A GOAL-SEEKING APPROACH TO COORDINATING THE DISCHARGE OF
A COLLECTION OF BATTERIES

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A GOAL-SEEKING APPROACH TO COORDINATING THE DISCHARGE OF
A COLLECTION OF BATTERIES

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Thesis

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The problem of accurately predicting the state-of-charge of a battery, and hence its lifetime - which is the duration of time over which a battery can supply energy to a load, has remained elusive. Because many modern applications are powered by a collection of batteries, new methods reported in the recent literature aim to extend the lifetime of a collection of batteries by using redundant batteries and scheduling a subset of the batteries for discharge.

To compare the effectiveness of different strategies for scheduling batteries, we defined a new metric called discharge efficiency. We show that both the lifetime and the discharge efficiency can be improved by using redundant batteries and scheduling strategies. To cope with the uncertainties that come to pass during the discharge process, we formulated the problem of coordinating the discharge of a collection of batteries in the goal-seeking paradigm. This formulation allowed us to design an effective decision-maker to coordinate the discharge. The results in this thesis show that the decision-makers based on the goal-seeking paradigm improve lifetime of a collection of batteries between 12.7% (when the collection supplies constant power to a load) and 21.6% (when the collection supplies constant current to the load). Similarly depending on the discharge scenario, the discharge efficiency is improved by 11.7% to 13.4%. In addition, the new decision-maker utilizes a small number of redundant batteries more effectively than the scheduling methods reported in the literature.
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<td>cap</td>
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<tr>
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<td>Battery Module</td>
</tr>
<tr>
<td>$c_d$</td>
<td>Diffusing Charge</td>
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A battery is an important source of energy that is used in a broad spectrum of applications from hybrid-electric vehicles to communication systems and inter-planetary space missions. At the surface, the problem of discharging a battery appears to be trivial – one connects a load and the battery supplies energy to the load by converting its stored chemical energy to electrical energy. Beneath such a surfaced view there is a collection of electro-chemical processes that interact in ways that are not well understood and cause the dynamic behavior of each battery to be unique.

Many applications utilize a collection of batteries that are often connected in series. In some situations, there may be redundant batteries or replacement batteries. For example, the International Space Station uses two sets of batteries. The problem of discharging a collection of batteries is complicated because of two factors. First, because the behavior of each battery is unique, it is difficult to estimate the duration of time over which a particular battery would discharge [11, 12]. Further, if the batteries are discharged without care or control, the lifetime over which the collection of batteries could have supplied useful energy to the load is reduced [11, 12, 18, 20]. These two factors motivate this investigation into methods to coordinate the discharge of a collection of batteries.
The objectives for coordinating the discharge are to extend the lifetime of the collection of batteries and to ensure that when the batteries can no longer meet the requirements of the load, each battery is completely discharged, i.e., all the energy it could have supplied to the load is actually supplied.

The benefit of coordinating the discharge of a collection of batteries can be illustrated with a simple example. Suppose there are 12 batteries available, each battery having a capacity of 0.86 Ah, and each battery can supply a nominal voltage of 1.35 V. Assume that these batteries are connected in series to a load that requires a minimum of 14 V. If these batteries are discharged by drawing a constant current of about 250 mA, then one expects that the batteries would be able to supply energy to the load for about 190 minutes. Now consider the situation that there are eight additional (spare) batteries available. The most common method of using spare batteries is to wait until one of the batteries is completely discharged; at that time, one of the spare batteries is used to replace the completely discharged battery. This method would be no use in the above scenario – because when one of the batteries is completely discharged, it is likely that there are other batteries that are discharged. If the number of spares is not sufficient to replace all the batteries that are completely discharged, then the spares are not useful to supply energy to the load. By coordinating the discharge of the collection of batteries, one can extend the lifetime of the batteries significantly. The results in this thesis show that by using eight spare batteries, and an efficient-decision maker that schedules batteries for discharge, and for rest, the collection of batteries, i.e., 12 batteries plus the eight spare batteries, can supply energy to the load for about 450 minutes.
1.1 Contribution of Thesis

The main contributions of this thesis are:

1. A goal-seeking formulation of the problem of coordinating the discharge of a collection of batteries.
2. A new decision-maker that is based on the goal-seeking formulation.
3. Simulation results that demonstrate the effectiveness of the new decision-maker in comparison with decision-makers that are based on selection strategies that were reported recently in the literature.

The goal-seeking formulation is a new approach to coordinating discharge because it explicitly represents the reasons about uncertainties that affect the discharge process. This formulation allowed us to use simple models of a battery and simple decision-making algorithms to achieve the coordination. The results demonstrate that the decision-maker based on the goal-seeking formulation consistently extends the lifetime of the collection of batteries by over 10% and ensures that the collection of batteries is discharged uniformly.

1.2 Outline of Thesis

Chapter 2 presents basic information about batteries and discusses the two important issues of estimating the State of Charge (SOC) of a battery and estimating the Capacity of a battery. Commonly used discharge scenarios, namely constant current discharge, constant power discharge and randomly varying current discharge, are described and their effects on the battery are discussed. After reviewing the literature in battery equalization and battery scheduling, which are two popular approaches to
extending the lifetime of a collection of batteries in the literature, this chapter concludes with a discussion of the computational foundation of a decision-maker that coordinates the discharge of a collection of batteries.

Chapter 3 presents a simple model of a battery that is used in all the simulation studies to support this investigation. Methods for estimating the SOC and capacity of a battery are discussed in detail. After emphasizing the uncertainties in determining or estimating the SOC of a battery this chapter concludes with a discussion emphasizing the limitations of the well-known state-transition paradigm when dealing with uncertainties.

Chapter 4 discusses and presents simulation results that demonstrate the effectiveness of three battery scheduling methods. These methods apply the principle of using redundancy to improve efficiency and are based on simple selection strategies that determine when a given battery should discharge or rest. A metric called discharge-efficiency is defined to allow one to compare the performance of different decision-makers.

In Chapter 5, we develop a formulation for coordinating the discharge of a collection of batteries based on the goal-seeking paradigm. A decision-maker that was designed based on this formulation is described. Because of the importance of estimating SOC and the difficulties associated with getting accurate estimates, this chapter explores four alternative goal-seeking decision-makers – each using a different approach to estimating SOC. Results that demonstrate the performance of these decision-makers are presented and compared to the results in Chapter 4.

Finally, Chapter 6 presents the conclusions of this investigation and suggestions for future work.
CHAPTER II
BACKGROUND

This chapter presents the background information necessary for the subsequent chapters in this Thesis. Section 2.1 discusses the structure of a battery and Section 2.2 discusses the important issue of capacity of a battery and methods and issues in determining or estimating the capacity of a battery. Three scenarios in which batteries are commonly discharged are discussed in Section 2.3. Section 2.4 discusses battery equalization which is the traditional approach used in the battery literature to ensure that individual batteries in a collection are uniformly charged or discharged. New methods for extending the discharge lifetime of a collection of batteries, called battery scheduling, are discussed in Section 2.3. These new methods also achieve a degree of equalization during the discharge process. Section 2.6 concludes this chapter with a brief discussion of decision-makers that would coordinate the discharge of a collection of batteries.

2.1 Structure of Battery

Collections of batteries are routinely used in a broad spectrum of applications that include electric or hybrid-electric vehicles, communication systems, uninterruptible power supplies and space missions. Various battery chemistries have been explored to minimize adverse effects to the operational environments and to meet the growing
demands of modern applications. Some of these chemistries are Nickel Metal-Hydride (NiMH), Nickel-Cadmium (NiCd), Lithium-ion (Li-Ion) and lead-acid.

A battery is comprised of one or more cells. Each cell can store a certain amount of charge which is defined as the cell’s *coulomb capacity* or *ampere-hour* (Ah) rating. When an application requires higher voltage, batteries that have a number of cells are used. Additional batteries may be used to meet the demands of an application. The capacity or amount of charge that a battery (or a collection of batteries) can store and deliver is an important consideration for applications.

### 2.2 Capacity of Battery

The discharge of a single battery is deceptively simple. At the surface, it appears that the problem is trivial – where one would connect a battery to a load and the battery would supply the stored chemical energy as electrical energy to the load. In reality, the electro-chemical processes that cause the transformation of the energy are quite complex and have been the focus of investigation by a large community of researchers [1, 3].

![Battery Voltage Discharge Time](image)

**Figure 2.1 Discharging Time Vs. Discharging Current.**

As is shown in Figure 2.1, the duration over which a battery can supply useful energy to a load is affected by the rate of discharge. A battery with a capacity of 1.87 Ah
would supply more energy if current is drawn from the battery at 0.187 A rather than at 1.87 A. The reason for this variation is that the usable capacity of a battery, which is a measure of the ampere-hours that can be delivered to a load at a specific discharge rate, is different from the theoretical capacity of the battery. Several models have been explored to estimate the usable capacity of a battery with varying degrees of success [4, 10]. One important metric that is commonly derived for usable capacity is the State of Charge (SOC) of a battery.

### 2.2.1 Estimating SOC

The SOC is a measure of the remaining capacity in a battery and can be represented as

\[
SOC = \frac{Q_{\text{present}}}{Q_{\text{max}}} \cdot 100.
\]  

(2.1)

Assuming that the amount of charge drawn from a battery is accurately represented by an ampere-hour counting method, the SOC can be estimated as

\[
SOC = \frac{Q(0) - \int_{0}^{t} i(t) dt}{Q_{\text{max}}} \cdot 100.
\]  

(2.2)

Accurately estimating the SOC of a battery has remained an elusive problem in battery research [4].

The method based on terminal voltage measurement is widely used. This method is applicable when the precision of the estimate is not critical to the application. This method is not relevant, however, for all battery chemistries. For example, Li-Ion batteries have poor correlation between terminal voltage and the SOC. Modern batteries use...
advanced technologies to maintain little change in the terminal voltage while the battery is discharging. Because the terminal voltage is susceptible to load conditions, a more accurate estimate of the SOC can be obtained from an open-circuit voltage measurement.

The open circuit voltage of a battery, in certain chemistries and operating ranges, is linearly proportional to the SOC [10]. Hence an SOC estimate based on an open circuit voltage measurement can be used. Two models to estimate SOC using this approach are discussed in Section 3.3. The SOC estimates in this approach reflect little dependence on temperature and the rate of discharge the battery was subjected to before the measurement [4]. It is, however, necessary to allow the battery to rest for a substantial duration of time to allow the electro-chemical processes to reach equilibrium.

Another method for estimating SOC is based on measuring the specific gravity of the electrolyte. While this method provides an accurate estimate of the SOC, it is not suitable for frequent measurement while a battery is discharging.

The method of ampere-hour counting is based on summing the total charge that is drawn out of the battery [4]. If we start charge calculation from a reference point such as full charge, we can measure the remaining capacity of the battery by integrating the current flowing out of the battery over time. However, this method assumes that the discharging process is 100 % efficient. This is not realistic and the method yields inaccurate estimates of SOC, especially after repeated discharging over long durations of time. Despite its inaccuracies, this method can be combined with a method based on the measurement of the open circuit voltage or the specific gravity to calibrate the calculations because of its convenience and simplicity. Section 3.4 discusses how this
method has been combined with open circuit voltage measurement to improve the accuracy of estimating battery capacity during the discharge process.

2.2.2 Battery Lifetime

The duration of time over which a battery could supply useful energy to a load is called the lifetime of a battery. When an application uses a collection of batteries, the idea of lifetime can be extended to the collection of batteries in the obvious manner. The SOC of a battery is a critical factor that influences the lifetime of a battery. Because of the inherent difficulties in estimating SOC, it is also difficult to estimate the lifetime of a battery (or a collection of batteries).

Other factors that affect the lifetime of a battery are depth-of-discharge, rate of discharge, temperature, shock and vibrations. A factor that is often overlooked is that when one battery in a collection is supplying a disproportionate amount of energy to compensate for some other weak battery, the lifetime of the good batteries are also shortened. This phenomenon is referred to as discharge equalization in the literature and it is known that improper equalization can reduce battery lifetime enormously.

Figure 2.2 shows the voltage profile of a NiMH battery through its lifetime when it is being discharged by drawing current at a constant rate. This figure shows that the voltage exhibits a sharp “knee” at the end of discharge where the voltage drops quickly. Given a constant current discharge, the sharp knee in voltage makes it possible to detect a battery that is about to become completely discharged.
2.3 Battery Discharge Techniques

Four common scenarios in which batteries are discharged are constant current, randomly varying current, constant power and pulse discharging [4]. Figure 2.3 shows the relationships between voltage, current and time in each of these scenarios.

![Figure 2.3 Battery Discharging Scenarios](image)

In Figure 2.3 (a), current is drawn from the battery at a constant rate. This is a common discharging scenario and can be achieved by an constant current source. Figure 2.3 (b) shows a scenario in which the voltage is held constant by using a voltage regulator. When
the load varies, current drawn from the battery will change correspondingly to keep the battery voltage constant. For certain applications, it is important for the batteries to deliver constant power. When the voltage decreases because of changes in battery capacity, the current would be increased to maintain power constant as shown in Figure 2.3 (c). Finally, Figure 2.3(d) shows a discharge scenario where the battery is provided a short duration of rest between two successive discharge periods.

2.4 Equalizing Batteries during Discharge

Whenever a collection of batteries is used to supply power to a load, the non-uniformity of individual batteries causes certain difficulties. Variations in temperature could affect individual batteries differently. Each battery could also be subjected to changes in internal resistance, self-discharge rate and inaccuracies in measuring devices. Finally, it is important to note that it is extremely difficult to predict the onset of damage processes, before the damage actually occurs.

If the batteries are re-chargeable, then further difficulties are introduced because damage process can be initiated when a battery is overcharged. Several techniques have been described in the literature to equalize the charge that is delivered to each battery in a collection during the charge process [5], [10], [11], [12], [13], [14], [18], [20].

The guiding principle in all these equalization methods is to maintain the SOC of each battery in a collection as close as possible to the SOC of the other batteries. The remainder of this section summarizes equalization methods that have been reported in the literature.
2.4.1 Equalization using Resistive Shunts

Consider a collection of batteries connected in series as shown in Figure 2.4(a). There is a bypass resistive shunt in parallel with every battery [5]. The amount of current drawn by the shunt resistance is proportional to the voltage of the battery. Batteries that have higher SOC will tend to dissipate larger amount of power through the resistive shunts. While this approach is useful in a charging scenario, it has little use in a discharging scenario. However, if each resistive shunt can be switched in or switched out, as shown in Figure 2.4(b), the configuration becomes useful in a discharging scenario [12]. During the discharge process, the resistive shunts can be switched on for batteries that have lower SOC than other batteries. This has the effect of reducing the current flowing through the battery and hence the healthy batteries in the collection would be compensating for a weaker battery while allowing the weaker battery to rest.

![Figure 2.4 Dissipative Resistive Shunts for Discharge](image)

2.4.2 Equalization using Flying Capacitor

Instead of dissipating power through resistive shunts, a flying capacitor represents a non-dissipative method in which the objective is to balance the voltage by moving
energy from one battery to another by using active voltage or current converter elements, [16]. A capacitor is connected to the collection of batteries and this capacitor can be charged selectively from one of the batteries in the collection that has a higher voltage. To select the source battery, a switch associated with the battery can be closed. Once the capacitor is charged, it is connected in parallel with a battery that has a lower voltage.

Figure 2.5 Flying Capacitor

While little power is dissipated in this method of equalization, it requires large number of switches. The capacitor must be large and the switching currents must be high. Charge equalization is only effective to the extent that voltage is correlated with SOC.

2.4.3 Equalization using Energy Converters

This method uses inductors and transformers for balancing the voltage of batteries and moves energy from one battery to another. The transformer can be connected to a weak battery in the collection by appropriately selecting switches. While the method is reliable, its complexity and cost restricts its utility in un-tethered applications [12].

13
In addition to the methods of equalization described in this section, other methods reported in the literature include the use of dc-to-dc converters and multi-winding transformers [12, 20].

### 2.5 Scheduling Batteries for Discharge

A new approach to extending the lifetime of a battery is based on scheduling batteries for discharge based on static or dynamic methods [1], [2]. This approach also utilizes switches and controllers to either switch in a battery for discharge or switch out a battery to rest. The scheduling algorithms use resting time and recovery advantages to equalize the battery and increase the energy efficiency. Basically, the controllers identify the weakest batteries in the collection and switch them out to rest while other batteries are discharging to supply power to the load. Chapter 4 presents different scheduling strategies.
2.6 Decision-Makers to Discharge Batteries

Irrespective of the method used to make the discharge process more efficient, current trends in the role of microcontrollers in engineering applications allows one to host a decision-maker on the microcontrollers. Depending on the number of batteries used and the decision-making algorithms used, several strategies may be used to host the decision-makers on microcontrollers [9].

Figure 2.7(a) shows a centralized approach where all the decisions are made by one controller. Figures 2.7(b) and (c) show alternate approaches where multiple controllers are used. The cost of microcontrollers, switches and the demands of the application would guide the choice of one approach over others. We selected the centralized approach in Figure 2.7(a) because of its simplicity.

(a)                                 (b)                                  (c)

Figure 2.7 Hosting Decision-Makers
2.6.1 Execution Architecture of Decision-Maker

The execution architecture in the microcontroller could either be event-triggered or time-triggered. In an event-triggered architecture, the activities of the microcontroller are selected based on the occurrence or non-occurrence of pre-defined events such as estimated SOC of a battery changed by 25%, or terminal voltage of the collection changed by 0.8V. An event-triggered architecture is used when the microcontroller must respond to such events without delay. It is, however, difficult to predict or analyze the behavior of a system that is executing under an event-triggered architecture because the occurrence (or non-occurrence) of events cannot be predicted \textit{a priori}.

In a time-triggered architecture, there is a time-base that guides the activities on the microcontrollers. We chose this approach because it is relatively simple and allowed us to inter-twine the functioning of the decision-maker with the discharge process as discussed in more detail in the next chapter.
CHAPTER III
SYSTEM VIEWS AND MODELS

The ultimate objective for this research was to design an effective decision-maker that could coordinate the discharge of a collection of batteries so as to extend the lifetime over which the collection could supply energy to a load. Since the lifetime of a battery depends on its State of Charge (SOC), we needed methods to estimate the SOC\(^1\) and the capacity of a battery. Because of the complex electro-chemical processes that interact to convert stored chemical energy into electrical energy, the dynamic behavior of each battery is unique. Consequently, there are uncertainties that any decision-maker must cope with during the discharge process. Before designing a new decision-maker that copes with uncertainties, it was necessary to understand why the popular state-transition paradigm is not suitable as a basis for designing the decision-maker.

Section 3.1 presents a systems view of the collection of batteries. This view is used to clarify the measurements and observations that must be made by a decision-maker to extend the lifetime of a collection of batteries that are discharging. Section 3.2 presents a model for Nickel Metal Hydride (NiMH) batteries that is based on the theory reported in [7]. Although the model is simple, it is used as the basis for all the simulation studies in this investigation. The decision-maker discussed in Chapter 5 used

\(^1\) Estimating SOC is known to be among the most difficult problems in battery charging and discharging literature [4].
this model both to select future actions and to assess the effectiveness of selected actions during the discharge process. Section 3.3 presents alternative approaches to estimating SOC and Section 3.4 discuss methods that are necessary to estimate the capacity of a battery during the discharge process. Finally, Section 3.5 discusses the well-known state-transition paradigm and a few limitations of this paradigm, especially to represent and reason about uncertainties.

3.1 Systems Views

To clarify the interactions between a decision-maker that would coordinate the discharge process and the collection of batteries in subsequent chapters, we considered the three views, namely System Level-1, System Level-2, and System Level-3 that are described in detail in the remainder of this section.\(^2\)

3.1.1 System Level-1

A single battery is a basic unit for storing and delivering charge. A battery may physically consist of a single cell or a fixed number of cells depending on the voltage it is expected to deliver. When connected to a load, either individually or as a part of a larger collection, a battery supplies energy to a load as long as it contains stored charge (chemical energy) that can be converted to electrical energy.

\(^2\) It is important to emphasize that all the studies in this research were based on MATLAB simulations and not on experimental systems.
3.1.2 System Level-2

At System Level-2, we considered a configuration as depicted in Figure 3.1. Each battery (which is itself at System Level-1) is associated with a switch that could be used either to switch in a battery to discharge or to switch out a battery to rest. There are a total of $n$ batteries in the collection and depending on the setting of each switch, an individual battery may either be resting or discharging.

![Figure 3.1 System Level-2](image)

In practical applications, it may be more appropriate to substitute each battery shown in Figure 3.1 with a group of batteries that are connected either in series or any other topology. In such a case, the associated switch would switch in or switch out the group of batteries.
3.1.3 System Level-3

At System Level-3 we included a microcontroller in which a decision-maker (software) could execute. The decision-maker determined values for each switch at System Level-2 based on the SOC of the battery, the needs of the load connected to the collection of batteries and the current operating environment. The decision-maker (via interfaces between the microcontroller and the collection of batteries) was assumed to be able to monitor the terminal voltage of the collection at System Level-3, the status of switches at System Level-2 and open circuit voltage at System Level-1.

![Figure 3.2 System Level-3](image-url)
3.2 Model for Batteries

We needed a battery model that captured the dynamic characteristics of real-life batteries. We used an essentialized model for NiMH batteries reported in [7]. This model is based on a linear-in-the-parameters approximate solution to the electrode-equation. The terminal voltage can be expressed as

\[ v = k_1 - k_2 \cdot i + k_3 \cdot c_d + k_4 \cdot \ln \left( \frac{c_s}{cap} \right) - k_5 \cdot \ln \left( \frac{cap - c_s}{cap} \right) \]  \hspace{1cm} (3.1)

where the terminal voltage is \( +v \), the current out of the battery is \(+i\), \( c_s \) is the stored charge, \( c_d \) is the diffusion charge and \( cap \) is the capacity of the battery. Parameters \( k_1, k_2, k_3, k_4, \) and \( k_5 \) are determined empirically by discharging one battery. \( k_1 \) represents the initial open circuit voltage \( (V_{oc}) \) and \( k_2 \) indicates the ohmic resistance \( (R_\Omega) \), measured from voltage drop when battery starts to discharging. Parameters \( k_3, k_4 \) and \( k_5 \) are necessary to accurately model the battery. Consistent with [7], we assumed that each NiMH battery has an average capacity of 0.86 Ah, the parameter values were

\[ v = 1.33 - 0.12 \cdot i + 20 \cdot c_d + 0.015 \cdot \ln \left( \frac{c_s}{0.87} \right) - 0.013 \cdot \ln \left( \frac{0.87 - c_s}{0.87} \right). \]  \hspace{1cm} (3.2)

The stored charge in the Equation 3.2 can be calculated from:

\[ \frac{dc_s(t)}{dt} = -(acc/60) \cdot i(t) - (4.83 \cdot 10^{-6}) \cdot c_s(t) \]  \hspace{1cm} (3.3)

and the diffusing charge is calculated as:

\[ \frac{dc_d(t)}{dt} = -0.35 \cdot c_d(t) - 0.001126 \cdot (acc) \cdot i(t). \]  \hspace{1cm} (3.4)

The charge acceptance, \( acc \), is a parameter that is important when the battery is being charged. Since we only considered the discharge process, we used the value \( acc = 1 \).
To represent variations in the dynamic behavior of individual batteries, we varied the parameters $k_i$ for each battery in the collection by substituting for each of them a random number within 10% of the initial values shown in Equation 3.2. Similarly, to represent the variations in the initial SOC of each battery, we varied the initial capacity of each individual battery by substituting a randomly selected number that was within 32% of 0.86 Ah. Figure 3.3 shows the voltage profile of each battery in a collection of 10 batteries that was discharged by simulating the model in MATLAB.

![Figure 3.3 Voltage Profiles of a Collection of Batteries](image)

3.3 Estimating SOC

Estimating the SOC of a battery is a critical necessity to coordinate the discharge of a collection of batteries. The accuracy of the SOC estimate is limited by uncertainties. The remainder of this section discusses two methods for SOC estimation that are based on measuring open circuit voltage and counting the ampere-hours drawn out of the battery.

---

3 The MATLAB code for simulating the battery is shown in Appendix A.
3.3.1 Open Circuit Voltage Measurement

An indicator of the SOC is open circuit voltage measurement. When a battery is discharging, its open circuit voltage cannot be measured. Even if the battery is allowed to rest, it is necessary to leave the battery at rest for a sufficient duration to ensure that the open circuit voltage measured is accurate. This phenomenon is shown in Figure 3.4 where a single NiMH battery is being simulated to alternate between a discharge and rest assuming that the load connected to the battery draws a constant 250 mA during the discharge period.

We used the simulation result shown in Table 3.1 to develop a relationship between SOC and open circuit voltage. For convenience, the open circuit voltage values that were measured are shown in Table 3.1. Since this experiment was based on a simulation, we also obtained the SOC of each battery in the collection as shown in Table 3.1.
Table 3.1 Open Circuit Voltage and Actual SOC Data from One Battery

<table>
<thead>
<tr>
<th>SOC</th>
<th>85.5227</th>
<th>78.2891</th>
<th>71.0567</th>
<th>63.8252</th>
<th>56.5948</th>
<th>49.3655</th>
<th>42.1372</th>
<th>34.9099</th>
<th>27.6837</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{oc}$</td>
<td>1.3511</td>
<td>1.3450</td>
<td>1.3401</td>
<td>1.3358</td>
<td>1.3318</td>
<td>1.3279</td>
<td>1.3239</td>
<td>1.3196</td>
<td>1.3148</td>
</tr>
</tbody>
</table>

For the data from each battery, we used linear regression to fit the data to an equation of the form $SOC = a \cdot V_{oc} + b$. The values for $a$ and $b$ for each of the 10 batteries were averaged to obtain a model as $a = 1.6616$ and $b = -2.1568$. Figure 3.5 (a) shows the relationship between the open circuit voltage and SOC for a collection of batteries obtained from this model. In the region when SOC is between 20% and 85%, the relationship is approximately linear. The maximum absolute value of error from the above fit was found to be 3.8436%.

![Figure 3.5 Voc vs. SOC Profiles Estimate](image)

To minimize the error in the linear model, we used least squares regression to fit the data to a third degree polynomial to obtain the model

$$SOC = 1.52654399115196 \cdot 10^6 - 3.44811832038858 \cdot 10^3 \cdot V_{oc} + 2.59488037693052 \cdot 10^3 \cdot V_{oc}^2 - 0.65058272525502 \cdot V_{oc}^3$$  \hspace{1cm} (3.6)

Figure 3.5 (b) presents a graph of the function shown in Equation 3.6.
3.3.2 Ampere-Hour Counting

When a battery is discharging, one can estimate the remaining SOC in the battery by counting the ampere-hours that have been drawn from the battery over the period of interest as

\[
C_{\text{rem}} = C_{\text{max}} - \int_0^t idt.
\]  

(3.7)

In principle, number of ampere-hours drawn out of a battery over a discharge period could be used to estimate the SOC remaining in the battery. The problem with this approach, however, is that the initial SOC of the battery is uncertain and difficult to predict. To compensate for this difficulty, we used a method for estimating SOC that used ampere-hour counting when batteries were discharging and open circuit voltage measurement when batteries were resting. In order to ensure accuracy of this method, we needed to estimate the maximum capacity of an individual battery and the remaining time over which the battery could supply energy to the load.

3.4 Estimation Maximum Capacity and Remaining Time

The method we used to estimate the maximum capacity \((C_{\text{max}})\) was based on using the relationships between open circuit voltage and SOC shown in Figure 3.5. During the discharge process, two open circuit measurements, \(V_{oc1}\) and \(V_{oc2}\) were obtained for each battery when the battery was resting. These measurements were used to obtain two SOC estimates, say \(SOC1\) and \(SOC2\). The ampere-hours drawn from the battery in the interval between the time of the two estimates is calculated as \(Q_{\text{out}} = I \cdot t_{\text{discharge}}\). This estimate was used to calculate \(C_{\text{max}}\) using \(SOC1\) and \(SOC2\) using the relationship [4]
\[
C_{\text{max}} = \frac{100}{\text{SOC}_1 - \text{SOC}_2} \cdot Q_{\text{out}}.
\]  

(3.8)

Assuming a constant discharge current of 0.25A, Table 3.2 shows a few example measurements and calculated values. Because this was a simulation based study, we could obtain the actual value of \(C_{\text{max}}\) from the internal model for the battery. We calculated the error between the estimated \(C_{\text{max}}\) and the actual \(C_{\text{max}}\) using the relationship

\[
E = \left(\frac{C_{\text{max,estimated}} - C_{\text{max,actual}}}{C_{\text{max,actual}}}\right) \cdot 100.
\]

(3.9)

Table 3.2 \(C_{\text{max}}\) Estimated using Linear Model

<table>
<thead>
<tr>
<th>(I_{\text{discharge}} (A))</th>
<th>(V_{\text{oc1}} (V))</th>
<th>(V_{\text{oc2}} (V))</th>
<th>(S_{\text{SOC1}}(%))</th>
<th>(S_{\text{SOC2}}(%))</th>
<th>(Q_{\text{out}}(\text{Ahr}))</th>
<th>(C_{\text{max,estimated}}(\text{Ahr}))</th>
<th>(C_{\text{max,actual}}(\text{Ahr}))</th>
<th>(E(%))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>1.3467</td>
<td>1.3235</td>
<td>80.841</td>
<td>42.292</td>
<td>0.375</td>
<td>0.973</td>
<td>0.968</td>
<td>0.48</td>
</tr>
<tr>
<td>0.25</td>
<td>1.3438</td>
<td>1.3229</td>
<td>76.023</td>
<td>41.295</td>
<td>0.375</td>
<td>1.080</td>
<td>1.056</td>
<td>2.3</td>
</tr>
<tr>
<td>0.25</td>
<td>1.3473</td>
<td>1.3426</td>
<td>81.839</td>
<td>74.029</td>
<td>0.062</td>
<td>0.800</td>
<td>1.001</td>
<td>20.0</td>
</tr>
</tbody>
</table>

As it is seen in Table 3.2, the error observed is between 0.48% and 20% when we used the linear model discussed in Section 3.3.1. However, when we used the model based on a cubic polynomial that was discussed in Section 3.3.1, the error decreased to a maximum of 10% as shown in Table 3.3.

Table 3.3 \(C_{\text{max}}\) Estimated using Cubic Polynomial Model

<table>
<thead>
<tr>
<th>(I_{\text{discharge}} (A))</th>
<th>(V_{\text{oc1}} (V))</th>
<th>(V_{\text{oc2}} (V))</th>
<th>(S_{\text{SOC1}}(%))</th>
<th>(S_{\text{SOC2}}(%))</th>
<th>(Q_{\text{out}}(\text{Ahr}))</th>
<th>(C_{\text{max,estimated}}(\text{Ahr}))</th>
<th>(C_{\text{max,actual}}(\text{Ahr}))</th>
<th>(E(%))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>1.3484</td>
<td>1.3269</td>
<td>83.666</td>
<td>47.941</td>
<td>0.375</td>
<td>1.048</td>
<td>1.071</td>
<td>1.9</td>
</tr>
<tr>
<td>0.25</td>
<td>1.3407</td>
<td>1.3280</td>
<td>70.872</td>
<td>49.769</td>
<td>0.250</td>
<td>1.185</td>
<td>1.113</td>
<td>6.4</td>
</tr>
<tr>
<td>0.25</td>
<td>1.3471</td>
<td>1.3383</td>
<td>81.506</td>
<td>66.884</td>
<td>0.125</td>
<td>0.855</td>
<td>0.978</td>
<td>10</td>
</tr>
</tbody>
</table>
Once $C_{max}$ is estimated, then $C_{rem}$ can be calculated using Equation 3.7. When the battery was discharged by drawing a constant current of $I$ amps out of the battery, the remaining time, $t_{rem}$, (in minutes) was calculated as

$$t_{rem} = \frac{C_{rem}}{I} \cdot 60.$$  \hfill (3.10)

When the battery was discharged by maintaining the power supplied to the load as constant, the remaining time, $t_{rem}$, was calculated as

$$t_{rem} = \frac{\int_{v_{bat}}^{v_{end}} C_{rem}(v) dv}{P} \cdot 60.$$  \hfill (3.11)

When the battery is discharged by varying the current at random to maintain constant voltage when the load resistance changes, it is not possible to estimate $t_{rem}$.

### 3.5 State Transition Paradigm

The state transition paradigm is an approach to modeling and representing systems that is based on the assumption that the given system is fully describable in terms of the state at a given time and transformations that determine how the system changes from one state to another state [16]. All the information that is necessary to accurately represent the system and define the transitions is assumed to be available.

Given that $Z$ is a set of states and $X$ is a set of inputs that can be presented to the system, a transformation called the *state transition*, $S_1$, determines how the system state changes in response to one or more inputs that are presented to the system.

$$S_1 : Z \otimes X \rightarrow Z$$  \hfill (3.12)
Another transformation called the output function, $S_2$, maps each state to an externally visible outcome (whenever such an outcome exists). It is customary to refer to such outcomes as consequences, each of which belongs to a set $\Psi$.

$$S_2 : Z \rightarrow \Psi$$

(3.13)

A fundamental assumption that is made while representing a system in the state transition paradigm is that the future behavior of the modeled system may be predicted if there is knowledge of the current state and the inputs that would be presented to the system. There is no room for uncertainty or indeterminism.

### 3.5.1 Modeling the Discharge of a Collection of Batteries

Consider a 4-tuple $CBD = (Z, S_i, \sigma, \tau)$ where $Z$ is a set of states, $S_i : Z \otimes X \rightarrow Z$ is set of transitions, $\sigma \subset Z$ is set of starting states, and $\tau \subset Z$ is a set of final states. It is possible to model a collection of batteries that are discharging using a CBD as discussed in the remainder of this section.

Let the SOC of an individual battery be represented as one of a finite set of levels, $l_i, l_2, \ldots, l_r, r < \infty$. Let us also consider without loss of generality that $SOC \ l_i < l_j$ whenever $i < j$. If there are $n$ batteries in the given collection, an $n$-vector that identifies the SOC level of each battery could represent a state of the collection of batteries. One initial state would be the $n$-vector in which all the batteries would have SOC level $l_r$. The final state would be the $n$-vector in which the SOC level of each battery would be $l_1$.

The transformation $S_1$ must identify the next state for every possible $n$-vector that is a valid state of the collection of batteries. Assuming, for simplicity, that the batteries
are connected in series, the transformation $S_2$ could be the terminal voltage of the collection of batteries.

Such an approach would have been feasible if we could estimate the SOC of a battery accurately and precisely understand the interactions between the complex electrochemical processes that are responsible for storing and delivering energy. However, it is well known that estimating the SOC is a difficult problem. Further, any arbitrary classification of the SOC of a battery into levels leads to further problems.

As an example, consider the states and transitions of a single battery that is shown in Figure 3.6. In this example, the SOC of the battery is considered to be one of $r = 4$ levels. Each level can be interpreted, intuitively, as representing 25% of the SOC. Thus, if a battery is in level $l_4$ then the battery is assumed to have 100% SOC and if it is in level $l_1$, it is assumed to have 25% SOC. If this is the representation that is used, an error in the estimate of the SOC may not be different from the actual state of the battery. However, a difference between the estimated SOC and actual SOC would cause a transition from one state to another at an unexpected time.

In contrast, suppose we considered the SOC of a battery at a finer resolution as shown in Figure 3.6, then an error in the SOC estimate would cause a decision-maker to assume that the battery is in a different state than its actual state. Once again, the time when a transition occurs and the resulting state would be unexpected. These two consequences, i.e., being unable to precisely identify the state of a battery and being unable to predict the time at which a battery would transition from one state to another, are caused by uncertainties that come to pass while the battery is discharging. The onset of damage processes in a battery is another cause for the discrepancy between the
estimated SOC and the actual SOC. Like SOC, it is also difficult to predict or recognize when damage, which is a reduced capacity to store and deliver charge, may occur.

Figure 3.6 Battery SOC Transitions
CHAPTER IV

METHODS TO DISCHARGE A COLLECTION OF BATTERIES

Using redundancy to improve the performance of a system is a well-known principle in engineering. When redundant batteries are used in a collection, a decision-maker must be designed to select some of the batteries for discharge while other batteries are resting. Such a decision-maker is often based on a strategy and we considered three strategies namely, Queued Selection, Sliding Window Selection and Random Selection. The results show that the sliding window selection strategy provides longer lifetime than the queued selection strategy. The results show that the random selection strategy extends the lifetime of the collection of batteries when compared to queued selection, which is the most commonly used approach in battery-powered applications.

Section 4.1 presents an overview of the architecture and objectives for the decision-makers. Three strategies, which are used by the decision-makers, are discussed. A metric, called discharge-efficiency, is defined and used to compare the performance of the decision-makers. Section 4.2 presents the scenarios and method used in the simulation studies and Section 4.3 presents the results obtained from the simulations. Finally, Section 4.4 concludes this chapter with a discussion of the results.
4.1 Designing a Decision-Maker

We assumed that the decision-makers are designed using a time-triggered architecture. This means that the decision-maker would observe the operational condition of the collection of batteries, which are discharging, in discrete time steps (or time ticks), \( t_1, t_2, \ldots, t_f \). At some time tick, \( t_f \), the collection of batteries would be unable to supply energy to a load that is connected. The duration between two successive time ticks is assumed to be uniform; this duration is referred to as a time-slice and is denoted as \( T_s \), i.e., \(|t_i - t_{i+1}| = T_s, \ 1 \leq i \leq f - 1\). For convenience, we assume that the first possible time tick is \( t_1 \) and that the switching transients (current and voltage) are small enough to be ignored.

Given a collection of \( n \) batteries, the basic problem in each time-slice is to identify the \( m \) batteries that must discharge and the \( k=n-m \) batteries that must rest during the time-slice. One can choose \( m \) batteries out of \( n \) batteries in \( \binom{n}{m} \) ways. Since this may represent a large number of choices, we focused on three selection strategies, namely Queued selection, Sliding window selection and Random selection that are described in detail in Section 4.3.

As described in Chapter 3, there are switches that can be set to include a particular battery in or exclude it from the discharge process. Given a collection of \( n \) batteries, the requirements of the application dictate the number, \( m \), of batteries that must be discharged. In all the simulation studies, we selected \( m \) based on the needs of a hypothetical application. For example, assuming that the application requires one to maintain a minimum terminal voltage of 12 V, if the designers specify that the battery
should be considered to be completely discharged when its terminal voltage is 1.25V, we would need at least 8 batteries to be discharging. We defined the discharge process to terminate when the selection process could no longer satisfy the demands of the application.

To evaluate the methods to discharge a collection of batteries we identified a metric, called Discharge Efficiency, which is considered the lifetime of the collection, energy used and capacity used. Because different applications could place different levels of importance on each of these factors, we used parameters to appropriately weight these factors to define the Discharge Efficiency as

\[
DE = c_1 \cdot \frac{\text{life\_time}}{\max\_\text{possible\_life\_time}} + c_2 \cdot \frac{\text{energy\_out}}{\max\_\text{possible\_energy\_out}} + c_3 \cdot \frac{\text{total\_used\_cap}}{\text{total\_possible\_cap}}
\]

(4.1)

where \( c_i > 0 \) and \( c_1 + c_2 + c_3 = 1 \).

4.2 Simulation Scenarios and Methods

In our research, we consider for a battery to supply energy to a load in one of the following three different discharge scenarios: Constant Current (CC), Constant Power (CP) and Randomly Varying Current (RVC). We examined each selection strategy in all the three discharge scenarios. All simulation experiments were conducted 10 times and all the results in this chapter are based on average values from these 10 trials.

For each experiment, \( n=20 \) batteries were used. The number of resting batteries, \( k \), was fixed for all the selection strategies considered in this chapter. The output voltage of the batteries was required to be a minimum of 14 V. When the terminal voltage of a
single battery was less than 1.25 V, we defined that battery to be completely discharged. We used this definition to determine that the number of resting batteries must be \( k = 8 \). Thus, the discharge process was terminated when the batteries could no longer supply a minimum of 14 V. Each experiment was conducted for two different values of the time-slice: 1 min and 15 min.

For the constant current discharge scenario, the discharging current was chosen to be 250 mA. In the constant power discharge scenario, the fixed power required was 3.75 W. To ensure constant power delivery, the current through the battery was increased as the battery discharged and the voltage decreased. For the randomly varying current scenario, we required variations within 10% of a nominal 238 mA load current.

As an example of the utility of the metric for discharge, for the collection of 20 batteries, and nominal values for the weight parameters, the discharge efficiency for this collection of batteries for the constant current discharge scenario is:

\[
DE = 0.25 \cdot \frac{\text{life\_time}}{454 \text{ min}} + 0.25 \cdot \frac{\text{energy\_out}}{29.17 J} + 0.5 \cdot \frac{\text{total\_used\_cap}}{22.72 Ah}
\]  

For randomly varying current and constant power discharge scenarios, the \textit{max\_possible\_lifetime} and \textit{max\_possible\_energy} would be different from the values showed in Equation 4.2. These values are obtained from MATLAB simulation using uniform batteries in the Sliding Window selection with very small time-slice, 0.1 min. The values for \textit{life\_time}, \textit{energy\_out}, and \textit{total\_used\_cap} in Equation 4.2 would be obtained either from a simulation or an actual implementation of a discharge method.
4.3 Simulation and Results

To establish a base line for comparing results, we first simulated the discharge of 12 batteries, which were connected in series, without allowing any battery to rest and \( T_s = 1 \) min is used. The discharge process was terminated when the batteries could no longer supply the voltage required by the load.

Figure 4.1 Voltage and SOC for CC Discharge, Base Line

Figure 4.1 shows the results from this base line simulation for the constant current discharge scenario. As seen in Figure 4.1 (a), the collection was able to supply energy to the load for about 189 minutes. Figure 4.1 (b) shows that the SOC that remains in each battery is different. This phenomenon occurs whenever a collection of batteries are discharged because the complex electro-chemical processes that convert the stored chemical energy to electrical energy are unique to each individual battery.

Figure 4.2 and Figure 4.3 show similar results for the constant power discharge scenario and the randomly varying current discharge scenario. As can be noted from Figure 4.4, the maximum lifetime for the base line discharge was observed in the
randomly varying current discharge scenario (215 minutes) and the worst lifetime was observed in the constant current discharge scenario (189 minutes).

Figure 4.2 Voltage and SOC for CP Discharge, Base Line

Figure 4.3 Voltage and SOC for RVC Discharge, Base Line

Figure 4.4 Base Line Terminal Voltage and Lifetime
4.3.1 Queued Selection

In this strategy, batteries $B_1, B_2, \ldots, B_m$ are initially selected for discharge. This selection is altered only when at least one of the selected batteries is completely discharged. In some time-slice $t_d$, if one of the selected batteries is recognized to completely discharged, it is replaced by one of the redundant $k$ batteries – by switching out the battery that is completely discharged and switching in one of the redundant batteries (in some pre-defined order). $T_s=1$ min is used and the discharging process is terminated when the collection can no longer supply the energy required by the load.

The queued selection method is simple and is commonly used in portable devices. It is necessary to monitor the voltage, or count ampere-hours drawn out, of each battery to terminate the discharge. A completely discharged battery is merely replaced with one of the redundant batteries. Because each battery is unique and displays considerable variation in lifetime, it is not possible to predict when the redundant batteries must replace one of the batteries. Even if we assume that the individual batteries would discharge uniformly, this approach will not extend the lifecycle of the collection because when $k<m$, and more than $k$ batteries are completely discharged, the application demands cannot be met. This state of termination is not desirable because there would be at least $k$ batteries that are not fully discharged.

Figure 4.5, Figure 4.6 and Figure 4.7 show the voltage and SOC of each battery in the collection under each of the three discharge scenarios. In each discharge scenario, it may be noted that there are several batteries in the collection that have remaining charge, and yet, this charge is not sufficient to supply energy to the load.
The terminal voltage observed from the collection of batteries the lifetime of the collection under different discharge scenarios are shown in Figure 4.8. Compared to the
base line lifetime, we see that the queued selection strategy improves the lifetime a little; in the constant current discharge scenario the collection now discharges for 230 minutes and in the randomly varying current discharge scenario, the lifetime is about 260 minutes.

![Total Voltage vs. Time Plot](image)

Figure 4.8 Terminal Voltage and Lifetime in Queued Selection

4.3.2 Sliding Window Selection

In this method, $B_1, B_2, \ldots, B_m$ are initially selected for discharge. This selection is changed at each time-slice, $t_i$, when the next $m$ resting battery modules $B_{m+1}, \ldots, B_{(m+m) \mod n}$ start to discharge while the remaining $k$ battery modules are allowed to rest. The sliding window selection process is terminated when the collection of batteries can no longer supply energy to the load.

Figure 4.9, Figure 4.10 and Figure 4.11 show the voltage and SOC of each battery during discharge in each of the three discharge scenarios. It may be noted that the sliding window selection strategy is not efficient because there is considerable charge remaining in the batteries at the end of discharge.
Figure 4.9 Voltage and SOC for CC Discharge, Sliding Window Selection

Figure 4.10 Voltage and SOC for CP Discharge, Sliding Window Selection

Figure 4.11 Voltage and SOC for RVC Discharge, Sliding Window Selection
The weakest battery in the collection, i.e., the battery with the lowest SOC, would always get completely discharged before the rest of the batteries under this strategy. When such a battery (or batteries) is (are) completely discharged, it affects the terminal voltage and energy that the collection can supply to the load. The lifetime of the collection depends on the number of weak batteries in the collection – and whenever there are weak batteries, a significant amount of SOC, which cannot be converted to useful electrical energy that can be supplied to the load, would remain in the batteries at the end of discharge.

Figure 4.12 shows the terminal voltage and lifetime of the collection of batteries when sliding window selection is used. Compared to the baseline and the lifetime of the queued selection strategy, this approach provides a battery lifetime. Further, it is not necessary to either monitor the voltage of each battery, or count the ampere-hours drawn out of each battery, in the collection. It is necessary to only monitor the terminal voltage of the collection to terminate discharge.

Figure 4.12 Terminal Voltage and Lifetime in Sliding Window Selection
4.3.3 Randomized Selection

In this method, in each time-slice, a subset of \( m \) batteries is randomly selected from the collection to discharge. Such a set is chosen from all possible subsets of size \( m \) with uniform probability. It is not required to monitor the voltage, or count the ampere-hours drawn out of, each battery in the collection when using this strategy. It is sufficient to monitor the terminal voltage of the collection to terminate discharge.

Figure 4.13 shows the voltage and SOC of each battery in the collection in the constant current discharge scenario. Figure 4.14 and Figure 4.15 show similar results for the constant power and randomly varying current discharge scenarios. It may be noted that in each discharge scenario, there is considerable SOC at the end of discharge that is not converted to electrical energy, and hence wasted.
The terminal voltage and lifetime of the collection under randomized selection are shown in Figure 4.16. It may be noted that randomized selection improves the lifetime of the collection over queued selection. However, lifetime observed under randomized selection is worse than the lifetime in sliding window selection. Further, the remaining SOC under randomized selection is considerably worse than the remaining SOC under sliding window selection.
4.4 Discussion of Results

The base line simulation results, which did not use any redundant batteries, are shown in Table 4.1. Each row shows the values for Life Time, Energy Out and Remaining Capacity in each discharge scenario.

Table 4.1 Base Line Experimental Results

<table>
<thead>
<tr>
<th>Discharge Scenario</th>
<th>Life Time (min)</th>
<th>Energy Out (J)</th>
<th>Remaining Capacity (A-Hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Current</td>
<td>189.3</td>
<td>12.191</td>
<td>2.461</td>
</tr>
<tr>
<td>Constant Power</td>
<td>195.9</td>
<td>12.216</td>
<td>2.457</td>
</tr>
<tr>
<td>Variable Current</td>
<td>214.8</td>
<td>12.621</td>
<td>2.316</td>
</tr>
</tbody>
</table>

Tables 4.2, 4.3 and 4.4 present a comparison of Life Time, Energy Out and Remaining Capacity for each of the selection strategies discussed in Section 4.3. Since the redundancy in each of these strategies was the same, we do not show that value.
Table 4.2 Life Time (min)

<table>
<thead>
<tr>
<th>Discharge Scenario</th>
<th>Queued Selection</th>
<th>Sliding Window</th>
<th>Random Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_s=1$ Min</td>
<td>$T_s=1$ min</td>
<td>$T_s=15$ min</td>
</tr>
<tr>
<td>Constant Current</td>
<td>231.9</td>
<td>348.5</td>
<td>346.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>337.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>285.42</td>
</tr>
<tr>
<td>Constant Power</td>
<td>240.5</td>
<td>361.02</td>
<td>358.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>352.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>308.58</td>
</tr>
<tr>
<td>Variable Current</td>
<td>259.3</td>
<td>384.65</td>
<td>383.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>375.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>321.84</td>
</tr>
</tbody>
</table>

Table 4.2 shows the lifetime (in minutes) for each selection strategy under each discharge scenario. It can be seen that the lifetime obtained from Queued Selection is the worst and the lifetime from the Sliding Window selection strategy is the best. The lifetime obtained from randomized selection is between these two extremes and is much better than the Queued Selection strategy. In the case of Sliding Window and Random Selection strategies, it can also be noticed that the lifetime is better when the time-slice is smaller, i.e, 1 min.

Table 4.3 Energy Out (J)

<table>
<thead>
<tr>
<th>Discharge Scenario</th>
<th>Queued Selection</th>
<th>Sliding Window</th>
<th>Random Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_s=1$ min</td>
<td>$T_s=1$ min</td>
<td>$T_s=15$ min</td>
</tr>
<tr>
<td>Constant Current</td>
<td>14.909</td>
<td>22.517</td>
<td>22.276</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>21.801</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>18.423</td>
</tr>
<tr>
<td>Constant Power</td>
<td>15.0252</td>
<td>22.557</td>
<td>22.427</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>22.012</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>19.280</td>
</tr>
<tr>
<td>Variable Current</td>
<td>15.196</td>
<td>22.662</td>
<td>22.510</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>22.128</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>18.944</td>
</tr>
</tbody>
</table>

Table 4.3 shows that the energy that is supplied to the application from the batteries is consistent with the lifetime of discharge. Table 4.4 shows that in all scenarios, the remaining capacity in the batteries at the end of discharge is worst for Queued
selection. In the case of Sliding window selection, the remaining capacity is close to that in the baseline scenario.

<table>
<thead>
<tr>
<th>Discharge Scenario</th>
<th>Queued Selection $T_s=1$ min</th>
<th>Sliding Window $T_s=1$ min</th>
<th>Random Selection $T_s=1$ min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Current</td>
<td>8.335</td>
<td>2.42882</td>
<td>3.10347</td>
</tr>
<tr>
<td>Constant Power</td>
<td>8.257</td>
<td>2.37084</td>
<td>2.93777</td>
</tr>
<tr>
<td>Variable Current</td>
<td>8.114</td>
<td>2.57213</td>
<td>3.3028</td>
</tr>
</tbody>
</table>

Each histogram in Figure 4.17 shows the duration of time for which each battery was discharged in the baseline simulation. As can be noted from Figure 4.17, all the batteries were used equally.

Figure 4.18, Figure 4.19 and Figure 4.20 show the battery usage in Queued Selection, Sliding Window Selection and Random Selection, respectively. The reason for remaining capacity in Queued Selection and Random Selection can be explained by noting that not all batteries are selected for discharge equally. For example, in Figure 4.18, batteries 11-20 are underutilized in Queued selection. Figure 4.20 shows that in the randomized selection strategy, some batteries are used more often than other batteries. In contrast, the batteries are utilized uniformly in the sliding window selection strategy as is shown in Figure 4.19.
Figure 4.17 Battery Usage in Base line

Figure 4.18 Battery Usage in Queued Selection

Figure 4.19 Battery Usage in Sliding Window Selection

Figure 4.20 Battery Usage in Randomized Selection

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None of the three selection strategies could minimize the remaining SOC in the collection. Only in the case of Sliding Window selection, the batteries were used equally. We believe that the reason for this difficulty is that these selection strategies are unable to cope with the uncertainties of SOC and battery parameters. Consequently, lifetime of the collection and the total energy of delivered to the load is not as efficient as we expected it to be. This notion is also captured by the discharge-efficiency metric that is shown in Table 4.5.

<table>
<thead>
<tr>
<th>Table 4.5 Discharge Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Queued Selection</td>
</tr>
<tr>
<td>Discharge Scenario</td>
</tr>
<tr>
<td>Constant Current</td>
</tr>
<tr>
<td>Constant Power</td>
</tr>
<tr>
<td>Variable Current</td>
</tr>
</tbody>
</table>
CHAPTER V

GOAL SEEKING APPROACH TO COORDINATED BATTERY DISCHARGE

This chapter presents a new approach to coordinating the discharge of a collection of batteries which is also based on using redundancy to improve performance. However, the selection strategy is based on the Goal Seeking Paradigm – which is a framework for modeling and describing systems in which one explicitly represents and reasons about uncertainties. The results in this chapter demonstrate that in comparison with the methods discussed in Chapter 4, this approach extends the lifetime of the collection of batteries and the discharge-efficiency by over 10%.

Section 5.1 presents an overview of the goal-seeking approach, its relevance for coordinating the discharge of a collection of batteries and a framework for designing a decision-maker that is based on the goal-seeking approach. Section 5.2 presents a detailed formulation of the coordination problem in the goal-seeking paradigm. Since estimating SOC is critical to the success of any decision-maker that would coordinate the discharge of a collection of batteries, Section 5.3 presents four approaches to estimating. Section 5.4 presents the results from simulation studies and Section 5.5 concludes this chapter with a discussion of the results.
5.1 Goal-Seeking Approach

The new approach to coordinating the discharge of a collection of batteries is different from other approaches that are based on classical control, optimal control, robust control or adaptive control, because the objective in this approach is not to find an optimal or precise solution to the problem of coordinating the discharge. In contrast, this approach is based on the principle of bounded-rationality [22] in which one is satisfied with any solution, as long as the solution is within an acceptable tolerance limit. The focus on a good-enough solution allowed us to use simple models for the batteries [7] and simple algorithms for decision-making. Such an approach is useful when it is not possible, or desirable, to construct a precise model of a system under consideration [8].

The goal-seeking paradigm overcomes certain difficulties of representing and reasoning about uncertainties that are inherent in the state-transition paradigm described in Chapter 3. The approach described in this chapter uses a decision-maker that is designed based on the following additional artifacts.

There is a set of Alternate Actions, $\Pi$. Each member of this set represents a decision or an action that can be selected by the decision-maker. The outcome of a selected action may be affected by one or more uncertainties that may occur. There is an explicitly represented set of Uncertainties, $\Delta$, that the decision-maker must consider when selecting actions. The set of Consequences, $\Psi$, specifies the interval of output terminal voltages that may be observed from the collection of batteries in response to a selected action if a given uncertainty comes to pass. A function called Reflection, $\Phi$, represents the decision-maker’s view of the environment. If a particular action is selected, the decision-maker uses this function to estimate what consequence may occur if a given
uncertainty comes to pass. The decision-maker uses a Performance Scale, $\Lambda$, that is used to compare outcomes of decisions and to select among alternate actions. An Evaluation Mapping, $\Omega$, is used to compare the outcomes of the decisions using the performance scale and the Tolerance Function, $\Gamma$, helps to determine the degree of the satisfaction with the consequence. Based on these artifacts, the functioning of a decision-maker, which is based on the goal-seeking paradigm, can be stated as:

$$\text{Select any action such } \pi \in \Pi \text{ that } \Omega(\Phi(\pi, \delta_i), \pi) > \Gamma(\delta_i, \pi) \text{ for any } \delta_i \in \Delta.$$ 

### 5.1.1 Discharging a Collection of Batteries

The interaction between the decision-maker and the collection of batteries is as depicted in Figure 5.1. The decision-maker establishes performance limits for the collection of batteries by selecting alternate actions, while considering uncertainties that may come to pass and its view of the environment (Reflection). The output terminal voltage obtained from the collection of batteries, $V_t$, (i.e., the “performance” of the collection of batteries) is monitored and evaluated using the performance scale and the Evaluation Mapping. As long as the $V_t$ is within an acceptable range, ($V_{\text{min}} < V_t < V_{\text{max}}$), the decision-maker does not select new alternate actions. Whenever the performance is outside the acceptable range, the decision-maker selects alternate actions to maintain the performance within the acceptable range. The discharge process is terminated when the decision-maker can no longer select alternate actions that would allow the collection of batteries to supply energy to the load.

The need to represent and reason about uncertainties during discharge was the primary motivation for considering this goal-seeking approach to coordinating the
discharge of a collection of batteries. In addition to the difficulty of estimating SOC (as discussed in Chapter 3), it is difficult to construct a precise model for a collection of batteries and it is not possible to precisely track the status of individual batteries and preserve the accuracy of such a model during the discharge process. By working within the framework of the goal-seeking paradigm, we could use a simple model for the batteries and simple decision-making algorithms that were adequate to extending the lifetime of the collection of batteries by coordinating the discharge of individual batteries. When compared to the selection strategies in Chapter 4, we observed an improvement in the discharge-efficiency metric in all the three discharge scenarios.

Figure 5.1 Coordinating Discharge of a Collection of Batteries

5.2 Goal-Seeking Formulation

The principal objectives for the decision-maker are to extend the lifetime of the collection of batteries and improve the discharge-efficiency. The decision-maker must select actions that would enable the collection of batteries to maintain the terminal voltage (and power) of the collection at System Level-3 within a tolerance limit specified
for the load. While the collection of batteries is discharging, a decision maker must cope
with variations in voltage and energy that can be delivered by each battery, which occur
either because of imprecise SOC information or because of the onset of some damage
process in a battery. Because the SOC is a critical factor in the discharge process and it is
inherently difficult to measure, we evaluated four alternative methods to estimate SOC
(discussed in Section 5.3) as different cases in this approach.

5.2.1 Alternate Actions, $\Pi$

The alternate actions represent the choices that are available to the decision-
maker. To appropriately select batteries for discharge or rest in each time-slice, $t_i$,
decision-maker must determine, for a given battery $b$, if it must discharge or rest in a
particular time-slice. Given a collection of $n$ batteries, it would suffice to have a vector of
$n$ elements such that element $i$ of this vector is a 1 if the corresponding battery $b_i$ is
discharging and 0 if $b_i$ is resting. If we denote the set of all possible binary vectors on $n$
elements as the set $\Pi_1$, then the decision-maker must select one member of this set at
each time tick so that the selected batteries would discharge in the time-slice following
the time tick. Because of the uncertainties associated with the SOC of an individual
battery and the discharge process, the choice of a member in $bv \in \Pi_1$ depends on several
other choices that are also available to the decision maker.

Clearly, the number of batteries that are discharging is directly related to the
terminal voltage and energy that can be delivered by the collection of batteries. The
decision-maker would select $m$ batteries to discharge and $k$ batteries to rest in each time-

---

1 Note that we use the notation $bv(i)$ to refer to the $i^{th}$ element of the member $bv \in \Pi_1$. 53
slice. To accomplish this, the decision-maker selects one member of the set \( \Pi_2 = \{(m,k) : m \leq n, k \leq n, m + k = n\} \). We use the notation \( \tau_{mk} \in \Pi_2 \) to denote that the ordered pair \((m,k)\) was selected by the decision-maker. Thus, the choice of \( bv \in \Pi_1 \) is restricted by the choice of \( \tau_{mk} \in \Pi_2 \).

In addition, we found that it was useful to allow the decision-maker to select a selection strategy. The available strategies are represented by the set \( \Pi_3 = \{\text{Queued Selection, Random Selection, Sliding-Window Selection, SOC Selection}\} \). The first three strategies have been discussed in Chapter 4 and the SOC selection strategy is discussed in detail in Section 5.3. The decision-maker can also select the time tick at which the selection strategy changed; this choice is represented by the set \( \Pi_4 = \{1,2,\ldots,f\} \). The set \( \Pi_5 = \{1,10,15\} \) represents the choices for the duration of the time-slice \( T_s \) in minutes. We included two methods for estimating SOC (as discussed in Section 3.3) and the set \( \Pi_6 = \{\text{ampere-hour measurement, open circuit voltage measurement}\} \) represents this choice.

Thus when the decision-maker selects \( bv \in \Pi_2 \) in each time tick, this choice actually depends on other choices, i.e., this choice is constrained by the member the decision-maker would select from the set \( \Pi_2 \otimes \Pi_3 \otimes \Pi_4 \otimes \Pi_5 \otimes \Pi_6 \).

5.2.2 Uncertainties, \( \Delta \)

Because the discharge process in each battery is unique and it is difficult to estimate SOC accurately, we vary the parameters of the battery model [7] arbitrarily within 10% of the starting values. Such a variation represents one source of uncertainty.
that we denote as $\delta_t$. Such changes in the parameters affect the stored charge, life time and voltage of each battery. For purpose of this research, we define a battery to be completely discharged when its terminal voltage falls below 1.20V. When a battery is completely discharged, the number of available batteries, $n$, is also decreased. To account for manufacturing variations in each battery, the initial SOC of a battery is assumed to vary by ±32%. This uncertainty is represented as $\delta_1$.

5.2.3 Consequences, $\Psi$

The consequences represent the output of the collection of batteries. We assume that the terminal voltage can be measured at System level-3, and this value represents the consequence of the selected actions. Clearly, if an action is selected and some uncertainty comes to pass, the measured voltage of the collection of batteries would reflect the effect of the uncertainty.

5.2.4 Reflection, $\Phi$

The Reflection, $\Phi$, is a function $\Phi : \Pi \otimes \Delta \rightarrow \hat{\Psi}$ that represents the decision-maker’s view of the environment. To emphasize that the consequence identified by the reflection is an estimated value, we used the symbol $\hat{\Psi}$ to denote the set of estimated consequences. Whenever the decision-maker has to select among a set of actions $\pi \in \Pi$, it uses the reflection to estimate the consequence if a given uncertainty, $\delta_t \in \Delta$, comes to pass.
As an example of the use of this reflection function, consider the scenario shown in Table 5.1. There are 7 batteries and four of these batteries (B4, B5, B6, and B7) are discharging in time-slice \( t_i \) and three are resting. The decision-maker must determine the batteries that would discharge in time-slice \( t_{i+1} \). Column 1 of Table 5.1 shows the estimated SOC for each battery. Batteries B1 and B2 are likely be selected because they have a higher estimated SOC value. The choice of the remaining two batteries is more difficult because the estimated SOC can significantly vary from the actual SOC. In Column 1 of Table 5.1, discharging batteries have a suffix “*”. SOC of discharging batteries are calculated by ampere-hour counting. SOC of resting batteries are calculated by open circuit voltage measurements.

<table>
<thead>
<tr>
<th>(\delta(SOC))</th>
<th>(\pi)</th>
<th>(\psi = \Phi(\pi, \hat{\delta}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1 %80</td>
<td>B1 B2 B3 B4</td>
<td>V1</td>
</tr>
<tr>
<td>B2 %80</td>
<td>B1 B2 B3 B5</td>
<td>V2</td>
</tr>
<tr>
<td>B3 %70</td>
<td>B1 B2 B3 B5</td>
<td>V2</td>
</tr>
<tr>
<td>B4 %70*</td>
<td>B1 B2 B4 B5</td>
<td>V3</td>
</tr>
<tr>
<td>B5 %70*</td>
<td>B1 B2 B4 B5</td>
<td>V3</td>
</tr>
<tr>
<td>B6 %65*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B7 %60*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that the actual consequence, which is the terminal voltage measured when the selected batteries are discharging, is not necessarily the same as the value indicated by the reflection. To emphasize, the expected consequence is computed based on the decision-maker anticipating some uncertainty, i.e., \(\psi = \Phi(\pi, \hat{\delta})\). If \(V_{avg}\) represents the
average voltage that can be delivered by a selected battery, then
\[ \hat{\psi}_{m,k}(t_i) = V_{avg}(t_i - 1) \cdot m \]
is a simple function for reflection. To account for the uncertainties, we decided to adjust
\( V_{avg} \) by the estimated SOC in the batteries selected as follows:

\[ \hat{\psi}_{m,k}(t_i) = \sum_{j=1}^{n} (V_{avg}(t_{i-1}) - 1) \frac{SOC_{j}^{t_{i}} b(j)}{SOC_{avg}^{t_{i-1}}} \] (5.2)

In Equation 5.2, \( m \) and \( k \) are the number of resting and discharging batteries as identified by alternate action \( \Pi_2 \). \( b(j) \) represents the \( j \)th element of the choice \( \Pi_1 \); which would be 1 if the corresponding battery is discharging and 0 otherwise. \( SOC_{j}^{t_{i}} \) is the estimated SOC for battery \( i \) for time-slice \( t_i \). To calculate \( SOC_{j}^{t_{i}} \), parameters for the battery model are first varied by 10% and the open circuit voltage is computed using the battery model discussed in Chapter 3 (Equation 3.1). If this voltage is less than \( V_{EoD} = 1.20 \) V, \( SOC_{j}^{t_{i}} \) is set to 0 to reflect that the battery is assumed completely discharged. \( SOC_{avg}^{t_{i-1}} \) is the average SOC of all batteries in time-slice \( t_{i-1} \).

5.2.5 Performance Scale, \( \Lambda \)

The performance scale is a metric used to compare outcomes of selected actions. This scale helps the decision-maker to determine which alternate action is preferable over other choices. We used the closed interval \([0, 1]\) on the real line as the scale – with 0 representing an undesirable choice and 1 representing a desirable choice.
5.2.6 Evaluation Mapping, $\Omega$

The Evaluation Mapping is a function that maps a selected action and its estimated consequence to a value on, $\hat{\lambda}_i \in \Lambda$, on the performance scale i.e., $\Omega : \Pi \otimes \Psi \rightarrow \Lambda$. The decision-maker uses this mapping both to select alternate actions using the estimated consequences and to evaluate the consequence of selected actions based on measured values.

When selecting batteries for discharge, the expected consequence, should be close the minimum battery collection voltage level, $V_{\text{min}} = 14 \text{ V}$, otherwise, there is needless energy dissipation. Similarly, we would like the measured consequence also to be close to $V_{\text{min}}$. These constraints are captured in the following evaluation mapping function:

$$
\Omega(\pi, \psi_{m,k}) = \begin{cases} 
\frac{-e^{-(\psi_{m,k} - V_{\text{min}})}}{1} \sum_{j=1}^{n} \text{SOC}_j \cdot b(j) \cdot \psi_{m,k} & \text{if } \psi_{m,k} \geq V_{\text{min}} \\
0 & \text{if } \psi_{m,k} < V_{\text{min}}
\end{cases} 
$$

(5.3)

Figure 5.2 demonstrates how the evaluation mapping changes assuming $V_{\text{min}} = 4.45 \text{ V}$ for various choices of $\Pi_1$.
When $\Psi = 4.4$ V, $\Omega(\cdot)$ gives a value of 0.90101 (for $\Pi_{3,4}$) and when $\Psi = 10.5$ V with a selection of $\Pi_{7,6}$ $\Omega(\cdot)$ yields a value of 0.0024 on the performance scale $\Lambda$.

5.2.7 Tolerance Function, $\Gamma$

The tolerance function, $\Gamma : \Pi \otimes \Delta \rightarrow \Lambda$, is used to quantify the degree of satisfaction with the consequence of the selected actions. The function of $\Gamma$ should define an acceptable bound on the value of $\Omega$. During the discharge process, we like the terminal voltage of the collection of batteries to remain greater than and close to 14 V. Thus, we used $V_{\text{min}}=14$ V and required selected actions that $\Gamma(\pi, \delta) \cong 1$.

Figure 5.3 shows how the artifacts of the goal-seeking formulation are used to coordinate the discharge of a collection of batteries.
5.3 SOC Estimation

Since the alternate actions selected by the decision-maker depend on the SOC of each battery in the collection, we explored four methods to estimate SOC that are described in the remainder of this section. Results that reflect the utility of these methods of estimation in the goal-seeking approach are presented for each method as a separate case in Section 5.4.

5.3.1 Using Voltage Derivatives (Case A)

The decision-maker starts the discharge process by selecting batteries using the sliding window strategy (in $\Pi_3$) and a time-slice duration of $T_s = 1$ minute (in $\Pi_4$). When the batteries are discharging, the decision-maker measures the terminal voltage of the collection at System Level-3. This measurement is used to calculate an estimated average terminal voltage of each battery that is discharging; when this average value drops below 1.275V, the decision-maker changes the selection strategy at a time tick (in $\Pi_4$) to SOC Selection with time-slice duration of $T_s = 10$ min.

To support the SOC selection strategy, the terminal voltage is measured at System Level-1 for each battery that is discharging. The rate of change of the terminal voltage is assumed to be an indicator of the SOC. A battery with a lower rate of change is assumed to have higher SOC than a battery with a higher rate of change. By ordering each battery in decreasing order of the SOC, say as $b_1, b_2, \ldots, b_n$. Decision maker then

---

Note that this indicator of SOC is only used to guide the decision-makers choice for the next time-slice. This indicator is not used to estimate the SOC of the battery.
identifies \( \pi_{mk} \in \Pi_2 \) with the smallest \( k \) such that the batteries, \( b_1, b_2, \ldots, b_k \), would be able to supply energy to the load even if some uncertainty comes to pass.

### 5.3.2 Estimation Table (Case B)

The method used in the preceding section did not estimate the SOC of the resting batteries. Initially, all batteries are assumed to have an SOC within 32% of each other. In this case, the decision-maker uses open circuit voltage measurement to estimate the SOC of resting batteries (using the mapping shown in Table 5.2) and ampere-hour counting to estimate SOC of the discharging batteries\(^3\) using

\[
\hat{\delta}_i(SOC) = \hat{\delta}_{i-1}(SOC) - \int_{t=1}^{i} i(\tau) d\tau
\]

#### Table 5.2 SOC Estimation Table using Open Circuit Measurements

<table>
<thead>
<tr>
<th>( V_{oc} )</th>
<th>( \hat{\delta}(SOC) )</th>
<th>( V_{ac} )</th>
<th>( \hat{\delta}(SOC) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.370</td>
<td>100%</td>
<td>1.323 &lt; Voc&lt;= 1.326</td>
<td>45%</td>
</tr>
<tr>
<td>1.360 &lt; Voc &lt;= 1.370</td>
<td>95%</td>
<td>1.320 &lt; Voc &lt;= 1.323</td>
<td>40%</td>
</tr>
<tr>
<td>1.354 &lt; Voc &lt;= 1.360</td>
<td>90%</td>
<td>1.317 &lt; Voc &lt;= 1.320</td>
<td>35%</td>
</tr>
<tr>
<td>1.348 &lt; Voc &lt;= 1.354</td>
<td>85%</td>
<td>1.313 &lt; Voc &lt;= 1.317</td>
<td>30%</td>
</tr>
<tr>
<td>1.344 &lt; Voc &lt;= 1.348</td>
<td>80%</td>
<td>1.309 &lt; Voc &lt;= 1.313</td>
<td>25%</td>
</tr>
<tr>
<td>1.340 &lt; Voc &lt;= 1.344</td>
<td>75%</td>
<td>1.303 &lt; Voc &lt;= 1.309</td>
<td>20%</td>
</tr>
<tr>
<td>1.337 &lt; Voc &lt;= 1.340</td>
<td>70%</td>
<td>1.295 &lt; Voc &lt;= 1.303</td>
<td>15%</td>
</tr>
<tr>
<td>1.335 &lt; Voc &lt;= 1.337</td>
<td>65%</td>
<td>1.285 &lt; Voc &lt;= 1.295</td>
<td>10%</td>
</tr>
<tr>
<td>1.332 &lt; Voc &lt;= 1.335</td>
<td>60%</td>
<td>1.230 &lt; Voc &lt;= 1.285</td>
<td>5%</td>
</tr>
<tr>
<td>1.329 &lt; Voc &lt;= 1.332</td>
<td>55%</td>
<td>Voc&lt;1.230</td>
<td>0%</td>
</tr>
<tr>
<td>1.326 &lt; Voc &lt;= 1.329</td>
<td>50%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^3\) The estimate based on open circuit measurement is more reliable than the estimate based on ampere-hour counting [4]
After estimating the SOC of the batteries, the decision-maker uses a process that is similar to the one described in Section 5.3.1.

Consider the example shown in Figure 5.4. There are 7 batteries in a collection in which three batteries (b3, b4 and b6) are discharging while 4 batteries (b1, b2, b5 and b7) are resting. The SOC of batteries b3, b4 and b6 are estimated using ampere-hour counting and the SOC of the remaining batteries are estimated using open circuit voltage measurement. These estimates are combined to select the batteries in decreasing order of the estimated SOC. In this example, b5, b3 and b1 would be selected for discharge in the next time-slice.

![Figure 5.4 Examples for Sorting SOC for Batteries](image)

### 5.3.3 Using Linear Regression (Case C)

In this case, the SOC estimate is made assuming that the open circuit voltage is related to SOC linearly when the SOC is between 80% and 20% of the original value. During the discharge process, the decision-maker obtains two open circuit voltage measurements for each battery in the collection. For each measurement, SOC is estimated
as \( SOC = 1.661613121427587 \cdot 10^3 \cdot V_{oc} - 2.156852559350275 \cdot 10^3 \) and the two SOC estimates are used to estimate \( C_{max} \) which is total capacity of each individual battery for each battery. This estimate of the total capacity is useful because it allows us to apply ampere-hour counting more reliably to track the change in SOC of the battery. After completing estimation of \( C_{max} \) for each battery in the collection, the selection time point will be set and the ampere-hour measurement will be applied to rest of discharging process.

### 5.3.4 Fitting a Polynomial Function (Case D)

Instead of using a linear function, we used a cubic function to estimate the SOC. Using a similar method as described in Section 5.3.3, the coefficients of the cubic function were obtained using the least squares method as:

\[
SOC = 1.52654399115196 \cdot 10^6 - 3.44811832038858 \cdot 10^3 \cdot V_{oc} + 2.59488037693052 \cdot 10^3 \cdot V_{oc}^2 - 0.65058272525502 \cdot V_{oc}^3
\]  

(5.5)

Table 5.3 presents a summary of the alternate actions selected in each of the preceding four cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>( \Pi_1 )</th>
<th>( \Pi_2 )</th>
<th>( \Pi_3 )</th>
<th>( \Pi_4 )</th>
<th>( \Pi_5 )</th>
<th>( \Pi_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A</td>
<td>✓</td>
<td>✓</td>
<td>Sliding Window and Voltage Derivatives</td>
<td>✓</td>
<td>( T_s = 1 ) min and ( T_{s_2} = 10 ) min</td>
<td>NO</td>
</tr>
<tr>
<td>Case B</td>
<td>✓</td>
<td>✓</td>
<td>SOC Estimation Table</td>
<td>NO</td>
<td>( T_s = 15 ) min</td>
<td>A-Hr and ( V_{oc} ) measurements</td>
</tr>
<tr>
<td>Case C</td>
<td>✓</td>
<td>✓</td>
<td>Linear Fit</td>
<td>✓</td>
<td>( T_s = 1 ) min and ( T_{s_2} = 15 ) min</td>
<td>A-Hr and ( V_{oc} ) measurements</td>
</tr>
<tr>
<td>Case D</td>
<td>✓</td>
<td>✓</td>
<td>Polynomial Function Fit</td>
<td>✓</td>
<td>( T_s = 1 ) min and ( T_{s_2} = 15 ) min</td>
<td>A-Hr and ( V_{oc} ) measurements</td>
</tr>
</tbody>
</table>
5.4 Simulation and Results

For the simulation studies, a total 20 batteries were used. We assumed that a hypothetical load would need a minimum of 14 V. The decision maker stopped the discharge process when it could no longer select batteries to satisfy the needs of the application. For the constant current discharge scenario, we assumed that the load would draw 250 mA. For the constant power discharge scenario, power consumption was assumed to be fixed at 3.75 W, inclusive of all losses. For the randomly varying current discharge scenario, the current drawn was randomly varied in each time-slice within 10% of 238 mA.

5.4.1 Results for Case A – SOC Estimation using Voltage Derivatives

Figure 5.6, 5.7 and 5.8 show the voltage of each battery and the SOC when the collection of batteries was discharged in each of the discharge scenarios. It may be noted that in all three discharge scenarios, the lifetime has been extended over the lifetime of the methods discussed in Chapter 4.

![Battery Voltages](image1)
![Actual SOC’s](image2)

Figure 5.5 Voltage and SOC for CC Discharge, Case A
Figure 5.6 Voltage and SOC for CP Discharge, Case-A

Figure 5.7 Voltage and SOC for RVC Discharge, Case A

Figure 5.8 shows the number of batteries that were discharging throughout the lifetime. It can be noted that towards the end of discharge, the number of batteries that are discharging is increases in all three discharge scenarios. Figure 5.9 shows the terminal voltage of the collection and the lifetime in each of the discharge scenarios.
5.4.2 Results for Case B – SOC Estimation Using a Table

Figure 5.10, Figure 5.11 and Figure 5.12 show the voltage and SOC for each battery when SOC is estimated based on an estimation table. In all the three discharge scenarios, the lifetime is extended and the remaining capacity at the end of discharged is reduced. Figure 5.13 shows the number of batteries selected for discharge and Figure 5.14 shows the terminal voltage of the collection and lifetime in all the discharge scenarios.
Figure 5.10 Voltage and SOC for CC Discharge, Case B

Figure 5.11 Voltage and SOC for CP Discharge, Case B

Figure 5.12 Voltage and SOC for RVC Discharge, Case B
5.4.3 Results for Case C – SOC Estimation using a Linear Function

Figures 5.15 through 5.19 show the voltage and SOC, the number of batteries selected for discharge, the terminal voltage and the lifetime in all the discharge scenarios. Contrary to our expectation, the results show that the lifetime and the remaining capacity are worse than those obtained in the preceding sections.
Figure 5.15 Voltage and SOC for CC Discharge, Case C

Figure 5.16 Voltage and SOC for CP Discharge, Case C

Figure 5.17 Voltage and SOC for RVC Discharge, Case C
5.4.4 Results for Case D – SOC Estimation using Cubic Function

Figure 5.20 and Figure 5.21 show the voltages of each battery in the collection for the constant current and constant power discharge scenarios, respectively. By comparing the SOC information in Figures 5.20 (b) and 5.21 (b) with the corresponding values in Figures 4.5, 4.6 and 4.7 it can be noted that the decision-maker has effectively ensured that all the batteries in the collection have been completely discharged and that the
batteries are all discharged to the same depth. This is the coordination behavior that we expected the decision-maker to achieve.

![Figure 5.20 Voltage and SOC for CC Discharge, Case D](image)

Figure 5.20 Voltage and SOC for CC Discharge, Case D

![Figure 5.21 Voltage and SOC for CP Discharge, Case D](image)

Figure 5.21 Voltage and SOC for CP Discharge, Case D

Figure 5.22 shows the voltage and SOC of the batteries in the randomly varying current discharge scenario. It can be noted that even in this discharge scenario, the decision-maker extends the lifetime and the remaining capacity is much less than the methods in chapter 4. Figure 5.23 shows that the number of batteries selected for discharge are not radically changed throughout the discharge process. Figure 5.24 shows the terminal voltages and the lifetime in each of the discharge scenarios.
Figure 5.22 Voltage and SOC for RVC Discharge, Case D

(a) Battery Voltages                             (b) Actual SOC’s

Figure 5.23 Batteries selected for Discharge, Case D

(a) CC Discharge                      (b) CP Discharge                      (c) RVC Discharge

Figure 5.24 Terminal Voltages and Lifetime, Case D
5.5 Discussion

The decision-maker based on the goal-seeking formulation is a dynamic decision-maker in the sense that the alternate actions selected in two executions with the similar set of initial conditions would be different. The decision-maker establishes performance objectives based on the Reflection function. At the same time, it monitors the terminal voltage of the collection to ensure that the actual performance is within an acceptable tolerance limit. Whenever the measured terminal voltage is outside the acceptable limit, the decision-maker selects additional alternate actions so as to ensure that the collection of batteries continues to supply energy to the load as required.

Table 5.4 shows the observed lifetime of the collection of the batteries in all the discharge scenarios. When compared to the sliding window selection, the lifetime is extended by 10-15% and when compared to the random selection, the lifetime is extended by 10-20%.

<table>
<thead>
<tr>
<th>Discharge Scenarios</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Current</td>
<td>422.9</td>
<td>421.4</td>
<td>409.2</td>
<td>423.9</td>
</tr>
<tr>
<td>Constant Power</td>
<td>406.9</td>
<td>404.9</td>
<td>395</td>
<td>405.9</td>
</tr>
<tr>
<td>Varying Current</td>
<td>453.9</td>
<td>463.4</td>
<td>442.5</td>
<td>443.9</td>
</tr>
</tbody>
</table>

As can be noted in Table 5.4, the lifetime obtained when estimating SOC using voltage derivatives is comparable with the lifetime obtained when estimating SOC using a cubic function in the constant current and constant power discharge scenarios. When the current is varying randomly, a method that uses voltage derivatives appears to perform
better. Table 5.5 shows that consistent with the increase in lifetime; the energy supplied to the load is also increased.

<table>
<thead>
<tr>
<th>Discharge Scenarios</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Current</td>
<td>25.501</td>
<td>25.252</td>
<td>24.688</td>
<td>25.066</td>
</tr>
<tr>
<td>Constant Power</td>
<td>25.427</td>
<td>25.299</td>
<td>24.687</td>
<td>25.365</td>
</tr>
<tr>
<td>Varying Current</td>
<td>24.9148</td>
<td>25.276</td>
<td>24.3479</td>
<td>24.4248</td>
</tr>
</tbody>
</table>

Table 5.6 shows the remaining capacity in the collection at the end of discharge. It can be noted that in the constant current and constant power discharge scenarios, a decision-maker that estimates SOC based on a cubic function performs well. In the randomly varying discharge scenario, a decision-maker that estimates SOC based on an estimation table performs well.

<table>
<thead>
<tr>
<th>Discharge Scenarios</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Current</td>
<td>0.42823</td>
<td>0.81365</td>
<td>0.9495137</td>
<td>0.2236928</td>
</tr>
<tr>
<td>Constant Power</td>
<td>0.63652</td>
<td>1.03781</td>
<td>0.8710661</td>
<td>0.1418271</td>
</tr>
<tr>
<td>Varying Current</td>
<td>0.99069</td>
<td>0.81322</td>
<td>1.5907581</td>
<td>1.5438736</td>
</tr>
</tbody>
</table>

The lifetime and energy efficiency of decision-makers in Case A, Case B and Case D are comparable. A decision-maker based on the goal-seeking formulation described in this chapter consistently performs better than the decision-makers that are based on selection strategies described in chapter 4. Table 5.7 shows the performance of the new decision-maker using the discharge-efficiency metric. In comparison with Table
4.5, it can be seen that the discharge-efficiency of the new decision-maker is significantly better than the discharge-efficiency of the methods in chapter 4.

Table 5.7 Discharge Efficiency (Goal-Seeking)

<table>
<thead>
<tr>
<th>Discharge Scenarios</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Current</td>
<td>0.9420</td>
<td>0.9306</td>
<td>0.9160</td>
<td>0.9434</td>
</tr>
<tr>
<td>Constant Power</td>
<td>0.9214</td>
<td>0.9104</td>
<td>0.9035</td>
<td>0.9312</td>
</tr>
<tr>
<td>Varying Current</td>
<td>0.9186</td>
<td>0.9304</td>
<td>0.8948</td>
<td>0.9053</td>
</tr>
</tbody>
</table>
CHAPTER VI

CONCLUSIONS

We developed a new formulation for coordinating the discharge of a collection of batteries in a manner that extends the lifetime of the collection. This new formulation was motivated by the need to cope with the uncertainties that come to pass because of the complex interactions between the electro-chemical processes in each individual battery. The formulation was based on the goal-seeking paradigm which is a method to represent and describe systems by explicitly representing and reasoning about uncertainties. The goal-seeking paradigm has its origins in systems science and has been successfully applied to address coordination issues in complex, large-scale systems such as economic systems [25], management systems [24], global climate-change [17], and water management [23]. While the goal-seeking paradigm has been recently used as a basis to address coordination issues in engineering systems [8, 21], this formulation is the first application of the goal-seeking paradigm to extend the lifetime of a collection of batteries for discharge process.

The formulation allowed us to use simple models for a battery and design a decision-maker that was based on two principles: bounded rationality [22] and using redundancy to improve performance. Consistent with recent trends in the literature that address scheduling batteries for discharge [1, 2, 3, 14], the decision-maker scheduled
certain batteries in the collection while scheduling other batteries to rest. Going beyond the well-known state-transition paradigm, this decision-maker considered the uncertainties that come to pass during the discharge process. Results from simulation in MATLAB convincingly demonstrate that the decision-maker extends the lifetime of the collection of batteries beyond the lifetimes that could be achieved with battery scheduling methods reported in the literature [1, 2, 3, 14]. The results in Figure 6.1 (a), Figure 6.1 (b) and Figure 6.1 (c) demonstrate that the decision-maker based on the goal-seeking formulation consistently performs better than the methods discussed in Chapter 4 in all three discharge scenarios, namely constant current discharge, constant power discharge and randomly varying current discharge.

(а) CC Discharge  (b) CP Discharge  (c) RVC Discharge

Figure 6.1 Battery Collection Life in the Different Discharge Scenarios

Consistent with the lifetime extension, Figure 6.2 (а), Figure 6.2 (b) and Figure 6.2 (c) show that the energy supplied to the load is also increased.

(а) CC Discharge  (b) CP Discharge  (c) RVC Discharge

Figure 6.2 Energy Supplied in the Different Discharge Scenarios
Further, because the decision-maker considered the uncertainties and selected alternate actions to cope with the uncertainties, the results in Figure 6.3 (a), Figure 6.3 (b) and Figure 6.3 (c) show that the remaining capacity in the collection of batteries is considerably less than the remaining capacity observed when using battery scheduling methods. In addition to reducing the remaining capacity, the decision-maker based on the goal-seeking formulation that the batteries in the collection are discharged uniformly, i.e., one battery is discharged to a depth that is comparable to the depth of discharge of other batteries in the collection. This phenomenon was observed in the results reported in Chapter 5 in the constant current and constant power discharge scenarios. It is however noteworthy that uniform discharge is not observed in the discharge scenario when current was varied randomly. We believe that this is because the decision-maker is unable to select appropriate actions when the load resistance is changing randomly. Nevertheless, when compared to the results in the randomly varying discharge scenarios for battery scheduling methods, it can be observed that the goal-seeking approach performs better.

(a) CC Discharge                                (b) CP Discharge                                (c) RVC Discharge

Figure 6.3 Remaining Capacity in the Different Discharge Scenarios

Figure 6.4 (a), Figure 6.4 (b) and Figure 6.4 (c) show that the discharge-efficiency metric that was defined in Chapter 4 is a useful metric to compare the different methods of discharging a collection of batteries. As expected the performance of the decision-
maker based on the goal-seeking formulation is significantly better than the discharge-efficiency observed in battery scheduling methods.

Finally, it is important to recognize that applying the principle of using redundancy to improve performance is central to all the methods for discharge considered in this investigation. Figure 6.5 (a) shows how the decision-maker based on the goal-seeking formulation utilizes the available redundancy. In contrast, Figure 6.5 (b) shows that the methods based on battery scheduling do not utilize the available redundancy as well as the goal-seeking approach. In particular, note that the decision-maker based on the goal-seeking formulation utilizes a small number of redundant batteries very efficiently.

![Figures](http://example.com/fig64.jpg)  
Figure 6.4 DE in the Different Discharge Scenarios

![Figures](http://example.com/fig65.jpg)  
Figure 6.5 The Effects of Increment Discharging Battery Number on
6.1 Next Steps

Based on the results reported in this thesis, we believe that the goal-seeking approach can be extended to coordinate the optimal charging of a collection of batteries. It would also be interesting to compare the coordination capabilities of the goal-seeking approach with the capabilities of other formalisms in distributed computing such as the pursuer-evader [6].

While it is believed that the goal-seeking paradigm would allow a system to continue to operate over a wider range of inputs [16], this phenomenon was not observed in this initial attempt to apply goal-seeking to coordinate the discharge of a collection of batteries. It would be very interesting to compare the effectiveness of managing uncertainty in a goal-seeking approach with the effectiveness of other methods that are based in classical control theory.
BIBLIOGRAPHY


APPENDIX

SUBROUTINE MATLAB CODES

Table A.1  Simulation of Batteries in the Collection using Euler Integration Method

% Randomization Battery Parameters for different batteries in the collection
for m=1:num_tot_bat
    kb1(m) = 1.33 + 0.0010*rand(1)/2;
    kb2(m) = -0.12 - 0.012*rand(1)/2;
    kb3(m) = 20 + 2*rand(1)/2;
    kb4(m) = 0.015 + 0.00015*rand(1)/2;
    kb5(m) = -0.013 + 0.0013*rand(1)/2;
    kb6(m) = 0.276*rand(1);
    cs(m,1) = 0.86 + kb6(m);
end

% Simulation battery model equations using Euler Integration Method
for t=1:N
    for m=1:num_tot_bat
        if i(n)<=0
            acc = 1-5*exp(-40.*(1-cs(m,n)/(cap+kb6(m))))+4*exp(-50.*(1-cs(m,n)/(cap+kb6(m))));
        else
            acc = 1;
        end
        csf(m) = -(acc/60)*i(n)-(4.83e-6)*cs(m,n);
        csd(m) = -0.35*cd(m,n)-0.001126*acc*i(n);
        v_value(m,n) = kb1(m) + kb2(m)*i(n) + kb3(m)*cd(m,n)+ kb4(m)*log(cs(m,n)/(cap+kb6(m)))+kb5(m)*log(((cap+kb6(m))-cs(m,n))/(cap+kb6(m)));
        cs_value(m,n) = cs(m,n);
        cd_value(m,n) = cd(m,n);
        cs(m,n+1) = cs(m,n)+ T*csf(m);
        cd(m,n+1) = cd(m,n)+ T*csd(m);
    end
end

end

% Loop end for BATTERY
% Loop end for TIME