in damage detection. Also, the first three lower modes (i.e., 1\textsuperscript{st} to 3\textsuperscript{rd}) of the clamped-clamped composite plates are able to be generated with less interference of noise and measurement error, as compared to the cantilever case, of which the three consecutive mode shapes from the 3\textsuperscript{rd} mode are valid to be used.

6.2 Recommendations for Future Research

In this study, an exploratory study of two measurement systems for damage detection of plate-type structures is conducted. Some observations are made during carrying out this study, and the further research should be conducted to refine/improve the proposed damage detection methodology. The following recommendations are provided for future research considerations:

- More research on different types of damage at the different locations using the proposed damage detection algorithms and sensor systems should be conducted;
• Repetition and averaging of the tests may provide better results. Thus, more repeated experiments should be carried out, and averaging the test results may produce better outcomes for some cases, not currently working well based on the existing tests and data.

• Further employment of numerical FE simulation may simplify experimental study and help guide the placement of sensors to achieve optimal dynamic response;

• Only limited tests are conducted using the combined static/dynamic experimental techniques. More study on the static/dynamic tests should be considered, from which a damage quantification method may be developed.
BIBLIOGRAPHY


